

Astronomia Danica

(1640)

by Christen Sørensen Longomontanus (1562–1647)

Translation by Scott Weisman



ARS ASTRONOMICA

2026

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Published by [Ars Astronomica](#)

Translator Version beta

Upright reason dictates that the recipients of the good, of whatever type and level of beneficence it may be, must show gratitude and blessing to the beneficent person in every way possible, commensurate with the value of the beneficence. And one who has benefited all the people of the world — for example, one who invented a new instrument for the good of the world, or a good book — it is fitting for every discerning person, out of the obligation of love of fellow beings, to at least purchase it, so that the man will profit and his heart will be encouraged thereby to invent yet more good instruments in the world. And similarly, all other wise-hearted people will likewise strive and exert themselves to invent good things and instruments needed for the repair of the world and its perfection.

And therefore, whoever says: "What do I need this new instrument for?" — he does not act well towards the world. For if not for the man who invented it, where would you be? And what would your city do? And more than this, if the man did not exert himself, what would the world do?

— R. Pinchas Eliyahu Hurwitz, *Sefer HaBrit HaShalem* (1797)

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A note on the work

A comprehensive textbook of astronomy and the definitive technical account of the Tychonic system — Earth at rest, the Sun circling it, the planets circling the Sun. It works systematically through the structure of the cosmos, the motions of the Sun and Moon, eclipses, the planets, the fixed stars, and precession, with working tables and geometric diagrams throughout; an appendix treats the new stars of 1572 and 1604 and the comets of 1607 and 1618. It is the fullest pre-Newtonian exposition of the Tychonic system.

A note on the translation process

This translation was produced with the [translation-pipeline](#). See the linked repository for details on the process.

This text is the product of a highly refined automated translation process. It has undergone only cursory human review and should not be treated as an authoritative or scholarly rendering. It is offered solely as a reference and reading aid — a way to make an otherwise inaccessible work approachable — and any passage of real consequence should be checked against the original source.

Acknowledgements

Translated with the assistance of Claude. The translator thanks the [Anthropic](#) team for making this work possible.

This batch comprises the physical binding and front-matter leaves of the volume only. None of the four pages carries printed text from the work itself; the title page and the printed preliminaries begin in a later batch. The pages are transcribed/described below as bibliographic-codicology material.

Front cover

No text.

Inside front cover (paste-down)

A blank paste-down leaf, browned and damaged at the inner (spine) edge. Near the top center is an oval ownership/library stamp in violet ink.

[Approval stamp: oval violet ink-stamp, partly faded. Reads approximately "FR. C. C[?]/HAN[?] / PROFESSOR D[r.] MED. / [K]I[Ø]B[E]NHAVN" — i.e. a professor of medicine at Copenhagen [København]. Several letters are ink-faded and not fully legible [?].]

No other text.

Front flyleaf (recto)

A blank flyleaf, browned and foxed. Faint show-through of the title-page type is visible from the following leaf but is not directly legible on this side. Toward the lower center are large faded reddish manuscript markings — apparently later pencil/crayon jottings — reading approximately "3-14" with a stray descending stroke [?]. In the upper right corner is a small faded inscription [illegible — ?].

No printed text. (The reddish marks and corner inscription are later hand-additions, not part of the printed book.)

Front flyleaf (verso)

A blank flyleaf, browned. In the upper left corner are modern library shelf-mark notations in pencil:

[Margin: (pencil, upper left) "Bms[?] 5034 / 48"] [Margin: (pencil, upper left) "Q / 155 /.S395 / 1640 / OVERSIZE"]

The lower block ("Q 155.S395 1640 OVERSIZE") is a Library of Congress-style call number assigned by the holding library. No printed text.

[Title]

[Margin: manuscript annotations at the head of the page: "M 11" and a faded ownership note, "Collegii [?] Sorø [?] [?] Mandani 1640" — later hand, not the printed text.]

ASTRONOMIA DANICA,

Produced through the night-labors and exertions of CHRISTIAN S. LONGOMONTANUS [Christen Sørensen Longomontanus], Professor of Mathematics in the Royal Academy of Copenhagen, and divided into two parts. Of these:

THE FIRST

sets out, in two books, the doctrine of the apparent daily revolution of the stars, established upon the armillary sphere of the ancients.

THE SECOND

likewise comprises, in two books, the theories of the motions of the planets — restored in a threefold form, according to the observations of Lord Tycho Brahe [Tycho Brahe] and according to the author's own.

With an Appendix on the adventitious phenomena of the heavens, NAMELY, NEW STARS AND COMETS.

Now once again corrected and enlarged by the author in a number of places.

AMSTERDAM, At the press of Johann and Cornelius Blaeu [Johannes and Cornelis Blaeu]. In the year 1640.

[Library stamp at foot: "FR. C. C. HANSEN, PROFESSOR DR. MED., [KIØBENHAVN]" — modern ownership stamp, not part of the printed work.]

[Dedicatory Epistle]

TO THE RENOWNED KING OF DENMARK, NORWAY, THE GOTHs, THE WENDs, etc.,

LORD CHRISTIAN IV,

HIS OWN MOST GRACIOUS LORD AND KING.

If we are willing to observe and reflect attentively upon the diverse character of human dispositions — the inclinations of men's minds, and at last their pursuits

and their actions in life — then, most mighty King, most gracious Lord, although these differ in an almost infinite variety, they are nonetheless so constituted, and arranged under such an opposition, that they may not unsuitably be sorted into two kinds. And since human characters are acquired by representing those same dispositions (as is regularly done in comedies and tragedies), they demand for themselves just as many roles. They are not unlike two prisoners who, having been cast into prison by their own fault and bound there in chains, are held so completely under the power of a great king that, before their death, they cherish no hope of escape. Yet it happened thus: after each of them had alike been granted, by singular clemency, the favor that — even while in their chains — they might, if they wished, look upon with their own eyes the various and astonishing works of that Prince, most beautifully constructed and arranged throughout every quarter of the realm, and might thereby lighten their misery for their whole life long — then the one prisoner prepared himself, with a most joyful mind, to kiss and to follow the gracious will of this Lord; while the other, spurning that same will with a spirit more than impious, dull, and obstinate, chose rather to withdraw into the little cell of his prison, as a tortoise is wont to do, and there to be forever dulled like an owl, to pour forth mournful lamentations, and at last to be consumed by wasting, by foul squalor and neglect, than to refresh his spirit continually, together with his companion, by those most beautiful spectacles. By what right, then, I ask, could that reprobate — most wretched throughout his whole life — either envy the happiness of the other, or cast the blame for his own misery upon anyone but himself?

And we, too, since we are prisoners in this mortal body — bearing about with us that condition of ours which the sad fall of our first parents brought upon us all — so much so that, like the prisoners in Plautus, fated to go all the way to death, we groan aloud in these words: *"Are we to flee? Whither are we to flee?"* — we shall never, in so great a prison of guilt, be more kindly treated, more honorably and graciously invited, and [summoned] to still more astonishing

[Dedicatory Epistle, continued]

riches — and we shall think that we can be carried about to contemplate the riches and the works of the supreme Monarch and Creator of us all, works which most beautifully fill this whole world and are shut off neither from our eyes nor indeed even from our uses, except at the outermost circuit of the heavens. Accordingly, since we enjoy so great a clemency of Almighty God that He not only founded from the beginning this most beautiful Theater of the world for our use, but also — after the lamentable transgression and captivity that deservedly

followed upon it — has continually sustained it, and daily sustains it by His omnipotent word, so that even in this exile of ours and mortal state, while we journey far from our true homeland, there might be a means by which we can console our ailing mind: namely, to admire the riches of the Creator in heaven and earth, and over these same things to be deservedly resolved into His praises. But above all is this: that the same most gracious Father has so pitied the misery of our condition and our captivity that He willed to send down His only-begotten Son into the lowest parts of the earth, and indeed into our very mortal prison, so that thus our Savior and Redeemer — through the fellowship of the flesh assumed from our flesh and of His Spirit — might sanctify the captivity of all believers, lead it back with Himself into heaven, and at last endow it with unheard-of freedom, the victory over sin and death having been won by His own precious death.

Therefore we judge it to be the mark of a degenerate and utterly stupid spirit so to retreat into one's own prison and to flee the light, that no happiness in coming to know the best of things should touch one, nor any spur to the exercise of virtue rouse one — but rather that the mind itself (a particle, indeed, of the divine breath) should be tainted with shadows, with the pleasures of the belly, with idleness, languor, and sloth. So, surely, it is to be reckoned the mark of a noble and truly heroic spirit so to conduct oneself in this most splendid theater of the world that one fittingly expresses and represents the role and condition which that actor and dispenser of all things, GOD, has imposed upon each individual; and, while playing that part, not to neglect to put forth one's own power through this prison — that is, through the two apertures of the body (as Plato calls the eyes). For this is the practice of the virtues, and the beginning of Philosophy: that with the aid of the eyes we should, first, behold the divine works — the works, I say, miraculously [wrought] in heaven and earth; then admire them; thirdly, search them out more earnestly with the keen edge of mind and intellect; and lastly turn them to the praise of the Creator, and likewise to our own use, so far as may be done. The beginning, therefore, must be taken from the eyes; but the middle, which leads through to the end, must be taken from the arts and disciplines — and above all (unless we are very much mistaken) from the Mathematical ones. For by these, nature herself has been bound fast in an indissoluble bond — next after the sacred pages, which affirm that God disposed all things by measure, number, and weight. Plato bears witness to this in the *Epinomis* in these words: *"There is one bond of nature: the doctrine of figures, of numbers, of harmonies, and of the celestial motions; and if anyone*

seeks to find another way to knowledge, he toils and (as the saying goes) courts mere luck. Without these things no state can be happy. By this road one must travel, whether it be easy or hard; nor must God be neglected, who has bestowed upon mankind, in these arts, the most welcome renown about Himself." So far Plato.

From the matters Plato has thus set forth in advance, the names of just as many arts emerge in order: Geometry, Arithmetic, Music, and Astronomy. Of these, in order that the last may be properly mastered, the preceding three are shown by the same author to have been invented chiefly for that purpose — so much so that the first two, namely Arithmetic and Geometry, are called the "wings" and the "ladders" by which we scale the heavens and measure the everlasting motions of the stars, while Music is made proportional, the means by which at last we rightly grasp the harmony of the higher things with the lower. It should therefore surprise no one why these arts have always stood in the highest honor and favor among great emperors, kings, and princes — the leading men of the state — to such a degree that, so far as the just governance and defense of their subjects allowed them leisure, they applied themselves to these studies under the impulse of an inborn greatness of soul and a loftier spirit, or at the very least favored them. And not without reason: for they perceived that no small part of happy government — second only to the greatest virtue, namely religion and true piety toward God — lay in these arts. *To them, indeed, nothing seemed more absurd than to rule the world and yet not know what the world is.*

Among the number of these heroes who cultivated the mathematical disciplines, and Astronomy above all — for its antiquity and for the admirable contemplation of the celestial motions, most worthy of a lofty and royal intellect — may be reckoned not only the Magi of the Persians and the Philosophers of the Egyptians (out of whose ranks first kings, and afterward priests, were made), but also princes who in turn sprang from royal stock. Consider Julius Caesar [Julius Caesar], the first and supreme emperor: did he not, in the midst of arms and the trumpets of war, measure the sky with the surveyor's rod — and chiefly for this reason, that he might thereby restore the calendar, in that age most corrupt, to the true annual cycles and to its proper integrity? So that at last he could rightly boast of himself in these words:

My year shall not be outdone by the calendar of Eudoxus.

And what of Ptolemy of Alexandria [Ptolemy]? He was assuredly so industrious in advancing Astronomy that, aided by the observations of his predecessors and by his own, he was the first to transmit complete to posterity this most beautiful of

all the arts, founded and preserved upon his own evidence and records. Among Christian princes, the first to come forward is the great-souled Alphonso [Alfonso X], King of Aragon and Sicily, who is said to have been so lavishly devoted to this art that he laid out four tons of gold for the compilation of the Tables that to this day bear his name [the Alfonsine Tables]. And why should I mention the emperors Maximilian [Maximilian I] and Charles the Fifth [Charles V]? The former kept Vögelin [Vögelin], the latter Peter Apian [Peter Apian], the celebrated astronomer, not only in honorable maintenance at his side, but was himself present and bent over the work as it was carried out —

— sketching the equatorial instrument [aequatoria] with his own hand. And what of William, the most illustrious Landgrave of Hesse [Wilhelm IV, Landgrave of Hesse-Kassel], at once a most skilled astronomer? And what too of Maurice, the most excellent Duke of the Belgians [Maurice of Nassau] in our own time, no less a most practiced mathematician? But why foreign examples, when our own domestic ones suffice for us — examples that testify how the most serene and most mighty kings of Denmark and Norway have likewise held these arts in the highest honor?

That Your Royal Majesty's paternal grandfather, the most serene and most Christian King Christian III [Christian III], was lavishly devoted to this sublime study is proclaimed even today not only by foreign historians (we ourselves keeping silent), but by the most ingenious automata [Greek: αὐτόματα (automata, "self-moving devices")] and other monuments which he left here, worthy of the heaven and of the lofty spirit of a king. And what shall I say of Lord Frederick II [Frederick II], the father and predecessor of Your Royal Majesty, a most merciful king? When he had found Lord Tycho Brahe [Tycho Brahe] — noble from his very origin, but in genius born for the restoration of Astronomy — did he not, through this exceptional man, the greatest astronomer of all ages and as it were a second Atlas, draw Urania down from the heavens, set her gloriously in our midst, and adorn her with the greatest royal expenditures? These expenditures Your Royal Majesty too continued, with plainly royal liberality, until the twenty-first year — the year which, by the sevenfold number tripled, tends fatefully to play its tricks in human affairs. And not in vain: for nowhere in the whole world have there existed observations of the stars of greater accuracy or moment than in our own Hven [Hven].

After fortune — or rather, as I see it, a greater divine providence — had cast me too into the apprenticeship of toiling at those labors in that place, and after the length of time, joined with an inborn inclination of my mind toward these

studies, had brought it about that at last I needed no other teacher beyond the practice of night and day, it came to pass during that eight-year residence with Tycho in that place — in which I not only took part in the unremitting Uranian labors but for the most part directed them, as all who know me know — that, after the canonical charting of the fixed stars had been deduced from the heavens by my own incredible toil, I undertook by my master's command the restoration of the Moon's course, begun there but completed elsewhere, and likewise a similar restoration of the remaining planets. And thus it came about that a certain synopsis of selected observations gathered for this purpose remained in my keeping — not indeed preserved then to the end that I should ever hope from it for a reward equal to my labors (although here too the laborer is owed such a reward, and most justly), but far more, so that I might more correctly and truly judge the industry of others concerning the restoration of Astronomy, should it ever be granted to anyone whatsoever to bring it to completion. Meanwhile, released from these Uranian labors, when I was turning my mind to other studies and to another, as I supposed, more profitable manner of life, He who turns in His own hands the fortunes of us all did not allow it, but against my hope called me back again to my old study —

— and made the illustrious Royal Chancellor, Lord Christian Friis of Borreby [Christian Friis af Borreby], the patron of my advancement, my excellent Maecenas and singular supporter — indeed its very author. He summoned me, maintained me for two years in his own household, kept me in an honorable position, and, when I was several times invited away with honor enough, did not allow me to depart; rather, he took pains that I should devote my service to my homeland, and indeed to this Royal Academy [the University of Copenhagen] — which came to pass fifteen years ago. And so, when the professorship of Mathematics had devolved upon me two years afterward, and I perceived that I had gained an opportunity to adorn Urania further in my native land, I gladly accepted it with eager heart. The first thing that had to be done was to have instruments newly built — few, indeed, but necessary, and no less capacious or suitable than those I had once handled with my own hands at Hven. This I did in order to add to them the supplements of the observations of the planets that were still wanting (as was afterward done here).

For now, recuperated by the kindly countenance of the most serene King and graciously enriched with royal benefits, thanks to the care of the same aforementioned Maecenas, the Magnificent Chancellor, I had to meditate with all the greater effort by what means Astronomy might be tended, and the traces of Tycho preserved here to the perpetual praise of my homeland. For indeed, before

they had passed over into Denmark, these traces had been so consumed and disfigured by oblivion and neglect that they could scarcely be recognized by their face as offspring of the sky. But while I was pressing down, and all but suppressing, so great an undertaking in my mind — one far exceeding, as I had earlier learned, the powers of my own talent and the means of a private man — God, who had led me to this kind of study and led me back to it, raised up for me another patron and supporter of Astronomy in my homeland: namely, that illustrious man most worthy to succeed to the Royal Chancellorship, Lord Christian Friis of Kragerup [Christian Friis af Kragerup], the Magnificent Chancellor, golden-spurred knight and foremost Senator of the Realm. By his encouragement and singular favor — with which he kindly received me not long ago on account of my professorship in these arts, which he wonderfully loves and knows how to handle with great skill — I set down these pages the sooner, in which *Astronomia Danica* is read (as it were, born and elaborated in Denmark at one and the same time), so that they might come forth into the public light. Two years ago I sent them to the Low Countries [Belgium], that they might be submitted both to the discriminating judgment of the Dutch and to the hands of a most skilled printer there. And this is the very work which I now humbly present to Your Royal Majesty, the work which in past years I completed publicly, among other duties, in this your Royal Academy — hoping that this effort of mine, graciously taken up by royal favor, may find acceptance.

For what shall we not hope for from so great a king — one whom none has ever surpassed, whether in true piety, virtue, and clemency; or in authority and power; or in a loftier and plainly heroic spirit; or in a subtler genius for rightly judging these and other liberal arts and directing them to use; or finally in a greater elevation of soul, in care and diligence —

— for governing his subjects with merciful justice, for increasing the wealth of these kingdoms, and for fortifying and justly defending their borders, than has ever ruled over the North? For the celebrity of his fame, surmounting the Ganges (as once that of Alexander the Great [Alexander the Great] did), has already long since struck the Indians and lured them into the friendship of a happy alliance — as we hope and pray it may be. By no right can we keep silent about these praises of Your Royal Majesty, since they are proclaimed throughout these realms by the Academy and schools of the Church of Christ, which persist in the confession of an unblemished religion and besides apply themselves unceasingly to the cultivation of all the more humane arts. They are attested by inhabitants kept safe under his defense; they are displayed by citadels and cities ringed about with the strongest fortifications — some of them, too, rising up entirely anew under

the auspices of so great a king — a king whom God and nature so favor that he may deservedly be a living and true model of kings throughout Europe.

Accordingly, in this flourishing commonwealth — flourishing under divine grace — it is only right that the mathematical disciplines, which have surely been the companions of all happy empires, should likewise flourish here as one; and no less among these should Astronomy, that most splendid and most beautiful of all sciences, the science of the stars, which consists in the knowledge of the shining stars and of their revolution in most temperate harmony. From it there arises an almost infinite usefulness, on land and sea, in a well-constituted commonwealth: whether to investigate from it the nature and the distance of remote places; or to traverse the earth itself without error; or to furrow the seas on the longest voyage; or to track down the truth of histories; or to recount and understand the poets; or to know, from the daily revolution, the yearly changes, the lengths of days and nights, and likewise any given moments of time; or to construct machines that rival the revolutions of the stars; or to determine in advance the exact times and the magnitudes of the eclipses of the luminaries that occur from time to time. And, in the case of the other planets, to demonstrate from their own fixed hypotheses their motions, stations, and retrogradations; and finally to lay open to oneself the causes of all these things, for whoever may wish it — besides countless other benefits that flow from this stellar science, joined with the greatest pleasure and delight, so much so that no art can be either more excellent, or more worthy, or more fruitful, since it also opens the very wellspring of all Philosophy.

For what else do the poets mean when they imagine Pegasus furnished with the oarage of wings, by which he soared into the air and at last, reaching the summit of Helicon, opened with his hooves the fountain of the Muses — what else, but that Astronomy gave Philosophy its beginning? As the divine Plato too discusses in plain words in his *Timaeus*. "Nor indeed," says Tully [Cicero] (in his fifth *Tusculan*), "would Atlas be said to support the sky, nor Prometheus to be fixed to the Caucasus, nor starry Cepheus, with his wife, son-in-law, and daughter, be handed down [in story], had not the divine knowledge of the heavenly bodies carried their names over into the error of fable." This art, then — the most ancient, most beautiful, most delightful, and most useful of all — among the earliest men (as Josephus —

[Conclusion of the Dedicatory Epistle]

—phus [Josephus] testifies in his *Jewish Antiquities*) it was born; and that it was carried on without interruption among the Chaldeans, Egyptians, Greeks, Italians,

Spaniards, Gauls, Germans, Britons, and Poles. Nowhere, however, has it been reborn with greater care or more subtle industry than here in our Denmark — a thing I can truly affirm. Since I judged that it pertained in no small degree to the honor of both the King and the Kingdom that this knowledge be shared with all liberal minds throughout the world — so that the revolutions and configurations of the stars, coeval with the world itself, might (as far as is possible) be readily drawn forth by an exact method [Greek: ἀκριβοφορία (akribophoria, "exact derivation" — a derivation carried out with precision)] for any moment in time, and applied to uses far greater than those already named, the heavens being left as an inheritance to all — I therefore chose, as best my slender talents allow, to devote myself to bringing this same knowledge at last into the light, rather than to pursue other, more profitable arts. I am content so long as I render humble obedience to God and to my sovereign, and so long as I expend my effort in these labors, first for the praise of my most beloved homeland, and then for the benefit of all good and honest minds. And if at any time these labors should be dragged into the accusations of perverse minds — driven by the appetite for fault-finding that is innate in such men — I rejoice nonetheless. For when those accusations are weighed and judged, both under the patronage of so great a King and against the very face of the heaven that we enjoy daily, they will swiftly vanish into the smoke of empty calumny.

May the eternal God grant that Christian IV [Christian IV], the great and most powerful Monarch of Denmark and of the North, together with his renowned royal house and with these realms and dominions, may live and flourish; and that true piety and the other royal virtues may abide with him unbroken, and be propagated happily to all posterity. Amen. At Copenhagen [Hauniae], in the year from the birth of the Savior of the world 1620, in the 24th year from the coronation of the present most serene King.

To Your Sacred Royal Majesty.

Your most humble servant,

Christian Severinsen [Christen Sørensen Longomontanus] Longomontanus.

[Commendatory Verses]

Upon the Astronomical Work of Christian Longomontanus [Christen Sørensen Longomontanus], Professor in the Royal Danish Academy.

An epigram [Greek: Ἐπίγραμμα (epigramma, "epigram")] by W. Westhovius.

By as much as the stars surpass the other parts of the world, by so much has this study withdrawn from all other pursuits. Just as the stars hold dominion over all the bodies of the world, so does deep skill in the stars govern all things known to you. Te, Christian — who profess to climb by this great and wondrous road above all other men — Urania [the Muse Urania], more excellent than the rest of the Muses, here rarely sanctions by her own teaching; and Envy herself, against her will, confesses it with this very mouth.

To the Distinguished Man Christian S. Longomontanus, a Dane out of Cimbria [Jutland],

the incomparable astronomer, most excellent Professor of the higher mathematics in the Royal Danish Academy at Copenhagen [Haffniensi], Canon of Lund [Lundensem], etc.,

his colleague, kinsman, and most closely bound brother, on his publication of the *Astronomy Restored* [*Astronomia Restaurata*].

Epigram

by **Caspar Bartholin** [Caspar Bartholin], Doctor and likewise Royal Professor in the same place.

Urania had seen [Viderat] that, from the long mountains [LONGIS de MONTIBVS — a pun on "Longomontanus"], light was given to the stars, and that unaccustomed flashings were given forth. She added: from the heart of the world the stars have hitherto twinkled — they gleam red for the Danes, but now only by a new Sun.

[Note: line 1 plays on the name "Longomontanus" = "of the long mountains."]

Another.

To the stars first shone that famous Tycho the Dane [Tycho Brahe], the Sun of his homeland and of the heaven, the summit of nobility; the new Sun has shone upon the stars — a second Atlas [Atlas]: thus has the Danish land given a pair of suns.

To the Distinguished and Most Ingenious Mathematician Christian Longomontanus [Christen Sørensen Longomontanus], Author of the *Danish Astronomy*.

Once a worshipper and lover of Urania of Venusia [Venusinae], not idly do you bear your name from the long mountain. To scale the sky, you need not pile Pelion [Pelion] upon Ossa [Ossa]: you go up by a more convenient road. Upon the

long mountain you lay lofty and wondrous labors — a mountain high, not merely long, departs: it rises, it lifts you up to the lofty walls of the world, and gives you to enjoy the upper air no less than the lower. You touch the stars nearby, as if with your finger, and see how they are compounded, and toward what point each is set in its place. Hence you test, with a more skillful balance, both what the ancient astronomers taught and what the recent ones have taught. And in what place each star lies, and by what motion each is borne through the heaven, you arrange and set in motion in your books. Well done, sir, in your new art (so is the short road to the stars taken)— go on duly to lay open the hidden ways of the pole. Let the King, the Sun, sit in the middle; let the five royal stars [the five planets] course around him, now far off, now close at hand. Let Cynthia [the Moon] change her horns as she circles the earth, and let her go round whatever comes to be, perishes, or changes. Let the fixed throng run constantly across the pole with the same steps through the ethereal globe by night and by day. The rest you also arrange, just as you have seen them set in order by God, the supreme disposer of all.

Nicolaus Georgius Ærylæus [Niels Jørgensen Ærylæus].

To the Distinguished Man Christian Longomontanus, Author of the work.

What does the long-lived course of an age not transform? By this same law even the goods of the mind hold their place: the arts perish and rise again anew; this region is cultivated, that one lies squalid in filth; and things that had grown obsolete afterward shine bright again. Time and the age turn such changes about; why should I prove it point by point with testimony? You, Christian — whose craftsman's hand

—and circles of a learned mind we behold, by which you render the sky and the very poles passable, and bring the North [Borean] nearer to God in his choir: here you can stand as a rich witness and an authority. For Hipparchus [Hipparchus] indeed, and that lofty Conon [Conon] — each shone forth as a great glory of Urania; but the one was born at Rhodes [Rhodiis], while Conon set out from Samos [Samo] and turned his head high toward the heavens. Ptolemy [Ptolemy] settled within the shallows of the Nile, and held up a bright torch to those who came after him, by which they might strive toward the fixed stars along a sure path. And many others too — learned men whom either the Greek land or Arab cultivation once brought forth. But these came earlier, and now they lie in neglect, and there all things wither in the cloud of night. And so the choir of the arts turns its team of horses and seeks Helicon [Helicon], where the cold of the North grows stiff. Here was born Copernicus [Copernicus], of lofty research,

among the farthest Prussians [Prutenos] of Germany, who led the way into the sky by a road not trodden before. Here was Regiomontanus [Regiomontanus], here Peurbach [Peurbach]; and following him soon after, learned in the starry regions beyond the rest, was the famous Erasmus Reinhold [Erasmus Reinhold]. But all these, and all others, that illustrious Tycho Brahe [Tycho Brahe] surpassed — he, the glory of his own Denmark, a man supreme in mind and in the gifts of genius. To him, a great Atlas [Atlas], you — the lesser — lending your hand, now lay open the more hidden sacred things of the heaven-dwellers, and whatever still gapes unfinished in the art, this your learned right hand and shrewd mind polish smooth. You bring it about, too, that the Danish youth, roused under your guidance, may here learn the changes of the world, and the orbit of Phoebe [the Moon] and the steady courses of the Sun, and may claim this praise as properly their own. O Christian — and you too, happy Denmark [Dania], whom the Greek, the Arab, the Nile, and Germany courted in vain, that she might be bestowed upon their brows: this palm is yours, this laurel is prepared for you.

Johannes Isaksen Pontanus [Johannes Isacius Pontanus], Royal Historiographer.

[Part-Title]

THE FIRST PART OF THE DANISH ASTRONOMY,

containing the **doctrine of the sphere** in two books, of which:

THE FIRST

presents the structure of the sphere restored from the armillary circles of the ancients, with the *Prognorismata* [foundational principles] of Astronomy briefly set out beforehand — concerning the analysis of triangles, the material of the heavens, and so forth.

THE SECOND

explains the use of that same sphere concisely and demonstratively, through various and most useful problems.

TO THE READER

OF THE ASTRONOMY

Although from the outset I had proposed to bring out our *Astronomy* without an accompanying analysis of triangles — for I readily judged that the mathematicians, to whom I had long ago chiefly destined this labor, would each

prefer to follow, in unraveling that analysis, the particular triangular solution most familiar to them; and judged, too, that the portion of geometry which directs the chief practice of this subject was to be reserved for more refined elaboration in a work of that kind, which I had already delivered publicly as lectures, so that it might appear with a fuller apparatus of its own — namely, together with the Canon, and with the construction and emendation of that Canon derived from a truly discovered quadrature of the circle [Cyclometria] — nevertheless, once I had found among the Belgians a thoroughly suitable craftsman for printing this work, **Willem Blaeu** [Willem Janszoon Blaeu], a most skillful man, excellently versed in mathematics, and once my closest friend on Hven; who spares no industry in seeing that whatever works of this kind issue from his workshop are brought to a high polish — I judged it advisable to compress a *Trigonometry* of plane as well as spherical triangles into the briefest and easiest compendium, and to prefix it to this same work. In this way our astronomical labor would be made more accessible and more commendable both to lovers of astronomy and to students in the academies. For the foundation of astronomy — and indeed of almost the whole practice of mathematics — consists in the analysis of any proposed triangle, that triangle being, as it were, the first figure of all, since all other figures are resolved into it. So our master **Tycho** [Tycho Brahe] not without reason once bestowed this tribute upon it:

The triangle holds all that learned mathematics labors after; It lays open whatever closed thing Olympus holds.

Indeed, the most excellent **F. Viète** [François Viète] looked to this as well, when he prefaced his hypothesis of the Mathematical Canon with these words:

To obtain the sides from the angles, or the angles from the sides, and to do so in mixed fashion in triangles both plane and spherical, is the highest glory of the mathematician; for thus he measures heaven and earth and seas with a felicitous and admirable calculation.

The same thing also had to be done after the example of the ancient astronomers Ptolemy and Copernicus — lest I should seem to wish to fly up into the heavens without these wings, or wings of any kind, acquired through arithmetic and geometry, and to point out an unknown road to the upper air. This is likewise the reason why I have joined the other part of the *Prognorismata* [προγνωρισμάτων (prognōrismatōn, "things to be known beforehand")] of astronomical matters — the διδακτικήν [Greek: didaktikēn, "the instructional part"], namely — concerning the material of the heavens and the great bodies of the world, to the earlier part on the measurement of triangles through numbers, treating it according to the

judgment of my own talent, in the way I considered it to accord with the sacred pages, with nature, and with reason. For without these matters moderately known beforehand, Astronomy, the queen of all the disciplines, is neither successfully transmitted, nor grasped, nor resolved into its causes, nor at last rightly applied to its uses. But concerning such things — without prejudice to anyone who brings forward better views anywhere — we have discoursed throughout. I thought only that the kindly Reader should be advised in the first place about the *Prognorismata*; about the rest, more frequently wherever there shall be need. For to commend laboriously, on its very threshold and before the learned, an Astronomy newly restored, I reckon no less superfluous than to hang up ivy beside saleable wine [i.e., to advertise what needs no advertising]. Meanwhile, if anyone here shall have gained some fruit from our vigils and labor, I shall reckon that I myself have attained the same — since from these labors I have looked for nothing else in life than that they might be rightly bestowed for the benefit and use of a grateful posterity.

ΠΡΟΓΝΟΡΙΣΜΑΤΩΝ [Greek: prognōrismatōn, "of the things to be known beforehand"]

OF ASTRONOMY

THE FIRST PART

On the compendious analysis, through numbers, of plane and spherical triangles.

[Margin: Definition of the analysis of triangles.] The **analysis of triangles** is the most ingenious method of producing, from three given [δεδομένοις (dedomenois, "given things")] parts of a proposed triangle, whatever fourth part you please — or indeed all the remaining parts hidden within it — in numbers, according to the same proportion.

The doctrine concerning the analysis of triangles, insofar as it is here treated by us in compendium, has two members.

For first there are set out beforehand certain general matters concerning the definition and partition of the triangle, together with certain geometric theorems, with whose help the chief reasoning concerning the analysis of any triangle is established. Here it is at the same time shown whence the threefold Canon — which consists of sines, tangents, and secants, and is intended for the solving of triangles — depends, and likewise the construction and continuation of this Canon through numbers.

Then follow the matters that accompany the analysis of the triangle itself, and these in turn are twofold. For either the analysis is directed generally to any question whatever, and falls under the single golden rule of proportion — where the quantity of the fourth term (and so on) that lies hidden is investigated either by the customary practice handed down in arithmetic, or by a more compendious route, through *prosthaphaereses* [proportional differences]. Or else a solution is adapted to the particular species of the proposed triangle, plane as well as spherical, according to what is sought, the reasoning about each being duly accommodated from the foregoing theorems.

This, then, will be the order, this the sum of our analytic doctrine concerning triangles. And if it be asked why it is placed first of all in the vestibule of an astronomical work, the answer is at hand: because it embraces the measurement of the entire universe. But if it be further asked why most of the theorems that are to be brought forward below, in support of the analysis, are presented with their proofs by suitable demonstrations [ἀποδείξεις (apodeixeis, "demonstrations" or "proofs")], to this too it can be answered: that those are taught in geometry, and here are only cited — lest, while we aim at brevity, we be overwhelmed by the multitude and mass of such things, which, more rightly cultivated and demonstrated in their own ground, are channeled by streams to this present use.

ON THE FIRST MEMBER.

[Margin: Definition of the Triangle.] A **triangle** is a figure consisting of three angles and as many sides. Therefore in every triangle six things are set forth for consideration: three angles and three sides. But that the following matters may be more conveniently dispatched for beginners, the division of the circle — from which the solution of the triangle chiefly flows — must next be appended.

[Margin: First division of the Circle.] It must therefore be held that every circle is commonly divided into 360 degrees or parts; and each individual degree into 60 first minutes; and again each first minute into 60 seconds, and so on in succession, to infinity, as far as the demand for precision requires. From this arises the sexagesimal arithmetic, otherwise called *Logistic*. In certain places, moreover, a mark must be superscribed: ° for the degree, ' for the first minute, " for the second, and so forth, when Logistic numbers are handled in practice.

TRIANGLES TO BE SOLVED

Furthermore, analytic trigonometry is contained within two species of this arithmetic: namely, **addition** and **subtraction**. Of each, very briefly here.

[Margin: Addition.] **Addition** is the gathering of the proposed numerical species into one sum. For example, the numbers $28^{\circ} 35' 23''$ are to be added to $36^{\circ} 25' 46''$. The order being kept, so that homogeneous species are placed beneath one another, the operation proceeds — as in other cases — from right toward left. Then, in each species, whatever falls below sixty is set down up to the highest place; and when a sixty is completed, a unit is to be carried over for it into the preceding species. The reason for this is established by distributing the prefixed degree into the smaller species by 60.

	gr. (deg.)	mi. (min.)	fec. (sec.)		gr.	mi.	fec.
thus	36	25	46	Likew ise	27	39	56
	28	35	23		56	28	27
The sum is	65	1	9	Sum	84	8	23

Parsing notes: Both sums carry correctly under sexagesimal rules. Left example: $23'' + 46'' = 69'' = 1' 9''$ (carry 1'); $25' + 35' + 1' = 61' = 1^{\circ} 1'$ (carry 1°); $36^{\circ} + 28^{\circ} + 1^{\circ} = 65^{\circ}$. Result $65^{\circ} 1' 9''$. ✓ Right example: $56'' + 27'' = 83'' = 1' 23''$; $39' + 28' + 1' = 68' = 1^{\circ} 8'$; $27^{\circ} + 56^{\circ} + 1^{\circ} = 84^{\circ}$. Result $84^{\circ} 8' 23''$. ✓

[Margin: Subtraction.] **Subtraction** is the taking-away of a smaller proposed number from a greater, so that the difference appears in the remainder. Accordingly, the order kept in Addition being preserved, the smaller number will conveniently be placed below the greater — and this with regard not to the lower species (which bear the name of minutes), but rather to the whole magnitude of each of the two numbers being compared. Then, the practice of subtraction having been begun here too from the right, if a higher species somewhere does not suffice for the lower one to be subtracted from it, a unit will be summoned from the next preceding species, distributing itself into 60 under the species immediately following — just as, if you wish to take several coins of a smaller denomination from a larger one, this cannot be done otherwise than by resolving the value of the larger into the smaller denomination. Let the preceding examples be repeated.

gr.	mi.	fec.	gr.	mi.	fec.
36	25	46	56	28	27

	gr.	mi.	fec.		gr.	mi.	fec.
	28	35	23		27	39	56
Difference	7	50	23	Diff.	28	48	31

Parsing notes: Left example $36^{\circ} 25' 46'' - 28^{\circ} 35' 23'' = 7^{\circ} 50' 23''$ ✓ (borrow: 25' becomes 85', minus 35' = 50'; 36° becomes 35°, minus 28° = 7°). Right example $56^{\circ} 28' 27'' - 27^{\circ} 39' 56'' = 28^{\circ} 48' 31''$ ✓ (27'' becomes 87'', minus 56'' = 31''; 28' becomes 27' then borrow makes 87', minus 39' = 48' — i.e. $28'-1' = 27'$, $+60' = 87'$, $-39' = 48'$; $56^{\circ}-1^{\circ} = 55^{\circ}$, then $55^{\circ}-27^{\circ} = 28^{\circ}$). ✓

These few remarks on Logistic Addition and Subtraction had to be inserted, for the sake of learners only; they will become easier by means of more examples below.

[Margin: Another division of the Circle.] Furthermore, there is another division of any circle — namely, into its four quadrants — which we follow while pursuing the practice, and which is especially apt for the following distribution of the triangle.

Diagram. A circle with center A. Two diameters, B D (horizontal) and C E (vertical), cross at the center A at right angles, dividing the circle into four equal quadrants. The cardinal points are labeled around the circumference.

- C (top): endpoint of the vertical diameter
- B (right): endpoint of the horizontal diameter
- D (left): endpoint of the horizontal diameter
- A (center): center of the circle
- E (bottom): endpoint of the vertical diameter

For when, in the adjoining circle B C D E, whose center is A, the diameters B D and C E meet at right angles in A, it is evident in what manner the said circle is split into four equal parts, all four angles around the center A being right angles. And since the measure of an angle issuing from the center of a circle is determined by reference to the periphery of that same circle, and is double that of the angle which, standing on the same periphery, begins from the opposite side (by *Elements* III, proposition 20) — therefore the individual quadrants, or 90 degrees, enclose a right angle in every triangle, and conversely. The angle that exceeds this measure is called **obtuse**; the one that falls short of it, **acute**.

[Margin: Division of the triangle from the consideration of angles.] From this consideration of the angles in a single triangle, the proposed triangle is termed either **right-angled**, which possesses some right angle, or **oblique-angled**, when there are no right angles in it but either all are acute, or one is obtuse and the rest acute.

[Margin: Another division of the triangle from the character of its sides.] Furthermore, another division of the triangle arises from the character of its sides, so that it is called either **equilateral** [Isopleurum], or **isosceles**, or **scalene**. The equilateral is a triangle of equal sides and angles. The isosceles is a triangle of equal legs. The scalene, finally, is a triangle of all unequal sides. But this division of the triangle was the more general one; the special division follows.

Astronomia Danica — Pars Prior (Trigonometric Compendium)

PARASCEVE [Preparation]

A triangle is either plane (rectilinear) or spherical (curvilinear). [Margin: A special division of the triangle into plane and spherical.] A rectilinear triangle consists of three straight lines. A spherical triangle consists of three circular lines on the surface of the same sphere. [Margin: The spherical.]

GENERAL THEOREMS Serving the Solution of Triangles

The geometric theorems that bear on the solution of triangles are considered in two classes. Some are external to the procedure, and these are called the prior and more general theorems. Others enter into the construction of the canons [tables] that carry out the analysis of triangles by numbers; these are more specialized, since they are restricted solely to sines, tangents, and secants. To mark them off from the former, I call them *Enunciations* [Enunciata].

THEOREMS

[Margin: General theorems.]

1. A line — whether straight or circular — falling upon a homogeneous line makes two angles together equal to two right angles. Therefore, given one of those angles, the other is the complement of the given one to a semicircle, that is, to 180 degrees.

2. Vertical angles are equal. Consequently, given any one of the four angles formed at the point of intersection of two homogeneous lines, the three remaining angles are determined: first in general, since they are the complements of the given angle to the measure of the whole circle; and then specifically, by the preceding theorem.
3. Every proposed triangle can be resolved into two right triangles by dropping a homogeneous perpendicular line from one of its angles to the opposite side — conveniently, that is, so that this perpendicular falls opposite the given angle.
4. In every plane triangle the three angles are equal to two right angles, that is, to 180 degrees. Therefore, given two angles of a rectilinear triangle, the third is the complement of the two given ones to two right angles, that is, to 180 degrees.
5. In a spherical triangle the three angles are greater than two right angles. Here it must be noted that, although this holds geometrically and ἀκριβῶς [Greek: akribōs, "exactly"] for any spherical triangle whatsoever, nevertheless in a very small spherical triangle — one in which no side exceeds a single degree — the excess of such three angles above two right angles is scarcely, if at all, detectable. For this reason the solution of a spherical triangle of this kind does not differ from the analysis of a plane one.
6. A straight line falling upon parallel lines makes the alternate angles equal on either side.
7. If, within a rectilinear triangle, a line is drawn parallel to one of the sides, it cuts the remaining sides proportionally. [Margin: *Elements* VI, Props. 2 & 4.] For in this way equiangular triangles are formed on both sides, and these therefore have proportional sides.
8. Since all reasoning proceeds by means of straight lines, it is clear that in the analysis of a plane triangle — for the angles — and in that of a spherical triangle — for the angles together with the arcs or sides — one substitutes their sines (that is, the halves of the chords in the circle), or their tangents, or even their secants. But what these lines are, and in what ways they ought to be fitted to the analysis, must now be briefly set out.

Definitions of Sines, Tangents, and Secants

About the center A (in the following figure) let a circle BC be described, and in one of its quadrants — sufficient for the whole business — let any point D be conceived; let the radius AD be drawn, and likewise the opposite diameters of the circle DE and DF.

Now, since the *right sine* [sinus rectus] of an angle or of some arc is defined as the straight line that descends perpendicularly into the radius from the endpoint of the proposed arc, beneath that same arc, it will be —

Diagram. A circle centered at A, drawn within a square frame whose corners lie on the circle. The quadrant at the upper right is the working quadrant. The point D lies on the circle within that quadrant; the radius AD is drawn from the center to D. Two perpendicular diameters cross at A: the horizontal one terminating at B (top) and C (bottom-left region), and structures showing the sine, versed-sine, tangent, and secant lines built on arc DC.

- A (center of circle): the center
- B (upper region, on circle): endpoint of one diameter; arc DB is the complement of arc DC
- C (left, on circle): endpoint of the other diameter; arc DC is the proposed arc
- D (in upper-right quadrant, on circle): endpoint of the proposed arc DC
- E (interior, near center): foot of the perpendicular from D onto the horizontal radius — DE is the right sine of arc DC
- F (interior): foot of the perpendicular relating to arc DB — DF is the right sine of the complement (arc DB)
- G (exterior, on extended line): point where the tangent CG meets the extended radius AD — AG is the secant of arc DC
- H (exterior, upper): point where the tangent BH meets the extended radius — AH is the secant of the complement

— it will be DE, the right sine of the conceived arc DC, that is, of the angle DAC. Likewise DF is the right sine of the complement of arc DC — that is, of arc DB, or of the angle DAB. [Margin: What the right sine is.] [Margin: What the right sine of the complement is.]

Further, since the *versed sine* [sinus versus] of an angle or some arc is defined as the portion of the radius from the endpoint of the right-sine line, continued to the circumference, the versed sine of arc DC will be the line EC. Likewise the versed sine of the complement of arc DC — that is, of arc DB — is the line FB. [Margin:

What the versed sine is.] It should be noted, however, that although the versed sines just described are in use among Magini and others, I, setting these aside, employ the right sines throughout. [Margin: Note. The versed sine is in no use with us.]

So much for the sines (or lines) within the quadrant of the circle. There follows the discussion of tangents — lines drawn outside the circle — and likewise of secants, drawn through the circle.

Further, from A, the center of the circle, through D, the endpoint of the proposed arc, let the line AD be extended indefinitely, and from the endpoints of the diameter let the perpendicular lines CG and BH be erected, meeting the indefinitely extended line AH. [Margin: What the tangent is.] Now, since the tangent of some arc of the circumference is the straight line, perpendicular to the radius at the end of the diameter (the point, that is, which the said arc reaches), continued through the endpoint of the given arc — CG will be the tangent of arc CD, or of the angle DAC; and BH the tangent of the arc of its own complement, namely BD, or of the angle BAD.

Finally, the secant of an arc of the circumference, since it is the straight line drawn from the center through the given endpoint of the arc into the tangent of that same arc, will be AG, the secant of the proposed arc CD, or of the angle DAC. And AH is the secant of the complement of the same — that is, of arc BD, or of the angle BAD. [Margin: What the secant is.]

Up to this point the definitions of sines, tangents, and secants have been given. There follow —

CERTAIN ENUNCIATIONS Premised to the Analysis of Triangles

1. Given the sine, tangent, or even the secant of some arc, there is given in the canon the sine, tangent, and secant of the complement of that same arc to 90 degrees. Thus, given DE, the right sine of arc CD, the arc CD itself is given, and therefore its complement DB, and consequently the right sine of this, DF. The same holds for the tangents and secants to be obtained from the canon, so that here nothing but the use of the table is required — and even for this the correction for the second minutes will be recommended, since the numbers extend at least to the first minutes, or to fractions of them, as was indicated above.
2. The right sine of every angle is proportional to the side subtended by that angle, and conversely. Hence also, by the conversion of terms in the rule of

proportion: as the sine of one given angle is to the sine of the other angle, so is the one given side to the other side sought — namely, of those sides that subtend the same angles of the triangle.

This enunciation, whose use in analysis is most frequent, is demonstrated by the seventh of the preceding theorems as follows. Let ABC be any triangle, and let the side AB be produced to E, so that AE becomes equal to BC. Then, AE and BC having been made equal and set up as radii, let the arc EH be described, which is the measure of the angle at A, whose right sine is EF; and the arc BG, which is the measure of the angle at C, whose right sine is BD. Therefore, within the right triangle AEF, since BD is parallel to the base EF, it will be: as AB, the side opposite the angle at C, is to BD, the right sine of that same angle, so is AE, the side — that is, by hypothesis —

Diagram. A triangle ABC with the side AB extended to E (where AE = BC), used to demonstrate the proportionality of sines to opposite sides. A large circular arc sweeps from E down to H along the baseline; the foot points G, D, F, and H lie along the base line AC extended.

- A (left vertex, on baseline): the angle at A; vertex of the construction
- B (upper vertex): the angle at B
- C (right vertex, on baseline): the angle at C
- E (upper point, above A): endpoint such that AE = BC, set as a radius
- F (on baseline): foot giving EF, the right sine of the angle at A
- G (on baseline, left): foot relating to arc BG / the angle at C
- D (on baseline): foot giving BD, the right sine of the angle at C
- H (on baseline, right): endpoint of arc EH (measure of the angle at A)

— that is, by hypothesis, BC, which is set opposite the angle at A, to the right sine of that same angle, namely EF. Which was to be shown.

3. If the given angle is obtuse, one must work with its complement to a semicircle, that is, to 180 degrees.
4. In every right triangle, when one of the sides about the right angle is set as the radius (or whole sine), the other side about the right angle will be the tangent of the angle opposite it — as in the next-to-last figure, recalled here. For if the side AE of the right triangle AED is set as the radius AE, then DE will be the tangent of the angle DAE; or if DE is set as the radius, then AE will be the tangent of the angle ADE — by the seventh theorem of this set.

Diagram. A small quadrant figure (a recalled portion of the earlier circle figure) showing the secant–tangent relationship. The center A is at the lower left; a quadrant arc curves to the upper right, with point D on the arc and point E nearby on the radius, and points G and H on the exterior tangent lines.

- A (lower-left, center): the center of the circle
 - E (interior, on radius near A): foot point; AE relates to the right sine
 - D (on the arc): endpoint of arc DC
 - G (exterior, right): point on the tangent CG
 - H (upper right, exterior): endpoint where the tangent BH meets the extended radius
5. By the same reasoning, the radius AC is the mean proportional between GC, the tangent of arc DC, and the tangent of its complement BH. For as CG is to AC, so is AB to BH.
 6. The radius is the mean proportional between the secant of an angle and the right sine of the complement of that same angle — as in the third-from-last figure. For as AG, the secant of the angle DAC, is to AC, the radius, so again is AD, the radius, to AE, the right sine of the complement of that same angle at A.

Therefore, in a right spherical triangle, as the radius AD is to the right sine of the angle DAE — namely DE — so is AH, the secant of the remaining angle BAD, to the secant of the side AE; that is, to the side opposite the remaining angle, at D.

Since the analysis of a proposed triangle is carried through almost entirely by these few enunciations — which serve, as it were, as a directory for the reasoning — and since I am aiming at brevity, the other proportions that could have been elicited in many further ways from the sines, tangents, and secants among themselves had to be passed over. So much for the first member [of the subject].

ON THE SECOND MEMBER

Up to this point the matters treated have been the παρασκευαστικά [Greek: *paraskeuastika*, "preparatory things"], that is, the preparatory parts of the analysis of triangles. Those that accompany the analysis itself are, in the first and general place, the διδόμενα [Greek: *didomena*, "things given"], namely the data required in solving any triangle, and the ζητούμενα [Greek: *zētoumena*, "things sought"], that is, the things sought, which arise from them.

Now, since the ἀνάλυσις [Greek: analysis] of triangles falls universally under the golden rule of proportion — in which, three terms being given, a fourth is sought — it is readily understood from this that, in every triangle to be solved into numbers, three things must be given, so that from these the fourth, or all the three others remaining in the said triangle, may be brought to light according to the analogy of the data.

That rule, then, which on account of its most excellent usefulness is called the *golden rule* of proportion, will doubtless be the more acceptable and commendable to us — since it is the easier — the more it provides a more compendious way of finding the fourth term sought in individual cases (by investigating it, without question, by repeated trials), and of avoiding fractions, than the way that the multiplication and division of those numbers in the customary manner requires. [Margin: The foundation of the golden rule is delivered in our *Arithmetic*, Prop. 1; and is demonstrated in *Elements* VI, Prop. 16, and Book V, Prop. 19.] On this account, the industry of those who have laid out their labor fruitfully on it is especially to be praised. Among them one may certainly count the most ingenious invention of Lord John Napier, Baron of Merchiston, the Scot [John Napier], set forth in his admirable *Logarithm* [*Mirifici Logarithmorum Canonis Descriptio*], published in the year 1614. Yet because such a procedure (in my judgment) is somewhat too far removed from the continual inspection of demonstrations — which is more necessary for beginners — for that reason I judge that this compendium [the prosthaphaeretic method] is rather to be chosen, which carries out that golden rule in the very transaction [διδομένων, the handling of the given terms], and no less by sheer number—

— by the addition and subtraction of numbers (which they call by a single word) [προσθαφαίρεσιν, Greek: prosthaphairesin, "addition-and-subtraction"]. This is especially so since such a practice proves most apt for the solution of certain spherical triangles — those, namely, whose use in the sphere is most frequent.

But if anyone should ask about the inventor of this compendium: neither the Arabs nor John Regiomontanus [Regiomontanus] were its authors — as their analemmatic writings declare — and I certainly have no one earlier than our own Tycho [Tycho Brahe] and Wittich of Breslau [Paul Wittich]. By their mutual efforts, indeed, certain spherical triangles were first worked out at Hven [Hveen] in the year 1582 by such a procedure, for the benefit of the students of Urania [astronomy].

Afterward Clavius [Christopher Clavius] made this prosthaphaeretic doctrine somewhat broader. But last of all, the most excellent geometer and

mathematician of Wittenberg, Master Melchior Jöstel [Melchior Jöstel], made it so universal that it could readily be fitted, whenever required, to all triangles — plane as well as spherical — to be solved into numbers. The first proof of this that most courteous man showed me in the year 1598, in person, as to an intimate friend. That doctrine, altered in only a few points, is the one I now bring forward — with all the brevity I can — for carrying out the said golden rule; and I commend it especially to the lovers of learning [philomathes], wherever they may prefer to use it. If they should desire to have this compendium for prosthaphaeresis demonstrated in every way, they will find it in due time — either in our *Geometry*, or with that most distinguished man, Master Ambrose Rhodius [Ambrosius Rhodius], mathematician of Wittenberg, once the genuine [γνήσιον, Greek: gnēsion, "genuine, true-born"] disciple of the late Master Jöstel, of blessed memory.

THE PROSTHAPHAERETIC DOCTRINE Fitted to Carrying Out the Golden Rule Everywhere

The foundation of prosthaphaeresis is this: that the radius (or whole sine) — namely the number 10,000,000 (or extended to still more digits) — be placed in the first position among the data of the golden rule of proportion.

Now, the radius is sometimes admitted among the data, as in right triangles, and sometimes not, as in the other cases that are content with a single proportion set up. Again, it appears whether the radius is given in the first position of the rule, as is required, or only in some one of the following positions — in which case it needs to be transposed here. Accordingly, under this consideration the doctrine is set out in order and is restricted to three general rules.

FIRST RULE OF PROSTHAPHAERESIS

[Margin: Theorem 8.]

When the radius occupies the first place of the rule, and the two following terms are right sines, prosthaphaeresis is carried out for finding the fourth term in this way:

1. Let the smaller of the remaining given arcs, and the complement of the larger, be added together and subtracted from one another; and let the right sines be sought of both the sums and the differences of the arcs.

2. If the smaller arc is greater than, or equal to, the complement of the larger, then half the aggregate of the sines will be the fourth term sought. So here the sines found are added to one another, etc. [Margin: Prior case.]
3. But if the smaller arc is less than the complement of the larger, then half the difference will be the fourth term sought. So here the sines found are subtracted from one another, etc. [Margin: Posterior case.]

Demonstration and example of the prior case.

The meaning of the preceding rule is made sufficiently clear by the present diagram and the worked example added here. Let the larger of the given arcs, DE, be 65° , and the smaller, FG, be 54° . The former is the angle BAC; the latter is the side BA of the right triangle BCA, whose measure is the arc FG — or rather, the right sine of that same arc — while the angle at C is right. The problem is, from these, to find the side BC and the arc corresponding to it. This is easily accomplished by Propositions 2 and 1.

Diagram. A circle with center A. Through it run several chords and radii defining the construction for the prior case. The vertical and horizontal diameters cross at A; labeled points sit around the circumference and along the interior lines.

- F, I (top): points near the top of the circle
- E (upper right): point on the circumference
- B (interior, upper right): vertex of the right triangle
- G (left): point on the circumference, end of a horizontal chord
- O (right): point on the circumference
- L (interior): point on an interior line
- R (interior, near center): point of the construction
- Z (left): point on the left horizontal line
- K (far left): point on the circumference at the horizontal
- M (interior, left of center): foot of a perpendicular
- A (center): center of the circle
- C (interior, right of center): point near center
- D (right): point on the circumference at the horizontal
- H (lower left): point on the circumference
- N (interior, lower center): point below center
- S (lower right): point on the circumference
- P (bottom left): point on the circumference

- Q (bottom): point at the bottom of the circle

The right sine triangle BCA is inscribed near the upper right; the dotted arc traces the relation between FG and the chord constructions.

The rule of three is set out thus:

S.T. [whole sine] : BA or FG :: BAC or ED : BC For as 90° is to 54° , so 65° is to $47^\circ 9'$. 10000000 [whole sine] | S.R. 8090170 | S.R. 9063078 | S.R. 7332184

(That is: as the right sine of 90° is to the right sine of 54° , so the right sine of 65° is to the right sine of $47^\circ 9'$, the fourth term sought.)

The same by prosthaphaeresis.

Quantity	Value
GF	54° (smaller arc)
GK	25° (complement of the larger arc)
FK	79° (sum)
KH	29° (difference)

Taking the right sines of the sum and the difference (Resp. S.R.):

Term	Sine	Label
sine of sum (FK)	9816272	FM
sine of difference (KH)	4848096	HZ or MN
aggregate of sines	14664368	FN
half	7332184	FR or BC (the quantity sought)

The demonstration is this: as FH — the chord of the arc FH formed from the sum and the difference, whose total is double the smaller given arc FG — is to FN, the aggregate formed from the sines of the sum and difference, so half of FH (that is, FL or BA) is to half of FN (that is, FR or BC, the quantity sought), by Theorem 7.

Demonstration and example of the posterior case.

First, let the greatest obliquity of the Sun be given as $23^\circ 32'$, which is represented in the adjoining diagram by the arc DG, or by the angle BAC. Let the larger given

arc also be EF, 50° , whose right sine is TF or AB. Therefore, in the right triangle ACB, the sine BC is sought from these — from which its arc is afterward established, namely the declination of the Sun corresponding to 20° of Taurus [∅]. This BC is easily found by Theorem 7 in the following way:

Diagram. A circle with center A, with interior chords and radii laying out the posterior-case construction. Labeled points run around the circumference and along the interior lines.

- I, F (top): points near the top of the circle
- E (upper left): point on the circumference
- T (interior, upper left): point on an interior line
- L (interior): point on an interior line
- B (interior, upper right): vertex of the right triangle
- G (right): point on the circumference at the horizontal
- O (left): point on the circumference at the horizontal
- R (interior, near center): point of the construction
- ϕ (interior, right of center): point marked with phi
- Z (right): point on the right horizontal
- K (far left): point on the circumference at the horizontal
- A (center): center of the circle
- M (interior, left of center): foot of a perpendicular
- C (interior, right of center): point near center
- V (right): point on the right horizontal
- H (lower right): point on the circumference
- N (interior, lower center): point below center
- S (lower left): point on the circumference
- P (bottom): point near the bottom of the circle
- Q (bottom): point at the bottom of the circle

AG : GV or GAD as S.T. : $23^\circ 32' 10000000$ | S.R. 3992826

EF : BC so $50^\circ : 17^\circ 48' 36''$ S.R. 7660445 | S.R. 3058682

But

Quantity	Value
GD	(smaller arc)
F ^d D	(sum of arcs)

Quantity	Value
DH	(difference of arcs)

But by prosthaphaeresis in this way:

Quantity	Arc	Sine	Label
FG (complement of the larger arc)	40°		
	23° 32'		
(sum)	63° 32'	S.R. 8951939	FM
(difference)	16° 28'	S.R. 2834574	HZ or MN
Difference		6117365	LM
Half		3058682	RM or BC

The demonstration here too is plain from the diagram itself, and from the figures attached to the operation of the numbers. For since RM must equal BC: first, as FH is to half of FB, so FN is to half of it, FR. But when MN is taken away from FM, there remains LM, whose half RM equals BC.

[Margin: When the remaining terms are not sines.]

Furthermore, when the whole sine occupies the first place of the rule, and one of the following terms — or even both — are not right sines but tangents, or any other numbers whatever (provided they are not larger than the whole sine), then to those terms that are not right sines, enough zeros [ciphers] are added at the end so that they are equal to sines. Then from the Table of Sines the arcs corresponding to these are sought, as if they were right sines, and they are handled according to the prescribed First Rule of Prosthaphaeresis. Finally, from the fourth term, the quantity that emerges, as many zeros are removed as had been added at the start.

Example 1.

Let there be given, in the plane right triangle ABC, besides the right angle at C, also the base BA = 351 [parts]. Likewise the angle at A = 24°. From these, the side BC is sought.

Diagram. A small right triangle. Vertex B at top, vertex A at lower left, vertex C at lower right, with the right angle at C. Dotted arcs at A and B mark the angles.

- A (lower left): vertex with the given angle 24°
- B (top): upper vertex
- C (lower right): vertex of the right angle

The rule, according to Proposition 2, will stand thus:

$$C : BA :: CAB : BC \text{ as } 90^\circ : (351 \text{ parts}) :: 24^\circ : 143 \text{ (nearly) S.T. } 10000000 \mid 351 \text{ p.} \mid \text{S.R. } 4067366$$

But prosthaphaeresis proceeds thus, according to the Second Case:

Quantity	Arc	Value
Complement of the given angle	$66^\circ 0'$	
BA (as a sine: 3510000)	resp. arc $20^\circ 33'$	
sum	$86^\circ 33'$	S.R. 9981877
difference	$45^\circ 27'$	7126385
difference of sines		2855492
Half		1427746

Therefore, when the last four zeros [ciphers] are removed, the side BC sought is 143 parts, nearly.

Example 2, where neither of the following terms is a right sine.

Let there be, in the adjoining right triangle ABC, the side CA given as 8 parts, and the angle at A = $36^\circ 53'$. Therefore, by Theorem 4, the angle at B will be $53^\circ 7'$, so long as the proposed triangle is plane. But if, without using this angle, the side BC is sought from the given data,

it is found by Proposition 4. For if CA is set as the radius, BC will be the tangent of the given angle at A. For:

$$C : BAC :: CA : BC \text{ as } 90^\circ : 36^\circ 53' :: 8 \text{ p.} : 6 \text{ p. S.T. } 10000000 \mid \text{T. } 7503663$$

The prosthaphaeresis, however, in this example follows from the foregoing instruction as follows:

Quantity	Value	Corresponding arc
CA 8000000 (S.R.)		resp. arc $53^\circ 7'$

Quantity	Value	Corresponding arc
CAB Tang. 7503663 (treated as S.R.)		resp. arc $48^{\circ} 38'$

These are handled according to the First Case of the rule, in this way:

Quantity	Arc	Sine
(smaller arc)	$48^{\circ} 38'$	
(complement of the larger)	$36^{\circ} 53'$	
sum	$85^{\circ} 31'$	S.R. 9969401
difference	$11^{\circ} 45'$	S.R. 2036418
aggregate of sines		12005819
half		6002909 (the quantity sought)

Diagram. A right triangle. Vertex B at top, vertex A at lower right, vertex C at lower left, with the right angle at C. Dotted arc marks the angle at A.

- A (lower right): vertex
- B (top): upper vertex
- C (lower left): vertex of the right angle

Therefore, when six zeros [ciphers] are removed from the end, the side BC comes out as 6 parts.

Lastly, when the following given terms are found to be larger than the radius — whether only one is larger, or both: if only one is larger than the radius, let that one be divided by the radius (retaining in it, of course, as many figures as there are degrees equal to the zeros in the radius), and, the quotient being preserved, one proceeds with the retained figures, which are to be set as the right sine. Then, once the prosthaphaeresis is carried out, the given smaller number is multiplied by the quotient preserved earlier; the product thence is added to the half found, and the fourth term sought emerges.

Example.

In the adjoining spherical right triangle ABC, there are given, besides the right angle at C, also the angle at A = 30° , with its opposite side BC = $23^{\circ} 30'$. Whence the side AC is sought, by Proposition 4. For:

Diagram. A spherical right triangle with a curved side. Vertex B at top, vertex A at right, vertex C at lower left, with the right angle at C. The hypotenuse curves (drawn as an arc) between A and B.

- A (right): vertex with the given angle 30°
- B (top): upper vertex
- C (lower left): vertex of the right angle

C (right angle) : T. compl. BAC :: T. BC : S.R. CA as $90^\circ : 60^\circ :: 23^\circ 30' : 48^\circ 51'$ (approximately) S.T. 10000000 | T. 17320508 | T. 4348124 | S.R. 7531081

By prosthaphaeresis the procedure is dispatched thus:

Quantity	Value	Corresponding arc
T. 17320508 (S.R.)		resp. arc $47^\circ 4'$
T. 4348124 (S.R.)		resp. arc $25^\circ 46'$
Complement of the larger arc		$42^\circ 56'$
smaller arc		$25^\circ 46'$

Quantity	Arc	Sine
sum	$68^\circ 42'$	S.R. 9316913
difference	$17^\circ 10'$	S.R. 2951523 (to be subtracted)
difference		6365390
half		3182695
Smaller multiplied by the quotient, namely 1		4348124
Sought		S.R. 7530819, nearly, as before.

But when both of the following given terms are larger than the radius, let each be divided separately by the radius, the quotients being kept; and from the table of sines let the arcs of the truncated values [the parts cut off] be sought, by which the prosthaphaeresis is completed according to the prescriptions. Then to the half

found are added the products that arise — both from the quotient of the third number into the

second remainder, and from the quotient of the second into the whole third — or vice versa, if one prefers. The aggregate of those three will give the fourth term sought.

Example.

Let there be, in the right triangle ABC with the right angle at C, the base AB given as 60° , and the angle at A = 30° .

Diagram. A spherical right triangle with a curved side. Vertex B at top, vertex A at lower left, vertex C at lower right, with the right angle at C. The hypotenuse curves (drawn as an arc) between A and B.

- A (lower left): vertex with the given angle 30°
- B (top): upper vertex
- C (lower right): vertex of the right angle

From these, the remaining angle at B is sought. Therefore, by Propositions 5 and 6:

BA : Compl. A :: B as $90^\circ : 60^\circ :: 60^\circ : 73^\circ 54'$ S.T. 10000000 | Sec. 20000000
| T. 17320508 | 34641016

The same by prosthaphaeresis, from the prescriptions:

Quantity	Value	Corresponding arc
20000000 (S.R.)		resp. arc $0^\circ 0'$
7320508 (S.R.)		resp. arc $47^\circ 4'$
Complement of this		$42^\circ 56'$
Smaller arc		$0^\circ 0'$

Quantity	Arc	Sine
Aggregate of arcs	$42^\circ 56'$	S.R. 6811470
Difference of arcs	$42^\circ 56'$	S.R. 6811470 (to be subtracted)
Difference of sines		0000000
Half		0000000

There is added the product of the quotient of the third — namely unity — into the second remainder, 0000000 (which is therefore 0000000, that is, nothing); and from the whole third, namely 17320508, by 2 — that is, the quotient from the second. Therefore in this example the same product results, namely 34641016, the fourth term sought, as before.

This calculation also succeeds in these cases if the numbers larger than the radius are subtracted from the end until they are reduced to right sines.

Likewise, if a tangent or a secant happening to fall into the proportion extends itself two zeros [ciphers] beyond the radius of the circle, let the first figure on the left be taken as the quotient, the last on the right be rejected, and the remaining figures be taken as the right sine. But the procedure in these cases will not yield the result so precisely, unless the arcs have been most rightly corrected for the seconds and drawn from the *Opus Palatinum* [of Rheticus and Otho]. And even if this should happen in the operation above, nevertheless the prosthaphaeresis in that case will still be more reliable, and will not differ perceptibly from the truth.

By these examples, then, which we have left, all the cases of prosthaphaeresis are included — wherever the radius occupies the first place of the rule. And indeed, although the prosthaphaeresis sometimes brings little or no shortcut, as in the last cases of this rule, it is nonetheless gratifying — and useful besides — to make it so universal that the product of any two numbers proposed for multiplication is obtained by addition and subtraction alone. This procedure is exceedingly expeditious, since right sines alone, as in the first examples, suffice.

SECOND RULE OF PROSTHAPHAERESIS.

When the radius, or whole sine, is placed in the second or third place of the rule, a transposition must be applied so that it occupies the first place — since every prosthaphaeresis demands that the radius be set in the first place, as was said above. The transposition rests on two precepts, but on a single demonstration.

First precept of transposition.

If any number greater than the whole sine occupies the first place of the rule, to this

the arc be sought from the table of secants, and its complement serves as the radius. By transposing this complement into the first place, the example will then conform to the first rule of prosthaphaeresis.

Second precept.

If the number found in the first place of the rule is smaller than the radius (the whole sine), add one or more zeros to it until an arc corresponding to it can be extracted from the table of secants. Its complement likewise takes the place of the radius and moves into the first place of the rule, and here too the example will conform to the first rule of prosthaphaeresis. Once this is done, append to the fourth term as many zeros as were previously added to the number smaller than the whole sine.

The demonstration of this rule rests on the sixth Enunciation. For when the diagram of that passage is repeated here, it was shown there that the radius AD is the mean between the secant AG — that is, of the arc DC — and DF, the right sine of the complement of that same arc DG. Thus, as AG is to AC, so again AC (or AD) is to DF. Therefore, once the radius (the whole sine) AD is set in place of the secant AG, FD will be the right sine of the complement of the given arc DC, derived through the given secant AG, which we suppose to have been placed in the first place of the rule in the position of the radius AD.

Diagram. A circle with two perpendicular radii drawn from center A, one running left (toward C) and one upward (toward B). The figure illustrates the relationship among a secant, the radius, and the right sine of the complement, as used in the demonstration above.

- A (center of the circle, lower right region): vertex
- B (top of the circle): point on the circumference
- C (left side, on the circumference): endpoint of the horizontal radius
- D (interior, near upper region): point on a radius
- E (interior, below D near A): point
- F (right side, on the circumference): endpoint
- G (interior left of D): point on the line toward C
- H (upper left, outside the inner construction): point

Example of the prior case, where the first number in the rule is the greater.

Let there be a right-angled spherical triangle ABC with a given angle at A of 30° , and the opposite side BC of $25^\circ 39' 32''$. From this, by the 6th Enunciation, the base AB is sought. The proportion is as follows:

Sec. compl. A	:	S.T.	::	sec. compl. BC	:	sec. compl. of base
as 60° 0'	:	90°	::	64° 20' 28"	:	30°
200000		100000		230940		115470
00		00		08		04

The prosthaphaeresis runs thus:

Sec. 2000000 corresponds to 60° 0'; its complement, 30°, is set in place of the radius, promoted to the first place, and the proportion stands: as S.T. is to 30°, so Sec. 23094008 is to its result, etc.

Because the third number also exceeds the whole sine, let it be divided by the radius according to the precept set out above, in this manner: 23094008 S.R. — the arc cut off is 18° 2' — [?]; the other was 30°.

	Degrees	Minutes		
Greater complement	60°	0'		
Lesser arc	18°	2'		
Sum	78°	2'	S.R.	9782684
Difference	41°	58'	S.R.	6686981
				(subtract)

Difference of sines: 3095703 Half: 1547851

The sine of the right angle at 30°, as the smaller number set above in the proportion, is 50000000; multiplied by the second quotient, the product yields 10000000, to be added.

Half: 1547851 And they become 11547851, nearly as before.

Example of the latter case, where the number set in the first place is smaller than the radius.

Let there be a right-angled spherical triangle ABC with the two sides about the right angle at C given, namely AC 60° and BC 30°; from these the angle at A is sought, by the 4th Enunciation, in this manner:

Diagram. A small right-angled spherical triangle with the right angle at C (lower right). Vertex B is at the top, A is at the lower left, and the hypotenuse AB is drawn as a curved (spherical) arc.

- A (lower left): vertex of the sought angle
- B (top): vertex
- C (lower right): vertex of the right angle

AC	:	rad.	::	BC	:	BAC
as 60°	:	90°	::	30°	:	33° 41'
						[?]
S.R.		100000		T.		T.
866025		00		577350		666666
4				2		5

The prosthaphaeresis runs as follows:

8660254 sec. corresponds to arc 83° 22' 10" Its complement: 6° 37' 50" T. 5773502, as if S.R., corresponds to arc 35° 15' 50" Its complement: 54° 44' 10"

	Degrees	Minutes	Seconds		
	54°	44'	10"		
	6°	37'	50"		
Sum	61°	22'	0"	S.R.	8777044
Differenc e	48°	6'	20"	S.R.	7443763

1333281 6666455 T. BAC, nearly as before.

THIRD RULE OF PROSTHAPHAERESIS.

When the radius (the whole sine) is found nowhere in the rule of proportions, then a double prosthaphaeresis must be applied.

First, by the preceding second rule, let it be done: as the first of the given terms is to the second, so is the whole sine to another.

Then, by the first rule: as the whole sine is to the term found, so is the third to the fourth sought.

Or, if it seems more convenient, proceed thus: as the first is to the third, so is the whole sine to another. Then: as the whole sine is to the term found, so is the second to the fourth.

The reasoning is that the first, together with the second or third, is reduced to two other terms, the first of which is the whole sine; and thus at last the example is fitted to the first rule.

The demonstration is offered clearly enough in a proportion of numbers, that is, of the four terms of the rule of proportion. Let the terms of the proportion be, apart from unity, with such zeros: 8. 12. 9. ($13\frac{1}{2}$). The quotient that stands in the same ratio to 10 as 8 does to 12 (for 9 and $13\frac{1}{2}$ stand to those in the same ratio, namely as $1\frac{1}{2}$, or in sesquialteral proportion) — 10, or the radius, is interpolated, and the rule is formed thus: as 8 is to 12, so is 10 to (15).

Secondly: as 10 is to 15, so is 9 to ($13\frac{1}{2}$), as before.

And so, with the prosthaphaeresis repeated, the fourth number found is to be combined, according to the first rule, with the one that fell in the first place of the radius; and thus every variety that can arise will be judged according to that same first rule.

But this rule too must be illustrated by an example:

Example.

Let there be a plane scalene triangle with two given angles: at A 20° , at B 60° , with the side BC 125 paces. The side CA is sought. Therefore, by the 2nd Enunciation, it will be:

Diagram. A plane scalene triangle. Vertex A is at the lower left, B at the lower right, and C at the upper right; the side BC is the right-hand side.

- A (lower left): vertex of the 20° angle
- B (lower right): vertex of the 60° angle
- C (upper right): vertex

CAB	:	CB	::	CBA	:	CA
as 20°	:	125 p.	::	60°	:	
S.R.				S.R.		317
342020				866025		
1				4		

By Prosthaphaeresis I:

Sec. 34202010 corresponds to arc $73^{\circ} 0'$, and its complement $17^{\circ} 0'$, the lesser arc. The greater arc is $60^{\circ} 0'$. Therefore:

	Degrees	Minutes		
its complement	30°	$0'$		
	17°	$0'$		
Sum of arcs	47°	$0'$	S.R.	7313537
Difference of arcs	13°	$0'$	S.R.	2249511

5064026 2532013 half, which is the first sought number.

By Prosthaphaeresis II. Then:

	:	90°	::	$14^{\circ} 41'$	:	$7^{\circ} 11'$
as	:		::		:	
S.T.				S.R.		S.R.
100000				253261		125000
00				3		00

	Degrees	Minutes		
Greater complement	75°	$19'$		
Lesser arc	7°	$11'$		
Sum	82°	$30'$	S.R.	9914449
Difference	68°	$8'$	S.R.	9280532

Difference of sines — — — 633917 Half: the fourth sought — — — 316958, nearly as above.

Now our customary prosthaphaeresis rests on these three rules, since it furnishes great assistance for seeking out the fourth term in a rule of proportion everywhere — especially with large numbers, where otherwise multiplication and division would be troublesome. Yet in comparing the numbers through the

examples given, this one thing produced a small discrepancy: that as for the sines, tangents, and secants, and the arcs arising from them, I have for the most part refrained from correcting them for the seconds adhering here and there, content to leave the worked examples to prosthaphaeresis alone. Accordingly, let anyone who insists on παρεξήγησιν [Greek: parexēgēsīn, "strict precision" — exactness of interpretation] turn to the canon of the *Opus Palatinum*, or correct the sines accurately for the seconds from other sources, and so he will test the truth of the comparison more precisely.

So much for the analysis of triangles in general by prosthaphaeresis alone, which is understood to be most conveniently applied to the foregoing cases — whenever the radius (the whole sine) can be had in the first place of the rule, and the remaining given terms are likewise right sines. There now follows the solution of triangles of the specific kinds proposed.

This applies specifically to individual triangles. And since from the corollary of the definition of a triangle it is understood that six elements are considered in it, and likewise at the beginning of this second part it was understood that three data are required for solving any triangle so that the remaining three hidden elements may be brought to light, the analysis rightly proceeds through the variety in the kinds of plane and spherical triangles concerning the δεδομένων [Greek: dedomenōn, "the given things" — the data] and ζητούμενα [Greek: zētoumena, "the things sought" — the unknowns].

Of triangles.

Triangles, when they are [given / sought — δεδομένα/ζητούμενα], are either:

Pure, as:

- the three angles — I.
- the three sides — II.

Mixed, as:

- two sides with an angle, either
- the included one — III.
- one opposite to either of them — IV.
- two angles with a side, either
- adjacent — V.
- opposite to one of the angles — IV.

Then, since right-angled triangles are of great use in themselves, and sometimes serve conveniently for solving the rest, their analyses — both in plane and in spherical triangles — will rightly come first. With these considerations in mind, we shall give the following doctrines, most suitably adapted for those learning the practical solution of triangles in both kinds.

DOCTRINES ADAPTED TO THE PRACTICAL SOLUTION OF PLANE TRIANGLES, REDUCED TO NUMBERS.

FIRST DOCTRINE.

To exhibit, from the three given angles of a right-angled plane triangle, the proportions of the sides to one another.

Since an angle in a semicircle is a right angle, by Proposition 31 of Book 3 of the *Elements* [Euclid, *Elements* III.31], let the side opposite [the right angle] be made the diameter of a circle. Then the remaining two sides will be the chords subtending the remaining opposite angles; therefore their right sines are sought and doubled, and the proportions of these sides arise, taking the diameter of the circle as 20000000.

Or more compendiously, by the 2nd Enunciation: if the side opposite the right angle is set as the radius of the circle (10000000), then the right sines of the remaining angles will be the sides severally opposite to those angles. Let the first example be the triangle ABC, having a right angle at B, and an angle at A of 30° , and therefore an angle at C of 60° (by the 4th Theorem). Further, about the same triangle let a circle be described, by Proposition 5 of Book 4 of the *Elements* [Euclid, *Elements* IV.5].

Diagram. A circle with an inscribed right-angled triangle ABC. The diameter AC runs horizontally with A at the left and C at the right; D marks the center of the circle on the diameter. Vertex B lies on the upper circumference, so that the angle at B is a right angle inscribed in the semicircle.

- A (left endpoint of the diameter): vertex of the 30° angle
- B (upper circumference): vertex of the right angle
- C (right endpoint of the diameter): vertex of the 60° angle
- D (center, on the diameter): center of the circle

The diameter of this circle is therefore AC; and if it is set as 20000000, then BC will be 10000000 and AB 17320508.

Further, if CA is supposed as the diameter 10000000, then BC will be 5000000 and AB 8660254 — namely, the halves of the former values, that is, the right sines of the given angles.

SECOND DOCTRINE.

From the three given sides of a right-angled plane triangle, the angles are obtained.

Since, by the 2nd Enunciation, the side opposite the right angle stands to the radius as the side opposite any of the remaining angles stands to the right sine of the angle to which it is opposite. Therefore:

Multiply the side opposite the sought angle by S.T., and divide the product by the side opposite the right angle; the result is the S.R. of the sought angle, and therefore also the sought angle itself, and the remaining one, which is the complement of this to 90° .

Example.

Let there be a right-angled plane triangle ABC with given sides, of which AC — opposite the right angle at B — is 15 p. Likewise let AB be 12 p. and BC 9 p. The angle BAC will then be $36^\circ 52'$, and therefore BCA will be $53^\circ 8'$, by the 4th Theorem.

Diagram. A right-angled triangle drawn with the apex B at the top, the right angle at B. The hypotenuse AC forms the base, with A at the lower left and C at the lower right.

- B (top vertex): the right angle
- A (lower left vertex)
- C (lower right vertex)

THIRD DOCTRINE.

Given two sides of a right-angled plane triangle together with the included angle, the remaining angles and the remaining side are obtained.

First, when the included angle is the right angle: since, by the 4th Enunciation, if one of the sides about the right angle is taken as the radius, the other of the given sides about the right angle is the tangent of the opposite angle. Therefore: multiply the side opposite the sought angle by the radius, and divide the product by the other side; the result is the tangent of the sought angle.

Example.

Let the given sides about the right angle in the right-angled triangle above be retained, namely AB 12 p. and BC 9 p. The tangent of the sought angle BAC will then be 7500000, and the angle BAC itself $36^{\circ} 52\frac{1}{2}'$. And therefore also its complement, namely the angle BCA, is $53^{\circ} 8'$.

Further, for the side (the base) one proceeds by the 2nd Enunciation. Multiply the side of one of the angles other than the right angle by the radius, and divide the product by the right sine of the remaining angle. The quotient then gives the side opposite the right angle, for:

					ABC (right angle)		
	BAC	:	BC	::		:	AC
as	$36^{\circ} 52\frac{1}{2}'$	to	9 p.	so	90°	to	15 p.
	S.R.				S.T.		
	60000				10000		
	00				000		

Second, when the given angle included by the given sides is acute, its complement to 90° is likewise obtained by the 4th Theorem, and so the third side too is obtained by the 2nd Enunciation.

There is no need of an example here.

FOURTH DOCTRINE.

Given two sides of a right-angled plane triangle together with the angle opposite one of them, the remaining parts are obtained.

Since, besides the given right angle, another angle is also granted in this triangle, all three angles are therefore obtained by the 4th Theorem.

If, then, the side opposite the right angle is the one given between the two, the third of the sides will be obtained by the second Enunciation:

As S.T. is to the opposite side, so the right sine of the given angle is to the sought opposite side.

But if the base — the side opposite the right angle — is sought, then: as the right sine of one angle is to the given opposite side, so S.T. is to the base.

FIFTH DOCTRINE.

Given two angles of a right-angled plane triangle together with the adjacent side, the remaining parts are obtained.

First, all three angles are thus given by the 4th Theorem.

Then the two sides sought are found by the 2nd Enunciation, as in the preceding Doctrine.

SIXTH DOCTRINE.

Last: given two angles of a right-angled plane triangle together with the side opposite one of the angles, the remaining parts are also obtained.

This Doctrine is solved by the 2nd Enunciation in just the same way as the fourth and fifth of the preceding ones.

So much for the solution of plane triangles, which follow most plainly from the premises. Next come the oblique-angled plane triangles.

SEVENTH DOCTRINE.

Given the three angles of an oblique-angled plane triangle, the proportions of the three sides to one another are obtained.

Here the given oblique-angled triangle is circumscribed by a circle, by the 5th Proposition of Book 4 of the *Elements* [Euclid, *Elements* IV.5].

Afterward the right sines of the given angles are found and each is doubled, whereby they become the chords subtending the angles. These are the sought proportions of the sides, expressed in numbers of the kind in which the diameter is 20000000. Or:

If instead the diameter is taken as 10000000, the right sines of the given angles themselves serve for the sides, preserving the same proportion among themselves as before.

Example.

Let there be an oblique-angled triangle ABC, all of whose angles (or at least two) are given. Let the angle at A be 80° , the angle at B 60° , and the angle at C 40° . The corresponding sines are then these:

Angle		Sine	Side	
80°	S.R.	9848078	BC	of the kind in which the diameter DE of the circle is 10000000
60°	S.R.	8660254	CA	
40°	S.R.	6427876	AB	

Or: of the kind in which the diameter DE is commonly taken as 20000000 — when these right sines are doubled — BC will be 19696156, CA 17320508, and BA 12855752.

Diagram. A circle with an inscribed (circumscribed-about) oblique-angled triangle. The triangle ABC sits inside the circle; a diameter DE is drawn. Vertex A is at the left, B at the upper right, C at the lower center; D is at the top of the circle, E at the bottom left. A dotted line (the diameter or an auxiliary chord) runs across the interior.

- D (top of circle)
- A (left vertex of triangle)
- B (upper-right vertex of triangle)
- E (lower-left point on circle, opposite end of diameter)
- C (lower vertex of triangle)

EIGHTH DOCTRINE.

Given the three sides of an oblique-angled plane triangle, the three angles are obtained.

The given sides of the oblique-angled triangle are either equal or unequal.

1. If the given triangle has equal given sides — that is, if it is equilateral — then, since equal angles are subtended by equal sides, and since the three angles (by the 4th Theorem) are equal to two right angles, that is, to 180°, each of the angles is allotted 60° by dividing this number into three parts.

2. Further, if the proposed triangle has unequal sides, it will either have two equal sides and be isosceles, or it will obtain all three sides unequal and be scalene.

The isosceles triangle is resolved into its ζητούμενα [Greek: zētoumena, "the things sought" — the unknowns] by the 3rd Theorem, by dropping a perpendicular from the angle contained by the equal sides (the legs) onto the base. For in this way the isosceles triangle is divided into two equal right-angled triangles, and in each of them, together with the right angle, two sides are given — namely half the base, and one of the legs, which is opposite the right angle — whence the solution is made by the first case of the fourth Doctrine of this section.

Example.

Let ABC be an isosceles triangle with given sides, namely AB and CB, each of 12 p. (which are therefore called equal legs), and AC the base, 16 p. A perpendicular BD dropped from the angle B divides the base into equal parts at AD and, on either side, at right angles about D. Therefore, in the one right-angled triangle ADB, when the angle ABD is sought, by the 2nd Enunciation:

	AB	:	D	::	AD	:	ABD
as	12 p.	to	90	so	8 p.	to	41° 48' 37"
	S.T.					S.R.	
	10000					66666	
	000					66	

The angle ABD having thus been found, all the remaining angles of the proposed triangle ABC are obtained; for by the 4th Theorem the angle BAD is given as 48° 11' 23" — namely the complement of the angle found, to 90° — to which the angle ACB is equal, on account of the equal subtended sides BC and AB. Therefore, adding these equal angles together and subtracting the sum from 180°, the angle ABC finally stands as the remainder, thus:

48° 11' 23"
48° 11' 23"

96° 22' 46" sum of the angles at A and C, subtracted from the semicircle.
180° 0' 0" the semicircle.

83° 37' 14" the angle ABC,

which also comes out in the same numbers by doubling the value ABC found at the outset, thus:

$$\begin{array}{r} 41^{\circ} \ 48' \ 37'' \\ 41^{\circ} \ 48' \ 37'' \\ \hline \end{array}$$

83° 37' 14" as before.

Diagram. An isosceles triangle ABC with apex B at the top and base AC at the bottom. A perpendicular BD is dropped from B to the midpoint D of the base. A is at the lower left, C at the lower right, D between them.

- B (top vertex)
 - A (lower left vertex)
 - D (foot of the perpendicular, midpoint of base)
 - C (lower right vertex)
3. Or finally the proposed triangle of given sides will be scalene, namely having all three sides unequal.

For this case, although several methods present themselves through geometry by which the solution of such a proposed triangle of given sides into its angles may be made, none is more convenient than the one indicated in Proposition 36 of Book 3 of the *Elements* [Euclid, *Elements* III.36]. This method may therefore be retained, especially since the present Doctrine chiefly serves a good part of geodesy [geodesia], where, for finding the area of a triangle of given sides, the perpendicular — conveniently dropped from the largest of the triangle's angles onto the base — is required to be known. But, to return more quickly to the practice: let the smallest of the given sides be made the radius of a circle, and with its center fixed at the largest of the angles of the given triangle, let a circle be described, which by its circumference cuts off the remaining sides in such a way that the parts left outside, continued together with the rest through the circle, observe the proportions for forming similar rectangles by the cited Proposition 36 of Book 3 of the *Elements*. Once the portion of the largest side outside the circle is known, the remaining base is found, and the solution devolves upon the preceding Doctrines concerning right-angled triangles.

Example.

Let ABC be a plane triangle of given unequal sides:

- AB 25 p.
- AC 28 p.

- BC 17 p.

Therefore, making the smallest of the sides, BC, the radius, let a circle CEG be described with center B, whose circumference cuts off the remaining sides at E and G.

Diagram. A scalene triangle ABC with a circle described about vertex B as center, radius BC. The circle cuts side AB (extended) and side AC. F is at the top right (the extension of AB to the circle), B is the upper-center vertex (center of the circle), G is at the lower left where the circle meets the extended side, A is at the lower left vertex, E is on the base between A and C, D is between E and C, C is the lower-right vertex. A dotted line runs from B down to D.

- F (top right, on the circle, extension of side AB)
- B (upper-center vertex and circle center)
- G (lower left, where the circle cuts the side)
- D (foot of the perpendicular on the base, between E and C)
- A (lower-left vertex)
- E (on the base, where the circle cuts AC)
- C (lower-right vertex)

Now, the side AB being extended to F: since point A is conceived as lying outside the circle, then by Proposition 36 of Book 3 of the *Elements* the rectangle under FA, AG will be equal to the rectangle under CA, AE. Accordingly: as AC 28 p. is to AF 42 p. (namely the sum of the sides AB and BC), so AG 8 p. (the difference of the same sides) is to AE 12 p. Therefore the remaining part of that same side AC is also given, namely EC 16 p., and its half DC (or ED) 8 p. A perpendicular dropped to that point D divides the proposed triangle BAC into two right-angled triangles BDA and BDC, by the 3rd Theorem, and divides it in such a way that on both sides the two sides together with the right angle at D are known. There: AB 25 p. and AD 20 p.; here: BC 17 p. and DC 8 p. Therefore, by the fourth preceding Doctrine and its first case, the angles become known: at A, $36^{\circ} 52'$; at C, $61^{\circ} 56'$; and therefore the third, at B, $81^{\circ} 12'$, by the 4th Theorem.

If the perpendicular BD is desired to be had at the same time (since the area of the proposed triangle BAC stands from multiplying it into half the side AC), this is most easily offered by the 2nd Enunciation, and also by the method of prosthaphaeresis [prosthaphaeresis], since the sides are given, including the given acute angle at C or at A.

NINTH DOCTRINE.

Given two sides of an oblique-angled triangle together with the included angle, to find the remaining two angles and the third side.

Here too the two given sides are either equal or unequal.

If the sides are equal, the proposed triangle is isosceles. Therefore, since the two unknown angles at the base are likewise equal, and their measure is the complement of the given angle contained by the sides, to 180° , by the 4th Theorem — then, by bisecting the said complement, the measure of each of the angles at the base becomes known.

If the sides are unequal, a twofold way remains for solving the proposed triangle. Either one of the given sides is extended beyond the triangle, as is more convenient, and a perpendicular is dropped from the opposite angle to that extension, so that by the construction of a single right-angled triangle two triangles are formed and the solution proceeds in the manner of the right-angled triangle. Or the sum of the hidden angles is divided up through the proportions of the opposite sides, the individual unknown angles are brought to light, and afterward the side opposite the given angle is disclosed by the 2nd Enunciation.

Example of the former method.

We use this method especially when one of the given sides is the radius. Thus: let ABC be a triangle of given sides, AB being the supposed radius 100000, but BC 3571, with the contained angle ABC of 150° . Here the prosthaphaeretic angle at the Sun, BAC, is sought from the side AC, the distance of the Sun at C from the Earth at A. Therefore, BC being extended to D, and a perpendicular dropped from A to D, the triangle ACD will be right-angled, having the right angle at D, and ABD the given angle of 30° . For it is the complement of the given angle ABC, by the 1st Theorem. The angle BAD is also given as 60° by the 4th Theorem. And since BA is the radius, the remaining sides will be the right sines of the opposite angles, and therefore

and therefore AD is 50000, and BD is 86602. Adding to this the side CB of 3571 gives the whole DC of 90173. With this side placed in the role of the radius, or whole sine, AD will be the tangent of the angle ACD, by the 4th Enunciation; and the solution is carried out by the 3rd Dogma above. Thus:

as	DC	to	D	so	AD	to	ACD

90173	90	50000	29° 0'
			28"
	whole		T.
	sine		55449
	10000		
	0		

Therefore, since in the proposed triangle ABC the angle ABC is given as 150° , and the angle ACB has now been found as $29^\circ 0' 28''$, the complement of these to 180° — namely the angle BAC — will be $0^\circ 59' 32''$. This is the solar prosthaphaeresis sought at a distance of 30° from apogee, according to its proper correction.

For the side AC, in the right-angled triangle ADC: as the right sine [S. R.] of the angle at A is to the side DC, so is the radius to the required AC, by the 2nd Enunciation.

Another method, which is easier and does not require the radius among the given values, rests on this problem.

If the sum of two angles is given, the ratio of whose right sines (or of whose opposite sides) is established between themselves, then those angles too are given separately.

In the semicircle BC, whose center is A, let the sum of the two angles DAE and EAF be given — namely DAE [recte: the sum] 80° . Let the ratio of the right sines HD of the greater angle and EI of the lesser — or, by the 2nd Enunciation, of the side DM opposite the greater angle and the side EM opposite the lesser angle — be as 86 to 34. I assert that the angles DAF and EAF are given separately.

For draw from the center A to the point bisecting the sum of the angles the line AG; then from the same center A describe an arc KN, which the chord subtending the sum of the angles — that is, the sum of the given sides DE — will touch at L. Therefore LD, half the sum of the given sides, 60 parts, is the tangent of the half-sum of the angles, 40° ; and LM, the difference between the lesser side and the half-sum of the sides (or between the half-sum of the sides and the greater side) — namely 26 parts — is the tangent of the difference of the angles. Therefore: as DL, 60 parts, is to the tangent of the angle DAL, 40° , which the table gives as 8390996, so is 26 parts to the tangent 3636098, whose arc is about $19^\circ 59'$. This is the angle GAF. Accordingly, when DAG of 40° is added here, the angle DAF of about $59^\circ 59'$ is produced — the angle subtended by DM, 68 parts, of the given

sides. Or, if it is subtracted from GAF [recte: from DAF] the 40° , it leaves the angle EAF of $20^\circ 1'$, subtended by EM, 34 parts, the lesser of the sides.

It should be noted, however, that since the subtended angles are here derived from the proportion of the given sides alone, the ratio of the sum of the sides to their difference is the same as the ratio seen in their halves. Thus the ratio that holds between 60 parts and 26 parts is likewise the ratio of the sum of the sides, 120, to their difference, 52. These terms too can be reduced to their smallest numbers, as is taught in arithmetic, or reduced to a prosthaphaeresis by transference [prosthaphaeresis translationis], as was set forth above and will shortly be shown below by an example.

The practice with an example follows.

Therefore the given sides are added together, and their sum is the first finding.

Next, these same given sides are subtracted one from the other, and the difference is the second finding.

Further, the given angle is subtracted from the semicircle, and the remainder is halved; the tangent of this is sought, and it is taken as the term, or third finding. Then the second finding is multiplied by the third (or the third by the second), and the product is divided by the first finding. When the operation has been performed, there results in the quotient the tangent of the arc of the difference of the proposed angles. This, then, being sought in the table of tangents, and added to the halved angle (whose tangent was previously taken as the third finding), the sum gives the greater of the sought angles; or, the same being subtracted, the difference gives the lesser of the sought angles.

Example.

Let the same example be repeated which was given before in the case of the Sun. Therefore:

AB 100000

BC 3571

103571 first finding

96429 second finding

ABC 150° given angle

30° difference

15° half T. 2679492 third finding

Therefore: as 103571 is to 96429, so is 2679492 to the tangent 249472, to which corresponds an arc of $14^{\circ} 0' 28''$.

This arc, subtracted from the half of the complement of the given angle to a semicircle — namely 15° — leaves the solar prosthaphaeresis entirely as before: $0^{\circ} 59' 32''$.

Furthermore, regarding the construction of the tables of prosthaphaereses for the planets — which proceeds almost from this single present dogma — in order that the labor may be lessened and the calculation made far easier: since the two first findings remain constant throughout (arising, namely, from the given radius and eccentricity within the orbit of each individual planet), the division here is most conveniently dispensed with by transference, and the multiplication, if it must be employed, is rendered far more expeditious, according to what is now appended in this same example.

First, for the transference by Rule 3 of the prosthaphaeresis [Reg. 3 Prosthaphaer.], it will be:

as	first	to	secon	so	whole	to	93104
	findin		d		sine		24
	g		findin		[S. T.]		
			g				
	10357		96429		10000		
	1				000		

Therefore, by transference:

as	10000	to	93104	so	26794	to	24947
	000		24		92		20, as
					(15° ,		before
					T.)		

But here only multiplication was needed, since the division had been carried out on the radius. The shortcut set forth in our revised *Arithmetic* makes multiplication by the same number throughout exceedingly easy. For if all the numbers of the second proposed finding are multiplied by every digit from unity up to 10, and the products are kept in order, the calculation is completed by

addition alone, when these are added singly each time in place of the corresponding digits of the multiplier, thus:

Let the second finding,

9310424

be multiplied by:

1	9310424
2	18620848
3	27931272 [recte]
4	37241696
5	46552120
6	55862544
7	65172968
8	74483392
9	83793816

These products, by each of the digits below 10, will be applied in the following manner.

Parsing notes: Row 3 of the multiplication table is printed as "28831272" in the source, but $9310424 \times 3 = 27931272$; flagged as a likely compositor error. Several individual digit-glyphs in this table column are ink-degraded (the "5" and "6" rows partly broken). The right-sine abbreviation S.R. and whole-sine S.T. appear in the proportion brackets; the bracketed-proportion layout is rendered here as pipe-tables.

Multiplication with application of the products, by addition alone.

9310424
2679492
2. 18620848
9. 83793816
4. 37241696
9. 8379 3816
7. 65172 968
6. 55862 544
2. 1862084 8

2494720 finding IV.

Or more concisely, from the right, in this manner:

9310424
2679492

2. 18620848
6. 5586254
7. 651729
9. 83794
4. 3724
9. 838
2. 18

2494720 finding IV.

Parsing notes: Both worked multiplications above are heavily ink-degraded in the source, with digits broken across the column rules; the digit-grouping (where each partial product is shifted by one place) is partly conjectural. The leading multiplier-digits in the left column (2, 9, 4, 9, 7, 6, 2) and right column (2, 6, 7, 9, 4, 9, 2) are the digits of the multiplier 2679492 read in two different orders (left-to-right vs. right-to-left). Several partial-product rows flagged [?] where glyphs could not be resolved at scan resolution

DOGMA X.

Given two sides of a plane oblique-angled triangle together with the angle opposite one of them, to find the rest.

The analysis is accomplished from these given quantities [Greek: διδομένοις (didomenois, "given quantities")] by the 2nd Enunciation alone — the 3rd Enunciation also being employed if the given angle is obtuse. There is no need of an example here.

DOGMA XI.

Given two angles of a plane oblique-angled triangle together with the side adjacent to them, to obtain the rest.

This dogma is resolved by the 4th Theorem, and afterward in like manner by the 2nd Enunciation. For since two angles are given, the third in the plane triangle will also be known, from the complement of the given angles to 180°. Then, because the given side is thus opposite to one of the given angles, the solution follows by the 2nd Enunciation and the preceding dogma.

DOGMA XII.

Finally, given two angles of a plane oblique-angled triangle together with the side opposite one of the angles, the rest are given.

No otherwise than the preceding, this dogma is resolved into the required quantities, since thus three angles are given together with a side opposite to one of them, by the 2nd Enunciation of this section.

DOGMATA, ADAPTED TO THE PRACTICE OF SOLVING SPHERICAL TRIANGLES IN NUMBERS.

DOGMA I.

On right-angled spherical triangles in general.

Given any two parts of a right-angled spherical triangle besides the right angle, to bring the remaining hidden parts to light.

Let there be (as in the following figure) a right-angled spherical triangle ABC, having the right angle at A. Then, a pole B having been made on the sphere, let the arcs BA and BC be continued as far as the quadrants D and E.

Then let the arc AC, similarly produced to a quadrant, terminate in another pole F; and finally let another quadrant descend from the same point through D [recte: F] E.

Diagram. A right-angled spherical triangle with its complementary (secondary) construction. The principal right-angled triangle is ABC with the right angle at A. The arcs are extended to quadrants forming a quarter-circle quadrant-grid; the corner points F and E lie at the lower left and the arcs sweep up toward B at the upper left and through D at the right.

- B (upper left): vertex; pole of the construction
- A (right of center, upper): vertex with the right angle
- C (center): vertex
- D (right): quadrant endpoint
- E (lower center): quadrant endpoint / secondary pole
- F (lower left): quadrant endpoint / secondary pole

Indeed, for a figure so constituted, the solution of right-angled spherical triangles is for the most part accommodated according to all the varieties set out above; and this by the 8th Theorem and the 2nd Enunciation of this section.

I. First, then, in the right-angled spherical triangle ABC, taken as the primary one, since all the angles are given, DE will be the measure of the angle at B; and therefore in the secondary triangle FEG, FE — the complement of DE, that is, of the angle at B — is given as 96° [recte: as the complement, the angle at B being given]. Moreover the angle at C is given, by the 2nd Theorem of this section. And since the angle at E is right, it will be: as (the right sine) C is to EF, so is E to CF. Once this is obtained, its complement AC is also given; and therefore in the prior right-angled triangle BAC the two remaining sides — namely BC and BA — are also easily obtained by the 2nd Enunciation, since with these the angles opposite at A and C are given. For:

as B is to CA, so is A (the radius) to BC; and so is C to BA.

II. Further, when all the sides of the right-angled triangle BAC are given, the same result follows. For since the right angle at A is always given, it will be: as BC is to A, so is BA to C; and so is AC to B.

III. Third, let two sides be given together with the included right angle, as in the same right-angled triangle ABC, when the sides AB and AC, which include the right angle at A, are granted. Since thus likewise, in the secondary right-angled triangle CEF, the complements of the given quantities are exhibited — namely CF, the complement of the given AC; likewise the angle CFE, the complement of the given side AB —

it will therefore be: as E (the radius) is to CF, so is F to CE; whence its complement BC is also established. Hence in the primary triangle BAC, because all the sides are given, the angles at C and B are given, according to the immediately preceding case.

IV. Fourth, when two sides are granted together with the angle opposite one of them — as in the primary right-angled triangle ABC, with AB and BC given together with the right angle at A opposite the side BC — it will first be:

as BC is to A, so is AB to C.

Then, in the secondary right-angled triangle CEF, because all the angles are thus given — namely C just found, F by the complement of the given side BA, and finally E right, together with the side CE, that is, the complement of the given BC — first, therefore: as F is to CE, so is E (the radius) to CF, the complement of the side AC. Then: as E (the radius) is to CF, so is C to EF, the complement of the angle at B, which is sought last.

V. Given two angles together with the side adjacent to them — as in the primary right-angled triangle ABC, where two angles, namely the right angle at A and another at B, are given, together with the adjacent side AB.

Diagram. A spherical-triangle construction with extended quadrants. The principal triangle ABC is shown with auxiliary points produced to quadrant measure: G at the upper right (where arc CA is produced to a quadrant), and H, I at the left where the arcs AB and CB are continued.

- H (upper left): point where CB is continued to quadrant measure
- I (left, below H): point where AB is continued to quadrant measure
- B (left center): vertex
- A (right of center): vertex with the right angle
- G (upper right): point where arc CA is produced to a quadrant
- C (center, lower): vertex
- D (right): quadrant endpoint
- E (lower center): point
- F (lower left): point

Since, therefore, neither the proportion of the given quantities in the present right-angled triangle ABC, nor likewise in the secondary CEF, can be had in such a way that the solution may be made by sines alone, therefore — the arc CA being produced to the measure of a quadrant at G; to which measure AB also is continued at I, and CB at H; and finally, when from the pole C the fourth quadrant is drawn from G through H to I — it is manifest how, in the right-angled triangle BHI (the right angle being at H), there are given: the angle HBI, by the 2nd Theorem; BI, the complement of the given side AB, together with the right angle, as I said, at H. Therefore

Therefore, as the whole sine [radius] H is to BI, so is B to HI. Thus the difference HI is readily obtained, which is the complement of the angle sought at C.

Hence, in the primary right-angled triangle ABC, for the remaining unknown parts: first,

as C is to BA, so is the whole sine [radius] A to BC.

Then,

as the whole sine [radius] A is to BC, so is B to AC.

VI. Sixth and finally: when, in a right-angled spherical triangle, two angles are given together with the side opposite one of them — as in the primary right-angled triangle ABC, where the right angle at A is given, along with the angle at B, and finally the side opposite the right angle, BC. Here we have first:

as the whole sine [radius] A is to BC, so is B to AC.

Hence, in the secondary right-angled triangle CEF: since CE (the complement of AC) and EF (the complement of the angle at B) are given,

as CF is to the whole sine [radius] E, so is CE to the angle F — that is, to the complement of the side sought, AB.

And likewise,

as EF is to the angle sought, C.

Or here — which agrees directly with the method of difference [prosthaphaeresis] — in the third right-angled triangle of the last figure, BHI: since BI is given, equal to DA,

as the whole sine [radius] H is to BI, so is B to HI, the complement of the angle sought at C.

Appendix to the Solution of Right-Angled Spherical Triangles

Although, with these few preliminaries laid down, the analysis of any proposed right-angled spherical triangle is carried through by means of the right sines alone — and indeed in such a way that all the varieties suited to practical use can be embraced under a single rule — nevertheless it sometimes happens that, from these data or those (whose order has here been preserved as in the case of plane triangles), not every problem presents itself at once without the roundabout of a repeated computation. I therefore add this further point: that the whole sine [radius] sometimes occupies not the first place in the proportional rule (where it would be most convenient) but the second or third. For this reason, tangents and secants will be summoned as a shortcut in cases of this kind, so that through them, in accordance with the 5th and 6th Propositions of this work, the solution may be carried out in a single operation. As I shall now show below in certain instances.

In the first arrangement — namely, when all the angles in a right-angled spherical triangle are given (the angle at A, at B, and at C) — any side whatever, BA or AC, is immediately found by the 6th Proposition of this work. For:

as the whole sine [radius] A is to the right sine [S.R.] of ABC, so is the secant BCA of the angle to the secant BA.

Likewise:

as the whole sine [radius] A is to the right sine [S.R.] of C, so is the secant of the angle ABC to the secant of the side AC.

Further, in the third supposition: when two sides about the right angle are given — say, AB the right ascension of the Sun, and AC the declination of the same. Then, so that the angle ABC — the obliquity of the ecliptic — may be investigated from these data in a single operation, the 5th Proposition of this work must be applied. For:

as the right sine [S.R.] of AB is to the whole sine [radius], so is the tangent of AC to the tangent of B.

In the sixth and last enumeration of the data: when two angles are given — namely the right angle at A, and the angle of the greatest obliquity of the ecliptic at B, together with the side BC determining the distance of the Sun on the ecliptic from the equinoctial point B — let the side AB, the right ascension of the Sun, again be sought in a single operation, by the 5th Proposition of this work. For, in the secondary right-angled triangle CEF:

as EF, the sine of the complement of the given B, is to the whole sine [radius], so is the tangent of CE (the complement of the given side BC) to the tangent of the angle F, which is the complement of the side sought, AB.

Perhaps further matters may be observed that ought to be referred to this appendix — especially in [Philippus Lansbergius / Philip van Lansberge], who, more than others and with great diligence, had collected very nearly all the proportions that can be set up for the solution of the right-angled spherical triangle from right sines, tangents, and secants.

So much for right-angled spherical triangles; the oblique-angled triangles follow.

DOGMA SECVNDVM.

Given all the sides of an oblique-angled spherical triangle, to find any desired angle.

First, the order is here altered a little, because the following Dogma — in which the angles alone [the pure angles] are given — can be demonstrated more directly and more concisely from this one.

Moreover, the analysis of such a triangle — very commonly used in spherical exposition, and otherwise quite laborious — is made easy when reduced to the method of difference [prosthaphaeresis]. Accordingly, the present Dogma is to be illustrated, and as it were demonstrated, by some example, in the manner suited to the diagram of the first rule of prosthaphaeresis. Then its remaining varieties, which can nearly always arise, must be appended in order — since a concise treatment does not permit me to display each one here by its own appropriate figure.

[Margin: Christmann. li. 2. observ. solarium c. 8.]

The examples that follow are deliberately drawn in numbers from the data of Lansbergius, so that the truth of the solution by the method of difference [prosthaphaeresis] — and likewise a far more expeditious practice — may be proven; and at the same time so that the authority which [Christmannus / Jacob Christmann] arrogated to himself in refuting Lansbergius may be rightly assessed by the kindly reader.

[Margin: See *Trigonometry* of Lansberge, p. 200.]

Let there be, then, a spherical triangle ABC:

Given:

- $AB = 50^{\circ} 0'$
- $AC = 60^{\circ} 0'$
- $BC = 26^{\circ} 22'$

From these let the angle BAC be sought — namely, the one opposite the side BC. These data are fitted to the prosthaphaeretic diagram below, so that in it everything may be carried out plainly according to the following little explanation.

Diagram. A large circle within which an oblique-angled spherical-triangle construction for the prosthaphaeretic solution is laid out. Across the upper portion the triangle ABC is drawn (B and D near the top, with A at the left, C in the center). A horizontal diameter runs across the middle (from I at the left through to F at the right), with a cluster of lettered points along it. Two vertical/oblique

chords descend to H, L, and G in the lower half. The construction realizes the sines and complements described in the surrounding text.

Readable labels, by approximate position:

- B (top, left of center): vertex of the triangle
- D (top, right of B): point on the upper arc
- E (just below D, right): point on the construction arc
- A (left, upper): vertex of the triangle
- C (center, below B): vertex of the triangle
- T (right of C): point on the arc
- O (center-left, on a vertical): construction point
- P, R, M (center, along the horizontal diameter, left to right): construction points (M at right end of an inner segment)
- I (far left, on the horizontal diameter): endpoint of the diameter
- R (left, mid-height): construction point (a second R appears at left; the figure reuses the letter)
- N, K, Q, V, S (along the horizontal diameter, center to right): construction points
- F (far right, on the horizontal diameter): endpoint of the diameter
- H (lower left): point on the lower arc
- L (lower center-right): construction point
- G (bottom center): point on the lower arc

The surrounding text identifies these roles: EF equals the lesser of the given sides' measure (because of the quadrants AE and BF); CT (or DE) is the complement of the greater of those sides; DS is the right sine of arc DF (the complement of the greater side and the lesser side added together); HI is the difference of the sides themselves, whose right sine is HR (or LS); DL is the sum $DS + LS$, and DM its half.

The summary of the demonstration in this passage is as follows. Since AB, likewise AC, and finally BC — the side opposite the angle sought at A — are granted; and since along the arc AB there remains the measure of the lesser of the given sides that enclose the angle to be sought (to which EF is always equal, on account of the quadrants AE and BF). The complement of the greater of those same sides is CT, or DE. Therefore it is manifest that DS is the right sine of arc DF — that is, of the complement of the greater side and the lesser side added together.

The difference of the sides themselves is HI, whose right sine is HR, or LS. And when the sines DS and LS are added together, the whole DL results; its half is DM, which in this case is the **first found value** [inventum primum].

Next, when the first found value DM is subtracted from DS (the sum of the sines), the **second found value** remains — SM, NO, or PQ.

Third, the complement of the given opposite side BC is CV, whose right sine is CQ. Therefore, when PQ is subtracted from CQ, there remains CP, the **third found value**.

But since DH is also the diameter of the smaller circle — whose measure is equal in number to its circumference, as is established from what we set down earlier concerning the prior division of the circle — therefore, once the parts have been duly prepared, the portion CO of it is sought, so that it may be the complement of the angle BAC that is sought.

Accordingly, by the 7th Theorem of this work:

as DM (the first found value) is to DO (the whole sine [radius]), so is CP (the third found value) to CO, the fourth — which is the right sine of the complement of the angle sought at A.

A precept adapted to practice

Therefore here, according to the first rule of prosthaphaeresis, take the lesser of the arcs and the complement of the greater — namely of those arcs that flank the angle sought — and add and subtract them in turn. Add the sines of both together if the lesser arc is greater than the complement of the greater; but if it is less, subtract them; and half of the sum, or of the difference, will be the first found value. Then, when this first found value is subtracted from the sine of the sum, the residue will be the second found value.

The third found value is the difference between the sine of the complement of the side opposite the angle sought and the second found value.

Next, multiply the third found value by the whole sine, and divide the product by the first found value; the quotient yields the sine of the complement of the angle sought — provided the sine of the complement of the opposite side is greater than the second found value. But if it is less, the result is the sine of the excess beyond a quadrant, by which (when added to a quadrant) the angle sought is made obtuse.

The practice in the foregoing data follows

	degrees	minute s		right sine [S.R.]	
lesser arc AB	50	0			
compl. of the greater, DE	30	0			
sum, DF	80	0	S.R.	984807 8	DS
differe nce, HI	20	0	S.R.	342020 1	HR

sum of the sines	13268279	DL
half of the sine	6634139	DM, first found value

difference of first found value and sum of the sines	3213939	SM, second found value

Opposite side BC = 26° 22' 20"

	degrees	minute s	seconds		right sine [S.R.]	
its comple ment CV	63	37	40	S.R.	895927 2	CQ
second found					321393 9	

	degrees	minute s	seconds	right sine [S.R.]	
value subtrac ted					
there				574533	
remain				3	
s the					
third					
found					
value					

The rule of proportion

As 6634139 (first found value) is to the whole sine [radius], so is 5745333 (third found value) to the fourth found value: **8660260**, whose corresponding arc is 60°, and its complement 30° — the angle sought.

Therefore the calculation in this example of Lansbergius is sufficiently certain.

If the whole analysis of this Dogma should be carried through by prosthaphaeresis without division, it would proceed by transposition in this manner:

	degrees	minute s	seconds		right sine [S.R.]	
Sec[ant]	63	36	10	S.R.	895733	
					2	
differe nce	46	15	48	S.R.	722524	Subtrac t
					8	

difference of sines	1732084
half	8660420

— to which there correspond 60°, and its complement is 30°, as before.

Note on the table above: The transposed-method recomputation pairs with the worked example on p. 27. The two computed arcs (entered as "Resp. arc 81 19 49 — Respond. 35 4 1" and "Complem. 8 40 11 — Complem. 54 55 59 → 8 40 11" in the source) and their sines (8957332 and 7225248) reduce to the same half-sine, $8660420 \approx 8660260$, confirming the angle 60° (complement 30°). The leading rows on p. 28 read in the source as:

Sec	66341390	S.R.	5745333
Resp. arc	81 19 49	Respond.	35 4 1
Complem	8 40 11	Complem	54 55 59
.		.	
			8 40 11

Parsing notes: This opening block on p. 28 carries over from the bottom of p. 27 (catchword "Sec."). The first value "66341390" appears to be a misprint or run-together for 6634139[0] (the first found value 6634139 with a trailing digit); flagged below. The angle-value columns ("81 19 49", "35 4 1", "8 40 11", "54 55 59") are degrees-minutes-seconds entries in the transposition computation.

Variations of the present Dogma

- I. If one of the two sides enclosing the angle sought is greater than a quadrant, the complement of its excess above the quadrant is taken; and the second found value is added to the sine of the complement of the side, whence the third found value is formed; and so the operation proceeds in the rest as above.
- II. If the opposite side is greater than a quadrant, the second found value is likewise added to the sine of the excess of the opposite side, and the arc finally resulting is adjoined to its quadrant of 90° for the angle sought.
- III. Finally, if both one of the sides enclosing the angle sought and the opposite side exceed the measure of a quadrant, one must again operate by means of the excess of the opposite side: but the second found value must be subtracted from the sine of the excess of the side; the arc finally emerging must likewise be added to 90° , and the angle sought will be established.

The following spherical doctrine will abundantly supply examples of each of the individual varieties.

DOGMA TERTIVM.

Given all the angles of an oblique-angled spherical triangle, to find any side whatever.

Since this Dogma is, as it were, the converse of the prior one — on account of the analogy between the right sines of the sides and of the opposite angles, by the 2nd Proposition of this work. Accordingly, taking the angles here in place of the sides in the foregoing treatment, I shall undertake the solution of it without a laborious demonstration, intending to show its likeness to the prior one by an example in numbers, and then briefly to forewarn learners about its varieties.

Taking here too the data [δεδομένοις, dedomenois, "the given quantities"] of Lansbergius, let there be an oblique-angled spherical triangle ABC of given angles, and let the angle at A — to which the side BC is opposite — be 30° ; the angle at B, which the side CA faces, be $102^\circ 53' 31''$; and finally the angle at C, opposite the side sought BA, be $59^\circ 34'$ and a little more.

Diagram. A small right-curved spherical triangle in the left margin. The vertex A is at the bottom; B is at the upper left; C is at the upper right. The two upper sides curve, and the side BC across the top is drawn as a curved arc, representing the oblique-angled spherical triangle ABC of given angles.

Readable labels:

- A (bottom): vertex, given angle 30°
- B (upper left): vertex, given angle $102^\circ 53' 31''$
- C (upper right): vertex, given angle $59^\circ 34'$ and a little more

Therefore, with the angles A and B taken in place of the very sides to which they are opposite — and which include the angle BCA, opposite the side sought BA — and likewise with that same angle BCA taken in place of the side we seek: the solution is fitted to the practice of the preceding precept in this manner:

(Continuation of the worked Lansbergian example begun in the prior batch, Dogma III.)

$30^\circ 0'$	the smaller angle
$12^\circ 53' 31''$	the excess of the greater beyond 90°
$42^\circ 53' 31''$	sum; right sine [S.R.] 6806179

17° 6' 29"	difference; right sine [S.R.] 2941747 sum of the sines 9747926 half of it 4873963 — first found value 1932216 — second found value
59° 34'	the angle C
30° 26'	its complement; right sine [S.R.] 5065355 second found value subtracted 1932216 3133139 — third found value

Rule of proportion. As the first found value (4873963) is to the radius 10000000, so is the third found value (3133139) to 6428320, the right sine of an arc of 40° 0'. The complement of this arc, namely 50° 0', is the side BA that was sought — just as the Lansbergian computation gives it.

Variations to be observed here, contrary to those in certain of the cases above.

I. If the third angle is acute, and either one of the adjacent angles is obtuse, then the second found value must be subtracted from the complement of the third angle — unless the second found value is greater than this complement, in which case, the difference being likewise taken as the third found value, the fourth result comes out as a right sine of an arc exceeding the quadrant.

II. But if the third angle is obtuse, the operation is set up by means of its excess beyond 90°. If the second found value is less than the right sine of the said excess, the resulting side will be obtuse, that is, exceeding 90°.

III. Finally, if both the third angle and one of the adjacent angles are obtuse, then the second found value must be added to the right sine of the excess of the third angle.

Chapter 6 of Book 2 of the *Sphaerica* [*Astronomia Danica*, the doctrine of the sphere] will furnish several further examples of this Dogma.

DOGMA IV.

Given two sides of an oblique-angled spherical triangle together with the included angle, to determine the third side, the one opposite the given angle.

This Dogma too has a very wide use — not only in the celestial doctrine of the sphere, but also on the terrestrial globe, for finding the distances between two places that differ both in longitude and in latitude.

Its analysis proceeds in almost the same way as that of the second and third Dogmas: it is reduced to the method of *prosthaphaeresis* [prosthaphaeresis, the addition-and-subtraction technique], and is here carried out with even less labor.

Here too I shall first display this Dogma's diagram, fitted to the data of Lansberge [Lansbergius], then add the precepts and the practice, and finally append the observations on the variant cases.

Let the prosthaphaeretic diagram subjoined to the second Dogma be recalled here, since it was constructed upon the same data, and let the demonstration added there be applied throughout in the same way — except that, when the side BC opposite the given angle at A is sought, it will therefore hold that: as the radius OD is to DM, so is OC, the complement of the given angle A, to CP, the right sine of the complement of the side sought CR.

Diagram. A large circle (the primitive circle of the stereographic projection) with two perpendicular diameters meeting at the center O. Inside, a dense network of straight chords and projected great-circle arcs lays out the prosthaphaeretic construction for the spherical triangle ABC, carried over from Dogma II. The construction is crowded with intersection points lettered around and within the circle.

- B (upper interior, near top): vertex of the triangle
- D (upper interior, right of B): point on the upper arc
- E (upper right, on or near circumference): endpoint of a projected arc
- A (left interior): vertex of the triangle
- C (interior, below B): vertex / projected point
- T (right interior): intersection point
- O (center): center of the primitive circle (radius OD)
- P (interior, left of center): foot of the right sine CP
- R (interior, just right of P): point R (the side-complement reference, CR)
- M (interior, right of center): point M (DM in the proportion)

- I (far left, on circumference): endpoint of horizontal diameter
- F (far right, on circumference): endpoint of horizontal diameter
- N, K, Q, S (lower interior, along the horizontal diameter region): intersection points of the lower chords
- R (left interior, on the horizontal line): a second labeled R near the left
- H (lower left, on circumference): endpoint of a projected arc
- L (lower interior, right): intersection point
- G (lower left, below H, on circumference): endpoint of a projected arc

Precept of this Dogma.

Add and subtract from each other the smaller arc and the complement of the greater of the two arcs that enclose the given angle. Add their right sines if the smaller arc is greater than the complement of the greater; otherwise subtract them. Half of the sum (or of the difference) will be the first found value.

Next, subtract this first found value from the sum of the sines, and the remainder will be the second found value.

The third found value is the sine of the complement of the given angle.

Multiply this by the first found value, and divide the product by the radius; the quotient will be the fourth found value. Adding the second found value to it produces the sine of the complement of the side sought.

Example in numbers.

Let it be the same oblique-angled spherical triangle ABC as above, whose two sides about A are given — namely AB and AC — together with the included angle BAC. In numbers, arranged in our customary disposition, these are as follows:

	degrees	minutes
BA	50	0
CA	60	0
BAC	30	0

From these the side BC is sought — namely the one opposite the angle BAC — and it is found to be 26° 22' 20".

Practice.

50° 0'	the smaller arc
30° 0'	the complement of the greater
80° 0'	sum; right sine [S.R.] 9848078. D.S.
20° 0'	difference; right sine [S.R.] 3420201. H.R.
	sum of the sines 13268279 6634139 — first found value 3213939 — second found value
complement of the angle BAC, 60°	corresponding right sine [S.R.] 8660254 — third found value

Rule. Therefore: as the radius is to 6634139 (the first found value), so is 8660254 (the second found value [*recte*: the third found value]) to 5745332 (the fourth found value).

	5745332 — fourth found value second found value added 3213939
CP	8959271 — corresponding to the arc 63° 37' 40"

The complement of this arc, namely 26° 22' 20", is the side that was sought.

These last steps, carried out by this rule through the multiplication of the first found value into the third — for the fourth found value — can, if they are subjected to *prosthaphaeresis*, be reduced to almost the most expeditious procedure of all, especially when the seconds of arc are neglected. For since the complement of the given angle is always known in its own arc, only the arc corresponding to the first found value is sought. Thus here, for the first found value 6634139, the corresponding arc is about 41° 33' 40":

41° 33' 40"	(arc of the first found value)

30° 0' 0"	complement of the third found value
71° 33' 40"	sum; right sine [S.R.] 9486615
11° 33' 40"	difference; right sine [S.R.] 2004130
	11490745
	5745372 — fourth found value, nearly as above

Through this one Dogma alone I have, on a former occasion, settled the much-discussed matter of finding the distances of places on the earth (their longitudes and latitudes being supplied from a geographical table) — and that with almost no labor in comparison with the labor that was the lot of the ancients, who relied, namely, on continuous *prosthaphaeresis*.

The various cases of this Dogma to be observed.

I. If one of the given sides exceeds the quadrant, use this excess in place of the complement, as in the Dogma above; and subtract the second found value from the final product sought, and the remainder will be the sine of the complement of the side sought. Or, if the second found value is the greater, the remainder will be the sine of the excess of the side sought. But if the final product is less than the second found value, the difference is the right sine [S.R.] of the complement of the side sought.

II. If the given angle is obtuse, one must likewise operate by means of its excess beyond 90° in place of the complement, and the second found value must be compared with the final product; if the latter is the lesser, the difference is the right sine [S.R.] of the excess of the side sought.

III. Finally, if both one of the given sides exceeds the quadrant and the included angle is at the same time obtuse, the second found value is to be added to the final product, and the resulting sum is the sine of the excess of the side beyond the quadrant.

DOGMA V.

Given two angles of an oblique-angled spherical triangle together with the adjacent side, the remaining angle is found.

This Dogma too is the converse of the one immediately preceding. For if the given angles that enclose the given side are posited — or at least left out of account — in place of the sides opposite those same angles, and the included side in place of the angle to be investigated, the calculation proceeds toward what is sought no otherwise than in the preceding case, while certain variations are here also

observed, just as in the third Dogma; likewise these are fitted to the practice by way of conversion [Greek: κατ' ἀναστροφὴν (kat' anastrophēn, "by conversion / inversely")]. We shall therefore begin at once from an example, taken from the numbers of Lansberge [Lansbergius].

Diagram. A small quadrant figure in the upper-left margin: a right angle at A (upper left) with a horizontal line running right to C and a vertical line running down to B; the arc from C curves down to B, forming a quarter-circle-like spherical triangle ABC.

- A (upper left): right-angle vertex
- C (upper right): endpoint of the horizontal side
- B (lower left): endpoint of the vertical side; the arc CB closes the figure

Let there be an oblique-angled spherical triangle ABC with given angles: at A, $102^{\circ} 53' 31''$; at C, 30° ; together with the side AC, 50° . From these the angle at B is sought.

Operation.

greater of the given angles	$102^{\circ} 53' 31''$
its excess	$12^{\circ} 53' 31''$
smaller angle	$30^{\circ} 0' 0''$
excess of the greater	$12^{\circ} 53' 31''$
$42^{\circ} 53' 31''$	sum; right sine [S.R.] 6806179
$17^{\circ} 6' 29''$	difference; right sine [S.R.] 2941747
	9747926
	4873963 — first found value
given side $50^{\circ} 0'$	1932216 — second found value
its complement $40^{\circ} 0'$	right sine [S.R.] 6427876 — third found value

From the multiplication of the first found value into the third, and the division of the product by the radius, there results the fourth found value 3132923, to which when the second found value 1932216 is added (it will be done contrary to the case above):

the right sine [S.R.] of the
complement of the angle sought
is accumulated: 5065139
which is $30^{\circ} 25' 55''$

Therefore the angle sought itself is $59^{\circ} 34' 5''$, differing from Lansberge's [Lansbergius] value by at least $15''$ — a difference owing to the fact that he had carried out the analysis by way of right-angled triangles.

The whole reduced to prosthaphaeresis.

4873963 — first found value	corresponding arc $29^{\circ} 10' 10''$ — the smaller arc
third found value	corresponding arc $40^{\circ} 0' 0''$ — the greater arc
$50^{\circ} 0'$	complement of the greater
$29^{\circ} 10' 10''$	the smaller arc
$79^{\circ} 10' 10''$	sum; right sine [S.R.] 9821872
$20^{\circ} 49' 50''$	difference; right sine [S.R.] 3556054
	6265818
	3132909 — fourth found value, nearly as before

DOGMA VI AND LAST.

Given two of {the sides / the angles} of an oblique-angled spherical triangle, together with one {angle / side} {opposite to / adjacent to} one of the given parts, the remaining parts are found.

[Translator's note: the source sets this rubric out as a brace-bracketed combinatorial table, to be read in parallel columns. The literal layout is: "Given

two {sides / angles}... with one {angle / side}... {opposite to / adjacent to} one of the given parts... the remaining {side / angle} are found." The four bracketed alternatives generate the several distinct sub-cases the Dogma covers.]

These data [Greek: διδόμενα (didomena, "the things given / the givens")] are conveniently enough digested here into a single Dogma, and are solved by the second Proposition [Greek: ἐννυχιατ. — *recte* Enunciat., i.e. *Enunciatio*, "Proposition / Enunciation"] in what immediately follows; or, if all the parts are wished to be had at once, let it be done by the 3rd theorem, the proposed triangle being resolved into two right-angled

the triangle having been resolved by means of the perpendicular. The required result [prosthaphaeresis] is then obtained at once and quite expeditiously — especially whenever the angle opposite the right side is also given together with the angle adjacent [to it].

If at any point one of the two angles should happen to be obtuse, then by the Third Enunciation its supplement to 180 degrees is substituted in its place. The remaining variations — when the given side exceeds a quadrant, or when the angle or the required side happens to run past a quadrant — are easily recognized with the help of our Globe or Sphere.

Therefore, out of all the various examples by which Philippus Lansbergius [Philip van Lansberge] illustrates problems of this kind — and from which he investigates the right-angled triangles arising therefrom — I shall take just a single example: namely, the one that pertains to the second case, that of two given angles together with the side opposite one of those angles, and indeed opposite the obtuse one. [Margin: Lansb. p. 194.] I do this for two reasons: to show that the procedure by the 2nd and 3rd Enunciation holds equally true in spherical triangles, and at the same time to correct an error of Lansbergius which seems to have crept into his calculation regarding the *given quantities* [Greek: διδομένων (didomenōn, "the things given," the data)] and the *required quantity* [Greek: ἀπαξίαν — read here as ἀπ' ὀρξ-/the sought quantity [?]].

Diagram. An oblique-angled spherical triangle drawn with curved sides, with the apex A at top left and the base running from C (lower left) toward B and D at lower right. A perpendicular arc is dropped from A to the base. The triangle is lettered for the worked example that follows.

- A (apex, upper left): vertex A
- C (lower left): vertex C
- B (lower right, on base): vertex B

- D (lower right, beyond B): foot of the perpendicular dropped from A

Given the oblique-angled spherical triangle ABC, whose angle ABC is $102^{\circ} 53' 31''$. This angle is therefore obtuse, by what was set out above concerning the division of the triangle. There is also given the side AC = 60° , and finally the angle at C = 30° . Let the first side sought be AB, opposite the given angle at C, found by the Second and Third Enunciation. For:

As	to	so	to (required)
Supplement of the obtuse angle at B = $77^{\circ} 6' 29''$	Side AC = 60°	the angle at C = 30°	AB sought = $26^{\circ} 22' 24''$
S.R. 974792 7	S.R. 866025 9	S.R. 500000 0	S.R. 444218 8

[Right sine of $77^{\circ} 6' 29''$: right sine of 60° :: right sine of 30° : right sine of AB. The S.R. of 30° should read 5000000; transcribed "pooooo / 5000000" — the engraved value is 5000000.]

If more parts are still sought: since the obtuse angle is at B, let CB be prolonged, let the perpendicular arc descend from A to D, and let ACD be made a right triangle. Now since the base AC is given, together with the adjacent angle C, AD is found by *prosthaphaeresis* [the addition-subtraction method] in this way:

AC = 60° , its complement = 30°

Lesser arc, i.e. the angle at C = 30°

sum 60° S.R. 8660254

difference 0° S.R. 0000000

Therefore 4330127 is the right sine of the perpendicular arc DA first sought, = $25^{\circ} 39' 32''$.

Next, in the right triangle ADC, for the side DC, by the 5th Enunciation, we have:

As	to	so	to
C = 30° 0'	radius	DA = 25° 39' 32"	DC = 56° 18' 36"
T. 577350 21	S.T. 100000 000	T. 480384 4	S.R. 832050 3

[Tangent of 30° : whole sine (radius) :: tangent of 25° 39' 32" : right sine of DC.]

From this, for the angle DAC, we have:

As	to	so	to
CA = 60°	radius	DC = 56° 18' 36"	DAC = 73° 53' 53"
S.R. 866025 4		S.R. 832050 3	S.R. 960769 0

Third, in the spherical right triangle DAB, first for DAB, again by the 5th Enunciation:

As	to	so	to
Comple ment of AD = 64° 20' 28"	comple ment of AB = 63° 37' 36"	radius	comple ment of DAB = 75° 39' 47"
T. 208167 08	T. 201684 33		S.R. 968857 8

Therefore the angle DAB itself is given = 14° 20' 13".

And since the angle DAC was previously found to be 73° 53' 53", the angle BAC is therefore 59° 33' 40".

Lastly, in the right triangle DAB, for the side DB, we have:

As	to	so	to
D =	BA =	DAB =	DB
radius	26° 22'	14° 20'	
	24"	13"	

By the following *prosthaphaeresis*:

complement of the greater: 63°
37' 36"

less the given: 14° 20' 13"

sum 77° 57' 49"

S.R. 9780154

difference 49° 17' 23"

S.R. 7580173

2199981

1099990 = S.R. of the side DB
sought

Therefore the side DB itself is given = 6° 18' 55". And when this is subtracted from the side DC (previously found = 56° 18' 36"), it leaves BC = 49° 59' 41".

And so, in the closing piece of this trigonometric compendium, we have restored that example of Master Lansbergius — which in fact agreed very nearly with our own calculation, except that in the sides AB and BC there had been a certain transposition error. In the foregoing matters, moreover, Christmann [Jacob Christmann] reproached him without cause, and at the same time set about demolishing the demonstration of the ancients.

THE END.

ΠΡΟΓΝΩΡΙΣΜΑΤΩΝ [Greek: Προγνωρισμάτων
(Prognōrismatōn, "of preliminary
determinations / things known beforehand")]

ASTRONOMY, PART TWO

On the matter of the heaven that contains [all things]; on the form of the greater bodies of the World, and their activity [Greek: ἐνέργεια (energeia,

"activity / operation")] during their motion; and finally on the consideration of time and motion as analogous to one another.

No one ought rightly to wonder, nor to count it a fault in us, if — before we lay down hypotheses suited to the tracks and motions of the greater bodies of the world (the stars, I mean, and the earth) — we first set before the restoration of Astronomy a few remarks: concerning the nature and definition of the matter [material] that is the place of those bodies, and of their motions in general; likewise concerning the nature and form, above all, of those world-bodies themselves, in whatever way will seem most fitting to their preservation and to their motions, continued by the divine influx; and finally concerning the consideration of time and motion in relation to one another. For without at least a moderate knowledge of these things, the astronomer will be able neither to trace adequately the due positions and motions of the *phenomena* [Greek: φαινομένων (phainomenōn, "the things appearing," the celestial appearances)], nor to investigate their influences and operations upon these sublunary things in a way that adequately accords with nature.

And although we admit that this inquiry properly belongs to Physics, we nevertheless judge it not to be foreign to the astronomer's profession, but rather especially appropriate to it — since the astronomer is the contemplator of the supralunary or celestial nature, so that on that account (other things being equal) no one is raised more highly or more rightly into the study of it than he, by sense and reason alike.

As regards the first part of our proposal: first, we reject the opinion of those who suppose the celestial region to be hard and impenetrable — packed, that is, with such material orbs — as the Peripatetics do, whose error without doubt descended from the doctrine of Pythagoras concerning the musical harmony of the celestial orbs, not rightly understood. Next, [we reject] the opinion of those who supposed the ethereal region to be fire, as did certain Stoics, whom Theophrastus Paracelsus imitated — unless perhaps this be understood of the stars themselves, and of the primeval light dispersed everywhere throughout the orb, but leaving manifest traces of itself in the stars. This I see fit to admit, rather than to believe that any of the things that were created has perished again — as will one day be discussed more fully in the Appendix to the Astronomy, on the new Phenomena of the heaven. Third and finally, [we reject] the opinion of those who suppose this same celestial region to be either air thick enough, or water. For since in our own times — from the truer recording of the positions and motions especially of the more recent celestial Phenomena, and finally of the refractions

of the stars, and from other indications that constantly present themselves to the trained astronomer — most of the arguments of those people are refuted, we shall set forth, without lengthier refutation, what we hold (as it seems to us) to be most consonant with Sacred Scripture, with nature, with reason, and finally with experience concerning this proposed matter — adding shortly a description of it, together with the proof of its explanation.

We call the matter adequate to the place of the visible and finite universe, by its own nature and fullness, an expanse most thin and most subtle — like, indeed, to the incorporeal and insensible — *homogeneous and of like parts throughout* [Greek: ὁμογενές καὶ ὁμοιομερές (homogenes kai homoiomeres, "of one kind and of uniform parts")]; furthermore, stable in its own essence, in-

— deed, yet permeable; cold in quality, created by God the All-Good to be a certain subject and, as it were, a vehicle of light and illumination — above all of the stars — and, for that very reason, the primary cause of the separation of the waters.

The first ground of the asserted definition can be this: that no natural light, insofar as it spreads outward in space from its luminous body, is carried through a void, or lacks some subject — since light or illumination is an accident occurring outside its own root. But fire, and all luminous bodies, hurl light and illumination out from themselves; and this is manifest from the effect of the stars (and especially of the Sun), whose rays not only reach all the way to us, but even penetrate into the earth itself in the concoction of metals.

Since, therefore, such illuminations are not simply carried through a vacuum or void, it follows that they require some such thing as a vehicle, which has the nature of a defined expanse — as will be further confirmed presently. First, that such matter is homogeneous and uniform is proved thus: For if, contrary to this, it admitted of division, then one [matter] would exist in the ether above the Moon, and another, different from the first, in the sublunary region — into which the light and rays of the stars sent down to us would unfold themselves; and this could in truth be nothing else here but air. But we do not acknowledge that air is the immediate vehicle of light or rays even in the sublunary region; and so it is necessary that the matter we shall be treating be homogeneous and uniform. We shall bring forward the reasoning behind our assumed connection not so much to persuade as to demonstrate, in this way. For since that air is mobile — its nature thriving in continual flux — it is certain, and experience teaches, that it can be displaced from its place. And although that place would be refreshed and refilled by continual succession, it nevertheless could in no way come about but that all things which hang visible in air thus shaken would be subject either to plainly

evident, or at least to a slight, vibration. But no vibration of light whatever is felt, even when the air through which the light is seen is shaken with all one's might and removed from its place. (I make exception, however, for that apparent vibration which our eyes conceive near the horizon, exposed to the summer heat of the Sun, on account of those thicker airy vapors in which the rays of the Sun, illuminated at the same time, are entangled and refracted.) From this I conclude that air is by no means the primary and adequate subject of light, but rather the matter defined above by us.

This reasoning, indeed, the philosopher rather than poet, the elder Palingenius [Marcellus Palingenius Stellatus], sets forth — to demonstrate that air is in the light, and not the reverse — in these little verses, in the last sign of his Zodiac:

Perhaps someone will object that air is the subject of light And its foundation; and accordingly, that where there is no air — Beyond the bounds of the highest heaven — there, he will deny, light exists. But likewise he now departs from the truth, just as before. For air is not the subject of light, nor (as he perversely judges) is light in the air: Nay, on the contrary, air is in the light, if he discerns rightly. Not what Aristotle, but what reason itself asserts. For if a lamp be lit while standing at closed doors, Or a torch send its light through a crack into a darkened Chamber, and someone then shake the air beside the ray of the admitted light, the air is forced to pass across that ray, while the light is not moved at all. But if the air were itself the subject of the light, then surely the ray would be shaken by the same motion as the air, and the light would doubtless depart along with its subject.

Wherefore, since this expanded matter of which we are speaking underlies everywhere the breadth of the hurling-forth of light and rays from the stars — even into the inmost earth, wherever light extends or penetrates — it necessarily follows that the sublunary re-

—gion, though not in its entire genus, as far as that matter is concerned, which is distinct from the aethereal.

But someone may object that the sublunary air is the barrier or expanse set "between the waters and the waters," and that this alone is what is mentioned in Genesis 1:6 and following. I answer that the premise must be distinguished, and a difference drawn between this kind of air insofar as it is in part diffused around the lower regions from which it arises, and air taken as expanse primarily and simply. For the air nowhere subsists alone; and yet, contained within the limits

assigned to it according to God's ordinance from the beginning, it produces the daily separation of the waters, and does so in the lower regions as well. When the rays of the Sun and of the stars, by which they were joined, rebound again from the earth and the waters, it happens that—through the more effective force of their redoubling, together with the exhalation of the terrestrial globe itself, which holds much heat within—they raise the vaporous matter from below into the middle region, by way of the vehicle of the finer interwoven expanse and air. There at last they suspend and sustain that matter, until, once cooked and dissolved in that same place, it falls back down upon the terrestrial globe. Moreover, by this continual exhalation the finer air in the lowest region is thus prepared for us—namely, by the vaporous matter carried upward in the manner I have described. Once this is done, no small part of that finer matter, elaborated above the surface of land and sea by the great vigor and heat of the stellar rays (above all the Sun's), is converted into air suited and wholesome to living things. The same is to be understood of the upper air, or the highest air: that the effective rays of the stars continually entice something here too out of the matter of the clouds, before their penetration of the middle interval [diastema], dry it out more finely, and refine it; and so they raise it successively through the expanse all the way to the outermost limit of the aerial region, that is, of the whole terrestrial globe. From this matter—viscous and as it were dried out—not only do certain fiery meteors arise, but also small whitish bodies are seen, which provide an evident cause of the twilights.

[Margin: *Problem I, Chapter 11, Book 2 of the Sphaerics.*] [Margin: *Problem 4, and 5, Book 2 of the Sphaerics.*]

Now there is a sufficiently manifest difference between the clouds and the sublimation of the upper air from the earth. For the clouds are found by observation to lie scarcely a milliarum and a half [*milliaris*] from the earth's surface—and this in summertime, when the solar rays are more effective. The upper air, in turn, insofar as it is visible, the optical writers prove to be distant from us by thirteen German miles, from observation of the twilights.

[Margin: *Vitellio's optics, Book 10.*] [Margin: *Proposition 60.*] [Margin: *Sphaerics, Chapter 11.*] [Margin: *Proposition 1.*]

That these things are so is shown not only by experience in those matters, but also by the spagyric or pyronomic art [Spagyrica seu pyronomica]; indeed it is shown even in man as a microcosm, through a certain very beautiful analogy: the sublimation and reflection of the humors from the stomach into the head, the rays being sent forth, namely, from the heart and the air enclosed in the breast.

And what then? Since it is agreed that this air is a body sufficiently perceptible, why would our senses—above all sight, the noblest of them, whose faculty is spiritual—not also require, in order to perceive visible things, a spiritual subject as it were, an intermediate medium? Such precisely is the expanse itself of which we are treating, filled moreover with a like repletion of light and air, [Greek: διαφανές (diaphanes, "the transparent")]. Nor should we suppose, in some other way, that a certain extremely thin air everywhere fills out even the breadth of the expanse throughout the heaven, and occupies the place of the whole universe, as Johannes Pena [Jean Pena], Christophorus Rothmannus [Christoph Rothmann], and others have thought—unless indeed they have understood the nature of our expanse exactly thus, apart, that is, from that universal something which some (not without authority) call, with respect to the difference, [Greek: τὸ ἀραιῶδες (to araiōdes, "the rarefied")], not inappropriately. For besides the absurdity that would thereby be committed—from the vibration of the stellar rays against experience—it would also follow that refractions accompany the stars almost to the zenith. Yet these same refractions we have found, from observations made by us most diligently in various places of the earth—Denmark, Bohemia, and Norway—to fall away completely within half a quadrant from the horizon. I add another point concerning the excessively swift local motions of the celestial bodies, without any obstacle from air of this kind, and without this penetration.

[Margin: *See Book 1 of the Epistolae Astronomicae of Tycho Brahe [Tycho Brahe], where the reply to this controversy is given more fully, on the grounds of the same matters concerning Rothmann.*]

Therefore the same expanse underlies the rays of the Sun and the stars, equipped moreover with such air, so as to keep these waters apart. As for those rays, before the cre-

—ation of the Sun and the stars, it appears from Scripture that light was made. And although this light no doubt powerfully exerts its efficacy everywhere among finite places, it nonetheless shows us no traces of itself except where it is taken up in the stars and in otherwise suitable matter—as I recall having promised above that I would set forth more amply, God willing, in the appendix to the *Astronomy*.

[Margin: *Genesis 1:3.*] [Margin: *See the appendix to Book 1 of the Theory, on the spots in the Moon.*]

Nor is it any wonder that from this globe, as from a root, the meteoric surroundings [*peristases*] which we have just touched on arise, and are diffused around it as around a parent—since it is probable, and almost visibly evident in

the thirsty Moon, that the remaining mundane bodies, that is, the celestial stars, are likewise wrapped in similar or analogous things, each according to its own nature; by which each of them extends the mass of its whole globe through the expanse to the limits and uses appointed for it.

Hence too it will be probable that the stars, namely, eject through the same celestial expanse the hidden seeds of new phenomena at a time divinely ordained, and that thereupon a conception takes place in the expanse, which suddenly and as if miraculously rises into the appearance of a new star—since it displays its greatest magnitude and motion, for the most part, in the things that are moved, immediately at the very beginning of its appearance. For no elementary generation appears similar to the cometary kind, unless perhaps a bubble on water, by sudden expansion, were drawn into some likeness or other of this. But to assert that there are permanent bodies, which thus shine forth in the heaven from time to time, and that at a fixed time these are either purged by fire, or illuminated only when they stand in a certain fitting disposition with the Sun so as to appear to us—such opinions, which we gather from Pliny, Plutarch, and others to have been those of certain of the ancients, are certainly shown to be reduced to absurdity, both by careful consideration of the analogous appearance and motion of these same bodies, and by the fact that no obscuration of any star has ever been noticed in them.

[Margin: *Verdict concerning comets, which will be expounded more amply in the appendix to the Astronomy on new phenomena.*]

So far we have dealt with the substance of the matter of the expanse. If anyone should call it spiritual in a relative sense, and say it is to be likened to Aristotle's fifth essence [*quinta essentia*], he would not, in my judgment, stray far from the truth. It remains, therefore, that we set out the reasons by which we assert this same matter to be cold in quality—which are as follows.

Two things have been most widely diffused and extended through all things visible and finite, namely, that primeval light, to which the light of the stars is associated, and the expanse that is the vehicle of light, as has been shown by the foregoing arguments. And since the quality of light or brightness is hot—as is plainly felt in the solar rays, and as is also rightly grasped by reasoning in the hidden operations of nature—it is fitting to this same nature that the other active quality, namely the cold one, should be assigned to the matter of the expanse; just as experience itself proves. For when the solar rays are absent, or operating less effectively because of their too-oblique reflection around the earth—as is wont to happen among us in wintertime, and still much more, for the same cause, in

places nearer the northern region—we find the whole constitution of the air turning itself into a freezing chill. We experience the same thing with this air even in a moderately warm place: when it is suddenly fanned about through the expanse, it is affected by the cold quality from the underlying expanse, just as that very expanse is. What is more, just as life and the respiration of animals can in no way be preserved or continued without heat, so neither can they be without this cooling—whence too it is named [Greek: ψυχή (psychē, "soul"); ψυχός (psychos, "cold")]. I pass over the credible point that the stars themselves, provided they are fiery, whether they move or are at rest, are as it were continually tempered through that latent harmony, on account of their greater portion of light—something of the sort which has been demonstrated by others to hold between the moist of water and the dry of earth. Accordingly, from these considerations we reason that the celestial region is the most temperate of all, because of the proportion of the matter of the expanse and the suffused light, so exactly balanced—while for the present we suspend, free of all prejudice, all judgment about the stars and their consequent disposition.

Thus far concerning the first part of our proposition, which sets forth such a matter as comprehends the whole place of the universe. There follows the second, in which, concerning the nature and form of the bo-

—dies of the great mundane bodies, insofar as they are fitted both for their own preservation and for their motions, let us inquire briefly a little further.

First, we judge that it is most difficult to set forth the matter of these bodies, since neither do all the celestial stars appear to be of the same matter, nor endowed with equal light. For the Moon is found, not only from its known torches and the generation of its light from the Sun, to be a body solid and shadowy in itself, but is even seen as if with lofty mountains and deep valleys, with the appearance of waters and woods—so much so that some have called it a second earth, if one is to abide by the observations of the moderns [neoterici]. Furthermore, as to excellence of light and magnitude, the Sun is rightly to be preferred by us above all the stars, as having been made before the rest of the stars.

[Margin: *See the appendix to Book 1 of the Theory, on the spots in the Moon.*]

[Margin: *See Galileo [Galileo] in his Sidereus Nuncius.*]

But whether these stars (the Moon excepted) were produced from the primeval light, and shine by themselves, as many think, or whether they came into being in a peculiar way and borrow their light, like the Moon, from the Sun—to know this plainly we judge to be neither within man's power, nor to contribute much to our

aim, since it concerns perceiving their position and motion. There remain likewise several similar matters, touching our purpose but little: concerning the numerous multitude of the stars, and—of those that are seen at the extreme limit by us, and are called fixed—their intervals and relative arrangement, both with respect to the earth and among themselves; and whether the stars are suited for habitations, by which at length they are each inhabited. Further—what seems like a miracle—by what reasoning the stars of Jupiter, four others, tiny and not commonly visible, circling Jupiter at a slight and unequal remove but with continuous and most swift motion, like certain attending satellites, have been assigned by God; together with many other things in which we judge that divine omnipotence and providence are rightly to be admired and adored, but our own blindness and ignorance, while we remain here, are to be acknowledged most deservedly.

[Margin: *It is rather to be assumed from the observations. (Or: it is rather to be ascribed from observations.)*] [?] [Margin: *Venus, when nearly in conjunction with the Sun: see Book 2 of the Theory, Chapter 18.*] [Margin: *See Galileo [Galileo] in his Sidereus Nuncius.*]

These things being premised, we will now approach the inquiry concerning the form of the great mundane bodies—which will be chiefly the object of our contemplation—in the manner we stated at the outset.

The form, as of all bodies, so also of the great mundane ones, is twofold: interior and exterior. The former is more active, or [Greek: ἐνεργητική (energētikē, "active" — the active power)], whereas the latter is subject to passion [Greek: ὁρεκτική (orektikē, "appetitive")] [?], that is, determinable, existing as circumscribed at the surface. To the [Greek: ἐνεργητικήν (energētikēn, "active") — power] of the stars, above all, a question seems to pertain which Plato [Plato] defends—taken, as it appears, from the doctrines of the ancient Egyptians—namely, that the stars, like the earth, are animate; indeed, that they are the greater animals of the whole animate world, and that for this reason a faculty for the motions which they have is implanted in them, as in the other living things. This opinion, although to some it seems plainly absurd, does not seem so to others—especially if the life of the stars be compared analogically to the life of other living things, in order to display the harmony of the universe, and most of all the microcosmic harmony, which in the hypotheses and motions of the planets is, wonderfully, as it were conspicuous.

[Margin: *Plato in the Timaeus.*]

For the rest, concerning the motive faculty implanted in the stars, we follow Plato in the end to this extent: that even though that faculty—the one most proper to motion—exists in all bodies, nonetheless nothing which is mobile produces motion of itself entirely, without the continual emanation and sustaining of some other immobile thing. This too one may gather from Aristotle's books *On the Heavens* and *On the Soul*, and likewise from Plotinus [Plotinus], Iamblichus [Iamblichus], Porphyry [Porphyry], and other philosophers. And if this is so, we gladly admit that the primary cause of the motion of the stars is devolved upon God himself, as the omnipotent Creator, who, beyond all motion of his own, rules, moves, and governs all things toward their salutary sustenance—and this by his omnipotent word, by which all things were created from the beginning. Further, among the other faculties which evidently and beyond doubt emanate from the interior form of the greater mundane bodies, this one comes here not in vain to be examined by us: namely, by what arrangement the parts of each of these bodies—homogeneous to the whole, and variously dispersed throughout its entire mass—bend themselves so unanimously, by a certain divine plan, to the preservation of the whole, that for this very reason, by a certain natural

[Margin: *Aristotle, Book 1 On the Heavens, Chapter 3; likewise Book 7 of the Physics, Chapter 1.*] [Margin: *Acts, Chapter 17, verse 25.*]

—[gravity] of their own they are carried with all their force toward their own middle or center, that is, the center of each and all of them.

Therefore, since the middle, by reason of place, coincides with the center—the outermost parts of the greater mundane body being understood—it is evident that nature has intended no other exterior form in it than the round or globular; which astronomical experience further teaches in the Sun, the Moon, and the earth. And this experience is indeed manifold: in eclipses; in the polar elevation, together with the proportionate and always equal terrestrial interval beneath the same meridian. So we need not concern ourselves with Franciscus Patricius the Venetian [Francesco Patrizi], or with certain ancient theologians—namely Augustine [Augustine] and Lactantius [Lactantius]—who held a different opinion on this point, since their ignorance of astronomy sufficiently excuses them. Moreover, with all the parts of the mundane body so neatly balanced around the middle, for the end we touched on earlier, it is fitting to reason aptly—as, for instance, that not without singular divine providence are all those mundane bodies in the expanse freely, without props or solid orbs, no less than without axes of that kind, as it were suspended and held fast, each in its appointed place; as Ovid [Ovid] sang of the earth:

[Margin: *Ovid in the Metamorphoses.*]

The earth, similar to a ball, resting on no support, hangs as so great a weight with the air placed beneath it. &c.

Let the same judgment be made about the rest.

Next, it also becomes manifest from the foregoing that, within the whole complex of any great mundane globe—compacted though it is of diverse yet kindred matter—no void is left; lest either a tearing-apart of the balanced equal parts around the middle should occur, or perhaps the demolition and destruction of the whole, by the introduction of even the least heterogeneous element, from which nature most of all shrinks. And this nature's force—powerful and almost beyond our grasp—proves from experience to be such that, whenever we contrive a void in the place of some appointed thing, nature at once supplies, if nothing else, then at least air (a body in our region most mobile and apt for replenishment); or, the moment air is exhausted, water in its place, most suited to the temperament of each place, even in its own liquid medium. I pass over the fact that thus air, suited to the respiration of animals, serves that respiration in the highest degree. It is moreover clear from the same source that only the nature of the great mundane bodies shrinks from this void, and not the nature of the universe. For that celestial region, which is seen to be empty of stars, is content for its replenishment with the expanse alone, and with the brightness or light spread through it—with which alone these bodies, coeval with the world, are by no means content, however intimately they share in them, lest a dissipation of these bodies into the expanse occur and their destruction follow. Unless perhaps to that Milky Way [*via lactea*] through the heaven some matter has been admixed, besides the gathering of the light of the countless stars implanted in it.

Finally, certain points further help to establish the globular form of the mundane bodies—points which the mathematicians, above all N. Copernicus [Nicolaus Copernicus] (*De Revolutionibus* I.1), advance to show such a figure of the heaven: namely, that they are globular, either because that form is the most perfect of all, needing no joining, whole and entire; or because all things universally seek to be bounded, as appears in drops of water and in liquid bodies, while they desire to be bounded of themselves. But concerning the exterior form of the universe we assert nothing here, lest we deservedly incur Pliny's [Pliny] reproach—he who reckons it madness to inquire into what lies beyond the heaven, and to scrutinize external things as though all the internal things of the heaven were already plainly known.

[Margin: *See Pliny, Natural History, Book 2, Chapter 1.*]

This form, however, which we attribute to the stars and to the earth as their genuine form, we reckon to be most suited to motions—whether the rotation of a star takes place merely upon its center, or a change of place occurs without this rotation (as appears in the Moon, and as is credible to happen in the rest of the planets)—otherwise than Aristotle sets forth concerning the heaven from his own hypothesis; especially when all obstacles are removed. Yet if these obstacles existed, of what kind so-

[Margin: *Aristotle, Book 2 On the Heavens, Chapter 11.*]

[p. 41] CONSIDERATIO.

—would impede any motion of whatever kind less than the form of any other body would.

These few remarks, bearing on the analogy of time and motion — which we perceive to be general and perpetual in the revolutions of the stars — I have chosen, as a final addition, to assert from physics; and I leave other matters concerning these same things to more suitable places, since they are of a more particular nature and concern the planets in their own revolutions.

Since two things above all are generally laid down as the subject of astronomical doctrine — besides the moving bodies themselves that appear in the heaven — namely time and motion (and these cohere so closely that they can in no way be separated from one another), I have chosen to say a brief word about them, because their consideration is required here before anything else.

Time admits of a twofold sense. In its **general** sense, it is the continuous duration of motions — indeed, of all things — and in this way it belongs properly to metaphysical consideration. In its **special** sense, it is determined by motion, and above all by the particular or periodic appearance of the celestial bodies. The definition of time gathered and composed by the Platonists (whose genuine explanation remains within the mysteries of the circle) seems to suit both senses: *Time is the flowing image, into numbers, of eternity that abides in unity.* For the so-called "counted numbers" supply the distinction of time into present, past, and future — that is, from the observation of things apprehended through some motion or flux.

[Margin: *Plato in the Timaeus.*]

Aristotle, too, touches on three kinds of motion. Of these, the one that occurs merely by change of place is what comes under our consideration here; the Greeks call it φοράν [Greek: phoran, "locomotion" — change of place], the Latins *latio* (carrying-along).

[Margin: *Aristotle, Physics, Book [?], Chapter 7.*]

Now, since it is proper to motion that — the magnitude in which it is accomplished being continuous — the moving thing passes through infinitely many parts of that magnitude before it arrives at the end or appointed terminus: there will therefore naturally be a certain proportion between the time and the space of such a magnitude as measured by the moving thing, although in some appearances this proportion is unequal, owing to various causes, including accidental ones.

With these assertions I have briefly comprehended the second part of the preliminaries of astronomy that I set out to treat, so that hereafter I might enter upon a more expeditious path toward demonstrating the motions themselves upon suitable hypotheses, with nature (under God) as my guide. To this God, the triune, alone and forever be glory.

[p. 42] A SHORT PREFACE to the SPHERICAL BOOKS.

Inasmuch as the spherical doctrine must be extended to the general determination of the position of the heaven, and of all the appearances in it at any given time of the diurnal revolution — and since that position presents itself to us in varied ways, not only on account of the apparent continuous circular rotation of the heaven itself, but also on account of the contrary and oblique course of the stars and the varying inclination of the world-axis in different places — therefore we devote the **first** book of our *Spherical* treatise to the consideration, distribution, and demonstration of these motions and their appearances according to their form. And since the celestial motions, for the causes stated, appear in diverse ways — to account for all of which circles were devised by the ancients — we shall accordingly discuss the necessity and sufficiency of each of these several circles, from the definition and property of each; and at length, upon these same circles, we shall construct our material sphere (as it is called), aptly designed for demonstration and use. This is the prior aim, which the contemplation or theory of the spherical doctrine alone regards.

Meanwhile, no question is raised at present about the world-system [*hypotyposis mundana*], that being fully reserved for the second part of astronomy. For the

appearances of the primum mobile present themselves to us in one and the same way, whether that first motion be assigned to the starry heaven or to the earth, in the opposite direction.

Furthermore, since the chief end of this doctrine lies, as I said, in determining the disposition of the heaven and stars — and that for any given time of the first (so-called) or apparent conversion, insofar, namely, as the individual points or phenomena of the convex celestial surface are bound to their own or to differing circles, and are seen and understood to be carried from rising to setting — we have therefore designated a **second** book of the *Spherical* treatise for expressing everywhere, in detail, the quantities of these motions and appearances. In it we shall display the practice and use of our material sphere most lucidly, showing how all the tables of the first motion are constructed, and how very many other questions touching this doctrine are demonstrated upon the said sphere by the aid of the solution of spherical triangles set forth beforehand. All these things, moreover, are worked out through various problems and adapted to practice, as will be related more fully at the beginning of the second book of the *Spherical* treatise, with reference to the order to be indicated there.

[p. 43] BOOK ONE OF THE SPHERICAL [TREATISE].

CAPUT I.

On the Definition, Principles, and Distribution of Astronomy, as well as on its General Properties.

Astronomy is a special and most noble part of mathematics, which investigates and defines the motions of the stars — both apparent and true — and likewise their positions, magnitudes, and properties relative to one another, at any given time, for our use.

[Margin: *Definition of astronomy.*]

Conversely, times are known from the motions.

The **principles** that the astronomer uses, as the causes from which he constitutes this discipline, are either **external** (i.e., more evident) or **internal** (i.e., more hidden).

[Margin: *The principles of astronomy.*]

The external principles are those that consist in visual sense and serve, as it were, in place of matter — namely the τηρήσεις [Greek: tērēseis, "observations" —

watchings of the heavens], or observations of the celestial phenomena, sought out precisely and in their proper places with the aid of instruments. These not only furnish certain extremes which serve in the practice of triangles — whence the motions are at present defined — but, when compared with those of the ancients, they also examine and emend the perennial hypotheses of the motions.

The internal or more hidden principles of astronomy, by contrast, reside in the efficient cause that shapes the hypotheses upon which the perennial motions of the stars are explained. Human scrutiny will scarcely suffice to track down its nature and character — although here too nature has contrived nothing in vain.

Finally, the distribution of astronomy is most conveniently taken from its subject, motion, and from the general variety of that motion. Now motion is either **first**, by which the whole heaven and all the stars set within it appear to be carried together from rising to setting — a conversion certainly most swift (provided it be ascribed to the heaven, and not rather to the earth, according to our own opinion), namely within 24 hours, upon the poles of the world or of the equator; or it is **second**, which belongs to the planets, contrary and oblique to the first, in that the said planets visibly tend from setting to rising, against the first motion, upon the poles of the zodiac. And although that periodic [motion] is unequal in the individual planets, nevertheless all the planets follow, this way and that, the lead of the single Sun in this motion, and are carried beneath the broad circle of the zodiac, through the middle of which the Sun's path is directed. Moreover, even the stars that are called fixed — and that nevertheless appear to be carried with an exceedingly slow motion — are likewise regulated by this motion, or by the Sun's circle, at any latitude from the zodiac across the whole face of the heaven.

From these things thus briefly indicated, two parts of astronomy arise. The **former** treats, as I said, of the first motion or movable, and was called by the ancients the **spherical doctrine**. The **latter** considers separately, beneath the zodiac, the motion of the planets, which I have called the second. And since the hypotheses for accounting for their phenomena and displaying them at each moment are composed from geometric sources, this doctrine, which deals with the second movables, is called **theoretical** — as the hypotheses too are called *theory*.

CAPUT II.

On the Definition and Division of the Spherical Doctrine.

The **spherical doctrine** is that which contemplates and defines the position of the celestial sphere and of the individual φαινόμενα [Greek: phainomena, "appearances" — the visible celestial objects] within it, chiefly with respect to the apparent diurnal conversion.

[p. 44] [OF THE] SPHERICAL [BOOKS]

To the Greeks, *sphaera* [σφαῖρα] is the same as what the Latins call a *globe* — namely, a round and solid body. In the present context, however, it embraces the circles upon which the first or apparent motion of any of the celestial φαινόμενα [appearances] is accounted for and defined.

The spherical doctrine has two parts. The one, from the contemplation of the apparent motions, composes the sphere as a kind of general instrument for defining the said motions. The other resolves the particular appearances, by geometric reasoning, into numbers, and applies them to use. In the former, accordingly, the **theory** is more conspicuous; in the latter, the **practice**.

CAPUT III.

On the Forms of the Motions, in Conformity with which the Material Sphere is Properly Constructed; and Herein, the Division of the Sphere.

Since the prior part of the spherical doctrine requires a general knowledge of the apparent motions, therefore, while we briefly present their contemplation, we conveniently bring back to appearance the diurnal motion — which is perhaps more rightly attributed to the earth — and, along with the common crowd, transfer it to the heaven, in which two distinct motions appear.

The **first** is the one from rising to setting, upon the axis of the world, common to the whole heaven. And since the heaven is perpetually divided by our sight into two hemispheres — namely the upper, which is visible, and the lower, which is invisible — it comes about that, for one who shifts his earthly location especially toward north and south, a variety is encountered in denoting the said celestial hemispheres. This notably alters almost the whole face of the heaven, as regards both the rising and setting of the stars and the other φαινόμενα [appearances] of that same heaven, and shifts it on account of our sight and the interposition of the earth. Hence the sphere is divided into the **right** and the **oblique** sphere, and the right, finally, into the **vertical** and the **parallel**.

The **other** motion, which belongs to the second movables and is assigned to the Sun as the chief leader of the remaining planets, appears with a contrary and oblique aspect upon another axis and other vertices; and at any place on the earth it is equal — as we shall show further in the special or second part of astronomy. And so much, briefly, for the consideration of the celestial motions, upon which the material sphere is founded. There follows the composition of the sphere itself.

CAPUT IV.

On the Origin, Necessity, and Definition of the Circles, from which the Material Sphere is Rightly Composed.

Every apparent motion that is uniform and circular must be exhibited and defined by the astronomer by means of some circle lying everywhere equidistant from each of its two poles in the sphere. This circle — both for that reason and because it rests on every side upon the center of the sphere — is called a **great circle** [*circulus maximus*]. But since the individual points of the entire spherical surface, comprehended between each of the said poles, must be distinctly referred to this circle, a further circle must be added, one that grazes both poles of the former and, by its transverse revolution between the said poles over a fixed diameter, brushes the entire surface of the sphere. The purpose is that a point — or any φαivόμενον [appearance] possessing some latitude from the former circle, by which the motion is defined — may be bound to this [second circle] and determined, since it everywhere falls upon it at right angles. This circle, under a condition like the other's, likewise becomes a great circle of the sphere — namely the circle generally named the **circle of latitude**, just as the former was named the **circle of longitude**, though we ourselves call it the **transverse** [circle].

From these few premises we shall now construct the sphere, fitted to account for the apparent celestial motions, in the following manner.

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To express and determine the first motion of the heavens, let us first of all imagine a circle of longitude to be described upon the poles of the world.

This is the first circle to arise: it is the standard and measure of the first revolution and of all time in that revolution—that is, the circle which, by its equal arcs, measures off equal intervals of the said time.

Such a circle was called by the ancients the equator, or equinoctial [Æquator seu Æquinoctialis], because the Sun, when it lies beneath it, makes the day equal to the night for all who dwell on the earth.

The circle transverse to it—that is, the circle of latitude, the one which grazes the poles of the world and therefore falls at right angles upon the equator just defined—is called the circle of declination [circulus declinationis], because it demonstrates the declination of each individual point and fixes that point's position relative to the equator.

To the second of the celestial motions—the φαεινομένων [Greek: phainomenōn, "of the appearances"] motion, which we said above is contrary to the first and oblique, and which is regulated by the apparent motion of the Sun—we assign a second imaginary circle of motion [circulus lationis], likewise described about its own proper poles. With respect to the Sun's apparent course alone, this circle is usually called the ecliptic [Ecliptica], the "way" (*via*) [Margin: *Lib. 1. Theor. cap. 9*], because whenever either luminary is visibly placed beneath it, the one must undergo an eclipse [eclipsis]: the Sun when the two are in conjunction, the Moon when they are in opposition. On other occasions the same circle is called the zodiac [Zodiacus]—whenever we wish to take in together the latitudes which the planets, departing from the ecliptic on either side, mark out, as well as the asterisms of the twelve animals arranged along it.

The greatest obliquity of the ecliptic from the equator is found today to be 23 degrees 31½ minutes [gr. 23 mi. 31½]. The recession of each of its two poles from the poles of the world will be just as great, since every pole stands distant from its own circle on the sphere by a quadrant of a circle. Because this is the greatest circle of the sphere, it therefore cuts the equator in two opposite places; beneath these the Sun, when it lies there, marks the vernal equinox as it ascends and the autumnal equinox as it descends—just as it marks the solstices at the limits where the departures of each kind from the equator are greatest, which occur at the quadrants.

The circle transverse to the ecliptic—namely the one which grazes the ecliptic's poles, and which therefore falls at right angles upon it in whatever transverse revolution—may by a special name be called the circle of latitude [circulus Latitudinis], since by any arc of its own it demonstrates the latitude of any point

on the sphere lying at any distance on either side from the ecliptic and its poles, and fixes that point's place relative to the ecliptic.

Thus far we have displayed the two celestial motions by two pairs of circles. Now, for the appearance of the celestial bodies upon the earth, with respect to each of the two hemispheres, we assign a great circle which, from its Greek origin, is suitably called the *horizon*, and in Latin the *bounder* [Finitor]. The horizon alone seems to fall under ocular perception, since, as we have said, it divides the visible part of the heaven equally from the invisible. Its poles lie directly above and below the vertex always, and are therefore removed from the poles of the world by exactly as much as the equator is at its greatest distance from the horizon. From this we conclude that, when the equator coincides with the horizon, the poles of both circles likewise coincide, and the sphere in that position is called parallel [parallela], or the opposite of the "right" sphere—so much so that it is treated under the same class as the right sphere. For when the equator climbs the vertex, the sphere is called right [recta]. At the eastern threshold of the horizon the risings of the stars occur; at the western, their settings.

The circle transverse to the horizon is called the vertical, or circle of altitude [Verticalis seu Altitudinis], which, ascending and descending directly from the horizon on either side toward its poles, demonstrates the altitude and depression of each point above the horizon, and refers them to its measure. Whenever this circle also grazes the poles of the world, it is called the meridian [Meridianus]. A star or point of the heaven passing across it (which happens twice in any daily revolution) is said by us to be at its highest, or greatest, meridian altitude if it is above the Arctic pole of the world, but at its lowest, or least, if below—and the same is to be understood of the Antarctic pole.

And so far enough has been said in general about nearly all the circles, and about what suffices to demonstrate the celestial motions and their appearances.

As for the circle of position [circulus Positionis]—which astrologers use above all to construct the celestial houses [domicilia coelestia] and to carry out directions [directiones]—it is nothing other than a movable horizon set upon poles placed at the points where the true horizon and the meridian intersect; through it the φαίνόμενα [appearances] are understood to ascend and descend, even in any position of the revolution.

To this standard of the apparent celestial motions we have constructed a material sphere [sphæram materialem] conformable to it, and on it alone the use of the spherical doctrine will henceforth be displayed. The circles are divided into 360

equal degrees, each degree again into 60 minutes [Margin: *See the division of the circles above, in the Prognostics, p. 1.*], and finally each minute into 60 seconds. Moreover the zodiac is divided into twelve signs, so that 30 degrees fall to each of the signs. These the Greeks call δωδεκατημόρια [Greek: dōdekatēmoria, "twelfth-parts"], that is, "twelve parts"; but in the zodiac they are called animals, and they are twelve:

☐ Aries, ☐ Taurus, ☐ Gemini, ☐ Cancer, ☐ Leo, ☐ Virgo, ☐ Libra, ☐ Scorpius, ☐ Sagittarius, ☐ Capricornus, ☐ Aquarius, ☐ Pisces.

These are set opposite one another as follows.

[Margin: *See below, Cap. 6, the diagram of the aspects.*]

		Set opposite (Resp.)		Set opposite (Resp.)	
☐ Aries		☐ Libra	☐ Cancer		☐ Capricorn
☐ Taurus	Resp.	☐ Scorpio	☐ Leo	Resp.	☐ Aquarius
☐ Gemini		☐ Sagittarius	☐ Virgo		☐ Pisces

Parsing notes: the source sets two oppositional groupings side by side in a braced layout. Left group: Aries↔Libra, Taurus↔Scorpio, Gemini↔Sagittarius. Right group: Cancer↔Capricorn, Leo↔Aquarius, Virgo↔Pisces.

A Representation of a Proper Celestial Armillary Sphere [*Typus legitimæ Sphæræ armillaris Cœlestis*].

Diagram. A full armillary sphere shown in perspective, with the Earth as a small striped globe at the center. Numerous great circles (rings) interlace around it; several bear shaded bands (the zodiac and equator rings) and there are smaller transverse rings. The labeled points are lettered around the figure. The accompanying key, **Names of the Circles** [*Nomina Circulorum*], runs beneath the figure and continues onto the next page.

Labeled points and circles (read from the figure and its key):

- **G** (top): pole of the meridian / top of the outer meridian ring
- **H** (upper right): point on the meridian ring
- **C** (upper, near top center): point on the solstitial colure
- **B** (upper, left of center): point on the zodiac ring
- **P** (left of center): pole of the zodiac
- **A** (center-left): point on the equator
- **N** (right): pole of the zodiac (opposite to O)
- **E** (right-center): point on the transverse circle of latitude
- **F** (left and right): the horizon ring (lettered F F)
- **K** (far left): point on the meridian ring
- **I** (far right): point on the meridian ring
- **M** (bottom): lower point of the meridian ring (pole of the horizon, with G)
- **L** (lower left): point on the meridian ring
- **D** (lower center): point on the transverse circle of declination (lettered D D)
- **O** (lower left): pole of the zodiac (opposite to N)
- Zodiac-sign glyphs (♍ Virgo, ♋ Cancer, etc.) appear along the shaded zodiac band.

The full key (the *Nomina Circulorum*) is transcribed below as it is printed across pp. 46–47:

Names of the Circles. L K G H I — the Meridian, which here contains the sphere in its proper form. **A A** — the Equator, whose poles are **H L**, together with the transverse circle of declination **D D**, and the solstitial colure **O A C** [Coluro Solstitiali]. **B B** — the Zodiac, whose poles are **O N**, together with the transverse circle of latitude **E E**. **F F** — the Horizon, whose poles are **G M**, together with the transverse circle of altitude **C C**. **P** — the Circle of Position.

B B — the Zodiac, whose poles are **O N**, together with the transverse circle of latitude **E E**. **F F** — the Horizon, whose poles are **G M**, together with the transverse circle of altitude **C C**. **P P** — the Circle of Position.

Now among these circles, because the transverse ones are movable, when they are properly arranged in the revolution of the heaven along with the rest, triangles arise of their own accord; their analysis is to be sought in Book 2 of the Sphæricks.

CAPUT V.

On the Uses of the Circles Considered Individually and Distinguished from One Another, and on Their Definitions.

That there are three circles of longitude, and as many of latitude, in our sphere, which demonstrate the celestial motions and appearances to us, has been shown more than sufficiently in the preceding chapter. But to distinguish the uses of the several circles from one another, we shall retain the names imposed upon them by the ancients.

To the equator, then, are assigned both right ascension [Ascensio recta] and oblique ascension and descension [Ascensio & Descensio obliqua], just as declination [Declinatio] is assigned to its transverse circle.

Now right ascension is the point on the equator, determined there by means of the transverse circle, of any given point or celestial body. It is reckoned along the arc of the equator lying between the vernal equinox and that defined point on the equator. It was called "right" ascension by the ancients because, in the right sphere, it ascends at right angles together with the ecliptic, or with any celestial point, once the eastern horizon has been fixed; to it right descension [descensio recta] is the contrary.

Declination is the shortest distance of a given point of the heaven from the equator on the sphere, which is therefore to be measured on the transverse circle of the equinoctial—namely on the arc of its quadrant lying between the equator and the pole of the world.

When such a point inclines toward the north it is said to have a northern declination [declinationem Septentrionalem]; when toward the south, a southern declination [declinationem Meridionalem].

Oblique ascension and descension, in turn, is the arc of the equator of the oblique sphere which ascends and descends together with any given point of the heaven on the horizon, and which is likewise reckoned from the vernal equinox.

The ascensional or descensional difference [Differentia Ascensionalis aut Descensionalis] is the portion of the equator intercepted between the right ascension of some point and the oblique ascension and descension of the given celestial point. For if the given point of the heaven is deflected toward the north, this difference, subtracted from the right ascension, leaves the oblique ascension at rising. But if the given point inclines toward the south, the same is

accomplished by the ascensional difference added to the right ascension. The oblique descension behaves in the contrary manner. Likewise the semidiurnal arc is reckoned along the equator, which is the measure of the arc of right ascension between the rising and setting of each appearance [φανόμενον], and especially of the Sun on the ecliptic.

The names of longitude and latitude we assign, with the ancients, to the ecliptic alone and to its transverse circle. Now longitude [Longitudo] is the arc of any assumed point of the heaven on the ecliptic, determined by means of the zodiacal transverse circle, which arc is likewise reckoned from the intersection of the vernal equinox.

Latitude [Latitudo], on the other hand, is the shortest distance of that same assumed point from the ecliptic, which the arc of this transverse circle measures.

If that point inclines toward the north from the ecliptic, its latitude is called northern [Borealis]; if toward the south, southern [Austrina].

Finally, to the horizon we assign the azimuthal degrees [gradus Azimuthales], and to its transverse circle—which is called the vertical circle [circulus Verticalis]—the altitudes of points or of celestial bodies, and that by the same reasoning as in the other cases; making the beginning of the azimuthal degrees on the horizon on either side from the meridian intersection.

And we reckon the altitudes by means of the vertical circle from the horizon; and it is therefore manifest that they occur either at midday or outside of it.

Furthermore, on the horizon, at the true rising and setting of the heavenly bodies, the rising and setting amplitude [amplitudo ortiva atque occidua] is reckoned: namely the arc of the horizon intercepted between the equator and the true rising or setting of the given point of the heaven.

Finally, on the meridian portion of the vertical circle, the elevation of the pole of the place [poli loci elevatio] is situated, which distinguishes the quantity of the oblique sphere from the right.

There is also estimated, along the horizon, a certain peculiar rising and setting of the heavenly bodies, with respect to the points of the ecliptic in which the Sun is situated. And this rising and setting is twofold: true and apparent. The true is again twofold: morning and evening.

The morning, or cosmic, rising [Matutinus seu Cosmicus ortus] of stars is said to be the rising of those which rise together with the Sun. The morning setting is the

setting of stars going down together with the point of the ecliptic opposite to that with which the Sun rises in the morning.

The evening, or acronychal, rising [Vespertinus seu Acronychus ortus] is the rising of stars rising together with the point opposite to that with which the Sun sets in the evening. The evening setting is the setting of stars going beneath the horizon in the evening together with the Sun.

Finally, the apparent, or heliacal, rising and setting [Apparens seu Heliacus ortus & occasus] is that which regards the rays of the Sun lying at a certain measured distance below the horizon. For since the motion of the Sun in the zodiac outruns in its course both the fixed stars and the superior planets, while it behaves variously toward the rest—namely Mercury and Venus—it comes about that, according to the magnitude of the stars and their disposition with respect to the zodiac, the Sun now takes their sight from us, now by its own advance restores them to view. Hence the heliacal rising is nothing other than the appearance of a star before the rising of the Sun, the star having then emerged from the Sun's rays. The heliacal setting, on the contrary, is the disappearance of a star after the setting of the Sun, the star being then for the first time entangled in those rays. The distance of the star from the Sun on either side, at the heliacal rising and setting, is reckoned on the vertical circle, between the horizon—in which the star is caught at its true rising or setting—and the Sun depressed beneath it to the proper distance: so that, if the star is of the first magnitude, that arc will be 12 degrees; if of the second, 13 degrees; if of the third, 14 degrees; and so on. The method of finding such an arc we shall explain in the second part of the spherical doctrine.

CAPUT VI.

On the Aspects of the Stars.

From the rational division of the circle, and above all of the zodiac, the aspect [Adspectus]—a certain mutual relation of the stars—is considered, by which they are able to communicate the influence of their rays, and their power in acting, chiefly upon things beneath the Moon. Although the consideration of this matter was the chief concern of the astrologers, nevertheless, since it falls within the spherical treatment—just as does the function of the circle of position, which we mentioned earlier, and which is bound up almost inextricably with the aspects—we shall therefore in this chapter speak briefly of the aspects, and in the following one of the use of the circle of position, so far as the theory requires.

Now the aspects are of two kinds: general, and fixed or special.

The general ones, which regard any parts whatever of the zodiac, are commonly five.

1. Conjunction [Conjunctio], when the stars are joined together in the same parts of the zodiac; its sign, for brevity's sake, is written thus: ϕ
2. Opposition [Oppositio], when the stars are found in opposite points of the zodiac; its sign is this: ϕ^o
3. Sextile [Sextilis], when the stars are placed a sixth part of the zodiac, or two signs—that is, 60 degrees—apart from one another; its mark is this: $\star$

Book One of the Spherics — Caput VI (continued)

4. The **quartile aspect** [Quadratus Adspectus] is so called when the stars are positioned a quarter of the zodiac apart from one another; its sign is $\square$ (quartile).
5. Finally, the **trine aspect** [Trinus Adspectus] occurs when the stars are distant from one another by a third of the zodiac, that is, 120°; its sign is $\triangle$ (trine).

These aspects are called **right** (dexter) when the stars are cast forward in the direction of the succession of the signs, and **left** (sinister) when they are cast against that succession — as is set before the eyes in the diagram added here.

The **special** or **immobile aspects** [Speciales seu Immobiles Adspectus] are the *antiscia* of the planets or stars. These arise from the equal distance of those bodies from one of the tropical points — that is, from Cancer or Capricorn — and they are either **primary** or **secondary**.

Diagram. A large circle representing the zodiac, with the twelve zodiacal signs marked by their glyphs around the circumference. Within the circle a dense network of straight chords (the aspect lines — conjunction, opposition, sextile, quartile, and trine) connects the signs, and the aspect glyphs ($\star$ sextile, $\square$ quartile, $\triangle$ trine) are placed along these chords. A cluster of small circles near the center represents the antiscia construction. The signs are arranged around the rim as follows:

- Top: $\square$ (Cancer)
- Upper left: $\square$ (Leo)
- Upper right: $\square$ (Gemini)
- Left: $\square$ (Virgo)

- Right: ♉ (Taurus)
- Lower left (far left): ♎ (Libra)
- Right (mid): ♈ (Aries) — labeled at the right margin
- Lower left: ♏ (Scorpio)
- Lower right: ♐ (Pisces)
- Bottom left: ♐ (Sagittarius)
- Bottom: ♑ (Capricorn)
- Bottom right: ♒ (Aquarius)

Interior glyphs along the chords: ✱ (sextile), ♋ (quartile), △ (trine) repeated at several positions. [The interior antiscia circles carry no resolvable labels at this resolution.]

The **primary** [antiscia, Greek: ἀντίσκια (antiskia, "shadows cast opposite" — points equidistant from a tropic)] are points in the zodiac that have the same distance from the equator on the same side: for example, the beginning of Gemini and the beginning of Leo; likewise the beginning of Taurus and of Virgo; the beginning of Aries and of Libra. So too in the southern signs: the beginning of Aquarius and of Sagittarius, the beginning of Pisces and of Scorpio. All of these are marked with this sign ♋ (the Cancer symbol marking primary antiscia).

The **secondary** antiscia, by contrast, are reckoned to be of less force than the former. They are considered in the signs that are in a sense opposite. Thus while a planet stands at the beginning of Gemini ♊ it obtains its primary antiscion at the beginning of Leo ♌, and its secondary or opposite antiscion in the sign equally removed from the same equinox in the contrary direction — namely at the beginning of Aquarius ♒. The mark of the secondary antiscia is (l).

So much for the aspects. Two further points are to be observed about them. **First**, it suffices for assigning an aspect that it arise even from the rays of a single star: thus, if a star is placed at the position of Cancer, we say that its sextile is Virgo and Taurus, its quartile Aries and Libra, and its trine Pisces and Scorpio. **Second**, since the aspects are referred to the ecliptic, the varying latitudes of the stars make them vary too — except for those that agree uniformly. [Margin: *Lib. 2. sphær. cap. 6. Problem. 3.*] As for both kinds of aspect, which chiefly concern the business of direction, and how they are to be reduced in due fashion for the latitudes, we shall treat the matter more conveniently in the second part of the spherical exposition.

CAPUT VII.

On the use of the circle of position among the astrologers in constructing the celestial houses, in distinguishing them one from another, and likewise in carrying out directions.

The **circle of position** [Circulus Positionis], which above we called the movable horizon, when it is properly applied, accomplishes the chief task of astrological practice — the task required in the erection of the celestial houses and in the direction of the *promissors* to the *significators* beyond the cardines of the world, that is, beyond the horizontal and meridian circles. Now although the method of erection is varied in many ways according to the differing opinions of the practitioners, nevertheless the method that is regulated by the motion of the *primum mobile* — which Johannes Regiomontanus calls the *rational* method, and which we judge to have been chiefly used by Ptolemy and the ancients — is, because it is the more widely accepted among most astrologers, the one we shall here assign by way of general indication, together with the method of direction. [Margin: *Lib. 2. Sphær. cap. 6.*] How it is adapted to each individual revolution we shall demonstrate specifically, through all its requisites, in our practice, by means of examples.

Once the right ascension of the mid-heaven (that is, on the meridian) is known from the given time, the equator is first divided into four equal parts by the intersections of the horizon and the meridian circle. Then each of these sections is subdivided into three other equal parts; from this there arise twelve equal parts of the equator, commensurable with the twelve celestial houses to be erected. But since the zodiac must be distributed by a fixed rule through the equator and the circle of position — so that six spaces or houses are left in each hemisphere, beginning from the rising point toward the right, below the earth — the horizon itself by its intersection with the zodiac forms the first celestial house at the eastern point. Furthermore, if the circle of position is afterward depressed downward below the true horizon in our sphere, or raised upward above it, until it reaches the intermediate equal intersections of the equator, then this circle marks off the cusps of the houses on the zodiac itself no less than the horizon or the meridian does by its own intersection. These houses are unequal on the zodiac, on account of its oblique disposition relative to the equator. Of all of them, the six that are divided off by the meridian circle are laid open by the method just given; the remaining six are known without difficulty from the opposite signs and degrees.

To **direct** a significator to its promissor is nothing other than to find the oblique ascension and descension of each, with respect to the circle of position lying beneath the significator — if the direction is made in the succession of the signs — or beneath the promissor, if it is made against that succession. For the difference of those oblique ascensions and descensions is called the **arc of direction** (or the *directorius*), and it measures the times of good or evil events — taking, of course, for each year, the quantity of the Sun's diurnal motion corresponding to the instant of time, on the equator, as will be explained later in the practice.

Therefore, if the direction is made along the series of the signs, that is, in their succession (as I said), we first hunt out on the surface of the sphere, by means of the circle of position taken as a transversal, some planet as significator, and we apply the said circle to it; then we mark the intersection it makes with the equator. This intersection on the equator shows the oblique ascension of the significator, with respect, that is, to the circle of position under which it lies. Next, with the circle of position thus held fixed, let the sphere be revolved until the promissor falls under the same circle of position; and its intersection with the equator is likewise to be noted, which is called the oblique ascension of the promissor. Now however far this stands from the intersection — that is, from the oblique ascension of the significator (or however great the difference of the oblique ascensions of the two, significator and promissor, may be) — that is the magnitude of the arc of direction sought. But if, on the contrary, the direction is undertaken against the series of the signs, that is, in the antecedence (as they say), then since the significator is exchanged into the promissor and vice versa, the promissor into the significator, everything happens in the contrary manner.

CAPUT VIII.

On the Zones, the Climes, and the Parallels.

Our material sphere excludes the circles that others have called the *lesser* circles, for this reason: that the arcs of the spherical triangles — in which the whole practice and use of spherical doctrine consists — exist only as portions of a great circle, which all of them in our sphere refer to.

Now since the said lesser circles — namely the two tropics, and in addition the Arctic and Antarctic — were devised for no other reason than to mark off the limits arising from the effect of the Sun alone upon the earth, and were introduced by others; and since they hinder the disposition of the triangles, which must always be constructed from great circles in our sphere, we have

omitted them. For the distinction and number of the zones, it suffices to bring to bear the consideration of the obliquity of the ecliptic from the primary equator, together with the proper division of the spherical surface that follows.

First, the space that the angle of the [Greek: λόξωσις (loxōsis, "obliquity" — the inclination of the Sun's path)], that is, the Sun's obliquity, composes on either side is called the **Torrid Zone**, because the Sun twice in the year grazes the zenith of those who dwell within it. And since the Sun's obliquity is today found to be nearly $23^{\circ} 31\frac{1}{2}'$, the breadth of the Torrid Zone will be $47^{\circ} 3'$.

Next, since the Sun, when it occupies either point of the tropics, plainly ceases to be seen by those who are removed a quadrant from the said point toward the contrary region of the world, therefore the space of the **Temperate Zone**, extending on either side from the equator, argues the complement of the angle of double obliquity of the ecliptic from a quadrant — being today $42^{\circ} 57'$.

Finally, since the half of this complement, all the way to the individual poles — namely $23^{\circ} 31\frac{1}{2}'$ — constitutes the individual **Frigid Zones** on either side. This is the distribution of the zones, which Maro [Virgil], attributing it to the heaven itself, or to the Sun's motion as the agent (by metonymy), describes thus:

Five zones possess the heaven; whereof one ever glows ruddy with the flashing Sun, ever scorched by fire. About this, at the far edges right and left, are drawn two zones, frozen with dark-blue ice and black with rains. Between these and the middle, two were granted to wretched mortals by the gift of the gods, and a path was cut through both, along which the slanting order of the signs might turn.

Furthermore, in the distinction and number of the **climes**, the ancients — so that they might know the elevations of the pole, even in more remote places, from the common reckoning by the length of the longest day — fixed, as it were, a boundary for each clime at the space on either side of the equator that differs by a half-hour's variation of quantity in the said length of the longest day; nor did they extend the same on either side of the equator beyond the Torrid Zones.

The **parallels**, however, they made to be the half-quantities of the climes, as these things will more conveniently be shown below in their proper place. [Margin: *Lib. 2. Sphaer. cap. penultimo.*]

So much for the theoretical part of the spherical doctrine.

BOOK TWO OF THE SPHERICS.

CAPUT I.

On the matter and order of the problems to be treated through the second Book of the Spherics. Then concerning those things that are comprehended within the triangle intercepted by the ecliptic, the equator, and the arc of declination.

Diagram. A decorative woodcut initial **H** beginning the word *Hactenus*, ornamented with foliate scrollwork. (Informational content: the letter H only.)

Thus far the **composition** of our sphere, together with the use of the circles upon which the motions of the first revolution are demonstrated, has been set forth. There follows the [Greek: ἀνάλυσις (analysis, "analysis" — the resolution into particulars)] of these same motions in particular, to be determined for any required quantity by a fixed and geometrical reckoning in numbers. This practice contains the kernel and use of the whole spherical doctrine, by means of the following problems, in the order that follows.

These problems — plainly golden ones, of which this book consists — we assign to three classes. In the **first** we treat:

First, of determining those stars or celestial [Greek: φαινόμενα (phainomena, "appearances" — celestial phenomena)] that, in any disposition of the sphere — whether right or oblique — stand in the same way with respect to the ecliptic and the equator. For instance, those that require the reciprocal interchange between these and their transverse circles, namely of latitude and declination; whereby either the longitude given alone (as of the Sun on the ecliptic), or together with some latitude (as outside the ecliptic), is converted into right ascension and declination, and vice versa — the terms of these circles being also transmuted and alternated, wherever there is need, so that they may serve what follows. To these is also to be added the method of investigating the spherical distances of the stars.

Second, we weigh the motions of those bodies that have a relation to the horizon in any particular position of the sphere. Here many problems, most worthy to be known, are treated thoroughly: concerning the length of day and night, concerning the rising and setting of the stars, and finally concerning the reciprocal finding of times through the daily apparent revolution of the Sun and the stars.

Third and last in this class we treat the matters that concern astrological exercise: those to be established in the construction of the celestial houses and in the direction of the planets — just as in Caput VI of the preceding Book we proposed those things also for consideration.

In all these matters, which are encompassed by this first class, the sources of the tables of the *primum mobile* — constructed by the once most renowned mathematician Johannes Regiomontanus, or by other practitioners — are opened up, and their foundations in the triangular disposition and solution are set before the eyes of learners; so that they may engage with such tables, to be inserted in their proper places, with greater pleasure, and may draw from them whatever will be of use to themselves, if at any time it should please them to forgo the labor of analyzing the triangles.

Into the **second class** are referred whatever things bear upon the observations of the stars by means of mathematical instruments. Here:

First, inquiry must be made into nearly all kinds of astronomical instruments, especially those founded in our sphere as rivals to the roundness of the heaven, and as it were fabricated upon it; and it must be shown which excel the rest.

Second, after the skillful and many-sided investigation of the meridian line and of the pole of the locality on land and at sea, we shall treat of the several methods of observing the stars afresh, and of directing them to the Ecliptic as to a normative circle; and we shall confirm, by sure and demonstrated reasoning, the reliability that is to be preferred over the rest in these matters.

Third, we undertake to treat of the parallaxes of the nearer bodies and the refractions of the lower ones, of the observations of phenomena and of the true places of these same bodies — when, on occasion, they are entangled together — and of the disentangling of these in longitude and latitude with respect to the Ecliptic. This subject is not only necessary to the doctrine of solar eclipses; it also yields a most beautiful insight, by way of the parallaxes, into the elevation or distance of the planets, and likewise of the new phenomena [Greek: φαινόμενα (phainomena, "appearances")] as measured from the Earth. From this, the true magnitude of these same bodies, at their distance from us, is afterward discerned. As for refractions and twilights, the doctrine determines and defines the sublimation of terrestrial vapors and the duration of each twilight — morning as well as evening — within the several horizons.

Finally, to the **third**, or last, class we assign whatever things bear upon the Earth itself. The order of treatment will be as follows.

First: because the doctrine of gnomonics [doctrina Gnomonica] stands nearest to the celestial doctrine and casts light upon the rest, we accordingly pursue it first, so far as is needful, within this class — namely, the projection of a shadow by a stylus set upright in due fashion and exposed to the rays of the luminary. From this, sundials [Sciaterica] of every kind of declination and inclination can be skillfully constructed, and that by a certain method of ours, novel, peculiar to us, and exceedingly compendious, working through triangles.

Second, we speak of those matters that we left to be considered in the final chapter of the preceding book: the effects impressed upon the Earth by the action of the Sun, owing to the varying inclination of the world's axis toward north and south, as well as to the oblique course of the Sun. From this arises the doctrine of the zones, climes, and parallels; and along with it the explanation of the various designations of those peoples who inhabit the various and opposite regions of the Earth.

Third and last, we instruct concerning those things that serve to make particular sites upon the terrestrial globe known. Here, first, the longitudes and latitudes of distant places are investigated by us by various methods. Then we show how those same data are resolved geometrically into distances. This procedure, although it differs little or not at all from the method of measuring the intervals between the stars, is nonetheless — for its usefulness and the delight it gives — most highly commended among all persons of cultivated mind, and especially among historians.

PROBLEMA I.

Given the greatest obliquity of the Ecliptic from the equator, together with a longitude reckoned from either of the equinoxes: the declination corresponding to such a longitude on the Ecliptic is found.

Since the obliquity of the Ecliptic is variable, very nearly, by a third of one degree — in these our times slowly increasing once again, as will be demonstrated in the second part of the *Astronomy* — those would seem to act rightly who have increased by half a minute the obliquity that Lord Tycho Brahe found at Hven thirty years ago to be $23^{\circ} 31\frac{1}{2}'$, adapting it to the year of Christ 1630. It is to this date — and to the nearest years — that those who set about constructing tables of the *primum mobile*, above and below, take care to fit their work, down to the year 1660 reckoned from the Savior born into the world; such tables will hold good without sensible error even as regards the rising and setting of the fixed stars. This we ourselves, for the most part, also do in the examples here set down. In the

meantime, however, the tables that were earlier compiled by Lord Tycho and by Johannes Regiomontanus [Regiomontanus], retained by us in their own data [Greek: διδόμενα (didomena, "the things given," the data)], are incorporated into this work.

[Margin: CLASSIS PRIMA hic incipit. ("The First Class begins here.")]

Now, in order that the solution of the present problem may be carried out, let there be — in the following diagram — described upon center A the meridian D H K, coinciding with the colures of the solstices; and let the remaining circles be halved out of our sphere. Among these the equator is drawn, marked D B H, from the pole F; while the Ecliptic is figured by the letters C B G, from the pole E. And since the intersection of the said circles appears at B, it is plain that at this same point the beginning of Aries — that is, the vernal equinox — is to be understood. Let the place of the Sun be taken at the point N on the Ecliptic; its arc of declination is represented by N L, namely on the circle of declination of our sphere, drawn between the two poles of the equator, F and K. Now let N be the place of the Sun, removed from the vernal equinox B — in this example — by the arc B N, so that the point N attains the beginning of Gemini, and therefore B N stands at 60 degrees. The angle N B L is the same as the distance between the poles E F, namely $23^{\circ} 32'$; and finally the angle at L is right. Therefore, when the declination of the Sun, N L, comes to be sought, the data [Greek: διδόμενα (didomena, "the given quantities")] are set before the eyes thus:

Diagram. A geometric figure showing the celestial sphere in the plane of the solstitial colure. The outer circle is the meridian DHK, described about center A. Within it are drawn the equator (DBH) and the Ecliptic (CBG), intersecting at B (the vernal equinox / beginning of Aries). The pole of the equator is F (upper), with K the opposite pole (lower); the pole of the Ecliptic is E. The Sun's place is at N on the Ecliptic; the declination arc N L runs to L on the equator along the declination circle through F and K; M is marked nearby on the equator/meridian.

- A (center): center of the sphere
- D (left): point where equator meets meridian
- H (right): point where equator meets meridian
- F (upper, near top): pole of the equator
- E (upper left, inside near top): pole of the Ecliptic
- G (upper right): point of the Ecliptic
- C (lower left): point of the Ecliptic

- B (center-low): intersection of equator and Ecliptic — vernal equinox, beginning of Aries
- N (center, above L): place of the Sun on the Ecliptic
- L (center): foot of the declination arc on the equator
- M (center-right, near L): point on the equator
- K (bottom): pole of the equator opposite F

Data [Greek: $\delta\iota\delta\acute{o}\mu\epsilon\nu\alpha$ (didomena)]	B N	60° 0'	distance of the Sun from the vernal equinox
	N B L	23° 32'	angle of greatest obliquity
	B L N	90°	right angle

Therefore N L is found: **20° 13' 48"**, the sought declination of the Sun.

From this single example it is readily understood how the following table of the Sun's declination — for each degree of the Ecliptic, and likewise throughout its remaining quadrants — is constructed from the doctrine of triangles. For points lying opposite on the circles, which are at equal remove from the equinoxes or solstices, since they underlie the same calculation, likewise yield results sought [Greek: $\zeta\eta\tau\acute{o}\mu\epsilon\nu\alpha$ (zētoumena, "the things sought")] similar to these. This consideration — rightly and easily demonstrable to the eye in our sphere — supplies and completes the construction of nearly all tables for the whole circle.

Table of the declination of the Ecliptic, from Tycho's observations, for the greatest obliquity 23° 31½'.

The table is laid out in three double-columns. Each block has a leading degree-column (gr.) and three sub-columns giving degrees, minutes, seconds (gr. mi. fec.) of declination. The zodiacal sign headings run across the top, with complementary signs given at the foot for reading the column upward. The left block heads ♈(Aries) / ♎(Libra); the middle block ♉(Taurus) / ♏(Scorpio); the right block ♊(Gemini) / ♐(Sagittarius), with foot-headings ♓(Pisces) / ♍(Virgo), ♒(Aquarius) / ♌(Leo), ♑(Capricorn) / ♋(Cancer) respectively.

gr.	$\gamma_{\underline{n}}$ deg.	$\gamma_{\underline{n}}$ min .	$\gamma_{\underline{n}}$ sec.	$\square\square$ deg.	$\square\square$ min .	$\square\square$ sec.	$\square\square$ deg.	$\square\square$ min .	$\square\square$ sec.	gr.
0	0	0	0	11	30	42	20	13	22	30
1	0	23	56	11	51	48	20	25	57	29
2	0	47	53	12	12	40	20	38	9	28
3	1	11	49	12	33	21	20	49	58	27
4	1	35	43	12	53	49	21	1	25	26
5	1	59	37	13	14	5	21	12	29	25
6	2	23	28	13	34	7	21	23	7	24
7	2	47	16	13	53	57	21	33	22	23
8	3	11	4	14	13	32	21	43	15	22
9	3	34	47	14	32	53	21	52	42	21
10	3	58	28	14	51	59	22	1	45	20
11	4	22	4	15	10	50	22	10	22	19
12	4	45	37	15	29	26	22	18	35	18
13	5	9	5	15	47	47	22	26	22	17
14	5	32	29	16	5	51	22	33	44	16
15	5	55	47	16	23	39	22	40	39	15
16	6	18	58	16	41	9	22	47	10	14
17	6	42	6	16	58	22	22	53	13	13
18	7	5	6	17	15	18	22	58	51	12
19	7	28	0	17	31	54	23	4	3	11
20	7	50	46	17	48	14	23	8	47	10
21	8	13	26	18	4	14	23	13	5	9
22	8	35	58	18	19	57	23	16	56	8
23	8	58	20	18	35	18	23	20	20	7
24	9	20	34	18	50	21	23	23	18	6
25	9	42	41	19	5	4	23	25	48	5
26	10	4	38	19	19	26	23	27	51	4
27	10	26	24	19	33	27	23	29	27	3
28	10	48	2	19	47	7	23	30	35	2

		ΥϚ		□ □		□ □				
	ΥϚ	min	ΥϚ	□ □	min	□ □	□ □	min	□ □	
gr.	deg.	.	sec.	deg.	.	sec.	deg.	.	sec.	gr.
29	11	9	27	20	0	26	23	31	17	1
30	11	30	42	20	13	22	23	31	30	0
gr.	□ □			□ □			□ □			gr.

Parsing notes: The three blocks are read top-to-bottom for the upper sign-pair (ΥϚ/ →) using the left gr. column 0□ 30□ and bottom-to-top for the lower sign-pair (□□ / □□ / □□) using the right gr. column. Several seconds-cells were typographically degraded in the scan; readings reconstructed from the smooth monotone progression of each column and flagged where genuinely doubtful. The row of sign-glyphs beneath the table is the foot sign-heading row (already given above); below the table frame there is only the catchword, no data-bearing glyph row.

PROBLEMA II.

Given the obliquity of the Ecliptic from the equator, and a longitude granted upon it: the right ascension corresponding to such a longitude is found.

From the things we defined in the preceding book, it is sufficiently clear that, in the preceding diagram, B L was the right ascension [ascensio recta] corresponding to the longitude B N on the Ecliptic. Therefore, for the same arc B L, in the right triangle B L N, the data [Greek: διδόμενα (didomena)] are as before:

Data [Greek: διδόμενα (didomena)]	side B N	60° 0' [recte: 60°; printed "66 min. o"]
	angle N B L	23° 32'
	angle B L N	90° right

Therefore B L is found: **57° 48'**, the sought right ascension.

For when the proportion is set up among the complements that appear in the triangle N F G, there results — from the analysis [Greek: ἀνάλυσις (analysis)] of the said triangle — the angle N F G of 32° 12', whose measure lies along the arc H L, namely the complement of B L.

This right ascension of one quadrant, likewise acquired by the doctrine of triangles (and which, in its remaining degrees, nowhere varies except by the longitude from the one premised), suffices for extension to the whole remaining circle; and how this is done we shall here indicate briefly. First, the right ascension just sought being subtracted from a semicircle, there remains the right ascension to the beginning of Leo ♌, 122° 12'. But when this same right ascension is added to a semicircle, the right ascension of the beginning of Sagittarius is made up: 237° 48'. So too for the beginning of Capricorn ♐, when to a semicircle is added the right ascension set down for the beginning of Leo ♌, they accumulate thus: 302° 12'. By such a device the following table of right ascensions is composed.

Moreover, through the columnar [arealem] entry of this table is found the point of mid-heaven [M.C., punctum medii caeli] of any star whose right ascension has been given beforehand; whence you will find the transit of 100 stars across mid-heaven, computed below. The remaining use of the table is impressed piecemeal in what follows.

LIBER SECVNDVS [Book Two of the Spherics] — Caput I

Table of Right Ascensions [Tabula Ascensionum Rectarum] (first half)

This table gives the right ascension (in degrees and minutes) corresponding to each degree of ecliptic longitude, computed to Tycho's maximum obliquity. The leftmost column ("grad.") is the degree within each sign; the six paired columns cover the first six zodiacal signs: Aries (♈), Taurus (♉), Gemini (♊), Cancer (♋), Leo (♌), and Virgo (♍). Each sign's pair gives degrees ("gr.") and minutes ("mi./min.").

grad.	gr.	mi.	gr.	mi.	gr.	mi.	gr.	mi	gr.	mi	gr.	mi
0	0	0	27	54	57	48	90	0	12	12	15	6
									2		2	
1	0	55	28	51	58	51	91	5	12	14	15	4
									3		3	
2	1	50	29	49	59	53	92	11	12	16	15	1
									4		4	
3	2	45	30	46	60	56	93	16	12	19	15	58
									5		4	

gra d.	° gr.	′ mi.	″ gr.	″ mi.	″ gr.	″ mi.	″ gr.	″ mi n.	″ gr.	″ mi n.	″ gr.	″ mi n.
4	3	40	31	44	61	59	94	22	12 6	20	15 5	54
5	4	35	32	42	63	3	95	27	12 7	22	15 6	51
6	5	30	33	40	64	6	96	32	12 8	24	15 7	48
7	6	25	34	38	65	9	97	38	12 9	25	15 8	44
8	7	21	35	37	66	13	98	43	13 0	26	15 9	40
9	8	16	36	36	67	17	99	48	13 1	27	16 0	37
10	9	11	37	34	68	21	10 0	53	13 2	28	16 1	33
11	10	6	38	33	69	25	10 1	58	13 3	28	16 2	29
12	11	2	39	33	70	29	10 3	3	13 4	29	16 3	25
13	11	57	40	32	71	34	10 4	8	13 5	29	16 4	20
14	12	53	41	31	72	38	10 5	13	13 6	29	16 5	16
15	13	48	42	31	73	43	10 6	17	13 7	29	16 6	12
16	14	44	43	31	74	47	10 7	22	13 8	29	16 7	7
17	15	40	44	31	75	52	10 8	26	13 9	28	16 8	3
18	16	35	45	31	76	57	10 9	31	14 0	27	16 8	58
19	17	31	46	32	78	2	11 0	35	14 1	27	16 9	54

gra	♏	♏	♏	♏	♏	♏	♏	♏	♏	♏	♏	♏
d.	gr.	mi.	gr.	mi.	gr.	mi.	gr.	mi	gr.	mi	gr.	mi
20	18	27	47	32	79	7	11	39	14	26	17	49
							1		2		0	
21	19	23	48	33	80	12	11	43	14	24	17	44
							2		3		1	
22	20	20	49	34	81	17	11	47	14	23	17	39
							3		4		2	
23	21	16	50	35	82	22	11	51	14	22	17	35
							4		5		3	
24	22	12	51	36	83	28	11	54	14	20	17	30
							5		6		4	
25	23	9	52	38	84	33	11	57	14	18	17	25
							6		7		5	
26	24	6	53	40	85	38	11	1	14	16	17	20
							8		8		6	
27	25	2	54	41	86	44	11	4	14	14	17	15
							9		9		7	
28	25	59	55	44	87	49	12	7	15	11	17	10
							0		0		8	
29	26	57	56	46	88	55	12	9	15	9	17	5
							1		1		9	
30	27	54	57	48	90	1	12	12	15	6	18	0
							2		2		0	

Parsing notes: The Cancer (♋) column row 30 reads "90 1" in the source; the corresponding Cancer row 0 reads "90 0". This is a minor compositor inconsistency (a sign-boundary value should be 90°0'; the "90 1" at the foot is the overlap-row carry from the Gemini→Cancer transition). Transcribed as printed. The table continues on the next page (signs Libra through Pisces).

Table of Right Ascensions [Tabula ascensionum rectorum] (second half)

Continuation. The six paired columns here cover Libra (♏), Scorpio (♏), Sagittarius (♏), Capricorn (♏), Aquarius (♏), and Pisces (♏).

gra d.	□ gr.	□ mi.	□ gr.	□ mi.	□ gr.	□ mi.	□ gr.	□ mi.	□ gr.	□ mi.	□ gr.	□ mi.
0	18 0	0	20 7	54	23 7	48	27 0	0	30 2	12	33 2	6
1	18 0	55	20 8	51	23 8	51	27 1	5	30 3	14	33 3	4
2	18 1	50	20 9	49	23 9	53	27 2	11	30 4	16	33 4	1
3	18 2	45	21 0	46	24 0	56	27 3	16	30 5	19	33 4	58
4	18 3	40	21 1	44	24 1	59	27 4	22	30 6	20	33 5	55
5	18 4	35	21 2	42	24 3	3	27 5	27	30 7	22	33 6	51
6	18 5	30	21 3	40	24 4	6	27 6	32	30 8	24	33 7	48
7	18 6	25	21 4	38	24 5	9	27 7	38	30 9	25	33 8	44
8	18 7	21	21 5	37	24 6	13	27 8	43	31 0	26	33 9	40
9	18 8	16	21 6	36	24 7	17	27 9	48	31 1	27	34 0	37
10	18 9	11	21 7	34	24 8	21	28 0	53	31 2	28	34 1	33
11	19 0	6	21 8	33	24 9	25	28 1	58	31 3	28	34 2	29
12	19 1	2	21 9	33	25 0	29	28 3	3	31 4	29	34 3	25
13	19 1	57	22 0	32	25 1	34	28 4	8	31 5	29	34 4	20
14	19 2	53	22 1	31	25 2	38	28 5	13	31 6	29	34 5	16
15	19 3	48	22 2	31	25 3	43	28 6	17	31 7	29	34 6	12
16	19	44	22	31	25	47	28	22	31	29	34	7

gra	□	□	□	□	□	□	□	□	□	□	□	□
d.	gr.	mi.	gr.	mi.	gr.	mi.	gr.	mi.	gr.	mi.	gr.	mi.
	4		3		4		7		8		7	
17	19	40	22	31	25	52	28	26	31	28	34	3
	5		4		5		8		9		8	
18	19	35	22	31	25	57	28	31	32	27	34	58
	6		5		6		9		0		8	
19	19	31	22	32	25	2	29	35	32	27	34	54
	7		6		8		0		1		9	
20	19	27	22	32	25	7	29	39	32	26	35	49
	8		7		9		1		2		0	
21	19	23	22	33	26	12	29	43	32	24	35	44
	9		8		0		2		3		1	
22	20	20	22	34	26	17	29	47	32	23	35	39
	0		9		1		3		4		2	
23	20	16	23	35	26	12	29	51	32	22	35	35
	1		0		2		4		5		3	
24	20	12	23	36	26	28	29	54	32	20	35	30
	2		1		3		5		6		4	
25	20	9	23	38	26	33	29	57	32	18	35	25
	3		2		4		6		7		5	
26	20	6	23	40	26	38	29	1	32	16	35	20
	4		3		5		8		8		6	
27	20	2	23	41	26	44	29	4	32	14	35	15
	5		4		6		9		9		7	
28	20	59	23	43	26	49	30	7	33	11	35	10
	5		5		7		0		0		8	
29	20	57	23	46	26	55	30	9	33	9	35	5
	6		6		8		1		1		9	
30	20	54	23	48	27	0	30	12	33	6	36	0
	7		7		0		2		2		0	

Parsing notes: The Sagittarius (♐) row 22 reads "261 17" in the source; row 23 reads "262 12." The progression 260 12 (row 21) → 261 17 (row 22) → 262 12 (row 23) → 263 28 (row 24) is slightly irregular at row 23 ("262 12" vs. an expected ~262 22),

see Numerical impossibles. Otherwise the table is clean and the columns close exactly on the round sign-boundary values.

PROBLEMA III.

Given the maximum obliquity of the Ecliptic together with the Sun's declination, the Sun's place on the Ecliptic corresponding to that declination is found.

This problem, being the inverse of the first, is of great use in investigating the Sun's motion when its declination has been observed by means of the meridian altitude — provided one takes care to eliminate parallax and refraction.

Example [Exemplum].

Take, in the diagram given above (which is retained here as well), the Sun's declination as observed from the meridian altitude — how this is done will be shown clearly in the following class [classis] — or otherwise given, as $20^{\circ}13'48''$, which is marked by the arc L N. Together with the obliquity of the Ecliptic, in the usual notation, the data [διδόμενα (didomena, "the given quantities")] appear thus:

- L N = $20^{\circ}13'48''$ — the given declination.
- N B L = $23^{\circ}32'$ — the obliquity of the ecliptic.
- B L N = $90^{\circ}0'$ — the right angle.

Therefore B N = $60^{\circ}0'$ is given — the Sun's longitude from the vernal equinox.

This yields the same distance from the vernal equinox as was previously assumed at the beginning of Gemini (♊). In the remaining quadrants the procedure is the same; one need only consider whether the Sun's declination was northern or southern, and in which quadrant of the circle the Sun stood. The table of the Sun's declination can likewise serve the same purpose — which is why we have retained it, computed to seconds of arc, from Tycho's [Tycho Brahe's] reckoning.

PROBLEMA IV.

Given the maximum obliquity of the Ecliptic together with a longitude, the angle is found which the meridian — or any circle of declination — makes with the Ecliptic.

In the preceding diagram this angle is represented by B N L. Finding it serves an outstanding purpose in what follows, when we come to investigate the rising of a point of the ecliptic, its inclination to the horizon, and finally the rising and

setting amplitude [amplitudo ortiva et occidua]. In the present example, for finding the said angle, the data appear thus:

- L B N = 23°32' — the obliquity of the ecliptic.
- B N = 60°0' — the given longitude on the Ecliptic.
- B L N = 90°0' — the right angle.

Therefore B N L = 77°42' is given — the angle of intersection sought.

After this example the rest is carried through to the compilation of the following table, which is invariable in each of the quadrants.

Table of the Angle of the Ecliptic with the Meridian or Circle of Declination [Tabula Anguli Eclipticae cum Meridiano seu Circulo Declinationis]

The table presupposes a maximum declination [obliquity] of 23°32'. It gives, for each degree of ecliptic longitude within a sign, the angle the ecliptic makes with the meridian (or any declination circle). The leftmost column ("gr. Ec.", degree of the ecliptic) reads top-to-bottom 0–30 for the signs heading each column-pair from the top (♈♎Aries-Libra, ♉♏Taurus-Scorpio, ♊♏Gemini-Sagittarius); the rightmost column ("grad.") reads bottom-to-top 0–30 for the signs labeled at the foot (♐♏Pisces-Virgo, ♏♏Aquarius-Leo, ♏♏Capricorn-Cancer). The foot labels are: under col. 1 "♏ ♏ G.", under col. 2 "♏ ♏", under col. 3 "♏ ♏ G."

gr. Ec. (↓)	♈♎ gr.	♉♏ mi.	♊♏ gr. mi.	♋♏ gr.	♌♏ mi.	♍♏ gr.	grad. (↑)
0	66	28	69	20	77	42	30
1	66	28	69	31	78	4	29
2	66	29	69	43	78	26	28
3	66	30	69	55	78	48	27
4	66	31	70	8	79	11	26
5	66	33	70	21	79	34	25
6	66	35	70	35	79	57	24
7	66	38	70	49	80	20	23
8	66	41	71	3	80	43	22
9	66	44	71	18	81	7	21
10	66	47	71	33	81	31	20

gr. Ec. (↓)	Υ _Δ gr.	Υ _Δ mi.	□ □ gr.	□ □ mi.	□ □ gr.	□ □ mi.	grad. (↑)
11	66	51	71	48	81	55	19
12	66	55	72	4	82	19	18
13	67	0	72	20	82	44	17
14	67	5	72	36	83	9	16
15	67	11	72	53	83	34	15
16	67	17	73	10	83	59	14
17	67	24	73	27	84	24	13
18	67	31	73	45	84	49	12
19	67	38	74	3	85	14	11
20	67	45	74	21	85	39	10
21	67	53	74	40	86	4	9
22	68	1	74	59	86	29	8
23	68	9	75	18	86	59	7
24	68	18	75	38	87	20	6
25	68	28	75	58	87	46	5
26	68	38	76	18	88	12	4
27	68	48	76	39	88	39	3
28	68	58	77	0	89	6	2
29	69	9	77	21	89	33	1
30	69	20	77	42	90	0	0

[Foot labels: col. 1 "□ □ G."; col. 2 "□ □"; col. 3 "□ □ G."]

Parsing notes: This is a back-to-back ("bidirectional") table — the first column-group is entered from the top by degree-within-sign for Aries/Taurus/Gemini (and, by quadrant symmetry, Libra/Scorpio/Sagittarius), and the same rows are read from the bottom for the complementary signs. The third column closes exactly: gr.Ec. 0 → 77°42' (= Problema IV result) and gr.Ec. 30 → 90°0'. Row 23 of col. 3 reads "86 59" in the source where the smooth progression (86 29 → 86 59 → 87 20) suggests an expected ~86°54'; see Numerical impossibles. The "Decl. max. 23°32'" in the heading differs from the obliquity 23°31½' used in the declination tables of the prior batch — but matches the rounded 23°32' used throughout Problemata III–IV here.

Astronomia Danica — Liber Secundus

Sphæricorum

Caput I (continued) — Problema V

PROBLEMA V.

Given the obliquity of the ecliptic together with a longitude, the arc of latitude intercepted between the ecliptic and the equator is found.

Such a latitude is shown in the diagram above by the arc NM. To investigate it, whenever its use is required, the data [διδόμενα (didomena, "the givens")] in the customary worked example are:

	Value	
Data [διδόμενα]	BN — 60° 0′	Longitude on the ecliptic.
	NBM — 23° 32′	Maximum obliquity.
	∠BNM — 90°	A right angle.

Therefore NM is given as 20° 40′ — the arc of latitude sought.

By this same method the arc of latitude can also be obtained for each individual point of the ecliptic, provided one takes into account the equal distance from some equinox or solstice (which likewise underlies equal data [διδόμενα]), as was noted earlier.

CAPUT II.

On the similar determination of those phenomena that wander aside from, and lie at any latitude away from, the ecliptic.

PROBLEMA I.

Given the latitude and longitude of some star, or of a point on the spherical surface, together with the maximum obliquity of the ecliptic, to find both the right ascension and the declination of that same star.

And conversely:

Given the right ascension and declination, to find the longitude and latitude from these.

Since the motions of the stars — nearly invariable in latitude (except for the wandering stars), but regular in longitude and determinate for any given time — must subsequently be considered in their declination and right ascension, and applied to various uses in these matters, this problem has the most frequent practical application. We shall dispatch it most conveniently and most concisely by means of oblique-angled triangles outside the ecliptic.

For the sake of explaining the present problem, let there be in the accompanying diagram a star, or a point of the heavens, whose given longitude is the arc BN and whose latitude is NO. Hence in the oblique triangle OEF the data [διδόμενα] are: EF, the distance between the poles; OE, the complement of the given latitude; and finally the angle FEO, which measures the arc NG of the given longitude, here reckoned from the beginning of Cancer $\square$ into the order preceding the signs. Therefore the side FO becomes known first, yielding the complement of the declination sought. Once this is obtained — since in the same triangle FEO all three sides are now evident — we bring to light the angle EFO, to be measured along the arc DL of the equator from the solstitial colure D. From this angle, in the present case, when the quadrant of the circle DB has been subtracted, there remains the arc BL, which measures the right ascension sought for the star (or point) at O.

Diagram. A sphere drawn in oblique projection, the construction for Problema I (the worked examples in Cap. II). The outer circle is the bounding meridian/horizon. Several great circles cross it (the ecliptic, the equator, the colures and a declination/latitude circle), defining the oblique triangle used in the spherical-triangle solution.

- F (upper area): pole region / vertex F (distance of poles EF)
- E (upper left, near F): vertex E
- G (upper right): point G (longitude arc NG terminus)
- O (center-upper, near N): the star or point whose coordinates are sought
- N (center): point N on the ecliptic (the star's longitude foot)
- H (right): point H on the bounding circle
- D (left): solstitial colure point D / equator reference
- A (center): center of the sphere
- L (center-low): point L on the equator (right-ascension foot)
- M (center, near L): point M
- B (center-low): point B (origin of measurement on the equator, foot of quadrant DB)

- C (lower left): point C on the bounding circle
- I (bottom): point I (lower pole / colure terminus)

Caput II, Problema I — Worked Examples

Exemplum I.

Let O in the immediately preceding diagram be a bright star above the head of Aries $\square$ [i.e., in the constellation Aries], whose longitude for the completed year 1630 — the epoch to which we refer in all examples of this kind, for the reason set out above — is given as $2^\circ 32'$ $\square$ [of Taurus], and its latitude as $9^\circ 57'$ north. With these presupposed, we investigated the said star's declination and right ascension, according to the preceding exposition and to our doctrines of triangles, in the triangle OEF, as follows.

I. For the Declination.

	Value	
Data [διδόμενα]	EF — $23^\circ 32'$	distance between the poles.
	OE — $80^\circ 3'$	complement of the star's latitude.
	$\angle$ FEO — $57^\circ 28'$	complement of the star's longitude from the vernal equinox.

Therefore FO is given as $68^\circ 17'$, whose complement OL — namely $21^\circ 43'$ — shows the very declination of the bright star of Aries $\square$ that was sought.

II. For the Ascension.

In the same triangle OEF, the following hold.

	Value	
Data [διδόμενα]	EF — $23^\circ 32'$	distance between the poles.
	FO — $68^\circ 17'$	complement of the declination just found.
	EO — $80^\circ 3'$	complement of the star's latitude.

Therefore the angle EFO is given as $116^{\circ} 39'$, the angle at the equator measured from the winter solstice, which is represented by the arc DL. So when the quadrant has been subtracted from the said arc, the remainder — namely $26^{\circ} 39'$ — gives the right ascension sought for the bright star of Aries $\square$.

Exemplum II.

Even one moderately versed in the spherical art can see from this single example set out above how the present problem may be carried through the remaining quadrants of the circle. (It was in this very manner that a table was once constructed at Uraniborg, with great labor, by Brahe's pupils [Tycho Brahe], but to an equal loss for astronomy it has not yet seen the light — a table from which we might at once derive either longitude and latitude from given right ascension and declination of the stars, or, conversely, right ascension and declination from given longitude and latitude.) Nevertheless, lest we seem to fail the learner's effort in the differing case where the latitude is north in the southern signs, here is another example, in the bright star, or eastern shoulder, of Aquarius $\square$. For the assumed year 1630 its longitude is given as $28^{\circ} 15'$ $\square$ [Aquarius], and its latitude as $10^{\circ} 42'$ north; to which a diagram from our sphere fits as follows.

The rest remaining as before, let the place of the bright shoulder be at O, whose latitude NO is given, and whose longitude is at N on the ecliptic. From these is sought first the declination LO, then the point L on the equator that determines the right ascension of the same star.

Diagram. A sphere in oblique projection, the construction for Exemplum II of Problema I, with the southern-latitude configuration. The bounding circle carries great circles (ecliptic, equator, colures, and a latitude circle through O), forming the oblique triangle OIK used in the solution.

- F (top): vertex F
- E (upper, near F): vertex E
- G (right): point G
- H (right): point H on the bounding circle
- D (left): solstitial colure / equator reference D
- A (center): center of the sphere
- L (center, near O): point L on the equator (right-ascension foot)
- O (center-low): the star (eastern shoulder of Aquarius)
- N (center-low, near O): point N on the ecliptic
- B (center): point B

- C (lower right): point C on the bounding circle
- I (lower right): point I
- K (bottom): point K (southern pole / colure terminus)

Exemplum II (continued)

I. For the Declination.

	Value	
In triangle OIK, data [διδόμενα]	IK — 23° 32'	southern distance between the poles.
	IO — 100° 42'	aggregate of latitude and a quadrant.
	∠OIK — 58° 15'	the star's distance from the beginning of Capricorn ♐, which the arc CN on the ecliptic measures.

Therefore KO, the complement of the declination, is given as 87° 55'. And accordingly the declination LO sought is 2° 5' south.

II. For the Right Ascension.

In the same proposed triangle OIK, for the angle OKI the three sides are as follows.

	Value	
	KI — 23° 32'	distance between the poles.
	KO — 87° 55'	complement of the declination.
	IO — 100° 42'	the star's latitude with a quadrant added.

Therefore the angle OKI is given as 123° 17', which defines the arc HL from the summer solstice on the equator. When this is subtracted from a whole circle plus a quadrant — namely from 450° — there remains the right ascension sought for the right shoulder: 326° 43'.

PROBLEMA II.

To convert a given right ascension and declination into longitude and latitude.

This problem is the converse of the prior one, as is plain in the penultimate diagram recalled here, where in the triangle EFO, for the complement of the latitude EO:

	Value	
the data are [διδόμενα]	EF — $23^{\circ} 32'$	distance between the poles.
	FO — $68^{\circ} 17'$	complement of the declination of the bright star of Aries α . [recte: $68^{\circ} 17'$; printed "68 mi. 27"]
	$\angle$ EFO — $116^{\circ} 39'$	distance of the bright star of Aries α on the equator from the winter solstice.

Therefore EO is given as $80^{\circ} 3'$, the complement of the latitude sought. And so ON, the latitude itself sought, is given as $9^{\circ} 57'$.

Next, for the longitude of the said star at N on the ecliptic — since in the proposed triangle FEO all the sides are known, namely:

	Value	
Data [διδόμενα]	EF — $23^{\circ} 32'$	distance between the poles.
	EO — $80^{\circ} 3'$	complement of the star's latitude.
	FO — $68^{\circ} 17'$	complement of the declination just found.

Therefore the angle FEO is given as $57^{\circ} 28'$, the angle of elongation on the ecliptic of the said star from the summer solstice, which the arc GN measures. And when

this arc, in the present case, is subtracted from a quadrant of the circle, there remains the longitude of the star sought: $2^{\circ} 32'$ [Taurus].

In a similar way the reciprocal computation occurs in the remaining cases. The examples of these two problems are of all the most frequent.

PROBLEMA III.

Given a longitude together with the declination of a star, to hunt out its right ascension and latitude.

Retaining the same diagram (namely the penultimate one), since in the triangle EFO:

	Value	
the data are [διδόμενα]	EF — $23^{\circ} 32'$	distance between the poles.
	FO — $68^{\circ} 17'$	complement of the declination.
	$\angle$ FEO — $57^{\circ} 28'$	distance on the ecliptic from the tropic of Cancer □.

Therefore EO is given as $80^{\circ} 3'$, the complement of the latitude; and consequently the latitude itself is $9^{\circ} 57'$.

Next, for the right ascension — since now in the same triangle FEO all three sides are granted:

	Value	
For [the sides are]	EF — $23^{\circ} 32'$	distance between the poles.
	FO — $68^{\circ} 17'$	complement of the declination.
	EO — $80^{\circ} 3'$	complement of the latitude.

Therefore the angle EFO is given as $116^{\circ} 39'$, the distance of the equator from the winter solstice. When the quadrant DB has been subtracted from this, there remains BL, the right ascension finally sought: $26^{\circ} 39'$, as before.

PROBLEMA IV.

Given the latitude and declination, to obtain the longitude and right ascension.

This problem renders its chief service in the neighborhood of the equinoctial points, where — the star's declination having been given by observation, and its latitude held invariably the same — we desire to obtain the longitude or right ascension from the data. In the triangle so proposed, the three sides are granted: two from the complements of the presupposed latitude and declination, and the third from the distance between the poles, which everywhere stands available.

But so that the analysis [ἀνάλυσις (analysis)] of spherical triangles which we set out earlier may be made more illustrious in this problem, and more welcome by comparison, let us borrow an example from *De Revolutionibus* III.2 of Copernicus (*Copernicus, De Revolutionibus* III.2).

[Margin: *Lib. 3, Revolut., cap. 2.*]

In Spica Virginis — whose declination Copernicus found at Frauenburg [Frauenburg / Frombork] in Prussia in the year of our Lord 1525 to be 8° 40' southern, while its latitude he presupposed from the Hipparchian Catalogue of Stars [al. Hipparchus] to be 2° 0' southern, and the maximum declination of the Sun (that is, the distance between the poles) to be 23° 28' — to these data the following diagram has been laid out in conformity, according to the arrangement of the circles in our sphere. For, the face of the sphere being merely turned about, everything else agrees with the indication of the earlier letters; and since in the triangle KIO:

[Margin: *De Revolutionibus* III.2.]

Diagram. A sphere in oblique projection, the construction for Problema IV using Copernicus's Spica Virginis data; the sphere's face is turned about relative to the earlier figures. Great circles cross the bounding circle forming the oblique triangle KIO.

- F (top): vertex F
- E (upper right, near F): vertex E
- G (upper left): point G
- H (left): point H on the bounding circle
- D (right): point D / equator reference
- A (center): center of the sphere

- L (center): point L on the equator
- B (center): point B
- O (center-low): the star (Spica Virginis)
- N (center-low, near O): point N on the ecliptic
- C (lower right): point C on the bounding circle
- I (lower left): point I
- K (bottom): point K (southern pole / colure terminus)

	Value	
Data [διδόμενα]	KI — 23° 28'	distance between the poles.
	IO — 88° 0'	complement of Spica's latitude.
	KO — 81° 20'	complement of Spica's declination.

Therefore the angle OIK is given as 72° 39', which NC measures — the distance of the star from the winter tropic on the ecliptic. When this is subtracted from a quadrant of the circle, BN is given as 17° 21', the longitude of the star from the autumnal equinox at B; which is accordingly 17° 21' [♎ Libra], just as Copernicus found it.

By a similar device we hunt out the right ascension of the said star. For since the three presupposed sides of the triangle KIO are given, the angle OKI will not lie hidden — the angle that points out the right ascension on the equator HL, from the summer solstice H. When a quadrant is added to it, the right ascension sought is produced.

PROBLEMA V.

Finally, given the longitude and the right ascension, to find out the latitude and declination.

In setting out this problem, recall the third-from-last diagram, together with its worked example for the bright star of Aries [γ Arietis], whose longitude was given as 2°32' ♈ [Taurus] and whose right ascension was 26°39'. Thus, in triangle E F O, since two angles are given (at E and at F) along with the interjacent side E F, both side E O — the complement of the latitude — and side F O — the complement of the declination of the star sought — become known; and these were the quantities proposed. Everywhere the prescribed analysis [ἀνάλυσις, analysis] and

the practice of spherical triangles dispatch the matter, even where it proves necessary to drop a perpendicular arc.

Diagram. A celestial sphere shown as a large circle with several great-circle arcs crossing through its interior, forming the spherical triangle used in Problems V and VI. The labeled points, reading roughly clockwise and across the interior, are listed below.

- E (upper left of interior): vertex of the triangle
- F (top): vertex; an arc descends from F through the interior
- G (upper right, on the circle): endpoint of an arc
- O (interior, upper middle): intersection point near the triangle's apex
- N (interior, just right of O): intersection point
- D (left, on the circle): pole/endpoint on the horizon-or-meridian circle
- A (interior center): central point
- L (interior, lower middle): intersection point
- M (interior, just right of L): intersection point
- H (right, on the circle): endpoint
- B (interior, below center): point on a crossing arc
- C (lower left, on the circle): endpoint
- K (bottom, on the circle): pole at the foot of the descending arc
- I (lower right, on the circle): endpoint

PROBLEMA VI.

Given the latitude and declination, to determine the angle subtended by the distance between the poles — namely, the poles of the ecliptic and of the equator.

This problem resembles the fourth. Here too three sides are given: the two that enclose the sought angle, obtained from the complements of the latitude and of the declination; while the third, which measures the arc of the interval between the poles, lies opposite the sought angle — as can be shown in the diagram attached to the fourth problem. Let the data $[\delta\iota\delta\acute{o}\mu\epsilon\nu\alpha, \text{data}]$ be IO , KO , and IK , from which the angle at O is given.

This problem has a special use in investigating the rising and setting of the stars, a matter treated below in its proper place.

PROBLEMA VII.

Given two stars — together with either the difference of their latitudes and longitudes, or the difference of their declinations and right ascensions — to find their distance from one another.

This noble problem is prized for its very wide use, not only on the celestial sphere but on the terrestrial globe as well. For by the same procedure and solution of a triangle, the intervals between stars in the heavens correspond to the distances between places on the earth. But we shall give the terrestrial examples in their proper place; for now we shall look up to the celestial stars.

It must be noted, however, that for the differences of longitudes or of right ascensions, one must know either the longitudes of both stars together, or the right ascensions of both. Once these have been subtracted one from the other — as the succession of the signs in their proper order requires — the said difference, which must be presupposed, is left over, as the following example will teach.

Exemplum. [Example]

Let it be required to find the distance between the bright star of Aries [γ Arietis] and Aldebaran, the bright eye of Taurus [α Tauri]. Then, according to the present problem, the data are these:

[Margin: For the year 1600.]

	Longitude	Latitude
Bright star of Aries $\square$	$2^{\circ}6' \square$	$9^{\circ}57' \text{ N.}$
Aldebaran	$4^{\circ}12' \square$	$5^{\circ}31' \text{ S.}$
Diff. of longitude	$32^{\circ}6'$	

These things being thus granted, let it now be, in the adjoining diagram, that A is the northern pole of the ecliptic and D the southern pole of the same. Let the arcs A B C D and A E F D [recte: A F E D] be half-circles of latitude drawn from our sphere. And since the portion of the ecliptic that represents the difference of the longitudes of the presupposed stars is understood by the arc E B, then — placing the position of the bright star of Aries at F, and that of Aldebaran at C — F C will be the arc of the interval that is sought. Therefore, in triangle F A C, since

[Margin: For the year 1600.]

Data [$\delta\iota\delta\acute{o}\mu\epsilon\nu\alpha$] are:

- $FA = 80^{\circ}3'$, complement of the latitude of the bright star of Aries
- $AC = 95^{\circ}31'$, the excess of Aldebaran's latitude beyond a quadrant
- $FAC = 32^{\circ}6'$, the difference of longitude

Therefore $FC = 35^{\circ}32'$ is given — the sought distance of the stars.

The same result follows from the assumed declinations of the stars, together with the difference of their right ascensions. For in the present diagram, if the arctic pole of the equator is set at A and the antarctic at D, then AHD and AGD will be the half-portions of the circle of declination of our sphere on which the said stars stand, with respect to the poles of the world: the bright star of Aries at G, and Aldebaran at H. And since the data are

	Right ascension	Declination
Bright star of Aries □	$26^{\circ}13'$	$21^{\circ}33'$
Aldebaran	$63^{\circ}16'$	$15^{\circ}38'$
Diff. of ascension	$37^{\circ}3'$	

From these data, then, since in triangle GAH

Data [διδόμενα] are:

- $GA = 68^{\circ}27'$, complement of the declination of the bright star of Aries
- $HA = 74^{\circ}22'$, complement of the declination of Aldebaran
- $GAH = 37^{\circ}3'$, the difference of the right ascensions

Therefore $GH = 35^{\circ}32'$ is given — the sought interval of the stars, as before.

Diagram. A pointed-oval (vesica) figure formed by two great-circle arcs meeting at the top point A and the bottom point D, the poles. Two further arcs cross the interior horizontally, marking the latitude/declination circles on which the two stars lie.

- A (top vertex): northern pole (of the ecliptic, or arctic pole of the equator)
- G (left side, upper): position of the bright star of Aries on the declination circle
- H (right side, upper): position of Aldebaran on the declination circle
- F (left side, middle): position of the bright star of Aries on the latitude circle
- B (right side, middle): point on the ecliptic arc
- E (left side, lower-middle): point on the ecliptic arc (endpoint of arc EB)
- C (right side, lower-middle): position of Aldebaran on the latitude circle

- D (bottom vertex): southern pole

CAPVT III.

On the rising and setting amplitude of an assumed point of the ecliptic under a given elevation of the pole; likewise, on the ascensional and descensional difference.

Having dispatched those matters that concern the motions bearing on the equator and the ecliptic in any position of the sphere, we shall now treat — in this chapter — those that determine the same motions of the stars relative to the horizon, the meridian, and the ecliptic, together with the preliminaries set out here that will serve, in what follows, for the rising and setting of the stars.

PROBLEMA I.

Given (besides the elevation of the pole of the place) the declination of any point of the ecliptic, or of any other rising or setting point, to determine the rising or setting amplitude of that point on the horizon.

Although this problem can be dispatched in various ways, we nevertheless follow the easiest method of all, drawing solely on this principle (already tasted in the preceding book): that the equator, in every position of the sphere, ascends and descends on both sides of the horizon at a remove of 90° from the meridian. Therefore, if the declination is northern, the portion of the rising or setting amplitude intercepted between the equator and the rising or setting point must be added to a quadrant. But if the declination is southern, it must be subtracted from the same [quadrant],

so that the rising amplitude, as defined in the prior book, is completed. Moreover, the said portion, for every equal declination given, comes out equal. Let it suffice to have given this warning once.

Exemplum I. in declinatione Borea. [Example I, in a northern declination]

Let there be sought the rising or setting amplitude of a point having declination $11^\circ 31'$ N. — such as is the Sun's at the start of Taurus $\square$ — together with the elevation of the pole of this place, $55^\circ 43'$. Therefore, in the adjoining diagram, let the whole meridian be C E H I, as it is represented on our sphere; let the horizon be C B G, whose pole is E, and the equator D B H, whose pole is F. And since the arc C M shows the rising or setting amplitude on the horizon, while C B is a

quadrant of a circle, the portion B M is sought in the right triangle B M L. For its investigation:

Data [διδόμενα] are:

- M B L = $34^{\circ}17'$, complement of the elevation of the pole
- M L = $11^{\circ}31'$, the first given declination
- B L M = $90^{\circ}0'$, right angle

Therefore B M = $20^{\circ}46'$ is given. To this is added the quadrant C B = $90^{\circ}0'$. The amplitude C M is made up = $110^{\circ}46'$.

Exemplum II. in data declinatione Meridionali. [Example II, in a given southern declination]

Let it be Sirius, the Greater Dog [α Canis Majoris], whose rising and setting amplitude in our horizon we intend to investigate. Therefore, its declination being given as $16^{\circ}12'$ — represented in the preceding diagram by the arc N O, with Sirius holding the point O on the horizon — in the right triangle, for the portion B O we shall have:

Data [διδόμενα] are:

- N O = $16^{\circ}12'$, the declination of Sirius
- N B O = $34^{\circ}17'$, complement of the elevation of the pole
- B N O = $90^{\circ}0'$, right angle

Therefore B O = $29^{\circ}42'$ is given, the sought portion of amplitude; which, taken away from the quadrant C B = $90^{\circ}0'$, leaves the sought rising or setting amplitude of Sirius as $60^{\circ}18'$.

Diagram. A celestial-sphere diagram (a circle with several great-circle arcs crossing the interior) showing the meridian, horizon, and equator circles together with the rising/setting amplitude construction used in both examples of Problem I.

- E (top): pole of the horizon
- F (top right, on the circle): pole of the equator
- D (upper left, on the circle): endpoint of the meridian/equator arc
- N (interior, upper middle): point on the declination arc (Sirius construction, arc N O)
- A (interior center): central point

- M (interior right): point marking the amplitude arc C M
- C (left, on the circle): endpoint (one end of horizon C B G and meridian C E H I)
- O (interior, lower-middle): point where Sirius meets the horizon (arc N O)
- B (interior, below center): intersection point on the horizon/equator
- L (interior, lower right): point of the right triangle B M L
- G (right, on the circle): endpoint of horizon C B G
- H (right, on the circle): endpoint of equator D B H and meridian C E H I
- K (lower left, on the circle): endpoint
- I (bottom, on the circle): endpoint of meridian C E H I

PROBLEMA II.

Given likewise the elevation of the pole of the place, to make known the ascensional and descensional difference; likewise the semidiurnal arc, and consequently the length of day and night at each season of the year from the motion of the Sun, when the present problem is fitted to the points of the ecliptic.

As an example of this most noble problem, we shall recall the nearest figure here. In its first triangle B L M, the ascensional or descensional difference is shown by the arc B L — namely, of the point existing at M — which arc, since for its investigation it presupposes:

Data [διδόμενα]:

- L M = $11^{\circ}30'$, the declination of the Sun
- M B L = $34^{\circ}0'$, the complement of the elevation of the pole
- M L B = $90^{\circ}0'$, right angle

Therefore B L = $17^{\circ}33'$ is given — the sought ascensional or descensional difference.

II. Pro Arcu Temporis Semidivni. [For the arc of semidiurnal time]

[Margin: For the remaining elevations, let the ascensional differences be sought from the direct tables of Johannes Regiomontanus [Regiomontanus], whence we too have drawn our numbers.]

Since it is established from our sphere that the semidiurnal arc of the equator, from the meridian to the horizon — namely D B — is always 90° , and converted into time gives 6 hours: thus, the ascensional or descensional difference once

found being likewise converted into time (dividing the degrees by 15, and for each 15' of a degree taking 1' of time), and 6 hours being added if the declination is northern, or subtracted if the declination is southern, there is left the semidiurnal arc of the rising or setting point in hours and minutes of hours. So, in the present example, where the converted ascensional difference of 17°33' into time gives 1h 10m 12s: if 6 hours are added to these, the semidiurnal arc of the ecliptic point at the start of Taurus ☉ is made up, with us, as 7h 10m 12s. This time doubled gives the diurnal quantity 14h 20m 24s; which time, finally, subtracted from 24 hours, gives the quantity of night as 9h 39m — which was suited to the proposed problem.

Now, since from this single example it is gathered how the ascensional or descensional difference, for any given declination and for the several elevations of the pole, is to be computed from the doctrine of triangles, it pleases us to leave here a table for the elevation of the pole of 56° (such as is the case in most places of Denmark) and for declination running continuously up to 34°, the obliquity of the ecliptic being now assumed as 23°30'. For the use of this little table will hereafter bring us singular help in calculating the rising and setting of the fixed stars.

Tabula differentiarum Ascens. ad Elevat. loci gr. 56. [Table of ascensional differences for the place's elevation of 56°]

d e.	g r.	m i.	d e.	g r.	m i.	d e.	g r.	m i.	d e.	g r.	m i.	d e.	g r.	m i.	d e.	g r.	m i.
1	1	2	7	1	2	1	2	1	1	3	4	2	4	4	3	6	5
		9		0	9	3	0		9	0	1	5	3	4	1	2	8
2	2	5	8	1	1	1	2	4	2	3	3	2	4	1	3	6	5
		8		2		4	1	2	0	2	9	6	6	8	2	7	3
3	4	2	9	1	3	1	2	2	2	3	4	2	4	4	3	7	1
		7		3	5	5	3	4	1	4	1	7	9		3	4	9
4	5	5	1	1	9	1	2	9	2	3	4	2	5	1	3	9	0
		7	0	5		6	5		2	6	8	8	2		4	0	
5	7	2	1	1	4	1	2	5	2	3	0	2	5	1			
		7	1	6	5	7	6	7	3	9		9	5	6			
6	8	5	1	1	2	1	2	4	2	4	1	3	5	5			
		8	2	8	2	8	8	8	4	1	8	0	8	2			

Parsing notes: The table is laid out as six paired columns ("de." = declination in degrees, then "gr. mi." = the ascensional difference in degrees and minutes),

reading 1–6, 7–12, 13–18, 19–24, 25–30, 31–34 across the page. Several cells are typographically degraded in the scan; flagged readings noted in QA report. Final two columns have only four rows (declinations 31–34). The row-13 difference "20°1'" and row-28 minutes "1" sit on the edge of legibility.

PROBLEMA III.

Given the declination, the right ascension, and the ascensional difference, the oblique ascension and descension of any point of the ecliptic under a given elevation of the pole is given.

Problem I of Chapter I of this [book] dispatched the declination of a point of the ecliptic; Problem 2 of the same chapter, the right ascension; and finally the problem immediately preceding, the ascensional or descensional difference. For the rest, it must be noted in this place that we accomplish the present problem by duly applying to examples the ascensional difference found in the foregoing problem, in this manner:

The rule for applying the ascensional difference is set out as a bracketed scheme:

If the declination of the given point is northern [Borea]: subtract the ascensional difference from the right ascension, and what remains is the oblique ascension sought; add it to the right ascension, and what remains is the oblique descension sought.

If the declination is southern [Austrina]: add the ascensional difference to the right ascension, and what remains is the oblique ascension sought; subtract it from the right ascension, and what remains is the oblique descension sought.

(Schematically: northern declination → subtract for ascension, add for descension; southern declination → add for ascension, subtract for descension. The result in each case is the oblique [obliqua] ascension or descension.)

Exemplum I. [Example I]

Let there be a point on the ecliptic at the beginning of Taurus [♉], whose oblique ascension and descension are to be found, under an elevation of the pole of 56°. To this point there correspond:

- Right ascension: 27° 54'
- Declination: 11° 30' N — therefore likewise
- Ascensional difference: 17° 33'

Therefore there results an oblique ascension of $10^{\circ} 21'$, and an oblique descension of $45^{\circ} 27'$.

Exemplum II. [Example II]

Let the oblique ascension and descension be sought of the ecliptic point at the beginning of Capricorn ♑ , for which the following data are given:

- Right ascension: $270^{\circ} 0'$
- Declination: $23^{\circ} 30' \text{ S}$
- Ascensional difference: $40^{\circ} 8'$

From these there results an oblique ascension of $310^{\circ} 8'$, and a descension of $229^{\circ} 52'$.

From these premises the following table of oblique ascensions and descensions for each degree of the ecliptic, under an elevation of 56° , was once constructed by Johannes Regiomontanus [Regiomontanus]. We, however, have laid it out separately for each individual sign, so that the table might serve more readily for the purposes that follow — for determining the rising and setting of the stars.

It should be noted, moreover, that its entire structure is the **διδόμενα** [Greek: didomena, "the things given / the data"], since they agree with a quadrant of the circle, once one considers both the points of opposite signs and those equally distant from the solstices or equinoxes. On this matter, although we gave warning above, we nonetheless leave its demonstration here, for the benefit of beginners, by means of examples in the table that follows.

Table of oblique ascensions and descensions for an elevation of the pole of 56° (continued)

[Signs: Aries ♈ , Taurus ♉ , Gemini ♊ , Cancer ♋]

The leftmost column "grad." [degrees] gives the degree within each sign (0–30). Each sign has paired columns "Ascen." (oblique ascension) and "Desc." (oblique descension), each in degrees (gr.) and minutes (mi.).

	♈	♉	♊	♋	♌	♍	♎	♏
	Asce	Desc.	Asce	Desc.	Asce	Desc.	Asce	Desc.
grad.	n.		n.		n.		n.	
0	0 0	0 0	10 21	45 27	24 44	90 52	49 52	130 8
1	0 19	1 30	10 44	46 58	25 21	92 21	50 58	131

grad.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.
								14
2	0 39	3 0	11 7	48 30	25 59	93 49	52 5	132
								16
3	0 58	4 31	11 31	50 1	26 38	95 16	53 13	133
								19
4	1 18	6 1	11 55	51 33	27 18	96 42	54 22	134
								22
5	1 38	7 32	12 19	53 5	27 59	98 7	55 32	135
								22
6	1 57	9 2	12 44	54 36	28 40	99 32	56 43	136
								23
7	2 17	10 33	13 9	56 8	29 22	100	57 55	137
						16		22
8	2 37	12 3	13 34	57 40	30 5	102	59 8	138
						32		18
9	2 57	13 33	14 0	59 12	30 48	103	60 22	139
						46		14
10	3 17	15 5	14 26	60 44	31 37	105	61 37	140 9
						10		
11	3 37	16 35	14 52	62 16	32 17	106	62 53	141 3
						33		
12	3 57	18 6	15 19	63 48	33 3	107	64 9	141
						15		17
13	4 17	19 37	15 46	65 19	33 50	109	65 26	142
						16		48
14	4 37	21 8	16 13	66 50	34 39	110	66 44	143
						37		41
15	4 57	22 39	16 41	68 21	35 29	111	68 3	144
						57		31
16	5 17	24 9	17 10	69 53	36 20	113	69 23	145
						16		21
17	5 38	25 40	17 39	71 24	37 12	114	70 44	146

grad.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.
						34		10
18	5 59	27 11	18 9	72 55	38 4	115 50	72 5	146 57
19	6 20	28 42	18 39	74 26	38 57	117 7	73 27	147 43
20	6 41	30 13	19 9	75 57	39 51	118 23	74 50	148 28
21	7 2	31 44	19 40	77 26	40 46	119 38	76 13	149 13
22	7 23	33 15	20 12	78 16	41 42	120 52	77 37	149 56
23	7 45	34 47	20 44	80 26	42 39	122 5	79 2	150 38
24	8 6	36 18	21 16	81 56	43 38	123 16	80 27	151 20
25	8 26	37 50	21 49	83 27	44 38	124 28	81 53	152 1
26	8 50	39 21	22 22	84 56	45 39	125 37	83 19	152 42
27	9 13	40 52	22 56	86 25	46 41	126 46	84 46	153 22
28	9 35	42 24	23 31	87 54	47 44	127 54	86 13	154 1
29	9 58	43 55	24 7	89 23	48 48	129 2	87 40	154 39
30	10 21	45 27	24 44	90 52	49 52	130 8	89 8	155 16

Parsing notes: At grad. 7, Gemini ascension reads "29 22" and the descension reads "100 16" — the descension figure is hard to read (the minutes may be 16); flagged [?]. At grad. 10, Aries descension minutes read "1" but the engraving is faint; flagged [?]. At grad. 12, Gemini descension "107 15" minutes uncertain; Cancer descension reads "141 17" but breaks the smooth progression (neighbors

141 3 / 142 48) — see Numerical impossibles. At grad. 22, Taurus descension reads "78 16" but breaks the smooth progression (neighbors 77 26 / 80 26) — see Numerical impossibles.

Table of oblique ascensions and descensions for an elevation of the pole of 56° (continued)

[Signs: Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏]

grad.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.
0	89 8	155 16	134 33	169 39	180 0	180 0	225 27	190 21
1	90 37	155 53	136 5	170 1	181 30	180 19	226 58	190 44
2	92 6	156 29	137 36	170 24	183 0	180 39	228 30	191 7
3	93 35	157 4	139 8	170 46	184 31	180 18	230 1	191 31
4	95 4	157 38	140 39	171 9	186 1	180 18	231 33	191 55
5	96 33	158 11	142 10	171 32	187 32	180 38	233 5	192 19
6	98 3	158 45	143 42	171 54	189 2	180 17	234 36	192 44
7	99 33	159 17	145 13	172 15	190 33	182 17	236 8	193 9
8	101 3	159 49	146 45	172 37	192 3	182 37	237 40	193 34
9	102 33	160 21	148 16	172 58	193 34	182 57	239 12	194 0
10	104 3	160 51	149 47	173 19	195 5	183 17	240 44	194 26
11	105 34	161 22	151 18	173 40	196 35	183 37	242 16	194 52
12	107 5	161	152	174 1	198 6	183	243	195

grad.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.
		12	49			57	48	19
13	108	162	154	174	199	184	245	195
	36	22	20	22	37	17	19	46
14	110 7	162	155	174	201 8	184	246	196
		51	51	43		37	50	13
15	111	163	157	175 3	202	184	248	196
	39	19	21		39	57	21	41
16	113	163	158	175	204 9	185	249	197
	10	48	52	24		17	53	10
17	114	164	160	175	205	185	251	197
	41	15	23	42	40	38	24	39
18	116	164	161	176 4	207	185	252	198 9
	12	42	54		11	59	55	
19	117	165 8	163	176	208	186	254	198
	44		25	23	42	20	26	39
20	119	165	164	176	210	186	255	199 9
	16	34	55	43	13	41	57	
21	120	166 0	166	177 4	211	187 2	257	199
	48		26		44		27	40
22	122	166	167	177	213	187	258	200
	20	26	57	23	15	23	57	12
23	123	166	169	177	214	187	260	200
	52	50	27	43	47	41	27	44
24	125	167	170	178 2	216	188 6	261	201
	24	16	58		18		57	16
25	126	167	172	178	217	188	263	201
	55	41	28	22	50	26	27	49
26	128	168 5	173	178	219	188	264	202
	37		59	41	21	50	56	22
27	129	168	175	179 1	220	189	266	202
	59	30	29		52	13	25	16
28	131	168	177 0	179	222	189	267	203

grad.	♈ Asce n.	♈ Desc.	♈ Asce n.	♈ Desc.	♈ Asce n.	♈ Desc.	♈ Asce n.	♈ Desc.
	30	52		20	24	35	64	31
29	133 2	169	178	179	223	189	269	204 7
		16	30	40	55	58	23	
30	134	169	180 0	180 0	225	190	270	204
	33	39			27	21	52	44

Parsing notes: Libra descension column near the top (grad. 3, 4, 6) reads "180 18 / 180 18 / 180 17," which interrupts the otherwise monotone increase (180 0 → 180 19 → 180 39 → 180 38 → 180 17 → 182 17); these minutes are faint and flagged [?] — see Numerical impossibles. At grad. 12, Leo descension reads "161 12" but breaks the monotone increase (neighbors 161 22 / 162 22), likely a compositor transposition of "162 12" — flagged. At grad. 27, Scorpio descension reads "202 16" but breaks progression (neighbors 201 49 / 203 31) — likely should be "202 56"; flagged. At grad. 28, Scorpio ascension reads "267 64" — minutes exceed 60, impossible; flagged.

Table of oblique ascensions and descensions for an elevation of the pole of 56° (continued)

[Signs: Sagittarius ♐, Capricorn ♑, Aquarius ♒, Pisces ♓]

grad.	♐ Asce n.	♐ Desc.	♐ Asce n.	♐ Desc.	♐ Asce n.	♐ Desc.	♐ Asce n.	♐ Desc.
0	270	204	310 8	229	335	269 8	349	314
	52	44		52	16		39	33
1	272	205	311	230	335	270	350 2	316 5
	20	21	12	58	53	37		
2	273	205	312	232 5	336	272 6	350	317
	47	59	16		29		25	36
3	275	206	313	233	337 4	273	350	319 8
	14	38	19	13		35	47	
4	276	207	314	234	337	275 4	351	320
	41	18	21	22	38		10	39
5	278 7	207	315	235	338	276	351	322

grad.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.
		59	22	32	11	33	32	10
6	279	208	316	236	338	278 3	351	343
	33	40	22	43	44		54	42
7	280	209	317	237	339	279	352	325
	58	22	21	55	16	33	15	13
8	282	210 5	318	239 8	339	281 3	352	362
	23		18		48		37	45
9	283	210	319	240	340	282	352	328
	47	48	14	22	20	33	58	16
10	285	211	320 9	241	340	284 3	353	329
	10	37		37	51		19	47
11	286	212	321 3	242	341	285	353	331
	33	17		53	21	34	40	18
12	287	212 3	321	244 9	341	287 5	354 1	332
	55		56		51			49
13	289	213	322	245	342	288	354	334
	16	50	48	26	21	36	22	20
14	290	214	323	246	342	290 7	354	335
	37	39	40	44	50		43	51
15	291	215	324	248 3	343	291	355 3	337
	57	29	31		19	39		21
16	293	216	325	249	343	293	355	338
	16	20	21	23	47	10	23	52
17	294	217	326	250	344	294	355	340
	34	12	10	44	14	41	43	23
18	295	218 4	326	252 5	344	296	356 3	341
	51		57		41	12		54
19	297 7	218	327	253	345 8	297	356	343
		57	43	27		44	23	25
20	298	219	328	254	345	299	356	344
	23	51	28	50	34	16	43	55
21	299	220	329	256	346 0	300	357 3	346

grad.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.	□ Asce n.	□ Desc.
	38	46	12	13		48		26
22	300	221	329	257	346	302	357	347
	52	42	55	37	26	20	23	57
23	302 5	222	330	259 2	346	303	357	349
		39	38		51	52	43	27
24	303	223	331	260	347	305	358 3	350
	17	38	20	27	16	24		58
25	304	224	332 1	261	347	306	358	352
	28	38		53	41	55	22	28
26	305	225	332	263	348 5	308	358	353
	38	39	42	19		37	42	69
27	306	226	333	264	348	309	359 2	355
	47	41	22	46	29	59		29
28	307	227	334 1	266	348	311	359	357 0
	55	44		13	53	30	21	
29	309 2	228	334	267	349	313 2	359	358
		48	39	40	16		41	30
30	310 8	229	335	269 8	349	314	360 0	360 0
		52	16		39	33		

Parsing notes: This page is somewhat fainter than the preceding two. Pisces descension column shows three values that break the otherwise smooth monotone increase and flag as impossible/suspect: grad. 6 "343 42" (neighbors 322 10 / 325 13 — likely should be ~323 42), grad. 8 "362 45" (exceeds 360°, and neighbors 325 13 / 328 16 — likely should be ~326 45), grad. 26 "353 69" (minutes exceed 60 — likely should be ~353 59). See Numerical impossibles. At grad. 18, Pisces descension "342 54" also breaks progression (neighbors 340 23 / 343 25) and is faint; flagged. At grad. 12, Sagittarius descension reads "212 3" but breaks monotone increase (neighbors 212 17 / 213 50, suggesting ~213 3); flagged. At grad. 9, Sagittarius ascension "283 47" faint; flagged.

Astronomia Danica — Liber Secundus

Sphæricorum, Caput IV

CHAPTER IV.

On the Rising and Setting of the Sun and of the Stars, in its Threefold Form — Cosmic, Acronychal, and Heliacal.

PROBLEM I.

Given the declination and right ascension of some star at its rising or setting, together with the elevation of the pole, to find the point of the ecliptic with which that same star rises and sets.

Oblique ascension and oblique descension stand in a reciprocal relation within the table of oblique ascensions. Consequently, once the oblique ascension of a star at its rising and setting is given, the point of the ecliptic with which it ascends and descends is likewise given. This matter has the greatest use in history, in poetry, and in astrology. We shall therefore dispatch the present problem concisely, drawing on what has gone before.

First, then, one must determine which stars rise and set at a given elevation of the pole. This is easily done by comparing the declination with the complement of the pole's elevation — northern complement in our northern region, southern in the southern. For if the complement of the pole's elevation is greater than the star's declination, the star rises and sets in the present horizon; if less, it does not.

Examples, at a pole elevation of $55^{\circ} 43'$.

I.

	degrees	minutes
Declination of the bright star of the Crown [Lucida Coronæ], north	28	0
Complement of the pole's elevation	34	17
Difference, which is the depression of the star below the	6	17

	degrees	minutes
horizon		

[Margin: *Anno 1630* — "in the year 1630"]

II.

	degrees	minutes
Declination of the bright star of the Lyre [Lucida Lyræ, Vega]	38	29
Complement of the pole's elevation	34	17
Difference	4	12

This [second result] proves that this star never rises or sets in our horizon. Rather, at its least altitude — namely at its lowest point, the *Nadir* (as the Arabs call it) — it stands $4^{\circ} 12'$ above the horizon, refraction not yet being included.

For the rest, let us proceed with the example of the rising and setting of the bright star of the Crown on our own horizon. We recall the calculation from what precedes and set it before the eyes in a single diagram.

Diagram. A sphere shown in orthographic projection, illustrating the rising of the bright star of the Crown. In it, **G A B** is the meridian and **G L** the horizon, whose pole is **A**; **F H** is the equator, whose pole is **B**; and finally **E R** is the ecliptic, whose pole is **C**. This pole **C** is continually carried around — in the apparent daily revolution — on a small circle (called the Arctic circle) about the pole of the world **B**, maintaining a uniform distance from it, namely $23^{\circ} 32' [?]$, which is the greatest obliquity.

- A (top): pole of the horizon
- B (upper right): pole of the equator (pole of the world)
- C (right of B): pole of the ecliptic
- E (upper left): on the ecliptic
- F (left): on the equator
- G (left, on horizon line): meridian/horizon intersection
- L (right, on horizon line): horizon
- K (center): point of the ecliptic with which the star ascends

- H (below K): point on the equator (oblique ascension)
- D (center-right): location of the star (bright star of the Crown)
- M, N (lower center): construction points
- P (lower right): on the ecliptic
- R (bottom right): ecliptic terminus

Since, then, in accord with the proper arrangement of the spherical globe, the bright star of the Crown at **D** rises together with the point **K** of the ecliptic, it is desired to find that stated point.

[Margin: *Problema 1, cap. 2.*] [Margin: *Probl. 2, c. 2.*]

In order, therefore, to obtain the ascensional difference **N H** from a fixed point **N** of the equator, we shall work — for the declination of the bright star of the Crown, **D N**, $28^{\circ} 0'$, and for its right ascension, $229^{\circ} 45'$ (this for the completed year 1630, which we here assume, and for which the star's longitude, $7^{\circ} 4'$ Scorpio [□], and its latitude, $44^{\circ} 23'$ north, are given closely enough from the canonical star catalog and from the reduction of longitude).

Further, the ascensional difference **H N** is given as $51^{\circ} 15'$ at the elevation of Copenhagen [Hafnia]. When this is subtracted from the right ascension, there remains the oblique ascension of the bright star of the Crown, $178^{\circ} 30'$ — namely the point **H** on the equator.

[Margin: *Probl. 2, cap. hujus.* — "Problem 2 of this chapter"]

Therefore, by the reciprocal use of the table of oblique ascensions, under the assumed pole elevation, the point corresponding to this oblique ascension is given as 29° Virgo [□] very nearly. With this point one knows the true (that is, morning) rising of the Crown together with the Sun — or the evening rising, when the Sun sets at the opposite point — according to what we set forth in Chapter 5 of the earlier book.

[Margin: *Probl. præcedens.* — "the preceding problem"]

Alternatively, by our own method of discovery, which proceeds without a given right ascension.

I. Keeping this same diagram, from triangle **B D C**, with all sides given through what precedes, the angle **B D C** is given.

For the *data* [Greek: δεδομένα (dedomena, "the givens")] of the said triangle are:

	degrees	minutes	
D C	45	37	complement of the Crown's latitude
B D	62	0	complement of the declination
B C	23	32	distance of the poles

Therefore the angle **B D C** is given as $21^{\circ} 10'$.

II. In the right triangle **B L D**, for the angle **B D L**, the givens are:

	degrees	minutes	
B L	55	43	elevation of the pole at this place
B D	62	0	complement of the declination of the star of the Crown
B L D			right angle

Therefore the angle **B D L** is given as $69^{\circ} 21'$. Subtracting from it:

	degrees	minutes
B D L	69	21
B D C	21	10
There remains C D L	48	11

and **C D L** is equal to **K D M**.

III. In the right triangle **K M D**, for the side **M K**, the givens are:

	degrees	minutes	
D M	44	23	latitude of the star of the Crown
K D M	48	11	

	degrees	minutes	
K M D	90	0	right angle

Therefore the side **M K** is given as $37^{\circ} 59'$. When this arc is subtracted from the given longitude of the Crown, which we assumed earlier to be $7^{\circ} 4'$ Scorpio [□], the point of the ecliptic is found with which the Crown ascends: $29^{\circ} 3'$ Virgo [□], differing slightly from the result above — and indeed differing for certain definite reasons.

Next, to demonstrate the setting of the stars — that is, the point of the ecliptic with which a star sets — let us take the same bright star of the Crown for our example.

The star of the Crown, then, having been brought back to the western, setting horizon — as happens roughly by inverting the previous diagram — let there be, as before, in the present case **A** the pole of the horizon, **B** the pole of the equator, **C** the pole of the ecliptic. Upon these let there be drawn, first, the horizon **L G**; then the equator **O F**; and third and last, the ecliptic **R E**, all just as they appear in our sphere.

Diagram. The same sphere inverted to show the setting (western) horizon. **A** is the pole of the horizon, **B** the pole of the equator, **C** the pole of the ecliptic; **L G** is the horizon, **O F** the equator, **R E** the ecliptic.

- A (top): pole of the horizon
- B (upper center): pole of the equator
- C (below B): pole of the ecliptic
- E (upper right): on the ecliptic
- F (right): on the equator
- L (left, horizon line): horizon
- G (right, horizon line): horizon
- D (center-left): location of the Crown on the western horizon
- K (center): construction point
- N (center): point on the equator
- O (left): equator terminus
- M (lower center): construction point
- R (lower left): ecliptic terminus
- Γ (right of K): construction point

Since, then, **D** is the location of the Crown on the setting horizon — just as before it was on the rising horizon — the same right ascension **N** and declination **N D** are retained as before, the longitude likewise remaining unchanged at **M**.

Hence, in the right triangle **D K N**, since the ascensional difference **N K** is not altered, all the givens [Greek: δεδομένα (dedomena, "givens")] being kept as before, that difference will be 51° 15'. By the precepts attached to the foregoing problem, this is here added to the star's right ascension at **N**, and thus the point **K** on the equator yields the oblique descension of the bright star of the Crown: 281° 0'.

For the rest, since the tables of Regiomontanus contain only oblique ascensions and the arcs of the ecliptic corresponding to them (and conversely), but not oblique descensions and the like — therefore, so that these same tables may serve our purpose, a semicircle of 180° must always be added to the oblique ascension found. Thus, whenever the sum grows beyond a full circle, the whole circle (360°) is to be cast off from the total, and what remains is the oblique ascension of the sign and of the sought point opposite on the ecliptic, as will appear in this example.

	degrees	
Oblique descension of the Crown	281	obtained
Semicircle	180	to be added
Sum	461	
Measure of the circle	360	to be subtracted
Oblique ascension of the opposite sign	101	

To this, in the table of oblique ascensions under a pole elevation of 55° 43' — with the individual minutes adhering to it suitably corrected — there correspond 7° 44' Leo [☐], whose opposite point on the ecliptic is 7° 44' Aquarius [☐], with which the bright star of the Crown sets.

Now, by the method we set out above, we shall obtain the same point of the Crown at its descent on the ecliptic.

First, then, since in the upper figure — as also in the present one — the angle **M D N** was found to be 21° 10' and the angle **B D L** 69° 21', therefore, by adding these

in the manner opposite to that used above, the angle **C D L** will be $90^\circ 31'$, to which the vertical angle **M D H** is equal.

Further, in the right triangle **M H D**, the givens [Greek: δεδομένα (dedomena, "givens")] are:

	degrees	minutes	
M D H	90	31	the angle just found
M D	44	23	latitude of the Crown
D M H	90	0	right angle

Therefore the side **M H** is given as $90^\circ 44'$.

If this side just found is added to the point **M** on the ecliptic — namely $7^\circ 4'$ Scorpio [□], where the star's longitude was first established — there results $7^\circ 48'$ Aquarius [□].

This sought result exceeds by a mere 4 minutes the point on the ecliptic found earlier with which the bright star of the Crown descends. There is, however, no lack of an explanation for that small discrepancy, so that our method may be the more reliable.

Another example, for the true rising and setting of Sirius — that is, the bright star in the mouth of Canis Major — on our horizon.

In the adjoining diagram, marked with the labels of the one above, let the point of the star's rising on the ecliptic be taken. The longitude of this star for the completed year 1630 is given at **M** as $9^\circ 1'$ Cancer [□]; its latitude **D M** is $39^\circ 30'$ south. From the foregoing one finds the star's declination **D N** to be $16^\circ 12'$, and its right ascension $97^\circ 13'$ at **N**.

Diagram. A sphere showing the rising of Sirius. The labels follow those of the preceding figures: **A** pole of the horizon, **B** pole of the equator, **C** pole of the ecliptic; the horizon, equator, and ecliptic are drawn as before.

- A (bottom): pole of the horizon
- B (lower left): pole of the equator
- C (upper left): pole of the ecliptic
- C (lower left, second label): construction point near pole
- R (upper left): on the ecliptic

- M (top center): location of Sirius on the ecliptic
- N (center): point on the equator
- D (center-left): construction point
- K (center): construction point
- H (right of center): construction point
- E (right): on the ecliptic
- F (lower right): on the equator
- G (right, horizon line): horizon
- L (left, horizon line): horizon

Therefore, in the right triangle **D N K**, for the ascensional difference **N K**, the givens [Greek: δεδομένα (dedomena, "givens")] are: the side **D N**, $16^{\circ} 12'$ (the star's declination); the angle **N K D**, $34^{\circ} 17'$ (the complement of the pole's elevation); and the angle **D N K**, a right angle. Therefore **K N** is $25^{\circ} 13'$.

When this ascensional difference is added to the given right ascension, the star's oblique ascension is gathered as $122^{\circ} 26'$. To this there corresponds, in the table of Regiomontanus — the calculation being directed to our pole elevation — the point of the ecliptic $21^{\circ} 53'$ Leo [☉], with which Sirius rises above our horizon when the Sun reaches that point. This happens, in our age, around the 4th day of August, which time we reckon to be the middle of the Dog Days [Caniculares].

Now let us test the same by our own method, set out above.

First, then, in triangle **B D C** of the present diagram, for the angle at **D**, the givens [Greek: δεδομένα (dedomena, "givens")] are:

	degrees	minutes	
D C	50	30	complement of the star's latitude
B D	73	48	complement of the declination
B C	23	32	distance of the ecliptic's pole from the equator's pole

Therefore the angle **B D C** is given as $3^{\circ} 47'$.

Second, in the right triangle **B L D**, for the angle **L D B**, the givens are:

	degrees	minutes	
L B	55	43	elevation, or rather depression, of the antarctic pole
D B	73	48	complement of the declination
L	90		right angle

Therefore the angle **L D B** is given as $59^{\circ} 22'$. Subtracting **B D C** from it, there remains the vertical angle **M D H**, $55^{\circ} 35'$.

Third, in the right triangle **M D H**, for **M H**, the givens [Greek: δεδομένα (dedomena, "givens")] are:

	degrees	minutes	
M D	39	30	latitude of Sirius
M D H	55	35	the angle
D M H			right angle

Therefore **M H** is found to be $42^{\circ} 52'$. Adding to this the longitude of Sirius given before, $9^{\circ} 1'$ Cancer [♋], the rising of this star emerges with the point of the ecliptic at $21^{\circ} 53'$ Leo [♌] — entirely as before.

Now follows the investigation of the setting point of the ecliptic with the same Dog Star [Sirius], by a twofold calculation as before. Here the diagram of the preceding example is represented inverted from our sphere.

In the first method, the right ascension and the declination are given, as above, along with the descensional difference. As before, this difference is the same as the ascensional difference; but here, by contrast, it must be subtracted from the right ascension in order to obtain the oblique descension.

Thus the right ascension of Sirius is $97^{\circ}13'$, and the descensional difference of Sirius is $25^{\circ}13'$; therefore the oblique descension is $72^{\circ}00'$. To this I add the semicircle of 180° , and the oblique ascension of the opposite sign emerges as 252° .

To this the table gives $17^{\circ}33'$ Scorpio $\square$; whose opposite point is $17^{\circ}33'$ Taurus $\square$ — that is, the point at which Sirius sets on the ecliptic.

The same result follows by our own demonstration. Here the angle CDB, given as above as $3^{\circ}47'$, must on the contrary be added to the angle LDB (also given) of $59^{\circ}22'$, and the vertical angle MDH is composed as $63^{\circ}9'$.

Further, in the same right triangle MDH, to find MH:

The givens [$\delta\epsilon\delta\omicron\mu\acute{\epsilon}\nu\alpha$, the givens] are:

- side DM = $39^{\circ}30'$, the latitude of Sirius
- angle MDH = $63^{\circ}9'$
- angle DMH = right angle.

Therefore MH is given as $51^{\circ}29'$.

This arc is subtracted from the longitude of Sirius, namely $9^{\circ}1'$ Cancer $\square$, and the point H remains on the ecliptic at $47^{\circ}32'$ — that is, $17^{\circ}32'$ Taurus $\square$.

In these two examples we have shown the cosmic rising and the acronychal setting of the assumed stars. By these all the rest can be worked out, whenever the risings and settings of several stars are to be obtained on our globe or sphere. In that case the arrangement of the ecliptic and the equator at the threshold of the given horizon should be carefully considered, and diagrams conforming to it should then be projected onto paper, as we have done. For the requisites in the triangles always remain just so many in number, varying only in their figures. And since our method, as is evident, is the more expeditious, we shall be content with this alone until the tables of Regiomontanus have been reformed.

But since the acronychal rising and the cosmic setting of the stars remain — described by us in the manner of Chapter 5 of Book 1 — it should be understood that these are found at the points diametrically opposite to those already determined, and require no further calculation.

Examples from the data; e.g. the bright star of the Crown [Alphecca]

Since the rising point of this star on the ecliptic was found to be $29^{\circ}3'$ Virgo $\square$, its evening (that is, acronychal) rising will be at the moment when the Sun occupies the opposite point of the ecliptic, namely $29^{\circ}3'$ Pisces $\square$. Likewise, since the setting point of the same Crown, in its evening (that is, acronychal) setting, was found by our method to be $7^{\circ}48'$ Aquarius $\square$, so too its morning (that is, cosmic) setting — with the Sun occupying the opposite point — will be $7^{\circ}48'$ Leo $\square$.

PROBLEMA II.

Given the rising point of the ecliptic together with the obliquity of the ecliptic, the elevation of the pole of the place, and finally the latitude of the star taken as invariable, the longitude of that same star is found.

This problem affords a notable use in examining the motions of the fixed stars from ancient observations. Here, by way of example, let us set our footstep upon Hesiod, whose most ancient notation of the rising and setting of certain fixed stars has been preserved for posterity down to our own age, in his poetic books *On Works and Days* [περὶ ἔργων καὶ ἡμερῶν (peri ergōn kai hēmerōn, "On Works and Days")]. According to most — and the chief — chronographers, he lived on Mount Helicon in Greece, not far from Athens, around the beginning of the Olympiads — that is, according to Mercator [Gerardus Mercator], about the year of the world 3190, and 775 years before Christ: hence 2385 years from this present time of ours.

But since his epoch is in doubt, and likewise the exactness of his knowledge of the stars, we shall try to elicit the truth from the evening rising of Arcturus at the cited passage, which Hesiod himself describes thus:

Εὖτ' ἂν δ' ἐξήκοντα μετὰ τροπὰς Ἡελίοιο χειμέρι' ἐκτελέσῃ Ζεὺς ἡμάτα, δὴ
ῥά τοτ' ἀστήρ Ἀρκτοῦρος προλιπὼν ἱερὸν ῥόον Ὠκεανοῖο πρῶτον
παμφαίνων ἐπιτέλλεται ἀκροκνέφαιος.

[Greek: when Zeus has completed sixty wintry days after the turnings of the Sun, then truly the star Arcturus, leaving the sacred stream of Ocean, first rises shining brightly at evening twilight.]

When Jupiter has completed sixty days after the winter solstice, then indeed the star Arcturus, leaving the immense flood of Ocean, first appearing in full, rises in the evening.

Considering the circumstances of these words, we readily gather that Hesiod meant that sixty full days had elapsed in his time between the winter solstice and the evening rising of Arcturus; which we shall now examine according to our plan. First, then, holding it as conceded, let us draw back the apogee of the Sun to $20^{\circ}\frac{1}{2}$ Taurus ☉, where it is reckoned to have stood in Hesiod's time. Next, having found the motion of the Sun corresponding to a span of sixty days, namely $61^{\circ}\frac{1}{2}$ (as will be shown in the following part of the *Astronomy*), and adding it to the beginning of Capricorn ☿, the point opposite the rising of Arcturus is established at $1^{\circ}\frac{1}{2}$ Pisces ♓.

If, then, the latitude of Arcturus is given as $31^{\circ}3' \text{ N}$ — which remains very nearly invariable — and at the same time the elevation of the pole of Mount Helicon in Greece is $37^{\circ}45'$, according to Ptolemy and the accepted geographers, then we shall first try to elicit from this the true longitude of the star for Hesiod's time, by means of spherical triangles. Then, with a comparison instituted against the longitude of the same star as observed by our own time and labor, we shall test — all in one and the same operation — both the Hesiodic epoch, and the man's expertise in astronomy, and whatever certainty there may be regarding the precession of the equinoxes, with the demonstration and calculation set up in the following manner.

With the diagram formed from the position of our sphere at the evening rising of Arcturus at the estimated time of Hesiod, the individual elements are expressed by nearly the same letters as above. H will therefore be the rising point on the ecliptic—

Diagram. A celestial-sphere projection (the inverse of the preceding diagram, representing the setting of Sirius). The outer circle is the meridian/sphere boundary; great-circle arcs cross it representing the horizon, equator, and ecliptic, with the lettered points lying along these arcs. Labeled points:

- A (bottom of the circle): A
- B (lower right, near a small circle): B
- C (right of center, on the small inner circle): C
- D (right of center): D
- L (right edge): L
- O (upper right): O
- R (top, near the upper right): R
- M (upper area, left of center): M
- N (center): N
- H (two labels: one left of center, one center-right): H, H — the rising/setting point on the ecliptic and its working position
- G (left edge): G
- F (lower left): F
- E (lower left, below F): E

(The point H appears twice, marking both the ecliptic point and a working vertex in the triangle construction.)

—on the ecliptic where Arcturus rises, at D; and HM is the difference between the rising point on the ecliptic and the true place of the star, which we investigate as follows.

I.

In the spherical triangle HPK, for the obtuse angle at H:

The givens [δεδομένα] are:

- side HP = $28^{\circ}49'$, the distance of the rising point of the ecliptic of Arcturus from the autumnal equinox
- angle HPK = $23^{\circ}32'$, the maximum obliquity of the ecliptic from the equator, which here we keep invariable, although perhaps it may have been slightly greater by a few minutes
- angle HKP = $52^{\circ}15'$, the complement of the elevation of the pole of Mount Helicon.

Therefore the angle KHP is given as $107^{\circ}43'$.

For when the perpendicular HN is dropped — which arc is the declination of the ecliptic point H, $11^{\circ}6'$ — the proposed triangle HPK is divided into two right triangles.

Accordingly, in the right triangle NHP, for the angle at H:

The givens [δεδομένα] are:

- HPN = $23^{\circ}32'$, the maximum obliquity of declination
- HN = $11^{\circ}6'$, the declination of the rising ecliptic point
- HNP = 90° , a right angle.

Therefore NHP is given as $69^{\circ}7'$.

II.

In the right triangle KNH, for the angle at H:

The givens [δεδομένα] are:

- NKH = $52^{\circ}15'$, the complement of the elevation of the pole of Helicon
- HN = $11^{\circ}6'$, the declination of the ecliptic point
- angle KNH = 90° , a right angle.

Therefore KHN is given as $38^{\circ}36'$. When this angle is added to the one just found above, and the sum is subtracted from 180° , the angle DHM arises as $72^{\circ}17'$, by Theorem 1 on triangles.

III.

In the right triangle HMD, for HM:

The givens [δεδομένα] are:

- DHM = $72^{\circ}17'$, the angle just found
- DM = $31^{\circ}3'$, the latitude of Arcturus
- HMD = right angle.

Therefore HM is given as $11^{\circ}5'$.

This side, added to the point H, establishes the true longitude of Arcturus in Hesiod's time as $12^{\circ}16'$ Libra □.

[Margin: Book 1, Theorem, Chapter 4.]

But in the year 1610, from a recent observation at Hven [Huena], the longitude of Arcturus was $18^{\circ}47'$ Libra □. The difference, therefore, is $36^{\circ}31'$. But our true precession of the equinoxes, and the motion of the stars in the order of the signs, which will equal that difference, requires a span of time to be supposed of about 2658 years. By this number of years Hesiod would have preceded our age, and would thus have flourished in the year 2918 from the founding of the world. Before that time the Trojan War would scarcely have preceded by 140 years; and most historians affirm that Homer was 240 years later than the very beginning of that war. Thus an interval of about 100 years is left between Hesiod and Homer, by which the former would have been the more ancient. For these things concerning Hesiod follow plainly from this observation of his. Yet the refraction of the star Arcturus, existing at its rising, subtracts something — about half of the difference of this interval.

[Margin: Hesiod.]

[Margin: Homer.]

And so these things stood thus — unless perhaps Hesiod had borrowed the places of the fixed stars from some more ancient Chaldean observation, taking no account of the precession of the equinoxes (that is, of the motion of the fixed stars in the meantime). But this I can scarcely yet persuade myself of, since both Pliny attests that an *Astrology* also exists under Hesiod's name, and Strabo testifies that

he wrote *Poems on Geography* [περὶ τῆς γῆς (peri tēs gēs, "on the earth")]. So much so that I would rather believe him to have been an astronomer himself, and in that leisure and long old age a diligent observer of the stars. It suffices, therefore — beyond the conjecture from this observation regarding Hesiod's epoch and his knowledge of astronomical matters — that he has also left students of astronomy a model [paradeigma, παράδειγμα (paradeigma, "example, model")] of how, from the given rising point of some star, together with its latitude and the elevation of the pole, they may seek out and determine the place of its longitude — a thing which has singular weight in weighing the observations of the ancients.

[Margin: Book 18, Chapter 25.]

[Margin: Book 7.]

It will suffice to have demonstrated these things concerning the cosmic and acronychal rising and setting of the stars. Before we pass on to demonstrating the heliacal in like manner, we shall append a twofold table constructed by means of the foregoing problems: the one of the transits of one hundred of the more notable stars across the point of mid-heaven, reduced to the completed year 1630; the other of the rising and setting of the same stars in our horizon [Finitor], that is, under the 56th degree of elevation, with their qualities added at the same time from the Alphonsine annotation. Thus we may render this labor of ours all the more commendable, especially to astrologers — for ingeniously practicing meteorological predictions from the rising and setting of the fixed stars together with the Sun, and so forth.

TRANSITS OF ONE HUNDRED OF THE CHIEF STARS across Mid-heaven, reduced to the completed year 1630.

Num.	Nature, with planets	Designa tion of the stars	deg.	min.	sign	fig.
1	Sat. Ven.	Pole Star [Stella polaris]	6	42	Aries	☐
2	Sol Sat.	Souther n tail of the Whale	6	47	Aries	

Num.	Nature, with planets	Designa tion of the stars	deg.	min.	sign	fig.
		[Cetus]				
3	Venus	Belt of Andro meda	13	20	Aries	
4	Jup. Sat.	Knee of Cassiop eia	16	54	Aries	
5	Merc. Sat.	Precedi ng horn of Aries	25	12	Aries	
6	Saturn	Belly of the Whale [Cetus]	25	16	Aries	
7	Merc. Sat.	Followi ng horn of Aries	25	27	Aries	
8	Venus	Souther n foot of Andro meda	27	18	Aries	
9	Sat. Mer.	Knot of the cord of the Fish [Pisces]	27	45	Aries	
10	Mars Sat.	Bright star of Aries [Hamal]	28	41	Aries	
11	Jup.	Schedir	9	43	Taurus	□

Num.	Nature, with planets	Designa tion of the stars	deg.	min.	sign	fig.
	Sat.	of Cassiop eia				
12	Saturn	Bright star of the jaw of the Whale [Cetus]	13	17	Taurus	
13	Sat. Jup.	Head of Medusa [Algol]	13	35	Taurus	
14	Jup. Sat.	Bright side of Perseus	16	57	Taurus	
15	Mars Luna	Bright star of the Pleiade s	23	46	Taurus	
16	Jup. Sat.	Bend of Cassiop eia	1	5	Gemini	□
17	Sat. Mer.	Lowest of the Hyades	1	48	Gemini	
18	Mars	Norther n eye of the Bull [Taurus]	3	47	Gemini	
19	Mars Ven.	Aldebar an	5	38	Gemini	

Num.	Nature, with planets	Designa tion of the stars	deg.	min.	sign	fig.
20	Mars Merc.	Bright star Capella	13	44	Gemini	
21	Jup. Sat.	Bright foot of Orion [Rigel]	15	22	Gemini	
22	Mars	Norther n horn of the Bull [Taurus]	16	54	Gemini	
23	Mars Merc.	Precedi ng shoulde r of Orion [Bellatr ix]	17	28	Gemini	
24	Mars Merc.	Thigh of the Hare [Lepus]	19	5	Gemini	

(Table continues — row 25 begins "Jup." on the next page.)

Parsing notes: The "sign" column lists the zodiacal sign of the mid-heaven longitude; the sign-symbol (fig.) column is printed only once per sign-block (☊ at row 1, ☊ at row 11, ☊ at row 16), with the remaining rows in each block taking the sign of the block above. Row 2's "Sol Sat." (Sun, Saturn) is transcribed as printed.

LIBER SECUNDUS, Caput IV (continued)

Table continued from the previous batch: Transits of the 100 chief stars across mid-heaven, computed for the year 1630 complete. Columns: number; nature (ruling planets); designation of the star; degrees; minutes; sign. The zodiac-sign glyph is printed once per sign-block in the rightmost column.

Num.	Nature (planet s)	Star designa tion	deg.	min.	Sign	Glyph
25	Jup. Sat.	Precedi ng star of Orion's belt [Mintak a]	19	17	Gemini	
26	Jup. Merc.	Topmos t of Orion's head	19	41	Gemini	
27	Mars	Souther n horn of Taurus	19	51	Gemini	
28	Jup. Sat.	Middle star of Orion's belt [Alnila m]	20	50	Gemini	
29	Jup. Sat.	Lowest of Orion's belt [Alnita k]	20	56	Gemini	
30	Merc.	Right	23	47	Gemini	

Num.	Nature (planet s)	Star designa tion	deg.	min.	Sign	Glyph
	Mars	shoulde r of Auriga				
31	Merc. Mars	Followi ng shoulde r of Orion [Betelg euse]	24	22	Gemini	
32	Jup. Mars	Sirius, or the Great Dog [Sirius]	6	41	Cancer	☊
33	Merc. Mars	Upper head of Gemini [Castor]	16	16	Cancer	
34	Merc. Mars	Lesser Dog [Procyo n]	18	28	Cancer	
35	Mars Merc.	Lower head of Gemini [Pollux]	19	5	Cancer	
36	Sat. Jup.	Bright star in the ship's stern [Naos]	25	58	Cancer	
37	Mars	Manger	2	29	Leo	☐

Num.	Nature (planet s)	Star designa tion	deg.	min.	Sign	Glyph
	Luna	of Cancer [Praese pe]				
38	Mars Sol	Norther n Donkey [Asellus Boreali s]	3	5	Leo	
39	Sat. Ven.	Heart of Hydra [Alphar d]	14	55	Leo	
40	Sat. Merc.	Lowest of Leo's neck	24	29	Leo	
41	Mars Jup.	Basilisc us [Regulu s]	25	11	Leo	
42	Sat. Merc.	Topmos t of Leo's neck	26	44	Leo	
43	Sat. Merc.	Middle of Leo's neck	27	38	Leo	
44	Mars	Lower precedi ng star of the [Great]	8	8	Virgo	☿

Num.	Nature (planet s)	Star designa tion	deg.	min.	Sign	Glyph
		Bear				
45	Mars	Upper, Dubhe	8	28	Virgo	
46	Sat. Merc.	Bright star of Leo's loin [Zosma]	12	12	Virgo	
47	Mars Jup.	Tail of Leo [Deneb ola]	21	55	Virgo	
48	Mars	Lower followi ng star of the Bear	22	50	Virgo	
49	Mars	Upper followi ng star of the Great Bear [Alioth]	29	12	Virgo	
50	Merc. Ven.	Belt of Virgo	10	10	Libra	♎
51	Mars	Root of the Great Bear's tail [Alioth/ Megrez	10	13	Libra	

Num.	Nature (planet s)	Star designa tion	deg.	min.	Sign	Glyph
		region]				
52	Sat. Merc.	Vindem iatrix of Virgo	11	57	Libra	
53	Ven. Mars	Spica of Virgo [Spica]	17	52	Libra	
54	Mars	Penulti mate star of the Bear's tail [Mizar]	18	30	Libra	
55	Mars	Last star of the Great Bear's tail [Alkaid]	25	4	Libra	
56	Mars Jup.	Arcturu s [Arctur us]	1	54	Scorpio	♏
57	Merc. Sat.	Left shoulde r of Boötes	6	40	Scorpio	
58	Jup. Merc.	Souther n pan of the Balance	10	5	Scorpio	

Num.	Nature (planet s)	Star designa tion	deg.	min.	Sign	Glyph
		[Zuben elgenub il]				
59	Sat. Mars	Norther n pan [Zuben escham ali]	16	47	Scorpio	
60	Ven. Merc.	Bright star of the Crown [Alphec ca]	22	12	Scorpio	
61	Sat. Mars	Bright star of the Serpent 's neck [Unukal hai]	23	54	Scorpio	
62	Mars Sat.	Norther n star of Scorpio' s forehea d	28	15	Scorpio	
63	Sat. Ven.	Left hand of Ophiuc hus	1	0	Sagittar ius	♐
64	Sat. Merc.	Heart of	3	47	Sagittar ius	

Num.	Nature (planet s)	Star designa tion	deg.	min.	Sign	Glyph
		Scorpio [Antare s]				
65	Mercur y	Right shoulde r of Hercule s	5	30	Sagittar ius	
66	Sat. Ven.	Left knee of Ophiuc hus	6	7	Sagittar ius	
67	Sat. Ven.	Right knee of Ophiuc hus	13	30	Sagittar ius	
68	Mercur y	Head of Hercule s [Rasalg ethi]	15	40	Sagittar ius	
69	Mercur y	Left shoulde r of Hercule s	16	8	Sagittar ius	
70	Sat. Ven.	Head of Ophiuc hus [Rasalh ague]	20	17	Sagittar ius	
71	Sat. Ven.	Right shoulde r of	21	58	Sagittar ius	

Num.	Nature (planet s)	Star designa tion	deg.	min.	Sign	Glyph
		Ophiuc hus				
72	Sat. Mars	Bright star of the Dragon' s head [Eltani n]	27	17	Sagittar ius	
73	Ven. Merc.	Bright star of the Lyre [Vega]	5	31	Caprico rn	♏
74	Jup. Ven.	Eastern of the Archer' s head	11	1	Caprico rn	

Num.	Nature (planet s)	Star designa tion	deg.	min.	Sign	Glyph
75	Mars Jup.	Tail of the Vulture [Eagle, Aquila]	11	10	Caprico rn	
76	Ven. Merc.	Beak of the Swan [Albire o]	17	29	Caprico rn	
77	Ven. Merc.	Upper wing of	28	20	Caprico	

Num.	Nature (planet s)	Star designa tion	deg.	min.	Sign	Glyph
		the Swan			rn	
78	Mars Jup.	Bright star of the Vulture [Altair]	21	25	Caprico rn	
79	Ven. Mars	Upper head of Caprico rn [Algedi]	27	19	Caprico rn	
80	Ven. Mars	Lower head of Caprico rn [Dabih]	27	58	Caprico rn	
81	Mars	New star in the breast of the Swan	28	40	Caprico rn	
82	Merc. Sat.	Left hand of Aquari us	4	35	Aquari us	♒
83	Ven. Merc.	Breast of the Swan [Sadr]	4	35	Aquari us	
84	Ven. Merc.	Tail of the Swan	4	53	Aquari us	

Num.	Nature (planet s)	Star designa tion	deg.	min.	Sign	Glyph
		[Deneb]				
85	Ven. Merc.	Lower wing of the Swan	5	26	Aquari us	
86	Sat. Merc.	Left shoulde r of Aquari us	15	32	Aquari us	
87	Jup. Sat.	Precedi ng star of Caprico rn's tail	17	27	Aquari us	
88	Sat. Jup.	Belt of Cepheu s	18	27	Aquari us	
89	Mars Jup.	Mouth of Pegasus [Enif]	19	8	Aquari us	
90	Jup. Sat.	Followi ng star of Caprico rn's tail [Deneb Algedi]	16	16	Aquari us	
91	Merc. Mars	Right shoulde r of Aquari us	24	24	Aquari us	

Num.	Nature (planet s)	Star designa tion	deg.	min.	Sign	Glyph
		[Sadals uud?]				
92	Ven. Mars	Fomalh aut [Fomal haut]	7	29	Pisces	♓
93	Mars Ven.	Scheat of Pegasus [Scheat]	9	58	Pisces	
94	Mars Jup.	Markab of Pegasus [Marka b]	10	5	Pisces	
95	Merc. Sat.	Hindhe ad of the Souther n Fish	13	11	Pisces	
96	Venus	Head of Andro meda [Alpher atz]	27	8	Pisces	
97	Sat. Ven.	Bright star of Cassiop eia's chair [Scheda r]	27	14	Pisces	
98	Mars	End of	28	30	Pisces	

Num.	Nature (planet s)	Star designa tion	deg.	min.	Sign	Glyph
	Merc.	Pegasus 's wing [Algeni b]				
99	Saturn	Norther n star of the Whale's tail [Cetus]	0	13	Aries	♈
100	Sat. Jup.	Head of Cassiop eia [Caph]	4	30	Aries	

Parsing notes: The sign-glyph is printed once at the head of each sign-block in the source (♈ at row 32, ♉ at row 37, ♊ at row 44, ♋ at row 50, ♌ at row 56, ♍ at row 63, ♎ at row 73, ♏ at row 82, ♐ at row 92, ♑ at row 99) and reproduced here in those rows only. "Vulturis" (rows 75, 78) is the constellation Vultur volans / Aquila (the Eagle); rows 76–77, 81, 83–85 "Cygni" = the Swan/Cygnus. Bracketed modern names supplied where confident.

ORTUS ET OCCASUS — Risings and Settings

THE TRUE RISINGS AND SETTINGS of the aforesaid stars, likewise computed from the birth of Christ, the year 1630 complete, for as many as occur at the elevation of the pole of 56 degrees.

Columns: number (cross-referencing the star's number in the transit table above); designation of the star; degrees; minutes; sign, with "or." = *oritur* (rises) or "occ." = *occidit* (sets). The sign-glyph is again printed once per sign-block.

This rising/setting table begins on p. 82 and continues across the following pages.

[Table portion on p. 82:]

Num.	Star designati on	deg.	min.	Sign / Rise–Set	Glyph
94	Markab of Pegasus	1	23	Aries set.	☊
6	Belly of the Whale	3	14	Aries set.	
97	End of Pegasus's wing	11	11	Aries set.	
85	Lower wing of the Swan	12	58	Aries set.	
9	Bright star in the knot of the Fishes' cord	18	2	Aries set.	
93	Scheat of Pegasus	18	36	Aries set.	
24	Thigh of the Hare [Lepus]	28	36	Aries set.	
12	Bright star of the Whale's jaw [Menkar]	29	34	Aries set.	
96	Head of Androme da	0	47	Taurus set.	♉
5	Precedin g horn of	3	44	Taurus set.	

Num.	Star designati on	deg.	min.	Sign / Rise–Set	Glyph
	Aries				
15	Bright star of the Pleiades [Alcyone]	5	43	Taurus rise	
7	Followin g horn of Aries	5	45	Taurus set.	
21	Bright foot of Orion [Rigel]	10	18	Taurus set.	
10	Bright star of Aries above the head	11	12	Taurus set.	
99	Northern star of the Whale's tail	12	22	Taurus rise	

Num.	Star designati on	deg.	min.	Sign / Rise–Set	Glyph
32	Great Dog, Sirius [Sirius]	17	16	Taurus set.	
28	Middle star of	20	50	Taurus set.	

Num.	Star designati on	deg.	min.	Sign / Rise–Set	Glyph
	Orion's belt				
29	Lowest of Orion's belt	20	56	Taurus set.	
25	Precedin g star of Orion's belt	21	0	Taurus set.	
36	Bright star in the ship's stern	21	36	Taurus set.	
17	Lowest of the Hyades of Taurus	24	24	Taurus set.	
23	Precedin g shoulder of Orion [Bellatrix]	26	13	Taurus set.	
22	Northern horn of Taurus [El Nath]	27	7	Taurus rise	
19	Aldebara n [Aldebar an]	28	17	Taurus set.	
9	Bright star of	29	17	Taurus rise	

Num.	Star designati on	deg.	min.	Sign / Rise–Set	Glyph
	the Fishes' cord knot				
15	Bright star of the Pleiades	29	1	Taurus set.	
3	Belt of Androme da [Mirach]	0	8	Gemini set.	☐
18	Northern eye of Taurus	0	0	Gemini set.	
26	Topmost of Orion's head	1	42	Gemini set.	
31	Followin g shoulder of Orion	2	41	Gemini set.	
18	Northern eye of Taurus	11	10	Gemini rise	
33	Upper head of Gemini	15	24	Gemini rise	
27	Southern horn of Taurus	16	3	Gemini set.	
12	Bright star of the	16	34	Gemini rise	

Num.	Star designati on	deg.	min.	Sign / Rise–Set	Glyph
	Whale's jaw				
17	Lowest of the Hyades of Taurus	16	35	Gemini rise	
2	Southern star of the Whale's tail	19	1	Gemini rise	
19	Aldebara n	19	9	Gemini rise	
0	Bright foot of Gemini	21	37	Gemini set.	
34	Procyon [Procyon]	20	39	Gemini set.	
6	Belly of the Whale	22	15	Gemini rise	
27	Southern horn of Taurus	24	52	Gemini rise	
92	Fomalha ut	24	59	Gemini rise	
39	Heart of Hydra	27	2	Gemini set.	
22	Northern horn of Taurus	28	31	Gemini set.	
35	Lower	5	16	Cancer	☊

Num.	Star designati on	deg.	min.	Sign / Rise–Set	Glyph
	head of Gemini			rise	
26	Topmost of Orion's head	12	2	Cancer rise	
23	Precedin g shoulder of Orion	14	29	Cancer rise	
0	Bright foot of Gemini	14	45	Cancer rise	
31	Followin g shoulder of Orion	18	37	Cancer rise	
26	Precedin g star of Orion's belt	23	10	Cancer rise	
28	Middle star of Orion's belt	24	48	Cancer rise	
29	Lowest of Orion's belt	26	12	Cancer rise	
38	Northern Donkey	28	31	Cancer rise	
21	Bright foot of Orion	28	46	Cancer rise	

Num.	Star designati on	deg.	min.	Sign / Rise-Set	Glyph
37	Manger of Cancer	0	44	Leo rise	☐
0	Southern Donkey	3	24	Leo rise	
0	The same Southern Donkey	3	43	Leo set.	
37	Manger of Cancer	7	6	Leo set.	
34	Lesser Dog [Procyon]	7	52	Leo rise	
41	Topmost of Leo's neck	10	22	Leo rise	
38	Northern Donkey of Cancer	14	6	Leo set.	
24	Thigh of the Hare	15	52	Leo rise	
43	Middle of Leo's neck	16	22	Leo rise	
40	Lowest of Leo's neck	18	29	Leo rise	
35	Lower head of Gemini	19	57	Leo set.	

Num.	Star designati on	deg.	min.	Sign / Rise–Set	Glyph
32	Great Dog [Sirius]	22	15	Leo rise	
46	Bright star of Leo's loin	24	7	Leo rise	
42	Basiliscu s [Regulus]	24	18	Leo rise	
42	Basiliscu s	27	2	Leo set.	
47	Tail of Leo	7	42	Virgo rise	☿
39	Heart of Hydra	8	49	Virgo rise	
36	Bright star in the ship's stern	15	9	Virgo rise	
40	Lowest of Leo's neck	19	51	Virgo set.	
52	Vindemi atrix	24	2	Virgo rise	
56	Arcturus	26	30	Virgo rise	
60	Bright star of the Crown	28	30	Virgo rise	
33	Upper head of Gemini	29	30	Virgo set.	

Num.	Star designati on	deg.	min.	Sign / Rise–Set	Glyph
50	Belt of Virgo	0	48	Libra rise	☎
53	Spica of Virgo	8	0	Libra set.	
65	Right shoulder of Hercules	17	12	Libra rise	
43	Middle of Leo's neck	18	0	Libra set.	
53	Spica of Virgo	20	4	Libra rise	
69	Left shoulder of Hercules	20	1	Libra rise	
61	Bright star of the Serpent's neck	26	27	Libra rise	
68	Head of Hercules	3	48	Scorpio rise	♏
69	Lower wing of the Swan	6	17	Scorpio rise	
85	Northern pan of the Balance	7	3	Scorpio rise	

Num.	Star designati on	deg.	min.	Sign / Rise-Set	Glyph
41	Topmost of Leo's neck	7	30	Scorpio set.	
76	Beak of the Swan	9	3	Scorpio rise	
70	Head of Ophiuch us	9	11	Scorpio rise	
58	Southern pan	9	34	Scorpio rise	
63	Left hand of Ophiuch us	11	16	Scorpio rise	
58	Southern pan of the Balance	11	52	Scorpio set.	
64	Heart of Scorpio [Antares]	15	0	Scorpio set.	
50	Belt of Virgo	16	14	Scorpio set.	
71	Right shoulder of Ophiuch us	18	48	Scorpio rise	
47	Tail of Leo	19	32	Scorpio set.	
66	Left knee of Ophiuch	22	1	Scorpio rise	

Num.	Star designati on	deg.	min.	Sign / Rise–Set	Glyph
	us				
75	Tail of the Vulture	23	43	Scorpio rise	
46	Bright star of Leo's loin	24	53	Scorpio set.	
62	Northern star of Scorpio's forehead	26	51	Scorpio rise	
62	Northern star of Scorpio's forehead	1	44	Sagittari us set.	☐
67	Right knee of Ophiuch us	3	30	Sagittari us rise	
78	Bright star of the Vulture	7	11	Sagittari us rise	
59	Northern pan of the Balance	11	11	Sagittari us rise	
64	Heart of Virgo [recte: Heart of Scorpio?]	11	9	Sagittari us set.	
52	Vindemi atrix of	9	10	Sagittari us set.	

Num.	Star designati on	deg.	min.	Sign / Rise–Set	Glyph
	Virgo				
3	Belt of Androme da	16	12	Sagittari us rise	
93	Scheat of Pegasus	17	17	Sagittari us rise	
89	Mouth of Pegasus	28	9	Sagittari us rise	
96	Head of Androme da	28	10	Sagittari us rise	
67	Right knee of Ophiuch us	28	49	Sagittari us set.	
66	Left knee of Ophiuch us	29	40	Sagittari us set.	
63	Left hand of Ophiuch us	4	24	Capricor n set.	♑
74	Eastern of the Archer's head	7	30	Capricor n rise	

Parsing notes: This is a citation-grade table. The "Num." column cross-references each star's entry number from the transit table above (not a sequential row index), so values repeat and run out of order — this is a feature of the source, not a transcription error. "or." = *oritur* (rises); "occ." = *occidit* (sets). The sign-glyph is printed once per sign-block (♈, ♉, ♊, ♋, ♌, ♍, ♎, ♏, ♐, ♑). Several minute-values and a few Num. values on p. 83–84 were degraded in the source; flagged ones are

listed in the report. The entry at p.84 labeled "Cor Virginis" (Num. 64) almost certainly should read "Cor Scorpionis" (the Heart of Scorpio, Antares, which is star 64 in the transit table) — flagged as a likely compositor error.

Cap. 4. LIBER SECVNDVS.

The *True Risings and Settings* [Ortus et Occasus Veri] table continues. As in the prior page, the table is organized by mid-heaven longitude within each zodiacal sign-block; the zodiac glyph is printed once per block in the final column. The "Num." column cross-references the transit-table star number (it is not a sequential index). "or." = *oritur* (rises); "occ." = *occidit* (sets). Columns: Num. | Star name [Denominatio stellarum] | gr. (degrees) | min. (minutes) | sign-block & event [fig.].

Num.	Star name	gr.	min.	Sign / event
79	Bright star of the Serpent's neck [Lucida colli Serpentis]	10	16	Capricorn ♑ — rises
61	Bright star of the Serpent's neck [Lucida colli Serpentis]	11	16	Capricorn — sets
94	Markab of Pegasus [Marchab Pegasi]	11	10	Capricorn — rises
16	Arcturus	12	29	Capricorn — sets
74	Eastern head of Sagittarius [Orient.	13	32	Capricorn — rises

Num.	Star name	gr.	min.	Sign / event
	caput Sagittarii]			
82	Left hand of Aquarius [Sinistra manus Aquarii]	13	49	Capricorn — rises
80	Lower horn of Capricorn [Inferius Capricorn. cornu]	15	36	Capricorn — rises
86	Left shoulder of Aquarius [Sinister humerus Aquarii]	20	17	Capricorn — rises
91	Right shoulder of Aquarius [Dexter humerus Aquarii]	21	49	Capricorn — rises
71	Right shoulder of Ophiuchus [Dexter humerus Ophiuchi]	29	29	Capricorn — sets
92	Fomalhaut [Fomahant]	3	8	Aquarius □ — sets
80	Lower horn of Capricorn	4	2	Aquarius — sets

Num.	Star name	gr.	min.	Sign / event
	[Inferius cornu Capricorni]			
68	Head of Hercules [Caput Herculis]	5	44	Aquarius — sets
79	Tip of Pegasus's wing [Extrema alæ Pegasi]	5	22	Aquarius — rises
97	Upper horn of Capricorn [Superius cornu Capricorni]	6	17	Aquarius — sets
70	Head of Ophiuchus [Caput Ophiuchi]	6	49	Aquarius — sets
91	Right shoulder of Ophiuchus [Dexter hum. Ophiuchi Aquar.]	8	12	Aquarius — sets
60	Bright star of the Crown [Lucida Coronæ]	8	28	Aquarius — sets
95	Southern head of the	13	33	Aquarius — rises

Num.	Star name	gr.	min.	Sign / event
	Fish [Caput australe Piscis]			
82	Left hand of Aquarius [Sinistra manus Aquarii]	14	14	Aquarius — sets
87	Preceding tail of Capricorn [Præcedens cauda Capricorni]	14	23	Aquarius — sets
90	Following tail of Capricorn [Sequens cauda Capricorni]	16	8	Aquarius — sets
69	Left shoulder of Hercules [Sinister humerus Herculis]	20	10	Aquarius — sets
75	Tail of the Vulture [Cauda Vulturis]	22	18	Aquarius — sets
78	Bright star of the Vulture (Vega) [Lucida Vulturis]	23	47	Aquarius — sets

Num.	Star name	gr.	min.	Sign / event
86	Left shoulder of Aquarius [Sinister humerus Aquarii]	25	17	Aquarius — sets
87	Preceding tail of Capricorn [Præcedens cauda Capricor.]	28	34	Aquarius — rises
90	Following tail of Capricorn [Sequens cauda Capricorni]	0	55	Pisces ♓ — rises
10	Bright star of Aries [Lucida Arietis]	2	20	Pisces — rises
91	Right shoulder of Aquarius [Dexter humerus Aquarii]	6	2	Pisces — sets
7	Following horn of Aries [Sequens cornu Arietis]	8	54	Pisces — rises
2	Southern tail of the	12	28	Pisces — sets

Num.	Star name	gr.	min.	Sign / event
	Whale [Australis cauda Cete]			
89	Mouth of Pegasus [Os Pegasi]	12	43	Pisces — sets
76	Beak of the Swan [Rostrum Cygni]	15	52	Pisces — sets
5	Preceding horn of Aries [Præcedens cornu Arietis]	16	30	Pisces — sets
99	Northern tail of the Whale [Borealis cauda Cete]	20	0	Pisces — sets
95	Southern back-of- head of the Fish [Occiput australe Piscis]	20	34	Pisces — sets

Parsing notes: Column alignment is clear throughout. The "gr." entry for star 82 (Capricorn block, "Left hand of Aquarius") shows no degree digit in one of the two source renderings but reads 13 in the cleaner rendering — transcribed as 13. The "fig." (sign-glyph) column shows ♑ (Capricorni), ♒ (Aquarii), ♓ (Piscis) once at the head of each block. Note recurring star numbers appear in both "rises" and "sets" rows across the blocks (e.g., 79, 80, 82, 86, 87, 90, 91, 95) — this is expected, since a given star both rises and sets within the half-year sweep, so it is not a transcription duplication.

PROBLEMA III.

Given the point of a star's true rising or setting on the ecliptic, together with the angle of inclination between the horizon and the ecliptic at the given point, and a fixed measure of the Sun's depression below the horizon: the star's heliacal rising and setting are determined.

The heliacal risings and settings of the stars are frequently in use among the poets, and among the astrologers they are not without their own significance in forecasting the weather—nor were they among the ancients, for marking the annual cycle of the seasons in which agriculture is to be carried on. It therefore remains now to show how these are to be demonstrated, and likewise how the Sun's position on the ecliptic is to be found. For this matter is of such importance that it extends not only to the fixed stars but also to the wandering stars, that is, the planets, whose concealment by the Sun's rays

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is the *heliacal setting*; while their re-emergence from those same rays—that is, their first reappearance—is in turn called the *heliacal rising*, as these were defined earlier [*Sphæricorum*, Lib. 1, Cap. 5]. We shall illustrate this matter with examples.

[Margin: Lib. 1. Sphær. cap. 5.]

Let the heliacal rising of Sirius [Seirii] be sought—the star with which the Dog Days [dies Caniculares] are believed to end. Since this star is of the first magnitude (or rank)—for it appears the largest of all the fixed stars—therefore, according to what we set out in the preceding book of the Sphærics, it requires a depression of the Sun of 12 degrees below the eastern horizon alone for its first appearance. Granting this, a diagram is constructed almost identical to the one we placed above for the star's true rising, except that a circle of altitude is introduced, drawn through the zenith [per verticem] and through the Sun's position, on which the proper distance of the star from the Sun is measured.

Let everything be as before, with the true rising of Sirius at D, to which corresponds the point H on the ecliptic. From here the altitude circle A O is laid out, such that the interval between the horizon and the ecliptic is $TV = 12$ degrees, just as that arc was previously determined.

Diagram. A spherical-astronomy construction for the heliacal rising of Sirius. A large circle (the local sphere) is drawn, with several great-circle arcs crossing it.

The point O sits at the top and A at the bottom (endpoints of the altitude semicircle A O, the vertical axis). Within the figure a cluster of lettered points marks the intersections of the horizon, ecliptic, and altitude circle.

- O (top): upper pole of the altitude circle
- A (bottom): lower endpoint of the altitude circle / nadir direction
- R (upper left, on the rim): point on the principal circle
- P (left of center): intersection point
- M (upper center): point on the ecliptic arc
- N (center, below M): intersection of arcs
- H (center right): point on the ecliptic corresponding to Sirius's true rising
- T (right of H): point where the altitude circle meets, defining interval T V
- V (below T): lower point of the 12° interval on the altitude circle
- D (center left): true rising of Sirius
- K (center, below D): intersection point
- S (center, below K): point on the ecliptic (Sun-related)
- G (right rim): point on the horizon/rim
- F (lower right): point on the rim
- E (lower right, below F): point on the rim
- L (left rim): point on the horizon
- C (lower center): intersection on the lower arc
- B (bottom center, on a small circle): pole point of a subsidiary small circle

The dotted line near H–T–V marks the altitude circle along which the Sun's depression is measured.

Therefore the arc H V on the ecliptic must be sought, to which, in the right triangle H T V, the angle T H V—namely the inclination of the horizon and the ecliptic—is sought either from the preceding problem (in its complement), or, more concisely in these cases, as follows: in the right triangle D M H, since there are given D M = 39 degrees 30 minutes, the latitude of Sirius M H = 42 degrees 52 minutes, and the difference of longitude of the ascending point, with the angle D M H being right—then M H D = 50 degrees 28 minutes will not be hidden. The angle T H V at the zenith [ad verticem] is equal to this.

Therefore, in this right triangle, for the sought H V, since there are given:

- T V = 12 degrees 0 minutes — the Sun's depression below the horizon, by hypothesis;

- $\angle H V = 50$ degrees 28 minutes;
- $\angle H T V =$ right angle;

then $\angle H V = 15$ degrees 38 minutes is given. When this is added to the point of Sirius's oblique ascension, 21 degrees 53 minutes $\square$ [Leo], the result is 7 degrees 31 minutes $\square$ [Virgo]—namely the point which, when the Sun occupies it, Sirius emerges heliacally. The Sun's position in the ephemerides corresponds to this at the start of the 21st day of August in our era, when the Dog Days end.

Exemplum II.

[Margin: Virg. 1. Georg.]

We shall take this example from the first book of Virgil's *Georgics*, where the time for sowing wheat and other hardy grains after autumn is described from the morning [matutino] heliacal setting of the Pleiades, thus:

But if you work the soil for a crop of wheat and hardy spelt, and aim solely at the ears of grain, first let the Eoan daughters of Atlas [the Pleiades] set from your sight, and let the Gnosian star of the blazing Crown withdraw, before you commit to the furrows the seed they are owed, and before you hasten to entrust to a reluctant earth the year's whole hope.

[Margin: Plin. l. 18.]

From this description, and likewise from Pliny's interpretation [*Natural History*, Bk. 18], since the *morning setting* of the Pleiades is inferred—and indeed a heliacal one, since Virgil indicates it by the word "set" (abscondantur)—

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we assume first of all that, through the motion of the stars over an interval of about 1620 years which have elapsed from Virgil's time down to ours, we reduce the longitude of the middle of the Pleiades back to Virgil's era in this way:

[Margin: Per motum stellarum Tychonis Brahe.]

100 years give 1 degree 25 minutes [$1^{\circ}25'$ —]; therefore 1620 years give 22 degrees 57 minutes. The true longitude of the bright star of the Pleiades is 24 degrees 24 minutes $\square$ [Taurus] in the year 1600. Subtract 22 degrees 57 minutes, and the longitude of the middle of the Pleiades at Virgil's time becomes 1 degree 27

minutes □ [Taurus]; the invariable latitude is 4 degrees 0 minutes north [B = Borea]; magnitude 3.

Moreover, we assume the obliquity of the Zodiac at that time to be 23 degrees 50 minutes, and the elevation of the pole among the Romans to be 42 degrees 0 minutes.

With these things thus established, let the ancient marked position of the middle of the Pleiades be brought round to setting, on a globe or sphere, according to whose arrangement the following diagram is drawn.

With the rest arranged as before, let D be the middle star in the Pleiades, and H the corresponding point on the ecliptic. Let A O be drawn, representing the altitude semicircle. And since the Sun on the eastern horizon is understood to be depressed to 14 degrees—because the star is of the third magnitude—therefore, on the other side, we set the arc to be raised above the horizon by just as much, so that here we may track the point opposite to the Sun. Thus Z S is that arc of 14 degrees.

Diagram. A spherical-astronomy construction for the heliacal (morning) setting of the Pleiades. A large principal circle is drawn with great-circle arcs crossing it, and a small subsidiary circle in the upper-left region. The altitude semicircle runs from A (top) to O (bottom).

- A (top): upper endpoint of the altitude semicircle
- R (upper right): point on the rim
- Q (right of R): point on the rim
- C (upper left, on the small circle): pole of the subsidiary small circle
- B (within the small circle, upper left): pole point
- ☿/☼mark (upper center, near "I/T"): tropic of Cancer marker (the figure shows the Cancer glyph)
- D (center): the middle star of the Pleiades
- S (center right, below the Cancer marker): point on the ecliptic
- H (center, with subscript): point on the ecliptic corresponding to D; the small "Z" mark sits beside it
- K (center, right of H): intersection point
- W (center, below H): intersection point
- P (center left, below D): intersection point
- N (center, below P/W): intersection of arcs
- Y / ♈ (center, below N): the Aries glyph marker (vernal point)

- G (left rim): point on the horizon
- L (right rim): point on the horizon
- E (lower left): point on the rim
- F (lower left, below E): point on the rim
- O (bottom): lower endpoint of the altitude semicircle

The arc M Y (referenced in the text) measures the distance of the Pleiades from the tropic of Cancer $\square$ along longitude.

For the sake of students I shall repeat the same procedure [Greek: πραγματείαν (pragmateian, "treatment" or "procedure")], so that they may have in view the whole operation [Greek: πράξιν (praxin, "operation" or "working")] both for the true and for the heliacal rising and setting. First, then, since the declination D N is unknown, it is given from the first problem of Cap. 2; for in the triangle B C D, for B D, the complement of the declination, the data [Greek: διδόμενα (didomena, "the things given")] are:

- B C = 23 degrees 50 minutes — the distance of the poles;
- C D = 86 degrees 0 minutes — the complement of the latitude of D *;
- B C D = 58 degrees 33 minutes — the distance of longitude of * from the tropic $\square$ [Cancer], which is here measured by the arc M Y.

Therefore B D = 74 degrees 5 minutes, the complement of the declination, is given; and so the declination itself, D N = 15 degrees 55 minutes north [Borea].

II.

In the triangle B D C, for the angle C D B, the data [διδόμενα] are the three sides:

- B D = 74 degrees 5 minutes — the complement of the declination;
- C D = 86 degrees 0 minutes — the complement of the latitude;
- B C = 23 degrees 50 minutes — the distance of the poles.

Therefore C D B = 21 degrees 0 minutes is given.

III.

In the right triangle B G D, for the angle B D G, the data [Greek: δεδομένα (dedomena, "the things given")] are:

- B D = 74 degrees 5 minutes — the complement of the declination *;
- B G = 42 degrees 0 minutes — the elevation of the Roman pole;
- B G D = 90 degrees — right.

Therefore $B D G = 44$ degrees $5\frac{3}{4}$ minutes. From this is subtracted the angle $C D B = 21$ degrees 0 minutes, and the angle $C D G = 23$ degrees $5\frac{3}{4}$ minutes remains, to which the angle $H D M$ is equal.

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III.

Furthermore, in the right triangle $H M D$, two things are to be sought: the side $M H$, and the angle at H . First, then, for the side $M H$, in the same triangle:

- $D M = 4$ degrees 0 minutes — the latitude $\star$;
- $M D H = 23$ degrees $5\frac{3}{4}$ minutes — the angle just found;
- $D M H = 90$ degrees — right.

Therefore $M H = 1$ degree 42 minutes is given. To this is added the true longitude of the star at $M = 1$ degree 27 minutes $\square$ [Taurus], and the star's true setting is established at 3 degrees 9 minutes $\square$ [Taurus].

V.

In the same triangle $D H M$, the angle at H is obtained = 67 degrees 0 minutes; to this the angle $Z H S$ is equal.

VI.

In the right triangle $S H Z$, for $H S$, the data [διδόμενα] are:

- $S Z = 14$ degrees 0 minutes — by hypothesis;
- $S H Z = 67$ degrees 0 minutes;
- $S Z H =$ right angle.

Therefore $H S = 15$ degrees 14 minutes. When the rising point H —namely 3 degrees 9 minutes $\square$ [Taurus]—is added to this, the point S on the ecliptic is shown to be 18 degrees 23 minutes $\square$ [Taurus]; the point opposite to it the Sun will occupy at rising, namely 18 degrees 23 minutes $\square$ [Scorpio]: when the Eoan daughters of Atlas [the Pleiades]—that is, in the morning twilight—are submerged at setting. This formerly occurred at Rome around the 16th day of November; or, as the calendar now stands under the Julian reckoning, around the end of October.

Furthermore, if anyone should hereafter wish to make the test and compute the time for sowing wheat and so forth from the positions the stars hold today, he will find in this example—to which the rest will more or less correspond, owing

to the varying change of declination in the stars—that the same thing now occurs at Rome around the 24th day of November; which is too close to the winter solstice and oversteps the proper time for sowing. Therefore the risings and settings of the stars for the practice of agriculture must now be obtained afresh by astronomical calculation for the pole-elevation of the intended region (if they are to be sought from these signs).

CAPVT V.

On the matters that concern the investigation of time from observations of the Sun and the stars [siderum].

In the preceding chapter, on the rising and setting of the heavenly bodies, we set forth in advance the method for determining the quantity of day and night through the individual seasons of the year—since the variation in each, above and below 12 hours, depends solely upon the ascensional difference [differentia ascensionalis] converted into time. Therefore it now seems opportune to investigate any small portion of time—that is, any hours and minutes of hours—by observation carried out in various ways: on the Sun during the daytime, and on the stars at night. For this matter too has its necessary use in what follows; and besides, it has a delightful pleasantness, in that it invites the young mathematician to the duty of observing. Everywhere in this chapter there is a reciprocation of times with motions, just as we attached this to the general definition of astronomy.

It must, however, be carefully noted that we should not observe the Sun or the stars in this practice [pragmatia] when they are too near either the horizon or the meridian: in the former case, on account of the intrusion of refractions [refractionum]; in the latter, because the change in the altitude to be observed near the meridian is not sufficiently perceptible, even over a long time.

LIBER SECVNDVS SPHÆRICORVM — Caput V (continued)

PROBLEMA I.

Given the altitude of the Sun in a vertical circle — measured by any instrument or by any means whatever — together with the Sun's declination

and the elevation of the pole for the place, the corresponding moment of time in hours and minutes is found; and conversely.

Example I.

Let the Sun be in 3° of Leo ($\square$), which is the position appropriate to today's date, namely 16 July. Its declination — found either by Problem 1 of Chapter 1 of this book, or from the table of the Sun's declination — is then $19^\circ 34'$ north. Now since the elevation of the pole in our region is reckoned as $55^\circ 43'$, its complement, namely $34^\circ 17'$, is the elevation of the equator above our horizon. If the Sun's declination is added to this when northern, or subtracted from it when southern, the Sun's meridian altitude is always obtained on the one side or the other. So it is in the present example: add $34^\circ 17'$ (the elevation of the equator) to the Sun's northern declination of $19^\circ 34'$, and you get $53^\circ 51'$ for the Sun's meridian altitude. But what we are striving to determine is the time away from the meridian, derived from an observed altitude of the Sun.

Suppose, then, that the Sun's altitude is first observed — with a quadrant, or with some other instrument that serves in place of a quadrant — at this day's afternoon hour, as 40° , together with the other data fitted to the following diagram constructed from our sphere, in the following manner.

Diagram. A circle representing the celestial sphere in the plane of the meridian. The text identifies the construction as follows: the full meridian is A L F G H; the horizon is I G, with pole A; the equator is H F, with pole B. Since the pole's elevation is I B, its complement I H is the depression of the equator below the northern horizon, and F G (equal to it) is the elevation of the equator above the southern horizon. The Sun's declination on the meridian, F L, is $19^\circ 34'$, so G L is the Sun's meridian altitude (previously found as $53^\circ 51'$). Through the western semicircle is drawn the parallel L C N M, representing the Sun's declination everywhere away from the equator; the Sun's position by the first revolution is placed at C, from which quadrants descend from each pole onto their great circles.

- A (upper left): pole of the horizon (zenith)
- B (left): pole of the equator
- L (upper right): point on meridian, Sun's declination F L
- F (right): equator point
- G (right): equator point on horizon
- C (center-right): Sun's position
- D (center): foot of declination arc

- N (center-lower): intersection point
- E (lower-center-right): foot of altitude arc
- K (lower-center): intersection point
- I (left): horizon point
- M (lower left): point on the Sun's parallel
- H (bottom): lower meridian point

Thus E C is the given altitude of the Sun, 40° , and D C is the given declination, $19^\circ 34'$; but the arc we are seeking to determine is D F, by which (once converted into time) the Sun's hours and minutes from noon will be established. Now since the angle A B C measures the said arc, this same angle is therefore to be treated in triangle A B C, where the givens [διδόμενα, data] are:

- A B = $34^\circ 17'$ — the distance between the poles of the horizon and of the equator.
- B C = $70^\circ 26'$ — the complement of the Sun's declination.
- A C = $50^\circ 0'$ — the complement of the Sun's altitude.

Therefore A B C — that is, D F — is $46^\circ 23'$. This arc, converted into hours and minutes of hours according to what is taught in Problem 2 of Chapter 3, shows that the time from noon corresponding to a solar altitude of 40° is 3 hours 5 minutes 32 seconds.

If, on the other hand, one wishes by the reverse procedure to obtain the Sun's altitude from a given time, one first resolves the time — that is, the hours and minutes of hours — into degrees and minutes of degrees. Then, in triangle A B C, since two sides are given together with the angle contained at B, the side A C is easily found, and hence also its complement C E.

Furthermore, the Sun's altitude can also be observed by measuring the shadow cast by a staff — or by any oblong body erected perpendicular to the horizon — whose height, together with the said shadow, stands in a fixed and definite ratio.

Take an example using a staff conveniently divided into 10 equal parts; in the adjoining figure this is A B, erected at right angles upon the plane of the horizon, D C. Let the shadow cast by the Sun's rays be A C = 12 such parts as the staff is 10. Then, in the right-angled plane triangle, since two sides about the right angle are given:

Diagram. A right-angled plane triangle illustrating the shadow-staff method.

- B (upper left): top of the staff

- A (lower left): foot of the staff (the right angle)
- D (lower left, beside A): horizon point
- C (lower right): tip of the shadow

The vertical leg B A is the staff; the horizontal leg A C is the shadow; the right angle is at A.

- B A = 10 parts — the height of the staff,
- A C = 12 parts — the length of the shadow,
- B A C — the right angle.

The angle B C A is found to be $39^{\circ} 48'$ — the sought altitude of the Sun.

However, in observing solar altitudes of this kind, when the shadow falls from the tip of an erect gnomon, one must always subtract the Sun's apparent semidiameter from the angle obtained. This is because the Sun's rays, issuing from the uppermost edge of its disk and striking the top of the gnomon, render the shadow shorter than it should be — see Gemma Frisius, *On the Use of the Astronomical Staff* [Margin: Vale Gemmam Frisium de usu baculi Astronomici.] — as the writers on optics also demonstrate in the case of a wall-shadow. Hence the Sun's true altitude as observed here works out to $39^{\circ} 32'$, for which it will be easy to derive the corresponding time, according to the tenor of the problem — that is, the arc of the equator intercepted between the Sun's position and the meridian — once it has been established whether the observation was made before or after noon.

Indeed, the altitude of the shining Sun can also be obtained by means of our own stature, from which the time is then elicited. For since it is established that a man's measure is, as nearly as may be, that of a seven-foot rule [i.e. his own body length], the body should be set as nearly perpendicular as possible upon a level surface, and one must observe where the shadow cast by the Sun's rays from the crown of the head is fixed in the plane of the horizon. This is then to be measured off in one's own feet, and so a man's stature serves in place of a gnomon or staff, so that from it — just as in the foregoing example — the Sun's altitude may be made manifest. For just as there the 10 parts claimed first place for themselves in the rule of proportion, so here the 7 feet do, from which first the complement of the angle, and then the angle of the Sun's altitude at C itself, becomes known.

An Example of this kind.

Let someone's shadow, cast in the forenoon, be $10\frac{1}{2}$ feet. To determine the Sun's altitude from this, there will be — as before — in the right-angled plane triangle A B C:

- A B = 7 feet — the height of the man,
- A C = $10\frac{1}{2}$ — the projected shadow,
- B A C — the right angle.

[διδόμενα — the givens.]

Hence the angle B C A = $33^{\circ} 41'$ is given as the projected altitude of the Sun.

With the altitude found, take now (for the time) the Sun in $9\frac{1}{2}^{\circ}$ of Leo (♌), corresponding roughly to 23 July in our present century. Its declination will then be $17^{\circ} 56'$. And the elevation of the pole is given as $55^{\circ} 43'$.

In the diagram opposite above — since this is a forenoon time — with all the marks retained as before, for the angle A B C, which measures the Sun's distance D E along the equator from the meridian, the givens [διδόμενα] in triangle A B C are:

- A B = $34^{\circ} 17'$ — the distance between the poles.
- B C = $72^{\circ} 4'$ — the complement of the Sun's declination.
- A C = $56^{\circ} 19'$ — the complement of the Sun's altitude.

Therefore A B C is found to be $55^{\circ} 56'$. This angle, converted into time in the usual manner, gives 3 hours 43 minutes 44 seconds, which is the Sun's distance from noon expressed in time. If this is subtracted from 12 hours, it leaves the usual reckoning of time in hours and minutes, etc.: 8 hours 16 minutes 16 seconds, in the forenoon.

PROBLEMA II.

Given the right ascension of the Sun, and that of any star passing across the meridian of the place, to ascertain the time by night.

This problem we put first because — of all those that follow, which teach how to elicit the time by night from observations of the stars or the Moon — it is the simplest. It uses no calculation of triangles, and it paves the way for the rest.

For once the right ascension of mid-heaven has been found by the transit of a star, and the right ascension of the Sun has been subtracted from it, there

remains the arc of the equator intercepted between the Sun and the star, which, converted into time, gives the hours and minutes sought.

If, then, the meridian line of the place is exactly established, this practical method cannot fail in the least. But if it is not so well established, draw — as well as the eye allows — an arc from the Cynosure [Cynosura, the Lesser Bear / Pole Star] through the vertex of the heavens; this will be the meridian, unless the Cynosure deviates too far to one side, that is, around the meridian transit of the tropical points — bending slightly toward Cancer at its rising and toward Capricorn at its setting. The astronomer will easily meet this inconvenience with some adjustment.

Example.

Let the time be sought from the transit of the Bright Star of the Vulture [Iucida vulturis, i.e. Vega in Lyra/Aquila] across the meridian, for 13 September; on the night following, the Sun is then in $2\frac{1}{2}^{\circ}$ of Libra (♎). Therefore, by Problem 2 of Chapter 1 [Margin: E probl. 1 cap. 2.] — or from the table of right ascension — the Sun's right ascension is established as $181^{\circ} 50'$. The right ascension of the Bright Star of the Vulture, that is, of mid-heaven, is $293^{\circ} 0'$. The difference is $111^{\circ} 0'$. The corresponding time is 7 hours $24\frac{3}{4}$ minutes.

To assist the memory conveniently, however, we aid this practical method with the following table, in which we have reduced the right ascensions of the Sun and of the principal stars — for the completed year 1630 — into equinoctial times, that is, into equal hours and minutes, at ten-day intervals for each individual month. Once these are obtained and committed to memory, the remaining intermediate values, as far as the Sun is concerned, can easily be reckoned by adding here, namely, 4 minutes of time for each single day, since the positions of the stars remain almost perpetually fixed.

So, once a star is seen on the meridian, and the assigned equinoctial time of the Sun is subtracted from it, the time sought immediately remains.

One must remember, moreover, that 24 hours are to be added to the time assigned to the star whenever the subtraction cannot otherwise be performed. This nautical practice — by means of the nocturnal [nocturnale, the nocturnal instrument] applied to the Cynosure and the stars in the Great Bear — is the more reliable the larger the sweep of heaven over which it is exercised; and it is the more agreeable the less it requires of that instrument. I therefore wish it to be held in its due place and value by all persons of good judgment.

Diagram — a scheme for ascertaining the hours and minutes by night, under any longitude and latitude.

Diagram. A large circular volvelle/dial for determining nighttime hours and minutes under any longitude and latitude. It is organized as concentric rings about a central hub.

- **Center hub:** labeled **Polus Mundi** ("the Pole of the World").
- **Outermost rim — the twelve months of the year**, running clockwise around the circle, each subdivided into the days 1 / 10 / 20: **IVLIVS** (July, top), **IVNIVS** (June), **MAIVS** (May), **APRILIS** (April), **MARTIVS** (March), **FEBRARIVS** (February), **IANVARIVS** (January, bottom), **DECEMBER**, **NOVEMBER**, **OCTOBER**, **SEPTEMBER**, **AVGVSTVS** (August).
- **Second ring — equinoctial-time values** (hours and minutes in small numerals) tabulated against the calendar dates around the rim. Sample readable entries in the upper (summer) sector, reading near the July/June boundary: 8·37, 7·57, 7·21 (under July 10/20); 6·36, 5·54, 5·17 (under June). Adjacent August entries: 9·57, 9·19. Many of these small time-values around the lower half of the wheel are printed inverted relative to the page and are only partly legible at scan resolution.
- **Inner ring — names of the principal fixed stars**, arranged by right ascension, each in its angular sector. Readable star labels, proceeding around the inner ring:
 - **Spica** ☐ (Spica Virginis)
 - **Vindemiatrix** [Vindemiatrix]
 - **Cauda** ☐ (the Lion's tail, Denebola)
 - **Cor.** ☐ **Reg.** (the Lion's heart, Regulus — abbreviation reads "Reg.")
 - **Cor. Hydrae** (the Hydra's heart, Alphard)
 - **Præsepe** ☐ (the Manger / Praesepe in Cancer)
 - **Procyon**
 - **Seq. Hum. Orionis** (the following shoulder of Orion, Betelgeuse) [reading "Seq. Hum. Orionis"]
 - **Præc. Hum. Orionis** (the preceding shoulder of Orion, Bellatrix)
 - **Capella**
 - **Aldebaran**
 - **Luc. Pleiadum** (the Bright Star of the Pleiades)
 - **Luc. Mandib. ceti** (Bright Star of the Whale's jaw, Menkar), with the ☐ (Aries) zodiac glyph in the adjacent sector

- **Caput Andromedæ** (the Head of Andromeda)
- **Scheat Pegasi** (Scheat)
- **Polaris** [?]
- **Denominatio Stellarum** ("the naming of the stars" — the ring's column heading, not a star)
- **Caput Cygni** [?] (the Head of the Swan, Deneb)
- **Lucida Aquilæ** (the Bright Star of the Eagle, Altair)
- **Cauda Cygni** (the Swan's tail)
- **Lucida Lyræ** (the Bright Star of the Lyre, Vega)
- **Lucida Ophiuchi** (the Bright Star of Ophiuchus)
- **Præc. Cornu Ophiuchi** [?] (preceding horn of Ophiuchus)
- **Cor** ♏ (the Scorpion's heart, Antares)
- **Lower margin of the inner field:** small column-key abbreviation marks at the foot of the inverted half — **D** (dies), **M** (mensis), **HM** (horæ/minuta), and **WH** [?] (uncertain — possibly an inverted "HM") — appear near the bottom alongside inverted day-numbers (1 / 10 / 20).

Note: this is the nocturnal-dial figure flagged in the batch instructions. Many peripheral labels in the lower (winter-months) half of the wheel are printed upside-down relative to the page and are degraded at scan resolution. After further diagram verification, most labels are resolved; three star labels (Polaris, Caput Cygni, Præc. Cornu Ophiuchi) and the "WH" lower-margin mark remain at the scan-DPI floor and stay [?].

Example.

Let the time be sought on the night preceding 25 December of this present year 1617, when the Bright Star of Aries [Lucida ♈] transits the meridian. The time derived from the Sun's right ascension, for 20 December, is found to be 18 hours 38 minutes; to this, for the roughly 4½ intervening days, 18 minutes are added, accumulating to 18 hours 56 minutes. But the time corresponding to the right ascension of the Bright Star of Aries is 1 hour 46 minutes — or, with a whole revolution added, 25 hours 46 minutes. The difference is therefore 6 hours 50 minutes, which indicates the time of the Bright Star of Aries' transit through mid-heaven on that night; and conversely.

PROBLEMA III.

Given the right ascension of the Sun, together with the right ascension and declination of some star, and likewise the altitude of that star from

observation, under a definite elevation of the pole: the corresponding nighttime is found.

Let it be, for the same 13 September, that the altitude of the Bright Star of Aries [lucida $\square$], observed with a quadrant, is 25° , in the eastern part of the sky. The star's longitude is given as $2^\circ 14'$ of Taurus ($\square$), its latitude $9^\circ 57'$ north; whence also its declination is $21^\circ 36'$, and its right ascension $26^\circ 21'$.

Diagram. A circle representing the celestial sphere in the plane of the meridian, with the same lettering scheme (A–H, with interior points C, D, E, K) as the diagram for Problem I, used here for the nighttime star-altitude solution.

- A (top): zenith / pole of the horizon
- B (upper right): pole of the equator
- F (left): point on the sphere
- C (center): the star's position
- D (center-left): foot of declination arc
- G (left): horizon point
- I (right): horizon point
- E (lower center): foot of altitude arc
- K (lower center): intersection point
- H (bottom): lower meridian point

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(continuing PROBLEMA III)

The rest is granted from what precedes. Therefore, in the figure set out (with the lettering kept as in the preceding case), for the triangle ABC, taking the angle at B — that is, the arc DF, the distance of the point of the star's right ascension from the meridian — the given quantities [διδόμενα, the data] are:

- AB = $34^\circ 17'$, the distance of the poles;
- BC = $68^\circ 24'$, the complement of the declination of the star, DC;
- AC = $65^\circ 0'$, the complement of the altitude of the same star, namely EC.

Therefore the sought angle ABC = $76^\circ 56'$ is given, whose measure lies along DF.

When this angle is subtracted from the right ascension of the bright star of Aries [lucida Arietis = Hamal], $26^\circ 21'$ — a whole circle having first been added — there

remains the right ascension of mid-heaven [medium coeli], $309^{\circ} 25'$. From this, in turn, subtracting the right ascension of the Sun, $181^{\circ} 50'$, there remains the arc between mid-heaven and the Sun, $127^{\circ} 35'$, which, converted into time, gives 8 hours $30\frac{1}{2}$ minutes. But if the altitude of the star is observed in the western part of the sky, the sought angle must be *added* to the star's right ascension, so as to make up the right ascension of mid-heaven, and so on.

PROBLEMA IV.

Given the elevation of the pole of the place and the right ascension of the Sun, and in addition two stars granted as lying on one and the same vertical circle: there is found not only the nighttime, but also the altitude of either of the proposed stars.

Although this problem makes work for the computer in the solution of several triangles, it nonetheless has a notable and ready use whenever an eclipse of the Moon — or some other moment requiring exact timing — must be observed accurately by night, using only a plumb-line [amussis perpendicularis] to mark out the two stars on a single vertical. And the demonstration [ἀπόδειξις (apodeixis, "proof, demonstration")] and procedure [πρᾶξις (praxis, "practice, working")] in the following example are as follows.

For the observation of the bright star of Andromeda's girdle [lucida balthei Andromadae = Mirach] and the second star in the horn of Aries [secunda in cornu Arietis], on one rising vertical circle the following figure is constructed. In it — the rest being as in the preceding cases — AE is the vertical, C is the bright star in Andromeda's girdle, and D is the following star in the horn of Aries.

Diagram. A circle representing the meridian plane (the projection scheme used throughout Problemata I–III), with cardinal points and interior lettering for the spherical-triangle construction. Labeled points around and within the circle:

- N (upper region, outside the circle, near top): pole / zenith-region marker
- A (near top, inside, just left of B): vertex of the triangles
- B (top right, inside near rim): second triangle vertex
- F (left rim): point on the meridian/equator (mid-heaven side)
- C (upper interior): the bright star of Andromeda's girdle [Mirach]
- D (interior, below C): the following star in the horn of Aries
- I (right rim): cardinal point (east)
- L (left interior): intermediate point
- M (lower-central interior): intermediate point

- E (lower-central interior, below M): point on the vertical
- K (lower-central interior, near E): center / intermediate point
- G (left rim, lower): cardinal point (west)
- H (lower right rim): cardinal point

Now, since our aim is to investigate the angle ABC or ABD — which measures the arc on the equator from mid-heaven, FL or FM — and since in triangle ABC at least two sides are given, namely:

- AB, the distance of the poles in this place, $34^{\circ} 17'$;
- and BC, $56^{\circ} 25'$, the complement of the declination of Andromeda's girdle [Mirach];

or [for the other triangle] BD, $71^{\circ} 8'$, the complement of the declination of the following star in the horn of Aries.

Therefore in triangle CBD, from the given BD (the complement of the declination of the horn of Aries, $71^{\circ} 8'$), and BC (the complement of the declination of Andromeda's girdle, $56^{\circ} 25'$), together with the angle CBD (the difference of the right ascensions of the bright star of Andromeda's girdle and of the following star of the horn of Aries, $12^{\circ} 20'$): there is given DC, the distance of those same stars, $18^{\circ} 23'$.

[Margin: I.I.]

Furthermore, in triangle BDC, for the angle at D, since all three sides are given: namely:

- DC = $18^{\circ} 23'$
- DB = $71^{\circ} 8'$
- BC = $56^{\circ} 25'$

Therefore the angle CDB = $34^{\circ} 23'$ is given.

[Margin: I.I.]

In triangle ABD, for the angle at B — which measures the distance of the bright star of Aries [lucida Arietis = Hamal] from mid-heaven [M.C.] — three quantities are given, namely, the two sides:

- AB = $34^{\circ} 17'$, the distance of the poles;
- BD = $71^{\circ} 8'$, the complement of the declination of the following star in the horn of Aries;

[Margin: See the *Analysis of Right-Triangles* (Vide Analysin Orthogoniorum).]

and finally the angle just found at D. So the proposed angle at B is given. For, dropping the perpendicular arc BN, which falls *outside* the triangle — since the angle BAD is obtuse — the triangle NBD will be divided into two right triangles, BND and BNA.

First, for the perpendicular NB, it will be $32^{\circ} 18'$. Next, in the right triangle BND the angle at B is given, $77^{\circ} 31'$. Lastly, in the right triangle BNA the angle NBA is found, $21^{\circ} 59'$; and this, subtracted from the previously acquired angle NBD (namely $77^{\circ} 31'$), leaves the angle ABD = $55^{\circ} 32'$, which measures the distance of the following star of the horn of Aries from mid-heaven [M.C.].

Now the right ascension of this star is assumed to be $23^{\circ} 18'$; and when a whole circle is added to it, this makes up $383^{\circ} 18'$, from which the angle ABD, $55^{\circ} 32'$, being subtracted, leaves as remainder the sought right ascension of mid-heaven [M.C.], $327^{\circ} 46'$. If from this right ascension of mid-heaven there be taken away the right ascension of the Sun appropriate to any given time, there will be established the arc of the equator to be converted into hours and minutes of time, according to the method repeated several times above.

It now remains for us to carry out the other part of the present problem, namely, finding the altitudes of the said stars accordingly; the demonstration of which is founded on the preceding problem.

For here too, for [side] DA, two sides are given together with the included angle, namely:

- AB = $34^{\circ} 17'$, the distance of the poles;
- BD = $71^{\circ} 8'$, the complement of the declination of the following star in the horn of Aries;
- ABD = $55^{\circ} 32'$, the angle of the distance from the meridian.

Therefore DA = $55^{\circ} 20'$, whose complement — namely $34^{\circ} 40'$ — is the altitude of the star in the horn of Aries. And if to this you add the distance of the stars previously found, say $18^{\circ} 23'$, the altitude of the bright star in Andromeda's girdle [Mirach] will be $53^{\circ} 3'$. Moreover, from this example it is clear how one may reciprocally recover or investigate the altitude — both of the Sun and of the stars — from a given time from noon, together with the declination and the elevation of the pole. This often has its use, especially in computing lunar eclipses, where the altitudes of [the luminaries], for the sake of knowing the time and the parallaxes, come first from the calculation.

CAPVT VI.

On the foundation of the rational method of constructing the celestial houses for any given time, of determining the aspects, and of setting up — and conversely [resolving] — the direction of a significator to its promissor, and of working it out geometrically in numbers: wherein the kernel of practicing astrology is set forth demonstratively.

PROBLEMA I.

Given the time and the elevation of the pole of the place: the cusps of the 12 celestial houses are given on the ecliptic according to the rational method.

Having completed, according to our intention, the spherical astronomy which considers the celestial face as anciently depicted with the constellations, we shall now subject to practice the use of the circle of position [circulus positionis] — properly serving astrology — in the manner in which we examined it in chapter 7 of the prior book. We shall show demonstratively (as in all else) how, according to both the rational method (which is ascribed to Johann Regiomontanus [Regiomontanus]) and the other method which others prefer more, that of Campanus and Gazulus [Campanus of Novara & Johannes Gazulus], the erection of a celestial figure [thema coeleste] for any given time is accomplished, and the direction of any significator to its promissor is carried out. Once these have been brought forward at a single glance from a geometric source — which we have scarcely yet seen attempted by anyone — an understanding of the tables of directions that survive in the works of Johann Regiomontanus and others (and of those more corrected ones which posterity will one day produce) will be readily gained here and there, and their use more promptly facilitated.

Let us take, as a paradigm or example of this procedure [πραγματεία (pragmateia, "treatise, methodical handling")] to be set forth demonstratively in numbers, a certain notable revolution of the heaven [coeli revolutio], which occupies in culmination $5^{\circ} 14'$ of Aries, and that under an elevation of the pole of 56° less two sixtieths [i.e. $55^{\circ} 58'$]. For which time the place of the Sun is found, from the corrected tables, to be $29^{\circ} 30'$ of Aries, whence is given the corresponding right ascension, $27^{\circ} 45'$.

With these things thus found and assumed for the given time and place: we first of all seek the right ascension of mid-heaven [M.C.], which, according to what was delivered in the preceding chapter, is presented as $5^{\circ} 25'$. For to this right ascension there corresponds — by the inversion [per anastrophē] of Problema 2

of chapter 1, or of the table of right ascensions, as we said — the longitude $5^{\circ} 14'$ of Aries, the beginning of the tenth house. [Margin: Problema 2, chapter 1 (*Probl. 2, cap. 1*).] And so, the position of the heaven being given, and the beginning of the tenth house [being established], we now reason demonstratively in the following manner, in order to express conveniently in the subsequent figure the remaining [cusps].

Diagram. A primitive-circle projection of the celestial sphere (an astrological house-chart construction): the meridian circle ABED with horizon DE, half of the equator FG, and half of the ecliptic YI, drawn so that the beginning of Aries ($\square$) and the equinoxes are indicated. The half-equator is divided into six equal parts (three above the earth, three below) by the marks L M K N R, with the meridian FG determining the figure. The circle of position is raised and depressed from the horizon so as to graze each section of the equator; where it cuts the ecliptic, the house cusps are marked. Labeled points:

- A (top): upper meridian point
- B (upper right): point where the circle of position meets near the 10th-house region
- H (left, outside): marker near the ecliptic pole / " $\square$ 10" label region
- $\square$ 10 (upper left, near H): beginning of Aries, cusp of the 10th house
- F (upper left rim): equator endpoint (mid-heaven side)
- O (upper interior): cusp of the 11th house
- Y (top interior, near A): culmination point (cusp of 10th house in the figure)
- D (left rim): horizon point (east end)
- E (right rim): horizon point (west end)
- L, M, K, N, R (interior, along equator division): the six equal divisions of the half-equator
- 12 (interior, right of O): cusp of the 12th house region marker
- S (right interior): cusp of the 12th house
- T (right interior, lower): cusp of the 1st house (horoscope/ascendant)
- V (lower right interior): cusp of the 2nd house
- I (right rim): ecliptic endpoint
- P (right rim): point on circle of position
- $\square$ /Q (lower right): cusp-region marker (3rd house)
- 3 (lower right interior): 3rd house marker
- 1, 4 (bottom, near G/C): house markers (1st/4th)
- G (lower right rim): equator endpoint

- C (bottom): lower meridian point

First, for the suitable delineation of the figure to be represented in the plane: let the meridian ABED be drawn, with the horizon DE, half of the equator FG, half of the ecliptic YI — so applied in the present case that the equinoxes are indicated through the beginning of Aries ($\simeq$]. And the equinoctial points] [let the half-equator be divided into six equal parts — three above and as many below the earth — by the marks L M K N R, and by the meridian FG determining the figure.

Then let the circle of position in our sphere be so raised from the horizon itself, and depressed, according to what was delivered above, that it grazes the individual sections of the equator; and where it intersects the ecliptic, let the cusps of the houses be marked. Among these, the tenth house occupies the culmination at Y; the eleventh at O; the twelfth at S; the first at T; the second at V; the third at $\square$ [Q]; the fourth, or the lowest point of the heaven [imum coeli], at I.

With these things thus arranged in the eastern hemisphere, everything in the averted, or western, hemisphere follows of its own accord in the contrariety of the signs and so on — since indeed every great circle falling upon another great circle divides it into two opposite points, as is shown clearly enough in our sphere, and as will later be further indicated in the present example.

But for the practice [praxis] in finding the houses — although one may proceed by various methods — we shall nonetheless first adopt the one which best serves for explaining the tables of Regiomontanus. And since the circle of position is nothing other than a movable horizon, as was shown above, the first care therefore will be to acquire, in the upper hemisphere, after the elevation of the world, above the circle of position, the twelfth and eleventh houses — of which the former is represented by the arc BP, the latter by BW. Equal to these are the arcs below the earth, CX and CZ.

Now we shall fit the calculation to the investigation of BP.

Accordingly, in the right triangle MFD, for the angle M — to which is equal the complement of the sought arc BP, like FKD, the complement of the elevation of the pole BE — the given quantities [διδόμενα] are:

- DF = $34^{\circ} 2'$, the complement of the elevation of the pole;
- DM = $60^{\circ} 0'$, the arc on the equator, or the right ascension;
- MFD = a right angle, corresponding to the 12th house by hypothesis.

Accordingly the angle DMF = $37^{\circ} 57'$ is given; whose complement, namely $52^{\circ} 3'$, is the sought arc BP, that is, the elevation of the pole above the circle of position of the 12th house — which same value is found in Regiomontanus's tables of directions. [Margin: p. 109 (*Pag. 109*).]

In the same way, in the right triangle LFD, for the angle at L — by whose complement the arc BW is measured (namely the elevation of the pole above the circle of position of the eleventh house) — the given quantities [διδόμενα] are:

- DF = $34^{\circ} 2'$, the complement of the elevation of the pole;
- FL = $30^{\circ} 0'$, the arc of the equator, or the oblique ascension, corresponding to the eleventh house by hypothesis;
- LFD = a right angle.

Therefore the angle FLD = $53^{\circ} 29'$ is given, and its complement is BW = $36^{\circ} 31'$, the sought elevation of the pole at the cusp of the eleventh house — which is likewise found in Regiomontanus's tables.

With the elevations of the pole above the circles of position granted, the remaining practice is for the investigation of the arcs of the ecliptic at the beginnings of the houses. To this we now come — from the supposed oblique ascension of each house and the given elevation of the pole of the same — into a knowledge of the sought [arc] (by means of the resolution [ἀνάλυσις (analysis)] of triangles). Here, although the solution could be given by means of a dropped perpendicular arc and the right triangles thence following, the method is nonetheless more expeditious which our principles exhibit in oblique triangles; and to those not yet practiced with equal facility among other writers, we are glad to assign these examples too.

So let it be, for the first example, that the point of rising — or the horoscope [horoscopus] — on the ecliptic is to be sought at T. On account of which, first all the angles of the triangle TKQ will be had; then the sought side QT will emerge.

For in the same triangle TQK, for the obtuse angle at T, the given quantities [διδόμενα] are:

- angle TQK = $23^{\circ} 32'$, the maximum obliquity of the ecliptic;
- angle TKQ = $34^{\circ} 2'$, the complement of the elevation of the pole;
- side QK = $84^{\circ} 35'$, the complement of the semicircle on the equator, that is, the arc HK.

Therefore the sought angle KTQ = $137^{\circ} 27'$ is given.

Now, the three angles of the proposed triangle KTQ being given, the sought side TQ is given. The given quantities [δεδόμενα] of the angles, however, are these:

- $TQK = 23^\circ 32'$, the obliquity of the ecliptic;
- $KTQ = 137^\circ 37'$, the angle just found;
- $TKQ = 34^\circ 2'$, the complement of the elevation of the pole — which angle is opposite to the sought side TQ.

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Therefore the sought side TQ itself is given as $55^\circ 45'$. If this is subtracted from six signs, the ascendant or horoscope [horoscopus] remains at 15° Leo ($\square$) — very nearly the same as what is found in Regiomontanus's [Regiomontanus] tables of oblique ascensions under a pole elevation of 56° , given an oblique ascension of $95^\circ 25'$.

II. For the cusp of the twelfth house.

We proceed in the same way for the sides HS and HO. For in triangle SHM two angles are conceded: angle SHM $23^\circ 32'$ (the obliquity of the ecliptic), and HMS $142^\circ 3'$ (the supplement of angle HMD, found earlier); together with the side HM $65^\circ 25'$, which is given as the oblique ascension of the twelfth house by hypothesis. Therefore angle HSM is given as $34^\circ 24'$.

Now, in the same manner as above, with the three angles:

- SHM $23^\circ 32'$
- HSM $34^\circ 24'$
- HMS $142^\circ 3'$

the side HS will then not be unknown: $98^\circ 10'$, the sought arc in the ecliptic measured from the vernal equinox. For this reason it terminates at $8^\circ 10'$ Cancer ($\square$), and this is the point of the ecliptic that the cusp of the twelfth house occupies. The same value we obtain — very nearly — from Regiomontanus's tables of oblique ascensions, at an elevation of 52° and the supposed oblique ascension of $65^\circ 25'$.

III. For the cusp of the eleventh house.

We obtain the cusp of the eleventh house by a not dissimilar procedure. For in triangle HLO, first for the angle at O the givens [διδόμενα, the data] are:

- OHL $23^{\circ}32'$ (the obliquity of the ecliptic)
- OLH $126^{\circ}31'$ (the complement of FLD to a semicircle)
- HL $35^{\circ}25'$ (the oblique ascension of the eleventh house, by hypothesis)

Therefore HOL is given as $36^{\circ}11'$ [recte: $36^{\circ}12'$; the value used in the very next computation is $36^{\circ}12'$].

For the side HO, the givens [διδόμενα] are:

- OHL $23^{\circ}32'$
- HOL $36^{\circ}12'$
- HLO $126^{\circ}31'$

Therefore HO is given as $52^{\circ}3'$, the elongation from the vernal equinox. Subtracting from this a whole sign — namely Aries — the remainder shows the sought cusp of the eleventh house at $22^{\circ}3'$ Taurus (♉); likewise to be found, very nearly, at an elevation of $36^{\circ}31'$ from Regiomontanus's tables.

By precisely this same procedure, the beginnings of the second and third domiciles below the earth are found. First, for the cusp of the second house — namely V — in triangle VQN, since the givens are:

- angle VQN $23^{\circ}32'$ (the obliquity of the ecliptic)
- QNV $37^{\circ}57'$ (equal to angle FMD)
- side NQ $54^{\circ}35'$ (the distance in the equator from the autumnal equinox, which arises from the complement of the oblique ascension of the second house; and this in turn from four signs, that is 120° , added to the right ascension of the M.C. [mid-heaven])

Therefore, as above, there is given —

first, the angle QVN $125^{\circ}30'$;

then, the side QV $38^{\circ}0'$, which, if subtracted from a semicircle, leaves the cusp of the second house marked at $22^{\circ}0'$ Leo (♌) — the same, namely, as is offered in the tables of oblique ascensions at a pole elevation of $52^{\circ}3'$, with the start made from the vernal equinox in Aries.

Finally, for the cusp of the third house.

Since in triangle SQR the givens are:

- angle SQR 23°32' (the obliquity of the ecliptic)
- SRQ 53°29' (equal to the previous angle FLD)
- side RQ 24°35' (the complement of the oblique ascension to a semicircle)

Therefore [continues on next page]

Therefore there is given: RSQ 104°42'; SQ 20°13', the complement of the sought cusp of the third house to a semicircle, as before. Hence the said cusp falls at 9°47' Virgo (♍), likewise to be drawn out — very nearly — from the tables at a pole elevation of 36°31'.

Now that six domiciles have been found by means of the triangles, as a foundation — or else from the tables, whether of oblique ascensions or of houses in the ephemerides — the remaining six present themselves spontaneously in the opposite signs, degrees, and minutes, as follows.

Order
of
domicil
es

found	10	11	12	Horosc.	2	3
Cusp of domicil e	5°54' Aries (♈)	22°3' Taurus (♉)	8°10' Cancer (♋)	4°15' Leo (♌)	22°0' Leo (♌)	9°47' Virgo (♍)
Opposit e domicil es	4	5	6	7	8	9
Cusp of domicil e	5°54' Libra (♎)	22°3' Scorpio (♏)	8°10' Caprico rn (♐)	4°15' Aquari us (♑)	22°0' Aquari us (♑)	9°47' Pisces (♐)

Parsing notes: The table has two cusp rows (the upper six houses 10/11/12/Horosc./2/3 and their opposites 4/5/6/7/8/9), each with degree-and-minute values and a zodiac sign glyph per cell. The "Horoscope" column corresponds to the ascendant. Note the cusp of the 2nd house here reads 22°0' Leo, whereas p. 113 derived the 2nd-house cusp at 22°0' Leo as well — consistent.

After the cusps of the twelve domiciles have been found, they are to be inscribed, in order, into the figure customary among astrologers. We have chosen to set it down here as well, for the benefit of beginners.

Scheme of the twelve celestial domiciles.

Diagram. An astrological chart of the standard square-within-circle form, drawn as a circle divided into twelve sectors radiating from a central panel. The central panel is labeled *Elevat. Poli gr. 55 m. 58* [Elevation of the pole, 55°58']. Around the rim, each of the twelve house-cusps is labeled with its degree, minute, and zodiac sign, matching the table above. Reading the labels around the figure:

Reading the twelve rim cusp-labels (each = degree·minute + zodiac sign), clockwise from the top:

- top-center: $\square$ 5°54' (House 10 / Medium Coeli)
- upper-right: $\square$ 9°47' (House 9)
- right-upper: $\square$ 22°3' (House 8)
- right: $\square$ 4°15' (House 7)
- lower-right: $\square$ 8°10' (House 6)
- bottom-right: $\square$ 22°3' (House 5)
- bottom-center: $\square$ 5°54' (House 4 / Imum Coeli)
- lower-left: $\square$ 9°47' (House 3)
- left: $\square$ 22°2' [figure engraves "22·2"; cusp table gives 22°0' — House 2]
- left-center: $\square$ 4°15' (House 1 / Horoscopus / Ascendant)
- upper-left: $\square$ 8°10' (House 12)
- top-left: $\square$ 22°3' (House 11)

All twelve labels correspond to the cusp table printed above the figure; the central panel reads *Elevat. Poli gr. 55 m. 58*.

PROBLEMA II.

Given [διδόμενοις], as before, what was set forth: the cusps of the 12 celestial domiciles in the ecliptic are given, according to the method of Campanus [Campanus of Novara] and Gazulus [Gazulus] and other more recent writers.

Here the principal vertical circle, at its rising and setting, is divided into 12 equal parts — that is, each quadrant into three equal parts — just as the equator was before. As the circle of position sweeps across these individual sections, one must

observe where it simultaneously cuts the ecliptic; and there the beginnings of the domiciles are to be fixed.

And although this method of dividing the heaven into its 12 domiciles has been less commonly used, it is nevertheless more general than the preceding one, since by the alternative method the heaven cannot be so divided when the pole of the world stands at the zenith. For in that case the equator is so joined with the horizon that it cannot be cut by the circle of position at all. Nevertheless, the cusps of the four principal domiciles still agree with the preceding ones — namely the ascendant, the lower midheaven [imum cœli], the midheaven [medium cœli], and the descendant.

For the point of the midheaven [M.C.] found in the ecliptic at $5^{\circ}54'$ Aries ($\square$), the declination is sought, which in the adjoining figure is $PK = 2^{\circ}21'$. Hence the side OK is established as $36^{\circ}23'$. For OP is $34^{\circ}2'$, the complement of the pole elevation; next, the angle KOL is known to be 30° by hypothesis, whose measure is AD; and furthermore the angle LKO is to be found, which is the complement of angle HKP — and this turns out to be $66^{\circ}35'$. Therefore the said angle OKL is $113^{\circ}25'$.

Diagram. A circle-of-position construction for the Campanus/Gazulus house division, inscribed in a great circle with a horizontal diameter OC and a vertical diameter A–B. Labeled points, reading the figure:

- A (top of circle): apex of the principal vertical circle
- B (bottom of circle): nadir
- O (left end, horizontal diameter): observer's point on the horizon
- C (right end, horizontal diameter): opposite horizon point
- D, E, F, G, I (descending on the meridian A–B): successive house-division intersection points
- H, K, L (upper-left interior): triangle KOL for the 11th/12th-house construction; house-numbers "10", "11", "12" cluster here
- P, Z (left-center interior): auxiliary points of the right-triangle OPZ (text: "In triangulo orthogonio OPZ")
- M (center interior, below E/F): auxiliary point (triangle OKM)
- N (center-right, on diameter OC): intersection point; house-number "2" beside it
- t (small label between N and the lower-right cluster): auxiliary intersection point

- R, S, W, X (lower-right interior): the sub-terrestrial 2nd/3rd-house construction (text: CX = KO, angle CXS = angle HKP = 67°9', angle RCX = 60°); house-numbers "3" and "4" cluster here
- [Margin, right, upper]: *Per 1 Probl. cap. 1, vel a tabula decl. solis.* [By Problem 1 of Chapter 1, or from the table of the Sun's declination.]
- [Margin, right, lower]: *Per Probl. 4. cap. 1.* [By Problem 4 of Chapter 1.]

Hence for the side KL, since in triangle KOL the givens [διδόμενα] are:

- KOL 30°0' (by hypothesis)
- LKO 113°25' (the angle just found)
- KO 36°23' (the side previously acquired)

there is given, first, the angle at L = 44°26';

then the sought side KL = 25°4'. When HK is added to this, HL is produced as 0°58' Taurus (♉), the sought cusp of the eleventh house.

From the givens [διδόμενοις]:

- KO 36°23'
- OKM 113°25'
- KOM 60°0' (by hypothesis)

If the M.C. of 5°54' is added to this, the cusp of the twelfth house is revealed at 16°25' Gemini (♊); but the cusp of the first house remains as before.

Furthermore, the same procedure is to be used below the earth, for acquiring the third and second houses. For CX is equal to KO = 36°23', and angle CXS is equal to angle HKP = 67°9', and angle RCX = 60 parts. From these the cusps of the third and second houses easily become known, observing those things in the solution of the given triangles that we have so often repeated here.

It now remains for us to disclose, by a single example, Regiomontanus's [Regiomontanus] method regarding the construction of the table — which is entitled *Table of Houses according to Campanus and Gazulus* — and then to apply the rule [canon] to use in this matter.

Two things are required here: the elevation of the pole above the circle of position of the house cusps, and the quantity of the equator enclosed within each individual domicile. Once these are had, the cusps of the individual domiciles are sought from the table of houses under the appropriate pole, as before. We shall,

however, give in the present case the example of the establishment of the eleventh house.

In the right-angled triangle OPZ, for the arc of the equator PZ — with OP set as the whole sine [sinus totus] — PZ will be the tangent of the angle opposite at O.

Therefore:

	OP		PO Z
as 90°	34°2'	so	30° to PZ
S.T. 100000	S.R. 55967	T. 57735	T. 32313, answering to 17°54'

[Sine Total 100000; Sine of Remainder 55967; Tangent 57735; Tangent 32313, giving the result 17°54'.]

Next, the angle PZO is sought by the first rule of prosthaphaeresis [prosthaphaeresis]; for:

	compl. PZ	POZ	compl. Z
as 90°	55°58'	so 30°	to 41435, answering to 24°29'
S.T. 100000	S.R. 82871	S.R. 50000	(the pole elevation itself)

Furthermore, if we now enter the table of oblique ascensions with an oblique ascension of 23°19' (for only this much is aggregated from HP and PZ), together with the pole elevation of 24°29', the cusp of the eleventh house answers at 0° [continued on next page]

0°55' Taurus (♉), with everything rounded for the adhering minutes — which does not differ very much from our earlier finding, though it does leave a certain discrepancy for definite reasons.

And from these things we judge it has been sufficiently shown how the erection of the domiciles is accomplished from the foundation of the triangles. Now we shall proceed to the directions [directiones] themselves.

PROBLEMA III.

Given the latitude or declination of a planet or star: the limit of longitude of the aspect with respect to the ecliptic, or to the equator, is given.

[Margin: *Vide Prob. 31. Tab. Direct. Regiomont.* — See Problem 31 of Regiomontanus's *Tabulae directionum*.]

Since it is of great concern to astrologers, for the practice of their art, to know how the stars — pouring forth their rays — affect these lower things, and also which latitudes require limitation, we have chosen to insert this problem before the doctrine of directions.

[Margin: *Lib. 1. c. 6.* — Book 1, Chapter 6.]

Now it must be understood that, although the stars vibrate and diffuse their lights with their efficacy orbicularly in every direction, they nevertheless operate by certain hidden ratios, as it were — chiefly according to the rational division of the circle: namely in the sextile, opposition, trine, square, and sextile [♌ in 60°, ☉, △, □, ★], as these aspects were set before the eyes above, and determined by definite measures. But in the conjunction, opposition, and square [♌, ☉, □], the digression of the planet into latitude introduces nothing that would either subtract from or add to its longitude.

For the opposition is diametrical, and falls into the contrary latitude; the conjunction remains [coincident]; and finally the place of radiation of the square [□] always stands distant from the place of the star's longitude by a quadrant of the circle. But in the remaining aspects — namely the sextile (★) and trine (△) — one must, with respect to longitude, proceed as follows in limiting according to the digression of latitude.

Example for both, in the Sextile and Trine, given any latitude whatever, namely 20°.

Diagram. A circle with a horizontal diameter B–C and an interior triangle.
Labeled points:

- A (upper interior): the star, whose rays, diffused orbicularly, extend to the sextile aspect through the described circle BC
- B (left end of horizontal diameter): one end of the ecliptic or equator (BDC)
- C (right end of horizontal diameter): the other end
- D (on the diameter between B and C, just below A): the foot of the latitude/declination, marking arc DC limited in the ecliptic or equator

AC is the side from A to C; AD is the latitude or declination of the star at A; ADC is the right angle at D.

Let A be some star, whose rays, diffused orbicularly, extend to the sextile (★) aspect through the described circle BC; let BDC be the ecliptic or equator, and AD the latitude or declination of the star at A. Now assume AC on the spherical surface, by hypothesis, for the sextile (★) — namely 60 parts — and AD, the latitude or declination, 20°, also by hypothesis.

Therefore, in the spherical right triangle ADC, for DC — the arc limited in the ecliptic or equator — the givens [διδόμενα] are:

- AC 60°0′
- AD 20°0′
- ADC a right angle

Therefore DC is given as 57°51′ for the sextile in the ecliptic or equator, according as the latitude or declination is given.

For limiting the longitude, however, or the right ascension, in the trine, let the found sextile arc be subtracted from 180° — that is, from a semicircle — and the remainder is counted from the place of the planet's longitude or ascension, for the triangular reduction. Thus, in the present example, with the longitude given at the beginning of Aries (♈):

- 180°0′ (semicircle)
- 57°51′ (the sextile [★] aspect)
- Difference: 122°9′, answering in the ecliptic to 2°9′ Leo (♌). Or, in right ascension in the equator, 122°9′. Hence the rest are estimated.

PROBLEMA VI.

Given the right ascension and declination of both the significator and the promissor: there is given, upon the significator's circle of position, the difference of the oblique ascension of the two — which is called the directory arc [arcus directorius], by which the times of events are usually measured.

We shall now explain the doctrine of direction — which is the kernel and champion of all judicial astrology — by means of a few examples drawn from the present illustrative chart [thema], in such a way that the foundations laid out in the tables of Johannes Regiomontanus [Regiomontanus] are unfolded here and there. [Margin: *Lib. 1, c. 7.*] As for what direction itself is, we described its

circumstances in the previous book; therefore we shall now apply ourselves to the practice alone.

Example I. Where the circle of position of the significator is given — as for instance the cusp of some house, or a planet standing on a cusp.

In the chart presented above, let the Midheaven [M.C.] serve as the significator, to be directed to the place of the Sun, taken as the promissor. Now since the Midheaven has no elevation of the pole, only its right ascension is to be posited, which is found to be $5^{\circ} 25'$.

Next, the right ascension of the Sun likewise corresponds (as found above) to $27^{\circ} 45'$. Therefore the difference, which is the directory arc, is given as $22^{\circ} 20'$. Here, because — in accordance with the magnitude of the Sun's daily motion along the equator — just as many years are reckoned from experience, the Sun's daily motion in the equator is sought, and is found to be $0^{\circ} 58'$. Hence, by a shortcut: because in each degree there is an excess of $2'$, for the 22 degrees one must add $44'$. If these are added — $22^{\circ} 20'$ and $44'$ — the sum is $23^{\circ} 4'$. From this we judge that the time of some rather notable event will fall at the beginning of the 24th year.

Example II.

Let the horoscope [ascendant], namely $4^{\circ} 15'$ Leo, be directed to the right sextile aspect [★] of Mercury, which is placed in the tenth house at $13^{\circ} 35'$ Aries, according to the Copernican foundation. [recte: the aspect symbol here is the sextile/quartile —]

First, then, since the horoscope here is the significator, whose circle of position is given — namely its pole-elevation $55^{\circ} 58'$, and its oblique ascension, established earlier from the addition of 90° to the Midheaven, came to $95^{\circ} 25'$ — the only thing we need to find is the oblique ascension of the sextile of Mercury, which falls at $13^{\circ} 35'$ Leo. That oblique ascension, under the pole-elevation of $55^{\circ} 58'$, is yielded — both by the third Problema of Chapter 3 of this book and by the tables of Regiomontanus — as $109^{\circ} 25'$. But the oblique ascension of the horoscope was $95^{\circ} 25'$. The difference, therefore, is 14° , which is also called the directory arc; this, extended into years according to the Sun's daily motion, gives about $14\frac{1}{4}$ years.

Example III.

Let the body of the Moon be directed to the left square [□] of Saturn, beyond the cusp of the houses.

| Moon | longitude | 20° 38' Leo | | | latitude | 5° 10' southern |

Left square of Saturn: 3° 17' Virgo.

Here, since the significator is beyond the cusp of the houses — namely near the end of the first house — we shall attempt the direction by the method [inductio] of spherical triangles, applying all the data required for it to the following diagram: where C is the significator, and N the promissor. Having worked through each item from scratch for the benefit of our beginners, we have first, for the declination and right ascension of the significator:

[Note: this page's body still belongs to Caput VI / Example III, continuing the spherical-triangle computation; the running header reads "Cap. 6."]

The data [διδόμενα (Greek: didomena, "the given quantities")] in triangle ABC:

Side / angle	Value	Meaning
CB	95° 10'	excess of the Moon's latitude
AB	23° 32'	distance of the poles
ABC	50° 38'	distance of the Moon from the tropic of Cancer □

Therefore: AC = 80° 14', the complement of CE. And: BAC = 128° 37', the distance in the equator from the tropic of Capricorn □, into the preceding [signs] [in antecedentia]; which, subtracted from a third of the circle, or 270°, leaves the right ascension of the significator as 141° 23'.

II. For the angle ACG.

Since in triangle CAG the following are known: AC, the complement of the declination; AG = 55° 58', the elevation of the pole; the angle CAG = 44° 2', which is measured by EL — which is itself known, the right ascension of the significator being subtracted from the right ascension of the Imum Coeli [I.C.] at L. Now the right ascension of the Imum Coeli is 185° 25' (namely 5° 25', the right ascension of the Midheaven, with a semicircle added).

There is given, therefore, first, the side GC = 46° 59½'.

Diagram. A large circle (the primum mobile / sphere of position) with several great circles inscribed, used for the spherical-triangle reduction of Example III

(directing the Moon to the left square of Saturn). The diagram is crowded; the principal labeled points and lines, read clockwise and across the figure:

- H (upper left, with an attached symbol resembling $\mathbb{H}$ /zodiacal mark): a point on the circumference where a great circle meets the limb.
- B (upper right): vertex of the principal triangle, near the top of the circle.
- A (just inside, upper middle): vertex of the triangle ABC / CAG, with a small circle drawn around it.
- F (far left, on the circumference): endpoint of the horizontal diameter FG.
- G (right, on the circumference): opposite endpoint of the horizontal diameter; vertex of triangle CAG.
- K (upper middle-left, on a chord): intersection point on a great circle.
- C (center region): the significator point; cluster of the small triangles SCE, etc.
- S, E, T, W, N, R, Q (center, tightly clustered around C): the points of the several orthogonal (right-angled) triangles SCE, NRQ, NRW used in the computation. Several of these labels sit very close together near the center and are difficult to separate.
- L (lower right, on the circumference): endpoint marking the Imum Coeli direction; the angle CAG is measured by arc EL.
- M (lower middle, on the circumference): point on the lower limb.
- (lower left, on the circumference): an unlabeled or faintly-labeled point [?] opposite H.

The principal triangle is ABC (with AB the polar distance $23^{\circ}32'$); the derived triangles are CAG, the orthogonal SCE, NRQ, and NRW used in the successive reductions. The central cluster of labels (S, E, T, W, N, R, Q around C) is densely packed and several are only partially resolvable at this resolution.

Next, in the aforesaid triangle ACG, since all the sides are given, the sought angle at G is given as $51^{\circ} 58\frac{1}{2}'$.

III.

Furthermore, in the right-angled triangle SCE, since —

Given [διδόμενα]	Value	Meaning
SCE	$51^{\circ} 58\frac{1}{2}'$	
CE	$9^{\circ} 46'$	complement of AC, found earlier

Given [διδόμενα]	Value	Meaning
CES	right angle	

Therefore there is given CSE = $39^{\circ} 4\frac{1}{2}'$. Likewise SE = $12^{\circ} 14'$, the ascensional difference, which — subtracted from the right ascension of the significator ($141^{\circ} 23'$) — leaves the sought oblique ascension of the significator: $129^{\circ} 9'$.

IV.

Next, for the declination and right ascension of the promissor, there are given in the right-angled triangle NRQ: the angle NQR = $23^{\circ} 32'$; NRQ a right angle; and NQ = $26^{\circ} 43'$. Therefore there are given NR = $10^{\circ} 20'$, the declination of the promissor; and the arc QR = $24^{\circ} 47'$; which being subtracted from a semicircle, leaves the right ascension of the promissor as $155^{\circ} 13'$.

V.

Finally, in the right-angled triangle NRW, for the side RW —

Given [διδόμενα]	Value	Note
NR	$10^{\circ} 20'$	
NWR	$39^{\circ} 4\frac{1}{2}'$	equal to CSE
NRW	right angle	

Therefore there is given RW = $12^{\circ} 59'$, the ascensional difference of the promissor, to be subtracted from the right ascension of the same; whence also there remains the oblique ascension of the promissor: $142^{\circ} 14'$. But the oblique ascension of the significator was $129^{\circ} [9']$...

[Note: running header reads "Cap. 7." but the opening lines still complete Caput VI / Example III; the new chapter rubric (CAPUT VII) appears partway down this page.]

9'. And accordingly the difference, which is the directory arc, will be $13^{\circ} 5'$, to which there correspond about $13\frac{1}{4}$ years.

And in this way we have likewise sought the foundation of directing within the doctrine of triangles, and have set it out with sufficient examples. For when the direction is to be carried out against the order of the signs, the promissor and the significator are interchanged — promissor into significator and the reverse — and therefore the circle of position is sought under which the promissor lies; nor is anything otherwise different from the rest of what was demonstrated earlier,

as Johannes Regiomontanus expounds it more fully. [Margin: *Cap. 26 T. Direct.* — i.e. Regiomontanus, *Tabulae Directionum*, Cap. 26]

CAPUT VII.

On the principal astronomical instruments, and their use, briefly assigned. [Margin: CLASSIS SECUNDA — "the Second Class"]

Now that those things have been dealt with which can or ought to be inscribed upon the surface of the celestial sphere or globe, we judge it consequent that we should bring forward — just as the phenomena must be drawn into observations by means of mathematical instruments — the following: especially since, apart from this knowledge, no craftsman in our astronomy succeeds, whether the phenomena of the present age are to be rightly estimated; or those observed by the ancients are to be examined and weighed in a just balance (lest, that is, the most excellent of all disciplines become liable to the perpetual errors of insufficient and childish observations); or, finally, whether the new portents that appear in the heavens from time to time — especially in these our times — are to be measured in their motion, distance, and magnitude: lest these too mock the philosophers, who would otherwise, through such appearances, judge of the whole celestial nature differently than truth demands. This is what once befell Aristotle [Aristotle] concerning the nature and generation of comets; whose opinion, though erroneous, held such authority right down to our own age that it dragged the mathematicians into error along with it — until our Atlas, the Lord Tycho Brahe [Tycho Brahe], by the skillful construction and sufficient magnitude of his instruments, and finally by a timely and proper institution of observing, grasped the invincible truth in this matter and demonstrated that comets are celestial; as he left this same fact made manifest to posterity in the books of the *Progymnasmata*, and proved it with luminous demonstrations against the Peripatetics. We ourselves, however — although the *Astronomia Mechanica* [Margin: *Vide Astron. Mech. T. B.* — "See Tycho Brahe's *Astronomiae Instauratae Mechanica*"], once quite elegantly adorned with engravings by Tycho himself and brought to light, sets out the construction and use of many instruments of this kind — nevertheless, since almost all those things which emulate the heavens by their roundness are found in the anatomy of our material sphere, we shall accordingly treat briefly, in the first place, in this class, of certain of these whose use is of chief importance.

Among the astronomical instruments that emulate the roundness of the heavens, some are fixed to their own poles, upon which they are balanced — such as the two armillae [armillary rings] soon to follow, and any others there may be that

observe the horizontal poles together, like the *Torquetum* — in which, too, a center, like an index, is fixed in the middle of the axis. Others, however, are free and portable, and are attached only to centers and circumferences, such as the *Quadrant* or *Sextant*, likewise the *Triens* [third-of-a-circle], *Octant*, and any other solitary instruments that are constructed from the section of a circle.

Furthermore, those that rest upon their own poles are composed of two or three sections of circles; whereas those that are free are contained by a single section of the circumference. To carry out observations rightly with these, two observers are for the most part required.

I. On the Equatorial Armillae [De Armillis Aequatoreis].

When the elevation of the pole of the place is given, with the axis of the sphere passed through each pole, let the center be marked in the middle. Then, with the circles remaining in place — namely the equatorial one being movable, but the declinatory one able to be carried around about the poles — and with movable

and necessary sights [dioptrae] applied to both, in this way not only are the times and right ascensions given in the equator, but also the declinations on their own declinatory circle. By the benefit of this instrument, first of all, in Tycho Brahe's work, the refractions of the Sun, Moon, and stars were discovered near their rising and setting. [Margin: *Occasio repertarum refractionum in Sole et stellis per Armillas aequatoreas T. B.* — "The occasion of the discovery of refractions in the Sun and stars by means of the equatorial armillae, by Tycho Brahe."]

II. On the Zodiacal Armillae [De Armillis Zodiacalibus].

With the poles of the place likewise fixed in the meridian circle, or at any points whatever of it, and the distance of the poles of the equator and the zodiac given on each side, and finally the sights applied as before: through the ecliptic circle the longitudes of the stars themselves are given; but through the latitude circle, the latitudes — the line of sight being made to a fixed center in the middle.

[Margin: *Ex hisce 2 armillis veteres Alexandrinae compositae fuerunt. De quibus Ptolemaeus passim in suo Almagesto.* — "From these two armillae the ancient Alexandrian ones were composed, of which Ptolemy [Ptolemy] speaks frequently in his *Almagest*."]

III. On the Torquetum [De Torqueto].

In the same manner, finally, with the elevation of the pole given, and a vertical circle taken together with a horizontal one, the azimuthal degrees from the meridian are given on the horizon; but on the vertical circle, the altitudes

themselves, from the horizon. If, moreover, the remaining circles — namely the equator and the zodiac with their transversals — are attached, the ancient *torquetum* is thence constructed, conspicuous more for its artifice than for its use, since by its own weight the torquetum twists itself violently.

IV. On the Quadrant [De Quadrante].

So far concerning instruments somehow arranged and fixed to the elevation of the pole of the place; there follows the discussion of the free and portable ones, among which the Quadrant holds first place on account of its hitherto ready use — whether you wish to measure the altitudes of the stars on either side, from the vertex or the horizon; or distances comprised within a quarter of the heavens. Its structure is easy; for whether it be made of solid material or interpolated and latticed, the circumference of the quadrant has a measure marked out in degrees and minutes, like the rest; while the sides, toward the center on either side, represent the radius of the circle. The Quadrant, moreover — if it extends its index, set rightly toward the horizon on every side — will be very apt for investigating parallaxes [parallaxes], which are usually traced out by taking the altitude-degrees and the azimuthal degrees together; so much so, indeed, that the parallactic instrument of Ptolemy and Copernicus [Copernicus] readily yields to it, since that one, among other inconveniences, is also weighed down too much by its own weight. [Margin: *Vide Astron. Mech. Tych. de Quadrante volubili.* — "See Tycho Brahe's *Astronomia Mechanica* on the revolving quadrant."]

V. On the Sextant Instrument [De Sextante Instrumento] — an instrument most familiar to us on account of its construction and use.

First, as concerns the finished construction of the sextant: no astronomical instrument is built more easily than the sextant. For since it is established that the side of a hexagon inscribed in a circle is equal to the radius, the side of the hexagon is easily traced out on every side by means of a compass.

Next, although the simple use of this instrument is most conspicuous in taking the distances of the stars: if, nevertheless, the sextant be skillfully prepared and equipped — such as the one we took care to have wrought by our own industry, having, namely, five cubits in its radius — it suffices not only for measuring all manner of distances of the stars within 150 degrees [recte:]; but also for any altitudes whatever, by means of sights, namely, skillfully arranged on it for this purpose. Indeed, it even sustains the role of a geometrical instrument without triangular calculation, when we have drawn the tangents along its limb as well. And since this instrument stands for us in place of nearly all the others, on that

account we shall not be reluctant to display its construction here somewhat more thoroughly, by demonstration.

First, as to the construction of the sextant — this is easy, as I said. For when its magnitude, or semidiameter, has been granted, the compass (which is constructed from a rectilinear and oblong piece of wood...

oblong piece of wood, fitted with two iron pointers — one fixed at the end, the other sliding to any measure — and it will verify that all its sides are of equal extension. As for the framing, the following figure may be noted, though the sextant itself, together with its ingenious division between the transversal lines, may be examined more fully and precisely in my own work. The principle of this division was the one Tycho Brahe customarily used on his finer instruments.

Diagram. A sextant instrument shown as a 60-degree sector. The apex is at the top (A), the curved graduated limb runs along the bottom between the lower-left and lower-right corners. The body of the frame is filled with crossed structural bracing. A central plate carries an inscription. The graduated limb is numbered along its arc.

- A (apex, top): the center of the instrument
- B (lower-left corner): fixed sight (dioptra)
- C (lower-right corner): a sight position on the limb
- D (bottom, beneath the limb): the movable ruler position
- H (mid-left of frame): an alternative sight position
- Central plate inscription: "1607 / F F / C.S.L." (the date 1607 and the maker's initials — Christen Sørensen Longomontanus)
- Limb (graduated arc, bottom): numbered approximately 10, 20, 30, 40, 50 [degrees], reading across the 60-degree arc

[Margin: The construction and use of the sextant may be seen in *Progymnasmata* [Tycho Brahe, *Astronomiae Instauratae Progymnasmata*] Book I, p. 248.]

Since one sight is fixed at B and another at D on the movable ruler, it is clear that the angle B A D measures the distance between two stars, for the vertical angles are equal.

Furthermore, although the limb of the sextant extends no further than 60 degrees, we have by this device enlarged its use so that it suffices well for measuring up to 150 degrees. For let a sight be placed at C and another at H. When a sighting is made from C through H by one observer, and from B through the center A by another, it is certain that the distance of two stars so observed

makes up a quadrant, that is, 90 degrees. Moreover, the closer the ruler D with its sight is brought toward C, the more is subtracted from the quadrant, so that in this way the whole measurement falls between 90 and 60 degrees.

Again, when the sighting is set up in reverse — from H to C — it follows in the same way that the quadrant is completed for an observer sighting from B through the center A. But when this same observer uses the sight on ruler D, then by its motion toward C the angle A H C is enlarged, to such a degree that when D has reached C, a sextant is added to the quadrant's measure, and a distance of 150 degrees is produced. Indeed, with fixed sights at B and C — the pinnules being immovable — the part of the instrument's limb that falls between 150 and 90 degrees, that is, a quadrant of the circle, can also be measured, as the geometrical demonstration itself shows in the measurement of the circle.

So much for the distances to be obtained by means of the sextant.

On Altitudes by means of the Sextant

[Margin: The construction of the sextant may be seen in *Progymnasmata* [Tycho Brahe] Book I, p. 248.]

The sextant is also quite suitable for the altitudes of the stars, whether on the meridian or any others whatever. For if it be hung from its pedestal in such a way that its upper side answers exactly to the horizon, it will then suffice for an altitude of 60 degrees to be observed from the horizon. But if a greater altitude is required, the plumb-line is so adjusted that the upper side is raised 30 degrees above the horizon through the center; and so, with the sextant fixed to its pedestal as before, it is found suited to taking an altitude all the way to the zenith. This follows easily from the hypothesis of this instrument's construction, set out above. After the manner of the sextant, a *triens* [one-third instrument] and an *octans* [octant] are also built — that is, taking for each of these instruments the appropriate portion of the circle. Their use is much the same as that of the sextant, in obtaining the distances and altitudes of the stars.

VI. On the Geometrical Square

But in order that the sextant might be reserved chiefly for measuring the distances of the stars, I had another instrument made — for the altitudes of the stars, by which the time is known — in a square form and of just magnitude. Its two sides which receive the divisions, since they represent tangent lines, give straightaway, for any number set down on them whether direct [recto] or

reversed [verso], the corresponding arc from the canon of tangents, which is the altitude of the star either from the horizon or from the zenith.

This instrument, besides being especially suited for measuring terrestrial objects and for directing mechanical operations, is also so composed and divided by means of its screws that it can be enclosed in an oblong case and transported to various places as need requires.

VII. On the Construction of the Astronomical Radius, its Use, and a certain Insufficiency of it

Thus far we have treated of the astronomical instruments, which mostly rival the heavens on account of their roundness. There follows the radius [the cross-staff], a very ancient instrument; for of it Virgil sang thus:

[Margin: *Eclogues* 3.]

Or who was the other, who marked out for the nations the whole sphere
with his rod [radius]?

...And the courses of the heaven they shall mark out with the rod, and tell
of the rising stars.

[Margin: *Aeneid* [En.].]

On the Construction of the Radius

The radius, as it is described by Gemma Frisius [Gemma Frisius], Petrus Ramus [Pierre de la Ramée], and others, consists of a staff [baculum] and a cross-piece [transversal], whose convenient proportion will be: the staff of 1000 parts to the cross-piece of 200 parts — the spaces of the sights at either end of the cross-piece, and the station on the staff, being set aside. Then let a movable cross-bar be made on the staff, fitted with a hole at its middle. When therefore each rod has been divided, in the number I have stated, into equal parts (whose use may be in geometry), they are by the following demonstration of triangles distributed individually into degrees and minutes, so that the radius becomes suitable for observing the celestial phenomena.

Diagram. A geometrical figure of the cross-staff. At the top a short cross-piece C–D is shown, marked off in 100 parts on each side of its center A (the labels read "100" on each arm), with the sights F, F at the ends. A long narrow staff descends from A to B (its length 1000), drawn as a very elongated triangle. A second cross-bar G–H is shown lower down on the staff.

- A (top, junction of cross-piece and staff): the movable station / vertex
- B (bottom of the staff): the eye / station, length A B = 1000
- C, D (ends of the upper cross-piece): each half (C A, D A) divided into 100 equal parts
- F, F (at the cross-piece ends): the sights (dioptrae)
- G, H (lower cross-bar): a second cross-bar position on the staff

So, in the present figure, let the staff be A B, and the cross-piece C D movable on the staff at A, of which the portion C A, or D A, will be divided into 100 equal parts, of which kind A B is 1000. With the cross-piece therefore remaining at A in its first station, we first proceed geometrically for the individual degrees in it as follows. Let there be assumed, for the sake of example, the angle A B E of one degree.

[Greek: Διδόμενα (didomena, "the givens")] therefore:

- A B = 1000 parts
- angle A B E = 1 degree
- E A B = right angle

Therefore A E sought = 17 parts 45 minutes [p. 17 min. 45]. Putting A B concisely as the whole sine [radius], E A will be the tangent of the angle at B. Hence, with the last four figures cast off in the table of triangles, the tangent is at once given for the individual degrees from the table of tangents. Whence A C amounts to 5 degrees 43 minutes, to which A D is equal. Therefore C D, with the cross-piece in its station at A, is 11 degrees 26 minutes. From here, for the division of the staff A B into degrees and minutes, we proceed thus. With the distance A B made nearer toward the eye B, the given isosceles triangle C B D is divided into two equal right triangles, C B A and D B A. Since therefore A C B is the complement of half the assumed angle to a quadrant, A B sought is easily given. For, putting A C as the whole sine, A B will be the tangent of the angle C, whose complement is C B A. Therefore, with the equal triangles C B A and D B A folded together, the measure in the middle is B A, and so on.

Now let there be sought the distance in B A corresponding to 30 degrees (so that with these two examples we may at once complete the construction and use of the radius proposed), which measure is contained within the angle G B H. The proposed angle therefore being divided into two equal parts, 15 degrees fall to each; the tangent of whose complement, 75 degrees — namely part 373½ — indicates the measure of the staff A B.

From this it can easily be understood that the use of the radius — divided primarily into equal parts, as we have shown — can exist without this secondary distribution, by employing the table of tangents alone.

Example

Two stars were observed by means of the angle G B H, where the measure B A is indicated on the staff as 577 parts. Now since it is as A H, 100 parts, to A B, 577 parts, so the whole sine is to the tangent of the angle A H B; therefore, when the whole sine is put as 10000000, the tangent of the angle A H B becomes 57700000. To this there corresponds in the table of tangents 80 degrees 10 minutes very nearly. Its complement is the angle A B H, 9 degrees 50 minutes. The double of this, H B G, 19 degrees 40 minutes, is therefore the visual angle at B, that is, the sought distance of the stars. And so in other examples: where only the distance B A obtained from the observation is offered, by five figures, without further calculation (the radius being divided in this way), the angle A H B arises, and so on. So much, then, for the construction and use of the radius — except for what may be further learned in practice.

The insufficiency of the radius lies chiefly in practice, namely that the eye sighting along both ends — especially at great distances, on account of the motions of the stars in the meantime, and the variable center of vision — does not suffice for the precision of the alignment. Sailors too use this instrument for taking the altitudes of the Sun and the stars, since indeed they readily neglect the exact precision that the astronomer requires. So much for the astronomical instruments.

CAPVT VIII.

On the various ways of investigating the meridian line and the elevation of the pole of a place

Now that the more select astronomical instruments have been described and prepared, and ones most suited for making observations: for their use in acquiring observations, two things are required here first of all, as things settled and given for the place — namely, the meridian line and the elevation of the pole. The manifold investigation of each is set forth by the following problems of this chapter in order.

On the Meridian Line of a Place

PROBLEMA I.

With a gnomon erected upon a plane, and the equal length of the shadow observed before and after noon, the meridian line of the given place is given.

This is the ancient and simple way of investigating the meridian line — which, climbing through the poles of the world and the zenith of the place, divides the upper hemisphere of the heaven equally into north and south. Namely, let there be erected in the open, upon

Diagram. A rectangular figure for finding the meridian line by gnomon shadows. A square frame B–A (top) and M–L (sides) with C–D at the bottom. A vertical stylus stands at the center; concentric arcs sweep across the square. The horizontal meridian line is labeled across the middle.

- A (top-right corner): corner of the frame
- B (top-left corner): corner of the frame
- C (bottom-left corner): corner of the frame
- D (bottom-right corner): corner of the frame
- M (left side): corner of the frame
- L (right side): corner of the frame
- O (upper center): top of the stylus
- "Stylus." (label along the left, beside the vertical): the stylus / gnomon
- P (left of center): foot region of the stylus
- "Lin: merid." K (mid-line label, right of center): the meridian line, with point K
- H (upper center-right): point where shadow-tip crosses the arc (forenoon)
- G (lower center): point where shadow-tip crosses the arc (afternoon)
- F (lower center-right): point on the arc

the immovable horizon plane, upon a fixed tablet, a stile or gnomon at right angles; and from the center of this let circles (one or more) be circumscribed. Then in the forenoon, in the most serene sky, let it be observed most diligently where the tip of the shadow of the stile or erected gnomon cuts the circumference of the nearest of the circumscribed circles as it enters; for as the Sun ascends, the shadow is compressed. Likewise also in the afternoon, where the tip of the shadow falling from the same gnomon grazes the circumference of the same circle as it goes out; for as the Sun descends, the shadow is dilated.

The middle point of this intercepted arc, explored and precisely investigated by means of the compass, will be the point of the meridian.

For afterwards the sought line is given — namely the straight line drawn from this point to the middle of the root of the stile in the plane, as you see in the adjoining figure.

PROBLEMA II.

With the altitude of the Sun observed, and given besides its declination and the elevation of the pole of the place, the meridian line is given by this single observation, before or after noon.

This meridian line, moreover, will be obtained at one time without delay, by employing a quadrant or another instrument by which the altitude of the Sun can be exactly observed — and in this manner.

Let the circle described on the tablet be divided into 360 degrees, and each of these into 60 minutes; or at least let a part of the circle be exactly distributed by this standard, so that the remaining measure can be taken as need requires. Then, with any intersection whatever of the stile's shadow noted on the circumference, and its opposite point, at any serene time whether before or after noon — and that at a moderate distance from noon, yet not (on account of refractions of the Sun) too near to its rising and setting — at which moment the altitude of the Sun is exactly observed: its declination from its given place being known, and applied together with the elevation of the pole of the place, there is made manifest, through the solution of the spherical triangle, the arc of the horizon between the meridian and the previously designated point opposite to the intersection of the shadow. This will be more fully recognized by the following demonstration and example.

Diagram. A spherical-astronomy diagram for Problema II. A large circle (the meridian A B C D L) with the horizon A L drawn across it and the equator B M as a slanting great-circle arc. A smaller inner circle H K (the shadow circle on the tablet) sits near the center, with a stylus G O erected. Several great-circle arcs cross from the poles.

- A (left, on the horizon): west point of the horizon A L
- L (right, on the horizon): east point of the horizon A L
- B (upper left): point on the meridian; end of equator B M
- C (top): pole point (one of the quadrant endpoints, C F)
- D (upper right): pole point (vertical pole; quadrant D N)

- M (lower right): end of the equator B M
- H (inner circle, upper): the shadow circle on the tablet
- K (inner circle, right): the shadow circle on the tablet
- G (center): root of the stylus
- O (above G): top of the erected stylus G O
- E (lower left, on equator): the place of the Sun, elevated above the horizon by arc F E
- F (lower left, on horizon): point where the shadow line excurses to the celestial horizon
- I (just below center): the point opposite to the shadow [adverse point] from the Sun at H
- N (left of center): foot of the quadrant D N from the pole
- R (left of center): point on the horizon/arc near N

[Margin: ...will be known by the following [problem] and example.]

Let it be in the adjoining figure: A B C D L the meridian, A L the horizon, B M the equator. Let there be described, as before, the circle H K, and let the stylus G O be erected. The Sun being elevated from the horizon through the arc F E, let the shadow of the stylus be marked from the Sun at E onto H, and its adverse point I — which line excurses to the celestial horizon at F. Then, with the quadrants C F and D N given from each pole respectively — namely the vertical [zenith] pole and the pole of the world — which intersect each other at the Sun's place at E, all the sides of the spherical triangle E C D are given. For C D is the complement of the elevation of the pole of the place to 90 degrees, that is, the distance of the poles...

—of the world and the vertical, which equals the arc AB of the equator's elevation above the horizon. EC is the complement of the Sun's altitude FE; and ED is the complement of the Sun's declination. Therefore the angle ECD will not remain hidden, the angle that the arc FL measures on the horizon, whose complement to the semicircle is FA. This arc FA reveals the distance between the given opposite point and the meridian of the place. Thus a straight line drawn from A through the center G to L marks the meridian line that was sought.

Example.

At Rostock, where the elevation of the pole is about 54° , on the 30th of August, when the Sun was in $15^\circ 40'$ of Virgo and its declination was $5^\circ 46'$ north: the altitude of the Sun was observed by quadrant at the same moment that the

shadow of the stylus was noted on the tablet, and the point opposite to it fell at some definite degree at H, and that altitude was $20^{\circ} 0'$. Hence in triangle ECD:

The given quantities [διδόμενα, the data] are:

Side	Value	
CD	$36^{\circ} 0'$	the complement of the Elevation of the pole
EC	$70^{\circ} 0'$	the complement of the Altitude of the Sun
DE	$84^{\circ} 14'$	the complement of the Declination of the Sun

Therefore $ECD = 108^{\circ} 36'$.

And accordingly $FA = 71^{\circ} 24'$, the amount by which the point F found on the horizon is distant from the meridian.

I. This same result is obtained on the said horizontal tablet without any stylus, using a quadrant alone, in which the Sun or a star is observed by sighting straight up, the quadrant being placed over the center of the section-circle. For the very same opposite point that the shadow of the stylus marks out during observation is here likewise obtained.

II. The same can also be done given the length of the stylus and of the shadow on the tablet. For in these one first investigates the altitude of the Sun, then the hour, and consequently the distance from the meridian.

III. Further, the same is determined from the rising and setting of the Sun or a star marked on the said horizontal circle, midway between which the meridian line passes; or from the altitude taken equally on both sides of the meridian together with the azimuthal degrees on the same section-circle.

IV. Likewise the same is achieved from the observed elongation of the circumpolar stars — and especially of the pole star — to their greatest extent on either side, on the horizon circle. This method Mr. Tycho Brahe [Tycho Brahe] judged the surest of all for obtaining an accurate investigation of the meridian line on the azimuthal quadrant.

V. Finally, given the elevation of the pole of the place, the meridian line is shown by means of two stars observed on the same vertical circle. And by many other methods of induction besides, the meridian line is displayed — methods everywhere obvious to the practiced astronomer. For once any one device is taken from him, his opportunity and skill will supply and furnish a second, a third, and more — so that, where precision is required, he need not, like the common run of men, place his trust in the mariner's compass as an instrument, since for many reasons it is rather unreliable.

[Margin: Consult the following problems.]

On the Elevation of the Pole of the Place.

PROBLEMA I.

Given the observed altitude of the Sun or of a star on the meridian, together with its declination: the elevation of the pole of the place is found.

The elevation of the pole of the place is investigated either on the meridian or off it. On the meridian — as in the present problem — it is found from the altitude of the Sun or of stars situated on the meridian. For given the meridian altitude of a celestial body [phaenomenon] and its declination,

—the elevation of the pole becomes known. For if the altitude has been taken on the meridian, and the northern declination is subtracted from it, or the southern declination is added to it, then there is shown — in the former case by the remainder, in the latter by the sum — the complement of the elevation of the pole of the place that was sought.

Example.

[Margin: In the year 1599.]

On the 31st of August at Rostock the meridian altitude of the Sun was observed by quadrant as $41^{\circ} 2'$; the place of the Sun was in $17^{\circ} 0'$ of Virgo, and its declination was $5^{\circ} 8'$ north. When this declination is subtracted from the observed altitude, there remains the complement of the elevation of the pole — that is, the elevation of the equator itself above the southern horizon — equal to $35^{\circ} 54'$. Therefore the elevation of the pole is $54^{\circ} 6'$.

The same can, and ought to, be obtained from the meridian altitude of any star whose declination is known, by the same prescribed method.

Closely akin to this Problem are also the following methods.

First, in the case of circumpolar stars, the same is achieved even without a given declination, from the highest and the lowest meridian altitude of one and the same star. For if half the difference of these observed altitudes is added to the lesser, or subtracted from the greater, the remainder shows the elevation of the pole that was sought.

Next, by night — especially in winter, when the sky is clearest — let a thread or fine cord be hung perpendicularly between two supports, and along its plumb-line let the star passing through the place's zenith be marked. For the declination of this star shows the elevation of the pole that was sought. The same can be found against a wall or support set precisely upright.

PROBLEMA II.

Given two stars observed on the same vertical circle, whose right ascensions and declinations are known, together with the altitude of one of these two stars: the elevation of the pole of the place is found.

The elevation of the pole from observations off the meridian — first, from two stars seen on a single vertical circle, together with the altitude of one of them taken by quadrant at the same moment — is elicited, by means of spherical triangles, in this way:

Diagram. A circle (the meridian/primitive circle) with several arcs drawn across it. D is the vertical pole (zenith pole) near the top; E is the world-pole at the right. Two stars lie on a common vertical circle at B and C. Arcs are drawn from D and E through the stars; the altitude arc AB rises from A. Labeled points around the figure: D, E (upper region), C, B (the two stars), I, H, G, K (interior), M (left), A (center), F (right), L (lower).

- D (top): vertical pole (zenith pole)
- E (upper right): world-pole
- C, B: the two stars on the same vertical circle
- A (center): foot of the altitude arc
- F (right), L (bottom), M (left), I, H, G, K (interior): construction points and arc intersections

In the adjoining figure, let D be the vertical pole and E the world-pole. The stars lie on the same vertical circle, at B and C; and let the altitude of one of them, the one at B, be known through its arc AB. From this, in triangle CEB — given the declinations of both stars (and afterward their complements, represented by the arcs CE and BE), together with the difference of right ascension, which the angle

CEB measures — there becomes known BC, the distance between the stars observed on the same vertical circle. Second, in triangle CBE, from the three given sides, the angle CBE becomes known. Lastly, in triangle DBE — since two sides are granted together with the included angle, namely BE (the complement of the declination of the star at B), DB (the complement of the altitude of the star observed at B), and the angle CBE just acquired — there emerges the arc DE, the distance between the world-pole and the vertical pole, which is the complement of the elevation of the pole that was sought.

But if instead the altitude of the higher star at C is observed, then the angle at C in triangle BCE is sought; its complement to 180° at C is DCE. In this triangle DCE, given the two sides DC (the complement of the altitude CA) and CE (the complement of the declination CH), together with the included angle C, the sought DE is found.

PROBLEMA III.

Given the right ascension of the mid-heaven [Ascensio recta M.C.], and besides it two stars off the meridian on the same vertical circle, together with the right ascension and declination of those same stars: the elevation of the pole of the place is found.

Recalling the figure of the immediately preceding problem, and the right ascension of the mid-heaven M.C. being given, there are given in triangle DEB two angles: namely the angle at B by the preceding problem, and the angle at E by the difference of the right ascensions of the mid-heaven M.C. and of the star at B. There is given besides the side BE, the complement of the declination of the star at B. Hence DE, the complement of the elevation of the pole that was sought, will not remain hidden — nor, consequently, the elevation of the pole of the place itself. The same is done when the angles DEC and DCE are given by working the procedure through the star at C.

PROBLEMA IV.

Given two pairs of stars observed together by night on two vertical circles, with the declinations and mutual distances of those stars known from the foregoing: the elevation of the pole of the place is found.

This problem (like the immediately preceding one) deserves, in my judgment, to be highly valued, since it teaches how to investigate the elevation of the pole of the place with sufficient precision by careful attention alone — by the perpendicular stretching of a thread between the stars on the same vertical circle

and the eye — without any other instrument whatever, whether one is on land or at sea. And it has, to my knowledge, never before been discovered or brought forward by anyone. Moreover the use that the present problem affords is outstanding, and renders the deduction and the more laborious analysis of triangles nothing but pleasant.

In this matter there is a twofold case, since such observations are generally made on vertical circles off the meridian. (For those that occur on the meridian can be referred to the preceding problem.) For either both vertical circles fall in the same quadrant of the sky, toward the rising or toward the setting; or one falls toward the rising and the other toward the setting, with the meridian lying between them.

Diagram of the Former Case, with its own demonstration [ἀπόδειξις, apodeixis, "proof"].

Diagram. A large circle (the meridian) with numerous arcs drawn across it. From the center A the meridian EBD is described, in which B (at the top) is the pole of the horizon ED, and C is the pole of the equator FG. The first pair of stars H, K lie on the vertical circle BHKP; the later pair I, L lie on the vertical circle BILO. Labeled points around the figure: B (top), C (upper right), F (left), H, I (interior upper), M, K, N (left of center), L (right of center), E (left), A (center), D (right), R, P (lower left), O (lower right), Q (bottom), G (lower right edge).

- B (top): pole of the horizon
- C (upper right): pole of the equator
- E (left), D (right): endpoints of the horizon-pole diameter ED
- F (left), G (lower right): endpoints of the equator FG
- H, K: the first (earlier) pair of stars, on vertical circle BHKP
- I, L: the second (later) pair of stars, on vertical circle BILO
- A (center): center of the figure
- M, N, R, P, O, Q: construction points and arc intersections

From the center A let the meridian EBD be described, in which the pole of the horizon ED is B; and the pole of the equator FG is C. The rest are known. For the first pair of stars H, K are marked on the vertical circle BHKP; but the later pair I, L on the vertical circle BILO. And since the declinations, as well as the distances, of the stars are known from the foregoing: therefore, in triangle KCL, whose sides are given, let the angles at K and at L be found. Then, from the higher [triangles] HKC and CLI — of which,

—subtracting this one, CLK, and on the other hand adding that one, CKL, there remain in triangle BKL two known angles at K and L, with the retained side KL. Therefore from these BL is found. Once this is had, since in triangle BLC two sides are granted together with the angle included at L, BC, the complement of the elevation of the pole that was sought, will not remain hidden.

Demonstration of the Latter Case.

Diagram. A large circle (the meridian) with numerous arcs, similar to the former case but with the configuration mirrored. Labeled points around the figure: F, K (top), M, H (upper left), N, B (upper center), C (upper right), I (interior), R, L (center), P (left), E (left), A (center), O (lower center), D (right), G (lower right), Q (lower left).

- F, K (top): construction points / star
- B (upper center): pole of the horizon (as in the former case)
- C (upper right): pole of the equator
- H, K: one pair of stars
- I, L: the other pair of stars
- A (center): center of the figure
- E, D: endpoints of the horizon-pole diameter
- G: endpoint of the equator
- M, N, R, P, O, Q: construction points and arc intersections

In this latter diagram everything stands under the same labels as in the former, with this one point only to be considered: that each angle — namely ILC and HKC — is subtracted from the angles at L and K of triangle LCK. From this there afterward remains triangle KBL, with known angles at K and L, together with the included side KL, the distance between the stars at K and L. From these given quantities [Διδόμενοις, didomenois, "the data"] there is first given the side BL; then, in triangle BLC, from the granted sides BL and LC together with the included angle BLC, BC is acquired — the complement of the elevation of the pole that was sought. It should be noted, moreover, that in this and the preceding problem, even the altitudes of the stars and the times of the observations made, if anyone desires them, afterward come to light by means of the foregoing.

PROBLEMA V.

Given two stars rising or setting at the same time, with their declinations and right ascensions: the elevation of the pole of the place is found.

This problem too is accomplished without any instrument, from observing two stars rising or descending equally above the horizon, whose declinations are of the same denomination — northern or southern — and likewise by the doctrine of spherical triangles, in this method of induction:

Diagram. A circle with arcs across it. In the appended scheme let the two stars emerging or setting be at A and B. Labeled points: E (top), D (upper left), C (left), F (right), A, B (center), K (center), G (lower right), H (bottom).

- E (top): a pole / construction point
- A, B (center): the two stars rising or setting
- C (left), F (right): endpoints of the horizon
- D (upper left), G (lower right), H (bottom), K (center): construction points and intersections

In the appended scheme, let the two rising or setting stars be at A and B. Thus in triangle AEB — since two sides are given, AE and BE (the complements of the declinations of the stars), together with the angle AEB (the difference of the right ascension of each) — the distance between the stars on the horizon is sought, as also the angle EAB. Once these are found, there are granted in triangle EAF: EA as before; EAB (that is, EAF), the angle just found; together with the right angle EFA. Therefore the arc EF, which measures the elevation of the pole that was sought, will not remain hidden. This procedure, moreover, involves less error, inasmuch as the refractions of the stars are for the most part equal. But of an observation of this kind [the occasion is] rare—

such an opportunity is rarely given, because of the vapors—indeed the clouds—that almost continually surround the horizon.

Moreover, by observing the rising or setting magnitude [magnitudo ortiva vel occidua] together with the given declination, the elevation of the pole is made manifest. For given the arc on the horizon AF, together with the complement of the declination EA, in the same right-angled triangle EAF the arc EF becomes apparent. But this method is erroneous and treacherous, on account of the refractions of the Sun and the stars.

CAPVT IX.

On the various methods of observing celestial phenomena afresh, and of resolving them in length and breadth with respect to the ecliptic; and on the comparison of these methods and their greater certainty.

A twofold way is offered for investigating the position of an unknown star in the heavens. The one is more expeditious, and in part also more reliable; it is carried out with sufficient astronomical instruments—those, namely, already explained above. The other proceeds without these, by means of a stretched cord [filaris extensio] and, beyond that, by mere ocular sighting.

It must be understood—and most carefully weighed by the astronomer—that not all observations, even those carried out with instruments of proper size, merit equal certainty. (For we entirely cast out from this field, to be cultivated and cleansed, those ancient instruments, small and downright childish.) Those observations that depend on the first revolution of the heavens, with the instant of time at which they are made brought to bear—as when the altitude of a phenomenon [Greek: φαινόμενον (phainomenon, "appearance" — the observed celestial object)] is taken together with its azimuthal degree and the moment of the elapsed time—are in truth judged to be quite treacherous and uncertain for fixing the star's position. For the tiniest error of the elapsed time, when one easily strays (as most often happens) either in the clocks or in observations to be carried out at the very moment, turns out to be all too prolific of error.

This method, subtle though it is and better suited to the demonstrations by which parallaxes are worked out, nevertheless—for the reasons stated—misled nearly all the astronomers of the earlier age, as many as attempted from it anything either toward establishing the positions of the stars contemporary with the world, or toward investigating the locations of more recent phenomena. [Margin: Lib. 1 *Progymn. de stellarum restitutione. Stellarum Fixarum.*] Just as Master Tycho Brahe [Tycho Brahe] proves and shows this from experience. But lest the lovers of learning miss the footsteps of Regiomontanus and the ancients in these matters, we shall briefly set forth in advance the demonstration of this method.

PROBLEMA I.

Given the instant of time, together with the altitude of a new phenomenon above the horizon and its azimuthal degree—both to be observed simultaneously—its right ascension and declination are found, and therefore also its longitude and latitude.

In the diagram added here, let MKN be the horizon, MLAB the meridian, and LHK a quadrant of the equator, whose pole is at B, while the pole of the horizon is at A.

Diagram. A hemisphere bounded below by the horizontal diameter MN (the horizon) and arched above by the meridian semicircle MLAB. Within it are drawn the equator-quadrant and a small right-angled triangle near the center marking the observed phenomenon.

- M (lower left): west point of horizon
- N (lower right): east point of horizon
- A (upper, near zenith): pole of the horizon (zenith)
- B (upper right): pole of the equator
- L (left, on meridian arc): point of mid-heaven on the equator
- H (center, below C): foot of the perpendicular
- C (center): position of the observed phenomenon
- G (on horizon, center): point on the horizon below the phenomenon, its distance from the meridian
- K (right of center): point on the equator quadrant

Now let there be given, from observation through the quadrant CG, the altitude of a certain phenomenon [Greek: φαινόμενον (phainomenon)] situated at C; likewise MG, its distance on the horizon from the meridian; and finally the point L, namely of mid-heaven on the equator. This last is known—by applying the right ascension of the Sun—from the lapse of time between the noon of the preceding or following day and the moment of the observation made, if you resolve the hours and minutes of the stars, known from a clock or from the altitude of some known star, into degrees and minutes of the equator, in the manner we have shown in several preceding examples.

Therefore, in triangle ABC there are given: AB, the distance of the poles—namely of the world and of the vertical—which is the complement of the elevation of the place, as we have often said; likewise CA, the

complement of the observed altitude of the phenomenon; together with the angle CAB, or GN, the complement of the observed azimuth MG to a semicircle. Hence CB is made manifest, measuring the complement of the sought declination, and thus the declination HC itself.

Then, since all the sides of triangle ABC have now become known, the angle ABC will not escape us—that is, the arc LH—indicating the distance of the sought phenomenon from the point M.C. [medium coeli, mid-heaven] at L. When this is added to the right ascension of the M.C., previously given from the time (if the phenomenon was given in the eastern region), or subtracted (if the same lay in the part west of the meridian), the right ascension of the said unknown star too is brought into the open. [Margin: *Probl. 1. c. 2.*] Once these have been acquired, the longitude and latitude of the same are determined from what precedes.

PROBLEMA II.

Given a point of mid-heaven, or some fixed star outside the meridian, on the equatorial armillary circle; given moreover the declination of the phenomenon on the declination circle of the same armillary: the right ascension of this is also found, together with the declination.

Likewise, on the zodiacal armillae rightly disposed, given a point of this kind—whether the M.C. or a known star outside the meridian—the longitude of the unknown phenomenon is found on the armillary circle of longitude, and its latitude on that of latitude.

There follow the observations to be made with armillary instruments—the equatorial, namely, and the zodiacal with their colures—set forth in some fashion earlier in our treatise on the sphere. Certainly they are to be held in nearly the same standing as those which the immediately preceding problem taught, and of no much greater certainty. For although armillae of this kind are gathered to have been very much in use among Hipparchus, Ptolemy of Alexandria [Ptolemy], and the ancients for taking the stars—because the positions of the Sun and stars could thereby be obtained at once in length and breadth, without the laborious computation of triangles, which to them was far more intricate than to us—nevertheless, on account of the more difficult combination and disposition of these same circles (which require the utmost precision), found troublesome both in their manufacture and in their use and application, these means of observing among the ancients are not free of suspicion and fault.

Accordingly, even though our Tycho Brahe once procured instruments of this kind for himself—conspicuous in size, and moreover wrought with the greatest industry and skill and at the greatest expense—nevertheless, when extreme precisions were being pressed for during observation, he eventually set these same aside, and all but entirely made them obsolete; just as he did the astrolabe and the torquetum, formed in the plane after the likeness of these.

As for this twinned problem, the things that are accomplished by it will be most correctly understood in practice alone, with the armillary instruments duly disposed to the meridian. These too can most conveniently be demonstrated on our material sphere, which represents both with its circles.

PROBLEMA III.

Given the meridian altitude, and the distance of an unknown star from some known fixed or wandering star: the position of the unknown is likewise found.

Among astronomical instruments, then, the quadrant is best suited for altitudes, while the sextant—which Tycho devised in place of the insufficient cross-staff [radius], and conveniently substituted in its

place—is best suited for observing the intervals or distances of the stars; from which afterward all that we desire and wish can be drawn out. That one or both of these will suffice for the astronomer who himself wishes to explore the positions and motions of the stars—since either can duly discharge the other's office when used singly—will appear in this and the immediately following problems. For by means of the meridian altitude of an unknown star, conveniently to be sought on the quadrant or quadrate, and afterward its declination; and by means of the distance of the same from some known star, taken on the sextant—we shall most easily and correctly grasp the rest, toward tracking down the star's position, by the following method.

An example, for the position of the bright star of the Lyre [Vega] thus acquired, drawn here from Book 2 of the Progymnasmata of Tycho Brahe.

The distance of the bright star of the Lyre from the upper and preceding star in the square of Pegasus—which is named Scheat—observed through the Tychonian sextant, was about $55^{\circ} 31'$.

The declination of the bright star of the Lyre, acquired either from the meridian altitude or on the declination circle in the year [15]77, was $38^{\circ} 26'$ N.

Now Scheat of Pegasus is known as having: right ascension $340^{\circ} 52'$; declination $25^{\circ} 50'$ N.

From these data [Greek: διδόμενα (didomena, "the givens")] the following triangle is constructed in this manner:

Diagram. A spherical triangle near the pole of the world. From pole A, two great-circle arcs (quadrants of colures, i.e., declination circles) descend to the equator-arc DE at the bottom. The third side BC connects the two stars.

- A (top): pole of the world
- C (upper left): the bright star of the Lyre [Vega]
- B (right): Scheat of Pegasus
- D (lower left, on equator): point on the equator
- E (lower right, on equator): point on the equator marking Scheat's right ascension

In the appended diagram, let quadrants of the colure of the equinox—or of the declination circles of our sphere—be drawn down from the pole of the world A; and let DE be the portion of the equator. Accordingly, in triangle ABC there are given: BA, $64^{\circ} 10'$, the complement of the declination of Scheat of Pegasus; AC, $51^{\circ} 34'$, the complement of the declination of the Lyre; CB, $55^{\circ} 31'$, the observed distance of the given stars from one another.

Therefore there emerges the angle CAB, about $65^{\circ} 14'$, which is called the angle of the difference of the right ascensions [angulus differentiae ascensionalis], whose measure lies along DE. Since, then, the right ascension of Scheat of Pegasus—placed at B—has been determined, through the point E on the equator, as $340^{\circ} 52'$: subtracting from this the arc DE found, $65^{\circ} 14'$ (since the Lyre lies to the west [in antecedentia]), there remains the sought right ascension of the bright star of the Lyre, $275^{\circ} 38'$.

This, together with its observed declination of $38^{\circ} 26'$, if it be presupposed, [Margin: *Per Probl. 1. conversionem.*] lets us determine from these its latitude of $61^{\circ} 43'$ N, and its longitude of $9^{\circ} 20'$ ♎ [Capricorn], in the same year [15]77. And by this method the positions in the heavens of all unknown phenomena [Greek: φαινόμενα (phainomena)] can be tracked down, given first some one [position] that will hold the place of a foundation—just as Tycho Brahe substituted the bright star of Aries ♈, by means of the Sun and the star of Venus. [Margin: *T. B. lib. 1. Progym. cap. 2.*]

For the rest, it must be noted here that two stars to be explored by distance in this way should hold a disposition toward the equator not too oblique; for otherwise the procedure will scarcely have assured certainty.

Furthermore, for the sake of testing and of further certainty, in discerning the position of an unknown star, the distance from two other known stars on either side can be explored, and weighed by this same standard, until the right ascension that is derived from it coincides as nearly as possible in a single point, so to speak. This can be done, provided the instruments employed for observing are of sufficient size in their structure.

PROBLEMA IV.

Given the distances of an unknown phenomenon from two stars whose longitudes and latitudes are known: the longitude and latitude of the unknown are likewise found.

The fourth and last method of observing the stars through instruments—by which we aim to fix their positions in fixed places of the heavens, and in the same way to know them in length and breadth—dispatches the business from the distances of the unknown phenomenon [Greek: φαινόμενον (phainomenon)] alone, from two stars known in latitude and longitude. This method, if the observations have been duly carried out and those three stars to be observed have formed a moderately well-shaped triangle, is no less certain than the immediately preceding one—although it is more laborious in the deduction and calculation of the triangles.

Let our example of this be the illustrious comet of the year [15]77, observed on the 13th day of November through the trigonal instrument, that is, the sextant of Tycho Brahe, at its distances from these two fixed stars, as follows. [Margin: *Vide lib. 2. Prog. pag. 35.*]

I. The comet was distant from the bright star of the Vulture [Altair, in Aquila] by $26^{\circ} 48'$. Then, at the same time, the distance of the same comet from the lower horn $\square$ [of Capricorn] was observed to be $21^{\circ} 19'$. The longitude of the bright star of the Vulture was $\square 25^{\circ} 52'$ [recte: $\square?$ — see note]; but its latitude $29^{\circ} 19'$ N. The longitude of the lower horn $\square$ was $28^{\circ} 16'$, and its latitude $4^{\circ} 37'$ N.

From these data [Greek: διδόμενα (didomena)] the adjoining figure is constructed in this manner, and resolved into numbers for the sought quantities:

Diagram. A spherical configuration with the pole of the ecliptic A at top. From A, three great-circle quadrants descend to the ecliptic arc EFG at the bottom, passing through the three observed bodies. The three bodies are joined to one another by great-circle arcs BC, BD, and CD.

- A (top): pole of the ecliptic
- B (middle, center): the bright star of the Vulture [Altair]
- D (middle right): the lower horn of Capricorn
- C (lower left): the comet
- E (bottom left, on ecliptic): foot of the quadrant through the comet
- F (bottom center, on ecliptic): foot of the quadrant through B
- G (bottom right, on ecliptic): foot of the quadrant through D

Let A be the pole of the ecliptic, from which the described arc EFG represents a portion of the ecliptic. Let the bright star of the Vulture be at B, the lower horn $\square$ at D, and the comet at C; and let three quadrants AC E, AB F, ADG descend through these three places onto the arc of the ecliptic; and let the three positions

of the stars be connected by arcs of great circles, namely BC, BD, and CD. With the figure thus ordered, let us consider in it the given quantities [Greek: διδόμενα (didomena)], and let us resolve the triangles in order for C—that is, for the sought position of the comet—both as to the latitude EC and as to the longitude E [F? — measured along the ecliptic].

I. In triangle ABD, then, there are given:

Side	Value	Description
A B	60° 41'	complement of the latitude of the bright star of the Vulture
A D	85° 23'	complement of the latitude of the horn □
B A D	2° 24'	difference of longitude of the Vulture and the horn □

Therefore BD, 24° 48', the distance of those same stars.

II. In triangle ADB, from all the sides granted:

Side	Value
A D	85° 23'
B D	24° 48'
A B	60° 41'

there emerges the angle ADB, 4° 59'.

III. Likewise in triangle BDC, since there are given:

Side	Value	Description
B D	24° 48'	distance of the stars previously found
C D	21° 19'	distance of the comet from the horn □
C B	26° 48'	distance of the

Side	Value	Description
		comet from the Vulture

Therefore there is found CDB, $72^{\circ} 42\frac{1}{2}'$; and adding to this the angle ADB, the total angle ADC, $77^{\circ} 31\frac{1}{2}'$, is composed.

IV. Then in the said triangle at C, since two sides are given about the angle found at D, namely:

- $AD = 85^{\circ} 23'$
- $CD = 21^{\circ} 19'$
- $ADC = 77^{\circ} 31\frac{1}{2}'$

it follows that $CA = 81^{\circ} 1'$, the complement of the comet's latitude; and therefore the latitude sought itself is $8^{\circ} 59'$ North.

And $CAD = 21^{\circ} 1'$, which measures the arc EG, that is, the difference in longitude of the comet from the lower horn of Capricorn $\square$.

Therefore, since the comet was ahead of that star, this arc of $21^{\circ} 1'$, subtracted from the longitude of the star in Capricorn $\square$ ($28^{\circ} 16'$), leaves the longitude of the comet sought as $7^{\circ} 15'$ Capricorn $\square$. And in this way — even by means of the distances alone of unknown bodies from two known stars — we are able to hunt out their positions. Note, however, that the three stars to be taken in this manner should be so suited to our procedure that the triangle, referred to the ecliptic, does not come out too distorted, nor oblique toward a single straight line; on which point I have advised before as well.

[Margin: See Tycho Brahe's *Progymnasmata*, Book 1, p. 551, in the example of Maestlin.]

Whoever desires more examples should seek them toward the end of Chapter 2 of Book 1 of Tycho Brahe's *Progymnasmata*, where, by a similar deduction from triangles, the 23 stars of Cassiopeia have been restored by observations.

PROBLEMA V.

When an unknown body has been detected, by its disposition along straight lines from two pairs of other stars whose positions are known, by simply laying a ruler or stretched thread crosswise [decussatim]: the longitude and latitude of the unknown phenomenon are given.

Among those methods which bring the positions of the stars into our knowledge without an elaborate instrument, the procedure carried out along straight lines crosswise toward four stars — to be explored by a ruler, or rather by a stretched thread — deserves the greatest certainty above the rest. For that reason I shall here set it out demonstratively in some example. For we had an observation of this kind, among others, in the year of our Savior 1610, on December 6, about the ninth hour of the evening, on the planet Mars, which at that time was found upon a straight line that ran very nearly first through the bright star of Aries γ , and the one further back on the back of Cetus. Then on another straight line through the outermost wing of Pegasus, and the star at the tip of the nostrils of Cetus, the first in number.

With these matters thus noted, a suitable configuration can be drawn on the globe or sphere in the following way:

Let A and C be the poles of the ecliptic, BD the portion of the zodiac running between them at the middle, K the star Mars. The fixed stars, with their positions, are marked with these symbols:

	Longitude	Latitude
E — outermost wing of Pegasus	3° 46' Aries γ	12° 35' B [North]
F — back of Cetus	6° 20' Aries γ	16° 55' A [South]
H — bright star of Aries γ	2° 14'	9° 57' B [North]
G — beak of Cetus	9° 39' Taurus γ	7° 50' A [South]

Parsing notes: The "B"/"A" markers denote Borealis (North) / Australis (South) latitude. The H-row longitude prints the sign symbol Aries γ in the running text above the table; printed as "gr. 2 mi. 14" with no repeated sign symbol in the table cell.

Practice.

Diagram. A spherical (zodiacal-lune) figure showing the two poles of the ecliptic joined by meridian arcs. The same figure recurs on the facing page (p. 134). It depicts the celestial sphere with the poles of the ecliptic at top and bottom and a cluster of points where the straight-line alignments cross.

- A (top): north pole of the ecliptic
- C (bottom): south pole of the ecliptic

- E (left): outermost wing of Pegasus
- F (lower left): back of Cetus
- H (upper right of cluster): bright star of Aries
- G (right): beak of Cetus
- D (right, on the mid-circle): point on the zodiac/ecliptic
- B (mid-circle): point on the zodiac
- K, L, M, N, P (central cluster): intersection points of the crossing alignment lines, including K = position of Mars

I. In triangle EAG, for EG [διδόμενα — Greek: διδόμενα (didomena, "the givens" / "the data")]:

- EA = $77^{\circ} 25'$, complement of the latitude of the wing of Pegasus
- GA = $97^{\circ} 50'$, excess of [the colatitude of] the beak of Cetus
- EAG = $35^{\circ} 53'$, the difference in longitude

Therefore GE is given = $41^{\circ} 5'$.

II. In the same triangle, for EGA, all the sides being given, AGE is given = $60^{\circ} 30'$.

III. In the right triangle GDL, for DLG and DL [διδόμενα — the givens]:

- DG = $7^{\circ} 50'$, latitude of the beak of Cetus
- LGD, that is, AGE = $60^{\circ} 30'$, just found
- D = the right angle

Therefore are given:

- DLG = $30^{\circ} 26'$
- DL = $13^{\circ} 33'$

IV. Likewise in triangle FCH, for FH, and HFC — or its supplement to a semicircle, HFA [διδόμενα — the givens]:

- FC = $73^{\circ} 5'$, complement of the latitude of the back of Cetus
- CH = $99^{\circ} 57'$, excess of the latitude of the bright star of Aries $\square$ beyond 90°
- FCH = $25^{\circ} 54'$, the difference in longitude of these stars

Therefore are given:

- FH = $37^{\circ} 7'$
- HFA = $45^{\circ} 29'$

V. In the right triangle MFN, for MNF and MN [διδόμενα — the givens]:

- FM = 16° 55', latitude of the back of Cetus
- MFN = 45° 29', the same as the HFA found above
- M = right angle

Therefore are given:

- MNF = 46° 59'
- NM = 16° 29'

Furthermore, let the sides DL and MN be added, and let the sum be subtracted from MD, and there remains the side NL = 3° 17'.

VI. In triangle NKL, for NK [διδόμενα — the givens of the angle at the side]:

- N = 46° 59'
- L = 30° 26'
- NL = 3° 17'

Therefore NK is given = 1° 38'.

VII. And finally, in the right triangle NKP, for KP and NP [διδόμενα — the givens]:

- KNP = 46° 59'
- KN = 1° 38'
- KPN = right angle

Therefore are given:

- KP = 1° 12', the very northern latitude of Mars
- NP = 1° 10', to which, if MN = 16° 29' is added, there is produced the arc MP = 17° 39'; and if this is added to the longitude of the star at F, which lies near M at 6° 20' Aries □, there emerges at the same time also the longitude of Mars □ = 23° 59' Aries □.

PROBLEMA VI.

Given the straight line of two stars together with the unknown phenomenon, and at the same time the distance of one known star from the unknown body: here the longitude and latitude are given.

Furthermore, from an observation along one straight line, through at least two stars [φαινόμενα — Greek: φαινόμενα (phainomena, "the appearances" / "the

phenomena observed")], together with the distance of the body from one of those same stars, we can investigate its position more concisely. As in the preceding diagram, given the straight line EK through the position of Mars at K, with the distance EK: there is sought first the whole distance EG, namely of the stars. Then next the angle AEK. Third and finally, in triangle AEK, since two sides are given about the given angle at E, AK will not lie hidden — the complement of the latitude of Mars — nor will EAK, the difference in longitude from the star at E.

As for the other methods of observing and thoroughly investigating the unknown position of a moving body without instruments, this can be referred to them whenever we see the same body, as it were partially, conjoined with some fixed star, or with another wandering star whose place in the heavens is known; or [when we see] how nearly [it approaches], and by how much it falls short of a partial conjunction, which we estimate as precisely as can be done. And afterward we mechanically fit it, by globe or by figures of the asterisms — especially those of Master Johann Bayer [Iohannes Baijerius] — duly to the position of the unknown.

[Margin: See the splendid work of the figures of the stars of Johann Bayer, V.I.D., formed from the verification of Tycho Brahe.]

And so much, thus far, concerning observations.

CAPUT X.

On the Parallaxes of a phenomenon, owing to the sensible [magnitude of the] semidiameter of the Earth relative to its orbit and position — the quantity of this [parallax], together with the principal methods of observing the parallaxes in altitude (whether the phenomenon be fixed, or have its own proper motion), from which afterward its distance from the Earth is given. Finally, by what methods chiefly it is fitting to separate and resolve the said parallaxes of altitude into longitude and latitude.

In the preceding chapter we set forth by various methods how the stars are commonly to be observed and reduced to order. In the present chapter, however, (by God's help) we shall treat of the most ingenious portion of all Astronomy — namely, by what device parallax (or aberration of sight) in the marking-down of a phenomenon may be recognized, through observations to be instituted in their own way precisely and opportunely, and afterward bound together by the methods of triangles — [parallax] arising on account of the distance of the Earth's center from the observer's position, if any such [parallax] be sensible in

comparison with the distance of the phenomenon. From this are deduced very many things in Astronomy, as necessary as they are pleasant, and as it were chiefly adapted to establishing the truth of Physics — namely, concerning the distances of phenomena from the Earth and their magnitudes. These make uniquely for [understanding] not only the computation of the eclipses of the luminaries, but also the generation and position of comets and of other new phenomena, against the opinion of the ancient Peripatetics. Moreover, so that the cause of parallaxes may be established at the outset, the following lemma must be set down first.

In it, let A be the Earth, B the station of the observer on its surface, CL a quadrant of the sphere of the fixed stars, which — on account of their too-great distance from the Earth — lacks all parallax, so much so that lines drawn from the Earth's center A and from B on the surface are understood to converge on that account. Let the orbit of the phenomenon possessing parallax be DM, inasmuch as relative to its semidiameter AM, a sensible magnitude of the side AB is found. Now the angle of parallax at the horizon is AMB, to which HMK is equal; but with the phenomenon placed in altitude, namely at E, the angle of parallax is AEB, to which FEG is equated. But when the phenomenon shall have been at D, the angle of parallax vanishes entirely, because A, B, D, C constitute a straight line. It is also easily perceived that the angle BMA comes out the greatest of all the parallactic angles, while the least is the one nearest to D, on account of the disposition of the angle at B. For when that angle [at B] is a right angle, as at the horizon, each of the angles at A and M will be greater than when the same angle at B is found to be obtuse — and this successively, as in the remaining positions of the parallactic phenomenon above the horizon. With these matters thus established, we shall now proceed to the general demonstration of parallax from observations to be set out.

[Margin: Formed from the verification of Tycho Brahe.]

Diagram. A quarter-circle (quadrant) figure illustrating the parallax lemma. The center of the quadrant is the Earth's center; the arc CL is the quadrant of the sphere of the fixed stars. Lines from the Earth's center and from the observer's surface position radiate to the phenomenon's positions on its orbital arc.

- A (lower left, center of quadrant): the Earth (Earth's center)
- B (left, on the radius): the observer's station on the Earth's surface
- C (top): upper end of the fixed-star quadrant
- L (lower right): lower end of the quadrant (CL = quadrant of the fixed-star sphere)

- D (left, on the phenomenon's orbit): position of the phenomenon at the zenith/overhead point where parallax vanishes
- M (center area): point on the phenomenon's orbit at the horizon
- E (center-left): position of the phenomenon in altitude
- F (upper, near C): point on the fixed-star sphere
- G (near F): point on the fixed-star sphere
- H (right): point on the fixed-star sphere
- K (right): point on the fixed-star sphere (angle of parallax at horizon HMK)

PROBLEMA I.

Given both meridian altitudes of a phenomenon that is always visible around the pole of the world [i.e., circumpolar]: the parallax aggregated in both altitudes — if it have any — or the difference, is given.

A celestial phenomenon whose parallax we are endeavoring to investigate (if it have any) will either be immobile with respect to the fixed stars, or it will move. We shall treat of both in this and the following problems. For if the phenomenon be mobile, it is reduced to immobility, by observing the change of its declination in its diurnal motion and proportionally accommodating it.

Now there are two cases of the present problem. For either the phenomenon goes around the pole of the world in such a way that it is always seen below the vertex of the place, toward the north, in both meridians; or in the upper meridian it rises above the vertex toward the south. The former case the adjoined diagram explains, in which — keeping for the most part the earlier letters, and with AN the half of the world's axis, P having been described from center A by the circle of the phenomenon — the highest meridian altitude of this [phenomenon] is set at E, and the lowest at M; and this with respect to the first revolution alone, the phenomenon being otherwise immobile. Therefore, the elevation of the pole at N or P being given, it is easily gathered, from both meridian altitudes of the phenomenon, by how much the arc NM exceeds the arc NE. Now for the aggregate of the parallaxes, a semicircle GQ is drawn from the pole of the world P; and since the arcs PF and NE are equal, PQ is made equal to PG. But the parallax of the lowest altitude of the phenomenon at M is [measured] by the angle HMK (or BMA); and if to this is added the arc QH, which is equal to FG, the parallax of the highest altitude, then the aggregate of both parallaxes of the phenomenon emerges in the arc QK.

Diagram. A quadrant figure (first case) showing a circumpolar phenomenon below the vertex toward the north. The world's axis is along AN; an inner circle of the phenomenon is described about center A. The highest meridian altitude is at E, the lowest at M.

- A (center, on baseline): center of the quadrant (Earth)
- R (lower left): end of the baseline
- L (lower right): end of the baseline
- C (top): top of the outer quadrant
- F (upper, near C): point on the phenomenon's circle
- G (near F): point on the phenomenon's circle
- P (upper right area): pole of the world
- D (left): point on the phenomenon's circle
- E (center-left): highest meridian altitude of the phenomenon
- N (center): point on the world's axis (NM, NE arcs)
- M (center): lowest meridian altitude of the phenomenon
- Q (right): point of the drawn semicircle GQ
- H (right): point (angle HMK)
- K (right): point (parallax arc QK)

In the other case, we no less extricate the difference of parallaxes — if there be any — having taken account, however, of refraction, if the phenomenon in the lowest meridian shall have been near the horizon. Let the phenomenon in the present diagram be in the highest meridian at E, surpassing the vertex ED or FC. Since it is established from the foregoing that the parallactic angle in the upper [meridian] was [measured] by BEA, but in the lowest meridian by BMA, therefore — the pole being P, if the semicircle FQ be drawn — KQ will be the difference of the parallaxes, or of the said angles BEA and BMA, which we had proposed to acquire.

Diagram. A quadrant figure (second case) showing the phenomenon in the highest meridian surpassing the vertex toward the south. A dotted curve (the phenomenon's path) crosses the figure; the refraction near the horizon is indicated.

- A (center, on baseline): center of the quadrant (Earth)
- L (lower right): end of the baseline
- C (top): top of the outer quadrant
- F (upper left, near C): point on the phenomenon's circle

- G (near F): point on the phenomenon's circle
- P (upper right): pole of the world
- D (center-left): point (vertex line ED)
- E (center-left): the phenomenon in the highest meridian
- N (center): point on the figure
- B (lower center): point (parallactic angles BEA, BMA)
- M (center): the phenomenon in the lowest meridian
- Q (right): point of the drawn semicircle FQ
- H (right): point
- K (right): point (parallax difference KQ)

Astronomia Danica — Liber Secundus Sphæricorum, Caput X (continued)

PROBLEMA II.

Given a phenomenon set with two fixed stars on the same vertical circle — whose positions are of course known, together with the elevation of the pole of the place — and given the altitude of the phenomenon taken at the same time as that of one of the fixed stars; then, in a second observation made without reference to the verticals, given the distances of the phenomenon from those same fixed stars, together with its own altitude and likewise that of one of the stars: to determine the parallaxes of the phenomenon appropriate to each time, if it has any.

This problem, which is general for finding all parallaxes — whether the phenomenon is permanently visible, or rises and sets — was devised most skillfully by the Englishman Digges [Thomas Digges], but given a new form by Thaddaeus Hagecius [Tadeáš Hájek]. We now set it out compactly, requiring in the first observation only the altitude, without observing the distances from the fixed stars, as shown in the following diagram.

[Margin: Vide problem. T. B. prim. lib. pag. 668. — See Problem in the first book of Tycho Brahe, p. 668.]

Everything else being known in itself, let it be that in the first observation the apparent place of the phenomenon is at I, on which vertical its true position at D is also to be understood, together with the fixed stars at C and B. Therefore, given the altitude of that point and likewise of one of the fixed stars, its distance from

each is given by the same operation, through the preceding problems. Then, in the second station, with the phenomenon shifted to H and the fixed stars accompanying it as it appears —

Diagram. A quadrant of a sphere with vertex A at the top and arc descending to the right and left. Within it a network of great-circle arcs (verticals and the diurnal circles of the bodies) is drawn, with the phenomenon and stars marked in two positions.

- A (apex): vertex / zenith point
- B (upper interior, near C): one fixed star (true positions of the stars)
- C (upper interior, left of B): the other fixed star
- C (right side, lower than the inner C): second-station position of a star
- D (center, below B/C): true place of the phenomenon
- I (left interior): apparent place of the phenomenon in the first observation
- H (below D): apparent place of the phenomenon in the second station
- K (lower left, on the arc): endpoint of the horizon/base arc
- L (lower right, on the arc): endpoint of the base arc
- G (bottom): lower intersection of the diurnal arcs
- D (right side, below the right C): second-station true place marker

— since here, by the tenor of the problem, sufficient given quantities [διδόμενα (didomena, "givens")] are present, its parallax is at length obtained from these for each altitude, and so on.

PROBLEMA III.

Given, over an interval of time, the distance of a phenomenon from two fixed stars — placed either above or below it — together with the altitude of the phenomenon on each occasion, from the lowest that can be had up to the highest: the path traced out by the orbit of the phenomenon is gathered, and from this the difference of parallax, if it has any.

This appears to be the most general method of unravelling parallax in those phenomena that admit it. As in the adjoining figure, let ABC be the meridian and ADC the horizon, whose pole is B; let the true track of the new phenomenon be EF, its apparent track DG; and let there also be two fixed stars from which, over an interval —

Diagram. A semicircle (the meridian) standing on a horizontal base (the horizon AC), with pole B at the top. A fan of great-circle arcs runs from B down to the

base, and several short oblique segments mark successive positions of the phenomenon and stars at three altitudes.

- A (left end of base): left endpoint of the horizon
- C (right end of base): right endpoint of the horizon
- B (apex): pole of the horizon (zenith)
- F (upper left, below B): point on the true track
- G (upper left, left of F): point on the apparent track
- I, L (left interior): track points
- N (interior, repeated): intermediate star/track marks
- M (interior, repeated three times down the right side): successive positions of a star
- H (lower interior): position at the first (lowest) altitude
- K (left interior, above H): position at the second altitude
- R, V, Q, D, E (along the base, left to right): foot-points of the verticals on the horizon

— let its distance from them be taken: M and N, at the first altitude at H, at the second at K, and finally at the third at L. Since, then, the greater parallax is always nearer the horizon, the distances of the fixed stars likewise come out wider here, but narrower the higher the phenomenon has climbed — provided it is placed below the stars, as in the present figure; and conversely, if the same phenomenon should be above the stars in its diurnal revolution. So much so that, if you wished to examine the track of a new phenomenon having parallax by means of such observations, you would never persist in keeping to the traces of a great circle in the sky.

[Margin: Amplius de enucleatione parallaxium novorum phænomenon legantur Tho. Digges. Angulus de stella anni 72. Thaddæus Hagæus in sua Dialexi. — On the further unravelling of the parallaxes of new phenomena, read Thomas Digges, *Alae* on the star of the year [15]72; Thaddæus Hagecius [Tadeáš Hájek] in his *Dialexis*.]

And so, concerning the investigation of the parallaxes of a new phenomenon that admits them, we have given certain problems that seemed to us most suited to this business. But whoever requires more and other things, let him consult Regiomontanus, Digges, Thaddæus [Hájek], Tycho [Brahe], and other astronomers for himself.

PROBLEMA IV.

Given the parallax of altitude of a phenomenon: its distance from the earth is given.

For the explanation of the present problem, let the first diagram of this chapter be recalled here, on which we explained the cause of the parallax that intervenes.

Diagram. A quadrant figure: a quarter-circle arc with center at the lower left (A), bounded by a vertical radius (A–B–C up the left side) and a horizontal radius (A–L along the bottom). The arc carries points F, G, H; interior lines run from the vertex region to the small right-angled triangle at M.

- A (lower left, center of the quadrant): center of the earth / vertex of the right angle
- B (left side, just above A): point on the vertical radius
- C (upper left, top of the vertical radius): top of the radius
- D (left side, between B and the arc): point on the vertical
- E (interior, near D): place of the phenomenon (true place, complement of altitude DAE)
- F (on the arc, upper): point on the arc
- G (on the arc, below F): point on the arc
- H (right side, on the arc): point on the arc
- K (lower right, on the base): endpoint of the base radius
- L (bottom, below K): endpoint of the horizon
- M (lower interior, near the base): apparent place of the phenomenon (small right triangle BMA / vertex of the parallactic angle BMA)

Let the parallactic angle at the horizon first be given, BMA. Since, then, in the same plane triangle AB is given — the semidiameter of the earth, which always remains 860 German miles, as is gathered from geometry — likewise the angle ABM is a right angle, and besides the parallactic angle BMA, and consequently also the angle at A: therefore the side BM, or also AM, will be given — that is, the distance of the phenomenon from the surface, or from the center of the earth, as one pleases.

The same is also accomplished in triangle BEA, given namely the complement of the apparent altitude of the phenomenon DBE, or the true one DAE, together with the parallactic angle BEA. For thus the three angles will be given, along with the

side BA, as was demonstrated before; and therefore either the side BE or AE will be given as well.

But since it helps to illustrate the proposed problem by examples — which are most conveniently drawn from the Moon, as she suggests to us parallaxes that are, as it were, known and familiar — let it be so taken from the restored lunar hypothesis. The horizontal parallax of the Moon, when she is at her mean distance from the earth, is $1^{\circ} 0' 51''$, which the angle BMA measures. And since the angle at B here is a right angle, the angle BAM will be $88^{\circ} 59' 9''$.

In the said right-angled triangle ABM, then, for the side AM as the distance of the Moon from the center of the earth, it will be:

	M		A B		B		A M
as	$1^{\circ} 0' 51''$	to	1 semidi amete r of the earth	so	90°	to	$56\frac{1}{2}$ semidi amete rs of the earth
S. R.	17699 81				10000 000		

Parsing notes: The four-column proportion is laid out in the source as a braced four-term ratio ($M : AB :: B : AM$), i.e. $\sin(1^{\circ}0'51'') : 1 \rightarrow \sin(90^{\circ}) : AM$, giving $AM \approx 56\frac{1}{2}$ earth-semidiameters. "S. R." = *sinus rectus* (the natural sine / log-sine entry). The sine entry under M reads 1769981 (sine of $1^{\circ}0'51'' \approx 0.0176998$, scaled); under B reads 10000000 (sine of 90°). [The entry 1769981 vs. the more usual 176998 form is transcribed as printed.]

The semidiameters of the earth by which the Moon is distant from it once found, it will be easy to extend them into miles. For:

	Semidiamet ers of earth	miles	Semidiamet ers of earth	miles
as	1	860	$56\frac{1}{2}$	48590

A second example, likewise taken from the Moon, in triangle BEA, also at her mean distance from the earth: let the true altitude of the Moon be 45° , which the angle BAE measures. But the parallactic angle AEB is $43' 35''$. Therefore, by the

complement of the sum of these to a semicircle — by which the operation must proceed — there is found $45^\circ 43' 35''$. Accordingly:

	Semidia meters of earth			Semidia meters of earth	
as	43' 35"	to	1	so	45° 43' — to 56½ [semidia meters]
S. R.	126776			S. R.	7160143

— almost as before.

Parsing notes: The proportion is $\sin(43'35'') : 1 \rightarrow \sin(45^\circ 43') : AE$, yielding $AE \approx 56\frac{1}{2}$ earth-semidiameters, agreeing with the prior result. "S. R." = *sinus rectus*. The minutes entry in the second sine argument is printed as "43¾" (or 43-and-a-fraction); transcribed here as "43'" with the fraction flagged. The first log-sine $126776 = \text{sine of } 43'35''$; the second $7160143 = \text{sine of } 45^\circ 43'$.

Further, for the given distance of the phenomenon from the earth it is clear, by the conversion of the solution of triangle ABM or ABE, in what manner a table of parallax from the horizon all the way to the vertex is to be constructed — namely, the sides AM (or AE) and AB being known — so that no further labor need be undertaken to demonstrate the matter. And once the distance of a heavenly body from the earth is given, together with its apparent magnitude, its true magnitude is easily given in comparison with the earth, after the example to be left by us near the beginning of Book I of the *Theorica*.

[Margin: Lib. 1 Theor. cap. 1. — Book 1 of the *Theorica*, chapter 1.]

PROBLEMA V.

To distribute a given altitude-parallax from the horizon into longitude and latitude.

Among the various methods of this procedure handed down by others, we shall bring forward two: one discovered by Tycho [Brahe] and inserted in Book 2 of the *Progymnasmata*, which is the more general and does not differ much from the following diagram; the other, which is more special and especially suited to a solar eclipse, devised by us.

[Margin: Lib. 2 Progymn. pag. 38. — Book 2 of the *Progymnasmata*, p. 38.]

Now the method produced by Tycho is demonstrated thus:

In the adjoining figure, let ABE be the meridian of the place, A the pole of the world, B the pole of the vertex of the place, and lastly C the pole of the ecliptic — this last set in the arctic circle about the pole of the world. From these poles are described DE, the horizon, whose pole is B; and FH, the ecliptic, from its own pole C. With the quadrant BR drawn, the parallax in the circle of altitude MQ is given from the hypothesis. Then, with CQ and CP drawn from the pole of the ecliptic C, and QP drawn perpendicular at P, it is evident that in the small right-angled triangle QPM every species of parallax is found. For QM is of altitude, as we defined above; here QM is of altitude, MP is of latitude, and QP of longitude — differing almost insensibly from the arc LK marked on the ecliptic. For knowing these latter quantities, one must proceed by this route.

Diagram. A full circle (the meridian) with a small circle (the arctic circle) near the top inside it. A horizontal diameter DE (the horizon) crosses the figure; the ecliptic FH and circle of altitude run obliquely; a small right-angled triangle QPM sits at the right, where the parallax species are read off.

- A (upper left interior, on the small circle): pole of the world
- B (top, above the small circle): pole of the vertex of the place
- C (interior, below A): pole of the ecliptic
- N (top, near B): point near the meridian/ecliptic intersection
- F (upper left, on the small circle): point on the ecliptic
- H (upper right): point on the ecliptic
- K (right interior, in the small triangle): point on the ecliptic arc (arc LK)
- L (right interior, below K): point on the ecliptic arc
- M (right interior, near K): apparent place (altitude QM)
- P (right interior, below M): foot of the perpendicular (longitude QP)
- Q (right interior, below P): true place / vertex region of the small triangle
- D (left, on the horizontal diameter): left endpoint of the horizon
- E (right, on the horizontal diameter): right endpoint of the horizon
- F (lower left, on the circle): lower marker

First, in triangle ABC, for BC: since AB, the distance of the poles of the world and of the vertex, is given; AC, the distance of the pole of the world and of the ecliptic, which is always equal to the maximum obliquity of the equator and the ecliptic; and besides, the angle BAC is also known from the remoteness of C, the pole of

the ecliptic, from the point of mid-heaven at N — for which the right ascension of the Sun and the time, in the diurnal revolution, must be presupposed. Therefore, from these three —

[Margin: Prob. 1 c. huius. Prob. 4. cap. 1. huius. — Problem 1 of this chapter; Problem 4 of chapter 1 of this [book].]

— the first sought arc CB is given. Hence, in triangle CBM, for the angle at M: since we have the said triangle with its sides given — namely BM, the complement of the altitude of the body above the horizon; CM, the arc defined from the pole of the ecliptic at M, which here exceeds the quadrant by the arc KM; and finally CB, just now found — therefore from these the angle CMB is given, to which QMP is equal. At last, in the small right-angled triangle QPM, from the given quantities — besides the altitude-parallax QM, two angles, namely QPM the right angle and QMP just found — there are given the quantities we sought: MP, the parallax of latitude, and QP, that of longitude.

This practice suffices well enough for moderate parallaxes, such as the Moon's; but if greater ones could be conceived, to be resolved into longitude and latitude according to the position of the ecliptic, the operation may be more correctly set up in the oblique-angled triangle CMQ. In it, since the two sides CM and QM, together with the contained angle at M, are known from what precedes, the side CQ — measuring the parallax of longitude beyond CM — will not be hidden, nor will the angle MCQ, which rightly contains within itself the parallax of longitude. An example of this problem is to be sought in Tycho [Brahe], at the place cited.

PROBLEMA VI.

Given a point on the ecliptic culminating [coelum mediante], together with its declination and the angle of intersection of the meridian and the ecliptic; given also the place of the luminaries on the ecliptic: the true altitude of these is given, to which — by the preceding problem — the corresponding parallax is likewise distributed into longitude and latitude.

We have devised this manner of disentangling and distributing parallaxes, suited (unless I am mistaken) especially to a solar eclipse — namely, since, having first obtained both the altitude of the luminaries above the horizon (which is required in this business) and the angle running between the vertex and the ecliptic, and finally the parallax of altitude, we may then quite easily distribute it through longitude and latitude.

Diagram. A full circle (the meridian ABDM) with the horizon AM as a diameter from pole D; the equator EN drawn from pole Q; and the zodiac EP oblique. The Sun's place is at F on the ecliptic, with its right ascension at R. A cluster of arcs and small triangles appears at the upper right and lower right.

- A (left, on the circle): point on the meridian / left end of the horizon
- B (upper left interior): point on the meridian
- D (upper interior, near B): pole of the horizon
- M (right, on the circle): right end of the horizon
- E (top): point of the equator/zodiac (locus of the Sun's ecliptic origin)
- N (right, on the circle): point of the equator
- Q (interior): pole of the equator
- P (lower right, on the circle): point of the zodiac
- C (upper interior, near B/D): mid-heaven declination point
- F (interior, near C): place of the Sun on the ecliptic
- R (interior, near F): right ascension of the Sun
- G (lower left interior): interior construction point
- L (lower interior): interior construction point
- T, S (small triangles, upper right): construction points

Let there be a diagram in which ABDM is the meridian, AM the horizon from pole D, EN the equator from pole Q, and lastly EP the zodiac. Let the place of the Sun be at E on the ecliptic, whose right ascension is known at R; and the time by which the Sun is distant from mid-heaven is measured by BR. Further, since AB is given — namely the elevation of the equator above the horizon — its complement BD is also given; likewise BC, the declination of mid-heaven, from what precedes; and similarly CF as well. And since CD too has been granted from the induction of the preceding, the angle DCF is acquired —

[Margin: Prob. 1 c. huius. Prob. 4. cap. 1. huius. — Problem 1 of this chapter; Problem 4 of chapter 1 of this [book].]

or rather it is given from its own table. In triangle DCF, therefore, we have as the given quantities [διδόμενα]:

- DF, the complement of the Sun's altitude
- CFD
- CD

From these, then, the following are obtained:

- DF
- CFD

Now if the angle CFD turns out to be obtuse, this is the indication that the luminaries are positioned beyond the ninetieth degree of the ecliptic, and that the Moon is inclining westward through the parallax of longitude.

Next, draw the arc or line ST. It is then clear that in the small right triangle FST all the parallaxes are given: the parallax of altitude by SF, the parallax of latitude by ST, and finally the parallax of longitude by FT. And because the triangle is so cramped that its longest side scarcely spans a single degree, we may treat it as a plane triangle in place of a spherical one without any appreciable error. In this triangle all the angles are given, since — besides the angle at F (at the ecliptic), which is given — the angle at T is always a right angle. The side SF, which measures the parallax of altitude, is therefore readily obtained; and once the parallax of altitude has been determined from SF, the side ST (the parallax of latitude) and the side TF (the parallax of longitude) follow easily.

This method seems to me by far the most concise and convenient of all in a solar eclipse, especially where the Moon has no appreciable latitude — quite apart from the fact that it supplies the foundation of Reinhold's tables, page 99 and following. Moreover, when the Moon has northern latitude, account can be taken of the increase of altitude when the eclipse calculation is set up. Anyone who wants a worked example will find it in Book I of the *Theorica*, in the solar eclipse computed for the year 1612, on the 20th day of May.

[Margin: *Tomo 1 Theor. cap. 8*; likewise *lib. 1 Theor. cap. 9*.]

Diagram. A spherical figure (the carry-over diagram from the preceding Problem VI, here used again) showing the celestial sphere with intersecting great circles.

- A (lower left, on the bounding circle): vertex of the horizon
- B (upper left): point on the equator/horizon construction
- C (near top center): point near the zodiac/equator intersection
- D (top): pole of the horizon
- E (top, with the ~~M~~ zodiac sign mark and ~~N~~ point on the zodiac and equator
- Q (upper right): point on the bounding circle
- M (right): the other horizon vertex (horizon arc AM)
- N (lower right): point on the equator (equator arc toward N)
- P (lower right, below N): terminus of the zodiac arc

- R (left of center, in the cluster): the right-ascension point
- S (center of cluster, right of R): endpoint of arc/line ST and of side SF (parallax of altitude)
- T (center of cluster, below B/right of S): right-angle vertex of the small triangle FST; side FT = parallax of longitude, side ST = parallax of latitude
- F (center of cluster, small raised glyph between R and S): position of the Sun on the ecliptic; common vertex of triangles DCF and FST
- G (left, below A): point on the construction
- K (lower center): point on the lower bounding arc
- L (center): point on the horizon arc

CAPVT XI.

On the Refractions of the Stars, on Twilights, and on the Lifting of Meteors above the Surface of the Earth.

The doctrine of the refraction of the stars, although in a sense already hinted at in the *Optics* of Witelo and of Alhazen [Ibn al-Haytham], was nonetheless first discovered in our own age — through our own Tycho Brahe — from celestial observation, and its causes were lucidly debated with Rothmann. Afterward Johannes Kepler [Johannes Kepler] attempted to explain these matters with numbers tied to a demonstration, but not in a sufficiently general way; for in maritime regions and those nearer the northern pole, the refractions vary enormously on account of the thicker constitution of the air, so much so that in those places they become twice or even three times as great as in inner Germany, or generally in places that have air purified by the more effective rays of the Sun.

Two things, then, present themselves as indicators of the variation in the refractions of the stars: the nature of the ground, and the twofold obliquity of the Sun's rays as they strike the surface of the earth — the one obliquity arising from the proximity of the pole, the other from the horizon at each individual place. Having considered these, and given that scarcely anywhere — in any of the places where I have so far stayed in Norway and elsewhere — are appreciable refractions detected beyond half a quadrant from the horizon, I shall set out their causes briefly, together with Lord Tycho, by means of an attached diagram.

[Margin: *Tomo 1 Epist. Astron.; Ioh. Keplerus in Opticis.*]

In it,

Diagram. A schematic of three concentric circles (orbs) drawn about center A, representing the structure of the heavens and the air enveloping the earth, used to explain why refraction increases toward the horizon.

- A (center): the center of the earth
- D (top): point on the outermost orb (the heaven of the fixed stars)
- E (upper right): point on the outermost orb
- F (right): point on the outermost orb, at the horizon level
- B (lower left): terminus of the lowest horizon line
- C (left): point on the outermost orb
- G (lower right): point at the horizon on the right
- H (center, near A): point on the terrestrial globe
- N (lower center, right of A): point on the terrestrial globe (terrestrial globe HN)
- M (center): point on the aerial sphere
- K (center, near M): point on the aerial sphere
- L (center, near K): position of a star existing in the horizon at G; refraction is greatest here

three orbs are described about center A. The outermost is the celestial orb, in which the stars are understood to be embedded — BCD. The terrestrial globe is HN, and the aerial sphere is MKL, poured out all around the earth on every side. Now let the ethereal region be that which is contained between this [aerial] sphere and the outermost orb — of so subtle a matter and essence (as was shown by us earlier in the preliminaries, and is shown further by the doctrine of parallaxes and of refractions) that it produces of itself no refraction or vibration of the stars.

It follows, then, that the cause of refraction is to be considered as lying with the aerial sphere alone, in the manner I described above — so that, by reason both of the thicker transparent medium [diaphanum] and of the more oblique projection of the rays, the stars are refracted more effectively, whether through our own sight or as the rays insinuate themselves toward us. Hence it comes about that a star is subject to the greatest refraction when it is in the most sloping position. For example, take the star L existing on the horizon at G: on account of the more oblique incidence of the ray at L, and the thicker transparent medium HL, the refraction at P [nearly] vanishes.

PROBLEMA I.

Given the depression of the Sun below the horizon of a place when the morning or evening twilight wholly ceases: the elevation of the topmost vapors above the earth is found.

We touched on the cause of twilights in the *Astronomical Preliminaries*: namely, tiny particles raised aloft among the vapors above the clouds, which, because they are illuminated by the Sun's rays, fade off into whiteness. As for the sublimations of these particles — at the highest boundary of which refractions doubtless begin to occur — let us investigate them from the last proposition of Alhazen's *Optics* and the 60th proposition of Witelo's first book, so that the difference between the side HL and the side HP (the refracting transparent medium, as it were, in the preceding figure) may become known to us, and so that afterward the limits of these twilights may be ascertained for each place on earth and each season of the year.

[Margin: *For Alhazen, the last [proposition]; and Witelo's 60th.*]

Now on account of the discrepancy of 7 degrees found between the observations of Tycho and of Rothmann in Book I of the *Astronomical Letters* — by which Rothmann raises the depression of the Sun below the horizon above Tycho's figure, with all twilight then ceasing: for Rothmann puts it at 24 degrees, while Tycho puts it at only 17 degrees — and furthermore because we in Norway find that twilights are more lasting even in winter, we do not doubt, as a general rule, that for the final disappearance of these [twilights] the Sun is placed 20 degrees below the horizon. From this, the demonstration of the proposed problem, upon the appended diagram, follows in this manner.

Let A (in the following figure) be the center of the earth; let BCEFG be the circle of the Sun, CHD the body of the Sun from center B, and KTLP the earth. Let the visible horizon be LM, above which straight line sight alone occurs. Let the shadow of the earth cast by the Sun be RPTQ, and let the part of the earth illuminated by the Sun be PKT — 180 degrees 28 minutes, owing to the fact that the diameter of the Sun contains the diameter of the earth

Diagram. A geometric construction (the "penultimate diagram" referred to in Problem II) showing the earth, the Sun's circle, the earth's shadow, and the twilight geometry. A large circle (the Sun's circle BCEFG) with center E–G as its horizontal diameter; a smaller circle at lower right represents the Sun's body; the shaded sliver QRM at upper center represents the shadow region.

- A (center): the center of the earth
- E (left, on horizontal diameter): point on the Sun's circle BCEFG
- G (right, on horizontal diameter): point on the Sun's circle
- F (top): point on the Sun's circle
- R (upper center): apex point of the shaded shadow sliver
- Q (upper left of R): point of the shadow region RPTQ
- M (center, above A): the point where twilight closes (LM the visible horizon)
- P (center, right of M): point of the illuminated region PKT
- L (center, near M): point on the visible horizon
- T (center, below A): point of the shadow region
- K (center, below A): point of the earth KTLP
- H (lower center-right): point of the Sun's body CHD
- C (bottom): point on the Sun's body CHD
- B (lower right): center of the Sun's body
- D (lower right): point on the Sun's body CHD

— [the earth's diameter] about $5\frac{1}{2}$ times over, with the Sun raised to its mean distance from the earth, that is, 1200 earth-radii [semidiameters] according to Ptolemy [Ptolemy], and even a little more according to our recent emendation. The angle PAB (that is, half the circumference PKT) will therefore be 90 degrees 14 minutes. But the angle GAB, by the hypothesis of the Sun's depression below the horizon — when, that is, the twilight wholly fails — was 20 degrees. Therefore the angle PAG is given as 70 degrees 14 minutes. And since LAG is a right angle, LAP will be 19 degrees 46 minutes.

Moreover, since the twilight closes at point M — where those whitening particles, which are caused as twilights by the terrestrial vapors, are no longer visible — the angle LAP is bisected at point M, at the point of contact of the circle where the sides MP and ML are equal. Thus MAP will be 9 degrees 53 minutes. But what was proposed was to find AM. Therefore, for this purpose, in the plane right triangle MPA — since all the angles are given, together with the side AP, the earth's radius, as 860 German miles — AM will not lie hidden: it is 873. The difference, then, between AL and AM — namely NM, about 13 miles — defines the elevation of the highest vapors above the earth, which we have been discussing.

PROBLEMA II.

Given the distance of the highest vapors from the earth: to bring to light, and to distinguish from one another, the quantity of the refracting transparent medium in each case — in the maximum and in the minimum apparent refraction.

For the elucidation of this problem, let the penultimate diagram (that is, the one placed on the page immediately following) be recalled, with straight lines drawn from the center of the earth A to L and to P. In the first right triangle AHL, then — with the right angle at H (since the starry heaven at G excludes appreciable parallaxes, as was shown above concerning the fixed stars) — we proceed as follows in seeking HL.

First.

as	AL : H	so	AH : HLA
	873 : 90°		860
			S.R. 98516.
			Answer:
			80° 7'

That is: as AL (873) is to H (90°), so is AH (860) to HLA; the sine of the result (S.R.) is 98516, giving the answer 80° 7'.

Then, in the same triangle.

as	H : AL	so	HAL : HL
	90° (S.T. 100000)		873
			gr. 9 min. 53 → 150 — German miles (S.R. 17164)

That is: as H (90°, whose total sine S.T. = 100000) is to AL in miles, so is the angle HAL (9° 53') to HL = 150 [recte: read with S.R. 17164] German miles.

Diagram. A geometric construction (a quadrant figure) showing the earth's center, the concentric arcs of the terrestrial globe and aerial sphere, and the lines for the altitude of 45° at which refraction is said nearly to cease. A semicircular/quadrant outline with horizontal base BAG and vertical radius AD.

- A (center of base): the center of the earth
- B (lower left): terminus of the horizontal base

- D (top): top of the vertical radius
- E (upper right): point on the outer arc, at the 45° altitude line
- C (left): point on the outer arc
- F (right): point on the outer arc
- G (lower right): point at the horizon on the base line
- M (center): inner point of the construction
- P (center, right of M): point of the construction
- H (center, below M): point on an inner arc (triangle HAP)
- L (center, near H): point of the construction
- K (center, near L): point on an inner arc
- N (center, on base right of A): point on the base line

Furthermore, at the phenomenal altitude of 45 degrees — where we said earlier that the refraction of the stars very nearly ceases — in triangle HAP, since the angle PHA is given as 135 degrees (by adding, that is, PHL = 45 degrees to the right angle), together with AH and AP, the computation for the angle at P will first run thus:

as	AP : AHP	so	AH : HPA
	873 (S.R. 70710) : 135°		860 → (69658), to which correspond s S.R. ...

Result:

- 44° 9' — to be added [for the angle at P]
- 135° 0' [angle AHP]
- 179° 9' (sum)
- 179° 60' (the semicircle)
- HAP = 0° 51', the difference — the angle HAP sought.

Then: as 135° (S.R. 70710) is to AP (873), so is the angle HAP (51', S.R. 1483) to M ... = about 150 [the superior measure], in miles (1483/18⅓ proximate).

[Margin: *On twilights, let be read: Petrus Nonius [Pedro Nunes], Johannes Kepler, the lord Ambrosius Rhodius, the lord Petrus Laurenbergius [Peter Lauremberg], etc.*]

Therefore the difference sought, between the thickest transparent medium and that in which the refractions cease, is $13\frac{1}{3}$ German miles.

From this comparison one may see how much a transparent medium that is thicker in quantity near the horizon contributes to increasing the refractions — over and above the two causes that concur as one: the one, namely, always self-similar, concerning the unequal obliquity of the incidence of the rays; the other concerning the change of the air with a change of place upon the earth. We have learned from experience that this last cause has so great a variability inherent in it, that we do not undertake to measure the refractions except through that same experience — imitating in this matter Lord Tycho Brahe, who in the first book of the *Progymnasmata* had published tables of refractions separately for the Sun, the Moon, and the stars, for the horizon of Denmark.

But as for the observation of the Dutch in Nova Zembla in the year of our Lord 1596 and the following [years], where the Sun returned into view 17 days before its lawful time — what shall we say, except that so great a thickness of the air was gathered, through the Sun's perpetual absence below the horizon from the 3rd of November to the 11th of February (new style), that it produced this entire discrepancy?

PROBLEMA III.

Given the elevation of the pole and the declination of the Sun: the moment of the hour of the twilight ceasing on the horizon — both the morning twilight and the evening — is found.

What twilight is, is gathered from the preceding causes: namely, the doubtful light that precedes the Sun's rising and follows its setting, and is not wholly shut off by the horizon until the Sun has descended 20 degrees below it, according to the primary motion. But in the first place, before the elucidation of the problem, by comparing the complement of the elevation of the pole and the northern declination of the Sun (together with the given depression of the Sun), on the near side — or, [in the case of] meridional declination, on the far side of the equator — it will be examined when and in what places the twilight is continuous. For when and where the aggregate (formed from the Sun's declination at any given time and its depression of 20 degrees below the horizon, by hypothesis) is greater than the elevation of the equator (that is, the complement of the pole), perpetual twilight reigns — as is easily recognized on our material sphere.

Astronomia Danica — Liber Secundus (Sphæricorum), Caput XI–XII

[continuation of CAPVT XI, PROBLEMA III]

We now proceed to the demonstration by means of an example fitted to the present time. On this present day [Margin: 6 Feb.], on which the Sun at dawn was in the 6th degree, 14' of [Pisces?] — its declination being $9^{\circ} 11'$ — the complement of the elevation of the pole of this place is $34^{\circ} 17'$, and finally the Sun's depression below the horizon is 20° , by hypothesis. We will fit the following diagram to our [material] sphere.

Diagram. A circle representing the celestial sphere with the meridian and horizon drawn as two great circles crossing near the center, plus the equator and oblique arcs marking the Sun's position below the horizon. In this example the meridian is F E H G; the horizon is F H, whose lowest pole is A; the equator is E G, whose antarctic pole is B. C is the place of the Sun below the horizon at the moment dawn first appears of all. Therefore L C is the southern declination of the Sun, $9^{\circ} 11'$; M C is the depression of the Sun below the horizon, 20° . The labeled points are:

- I (top): upper intersection point of the oblique circles
- E (upper left): pole of the equator side / equator endpoint
- F (left): horizon endpoint
- K (center): center of the sphere
- M (right of center): point on the horizon-depression line
- L (just below M): point marking the Sun's declination arc
- M (right edge): horizon endpoint
- C (lower center-right): the Sun's place below the horizon
- G (right, below M): equator endpoint
- B (lower left): antarctic pole of the equator
- A (bottom): lowest pole of the horizon

Therefore, in triangle A B C, for the angle at B [Greek: διδόμενα (didomena, "the things given" — the given quantities)] are:

- A B = $34^{\circ} 17'$, the distance of the poles
- B C = $80^{\circ} 49'$, the complement of the Sun's declination
- A C = $70^{\circ} 0'$, the complement of the Sun's depression, continuously [taken].

From these the angle C B A is given as $67^{\circ} 48'$, which the arc of the equator G L measures. Converted into time, this yields 4h 31m 12s — the first morning daybreak. From this the evening twilight's extreme moment is likewise obtained: by subtracting the same sought-for time from 12 hours, namely 7h 28m 48s.

Another example.

The moment to be sought is that of the evening twilight ending on 24 March of the year 1611, on which the festival of Easter falls. Around this time the place of the Sun is in $13\frac{1}{2}^{\circ}$ of Aries [Margin: □], to which corresponds a solar declination of $5^{\circ} 21'$ north.

Diagram. A second circle of the celestial sphere with meridian, horizon, equator, and oblique arcs, oriented for a northern-declination Sun. The labeled points are:

- A (top): pole
- B (upper left): point on the meridian / equator side
- G (upper right): equator endpoint
- D (left): horizon endpoint
- I (left of center): point on an oblique arc
- M (center): point near the center
- K (center-right): center of the sphere
- H (right edge): horizon endpoint
- C (below center): the Sun's place below the horizon
- I (lower left): lower oblique-arc point
- F (bottom): pole

Again, in the appended diagram, the angle C B A is to be sought. Thus, given in triangle A B C: A B = $34^{\circ} 17'$, the distance of the poles; B C = $84^{\circ} 39'$, the complement of the Sun's declination; A C = $110^{\circ} 0'$, the excess of the Sun's depression below the horizon, 20° beyond the quadrant. Therefore the oblique angle A B C is given as $138^{\circ} 20'$, to which corresponds in time 9h 13m 20s. Subtracting these from 12 hours leaves 2h 46m 40s — a time which on the same day also marks the first morning dawn.

PROBLEMA IV.

Given the direct interval on the earth between two observers viewing the same small cloud at [a given] altitude above the horizon: the height of that cloud above the surface of the earth is found.

This problem investigates for us the distance of the middle region of the air, by means of the elevation of clouds. Here it should be noted in passing — something we have seen confirmed by experience here too — that in winter, because of the lesser efficacy of the solar rays, clouds and meteors of this kind are far nearer to us, by nearly double, than in summer; that is, in proportion to how high the Sun itself is elevated above the horizon of the place in its daily revolution. With us, therefore, in winter they are elevated scarcely $\frac{1}{2}$ mile, while in summer about $1\frac{1}{2}$ miles above the surface of the earth.

But to add a demonstration to this problem: let the horizon $G B$ be drawn, below the appearance of the suspended cloud at C ; and from two stations A and D , the interval between which can be ascertained, let the altitude of that same cloud be taken by both observers simultaneously to a given mark, [measured] from the horizon on the same side, lest the cloud's motion or breadth in the meantime impede this procedure. Then, with lines drawn from A and D to the small cloud C , and also the perpendicular $C B$, it is established that these measure the sought-for elevation of the cloud above the earth, in this way:

Diagram. A woodcut semicircular figure: a horizon line $G — A D B — E$ along the base, with a billowing cloud drawn near the top center. Lines run from the two observer stations A and D up to the cloud at C , with a vertical perpendicular $C B$ dropped to the horizon. The labeled points are:

- G (left end of horizon)
- A and D (two observer stations on the horizon, close together near center)
- B (foot of the perpendicular from the cloud)
- E (right end of horizon)
- C (the cloud, at the apex; the cloud body is shown as a woodcut puff)

First, in triangle $A D C$, since all the angles are given together with the side $A D$ — inasmuch as A is measured by the altitude of the prior [observation], D by the complement of the altitude of the latter [observation] to a semicircle, and the side $A D$ is granted by hypothesis — the side $A C$ is therefore given. Hence, in the right triangle $A B C$, from the given angles together with the side $A C$, the sought-for elevation of the cloud $C B$ is given.

PROBLEMA V.

Given the interval between the shadow of a cloud cast perpendicularly by the solar rays onto the surface of the earth or water, and the Sun's altitude

at the same time from the place of the observer: the height of a cloud seen from the earth is found.

This problem adds to the foregoing one concerning the determination of the altitude of clouds the experience of Mr. David Pastor [David Herlitz?] of Resterhave in East Frisia, an outstanding cultivator of the renewed astronomy at this time, to be demonstrated by the following diagram thus.

Diagram. A woodcut semicircular figure: a horizon/diameter line $D — A B K G — E$ at the base, with the Earth's center at A. A billowing cloud is drawn in the upper portion near C/H. The radius line $A F$ runs to the Sun at F; the cloud shadow $C H$ falls directly onto the earth's surface below at B G. The labeled points are:

- D (left end of the base line / meridian)
- A (center, the Earth)
- B, K, G (points on the base line near the center, marking the shadow's foot region)
- E (right end of the base line)
- F (upper right, the Sun, toward which radius $A F$ is drawn)
- H (the cloud, upper region)
- C (the nearer, facing part of the cloud)

Let $D F E$ be the meridian; from the center — namely the earth, A — a small cloud is seen at C. Through its nearer and facing part A, let $A F$ be drawn, leading the Sun's rays down from the Sun at F. But since the whole region $H F$ above the clouds is illuminated by the rays of that same Sun, the shadow of the cloud $C H$ accordingly besets the earth's surface directly below, namely B G; and therefore the angle at B is made a right angle. The angle $C A B$ measures the Sun's altitude itself above the horizon, whose complement the angle $A C B$ exhibits; and the side $A B$ has likewise become known from the distance of the shadow — at a fixed place on the surface of the earth or water — as it appears from the observer's station A. Hence $C B$ itself, the sought-for elevation of the cloud, is given.

Moreover, this method of ours too is among those contemplations born of the occasion: for if the observer's station is fixed beneath the shadow of the cloud, between B and G — namely at K — and a line is drawn from K to C, through whose end at C the observation of the cloud's altitude above the horizon is made by a quadrant or other instrument from K, and at the same time the moment is noted

[CAPVT XI continued, p. 147]

when the Sun's ray first descends perpendicularly from C — namely from the same cloud — to B; something which can often be observed in the open field. For this reason, in the right triangle K B C, likewise from all the angles given together with the side K B, B C is given as before.

CAPVT XII.

On the doctrine of Gnomonics in the construction of sundials.

[Margin: THIRD CLASS.]

The doctrine of Gnomonics I here, too, do not assign to Altimetry — that is, to the measurement of objects — along with the ancients, who for the most part measured the height of objects from the straight and reversed shadow of the Sun; since indeed we more correctly use, as our Gnomon, instruments [such as] the quadrant, sextant, or another sufficient in geometry. Rather, I direct the principal part of this doctrine toward the skillful construction of sundials [sciathericorum] — and indeed of every kind — upon the [armillary] sphere, intending to show by shortcuts what I can accomplish in this part above what others have hitherto discovered. These things I devised at the castle of Roskilde [Margin?: Rosenholm] in the year of the Lord 1601, and in that lofty leisure which then fell to me — having just first returned from Bohemia — through the kind invitation of a noble man, Senator of the realm, and a pillar of all solid erudition and of the erudite: Mr. Holger Rosenkrantz [Holigerus Rosæ Krantzius]; before, in the same year, I set out for Norway, as far as Nidaros [Nidrosia, Trondheim], together with the most noble and at the same time most learned and humane man, Mr. Steen Bille of Billesholm [Stheno Bille de Billisholmio].

Now this doctrine is called Gnomonics from [Greek: γνώμων (gnōmōn, "gnomon" or "indicator")] — that is, from the *style* [stilus, the dial's pointer] — because it embraces a fixed reckoning of the shadow of the style cast by the solar rays onto a surface, chiefly a plane one. Such a shadow is marked either from the root of the style fixed in the surface or from its extremity. Then this — [Greek: σκιαθηρικόν (skiathērikon, "shadow-catcher," i.e. sundial)], from [Greek: σκιά (skia, "shadow")], that is, [the shadow], the said instrument — is very ancient, inasmuch as it was chiefly destined by the ancients for marking out times in hours and the sixtieths of hours [Margin: 2 Kings ch. 20]. This can be fashioned in diverse forms: either it will be adapted to the plane of the horizon, whence it is

called *horizontal*; or it will be erected perpendicular to a wall facing the Sun, and is thence called *mural*. Both kinds, however, either indicate the times simply in hours and the minutes thereof; or, with more elaborate apparatus — namely with the inscription of the signs of the zodiac — whence the course of the Sun through the whole year, likewise the quantity of day and night together with the immovable feasts, the twilights, and the rest, are known for each individual day.

Finally, it should be indicated in general that sundials can be constructed for any plane whatever, likewise for a spherical or even a concave surface. And although others have completed this doctrine mechanically through many windings, we nevertheless will demonstrate it more correctly and with greater profit by far, by the doctrine of triangles, briefly in what follows — first in sundials which are to be constructed upon the plane of the horizon, then in those upon the vertical [Margin:... of Gnomonics]. [Margin: There exist among many other things of this kind the eight books of Clavius written *de Gnomonicis*.]

PROBLEMA I.

Given the elevation of the pole of the place, there are obtained — both in the plane of the horizon and in the vertical — the intersections of the equal hours, together with the arcs intercepted between the equator and the horizon, and likewise between the equator and the vertical of the place.

What we called to mind in the preceding book concerning the equator — namely that it is the very regulator of all equal times [Margin: Bk. 1 Sphær. cap. 4] — which was also afterward shown by many examples in this book: this too will now, when we are dealing with sundials, especially be recalled to memory.

[CAPVT XII continued, p. 148]

But, to the exposition of the problem: let A B O H be the meridian in the added diagram, E H the horizon, whose pole is B; and K B the principal vertical, in the ninetieth degree from the meridian, at K. At the rising or setting [point] is O G, the equator, whose pole is A; let its quadrant O K be divided into six equal parts for the six hours on either side of noon, equal in the equator.

Diagram. A circular dialling-construction figure: the meridian circle A B O H drawn with the horizon E H as a horizontal diameter, the vertical K B, and the equator quadrant O G to the upper right. A fan of lines runs from the pole A (upper left) like shadow-lines, cutting the equator quadrant into equal 15° segments and projecting onto the horizon and the vertical. The labeled points are:

- A (upper left): pole of the equator (the mundane axis / style root)
- B (top): pole of the horizon
- Q, R, S, T, V (descending along the vertical/upper-left arc): successive hour-intersection points on the vertical K B
- O (right): rising/setting point, top of the equator quadrant
- C, D, M, N, P (along the equator arc descending to the right): the equal divisions of the equator quadrant (15° each per hour)
- E (left): horizon endpoint
- K (center): the place / foot of the principal vertical, and center
- H (right): horizon endpoint
- G (lower right, and a second G lower down): equator endpoint(s) at rising/setting
- The vertical fan points along the horizon near center: V, T, S, R, Q reading toward K

The style must be erected at K toward A, so that it is always made conformable to the axis of the world, [and] will indicate, by its own shadow cast by the solar rays in the daily revolution [of the Sun], those intersections — which, when they are in the equator (as I said) equal, must be unequal in the horizontal quadrant K H and the vertical B K. They are understood, from the same place, by the same marks which we have affixed on either side, just as on our [material] sphere all these things are perceived to the eye. Thus, with circle-proportions drawn from the pole of the world A — as it were the shadows of the style — which divide the equator into six equal parts O, C, D, M, N, P, computed for individual intervals of 15° each, as agreeing with one hour, it is to be seen how much space the individual intervals — corresponding to the equal spaces of the equator — claim for themselves, both in the horizon and in the principal vertical; which the doctrine of spherical triangles makes manifest thus.

First, in the horizon, for the eleventh hour before noon and the first hour after noon: since in the right triangle Q C K there are given K C = 75° by hypothesis [recte: read with the construction; printed 7C], the angle Q K C = $34^\circ 17'$ (the elevation of the equator of this place), together with the right angle at C — therefore from these K Q is given as $77^\circ 31'$. Further, in the same way, supposing K D = 60° by hypothesis, and the angle at K retained, there emerges K R, or rather H R, the interval of two hours from noon. And so in the rest, diminishing on each occasion the arc of the equator intercepted between the meridian and the rising or setting [point] by 15° .

For the vertical sundial, in just the same way, both B Q and the rest of the hour-intersections on the wall (i.e. the principal vertical) become known, putting here the angle at K [recte: the angle B K O] equal to the elevation of the pole itself; as in this example. Let the interval on the wall K B be given, agreeing with three hours from noon. Since the demonstration is closed within the right triangle K M S, it will be, for K S, as follows:

M	(angle)	K compl.	K M compl.	K S compl.	B S
grad. 90	to gr. 34 m. 17	so gr. 45 to	(proporti on)	T	
S. T. 100000	S. R. 56329	T. 1000000	56329	Resp. gr. 29 m. 24	

Parsing notes: This is a rule-of-three (proportion) layout, not a true tabular grid. As printed it reads: "as grad. 90 is to gr. 34 m. 17, so gr. 45 is to [the sought quantity]," with the trigonometric values (sine/tangent) shown beneath: S.T. (sine total) 100000, S.R. 56329, T. (tangent) 1000000, yielding the answer gr. 29 m. 24. The bracketing braces { in the source group "K compl. / K M compl. / K S compl." against "M /... / B S." The column alignment is approximate; the load-bearing result is **Resp. gr. 29 m. 24** (the intersection B S = 29° 24').

And so in the rest, until tables are drawn up for fixed pole-elevations, and the intervals from the meridian up to the sixth hour — equal on either side in the equator — have gone out [i.e. been computed]; [those] we obtain for the horizontal arise with contrary intersections. The hours, however, which lie beyond [the sixth] in the horizontal are indicated by contrary intersections.

Now, with a quadrant of a circle prepared — in which both degrees and minutes can be had — it will be easy to delineate a sundial of either kind through its own hours, whether it be a large or a small one. For since all circles are divided equally, therefore, if several circles are described from the same center, a ruler marked through the degrees and minutes on the quadrant cuts off equal portions in all of them, [continues onto next page]

for the individual hours — but this is a mechanical matter, and so it must be left to each practitioner's own ingenuity.

Finally, as to the placement of the style [stilus]: since it must take on the disposition of the world's axis, it will, on a horizontal dial, be inclined toward the north by an amount corresponding to the elevation of the pole above the plane of

the horizon; whereas on a mural (vertical) dial, the style will stand off from the wall toward the south by an angle corresponding to the elevation of the equator at the place. In either case the correctness of the arrangement can be tested with the small quadrant. The length of the style here is purely arbitrary. And with this it will be enough to have treated briefly the simple sundials laid out with respect to the axis of the world (that is, of the earth), without any declination.

Lastly, regarding the determination of the intercepted arcs — which was the second part of the problem — this too is readily dispatched within the preceding diagram. For in triangle CKQ (to dwell, by way of example, on the eleventh, or the first, hour from noon): KC is given, as is the angle CKQ, the elevation of the equator; and furthermore the angle at C in the equator is right. Therefore CQ is given for the horizontal sundial, and likewise CQ for the mural, together with its complement AQ — that is, when the arc BO in the meridian is taken for the angle at K, the elevation of the pole at the place itself. The use of these arcs is required later in connection with the skillful inscription of the signs of the zodiac, where worked examples of finding these arcs in numbers are also presented more fully. And although tables for this problem will be found below — for nearly every elevation of the pole, from which sundials can be quickly produced — we nevertheless leave this particular one for the elevation of Copenhagen [Hafnia] in this place, at least for the hourly intervals on each kind of sundial.

A little table of the hourly intervals, with their differences, for the elevation of the pole 55° 43', on the Copenhagen sundial, both horizontal and mural.

Hours from noon (AM / PM)	In the horizontal:		In the mural: Arc	
	Arc in horiz. (deg. min.)	Difference (deg. min.)	in wall (deg. min.)	Difference (deg. min.)
12 / 12	0 0		0 0	
11 / 1	12 29	13 1	8 36	9 25
10 / 2	25 30	14 4	18 1	11 23
9 / 3	39 34	15 30	29 24	14 54
8 / 4	55 4	16 58	44 18	20 16
7 / 5	72 2	17 58	64 34	25 26
6 / 6	90 0		90 0	

Parsing notes: The difference columns are printed offset between the rows they bridge (each difference sits between two adjacent arc values); I have aligned each difference to the lower of the two rows it spans, matching the engraving's vertical placement.

PROBLEMA II.

Given the inclination of a wall to the east and to the west from the meridian: the hourly intersections on the mural sundial are found.

[Margin: Prob. 1, c. 8, of this (book).]

For the mural sundial this case is a special one, because the walls or partitions on which sundials are hung are rarely set exactly opposite the point of the meridian. This inclination, however, is determined either by compass, or by a meridian line, or — better still — by celestial observation, namely the shadow of the Sun or the apparent position of a star tested against the planeness of the wall. Once it has been found, the procedure is as follows.

In the diagram added here, let there be, upon center A, a wall KED inclined to the east or west by 30 parts (degrees) from the principal vertical circle FHLC — itself first drawn about B, the south (antarctic) pole of the world. From this point let the line EAD descend perpendicular to the wall; and from it the aforesaid inclination is indicated both by the arcs KF and HI, and by the distance of the pole B of the equator from the said perpendicular line. And since the arc EBDC, lying upon the said perpendicular along the surface, represents the meridian, it is plain that the point of noon — that is, of the twelfth hour — rests at DC. Suppose, then, that the wall here inclines toward the west: the Sun's rays will therefore at last appear upon it in the morning at F, but will likewise depart so much the later in the evening at H. For the mural plane can receive the Sun's rays for no more than 12 hours, as the half of the equator HLF demonstrates. But before all else, in order that the angle DFC and the arc CF may be known, the procedure is as follows.

Diagram. A circle drawn about center A, representing the celestial sphere projected for an inclined mural dial. A horizontal diameter runs from K (left) through A to the right; a vertical diameter runs from E (top) through A and B down to C (bottom). The inclined wall and the equator are shown as great-circle arcs, with the hours of the afternoon (numbered 1 through 12) marked along the lower-right arc and the morning rays meeting the wall at F (lower left).

- E (top): endpoint of the perpendicular EAD to the wall; top of the principal vertical circle

- A (center): center of the projection
- B (just below A, center-lower): the south (antarctic) pole of the world, about which the principal vertical circle FHLC is drawn
- K (left): left endpoint of the horizontal diameter; one terminus of the inclined wall KED
- D (lower center, below B): lower endpoint of perpendicular EAD; with C marks the meridian / noon point (DC)
- C (bottom): foot of the vertical diameter; noon (twelfth-hour) point
- F (lower left): point where the Sun's rays first strike the wall in the morning
- H (right, upper): point where the rays depart in the evening
- I (upper right, near 8/9): point on the wall arc; arc HI marks the inclination
- L (lower right, near hour 3): point on the equator arc HLF
- 9, 8, 7, 6, 5, 4, 3, 2, 1 and Roman X/XI/XII (around the right and lower arc): the afternoon hour-lines radiating from B
- 9, 10, 11, 12 (lower left): the morning hour-lines near F

In the right triangle DFC, since the data [διδόμενα (Greek: didomena, "the givens")] are: DC, the elevation of the pole at Copenhagen, $55^{\circ} 43'$; the angle FDC, falling short of a right angle by as much as the wall inclines on the other side — so in this example it is 60° ; and the angle at C is right — therefore DFC is given as $60^{\circ} 48'$, and FC as $55^{\circ} 3'$. Furthermore, once FC and the angle at F have been found, that arc of the equator CF must be divided, as far as can be done, into equal hourly parts. So in this example the arc CF, $55^{\circ} 2'$, makes 3 hours 40 minutes 12 seconds. From this it is understood that the entry of the Sun's rays first onto this declining sundial occurs at F at about 8 hours 20 minutes — at which morning hour the first illumination of that sundial occurs. In the same way the operation can also be set up on the eastern side. For since, between the end of the quadrant at L and the 3rd hour PM — by reducing the hour-arc on the equator into hours, and so on, and its resolution [ἀνάλυσις (Greek: analysis, "unraveling, resolution")] — there are gathered $10^{\circ} 3'$, it follows that arc H falls short, by just that much, of the 3rd hour on the equator. Thus H is removed on the equator from the 3rd hour PM by $79^{\circ} 57'$; and likewise for the remaining hours up to the last, and so on. With these duly divided and distributed across the equator, the found angle DFC of $60^{\circ} 48'$ is assumed as if it were the elevation of the pole; and so for the hourly arcs and the rest, the procedure follows by the immediately preceding problem.

[Margin: Prob. 1, cap. of this (book).]

PROBLEMA III.

Given the declination of the Sun, and the arc intercepted between the horizon and the equator on the horizontal sundial — or the arc intercepted between the equator and the vertical on the mural sundial — together with the defined length of the style: the elongation of the style's shadow is found for any hour and at any time of the year. From this follow the entries of the Sun into any twelfth-part [dodecatemorion], that is, sign of the zodiac; the length of day and night; the rising and setting of the Sun; likewise the months and days of the year, and the other things that are customarily inscribed with art on sundials.

This problem is a special discovery of our own, which will first be illustrated on the horizontal sundial.

By way of example, let there be sought, in the diagram of the first problem of this chapter, the arc intercepted ΩC at the 1st hour PM at the elevation of our pole — which is found by a very easy prosthaphæresis [prosthaphæresis]. For in the right triangle:

	S.T.	QCK	S.R. KQ	S.R. CK	Resp. S.R. QC
as	90°	to 77° 32'	so 34° 17'		to 33° 22'

— and so on for the rest of the arcs, which the table below will contain.

With all the arcs for the horizontal thus obtained — which on either side of the meridian are equal on a non-declining sundial — let the style be erected above the plane of the horizon to the elevation of the pole at the place, and let it always be 10 parts, so that the operation may be carried out the more easily. Now in the following figure AB, since the length of the style's shadow at each hour is sought, on a non-declining sundial let the quadrant of the plane of the horizon HD be drawn, together with the equator HE. From here, from center A, straight lines are drawn to the individual hours on the plane of the horizon; but from the summit of the style they are drawn to the point of the Sun's declination from the equator — all of them considered on the northern, and opposite, part of the sphere.

Diagram. A quarter-circle (quadrant) construction for the horizontal sundial, drawn about center A at lower left, opening toward the lower right at V. The outer arc is labeled, in cursive engraving, *Limbus Horizontalis* ("the horizontal limb"); inner concentric arcs are labeled *Tropicus* ☐ (Tropic of Cancer), *Æquator*

(Equator), and *Tropicus* $\square$ (Tropic of Capricorn). Hour-lines radiate from A; the style and shadow geometry are shown near center.

- A (lower left, center of construction): the center; foot of the style
- B (center, below the hour cluster): summit-of-style reference / shadow point
- H (lower left, on baseline): one terminus of the quadrant HD and of the equator HE
- D (top): upper terminus of the quadrant of the horizon plane HD
- V (lower right): right terminus of the horizontal baseline
- E (upper right area): terminus of the equator HE
- I, N (upper arc, near D): points on the horizontal limb
- P, R (left interior): intercepted-arc points
- F, G (interior near B): shadow / hour points
- K, L, M (lower interior): hour-line and arc-intercept points
- 10, 9, 8, 7, 6 and 2, 1 (interior numerals): hour markers along the radiating lines
- *Limbus Horizontalis* (outer arc): "horizontal limb"
- *Tropicus* $\square$ (inner arc): "Tropic of Cancer"
- *Æquator* (inner arc): "Equator"
- *Tropicus* $\square$ (inner arc): "Tropic of Capricorn"

Therefore, in the plane triangle ABG, for the length of the style's shadow AG in the plane of the horizon at the 11th or 1st hour, when the Sun shall have been at the beginning of $\square$ [Cancer] and shall have obtained a declination of $23^{\circ} 32'$:

The data [$\delta\iota\delta\acute{o}\mu\epsilon\nu\alpha$] are:

- GBA: $66^{\circ} 28'$, the complement of the Sun's declination.
- AGB: $56^{\circ} 54'$, composed of the arc intercepted between the horizon and the equator at the 11th or 1st hour, and of the Sun's maximum declination. [Margin: see Prob. 1 of this chap.]
- AB: 10 parts, the length of the style.

Hence: as GAB (the angle subtended) at the 1st hour PM, $56^{\circ} 54'$, is to 10 parts, so GBA $66^{\circ} 28'$ is to 10... — to find the sought shadow-length AG.

- S.R. (sine) 83772... S.R. 91683

Exemplum I. (Example I.)

Let the projection of the shadow be sought, with a style of 10 parts, at the elevation of the pole $54^{\circ} 43' [?]$, at the third hour, when the Sun shall have been at the beginning of $\square$ [Taurus] or $\square$ [Virgo].

[Note: the elevation here is printed "54. 43"; in context this work's Copenhagen elevation is $55^{\circ} 43'$ — a probable compositor slip of 54 for 55. Flagged in the report.]

At that time the Sun's declination is $11^{\circ} 31'$ northern. The intercepted arc corresponding to the 3rd hour is $25^{\circ} 44'$.

	deg.	min.
Declination of the Sun	11	31
Aggregate (sum)	37	15

Now: as S.R. $37^{\circ} 15'$ is to 10, so S.R. $78^{\circ} 29'$ is to 16... 11 (10 minutes making one little part — for thus the operation becomes easier): the sought length of shadow.

Exemplum II. (Example II.)

Let the quantity of the shadow be sought on the horizontal dial, likewise at the 3rd hour, when the Sun shall have been at the beginning of $\square$ [Scorpio], having again a declination of $11^{\circ} 31'$. And here, because the declination is southern, it must be subtracted from the intercepted arc corresponding to the 3 hours, namely $25^{\circ} 44'$.

And there remains the difference $14^{\circ} 13'$. Then the same declination must be added to the right angle, and one must operate, as before, through the complement of this aggregate to 180° , namely $78^{\circ} 29'$.

As S.R. $14^{\circ} 13'$ is to 10, so S.R. $78^{\circ} 29'$... PM

- S.R. 24559... S.R. 97987... to 39. 54. — the sought length of shadow; and so for the rest, until the calculation has been carried through for every hour, and likewise for the beginnings of the signs of the zodiac.

Furthermore, in the plane of the horizontal sundial, through the extremities of the lines found for the individual hours and for the beginnings of the signs — as we have said — arcual (curved) lines are to be drawn, along which the shadow

from the gnomon or style will travel, from sunrise to sunset, throughout the whole day, at an equal remove.

On the sides, on either hand, a calendar can be inscribed, as also on the meridian line the length of day and night, and so forth.

In just the same manner, the quantity of the shadow's extension from the style on the mural sundial is to be investigated for the individual hours, by means of the given style-length of 10 parts (most conveniently), and the arc intercepted between the equator and the vertical at the individual hours — as in the present quadrant, where the mural quadrant EC is described from A:

Diagram. A quarter-circle construction for the mural (vertical) sundial, drawn about center A at upper right, opening toward the lower left. The style AS projects from the wall; lines radiate from pole B; the equator and quadrant arcs are subdivided into hourly parts.

- A (upper right, center of construction): the center; foot/origin of the mural quadrant
- B (right, below A): pole from which the hour-lines are drawn; the angle BAS represents the complement of the pole-elevation
- S (upper right, between A and B): summit of the style AS projecting from the wall
- E (upper left): one terminus of the mural quadrant EC and of the equator ED
- C (lower right): lower terminus of the mural quadrant EC
- D (lower right, below C): terminus of the equator ED
- F, G, H, K, L (lower-left arc): the six equal parts of the equator quadrant for the six hours (F G H K L E)
- M, N, P, Q, R (interior): the unequal hourly parts (M N Q P R E) cut on the quadrant EC
- 2, 3, 4, 5, 6 (left margin numerals): the hour markers along the arc

[Margin: Prob. 1, cap. of this (book).]

— but from B the equator ED. Let the style project from the wall, namely AS, in such a way that the angle BAS represents the complement of the elevation of the pole at the place. Let the quadrant of the equator be divided into six equal parts, for the six hours before noon or after noon, which are F, G, H, K, L, E; the lines drawn from these to the pole B likewise cut the quadrant EC into just as many unequal hourly parts — namely M, N, Q, P, R, E — which (just as with the

intercepted arcs FM, GN, etc.) are found in the way shown before. But the complements of these arcs to the dodecatemoria (twelfth-parts)...

the inscription of the twelfth-parts [dodecatemoria], and so on, are required, because the measure of the angle at A — that is, of the center of the axis on a mural dial — is comprehended by these arcs themselves. Just as within our sphere, this can be demonstrated most thoroughly and most plainly by string-extensions [filares extensiones]. For now, however, we will teach this inscription of the signs [Greek: πραγματείαν (pragmateian, "the procedure" or "treatment")] by means of a single example, namely, under our pole-elevation of $55^{\circ}43'$, for the first hour after noon, F M, and for the Sun's entry into the beginning of Aries (♈) and Libra (♎).

Here, first, since the arc F M is given from what precedes as $54^{\circ}47'$, its complement M B — that is, the angle M A B — will be $35^{\circ}13'$. Next, the Sun's declination at the beginning of Taurus (♉) and Virgo (♎) is taken as $11^{\circ}31'$; and since here, in the northern signs, this is added to the quadrant, the angle A S B comes out to $101^{\circ}31'$. Therefore, since two angles of triangle A B S are given — namely, that at A and that at S — the angle at B in the rectilinear triangle is also given as $43^{\circ}16'$. Moreover, the length of the style A S is given by hypothesis, conveniently, as 10 parts.

Therefore, as A B S is to A S B:

A B S	$43^{\circ}16'$, 10 p.

sine 685.39	

A S B	

complement $78^{\circ}30'$	

sine 97987	

p. M	

14 17, the sought-for extension of the shadow [umbra].	

And so likewise in the remaining cases, for any pair of zodiacal signs having one and the same declination at their beginning — provided only this is observed: that in the southern signs, the work by the rule of proportion is set up here with the complement of the Sun's declination placed in the third position.

Nor is the procedure any different for declining dials [sciathericis declinantibus], once the intercepted arcs for the individual hours have been determined. This

alone must be noted: that if a mural dial directly opposite to the meridian nonetheless declines toward the north above, or toward the south — and however much it does so (which can also be explored with a quadrant) — then if toward the north, that amount must be added to the pole-elevation of the place; but conversely, if the wall is found to lean toward the south by its inclination. Once all these things are understood together, and once — by string-projections to the center within our sphere, mimicking the solar rays — they have been explored, a sundial [sciathericon] for any plane whatever is constructed from the doctrine of triangles, and most conveniently and most skillfully by the demonstrations and indications we have set out above.

There now follows a twofold table, in which, namely, both the hourly interval on each kind of dial — horizontal and mural — is computed for the individual degrees, and the arc between the equator and the horizon on the horizontal dial is computed for every fourth degree, for pole-elevations from 30° up to 70° , so that they may be ready to hand when needed. Indeed, in the complements of the pole-elevations, the intercepted arcs between the equator and the vertical are also given, whence afterward, from their complements, the angle at A for the mural dial.

For example: let the said angle at A be sought from the lower of the following tables, for the first hour on the mural dial — the angle we found a little earlier to be $35^\circ 13'$ under this pole-elevation, namely, $55^\circ 43'$. With its complement, then, $34^\circ 17'$, I find the intercepted arc for the first hour to be $54^\circ 47'$ (when the correction has been duly made for the 17 minutes adhering to it); and the complement of this is $35^\circ 13'$ — namely, the sought-for angle at A.

Table of the measure of the horizontal arc from the meridian for the individual hours; and of the mural arc in the complement of the former, for pole-elevations of single degrees from 30 to 70.

Parsing notes: The leftmost narrow column header (rotated) reads "Horiz. Sciath. poli elevatio" (horizontal-dial pole-elevation); the rightmost narrow column reads "Murali Sciath. compl. El. poli" (mural-dial complement-of-elevation). The six paired hour-columns are headed (left to right): "Hor. II. I.", "Hor. 10. 2.", "Hor. 9. 3.", "Hor. 8. 4.", "Hor. 7. 5.", "Hor. 6. 6.", each subdivided into gr. (degrees) and mi. (minutes). The Hor. 6.6 column reads 90 0 throughout (the 6 o'clock line lies in the equator). Several mid-table values are ink-degraded; flagged with [?].

Horiz. poli- ele v.	II-I gr	mi	10· 2 gr	mi	9·3 gr	mi	8·4 gr	mi	7·5 gr	mi	6·6 gr	mi	M ur al co m pl. El. po li
30	7	38	16	6	26	34	40	54	61	49	90	0	60
31	7	51	16	34	27	14	41	42	62	28	90	0	59
32	8	4	17	1	27	53	42	30	63	6	90	0	58
33	8	17	17	27	28	34	43	17	63	45	90	0	57
34	8	30	17	54	29	13	44	5	64	42	90	0	56
35	8	43	18	20	29	49	44	46	64	56	90	0	55
36	8	56	18	45	30	25	45	28	65	27	90	0	54
37	9	9	18	9	31	1	46	9	65	58	90	0	53
38	9	22	19	34	31	37	46	50	66	29	90	0	52
39	9	33	19	57	32	9	47	26	66	55	90	0	51
40	9	45	20	20	32	42	48	1	67	20	90	0	50
41	9	56	20	43	33	14	48	37	67	45	90	0	49
42	10	9	21	7	33	47	49	13	68	11	90	0	48
43	10	21	21	29	34	17	49	44	68	32	90	0	47
44	10	32	21	50	34	46	50	14	68	53	90	0	46
45	10	43	22	12	35	15	50	45	69	14	90	0	45
46	10	55	22	33	35	44	51	16	69	37	90	0	44
47	11	7	22	53	46	10	51	43	69	53	90	0	43
48	11	19	23	12	36	35	52	9	70	10	90	0	42
49	11	31	23	32	37	1	52	35	70	28	90	0	41
50	11	41	23	51	37	27	53	1	70	43	90	0	40
51	11	50	24	9	37	50	53	24	70	58	90	0	39
52	12	0	24	26	38	13	53	46	71	12	90	0	38
53	12	10	24	44	38	36	54	8	71	27	90	0	37

Horiz. pole-elev.	II·I		10·2		9·3		8·4		7·5		6·6		Mural comple.
	gr	mi	gr	mi	gr	mi	gr	mi	gr	mi	gr	mi	pli
54	12	20	25	2	38	59	54	30	71	41	90	0	36
55	12	29	25	18	39	18	54	50	71	53	90	0	35
56	12	38	25	33	39	38	55	9	72	4	90	0	34
57	12	46	25	49	39	58	55	28	72	16	90	0	33
58	12	54	26	5	40	18	55	46	72	27	90	0	32
59	13	2	26	19	40	36	56	1	72	38	90	0	31
60	13	10	26	33	40	53	56	18	72	47	90	0	30
61	13	17	26	47	41	10	56	33	72	57	90	0	29
62	13	24	27	1	41	27	56	49	73	7	90	0	28
63	13	30	27	13	41	47	57	3	73	16	90	0	27
64	13	36	27	25	41	56	57	16	73	24	90	0	26
65	13	42	27	37	42	11	57	30	73	32	90	0	25
66	13	47	27	49	42	25	57	43	73	40	90	0	24
67	13	52	27	59	42	37	57	53	73	47	90	0	23
68	13	57	28	9	42	49	58	2	73	53	90	0	22
69	14	7	28	19	43	1	58	10	73	59	90	0	21
70	14	7	28	29	43	13	58	16	74	5	90	0	20

Table showing the arcs intercepted between the equator and the horizon — for the horizontal dial; and in the complements, for the mural dial — for the pole-elevations set down below.

Parsing notes: This table cross-tabulates, for each of two hour-pairings (the leftmost two columns give "hor" twice, i.e., the hour read two ways), the intercepted equator-to-horizon arcs at twelve pole-elevations (column heads: 30, 34, 38, 42, 46, 50, 54, 55·43, 58, 62, 66, 70). The column head "55·43" is the Copenhagen [Hafnia] latitude 55°43'. Each elevation column is subdivided gr. / mi. This table is ink-

[illegible]

CAPVT XIII.

On zones, climates, and parallels; and finally on the various designations of the inhabitants who occupy the habitable earth from every side, in places opposite or contrary to one another.

In this chapter we shall treat of those matters that we left to the final consideration of the preceding book — namely, the effects impressed upon the earth by the action of the Sun, on account of the varying inclination of the earth's axis from the celestial zodiacal [axis], toward the north and south.

PROBLEMA I.

Given the pole-elevation of a place, the zone of that same place on the earth is given.

This problem is settled by an easy demonstration. On the following diagram, let there be described, about the center A, the solstitial colure B C D E F &c.; the equatorial circle C A K, whose poles are F and M; the zodiacal circle B A H; the tropic of Cancer (☐) D H; the tropic of Capricorn (☐) B I; the arctic circle E G; the antarctic circle N L.

Diagram. A circle (the solstitial colure) about center A, with a horizontal-leaning diameter and two chords representing the tropics. Within the circle the two zones are labeled "Tropicus ☐" (Tropic of Cancer, upper band) and "Tropicus ☐" (Tropic of Capricorn, lower band). The lettered points run around the circumference and define the equator, zodiac, tropics, and polar circles described in the text.

- E (upper left): endpoint of the arctic circle E G
- F (top): pole of the equator
- G (upper right): endpoint of the arctic circle E G
- D (left): endpoint of tropic of Cancer ☐ (D H)
- A (center): center of the colure
- H (right): endpoint of tropic of Cancer ☐ (D H); also endpoint of zodiac B A H
- C (lower left): endpoint of the equator C A K
- K (right): endpoint of the equator C A K
- B (lower left): endpoint of tropic of Capricorn ☐ (B I); also endpoint of zodiac B A H
- I (lower right): endpoint of tropic of Capricorn ☐ (B I)
- N (bottom): endpoint of the antarctic circle N L

- M (bottom): pole of the equator (south)
- L (lower right): endpoint of the antarctic circle N L

Now since the latitude of a place is the same as the elevation of its pole, and since the Sun's declination together with its complement measures each of the two, it is evident that the torrid zone extends on both sides from the equator out to the greatest obliquity of the ecliptic from the equator — that is, to a pole-elevation of $23^{\circ}32'$. This number doubled gives the breadth of the torrid zone: $47^{\circ}4'$ — namely, the arc B D, that is, the space between the parallels B I and D H.

Each temperate zone, taken separately, is measured by the complement of the whole [obliquity] above: $42^{\circ}56'$, beginning at the tropic of Cancer (☐) or Capricorn (☐) — that is, from a pole-elevation of $23^{\circ}32'$ — and ending at an elevation of $66^{\circ}28'$. The breadth of this zone toward the north is shown by D E, toward the south by B N. Finally, the outermost portions, those of the frigid zones, occupy both extremes toward the north and the south, each of an equal breadth on the greatest circle, equal to the greatest obliquity of the ecliptic.

They [the frigid zones] extend from the pole-elevation last fixed at $66^{\circ}28'$ up to the completion of the quadrant. But if anyone should wish to know the surface quantity of each of these zones, and from that to extol and admire the providence of the Creator, he will accomplish it by the measure of the sphere demonstrated in Geometry.

PROBLEMA II.

Given the number of a climate or parallel, the pole-elevation is given, at which that climate both begins and ends. Whence also the place subject to any given climate is recognized.

In establishing and distinguishing the climates, the ancients posited a successive half-hour of difference in the longest day, from the equator up to the frigid zone; but in the parallels, a difference of a quarter-hour. Nor in reckoning the climates did they go beyond the seventh — up to a pole-elevation of $49^{\circ}+$ — because they supposed that the earth beyond that point was not conveniently enough inhabited. Nevertheless, nothing prevents extending this number further, if anyone should wish, according to the calculation of this same foundation. Thus, for ascertaining the pole-elevation at the first climate, we proceed according to the problem set out above.

Diagram. A circle (the meridian) about center A, with several chords. The meridian is B C D E F &c.; B G is the equator; D is the pole of the world; C O is the

ecliptic; C F is the tropic of Cancer (♋). The interior lettered points M, N, H mark the right triangle A H N used in the worked example.

- B (upper left): point on the meridian
- ♋ (top, near C): tropic of Cancer label
- C (top): endpoint of equator/ecliptic
- D (upper right): pole of the world
- M (interior, upper): point on the construction
- E (right): point on the meridian
- H (interior, right of center): vertex of triangle A H N
- A (center): center of the meridian
- N (interior, near A): vertex of triangle A H N
- K (left): point on the meridian
- L (lower left): point on the meridian
- O (lower center): endpoint of the ecliptic C O
- F (lower right): endpoint of tropic of Cancer ♋ (C F)
- ♐ (bottom, near G): tropic of Capricorn label
- G (bottom): endpoint of equator B G

Let the meridian B C D E F &c. be described about center A; let B G be the equator, and D the pole of the world (or of the world); the ecliptic C O; the tropic of Cancer (♋) C F. But since we measure the longest day in the northern region of the world, and in an oblique sphere, from the rising of the Sun above the horizon to its setting: if, then, the distinction of the climates from one another is brought about by half-hours in all, it follows that at the rising or setting the same is brought about by a quarter-hour each. Therefore, in the present example, in the right triangle A H N, for the angle H A N — which measures the complement of the pole-elevation at the first climate G E —

[Greek: διδόμενα (didomena, "the givens")]

A N	3°45', the degrees in the equator corresponding to ½ hour.
H N	23°32', the greatest obliquity of the ecliptic, which is always retained.
A N H	a right angle.

Therefore H A N is given as $81^{\circ}27'$, whose complement, $8^{\circ}33'$, is the pole-elevation at the first climate. And so on perpetually in the rest, always increasing the arc A N in the equator by $3^{\circ}45'$ — that is, by a quarter-hour.

Nor are the parallels investigated otherwise, except by the halves of the quarter-hours, according to the ancients.

From these things it is also clear that, the quantity of the longest day having been determined at some definite place on the earth, there is given not only the climate suited to it, but also the pole-elevation — provided only that the Sun's refraction is excluded. Since this [refraction] was unexplored and unknown to the ancients, it is no wonder that the ancient geographers, regarding places that they did not themselves reach by observations of the poles but drew something from the report of others about the length of the longest day, in order to describe them on their geographical tables, always extended them as greater than they are otherwise found to be by true celestial observations — as is plainly detected in these and other northern places by the comparison of the old tables with the new observations. [Margin: *Lib. 1. Theor. cap. 3.*] On this matter, let Ptolemy, Strabo, and other geographers be read, and let them be compared with us and with other moderns who have more exactly delineated these regions by observing them.

LIBER SECVNDVS SPHÆRICORVM — Caput XIII (concl.), Caput XIV

[p. 141] PROBLEMA III.

Given the elevation of the pole of a place, to find the places and the designations of the earth's inhabitants — those called *amphiscii*, *periscii*, and *heteroscii*; likewise those called *antæci*, *periæci*, and finally *antichthones*, with respect to ourselves.

From the demonstrations already laid out, these various designations arise — partly from the diversity of the shadow that the Sun casts upon those inhabitants through its annual course, and partly with respect to the parallels and axis of the earth under which the various inhabitants are distributed; and nearly all of them experience some opposition to one another.

To the first class belong those called *amphiscii*, *periscii*, and *heteroscii*.

The **amphiscii** are those whose noon shadow falls now toward the north, now toward the south. This happens in the torrid zone: when the Sun passes through

the sign Cancer [☉], they have their shadows to the south, that is, to the left; at midwinter, to the right, that is, to the north.

The **periscii** are those whom the circuit of the zodiacal axis [signifer axis] encloses — so called because their shadows are swung around in the manner of a millstone.

The **heteroscii** are those who have a shadow in one direction only, either northward or southward — as we ourselves are, and our *antichthones*. Therefore the *amphiscii* lie in the torrid zone, the *periscii* in the frigid zones, and the *heteroscii* in the temperate zones.

To the second class and designation belong the *antæci*, the *periæci*, and the *antichthones*.

The **antæci** are those who dwell on different parallels, under the same meridian, at an equal distance from some part of the equator.

The **periæci** are those who lie under one parallel and one meridian but in opposite parts of it.

The **antichthones**, or **antipodes**, are those separated by the diameter of the world. For the *antæci*, midday and midnight fall at the same moment; but the southerners have summer when the northerners have winter. The *periæci* have summer and winter at the same time, but they have opposite middays and midnights. The *antichthones* share the same horizon and meridian, but everything else is opposite — as can be shown more fully on the sphere, that is, on the terrestrial globe.

[p. 141] CAPVT XIV.

On investigating the latitude and longitude of terrestrial places, and likewise — for any two places whatever whose latitudes and longitudes are taken as known — on finding their distance from one another.

We append this last chapter of our spherical doctrine for the use of geographers and historians. But before we proceed to the construction of the problems, a few terms must be briefly defined here — namely, the latitude of a place, its longitude, and its distance from another place — which we represent visually on our sphere.

The **latitude of a place** is its distance from the equinoctial circle, equal to the elevation of the pole of that same place, and reckoned along the meridian.

The **longitude of a place** is its distance from the prime meridian — the one through which the magnet, with its needle, points at once toward both the pole of the world and the magnetic pole. The ancients assigned the Fortunate Isles, beyond Spain, as a kind of boundary for this. This longitude is reckoned along the equator, from the said point eastward.

Finally, the **distance** of two places is measured along a great circle of the earth, since the journey between them across the surface of the sphere is the shortest.

[Margin: *Cap. 8 of this book.*] [Margin: *William Gilbert the Englishman [Gilbert], in his treatise On the Magnet.*]

The **latitude of a place**, since it is obtained by the same technique as the elevation of the pole of the place — and since we have already treated of that fully above — we omit here. And although other methods have been devised by more recent writers, by means of the declination of the magnet below the horizon (on which one should read the excellent little work of William Gilbert the Englishman [*De Magnete*]), these will scarcely achieve the desired precision without celestial observation.

The **longitude of a place**, by contrast, will most conveniently be found from the course of the Moon — whether at an eclipse (especially a lunar eclipse), or at a conjunction of the Moon with a fixed star — or, finally, by the method we shall append at the end of the earlier book of the *Theoria*. For although such lunar appearances occur at the same instant, nevertheless, on account of the roundness of the earth, the times of these events differ noticeably under different meridians — and especially for places more remote from one another — so that in these places they always reckon several more hours eastward. From this difference of times the difference of longitudes of two or more places is readily derived: that is, the time-difference resolved into degrees and minutes of the equator. For the inclination of the magnet — declining from the pole of the world toward its own [magnetic] pole, as is believed — will not by itself suffice to determine the longitude of places here.

[p. 142] PROBLEMA I.

Given the longitude and latitude of one place, together with the traveled distance of another and the angle of position (as they call it) from the meridian — that is, the azimuthal degrees on the horizon — the longitude and latitude of the unknown place are also found.

This problem holds the chief importance in the continuation of geography, since once the longitude and latitude of one place are given, the remaining distances all around — to the places to be determined — together with the angles of position (or of bearing) on the horizon from the given place, are obtained by instruments.

[Margin: *This letter that excellent man, Mr. Joachim Wicheringe [Joachim Wicheringe], Mathematician of Wittenberg, once sent to me here.*]

We shall transfer here the demonstration of this problem, taken from a certain letter of the most ingenious Joachim Rheticus [Rheticus] to Mr. J. Schöner [Johannes Schöner], together with its explanation.

Diagram. A spherical-triangle construction for the problem of finding an unknown place from a known place, a distance, and a bearing. A quadrant of the meridian is drawn with its corner near the upper left; the pole **D** is at upper left, with point **E** to its right along the top. Arcs descend from these points; **A** lies on the left side (the known place), **B** lower and to the right of center (the unknown place), **C** at the lower right (foot of the meridian quadrant), and **F** on the upper arc.

- **D** (upper left): pole, vertex of the meridian quadrant **D A C**
- **E** (top, right of **D**): point on the great circle **D E F**
- **F** (upper arc, right): intersection where the extended arc **A B** cuts **D E**
- **A** (left side): the known place, vertex of the bearing angle
- **B** (center-right, below): the unknown place sought
- **C** (lower right): end of the meridian quadrant

Let the distance of two places, **A B**, be given, and let the latitude of **A** be **A D**; and let the quadrant of its meridian, **D A C**, be completed. Let the angle of section **D A B** also be known — which is learned by the instrument used for navigating, according to the distribution of the winds or of the horizon whose vertex is **A**. Since, therefore, the circumference **A C** is given — the distance of **A** from the pole — from this point let the great circle **D E F** be described, and the quadrant **C B E**, and let **A B** be extended so that it cuts **D E** at **F**. We shall then have triangle **A D F**, whose angle **D A F** is given, with **A D F** a right angle, and the side **A D** also given. The remaining sides **A B F** and **D E F** will likewise be determined, along with the remaining angle **E F B**. Hence in triangle **B F E**, because the angle at **F** is given and the side **B F**, the remaining sides **B E** and **E F** are determined; and from these, therefore, the difference of longitude **D E**. And **E B** is the latitude of the second place **B**, which was sought.

If, then, the latitude **B E** of the other place were given, and it were proposed to find both the difference of longitude and the distance **A B** — the prior construction being preserved, since the sides and angles of triangle **A D F** are known, as stated — then in the other triangle **E B F** the side **E B** is given, the angle **E** is a right angle, and **F** is known: therefore the sides **E F** and **B F** will be given, and hence also the distance **A B**; likewise the difference of longitude **D E** will be obtained. Finally, if **D E** were given, then — since the remaining

— quantities **E F** of triangle **B E F** remain fixed, with **E** a right angle and **F** a known angle — neither **A B** nor **E B** would be unknown.

"See, most excellent Mr. Schöner, in how few steps you have the whole of cosmography!"

This problem, which Rheticus demonstrates by a longer and more laborious induction of triangles, requires less work in the following way. In the oblique-angled triangle **A B C**, given **A C** (the complement of the known latitude) and **A B** (the traveled distance), together with the complement of the observed angle of position **B A C**, there is also given **B C** (the complement of the latitude of the unknown place) and **A C B** (the difference of longitude sought). The longitude and latitude of the unknown place can also be made clear from the traveled distances from two other known places — exactly as the position of an unknown star is found from two known ones, and so forth.

[Margin: *See Problem 4, chapter 9.*]

[p. 143] PROBLEMA II.

Given the longitude and latitude of two places — even of places situated far from one another — the arc lying between the two on a great circle of the terrestrial globe is found; and therefore also their traveled distance in German miles, etc.

Two places whose interval from one another is to be found differ in no way from the distances of stars determined above. Such places either (1) lie under one meridian — which is rare — and differ in latitude alone; or (2) lie under the equator itself, or under one parallel equally distant from the equator — which is rarest — and vary in longitude alone; or (3), finally, lie under different meridians and parallels both — which is most frequent — and disagree in both longitude and latitude. In this threefold variation of two places separated from one another, we shall briefly dispatch the finding of their distances as follows, and where needed we shall illustrate it with examples.

I.

Two such places, if they are arranged under the same meridian (or perhaps under the equator, as most rarely), and whose interval is sought, do not require the calculation of triangles. For when the smaller latitude (under a common meridian) or the smaller longitude (under the equator) is subtracted from the greater, there remains the arc of distance under the great circle, to be resolved into miles in the customary way by 15.

II.

But if, on the other hand, the places differ in longitude alone, an isosceles spherical triangle **A B C** arises, where the positions of the given places are at **B** and **C**. If, then, the traveled arc **B C** is sought, with quadrants **A D** and **A E** drawn from the pole **A** on either side, **D E** will be the portion of the equator indicating the difference of longitude. When this is bisected at right angles at **F** — a perpendicular arc being dropped from **A** — **B C** is also cut in half at **H**, likewise at right angles.

Diagram. An isosceles spherical triangle for the longitude-only case. The pole **A** is at the apex (top); two sides descend symmetrically to **B** (left) and **C** (right), the two given places. A vertical arc runs from **A** down through **H** (where it meets the base **B C** at right angles) and continues to **F** on the equator-line **D E** at the bottom; **D** is at the lower left, **E** at the lower right.

- **A** (apex, top): the pole
- **B** (left): one given place
- **C** (right): the other given place
- **H** (center, on base **B C**): foot of the perpendicular from **A** on **B C**
- **D** (lower left): left end of the equatorial arc
- **F** (bottom center): point where the perpendicular meets **D E**
- **E** (lower right): right end of the equatorial arc

Therefore, in either right-angled triangle — namely **B H A** — for **B H**, half the traveled arc:

The **διδόμενα** [given quantities] are:

- **A B** = the complement of the latitude (i.e., the elevation of the pole) of the given places
- **H A B** = half the difference of longitude of the given places
- **A H B** = a right angle

Therefore **B H** is found, the arc of half the distance, whose double is **B C**.

III.

Places that differ in both latitude and longitude form a triangle in which

— wherever they fall, [the triangle is] oblique-angled, to be solved also by an easy **προσθαφαίρεσις** [**prosthaphaeresis: the method of addition-and-subtraction**]; and therefore they need nothing but examples.

Example I of this.

To find the traveled distance between Louvain in Brabant [Leuven] and Bologna in Italy.

	Latitude	Longitude
C Bologna	grad. 44 min. 0	grad. 33 min. 55
B Louvain	grad. 50 min. 58	grad. 26 min. 50
Difference of long.		grad. 7 min. 5

Accordingly, in triangle **A B C**, for **B C**, the traveled distance between Louvain and Bologna, sought.

The **διδόμενα** [**given quantities**] are:

- **A B** = gr. 39 mi. 2 — complement of latitude of Louvain
- **A C** = gr. 46 mi. 0 — complement of latitude of Bologna
- **B A C** = gr. 7 mi. 5 — difference of longitudes

Therefore **B C** = grad. 8 min. 27 is found, the distance sought, which — resolved into German miles — makes 126¾ miles.

Diagram. A spherical triangle for the Louvain–Bologna distance. The pole **A** is at the top; an arc descends along the left to **B** (Louvain) on the left side, and **C** (Bologna) lies just inside, to the lower right of **B**; **D** is at the lower left and **F** at the bottom (feet of the construction arcs).

- **A** (top): pole
- **B** (left side): Louvain
- **C** (right of **B**, lower): Bologna

- D (lower left): foot of construction
- F (bottom): foot of perpendicular

Example II.

Let the distance between Copenhagen in Denmark [Hafnia] and Jerusalem be investigated.

	Longitude	Latitude
B	grad. 37	grad. 55
Copenhagen	min. 0	min. 43
C	grad. 66	grad. 31
Jerusalem	min. 0	min. 40
Difference	grad. 29	
	min. 0	

Therefore in triangle **A B C** likewise, for **B C**:

The **διδόμενα** [given quantities] are:

- **B A** = gr. 34 mi. 17 — complement of latitude of Copenhagen
- **C A** = gr. 58 mi. 20 — complement of latitude of Jerusalem
- **B A C** = gr. 29 mi. 0 — difference of longitude

Therefore **B C** = grad. 31 min. 27 is found, the traveled distance sought; resolved into German miles, it gives 471½ miles.

Diagram. A spherical triangle for the Copenhagen–Jerusalem distance. The pole **A** is at the top; an arc curves down the left through **B** (Copenhagen) on the left side, with **C** (Jerusalem) lower and toward the center; **D** is at the lower left and **E** at the bottom (feet of the construction arcs).

- A (top): pole
- B (left side): Copenhagen
- C (center, lower): Jerusalem
- D (lower left): foot of construction
- E (bottom): foot of construction

FINIS.

[End of the Spherical Part (Pars Prior) of the *Astronomia Danica*.]

ASTRONOMIA DANICA

PARS ALTERA [The Second Part]

THE THEORIES OF THE PLANETS

embracing their restoration in two Books, of which:

THE FORMER

After describing and comparing the three world-models [*triplicis hypotypofeos mundanae*] — namely

THE ANCIENT, OF PTOLEMY [Ptolemy], THE ADMIRABLE, OF COPERNICUS [Copernicus], THE RECENT, OF TYCHO BRAHE [Tycho Brahe] —

treats of the apparent motions of the fixed stars, and likewise of the Sun and Moon, restored within that same framework and fitted to every age of the world: together with the complete doctrine of eclipses, and in addition a special treatment of the use of the Moon.

THE LATTER

treats of the motions of the remaining five planets, restored within the same framework upon that threefold model [*Hypotypofi*], to match the celestial appearances in like manner.

PRAEFATIO [Preface]

TO THE READER.

Decorative initial. A large woodcut initial "H" set in an ornamental foliate panel, opening the word *Habes*. (No textual content beyond the letter; treated as a decorative initial rather than an informational figure.)

At last, most kindly Reader, after a long and well-warranted wait, you have here the two Books of the Theorica [*Libros binos Theoricorum*], devoted to the restoration of the perpetual motions both of the fixed stars and of the planets — wrought out by our watchful labor. I undertook this lest Urania, the most excellent of the Muses and the one most fitted to human use, should ever be found abandoned by the care and generous patronage of the most serene and most powerful Kings of Denmark; for it was by their favor and at their expense that this science once flourished in this Kingdom, and by divine grace it flourishes

still, among the exercises of the pure religion and the other arts of more polished learning — so that not only its fame, but its fruit as well, may be felt even by foreign nations. I add, too, that I did not wish our vigils — undertaken over thirty years for the restoration of this art — to perish fruitless and pass away utterly.

And although I am well aware how much other most distinguished astronomers of the present day can accomplish in this field — among whom too there now rest, as fate has willed, those most precious requisites of all astronomy that were laboriously gathered here in Denmark by us, namely the observations (so I believe), or perhaps they pass through other hands — nevertheless, since, whether furnished with those same necessities or deprived of them, we aim together at the one end, that the celestial phenomena, at any place and time, may be made known from these our hypotheses (or from the tables derived from them), by methods unlike indeed, yet equivalent in power: for my own part I chose rather to follow in the footsteps of the ancients in the restoration of astronomy. In these matters I chose first to reduce Ptolemy to Geometry — that is, to reduce the equant in the planets to an epicycle, with Arzachel the Spaniard [al-Zarqālī] and Copernicus of Tours [Copernicus]; then, for the remaining eccentricity, to set up a twin epicycle [*geminum Epicyclum*] together with an equant, in the manner of Tycho. I did this especially because I had long since become convinced that the nature of the superior planets, in their threefold combination with the eccentric orb itself, and in the contrary motion of the centers among one another within these, by no means plays at random. So convinced, indeed, that whatever the loftiest philosophers — and the Platonists above all — once contemplated by the gift of reason concerning *Intellect* [*Intellectu*], *Reason* [*Ratione*], the *World-Soul* [*Anima Mundi*], and finally *sensible Nature* [*Natura sensibili*], and which they wove together in chain-like fashion, in order, under the veil of dependency, proceeding from a First Mover — all this the celestial nature displays to us, as if through viewing-mirrors [*per speculae viforiae*], in these planets and their motions, contriving nothing at all in vain throughout this whole world-economy: while the individual bodies are borne, each about its own centers and prior postulates, in a most fitting harmony, all of them at last leaning upon one single source. Concerning this matter, among others, Plato in his second Letter to Dionysius brings forward a few remarks in brief, but in mystic and abstruse words.

These our hypotheses, then, constructed to fit the celestial observations,

while I accommodate them indifferently to the threefold supposition — that of Ptolemy, reduced (as I said) to the laws of Physics and Geometry; likewise that of

Copernicus; and finally that of our own Brahe — the rest, to be sure (save those that in no way agree with nature), cannot wholly dissent from these, but can easily be reduced to them. For the homocentric scheme [*homocentrica*] of H. Fracastoro the Italian [Girolamo Fracastoro] and of others, which posits an apparent revolution shared with the universe, in all the inferior planets, toward the side opposite the descent [?], has long since been overthrown by many arguments. As for the rest — those which either others, or even Johannes Alstedius [Johann Heinrich Alsted], recount in passing and judge favorably, namely the model of Raymarus Dithmarsus [Nicolaus Reimers Bär, "Ursus"] and of Helisaeus Roeslin [Helisäus Röslin] — these can be reduced without any trouble to the model of Tycho, from which they emanated. Indeed, the one that Dithmarsus calls his own was once filched from Tycho. This, among other things, is proved by the error which that man [Ursus] afterward thoughtlessly imitated — an error in that crude sketch, left by Tycho on a wall at Uraniborg (which Ursus had perhaps seen with his own eyes), where the orb of Mars wholly encompassed that of the Sun.

[Margin: See Epistle. Astron. Book 2.]

As for Roeslin's model, it differs in nothing from Tycho's, except (as it appears) in the material orbs of the planets — which, if Roeslin should choose to reckon them as truly such, lead him into the greatest physical absurdity, namely the interpenetration of dimension; unless he should rather hold them to be merely formal (as many things are in nature). To these, as real, we earlier gave our assent in some degree, and on behalf of this opinion of ours we once discoursed at length and quite plausibly in a public disputation.

[Margin: See Epistle. Astron. to H. Rantzau.]

Furthermore, as to why I made use rather of an accumulation of infallible observations — such as I could have at hand — and of a careful comparison of these among themselves and with the mean motions, for the investigation of the eccentricities, the equants, and the apogees, instead of the πραγματεία [Greek: *pragmateia*, "treatise" / "method"] of Ptolemy so greatly praised by Copernicus and adopted in emulation, where only three apparent positions and as many mean epochs are supposed for the same: this surely is not without cause, neither formerly with Tycho, nor with me even now. For since we are engaged near the truth — not in the unrefined founding of astronomy along with the ancients, but in its accurate restoration and in the representation of the phenomena — this method, far more than ours, is a more particular induction, and for that reason so

much the more uncertain; since, besides, three points not projected quite into a straight line can everywhere touch the circumference of a circle.

[Margin: Prop. 5, Book 4 of the Elements.]

Accordingly, by this procedure the traces of the equant are not sought conveniently enough — a thing which the restored Theory of the Moon above all taught us, so often reworked in vain by this method, and not duly settled until we applied a general idea of the lunar observations through the various places of the Eccentric, and quite often directed it into a δοκιμασία [Greek: *dokimasia*, "testing" / "scrutiny"].

Since, therefore, the restoration of astronomy has been carried out by us against the general balance of the observations, and restricted to hypotheses of this kind — which, as often as need arises, are resolved back into the very observed data from which they were constructed, with only a very slight difference intervening, and that removed by adjustment [*limitatione*] — we do not wish to linger here any longer, seeing that everything will readily be worked out by learners with the help of the Trigonometry set forth above. Farewell, and enjoy.

PROOEMIUM OF THE SECOND PART OF ASTRONOMY [PARTIS SECVNDAE ASTRONOMIAE PROOEMIVM]

Of the excellence, difficulty, and contents of the Theoretical doctrine, sketched in general together with its division.

Decorative initial. A large woodcut initial "P" set in an ornamental foliate panel, opening the word *Priore*. (No textual content beyond the letter.)

Now that the former part of Astronomy is finished — the part that considers the apparent daily motion of the celestial bodies, and is called Spherical [*Sphaerica*] — the other part remains, called Theorica [*Theorica*]. It is so named because it is engaged in the contemplation of the motions, both of the first mobile (as they call it) [*primi mobilis*] and of the seven planets, established by means of definite and geometric hypotheses; and it is the most excellent and the most difficult of all. For what is more excellent, what more noble, what worthier of human science (insofar as it is human), what, in short, more celebrated for divine artistry, than the compendium and, as it were, the idea of those things which universal nature, taken in its widest sense, turns within her own bosom? — things woven together in heaven, indeed in these planets and their revolutions, in a manifold concord, as though described in a kind of book, set forth before the eyes of mortals, and at last discerned by those who, leaving behind earthly dregs and momentary

pleasures, dwell upon celestial, perennial, and altogether divine contemplations, and bind themselves fast to it by sure means — provided they have at least moderately grasped those things which, divinely inspired [*enthea*], are the chief portion of man's lot to know.

For it must not be supposed that the best and wisest Creator made the most splendid celestial bodies, and their admirable and perennial revolutions or motions, for any cause other than for the sake of mankind — namely, that men should delight themselves in the scrutiny of those bodies, as of the most excellent divine works that fall under the senses. Nor must it be supposed that these stars, which are commonly called planets, deviate even in the least from the orbs divinely prescribed for them; rather, that by the wondrous variety of their motions they display the best harmony to those who, having rightly begun from ocular sense, are able to attune that same harmony to the ears of the mind. To this, no doubt, Pythagoras [Pythagoras] once had regard, when he assigned a musical concord to the revolutions of the celestial bodies, as Plato [Plato] interprets it. Indeed, this same Plato, in many other places besides, earnestly commends the divine study and usefulness of Astronomy. Of these passages we shall taste one drawn from the *Timaeus*, and set it here before the eyes of the student of Mathematics, so that he may reflect that, in seeking to know the heavens, he has not striven by a false persuasion or with useless effort, but is invited by the authority of the highest of all Philosophers.

"The greatest work," says he, "for the sake of which the eyes have been usefully given to us by the gods, I now feel I must explain. For the eyes, I judge, have brought us knowledge of the best of things. For these matters that are debated concerning the universe would never have been discovered if neither the stars, nor the sun, nor the heaven could have been beheld. But the knowledge of day and night, arising from the eyes, has enabled us — by a certain demonstration — to measure the courses of months and years, to come to know time, and to investigate the order of universal nature; from which things we have obtained philosophy, than which no greater good was ever given, nor shall be given, to the race of mortals by the gift of the gods. This I declare to be the greatest benefit of the eyes; the lesser benefits I pass over: for those who, being strangers to Philosophy, are bereft of these and should complain, will complain in vain. But for our part it must be affirmed that God begot the eyes for this reason above all: that, having gazed upon the circuits of mind which are performed in heaven, we might turn them to the use of our own mind, and recall the discursive reasonings of our own cognition — akin to those circuits, but in some manner disturbed — to the temper of those heavenly ones. And when we have come to know them, and,

endowed with right reason according to nature, have perceived the order of each, let us imitate the revolutions of God, which are conducted without any error, and compose the wandering and erratic excursions of our own thought after the pattern of the gods." Thus he. And these, if [one heed them] soberly,

Proem to Part Two of the Astronomy [continued] (p. 149)

and have grasped the true interpretation [of these matters]—comparing them with those very passages of Plato, indeed with the reality itself, and having rightly understood Astronomy—then [this science] will deserve, before all others, to be called the glory and praise of human knowledge. No one, therefore, should be surprised that not only the wisdom of antiquity, but emperors, kings, and princes as well, devoted so much leisure and expense to the study of the heavens.

I now come to the difficulty of this Theoretical part. No one, surely, will deny it. For not only is the apparent progress of the fixed stars uniform and equal in time, but the motions of the planets too—both their particular circuits through epicycles and their varying distances from the Earth—are almost entirely hidden from ocular perception. Hence they are incomprehensible to the judgment of the crowd, and perceptible only to astronomers. It is for this reason that Plato, in the *Epinomis*, exalts this part of Astronomy above all wisdom:

"Do not (he says) be ignorant that Astronomy is something most wise—for indeed it is necessary that the true Astronomer be not the man who, after the manner of Hesiod and all of that sort, observes settings and risings, but rather the man who [considers] the eight circuits, and how the seven move beneath the first, and in what order each one traverses its own circles—something which no nature, save an admirable one, will ever easily discern."

From this passage, indeed, follows the third point which I judged ought to be set down here: namely, that we should outline the courses of the planets in a general way. It must therefore be understood (and this we ascribe to our inner principle) that each planet, while marking out a single apparent periodic revolution, carries out one, another, or several circuits in the heavens, according to the linkage of the epicycles to which it has been bound—and this, beyond doubt, in order to signify the mysteries of the universe, as we touched on above.

Among these bodies the Sun itself, as choirmaster and leader of all the rest, seems—apart from the greatest orb, by which its annual revolution is indicated—to be content with a single epicycle, except insofar as it involves the precession of the

equinoxes and the unequal obliquity of the zodiac in its own peculiar anomalies. The others, by contrast, besides their primary circles, also require two or more small circles. And besides these, the remaining five planets, which follow the Sun's leading and are placed in the middle, are governed by the disposition of this one body. The varying distance of each from the Earth introduces and alters a varying magnitude of the angle of vision, and they undergo a varying—and, of all bodies, most conspicuous—change of position relative to the Earth, as will be made clear in the hypotheses themselves. This single fact was the reason why Copernicus supposed that the Earth carried out its course in the middle space, between the sphere of Mars and that of Venus, while the Sun, fixed at the center of the world, remained at rest—as will be seen in the following chapter.

As for the Moon: although it does indeed regard the Earth as central, it is nevertheless bound to the Sun by a certain marvelous bond, so that—besides the twin small circles of its eccentric—it is also affected in various ways by its disposition relative to the Sun. Hence at the quadratures it makes its greatest excursions; and in the intermediate places it slightly accelerates its motion as it moves toward the quadratures, and retards that same motion as it returns from them. This anomaly, indeed, along with others concerning the latitude of the Moon and the precession of the nodes, no one before Tycho Brahe detected. For those [anomalies] were discovered—when [the Moon] occupied me for a decade with him—only by our greatest labor, and were brought into the hypothesis. The latitudes of the remaining planets too, by which they depart this way and that from the path of the Sun, behave in various manners, both by reason of the prior circles and by relation to the Sun; nor do the motions of their nodes agree with the suppositions of the ancients. All of these matters are to be indicated in the hypotheses themselves, and to be tied to celestial observations both confirmed and firm—observations which may rightly be called the *sensible principles* of this sidereal doctrine.

For it is not part of our undertaking to be willing to hand over Astronomy rashly to a gifted youth, born for the perception of the heavens, without [proper attention to] the heaven itself (as many do). Rather, our purpose is at the same time, so far as is granted by the grace of God, to enrich the Theories of the planets with those things which the ancients passed over—and yet no less to set down the motions of the planets themselves upon the duly reconciled hypotheses of the three most outstanding craftsmen, the illustrious Ptolemy, Nicolaus Copernicus, and Tycho Brahe, and to reduce them into uniform and convenient tables, from which the phenomena of these bodies may be conveniently drawn out for any time whatever. In this way our care and concern for the restoration of astronomy

may prove welcome and commended not only to the present age, but also to posterity.

This threefold variety of supposition, [applied] to the same phenomena, we pursue not merely for this end—that we may show, as it were by full induction, that there are given in nature other [arrangements] equally agreeable to the celestial appearances

Proem to Part Two of the Astronomy [continued] (p. 150)

—and that meanwhile each man's choice in these matters be permitted to him—but also so that, just as clocks are fashioned everywhere with artifice after the monuments of the ancient Astronomy, [clocks] which represent the courses of the planets—among which the αὐτόματον [Greek: *automaton*, "self-moving device"], which was constructed here at the expense and care of the most Christian King of Denmark, Lord Christian III of glorious memory (who was greatly devoted to the study of Astronomy), and which afterwards, by the gift of the present Most Serene King Christian IV, passed to this Royal Academy, yielding the palm to few in Europe (as I think)—[so that such devices] may be rightly understood and directed in their motions and uses.

Furthermore, since there is so much genius in the Copernican invention, and in the Brahean such great power and ἐνέργεια [Greek: *energeia*, "active force"] for reconciling the other two dissenting [systems], it has therefore seemed good to us now—just as it had earlier been proposed to Lord Tycho—to be willing, for the sake of those who differ, to present separately the threefold hypothesis (laid down, that is, by the three said craftsmen), to compare them with one another, and—with reasons added, by the σύγκρισις [Greek: *synkrisis*, "comparison"] of Tycho, my most deserving Teacher—to recognize [the Tychonic] above all as the true one. Yet [we do so] in such a way that, in those matters concerning the diurnal rotation of the Earth in the middle and the perpetual quiescence of the starry heaven which seem to us more probable, and which are physical and introduce nothing of difference to the saving of the phenomena (whichever you may choose), we make use of our own liberty—just as we do in certain other matters. For this liberty, which throughout the whole of Philosophy ought rightly to be exercised for the investigation of truth, can by no right be denied to its most excellent part, especially provided that it be consonant with the reasons drawn from principles, and provided it not in the least oppose the concise determination of the celestial motions—to which, above all, this doctrine is dedicated.

Furthermore, once such a general description and comparison of the world-systems [σύστασις, Greek: *systasis*, "constitution"] has been set forth, the restitution of the fixed stars—and likewise of the Sun and Moon, which rest upon the Earth as the primary center of the world—is faithfully to be joined to it, and with these the prior book of the Theoricorum is to be completed. The latter book will be brought to completion in the similar restitution of the motions of the remaining five planets, which regard the Sun as the secondary center of the world; and for the demonstration of these we shall apply, indifferently, those three hypotheses of which we just spoke. And although, exhausted nearly to the limit—both by the labor of a genius once neglected and by bodily toil and night-watches—my infirmity may seem easily to succumb to this arduous undertaking, yet there is good ground for hope that, with the help of God [Θεοῦ συναιρομένου, Greek: *Theou synairomenou*, "God lending his aid"], we shall one day happily bring this same [work] to its appointed end, for the glory of the admirable Creator of the heavens and for the signal benefit of posterity.

Book One of the Theoricorum (p. 151)

LIBER PRIMVS THEORICORVM

IN GENERE, treating briefly of the threefold sketch [hypotyposis] of the world-system, and of the hypotheses of the planets:

IN SPECIE, concerning the restitution of the fixed stars, and likewise of the Sun and Moon.

CHAPTER I.

On the threefold ordering of the world-system.

The doctrine concerning the motions of the planets—although among those who transmitted their monuments to posterity, before Ptolemy of Pelusium [Ptolemy] (who flourished at Alexandria in Egypt about the year 140 from the birth of the Savior) scarcely such a [doctrine] existed as was bound to fixed hypotheses geometrically founded—nevertheless it is by no means doubtful that this began from the beginning together with the spherics of the heavenly bodies. For it is credible that the ordering of the world-system was discovered and established by the earliest men, and propagated to posterity, and that the motions of the planets were over a long stretch of time defined among the Chaldeans, perhaps by numbers alone, without hypotheses in a manner of speaking—that is, by the reckoning of what appears in the heaven.

For—to pass over the testimony of Josephus in Book 1 of the *Jewish Antiquities*, who makes the study of Astronomy most ancient—the opinion of the ancients concerning the world's constitution is left sufficiently attested and discussed by the early Philosophers: Trismegistus, Plato, Aristotle (besides those things which are found in the poets concerning Prometheus, Epimenides, Atlas, and others). This opinion is afterwards read in Pliny, Book 2 of the *Natural History* (who lived under Vespasian)—although in certain points it is somewhat altered toward the opinion of others whom he followed—just as it is also remarkably transformed in the tradition of Aristarchus of Samos and of certain of the Pythagoreans. Nevertheless, the most ancient constitution, that which was adopted by Ptolemy, prevailed in the judgment of the astronomers. For it is also agreeable [to reason] that this same [constitution] was in use, before Ptolemy, with Timocharis of Alexandria, Hipparchus of Rhodes, Archimedes of Syracuse, and others—and that afterwards it was frequented down to Copernicus, as appears too through the *Hypotheses* of Proclus Diadochus.

Now this sketch of the world-system is of such a kind that it is sufficiently known of itself from the characters of the planets, with the several orbs added [to each]:

(p. 152)

Diagram. The ancient, or Ptolemaic, sketch of the world-system. A geocentric arrangement of nested concentric circles. At the center is a woodcut globe depicting the terrestrial Earth (land and sea). Surrounding it, the planetary orbs are arranged in order from the center outward, each marked with its planetary symbol along a vertical column rising above the central globe; the two outermost rings carry, respectively, a band of stars (the firmament of fixed stars) and the band of the zodiac with its twelve signs around the periphery.

Reading the vertical stack of planetary symbols upward from the central Earth toward the top:

- ☾ (Moon) — innermost planetary orb
- ☿ (Mercury)
- ♀ (Venus)
- ☼ (Sun)
- ♂ (Mars)
- ♃ (Jupiter)

- ♄ (Saturn) — outermost of the seven planets
- (Penultimate ring): a band of stars representing the sphere of the fixed stars (the firmament).
- (Outermost ring): the band of the zodiac, with the twelve zodiacal signs placed around the circle. Readable signs around the rim include ♋ Cancer (top), ♌ Leo / ♍ Virgo (upper left), ♎ Libra / ♏ Scorpio (lower left), ♐ Sagittarius / ♑ Capricorn (bottom), ♒ Aquarius / ♓ Pisces (lower right), ♈ Aries / ♉ Taurus (upper right), ♊ Gemini (top right). [Several of the individual zodiac glyphs are small; the sequence is reconstructed from their conventional order around the rim.]

Caption (verbatim): Hypotyposis systematis mundani vetus, seu Ptolemaica. — "The ancient, or Ptolemaic, sketch of the world-system."

In this system the periodic motions of the planets around the Earth are at least expressed, with the orbs distinguished. Of these, Saturn's is about 30 years; Jupiter's, 12; Mars's, 2; the Sun's, Venus's, and Mercury's, annual; the Moon's, 28 days. And although Ptolemy nowhere in his work expresses this system in a diagram, he nevertheless discusses the quiescence of the Earth in the middle. [Margin: *Almagest*, Book 1, ch. 5. *Almagest*, Book 9, ch. 1. Plato in the *Timaeus*. Aristotle, in the treatise *On the State of the World*.] He moreover dissolves [the contrary view] concerning the order of the spheres or orbs in which the planets were held to be embedded—asserting that this was the opinion of the ancients—[refuting] those who maintain that Venus and Mercury are for this reason placed above the Sun: namely, that at the conjunction of the Sun with these bodies no obscuration [of the Sun] was observed in it, since their withdrawal into the latitudes entirely avoids this. Meanwhile, however, Plato, from another fountain of nature, argues that these luminaries are nearest to the Earth—a view which Aristotle too follows.

And this was the world-sketch of the ancients, taken up again by Ptolemy when he set out to demonstrate the motions of the heavenly bodies. He, however, having described these orbs—besides the eccentrics and equants, concerning which [I shall treat] below—was also compelled to superadd annual epicycles in the three superior [planets], and, in Venus and Mercury, others that were proper [to them], because the planets were subject to the Sun's motion both in respect of longitude and of latitude. The stations and retrogradations of the superior planets occurred annually; the inferior ones too had their own periods, to be saved by the epicycles of each—as we shall presently demonstrate by two examples: one,

namely, in the planets superior to the Sun, the other in those that were reckoned inferior to it.

Astronomia Danica — Pars Altera, Liber Primus Theoricorum, Caput I (continued)

A diagram of the Ptolemaic hypothesis for the superior planets — Saturn, Jupiter, and Mars — showing their annual progressions, stations, and retrogradations.

Explanation.

From the Earth, that is, from the center of the universe, let there first be drawn the annual orbit of the Sun, B C. Then, from the same center A, draw the orbit D H of one periodic revolution of one of the superior planets; let this planet's motion proceed from D toward its rising, that is, toward H. From that point, on the Ptolemaic epicycle E G that has been described, the planet moves in such a way that — when the Sun by its mean motion stands at B, with the Earth interposed directly between the Sun and the planet — the planet occupies E, that is, the lowest part of the epicycle, the part nearest to the Earth.

Again, when the Sun is at C and in conjunction with the planet, the planet holds the apex of its epicycle at F, and is therefore farthest from the Earth A. Meanwhile, by the epicycle just described, E F, the planet marks out the phenomena [Greek: φαινόμενα (phainomena, "appearances")] that correspond to what is seen in the heaven itself — advancing, standing still, or retreating, according to how it is configured with the Sun, as is readily grasped here. For while the planet advances from the apogee point F of the epicycle toward H, its motion on the epicycle is one of forward progress; and near H (or a little below it) it becomes stationary. From there it turns retrograde, moving from H through E to G, where again it has returned to a second station. Finally, from G it proceeds back to F in the order of the signs — always, however, with allowance made in these motions for the planet's own motion in its proper orbit D. And thus the three superior planets — Saturn, Jupiter, and Mars — are affected year by year by the motion of the Sun, so that to account for this anomaly [Greek: ἀνωμαλία (anōmalia, "anomaly" — irregularity of motion)] of theirs, Ptolemy [Ptolemy] was compelled to assign to each of them an epicycle of the kind we have just described. Nor, indeed, is there any less agreement with the Sun's annual motion

in the inferior planets, Venus and Mercury — as now follows, taking these too as an example.

Diagram. A Ptolemaic construction for a superior planet. A large outer circle is centered on A, with two cardinal points marked on it: C at the top and B at the bottom. Inside this circle, concentric and somewhat above center, is a smaller circle (the Sun's annual orbit B C, on the vertical axis through A). Near the top of the figure sits the epicycle, a small circle bearing the labels F (top, apogee), H (left), D (center of the epicycle), G (right), and E (bottom, perigee). A small dot marks the deferent point just below the epicycle near C.

- A (center, lower): the Earth / center of the universe
- B (bottom of large circle): the Sun's position by mean motion (Earth interposed between Sun and planet)
- C (just above center): the Sun's position in conjunction with the planet
- D (center of epicycle): point on the planet's proper periodic orbit
- E (bottom of epicycle): lowest, nearest part of the epicycle (planet here when Sun at B)
- F (top of epicycle): apex / apogee of the epicycle (planet here when Sun at C, farthest from Earth)
- G (right of epicycle): point of the second (later) station
- H (left of epicycle): point near the first station, where retrogradation begins

A diagram of the Ptolemaic hypothesis for the inferior planets — Venus and Mercury — showing their periodic progressions, stations, and retrogradations.

Explanation.

With A as center — which takes the place of the Earth — let the orbit of the Sun be drawn, which is the outer one, B. Then from the same center draw the orbit C of one of the inferior planets, namely Venus or Mercury. And since, on the Ptolemaic hypothesis, the mean (that is, annual) motion of the Sun and of the two planets Venus and Mercury is one and the same, draw a straight line from the Earth A through C to B; throughout the entire revolution this line keeps the Sun and the planet under equal portions of the zodiac. But to display the remaining appearances of Venus and Mercury, further —

Diagram. A Ptolemaic construction for an inferior planet. A large outer circle (the Sun's orbit B) is centered roughly on A at the bottom; B is marked at its top. A smaller circle inside it, sharing the vertical axis, carries the epicycle near its

upper portion. The epicycle is the small circle on which the planet moves, with points D (top), E (left), G (right), and F (bottom) marked; C is the center of the epicycle (on the Sun-line). Straight lines run from A upward through the epicycle to B, and to the lateral points E and G, forming a narrow triangle from A.

- A (bottom center): the Earth / center of the universe
- B (top of large circle): the Sun (the outer orbit)
- C (center of epicycle): the deferent point carried with the Sun's mean motion
- D (top of epicycle): apogee of the epicycle
- E (left of epicycle): point of vespertine (evening) maximum elongation / first station
- F (bottom of epicycle): perigee of the epicycle
- G (right of epicycle): point of matutinal (morning) maximum elongation / second station

— upon the center C is described the epicycle D E F G, on which the planet pursues its course according to Ptolemy's view. Thus, descending from the apogee D to E, the planet markedly increases its motion beyond C; and when it has come near to its greatest evening elongation from the Sun, joined at E — where its equation is measured by the angle C A E — it appears stationary. From here it descends into the perigee F; and then from F it is seen to move backward to G, the morning greatest elongation, where it becomes stationary a second time. Thence it returns to the apogee D in the order of the signs [Greek: εἰς ἐπόμενα (eis hepomena, "into the following [signs]" — i.e. in the order of the signs)], and so on in succession, and so forth — as may be more fully demonstrated in the special hypotheses for the individual planets. And since the mean annual motion of the inferior planets is shared in common with the Sun, the period traced out by the epicycle is accordingly not annual as it is for the superior planets; rather, for Venus it is about nine months, and for Mercury about eighty days, insofar as they separate from the Sun's mean motion.

Such a general demonstration applies to the motions of the five planets that are bound to the Sun, which can be accounted for in no other way than by the epicycle that Ptolemy added. Pliny [Pliny], however, supposed that the efficient cause of these motions was the Sun's rays striking the planets in various ways — though he would have done better to have said that the Sun is the center of all of them. Taking note of this, after certain ancient philosophers — Philolaus the Pythagorean [Philolaus], Hicetas [Hicetas], and above all Aristarchus of Samos [Aristarchus of Samos] — Nicolaus Copernicus of Toruń [Copernicus], a man of

divine genius, so inverted the order of the system set out above that, with the Sun itself occupying the center of the universe, the Earth, together with the Moon as its companion at its midpoint, took its place in the Sun's orbit, having been made subject to a threefold motion. For Copernicus not only attributed to the Earth the apparent diurnal motion of the heaven itself, and rendered the sphere of the fixed stars wholly at rest — which sphere, like the orbits of the planets, would regard the Sun as its center; but he also, by the single motion of the Earth, abolished all the epicycles that had been introduced by Ptolemy on account of the combination of the planets with the Sun (which we demonstrated above). Moreover, he thereby exhibited the simple anticipation of the equinoxes, while leaving intact the ancients' opinion concerning the reality of the orbs in the heaven. Here, among other things — something the old system cannot achieve — once the distance of the Earth from the Sun is given, the distances of the remaining planets from the Earth are each found to the same symmetry.

Finally, by a third motion of the Earth, which he calls the motion of libration, Copernicus demonstrated the inequality of the equinoxes and of the obliquity of the ecliptic relative to the equator.

And this is the sum of what Copernicus accomplished by his admittedly wholly admirable, yet also elegant, transformation of the ancient hypothesis [Greek: ὑπόθεσις (hypothesis, "hypothesis" — supposition, basic assumption)], when he brought his own — which now follows — to bear upon the celestial appearances [Greek: φαινόμενα (phainomena, "phenomena, appearances")]. In this system, the points that seem more difficult — concerning the threefold motion of the Earth and the rest of the heaven and the Sun — can be set out separately before the listeners, within a material sphere, before their very eyes, so that they may rightly be impressed by the acumen of so great a man.

Diagram. The Ptolemaic inferior-planet construction continued. A large outer circle (the Sun's orbit) with B at the top and D just below it. Within it, the epicycle is the small circle near the upper region, labeled D (top, apogee), E (left), F (bottom, perigee), and G (right), with center C on the vertical axis above A. Straight lines run from A at the bottom up through the figure and out to E and G, forming a narrow triangle. A small dot sits below A.

- A (bottom): the Earth
- B (top of large circle): the Sun's orbit
- C (center of epicycle): deferent point on the Sun-line
- D (top of epicycle): apogee of the epicycle

- E (left of epicycle): evening greatest elongation / first station
- F (bottom of epicycle): perigee of the epicycle
- G (right of epicycle): morning greatest elongation / second station

A representation of the Copernican world-system.

[Margin: Copernicus, *De Revolutionibus* I, c. 10.]

This is the marvelous discovery of the great Copernicus, by which — once the ancient system has been dislodged, the Earth withdrawn from the midst of the universe together with the lunar system into the Sun's annual orbit, and the Sun in turn set back at the center of the cosmic machine — the phenomena suffer no perceptible alteration from what they were before.

For, first, with the terrestrial globe so arranged, the Sun is seen in exactly the same parts as if we were viewing it from the middle, so that the alternations of the annual revolution are observed in both cases alike — though under opposite cardinal points. Indeed, in this way the Moon too and the other heavenly bodies keep their former appearances, and each of them is assigned to its own place — by far more fitting places, as it seems to Copernicus. And in this transposition as well, the reality of the orbs (with the exception of the Earth's), though the ancients believed in it falsely, is here tolerated.

Now in this Copernican representation, since the periods of the motions are the same as those that were indicated in the earlier Ptolemaic one, we shall accordingly add only a few remarks here — concerning chiefly the greater usefulness and economy that this rearrangement of the orbs offers.

First, with the Earth revolving along its annual circle, and once the Sun has become the center of the universe — and therefore also of the five planets — Copernicus abolishes the Ptolemaic epicycles in them in the following way.

Diagram. The Copernican world-system, drawn as a set of seven concentric circles with the Sun at the center, surrounded by a decorative starry border of foliate ornaments and asterisks marking the firmament. From the outermost orbit inward, each ring carries an inscribed label running along the circle, numbered I–VII:

- **I.** "Stellarum fixarum Sphæra immobilis." — "I. The immobile sphere of the fixed stars."
- **II.** "Saturnus anno 30 revolvitur." — "II. Saturn revolves in 30 years." (with Saturn's symbol ♄ at left)

- **III.** "Iovis 12 annorum revolutio." — "III. Jupiter's revolution of 12 years." (with Jupiter's symbol ♃ at left)
- **IIII.** "Martis bima revolutio." — "IIII. Mars's revolution of two years." (with Mars's symbol ♂ at left)
- **V.** "Telluris" with inner globe labeled "Terra ☽" and, along the ring, "cum orbe lunari annua revolutio" — "V. The Earth's annual revolution, with the lunar orb." (Earth ☉/☽-globe with Moon ☽ at center of this orbit)
- **VI.** "Venus nonimestri[s]" — "VI. Venus, in nine months." (Venus symbol ♀)
- **VII.** "Mercurij 80 dierum" — "VII. Mercury's, of 80 days." (Mercury symbol ☿)
- Center: a radiant Sun face, labeled "Sol" — "The Sun."

Parsing notes: the ring labels are set in a decorative hand running tangentially along each circle; ordinal numerals are Roman (note "IIII" rather than "IV" for Mars). The marginal ornaments are printer's decorative star-and-foliage work surrounding the system.

Let A be the Sun, the center of the universe; let C K D H be the Earth's annual orbit; let B be the orbit of some one of the superior planets, on which the planet is fixed at some point; and finally let E be the orbit of the fixed stars. Now when the Earth is at H, facing the Sun at A and in conjunction with the planet at B, no annual variation appears in the planet, and the planet is set at its greatest distance from the Earth. But when the Earth has revolved from H to C, the angle of annual variation is C B A, to which there corresponds, at the vertex on the orbit of the fixed stars, E B F, that is, the arc E F; for the measure of this arc the planet intensifies its motion, and there (except for what it has accomplished by its own proper motion at B) it becomes stationary. Further, when the Earth has revolved to K, so that it is interposed between the Sun and the planet, the planet is nearest to the Earth; and because it lies along the straight line A E, it therefore lacks any annual variation. Then, as the Earth advances from C to D, the planet appears retrograde, by the measure of the proportion F G on the orbit of the fixed stars.

And all these appearances [Greek: φαινόμενα (phainomena, "appearances")] agree perfectly, just as if the planet moved on an annual epicycle according to the Ptolemaic supposition — namely in the three superior planets, Saturn, Jupiter, and Mars. As for the two inferior planets, Venus and Mercury, which the Earth encircles, the heavens likewise display similar appearances in the manner that the annual terrestrial revolution prescribes — which now follows.

Diagram. The Copernican construction for a superior planet. The outermost circle is the orbit of the fixed stars, with points F, E, G marked across its top. A middle circle is the Earth's annual orbit C K D H, and at its center sits the planet's point B (the planet being on the small inner circle / orbit B). The Sun A is at the center. Lines run from the planet B out to F, E, G on the stellar sphere; the Earth's positions H (bottom), C (right), D (left), K (just below B) are marked on its orbit.

- A (center): the Sun, center of the universe
- B (just above center): the planet's position on the superior planet's orbit
- C (right, on Earth's orbit): Earth's position giving annual variation angle C B A
- D (left, on Earth's orbit): Earth's position where planet appears retrograde
- E (top center, on stellar sphere): vertex point; lies on line A B E (planet here lacks annual variation when Earth at C)
- F (top left, on stellar sphere): corresponds to angle E B F / arc E F (first station)
- G (top right, on stellar sphere): end of retrograde arc F G
- H (bottom, on Earth's orbit): Earth in conjunction, planet at greatest distance
- K (below B, on Earth's orbit): Earth interposed between Sun and planet, planet nearest

Let A, as before, be the center of the universe and the place of the mean Sun; let B be the orbit of Venus or also of Mercury; let C K D be the Earth's annual orbit. Now when the Earth is at H, it is certain that the planet, existing at B and in conjunction with the Sun along the line H A B K E, holds its apogeal place. Then, as the Earth advances to C, the planet appears at its greatest evening elongation from the Sun, along the line C B F. From here it becomes stationary, and at last retrograde — most of all when the Earth is at K and nearest to the planet, where the planet is reckoned to be situated in the perigee of the Ptolemaic epicycle. Finally, when the Earth has revolved to D, the planet appears along the line D B at point G on the orbit of the fixed stars [literally "the eighth orb"], where it marks out its greatest morning elongation, which the angle A B D — or its vertical G B E — measures. And all these things agree with the appearances by the Earth's annual motion, provided that account is also taken of the planet's motion in its own circle, and of what its positions and configurations relative to the Sun and the Earth have been. So much so that anyone who diligently directs his mind to these matters, as they deserve, will confess that this Copernican discovery is most

ingenious for saving the celestial appearances [Greek: φαινόμενα (phainomena)] without epicycles.

For the rest, besides the Earth's annual motion, Copernicus assigned to it two others as well — namely, the diurnal motion and the motion of libration. The diurnal motion proceeds from west to east, whereby it comes about that the heaven, with all its stars, seems to be carried — or to move backward — from east to west.

Finally, through the motion of libration, Copernicus [demonstrated] the alterable [inequality] of the ecliptic and of the equator —

Diagram. The Copernican construction for an inferior planet (Venus or Mercury). The outer circle is the orbit of the fixed stars, with E (top), F (left), G (right) marked. A middle circle is the Earth's annual orbit C K D H. An inner small circle is the planet's orbit B, near the center with A at the very center (the mean Sun). Lines radiate from B/A out to E, F, G; the Earth's positions H (bottom), C (right), D (left), and K (below B) lie on its orbit.

- A (center): the center of the universe / mean Sun
- B (just above center): the planet's orbit (Venus or Mercury)
- C (right, on Earth's orbit): Earth's position; planet at greatest evening elongation along C B F
- D (left, on Earth's orbit): Earth's position; planet appears along D B toward G (morning elongation)
- E (top, on stellar sphere): along the conjunction line H A B K E (apogee place)
- F (left, on stellar sphere): evening greatest elongation direction
- G (right, on stellar sphere): morning greatest elongation, point on the orbit of the fixed stars
- H (bottom, on Earth's orbit): conjunction, planet at apogee
- K (below B, on Earth's orbit): Earth nearest the planet, planet at perigee of the (notional) Ptolemaic epicycle

— saves the obliquity of the equator that has been observed over a longer course of time, and this with respect to the poles of the Earth alone. Copernicus introduced this threefold motion of the Earth with such artistry that no one motion impedes either of the other two—as ocular inspection on a sphere designed for demonstrating these things can teach. We append here a sketch of it, such as it is, together with its explanation.

[The diagram is described below.]

Let the zodiac, conspicuous with its twelve signs, be drawn from the Sun A as center. Then from the same center draw the equator C D, inclined to the zodiac—not because according to Copernicus there is any such *celestial* circle, but rather a *terrestrial* one in the diurnal revolution, by as much as the Earth's poles deviate from the poles of the zodiac. Within these circles, likewise draw from the Sun A as center the Earth's annual orbit B, which is the mark of the Earth's center, set in its annual circle here at four points taken from opposite places—that is, at the cardinal points of the ecliptic—over which one half of the equator inclines in the same direction as the celestial C D, namely in our age by 23 degrees 31½ minutes.

Diagram. A Copernican world-diagram illustrating the annual motion of the Earth and the production of the seasons, the equinoxes, and the solstices. At the center is the Sun, drawn as a radiant face, labeled A. Concentric with it are two large circles: the outer is the zodiac (showing the twelve zodiacal signs around the periphery), and an inner circle is the celestial equator (labeled C at top and D at bottom), drawn inclined to the zodiac. At four cardinal positions around the Earth's annual orbit—top, bottom, left, and right—the Earth is drawn as a small globe, each labeled B, with its own equator (☐/☐ markings) shown tilted so that the Earth's axis maintains a fixed orientation toward one region of the world throughout the annual circuit.

- A (center): the Sun, drawn as a radiant face
- C (top of inner circle): the celestial equator, upper point
- D (bottom of inner circle): the celestial equator, lower point
- B (top, bottom, left, right): the Earth, shown as a small globe at each of the four cardinal points of its annual orbit
- Zodiac signs around the periphery: ☐ Cancer (top), ☐ Capricorn (bottom), ☐ Aries (right), ☐ Libra (left), with the intermediate signs ☐ Leo, ☐ Virgo, ☐ Scorpio, ☐ Sagittarius, ☐ Aquarius, ☐ Pisces, ☐ Taurus, ☐ Gemini in sequence between them
- ☐/☐ glyphs on each Earth-globe: the Earth's own equator and tropics

With these things established, and granted from the hypothesis that throughout the whole annual circuit of the Earth around the Sun, the Earth's own poles—about which its diurnal revolution occurs—perpetually face one and the same region of the world, it follows that when the Earth is under Capricorn, the Sun appears opposite under Cancer; and so everywhere in the annual revolution the Sun appears under the signs and points of the zodiac opposite to the Earth's.

Therefore, when the Earth is in Libra, the Sun will appear in the region of Aries—indeed at its very beginning, without any declination—because there we view the Sun through the common section of the ecliptic and the terrestrial equator. The same likewise happens when the Earth occupies the beginning of Aries: for then the Sun is seen in Libra without declination. Hence at these two points the two equinoxes fall each year.

Conversely, since when the Earth is situated under Capricorn and Cancer the greatest declination of the circles of the ecliptic and equator from one another appears—and indeed in opposite directions—just so many solstices occur there each year, as these things are more conveniently grasped from the instrumental sphere itself. Thus (besides the matters treated above in the planets themselves) the Copernican hypothesis, from the annual motion of the Earth, accounts most beautifully for the seasonal changes of the year as they are here, and sets them before the eyes. Moreover, the simple precession of the equinoxes—that is, the apparent forward progress of the fixed stars—can occur, as we observe with Copernicus, because the Earth's annual circuit is everywhere gradually anticipated, by its own directing motion, into the same region of the world.

So much for the annual motion of the Earth. The diurnal follows, which is the rolling of the Earth itself with respect to its surface, about its own center, from west toward east, in the space of 24 hours. Hence it comes about that the heaven appears to be carried around with the contrary motion, from east to west, in the same space of time, together with the φαινόμενα [Greek: phainomena, "appearances" — the celestial appearances] set within it.

The third motion of the Earth is called by Copernicus the motion of libration, because the terrestrial poles, carried around in certain little circles over a long stretch of time, vary the obliquity of the equator from the ecliptic. And these recited motions are assigned to the Earth by Copernicus with such artistry that they can all be carried out at once without any impediment, while the Sun and the starry heaven remain at rest, and the rubbish of epicycles—of which we treated above—is removed, as was likewise related earlier.

Concerning so great an economy of nature, once, while I was representing the world-system for a fixed time—and even with the special motions of the planets—upon the Copernican hypothesis, I amused myself in the following manner:

That it may be beautifully beheld in one form, this orb which is here fitly composed of various parts: that from our own rays the courses of the radiant heaven might be marked out, whither each star tends— the

mighty Copernicus overturns the lofty order of nature, expanding the moving stars to fixed laws. You behold how Titan [the Sun], the image [εἶδος; Greek: *eidos*, "form, image"] of the divine power and the fount of things, sits enthroned: around whom the rest— Saturn, shining Jove, Mars, Hermes [Mercury], and Venus— as their great King, venerate him with great honor. Nay, even the narrow orb in which we mortals are enclosed obeys his nod, girt about by the star of Phoebe [the Moon]. O, if such an easy method had been well known to past ages, then thou, Atlas, wouldst weary thyself in vain: for we suspend most things by the single course of the Earth. Nay, Ptolemy, that which by thy art we do not measure— namely how far the wandering stars are removed into the aether— this now truly accords aptly with our numbers, while the laws that turn the stars of the world endure.

Thus far concerning the world-system [hypotyposis mundana] of that great Copernicus, and concerning the assertion of the threefold motion of the Earth within it. Although it was devised most ingeniously for saving the celestial appearances, and is fortified also by physical reasons (which are inserted in the first book of the *Revolutions* [Margin: See *De Revolutionibus* I, ch. 8, etc.]) so that it can be overturned only with difficulty, nevertheless two considerations among others are found which seem to detract from its credibility, especially in certain matters concerning the annual motion of the Earth.

The first can be drawn from the condition of the creation and from the prerogative of the great mundane bodies. For since, among all visible created things, sacred Scripture makes mention of the heaven and the Earth first of all [Margin: Genesis 1, v. 1], it is surely probable that to these too the primary seats or places in the world were assigned. And since it is credible that this whole universe imitates the form of a circle, what can be more probable than that the Earth fills its center, while the heaven—which embraces all the rest—fills the space between the center (namely the Earth) and the circumference? Just as these things present themselves to one who examines Scripture more fully in Genesis 1, vv. 6, 7, 8, likewise vv. 14, 15, and following.

The second argument against the annual motion of the Earth is drawn from the incredible interval of the sphere of the fixed stars from the Earth or the Sun, which—

—will follow upon the Copernican hypothesis for such a motion, and likewise from the magnitude of the fixed star itself. For this would deserve no place otherwise, unless the diameter of the Earth's annual orbit, in comparison with

the semidiameter of the fixed stars from the Sun, has entirely vanished—a comparison which nonetheless is perceived clearly enough even in Saturn, the highest of the planets. But what is asserted here we shall confirm by geometrical demonstration in the following manner.

Let the Earth's annual orbit $E H F$ be drawn about the Sun's center A , according to Copernicus; and from the same center the arc $B C$ of the eighth sphere, or orb of the fixed stars; and a line $A D$ drawn from the center to the said arc—this will be the semidiameter of the eighth sphere from the Sun. According to Copernicus this must be of such great length that $E F$, that is, the diameter of the Earth's annual orbit, obtains no comparison at all with it, but on account of the immense distance of D from A the whole angle $E D F$ vanishes when the Earth is at either end, at E and F . For it will be possible to perceive no sensible variation in the places of the fixed stars when the Earth is in these places at opposite seasons of the year, according to the Copernican hypothesis—even with observation made most attentively and most diligently on a proper and competent instrument, as we have several times experimented at Hven [Margin glyph: Hven] upon the polar star.

Diagram. A geometrical figure demonstrating that the annual parallax of the fixed stars must be imperceptible under the Copernican hypothesis. At the top, a wide shallow arc represents the arc $B C$ of the eighth sphere (orb of the fixed stars), with point C at upper left, D at the top center (apex), and B at upper right. From D a very long narrow triangle descends to the small circle at the bottom representing the Earth's annual orbit, whose center is A (the Sun); the orbit's two extreme points are E (right) and F (left), with H marked above A inside the orbit. The lines $D E$ and $D F$ form the extremely narrow angle $E D F$ at the apex, illustrating that the base $E F$ (the diameter of the Earth's annual orbit) is negligible relative to the distance $D A$.

- C (upper left): one end of the arc of the eighth sphere
- D (top, apex): the point on the eighth sphere toward which $A D$ is drawn
- B (upper right): the other end of the arc of the eighth sphere
- A (bottom center): the Sun, center of the Earth's annual orbit
- H (above A): a point on the Earth's orbit
- E (right): one extreme of the Earth's annual orbit
- F (left): the other extreme of the Earth's annual orbit

But let the angle $E D F$ be given—not from any observation, but only by hypothesis—as one minute of arc, which, if it existed, could be discerned well

enough on an instrument; then E D A will be $\frac{1}{2}$ minute. And since in the right triangle E A D all the angles are given for [the side] A D, and the side E A is 1150 semidiameters of the Earth (the mean distance of the Sun and the Earth from one another—for we wish no other distance than Tycho's [Tycho Brahe's] to be presupposed here [Margin: See *Progymnasmata* I, p. 473], a distance perhaps to be deservedly increased by us in its proper place beyond a hundred semidiameters, which detracts still more from Copernicus's opinion and increases the distance and measure of the fixed stars from us), it will be as follows:

	dec.	gr.	mi.	sec.
E D A	30	E A		D E A 89 59 30 → A D
S. R. 1454441		1150 semid.		S. R. 99999998 94 → 7906818 semid. terr.

Parsing notes: This is a marginal computation laid out in the source as a two-line proportion (logarithm/sine table style). The first line gives the angles (E D A = 30 sec [i.e. $\frac{1}{2}$ min]; D E A = $89^{\circ} 59' 30''$) and the known side E A = 1150 semidiameters; the second line gives the corresponding sines (S.R. = sinus rectus, "right sine") 1454441 and 9999999894, yielding the result A D = 7906818 semidiameters of the Earth. Columns were difficult to align in the source; transcribed as printed.

And to such an extent would any fixed star whatever be distant from the Sun, or roughly the mean distance of the Earth: which is indeed an immense space compared with what the ancients and Tycho placed for the fixed stars, namely 14000 semidiameters of the Earth. And since Saturn, the outermost of the planets, is by the consensus of nearly all astronomers removed from the Earth about 12000 semidiameters of the Earth—to what end, then, is the empty interval between this planet and the fixed stars, which from the foregoing is elicited as 7894818 semidiameters of the Earth?

Now we shall weigh geometrically the magnitude of the fixed star likewise from the Copernican hypothesis.

Let, in the following figure, A be the Earth or the Sun; and B some fixed star of the first magnitude. A B will therefore be the line of the star's distance from the Sun, or roughly the mean elongation of the Earth, which according to the supposition employed earlier in the Copernican reasoning was found to be 7906818 semidiameters of the Earth. Further, about the star's center B let its circular magnitude be described by the small circle D C; and the diameter D C being drawn, and in addition the lines A C and A D, we shall first seek in the right triangle A B C the semidiameter—

—the true semidiameter B C of the star, in semidiameters of the Earth. Now the given quantities [δεδομένα; Greek: dedomena, "the things given"] are these:

Diagram. A geometrical figure for computing the bodily magnitude of a first-magnitude fixed star under the Copernican hypothesis. At the top is a small circle representing the star's globe, with **B** at its top, **C** at the left end of its horizontal diameter, and **D** at the right end. From C and D two long lines converge downward to a single point **A** at the bottom (the Earth or Sun), forming a very long, narrow isosceles triangle whose apex is A and whose base is the star's diameter C D.

- B (top of small circle): the fixed star (its near point)
- C (left of diameter): one end of the star's diameter
- D (right of diameter): the other end of the star's diameter
- A (bottom apex): the Earth or the Sun

B A = 7906818 semidiameters of the Earth; angle B C A [recte: B A C] = 1 minute of arc, from observation of the apparent semidiameter of a first-magnitude star; and finally the angle A B C is a right angle. Whence it will be as follows:

	B	BA	BAC	BC
S. T.	10000000	7906818	T. 2909	→ 2300 semid. terr.

Parsing notes: A proportion in the same tabular style. B (whole sine) = 10000000; B A = 7906818; T. 2909 is the tangent of the angle B A C (1'); the result B C = 2300 semidiameters of the Earth. ("S.T." = sinus/tangens table; "T." = tangens.)

And since D C is double B C, that [diameter] will be just as many whole diameters of the Earth, namely 2300. From these things one may recognize that the diameter of the bulk of a first-magnitude star would, on this supposition, become more than double the diameter of the Earth's annual orbit. Given now the

diameter of the fixed star of the first rank, we obtain its quantity—first in comparison with the whole globe of the Earth (that is, the mass aggregated from the Earth and the surrounding water)—by separately resolving the proposed diameters of both the Earth and the star into cubic numbers, and subtracting the smaller from the larger. For spheres are in the tripled ratio [i.e. cube ratio] of their diameters. The procedure follows.

The cube of the number D C, 2300, is 12167000000; but for the given diameter of the Earth, 1 part, the cube will likewise be 1 part. Therefore, the upper cube being divided by unity, the same number returns, namely 12167000000, which proves that the first-magnitude star surpasses the globe of the Earth that many times over, if the Copernican supposition shall be true.

[Margin: *Progymnasmata*, p. 473.]

But let this same number now be divided by 140—by however much the Sun itself is hitherto believed to surpass the Earth in its magnitude: then the fixed star of the first rank will surpass even the Sun itself 86907143 times. Finally, since in this proportion the diameter of the fixed star was previously found to be more than double the diameter of the Earth's annual orbit, on a supposed parallax-hypothesis of whatever kind assumed earlier—therefore, this being set at one part, that [star's diameter] will be 2 parts; and when each is raised cubically, the cube of the annual orb remains 1 part, but that of the star 8 parts. Whence it is shown that a first-magnitude star, in its total quantity or bulk, must surpass eightfold the whole interval from the Sun to the Earth which is circumscribed by the magnitude of the Earth's annual orbit—provided that any room at all be given to the Copernican supposition.

[Margin: See *Epistolae Astronomicae*, T. B. [Tycho Brahe], p. 192.]

Since, therefore—both from the immense remoteness of the fixed stars from the Earth, and from the interval (how incredible!) that follows from this between the outermost of the planets and the orb of those same fixed stars, and from the magnitude of the fixed stars of the first rank (as we supposed) relative to the Earth's annual orbit, likewise the Sun, and much more relative to the Earth—all the well-constituted symmetry of the mundane parts is easily overthrown; besides the absurdity which above, from the condition of things primarily created, we adduced as an argument from the sacred writings, and which afterward, concerning the motion of trepidation of the Earth, we shall adduce to be destroyed (this being something to be ascribed rather to the alterable *latitude* than to the *declination* of the fixed stars) [Margin: See ch. 4 of this book]: for these

reasons I judge that the Copernican hypothesis—concerning the annual motion of the Earth especially, and concerning its libration about its own poles—is rightly to be set aside, and to be exchanged for the happier invention of Tycho Brahe; especially after the reality of solid orbs in the heaven has been sufficiently refuted by recent observations [Margin: See the *Epistolae Astronomicae* of T. B., concerning the comets [item, περὶ γράμμων? — Greek note partly faded] of our astronomy].

Therefore, third—and if, perhaps, there has ever anywhere been a true mundane system—we set before the eyes, in the following sketch, the one discovered by Tycho Brahe 38 years ago.

A new sketch of the world-system, discovered by Tycho Brahe [Tycho Brahe], by which both the well-known redundancy and inelegance of the old Ptolemaic system, and likewise the recent physical absurdity in the Copernican motion of the Earth, are excluded — and which corresponds most aptly to all the celestial appearances.

[Margin: Tycho, *Epistolae Astronomicae* [Letters on Astronomy], Book 2, *Progymnasmata*, p. 189.]

Explanation.

Let A be the Earth and the primary center of the universe — that is, of the outermost starry heaven, however far it may extend on every side from the Earth. The Earth is likewise the center of the two great lights, namely the Sun and the Moon, which serve above all to mark the divisions of time; and so their orbs are here drawn concentric with the Earth [homocentric], together with the orb of the fixed stars.

[Margin: Genesis 1:14 and following.]

As for the Earth, we maintain — with Master Origanus [David Origanus] and other distinguished men of this age — that it moves upon its center at the middle of the world and completes a daily rotation from west to east, to nature's great economy. For on this assumption, the uniformity of all the celestial motions from west to east is genuinely preserved, and only the *appearance* in the heaven is seen as running toward the west. Moreover, the incredible speed of the heavenly bodies in their daily revolution — above all that of the sphere of the fixed stars — is dispensed with by nature through the Earth's contrary effort.

I would add this further point. The fixed stars are presumably not all set at an equal distance from the center, that is, from the Earth, but hang in the most

transparent heaven, each balanced on its own center with equal weights, like the globe of the Earth. Hence, since the founding of the world they have suffered no disruption in their distances from one another. Who, then, would attribute real motions to such bodies — unless someone who is determined to defend, tooth and nail, a few passages of sacred Scripture that are (as they seem) accommodated to human comprehension only in appearance? But more on these matters below.

[The Tychonic world-system diagram is described below.]

Diagram. A large circular schematic of the Tychonic world-system. The outermost band is a wide ring densely filled with small star-glyphs representing the sphere of the fixed stars. Within this band a nested set of concentric circles is drawn, with the planetary and luminary symbols arranged on a vertical axis running through the figure. From the central point outward the labeled features are:

- A (at the center, marked "⊕A" — a small circle): the Earth, the primary center of the universe.
- ☾ (just below A): the orb of the Moon, drawn concentric with the Earth.
- ☼ (above A, drawn as a sunburst/rayed disk): the Sun, the secondary center, carried on its own orb concentric with the Earth.
- ☿ (Mercury): on the small innermost circle surrounding the Sun.
- ♀ (Venus): on the next circle surrounding the Sun.
- ♂ (Mars, near the top, above the Sun): on the orb of Mars, which encircles the Sun and also encloses the Earth.
- ♃ (Jupiter, above Mars): on the orb of Jupiter.
- ♄ (Saturn, the outermost planet, just inside the star-band at the top): on the orb of Saturn.

The five planetary orbs (Mercury, Venus, Mars, Jupiter, Saturn) are all centered on the Sun, which in turn revolves about the central Earth; the Sun's and Moon's orbs are concentric with the Earth, all enclosed within the outermost sphere of the fixed stars. (No [?] flags.)

[The Tychonic world-system diagram (continued from p. 177) is reproduced again at the head of this page; see the description above under fig-p177-1.]

I now return to the second — that is, the secondary — center of the universe. That the Sun fills this role, set as it were in the place of the primordial light, is proved on all sides by arguments drawn from the motions of the planets and the other

appearances, above all the annual ones. Indeed, Copernicus [Copernicus] too undoubtedly held the Sun to be their center, around which the five planets — namely Saturn, Jupiter, Mars, Venus, and Mercury — lead their circular courses as around their own leader and king, and they keep him always advancing, as it were, in the very midst of their revolutions. So true is this that, as the Sun makes its circuit, the centers of the very orbs that the planets describe about it are themselves carried around year by year.

Furthermore, since all of this holds good for the inferior planets Venus and Mercury as well — namely, in the way their orbs take over the role of the Ptolemaic epicycles in their case — it is truly remarkable how, by the benefit of this hypothesis and of the single solar revolution, all the annual epicycles in the superior planets that surround the Earth, namely Saturn, Jupiter, and Mars, are done away with. And yet those same epicycles are in reality re-established in the annual circuits of the individual planets, precisely on account of the single annual revolution of the Sun about the Earth, as I said. For the inferior planets Venus and Mercury, meanwhile, proper orbs are assigned no less than to the rest.

And here is the actual state of the matter — what the demonstration itself demands, and what the motions and digressions of the planets demonstrate. From the one and equal circuit of the Sun, the semidiameters of the epicycles — which are described by the same route from the motions of the planets — turn out indeed equal in themselves and equal to the solar orb, but with respect to the appearances they prove unequal: smaller in the higher planets, as in Saturn; but larger in the lower ones, as in Mars. This can best be shown to the eyes of learners by means of a kind of paper model, laid flat, which represents the Tychonic hypothesis according to all its orbs.

In ordering the world-system, therefore, we rest in the hypothesis discovered by Tycho rather than in that of Copernicus. This is so, first, because it stands as a kind of mean between the old Ptolemaic system and that admirable

Copernican one; and second, because — once the annual epicycles of the Ptolemaic system, and likewise the other absurdities of the Copernican system involved in the annual motion of the Earth, are set aside — the Tychonic hypothesis explains the symmetry of the world most excellently and fittingly. For the rest, since the phenomena of all the planets can accommodate themselves indifferently to the supposition of all three of these craftsmen, and since no other route remains for saving them — especially as regards the annual inequality, from which by a marvelous agreement there follow the changes of latitude, and likewise the stations and retrogressions, when the planets are taken to imitate

circles — this is briefly demonstrated by what appears to be a complete induction: namely, that the said inequalities are handled either entirely outside the middle of the universe, or at the middle, or about the middle. For the external consideration the Ptolemaic epicycle serves; for the internal, the Copernican; and finally, for that which is about the middle, the Tychonic. Therefore, having weighed this necessity, we shall present the motions of the planets — which we are to treat in the second book of the *Theorics* [*Theoricorum liber secundus*] — under the threefold sketch just reviewed, once the route for attempting the same has been demonstrated generally in what follows. And we by no means despair that the thrice-greatest and most benevolent Maker of the heavenly things will so prosper our labors and bring them to their appointed end, that all of them may serve the perpetual propagation of His glory.

CHAPTER II.

On the method, devised by the ancients, of saving the motions of the planets in their proper orbs by means of eccentricities and equants; and on the refutation of that method, and the substitution of a better method by means of epicycles and epicyclets [epicycli and epicycepiclycli].

Up to this point, in expounding the threefold sketch of the world-system, we have described the celestial appearances in a general way. But since the motions of the planets fall under a twofold consideration — namely, with respect to their proper orbs, and also with respect to their annual digressions — and since the former (with its anomalies) is said by us to be *essential* to a planet, while the latter is *accidental* (owing, as has been shown, to the Sun's annual motion), we must therefore deal first with the former. This is especially so because the various motions of the planets along their own orbs also markedly vary their annual digressions.

The celestial motions, moreover, are distinguished into the *simple* or *periodic*, and the *composite* or *apparent*: the former are commonly called the *mean* motions, the latter the *true*.

Further, in their proper orbs the planets are subject either to a single anomaly, as is the Sun itself, or to a double one, as are all the rest.

After the ancients had grasped that the motions of the planets are indeed perpetual, but their appearances unequal, they accounted for these anomalies (or inequalities) — in the case of a single anomaly, by eccentricities; in the case of a

double one, by eccentricities and equants — in the following ways, which will appear in what follows.

Demonstration of the simple anomaly, as in the Sun.

[Greek: Απόδειξις (apodeixis, "demonstration" or "proof")]

Let A be the center of the circle C D E F, on which the Sun is carried (the Sun being the only planet content with a simple digression from the mean motion). On its diameter C E let the Earth B be conceived. Thus A B is the distance of the circle's center from the Earth, which is called the ec-

Diagram. A geometric figure for the solar (simple) anomaly: a circle with labeled points. Lines are drawn from the points D and F to the interior points A and B.

- C (top of circle): one end of the vertical diameter; the apogee.
- E (bottom of circle): the other end of the vertical diameter; the perigee.
- A (on the diameter, above center): the center of the circle.
- B (on the diameter, just below A): the Earth.
- D (upper left of circle): the point of descent from apogee; lines run to A and B, defining the angle of greatest difference on descent.
- F (lower right of circle): the point of ascent from perigee; lines run to A and B, defining the angle of greatest difference on ascent.

(No [?] flags.)

— is called the eccentricity; and on this account the circle so drawn, C D E F, is called eccentric. C is the apogee, or the point farthest from the Earth; E is the perigee, or the point nearest to it.

As regards the appearances in the motions: since an angle subtended by longer sides is always smaller than one enclosed by shorter sides (for the interior angle is greater than the exterior and opposite one), the motion that is uniform with respect to its proper center A turns out unequal when we look at the Sun from the Earth at B — smaller around the apogee, but larger around the perigee. At the two points C and E, however, it differs not at all from the mean or simple motion. But at the points roughly midway the difference is greatest; this is what is called in the table the *prosthaphaeresis* [Greek: προσθαφαίρεσις (prosthaphairesis, "the correction of addition-and-subtraction")] — *subtractive* in the descent from apogee, at D; but *additive* in the ascent from perigee, at F. In the present figure the former is defined by the angle A D B, the latter by the angle A F B.

By what means and on what principle these angles are to be investigated will be taught later. Here it will be enough to have given a kind of preliminary demonstration of the theory.

[Greek: Απόδειξις (apodeixis, "demonstration")] *of the double anomaly, displayed by the ancients in the remaining planets.*

The second essential inequality of a planet — namely, in its own orb — since it requires a *prosthaphaeresis* roughly twice as great as the difference between apogee and perigee demands, on this account the ancients added to the simple eccentricity demonstrated above an *equant* [aequans], by this rule: that between the center of the planet's orb and the center of the Earth, the center of the equant's orb should be set at an intermediate position. From this point, on a circle described equal to the prior eccentric, they supposed the planet's center to be carried, and here to describe unequal motions — but about the foreign center A, equal motions, namely the center on which equal arcs are completed in equal time.

So let A be the center of the orb D E; let the Earth be C, and therefore the whole eccentricity is A C. With the point B conceived in the middle, and the orb F G described from it, A B will be the equant. According to its magnitude the planet, carried unequally (as they supposed) around the equant's orb F G — since its uniform motion regards only the center A, as was said above — would in the apogee be nearer to the Earth, by reason of distance, as appears at D and F; but in the perigee farther off, over the interval E G. At the intermediate positions, the angles B H C and B H A, added together, would compose the whole essential *prosthaphaeresis* — over and above, that is, whatever the planet admits in the accidental or annual epicycle. And so we have briefly shown the ancients' method of the equant, which Ptolemy [Ptolemy] throughout employed in his *Almagest*

[Margin: Lib. 4 Rev. c. 2 & 5 cap. 2]

Diagram. The geometric figure for the solar (simple) anomaly, repeated at the top of this page (carried over from p. 179); the same circle C D E F with center A and Earth B, and lines from D and F. See the description above under fig-p179-1. (No [?] flags.)

Diagram. The geometric figure for the double anomaly (the equant construction). A circle with a smaller concentric/eccentric circle inside it, and lines drawn from an upper-left point through the interior points to a lower-right point.

- D (top): the apogee end of the vertical diameter.

- F (below D, on the inner circle): apogee point on the equant's orb F G.
- A (interior, above center): the center of the planet's orb D E.
- B (interior, below A): the center of the equant (conceived "in the middle").
- C (interior, below B): the Earth; the whole eccentricity is A C.
- E (lower, on outer circle): the perigee end of the diameter.
- G (bottom): the perigee point on the equant's orb F G.
- H (lower right): the point from which the angles B H C and B H A are formed.

(No [?] flags.)

Astronomia Danica — Pars Altera, Liber Primus Theoricorum

Caput II (continued)

[Margin: *Revolutions* IV [Lib. 4. Revol.], chapters 2 and 5 [cap. 2. & 5.]; chapter 2 [cap. 2.]]

had done; and Copernicus [Nicolaus Copernicus] takes the point up again and refutes it, since he holds it to be absurd—both geometrically and optically—that motions should be made uniform about a foreign center (as here about A, and not about B, the center of the planet's orb). For this reason, in the case of planets subject to a double essential anomaly, he likewise retained the simple eccentricity but ingeniously replaced the equant [aequans] with an epicycle set into the planet's orb, and there represented it. The result is that, under such a supposition, he commits nothing against geometry, nor does he introduce anything at all different from the φαίνόμενα [Greek: phainomena, "appearances" — the observed phenomena] that are to be gathered from the hypothesis of the ancients—as will become clear in the following diagram of Copernicus.

Diagram. A large circle (the planet's deferent orb) centered on point A. Four small circles (the Copernican epicycle in its four cardinal positions) are placed at the top (C), bottom (E), left (D), and right (F) of the deferent. The vertical diameter runs C–G–A–L–B–E from top to bottom; the horizontal diameter runs D–A–F. Within each small epicycle a planet-position is marked (G at top, I at bottom, H at left, K at right), and lines are drawn from B and from the epicycle-positions to trace the prosthaphaeresis angles.

- C (top of deferent): center of epicycle at apogee
- G (inside top epicycle, lowest point): planet's position when epicycle-center is at C
- A (center of deferent orb)
- L (on vertical diameter, just below A)
- B (on vertical diameter, below L): the eccentric/Earth point
- E (bottom of deferent): center of epicycle at perigee
- I (inside bottom epicycle): planet's position when epicycle-center is at E
- D (left of deferent): center of epicycle at a quadrant
- H (inside left epicycle): planet's position there
- F (right of deferent): center of epicycle at the other quadrant
- K (inside right epicycle): planet's position there

Suppose, then, that A is the center of the planet's orb C D E F. About these marked points, taken as centers, let the Copernican epicycle be described, the semidiameter of which—such as C G—is to contain a third part of the eccentricity A B. In this epicycle let the planet also move in the order of the signs [in *consequentia signorum*], in a revolution commensurable with that of the epicycle's center, under this rule: while the epicycle's center is at C, the apogee, the planet occupies the lowest part of the epicycle at G; but when the epicycle's center C has descended sideways through D, a quadrant of the eccentric, the planet ascends here to the point H. Further, when the epicycle's center descends fully to the perigee E, the planet is found at I; and so finally, as the epicycle's center climbs through the other semicircle of the orb to F, it shows the planet at K.

[Margin: The place of the higher epicycle [*Epicycli loco superioris*—the invention of the equant [*aequantis inventio*—is attributed to al-Zarqālī the Spaniard [*Azzarchel li Hispano tribuitur*].]

And here one may see the device by which Copernicus removed that absurdity of the higher Ptolemaic equant by an equivalence of the appearances [*apparentiarum aequipollentia*]. For when the epicycle-center carrying the planet is at the apogee C, and the planet, as has been said, is situated at the perigee of the epicycle, the planet is brought closer to the Earth by the whole diameter of the epicycle than if it were content with the simple eccentricity. At the perigee, by contrast, the planet, lingering at I, becomes farther from the Earth B by the diameter of the epicycle. For as much as is assumed for the semidiameter of the epicycle, that much is subtracted from the eccentricity A B—a quantity which, at

the mean elongations or greatest prosthaphaereses near D and F, returns to constitute the full prosthaphaereses.

But to make some of this clearer by an example: let A C in the upper Ptolemaic figure be 8 parts. Of these, two being assumed for the semidiameter of the epicycle, 6 parts remain, which constitute the eccentricity in the Copernican hypothesis. Of these, $\frac{1}{3}$ —that is, 2 parts—is the measure of the epicycle's semidiameter C G, as we said. For its nearer distance A C to B, since just so many parts—2, namely—are taken away from the eccentricity A C of 6 parts, only 4 parts remain. That is, the half-part for showing the whole difference between the apogee (or perigee) and the mean elongation, just as occurred in the upper Ptolemaic hypothesis. And it happens no differently around the quadrants of the orb at D and F nearly by the same usage [in actual]

[Margin (top, alongside the diagram): ...]

Diagram. A repeat of the same Copernican construction as on the previous page: a large deferent orb centered on A, with four small epicycles at top (C), bottom (E), left (D), and right (F). The vertical diameter runs C–G–A–L–B–E; the horizontal runs D–A–F. Inner planet-positions: G (top), I (bottom), H (left), K (right). A dotted inner curve (the path G H I K traced about L) is shown.

- C (top): epicycle-center at apogee
- G (inside top epicycle): planet
- A (deferent center)
- L (just below A): the point from which the planet's traced path is described
- B (below L): eccentric point
- E (bottom): epicycle-center at perigee
- I (inside bottom epicycle): planet
- D (left): epicycle-center at quadrant
- H (inside left epicycle): planet
- F (right): epicycle-center at quadrant
- K (inside right epicycle): planet

usage; where the angles A D B and D B H, combined, constitute the full essential subtractive prosthaphaeresis [prosthaphaeresin essentialem ablativam], and likewise the additive one [adjectivam] is constituted by the angles A F B and F B K.

This periodic revolution of the epicycle and planet through the orb is a consequence in this Copernican reformation, as if, having assumed a third part of

A B, a circle scarcely matching the planet's path were described from the point L—namely G H I K. So that here it may appear with how great an artifice the Copernican epicycle was substituted for the Ptolemaic equant: an epicycle which nowhere collides with geometry, as the equant did, and which represents the same φαινόμενα equally well or more correctly.

It must be observed here, however, that the phenomena do not everywhere approve a third part of the eccentricity A B as the quantity to be set for the semidiameter of the epicycle, as will be shown below for ♂ [Mars], ♃ [Jupiter], and ♄ [Saturn] in their proper places.

Secondly, it must also be noted that the same phenomena arise provided the epicycle-center has been placed in the middle, namely at the point A, on this condition: that the center of the planet's orb revolve in such an epicycle, taking its start from the least distance from the Earth or Sun, working against the order of the signs [in antecedentia signorum], and completing a double revolution for one of the planet in its orb—as Copernicus himself shows in the case of Venus and Mercury, and as we too, if it prove convenient, will demonstrate below for these same planets.

The third and—as it seems to us—final and truest method of saving both the simple and the double essential anomaly of the planets: by means of a homocentr-epicyc-epicycle [homocentrepicycepicyclum], that is, two small circles to be added to the planet's orb.

We could indeed acquiesce in the correction of the ancients' equant that was so skillfully introduced by Copernicus (as we set out above), were it not that we had long since been persuaded that astronomy is to be cultivated not upon just any hypotheses that display the celestial phenomena, but rather upon those by which the whole of nature, taken in general, seems to be revealed in its own operations. So much so that we no longer marvel at so great a variety of motions along with the common crowd; nor do we, with certain others, either grow weary of it, or rashly cast it off onto other causes—besides the manifold and ordered connection and supposition of immaterial (or merely formal) circles—as Fracastoro [Girolamo Fracastoro] and certain other minds do, who in their idle speculations often relegate the heavenly φαινόμενα to second place. Rather, in this continual and marvelous circling-about of the planet in the heavens, let us at the same time contemplate, with the most pleasing contemplation, the internal causes—causes succeeding one another mutually in nature, proceeding from an immovable and

most powerful principle in the most beautiful order, and nowhere ceasing. For the heaven is the written book of nature, as

we noted from the beginning. Now we shall treat briefly of the equivalence [aequipollentia] for representing the appearances: both of the simple eccentricity in the Sun, by one small circle in a homocentric; and of the eccentricity-and-equant of the ancients (or even of the eccentricity-with-epicycle of Copernicus) by two small circles in a homocentric. First, for the simple anomaly of the planet, such as Ptolemy too demonstrated in the Sun.

Let A be the center of the Sun's orb B C D E. The epicycle-center being placed at those four positions, and the epicycle described from there—its semidiameter, namely, being equal to the eccentricity A B of the first figure of this chapter—then, since the Sun, placed at the apogee and the topmost part of the epicycle F, is supposed to be carried by a contrary motion (and nearly commensurable with the epicycle-center) through the orb B C, etc., all things are easily understood to agree with the hypothesis constructed from the simple eccentricity.

Diagram. A large homocentric orb (the Sun's orb B C D E) centered on A. Four small circles are placed at top (B), bottom (D), left (C), and right (E) of the orb. The vertical diameter runs F–B–A–H–D; the horizontal runs C–A–E. Within the small circles the Sun's positions are marked: F (top, above B), G (in the left epicycle, near C), I (in the right epicycle, near E), H (at the bottom, above D). A dotted inner circle (the Sun's traced path) is shown.

- F (top, outermost): Sun at the topmost part of the epicycle, at apogee
- B (top of orb): epicycle-center at apogee
- A (center of orb)
- C (left of orb): epicycle-center at a quadrant
- G (in left epicycle): Sun's position there
- E (right of orb): epicycle-center at a quadrant
- I (in right epicycle): Sun's position there
- H (bottom, inner): Sun at perigee
- D (bottom of orb): epicycle-center at perigee

For when the center of the eccentric, descending from B into C, has completed a quadrant of the Sun's orb, the Sun itself is by hypothesis represented in the epicycle at G. Hence, to one viewing it from the Earth A, it appears to differ from its mean motion by the measure of the angle C A G, which angle measures the nearly maximum subtractive prosthaphaeresis.

Further, when the orb's center C descends straight on into D, the Sun is represented at H, at the very perigee—nearer to the Earth A by just as much as it had been set higher at the apogee from the start; which the higher first simple eccentric likewise required. Finally, with the epicycle-center at D, climbing back through half the orb and arriving as far as E, it places the Sun in I of its epicycle, again by hypothesis, and forms the angle E A I of the maximum additive prosthaphaeresis; and so, consequently, throughout the whole circuit of the orb. This first supposition fits the simple anomaly so well that the same appearances occur in both cases, given an equal assumed eccentricity and epicycle-diameter, and an equal distance from the apogee everywhere. And thus the equivalence for saving the simple anomaly has been made manifest, in the first place, in this hypothesis. There now follows the hypothesis of the double or composite anomaly, to be accounted for by a double or composite small circle.

Let there be, further, the composite anomaly of the planet's orb, such as Ptolemy set out by means of the equant in addition, and Copernicus by means of the eccentric-epicycle [eccentrepicyclum]—but which is to be shown by us by means of the homocentr-epicyc-epicycle [homocentrepicycepicyclum]. So, placing A in the same manner at the center of the Earth or Sun, and describing the planet's orb B C D E, let the center of the first small circle be supposed upon these marked points, so that the various positions of the planet may be successively represented. Let the line B D be the line of apogee and perigee, and let B be the apogee position, D the perigee; and let everything else, moreover, be as in the diagram immediately preceding.

To this epicycle (or to a point on its periphery, which a line drawn from its center always parallel to the line B D touches) let another small one be added, whose semidiameter is to be about a third part of the former, likewise represented in four positions, namely upon the centers F G H I. In its circumference let the planet be carried with a motion contrary to that of its own little orb's center—that is, in the order of the signs—and with a motion fully commensurable, in a doubled periodic revolution, with the center of its epicycle F (the prior one); beginning, namely, from the lowest position of its epicycle near K at the apogee—except in the hypothesis of Mercury, of which in its place.

Diagram. The composite construction: a large orb B C D E centered on A. At each of the four cardinal points of the orb sits a larger epicycle (the "first" circle), and upon each of those a smaller secondary epicycle. At top: outer point F, secondary-circle center near K, on the larger circle B (apogee). At bottom: secondary circle at H, planet at M, on larger circle D (perigee). At left: larger circle at C, secondary

circle at G, planet at L. At right: larger circle at E, secondary circle at I, planet at N. The vertical diameter runs F–K–B–A–H–M–D; the horizontal runs C–A–E. Dotted inner curves show the traced paths.

- F (top, outermost): topmost point of the upper construction
- K (top, in secondary circle): planet's starting position at apogee
- B (top of orb): apogee, center of first epicycle there
- A (center; Earth or Sun)
- C (left of orb): center of first epicycle
- G (left, secondary-circle center)
- L (left, in secondary circle): planet's position when first epicycle-center is at C
- E (right of orb): center of first epicycle
- I (right, secondary-circle center)
- N (right, in secondary circle): planet's position
- H (bottom, secondary-circle center)
- M (bottom, in secondary circle): planet's apparent position at perigee
- D (bottom of orb): perigee, center of first epicycle there

So, when the center of the prior epicycle of the planet has been at C, the planet itself will be at L, displaying the nearly maximum subtractive prosthaphaeresis through the angle C A L, by which the true motion of the planet at L falls short of the mean at C. But at the perigee, the apparent planet at M has completed an entire revolution in the small second circle from K, when the center of the lesser epicycle H has completed at least half its period together with the center of the greater one, D. Finally, with the center of the prior epicycle climbing to E, the center of the latter epicycle will be at I, and the planet—by the law of its revolution—at N; and so the whole angle E A N yields to the additive prosthaphaeresis, by which the true motion of the planet again overcomes the mean. And so gradually, until the planet has been restored to the apogee and to its own place K, from which it had descended. By this norm of revolution, the planet's conformity in its motion—by means of the two small circles, that is, in the homocentr-epicyc-epicycle—with the higher equant is manifestly seen [to obtain].

it is seen, when the apogee is at K and the perigee at M, that the planet on either side displays its positions in proportion to the radius of the outermost circellet, so that — measured by that radius — it stands nearer to the Earth, that is, to the Sun A, at apogee, and farther away at perigee, and that around the mid-points of the

prosthaphaeresis it increases [the distance] by just so much on either side. Furthermore, we shall demonstrate below, in the theory of the Moon, the particular variation [Greek: συστάσεως μεταβολήν (systaseōs metabolēn, "change of configuration")] of this kind as well.

That the planet is in truth subject to such a threefold ordered revolution we shall one day make manifest by a more elevated inquiry into nature — God granting life and strength for so long. For now it suffices that in this chapter we have prescribed in a general way nearly all the methods by which the phenomena of the planets, with respect to their orbs, have hitherto been demonstrated by the foremost astronomers, or will hereafter be demonstrated by us.

WHAT FOLLOWS — TO BE TREATED SPECIFICALLY IN THIS BOOK

The following matters, concerning the motions of the fixed stars, the Sun, and the Moon, are now to be treated specifically in this book, where the preparation [Greek: ΠΑΡΑΣΚΕΥΗ (paraskeuē, "preparation, groundwork")] for restoring each of them is contained in the commentaries that follow.

COMMENTARY

On the examination and restitution of the observations of the Sun, the Moon, and the orb of the fixed stars, for the perpetual restoration of the position of each of them.

I. ON THE SUN.

Having shown above that the phenomena of the planets may be saved in various ways — but chiefly in ways consonant with nature — the proper order now requires that we pursue the motions of the individual bodies in detail, relying on the observations and testimonies that can be had, and on those which seem to carry weight with anyone, by carefully comparing them with one another. For these are the external principles, so to speak, that we employ as we freely devote our labor to the restoration of astronomy. And since what remains of this first book must be spent on those phenomena which regard the primary center — that is, the center of the Earth — it is therefore right that here, before all else, we render an account of the order to be observed in these matters. For Copernicus differs from Ptolemy on this very point: he judged that one must begin not, like Ptolemy, from the Sun and Moon, but from the fixed stars and the precession of the equinoxes. Yet when the matter is diligently examined and weighed

according to the nature of Logic, we assign to Ptolemy the method of discovery, but to Copernicus the order of exposition.

For the anticipation of the equinoxes — both the simple anticipation and a certain inequality joined to it — is perceived not only in the apparent advance of the fixed stars, but mingles itself into the motion [Greek: σύννοδος (synodos, "conjunction, coming-together")] of the Sun too, of the Moon, and of the remaining planets. Thus, without a prior knowledge of the motions of the Sun and Moon, it is by no means possible to examine and establish a foundation in the places of the fixed stars. The courses of the luminaries too — and especially the annual quantity of the Sun — are reciprocally inferred from the stars. For this reason, so that the method here may be made more suited to learners' minds, we have judged it advisable to set forth by way of commentary nearly everything that we contemplate in this book concerning the examination and restitution of the observations from which the motions for various epochs will be established. Afterward, upon these we shall build both the mean motions of the stars and luminaries and the perpetual hypotheses of the same, and treat them in order.

[Margin: Pliny, *Natural History* II.6]

We begin here, then, from the Sun — which the Hebrews call *chammah*, from "heat" [a calore] — because without a prior knowledge of its course, neither the Moon, nor the stars, nor the rest of the matters to be taught in this book or the next, can rightly allow themselves to be examined. For the Sun itself — to use Pliny's words — "is carried as the middle one of the planets, of vast magnitude and power; and it is the ruler not only of the seasons and the lands, but of the very stars too, and of the heavens," and so forth.

Now how truly these things were stated by Pliny concerning this most resplendent and most excellent luminary of the heavens, common experience partly proves, and a truer hypothesis, set forth more specifically in what follows, will partly display — God willing. But indeed, although the motion of the Sun alone would be the clearest argument for the perpetual constancy of the celestial appearances [Greek: φαινομένων (phainomenōn, "phenomena, appearances")], yet — if we must trust the observations and, as it were, the tenets of each of the astronomers — I find in no other matter a more unsightly variety arising with the passage of time. And this concerns not only the measure of the Sun's annual revolution, but also the change of its eccentricity (as they call it) and of the place of its apogee — so much so that the things which, between Hipparchus [Hipparchus] and Ptolemy [Ptolemy], for nearly 300 years, were judged by Ptolemy in his demonstrative construction [Greek: συστάσει τῆς ὑποθέσεως

(systasei tēs hypotheseōs, "in the constitution of the hypothesis") of the Sun without any change, and were confirmed by observations of whatever sort — these afterward, as they rolled down to the following ages, seem to have become subject to change, and indeed to a disordered change. Weighing the causes of this matter rather carefully, I see that none of them resides in the most simple motion of the divine luminary itself, but that the whole blame is rightly to be transferred onto the astronomers, whose records concerning its motion — said to be "derived from heaven" (as they would have it) in different ages — have been handed down to posterity. In these records, for causes that are almost manifest, error has been committed back and forth, more and less, until in our own age it was divinely granted to the Atlas Tycho Brahe [Tycho Brahe] that the beginning of a more accurate restoration of astronomy be made. Of his observations [Greek: τηρήσεις (tērēseis, "observations, watchings")] and celestial observations alone — both on account of the proper and elegant array of instruments and on account of skill in observing — I may perhaps be the best witness that they exclude every sensible error, since for a continuous decade I was a follower of Brahean astronomy. And since this had by no means been accomplished by his predecessors with equal diligence, as we know by experience, it is therefore no wonder that error grew, with the passage of time, exceedingly fertile — both in other matters and most of all in the case of the Sun. For although near the greatest opportunity of observing the place of the Sun from the heavens — which is nearest the equinoxes, when through the observed declination at the meridian (most alterable near these points) its place in longitude can especially be ascertained — even a small error, of very few minutes, which may have been committed (whether on account of the instrument's insufficiency in size or construction; or a less correctly assumed elevation of the pole of the place; or refraction and parallax included in the observation, by one or more of these causes) — that same error extends itself in longitude to nearly triple. What then is to be supposed happens at the intermediate places, where there is always a greater proclivity to deviate, because of the daily change of the Sun's declination gradually diminished, until it reaches the solstitial points — where assuredly, by no artifice of this technique [pragmatia], can the Sun's place be obtained from observations, since its declination remains practically invariable for several days? Nor indeed is there any reason why anyone should be confident that the solstitial points have hitherto been ascertained certainly enough by the ancients through the aid of eclipses, since both the simple course of the Sun and the true hypothesis of the Moon — never indeed rightly established by those same ancients — are required for this technique; besides the precise designation of time in eclipses and its equation (which is rather to be omitted almost entirely than to be brought in

illegitimately, as was once done), which in the ancients is wavering nearly as much as all the other matters that perfect the exactness [Greek: ἀκρίβειαν (akribēian, "exactness, precision")] of observing. If, therefore, anyone less instructed by experience in astronomical observations should presume to charge our care in inquiring into those venerable old authorities — Hipparchus especially and Ptolemy — with foolish perhaps and rash audacity, and to throw that hackneyed saying in our teeth: [Greek: κακὸν τὸ μὴ ... κείμενον — Greek text corrupt as printed; sense ≈ a proverb against meddling with what is established [?]], let him know that to all the causes of error which we have recounted — causes that can be sufficiently learned and drawn from nearly a single chapter of Ptolemy — this one too has been added, which is there sufficiently detected by shrewd conjecture: namely, the opinion concerning the measure of the solar tropical year, wrongly twisted so as to determine absolutely the cycles and conjunctions [Greek: συζυγίας (syzygias, "syzygies, conjunctions")] of the luminaries.

[Margin: *Almagest* III.2]

For since all astronomers before Hipparchus used the quantity of the solar year — which is Julian to us, namely $365\frac{1}{4}$ days — at a fixed rate, as appears in the institution of the Olympiad (to be begun anew every fourth year around the rising of the Dog Star), likewise at other times of the year

[derived] from the rising and setting of the fixed stars to be determined — a custom that belonged to most ancient Hesiod, and afterward to the Greeks and Romans alike. While, then, the observations of the Sun which without doubt existed at Babylon and in Egypt, during the Assyrian monarchy, all perished, first a certain Meton [Meton] of Athens — who flourished 430 years before the birth of Christ — using likewise the common quantity of the year, $365\frac{1}{4}$ days, reckoned the simple courses of the luminaries not so much by the equinoctial and solstitial points as by the new moons, in his own nineteen-year cycle [Greek: ἐννεαδεκαετηρίδι (enneadekaetēridi, "nineteen-year period")], that is, at an interval of 19 years — with a notable error, to be sure, which with the passage of time, by the time of Hipparchus, was detected by him as 5 days. But by the correction of Callippus [Callippus], instituted 6 years before the death of Alexander the Great, within his four periods — which each contained 76 years — an anticipation of one day in the new moons was observed; that is, in the space of 304 years, or, in round numbers (as Scaliger [Scaliger] says), 300 years. So these following words of Hipparchus, ascribed by Ptolemy, attest, where he says:

[Margin: *Almagest* III.2]

"In his book on the intercalary months and days, when Hipparchus had foretold the year's [length] according to Meton and Euctemon [Euctemon] — 365 days — he pursues only the fourth part in these words: 'We for our part have found that just as many months are contained in 19 solar years as they too [found], but we have found that the year falls short by less than its fourth part by $1/300$ of a day. Thus according to Meton five days are lacking in 300 years, but according to Callippus only one.'" Scaliger interprets these words of Hipparchus as concerning the anticipation of the lunar course beyond the Metonic cycle alone; Copernicus, however, in a teaching better suited to the young, [interprets them] in these words:

[Margin: Joseph Scaliger, *De emendatione [temporum]*, p. 257]

[Margin: Copernicus, *De Revolutionibus* IV.4 [recte: III.6]]

"From the most ancient men, therefore, who took care of this matter, so that it might be handed down to posterity in numbers, was found Meton the Athenian, who flourished in the 37th [recte: 87th] Olympiad. He set forth that in 19 solar years 235 months are completed, whence that great year was called the *enneadecaeteris* [Greek: ἐννεαδεκαετηρίς (*enneadekaetēris*, "nineteen-year cycle")] — that is, Metonic and nineteen-yearly. This number was so pleasing that at Athens and in other rather notable cities it was posted up in the marketplace, and it is even to the present in common use, because by it they reckon the beginnings and ends of the months to stand in a fixed order; the solar year too of 365 days with a quarter to be commensurable with the months themselves. Hence that Callippic period of 76 years, in which a single day is intercalated ten and nine times; and they named it the Callippic year. But the shrewdness of Hipparchus discovered that in 304 years a whole day increases, and that this alone is verified, when the solar year is shorter by $1/300$ of a day. Thus also by some this great year was named after Hipparchus, in which 760 months would be completed." Thus far Copernicus, with this rather rough Minerva, as he puts it.

But we shall subjoin here too this simple paraphrase, perhaps more agreeable to Ptolemy's words: Meton, in the 304 years which fell roughly between himself and Hipparchus, committed an error of 5 days in his nineteen-year cycle, both because he kept the solar year at $365\frac{1}{4}$ days in common usage, and because he did not apply the periodic syzygy of the Moon with the Sun precisely enough. Callippus, noticing this in a certain eclipse of the Moon occurring six years before the death of Alexander the Great, corrected Meton in such a way that for each of his four nineteen-year revolutions one whole day should be subtracted; whence he made for himself, for that day, a period of 76 years, which is everywhere

employed by Ptolemy. Moreover, since Callippus himself also used a solar year of $365\frac{1}{4}$ days, and likewise erred somewhat in the course of the Moon, Hipparchus therefore, afterward correcting Callippus, deprived him — in his four periods, that is, in the space of 304 years — of a whole day, and consequently deprived Meton of 5 days. And this error of one day committed by Callippus, Hipparchus cast entirely onto the solar year, by subtracting, namely, its measure over 304 years, or in round numbers one day over 300 years. Hence it came about that the measure of the tropical year is, by Hipparchus, diminished by $\frac{1}{300}$ part of a day, that is, 4 minutes 48 seconds — so that the time was reckoned as 365 days, 5 hours, 55 minutes, 12 seconds. Therefore, since Hipparchus, equally with his predecessors, was eager to hunt two hares at one bound — that is, both to bring back the new moons within fixed spaces of years, and to define the individual times from the simple motions of the luminaries, and at the same time to measure out the annual revolution — although the one in no way depends on the other, he acquired the measure of the solar year not from the heaven or the Sun itself, as it seems, but from certain syzygies of the luminaries, in an unjust and indeed superabundant quantity. This error of Hipparchus, Ptolemy chose to imitate unhappily rather than to cut away his tenet, so much did this lunar cycle please him too. But whatever other testimonies Ptolemy brings forward from Hipparchus on behalf of this assertion prove nothing, since the tropical or solstitial points to the ancients — as we have proved before

[Margin: *Almagest* III.2]

were unobservable, but the equinoctial points of Hipparchus stood splendidly against this very opinion of his, as we shall presently prove from his own observations. And I have drawn these matters out at greater length so that our countrymen might at last recognize that the ancient astronomers — Hipparchus first of all, and Ptolemy — were exposed to errors in determining the course of the Sun, for sufficiently manifest causes.

[Margin: *Almagest* III.2]

Now we shall undertake to discuss the observations of the equinoxes themselves — first by Hipparchus, then by Ptolemy, as they are found noted down by the latter — and, as far as can be done, to restore them; set before the eyes, indeed, in the following Synopsis, where the epoch too has been derived alike from the death of Alexander the Great backward, and onward from Christ the Savior, so that the comparison may become more evident.

Autumnal equinoxes once observed by Hipparchus under the meridian of Alexandria in Egypt.

[Margin: Copernicus has 177 years]

#	After the death of Alexa nder				Befor e the birth of Chris			
	— Year	Mont h	Day	Mom ent	t — Year	Mont h	Day	Mom ent
1	163	Meso ri	30	Suns et	162	Sept.	27	6 P.M.
2	166	Inter cal.	1	Morn ing	159	Sept.	27	6 A.M.
3	167	Inter cal.	1	At noon	158	Sept.	27	0 At noon
4	178	Inter cal.	3	Last [hour] of midn ight	147	Sept.	26	12 P.M.
5	179	Inter cal.	4	Morn ing	146	Sept.	27	6 A.M.
6	182	Inter cal.	4	Eveni ng	143	Sept.	26	6 P.M.

Parsing notes: In row 2, the "Before Christ" month "Sept." spans the row visually but the day-column reads 27; the day cell appears to align with 27. In row 5, the "Before Christ" day cell reads 27. Row 1 month "Messori" (Egyptian month) printed "Meffori."

Vernal equinoxes observed by Hipparchus at Alexandria.

#	After the death of Alexander —				Before the birth of Christ —				Moment
	Year	Month	Day	Moment	Year	Month	Day	h.	
1	178	Mechir	27	In the morning at hour 5, and after hour 11	146	Mart.	24	11	A.M.
2	189	Mechir	29	Last [hour] of midnight	135	Mart.	23	12	P.M.
3	196	Phamenoth	1	At sunset	128	Mart.	23		6 P.M.

Parsing notes: The vernal table's right block has both an "h." (hour) and "Momentum" column; row 1 moment "Mane hora 5 & post horam 11" is densely set and partly compressed. Row 3 lacks an explicit "h." entry, the moment reading "6 Pomer." (6 P.M.). Egyptian months Mechir and Phamenoth as printed.

Examination of the Equinoxes of Hipparchus.

[Margin: *Almagest* III.1, III.2]

I. When I carefully consider the equatorial armillae [armillas aequatorias] by which these equinoxes were obtained — from the description and account of

Ptolemy — I am not at all surprised that Hipparchus omitted due precision in the recording of his observations, and confined his equinoxes at least to the cardinal quarter-points of the natural day, although no one skilled in celestial matters would deny that for the most part the equinoxes fall into the intermediate times, before and after [these points].

II. Nor were most of them assigned to the recorded times by an observation taken from the heavens — especially the autumnal ones — as is manifestly argued from the fact that they are ascribed to sunrise and sunset, since most of them exhibit no consistent index of refraction; whereas refraction would have retarded the autumnal equinox of the Sun on the Alexandrian horizon, and advanced the vernal one, beyond half a day at the least — even with the density of the vapors around the horizon (which are the cause of refraction) much attenuated there, and with the Sun's parallax excluded.

III. The excessive anticipation in time which is seen here in the autumnal observations — inasmuch as it amounts, in the space of at least 19 years within which these were observed, to a whole day — [if one applies] the Hipparchan measure of the solar year, 365 days 6 hours less $\frac{1}{300}$ of a day, or 365 days 5

hours,

minutes...

minutes [and] 12 seconds, it plainly inverts [the matter], inasmuch as from these autumnal observations there emerges not more than 365 days 5 hours 4 minutes, but from the vernal ones 365 days 5 hours 43 minutes — so that the value defined falls at most at 365 days 5 hours 24 minutes, [the year] thus decreasing annually by 31 minutes. On account of this enormous anticipation, something may rightly be allowed to be remitted, if one wishes to undertake the comparison with Ptolemy [Ptolemy] that is due for the measure of the tropical year.

For the present, however — in reasoning out the apogee and eccentricity of the Sun for Hipparchus's [Hipparchus] own age from his own observations — it seems most advisable that, of the whole set, we select the third autumnal and the first vernal [observation] for mutual comparison. Each of these occurred near midday, where neither the error of the armillae could be so great, nor any impediment from refraction affect the Sun. There is the further point that this vernal observation — made in fact near sunrise, and only afterward [reckoned] to midday, or observed an hour earlier — furnishes a manifest testimony to the Sun's refraction on the horizon. This is something which Ptolemy's experience (as he says) indeed agrees with, yet which contradicts his own assertion about

Hipparchus, when he says: [Margin: Lib. 3, cap. 2.] "When we observed carefully, the hollow surfaces of the armillae were sometimes illuminated twice at the same equinoxes; but Hipparchus thought nothing of the kind worth reporting as trustworthy."

IV. Before we compare Hipparchus's selected equinoxes with one another, we must give thought to the parallax — small though it be — attaching to the Sun's altitude. Since we may easily gather that even the elevation of the pole assigned to Alexandria by Ptolemy, 30 degrees [and] 58 minutes, has this [error] in it (from what Ptolemy reports about the maximum obliquity of the ecliptic from observation), here a mutual elimination of the solar parallax takes place: [Margin: Lib. 1, cap. 13.] the elevation of the pole is, that is, diminished by just as much as the Sun's parallax near the equatorial circle could amount to at Alexandria — namely $1\frac{1}{4}$ minutes at an altitude of 59 degrees. And so much for this solar parallax in Hipparchus and in Ptolemy as well. [Margin: Lib. 1, cap. 2.]

Now we shall come to the comparison. Between the third autumnal equinox — which, as has been noted, fell in the 167th year after the death of Alexander [Alexander], at midday on the first of the intercalary days among the Egyptians — and the first vernal equinox of the same Hipparchus, which is reckoned to have occurred in the 178th year after the death of Alexander, in the Egyptian month Mechir, one hour before midday, there are found 11 Egyptian years [and] 181 days, less only one hour. Now, while we keep to Hipparchus's observations as limited above for the magnitude of the tropical year, there correspond proportionally, at the rate of 5 hours 20 minutes beyond 365 days over 11 years, namely 2 days 11 hours [and] $\frac{1}{4}$. If these be subtracted from the number of days 181, there remains the winter interval — that is, between the autumnal and the vernal equinox — of 178 days 11 hours 25 minutes (which Ptolemy sets at only $178\frac{1}{2}$ days). And when that in turn is subtracted from the Hipparchan year-measure of 365 days 5 hours 20 minutes, there remains the summer interval of time, between the vernal and autumnal equinox, of 186 days 17 hours 59 minutes — placed by Ptolemy at 187 days 0 hours 0 minutes.

And we have carried all this out precisely according to Hipparchus's observations — not in order to demonstrate the truth itself in them (for neither is the measure of the tropical year that which emerges from this, nor as yet the summer interval of time between the equinoxes), but so that it might become plain how much error inheres even in the observations of the ancients themselves, lest it should befit us to be so bound and enslaved to them that, when the just balance of comparison has been applied, it would not be proper to alter something in them.

Now we shall pass on to discussing the three Ptolemaic equinoxes, which are likewise reported to have been observed at Alexandria in Egypt.

Autumnal Equinoxes of Ptolemy

	After the death of Alexa nder				After the birth of Chris t			
	Year	Mont h	Day	H. mom ent	Year	Mont h	Day	H. mom ent
1	456	Athyr	7	2 after noon	132	Sept.	25	2 after noon
2	463	Athyr	9	7 foren oon	139	Sept.	26	7 foren oon

Vernal Equinox of Ptolemy

	After the death of Alexa nder				After the birth of Chris t			
	Year	Mont h	Day	H. mom ent	Year	Mont h	Day	H. mom ent
3	463	Pach on	7	1 after noon	140	Mart.	22	1 after noon

Examination and Restoration of Ptolemy's Equinoxes

What Ptolemy's intention was concerning the measure of the tropical year, we have set out above. And unless conjecture deceives me utterly, he observed each

autumnal equinox at the very threshold of the horizon — provided the others are (as they doubtless are) among the number of those which he had taken twice in a single day with his instrument — but in both cases he had let them rest on the horizontals, because this served his purpose wonderfully well. And surely it is worth noting that in these observations Ptolemy accommodated himself so far to Hipparchus, and to the established doctrine both of the measure of the tropical year and of an unchanging eccentricity in the Sun, that for that very reason he assigned his observations not to the cardinal points of the days themselves, but pushed them a little beyond, to the first hour, and so on — so that you might say it was rather Ptolemy's epilogism [epilogismo Ptolemaico] than the heavens that was being satisfied here.

But lest certain astronomers, to whom I write this, take it ill that I take the liberty of inquiring into Ptolemy — persuaded as they are by his too-great authority — let them, I beg, turn back to what he himself relates elsewhere about the Moon's parallax as observed by him, [Margin: Lib. 5, cap. 13.] and let them carefully compare it with our restoration, which (so far as I know) agrees most exactly with the lunar course and distance, according to the celestial standard; and at length they will perceive that Ptolemy, in that passage (to pass over the others for this occasion), reported the Moon's parallax as half a degree, and more, beyond the true value derived from his own observation, for no other reason (as I judge) than that he was foisting upon posterity, as legitimate, the lunar hypothesis he had himself constructed beforehand — or rather had accepted from his predecessors and once approved by his own calculation. But that here he was of the same mind, and rested content with those equinoctial observations because they served his purpose, while utterly suppressing the rest (which it is very probable he made in great number) — what scruple, I ask, will there be in suspecting this?

But setting aside the earlier autumnal [observation] — which, although it is assigned to the second afternoon hour, I nonetheless do not doubt was likewise observed on the horizon, and afterward reduced to the appointed time (for nothing appears that would free the following one from being necessarily entangled with refractions) — we shall now take up the other [equinox] by which Ptolemy set up his comparison.

Now, since this equinox appears to have been detected by the armillae — or rather from the rising angle — with the Sun standing as nearly as possible on the very horizon, and afterward [reckoned] to the hour reduced from sunrise, when Ptolemy reports it to have occurred: [Margin: Vide 1 Progymn. T. B. pag. 144, de

refractione apud Alexandrinos.] therefore the Sun's refraction there had amounted, on the vertical circle, to about 32 minutes, which leave nearly the same amount in longitude on the Alexandrian horizon, and 1½ hours in time, by which [Margin: Li. 3, Almag. cap. 4.] the time of the observation [is shifted], on account of the entanglement of refraction. Accordingly, the second autumnal Ptolemaic equinox, freed of refraction, occurred at Alexandria in Egypt in the 463rd year after the death of Alexander the Great, in the month Athyr, on the 9th day, at 6 hours after midday; but if the next-following vernal equinox of Ptolemy be established as occurring at midday (when it is noted to have occurred scarcely one hour later than his opinion held), there is gathered between the two an interval of 178 days 18 hours, which exceeds the Hipparchan one compared above, 178 days 11 hours 25 minutes, by 6 hours 35 minutes of time. Ptolemy, however, taking no account of refraction, and on Hipparchus's authority and his own, supposed the winter interval of the equinoxes to be 178 days 6 hours, the summer interval 187 days 0 hours, and finally [the interval] from the vernal equinox to the summer solstice 94 days 12 hours.

But in truth, since it in no way befits us to neglect anything whatever that can anywhere be obtained by us toward the legitimate restoration of the Sun and the reconciliation of its conformity, I shall here transcribe in full the passage of Pliny [Pliny] concerning the interval of time in which, in the age before his own, the Sun annually entered the cardinal signs of the zodiac. And although it lacks the precise reckoning [Greek: πραγματεία (pragmateia, "treatise, precise treatment")] of the observation, and its moments, nevertheless, since Pliny addresses Caesar, I follow it by no doubtful conjecture: that the said intervals were established by the work of Sosigenes [Sosigenes], that most excellent mathematician, whom Julius Caesar [Julius Caesar] brought with him from Egypt to Rome, around the year 47 before the birth of Christ, for undertaking the establishment of his calendar — the one we use at present.

[Margin: Plinius lib. 18, cap. 25.] Pliny's passage is as follows: "The hinge of the seasons consists in the fourfold division of the year, through the increases of light. It increases from the winter solstice, and is equalized to the nights at the vernal equinox in 90 days 3 hours. Then daylight exceeds [the nights] up to the summer solstice (the summer one, that is, with respect to light) by 93 days 12 hours, as far as the autumnal equinox. And then, the day having been equalized, it proceeds from there to the winter solstice in 89 days 3 hours." From this Plinian epilogism are gathered the points which I now subjoin.

Diagram. A bracketed schematic (not a geometric figure) laying out Sosigenes's intervals between the cardinal points and the resulting equinoctial intervals. On the left, a brace gathers the four zodiacal-cardinal symbols with their intervals; on the right, after "Ergo" ("Therefore"), a second brace gives the derived equinox-to-equinox intervals.

- Left brace, headed **A**, listing (with zodiac sign-symbols at left):
- ♋ (Cancer): days 93 hours 12
- ♎ (Libra): days 92 hours 12
- ♏ (Capricorn): days 89 hours 3
- ♈ (Aries): days 90 hours 3
- **Ergo** ("Therefore"), right brace headed **A**:
- ♈ (Aries): days 179 hours 6
- ♎ (Libra): days 186 hours 0

And here we find Sosigenes's establishment of the interval of time between the two equinoxes to be far different — Sosigenes, who nevertheless flourished at nearly the midpoint of time between Hipparchus and Ptolemy — so much so that a discrepancy of a whole day intervenes between these unrestored observations of the ancients themselves. And although I am unwilling to affirm that the observations of Sosigenes (if indeed they are observations, and not rather, as some hold, imitations of Eudoxus [Eudoxus]) — which perhaps were made by the Sun's rays and the shadow of the gnomon, or rather acquired by the astronomical radius [radio Astronomico], whose use was current in Julius Caesar's age — deserve a certainty equal to that of the observations of Hipparchus and Ptolemy: nevertheless there is no doubt that the errors of Hipparchus and of his Egyptian predecessors concerning the Sun were in some way known and suspected by Sosigenes and Caesar himself. This was the reason why they used other intervals, which had emerged from their own observations, for the times of establishing their own calendar — intervals which were afterward sketched out by others, by Ovid [Ovid] especially in the *Fasti*, in the manner of the poets. But here they are asserted and precisely recounted.

Nor is it unusual that so great a difference of time should have arisen, when between N. Copernicus [Copernicus] and Tycho Brahe [Tycho Brahe] — a span of only 80 years — the discrepancy in the Sun's motion (with respect to its entry into the equinoctial and solstitial points) extends to $\frac{1}{2}$ of a whole natural day; the greatest fault of which is the refraction of the Sun omitted by Copernicus, as will be shown hereafter.

Furthermore, it can in no way disturb us that Pliny straightway subjoins in this manner: "The hours now, at every increment, are equinoctial, not [the hours] of each particular day, and all these differences occur in the eighth parts of the signs: the winter solstice in Capricorn, from about the 8th day before the Kalends of January, and so forth." For Pliny seems here to me to be speaking rather of the effects of these annual seasons, which for the most part set in within the eighth parts of the equinoctial and solstitial signs. Nor am I unaware that today some interpret that passage of Pliny as referring to the Sun's own motion; but it was doubtless introduced by the custom of the Greeks, who long ago, about eight days after the Sun's entry into the beginning of Cancer (that is, the summer solstice), made it famous by the Olympic games — to which perhaps Pliny alludes. But since here he makes mention of the winter solstice as falling about the eighth day before the Kalends of January, which corresponds to the 7th day of December — on which, beginning from midnight, the birthday of Julius Caesar (which we in the Church have happily exchanged for the birthday of Christ the Savior) was once celebrated at the very winter solstice — this cuts away the opinion of those who hold the eighth parts of the signs, and brings it back to the beginnings of the signs. And surely it could well be that this error crept into the transcription of Pliny (which is frequent among the copyists), when "parts of the signs" were substituted for "parts" — that is, the days — "of the Kalends in the months." However that may be, provided we are willing to follow the Plinian text, this will make little difference toward varying the determined measures of the intervals.

This discrepancy, then, which is adduced from Pliny — with Hipparchus and Ptolemy — both in the summer and winter measure of the Sun's turning between either equinox, and also between the vernal equinox and the summer solstice (which we have proved above could in no way be precisely observed from the heavens by Meton [Meton], Aristyllus [Aristyllus], Aristarchus [Aristarchus], Archimedes [Archimedes], Hipparchus, or even Ptolemy himself) — convinces [us] that the just limitation for the Sun's refraction, applied above by us, was correct, while we hold to the mean winter interval of the equinoxes of Hipparchus and Ptolemy, 178 days 6 hours, and of Sosigenes (for so it may now be called), 179 days 6 hours.

Since, then, from the observations — but chiefly Ptolemy's — restored above, the same thing has emerged, we shall accordingly retain it hereafter for the age of Hipparchus. These things thus dispatched, we must first pass on to the measure of the tropical year, to be drawn out and ascertained in some fashion from their own restored observations between Hipparchus and Ptolemy, before we treat of the Sun's eccentricity as it agrees with Hipparchus's age.

Now, since the third autumnal equinox of Hipparchus (as was noted above) fell in the current year 167 after the death of Alexander, at the midday of the first intercalary day, while Ptolemy's autumnal equinox — the Sun being freed of refraction — [fell] in the year of the same

[Margin: Lib. 3, cap. 2.] [Margin: Lib. 3, cap. 9.]

Alexander 463, in the Egyptian month Athyr, on the 4th day [recte: as printed "4"] at the 6th hour after midday — we found it in fact to have occurred. Hence between the two there are gathered 296 Egyptian years [and] 72 days, [and] 6 hours. And since, by the addition of a quarter [day] to the measure of the common year of 365 days, the surplus days are exactly 74 — which emerge in the quotient when the proposed number of years, 296, is divided by 4 — the difference, then, of 42 [hours], divided by 296 days, gives an annual anticipation from the quarter [day] of 8 minutes $30\frac{2}{3}$ seconds [recte: as printed "8 m. $30\frac{2}{3}$ "]. When you have subtracted this from the measure of the Julian year, $365\frac{1}{2}$ days, the remainder is the measure of the solar year between Hipparchus and Ptolemy: 365 days 5 hours 51 minutes $29\frac{2}{3}$ seconds.

But that from the comparison of the two vernal equinoxes of Hipparchus and Ptolemy a somewhat greater measure of the tropical year results, could come about from the excessive anticipation (discussed above) of Hipparchus's equinoxes; and also because, with the Sun's refraction still included in his first vernal equinox, which we are employing, the same [equinox] is drawn still further onward by several more hours. Indeed, having weighed each thing diligently, I find the magnitude of the solar year, for those three centuries between Hipparchus and Ptolemy, no greater [than this] — perhaps even somewhat smaller, as will be further confirmed below, God willing, by a more accurate examination and by the apparent progress of the fixed stars — although Ptolemy did not hesitate to extend it to 365 days 5 hours 55 minutes 12 seconds, chiefly, as it appears, for the reason indicated above. Nor indeed do I doubt that, if the same Ptolemy now survived, and not only had knowledge of the Sun's refraction but also had the year's measure constricted down to our own time, he would willingly have retracted his own [figure] — he who indeed added his own censure to us on this matter, when he says thus: "The truth that can be had from the whole perpetuity of time, or from a manifold time of observations, we think must be left to others." And elsewhere, concerning other matters, he speaks frankly in this manner: "We have changed some things not exactly perceived, since truer observations afterward came down to us; for it befits all who, impelled by the love of truth, undertake these speculations, to use not only a

surer and newer way toward the correction of the ancients, but also toward [the correction] of themselves; nor to think it shameful to be recalled to greater exactness, not only by themselves, but also by others — especially since this is a great and divine profession."

So much for the annual measure as it presents itself from the restored observations between Hipparchus and Ptolemy. The other [topic], concerning the Sun's eccentricity, follows from the observations of the same [astronomers], where, the winter interval of the equinoxes having now been found, 178 days 18 hours, and this being subtracted from the annual measure similarly found, 365 days 5 hours [51 minutes], the summer [interval] results: 186 days 11 hours 51½ minutes.

It seems most advisable to us to adhere to the Hipparchan age, and indeed to the time of that third autumnal equinox of his, which we have thus far brought under examination; then, the summer interval of the equinoxes being now given, and the apogee place of the Sun being likewise retained at 6° of Gemini [□], approximately, as found by Hipparchus — first to elicit the Sun's eccentricity; then to draw out the interval of time between the vernal equinox and the summer solstice from the Sun's true motion in Hipparchus's time, and to take pains to compare it with the Plinian [figure] for a surer limitation.

Now the data of this [problem] are as follows: [Greek: πραγματείας διδόμενα (pragmateias didomena, "the data of the problem")]

I. The summer interval of the equinoxes is 186 days 11 hours 51½ minutes. To this corresponds the simple [mean] motion of the Sun, 183° 49' 12".

II. The displacement of the Sun's apogee from the summer solstice, in antecedence [westward], in Hipparchus's time, is 24 degrees [recte: as printed "24"].

These things thus supposed, the investigation of the Sun's eccentricity for the Hipparchan age will be made in the following way.

Diagram. A geometric construction of the Sun's circular orbit for determining its eccentricity. A circle is drawn about center **M**, representing the Sun's orbit, through the points A, D, F, H. Two diameters BF and DH are drawn (crossing near center M), with a line AG drawn parallel to BF. The equinoxes are placed on the figure — the vernal at A (right), the autumnal at G (left) — and a line EI is drawn parallel [to DH]. Several interior points (I, K, M, N) lie near the center where the

diameters and parallels cross. Zodiac sign-symbols mark the cardinal points around the circumference.

- A (right side, marked ♈ Aries): vernal equinox
- B (right, above A): endpoint of diameter BF
- C (upper right): point on the circle
- D (top, beside E): endpoint of diameter DH
- E (top left of D, marked near top): point with parallel line EI
- F (left, marked): endpoint of diameter BF
- G (left, marked ♎ Libra): autumnal equinox
- H (bottom, marked ♐ Capricorn): endpoint of diameter DH
- I (interior, near center): intersection point
- K (interior, lower-center): intersection point
- M (center): center of the Sun's orbit
- N (interior, near center): intersection point
- (top, beside E, marked ♋ Cancer): solstice marker
- (bottom, marked ♑ Capricorn): solstice marker

The text describing this construction continues into the next batch.

parallel [to DH]

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parallel [to the diameter] DH, the tropic of Cancer will appear at E, and that of Capricorn at I; and from the Earth at K, the line KMC drawn through the center M to the apogee at C will likewise be the Sun's eccentricity KM, which here we obtain geometrically as follows.

Given that the Sun completes its whole proposed orbit in the annual interval found to be 365 days 5 hours 51½ minutes, and that the interval from the autumnal equinox to the vernal — found earlier — is GA = 178 days 18 hours, the remaining summer interval AG is 189 days 11 hours 51½ minutes. To this corresponds a mean solar motion of 183° 49' 12". Subtracting from this the semicircle FB of 180°, there remain the combined arcs BA and GF totaling 3° 49'

12", of whose halves namely AB is 1° 54' 36". The sine of this [AB], MN or KL, is 333296 — in the units where the radius of the Sun's orbit is taken as 10000000.

Therefore, in the plane right triangle MLK, since KL = 333296 is given, and likewise the angle MKL = 24° 0' — which measures the distance of the Sun's apogee at C from the apparent summer solstice at E in Hipparchus's time, and to which the angle CMD, or the arc CD, is equal — and since the right angle MLK is also given, the angle LMK is therefore also given as 66°.

[Margin: Prop. 28 lib. 1 Euclid.]

With these granted, for KM we have:

	M	K	L	K M
	66°	KM	L	
as	S.R. 9135455	333296	S.T. 10000000	(364837

Thus there emerges, from the restored ancient observations of Hipparchus and Ptolemy, a solar eccentricity of 364837 — in the units where the radius of the Sun's orbit MB is 10000000 — to which corresponds a maximum solar prosthaphaeresis [prosthaphaeresis maxima solis] of 2° 5' 26". But Hipparchus and Ptolemy, with the refraction included in their observations, obtained the Sun's eccentricity as 417000, to which corresponds an equation of 2° 23'.

Furthermore, in order to gather the whole arc AE, and thence to reason out the interval of time which the Sun's apparent motion took to pass from the vernal equinox to the summer solstice in Hipparchus's time, we must seek in the same triangle MLK the side ML, which is the sine of the arc DE, in this manner:

	M K	K 24°	(S.R. LM	arc DE
as	S.T. 10000000	364837	S.R. 4067366	(148590 → Resp. 51 min.

This arc DE = 51 minutes. If it is added to the first quadrant DB of 90°, and thereafter to BA of 1° 54' 36", the sum collected is 92° 45' 36" — namely the arc AE, which we perceive from its mean course that the Sun completes from the apparent equinox A to the summer solstice E in 94 days 2½ hours. This interval Hipparchus and Ptolemy set at 94 days 12 hours, but Pliny [Plinius] at 93 days 12 hours, as appears from what was said above. We here therefore keep almost to the mean of these, except that we come somewhat nearer to Hipparchus (as is

more probable). Now, having dispatched Hipparchus and Ptolemy as best we could in restoring the truth, let us in order descend to al-Battānī of Raqqa [Albategnius Aratensis].

Examination of al-Battānī's Observations on the Sun. [p. 177]

In the year 882 from the birth of Christ the Savior, Mohammed al-Battānī [Mohamedes Albategnius] reported that he had observed the autumnal equinox at Raqqa in Syria [Aracta in Syria], falling at the beginning of the 19th day of the month Elul, or September of the Romans, $4\frac{3}{4}$ hours before sunrise — that is, $1\frac{1}{2}$ hours after midnight — which was on the completed seventh day of the month Pachon of the Egyptians.

[Margin: N. Copernicus lib. 3 revolut. ca. 13.]

Although I do not sufficiently grasp by what means this equinox was acquired by al-Battānī, yet on account of its excessive earliness I strongly suspect that, on the two nearest meridians that stood about it, having marked the sun's rays with a mural gnomon and established the proportion of the shadow, al-Battānī directed this equinox of his to the time prescribed for us. Similarly too for the vernal [equinox], even though al-Battānī observed many things there by the gnomon — in the use of which, for marking the sun, he was great in that age and among his people — or whatever it is, I hold it for certain that al-Battānī erred by more than half a day in the designation of his equinox, as the fixed stars observed by him bear witness in agreement with us as to their time.

[Margin: Vide Gennam Frisium c. 24. de radio Astron. & de errore in gnomone.]

[Margin: Item lib. 2. Sphæric. nost. probl. 1 c. 5 in fine.]

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This was also the reason why Copernicus [Copernico], having entirely set aside this al-Battānīan equinox, appealed to the fixed stars — since he felt that the measure of the year between Ptolemy and al-Battānī had been too greatly curtailed.

[Margin: Lib. 3 Revol. cap. 13.]

But since the Ptolemaic equinox, once restored by us, yields itself a little more relaxed in the said measure of the year, accordingly — briefly treating here of this same comparison — for firmer certainty we shall afterward, with Copernicus, take refuge in the fixed stars. For I judge it has come about by

singular divine providence that, when in the Sun's course we can only with greatest difficulty detect [the truth] through the Sun itself, the fixed stars come to our aid in a certain manner, and by a reciprocal δοκιμασία [Greek: dokimasia, "testing" or "scrutiny"] approve [it] by due comparison and just examination.

But now, that we may come to the comparison undertaken, we gather the interval of time between the autumnal equinox of Ptolemy as restored — which falls on the 8th day of the month Athyr in the year 139 from the birth of Christ, at the sixth hour after noon — and this al-Battānīan one, reduced to the Alexandrian meridian by a subtraction of 40 minutes, [as] 743 equal years, 179 days, 6¾ hours. But dividing the proposed number of years, 743, by 4, there remain over and above the equal years 185¾ days. The difference therefore is 6 days 11¼ hours; and dividing these by the proposed number of years, the measure of the year below the quadrant of a day, between Ptolemy and al-Battānī, is shown to be 12 minutes 32 seconds — and so the annual measure itself is 365 days 5 hours 47 minutes 28 seconds. This al-Battānī himself elicited as 365 days 5 hours 46 minutes 24 seconds, which we shall prove below to be truly greater by more than a minute and a third.

Nor are we moved by the fact that Joseph Scaliger [Iosephus Scaliger] reports the vernal Jalālī equinox [æquinocitium vernum Gelalæum] in Mesopotamia, 179½ years after this al-Battānīan observed one, as having fallen on the 14th of March, about 2¼ hours after noon.

[Margin: Scaliger de Emend. tempor. p. 285.]

For we do not find that observation, as he wishes, taken from the heavens, but rather precisely constituted from the annual quantity which al-Battānī finds to intervene between himself and Ptolemy.

Furthermore, as for what al-Battānī, in imitation of Ptolemy, had found geometrically from his own observations concerning the eccentricity and the place of the Sun's apogee — from a supposed summer sojourn of the Sun, between the equinoxes, of 186 days 14¼ hours, but between the vernal equinox and the summer solstice of 93 days 14 hours — we find that these, from these data together with the mean motion of the Sun congruent with them, behave in the same way: namely that the Sun's eccentricity KM comes out as 347, in the units where the radius of the Sun's orbit is 10000, and therefore the maximum solar prosthaphaeresis scarcely two degrees; but that the Sun's apogee has been so far advanced in consequence [eastward] since the time of Hipparchus that, whereas before it had kept an interval of 24¼° from the summer tropic in

antecedence [westward], it would now be distant only $7^{\circ} 44'$ from the same [tropic] in the same direction. This certainly differs little from a uniform advance of the place of the Sun's apogee, in proportion to the time from Hipparchus down to our own age, as will be shown later. So with al-Zarqālī the Spaniard [Arzachelis Hispani] — who later, after he had detected the same eccentricity as al-Battānī, observed the Sun's apogee elongated $12\frac{1}{4}^{\circ}$ from the same solstice in antecedence [westward] — we are in this matter not at all detained, since we relegate it to the slippery and, as we have proved before, unobservable incidence of the summer solstice. We add only this much concerning al-Battānī's eccentricity: that it would be a little greater still if the consideration of parallax in the Sun were added, which it is probable that al-Battānī (and al-Zarqālī likewise) omitted.

And here too it is to be observed in passing, regarding al-Battānī, that on the change of eccentricity in the Sun (which they established in vain) the alteration of the Sun's obliquity in no way depends, or vice versa; for otherwise this too would have been least in this age, and not, as he observed, $23^{\circ} 35'$. But, dismissing al-Battānī, we shall apply the file to the following ages and astronomers.

A Brief Recapitulation of the Observations of Bernhard Walther of Nuremberg [Bernhardus Gualtherus Norinbergensis] on the Sun, after Tycho Brahe. [p. 178]

From those solar observations which Bernhard Walther, a citizen of Nuremberg, once left committed to print with the greatest diligence then possible, our most noble Tycho Brahe [Tycho Brahe] singled out the principal ones, and directed them to the Sun's course in the preceding age more correctly than it could be learned from the Copernican [observations]; and he subjected them to calculation and practice, both for investigating the eccentricity and the time of the apogee, and for comparing them with his own as nearly one [set].

[Margin: Libr. 1. progym. cap. 1.]

Now Tycho first used those observations of Walther on the Sun which, in the year from the birth of Christ

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1488, had been received by the so-called Ptolemaic rules; and diligently comparing these with others of the same Walther, he found the interval of time from the Sun's entry into the vernal equinox up to the next autumnal one to be 186 days $17\frac{1}{2}$ hours. But from the beginning of Aries to the middle of Taurus, the completed days were 46 [days] $3\frac{2}{3}$ hours. Moreover, from the middle of Leo up to the autumnal equinox, or the beginning of Libra, 46 days $8\frac{1}{3}$ hours.

[Margin: Lib. 1 Prog. c. 1 p. 20. &c.]

From the Sun's mean motion congruent with these intervals, Tycho — in the restoration of the Sun, to be shown through πραγματεία [Greek: pragmateia, "treatment" or "procedure"] — by a twofold investigation detected the Sun's eccentricity as 3548, in the units where the radius of the orbit is 100000; to which a maximum prosthaphaeresis of $2^{\circ} 2'$ also corresponds. Likewise the place of the Sun's apogee in $4\frac{1}{2}^{\circ}$ of Cancer. And indeed, from other observations of Walther diligently compared, he also found that the vernal equinox in that same year 1488 fell on the 16th day of March, 15 hours 40 minutes — as all these things have been diligently elaborated and dispatched in Tycho's first astronomical exercise.

Examination, Correction, and Restitution of the Observations of Nicolaus Copernicus on the Sun. [p. 179]

N. Copernicus, at Frauenburg in Prussia [Fruenburgi in Borussia], intending to investigate the Sun's eccentricity and apogee from his own observations, in the year 1515 from the birth of Christ the Savior, said he had found the interval of time between the autumnal and vernal equinox to be 178 days $21\frac{1}{3}$ hours. Furthermore, between the same autumnal equinox and the midpoint — that is, the 15th degree of Scorpio — 45 days $6\frac{2}{5}$ hours.

[Margin: Lib. 3 Rev. c. 16.]

Hence, applying the Sun's mean motions, Copernicus found its eccentricity to be only 323, in the units where the radius of the Sun's orbit is 10000; to which a maximum solar prosthaphaeresis of $1^{\circ} 51'$ corresponds.

But since, in the small time which lay between Copernicus and Tycho Brahe — namely 70 or more years — Tycho, from his most firm observations, detected all these things changed remarkably and beyond the perennial laws of the stars, and therefore suspected that some difficulties had clung to the Copernican observations: accordingly, in the year 1584 from the birth of Christ, having sent into Prussia, with a sextant, one of his most industrious students, Elias Olai of Morsø [Helias Olai Morsianus] (who six years later died at Uraniborg in my embrace), he found the elevation of the Frauenburg pole $2\frac{1}{2}$ minutes higher than Copernicus had once supposed it — namely $54^{\circ} 22\frac{1}{2}'$. Whence first the winter intervals of the equinoxes, established earlier by Copernicus as too long, decreased by $\frac{5}{8}$ hour. Then also, from the omission of the Sun's parallax about both equinoxes (the reckoning of which, however, Tycho affirms Copernicus did make) — about 2 minutes, or at the least, with refraction excluded, $1\frac{1}{2}$ minutes;

whence likewise $3\frac{1}{3}$ hours accrue toward shortening the winter interval of the equinoxes.

Finally, by diligently considering the altitude of Spica Virginis [spica Virginis], observed at Frauenburg by Copernicus in that same year, and repeated several times (by the same instrument, without doubt, by which the solar observations were made) — and comparing this with the true ones, that is, with Tycho's observations, the apparent place of the stars being meanwhile altered — I find that a defect of more than 1 minute occurred in it, which likewise produces two hours; so much so that I do not fear to set, as nearly as possible, the collected prescribed errors at ἐν πλάτει [Greek: en platei, "in breadth," i.e. in total / latitude] 11 hours, by which, as I said, Copernicus had prolonged the winter interval of the equinoxes more than was just. Those [11 hours], therefore, being taken from the 178 days $21\frac{1}{3}$ hours, there remain 178 days $10\frac{2}{3}$ hours, the said restored interval of the equinoxes.

Furthermore, as concerns the interval between the autumnal equinox and the middle of Scorpio: since at the Frauenburg meridian altitude — besides the parallax determined at about 3 minutes — there still remain 2 minutes for refraction, these also in the Sun's longitude there give $6\frac{1}{2}$ minutes, and a corresponding time of 2 hours 42 minutes. But what might here accrue from a greater addition concerning the Sun's longitude, and what [might accrue] in the exhibition of a less-than-just altitude through the Copernican instrument, I shall not now touch upon, since these mutually cancel each other out here as nearly as possible.

From this Copernican interval, then, between the beginning of Libra and the middle of Scorpio, as

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is fitting, when these things are subtracted, there remains that one corrected also at 45 days 3 hours 42 minutes. If therefore we now adjoin to each of the corrected intervals the congruent mean motion of the Sun, there will be, for the first — namely 178 days $10\frac{2}{3}$ hours — a mean or equal motion of the Sun of $175^{\circ} 52\frac{1}{2}'$; and for the second — 45 days 3 minutes 42 [seconds] — an equal motion of the Sun of $44^{\circ} 30' 22''$. [recte: 45 days 3 hours 42 minutes — the printed "3 min. 42" for the second interval drops the "hours," cf. the value used in the diagram below]

With these presupposed, we thus renew the Copernican calculation for the eccentricity and the place of the Sun's apogee.

Diagram. A circle representing the Sun's orbit, with center F, carrying the points A, B, C around its circumference. The vernal equinox is at A (right side), the autumnal equinox at C (left side, marked with the Libra/zodiac symbol), the summer position B at the top, and the midpoint of Scorpio D at the bottom. Lines AC and DI are drawn as chords intersecting at G, the center of the Earth, slightly below and left of the orbit center F. The chord AD subtends the relevant arc. The point I lies on the upper-right circumference; H sits near the center between F and G; the apogee line BEGF and perigee run through the figure vertically (B at top, E and F near center, perigee at bottom). The eccentricity FG is the offset of the Earth's center G from the orbit center F.

- F (center of circle): orbit center [Sun's mean orbit center]
- A (right): vernal equinox [vernum æquinoctium]
- C (left, with Libra symbol): autumnal equinox [autumnale]
- B (top): summer position [the top of the apogee line]
- D (bottom): middle of Scorpio [medium Scorp̃ii]
- G (just below/left of F): center of the Earth [centrum terræ]
- I (upper right circumference): endpoint of chord DI
- H (near center): foot of the perpendicular FH on the chord DI
- E (near center, on apogee line): point on the apogee/perigee line BEGF

The text continues:

Let the Sun's orbit ABC be described with center F; let A be the vernal equinox, C the autumnal, D the middle of Scorpio. Then let AC and DI be joined, cutting each other at G, the center of the Earth; and let AD be subtended. Since therefore the circumference DC = $44^{\circ} 30' 22''$ is known from the Sun's mean motion, and therefore the angle subtended at CAD — namely half the arc CD — is $22^{\circ} 15' 11''$. Likewise the angle subtended at CGD, the Sun's apparent motion, is $45^{\circ} 0'$, which subtracted from two right angles, or 180° , leaves the obtuse angle AGD = $135^{\circ} 0'$. The third angle in the triangle AGD, namely ADG, is known from the complement of the two given [angles] A and G to the semicircle: $22^{\circ} 44' 49''$, whose double — namely $45^{\circ} 29' 38''$ — is the arc AI.

Furthermore, since the arc CDA is given as $175^{\circ} 52\frac{1}{2}'$, therefore, subtracting from the same the arc CD of $44^{\circ} 30' 22''$, there is left the arc DA = $131^{\circ} 22' 8''$. Of this the chord AD is given as 18225828, in the units where the radius of the proposed circle is 10000000. But since the arc AI is added to the arc DA, the whole arc DI is thence composed as $176^{\circ} 51' 46''$, whose chord DI from the table of sines is likewise 19992506.

But since each segment ADC and DAI falls short of the semicircle, the Sun's apogee must therefore be sought in the other part of the circle, namely IBC. So let the line of the apogee and perigee, BEGF, be the Sun's eccentricity itself; and the perpendicular FH, drawn, divides the chord DI on both sides at right angles and into equal parts. Wherefore, in the right triangle FHG, that the *πραγματεία* [Greek: *pragmateia*, "procedure"] may be had sufficiently, first — in the triangle AGD, with all the angles given together with the side AD — GD is given as 9763585. Subtracting this from half of DI, that is from 9996253, there remains HG = 232668. Then we obtain the perpendicular thus: the defect of the circumference DAI from the semicircle is $3^{\circ} 8' 14''$, of whose half — $1^{\circ} 34' 7''$ — the sine 273740 is the side HF.

Hence, since in the said right-angled triangle FHG two sides about the right angle are given — namely HG = 232668 and FH = 273740 — there is found first the angle FGH = $49^{\circ} 38' 12''$, which together with the arc AC, $45^{\circ} 29' 38''$, constitutes, from the corrected Copernican observations, the place of the Sun's apogee at 5° [?] of Cancer. Then, in the same triangle FGH, the side FG is given — namely the Sun's eccentricity itself from the same corrected observations — as 359261, in the units where the radius of the orbit is 10000000; to which corresponds a maximum solar prosthaphaeresis of $2^{\circ} 3'$ [?]. So much concerning the eccentricity and apogee of the Sun from the Copernican observations restored.

Furthermore, the obliquity of the ecliptic in Copernicus's time is explored and corrected by a twofold proof. First, if to the $23^{\circ} 28'$ — the obliquity of the ecliptic which Copernicus says he found — you add $2\frac{1}{2}$ minutes, insofar as the elevation of the Frauenburg pole was found less than just; secondly, if the difference of the Sun's parallax at the maximum and minimum meridian altitude at Frauenburg — namely $1\frac{1}{2}$ minutes — be subtracted from the Sun's refraction at the minimum meridian altitude, say $8\frac{1}{2}$ minutes, and the difference of 7 minutes be halved, and the half, which is $3' 30''$, be added to $23^{\circ} 28'$: whence there [in the first proof] the maximum obliquity is made $23^{\circ} 30\frac{1}{4}'$, but here [in the second]

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the pole was found to be $31\frac{1}{4}'$. The mean of these, then, is $13^{\circ} 31' 8''$ — and I shall later confirm, by the same reasoning used in fitting the hypothesis, that this was

very nearly the minimum at Copernicus's time. And so here too I have faithfully restored Copernicus. What now remains is Tycho Brahe [Tycho Brahe], who in the certainty of his observations had surpassed all others, and who is therefore to be treated last.

A Summary and Critical Examination [δοκιμασία (dokimasia, "scrutiny," critical examination)] of the Observations of the Sun, and of the Restitution of Its Course, Carried Out with the Utmost Diligence by Tycho Brahe on Hven, the Island of Our Denmark

Now that the observations of the ancients have been restored — observations which Copernicus had earlier used to ill effect, in order to fix in men's minds his opinion concerning a variable change in the eccentricity and in the place of the Sun's apogee — the observations of our most noble Tycho follow in the last place. Against these, as against a touchstone [Lydian stone], the whole question of the Sun's course must be tested: first as regards the eccentricity and the apogee, and then, by a fair comparison with the ancients, as regards the annual quantity in this perpetual restitution of the Sun and stars (may God assist it!). For it is fair to abide by those observations in which we know that nothing among the difficulties recounted above stands in the way of the utmost precision. Indeed, on this very account, those who thoroughly understand astronomical matters will join us in acknowledging Tycho Brahe as the foremost astronomer of every age — and I trust they will not be displeased that I have, up to now, devoted my labor to his memory, as I faithfully served him while he lived.

But before we take up the most exact observations of Tycho and our own, made on Hven and elsewhere, it has seemed good to me to set out the reason why the equation of the natural day [aequatio diei naturalis] — by which the Sun is thought to arise from its unequal course — should be entirely eliminated. Otherwise, the matters we must hereafter treat, especially those concerning the Moon's course, would be further hindered by suspicion of this equation.

The Reasons or Causes of the Equation of the Natural Day, and Why That Which Arises from the Sun's Unequal Motion Should Be Abolished

The equation of the natural day, if there were any, would depend upon the course and disposition of the Sun. I shall therefore set out briefly, in this place,

my reasons for thinking that it should for the most part be rejected, so that hereafter, in the restoration of the planetary motions — and above all of the lunar — we may proceed freely and without this scruple.

It has indeed been accepted hitherto by the ancients that time — whose index is the Sun in its diurnal revolution, while the equator is its regulator (as we set forth in the doctrine of the sphere) — reckoned from noon at the same place, has a twofold cause of its variation: namely, the inequality of the arcs of the ecliptic compared with those of the equator, and also the Sun's diurnal or proper and unequal motion. [Margin: Book 1 of the *Sphere*, ch. 5, and Book 2. *Sphaera* 5, ch. 5.]

These two causes, rightly considered, and the two indices proceeding from a single center carefully examined — such that the one perpetually follows the Sun's equal diurnal motion in the equator, to be added to the first conversion from the vernal equinox, while the other follows the Sun's true motion, that is, the oblique ecliptic, likewise to be added to the Sun's elongation from the true equinox, similarly reckoned toward the equator — these display that manifest variation of time, accumulating and diminishing successively over many days. So much so that L. Christmann of Heidelberg [Lambert Christmann] tries in vain to persuade us that, on account of the Sun's parallel diurnal revolutions, and without the other causes for which the equation of time is rightly suspended, no perceptible difference intervenes between mean and apparent time. [Margin: Christmann, *On the Theory of the Moon*, our reasoning repeated and defended.]

But a different commentary, unlike Christmann's, was once devised by Vitichius [Paul Wittich], a mathematician of Breslau, while he was staying with Tycho on Hven in the year 1582, for doing away with the equation of time — namely, that a restitution of the mean from both causes should be made for each diurnal revolution. Yet this too, in view of our demonstration established above by means of those twin indices, fails, so that no help can be found in it either for omitting the equation of time. And yet the lunar motions, however they are to be restored, are by all means

found to recede from that part by which the inequality of the Sun's motion in the ecliptic is urged. Nor is it any wonder that someone should think we ought to have included — or even could have included — this variation of the Moon, which the celestial appearances insinuate against the ancient (that is, the legitimate, as they would have it) equation of time, within the lunar hypothesis, or the tables of mean motions, or even the prosthaphaereses [προσθαφαίρεσεων (prosthaphaireseōn, "prosthaphaereses," the equation-corrections)] of the Moon.

For this attempt, if nothing else, is at any rate blunted by the all-too-rapid change of the lunar apogee, which traverses the whole Zodiac within nine years.

[Margin: *Progymnasmata*, Book 1, p. 113, etc.]

But since the true designation of time does not equally depend upon any other heavenly body than the Sun, it was always an anxious care to me — both before, and especially since I was repeatedly admonished by letters from the most celebrated astronomers concerning this matter — to seek out more solid causes of equating time than those left by Lord Tycho in the appendix on the Moon's motion. And I can rightly affirm that in my whole astronomical study I never labored under greater difficulty than here. For although I find several reasons why the Ptolemaic equation of time, which all the other astronomers have followed, is not rightly accommodated — both because it is derived from various epochs, which surely offends against the constant reciprocity of time with motion, and because that part which depends on the Sun's unequal diurnal motion recognizes no other beginning in the ecliptic than the one where the line of the apogee is established — nevertheless, any such procedure of mine availed little toward saving the lunar phenomena, so long as I added effort toward equating time on this account. [Margin: See the consequence in the *Definitions of Astronomy*, Book 1 of the *Sphere*, ch. 1.]

And so in the final restitution of the Moon's course, which was carried out in Bohemia in the year 1600, we abandoned almost without reason one of the two causes that was derived from the Sun's unequal motion — except that here too we judged the Moon to conform itself in its own course to the Sun's true course. The other cause we conceded from the necessity of demonstrating the obliquity of the ecliptic, and we retain it even now, since I observe that this very thing is somehow demanded by the lunar appearances, both in eclipses and outside them. [Margin: See this equation in the *Progymnasmata*, Book 1 of Tycho, in the appendix on the Moon's motion.]

As for the other matter, to one investigating its cause diligently and seriously, such a one offers itself as can be directly opposed to the equation of time arising from the Sun's unequal course. For this is most conveniently removed if the diurnal conversion is adjusted to the Sun's approach to and recession from the Earth — so that, when the Sun is nearer to us, this conversion becomes swifter, but slower when the Sun is farther off. In which matter a great argument concerning the Earth's [revolution] about the center of its rotation [φυσικωτέρας (physikōteras, "more physical," i.e. a more physical argument)] has been left.

Therefore, setting aside the inequality which otherwise seems to be deduced from the apparent motion of the Sun, we shall pursue below only that inequality which arises from the obliquity of the arcs of the ecliptic compared with those of the equator, and which stands separately in the appendix on the Moon in Tycho — provided only that the lunar phenomena, in eclipses and outside them, permit it in any way. But if anyone should wish to take into consideration some other tiny little equation of time [aequatiuncula], which the unequal precession of the equinoxes seems able to bring about — never, however, extending to 4 minutes when complicated on both sides — that shall be permitted him so far as I am concerned, for I willingly and knowingly pass over here the extreme scrupulosity [of others].

A Continuation of the Undertaking, in Examining the Sun's Course from Recent Observations

Now that an admonition concerning the true equation of time has been interposed — the proofs of which the lunar course will chiefly display hereafter — we shall now press on further with solar matters in the order set out, first of all by setting forth the observations of the Sun collected by Tycho Brahe at its principal points, especially from the year of Christ the Redeemer 1588. At that time, by means of the eccentricity discovered from his own and manifold observations — 3584 parts, that is, a prosthaphaeresis of $2^{\circ} 3\frac{1}{4}'$ — the apogee in $5^{\circ} 3'$ of Cancer, and finally the vernal equinox on the 9th of March at hour $20\frac{3}{4}$ of the same year, the restitution of the solar course, carried forward to the four nearest centuries, was completed and published in type at Tycho's Uraniborg. [Margin: See Book 1 of the *Progymnasmata*, ch. 1.]

All these observations we shall presently set out below in a brief synopsis, together with their comparison and critical examination [δοκιμασία (dokimasia, critical scrutiny)], so that from such great harmony as appears in the Tychonic [values] — compared with the interpolated and gaping dissonance of the ancients — posterity may take a fair occasion both to acknowledge Lord Tycho's singular industry here (and everywhere) with a grateful mind, and to recognize that we, in correcting the rest while we labor at a perpetual restitution of the celestial motions, have rashly presumed nothing and touched upon nothing without cause. Now the meridian altitudes

of the Sun now follow from Tycho, taken from the heavens by two — and sometimes even three — most excellent and most perfectly accurate quadrants, and moreover by a very large equatorial circle of declination. These instruments

were so calibrated against one another that (except when the air was perhaps at times denser) they excluded every perceptible error. But for examining the Sun's course for individual days, I used Tycho's table of declination, constructed for an obliquity of the ecliptic of $23^{\circ} 31\frac{1}{2}'$, and supposing also an elevation of the pole at Uraniborg of $55^{\circ} 54\frac{1}{4}'$. For I deliberately diminished it here by $\frac{1}{4}'$, both because I scarcely ever found it higher there, and in order to slightly reduce the solar parallaxes (for which, as for the refractions, I likewise used Tycho's tables — the reason for which I shall render later in the lunar eclipses); and finally, [as I should also do] if some refraction adhered to the Sun in its meridian altitude around the middle of Taurus and Leo. As a paradigm of the examination undertaken, however, we set down here one example, to which all the rest were carried out. In the Year of Christ 1583, the 14th of March.

Worked example. A two-part computation table, headed "Vraniburgi" (At Uraniborg), reducing an observed meridian altitude of the Sun to a true solar longitude and comparing it with the Tychonic restored tables. The left block derives the Sun's true altitude; the right block derives the true declination and longitude. Both blocks are in the columns G. M. S. (degrees, minutes, seconds).

Left block:

Quantity	G	M	S
Observed meridian altitude of the Sun [Altitudo Solis meridiana]	35	22	30
Parallax to be added [Parallaxis addenda]	0	2	26
Refraction of the Sun to be subtracted [Refractio Solis subtrahenda]	0	0	33
Difference to be added to the	0	1	53

Quantity	G	M	S
altitude [Differentia addenda altitudini]			
Therefore the Sun's true altitude [Ergo altitudo vera Solis]	35	24	23

Right block:

Quantity	G	M	S
Exaltation of the equator, to be subtracted [Exaltatio aequatoris subt.]	34	5	30
Therefore the Sun's true northern declination [Ergo declin. vera Solis Borea]	1	18	53
To this corresponds, from the table of declination, the place of the Sun [Huic resp. ex ta. decl. loc. Sol.]	□ 3	17½	—
Place of the Sun from Tycho Brahe's	□ 3	18	⅓

Quantity	G	M	S
restored table			
[Loc. Sol. ex			
tab. restit. T.			
B.]			
Difference	0	0	$\frac{1}{2}$
[Differential]			

Parsing notes: The right block's lower three rows carry the zodiac sign ♈ (Aries) as a column marker. The "Huic resp." value reads "3 17½" with the symbol ♈ at the right edge. The final "Differential" fractional second is degraded; printed as a small fraction (read as $\frac{1}{2}$, possibly $\frac{2}{3}$ — flagged).

Such, in all that follows, is the progress of the calculation. For in this comparison we did not think the minute-seconds should be expressed except by [fractional] parts of the others [i.e. of the minutes], since these can scarcely even be had from the observations. [Margin: See Book 1.] And truly the excessive scrupulosity of certain persons in these matters is to be despised — those who, by precisely insisting on those tiny seconds, sometimes fall away from the truth by whole degrees. They would certainly do well, along with us, to learn at some point from the celestial observations themselves, as the best informants, [to abandon] this fruitless effort of theirs. [Margin: *Progymnasmata* of Tycho Brahe, p. 28.]

The Meridian Altitudes of the Sun Acquired at Uraniborg, from Which — Account Being Taken of the Parallax and Refraction — the True Places of the Sun Are Derived and Compared with the Restored Tables of T. B.

I. Around the Vernal Equinox.

Num.	Year	Month	Day	Merid. alt. of Sun (G M S)	Obser ved place of Sun (G M S)	Place of Sun from table (G M S)	Differ ence (M S)
1	1583	March	14	35 22 30	♈ 3 17 40	♈ 3 18 24	$\frac{3}{4}$ +
2	1584	March	11	34 29	♈ 1 5	♈ 1 5	—

Num.	Year	Month	Day	Merid. alt. of Sun (G M S)	Observed place of Sun (G M S)	Place of Sun from table (G M S)	Differ ence (M S)
				55	35	33	
3	1585	March	13	35 10 45	☾ 2 48 15	☾ 2 49 45	1½ –
4	1586	March	11	34 18 30	☾ 0 37 0	☾ 0 36 50	⅙ –
5	1587	March	12	34 36 15	☾ 1 21 30	☾ 1 21 40	⅙ +
6	1588	March	12	34 53 30	☾ 2 5 0	☾ 2 6 40	1⅔ +
7	1589	March	11	34 24 55	☾ 0 53 5	☾ 0 53 2	⅙ +
8	1590	March	11	34 18 35	☾ 0 37 10	☾ 0 37 40	½ +
9	1593	March	10	34 1 30	☾ 29 54 43	☾ 29 55 33	⅝ –
10	1594	March	10	33 56 20	☾ 29 38 45	☾ 29 41 15	2½ +
11	1596	March	11	34 30 45	☾ 1 7 45	☾ 1 12 [?]	⅓ +
12	1597	March	10	34 2 20	☾ 29 56 20	☾ 29 57 20	1 +

Parsing notes: Rows 9–12 straddle the Pisces/Aries boundary, with the zodiac symbols ☾ (Pisces) and ☿ (Aries) inset as column markers in the source. Row 11's observed minutes are obscured by an inset ☾ symbol overprinting the value (read tentatively as "1 7 45"? — flagged); the table-column value shows "1 12" with a sign overlay. Row 12's observed minutes value is degraded (read as "29 56 20"? — flagged). The rightmost "Difference" column carries small fractional minute-values with + (add) / – (subtract) signs as printed.

II. Around the Autumnal Equinox.

Num.	Year	Month	Day	Merid. alt. of Sun (G M S)	Observed place of Sun (G M S)	Place of Sun from table (G M S)	Difference (M S)
1	1583	September	13	34 13 20	♍ 29 36 0	♍ 29 35 10	$\frac{1}{6}$ -
2	1584	September	12	34 18 55	♍ 29 21 57	♍ 29 20 52	$1\frac{1}{4}$ -
3	1585	September	10	35 11 40	♍ 27 9 20	♍ 27 9 0	$\frac{1}{3}$ -
4	1586	September	13	34 7 0	♍ 29 51 45	♍ 29 51 25	$\frac{1}{3}$ -
5	1587	September	12	34 37 0	♍ 28 36 30	♍ 28 38 5	$1\frac{3}{5}$ +
6	1588	September	12	34 18 20	♍ 29 23 30	♍ 29 22 45	$\frac{1}{2}$ -
7	1589	September	9	35 35 0	♍ 26 11 15	♍ 26 12 0	$\frac{3}{4}$ +
8	1590	September	12	34 30 15	♍ 28 53 30	♍ 28 54 30	1 [?]
9	1592	September	10	35 5 5	♍ 27 26 15	♍ 27 26 45	$\frac{1}{2}$ +
10	1594	September	13	34 6 20	♍ 29 56 30	♍ 29 55 0	$1\frac{1}{2}$ -
11	1596	September	14	33 30 10	♌ 1 24 10	♌ 1 24 20	$\frac{1}{6}$ +

Parsing notes: Column 5 ("Place of Sun from table") for row 10 reads "29 55 0" with the seconds value partly degraded. The zodiac column markers are ♍ (Virgo) for rows 1–10 and ♌ (Libra) for row 11. Row 9's observed altitude seconds ("5 5") is uncertain — flagged. The fractional + / - signs in the Difference column are as printed.

III. According to the Middle of Taurus.

Num.	Year	Month	Day	Merid. alt. of Sun (G M S)	Observed place of Sun (G M S)	Place of Sun from table (G M S)	Differ ence (M S)
1	1582	April	27	50 52 0	□ 16 25 30	□ 16 24 45	$\frac{3}{4}$ -
2	1583	April	26	50 30 45	□ 15 10 40	□ 15 13 30	$2\frac{2}{5}$ +
3	1584	April	30	51 48 30	□ 19 46 45	□ 19 47 45	$1\frac{1}{2}$ +
4	1585	April	26	50 39 20	□ 15 41 10	□ 15 42 50	$1\frac{1}{2}$ +
5	1586	April	27	50 52 30	□ 16 26 50	□ 16 26 40	$\frac{1}{4}$ -
6	1587	April	29	51 20 35	□ 18 8 10	□ 18 6 0	$2\frac{1}{6}$ +
7	1588	April	27	51 0 20	□ 16 56 $\frac{1}{2}$	□ 16 54 0	$2\frac{1}{2}$ +
8	1590	April	26	50 45 30	□ 16 2 20	□ 16 2 25	$\frac{1}{3}$ [?]+

Parsing notes: All rows carry the zodiac marker □ (Taurus). Row 7's observed place seconds is printed as a fraction " $\frac{1}{2}$ " rather than an integer count. Source numeral "1582 April. 27" in row 1: the day "27" is read with some uncertainty against the page's earlier "30" — flagged as a possible misread vs. the autumnal section's day conventions.

IV. Beside the Middle of Leo.

Num.	Year	Month	Day	Merid. alt. of Sun (G M S)	Observed place of Sun (G M S)	Place of Sun from table (G M S)	Differ ence (M S)
1	1582	July	28	30 40 35 [recte: 50 40 35]	□ 14 14 35	□ 14 15 ¼	½ +
2	1584	July	28	50 31 51	□ 14 46 45	□ 14 45 0	1½ –
3	1585	July	27	50 6 30	□ 13 19 10	□ 13 19 40	½ +
4	1587	July	27	51 0 50	□ 13 4 15	□ 13 5 45	1¼ +
5	1589	July	29	50 18 15	□ 15 30 35	□ 15 30 20	¼ +

Parsing notes: All rows carry the zodiac marker □ (Leo). Row 1's day "28" and the first column's altitude "30 40 35" appear inconsistent with the surrounding ~50° altitudes (see Numerical impossibles below). The Difference-column fractions and +/- signs are as printed.

Besides these, I add some choice observations made near each solstice, so that from these the maximum obliquity of the ecliptic in our time may be shown at the higher tropic [tropo sublimiore], and the refraction of the Sun at the lower [tropic in decliviore]. These too, just as they were carefully acquired at Uraniborg with several instruments, the following little table comprises.

Table. Meridian altitudes of the Sun at Uraniborg near each of the two solstices, drawn from Tycho's observations. The table is split into two halves — the summer solstice (Æstivum) on the left and the winter solstice (Hybernum) on the right — each giving an observation number, year, month, day, and the Sun's meridian altitude in degrees, minutes, and seconds.

Meridian altitude of the Sun at Uraniborg near each solstice.

Summer solstice (Æstivum):

No.	Year	Month	Day	Alt. Sun Merid. (G. M. S.)
1	1586	June	12	57 35 $\frac{1}{3}$
2	1587	June	14	57 34 $\frac{1}{2}$
3	1593	June	11	57 35 0

Winter solstice (Hybernum):

No.	Year	Month	Day	Alt. Sun Merid. (G. M. S.)
1	1589	December	15	10 44 $\frac{4}{5}$
2	1590	December	10	10 43 $\frac{1}{4}$
3	1594	December	11	10 41 $\frac{1}{4}$

Parsing notes: The seconds column is printed as fractions of a minute (e.g. $\frac{1}{3}$, $\frac{1}{2}$, $\frac{4}{5}$, $\frac{1}{4}$) rather than as whole seconds; transcribed as printed. The "G. M. S." header spans the altitude column in both halves.

Beyond these exact and truly incomparable observations of the Sun by Tycho Brahe — whose worth their very precision [δοκιμασία, Greek: dokimasia, "testing, proof"] amply attests — he also brought other considerations to bear in testing the perfect restitution. In particular, he used the observations of the most illustrious Wilhelm, Landgrave [of Hesse] [Guilielmi Lantgravii]; and on top of that he investigated the matter with still further certainty by another kind of observation [πραγματεία, Greek: pragmateia, "procedure, treatment"] concerning the Sun's perigee [perigaeum Solis]. [Margin: *Lib. 1. Prog. pag. 77.*] These observations support one another remarkably well, since they can be derived from the established fixed stars and from the distances of Venus from the Sun [□ à □ distantiis] near the Sun's perigee. I submit them to the judgment of anyone who understands these matters. Together with the eclipses in both luminaries, they leave no doubt that Tycho, in restoring the Sun's course for our own age, has tracked the celestial truth in its apparent motions as closely as a ruler's edge, and that in skill, reliability, and diligence he has far surpassed all his predecessors.

Indeed I have confirmed this for myself, twelve years after Tycho completed his restitution of the Sun. After being trained in the divine discipline of mathematics

— and especially after the office of publicly teaching and practicing this Queen, Astronomy, was conferred on me — I undertook it. For from the year of our Lord 1607, around the same time when the bearded comet [cometa caudatus] blazed forth, I had a sextant of nearly five cubits in its semidiameter built precisely in imitation of Tycho's, so that with that instrument, together with a geometric quadrant of sufficient size, I might assemble what I judged had still to be drawn from the heavens to supplement the astronomical restitution. In this way I hoped to revive in my own homeland a fresh offshoot of Urania (in case she should perhaps be languishing in Bohemia).

Therefore, among other observations from which the continuation of the restored solar motion since Tycho can be judged: in the year 1610, around the autumnal equinox — namely on 11 September — using this same new sextant of ours, carefully calibrated beforehand against the fixed stars, I observed the Sun's meridian altitude as 35 degrees 1½ minutes. And since the elevation of the pole at Copenhagen [poli Hafniensis] extends very nearly to 55 degrees 43 minutes, once parallax and refraction were properly eliminated (as above), I found the Sun's true declination to be 0 degrees 46½ minutes N. To this corresponded a solar longitude of 28 degrees 3½ minutes [in Virgo, ♍]; Tycho's restored tables likewise give it as 28 degrees 4¼ minutes ♍. The difference, as you see, does not amount to even 1 minute — which is certainly imperceptible, and it commends the Tychonic prosthaphaeresis [prosthaphaeresin] of the Sun as rightly established above all others.

Furthermore, in the same year 1610, on 12 December and for several following days, with great diligence I tested through this same sextant of ours the Tychonic restitution of the solar perigee, using the distance of the Sun from Venus by daylight. This distance I found on 12 December, at hour 2½ after noon, to be 47 degrees 2 minutes. On the following day, around the same time, 47 degrees 1 minute; likewise on the 15th, at the same altitude, 46 degrees 59 minutes; and finally on 16 December the same distance, 46 degrees 55 minutes — this from observations repeated and confined on each day, often enough, and carefully limited. But in order to bring into examination here the distance between the Sun and Venus on 12 December at the hour I mentioned (2½), with the Sun at an apparent altitude of about 4 degrees and Venus at 17 degrees — namely 47 degrees 2 minutes — it was also necessary that the position of Venus around that same time should be acquired from the heavens.

Accordingly, I found this a little later on the same evening from the distances of Venus from the fixed stars. That is: with Venus at an altitude of 15½ degrees

above the horizon, Venus was distant from the Bright Star of the Vulture [Lucida Vulturis, i.e. Vega] by 37 degrees 8½ minutes, and from Markab Pegasi [Marchab Pegasi] by 36 degrees 13¼ minutes. Hence, from the verified positions of the fixed stars, the apparent longitude of Venus was found to be 17 degrees 56½ minutes □ [Aquarius], with a northern latitude of 1 degree 29 minutes. But in order that we may avoid both refractions and parallaxes in both Venus and the Sun, since the eleva-

—tion of Venus from the Earth was at that time, by hypothesis, nearly 850 Earth-semidiameters, the parallax in the altitude circle from the horizon at 15½ degrees was about 4 minutes. But the refraction of Venus too, at the same altitude (which in the brightness of Venus we apply, following Tycho, not unreasonably in a way midway between that assigned to the Sun and that assigned to the fixed stars), was found to be about 5 minutes. Thus the difference between the parallax and the refraction of Venus, scarcely 1 minute here, distributes to the longitude only half a minute to be subtracted. The reduction for the motion of Venus over the 2½ hours that elapsed between its observation and that of the Sun subtracts 6 minutes. The true longitude of Venus at hour 2½ on 12 December — when it was observed from the Sun — was therefore 17 degrees 50 minutes □.

Now, since the distance between Venus and the Sun (namely 47 degrees 2 minutes) was found to differ in no nearly perceptible way from the distance lying between the longitudes of both with respect to the ecliptic — on account of the small latitude of Venus at this time — therefore, once the said distance of 47 degrees 2 minutes was subtracted from the longitude of Venus just substituted, the apparent position of the Sun comes to 0 degrees 48 minutes [□ Sagittarius? — recte: read in context as the Sun's longitude near the winter perigee]. And because the parallax was very nearly the same for the Sun and for Venus during the observation, but in addition there was refraction in the Sun — with this parallax excluded (while you restore the true altitude of about 3½ degrees) — it was 14 minutes in the vertical circle. This, distributed at this location according to longitude, extends to only about 5 minutes to be subtracted from the apparent position of the Sun. The true position of the Sun in the year 1610 on 12 December at hour 2½ was therefore 0 degrees 43 minutes □ [Capricorn], which the calculation from Tycho's tables shows as 0 degrees 42½ minutes □ — only ½ minute intervening. Nor do I doubt that, if a similar test [δοκιμασία, dokimasia] from our observations should be made for the remaining days, the course of the Sun about its perigee would hold itself to very nearly the same agreement — so much so that the Tyconic restitution of the apogee in the Sun excludes very nearly all perceptible error even down to our own time. This is so, even though

the excessive slowness of this motion, on Tycho's supposition, may introduce some discrepancy with the passage of time, as we shall demonstrate a little further on.

Συστάσις [Greek: systasis, "constitution, determination"] of the eccentricity of the Sun from the observations of Tycho Brahe, mutually limited.

Now that the limitation of the Tychonic observations of the Sun has been diligently carried out, it appears from the differences added that the motion of the Sun around the vernal equinox comes out a little slower, and around the autumnal a little faster, than Tycho's tables present it. And although the $\frac{1}{8}$ minute previously neglected by us in the pole's elevation may contribute something toward removing these differences, it is nevertheless not so great but that something still remains — namely about one and a half minutes to be subtracted from the longitude, and half a minute from the maximum prosthaphaeresis [προσθαφαηρες., Greek: prosthaphairesis] of the Sun. This last we have now relaxed by only 27 seconds, and we have found that a divine and indeed unalterable numerical proportion arises from this — that of the second number in the order of perfect numbers. [Margin: *Plato de re/pub. lib. 1.*] [Margin: *Arithm. solut. cap. 12.*] So that the eccentricity of the Sun stands to the radius of its orbit as unity to twenty-eight, or as 35714 to 1000000, very nearly. Persisting in this invariable ratio — which is the nature of perfect numbers — concerning the Sun's eccentricity, we will now add, through a fuller comparison, how far those craftsmen previously corrected by us diverged this way and that, to either side. We do this so that how much room for changing the Sun's eccentricity had been left to Copernicus may be plainly judged by the discerning from the differences.

A collection of the maximum solar prosthaphaeresis from the observations of various astronomers, here recognized and corrected by us above.

	deg.	min.	sec.
Hipparchus & Ptolemy [Hipparchi & Ptolemaei]	2	5	26
al-Battānī & al-Zarqālī	2	0	0

	deg.	min.	sec.
[Albategni & Arzachelis]			
Walther of Nuremberg [Gualteri Norinbergensis]	2	2	0
N. Copernicus of Toruń [N. Copernici Turonensis]	2	3	0
Tycho Brahe the Dane [Tychonis B. Dani]	2	2	48

From the small differences that appear in this comparison — nowhere extending more than 3 minutes either way from the value we have found, since everything was duly provided for above, each in its proper place — let anyone practiced in astronomical observation and in its application now judge how rightly Copernicus had introduced his change of eccentricity into the theory of the Sun. That change, had it indeed been true, would also have introduced a quite manifest and perceptible inequality into the motion of the Sun's apogee, if any standing were to be granted to Copernicus's supposition. For this reason I shall show by the following demonstration [δοκιμασία, dokimasia] that this too provided no standing against the same man.

First, I think it should be noted that, although the matter comes to the same result from whatever epoch of a given interval the simple motion of the Sun's apogee is begun, I can nevertheless no longer keep silent: this illustrious testimony about the beginning and duration of the world, derived from the changed position of the Sun's apogee, I shall here set forth. [Margin: *Plura de epo-/chis infra.*] For, given the Sun's apogee placed at the beginning of Aries ☊ at the first creation of the world, and its perigee at the beginning of Libra ☎ — from which the Sun first appeared to the world by divine working — then, by the apogee of the Sun thus shifted in the meantime, I find (as I said) that from the beginning of the world to the redemption of the human race, that is, to the passion of Christ Jesus on the cross, 4000 years elapsed, save only for half a year. And therefore from the

beginning of the world to the start of the year 1588 after the birth of Christ, we rightly count 5554 Julian years — assigning, of course, 33 complete years to the life of Christ in this world, plus whatever runs beyond his nativity to the time of the passion. We judge that no other exactitude [ἀκρίβειαν, Greek: akribelian, "precision"] is necessary here.

And since, from the most accurate observations of Tycho Brahe, we gather that from the founding of the world to the said year 5554 — with respect to the true equinox (from which the mean scarcely ever differs by more than $\frac{1}{2}$ a degree, as we shall prove later) — the Sun's apogee progressed by $95\frac{1}{2}$ degrees in the consequence of the signs [in consequentia signorum]: we have deduced this same value proportionally from the beginning for each of the centuries in which it was observed, and set it in comparison with what was observed, as follows.

Table. The position of the Sun's apogee from the beginning of the world and from the true vernal equinox, for various astronomers across the centuries. Columns: years from the founding of the world; the value reduced (deduced proportionally) in degrees and minutes; the observed value in degrees and minutes; and the difference in minutes (with sign).

The position of the Sun's apogee from the beginning of the world and from the true vernal equinox.

Across various centuries	Years of the World (Annorum Mundi)	Reduced (Reductus) G. M.	Observed (Observat.) G. M.	Difference (M)
Hipparchus of Rhodes [Hipparchi Rhodii]	3810	65 16	65 30	14 —
C. Ptolemy of Alexandria [C. Ptolemaei Alexandrin i]	4099	70 3	—	—
al-Battānī son of	4849	82 53	82 16	37 +

Across various centuries	Years of the World (Annorum Mundi)	Reduced (Reductus) G. M.	Observed (Observat.) G. M.	Difference (M)
Muhamma d [Albategnii Mahumetis]				
Walther of Nuremberg [Gualteri Norinberge nsis]	5454	93 43	94 15	32 —
N. Copernicus of Toruń [N. Copernici Turonensis]	5492	94 23	95 8	45 —
Tycho Brahe the Dane [Tychonis Brahe Dani]	5554	95 30	95 30	0

Parsing notes: The Ptolemy row gives only the reduced value (70 3); the observed and difference cells are blank in the source. The sign on each difference (— or +) is printed to the right of the value; transcribed as printed.

From this it appears that the apogee's motion, deduced proportionally for the Sun across each of the centuries in which it was observed by astronomers, nowhere differs by more than $\frac{1}{3}$ of a single degree from their own observed values — which in the Sun's own motion scarcely causes a difference of one and a half minutes — Ptolemy excepted, who too confidently affirmed that Hipparchus's supposed position for the Sun's apogee endured down to his own age. [Margin: *Vide Prog. 1. T. B. pag. 32.*] His observations were doubtless violently

accommodated to such a purpose. From all the others to be censured concerning the discovery of the Sun's apogee, I have withheld only Copernicus; for his observations, free of refractions and other inconveniences, yielded the position of the Sun's apogee that I have placed here, as set forth above.

Furthermore, as for the fact that for this same apogee motion the interval of time between Walther of Nuremberg and Tycho Brahe does not correspond precisely to the proportion we have established, we answer nothing other than this: it was by no means possible for Tycho, from such observations of Walther's — and with the pole's elevation at Nuremberg not sufficiently precisely known — to arrive at a thoroughly accurate measurement of the solar motion in Walther's time, and hence, by the single interjected interval of a hundred years, to extend the Sun's course precisely over four centuries. For although the eccentricity discovered by Tycho himself would suffice very nearly for all the centuries, as we have shown, it nevertheless diminished the apogee's motion in such a way that it could scarcely agree with this century of ours. This will readily be established by comparison with the observations of the ancients, which we have just set forth.

A common opinion had taken hold of Tycho concerning the certainty or truth of the solar hypothesis introduced by Copernicus: that some change of eccentricity — like an anomaly in the position of the apogee — would necessarily entail a small Copernican circlet [circellum Copernicaeum]. But this was in vain, as is now clear from the constant, simple, and proportional change in the Sun's apogee, which never varied beyond three-quarters of a degree, much less by $7\frac{1}{2}$ degrees (a value which Copernicus's supposition sometimes even demands double of). Indeed, as I write this, there comes to mind an admonition once given to me by the noble Lord Holger Rosenkrantz [Holigeri Rosekrantzii] concerning this kind of alteration of eccentricity in the Sun and the planets, introduced by Copernicus — which, that man of lofty judgment foresaw, plainly conflicts with the everlasting nature of the celestial revolutions, and cannot be derived except from false principles, that is, from worthless observations (as it appears).

I therefore gladly submit to the fair judgment of the discerning the restitution of the Sun's eccentricity and the motion of the apogee — accomplished in this commentary of ours no less laboriously than truly. And what remains concerning the mean annual quantity, from which the mean motion of the Sun is constituted, I shall touch on briefly — only so far as the examination of the lunar course and the recognition of the apparent motion of the fixed stars in former ages requires — reserving a fuller and more precise knowledge of it for its proper place below, where I shall hand it down with many testimonies duly collected.

On the mean annual measure, from which the mean motion of the Sun within certain epochs of the ancients depends — at least for the examination of the Moon's motion and the appearance of the fixed stars in what follows.

Thus far, in comparing Hipparchus's equinoxes — especially the first Vernal one — with Tycho's, and in reducing each to the moments of their mean motions, as is fitting and as best I could, I found the anticipation over 1732 years to be 12 days, 20 hours very nearly. For that Vernal equinox of Hipparchus occurred in the year 146 before the birth of Christ, in the month of March, on the 24th day, at hour 11 before noon — if we reckon Julian years backward, which year corresponds to the third in the bissextile [leap-year] cycle, and into which also falls the year after the birth of Christ 1587. In that year the Vernal equinox was likewise observed by Tycho on 10 March, at hour 14, minute 56, at Uraniborg. Therefore, when you have first added 1 hour 34 minutes for the difference of meridians relative to our equinox, then 3 hours 50 minutes for the Sun's inequality, then also about 7 hours for the unequal precession of the equinoxes (so far as it is yet known to us), this equinox is reduced to the Meridian of Alexandria on 11 March at hour 33 [i.e. hour 9 of the following day]. Thus the anticipation of this equinox from the earlier Hipparchan one comes to 12 days 19½ hours. This, however, when deduced from the Autumnal equinoxes, does not certainly agree — nor does the annual quantity of Hipparchus, derived from his own observations, as I touched on above.

But retaining the anticipation we found of 12 days 20 hours, and dividing it by the 1732 years, we find that each Julian year falls short by 10⅓ minutes, and that the year-length we seek is 365 days 5 hours 49⅓ minutes very nearly. Although this may perhaps be altered in small part after the examination of the fixed stars has been added, nevertheless, while we are concerned with restoring the course of the Moon among the ancients, we assume the same to be still a little greater, on account of the ancient supposition — namely 365 days 5 hours 49½ minutes. From such a given annual quantity there results a motion of the Sun in one Egyptian year, which consists of 365 whole days, of 11 signs 29 degrees 45 minutes 38 seconds 46 thirds. From this Ptolemy falls short by 14 seconds; while the Alphonsine and Copernican values exceed it by about ⅔ of one second, according to the mean annual quantity found by them to be a little smaller.

It now remains for us to derive the simple longitude of the Sun and the motion of its apogee from the mean equinox for the two ancient epochs that the ancients

used, namely those of Nabonassar [Nabonnassari] and of Alexander the Great. [Margin: *Ptolem. lb. 3. cap. 8.*] Both of these begin alike from noon on the first day of Thoth, the Egyptian month. Since 424 complete Egyptian years intervene, therefore, when we have begun our reckoning from the first Vernal equinox of Hipparchus mentioned above — most rightly observed, as it appears — to the epoch of the reign of Nabonassar, there are found 601 complete Egyptian years (or our common years), 176 days, with only one hour falling short. But with the Sun's apogee placed, in Hipparchus's time, at $5\frac{1}{2}$ degrees Gemini $\square$, and with the others — both for the inequality of the equinox (as we estimate it) — in Hipparchi [Hipparchi — i.e., the Hipparchan vernal equinox] $9\frac{1}{2}$ minutes subtracted from the true [position], by the prosthaphaeresis [προσθαφαίρεσις] of the Sun, I find that the mean motion of the Sun, relative to the mean equinox at that same time, falls short of the true [position] in the order of preceding signs by only $1^{\circ} 42'$.

Now, in the interval stated between this observation of Hipparchus [Hipparchus] and the epoch of Nabonnassar [Nabonnassar], there accumulate — according to the Sun's motion in the Egyptian year established above — $29^{\circ} 39\frac{1}{3}'$ beyond the complete revolutions. When these complete revolutions are subtracted from that epoch of the Hipparchan equinox, there remains the mean course of the Sun, reckoned to the epoch of Nabonnassar and likewise to the Alexandrian meridian, of $28^{\circ} 38\frac{1}{3}' \square$ [in Aquarius]. In the same way, the mean motion of the Sun reckoned to the epoch of the death of Alexander the Great is found to have stood, from the mean equinox, at $17^{\circ} 12\frac{1}{3}' \square$ [in Scorpio].

As for the Sun's apogee at these ancient and remaining epochs, its derivation from the beginning of the world is straightforward, following those things which we set out and demonstrated above by the same method.

And these matters we have necessarily had to put forward as a foundation concerning the restitution of the Sun — namely, the absolute and unchangeable character of its eccentricity, the simple motion of its apogee, and also the yearly quantity [of solar motion], roughly worked out [crassiore Minerva]. Our purpose was to have ready, for what follows, the solar motions corresponding to the ancient epochs named, and so to descend properly and directly to the examination of the lunar appearances in these commentaries.

§I.

COMMENTARY: ON THE MOON [DE LVNA].

In the preceding commentary we obtained two results — by labor strenuously spent against Copernicus [Copernicus] — bearing on the restitution of the Sun's motion: namely, the immutability of its eccentricity (the ἀκίνησία [Greek: akinēsia, "immobility" — i.e., that the eccentricity does not change]), and consequently also the simple progression of the apogee from ancient times. These two cannot, given the conditions of the hypothesis, be torn apart from one another. Now the second of our truly Herculean labors is at hand: to set forth the course of the Moon — called *Lebanah* by the Hebrews, from its whiteness — in conformity, so far as can be done, with the celestial appearances, even back to ancient ages.

This I judge necessary not only for establishing the hypothesis of each of the two luminaries, to be constructed below, but also for displaying the apparent motion of the fixed stars successively, in the order of the signs, and consequently the unequal anticipation of the equinoxes [the precession of the equinoxes]. All these things are properly grounded once the positions of the luminaries have been verified. There can be no doubt that the final restitution of the lunar hypothesis — which I completed in Bohemia in the Jubilee year 1600 in the company of the noble Tycho Brahe [Tycho Brahe] — so conforms to the Moon's celestial phenomena, in all respects and through all ages (the mean motions being adjusted within narrow limits, and besides restored for ancient ages), that nothing further remains by which anyone could call it into doubt. Unless, that is, someone prefers — like the Christmann of Heidelberg [Christmann von Heidelberg], with wholly novel, inept, and useless labor — to seek the disgrace of his own name rather than, as is fitting, to emulate the celestial phenomena here and elsewhere.

That the Moon has never undergone any change as regards its prosthaphaereses, a simple comparison will readily prove — provided we first observe that the ancients investigated the eccentricity of the Moon by no more than sets of three eclipses at each occasion. From this it came about that they rashly and undeservedly excluded the equant [aequans] from it.

Now from such a triad of eclipses — a method introduced more for the sake of brevity than of prudence, by Ptolemy [Ptolemy] or perhaps by his predecessors, and afterward frequented by Copernicus with inept applause (although it can be

employed only in the case of the Sun, which is content with a single equation; whereas in our own resolution, were there need of it, [the eccentricity] could be set out far more concisely, without Euclid's [Euclid] penultimate proposition of Book 3) — a heavy obligation has been laid upon us. [Margin: Euclid, Bk. 3, prop. 5.] For it will be easy to enclose within the circumference of a circle any three points whatsoever, projected in any manner outside a straight line. But how greatly this apparent ease has damaged the truth, modern eclipses show, and the remaining lunar phenomena show it far more — phenomena which we shall later prove more fully not to have agreed even with the ancient constitution. From this we shall produce a cause why an evident error crept into the ancients' registration of the fixed stars, which was made by means of the Moon — [an error] transmitted, with most unjust jurisdiction, all the way to the Sun and to the yearly quantity. Hence I judge that I have a just cause for recalling everything to the anvil, and, according to the Sun's yearly Egyptian motion —

— together with what was found in the preceding commentary, of re-instituting afresh the δοκιμασία [Greek: dokimasia, "testing" or "scrutiny"] of the ancients' eclipses; and indeed [employing the eclipses] in triads for each age, which for brevity's sake we shall set out in the following Table, taking the motions of both the Sun and the Moon not from the true but from the mean equinox — which in this comparison makes no difference. In this comparison, with our more recent prosthaphaereses applied, only the mean anomalies, corresponding to the mean eclipses according to the modern hypothesis, are to be set before the eyes first, so that the difference in each case may appear. The slight equation of time, which we noted above arises from the oblique passage of the Sun relative to the equator, has been deliberately omitted by us in this commentary, so that it might be the more clearly explored what other [discrepancy] there may have been, through the eclipses that follow — chiefly those occurring in our own age.

Motions of the luminaries to the epoch of Nabonnassar and the Alexandrian meridian

— those of the Sun indeed established by us above, those of the Moon reduced from the Prutenic Tables [Prutenic Tables] — sought as follows:

	Sig.	Gr.	Min.
Simple longitude of the Sun from the mean	10	28	38⅓

	Sig.	Gr.	Min.
equinox			
Simple longitude of the Sun's apogee	1	25	22
Longitude of the Moon from the Sun	2	10	18
Anomaly of the Moon	8	28	43

[Margin: Ptolemy, Bk. 4, ch. 6.]

Three Lunar Eclipses of the ancient Assyrians [Ternae Eclipses Lunae veterum Assyriorum]

						Sim p. Soli s Lon . □ Sig. G. M.	Lon . □ a □ Sig. G. M.	An om al. □ Sig. G. M.	Ver us lo. □ Sig. G. M.	Ver us lo. □ Sig. G. M.	Diff ere nt. M.
Nu m.	An no	Me nse	Die	H.	M.						
1	27	Thoth.	29	8	40	□ 20 23	6 6 14	2 6 6	□ 22 13	□ 22 7	6 –
2	28	Thoth.	18	11	10	□ 9 23	6 3 2	0 12 17	□ 11 22	□ 11 24	2 +
3	28	Phame.	15	7	50	5 3 44	5 29 7	5 13 3	□ 1 40	□ 1 25[?]	15 –

Parsing notes: This is a citation-grade eclipse table. Column headers: Num. (number), Anno (year, reckoned from Nabonnassar), Mense (month), Die (day), H. (hours), M. (minutes), then the Sun's simple longitude (Sig./Gr./Min. with zodiac

sign), the Moon's longitude from the Sun, the Moon's anomaly, the true place of the Sun, the true place of the Moon, and the difference in minutes (with + / - sign). In row 3, the "Verus lo. ☐" entry is printed as a bare "5 3 44" with no sign-glyph, and the "Verus lo. ☐" cell ✕ 1 25 is ink-degraded — flagged. Several sign-glyphs are faint at scan resolution.

[Margin: Ptolemy, Bk. 4, ch. 11.]

Three Lunar Eclipses observed by Hipparchus [Tres Lunae Eclipses ab Hipparcho observatae]

						Sim p. Soli s Lon . □ Sig. G. M.	Lon . ☉ a □ Sig. G. M.	An om al. ☉ Sig. G. M.	Ver us lo. □ Sig. G. M.	Ver us lo. ☉ Sig. G. M.	Diff ere nt. M.
1	547	Me sori	16	7	0	□ 28 21[?]	5 23 46	10 0 23	□ 26 27	□ 26 22	5 -
2	548	Me chy r	9	13	15	□ 24 3[?]	6 55	3 19 33	□ 26 8	□ 26 14	6 +
3	548	Me sori	5	14	36	3 17 34	5 23 11	8 38	□ 15 34	□ 15 28	6 -

Parsing notes: Same column structure as the preceding table. Month names: Messori and Mechyr (Egyptian months). Several leading sign-glyphs and values in the "Simp. Solis Lon." column are faint/ambiguous at scan resolution (rows 1–2 flagged). Row 3 "Verus lo. ☐" printed without a sign-glyph as "3 17 34" likewise ambiguous.

[Margin: Ptolemy, Bk. 4, ch. 6.]

Ptolemaic Triad of Lunar Eclipses [Trias Eclipsium Lunae Ptolemaica]

Nu m.	An no	Me nse	Die	H.	M.	Sim p. Soli s Lon . □ Sig. G. M.						Diff ere nt. M.
						Lon . □ Sig. G. M.	Lon . □ Sig. G. M.	An om al. □ Sig. G. M.	Ver us lo. □ Sig. G. M.	Ver us lo. □ Sig. G. M.		
1	880	Pau ni	20	11	15	1	5	10	□	□	15	
						13	27	14	14	14	+	
						38	41	19	33	48		
2	882	Chi ac	2	11	0	6	6 2	2 4	□	□	0	
						28	57	41	26	26		
						1[?]		37	37			
3	884	Pha rm.	19	16	0	□	6 5	4	□	□	26	
						12	18	26	14	15	+	
						47		40	49	15		

Parsing notes: Same column structure. Month names Pauni, Chiac (= Choiak), Pharmuthi (Egyptian). Row 1 "Simp. Solis Lon." printed as bare "1 13 38" without sign-glyph; row 2 "6 28 1" ambiguous and flagged. Years reckoned from Nabonnassar (880, 882, 884). Several cells degraded at scan resolution.

EXPLANATION [EXPLICATIO].

We have drawn these foregoing lunar eclipses from Ptolemy's record, so that they might, by the test undertaken, in some measure corroborate our restitution of the luminaries from the lowest traces of those very eclipses. Concerning them I have thought it right to give a general warning: first, that no special accuracy [ἀκρίβεια — Greek: akribēia, "exactness"] is reported in those observations made by the ancients, since the beginnings of the Moon's obscuration are at best merely indicated in some of them — and hence the reduction to the mean moments is not sufficiently true or fitting. But in Ptolemy's own [observations] —

— neither beginnings nor ends are assigned, but only a computation of the mean times. Now besides this uncertainty, it may also be doubted whether the moments of the eclipses reported by the ancients were derived from the celestial

revolution, or from clocks or water-clocks [clepsydrae], which are wont to deceive. But since we rest chiefly upon the Hipparchan [observations], we have restored the last of these as follows. Ptolemy reports that its beginning was 40 minutes after midnight at Alexandria; and since the Sun was in the middle of ♍ [Virgo], nothing of any sensible amount is given in equal hours on account of the distance of 40 minutes from midnight. Hence the half-duration, according to the calculation of Copernicus — who in other respects does not greatly err in this matter — is 1^h 56′ after midnight, as we have noted.

Moreover, since the error of Ptolemy is found to be more enormous than the rest — whether because he conceded something of his own accord to the time, so that, the mean motions of the luminaries being retained in their uniform constancy, he might uphold for the Sun alone a prosthaphaeresis equal to the supposed Hipparchan one; or whether the observations were carelessly carried out by novices, as appears now and again in the case of the Sun and in other matters too in Ptolemy — however it may be, we, recalling this Ptolemaic eclipse to the anvil through the motion of latitude, have found from the tables of Reinhold [Erasmus Reinhold] (which are found to behave tolerably in the apparent eclipses among the ancients) that the true latitude of the Moon was at least 46′ above the tail of the Dragon [the Moon's descending node, the tail of Draco] at the time noted by Ptolemy. From this, and from the half-diameter of the Earth's shadow at 49′, the eclipse turns out to be 3′ greater than Ptolemy's record — whereas he has it at only half [that magnitude]. From which it is easily inferred that he extended the mean time of this eclipse longer than was right.

We therefore, having almost left Ptolemy aside, shall remain alike with the ancient and the Hipparchan [observations]. In these the beginnings of the eclipses are in some measure handed down, and for that reason we shall make the greatest use of them hereafter for the ancient ages, whenever afterward either the mean motions are elicited or examinations of the [fixed] stars are instituted. Now we pass over, similarly, to the observations of Tycho and our own in the lunar eclipses.

Synopsis and δοκιμασία [Greek: dokimasia, "scrutiny"] of the Lunar eclipses observed in our own age

— and reduced as they appear to the Uraniborg [Uraniborg] meridian: with the epoch fixed from the birth of Christ.

Num .	Ann o	Men se	Die	H.	M.	Ano mali a 》 Sig.G .M.	Veru s locu s Solis Sig.G .M.	Veru s luna e locu s Sig.G .M.	Diffe rent. M.
1	1573	Dece b.	8	8	3	7 24 24	□ 26 50	□ 26 51	1 +
2	1576	Octo b.	7	11	25	2 5 31	□ 24 30½	□ 24 33	2⅔ +
3	1577	Apri li	2	8	50	7 6 34	□ 22 47½	□ 22 52	4⅙ +
4	1577	Sept eb.	26	13	3	5 28 56	□ 13 23½	□ 13 24½	1⅓ +
5	1578	Sept eb.	15	13	24	10 16 34	□ 2 19	□ 2 21¼	2¼ +
6	1580	Ianu ar.	31	10	10	1 6 29	□ 21 28½	□ 21 25½	3 -
7	1581	Ianu ar.	19	10	0	11 21 23	□ 10 4½	□ 10 2	2½ -
8	1581	Iulio	15	17	0	4 27 43	□ 3 40½	□ 3 37½	3½ -
9	1584	Nov ēb.	7	13	9	4 7 19	□ 25 49½	□ 25 50½	1⅓ +
10	1587	Sept eb.	6	9	30	10 1 21	□ 23 8⅓	□ 23 10⅔	2 -
11	1588	Mart io	2	15	2	3 20 2	□ 22 49	□ 22 48	1 -
12	1590	Dece b.	30	7	0	9 11 47	□ 19 1⅓	□ 19 4½	3⅓ +
13	1592	Iuni o	14	10	20	1 4 10	□ 3 15	□ 3 16	1 -

Num.	Anno	Mense	Die	H.	M.	Anomalia a ☾ Sig.G .M.	Verus Solis locus Sig.G .M.	Verus lunae locus Sig.G .M.	Different. M.
14	1592	Decemb.	8	7	41	6 5 15	☐ 27 15½	☐ 27 16½	⅞ +
15	1594	Octob.	19	16	2	2 2 40	☐ 5 29½	☐ 5 31¼	2 +
16	1595	April	13	16	30	7 3 43	☐ 3 24½	☐ 3 29	4½ +
17	1595	Octob.	7	20	29	0 8 20	☐ 24 15⅔	☐ 24 18⅔	2⅔ +
18	1596	April	2	9	25	5 17 32	☐ 23 9⅔	☐ 23 13¼	3⅔ +
19	1598	Febru.	10	18	7	1 13 44	☐ 2 32	☐ 2 30½	1⅓ -
20	1598	August.	6	7	37	6 10 31	☐ 23 12½	☐ 23 8⅓	4 -
21	1599	Januar.	30	17	56	11 18 37	☐ 21 11½	☐ 21 10½	1 -
22	1609	Julio	6	12	10	2 23 5	☐ 24 10	☐ 24 12½	2⅓ +
23	1613	Octob.	18	3	57	0 5 19	☐ 5 13¼	☐ 5 13¼	⅔ +

Parsing notes: Column headers (left to right): Num. (margin column, "N" with subscript), Anno (year AD), Mense (month, abbreviated), Die (day), H. (hours), M. (minutes), Anomalia ☾ (anomaly of the Moon, in signs/degrees/minutes), Verus locus Solis (true place of the Sun, with zodiac sign), Verus lunae locus (true place of the Moon, with zodiac sign), Different. (difference in minutes, with + / - sign). Fractional minutes (½, ⅓, ⅔, ¼, ⅙, ⅕, ⅞) are printed as small superscript/subscript fractions and are read as best resolved at scan DPI. The

"Differentia" column for rows 14 and 23 shows sub-minute fractions ($\frac{7}{8}$, $\frac{2}{3}$) — read with care. The table is bordered by a decorative double-rule frame; this is transcribed as a native pipe-table per rule 6, not a diagram. The last printed line of the page is a row of decorative type ornaments below the frame.

DECLARATIO. [Statement]

These 23 lunar eclipses were for the most part observed at Huena [Uraniborg], with two, three, and sometimes four of Tycho Brahe's most accurate instruments — recording the beginnings, the ends, and the other notable phases, so far indeed as the sky permitted. They were acquired with such diligence, and reduced to the true opposition of the luminaries, as no predecessor had ever achieved. The remainder were for the most part taken here at Hafnia [Copenhagen], whose meridian is close to that of Huena, observed with no less care and skill — just like those that are to be drawn upon below as worked examples of the doctrine of eclipses, and which are therefore omitted here.

And although solar eclipses are not added — both for brevity's sake and because of the intricacy of parallax — nevertheless experience in observing them and duly restoring them had taught us that they differ in nothing from the lunar ones. Indeed, when a proper examination is set up, no greater discrepancy in the motions of the luminaries ever intervenes, whether at conjunction or at opposition, than the discrepancies recorded here. From this one may see not only that we have carried out the restoration of the courses of the luminaries as exactly as possible for our own age, but also that the equation of time [aequatio temporis] which we abolished above is granted no place here whatsoever. Nor indeed does anything seem to be insinuated to us from the cause of the obliquity of the zodiac through the comparison of these eclipses, except some trifling and almost unstable quantity — as anyone skilled in the matter will easily detect from comparing the obliquity of the ecliptic with the equator in the various signs of the zodiac through which the Sun passes year by year.

For our part, accommodating to this same factor a limitation — after carefully sifting among themselves the other small differences besides — we reject only a single minute from the motion of the Moon's simple longitude reckoned from the Sun, the motion which is inserted into Tycho's final restoration, while retaining the anomaly and the prosthaphaereses [prosthaphaereses] in full, just as the Tychonic tables have them. And so the luminaries have been restored in conformity with their celestial motions, to such a degree that you could scarcely ever hope for greater precision. This is what both the eclipses and likewise the other celestial phenomena of the luminaries will readily prove to us — to the few,

that is, who deign to give any care to these matters amid the sordid affairs of the world. But, taking thought for brevity, we shall now complete the remaining web of our intended commentaries on the fixed stars.

III. COMMENTARIVS DE STELLIS FIXIS. [Commentary on the Fixed Stars]

Up to this point we have examined the courses of the Sun and Moon, for the ancient age as well as for our own, from observations of their own — examined and restored them to this end: that not only might a path lie open, in a manner, toward constructing their hypotheses and tables afterward in their proper places, but also that the foundation of the fixed stars might thereby be laid here and there — the foundation which among the ancients was for the most part established by means of the Moon.

First, then, with regard to the moments of conjunction of the Moon with certain fixed stars, observed by Timocharis [Timocharis], assigned in Ptolemy [Ptolemy], and now diligently revised and corrected by us, the place of the Moon according to its visible longitude must be computed by us accurately from our recent restoration. Thus, with the limitation duly established, the apparent places and seats of those same stars may be predetermined for a definite year of the age of Timocharis. This foundation will afterward, in conformity with the absolute restoration of Tycho Brahe — derived from his own observations, which we once drew in with our own eyes — suit the description of all the fixed stars equally, since they have never suffered any displacement from one another. [Margin: *Almagest* [Almagest] VII.4.]

Next, descending through Hipparchus [Hipparchus] to Ptolemy, we shall also weigh this latter's observation among the Sun, the Moon, and Basiliscus [Regulus], and compare it with Timocharis and with Hipparchus. Here, without doubt, the more rapid motion of the fixed stars will reveal itself than the opinion of those ancients allowed — confirmed likewise by the declination of certain fixed stars determined by the various ancients, Timocharis, Hipparchus, and Ptolemy, although the date is not sufficiently indicated in them (just as Ptolemy reviews these observations too). [Margin: *Almagest* VII.3.] Afterward, descending to al-Battānī [Albategnius] — since for the present we are without him directly — we shall rest content with the Copernican report concerning him. Finally, with the Tychonic infallible foundation of the fixed stars appended below — which the comparison of all these, most skillfully and most accurately to be set up, supplies

to us for the precession of the equinoxes [praecessio aequinoctiorum], both mean and true — we shall carry it out into a hypothesis of the equinoxes to be arranged in the first place.

I. As regards the observations of Timocharis on the conjunction of the Moon with the stars, we have selected three of the fixed stars — namely Lucida of the Pleiades, Spica Virginis [Spica], and the Northern Brow of Scorpius — each observed once. Among these Spica holds the chief place, and for that reason we have taken up the others on either side of it, so that the limitation of this one may be more rightly insinuated. To it the place of the Moon, from our restoration computed for the individual corrected moments of the observations — corrected among other things also on account of the reduction of seasonal time to equal time, which is found in Ptolemy in a less suitable form — is set forth beforehand in the following synopsis, and is compared with Timocharis, the Epoch being everywhere deduced from Nabonassar.

Name s of the fixed stars	Year	Month	Day	H. M. Timoc har.	H. M. Correc t.	Loc. ☾ Timo. S. G. M.	Loc. ☾ restit. S. G. M.
North ern Brow of Scorp.	454	Phaop hi	16	15 24	15 30	☾ 2 0	☾ 1 58
Lucid a of the Pleiad es	465	Athir	29	8 40	8 40	☾ 29 20	☾ 29 32
Spica Virgin is	466	Thoth	7	16 7	15 54	☾ 22 30	☾ 22 40

Parsing notes: Columns are: star name; year (from Nabonassar); Egyptian month; day; hour/minute per Timocharis; corrected hour/minute; Moon's place per Timocharis (sign, degree, minute); Moon's place per the restoration (sign, degree, minute). Zodiac glyphs: ☾ Scorpius, ☾ Aries, ☾ Virgo.

This restoration of ours, since it does not differ much from the observation of Timocharis — provided his true equinox be everywhere reduced to the mean by 6 minutes — let us therefore set Spica $\square$ at the beginning of the year 466 from Nabonassar, at $22^{\circ} 30'$. This holds especially when the true celestial distance between Spica itself and the star in the Brow of Scorpius (whose observation I judge to have been more certain than that of the Pleiades) requires it.

Next, since there can be doubt concerning Hipparchus's observation up to the Heart of the Lion [Cor Leonis], which Ptolemy reviews, at $29^{\circ} 50'$ of Cancer $\square$ (that is, at the beginning of Leo $\square$, near the mean equinox) — because it is most uncertain by what method of observation he hunted out its place — we therefore use a position established with the declination of Spica supposed at $0^{\circ} 36'$ B[oreal] for Hipparchus's age. [Margin: *Almagest* VII.2.] From this, according to the requirement of the triangles, we find Spica at $20^{\circ} 19'$ $\square$, and at $24^{\circ} 29'$ $\square$ with respect to the mean equinox. [Margin: *De Revolutionibus* (Copernicus) I.3.]

Furthermore, we arrive at Ptolemy, who in the second year of the emperor Antoninus Pius — that is, 886 from Nabonassar, on the 9th of Pharmuthi, at sunset, by the following method (constrained to our emendation) — explored the distance of the Heart of the Lion $\square$ from the summer solstice. At $5\frac{1}{2}$ hours after midday, by means of an astrolabe — as he reports — he took the centers of the luminaries at a distance from each other of $92\frac{1}{2}^{\circ}$. But since the Sun's place reckoned from the mean equinox, according to our restoration, was then at $4^{\circ} 3'$ $\square$ — and since the Sun moreover would have a refraction, apart from parallax, in the ecliptic and the Alexandrian horizon of no less than $20'$, perhaps even somewhat greater (while we subtract nearly a third part from that which occurs with us at the Sun) — there results a visible longitude of the Sun of $4^{\circ} 23'$ $\square$ from the mean equinox, as I said. If to this be added the observed elongation of the Moon, the visible place of the Moon will be found at the 5th hour at $6^{\circ} 41'$ $\square$ very nearly. But afterward, around the half-hour, the Heart of the Lion $\square$ is reported, detected from the center of the Moon by the same astrolabe, to be distant $37^{\circ} 10'$ in the direction following the ecliptic. And since the Moon over the half-hour (excluding the increment of the parallax of longitude) was advancing by $12'$, therefore, if we add all these to the prior apparent observed place of the Moon, the longitude of Basiliscus [Regulus], or the star in the Heart of the Lion $\square$, is found at $3^{\circ} 53'$ $\square$. Ptolemy, by his own induction, asserted that it had been placed — from the true equinox, both on account of the defect in his solar motion (which was refuted above) and on account of the neglect of refraction — at $2\frac{1}{2}^{\circ}$ $\square$.

At last, with that earlier antiquity thus passed over and much time interposed, let us proceed to al-Battānī, who is reported to have found Basiliscus [Regulus], at Aracta [ar-Raqqa] in Syria in the year 1626 from Nabonassar, at $14^{\circ} 1/12'$ □, and the Northern star in the Brow of Scorpius at $17^{\circ} 50'$ □. [Margin: Copernicus, *De Revolutionibus* III.2.] These, in comparison with the true distance of the stars in Tycho's table, admit a difference of 26 minutes, which argues an uncertainty in the observation. But when we refer Basiliscus to the mean equinox (which here was 22' more remote from the true in the direction of the following signs, as will afterward be shown), drawing it out from al-Battānī's observation, it will be at $14^{\circ} 27'$ □.

But since the Alfonsine astronomers [Alphonsini] were deceived in this matter and almost blinded — as the reader will likewise perceive in Georg Peurbach [Georg Peurbach] and in Johannes Regiomontanus [Regiomontanus] — and since Copernicus's [Copernicus] errors at the Sun were earlier demonstrated from Tycho and by our own observation, therefore in the last place [we shall descend] to the [solid foundation]—

—to the solid foundation which our master D. Tycho had laid in the fixed stars, before all others, for the end of the Julian year 1600 from the birth of Christ (which, as Egyptian or equal reckoning from Nabonassar, he set at 2350). Here, from the manifold and sufficient observation of Venus — on either side of the Sun by day, and then of the same Venus by night against various fixed stars, and at various seasons of the year — he set forth the undoubted limit of their motion, just as he himself most fully and most skillfully demonstrates in book I of the *Progymnasmata*, chapter 2, to which therefore we refer the reader. [Margin: Book I of the *Progymnasmata*, chapter 2.] At the aforesaid year Basiliscus [Regulus] was found at $24^{\circ} 17'$ □ from the true equinox, and Spica □ at $18^{\circ} 16'$ □. But because the inequality of the equinoxes, for our age, subtracts about 8 minutes, Basiliscus will be at $20^{\circ} 9'$ □, and Spica at $18^{\circ} 8'$ □ with respect to the mean equinox. These values, having thus been correctly obtained, are placed in view in a brief synopsis, and afterward, for eliciting the annual motion of the stars, are compared among themselves, with the intervals of time reduced backward from our age.

Ti m oc ha ri di s			Hi pp ar ch i			Pt ol e m ae i			Al ba te gn ii			Ty ch . Br ah e		
Ye	N	Pl	Ye	N	Pl	Ye	N	Pl	Ye	N	Pl	Ye	N	Pl
ar	a	ac	ar	a	ac	ar	a	ac	ar	a	ac	ar	a	ac
	m	e		m	e		m	e		m	e		m	e
	e	of		e	of		e	of		e	of		e	of
	of	st		of	st		of	st		of	st		of	st
	st	ar		st	ar		st	ar		st	ar		st	ar
	ar			ar			ar			ar			ar	
					□			□			□			□
18	Sp	G.	17	Sp	G.	14	Ba	G.	72	Ba	G.	0	Sp	G.
84	ic	M.	20	ic	M.	64	sil	M.	4	sil	M.		ic	M.
	a	22		a	24		isc	3		isc	14		a	18
	□	30		□	25		.	53		.	27		□	16

Parsing notes: Five paired column-groups, one per authority (Timocharis, Hipparchus, Ptolemy, al-Battānī, Tycho Brahe). Each group gives Year, Name of star, and Place of star (degrees G. and minutes M.). The "Year" values here are years reckoned backward from Tycho's epoch (Tycho = 0). The sign headers placed above the place values are: Timocharis □, Hipparchus □, Ptolemy □, al-Battānī □, Tycho □. The first two groups give Spica's place; the middle two give Basiliscus/Regulus's place; the Tycho group gives Spica again at 18° 16' □.

From this the mean annual motion of the fixed stars — that is, the simple anticipation of the equinoxes [anticipatio aequinoctiorum] — is elicited proportionally, as appears in what follows.

Annual motion of the stars.

Sec.	Tert.	
51	0	from Ptolemy to al-Battānī
49	46	from Hipparchus to Tycho

Sec.	Tert.	
49	37	from Ptolemy to Tycho
49	10	from Timocharis to Tycho
48	14	from al-Battānī to Tycho
46	0	from Timocharis to Ptolemy
44	0	from Timocharis to Hipparchus

Parsing notes: Columns are seconds (Sec.) and thirds (Tert.) of arc per year, with the interval between authorities named at right.

From these, after the foregoing emendation has been diligently collated among them — with the limitation always carefully observed, both as regards the longer interval of time and the superior observation — there emerges no greater mean advancement of a fixed star, appearing in the sky in one equal year, than 49 seconds 45 thirds, which over $72\frac{1}{3}$ years complete one degree. Nor do we trouble ourselves that some fairly perceptible difference intervenes between Timocharis and Ptolemy, and a still greater one between Timocharis and Hipparchus — since here, on account of the small intervals of time, that difference is nowhere greater than 20 minutes. Indeed, this difference is to be attributed rather to the observations of those men than to any greater inequality of the equinoxes, apart of course from that which will shortly be shown below to agree well enough with the hypothesis.

But although the supposition of both our Copernicus and our Tycho regarding the annual advancement of the stars in the direction of the following signs — or rather of the equinoxes in the direction of the preceding signs — surpasses ours by a little (since the former had set it at $50''\ 12'''$, while the latter set it precisely at $51''$), nevertheless, because I see that this happened to them chiefly through their having used the observations of Hipparchus and Ptolemy without the emendation of refraction and of the inequality of the equinoxes, therefore it was by no means right for us to depart further from the foundation of Timocharis, or even from Ptolemy's restored observation. But concerning al-Battānī, this must be reported here: that it is clearly established that the precession of the equinox from Ptolemy to him, and from him to us up to the present, agrees more correctly

with the stars than with the Sun's course and with the annual quantity deduced here and there from him — which will doubtless appear, as I remember having warned of earlier, around the weighing of al-Battānī's observation at the Sun.

It still remains, for the completion of this our commentary, to treat of the obliquity of the path of the Sun from the equatorial circle, as it is varied according to the intervals of time — and not, as Copernicus reports, [the obliquity] of the equatorial [circle] to the Ecliptic. [Margin: Book III, chapter 3, and following.] This is most manifestly clear from those things which Lord Tycho Brahe had adduced, by means of a demonstration of comparison, from the observations of former times and of his own time made against the fixed stars — namely, how the latitudes themselves vary in conformity with the alteration of the Sun's obliquity. [Margin: *Progymnasmata* Book III, p. 233 and following.] But as for us, regarding this mutation of the obliquity of the Ecliptic — when the individual findings of our predecessors have been diligently examined, and corrected for the parallax of the Sun (Copernicus reporting on this matter last after Ptolemy, whose supposition too we have restored from Tycho's emendation and our own) — [Margin: Copernicus, Book III, chapter 2, near the end.] we have found that the maximum obliquity was restored, around the true year of the World 3600, to be $23^{\circ} 53'$; and the minimum, around $23^{\circ} 31'$ very nearly, from the year 5400 of the same created world, which is A.D. 1434. Hence the mean is gathered to be $23^{\circ} 42'$, which Copernicus at least set at $23^{\circ} 40'$. And how this our constitution of the obliquity answers to the celestial truth also from the intermediate observations — together with the decrement and increment, likewise the inequality of the equinoxes, and finally the variation of the latitude of the Stars — we shall hereafter demonstrate in one congruent hypothesis. Now, with the Lernaean Hydra slain by means of these commentaries, which lay the foundation of the second part of the first book of the Astronomy, we ascribe praise and triumph with most humble heart to God the Archduke, and gladly take up what remains under His auspices also.

CAPVT III. [Chapter 3]

On the Epochs of the celestial motions henceforth employed by us, and on the discrimination of Times — both on account of the Sun's oblique progress relative to the diurnal revolution, and on account of the differences of meridians — together with appended tables.

Although various Epochs, to which the mean or equal celestial motions are fixed, are found among our predecessor Astronomers — for instance, the Epoch of the

Olympiad, of Nabonassar, of Alexander the Great, of Julius Caesar, of Christ in the flesh, of Antoninus, and of many others — [Margin: See Reinhold's [Erasmus Reinhold] 4th precept, *Prutenic Tables*.] nevertheless for us Christians two beginnings ought to be most illustrious above the rest, surely by divine favor toward us:

The first, when this most beautiful theater of the world began to exist by the word of almighty God; the other, when the only-begotten Son of God Himself deigned to clothe Himself in our human flesh and to be born, that He might restore the fallen world and free us from the power of the devil and from eternal death. On these beginnings — set apart from each other by a sufficiently long interval — we suspend and number the mean celestial motions in this perennial restoration of those motions which (with the good God's help) we are laboring at. Nor does it impede our undertaking here that each beginning is today called into doubt by various Chronologists, both as to the number of the years and as to the time of the year: for this dissension entangles the celestial motions with no difficulty, since they have been duly derived from other intervals well secured by celestial observation.

I hope, moreover, that it will prove most welcome to the Chronologists of truth that, from our observation, they hear for the very first time that they are strongly confirmed in rightly establishing — in their annals, around the beginning of the world — the apparent motion of the Sun's apogee, likewise of the maximum obliquity, as well as of the inequality of the equinoxes. [Margin: See above, near the end of the Commentary on the Sun.] For hence, from the time of creation up to the passion of the Son of God on the cross and the restoration of the world through Him, there are correctly gathered 4000 solar years, with a half-year as it were lacking, as we showed above in the commentaries — since the perigeal place of the Sun at the time of the world's founding had stuck fast in the beginning of Libra ♎, without any inequality of the equinox, and the maximum obliquity of the Sun then existed; all of which agree marvelously with the stated time and with these φαινόμενοις [Greek: phainomenois, "phenomena" — the appearances] compared among themselves.

This Epoch of the world's founding, then, fixed as to the number of Julian years, we shall retain — especially because by various more recent critics of the histories either this very number is closely touched upon, or another differing from it by a few years is [proposed] back and forth—

Astronomia Danica — Pars Altera, Liber Primus Theoricorum, Caput III (continued) — Caput IV (opening)

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— differing back and forth. Nor do we doubt that the world was founded by God at the vernal equinox, on account of various conjectures and testimonies. For not only have the Hebrews, from the very earliest time of the world down to the present, taken the beginning of their year from this point [Margin: Exod. 12, and following]; but also, when God wished to change the form of the year and to fit everything to the time of His Son's Passion, He compelled His people to it by a new command. There is much to read about this controversy in other writers, and much to weigh on behalf of the truth — which without doubt stands by this opinion. Here I refrain from going further, and refer the reader to my own *Disputation on the Epochs*. [Margin: Josephus Scaliger [Joseph Justus Scaliger]; Calvisius [Sethus Calvisius / Seth Calvisius]]

Furthermore, the second epoch I mentioned — that of the Messiah sent in the flesh [the Incarnation] — I retain, together with the ancient fathers, and I assign to the age of Christ on this earth only about 33½ solar years. This is so both on account of the prophecy of Daniel [Margin: Daniel, ch. 9, v. 17], and on account of the types which are found here and there in the sacred scriptures concerning the number of the 40 days, which prefigured in equal beauty just so many solar months from the baptism of Christ to His Passion [Margin: 1 Samuel 17; Matthew 4; Acts 1]. In these types He began His contests with the devil and with the world, after — at the age of 30 years and baptized by John [Margin: Luke 3, v. 23] — He had entered upon His office and ministry, for the sake of which He had been sent by the Father. And certainly I see no absolute necessity, urged by the more recent critics of the histories, for prolonging the interval of Christ's age, since most of them disagree among themselves by several years. [Margin: See, among others, Mercator; Bunting; Scaliger; L. Suslyga; Polanus; Johannes Kepler; Joseph Scaliger; Sethus Calvisius] Nor, even were it so, could the celestial motions which we pursue in this work be disturbed by it — any more than they are by the fact that the ancients placed Christ's birth in the flesh near the winter solstice, and others in October. For we begin our epoch from the first noon immediately following the Kalends of January, in the form of the Julian year (which we use throughout here, as the most common of all, in numbering the motions); and

these Kalends, in turn, closely follow the year of Christ as I have just asserted it, and the year hitherto in use in astronomical reckoning among all Christians — and indeed rightly so, as is discussed more fully in the said *Disputation on the Epochs*.

For the rest, although between the rising of the world and the Passion of Christ on the cross there had elapsed nearly 4000 solar (or Julian) years, as I said above — and therefore 3966 years between the Epoch of the world and that of Christ Incarnate — yet I see that, for numbering the celestial motions, the Epoch can most conveniently be applied which agrees with the noon of the 3rd Kalends of January [i.e. 3 January reckoned by Kalends] of the current year from the world's beginning, because it coincides with the first running [year] of Christ in being a leap year. Let there therefore be counted, from the world-Epoch to the Christian one, only 3964 complete Julian years, so that — with the mean motions prefixed to each individual Epoch — we may move gracefully in the proper extraction of these, in their δοκιμασία [Greek: dokimasia, "examination" or "verification"], as the examples below will show.

Now the connection is easy between the commonly assumed Olympiad (for about the true one there is doubt), likewise the [Epochs] of Nabonassar, of Alexander the Great, and the Julian Epochs, with our world-Epoch — as also the Greek or Egyptian [reckonings] with the Julian months. But concerning all these, besides the examples which we leave here and there, let the most learned Erasmus Reinhold be consulted. [Margin: On the assertion of the true Olympiad see also my *Disputation on the Epochs*, Precept 4, and following] So much for the Epochs.

As for what concerns the equation of the natural day, we establish it — as before, following Tycho [Tycho Brahe] only — universally from the Sun's oblique progress relative to the equator, the cause having been given above in the commentary. [Margin: Table from the *Prutenic [Tables]*] To this [equation], in its former [aspect], we assign the table of those next following; and the latter we assign to the differences of meridians upon the earth. [Margin: See above in the *Commentary on the Sun*, the account of this matter] For time too, being begun from the noon of one fixed place to which the Epoch of the celestial motions is bound, is altered at other places because of the earth's roundness, so that the eastern places always reckon more, and the western less, time — the beginning being taken under that meridian where no inclination of the magnetic needle from the terrestrial pole is felt, which the geographers call the Prime [Meridian], and which some hold fixed at the Fortunate Isles in the Ocean. How much that time will be can easily be drawn out from the latter table (which, by Tycho's

correction concerning the longitude and latitude of the more notable places, with my own assistance, I obtained) in the following column — once the degrees and minutes of the equator, which fall in the difference of the remaining places from our Copenhagen [Hafnia] meridian, have been duly converted into time, and rightly applied to each of the other places, as has been done here in accordance with what we set forth in the doctrine of the Sphere.

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Table of the Equation of the Natural Day [Tabula aequationis Diei naturalis]

The table is read by zodiac sign across three paired sign-columns. Each pair of columns shares the same equation values: the left member of each pair runs top-to-bottom by degree (Aries / Taurus / Gemini at the head), the right member runs bottom-to-top (Libra / Scorpio / Sagittarius at the head, with Pisces/Virgo, Aquarius/Leo, Capricorn/Cancer at the foot). The value columns give Minutes (M.) and Seconds (S.) of time. Each block has an outer degree column (0–30) at left and a complementary degree column (30–0) at right.

Block 1 — head signs ♈ Aries / ♎ Libra; foot signs ♊ Pisces / ♍ Virgo:

Deg (□/□)	M.	S.	Deg (□/□ foot)
0	0	0	—
1	0	20	—
2	0	40	—
3	0	59	—
4	1	19	—
5	1	39	—
6	1	59	—
7	2	18	—
8	2	38	—
9	2	57	—
10	3	16	—
11	3	35	—
12	3	51	—
13	4	12	—

Deg (□/□)	M.	S.	Deg (□/□ foot)
14	4	30	—
15	4	47	—
16	5	5	—
17	5	22	—
18	5	38	—
19	5	55	—
20	6	11	—
21	6	26	—
22	6	41	—
23	6	56	—
24	7	10	—
25	7	24	—
26	7	37	—
27	7	50	—
28	8	2	—
29	8	14	—
30	8	25	—

(Foot label of this block: **A** — sign symbols ♓ Pisces / ♍ Virgo. The outer right-hand degree column runs 30 down to 0, reading this block in the ascending sense.)

Block 2 — head signs ♉ Taurus / ♏ Scorpio; foot signs ♒ Aquarius / ♌ Leo:

Deg (□/□)	M.	S.	Deg (□/□ foot)
0	8	25	30
1	8	36	29
2	8	45	28
3	8	55	27
4	9	4	26
5	9	12	25
6	9	19	24
7	9	26	23
8	9	32	22

Deg (□/□)	M.	S.	Deg (□/□ foot)
9	9	37	21
10	9	42	20
11	9	46	19
12	9	50	18
13	9	52	17
14	9	54	16
15	9	56	15
16	9	56	14
17	9	56	13
18	9	55	12
19	9	53	11
20	9	51	10
21	9	48	9
22	9	44	8
23	9	40	7
24	9	34	6
25	9	28	5
26	9	21	4
27	9	14	3
28	9	6	2
29	8	57	1
30	8	47	0

(Foot label of this block: **A** — sign symbols ♒ Aquarius / ♌ Leo.)

Block 3 — head signs ♊ Gemini / ♐ Sagittarius; foot signs ♐ Capricorn / ♋ Cancer:

Deg (□/□)	M.	S.	Deg (□/□ foot)
0	8	47	30
1	8	37	29
2	8	26	28
3	8	15	27

Deg (°/′)	M.	S.	Deg (°/′ foot)
4	8	3	26
5	7	50	25
6	7	36	24
7	7	22	23
8	7	7	22
9	6	52	21
10	6	36	20
11	6	20	19
12	6	3	18
13	5	46	17
14	5	28	16
15	5	10	15
16	4	51	14
17	4	32	13
18	4	12	12
19	3	53	11
20	3	33	10
21	3	12	9
22	2	51	8
23	2	31	7
24	2	9	6
25	1	48	5
26	1	27	4
27	1	5	3
28	0	43	2
29	0	22	1
30	0	0	0

(Foot label of this block: **A** — sign symbols ♑ Capricorn / ♊ Cancer.)

Parsing notes: The table is laid out as three side-by-side double-entry blocks, each with two narrow degree columns flanking a central M./S. pair, the second and third blocks sharing a boundary value (8 25 ends block 1 and begins block 2; 8 47

ends block 2 and begins block 3) confirming continuity. Block 1's value at degree 12 prints as 3 51 but smooth interpolation between 3 35 (deg 11) and 4 12 (deg 13) suggests it should be near 3 53; transcribed as printed and flagged (see Numerical impossibles). The single header letters A at each block foot mark the ascending/descending reading direction.

Use of the Present Table [Vsus praesentis Tabellae]

The use of this table is as follows: by entering with the sign and degree of the Sun's place (descending or ascending, according as the sign is found at the head or at the foot of the table), the minutes and seconds which the area displays are, according to the requirement of the heading, added to or subtracted from the apparent time, so that equal [mean] time is produced. But the contrary procedure is to be followed if equal time is to be reduced to apparent.

Cap. 3, LIBER PRIMVS

There follow the longitudes and latitudes of the more notable places in Europe, [taken] from Tycho Brahe, corrected partly by his own observation, partly by itinerary distances and by corrected maps.

Table of geographic coordinates (part 1 of 2). Columns: Place name; Time [Tep. = differentia temporis] in Hours and Minutes (H. M.); Longitude in Degrees and Minutes (G. M.); Latitude in Degrees and Minutes (G. M.). The single-letter column before the Time gives the direction of the meridian difference from Copenhagen (S = subtract / west? / "Subtrahendum"; A = add / "Addendum"). The pages run two place-lists side by side; transcribed here as a single column-pair in reading order (left list, then right list).

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
Aarhu sia Cimb. [Aarh us]	S	0	9	34	30	56	16
Agrige ntum [Agrig ento]	A	0	18	41	5	38	31
Alba	A	0	28	43	35	47	18

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
Regalis [Székesfehérvár]							
Alcmarr. Holl. [Alkmarr]	S	0	36	27	45	52	36
Aldenburg [Oldenburg]	A	0	1	36	50	51	0
Alexandria Ægy. [Alexandria, Egypt]	A	1	35	60	30	30	58
Amberg	S	0	2	36	15	49	28
Amiens	S	0	47	24	50	49	20
Amsterdam [Amsterdam]	S	0	34	28	0	52	20
Ancona Italiae [Ancona]	S	0	1	36	25	43	42
Annæ	A	0	3	37	25	50	43

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
burg [Anna berg]							
Antve rpia [Antw erp]	S	0	36	27	35	51	12
Argen tina [Stras bourg]	S	0	21	31	30	48	30
Arimi num [Rimi ni]	A	0	7	38	30	43	53
Aveni o [Avign on]	S	0	38	27	15	42	35
Augus ta Vindel i. [Augs burg]	S	0	6	35	10	48	22
Aureli a [Orléa ns]	S	0	53	23	30	47	15
Babyl on	A	2	25	73	0	35	0
Baere nburgi um	S	0	16	32	40	52	40
Baden	S	0	19	31	50	48	42

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
a [Bade n]							
Barby e [Barb y]	S	0	1	36	20	52	2
Basila ea [Basel]	S	0	21	31	15	47	38
Bamb erga [Bamb erg]	S	0	6	35	0	49	58
Bautz en	A	0	9	38	50	51	9
Berlin um [Berli n]	A	0	6	38	10	52	29
Bern	S	0	22	31	0	46	50
Bonon ia [Bolog na]	S	0	1	36	30	43	57
Brand enbur gum [Bran denbu rg]	A	0	2	37	20	52	23
Brans vigum [Brau	S	0	3	35	45	52	29

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
nschw eig]							
Brema [Brem en]	S	0	18	32	15	53	12
Bresla w [Bresl au / Wrocł aw]	A	0	20	41	40	51	7
Brisac h [Breis ach]	S	0	22	31	15	48	4
Brens berg [Brau nsber g]	A	0	24	43	35	52	46
Brund usium [Brind isi]	A	0	34	45	13	41	27
Bruxel la [Bruss els]	S	0	37	27	30	50	50
Berda [Breda]	A	0	31	44	20	47	19
Camp en [Kamp en]	S	0	32	28	40	52	47

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
Capua	A	0	17	40	55	41	42
Caffill ae	S	0	13	33	25	51	18
Casso via [Košic e]	A	0	38	46	5	48	31
Cobur g	S	0	6	35	10	50	20
Colber g [Kolbe rg]	A	0	16	40	40	53	45
Colma r	S	0	22	31	5	48	5
Coloni a [Colog ne]	S	0	25	30	30	50	55
Compl utum [Alcal á de Henar es]	S	1	12	18	35	41	0
Comp ostella [Santi ago de Comp ostela]	S	1	40	11	45	43	0
Cordu ba [Córdo ba]	S	1	20	16	45	38	0

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
Costen tz [Konst anz]	S	0	14	33	5	47	36
Creuti a	A	0	33	45	0	49	57
Crema na	S	0	8	34	40	44	15
Cuben bach	S	0	4	35	40	50	8
Dantis cum [Danzi g / Gdańs k]	A	0	31	44	20	54	0
Dessa [Dessa u]	S	0	0	36	40	51	53
Daven tria [Deve nter]	S	0	29	29	27	52	14

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
Docku m [Dokk um]	S	0	31	29	0	53	15
Dresd a [Dresd en]	A	0	6	38	5	51	6

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
Edenb urg Ger.	A	0	2	37	15	51	26
Elbing a [Elbin g / Elbląg]	A	0	34	45	10	53	55
Elfenb urgum	A	0	0	36	45	56	3
Emde n	S	0	26	30	15	53	19
Enchu sa [Enkh uizen]	S	0	33	28	20	52	42
Erfurd ia [Erfur t]	S	0	5	35	20	51	7
Ferrar ia [Ferra ra]	S	0	1	36	20	44	18
Flensb urgum [Flens burg]	A	0	3	37	25	55	8
Floren tia [Flore nce]	O	0	0	36	40	43	27
Franc of. ad	S	0	18	32	15	50	2

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
Moen. [Frank furt am Main]							
Franc of. ad Oder [Frank furt an der Oder]	A	0	12	39	40	52	51
Frerbe rga [Freib erg]	A	0	5	37	50	50	52
Fribur gum [Freib urg]	S	0	24	30	40	46	40
Genev a [Gene va]	S	0	10	29	5	45	25
Genua [Geno a]	S	0	12	33	40	43	12
Gent Fland. [Ghen t]	S	0	40	26	45	51	4
Gorlit z [Görlit z]	A	0	12	39	35	51	9

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
Goslar	S	0	9	34	30	52	0
Gota [Gotha]	S	0	8	34	45	51	0
Grana ta [Gran ada]	S	1	14	18	15	37	30
Gran [Eszte rgom]	A	0	29	43	50	47	56
Graitz [Graz]	A	0	17	40	50	47	22
Gryph iswald um [Greifs wald]	A	0	5	38	0	53	54
Grunb erg [Grün berg]	A	0	13	40	0	51	50
Groni ngen	S	0	28	29	40	53	9
Grupe nhage n	S	0	9	34	25	51	40
Guster opium	S	0	0	36	35	53	42
HAFNI A Dan. [Cope nhage n]	O	0	0	36	40	55	43

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
Hamb urgum [Ham burg]	S	0	12	33	35	53	42
Hall	S	0	1	36	25	51	43
Halbe rstadi um [Halbe rstadt]	S	0	5	35	20	52	0
Harde rwick [Hard erwijk]	S	0	32	28	45	52	18
Haerle m [Haarl em]	S	0	36	27	40	52	22
Harlin gen	S	0	33	28	25	53	6
Helms tadiu m [Helm stedt]	S	0	5	35	25	52	23
Heidel berga [Heide lberg]	S	0	18	32	15	49	20
Hertz ogenb usch ['s- Herto	S	0	33	28	30	51	31

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
genbo sch]							
Hispal is, Sevilla [Sevill e]	S	1	26	15	15	37	25
Iena [Jena]	S	0	3	35	55	51	2
Ingols tadiu m [Ingol stadt]	S	0	4	35	40	48	46
Iliad. sed. Ep. sep	S	1	28	14	45	65	44
Insbru gk [Innsb ruck]	S	0	3	36	0	47	13
Katze nelbog en [Katze nelnb ogen]	S	0	21	31	30	50	12
Keiser berg [Kayse rsberg]	S	0	23	31	0	48	8
Kitzni ng	S	0	10	34	15	49	43

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
[Kitzin gen]							
Knuds trup. praed [Knud strup]	S	0	2	37	10	55	58
Lande strum	S	0	0	36	35	48	30
LadzK ron Dan. [Land skron a]	A	0	2	37	15	55	53
Laugi ngen [Lauin gen]	S	0	8	34	35	48	36
Lausa nna [Lausa nne]	S	0	27	30	0	47	40
Lipsia [Leipz ig]	A	0	0	36	45	51	22
Longb erg Patria [Lomb org, Longo monta nus' birthp	S	0	15	32	50	58	40

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
lace]							

Parsing notes (p. 199): The table is a two-column-block layout, each block carrying its own header row (Place | Tep. H.M. | Long. G.M. | Latit. G.M.). Several cells are ink-degraded in the gutter and at line-ends; flagged inline. The Basel (Basilaea) latitude line is partly broken (38 with no clear minutes — possibly a column slip); flagged. The direction letter for Florentia and HAFNIA reads ambiguously (o printed where S/A expected — Copenhagen is the reference meridian, so o = origin/0 is plausible for HAFNIA; flagged for Florentia). The place "Iliad. sed. Ep. sep" with latitude 65°44' is a far-northern site (possibly an Icelandic/Hólar episcopal see); name partly illegible, flagged.

Cap. 3 → Caput IV, THEORICORVM

Table of geographic coordinates (part 2 of 2 — continued from p. 199). Same columns. Left list then right list.

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
Lubec a [Lübeck]	S	0	7	34	10	53	57
Lucerna [Lucerne]	S	0	19	31	55	46	53
Lunenburgum [Lüneburg]	S	0	9	34	20	53	19
Lutetia a Parisi or. [Paris]	S	0	49	24	25	48	48
Lutze	S	0	29	29	30	49	42

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
nburg [Luxe mbou rg]							
Londi num Angl. [Lond on]	S	0	50	24	20	51	32
Lundu m Scania e [Lund]	A	0	5	37	50	55	45
Lugdu num Bat. [Leide n]	S	0	37	27	30	52	7
Lymvi ga Cymb. [Lemv ig]	S	0	15	32	52	56	42
Magde burgu m [Magd eburg]	S	0	2	36	10	52	10
Mantu a	S	0	6	35	15	44	33
Mansf eld	S	0	3	35	50	51	40
Marsil	S	0	34	28	20	42	45

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
ia [Mars eille]							
Mentz [Main z]	S	0	20	31	45	50	0
Messa na [Messi na]	A	0	24	42	45	39	12
Midde lburg	S	0	40	26	40	51	31
Misni a [Meiss en]	A	0	4	37	45	51	9
Mons Pessul anus [Mont pellier]	S	0	42	26	5	43	0
Mons Reg. Boruf. [König sberg]	A	0	40	46	45	54	21
Morsb urg [Meer sburg]	S	0	1	36	25	47	23
Munc hen [Muni ch]	S	0	3	36	0	48	6

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
Munster	S	0	22	31	10	51	54
Marpurgum [Marburg]	S	0	16	32	35	50	43
Marientburg	A	0	33	45	0	53	43
Narborna [Narbonne]	S	0	46	25	5	41	50
Naffow [Nassau]	S	0	21	31	20	50	16
Neapolis [Naples]	A	0	17	40	55	41	37
Nidrosia Norve. [Nidaros / Trondheim]	S	0	11	34	0	63	27
Nycoping Cymb. [Nykøbing]	S	0	13	33	30	56	50
Nurnberg [Nuremberg]	S	0	6	35	15	49	27

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
mberg]							
Odens ee Dania e [Oden se]	S	0	10	34	15	55	21
Ofenb rug [Osna brück ?]	S	0	20	31	35	52	17
Oster wick [Oster wieck]	S	0	7	35	0	52	3
Panor mus [Paler mo]	A	0	15	40	30	38	59
Patavi um [Padu a]	S	0	1	36	25	45	6
Pisa	S	0	4	35	45	43	6
Pisaur um [Pesar o]	A	0	10	39	10	43	51
Praga [Pragu e]	A	0	7	38	30	50	6

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
Saltzberg [Salzburg]	A	0	5	37	50	47	47
Sena [Siena]	A	0	3	37	20	43	3
Solotum [Solothurn]	S	0	22	31	5	47	8
Spira [Speyer]	S	0	18	32	15	49	10
Steenvig [Steenwijk]	S	0	30	29	15	52	45
Studgardium [Stuttgart]	S	0	14	33	5	48	39
Spandaw [Spendau]	A	0	5	37	50	52	30
Sivertslaw	S	0	17	22	25	56	2
Swinfurd [Schweinfurt]	S	0	9	34	20	50	3
Syrac	A	0	23	42	30	38	9

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
usae [Syrac use]							
Tyro	S	0	6	35	10	46	28
Tholc ha	S	0	59	21	55	43	10
Thorn [Toru ń]	A	0	31	44	25	52	34
Toletu m [Toled o]	S	1	16	17	40	40	10
Torga [Torga u]	A	0	3	37	30	51	33
Trien [Trent / Trento]	S	0	27	30	0	49	50
Tubin ga [Tübin gen]	S	0	15	33	55	48	24
Valent ia [Valen cia]	S	0	58	22	5	39	55
Veneti ae [Venic e]	A	0	6	37	15	45	18

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
Verona	S	0	5	35	20	44	51
Vienna Austria [Vienna]	A	0	19	41	30	48	28
Ulm [Ulm]	S	0	10	34	5	48	23
Lisbon [Lisbon]	S	1	39	11	55	38	50
Uraniborg [Uraniborg, Hven]	A	0	0[?]	36	45	55	54
Weimar [Weimar]	S	0	4	35	40	51	8
Wesel	S	0	27	29	50	51	34
Wismar	S	0	4	35	40	53	54
Wittenberg [Wittenberg]	S	0	2	37	15	51	52
Wurtzburg [Würz]	A	0	11	33	55	49	44

Place	Dir	Tep. H	Tep. M	Long. G	Long. M	Lat. G	Lat. M
burg]							
Wolffe nbutte l [Wolfe nbütte l]	S	0	7	34	55	52	23
Wolm erstat [Wol mirste dt]	S	0	3	36	0	52	19
Worm bs [Wor ms]	S	0	14	32	5	49	33
Wibur gu Cimb. [Vibor g]	S	0	12	33	35	56	30
Zeitz	S	0	1	36	20	51	0
Zerves ta [Zerbs t]	S	0	0	36	35	52	0
Zirixz ee [Zierik zee]	S	0	39	26	55	51	40
Zurich	S	0	17	32	20	47	9
Zyges	A	0	30	44	15	46	34

Parsing notes (p. 200): Layout identical to p. 199 (two place-blocks, each with its own header). Several latitude-minute cells in the gutter are ink-bled and flagged.

"Pisa" longitude reads 41 43 where neighbors cluster near 36–39 for the longitude-degree column; the printed 41 in the long.-deg position with 43 immediately following looks like a column-merge artifact and is flagged. Direction-letter column is mostly legible; a few (e.g. Vraniburg's Tep. minutes) bled to near-zero.

CAPVT IV (opening)

CAPVT IV.

[Heading, italic:] On the canonical setting-out of the fixed stars observed at Uraniborg in Denmark; on the precession of the mean and true equinox; on the maximum obliquity of the Sun; together with the variation of the latitudes of the fixed stars; and finally on the computation of all these.

This chapter, intended to embrace everything that pertains to the places and apparent motions of the fixed stars, we have arranged in order into four parts. The first [will give] the canonical description of the fixed stars which, in our horizon, [are] nearly... [text continues in next batch]

PARS ALTERA — LIBER PRIMVS THEORICORVM — Caput IV (continued)

are nearly conspicuous. The second [part] presents the mean motions of the precession of the equinoxes and of the obliquity of the ecliptic. The third [gives] the hypothesis and tables of the prostaphaereses [profthaphaerefeon — the equations] of these same quantities, from which the latitudes of the stars are also derived for all the ages of the world. The fourth, finally, contains the precepts, together with examples, for applying each [of these] to use.

I. As regards the determination of the fixed stars, this labor — [a labor expended] in those [stars] long ago observed and computed at Uraniborg on Hven — I rightly call **the Uraniborg nights**. For after Tycho [Tycho Brahe], in years now past, had most successfully laid their foundation by means of the Sun and Venus (as is plain from Book 1 of the *Progymnasmata*) [Margin: *Progym.*

[*Progymnasmata*] Bk. 1, ch. 2], I afterward — from the year of the Lord 1590, over the course of a five-year period — not only took part in this business and its execution, [the work] of restoring the fixed stars from the heavens, but indeed directed it. [I directed it] to such a degree that I can affirm in good faith that the utmost diligence which can ever be expected from human industry was applied by us to the restitution of these [stars] along [both] length and breadth; and that

every doubt was removed by reciprocal observations on both sides, often repeated where there was need, until they settled themselves to within the same minutes as nearly as possible. [This holds] even though now — on account of the mean longitude of the Sun, which we have diminished by more than a minute — we have correspondingly subtracted just as much, item by item, from the longitude of the stars in the following canon of Tycho. There too we have deliberately eliminated the tiny fractions of minutes — those, namely, which were arising from the limitation [of precision] — wherever they clung to the longitude and latitude, judging that a precision greater than [that] in whole minutes would not be necessary.

As for the apparent magnitude of each individual [star], which one may observe by the eye alone at each [observer's] discretion: even if perhaps it is not found to be equal — on account of the thicker air with us, which is now and then variable — nevertheless, wherever there has been a departure here also from the notation of the ancients, the cause was [some] evidence appearing in the sky. Now, two dots [Margin/note: the symbol "..."] added to the magnitude of a star in the outermost column of the table indicate it [to be] a little greater [in magnitude]; but a single [dot] everywhere argues [it to be] smaller; and finally an asterisk * [indicates] a star which the Ptolemaic canon does not have.

How useful, and how necessary, the effort of our Lord Tycho was here, there is no need to explain at length among the intelligent — since without a due restitution of the fixed stars no legitimate [determination] of the planets could be made. For of these [planets] no measurement is given equally as certain as [their] measurement from the fixed stars in the heavens. Accordingly, by however much one errs here, this way or that, in longitude and in latitude, by just so much is one in error there [in the planets] — even when instruments, even when observations, and finally the calculation and all the requisites are duly enough in order.

This cause, then, among others, shines forth manifestly enough: why the ancients — who all followed, down to Tycho, the antiquated and uncertain designation of the fixed stars by Hipparchus [Hipparchus] (which was made, namely, by means of the Moon's [position] crossing the meridian in its course, the Moon itself being not yet rightly known beforehand) — boast in vain to posterity of an astronomy solidly restored by them. And the outcome today demonstrates this [boast] to be all too true in the sky [Margin: See Ptolemy [Ptolemy], Bk. 1, ch., likewise *Progym.* p. 234, & following]: [the heaven] convicts [them], by comparison, not only of error in the planets but in the fixed [stars] too — the [stars] whose distance from

one another and whose longitude it teaches, among other things, to have been unchangeable, because they still gaze upon one another in the sky along the same straight lines as in the time of the ancients.

Lastly, as for the primeval naming of the asterisms [afterifmorum] which the fixed stars occupy spread out across the face of the sky: even though I should dare to affirm, by conjectures plainly not doubtful, that among Adam and the Patriarchs there was another [naming], in many cases far more significant — yet it is preferable (as our Lord Tycho also advises) [Margin: Bk. 1 *Progym.*; Cicero [Cicero], *Tusculans* 5 [*Tusc.* 5]] to indulge for the time being in the fables of the heathen, which strongly commend their [the ancients'] outstanding virtue and industry — against the sloth of certain of our own [contemporaries] in the contemplation of celestial things — until we shall have been solidly instructed about all things in the celestial school. These things, having been briefly set out by way of declaration of the first part, the canon of the stars itself now follows. [Margin: *Progym.* ch. 2 [Prog. cap. 2]; *Progym.* [E lib. 2]]

(Note: page top shows the trailing letters "...HORICORVM" of the display title; the page number 202 is at upper left.)

THE CANONICAL DETERMINATION OF THE FIXED STARS

most numerous and most principal, [made] according to the accurate observations of **Tycho Brahe** [Tycho Brahe] drawn down from the heavens —
the canonical determination [CANONICA DETERMINATIO], completed to the year of our Savior

1600.

FIRST, ON THE STARS THAT ARE SEEN ABOUT THE ZODIAC.

ARIES [♈]

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
The southe	♈	27	36	7	8	B	4

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
rn [star] in the prece ding horn							
The northe rn and follow ing [star] in the same horn	□	28	22	8	29	B	4
The bright [star] on the crown of the head: princi pal [Hama l]	□	2	5	9	57	B	3
In the gape, the northe rn of two	□	2	33	7	23	B	6
The [star]	□	3	19	5	42	B	6

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
more to the south							
The [star] in the neck	□	27	56	5	24	B	5
In the loins	□	8	35	6	7	B	6
The [star] at the spring ing of the tail	□	12	56	8	8	B	5
The prece ding of three in the tail	□	15	14	1	46	B	4
The middl e [one]	□	16	23	2	50	B	5
The last [one]	□	17	49	2	36	B	6
In the thigh	□	11	21	1	12	B	6
In the	□	9	34	1	7	B	6

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
ham [knee- back]							
In the left knee *	☐	9	22	1	30	A	6
In the right knee *	☐	7	51	0	39	A	6
The little [star] in the belly *	☐	8	45	4	1	B	6
The [star] that is below the bright one of the head *	☐	1	40	9	13	B	6
Above the back, the prece ding of 4	☐	10	34	10	50	B	5

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
unfor med [stars]							
The follow ing, namel y to the base of the wester n $\triangle$ [trian gle] of the follow ing	$\square$	11	22	11	16	B	4
The easter n [star] in the base of the triangl e	$\square$	12	39	10	24	B	3
In the apex of the same triangl e towar	$\square$	12	50	12	25	B	4

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
d the north							

TAURUS [♉]

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
The upper most [star] in the sectio n	♈	17	59	5	57	A	5
The next after it	♈	17	29	7	29	A	6
The third	♈	16	17	8	50	A	4
The fourth , most southe rn	♈	15	34	9	23	A	4
In the right should er	♈	21	45	8	41	A	5
In the breast	♈	25	0	8	3	A	4
In the right	♈	27	58	12	14	A	4

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
knee							
In the right hock [paste rn]	□	24	18	14	31	A	4
In the left knee	□	4	8	9	32	A	5
In the left hock [paste rn]	□	3	10	11	48	A	5
In the face, the first of the Hyade s [fucul ae], in the nostril s	□	0	11	2	47	A	3
Betwe en this one and the northe rn eye	□	1	15	4	2	A	3

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
The [star] betwe en the same and the southe rn eye	□	2	21	5	53	A	4
In the southe rn eye, Aldeb aran, Palilic ium [Aldeb aran]	□	4	11	5	31	A	1
In the northe rn eye	□	2	52	2	37	A	3
At the root of the southe rn horn	□	8	11	3	40	A	6
In the same horn, the more southe	□	12	12	2	31	A	6

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
rn of two							

TAURUS (continued)

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
The [star] more towar d the north	□	11	3	1	50	A	4
In the extre mity of the same	□	19	11	2	14	A	3
In the origin of the northe rn horn	□	6	34	0	40	B	5
In the extre mity, or the right foot, of Aurig a	□	16	58	5	20	B	2

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
[Hen ochus]							
In the ear, the northe rn of two	□	2	53	1	4	B	5
The more southe rn	□	2	37	0	35	B	4
In the neck, the prece ding of two	□	27	50	1	12	B	5
The [star] that follow s	□	0	27	0	47	A	6
In the neck of the quadri lateral , the southe rn of the prece ding	□	0	3	5	16	B	6

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
[stars]							
The northe rn of the same side	□	29	44	7	55	B	5
The southe rn of the follow ing side	□	2	33	3	57	B	5
The northe rn of this side	□	2	24	5	45	B	5
The wester n of the three bright er [stars] in the Pleiad es [Pleia des]	□	23	12	4	11	B	5
The lowest	□	24	2	4	2	B	6

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
and neares t to the wester n [one]							
The middl e and bright [star] of the Pleiad es	□	24	23	4	0	B	3
The [star] that is at the cusp towar d the rising [east]	□	24	46	3	55	B	5
In the hoof of the left foot *	□	19	56	13	30	A	6
The little star in the heel of	□	0	9	12	2	A	5

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
the follow ing foot *							
The [star] in the right should er *	□	1	57	8	41	A	5
The prece ding of three below the Hyade s [fucul ae] *	□	1	41	6	57	A	5
The middl e of the same *	□	3	27	7	5	A	5
The follow ing [one] *	□	4	54	6	18	A	5
The little	□	15	1	1	4	A	6

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
[star] in the southe rn horn							
The follow ing [star] in the same horn	□	16	54	1	20	A	6
The little [star] follow ing, the fourth in the sectio n *	□	17	32	9	35	A	6
The [star] that is betwe en the two prece ding, in the □ [squar e/grou	□	29	21	6	33	A	5

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
p] of the neck [coli] *							

GEMINI [♊]

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
In the upper head, Casto r , Apollo [Casto r]	♊	14	40	10	2	B	2
In the lower head, Pollu x , Hercu les [Pollu x]	♊	17	42	6	38	B	2
In the left hand of the prece ding	♊	5	31	10	58	B	5

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
twin							
In the left arm	□	9	53	7	43	B	4
In the should er- blades of the same	□	13	23	5	42	B	4
In the right should er	□	15	46	5	10	B	5
In the left should er of the follow ing twin	□	18	5	3	3	B	4
In the right side of the forego ing [twin]	□	13	17	2	56	B	6
The little star in the	□	14	9	6	0	B	6

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
left elbow of the upper [twin]							
In the northe rn and upper most knee	□	4	21	2	11	B	3
In the left knee of the follow ing [twin]	□	9	25	2	7	A	3
The [star] that is in the belly of the southe rn twin	□	12	55	0	14	A	3
In the ham of the lower twin	□	13	12	5	41	A	4
In the foot of	□	27	52	0	58	A	4

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
the prece ding twin, the forem ost							
The follow ing [star] in the same foot, called the heel [calx]	□	29	43	0	53	A	3
In the extre mity of the right foot, the prece ding	□	1	13	3	8	A	4
The bright [star] of the foot	□	3	30	6	49	A	2
In the lowest	□	5	28	10	9	A	4

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
[part of the] follow ing foot							
In the heel of the same foot	□	7	55	9	41	A	6
The [star] that is above the knee of the lower twin	□	6	22	1	12	A	6
In the thigh of the upper twin	□	8	36	1	31	B	6
The [star] which is below the lower head, in the hand	□	19	41	5	44	B	6

Designation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
* The little [star] betwe en both heads	□	17	3	7	24	B	5
* At the ear of the upper twin	□	13	28	9	42	B	5
The prece ding [star] at the top of the foot: Propu s in Greek [Margi n: Propu s Græce — Greek: πρόπο υς	□	25	21	0	13	A	4

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
(propo us, "foref oot")]							

CANCER [ꣳ]

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
The nebul ous [star] in the breast , which they call the Mange r [Praes epe]	ꣳ	1	45	1	14	B	nebul a [nebul osa]
The northe rn of the prece ding [stars] in the ꣳ	ꣳ	29	48	1	31	B	5

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
[square] of Cancer							
The more southern	□	0	8	0	48	A	5
The northern Ass [Asellus borealis]	□	1	56	3	8	B	4
The southern Ass [Asellus australis]	□	3	7	0	4	A	4
In the southern claw [brachium]	□	8	2	5	8	A	3
In the northern claw	□	0	43	10	23	B	5
In the	□	23	55	1	15	B	5

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
extre mity of the northe rn foot							
In the end of the southe rn foot	□	25	3	7	5	A	5
The bright er [star] that is at the root of the tail *	□	25	44	2	19	A	4
Closel y follow ing, in the back *	□	28	11	1	4	A	6
The northe rn of three in the southe rn	□	6	46	1	54	A	6

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
claw							
The southe rn in the same	☐	10	35	5	36	A	5
The northe rn of two in the muzzl e	☐	5	26	7	14	B	6
The lower and southe rn	☐	7	35	5	20	B	6

LEO [♌]

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
In the nostril s	☐	9	40	10	23	B	4
In the open jaw	☐	12	15	7	52	B	4
In the head, the	☐	15	50	12	21	B	4

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
more northe rn of two							
The	□	15	4	9	40	B	3
more southe rn							
In the neck, the northe rn of three	□	21	56	11	50	B	3
The middl e and bright [star] of the neck	□	23	58	8	47	B	2
The southe rn	□	22	19	4	52	B	3
The Heart, Regul us , Basilis cus [Regul us]	□	24	16	0	26	B	1
In the	□	24	49	1	26	A	5

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
breast , the more southe rn							
Prece ding Regul us closel y	□	21	42	0	0	B	4
The [star] which prece des this one, in the right knee	□	17	53	0	16	B	5
In the right paw [draca]	□	16	6	3	50	A	4
Follo wing, in the other foot	□	18	39	3	47	A	4
In the left paw	□	23	45	3	55	A	4

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
[draca]							
In the left armpi t	□	0	47	0	8	B	4
In the belly, the forem ost of three	□	22	23	2	10	B	6
The more northe rn of the follow ing	□	2	5	5	56	B	6
The more southe rn	□	4	4	2	49	B	6
The prece ding of two in the loins	□	3	13	12	53	B	5
The [star] that follow	□	5	40	14	20	B	2

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
s, bright							
In the haunc h, the prece ding and northe rn of two	□	7	49	9	41	B	3
The follow ing and southe rn	□	9	7	7	50	B	6
In the thigh	□	11	57	6	7	B	3
In the hinder knee	□	13	7	1	40	B	4
The middl e [star] in the foot	□	15	56	0	33	A	4
The lowest in the foot	□	19	26	3	3	A	4
The	□	16	2	12	18	B	1

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
bright [star] in the end of the tail [Dene bola]							
The outer most [star] in the hoof of the left foot *	□	16	31	4	48	A	6
In the hoof of the other foot, the prece ding *	□	16	0	5	43	A	5
The [star] which is nearly in the middl e of the	□	0	13	10	17	B	6

Design ation of the stars body *	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.

VIRGO [m]

Design ation of the stars	Long. Sign	Long. G	Long. M	Lat. G	Lat. M	N/S	Mag.
The northe rn of the prece ding [stars] in the □ [squar e] of the head	□	17	43	6	6	B	5
The southe rn	□	18	32	4	57	B	5
The northe rn of two follow ing, in the face	□	22	6	8	33	B	5

(Table continues into the next batch — VIRGO is incomplete; catchword "Austra-" [the southern].)

Astronomia Danica — Pars Altera, Liber Primus Theoricorum, Cap. 4

The Fixed-Star Catalogue (1600), continued

[The catalogue continues constellation by constellation. Columns are: designation | long. sign (S.) | long.° | long.' | lat.° | lat.' | N/S latitude (B = Borea/northern, A = Australis/southern) | magnitude. The zodiac-sign glyph in the "S." column is small; uncertain glyphs are flagged. B/A appears here in the latitude block as the original places it.]

VIRGO [♍], continued — **

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
The southern (Australis)	♍	21	57	6	10	B	5
At the tip of the southern and left wing	♍	21	31	0	43	B	3
Preceding, fourth in the left wing	♍	29	15	1	25	B	4
The next	♍	4	34	2	50	B	3

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
following							
The small next-to-last	□	9	27	2	23	B	6
The last	□	12	36	1	45	B	4
On the right side, under the girdle	□	5	54	8	41	B	3
In the right and northern wing, of the three preceding	□	29	52	13	36	B	5
The southern of the remaining two	□	1	51	11	37	B	6
The more northern,	□	4	22	16	15	B	3

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
called <i>Vinde miatri</i> x [the Grape- Gathe rer]							
In the left hand, Spica	□	18	15	1	59	A	1
Under the apron, on the right hip	□	15	21	8	10	B	3
On the left thigh, the most northe rn	□	17	57	3	11	B	6
The more northe rn of the two follow ing	□	21	8	1	45	B	6
The more southe rn	□	19	43	0	20	A	6

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
In the left knee	□	24	43	2	24	B	6
The more northern of the two in the upper hem	□	27	48	11	2	B	5
The middle of the three in the hem	□	28	8	7	18	B	4
The lowest and southern	□	28	50	2	57	B	4
The more southern of the two in the upper hem	□	29	50	11	48	B	4
[*]							
In the southe	□	1	21	0	31	B	4

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
rn foot							
In the northe rn or right foot	□	4	29	9	49	B	4
The lower of two, betwe en the Grape- Gathe rer and the girdle [*]	□	1	20	10	26	B	6
Follo wing the one on the right hip [*]	□	21	36	9	40	B	6
The one on the neck [*]	□	27	44	4	59	B	6
The little one follow	□	8	24	16	14	B	6

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
ing the Grape- Gatherer [*]							
Preceding, of the three in a straight line on the northern wing [*]	□	10	10	12	40	B	5
The middle of the same [*]	□	14	45	12	34	B	6
Following [*]	□	22	10	13	7	B	5
The one between the fourth and the fifth [*]	□	22	55	3	22	B	6

Note: the asterisk () in the source marks inform' stars (stellae informes — those outside the figure of the constellation).**

LIBRA [♎] — **

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
The southern pan [of the balance]	☐	9	30	0	26	B	2
The one above the southern pan	☐	8	41	1	55	B	5
The northern pan	☐	13	47	8	35	B	2
The one above the northern pan, to the west	☐	9	39	8	19	B	4
The first from the southern pan, toward	☐	12	25	1	14	B	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
d the east							
The second from the same pan, toward the east	□	16	18	2	59	B	6
The third from the same pan, toward the east	□	19	32	4	28	B	3
The one below this, toward the east	□	21	47	4	4	B	4
The one below the same, toward the west	□	19	26	2	21	B	4

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
The one below the northern pan, towards the east	□	15	45	8	7	B	4

SCORPIUS [m] — **

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
The topmost in the brow	□	27	35	1	5	B	2
The middle in the brow	□	26	58	1	54	A	3
The southern of the three brighter ones in the brow	□	27	24	5	22	A	3
The one	□	27	42	8	27	A	4

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
still more to the south, in the foot							
The most northe rn of the brow	□	29	2	1	42	B	4
The little one in the triangl e [△] with the bright star of the brow and the fifth	□	28	6	0	14	B	5
Prece ding the heart, to the north	□	2	10	3	55	A	4
Glowi ng red in the	□	4	12	4	27	A	1

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
middle, called Antares or the heart							
The one that follows the heart, to the south	□	5	52	5	50	A	4
In the preceding lower feet	□	0	45	6	37	A	5

SAGITTARIUS [↗] — **

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
The more southern of the two in the northern part of the bow	□	0	46	2	0	A	4

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
The more northern in the same part of the bow	□	27	40	2	28	B	4
In the left shoulder	□	6	50	3	31	A	4
Preceding this, in the javelin	□	4	39	3	50	A	5
Preceding, of the three in the head	□	7	55	1	45	B	4
The middle	□	9	27	0	59	B	4
The last	□	10	42	1	31	B	4
The first in the [northern]	□	12	43	3	7	B	4

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
junction							
In the northern junction, the middle	□	13	53	4	17	B	6
Following and higher	□	14	10	6	10	B	5
With this to the east, joined by two faint ones in a triangle [△] form	□	19	7	5	8	B	6
The eastern and last in the upper junction	□	22	51	5	12	B	6
The	□	19	23	1	25	B	6

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
faint one in the lower junction, to the east							
The faint one in the right elbow	□	16	25	3	8	A	6

CAPRICORNUS [♑] — **

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
The northern of the three in the preceding horn	□	28	17	7	3	B	3
The middle	□	28	50	6	53	B	6
The southern	□	28	30	4	41	B	3
The nebulous	□	27	7	7	16	B	6

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
star preceding the upper horn							
The western nebulous star of the base, in the triangle [△] in the front	□	28	56	0	49	B	neb.
The eastern nebulous star	□	29	40	0	28	B	neb.
The topmost in the same triangle	□	29	36	1	20	B	6
The nebulous star	□	27	12	0	24	B	neb.

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
preceding, in the front							
The northern of the two in the neck	□	2	48	3	25	B	6
The southern	□	2	5	0	15	B	6
The faint one preceding, in the right knee	□	1	46	6	58	A	6
Following, in the left knee	□	2	27	9	2	A	6
In the left shoulder	□	6	12	8	8	A	6
The lowest, in the	□	11	23	6	56	A	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
belly							
The northern, following, of two touching under the belly	□	11	59	6	29	A	6
The more eastern of the three in the middle of the belly	□	9	22	4	25	A	6
The lowest of them	□	7	30	4	27	A	6
The northern of the three	□	7	17	3	1	A	5
The forward of the	□	8	20	0	29	A	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
two on the back							
Following the same, on the back	□	12	6	1	16	A	5
Preceding, of the two at the haunches	□	14	24	4	48	A	4
Following the same	□	16	5	4	49	A	4
Preceding, of the two bright ones in the tail	□	16	13	2	26	A	3
Following	□	17	59	2	29	A	3
Preceding, in the upper tail	□	18	13	2	22	B	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
The southern of the rest in the upper tail	□	20	26	0	14	A	5
Preceding this, to the north	□	20	15	0	10	A	6
The northern at the tip of the tail	□	19	53	4	17	B	6

AQUARIUS [♒] — **

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
In the head	□	22	25	15	23	B	6
In the right shoulder, the brighter	□	27	48	10	42	B	3
The fainter and more southe	□	26	35	9	12	B	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
rn							
In the left shoulder	□	17	50	8	42	B	3
The one on the back, under the armpit	□	18	37	6	1	B	5
Following, and the lower of the three in the left hand	□	10	50	4	50	B	5
The middle	□	7	27	8	19	B	5
Preceding, the brighter	□	6	11	8	10	B	4
In the right elbow	□	1	9	8	18	B	3
In the	□	3	3	10	31	B	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
right hand, the more northern							

AQUARIUS [♒], continued — **

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
Preceding, of the two southern remaining ones	♒	3	22	8	53	B	4
Following	♒	4	52	8	10	B	4
Preceding, of the two in the right palm	♒	27	44	2	46	B	4
Following them	♒	28	30	2	30	B	6
In the right thigh	♒	29	52	1	10	A	5
The	♒	23	12	2	0	A	4

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
one at the haunches							
The southern in the right shin, Scheat	□	3	21	8	10	A	3
The northern, or the one at the knee	□	3	4	5	37	A	5
In the left hip	□	29	39	5	40	A	6
The more southern of the two in the left knee	□	26	54	10	48	A	5
The more northern	□	29	49	9	57	A	6

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
In the pouring-out of the water, the first from the hand	□	3	51	4	9	B	4
The southern, succeeding	□	6	3	0	19	A	4
Following, in the first bend of the water	□	8	59	1	24	A	6
The one accompanying it	□	11	37	1	0	A	5
In the other, southern bend	□	11	32	2	49	A	5
Preceding and the	□	10	42	3	58	A	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
more northern of the two following							
Following and the more southern	□	11	10	4	10	A	5
Near this, declining toward the south	□	11	13	4	44	A	5
After this, preceding, of two contiguous ones	□	14	6	10	59	A	5
Following the same contiguous ones	□	14	37	11	33	A	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
In the third bend of the water, the northern of three	□	13	2	14	29	A	5
The middle in the third bend of the water	□	13	45	15	16	A	6
Following, of the three, and the southern	□	14	43	16	23	A	6
The northern of the three following	□	7	53	14	45	A	5
The middle of the	□	8	20	15	30	A	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
same three							
The southern of these three	□	9	49	16	31	A	5
In the last bend, the upper of three	□	4	24	14	25	A	5
The middle	□	4	1	15	40	A	5
The lowest, nearest to Fomahant [Fomahaut]	□	3	16	15	53	A	5
The last in the pouring-out, Fomahant [Fomahaut]	□	28	10	21	0	A	1

PISCES [♓] — **

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
In the mouth of the southern fish	☐	13	1	9	4	B	5
The southern of the two on the back of the head	☐	15	49	7	18	B	4
The northern on the back of the head	☐	17	29	8	55	B	6
Preceding, of the two on the back	☐	19	41	9	3	B	5
Following, on the back	☐	21	55	7	14	B	5
Preceding, on the	☐	17	20	4	27	B	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
belly							
Following, on the belly	□	21	4	3	25	B	5
In the tail	□	27	1	6	24	B	5
Above this, toward the east	□	28	26	7	27	B	6
Following	□	2	28	5	28	B	6
Preceding, of the three brighter ones in the southern cord	□	8	35	2	11	B	4
The middle of the same	□	11	57	1	6	B	4
Following	□	14	18	0	58	B	4
In the bend of the	□	12	24	1	31	A	6

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
cord, prece ding and the northe rn of two small ones							
Follo wing the same, to the south	□	13	45	4	19	A	6
After the bend, prece ding of three	□	17	32	3	3	A	5
The middl e	□	19	55	4	40	A	5
Follo wing and last	□	21	56	7	56	A	5
The bright er, in the knot of	□	23	46	9	4	A	3

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
both cords							
In the northern cord, preceding from the knot	□	22	11	1	39	B	5
After this, the southern of three	□	21	15	1	52	B	5
The middle and the brighter, in the northern knot	□	21	15	5	21	B	4
The northern of three and last in the cord	□	21	35	9	24	B	5
The	□	23	14	22	0	B	6

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
northern of the two in the mouth of the northern fish							

PISCES [□], continued — **

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
The southern	□	22	48	20	43	B	5
The northern of the triangle, in the head	□	19	21	20	55	B	6
The southern of the same triangle	□	18	5	19	24	B	6
The middle and forward of	□	17	2	20	24	B	6

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
the triangle							
Preceding, of the three in the southern backbone, near the left elbow [of Andromeda]	□	17	55	13	21	B	5
The middle	□	18	1	12	22	B	6
The lowest of the three	□	18	8	11	21	B	6
The northern of the two in the belly	□	23	17	17	26	B	5
The one more	□	20	57	15	30	B	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
toward the south							
Following the middle of the three in the southern backbone	□	17	59	12	28	B	5
Following the northern one in the belly, to the north [*]	□	24	10	18	31	B	6
In the back of the head of the northern fish [*]	□	21	25	23	3	B	6

SECONDLY, ON THE STARS THAT APPEAR TOWARD THE NORTH

URSA MINOR, CYNOSURA [the Lesser Bear, Cynosura] — **

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
At the tip of the tail, commonly the Pole Star (<i>polaris</i>)	□	23	1	66	2	B	2
The next-to-last of the tail	□	25	35	69	50	B	4
The one at the root of the tail	□	3	23	73	50	B	4
The upper of the two following	□	21	28	75	0	B	4
The lower of the	□	24	51	77	38	B	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
same							
The upper of the two in the square of the preceding ones	□	7	15	72	51	B	2
The lower of the same	□	14	40	75	23	B	3

URSA MAIOR, HELICE [the Greater Bear, Helice] — **

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
The one on the muzzle	□	17	35	40	2	B	4
Under the left eye	□	17	9	43	55	B	4
Contiguous below this [*]	□	16	7	44	22	B	5
Above	□	18	24	47	50	B	4

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
the right eye							
Above the left eye	□	19	43	47	44	B	4
At the left ear	□	24	41	51	36	B	5
The lowest and preceding in the small triangle of the neck	□	23	49	42	30	B	5
Following, in the same triangle	□	25	1	45	3	B	4
The topmost at the apex of the same triangle [*]	□	27	59	46	21	B	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
On the neck, succeeding the said triangle	□	0	37	42	36	B	4
Following below this	□	3	37	38	15	B	4
In the front left knee	□	0	31	34	34	B	3
The more northern of the two in the right foot	□	25	55	29	15	B	3
The more southern	□	27	9	28	38	B	3
Below the right knee	□	27	6	33	30	B	5
In the right	□	27	25	36	6	B	5

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
knee itself							
The upper of the preceding ones in the larger square	□	9	33	49	40	B	2
The inner of the same square	□	13	42	45	3	B	2
The upper of the following ones of the square	□	25	24	51	37	B	2
The lower of the same	□	24	44	47	6	B	2
The upper of the left of the	□	13	55	29	51	B	4

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag
hind feet							
Following and the more southern	□	15	3	28	45	B	4
In the knee of the preceding of the hind feet	□	22	32	35	14	B	4
Preceding, of the two in the right hind foot	□	0	54	26	14	B	4
Following and the more southern	□	1	35	24	54	B	4
The third-from-last of	□	3	9	54	18	B	2

Designation	S.	L°	L'	Lat°	Lat'	N/S	Mag

the
tail

Pars Altera, Liber Primus Theoricorum, Caput IV — Fixed-star catalogue (1600), northern constellations continued.

The catalogue columns are: **Designation** | **Sign** (zodiacal sign of the longitude) | **Long.°** | **Long.′** | **Lat.°** | **Lat.′** | **B/A** (Borea/Australis — north or south of the ecliptic; here all **B** = north) | **Mag.** (magnitude). All entries in this batch carry **B** (northern latitude).

URSA MAIOR (continued), DRACO, CEPHEUS, BOÖTES

The catalogue continues from URSA MAIOR (Helice), with the final three rows of that constellation before DRACO begins.

URSA MAIOR [Ursa Major] (concluding rows)

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.

The	♈	9	55	56	22	B	2

next-
to-last
of the
tail

The	♈	21	11	54	25	B	2

last of
the
tail

The	♈	17	42	40	6	B	2

unfor-
med
star
betwe-
en the
tail of
this
[Bear]

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.

and
Leo

DRACO [Draco]

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.

The one on the tongu e	♈	18	55	76	17	B	4

In the mouth	♈	4	13	78	15	B	4

Of the two bright ones in the head, the prece ding	♈	6	18	75	21	B	3

The one at the jaw	♈	19	2	80	22	B	4

The follow ing of the bright ones	♈	22	23	75	4	B	3

In the first bend	♈	17	3	81	53	B	5

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
of the neck, of three northern ones, commonly [called] the bright one of the head							
The southern one	□	24	30	77	57	B	5
The middle of the same	□	20	32	79	52	B	5
The one that follows toward the east	□	9	28	80	54	B	4
The one near the second	□	28	32	81	51	B	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
d bend							
The northern one of the second bend [Boreae II flexurae]	□	12	25	82	49	B	3
Of the following side, the northern one	□	15	20	78	9	B	4
The southern one of the same side	□	27	46	79	25	B	4
The preceding of the following triangle	□	15	17	83	5	B	4
The one	□	19	39	80	38	B	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
that follow s toward the south							
The one above this	□	26	43	80	54	B	4
In the remaining triangle, the following	□	6	33	83	4	B	4
The southern one of the same	□	1	27	83	28	B	4
The preceding and northern one of the triangle	□	5	30	84	48	B	4
The one at the bend of the	□	29	43	81	4	B	3

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
third coil							
The one nearest the pole of the zodiac	♈	6	25	86	53	B	4
The one that follows by 2°4′ [?]	♈	28	20	83	18	B	5
The one succeeding this	♈	28	21	81	41	B	5
Nearest the pole, moderately bright	♈	26	50	84	46	B	3
The preceding antepenultimate from the outermost	♈	7	54	78	32	B	3

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
bend							
The antepenultimate, preceding the bend	□	12	27	74	12	B	3
The next-to-last at the bend	□	29	21	71	4	B	3
The one that follows the bend, the second	□	29	16	65	18	B	5
The one that next follows the bend	□	2	9	66	36	B	2
The next-to-last of the tail	□	10	25	61	33	B	3

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
The last of the tail	♏	4	36	57	7	B	3
Between [Draco] and the arm of Cepheus, unformed	♏	1	3	77	31	B	5

Parsing notes: The "Quæ 2,4 sequitur" row (now-marked ♏ 28 20) is ambiguous — both this and the next row read 28°+ in ♏ with very similar values; the "2,4" likely denotes an offset. Longitudes flagged where the sign glyph was small.

CEPHEUS [Cepheus]

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
In the girdle	♏	0	12	71	7	B	3
The bright one in the shoulder	♏	7	12	68	54	B	3
The one in the left shoulder	♏	27	52	62	35	B	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
The one that follows in the tiara, toward the north	□	8	28	61	3	B	4

BOÖTES, OR ARCTOPHYLAX [Boötes]

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
In the left hand, the preceding of three	□	24	8	58	53	B	4
The second	□	25	32	58	51	B	4
The third	□	26	58	60	5	B	4
In the left elbow	□	1	17	54	40	B	4
In the left shoulder	□	13	4	49	34	B	3

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
In the head	□	18	42	54	16	B	3
In the right shoulder above the crown	□	27	28	49	1	B	3
The one at the hip beneath the right arm	□	22	28	40	40	B	3
The lower of two contiguous ones on the back	□	18	15	42	11	B	4
The upper of the same	□	17	16	42	36	B	4
The one in the right leg	□	27	25	27	57	B	3

BOÖTES (concluded), CORONA BOREALIS, HERCULES, LYRA

BOÖTES (concluding rows)

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
The uppermost in the left shin	♈	13	41	28	9	B	3
The middle	♈	12	24	26	33	B	4
The lowest	♈	13	36	25	14	B	4
On the hem of the tunic, called Arcturus	♈	18	38	31	3	B	1
The lowest of three unformed ones, near the right knee	♈	26	12	30	28	B	4
The middle	♈	27	10	31	22	B	4

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
The upper most	□	27	51	33	52	B	4

CORONA BOREALIS, GNOSSIA [Corona Borealis]

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
The bright one of the crown [Alphe cca]	□	6	37	44	23	B	2
The prece ding	□	3	36	46	8	B	4
Above this	□	3	9	48	25	B	5
The one that succee ds, more towar d the north	□	—	—	—	—	B	6
The one that follow s the bright one	□	9	13	44	33	B	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
The next following	□	11	24	44	52	B	4
The one that again accompanies this	□	13	31	46	10	B	4
The last of all	□	—	—	—	—	B	6

Parsing notes: Two rows (the 4th and the last) have blank or illegible longitude/latitude columns in the source — only the sign □ and magnitude 6 are legible. Flagged.

HERCULES [Hercules]

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
In the head	□	10	30	37	23	B	3
In the right shoulder	□	25	26	42	48	B	3
The next-to-last of the right arm	□	23	35	40	6	B	3
The lowest	□	20	5	37	19	B	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
in the right arm							
In the left shoulder	□	9	9	47	47	B	3
In the left arm	□	14	21	49	23	B	4
The preceding in the spoils of the Lion [exuviae Leonis]	□	19	35	51	17	B	4
The following in the triangle of the spoils	□	27	18	52	19	B	4
In the base of the triangle, toward the	□	23	56	53	46	B	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
north							
The middle of those in the spoils	□	23	37	52	47	B	4
The one in the left hip	□	26	1	53	11	B	3
More easterly than this, in the left thigh	□	2	44	53	21	B	3
The preceding of three contiguous ones in the thigh	□	6	20	59	38	B	4
The middle	□	7	18	60	12	B	4
The following	□	9	46	60	14	B	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
In the left knee	□	22	55	60	47	B	3
The one in the left calf near the head of Draco	□	14	16	69	22	B	3
The preceding of three faint ones in the left foot	□	7	4	71	20	B	6
The middle of the same	□	11	6	71	14	B	6
The last	□	17	59	71	5	B	nebulosa [ne]
In the upper right thigh	□	23	7	60	23	B	3

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
More northerly in the same thigh	□	17	38	63	14	B	4
The one in the right knee	□	8	42	65	55	B	4
The one in the upper calf	□	5	56	63	51	B	4
The one in the lower leg	□	2	42	64	23	B	4
The preceding in the right lower leg	□	16	31	62	29	B	5
The one in the shin of the right foot near	□	2	27	60	16	B	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
the ankle							
The outermost in the right foot	□	27	5	57	16	B	4

Parsing notes: The magnitude of "The last" (□ 17 59 / 71 5) reads "ne" in the source, an abbreviation; interpreted as *nova* or a faint/unrecorded magnitude. Flagged.

LYRA [Lyra]

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
The bright one of the Lyre [Vega]	□	9	42	61	48	B	1
The one above the bright one, toward the north	□	13	13	62	27	B	5
The one below the bright	□	12	25	60	26	B	5

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
one, toward the Wain							
The one in the middle of the issuing- forth of the horns	☐	16	9	59	26	B	4
Of two contiguous ones, toward the north	☐	24	3	60	46	B	5
The one toward the south	☐	25	1	59	41	B	5
Of two preceding ones in the cross- bar [in jugol],	☐	13	15	56	5	B	3

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
the northe rn							
The small one benea th this	□	13	2	55	16	B	6
In the cross- bar, of two follow ing ones, the northe rn	□	16	10	55	6	B	3

Parsing notes: The longitude-minutes of "Of two contiguous ones, toward the north" read "24 3." with a trailing dot — taken as 24°3′. The latitude of "Of two preceding ones in the cross-bar" reads with the degrees column blank in some scans; transcribed 56°5′ on best reading.

LYRA (concluded), CYGNUS, CASSIOPEIA, PERSEUS

LYRA (concluding rows)

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
The small one which lies benea th this	□	16	19	54	32	B	6

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
The one nearly in the middl e of the body	□	20	51	58	6	B	5

CYGNUS [Cygnus]

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
In the beak	□	25	43	49	2	B	3
In the head	□	29	19	50	42	B	5
In the middl e of the neck	□	7	32	54	19	B	4
In the breast	□	19	24	57	10	B	3
In the tail [Dene b]	□	29	52	59	57	B	2
The first and bright est in the bend of the	□	10	52	64	28	B	3

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
upper wing							
Of three in the upper hollow [of the wing], the southern	□	—	—	—	—	B	4
The next-to-last of the upper wing	□	12	38	71	31	B	4
The outermost of the upper wing	□	9	35	73	51	B	4
The one in the bend of the lower wing	□	22	8	49	26	B	3
In the middle of it	□	—	—	—	—	B	4

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
The outer most of the lower wing	□	27	42	43	44	B	3
The prece ding in the lowest foot	□	0	31	54	59	B	4
The one that follow s in the lowest knee	□	5	20	56	36	B	4
The southe rn and prece ding of two contig uous ones in the upper foot	□	22	49	63	37	B	4
The follow ing of the	□	24	33	64	18	B	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
same, and more northe rly							
The lower of two unfor med ones follow ing the right wing	□	3	2	50	33	B	4
The upper of the same	□	4	52	51	31	B	4
The new [star] first seen in the year of the Lord 1600 [Nova primu m anno Dn. 1600	□	16	18	55	30	B	3

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.

app.
consp.
]

[Margin: See Book 1, *Progymnasmata* of Tycho Brahe [Tycho Brahe], at the end.
(*Vide lib. 1. Prog. T. B. in fine.*)]

Parsing notes: Three rows (the "southern of three in the upper hollow," and "In the middle of it") have blank/illegible longitude and latitude columns; only sign and magnitude legible. Flagged.

CASSIOPEIA [Cassiopeia]

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
In the head	☐	29	34	44	40	B	4
In the breast	☐	2	16	46	35	B	3
, Schedi r [Sched ar]							
In the girdle	☐	4	37	47	5	B	4
At the bend towar d the hips	☐	8	26	48	46	B	3
At the knee	☐	12	20	46	22	B	3
In the lower leg	☐	19	12	47	29	B	3
The	☐	26	38	48	54	B	4

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
outer most of the foot							
In the left arm	□	6	13	43	6	B	4
In the left elbow	□	5	15	43	28	B	5
In the right elbow	□	24	38	49	24	B	6
In the raisin g of the seat	□	7	5	52	14	B	4
The bright one of the throne [cathe dra]	□	29	34	51	14	B	3
The outer most of the throne	□	25	33	51	8	B	6
The one beside this, at the	□	25	31	52	39	B	6

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
extre mity of the star [group ing]							
The one nearly in a straig ht line with the 11th & 17th [stars]	□	19	27	52	48	B	6
The outer most of the footst ool [scabe llum]	□	22	10	56	13	B	6
The middl e of the footst ool	□	22	32	54	27	B	6
In the footst ool, neares t the	□	21	57	52	8	B	6

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
sole of the foot							
The one that follows the knee	☐	12	56	44	57	B	6
The one that precedes the knee	☐	9	59	45	4	B	6
The whorl of the Navel [Gyrus Umbilici]	☐	6	51	47	31	B	6
The tiny one at the locks of Cassiopeia	☐	29	9	45	38	B	6
The following of two northe	☐	29	31	41	15	B	6

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
rn ones in the rod [virga]							
The prece ding of the same	□	27	56	41	25	B	6
The next- to-last of the rod	□	26	55	39	15	B	6
The outer most of the rod	□	25	53	38	9	B	6

Parsing notes: Several CASSIOPEIA rows carry an asterisk (*) in the margin before the sign glyph in the source — likely a stellar-novelty or cross-reference mark, not part of the coordinates. Latitudes/longitudes in this section are dense and faint; the 6th-magnitude rows are best-reading transcriptions.

PERSEUS [Perseus]

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
In the outer most coil of the right hand	□	18	30	39	0	B	6

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
In the right elbow	□	23	8	37	28	B	4
In the right shoulder	□	24	25	34	30	B	3
The one in the left shoulder	□	19	3	31	34	B	4
The one in the crown of the head	□	21	49	34	26	B	5

PERSEUS (continued), AURIGA (ERICHTHONIUS), COMA BERENICES, OPHIUCHUS

PERSEUS (continued rows)

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
The one that is in the back	□	23	32	30	36	B	4
The shining one in the	□	26	16	30	5	B	2

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
right side							
The one that next follows below	□	27	3	27	59	B	5
The small one following this	□	28	12	27	55	B	5
The one that is at the bend of the same side	□	29	14	27	14	B	5
The one in the left elbow	□	22	5	26	4	B	4
The Head of Medusa, or Algol	□	20	36	22	22	B	3
The	□	20	30	20	54	B	5

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
one beneath Algol							
The one preceding this	□	19	17	20	33	B	4
The preceding toward the north in the same head	□	18	19	21	35	B	4
In the right ham	□	6	12	28	22	B	5
The one that precedes the right knee	□	4	10	28	50	B	4
Going before the bend of the knee	□	3	54	26	11	B	5

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
The middle in the right knee	□	5	13	26	39	B	4
The one below the right knee	□	5	59	24	35	B	6
The one that is in the sole of the right foot	□	8	0	18	56	B	5
The one in the left thigh	□	28	10	22	6	B	4
The one in the left knee	□	0	7	19	4	B	3
In the left lower leg	□	29	22	14	53	B	5
The	□	25	32	12	8	B	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
one in the left heel							
The following of the left foot	□	27	35	11	17	B	3
The unformed one above the head	□	26	44	42	26	B	5
The one in the upper part of the right thigh	□	2	31	29	31	B	5
The unformed one, preceding the head of Medusa	□	16	15	20	53	B	4

Parsing notes: One PERSEUS row ("The unformed one above the head," □ 26 44) carries a marginal asterisk in the source.

ERICHTHONIUS [Auriga] [Erichthonius — the charioteer Auriga]

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
The upper and preceding of two in the head	□	23	37	32	15	B	6
The lower and following	□	24	13	30	50	B	4
The shining one in the left shoulder, the She-goat [capella]	□	16	15	22	51	B	1
The bright one in the right shoulder	□	52 [recte: 22; printed 52 a compositor error	51	21	27	B	2

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
		for 22					
		—					
		Menk alanan , β Aur, □ 22°51′]					
In the right arm	□	23	58	13	44	B	4
The one in the left elbow	□	13	8	20	52	B	4
The prece ding Kid [haed us]	□	13	4	18	8	B	4
The follow ing Kid	□	13	48	18	11	B	4
In the upper foot	□	11	3	10	22	B	4

Parsing notes: "The bright one in the right shoulder" reads "□ 52 51" for longitude — the degrees value 52 is impossible within a sign (max 30). Flagged as a numerical impossible (likely a compositor error or misread; possibly the minutes column shifted). Transcribed as printed.

COMA BERENICES [Coma Berenices]

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
In the cusp of the first and northern triangle	♈	18	16	28	25	B	3
The upper one contiguous to this, following towards the south	♈	18	41	27	23	B	4
The lower of the same	♈	18	45	27	20	B	4
The one that follows the two contiguous ones	♈	19	18	27	7	B	4
The	♈	18	24	25	51	B	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
preceding of two southern contiguous ones							
The other contiguous one toward the east	□	18	47	26	7	B	4
The preceding of all toward the south	□	17	59	23	30	B	4
The uppermost of three following contiguous ones	□	21	9	25	16	B	4
The other and	□	20	50	24	56	B	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
going-before							
The lowest and following	□	22	51	24	0	B	4
The hindmost in the extension of the hair [coma]	□	28	57	32	46	B	4
The one that precedes this	□	27	48	31	41	B	4
The one between these and the first in the cusp	□	24	16	30	16	B	4
The one that is	□	28	14	28	32	B	5

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
in the southe rn cusp of the small triangl e [△ parvi]							

Parsing notes: Several COMA BERENICES rows carry marginal asterisks in the source.

OPHIUCHUS [Ophiuchus]

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
In the head	♏	16	49	35	57	B	3
In the right should er	♏	19	44	28	1	B	3
The lower and follow ing in the right should er	♏	21	4	26	11	B	3
The prece ding in the left	♏	4	58	32	36	B	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	B/A	Mag.
should er							
The follow ing in the same should er	♈	6	15	31	56	B	4

Pars Altera, Liber Primus Theoricorum, **Caput IV** — the fixed-star catalogue for the epoch 1600. This batch completes the northern constellations (the close of Ophiuchus, then Serpens, Sagitta, the Vulture/Aquila with Antinous, Delphinus, Equuleus, Pegasus, Andromeda, Triangulum) and opens the **THIRD division** — **the southern constellations** (Cetus, Orion, Eridanus, Lepus, the beginning of Canis Maior).

Column key for every table below: **designation** | **long. sign** | **long.°** | **long.′** | **lat.°** | **lat.′** | **N/S (B = Borea/northern, A = Australis/southern)** | **magnitude**.
Longitudes are given in zodiacal signs (the printed sign-glyph precedes the degrees); latitudes are ecliptic latitude. Where the source itself leaves a cell blank, the cell is left blank below and noted. The latitude direction-letter is printed as **B** (borealis, north of the ecliptic) throughout the northern constellations and as **A** (australis, south of the ecliptic) throughout the southern division.

OPHIUCHUS (continued from prior batch)

Designation	Sign	°	′	lat °	lat ′	B/A	Mag
Those in the left elbow [Quæ in sinistr o cubito	♏	0	2	23	40	B	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
]							
In the left hand, the more northern [In sinistra a manu borealior]	□	26	43	17	19	B	3
Following and more southern [Sequens & australior]	□	27	56	16	31	B	3
In the right elbow [In dextro ancone]	□	[blank]	[blank]	[blank]	[blank]	B	4
More southern and leading, in the	□	24	12	13	47	B	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
right hand [Australior & præcedens in dextra manu]							
More northern and following, in the same hand [Borealiior & sequens in eadem manu]	☐	25	13	15	20	B	5
In the right knee [In dextro genu]	☐	12	23	7	18	B	3
That in the left knee [Quæ in	☐	3	38	11	30	B	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
sinistr o genu]							
In the right shin [In dextra tibia]	□	[blank]	[blank]	[blank]	[blank]	B	3
The fifth of the unfor med stars, in the Milky Way [Quint a infor mium in via lactea]	□	26	30	33	3	B	4

Parsing notes: three Ophiuchus rows ("In the right elbow," "In the right shin," and the long.°/' of those rows) are printed with the longitude and/or latitude cells blank in the source — transcribed as [blank]. These are not my misreads; the cells are empty on the page.

SERPENS

Designation	Sign	°	'	lat °	lat '	B/A	Mag
Leadi ng, in the	□	11	34	38	12	B	5

Designation	Sign	°	'	lat °	lat '	B/A	Mag
mouth [Præcedens in ore]							
That which is in the mouth [Quæ in ore est]	□	14	23	39	7	B	3
That at the temples [Quæ in temporibus]	□	17	5	35	25	B	3
At the springing of the neck [In eductioe colli]	□	14	20	34	28	B	3
That toward the left eye [Quæ ad	□	15	9	37	29	B	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
sinistrum oculum]							
That toward the nostrils [Quæ ad nares]	☐	16	31	42	37	B	4
The second on the neck, below the head [Secunda in collo infra caput]	☐	12	45	28	58	B	3
In the middle coil of the neck [In medio nexu colli]	☐	16	49	25	36	B	2
The more	☐	18	45	24	6	B	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
southern of three [Australior trium]							
That which is in the second bend [Quæ est in secunda flexione]	☐	20	25	16	27	B	4
Third from the end of the tail [Antepenultima caudæ]	☐	24	33	19	57	B	3
Second from the end [Penul	☐	0	11	20	38	B	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
tima]							
Last [Ultima]	□	10	9	26	59	B	3

SAGITTA

Designation	Sign	°	'	lat °	lat '	B/A	Mag
Upper and more eastern [Superior & oriental]	□	1	31	39	13	B	4
The middle one, or that which leads it [Media seu hanc præcedens]	□	27	54	38	59	B	5
The little one that is above the middle	□	28	30	39	31	B	6

Designation	Sign	°	'	lat °	lat '	B/A	Mag
e [Parvula quæst supra media m]							
The upper of two adjacent stars in the nock [Superior duarum contig uarum in Glyphide]	☐	25	29	38	53	B	4
The lower of the same two [Inferior earundem]	☐	25	38	38	18	B	4

VULTUR [Aquila / Antinous group]

Designation	Sign	°	'	lat °	lat '	B/A	Mag
That which is in the head [Quæ est in capite]	☐	29	27	27	9	B	6
In the neck [In collo]	☐	26	52	26	50	B	3
The bright one on the shoulders [Lucida in scapulis]	☐	26	8	29	22	B	2
The small one above the bright one [Parva quæ supra lucida]	☐	25	32	30	55	B	6

Designation	Sign	°	'	lat °	lat '	B/A	Mag
m]							
That in the left shoulder [Quæ in sinistr o humer o]	□	25	25	31	18	B	3
The small one that follows [Quæ sequitur, parva]	□	26	7	31	59	B	5
Upper and leading, in the lower wing [Superior & præcedens in inferiori	□	21	15	28	47	B	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
ala]							
Lower and following, the wing [Inferior & sequences ala]	☐	22	13	26	35	B	5
Tail of the Vulture [Cauda vulture is — Altair]	☐	14	14	36	17	B	3
The unformed star next preceding the tail [Quæ proxime caudam præcedit	☐	12	43	37	40	B	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
infor mis]							
The middl e of the unfor med stars above the tail [Medi a infor mium supra cauda m]	□	9	11	43	33	B	4
The sevent h of the unfor med, namel y the one that follow s from the three [Septi ma infor	□	9	16	41	5	B	4

Design ation	Sign	°	'	lat °	lat '	B/A	Mag
mium quæ sc. ex tribus sequit ur]							

ANTINOUS

Design ation	Sign	°	'	lat °	lat '	B/A	Mag
Left hand [Manu s sinistr a]	☐	29	20	18	48	B	3
Right side [Latus dextru m]	☐	20	16	20	15	B	3
Knee [Genu]	☐	19	16	14	28	B	3

DELPHINUS

Design ation	Sign	°	'	lat °	lat '	B/A	Mag
The bright star of the tail [Lucid a	☐	8	31	29	8	B	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
caudæ]							
That which follows the tail [Quæ cauda m sequitur]	□	9	47	28	53	B	6
That which is below the tail [Quæ infra cauda m]	□	9	41	27	34	B	6
In the rhombus, the more southern of the leading side [In rhomboide præce	□	10	55	31	58	B	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
d. lateris australi- or]							
The more northern of the same side [Ejusd- em lateris boreali- or]	□	11	49	33	5	B	3
The more southern of the follow- ing side [Seque- ntis lateris australi- or]	□	13	35	32	0	B	3
That which is in the head [Quæ- est in	□	13	51	32	47	B	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
capite]							
That in the leadin g side, imme diatel y prece ding the fourth adjace nt star [Quæ in præce d. latere, III contig. anteit.]	□	10	16	32	9	B	5
Leadi ng of two, the lowest , in the rhom bus [Proce dens duaru	□	9	17	30	42	B	6

Designation	Sign	°	'	lat °	lat '	B/A	Mag
m infima , in rhomboides]							
Following of the same two [Sequences earundem]	□	10	41	30	41	B	6

EQUULEUS

Designation	Sign	°	'	lat °	lat '	B/A	Mag
Leading of the head [Præcedens capitis]	□	17	31	20	13	B	4
Following of the head [Sequences capitis]	□	19	53	21	6	B	4
Leading of	□	17	53	25	16	B	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
the muzzle e [Præcedens oris]							
Following of the muzzle e [Sequens oris]	☐	18	53	24	52	B	4

PEGASUS

Designation	Sign	°	'	lat °	lat '	B/A	Mag
The mouth of Pegasus [Oris Pegasi]	☐	26	21	22	8	B	3
The head [Caput]	☐	1	14	16	25	B	4
That toward the south, in the head [Quæ]	☐	29	44	15	43	B	5

Designation	Sign	°	'	lat °	lat '	B/A	Mag
ad austrum in capite]							
Lower and following, in the mane [Inferior & sequens in juba]	☾	12	59	14	31	B	6
Upper and leading, in the mane [Superior & procedens in juba]	☾	12	43	15	44	B	6
The bright star of the neck [Lucida colli]	☾	10	38	17	41	B	3
Follo	☾	12	24	18	29	B	5

Designation	Sign	°	'	lat °	lat '	B/A	Mag
wing, on the neck [Seque ns in collo]							
Left leg [Sinist rum crus]	☐	3	22	36	43	B	4
Left knee [Sinist rum genu]	☐	8	49	34	19	B	4
Right leg [Dextr um crus]	☐	14	2	41	1	B	4
Leadi ng of two, on the chest [Præc edens duaru m in pector e]	☐	17	28	28	49	B	4
Follo wing [Seque ns]	☐	18	52	29	25	B	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
Right knee [Dextrum genu]	□	20	9	33	8	B	3
In the same knee, toward the south [In eodem genu ad austrum]	□	19	24	34	[?]	B	5
Leading of two, in the wing [Procedens duarum in ala]	□	25	32	25	35	B	6
Following, in the wing, and more southern [Seque]	□	27	5	24	51	B	6

Designation	Sign	°	'	lat °	lat '	B/A	Mag
ns in ala, & austra lior]							
First of the wing, Marka b [Prim a alæ, March ab — α Peg]	☐	17	55	19	26	B	2
The spring ing of the leg, Scheat [Educt io cruris, Scheat — β Peg]	☐	23	48	31	8	B	2
The end of the wing [Extre ma alæ — γ Peg / Algeni b]	☐	3	37	12	35	B	2

Parsing notes: "In eodem genu ad austrum" — the latitude-minutes cell is illegible (ink-bled); degrees read 34, minutes flagged [?].

ANDROMEDA

\.]

Designation	Sign	°	'	lat °	lat '	B/A	Mag
The head [Caput]	☐	8	46	25	42	B	2
The lowest, and the right should er-blade [Infima & scapula dextra]	☐	17	5	27	6	B	5
Lower, in the left should er [Inferior in sinistro humero]	☐	15	24	23	3	B	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
The more southern of three, in the right arm [In dextro brachio trium australior]	☐	14	57	31	33	B	5
The northern [Borealis]	☐	15	44	33	20	B	4
The middle [Medialis]	☐	16	6	32	14	B	5
The more southern, in the upper hand [Australior, in superi	☐	10	27	40	56	B	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
ori manu]							
The more northe rn [Borea lior]	□	11	45	41	44	B	4
The faint one in the same place [Obsc ura ibide m]	□	14	22	42	8	B	5
The highes t of all, in the northe rn hand [Supre ma omniu m in boreal i manu]	□	12	46	43	49	B	4
Leadi ng and	□	15	8	17	48	B	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
upper of two, in the left arm [Præcedens & super. duarum in sinistro brachio]							
That which is in the left elbow [Quæ in sinistro cubito]	☐	16	52	15	58	B	5
The more southern, in the girdle [Australior]	☐	24	48	25	59	B	2

Designation	Sign	°	'	lat °	lat '	B/A	Mag
in cingulo]							
The middle [Media]	□	24	5	30	33	B	4
The northern [Borea]	□	23	35	32	30	B	4
The bright star in the southern foot [In australi pede lucida]	□	8	38	27	46	B	2
The end one, in the upper foot [Extrema in superiori pede]	□	9	5	36	49	B	5

Designation	Sign	°	'	lat °	lat '	B/A	Mag
[★ marg.] The bright er and leadin g, in the right foot [Lucid ior & præce dens in dextro pede]	□	9	5	36	49	B	5
The highes t in the left calf [Supre ma in sinistr a sura]	□	6	51	35	21	B	4
Lower [Inferi or]	□	5	5	28	59	B	5
That towar d the right knee	□	3	22	27	54	B	5

Designation	Sign	°	'	lat °	lat '	B/A	Mag
[Quæ ad genu dextru m]							
[★ marg.]	□	23	59	36	20	B	5
That which is in the last ring of the chain							
[Quæ est in extre mo caten æ annul o]							
The cleare r and upper, in the left should er- blade	□	16	18	24	20	B	3
[Clario r & superi or in							

Design ation	Sign	°	'	lat °	lat '	B/A	Mag
sinistr a scapul a]							

Parsing notes: this is a difficult, heavily-abraded column. Several cells in the Andromeda block are faint at scan DPI. In particular: the magnitude column for the "right foot," "left calf," "lower," "right knee," and "last ring" rows is hard to resolve (the printed digit reads like P in several places, which is a long-s/5 ligature artifact — most likely 5); I have transcribed the most probable reading. The "Lucidior & præcedens in dextro pede" row appears to duplicate the long./lat. of the row above it in the print (both $\square 9\ 5 / 36\ 49$) — flagged below as a possible compositor repetition. **Andromeda block** for the magnitude column and the two near-duplicate foot rows.

TRIANGULUS [Triangulum]

Design ation	Sign	°	'	lat °	lat '	B/A	Mag
At the apex of the triangl e [In apice triang uli]	$\square$	1	18	16	49	B	4
In the base, towar d the north [In basi ad borea m]	$\square$	6	48	20	33	B	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
The middle [Media]	☐	[blank]	[blank]	[blank]	[blank]	B	5
The more southern, in the base [Australior in basi]	☐	7	57	18	57	B	4

Parsing notes: the "Media" row of Triangulum has its longitude and latitude cells blank in the source — transcribed [blank], not a misread.

THIRD DIVISION — OF THE STARS THAT APPEAR IN THE SOUTHERN PART

TERTIO, DE STELLIS QVÆ IN MERIDIONALI PARTE VIDENTVR. ("Thirdly, concerning the stars that are seen in the southern part [of the heavens].")

From this heading onward the latitude direction-letter is **A** (australis — south of the ecliptic) throughout.

CETUS [Cete]

` "Lucida mandibulæ Cete" = Menkar (α Ceti).]

Designation	Sign	°	'	lat °	lat '	B/A	Mag
That which is in	☐	9	30	7	50	A	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
the snout [Quæ in rostro]							
The bright star of the jaw of Cetus [Lucid a mandi bulæ Cete — Menk ar]	☐	8	46	12	37	A	2
The middl e, in the mouth [Medi a in ore]	☐	3	52	12	3	A	3
Leadi ng of three, towar d the cheek [Præc edens	☐	2	1	14	32	A	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
trium ad gena m]							
That below the eye [Quæ infra oculu m]	□	1	53	5	52	A	4
That which is above the eye [Quæ est supra oculu m]	□	6	6	5	36	A	4
In the back of the head [In occipit e]	□	28	28	4	19	A	4
In the chest of the quadri lateral , the	□	24	8	25	17	A	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
leading northern one [In pector e quadri lateri præce d. borea]							
Of the two lower leading stars, the one towards the south [Duarum infer. præcedentium ad austrum]	♏	24	30	28	31	A	4
The southern of those follow	♏	28	10	28	17	A	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
ing, in the chest [Sequentium in pectore australe]							
Leading and northern [Præcedens & borealis]	☐	27	46	25	58	A	3
The middle, in the belly [In ventre media]	☐	12	24	25	1	A	4
The lowest, in the belly [Infima in]	☐	13	49	31	4	A	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
ventre]							
The northern of the belly [Boreas ventris]	♈	16	24	20	19	A	3
Of the two brighter on the back, the more eastern [Duarum lucidorum in dorso orient alior]	♈	10	41	15	47	A	3
The more western of the same [Occidentali	♈	6	10	16	55	A	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
or earundem]							
The northern of the tail [Borealis caudæ]	☐	25	22	10	1	A	3
The southern, or bright star, of the tail [Australis seu lucida caudæ — Deneb Kaitos]	☐	26	55	20	47	A	2
[* marg.] Following the bright jaw-star	☐	12	44	14	30	A	5

Designation	Sign	°	'	lat °	lat '	B/A	Mag
toward its rising, an unformed star [Lucidamandi b. ad ortum sequens infor.]							
[* marg.] Leading the northern of the belly, toward the south [Boream ventris præcedens austrum]	♏	15	3	21	55	A	5
[*]	♏	2	48	9	13	A	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
marg.] That which is in a straig ht line with the 3rd and 5th of the chapte r [Quæ in recta linea cum III. & V. cap.]							

Parsing notes: the "In ventre media" and "Infima in ventre" longitude signs read $\square$ in the print, but their longitude values (12°24', 13°49') sit oddly against the surrounding $\square/\square$ block — transcribed as printed per rule 8. The middle-belly latitude (25°1') and lowest-belly latitude (31°4') are clear.

ORION

` "Sequens seu lucidus humerus" = Betelgeuse; "Lucida in sinistro pede, Rigel" = Rigel (named in the source on p. 232).]

Designation	Sign	°	'	lat °	lat '	B/A	Mag
The highes t of	$\square$	18	10	13	26	A	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
three joined together, in the head [Suprematium conjunctarum in capite]							
[* marg.] The more western [Occidentalior]	☐	18	5	13	54	A	5
[* marg.] The third, which is toward the rising [Tertium quæ ad ortum]	☐	18	32	14	5	A	5

Designation	Sign	°	'	lat °	lat '	B/A	Mag
]							
The following, or brighter should er [Sequences seu lucidus humerus — Betelgeuse]	□	23	11	16	6	A	2
The left, or leading should er [Sinister, seu praecedens humerus — Bellatrix]	□	15	22	16	53	A	2
Following, in the left should er [Seque	□	16	46	17	22	A	5

Designation	Sign	°	'	lat °	lat '	B/A	Mag
ns in sinistr o humer o]							
That which is in the right arm [Quæ in dextro brachi o]	□	25	3	14	51	A	4
In the right forear m [In dextra ulna]	□	28	29	11	30	A	6
In the right hand, the more southe rn [In manu dextra austra lior]	□	27	22	9	15	A	4
Leadi ng, in the	□	26	20	8	44	A	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
right hand [Præcedens in dextra manu]							
Next to the highest, in the right hand [Proxima suprema in dextra manu]	□	27	21	7	21	A	6
The highest and last of those in the hand [Suprema & ultima earum quæ in manu]	□	28	7	7	19	A	6
Leading of	□	23	8	3	13	A	5

Designation	Sign	°	'	lat °	lat '	B/A	Mag
two, in the club [Præc edens duaru m quæ in colobr o]							
Follo wing of the same [Seque ns earun dem]	□	25	20	3	21	A	5
That which is below the right should er, towar d the west [Quæ est infra dextru m humer	□	18	55	19	18	A	5

Designation	Sign	°	'	lat °	lat '	B/A	Mag
um ad occas.]							
Of two faint stars on the back, the following [Ex duabus obscuris in dorso sequens]	□	17	39	19	37	A	6
Leading of the same [Præcedens earundem]	□	16	45	19	53	A	6
That which leads, of four on the back [Quæ ex quatuor]	□	15	33	20	9	A	5

Designation	Sign	°	'	lat °	lat '	B/A	Mag
or in dorso præcedit]							
On the shield, the most northern of nine [In clypeo novem borealissima]	☐	7	52	8	17	A	4
The second [Secunda]	☐	8	47	9	7	A	4
The third [Tertia]	☐	8	9	11	6	A	6
The fourth [Quarta]	☐	7	59	12	26	A	4
The fifth [Quinta]	☐	6	48	13	4	A	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
The sixth [Sexta]	□	6	22	15	27	A	4
The seventh [Septima]	□	6	32	16	50	A	4
The eighth [Octava]	□	6	57	20	2	A	4
The last [Ultima]	□	7	56	20	56	A	4
First of the Belt [Prima Balthei]	□	16	49	23	38	A	2
The middle [Mediana]	□	17	53	24	34	A	2
The last [Ultima]	□	19	5	25	22	A	2
That which	□	14	36	25	37	A	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
is in the hilt of the sword [Quæ in manu brio ensis]							
The highest of three on the sword [Suprematium in ense]	☐	17	27	28	10	A	5
The middle of the sword [Mediana ensis]	☐	17	23	28	45	A	3
The southern [Australis]	☐	17	26	29	17	A	3
Leading of	☐	16	19	30	38	A	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
two below the sword [Præc edens duaru m infra ensem]							
Follo wing of two below the sword [Seque ns duaru m infra ensem]	☐	18	22	30	38	A	5
The bright star in the left foot, Rigel [Lucid a in sinistr o pede,	☐	11	16	31	12	A	1

Designation	Sign	°	'	lat °	lat '	B/A	Mag
Rigel — β Ori]							
That which is in the left heel [Quæ in sinistr o calcan eo]	☐	12	14	29	53	A	4
That which is in the calf of the left foot [Quæ in sura sinistr i pedis]	☐	14	1	31	0	A	5
In the right knee [In genu dextro]	☐	20	48	33	8	A	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
[★ marg.] That which prece des the last star of the Belt, towar d the south [Quæ ultima m balthe i præce dit ad austr.]	☐	18	38	25	1	A	4
[★ marg.] That which is towar d the back, prece ding this one [Quæ ad dorsu	☐	14	33	19	40	A	6

Designation	Sign	°	'	lat °	lat '	B/A	Mag
m est, hanc præce dens]							

Parsing notes: the four-star and nine-star "shield" sequences and the belt/sword cluster are dense; magnitudes for "Quæ in sura sinistri pedis" (lat. 31°0') and "In genu dextro" (33°8') read with moderate confidence. The Rigel magnitude reads **1** (the source's brightest non-Sirius mark). No impossible values detected in the Orion block.

ERIDANUS

Designation	Sign	°	'	lat °	lat '	B/A	Mag
That at the left foot of Orion, at the begin ning of the river [Quæ ad sinistr um pede m Orioni s in princi pio fluvii]	♏	9	39	31	36	A	4
Above	♏	9	41	27	55	A	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
Orion's foot, in the river [Suprapedem Orionis in fluvio]							
Following, of two others [Duarum aliarum sequens]	☐	7	38	29	52	A	5
Leading [Præcedens]	☐	5	28	27	52	A	5
Following of the two upper ones [Sequens duarum superiorum]	☐	3	44	25	34	A	5

Designation	Sign	°	'	lat °	lat '	B/A	Mag
Leading of the same [Præcedens earundem]	□	1	13	25	12	A	4
After an interval, the following of four [Post intervallum sequens equator]	□	18	17	33	14	A	3
That which leads this one [Quæ præit hanc]	□	[blank]	[blank]	[blank]	[blank]	A	[blank]
That which is toward the north,	□	15	6	28	47	A	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
or the third leading g [Quæ ad septent. est, seu tertia præcedens]							
That which precedes all four [Quæ omnes quatuor antecedit]	☐	12	44	27	47	A	3

Parsing notes: the "Quæ præit hanc" row has all of its longitude, latitude, and magnitude cells blank in the source (only the sign ☐ is printed) — transcribed [blank], not a misread.

LEPUS

Designation	Sign	°	'	lat °	lat '	B/A	Mag
The upper of the leading ear	☐	10	13	34	34	A	5

Designation	Sign	°	'	lat °	lat '	B/A	Mag
[Superior præcedentis auris]							
The lower of the same ear	□	10	19	35	54	A	5
[Inferior ejusdem auris]							
The upper of the follow ing ear	□	12	26	35	18	A	6
[Superior seque ntis auris]							
The lower of the follow ing ear	□	12	13	36	14	A	5
[Inferior seque ntis							

Designation	Sign	°	'	lat °	lat '	B/A	Mag
auris]							
That which is in the head [Quæ est in capite]	☐	9	48	39	4	A	5
The end star of the forefeet [Extrema anteriorum pedum]	☐	6	24	45	0	A	4
That on the back, or middle of the body [Quæ in dorso, seu medio corpor	☐	15	48	41	6	A	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
e]							
In the left shoulder [In armosinistro]	□	14	9	43	55	A	3
The more southern of two in the hind feet [Australior duarum in posterioribus pedibus]	□	19	20	45	50	A	3
The more northern of the same [Borealior earundem]	□	21	35	44	18	A	3
Leadi	□	20	25	38	16	A	4

Designation	Sign	°	'	lat °	lat '	B/A	Mag
ng, on the back [Præcedens in dorso]							
Follo wing, on the back [Sequens in dorso]	☐	23	26	37	41	A	4
The last, in the tail [Ultima in cauda]	☐	26	21	38	26	A	4

CANIS MAIOR

Designation	Sign	°	'	lat °	lat '	B/A	Mag
The most splendid star in the mouth , called Sirius [In ore]	☐	8	34	39	30	A	1

Designation	Sign	°	'	lat °	lat '	B/A	Mag
splendidissima Sirius vocata — α CMa]							
That on the forehead, toward the right ear [Quæ in fronte ad dextram aurem]	☐	19	0	34	50	A	4
In the middle of the forehead [In media fronte]	☐	11	26	36	43	A	5
That below the left	☐	14	5	38	3	A	3

Designation	Sign	°	'	lat °	lat '	B/A	Mag
ear [Quæ sub sinistr a aure]							
In the neck [In collo]	□	12	2	39	30	A	4
In the right should er of the forefe et [In armo dextro anteri orum pedu m]	□	6	31	42	13	A	5
That at the tip of the front foot [Quæ in extre mitate pedis prioris]	□	1	41	41	19	A	2

Designation	Sign	°	'	lat °	lat '	B/A	Mag
That on the back [Quæ in dorso]	☐	15	29	46	10	A	5

The Canis Maior table continues into the next batch (catchword "Media" — the following row begins "Media ...").

Astronomia Danica — Pars Altera, Liber Primus Theoricorum, Caput IV (continued)

Fixed-star catalogue (southern constellations, continued)

The catalogue resumes with CANIS MAIOR (the Great Dog), continuing from the prior batch. Columns: designation | longitude sign | longitude° | longitude' | latitude° | latitude' | N/S (Borea/Australis; here all A = Australis/southern) | magnitude.

CANIS MAIOR (the Great Dog, continued)

Designation	Sign	Long.°	Long.'	Lat.°	Lat.'	N/S	Mag.
The middle one on the breast [Medi a in pector e]	☐	12	35	46	40	A	5
The one on the belly	☐	17	54	48	30	A	3

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
[Quæ in ventre]							
On the belly, between the hind thighs [In ventre inter poster iora femor a]	□	15	20	51	25	A	3
The lower of the front feet, on the right [Inferi or dextri pedis prioru m]	□	1	6	51	47	A	3
The one on the tail [Quæ in	□	24	10	51	25	A	3

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
cauda]							

Parsing notes: The Canis Maior longitude column has the structure Sign ♎ | Long.° | Long.′ throughout; the longitude-degree values are 12, 17, 15, 1, 24 and the longitude-minutes are 35, 54, 20, 6, 10. The latitude of the dextri-pedis row reads 51° 47′.

CANIS MINOR (the Little Dog)

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
On the neck [In collo]	♋	16	38	13	34	A	3
On the thigh, Procy on [In femor e, Procy on]	♋	20	17	15	57	A	2

Parsing notes: The longitude-degree column for the two Canis Minor rows reads 16 and 20 respectively (sign ♋ Cancer).

ARGUS (the ship Argo) [ARGVS]

Design ation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
The one on the highes t part of the stern	♋	5	52	43	19	A	3

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
[Quæ in supre ma puppi]							
The highes t of the ship's shield [Supre ma clypei navis]	☐	0	34	44	59	A	3
The one prece ding the shield [Præc edens clypei]	☐	27	59	47	28	A	3

HYDRA

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
The prece ding one on the head [Præc edens	☐	5	38	14	37	A	5

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
in capite]							
The one above the first, toward the north [Supra prima m ad aquilonem]	☐	6	45	14	17	A	4
The more northerly one on the back of the head [Borealiior in occipite]	☐	6	47	11	8	A	5
The one that precedes the third	☐	7	21	11	36	A	5

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
toward the south [Quæ tertia m ad austrum præit]							
The most easterly of all on the head [Omnium in capite orient alior]	☐	8	59	11	1	A	4
The one that precedes on the neck [Quæ in collo præcedit]	☐	11	50	11	6	A	6
The one that follow	☐	14	40	13	5	A	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
s in the extens ion of the neck [Seque ns in educti one colli]							
The middl e one of the neck, prece ding the three at the coil [Medi a colli, & præce dens trium in nexu]	□	20	10	15	0	A	5
The northe rn of the three at the	□	22	3	14	18	A	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
bend of the neck [Boreas triumph in flexu colli]							
The southern one at the coil [Australis in nexu]	♏	19	52	16	46	A	5
The bright star of Hydra, or the heart [Lucida hydræ, sive cor]	♏	21	44	22	24	A	1
The one that next follows the heart [Quædam]	♏	27	11	26	34	A	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
proxime cor sequitur]							
The one that next follows this [Quæ hanc deinde sequitur]	□	0	8	26	12	A	5
The preceding of two contiguous ones above this [Præcedens ex duabus contigu is supra hanc]	□	—	—	—	—	A	5
The	□	—	—	—	—	A	4

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
one that follows the same [Sequences earundem]							
The fifth one from the heart [Quæ à corde quinta est]	□	9	30	24	38	A	4
The one in a straight line with this and the next [Quæ in recta linea cum hac &	□	12	40	23	31	A	5

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
seque nte]							
The one next prece ding the Crater [Crate ra proxi me præce dens]	□	14	50	21	49	A	4
Unfor med, next prece ding the head [Infor mis, caput proxi me præce dens]	□	4	44	12	27	A	4

Parsing notes: The two rows "Præcedens ex duabus contiguis supra hanc" and "Sequens earundem" have their longitude and latitude columns blank in the source (only the sign □, the N/S marker A, and magnitude are printed).
Transcribed as printed.

CRATER (the Cup)

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
The one that is at the base of the Crater [Quæ est in basi crateris]	☐	18	12	22	41	A	4
The following of two in the middle [Sequens duarum in medio]	☐	23	42	19	29	A	4
The preceding of the same [Præcedens earundem]	☐	21	9	17	25	A	4

CORVUS (the Crow) [CORVVS]

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
The one at the eye [Quæ ad oculum]	♄	6	7	19	39	A	4
The preceding of the two upper ones, in the square [Præcedens duarum superiorum in ♄]	♄	5	12	14	25	A	3
The following of the same [Sequens earundem]	♄	7	54	12	7	A	3
The	♄	11	48	17	59	A	3

Designation	Sign	Long.°	Long.′	Lat.°	Lat.′	N/S	Mag.
following of the lower ones, in the square [Sequences inferiorum in quadrato]							

Part II — On the mean motions of the fixed stars

II. Now that the canonical designation of the stars has been set out, it follows that we should determine here, in a straightforward way, the motions of those same stars — motions that appear exceedingly slow in the heavens, as does the shifting of the oblique path of the Sun — by comparing the ancient observers both among themselves and with our own. From this comparison arise the mean motions [medii motus] that are chiefly in use in astronomy. In the following section of this same chapter we shall reduce these mean motions, by means of a suitable hypothesis and the tables to be derived from it, to true or apparent values.

In the earlier commentary on the stars, when occasion offered, I showed accurately — unless I am mistaken — from a comparison across various ages, beginning with Timochares [Timocharis] and using the observations of the practitioners as mutually checked against one another, that the mean motion by which the fixed stars are seen [Margin: *Almagest* lib. 1, cap. 2] to be carried in the heavens, in one equal year, is

(p. 217 running header: "Cap. 4. LIBER PRIMVS. 217"; the prose carries over to p. 218.)

49 seconds 45 thirds [49" 45"]. Therefore, now, without further delay or tedious repetition, we shall arrange the part of the following table dealing with the

simple precession of the equinoxes [praecessio aequinoctiorum] into the antecedence of the signs — or, what amounts to the same thing, the motion of the fixed stars into the consequence of the signs — for the ages of the world.

As for what concerns the anomaly of the equinoxes [anomalía aequinoctiorum], of the obliquity of the Sun [obliquitas Solis], and hence the change of latitude in the stars: having observed all these phenomena through various ages, restored them, and weighed them with the utmost diligence, we find that this revolution is completed in a period of 3600 solar years. Its beginning is rightly reckoned from the creation of the world, according to our determination of that epoch — namely, when the maximum obliquity of the Sun coincided most exactly with the mean and true equinox at the world's beginning. For from this assumption everything else that we require for accounting for these inequalities follows from a single, common hypothesis — and not, as Copernicus left it, from a double hypothesis resting moreover on a straight line.

It has seemed fitting to report here the observations from the heavens that must be brought to bear, both for establishing the mean motion of this anomaly and for treating its **προσαφάσεις** [Greek: προσαφάσεις (prosaphairesis, "subtractive correction" — the subtraction applied to obtain the true from the mean)] in the following section of this chapter. Concerning the alteration of the obliquity of the ecliptic, then: when we first established from our own examination that its mean value was $23^{\circ} 42'$, we found (to repeat here what we noted before) that the maximum, in the year of the world 3600 — a little before the beginning of the Monarchy of the Greeks — was $23^{\circ} 53'$, but that the minimum, in the year of the world 5400, was $23^{\circ} 31'$ and a half, and thus that the difference falls within 22 minutes. [Margin: *Almag.* lib. 1, cap. 2] Hence, down to the times of Eratosthenes and Hipparchus — whose observations on this matter Ptolemy [Ptolemaeus] cites — the maximum obliquity of the Sun had duly decreased to about $23^{\circ} 51\frac{1}{4}'$.

And although Ptolemy himself asserts that he found this same value in his own time, and apparently held it to be fixed along with the eccentricity of the Sun, nevertheless, when we weigh the fact that he neglected the difference of the Sun's parallax at the limit of each solstice, the true obliquity in Ptolemy's time — once this is added — will be about $23^{\circ} 49'$, as our supposition implies. Likewise, in the age of al-Battānī [Albategnius], that maximum obliquity comes out, on our supposition, as $23^{\circ} 35\frac{1}{2}'$, which agrees nicely with his observation. The minimum obliquity of the Sun, however, was around the year of the world 5400, as I just said above [Margin: Lib. 1, Prog. pag. 38, and in the commentary], $23^{\circ} 31'$.

Afterward, in Copernicus's time, the same obliquity, increasing once more, was $23^{\circ} 31\frac{2}{3}'$. [Margin: *Solis praemiss.* Lib. 1, Prog. pag. 47] For although Copernicus himself set it at $23^{\circ} 28'$ — a difference of $3\frac{1}{3}'$ — Tycho's refinement [Tychonis lima] had nonetheless removed this, since the observation of Walther of Nuremberg [Gualtherus Noribergensis], made 60 years before Copernicus, when duly applied by Tycho [Tycho Brahe], extended to $23^{\circ} 31'$. [Margin: See the observations of the Sun put forward in the commentary]

Finally, in Tycho's time and our own, the maximum obliquity — on this same supposition, by which the maximum altitude of the Sun was each year determined at Wandsbeck [Vraniburgi/Uraniborg] by manifold and undoubted observation [Margin: in the commentary on the same] as $57^{\circ} 35\frac{1}{2}'$ (the parallax added and the elevation of the equator subtracted) — was found to be $23^{\circ} 31\frac{2}{3}'$.

From all this it is sufficiently clear that the increase and decrease of the maximum obliquity of the ecliptic has been correctly established by us, together with its anomaly and period. And although we here employ Julian years in place of solar years, nonetheless — on account of the difference which, over the span of 6000 years, amounts by anticipation to 46 days, but in the true motion of the anomaly amounts to scarcely as many seconds — no error sensible either here or to the inequality of the equinox can arise. As for that inequality, just how it conforms to this single supposition will be clear to anyone who examines it by comparison, both above in the commentary on the Sun and on the fixed stars, and still more evidently it will be made manifest when, in the chapter immediately following, we shall treat attentively of those things which establish the measure of the tropical year, to be compared with one another absolutely across several intervals of ages.

(p. 218 running header: "218 THEORICORVM Cap. 4.")

Table of mean motions of the equinoxes and of the obliquity of the ecliptic, in Julian years

[Tabula mediorum motuum aequinoctialium & obliquitatis eclipticae in annis Iulianis.]

(p. 219 running header: "Cap. 4. LIBER PRIMVS. 219")

Left block — by epochs and by centuries. Columns: Years (Epoch) | Precession of the equinox [S° | G(deg) | M | S(sec)] | Anomaly of the equinox & obliquity [S° | G | M].

Years (Epoch)	Prec. S	Prec. °	Prec. '	Prec. ''	Anom. S	Anom. °	Anom. '
Mundi (World)	0	0	1	49	0	0	13 ½
Christi (Christ)	1	24	50	53	1	6	37 ½
20	0	0	16	36	0	0	2
40	0	0	33	11	0	0	4
60	0	0	49	47	0	0	6
80	0	1	6	23	0	0	8
100	0	1	22	58	0	0	10
200	0	2	45	57	0	0	20
300	0	4	8	55	1	0	0
400	0	5	31	54	1	0	10
500	0	6	54	52	1	0	20
600	0	8	17	50	2	0	0
700	0	9	40	49	2	0	10
800	0	11	3	47	2	0	20
900	0	12	26	46	3	0	0
1000	0	13	49	44	3	0	10
1100	0	15	12	42	3	0	20
1200	0	16	35	41	4	0	0
1300	0	17	58	39	4	0	10
1400	0	19	21	38	4	0	20
1500	0	20	44	36	5	0	0
1600	0	22	7	34	5	0	10
1700	0	23	30	33	5	0	20
1800	0	24	53	31	6	0	0
1900	0	26	16	30	6	0	10
2000	0	27	39	28	6	0	20

Years (Epoc h)	Prec. S	Prec. °	Prec. '	Prec. "	Anom. S	Anom. °	Anom. '
2500	1	4	34	20	8	0	10
3000	1	11	29	12	10	0	0
3500	1	18	24	4	11	0	20
4000	1	25	18	56	1	1	10
4500	2	2	13	48	3	0	0
5000	2	9	8	40	4	0	20
5500	2	16	3	32	6	0	10
6000	2	22	58	24	8	0	0
6300	2	27	7	19	9	0	0

Parsing notes: The anomaly leading "sign" figures for rows 4000–6000 are 1, 3, 4, 6, 8, consistent with the anomaly's advance of one sign per 300 years (one revolution of 12 signs in ~3600 years). The "Christi" anomaly-minutes entry reads "37 ½".

Right block — "In single years, up to 20" [In annis sing. usque ad 20]. Columns: Years | Precession of equinox [M | S] | Anomaly of equinox & obliquity [° | '].

Years	Prec. '	Prec. "	Anom. °	Anom. '
1	0	50	0	6
2	1	40	0	12
3	2	29	0	18
4	3	19	0	24
5	4	9	0	30
6	4	59	0	36
7	5	48	0	42
8	6	38	0	48
9	7	28	0	54
10	8	18	1	0
11	9	7	1	6
12	9	57	1	12
13	10	47	1	18

Years	Prec. '	Prec. "	Anom. °	Anom. '
14	11	37	1	24
15	12	27	1	30
16	13	17	1	36
17	14	6	1	42
18	14	56	1	48
19	15	46	1	54
20	16	36	2	0

Sub-table — "In months" [In menfibus]. Columns: Month | Precession [M | S] | Anomaly [M | S].

Month	Prec. '	Prec. "	Anom. '	Anom. "
January [Ianuar.]	0	4	0	30
February [Februar.]	0	8	1	0
March [Marti.]	0	12	1	30
April [April.]	0	17	2	0
May [Majus]	0	21	2	30
June [Iunius]	0	25	3	0
July [Iulius]	0	29	3	30
August [August.]	0	34	4	0
September [Septemb.]	0	38	4	30
October [Octob.]	0	42	5	0
November [Novemb.]	0	46	5	30
December	0	50	6	0

Month	Prec. '	Prec. "	Anom. '	Anom. "
[Decemb.]				

Part III — The hypothesis of precession and of the inequalities of the equinoxes

III. At last, the foregoing parts of this chapter being thus dispatched — since we have advanced to the constitution of a hypothesis by which the inequalities of the equinoxes and the maximum obliquity of the Sun can be legitimately accounted for — I thought it should first of all be pointed out that Lord Tycho's [Margin: Lib. 1, Prog. pag. 250] conjecture about these matters is without doubt the true one: namely, that they arise from some external cause and by accident.

And although they can be demonstrated in various ways even with the Earth at rest, in the orbits of the Sun and of the fixed stars (as we shall suggest below), nevertheless — since I have long since, from many and varied arguments, persuaded myself that the Earth, set in the middle of the visible world, not only most conveniently exhibits the reality of the apparent daily revolution in the heavens by its own rotation eastward about its center, but is also most rightly assigned, with Copernicus, the precession of the equinoxes into the antecedence [of the signs] — lest, among other things which I recall mentioning above on behalf of this argument [Margin: Cap. 1 of this book], there be introduced a contrariety of motions or an excessive violence, from which nature shrinks; and lest, at the same time, the stars fixed in the most limpid heaven should suffer the inconvenience of being drawn apart from one another (which has never been observed), if you should presume each one to be set in motion separately by some carrying-motion — although perhaps this too might somehow be conceded in an expansive celestial firmament.

For this reason we represent the regressions of the equinoxes in this manner: namely, by laying down that the poles of the Earth gradually, and with the slowest motion, regress into the antecedence [of the signs] about the axis of the ecliptic — an axis imaginary even within the Earth. Whence such an anticipation arises at the equator, from the ecliptic (or apparent path of the Sun) as they apparently intersect it. For neither on this supposition, nor on the Copernican one, is it necessary to recognize an equator anywhere in the heaven outside the Earth's globe, but only an ecliptic — whose course the Sun above all, and then all the remaining planets following back and forth (a thing marvelous to behold), do venerate throughout all the periods of their courses, [thereby honoring] the power and majesty given to the Sun itself by the Creator.

Furthermore, the motion of the inequality of the equinoxes, and also that of the obliquity of the ecliptic — whose effects we perceive in the heavens, namely in the apparent course of the Sun and in the positions of the stars — we attribute to the mobility of the axis of the ecliptic that is fixed through the Earth, by way of its poles on the surface of the globe. Whence it comes about that the Earth, immune from this motion (and not, as Copernicus supposed, entangled in another libration than that which simply proceeds through its poles, in excuse of the simple anticipation of the equinoxes — as I said before), has these inequalities of the Sun's orbit alone introduced [to it], as the following supposition will show.

The hypothesis of precession, likewise of the inequality of the equinoxes, and of the obliquity of the ecliptic, all together

[Hypothesis praecessionis, item inaequalitatis aequinoctiorum, & obliquitatis eclipticae simul.]

Diagram. A geometric construction (referred to in the text as "the following figure," but the figure itself falls on the next page beyond this batch; only its lettered description is given here in the prose). The construction is built up as follows from the text:

- A (center): the center of the underlying [subject] pole of the northern terrestrial ecliptic. About A two homocentric circles are first described.
- B C (arc): represents part of the little arctic circle of the ecliptic — the arc which the Earth's pole has traversed from C to B about the center of the ecliptic, by simple motion from the creation of the world into the antecedence of the signs. This arc measures the anticipation of the equinoxes and solstices, and the apparent motion of the fixed stars into the consequence [of the signs].
- N H M: the mean ecliptic, described from the same subject pole of the ecliptic A.
- N L M: the celestial equator, described (for the sake of demonstration) about center B.
- F E D G (little circle): described about center A; the true pole of the ecliptic, on the surface of the terrestrial globe, movable about its own axis into the consequence of the signs, is supposed to have changed its apogee in accordance with the motion C B.
- F (position): when the true pole of the terrestrial ecliptic is set at F, the maximum distance of the ecliptic from the equator occurs; this is known

through the interval K L, to which F B is equal (as demonstrated in the doctrine of the sphere).

- D (position): as the subject of the movable ecliptic pole ascends from F through E into D, there will be the minimum obliquity L I, to which B D corresponds.
- E and G (lateral positions): when the same [pole] is situated to the side at E or at G, the mean obliquity is indeed maintained, but the **προσαφάιρεσις** [prosaphairesis] of the equinoxes and solstices is at its maximum — there at E subtractive [ablativa], here at G additive [adjectiva] — at the tropical and equinoctial points.

K L, L I (intervals): the intervals on the equator/ecliptic figure marking, respectively, the maximum obliquity (K L = F B) and the minimum obliquity (L I, corresponding to B D).

(p. 220 running header: "220 THEORICORVM Cap. 4.")

The text breaks off mid-sentence at "in the tropical and equi-[noctial points]"; it continues in the next batch.

—at the equinoctial points, and so throughout the whole ecliptic, just as is plain enough from the very diagram of the hypothesis. Indeed, in that diagram there is comprehended far more than can be uttered here, concerning the universal condition and transformation of the nations, embraced through all the wondrous ages of the world.

Now, since we are here dealing with the surface of the globe, where it is appropriate to derive the equations geometrically by means of a spherical triangle, we determine the semidiameter, or radius, of the little circle AF as 10' 53", for the purpose of the foregoing examination of the obliquity of the ecliptic and the inequality of the equinoxes, which was established above from the observations. This is in the ratio that the third number in the series of perfect [values], namely 496, bears to a quadrant of the circle on the surface, 5400 — both understood in arcminutes.

We elicit the prosthaphaereses of the obliquity and of the equinoxes by means of the spherical triangle in the following way. In the spherical triangle ABG, as I said, there is given AB = 23° 42', the mean distance of the poles; and likewise AG = 10' 53"; and finally the angle BAG, the complement of the anomaly of the semicircle, that is, the arc DG. Therefore, from the calculation, the arc BG is obtained, which gives the measure of the obliquity of the ecliptic; and the angle ABG, to which an equal angle in the ecliptic HBS will be opposite, here

determines the subtractive inequality of the equinoxes — by the arc HS at the summer solstice, or by the arc NV at the vernal equinox. Proceeding likewise in the remaining cases, we constructed the first two columns of the following table.

Furthermore, as regards the proportional minutes [scrupula proportionalia] inserted into the same table, for the compact investigation of the latitudes among the fixed stars, these too we obtained by the following method. From the equinoctial point let two arcs run out to the completion of a quadrant, AB and AC, forming the angle BAC. From the demonstration above, the maximum alteration of the ecliptic is determined as 21' 46"; in place of which, if you reduce it to 1° in 60', and apportion the angle at A, then easily, in the right-angled spherical triangle ADE, you will hunt out with us the side ED itself and its double EF — which indicates the proportional minutes at any supposed value within the quadrant.

These are the things that constitute our hypothesis of the equinoxes and solstices, arising from the motion of the Earth and of the pole upon the ecliptic. But if anyone should wish to exhibit the same phenomena with the Earth either resting from all motion, or content with the diurnal revolution alone, he will necessarily separate the simple progression of the stars from the inequality of the equinoxes and from the maximum solar alteration of the obliquity — attributing the former to the starry heaven in the consequence of the signs, and the latter to movable poles of the solar orbit upon the surface of the sphere itself. Thus he will proceed by the same path of demonstration as we have, and he will change nothing of the tables we have constructed beforehand.

Diagram. A large schematic of the precession / obliquity hypothesis, showing two homocentric circles about the ecliptic pole and, at the center, the little circle of the movable true ecliptic pole, with the zodiacal signs ranged around the outer circle.

- B (center): common center of the construction; the pole's mean position.
- Little central circle about the ecliptic pole: C (left), D (top, near B), A (center label), E (right), G (lower-left), F (bottom) — the points of the small circle F E D G (F = maximum obliquity, D = minimum; E/G = lateral positions of maximum prosthaphaeresis); triangle ABG, arc DG per the text.
- Upper colure stack (top to bottom): L (top of outer circle); a box with Q (upper-left) and P (upper-right) enclosing ☊ Cancer; then R (left), H (center), S (right); then K — the summer-solstice colure markers (arc HS, "penes solstitium aestivum").

- Left arc (top to bottom): ♊ Gemini + X (on the inner circle); ♌ Leo + M; ♎ Libra (autumnal-equinox marker).
- Right arc (top to bottom): ♉ Taurus + O; ♈ Aries + N; ♐ Pisces + V (Aries and Pisces straddle the vernal equinox; arc NV); ♒ Aquarius [?] (lowermost right, faint).
- Bottom of vertical axis: ♑ Capricorn (winter-solstice marker, opposite Cancer).

Parsing note: the peripheral zodiacal glyphs are read against the cardinal frame (Cancer at top, Capricorn at bottom, vernal equinox on the right, autumnal equinox on the left). Only the lowermost-right glyph (read ♒ Aquarius by sequence) remains uncertain at scan resolution.

[Table: Prosthaphaereses of the Equinoxes]

Tabula Prosthaphaereseon Æquinociorum [Table of the Prosthaphaereses of the Equinoxes]

This page presents the first half of a six-figure table. Each "fig." block has three columns: *Æquator. æquinoc. S* (Equator of the equinox, S), *Excessus obliquit. A* (Excess of obliquity, A), and *Scrupula proportionalia* (Proportional minutes). The leftmost and rightmost columns give the degree argument (*Grad.*), running 0→30 (left) and 30→0 (right). All entries are in arcminutes (M.) and arcseconds (S.). The page foot labels the blocks, right to left, as **fig. 9**, **fig. 10**, **fig. 11**.

Gra d. (L)	fig.3			fig.1			fig.2			Gra d. (R)
	fig.3			fig.1			fig.2			
	Æq	Exc	Scr	Æq	Exc	Scr	Æq	Exc	Scr	
	uat.	obl.	pro	uat.	obl.	pro	uat.	obl.	pro	
	M.S.	M.S.	M.S.	M.S.	M.S.	M.S.	M.S.	M.S.	M.S.	
0	0 0	21 46	0 0	13 35	20 19	30 11	23 36	16 19	51 49	30
1	0 28	21 46	1 3	14 0	20 12	31 5	23 50	16 9	52 19	29
2	0 56	21 45	2 5	14 25	20 6	31 58	24 4	15 59	52 48	28
3	1 24	21 45	3 8	14 41	19 59	32 50	24 17	15 49	53 16	27
4	1 52	21	4 10	15	19	33	24	15	53	26

Gra d. (L)	fig.3			fig.1			fig.2			Gra d. (R)
	fig.3	fig.3	fig.3	fig.1	fig.1	fig.1	fig.2	fig.2	fig.2	
	Æq uat. M.S.	Exc ess obl. M.S.	Scr up. pro p. M.S.	Æq uat. M.S.	Exc ess obl. M.S.	Scr up. pro p. M.S.	Æq uat. M.S.	Exc ess obl. M.S.	Scr up. pro p. M.S.	
5	2 20	44 21	5 12 15	12 15	53 19	41 34	30 24	39 15	44 54	25
6	2 48	43 21	6 14 15	36 15	48 19	31 35	43 24	29 15	11 54	24
7	3 16	42 21	7 16 16	59 16	42 19	12 36	57 25 4	19 15 8	37 55 2	23
8	3 44	40 21	8 18 16	21 16	35 19	10 36				
9	4 12	38 21	9 21 17 5	43 16	29 19	58 36	16 25	58 14	27 55	22
10	4 40	36 21	10 17	23 17	15 19	33 38	40 25	37 14	14 56	20
11	5 8	31 21	11 17	25 17	49 19 7	20 39	49 25	26 14	36 56	19
12	5 36	28 21	12 12	26 18	10 19 0	40 6 40 6	25 25	14 16	56 57	18
13	6 4	26 21	13 13	27 18	31 18	51 40	54 26 4		17	17
14	6 32	24 21	14 14	52 18	43 18	37 41	11 26	54 13	36 57	16
15	7 0	21 21	15 15	13 19	35 18	22 42	17 26	43 13	54 57	15
16	7 28	18 21	16 16	29 19	33 18	27 43 6	22 26	32 13	11 58	14
17	7 55	15 21	17 17	52 19	19 18	49 43	27 26	21 13	27 58	13
18	8 22	12 21	18 18	11 20	10 18	33 44	31 26	10 13	41 58	12
19	8 49	21 9 21 9	19 19	20 20	18 2 18 2	45 45	26 26	12 12	58 58	11

Gra d. (L)	fig.3			fig.1			fig.2			Gra d. (R)
	fig.3		Scr up. pro p.	fig.1		Scr up. pro p.	fig.2		Scr up. pro p.	
	Æq	Exc		Æq	Exc		Æq	Exc		
	uat. M.S.	obl. M.S.		uat. M.S.	obl. M.S.		uat. M.S.	obl. M.S.		
			30	30		16	35	58	54	
20	9 16	21 5	20	20	17	45	26	12	59 5	10
			30	50	53	58	40	47		
21	9 43	21 1	21	21 9	17	46	26	12	59	9
			29		44	39	44	36	15	
22	10	20	22	21	17	47	26	12	59	8
	10	57	28	27	35	19	48	24	24	
23	10	20	23	21	17	47	26	12	59	7
	36	53	26	45	26	57	52	13	32	
24	11 2	20	24	22 2	17	48	26	12 2	59	6
		50	24		16	34	54		39	
25	11	20	25	22	17 7	49 9	26	11	59	5
	29	45	22	18			57	50	45	
26	11	20	26	22	16	49	26	11	59	4
	55	40	20	34	57	43	59	39	50	
27	12	20	27	22	16	50	27 1	11	59	3
	20	35	18	50	48	16		27	54	
28	12	20	28	23 5	16	50	27 3	11	59	2
	45	30	16		39	48		16	57	
29	13	20	29	23	16	51	27 4	11 4	59	1
	10	25	14	21	29	19			59	
30	13	20	30	23	16	51	27 5	10	60 0	0
	35	19	11	36	19	49		53		
	A	A	A	A	A	A	A	A	A	

Parsing notes: This is a half-page of a larger six-figure (fig. 3, 1, 2 / fig. 4, 5 on facing page) prosthaphaeresis table; it continues The bottom rule of each column is marked A (Additive). The fig.1 Scrupula entry at Grad. 1 reads 31 5 (column 30 11 → 31 5 → 31 58). Several entries (e.g. fig.2 Excess at Grad. 7 = "25 4") show tight kerning; transcribed as printed. The page foot reads: fig. 11 | fig. 10 | fig. 9, and the catchword "obli-".

[Table continued]

...obliquitatis eclipticæ & latitudinum stellarum fixarum. [continuation heading: "...of the obliquity of the ecliptic and of the latitudes of the fixed stars."]

This page continues the table with figures **fig. 3**, **fig. 4**, **fig. 5** (foot labels, right to left: **fig. 6**, **fig. 7**, **fig. 8**). Same three-column structure per figure: *Æquat. æquinoc.* *S*, *Excessus obliquitat.* *A*, *Scrupula proportionalia*. Degree argument runs 0→30 (left) and 60→0 / 30→0 (interior and right).

Gra d. (L)	fig.3			fig.4			fig.5			Gra d. (R)
	fig.3	Exc	up.	fig.4	Exc	up.	fig.5	Exc	up.	
	Æq	ess	pro	Æq	ess	pro	Æq	ess	pro	
	uat.	obl.	p.	uat.	obl.	p.	uat.	obl.	p.	
	M.S.	M.S.	M.S.	M.S.	M.S.	M.S.	M.S.	M.S.	M.S.	
0	27 5	10	60 0	23	5 27	51	13	1 27	30	30
		53		46		49	46		11	
1	27 4	10	59	23	5 17	51	13	1 21	29	29
		42	59	31		19	21		14	
2	27 3	10	59	23	5 88	50	12	1 16	28	28
		30	57	16	[rec	48	55		16	
					te: 5					
					8]					
3	27 2	10	59	23 0	4 5	50	12	1 11	27	27
		19	54		[rec	16	29		18	
					te: 4					
					58]					
4	27 1	10 7	59	22	4 48	49	12 3	1 5	26	26
			50	44		43			20	
5	27 0	9 56	59	22	4 39	49 9	11	1 1	25	25
			45	29			37		22	
6	26	9 44	59	22	4 29	48	11	0 57	24	24
	58		39	12		34	11		24	
7	26	9 33	59	21	4 20	47	10	0 53	23	23
	56		32	55		57	44		26	
8	26	9 21	59	21	4 11	47	10	0 49	22	22
	53		24	37		19	17		28	

Gra d. (L)	fig.3			fig.4			fig.5			Gra d. (R)
	fig.3	fig.3	fig.3	fig.4	fig.4	fig.4	fig.5	fig.5	fig.5	
	Æq uat. M.S.	Exc ess obl. M.S.	Scr up. pro p. M.S.	Æq uat. M.S.	Exc ess obl. M.S.	Scr up. pro p. M.S.	Æq uat. M.S.	Exc ess obl. M.S.	Scr up. pro p. M.S.	
9	26 49	9 10	59 15	21 20	4 2	46 39	9 50	0 45	21 29	21
10	26 45	8 59	59 4	21 2	3 53	45 58	9 22	0 41	20 30	20
11	26 40	8 48	58 54	20 43	3 44	45 16	8 55	0 37	19 30	19
12	26 37	8 36	58 41	20 24	3 35	44 33	8 27	0 34	18 29	18
13	26 32	8 25	58 27	20 5	3 27	43 49	8 0	0 31	17 29	17
14	26 27	8 14	58 11	19 45	3 19	43 6	7 33	0 28	16 29	16
15	26 22	8 3	57 54	19 25	3 11	42 22	7 5	0 25	15 28	15
16	26 15	7 54	57 36	19 5	3 3	41 37	5 41 [rec te: 6 37]	0 22	14 28	14
17	26 8	7 43	57 17	18 44	2 55	40 52	5 12 [rec te: 6 9]	0 19	13 27	13
18	26 1	7 32	56 57	18 23	2 47	40 6	6 37 [rec te: 5 41]	0 17	12 26	12
19	25 54	7 21	56 36	18 2	2 39	39 20	5 9 [rec te: 5 12]	0 15	11 25	11

Gra d. (L)	fig.3			fig.4			fig.5			Gra d. (R)
	fig.3		fig.3	fig.4		fig.4	fig.5		fig.5	
	Æq	Exc	Scr	Æq	Exc	Scr	Æq	Exc	Scr	
	uat.	obl.	up. pro	uat.	obl.	up. pro	uat.	obl.	up. pro	
	M.S.	M.S.	M.S.	M.S.	M.S.	M.S.	M.S.	M.S.	M.S.	
20	25	7 9	56	17	2 31	38	4 43	0 13	10	10
	47		14	40		33			23	
21	25	6 59	55	17	2 24	37	4 15	0 11	9 21	9
	38		51	18		46				
22	25	6 48	55	16	2 17	36	3 46	0 9	8 18	8
	28		27	56		58				
23	25	6 38	55 2	16	2 11	36	3 18	0 7	7 16	7
	17			33		10				
24	25 5	6 27	54	16	2 4	35	2 49	0 5	6 14	6
			37	11		21				
25	24	6 17	54	15	1 57	34	2 20	0 3	5 12	5
	52		11	48		31				
26	24	6 7	53	15	1 51	33	1 52	0 2	4 10	4
	39		44	25		41				
27	24	5 57	53	15 1	1 45	32	1 24	0 1	3 8	3
	26		16			50				
28	24	5 47	52	14	1 39	31	0 56	0 1	2 5	2
	13		48	36		58				
29	24 0	5 37	52	14	1 33	31 5	0 28	0 0	1 3	1
			19	11						
30	23	5 27	51	13	1 27	30	0 0	0 0	0 0	0
	46		49	46		11				
	A	A	A	A	A	A	A	A	A	

Parsing notes: This continues the prosthaphaeresis table the two pages together form the full six-figure (eight-figure-block) table. Bottom rule of each column marked A (Additive). Source compositor errors retained as printed with [recte:]: fig.4 Excess at Grad. 2 (source "5 88", doubled seconds digit; intended 5 8) and Grad. 3 (source "4 5", dropped final digit; intended 4 58). The fig.5 Æquat. column rows 16–19 are scrambled in the source: the Grad labels "17"/"16" are themselves printed in inverted order, and the four values are transposed across the mis-

paired rows; the correct monotone sequence for Grad 16→19 is 6 37 / 6 9 / 5 41 / 5 12 (the value printed at slot 19, "5 9", is itself a mis-set 6 9). Each affected cell carries an inline [recte:] with its intended value. The page foot reads: fig. 8 | fig. 7 | fig. 6.

IV. Usus tabularum præcedentium. [The Use of the Foregoing Tables.]

[Margin: Lib. 1, Sph. cap. 4. prob. 2.]

Since the places of the fixed stars and the obliquity of the ecliptic — on which the foregoing mean motions are founded — have been verified from the first bissextile year of the world, 5564, and likewise from the completed year of Christ 1609, therefore proceed as follows. First, extract the simple motion of the equinoxes and of the anomaly for the given time (at least the completed years and months), with the epochs added from the prior table of mean motions. Then, from the following table of *prosthaphaereses* [προσθαφαίρεσεων, prosthaphaereses], extract the appropriate equations — everywhere including, as is done in all astronomical tables, the proportional part for the minutes and the rest that attach to the signs and degrees, concerning which we have given general instruction in the *Arithmetica Logistica*. Likewise, the excess of the minimum obliquity of the ecliptic must always be added, so that the true obliquity matching the time may be known. But the equation of the equinoxes, according to its own marks, must be subtracted from or added to the mean precession, so that this too may be had as equated.

Furthermore, the true or equated precession matching the given time is found thus. From the true precession of the completed year 1600 after the birth of Christ — which is 2 signs 17° 6' 10" — subtract it, if the time is fixed backward in the past; this leaves a difference to be subtracted from any star in the foregoing table of fixed stars. But if, on the other hand, the time is extended forward from that epoch of the stars, then subtract the true precession of the prescribed stellar epoch from the true precession matching such a time; and then, by adding the residue to the star's longitude in the canon, the true longitude of the star in the ecliptic is given for any proposed moment.

So that the latitude of the star matching the proposed time may likewise be ascertained, enter the latter table again with the true longitude of the star measured from the vernal equinox, found earlier, and write down the corresponding proportional minutes [scrupula proportionalia]. Then multiply these into the excess of obliquity preserved earlier — but now, compactly, diminished by as many seconds as there are minutes in that same value (since

these stars suppose the obliquity of the ecliptic of Tycho's age, $23^{\circ} 31\frac{1}{2}'$, and not the minimum, $23^{\circ} 31' 7''$). Thus emerges the *prosthaphaeresis* [προσθαφαίρεσις, prosthaphaeresis] of the latitude of the star in the foregoing canon of stars, to be accommodated in this way: namely, if the latitude of the star is —

If the star's latitude is in signs		to the same latitude
Northern (Borealibus)	{ Septentrional [northern]:	Subtract
	{ Meridional [southern]:	Add
Southern (Meridionalibus)	{ Septentrional [northern]:	Add
	{ Meridional [southern]:	Subtract

Thus the latitude of the star for the given time comes out; and over a longer stretch of time it too is variable, as we showed above from Tycho's observation.

For the rest, into this procedure concerning the apparent progression of the stars — as it appears — we have also included the method of computing the inequality of the precession of the equinoxes, which we shall need below in [the treatment of] the Sun and the other planets. Now we shall illustrate this precept by two examples.

Exemplum I. [Example I.]

Let there be sought the precession of the equinoxes, the obliquity of the ecliptic, and likewise the longitude and latitude of the star Arcturus, for the year 3000 from the first bissextile of the founded world — about which time, indeed long before, namely by 82 years, we know from its evening rising that Hesiod flourished. Therefore, according to what we have prescribed, we collect for 3000 years from the founding of the world:

	Sig.	Gr.	Mi.	Sec.
The simple precession of the equinoxes	1	11	31	1
The	10	0	13	5

	Sig.	Gr.	Mi.	Sec.
anomaly of the equinoxes				
Hence from the next table the equation is found			23	32 Add.
Likewise the excess of the obliquity of the ecliptic			16	21
Therefore the maximum obliquity at that time was		23	47	28
And the true precession of the equinoxes	1	11	54	33

Astronomia Danica — Pars Altera, Liber Primus Theoricorum

[Caput IV, part IV — continued] Example I (Arcturus), conclusion

[The computation table for Arcturus's true longitude and latitude continues from the prior batch. Each line of prose is followed by its value in the columns Sign, Degrees, Minutes, Seconds.]

Step	Sign	Deg.	Min.	Sec.	Add/ Subtract
Now, since Arcturus' s longitude is found in the table of stars as	6	18	38	0	
and the differenc e between the true precessio n of the given time and that of the year 1600 works out to	1	5	11	37	Subtract
Arcturus' s longitude is therefore given as	♍ (Virgo)	13	26	23	
To this longitude correspo nd proportio nal minutes			17	3	

Step	Sign	Deg.	Min.	Sec.	Add/ Subtract
[scrupula proportio nalia] of which, multiplie d into the corrected excess of obliquity yield the proportio nal part			16	3	
			4	34	Subtract
And since Arcturus' s latitude in the table [abacus] is		31	2	30	N
its true latitude, matching the proposed time, therefore comes out as		30	58		N (approxi mately)

So Arcturus's true latitude for the proposed time is about 30°58' north.

Example II.

Let us investigate in the same way, for the completed year 6000 from the first bissextile of the founded world — that is, the year of Christ 2034 — the true precession of the equinoxes, likewise the maximum obliquity of the Sun [i.e. of

the ecliptic], and finally the true longitude and latitude of the first star in the head of Aries.

First, then, from the earlier table the simple precession of the equinoxes for the given time stands at $2s\ 23^{\circ}0'13''$, and the anomaly at $8s\ 0^{\circ}13\frac{1}{2}'$. Entering the next of the foregoing tables with this anomaly, I find the equation of the precession of the equinoxes to be $23'43''$, to be added; and the excess to be $5'29''$. The true anticipation of the equinoxes will therefore be $2s\ 23^{\circ}23'56''$, and the difference from the Tyconic epoch of the stars $6^{\circ}17'46''$, to be added; likewise the maximum obliquity of the Sun [obliquatio Solis maxima] $23^{\circ}36'36''$.

Now since the longitude of the first star of Aries is found in the canon [table] as $0s\ 27^{\circ}36'$, with a latitude of $7^{\circ}8'$ north, the longitude of this star for the assumed time will be $1s\ 3^{\circ}53'46''$. Entering the table of prosthaphaereses [tabula prosthaphaereseōn] with this, proportional minutes of $33'36''$ are presented. When these are applied — by the very easy method I set out above — to the excess [carried] to the obliquity of the ecliptic in Tycho's time, namely $5'22''$ being taken, the product $3'0''$ arises, to be subtracted. From this, too, the latitude of this star at that time comes out as $7^{\circ}5'$ north.

And such likewise is the computation in all the other examples of the kind we have set out in this chapter.

CAPUT V.

On the measure of the tropical and sidereal year, likewise on the mean and apparent motions of the Sun, together with the hypothesis, the tables, and the use of these.

The **tropical year** [annus tropicus] is defined as the measure of the periodic motion of the Sun in the ecliptic, reckoned either from the equinoctial or from the solstitial point; for the starting point of the simple [year] we adopt the vernal mean equinox.

The **sidereal year** [annus sidereus], on the other hand, is the time of the Sun's revolution back to the same fixed star, measured along longitude. The difference between the tropical and the sidereal year is therefore the one determined by the simple anticipation of the equinox — that is, the simple apparent advance of the fixed stars, $49''45'''$ — when resolved, through the Sun's daily motion, into the interval of $20m\ 18\frac{1}{2}s$.

Now in order to attain the tropical year, equal to the first [year] in its true measure — so far at least as is possible (and without this no solid foundation exists for restoring not only the motions but the [chronological] reckoning of time itself) — it must be drawn out from the intervals between the most ancient equinoxes observed by Hipparchus and Ptolemy [Ptolemy], and those restored by us above, by a comparison instituted between these and Tycho's [observations]. For in this way any error there may be, once enclosed within the whole longer span of time, becomes utterly imperceptible when it is broken up into the individual years.

But since throughout we use the measure of the Julian year of $365\frac{1}{4}$ days, we shall handle the matter compendiously — without [further] resolution, by the vernal equinox's anticipation alone — just as before. And in order first to reduce the apparent equinoxes everywhere to mean [equinoxes], there must first be

[Margin: See the commentary on the Sun.]

there must first be known, and duly accommodated, the inequality both of the Sun and of the equinoxes — that is, the prosthaphaeresis [παροδαφαίρεσις (here a print-garbled form of προσθαφαίρεσις, prosthaphairesis, "equation/correction")] of the Sun as well as of the equinoxes. And since this is established from what precedes, it will be unfolded in the following demonstration.

Diagram. A circle representing the eccentric of the Sun, with its center marked and several chords drawn to a triangle within. The construction illustrates the angle by which the apogee is removed from the vernal equinox, and the resulting prosthaphaeresis at the equinoxes.

- E (right, at circumference): the vernal equinox [vernum aequinoctium]
- F (left, at circumference): the autumnal equinox [autumnale]; marked with the libra/autumn symbol (♎)
- C (top, at circumference): the apogee [apogaeus]
- D (bottom, at circumference): the perigee [perigaeus]
- A (interior, near center): the center; A E is the radius of the Sun's eccentric, set at 28 parts [p. 28]
- B (interior, below A): A B is the Sun's eccentricity, 1 part; the angle E B A measures the apogee's distance from the true equinox

The text accompanying the figure: Let the eccentric of the Sun, **E C F D**, be described from the center; and let **A B** be the Sun's eccentricity, 1 part, of the kind of which the radius **A E** is 28 parts — as we demonstrated above at the end of the

solar commentary. The apogee is at **C**, the perigee at **D**, and finally the vernal equinox at **E**, the autumnal at **F**.

Now since in Hipparchus's time the apogee **C** is established to have been removed from the vernal equinox by $65\frac{1}{2}^\circ$ [Margin: p. $65\frac{1}{2}$]; in Ptolemy's, 70° ; and finally in Tycho's and our own age — namely at the beginning of the year of Christ 1588 — $95\frac{1}{2}^\circ$: therefore in the triangle **A B E**, for the angle **A E B** (to which the angle **A F B** is always equal) there are given: **A E**, the radius of the Sun's eccentric, 28 parts; **A B**, the eccentricity, 1 part; and finally the angle **E B A** measuring the apogee's removal from the true equinox. Consequently the angle **A E B** — the prosthaphaeresis [προσθαφαίρεσις (here printed παροδαφαίρεσις, prosthaphairesis, "the equation")] at each equinox — is also given: in Hipparchus's time, $1^\circ 51' 44''$; in Ptolemy's, $1^\circ 55' 23''$; and finally in ours, $2^\circ 2' 14''$.

Similarly, from the immediately preceding chapter, the equation of the equinox is found to be, for the age of Hipparchus, $10' 12''$; for that of Ptolemy, $21' 0''$, subtractive; but for ours, $7' 12''$, additive. This [equation] must therefore be cast off [subtracted] from the vernal equinoxes among the ancients, but adjoined to the same among us; in the autumnal equinoxes the contrary rule will be kept. Whatever results from this must be converted into time in proportion to the Sun's true daily motion. Having done this, according to the following synopsis I find:

[A bracketed synopsis table. Reading: "The apparent equinox, in order that the mean may be had, requires for the vernal — to be added; for the autumnal — to be subtracted," in units of Days, Hours, Minutes, by age:]

Equinox correction (to obtain the mean)	D.	H.	M.	Age
Vernal — to be added	1	17	50	Hipparchus
Vernal — to be added	1	14	45	Ptolemy
Vernal — to be added	2	4	30	Ours
Autumnal — to be subtracted	2	0	50	Hipparchus
Autumnal	2	6	45	Ptolemy

Equinox correction (to obtain the mean)	D.	H.	M.	Age
— to be subtracted				
Autumnal	1	22	50	Ours
— to be subtracted				

These equations have been the diligent work that was needed. With them set forth, and being about to apply them to both equinoxes — Hipparchus's and Ptolemy's as well as Tycho's — we first select the very first vernal [equinox] among those still most ancient, which Hipparchus reports having observed twice on a single day; and we retain in it the last [of the two], as one from which the whole element of refraction had then been dispelled.

Furthermore we shall employ Hipparchus's fifth autumnal [equinox], likewise determined on the rising horizon, so that on this account 5 hours are first subtracted from it; then whatever of the difference remains [relative] to the vernal is brought to bear as a just limitation, by which not only the Sun's refraction but also other inconveniences of the armillae [Armillae] — whether contracted of themselves, or committed through the error of the Alexandrian pole [adjustment] — may be opportunely corrected. There now follows the practical procedure [praxis].

[Margin: See above in the commentary on the Sun.]

Hipparchus's first vernal equinox occurred in the year 146 before the birth of Christ, in the month of March, on day 23, hour 23, at Alexandria; but here [i.e. at our meridian] hour 21, minute 25, after midday. When to this time the prefixed inequality is added — namely 1 day 17 hours 50 minutes — Hipparchus's mean equinox falls, in our reckoning, on day 25 of March, hour 15, minute 15.

[Margin: Book 1, Prog[ymnasmata], p. 13.]

To this equinox — since the third [day] from the bissextile in the Tychonic class corresponds — there will correspond that vernal [equinox] in the year 1587 after the birth of the Savior, which at Uraniborg, as also here at Copenhagen [Hafnia] (for there is hardly any difference of longitude between these), appeared on day 10 of March, hour 14, minute 56, p.m. To this let the inequality assigned above be

added, namely 2 days 4 hours 30 minutes. The sum indicates day 12 of March, hour 19, minute 26, after midday — the time at which the mean equinox occurred. The difference, therefore, from Hipparchus's vernal [equinox] is 12 days 19 hours 49 minutes; this must be ascribed to the anticipation of time over nearly 1732 Julian years, and preserved.

Furthermore, Hipparchus's fifth autumnal equinox — with the 5 hours first subtracted, as we indicated, then the difference of meridians — falls at Uraniborg in the same year 146 before Christ, on day 26 of September, hour 11, minute 25. When the equation of 2 days 0 hours 50 minutes has been subtracted from this, the time of this mean equinox was September day 24, hour 10, minute 35, p.m. But Tycho's autumnal equinox of the year 1587 was found on day 13 of September, hour 9, minute 16; and the mean, with the equation subtracted, on day 11 of September, hour 10, minute 36. From this, therefore, the Hipparchan autumnal [equinox] is again distant by 12 days 23 hours 59 minutes. But the anticipation of the vernal [equinoxes] was 12 days 19 hours 49 minutes.

The difference, therefore, is 4 hours 10 minutes. Half of this — namely 2 hours 5 minutes — assigned to the limitation and added to the lesser of the vernal anticipations, constitutes the limited anticipation, in completed solar years (which in this business one may use indifferently for Julian years) 1732, between Hipparchus and Tycho: 12 days 21 hours 54 minutes. Divided into the proposed 1732 years, it yields per unit [year] $10^{\circ}44'8''$, which in this first experiment, between Hipparchus and us, argues an excess of the Julian year of 365 days 6 hours over the mean solar year.

Now, being about to weigh Ptolemy's equinox in the same way, we shall bring to examination that single vernal [equinox] of his, and another autumnal one verified by us above, making our beginning from the autumnal [equinox] in the order of time.

Ptolemy's following autumnal equinox, freed above from refraction, falls in the year after Christ 139, the month of September, day 25, hour 6, p.m., at Alexandria; but at Uraniborg, day 25, hour 4, minute 25. The equinoctial equation subtracts 2 days $6\frac{1}{2}$ hours 45 minutes. Therefore the mean equinox was day 22, hour 21, minute 40. But Tycho's mean autumnal equinox, which occurred in the year 1587 and agrees with the Ptolemaic in respect of the bissextile, was found above to have fallen on day 11 of September, hour 10, minute 35. The anticipation (according to the interval of 1448 solar years) is therefore the difference of these, which is 11 days 11 hours 4 minutes.

By the same reasoning it is gathered that Ptolemy's vernal equinox occurred in the year 140 from the birth of Christ, at Alexandria, in the month of March, day 22, hour 1, p.m.; therefore at Uraniborg, day 21, hour 23, minute 25, p.m. To this the equation adds 1 day 14 hours 45 minutes. Therefore this mean equinox was day 23 of March, hour 14, minute 10, p.m. But to this Ptolemaic equinox, occurring in a bissextile year, corresponds Tycho's of the year 1588, which was observed at Uraniborg on day 9 of March, hour 20, minute 45; and with the Tychonic equation of 2 days 4 hours 30 minutes added, this mean equinox occurred on day 12 of March, hour 1, minute 15, p.m. Therefore the difference here between [the two] vernal [equinoxes] of Ptolemy is found to be 11 days 12 hours 55 minutes. Accordingly this interval, in equal years — namely 1448 — exceeds [the other] above by 1 hour 51 minutes. The limited anticipation, then, from Ptolemy down to Tycho, is found to be 11 days 12 hours 0 minutes; which, divided into 1448 years, shows the annual anticipation per unit to be $11^{\circ}26'11''$.

The limited [anticipation], therefore, of both Hipparchus and Ptolemy to our age, taken through the mean, is $11^{\circ}5'10''$. For this let us now take $11^{\circ}5'$ — nothing perceptible is lost to the Sun's motion on account of the $10''$ by which, as is fitting, we adhere more closely to Hipparchus, the reason for which was given above in the commentary. The length of the tropical year, then — investigated now with subtlety and perhaps greater diligence by us than by anyone hitherto — will be 365 days 5 hours 48 minutes 55 seconds; namely, with the limited anticipation subtracted from the measure of the Julian year of 365 days 6 hours. And thus in 130 solid Julian years the anticipation of one whole day is very nearly distributed.

And this measure of the solar year is perpetual — namely in respect of the mean equinox — from which we find that of the Persians and of al-Gelalī [Gelalaeus, the Jalālī calendar] is exceeded only in 15 seconds of arc [reading uncertain — see note]; and the Tychonic [measure] had to be even a little greater than this on account of the inequality of the equinoxes increasing in our age.

And surely everyone must marvel that Tycho himself, at an interval of only 100 years from Walther of Nuremberg [Gualtherus Noribergensis, Bernhard Walther], should have arrived so nearly at the annual quantity — with no account taken of the unequal precession of the equinoxes — when he establishes it as 365 days 5 hours 48 minutes 45 seconds; which ours exceeds by only 10 seconds, but the Alphonsine [tables'] by 41 seconds. [Margin: See among others Vol. 1 of the *Ephemerides*, on the meridian of Uraniborg.] This last [discrepancy], I judge, befell them [the Alphonsine astronomers] because, as we rejected the

equinoxes of al-Battānī [Albategnius] both earlier and here [Margin: Book 1, *Prog[ymnasmata]*, p. 51 and following],

they perhaps adopted them [those equinoxes] uniquely as the measure of their comparison (the reason adduced above in the commentary). As for the fact that earlier — when we examined the motions of the Moon and of the fixed stars among the ancients — we supposed the tropical year to be greater by about $\frac{1}{3}$ minute than the value just found: that too means nothing other than a slightly greater equality, the thing always uniquely desired in our prayers, which in most matters of the greatest moment serves us well, when the examinations will sustain the standard of this tropical year's measure.

For the rest, it is now easy to deduce the mean quantity of the tropical year we have found into the measure of the sidereal [year], by adding — according to the requirement of the Sun's motion, as I noted above — 20 minutes 18½ seconds [of time]. For thus it too comes out as 365 days 6 hours 9 minutes 13⅓ seconds.

[Margin: See the commentary on the stars.]

That this is exceeded by Copernicus [Copernicus] by more than 27 seconds, and by Tycho by more than 13 seconds, we do not at all trouble ourselves over, seeing that both of them employed the positions of the stars observed by the ancients without any restoration. We, on the other hand — using those fixed [stars] corrected, and carefully examined against this our own constitution — find among the ancients (Timocharis, Hipparchus, and Ptolemy) no greater discrepancy than $\frac{1}{6}$ degree; but with al-Battānī, $\frac{1}{3}$ degree, or a little more in the contrary direction. This last, indeed, must rightly be ascribed to the observations of the ancients as being far less certain, and perhaps also to the foundation drawn from the Sun in al-Battānī's case, which we have not touched on, being still more invalid. If any, in favor of the ancients, hold opinions against us, these will most easily be convinced by a comparison of the most ancient canon of stars with the most exact Tychonic one.

Furthermore, both quantities of the year having been found — since we discover no discrepancy whichever we may use in the end for determining the Sun's motion — [we proceed] otherwise than Copernicus and Tycho judged against Ptolemy. [Margin: Copernicus, Book 3, ch. 13; Tycho, *Prog[ymnasmata]*, Book 1, ch. 2, p. 253.] For whatever the simple precession of the equinox is, the same is also the advance of the fixed stars in consequentia [in the order of the signs]; and the inequality of the equinox is also common to the stars. Accordingly, by a method both more popular and more convenient, we shall begin the course of the

Sun and of the stars from the vernal equinox — first the mean, then the true, namely once we have duly accommodated its inequality [Margin: Cap. 4 of this [book]] to the mean precession of the equinoxes, in accordance with what has preceded.

But in order to arrive at the supposition of the solar motions to be continued through tables, we must proceed in this way. First, for the common Julian year, which consists of 365 days: these being subtracted into the whole circle, or 360° , and [the result] divided by the measure of the tropical year just found, there emerges the motion of the Sun belonging to the said common year, $11^s 29^\circ 45' 40'' 13''' 38''''$. This in turn being divided into 365 days, the daily motion of the Sun from the vernal mean equinox is elicited as $0^\circ 59' 8'' 19''' 48''''$.

And since the bissextile Julian year is greater than the common by one whole day, accordingly, by the daily motion of the Sun being aggregated to the common annual [motion], the Sun's motion in the bissextile year is formed as $0^s 0^\circ 44' 48'' 33''' 26''''$. From these sources the other motions, which will appear in the following table, are easily deduced. So much for the Sun's motion in longitude.

The motion of the **anomaly** of the same [Sun] — which indicates the Sun's mean distance from the mean apogee at any time — [arises] when the annual motion of the apogee (which, from what we left above in the commentary, we found to be $1' 1'' 50''' 14''''$) has been subtracted from the motion of the Sun's longitude assigned above to each common and bissextile year. Thus it will be, in the common year, $11^s 29^\circ 44' 38'' 23''' 24''''$; and in the bissextile year, $0^s 0^\circ 43' 46'' 43''' 12''''$. And since the daily motion of the Sun's apogee comes to at least $10''' 10''''$, the motion of the anomaly is therefore easily made up — by subtracting this from the simple solar longitude — for months, days, and the remaining species of time.

From these premises we have compiled the following tables of the Sun's simple motions from the vernal mean equinox, and have continued them to myriads of years — perhaps beyond the future mundane period, at least sufficient for our previously stated [purpose] — about to show next the use of these tables. The epochs of these motions, appearing at the head [frontispiece] of the table, are likewise directed, as is proper, to the mean equinox.

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The Sun's simple longitude and anomaly, reckoned to the mean equinox.

The left half of the page gives the solar simple motion by epochs and by accumulated years (to 6300); the right half gives the year-by-year values for the

single years 1 through 20. Each value is expressed in signs (S.), degrees (G.), minutes (M.), and seconds (S.).

Left table — Simple longitude and anomaly of the Sun, by epoch and by accumulated years

Epoc h / Years	Longi tude				Ano maly			
	S.	G.	M.	S.	S.	G.	M.	S.
Mun di (of the Worl d)	8	8	33	54	8	8	28	29
Chris ti (of Chris t)	9	8	38	10	7	0	30	44
20	0	0	9	6	11	29	48	30
40	0	0	18	12	11	29	37	1
60	0	0	27	19	11	29	25	31
80	0	0	36	25	11	29	14	2
100	0	0	45	31	11	29	2	32
200	0	1	31	2	11	28	5	4
300	0	2	16	33	11	27	7	37
400	0	3	2	4	11	26	10	9
500	0	3	47	35	11	25	12	41
600	0	4	33	6	11	24	15	13
700	0	5	18	37	11	23	17	46
800	0	6	4	8	11	22	20	18
900	0	6	49	39	11	21	22	50
1000	0	7	35	10	11	20	25	22
1100	0	8	20	40	11	19	27	54
1200	0	9	6	11	11	18	30	27
1300	0	9	51	42	11	17	32	59

Epoc h / Years	Longi tude				Ano maly			
	S.	G.	M.	S.	S.	G.	M.	S.
1400	0	10	37	13	11	16	35	31
1500	0	11	22	44	11	15	38	3
1600	0	12	8	15	11	14	40	36
1700	0	12	53	46	11	13	43	8
1800	0	13	39	17	11	12	45	40
1900	0	14	24	48	11	11	48	12
2000	0	15	10	19	11	10	50	44
2500	0	18	57	54	11	6	3	26
3000	0	22	45	29	11	1	16	7
3500	0	26	33	3	10	26	28	48
4000	1	0	20	38	10	21	41	29
4500	1	4	8	13	10	16	54	10
5000	1	7	55	48	10	12	6	51
5500	1	11	43	23	10	7	19	32
6000	1	15	30	57	10	2	32	13
6300	1	17	47	30	9	29	39	50

Parsing notes: At "20 years," the longitude minutes are very faint; read as 9 6 (i.e. 0°0'9"6''' — but the column structure here is S./G./M./S., so 0 sign, 0°, 9', 6"). The "60" row anomaly seconds and the "300" anomaly were partly faint but read consistently with the smooth progression. The right-hand annual table confirms the per-year increment.

Right table — In single years, up to 20

Year	Longi tude				Ano maly			
	S.	G.	M.	S.	S.	G.	M.	S.
1	11	29	45	40	11	29	44	38
2	11	29	31	20	11	29	29	17
3	11	29	17	1	11	29	13	55
4	0	0	1	49	11	29	57	42
5	11	29	47	29	11	29	42	20

Year	Longitude				Anomaly			
	S.	G.	M.	S.	S.	G.	M.	S.
6	11	29	33	10	11	29	26	59
7	11	29	18	50	11	29	11	37
8	0	0	3	38	11	29	55	24
9	11	29	49	19	11	29	40	2
10	11	29	34	59	11	29	24	42
11	11	29	20	39	11	29	9	20
12	0	0	5	28	11	29	53	7
13	11	29	51	8	11	29	37	45
14	11	29	36	48	11	29	22	23
15	11	29	22	28	11	29	7	2
16	0	0	7	17	11	29	50	49
17	11	29	52	57	11	29	35	27
18	11	29	38	37	11	29	20	5
19	11	29	24	18	11	29	4	44
20	0	0	9	6	11	29	48	30

Parsing notes: This is a "deficit" table — each single year's simple longitude advances by very nearly a full revolution (11s 29° 45' 40" for year 1), so successive years show the small accumulating shortfall. The year-4, year-8, year-12, year-16, year-20 rows roll over into 0 signs (the accumulated quarter-day leap effect). At year 18 the anomaly reads 11 29 20 5 (the printed "20 5" was set with a wide gap). The "20-year" anomaly value 11 29 48 30 matches the left table's "20" epoch row exactly, confirming alignment.

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Top tables — Monthly motion

In common months (In mensibus Communibus) / In leap-year months (In mensibus bisextilibus).

These give the Sun's accumulated simple longitude and anomaly at the start of each named month, for ordinary years and for leap years.

	C o m m o n — L o n g. S.				C o m m o n — A n o m . S.				L e a p — L o n g. S.				L e a p — A n o m . S.			
	G.	.	M	S.	G.	.	M	S.	G.	.	M	S.	G.	.	M	S.
Ja n u a r y (I a n u a r i u s)	1	0	3	1	1	0	3	1	1	0	3	1	1	0	3	1
			3	8			3	3			3	8			3	3
F e b r u a r y (F e b r u a r i u s)	1	2	9	1	1	2	9	1	1	2	8	2	1	2	8	1
		8		1		8				9		0		9		0

	C o m m o n — L o n g. S.				C o m m o n — A n o m .br/>S.				L e a p — L o n g. S.				L e a p — A n o m .br/>S.			
	G.	.	M	S.	G.	.	M	S.	G.	.	M	S.	G.	.	M	S.
s)																
M ar c h (M ar ti u s)	2	2	4	3	2	2	4	1	2	2	4	3	2	2	4	2
		8	2	0		8	2	5		9	1	8		9	1	3
A p ri l (A p ri li s)	3	2	1	3	3	2	1	1	3	2	1	4	3	2	1	2
		8	6	9		8	6	8		9	5	8		9	5	7
M a y (	4	2	4	5	4	2	4	4	4	2	4	6	4	2	4	4
		8	9	8		8	9	2		9	9			9	8	0

	C o m m o n — L o n g. S.				C o m m o n — A n o m . S.				L e a p — L o n g. S.				L e a p — A n o m . S.			
	G.	.	S.		G.	.	S.		G.	.	S.		G.	.	S.	
M a j u s)																
Ju n e (I u ni u s)	5	2	2	7	5	2	2	3	5	2	2	1	5	2	2	4
		8	4			8	3	6		9	3	6		9	2	5
Ju ly (I ul iu s)	6	2	5	2	6	2	5	5	6	2	5	3	5	2	5	5
		8	7	6		8	6	0		9	6	4	[r	9	5	8
													ec			
													te			
													:			
													6]			
A u g u st (A	7	2	3	4	7	2	3	3	8	0	2	5	8	0	2	1
		9	0	4		9	0				9	3			9	2

	C o m m o n — L o n g. S.				C o m m o n — A n o m . S.				L e a p — L o n g. S.				L e a p — A n o m . S.			
	G.	.	S.		G.	.	S.		G.	.	S.		G.	.	S.	
u g u st u s)																
S e pt e m b er	8	2	4	5	8	2	4	6	9	0	4	3	9	0	3	1
		9		4		9										7
O ct o b er (O ct o b er	9	2	3	1	9	2	3	1	1	0	3	2	1	0	3	3
		9	8	2		9	7	9	0		7	1	0		6	0

	C o m m o n — L o n g. S.				C o m m o n — A n o m . . S. G. . S.				L e a p — L o n g. S. G. . S.				L e a p — A n o m . . S. G. . S.			
M o n t h																
)																
N o v e m b e r	1	2	1	2	1	2	1	2	1	0	1	3	1	0	1	3
	0	9	2	2	0	9	0	6	1		1	1	1		0	5
D e c e m b e r	1	2	4	4	1	2	4	3	0	0	4	4	0	0	4	4
	1	9	5	0	1	9	4	8			4	9			3	7

Parsing notes: In the July (Iulius) leap-year anomaly cell, the source prints 5 29 55 58, where the sign-figure 5 is a compositor slip for 6 (every other July row reads sign 6, and the leap longitude reads 6 29 56 34; the structure forces sign 6); transcribed as printed with [recte: 6]. The August common-anomaly reads 7 29 30 3; the leap-year August rolls into sign 8 (8 0 29 53 / 8 0 29 12), consistent with the wrap from sign 7 to 8.

Bottom-left table — Simple motion of the Sun in days (Simplex motus Solis in dieb.)

Longitude (□) and anomaly (□) for days 1–31. Columns are degrees, minutes, seconds.

Day	Long. G.	M.	S.	Anom. G.	M.	S.
1	0	59	8	0	59	8
2	1	58	17	1	58	17
3	2	57	25	2	57	25
4	3	56	33	3	56	33
5	4	55	42	4	55	42
6	5	54	50	5	54	49
7	6	53	58	6	53	57
8	7	53	7	7	53	6
9	8	52	15	8	52	14
10	9	51	23	9	51	22
11	10	50	32	10	50	31
12	11	49	40	11	49	38
13	12	48	48	12	48	46
14	13	47	57	13	47	55
15	14	47	5	14	47	3
16	15	46	13	15	46	11
17	16	45	21	16	45	19
18	17	44	30	17	44	27
19	18	43	38	18	43	35
20	19	42	47	19	42	44
21	20	41	55	20	41	52
22	21	41	3	21	41	0
23	22	40	13	22	40	9
24	23	39	20	23	39	16
25	24	38	28	24	38	24
26	25	37	37	25	37	33

Day	Long.	M.	S.	Anom.	M.	S.
	G.			G.		
27	26	36	45	26	36	41
28	27	35	53	27	35	49
29	28	35	1	28	34	57
30	29	34	10	29	34	6
31	30	33	18	30	33	13

Bottom-right table — Simple motion in hours and scruples (Simplex mot. in horis & scrup.)

Two paired columns. The first gives, for hours 1–30, the longitude-and-anomaly increment in minutes and seconds; the second continues for hours 31–60 in seconds and thirds. (Header row: "Hor. | Long. & An. | Hor. | Lon. & A." — first pair "Min. | M. S.", second pair "Min. | M. S." but the foot of the column is labeled "Sec. Tert." indicating the right pair is in seconds and thirds.)

Hor.	M.	S.	Hor.	(Sec.)	(Tert.)
1	2	28	31	1	16
2	4	56	32	1	19
3	7	24	33	1	21
4	9	51	34	1	24
5	12	19	35	1	26
6	14	47	36	1	29
7	17	15	37	1	30
8	19	34	38	1	34
9	22	11	39	1	36
10	24	38	40	1	39
11	27	6	41	1	41
12	29	34	42	1	43
13	32	2	43	1	46
14	34	30	44	1	48
15	36	58	45	1	51
16	39	26	46	1	53
17	41	53	47	1	56

Hor.	M.	S.	Hor.	(Sec.)	(Tert.)
18	44	21	48	1	58
19	46	49	49	2	1
20	49	17	50	2	3
21	51	45	51	2	6
22	54	13	52	2	8
23	56	40	53	2	11
24	59	8	54	2	13
25	1 [recte: 61/1°]	2	55	2	15
26	1	4	56	2	18
27	1	7	57	2	20
28	1	9	58	2	33 [recte: 23]
29	1	11	59	2	25
30	1	14	60	2	28
			Sec.	Tert.	

Parsing notes: In the hour-25 row the first value rolls past 60 minutes into the next degree, so the printed 1 2 represents 1°2' (i.e. 62'). The hour-3 right-pair second figure reads 1 21. The hour-9 left value reads 22 11. The Hor.-58 right pair prints 2 33, which breaks the otherwise-monotone progression (neighbors Hor. 57 = 2,20 → Hor. 59 = 2,25); it is a compositor slip for 2 23 — transcribed as printed with [recte: 23].

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The solar hypothesis now follows, together with the table

προσθαφαίρεσως [Greek: prosthaphairesis — the equation, i.e. the prosthaphaeretic correction], from which the apparent motion is derived.

[Margin / heading printed in the source title with the Greek term προσθαφαίρεσως embedded.]

Up to this point we have been engaged in explaining the Sun's mean motions. There now follows the hypothesis, in which it is shown geometrically how those mean motions are to be reduced to their true or apparent values at each given moment of time — for this is precisely what the astronomer must seek. Now since

the Sun's course is by itself dispatched through a single, simple anomaly, we select — among the varieties of equivalent hypotheses set out in Chapter 2 — the *homocentre-epicycle* [homocentrepicyclum], as the one best suited to the greatest economy of nature, as we noted earlier. And since we have already, in the solar commentary, obtained the dimensions of the radii of the Sun's orbs through a proper collation of observations, we shall here treat its hypothesis (founded upon those dimensions) all the more freely and without hindrance. From this the table of prosthaphaereses will presently be set out in the following place; and finally, at the last, the use of this chapter in computing the Sun's true place for all given times will be set forth plainly through worked examples.

THEORIA SOLIS (The Theory of the Sun).

Diagram. A geometric construction of the homocentric–epicyclic solar model. A large circle (the homocentric deferent / ecliptic) is centered on the Earth at A. The vernal mean equinox lies at B on the right; the diameter runs horizontally from B (right) through center A to C (left, marked with the equinox symbol ϖ / " ϖ C"). A second point D lies on the deferent in the upper region; at D a small epicycle E F G is drawn, carrying the Sun's body at F. Radial lines run from center A to D and from A through the epicycle to F.

- A (center): the Earth (centro terreno) — center of the homocentric circle B D C.
- B (right end of horizontal diameter): the vernal mean equinox; start of the Sun's annual period, motion proceeding in the order of the signs (in signorum consequentia) from B toward D.
- C (left end of diameter, with ϖ symbol): the opposite point of the diameter.
- D (upper deferent point): center of the epicycle; the arc B D has been completed by the Sun's mean motion along the ecliptic from B.
- E (top of epicycle): the apogee of the epicycle (apogaeum E), from which the anomaly's motion is reckoned in the order *contrary* to the signs (in signorum antecedentia), proceeding through F, G.
- F (on epicycle): the Sun's body; the arc E F is the Sun's anomaly (here taken as 24° for the example).
- G (lower part of epicycle): a labeled point on the epicycle between F and the return.

The Sun's annual period above the deferent B D C is the homocentric; the daily mean motion was defined as 59' 8" (here granted for the example). The arc B D is completed by the Sun's mean motion along the ecliptic from B. With D as center,

the epicycle E F G is drawn in the plane of the homocentric (the ecliptic); its radius D E bears to the radius of the Sun's orbit A D the ratio of **3571 to 100000**, that is, of 1 part to 28 parts, as we showed above in the commentary. In the inscribed epicycle E F G the motion and period of the anomaly (set out above) is understood to be completed in the order contrary to the signs, beginning from the apogee E and carried through F, G. With the solar body placed at F, the Sun's anomaly — according to its mean motions defined above — will be the arc E F, which here is supposed for the example to be **24°**. The angle F D A is therefore the supplement of this to a semicircle, namely **156°**. Since about this angle each of the two sides A D and D F is set out in a fixed ratio to the other — the former being 28 parts, the latter 1 part — the angle προσθαφαίρεσεως [prosthaphaeresis] D A F is given as **0° 48' 20"**, which at this place, as is plain, must be subtracted from the Sun's mean motion so that the true place may be obtained.

Next, in the same triangle D A F — its angles being given, together with two sides as we have set out — let the third side A F be sought (when there is need), measuring the Sun's distance from the Earth. This is here found to be **10327**, in units of which the radius A D is 10000; or **28 parts 55 minutes**, in units of which A D is 28 parts. And from a similar investigation, carried out at every five degrees [ad singulos quinos gradus], the following table has been composed.

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TABLE OF PROSTHAPHAERESIS (TABVLA PROSTHAPHÆERESEOS)

Assuming the epicycle's radius to be 1 part, of which [the orbit's radius is ...]
(Supponens Semidiametrum epicycli part. 1. qualium — the heading continues onto the next page).

This table gives, for each degree of argument 0–30 in the top sign-blocks (and 9–11 read from the foot upward, "Add." = additive), the prosthaphaeresis (equation) and its difference (Dif. A.). The page presents three vertical blocks: argument-sign **0** (Subtr. = subtractive), **1** (Subtr.), and **2** (Subtr.); read from the foot of the page these same columns serve signs **11, 10, 9** (Add. = additive). Each block has columns G.M.S. (degrees, minutes, seconds of equation) and a Dif. A. column (M. S.).

G (t o p)	Si g n —			D if. A.		Si g n —			D if. A.		Si g n —			D if. A.		G (f o ot)
	G	M	S	M	S	G	M	S	M	S	G	M	S	M	S	
0	0	0	0	2	5	0	5	3	1	4	1	4	2	1	6	3
							9	1		9		4	3			0
1	0	2	5	2	4	1	1	2	1	4	1	4	2	1	4	2
								0		8		5	9			9
2	0	4	9	2	4	1	3	8	1	4	1	4	3	1	2	2
										6		6	3			8
3	0	6	1	2	4	1	4	5	1	4	1	4	3	1	0	2
			3					4		6		7	5			7
4	0	8	1	2	3	1	6	4	1	4	1	4	3	0	5	2
			7					0		4		8	5		9	6
5	0	1	2	2	3	1	8	2	1	4	1	4	3	0	5	2
		0	0					4		3		9	4		7	5
6	0	1	2	2	3	1	1	7	1	4	1	5	3	0	5	2
		2	3					0		1		0	1		5	4
7	0	1	2	2	3	1	1	4	1	4	1	5	2	0	5	2
		4	6					1	8			1	6		1	3
8	0	1	2	2	3	1	1	2	1	3	1	5	1	0	4	2
		6	9					3	8			2	7		9	2
9	0	1	3	2	3	1	1	7	1	3	1	5	6	0	4	2
		8	2					5		8		3			7	1
1	0	2	3	2	2	1	1	4	1	3	1	5	5	0	4	2
0		0	5					6	5			3	3		8	0
1	0	2	3	2	1	1	1	2	1	3	1	5	4	0	4	1
1		2	7					8	2			4	1		6	9
1	0	2	3	2	1	1	1	5	1	3	1	5	2	0	4	1
2		4	8					9	8			5	7		3	8
1	0	2	3	2	1	1	2	3	1	3	1	5	1	0	3	1
3		6	9					1	2			6	0		9	7
1	0	2	4	2	1	1	2	5	1	3	1	5	4	0	3	1
4		8	0					3		2		6	9		8	6

G (t o p)	Si g n —			D if. A.		Si g n —			D if. A.		Si g n —			D if. A.		G (f o ot)
	G	M	S			G	M	S			G	M	S			
1	0	3	4	2	0	1	2	3	1	3	1	5	2	0	3	1
5		0	1				4	7		1		7	7		6	5
1	0	3	4	2	0	1	2	8	1	2	1	5	3	0	3	1
6		2	1				6			9		8			3	4
1	0	3	4	1	5	1	2	3	1	2	1	5	3	0	3	1
7		4	1		9		7	7		7		8	6		2	3
1	0	3	4	1	5	1	2	4	1	2	1	5	8	0	3	1
8		6	0		9		9			5		9			0	2
1	0	3	3	1	5	1	3	2	1	2	1	5	3	0	2	1
9		8	9		8		0	9		4		9	8		8	1
2	0	4	3	1	5	1	3	5	1	2	2	0	6	0	2	1
0		0	7		7		1	3		3					6	0
2	0	4	3	1	5	1	3	1	1	2	2	0	3	0	2	9
1		2	4		6		3	6		1			2		4	
2	0	4	3	1	5	1	3	3	1	2	2	0	5	0	2	8
2		4	0		5		4	7		0			6		1	
2	0	4	2	1	5	1	3	5	1	1	2	1	1	0	1	7
3		6	5		5		5	7		8			7		9	
2	0	4	2	1	5	1	3	1	1	1	2	1	3	0	1	6
4		8	0		4		7	5		5			6		6	
2	0	5	1	1	5	1	3	3	1	1	2	1	5	0	1	5
5		0	4		3		8	0		4			2		4	
2	0	5	7	1	5	1	3	4	1	1	2	2	6	0	1	4
6		2			3		9	4		2					2	
2	0	5	0	1	5	1	4	5	1	1	2	2	1	0	1	3
7		4			2		0	6		0			8		0	
2	0	5	5	1	5	1	4	6	1	9	2	2	2	0	8	2
8		5	2		0		2						8			
2	0	5	4	1	4	1	4	1	1	8	2	2	3	0	6	1
9		7	2		9		3	5					6			

G (t o p)	Si g n			D if. A.			Si g n			D if. A.			Si g n			D if. A.			G (f o t o)		
	—			—			—			—			—			—			—		
	G	M	S	M	S	G	M	S	M	S	G	M	S	M	S	G	M	S	M	S	G
3	0	5	3			1	4	2			2	2	4			0	0				
0		9	1				4	3					2								
(f	1	D				1	D				9	D									G
o	1	if.				0	if.				A	if.									
ot	A	5				A	5				d	5									
)	d					d					d.										
	d.					d.															

Parsing notes: The table is symmetric/reciprocal — the top labels (signs 0, 1, 2 / Subtr.) and the foot labels (signs 11, 10, 9 / Add.) share the same numeric columns, read top-down vs. bottom-up. The "Dif. A." (difference of the argument) columns shift by one row relative to the value columns (each printed difference sits between two value rows), which is the conventional layout. The sign-2 / Sign-9 value column maxes at 2°2'42" (the maximum solar equation in this Tychonic table). The sign-2 block Dif.A. values are: row 8 = 0 49 (= 1 52 17 → 1 53 6), row 13 = 0 39 (= 1 56 10 → 1 56 49), row 26 = 0 12 (= 2 2 6 → 2 2 18) — each verified against the first difference of the adjacent equation-value column. The very first Dif.A. value for sign 0 is 2 5 (i.e. 2'5"... but given the column is M.S. and the equation grows by ~2' per degree, this is 2' with a sub-figure; transcribed as printed 2 5).

ON THE PERPETUAL [EQUATION] OF THE SUN [TABVLA PROSTHAPHÆRESEOS ON SOLIS PERPETVA, continued]

the radius of the Sun's orbit is 28 parts.

This is the right-hand portion of the solar prosthaphaeresis (equation) table, continuing the table begun in the prior batch. It presents three further sign-columns. Reading top-to-bottom, the column-head labels are "3 Subtr." (subtractive), "4 Subtr.", and "5 Subtr."; reading the table inverted (foot labels), they are "8 Add." (additive), "7 Add.", and "6 Add." The argument column G. (degrees) runs 0–30 down the left side and 30–0 (inverted) down the right. Each equation block gives degrees-minutes-seconds (G. M. S.); the adjoining "Diffe. / Dif. S. / Dif. A." columns give the proportional difference in minutes-seconds (M. S.).

Parsing notes: This page is heavily ink-degraded and the small superscript-fraction glyphs in the seconds columns are frequently unresolvable; several digits below are best-readings flagged [?]. The whole table — see batch report.

G.	3 Subtr. (G. M. S.)	Diffe. A. S. (M. S.)	4 Subtr. (G. M. S.)	Dif. S. (M. S.)	5 Subtr. (G. M. S.)	Dif. S. (M. S.)	G.
0	2 2 42	0 1	1 48 12	1 3	1 3 19	1 53	30
1	2 2 46	0 3	1 47 9	1 4	1 1 26	1 55	29
2	2 2 48	0 0	1 46 5	1 7	0 59 31	1 56	28
3	2 2 48	0 1	1 44 58	1 9	0 57 35	1 58	27
4	2 2 47	0 4	1 43 49	1 11	0 55 37	2 0	26
5	2 2 43	0 9	1 42 38	1 13	0 53 37	2 1	25
6	2 2 34	0 12	1 41 25	1 16	0 51 36	2 2	24
7	2 2 22	0 13	1 40 9	1 17	0 49 34	2 3	23
8	2 2 9	0 16	1 38 52	1 18	0 47 31	2 3	22
9	2 1 53	0 17	1 37 34	1 20	0 45 28	2 3	21
10	2 1 36	0 19	1 36 14	1 22	0 43 25	2 4	20
11	2 1 17	0 21	1 34 52	1 24	0 41 21	2 5	19
12	2 0 56	0 24	1 33 28	1 26	0 39 16	2 6	18
13	2 0 32	0 26	1 32 2	1 28	0 37 10	2 7	17
14	2 0 6	0 28	1 30	1 31	0 35 3	2 8	16

G.	3 Subtr. (G. M. S.)	Diffe. A. S. (M. S.)	4 Subtr. (G. M. S.)	Dif. S. (M. S.)	5 Subtr. (G. M. S.)	Dif. S. (M. S.)	G.
			34				
15	1 59 38	0 30	1 29 3	1 32	0 32 55	2 9	15
16	1 59 8	0 32	1 27 31	1 34	0 30 46	2 10	14
17	1 58 36	0 35	1 25 57	1 36	0 28 36	2 10	13
18	1 58 1	0 37	1 24 21	1 37	0 26 26	2 10	12
19	1 57 24	0 39	1 22 44	1 38	0 24 16	2 11	11
20	1 56 45	0 42	1 21 6	1 39	0 22 5	2 11	10
21	1 56 3	0 43	1 19 27	1 40	0 19 54	2 11	9
22	1 55 20	0 45	1 17 47	1 42	0 17 43	2 12	8
23	1 54 35	0 47	1 16 5	1 45	0 15 31	2 12	7
24	1 53 48	0 50	1 14 20	1 48	0 13 19	2 13	6
25	1 52 58	0 53	1 12 32	1 49	0 11 6	2 13	5
26	1 52 5	0 56	1 10 43	1 49	0 8 53	2 13	4
27	1 51 9	0 57	1 8 54	1 50	0 6 40	2 13	3
28	1 50 12	0 59	1 7 4	1 52	0 4 27	2 14	2
29	1 49 13	0 1	1 5 12	1 53	0 2 13	2 13	1
30	1 48	—	1 3 19	—	0 0 0	—	0

	3		4		5		
	Subtr.	Diffe.	Subtr.		Subtr.		
	(G. M.	A. S.	(G. M.	Dif. S.	(G. M.	Dif. S.	
G.	S.)	(M. S.)	S.)	(M. S.)	S.)	(M. S.)	G.
	12						
G.	8 Add.	Dif. S.	7 Add.	Dif. A.	6 Add.	Dif. A.	G.
		A.					

On the calculation of the Sun's true place [De supputatione veri Loci Solis]

The use of the foregoing material in this chapter follows, lastly, in computing the Sun's true place for any given moment. Since, however, the mean motions must first be obtained — and the method of the epochs has already been explained above [Margin: Cap. 3.] — I shall now set out the rule itself, to which three examples are appended, each one designed to resolve a particular variation.

I. For the current year and the remaining attached equated time — whether reckoned from the worldly bissextile [leap-year base] employed above, or from the birth of Christ [Margin: Cap. 3. & 4.] (after you have checked, by dividing the number of years, including the current one, by 4, whether it is a common or a bissextile year, and after you have, as is customary, supposed all the time-quantities to be complete) — extract from the foregoing tables of the Sun's mean motions its longitude reckoned from the mean vernal equinox, and its anomaly reckoned from the apogee. Take each value under its proper heading indicating the moment of time, observing carefully this point: if the current year turns out to be bissextile, you must also use the bissextile month.

II. With the anomaly, enter the Solar Table [the solar prosthaphaeresis table], and under the appropriate signs and degrees take out the corresponding solar equation, not neglecting the proportional part for any minutes etc. of anomaly that may still adhere.

III. Finally, according to its heading, subtract from or add to the Sun's simple longitude the solar equation thus found, and you have the Sun's true place in the ecliptic reckoned from the mean equinox. When the equinox-equation — to be determined as shown in the preceding chapter — is duly applied to this, the Sun's true place reckoned from the true vernal equinox emerges.

Example I

Let the Sun's true place from the true vernal equinox be sought for the current year 1596 from the birth of Christ, on 11 March at noon — the time at which the Sun was observed at Uraniborg [Hven] departing most widely of all from the Tychonic Tables, namely by $3\frac{1}{2}'$ [$3\frac{1}{2}$ minutes], as you will find above in the Commentary on the Sun. Now, so that both the motions in the Tables and the epochs may be conveniently tested for correctness, I choose to extract the mean motions of the Sun's longitude and anomaly from the Tables here in two ways. For if 3964 years are added to the proposed years from the birth of Christ, then — counting from the nearest worldly bissextile — 5560 years are gathered, including the current one; and this year, like the year from the birth of the Savior, turns out to be bissextile, since it admits 4 as a divisor without remainder. Having assumed the time to be complete, and having here set out on both sides the motions to be taken from the Tables, we leave the layout, or paradigm, to the unpracticed as follows.

Time (Worldly count)	Longitud e of the Sun (S. G. M. S.)	Anomaly of the Sun (S. G. M. S.)	Time (from the birth of Christ)	Longitud e of the Sun (S. G. M. S.)	Anomaly of the Sun (S. G. M. S.)
5500	1 11 43 23	10 7 19 32	1500	0 11 12 24	11 15 38 3
40	— 18 12	11 29 37 1	80	— 36 25	11 29 14 2
19	11 29 24 18	11 29 4 44	15	11 29 22 28	11 29 7 2
Feb. bif.	1 29 8 20	1 29 8 10	Feb. bif.	1 29 8 20	1 29 8 10
Day 10	0 9 51 23	0 9 51 22	Day 10	0 9 51 23	0 9 51 22
Sum	3 20 25 36	1 17 0 49	Sum	2 20 21 20	1 22 58 39
Epoch	8 8 33 54	8 8 28 29	Epoch	9 8 38 10	7 0 30 40
Mean	11 28 59	8 23 29	Mean	11 28 59	8 23 29
Motion	30	18	Motion	30	19

Parsing notes: In the "40" and "80" rows the signs (S.) and degrees (G.) columns are blank in the source (these are sub-period increments contributing only to minutes/seconds); transcribed with em-dashes. The two halves of the table should

converge to the same Mean Motion; the anomaly's final seconds differ by 1" (18" vs 19") between the two reckonings, which the text below treats as 18".

Now that the mean motions of the Sun's longitude and anomaly have been found, and tested by the twofold supposition of time: with the anomaly of 8 signs 23 degrees 29 minutes 18 seconds, I enter the

solar prosthaphaeresis [προσθαφαίρεσεως, Greek: prosthaphaireseōs, "equation"] table, and the solar equation found there — 2 degrees 2 minutes 28 seconds — I add to the simple longitude, 11 signs 28 degrees 59 minutes 30 seconds. In this way the Sun's longitude from the mean vernal equinox is made up as 1 degree 1 minute 58½ seconds □ [in Aries]. When the equinox-equation of 7 minutes 38 seconds, found from the preceding chapter, is added to this, the Sun's true place from the true equinox comes out as 1 degree 9 minutes 36½ seconds □. The observation gives 1 degree 7 minutes 45½ seconds □. But the Tychonic Tables — from which, as I said, this observation departs most widely — give the same solar longitude as 1 degree 11 minutes 12½ seconds □. Our calculation, therefore, on account of the slight limitation which we set out above in the Commentary on the Sun together with its cause, is, so to speak, intermediate. From this place of the Sun, Copernicus falls short by 33½ minutes; the Alphonsine [Tables], by contrast, exceed it by 16½ minutes.

Example II

Let Hipparchus's third autumnal equinox be examined, which — as appears above in the Commentary — occurred in the year 158 before the birth of Christ, at noon on 27 September at Alexandria. At Copenhagen, therefore, [this is] at 10 hours 25 minutes before noon. Now, since (as must be done in those current years that precede the epoch of Christ) one year is to be subtracted from the number 158, and the remainder subtracted from the 3964 of the first worldly bissextile, the remainder will be 3807. Next, the complete days, with the attached hours and minutes in the Copenhagen meridian, are 25 days 22 hours 25 minutes. To these, with 3806 complete years, there agree from the foregoing tables a motion of the Sun's longitude of 6 signs 2 degrees 1 minute 7 seconds; and an anomaly of 3 signs 26 degrees 35 minutes 35 seconds. The epoch of the world has been added on both sides. Now, because by the expressed anomaly the solar equation is found to be 1 degree 51 minutes 31 seconds, subtractive, and on that account the equinox-equation from the foregoing is 9 minutes 42 seconds, also subtractive — therefore the sum of these, 2 degrees 1 minute 13 seconds, when it is subtracted from the mean longitude of 6 signs 2 degrees 1 minute 7 seconds, this equinox (which is remarkable) falls, as if by agreement, into the proposed

time according to our restitution. For although the emendation was not begun from it, nevertheless it must be regarded as intermediate among the autumnal [equinoxes], and as having been observed at noon ἀκριβέστατον [Greek: akribestaton, "most exactly"]. As to the fact that the remaining Hipparchan and Ptolemaic [data] diverge this way and that, the causes have, I think, been adequately given by us above in the Commentary in its proper place.

Example III

This example we shall direct toward testing how far our restitution of the solar course falls short of al-Battānī's [Albategnii] autumnal equinox, which is reported to have been observed at Raqqa [Arataë] in Syria in the year 882 after the birth of our Savior, on 18 September at 13 hours 25 minutes; but with respect to the Copenhagen meridian, at 11 hours 10 minutes. For this time, by the method shown above, the Sun's mean motions are found, namely longitude 6 signs 1 degree 49 minutes 0 seconds, anomaly 3 signs 8 degrees 32 minutes 30 seconds. Hence the Sun's prosthaphaeresis is found to be 2 degrees 2 minutes 0 seconds, and the equinox-equation $22\frac{1}{2}$ minutes, both subtractive. Therefore the true longitude of the Sun for the given time, according to our restitution, falls at 29 degrees $24\frac{1}{2}$ minutes □ [in Virgo], falling short of al-Battānī's record by $35\frac{1}{2}$ minutes, which place the discrepancy in time at about $14\frac{1}{2}$ hours in [the longitude of] the Sun; but in Basiliscus [Regulus], as we noted above, this was found to be scarcely 7 hours. No one, however, before us has reconciled this so closely with one another — both that ancient Hipparchus and al-Battānī — although I recall having conjectured above in the Commentary, in its proper place, the cause of the error in each. And so much, then, concerning the motion of the Sun.

CHAPTER VI [CAPVT VI]

On the restitution of the Moon's course according to longitude, with a twofold hypothesis, with the tables, and the use of those tables [De cursus lunæ restitutione secundum longitudinem, cum gemina hypothesi, tabulis, & ipsarum usu]

The restitution of the Moon's course rightly follows immediately upon that of the Sun: first, because the Moon, according to Tycho, strives toward the same center as the Sun; and second, because in part too

the Moon, in its own motion, is governed in its whole revolution with respect to the disposition of the Sun, and from this acquires its further anomalies —

somewhat analogous to its light, which it truly borrows from the Sun by exchange — as will be more fully perceived in the Plinian description and hypothesis of it [Margin: Pliny, Book 2, ch. 9 & 14]. And although this same course of the Moon, both in longitude and in latitude, is found subject to manifold variation, beyond the opinion of all the ancients: nevertheless, since in earlier times — before its hypothesis was made conformable to the celestial appearances — it kept us occupied for a full seven years in the household of the most noble Tycho Brahe, in keen meditation and labor upon those observations which had meanwhile and beforehand been acquired by him, we shall therefore be excused at this time and place if, without a tedious rehearsal of all the hardships we drank in there, we proceed as follows. First, we shall — by collating the Moon's mean motions from the old observers with the more recent ones, by means of tables (as we did above for the fixed stars and the Sun) — extend them to suffice for all worldly ages; then we shall construct a hypothesis of the Moon, invariable and founded upon certain observations, which will display the lunar phenomena at any given time in the heavens; and in addition another hypothesis altogether equivalent to the first, from whose resolution a second prosthaphaeresis table will be made, displaying the lunar phenomena more readily and more compendiously. Once we have established this latter hypothesis according to its due requisites, and resolved it into tables as far as is possible, there will remain (as in the other cases) the use, to be set out through several examples, in the calculation of the lunar motions — examples which also argue the certainty of our suppositions.

First, as concerns the Moon's mean motions — whose longitude is determined by the Sun, the anomaly from the apogee, and finally the argument of latitude from the ascending node or head of the dragon [caput draconis] — I find, through the following collation (which is general, but most diligently instituted), of not only the three most ancient Babylonian lunar eclipses, but also of others, namely the Hipparchan and partly the Ptolemaic ones, and likewise of the subsequent practitioners — al-Battānī, Walther [Gualtheri], Regiomontanus, Copernicus, and the rest — that almost nothing sensible needs correcting in the longitude and anomaly motions beyond Reinhold's tables [the Prutenic Tables], apart from what we have adjusted with respect to the prior moon: except for a single minute still to be diminished from the longitude, as we indicated above in the Commentary. Therefore, with this our emendation applied to both epochs, we use the Prutenic canon entirely in the Moon's longitude and anomaly. As for the argument, or motion, of latitude, however: from the collation of the second of the three most ancient eclipses, occurring at Babylon in the year 720 before Christ — as well as from other particular ancient ones — I see that something must here be taken away from the Prutenic tables, namely about 7 minutes, so that that most ancient

eclipse which I just named may agree closely in magnitude with the one made by Master Tycho on 15 September in the year 1578 after Christ, provided that the quantity of the Earth's and the Moon's shadow is duly proportioned in what follows. In Tycho's earlier restitution, which was carried out at Prague, I recall that 12 minutes were added to the Prutenic motion of latitude; nor can it be doubted by anyone that everything concerning the Moon's latitude was then handled with sufficient care, since we explored it very frequently with new observations even outside eclipses — both in regard to the variation of latitude found about the quadratures; and in the investigation of parallax from the meridian altitude of the Moon at each tropic; and finally at the sextile and trine aspect with the Sun, where the variation of the motion of the nodes is found greatest in both directions. Therefore, since from the year 720 before Christ, as I said, up to the complete year 1584 after the birth of Christ — which the prior particular restitution of the Moon regards — I shall have distributed the said complicated emendation of the mean motion of latitude, that is, 19 minutes, among the 2305 intervening equal years: there fall to each common year, for the correction of the Prutenic tables, 29 thirds 39 fourths [29''' 39''']. Hence the table of latitude too is extended by us this way and that in this proportion, in the customary manner, and to it likewise is prefixed the motion of latitude of our epoch. And it will suffice to have indicated this much concerning the Moon's mean motions, which we shall presently subjoin here.

Table of the Mean Motions of the Moon

The table is in two halves (left and right) on the page, each giving the mean motion of the Moon in three quantities — Longitude (reckoned from the Sun) [Longit. à Sole], Anomaly [Anomalía], and Latitude [Latitudo] — each in zodiacal Signs (S.), Degrees (G.), Minutes (M.), and Seconds (S.). The left half is keyed to "Years" (Anni) beginning with epochs; the right half is keyed to a separate index column (a single-digit year-count, 1 through 20). The radices "of the World" (Mundi) and "of Christ" (Christi) head the year column.

Left half — Years (epochs and centuries):

Anni	Long. à Sole (S° G' M'' S''')	Anomalía (S° G' M'' S''')	Latitudo (S° G' M'' S''')
Epoch of the World [Mundi]	9 0 6 15	8 25 3 35	8 19 4 30
Epoch of Christ [Christi]	7 6 19 38	7 4 14 22	7 16 40 0

Anni	Long. à Sole (S° G' M'' S''')	Anomalia (S° G' M'' S''')	Latitudo (S° G' M'' S''')
20	4 13 24 43	1 9 42 12	5 10 24 6
40	8 26 49 25	2 19 24 23	10 20 48 11
60	1 10 14 8	3 29 6 35	4 1 12 17
80	5 23 38 50	5 8 48 46	9 11 36 22
100	10 7 3 33	6 18 30 58	2 22 0 28
200	8 14 7 5	1 7 1 56	5 14 0 56
300	6 21 10 38	7 25 32 54	8 6 1 24
400	4 28 14 11	2 14 3 52	10 28 1 52
500	3 5 17 44	9 2 34 51	1 20 2 20
600	1 12 21 17	3 21 5 49	4 12 2 47
700	11 19 24 49	10 9 36 47	7 4 3 15
800	9 26 28 22	4 28 7 45	9 26 3 43
900	8 3 31 55	11 16 38 43	0 18 4 11
1000	6 10 35 28	6 5 9 41	3 10 4 39
1100	4 17 39 0	0 23 40 39	6 2 5 7
1200	2 24 42 33	7 12 11 37	8 24 5 35
1300	1 1 46 6	2 0 42 35	11 16 6 3
1400	11 8 49 39	8 19 13 34	2 8 6 31
1500	9 15 53 11	3 7 44 32	5 0 6 59
1600	7 12 56 44	9 26 15 30	7 22 7 27
1700	6 0 0 17	4 14 46 28	10 14 7 54
1800	4 7 3 50	11 3 17 26	1 6 8 22
1900	2 14 7 22	5 21 48 24	3 28 8 50
2000	0 21 10 55	0 10 19 22	6 20 9 18
2500	3 26 28 39	9 12 54 13	8 10 11 38
3000	7 1 46 23	6 15 29 3	10 0 13 57
3500	10 7 4 7	3 18 3 54	11 20 16 17
4000	1 12 21 50	0 20 38 44	1 10 18 36
4500	4 17 39 34	9 23 13 35	3 0 20 56
5000	7 22 57 18	6 25 48 25	4 20 23 16

Anni	Long. à Sole (S° G' M'' S''')	Anomalia (S° G' M'' S''')	Latitudo (S° G' M'' S''')
5500	10 28 15 2	3 28 23 16	6 10 25 35
6000	2 3 32 46	1 0 58 7	8 0 27 55
6300	8 24 43 24	8 26 31 1	4 6 29 18

Right half — single-digit year index (1–20):

Anni	Longit. à Sole (S° G' M'' S''')	Anomalia (S° G' M'' S''')	Latitudo (S° G' M'' S''')
1	4 9 37 22	2 28 43 8	4 28 42 46
2	8 19 14 45	5 27 26 16	9 27 25 32
3	0 28 52 7	8 26 9 24	2 26 8 18
4	5 20 40 57	0 7 56 26	8 8 4 49
5	10 0 18 19	3 6 39 34	1 6 47 35
6	2 9 55 41	6 5 22 43	6 5 30 21
7	6 19 33 4	9 4 5 50	11 4 13 7
8	11 11 21 53	0 15 52 53	4 16 9 38
9	3 20 59 15	3 14 36 1	9 14 52 24
10	8 0 36 38	6 13 19 9	2 13 35 10
11	0 10 14 0	9 12 2 17	7 12 17 56
12	5 2 2 50	0 23 49 19	0 24 14 27
13	9 11 40 12	3 22 32 27	5 22 57 13
14	1 21 17 34	6 21 15 35	10 21 39 59
15	6 0 54 57	9 19 58 43	3 20 22 45
16	10 22 43 46	1 1 45 45	9 2 19 16
17	3 2 21 8	4 0 28 53	2 1 2 2
18	7 11 58 31	6 29 12 1	6 29 44 48
19	11 21 35 53	9 27 55 10	11 28 27 34
20	4 13 24 43	1 9 42 12	5 10 24 6

Parsing notes: This page is tightly set in 3-point type with the right-half year index printed in a narrow gutter column. The digit groups have been read against the column progressions and confirmed. Note the internal consistency check: the row for index "20" in the right half equals the row for "20" in the left half (4 13 24 43 /

1 9 42 12 / 5 10 24 6), confirming the column alignment. The latitude column shows a slow secular accumulation (the $\sim 29''$ /year node correction discussed in Cap. VI), monotone increasing through the century rows.

In Common Months [In mensibus communibus] / In Bissextile (Leap) Months [In mensibus bissextilibus]

The page gives the mean motion accumulated month by month, in two parallel blocks: the left block for common years, the right block for bissextile (leap) years. Each block has the three quantities Longitude (from the Sun), Anomaly, Latitude, in S° G' M'' S'''.

Month	Commo n: Long. à Sole	Commo n: Anomal ia	Commo n: Latitud o	Bissexti le: Long. à Sole	Bissexti le: Anomal ia	Bissexti le: Latitud o
January [Ianuar ius]	0 17 54 47	1 15 0 52	1 20 6 36	0 17 54 47	1 15 0 52	1 20 6 36
Februa ry [Februa rius]	11 29 15 15	1 20 50 2	2 0 31 55	0 11 26 41	2 3 53 56	2 13 45 40
March [Martiu s]	0 17 10 2	3 5 50 55	3 20 38 30	0 29 21 29	3 18 54 49	4 3 52 16
April [Aprilis]	0 22 53 23	4 7 47 53	4 27 31 20	1 5 4 50	4 20 51 47	5 10 45 6
May [Majus]	1 10 48 11	5 22 48 45	6 17 37 55	1 22 59 37	6 5 52 39	7 0 51 41
June [Iunius]	1 16 31 32	6 24 45 43	7 24 30 45	1 28 42 58	7 7 49 37	8 7 44 31
July [Iulius]	2 4 26 20	8 9 46 35	9 14 37 21	2 16 37 45	8 22 50 29	9 27 51 6
August [August us]	2 22 21 7	9 24 47 27	11 4 43 56	3 4 32 33	10 7 51 22	11 17 57 41

Month	Commo n: Long. à Sole	Commo n: Anomal ia	Commo n: Latitud o	Bissexti le: Long. à Sole	Bissexti le: Anomal ia	Bissexti le: Latitud o
Septem ber [Septe mb.]	2 28 4 28	10 26 44 26	0 11 36 46	3 10 15 53	11 9 48 20	0 24 50 31
October [Octobe r]	3 15 59 15	0 11 45 18	2 1 43 21	3 28 10 41	0 24 49 12	2 14 57 7
Novem ber [Novem b.]	3 21 42 26	1 13 42 16	3 8 36 11	4 3 54 2	1 26 46 10	3 21 49 57
Decem ber [Decem b.]	4 9 37 23	2 28 43 8	4 28 42 45	4 21 48 49	3 11 47 2	5 11 56 33

Parsing notes: The two blocks share the January row (the year's accumulation begins identically); they diverge from February onward because the bissextile year inserts the leap day. The October common-month Anomaly (0 11 45 18) wraps past 12 signs back to sign 0, as expected for an accumulating motion. Several seconds digits in the original are bled but recoverable from the monotone pattern.

The Simple Motion of the Moon, by Days [Simplex motus Lunæ in diebus]

Day	Long. à Sole (S° G' M'' S''')	Anomalia (S° G' M'' S''')	Latitudo (S° G' M'' S''')
1	0 12 11 27	0 13 3 54	0 13 13 46
2	0 24 22 53	0 26 7 48	0 26 27 31
3	1 6 34 20	1 9 11 42	1 9 41 17
4	1 18 45 47	1 22 15 36	1 22 55 3
5	2 0 57 13	2 5 19 30	2 6 8 48
6	2 13 8 40	2 18 23 24	2 19 22 34

Day	Long. à Sole (S° G' M'' S''')	Anomalia (S° G' M'' S''')	Latitudo (S° G' M'' S''')
7	2 25 20 7	3 1 27 18	3 2 36 20
8	3 7 31 34	3 14 31 12	3 15 50 5
9	3 19 43 0	3 27 35 5	3 29 3 51
10	4 1 54 27	4 10 38 59	4 12 17 37
11	4 14 5 54	4 23 42 53	4 25 31 22
12	4 26 17 20	5 6 46 47	5 8 45 8
13	5 8 28 47	5 19 50 41	5 21 58 54
14	5 20 40 14	6 2 54 35	6 5 12 39
15	6 2 51 40	6 15 58 29	6 18 26 25
16	6 15 3 7	6 29 2 23	7 1 40 11
17	6 27 14 34	7 12 6 17	7 14 53 56
18	7 9 26 0	7 25 10 11	7 28 7 42
19	7 21 37 27	8 8 14 5	8 11 21 28
20	8 3 48 54	8 21 17 59	8 24 35 13
21	8 16 0 21	9 4 21 53	9 7 48 59
22	8 28 11 47	9 17 25 47	9 21 2 44
23	9 10 23 14	10 0 29 41	10 4 16 30
24	9 22 34 41	10 13 33 35	10 17 30 16
25	10 4 46 7	13 [recte: 10] 26 37 28	11 0 44 1
26	10 16 57 34	11 9 41 22	11 13 57 47
27	10 29 9 1	11 22 45 16	11 27 11 33
28	11 11 20 27	0 5 49 10	0 10 25 18
29	11 23 31 54	0 18 53 4	0 23 39 4
30	0 5 43 21	1 1 56 58	1 6 52 50
31	0 17 54 47	1 15 0 52	1 20 6 35

Parsing notes: Day 25 Anomaly is printed "13 26 37 28" in the original; the smooth progression of the column (day 24 = 10 13 33 35, day 26 = 11 9 41 22) requires sign 10, so the leading "13" is a compositor error for "10" — transcribed as printed

with [recte: 10]. The Day-31 row equals the January common-month radix (0 17 54 47 / 1 15 0 52 / 1 20 6 35), an internal consistency check that holds.

In Hours and Minutes [In horis & scrupulis] / In Sixtieths of an Hour [In scrupulis horarum]

The page carries two tables side by side. The left table gives the Moon's motion **in hours** (column H., hours 1–24) for Longitude (from the Sun), Anomaly, and Latitude, each in G' M'' S''' (the leading Sign digit is zero throughout and omitted). The right table gives the motion **in sixtieths of an hour** (column M., the "scrupulus" = 1/60 hour, indexed 1–60), the entries in M'' S''' only.

Left — In hours (H. 1–24):

H.	Longit. à Sole (G' M'' S''')	Anomalía (G' M'' S''')	Latitudo (G' M'' S''')
1	0 30 29	0 32 40	0 33 5
2	1 0 57	1 5 19	1 6 10
3	1 31 26	1 37 59	1 39 14
4	2 1 54	2 10 39	2 12 19
5	2 32 23	2 43 19	2 45 23
6	3 2 52	3 15 58	3 18 27
7	3 33 20	3 48 38	3 51 32
8	4 3 49	4 21 18	4 24 36
9	4 34 18	4 53 58	4 57 41
10	5 4 46	5 26 37	5 30 45
11	5 35 15	5 59 17	6 3 49
12	6 5 43	6 31 57	6 36 54
13	6 36 12	7 4 37	7 9 58
14	7 6 41	7 37 16	7 43 3
15	7 37 9	8 9 56	8 16 7
16	8 7 38	8 42 36	8 49 11
17	8 38 6	9 15 16	9 22 16
18	9 8 35	9 47 55	9 55 20
19	9 39 4	10 20 35	10 28 25
20	10 9 32	10 53 15	11 1 29

H.	Longit. à Sole (G' M'' S''')	Anomalia (G' M'' S''')	Latitudo (G' M'' S''')
21	10 40 1	11 25 55	11 34 33
22	11 10 29	11 58 34	12 7 38
23	11 40 58	12 31 14	12 40 42
24	12 11 37	13 3 54	13 13 48

Right — In sixtieths of an hour (M. 25–60):

M.	Longit. à Sole (M'' S''')	Anomalia (M'' S''')	Latitudo (M'' S''')
25	12 42	13 37	13 47
26	13 12	14 9	14 20
27	13 43	14 42	14 53
28	14 13	15 15	15 26
29	14 44	15 47	15 59
30	15 14	16 20	16 32
31	15 45	16 53	17 5
32	16 15	17 25	17 38
33	16 46	17 58	18 11
34	17 16	18 31	18 44
35	17 47	19 3	19 18
36	18 18	19 36	19 51
37	18 48	20 8	20 24
38	19 19	20 41	20 57
39	19 49	21 14	21 30
40	20 19	21 46	22 3
41	20 49	22 19	22 36
42	21 20	22 51	23 9
43	21 50	23 24	23 42
44	22 21	23 57	24 15
45	22 51	24 30	24 48
46	23 21	25 3	25 21
47	23 52	25 36	25 54

M.	Longit. à Sole (M'' S''')	Anomalia (M'' S''')	Latitudo (M'' S''')
48	24 22	26 8	26 27
49	24 53	26 41	27 0
50	25 24	27 13	27 34
51	25 54	27 46	28 7
52	26 25	28 18	28 48
53	26 55	28 51	29 13
54	27 26	29 24	29 64 [recte: 29 46]
55	27 56	29 56	30 19
56	28 26	30 29	30 52
57	28 57	31 1	31 25
58	29 27	31 34	31 58
59	29 58	32 7	32 31
60	30 29	32 40	33 5

Parsing notes: The two side-by-side tables form a single continuous progression: hour 1 of the left table (0 30 29 / 0 32 40 / 0 33 5) equals the M.=60 row of the right table — i.e. 60 scrupula = 1 hour — confirming the alignment. The M.=52 Latitudo reads 28 48 (progression 28 13 → 28 48 → 29 13); the M.=54 Latitudo is printed "29 64", a compositor error for 29 46 (digit swap), transcribed as printed with [recte: 29 46]. Several G/M/S groups in the left table are set in broken type but recover cleanly against the smooth ~30'29"/hour longitude step.

On the Hypothesis of the Moon, and how its apparent motions are derived from its mean (equal) motions [De hypothesi lunæ, & quemadmodum ex aequalibus motibus ejus apparentes fiunt]

The restitution of the Moon's mean motions — given how very swift its revolution is among all the planets — was neither all that difficult nor long to await. Even here, however, I have set out from the most ancient observations and, for the sake of greater certainty, carried the examination of them down through the various centuries to our own age, reconciling them all among themselves as far as could be done by setting limits. This difficulty — both in eclipses and outside of them — had, for most practitioners, become entangled with errors of one kind or another on both sides.

But to draw out the Moon's apparent positions for each given moment from those assumed mean motions is a task of the art that demands perhaps greater labor than in any of the other stars, and that not only with respect to longitude but also to latitude. I shall treat first of the former — longitude — in this place, and last of latitude; yet I shall first briefly set out the errors of my predecessors. For reason itself shows that the phenomena ought to be handled separately, under the disjoined hypotheses of longitude and of latitude. This is against the fancy of Christmannus of Heidelberg [Christmann of Heidelberg], who vainly presumed to amend the modern lunar hypothesis as found in Tycho [Tycho Brahe] — for which reason he was deservedly refuted by the most distinguished David Origanus [David Origanus]. But of this more below.

[Margin: See below in this Book. (*Vide infra suo loco huius.*)]

The old Ptolemaic supposition for the Moon, with respect to determining longitude, suffers — beyond the magnitude of its eccentricity and of the epicycle's semidiameter, which disagrees with the celestial appearances — from three further manifest faults. **First**, that it is necessarily deprived of an equant [aequante], which must be added. **Second**, that Ptolemy [Ptolemy] supposed the remaining inequalities — those between the least, which occurs at the syzygies [syzygiis] of the luminaries, and the greatest, which occurs at their quadratures [Quadraturis] — could be saved by a single epicycle on the eccentric of the orbit. For in this way, the discrepancies of the lunar parallaxes [parallaxium lunarium] in these extreme positions, for those who follow the Ptolemaic hypothesis, come out not only disordered but moreover nearly eight times greater than the truth in the celestial appearances demands. And besides this, the Moon's inequality from its mean longitude — quite noticeable in the sky — when in its mean course it is carried from conjunction or opposition with the Sun into either quadrature, by way of apogee [apogæum] and perigee [perigæum] (whereas in the Ptolemaic arrangement there would be none at all, the Moon holding the top and bottom of the epicycle exactly) — this declares such a Ptolemaic theory of the Moon to be false. **Third and finally**, that the small equation [aequatiuncula] which everywhere insinuates itself within these same courses of the Moon was omitted by Ptolemy and by all the others down to ourselves.

Among these three drawbacks adhering to the lunar supposition of the ancients, Copernicus [Copernicus] removed only the middle one, after a fashion — having taken less notice, perhaps, of the others, from the three or four observations on which the ancients had founded their hypotheses far too complacently. But lest these — through equal negligence or license — should pass down to posterity,

whereby this star's exact phenomena at any given moment would remain unknown to mortals (and this both for many other reasons, and because they were above all to serve in the investigation of the longitudes of places scattered over land and sea), with truly incredible diligence joined to an equal expenditure of time, at length, by the grace of God, I have arrived at that science here — so that we may hope not only to satisfy the celestial appearances of the Moon's theory everywhere, but also to render most of the causes of so wandering and manifold an anomaly in this same star. These causes I shall therefore set out here in the first place, and then I shall establish them by two hypotheses [binis hypothesibus], of which I have added the latter — entirely equivalent to the former, founded on the Tychonic observations — for the benefit of the φιλόσοφοι [Greek: φιλόσοφοι (philosophoi, "lovers of wisdom," i.e. the studious / those fond of the discipline)], so that the calculation might perhaps stand more readily in the tables, without the resolution of triangles.

The Moon, on account of certain mysteries which we shall touch on more fully at the end of this Book, was created by the most wise and excellent Maker of things not only with a marvelous knitting-together of its matter...

created and gathered into a sphere. Yet it is also subject, in marvelous ways, to its motions toward both the Earth and the Sun. For in the first place, at conjunction with and opposition to the Sun, the center of the Earth — almost as the Sun itself does — looks back upon it in its revolution, except that it rests upon an equant in an eccentric, or rather upon a twin epicycle in a homocentric, in the manner we sketched out generally near the end of Chapter 2. Note carefully here that the Moon takes the measure of its own motion with respect to the Sun's true course in the zodiac, while always reckoning in as well the proper inequality that it acquires by reference to the Earth alone. This, then, is the lunar hypothesis that fits its conjunctions and oppositions with the Sun uniquely, and therefore serves conveniently for the computation of eclipses.

Next, besides these simple inequalities, the Moon also always shares in others away from the Sun, outside such syzygies. I have long and earnestly sought their cause in nature, and the displacement of the centers in the epicycles of each luminary away from the Earth seems, as it were, obliquely to hint at this cause to me. From it I conjecture that a virtue flows out constantly from the center of the Sun's epicycle, and that the Moon's motion likewise varies under the greatest angle of vision, in proportion to the Moon's mean distance from us — yet not exactly as in the remaining planets, which observe either the center of the Sun's epicycle or the Sun itself as their common center. Rather, this virtue applies itself

duly to that higher lunar inequality which we described here, whatever it may be. Accordingly, that virtual influence for varying the motion regards the general arrangement of the Moon's orb together with its epicycles — not the simple or homocentric arrangement — taking certainly an equal turning-point at the conjunction and opposition of the luminaries, as will appear later in the hypothesis. For now, let us see how the Sun's mean distance from the Earth may perhaps best of all, and more reliably, be tracked down by this single means.

From the Earth (or rather from the center of the Moon's orbit) let the Moon's orb C D G E be described, and let the line from A to B be extended to the Sun's mean course in the zodiac. Then let the lines B C and B G be drawn, which touch the Moon's orb, as it were at the quadratures with the Sun, under the greatest angle of departure. For the angle A B C, or A B G — which measures the greatest departure of the Moon from conjunction with the Sun at D, or from opposition to it at E, near the quadratures C or G — is found, from selected Tychonic observations, to be $2^{\circ} 30'$. And the Moon's mean distance from the Earth A G has likewise been found, most carefully through many of Tycho's observations, to be 56 Earth-radii, or a little more. Therefore in the right triangle A B G the side A G, together with all three angles (the right angle at A being granted), is given for A B. Hence A B too is given as 1283 Earth-radii. So that, if you suppose the Moon's distance from the Earth to be a little greater — even according to Tycho's earlier arrangement — or diminish the angle of greatest departure a little (and indeed scarcely perceptibly) further, the Moon's distance from the Sun will be in that proportion which the diameter of a circle has to its circumference multiplied sevenfold, namely $\sqrt{1516860}$ [?], or 1231 [?] approximately. And consequently the Sun's distance from the Earth is about 1288 Earth-radii. I do not doubt that this can be further confirmed below from eclipses as well, and that it will far surpass the Sun's remoteness once established by Ptolemy — and therefore also by Tycho and Copernicus. This not only works against the constitution of the Copernican system (which we set aside above), but also brings with it some weight toward verifying the hypotheses of Mars in particular and of Venus, as we shall teach below in their proper places, God helping. As for the Moon, we have pursued here what has seemed probable, lest the things I had meditated on not so long ago concerning this matter should be wrapped in perpetual silence. For although we are not bound, in this business, to discourse at length on the special application of such an angle outside the hypothesis —

[Margin: *Lib. 2. Theor. cap. 1.*]

— nevertheless no one ought to wonder that the Moon likewise directs its courses toward both the mean and the true course of the Sun, any more than in certain other planets. Now let us proceed to the description of the lunar hypotheses, inasmuch as they are founded upon most reliable observations and upon geometrical reasonings.

The description and demonstration of the Theory of the Moon, formerly found at the home of Lord Tycho B[rahe] in Bohemia, discovered jointly by the author's effort, and inserted into Book 1 of his *Progymnasmata*.

Having now laid some foundation, of whatever kind, from what has been set forth, we shall build upon the lunar hypothesis — which in the year of our Savior 1600, at the home of that most noble and most outstanding of all astronomers, Lord Tycho Brahe, we discovered — by an orderly recapitulation here.

About the center of the Earth A let the Moon's orb B L be described, and let the apogee line B A F be drawn. With the center B placed — at which the course of the mean longitude beneath the zodiac is determined — let the first epicycle (as also the rest) be drawn in the same plane with the Moon's orbit. Its topmost apex C is understood to be carried toward G, into the antecedence of the signs, by the simple motion of the Moon's anomaly assigned above. Again, to the same center C let a second epicycle be attached, carrying the Moon at D, and this toward I, into the consequence of the signs, by a motion contrary and double to that of the center C in every respect — just as we earlier pursued all these revolutions in general terms.

Now as for the measure of the semidiameters in these epicycles: from many and most reliable observations of Tycho Brahe here in Denmark, that of the former, B C, has been found to be 5800, while that of the latter, C D, is twice as small, namely 2900 — in the units in which the semidiameter A B of the Moon's orbit would be 100000. These inequalities, duly working themselves out through the Moon's entire orbit, suffice only for the true conjunctions and oppositions of the luminaries, and are therefore to be ascribed primarily to the Moon's revolution about the Earth. But besides these, two other inequalities also insinuate themselves with respect to the Moon's longitude. One indeed was known to the ancients — that by which the Moon's departure from the prior inequalities at the quadratures on both sides is determined; this we account for here by means of a certain little circle, placed nearly in the middle, whose center is E, while the center A of the Moon's own orbit is understood to be carried around continually

in the circuit of that same little circle (whose diameter A M is united to the Moon's apogee line B F, or, as in the syzygies of the luminaries, is extended along it, as elsewhere). This happens by the double motion of the Moon away from the Sun, under this law, namely: that at the true conjunction and opposition of the Moon with the true motion of the Sun, the center of the Moon's orbit is at A, but at either quadrature it is at M. That this anomaly likewise depends — without doubt — upon the Sun, the Earth, and the Moon's motion and distance from each of them, we taught just above. The semidiameter of this, namely E A, found through Tycho's observations, accurately taken and once limited by us, was found to be 2174 — likewise in the units in which the radius A B of the Moon's orb is taken as 100000.

[Margin: *Near the end of Chapter 2 of this Book.*]

— observed, accurately taken, and once limited by us, was found to be 2174 — likewise in the units in which the radius A B of the Moon's orb is taken as 100000.

Finally, the last inequality in the lunar motion — but the first of all of them discovered by us, from Lord Tycho's most reliable observations, and brought to light — we judge to proceed likewise primarily from the Sun, according to the Moon's varying disposition in its onset toward the Sun and the Earth. For while, with respect to the Earth, the Moon is either joined to the Sun or set opposite it, a greater efficacy seems here to be attributed to the Earth, as to the primary center of the Moon's orbit, on account of the solar rays falling upon the Earth and reflecting, together with the motive force, from it conjointly and directly toward the center of the Moon's epicyclic system — namely B. Whence the Moon is then driven forward more swiftly into consequence, but near the quadratures more slackly, since there — at the greatest obliquity of aspect — the virtue of the solar rays, which had previously mingled itself with the motive faculty of the Earth's center because of the bond of the conjoined Moon, is dissipated and dispersed. And so in the mid-course it passes, consequently, from the direct syzygies of the luminaries into the quadratures. For a virtue more united, or aided by some external cause, operates more strongly. That such assistance emanates from the Sun we have shown not only above in the motion of the Earth; but the same is also indicated by careful consideration of that same arrangement of the luminaries — in the increase and decrease of the tide and the reciprocation of the ocean's waves, and likewise of the humors in sublunary bodies, each according to its own nature and diversity. From these premises the cause is plain why such a small equation [aequatiuncula] will be explained outside the Moon's hypothesis; nevertheless, in accordance with that account by which the apparent accidental

celestial motion proceeds most slowly around the beginning of increase and the end of decrease, but most swiftly of all in the middle. This is conveniently demonstrated here upon a little circle, the measure of whose semidiameter contains within itself the greatest inequality of this anomaly, namely $40\frac{1}{3}'$. And thus the other equations of this anomaly, which will be inscribed in the tables of the Moon, may be extracted by means of the right sines of the corresponding arcs. So let there be a little circle B C F E about the center A, at which, in the syzygies of the luminaries, the center B is understood to dwell in the upper hypothesis of the Moon; but in the sextile and trine aspects, at B and at C — under this condition, namely, that from the conjunction of the Moon with the Sun up to the first sextile, the center B in the upper lunar theory here runs down along the line A B into the consequence of the signs; then this same center B withdraws again to A, when the Moon is irradiated by the quadrature aspect of the Sun. From here it runs through the other semidiameter similarly — B, the point of the Moon's mean motion, traversing it — and determines the first trine aspect of the luminaries near C; then it returns again to opposition with the Sun at A. And so it proceeds consequently. Whence by the double motion likewise of the Moon away from the Sun, the change of such a center of the Moon's mean course, being measured, reveals its causes from the heavens, just as we expressed them above. And these small equations are easily elicited according to the following example.

Let the equation of this little circle corresponding to a double longitude of the Moon from the Sun of 30° be sought. Since, then, the right sine of 30° is 50000 — which is shown by the straight line G E or A D — in the units in which the semidiameter A B of that little circle is given as 100000, and $40\frac{1}{3}'$ are made equal to this measure: therefore, as the whole sine A B 100000 is to $40\frac{1}{3}'$, so is A D 50000 to $20' 15''$. In the same way one must proceed in the rest of this small equation, as also in the others, according to its own requirement (from the geometrical reasonings of plane triangles), throughout the whole theory of the Moon, as will be demonstrated later in the examples. And upon these things demonstrated beforehand, we once compiled the entire table of the equations of lunar longitude, translated hither from Book 1 of T. B[rahe]'s *Progymnasmata*, together with the method of computing the lunar motions, and with a new example besides, by which this hypothesis is more fully illustrated and verified.

TABLE OF PROSTHAPHAERESSES

0 Sig. [Sign 0]

Diagram. A geometric construction for finding the Sun's mean distance from the Earth. A large circle (the Moon's orb) is drawn with center A. A vertical apogee line runs up from A to the point B at the top, far above the circle. From B two lines descend tangent to the circle on the left (at G) and the right (at C), meeting it near the horizontal diameter. The point D lies on the circle near the top (toward B), and E at the bottom; the horizontal diameter runs G–A–C.

- B (apex, far above circle): the Sun's mean position; vertex of the maximum-departure angle A B C / A B G
- D (top of circle, below B): position of the Moon at conjunction
- E (bottom of circle): position of the Moon at opposition
- C (right of circle, on horizontal diameter): quadrature point (tangent point of B C)
- G (left of circle, on horizontal diameter): quadrature point (tangent point of B G)
- A (center): the Earth (or center of the Moon's orbit)

Diagram. The Tychonic lunar hypothesis. A large circle (the Moon's orbit B L) is drawn about the Earth at center A. The vertical apogee line B A F runs through the center, with F at the bottom of the large circle and B inside, near the top. At the top of the figure sits the first (large) epicycle, with its apex labels; within it a smaller second epicycle carries the Moon. Near the center A is the small circle of the variation, with center E and a vertical diameter through M and A.

- A (center of large circle): the Earth, primary center of the Moon's orbit
- F (bottom of large circle): foot of the apogee line B A F (perigee direction)
- B (on apogee line, above center): center at which mean longitude is determined; center of the first epicycle
- N (top, above the epicycle): apex region of the upper epicycle assembly
- C (within upper epicycle): topmost apex of the first epicycle, carried toward G
- I (upper epicycle, right): direction (consequence of signs) of the second epicycle's motion
- D (within second epicycle): position of the Moon carried by the second epicycle
- H (upper epicycle, left): labeled point on the epicycle assembly

- K (upper epicycle, left): labeled point on the epicycle assembly
- G (upper epicycle, right): direction (antecedence of signs) toward which apex C is carried
- M (small central circle, top): position of the orbit-center at quadrature (top of the variation circle, diameter A M)
- E (center of small central circle): center of the little circle of variation
- A (small central circle, bottom): position of the orbit-center at syzygy

Diagram. The little circle of the last lunar inequality (the small equation / aequatiuncula). A circle is drawn about center A with a horizontal diameter C–A–D–B and a vertical diameter through A down to F, with E and G near the bottom. The right sine G E (= A D) for a doubled solar elongation of 30° is shown as the construction for extracting the equation values.

- A (center): center of the little circle; position of B at syzygy (opposition)
- B (right end of horizontal diameter): point of the Moon's mean motion in the upper theory
- C (left end of horizontal diameter): position determining the trine aspect
- D (on horizontal diameter, between A and B): foot of the sine A D
- E (lower portion): endpoint of the right sine G E
- F (bottom): foot of the vertical diameter
- G (lower left): point from which the sine G E is measured

The following table is the **Table of Prosthaphaereses** for the Moon's longitude (translated from Tycho's *Progymnasmata*). This page shows the column block headed "0 Sig." (Sign 0) at the top, running for degrees 0–30, with the complementary "11 Sig." (Sign 11) read from the bottom (the bottom operation-row reverses Add/Subtract). Column headers transcribed verbatim:

Gradus (Degrees)	Subtr. Prosth. Epicycl. (G. M. S.)	Add. Diff. (M. S.)	Elongat. à centre [Elongation from centre] (Partic.)	Sub. Diff.	Ecce- ntrici- tas [Ecce- ntrici- ty] (Partic.)	Ad. Dif.	Adde- Vari- tio [Add: Vari- ation] (M. S.)	Adde- Diff. (M. S.)
0	0 0 0	—	1029 00	1	00	38	0 0	—
1	0 5 4	5 4	1028 99	1	38	38	0 43	0 43
2	0 10 8	5 4	1028 98	2	76	38	1 26	0 43
3	0 15 12	5 4	1028 96	2	114	38	2 8	0 42
4	0 20 16	5 4	1028 94	3	152	38	2 50	0 42
5	0 25 20	5 4	1028 91	3	190	38	3 32	0 42
6	0 30 23	5 3	1028 88	4	228	37	4 14	0 42
7	0 35 26	5 3	1028 84	5	265	38	4 56	0 42
8	0 40 28	5 2	1028 79	6	303	38	5 38	0 42
9	0 45 29	5 1	1028 73	6	341	38	6 20	0 42
10	0 50 30	5 1	1028 67	7	379	38	7 2	0 42
11	0 55 30	5 0	1028 60	7	417	37	7 44	0 42

Gradus (Degrees)	Subtraction Prosthaphaeretic		Elongation from center [Particula]		Eccentricity [Particula]		Addition [Addition]		Addition Difference (M. S.)
	h. Epicl. (G. M. S.)	Add. Diff. (M. S.)	(Particula)	Sub. Diff.	(Particula)	Ad. Dif.	(M. S.)		
12	1 0 28	4 58	1028 53	8	454	38	8 26	0 42	
13	1 5 25	4 57	1028 45	9	492	38	9 7	0 41	
14	1 10 21	4 56	1028 36	10	530	38	9 48	0 41	
15	1 15 16	4 55	1028 26	10	568	37	10 29	0 41	
16	1 20 10	4 54	1028 16	11	605	38	11 10	0 41	
17	1 25 3	4 53	1028 05	12	643	37	11 51	0 41	
18	1 29 55	4 52	1027 93	12	680	38	12 31	0 40	
19	1 34 44	4 49	1027 81	13	718	37	13 11	0 40	
20	1 39 32	4 48	1027 68	13	755	37	13 51	0 40	
21	1 44 18	4 46	1027 55	14	792	38	14 31	0 40	
22	1 49 30 [recte : 1 49	4 45	1027 41	15	830	37	15 10	0 39	

Gradus (Degrees)	Subtraction Prosthaphaeretic		Elongation from center (Partic.)	Sub. Diff.	Eccentricity [Eccentricity] (Partic.)		Ad. Dif.	Addition [Addition] (M. S.)	Addition Diff. (M. S.)
	h. Epicl. (G. M. S.)	Add. Diff. (M. S.)							
	4]								
23	1 53 47	4 44	1027 26	15	867	37	15 49	0 39	
24	1 58 29	4 42	1027 11	16	904	37	16 28	0 39	
25	2 3 8	4 39	1026 95	17	941	37	17 7	0 39	
26	2 7 44	4 36	1026 78	18	978	37	17 45	0 38	
27	2 12 18	4 34	1026 60	18	1015	37	18 23	0 38	
28	2 16 50	4 32	1026 42	19	1052	37	19 1	0 38	
29	2 21 20	4 30	1026 23	19	1189 [recte : 1089]	36	19 38	0 37	
30	2 25 47	4 27	1026 04	—	1125	20	20 15	0 37	
(footnote operation row)	Addition [Add]	Subtraction .		Add.		Sub.	Subtraction .	Subtraction .	

11 Sig. [Sign 11]

Parsing notes: This is a classic prosthaphaeresis table with the top read top-down for "0 Sig." and the bottom read bottom-up for "11 Sig." (the footer operation-row gives the reversed Add/Subtract sense for the complementary sign). The eight column heads as printed are: *Gradus*; *Subtr. Prosth. Epicycl.* (degrees-minutes-seconds, the epicyclic prosthaphaeresis to subtract); its *Add. Diff.* (minutes-seconds); *Elongat. à centro in Partic.* (parts of the elongation from center); its *Sub. Diff.*; *Eccentricitas in Partic.*; its *Ad. Dif.*; *Adde Variatio* (minutes-seconds, the variation to add); its *Adde Diff.* In the "Eccentricitas" column the deg-29 value is printed "1189", a compositor error for 1089 (medial 0 misset as 1; the column runs 1052, 1089, 1125), transcribed as printed with [recte: 1089]; deg 30 (1125) is correct and is the column maximum once deg 29 is read as 1089. The "Elongat. à centro" column is a smoothly-decreasing 5-digit radius-like quantity (102900 → 102604); transcribed as printed. In the "Subtr. Prosth. Epicycl." column the deg-22 value is printed "1 49 30", a compositor error for 1 49 4 (the Add.Diff neighbors 4 46 / 4 45 force seconds ≈ 4), transcribed as printed with [recte: 1 49 4].

Astronomia Danica — Pars Altera, Liber Primus Theoricorum, Caput VI (cont.)

Lunar Prosthaphaeresis Table (continued)

These four pages continue the single great **Table of Lunar Prosthaphaereses [Tabula Prosthaphaereses Resium Lunarium]** begun in the prior batch. Each page carries one zodiacal-sign block read top-down (1, 2, 3, 4 Sig.) and the complementary block read bottom-up (10, 9, 8, 7 Sig.) using the reversed operation-labels in the foot row. The column headings are, left to right:

- **Gradus** — degree of argument (0–30; on the recto pages the degree column for the top-down block runs down the *right* margin, and the bottom-up block runs down the left).
- **Subtr. Prosth. Epicycl.** — Subtractive Epicyclic Prosthaphaeresis [G.M.S. = degrees, minutes, seconds]
- **Add. Differ.** — Additive Difference [M.S. = minutes, seconds]
- **Elongat. à centro** — Elongation from center [in particles, *Partic.*]
- **Sub. Diff.** — Subtractive Difference
- **Eccentricitas** — Eccentricity [in particles, *Partic.*]

- **Ad. Diff.** — Additive Difference
- **Adde Variatio** — Add: Variation [M.S.]
- **Adde Diff.** — Add: Difference [M.S.]

The foot row of each page restates the operation to apply when the table is read bottom-up for the complementary sign.

Throughout, I transcribe the numerals exactly as printed (with their compositor-era forms y/f for the long-s ligature of 5, r/r etc.), normalizing only obvious typographic glyph-shapes to Arabic digits where the digit is unambiguous; cells I cannot resolve are flagged [?].

1 Sig. (top-down) / 10 Sig. (bottom-up)

Grad us	Subt r. Pros th. Epic ycl. (G.M .S.)	Add. Diffe r. (M.S.)	Elon gat. à cent ro (Part ic.)	Sub. Diff.	Ecce ntric itas (Part ic.)	Ad. Diff.	Add e Vari atio (M.S.)	Add e Diff. (M.S.)	Grad us (10 Sig.)
0	2 25 47		1026 04	20	1125	37	20 15		30
1	2 30 12	4 25	1025 84	21	1162	36	20 51	0 36	29
2	2 34 34	4 22	1025 63	21	1198	37	21 27	0 36	28
3	2 38 54	4 20	1025 42	22	1235	36	22 3	0 36	27
4	2 43 11	4 17	1025 20	23	1271	36	22 38	0 35	26
5	2 47 25	4 14	1024 97	23	1307	37	23 13	0 35	25
6	2 51 37	4 12	1024 74	24	1344	36	23 48	0 35	24
7	2 55 46	4 9	1024 50	24	1380	36	24 22	0 34	23
8	2 59	4 6	1024	25	1416	35	24	0 34	22

Grad us	Subt r. Pros th. Epic ycl. (G.M .S.)	Add. Diffe r. (M.S.)	Elon gat. à cent ro (Part ic.)	Sub. Diff.	Ecce ntric itas (Part ic.)	Ad. Diff.	Add e Vari atio (M.S.)	Add e Diff. (M.S.)	Grad us (10 Sig.)
	52		26				56		
9	3 3 54	4 2	1024 01	26	1451	36	25 29	0 33	21
10	3 7 53	3 59	1023 75	27	1487	36	26 2	0 33	20
11	3 11 49	3 56	1023 48	27	1523	35	26 34	0 32	19
12	3 15 42	3 53	1023 21	28	1558	36	27 6	0 32	18
13	3 19 31	3 49	1022 93	28	1594	35	27 39	0 32	17
14	3 23 17	3 46	1022 65	29	1629	35	28 8	0 31	16
15	3 26 59	3 42	1022 36	30	1664	35	28 38	0 30	15
16	3 30 38	3 39	1022 06	30	1699	35	29 8	0 30	14
17	3 34 13	3 35	1021 76	31	1734	35	29 37	0 29	13
18	3 37 44	3 31	1021 45	31	1769	35	30 6	0 29	12
19	3 41 12	3 28	1021 14	32	1804	34	30 34	0 28	11
20	3 44 36	3 24	1020 82	33	1838	34	31 1	0 27	10
21	3 47 56	3 20	1020 49	33	1872	34	31 28	0 27	9
22	3 51	3 16	1020	34	1906	34	31	0 27	8

Grad us	Subt r. Pros th. Epic ycl. (G.M .S.)	Add. Diffe r. (M.S.)	Elon gat. à cent ro (Part ic.)	Sub. Diff.	Ecce ntric itas (Part ic.)	Ad. Diff.	Add e Vari atio (M.S.)	Add e Diff. (M.S.)	Grad us (10 Sig.)
	12		16				55		
23	3 54 24	3 12	1019 82	34	1940	34	32 21	0 26	7
24	3 57 32	3 8	1019 48	35	1974	34	32 46	0 25	6
25	4 0 36	3 4	1019 13	36	2008	34	33 11	0 25	5
26	4 3 35	2 59	1018 77	36	2042	33	33 35	0 24	4
27	4 6 30	2 55	1018 41	37	2075	33	33 58	0 23	3
28	4 9 21	2 51	1018 04	37	2108	33	34 21	0 23	2
29	4 12 8	2 47	1017 67	38	2141	33	34 43	0 23	1
30	4 14 51	2 43	1017 29		2174		35 4	0 21	0
<i>foot</i>	Add e	Subt r.		Ad.		Sub.	Subt r.	Subt r.	

Parsing notes: degree 0 of the 1 Sig. block has no Add. Differ. / Adde Diff. entry (it is the continuation point from the 0 Sig. block of the prior batch). The first (degraded) scan of this page renders many cells garbled; the values above are taken from the second, clean rendering of p. 245.

2 Sig. (top-down) / 9 Sig. (bottom-up)

Grad us	Subt r. Pros th. Epic ycl. (G.M .S.)	Add. Diffe r. (M.S.)	Elon gat. à cent ro (Part ic.)	Sub. Diff.	Ecce ntric itas (Part ic.)	Ad. Diff.	Add e Vari atio (M.S.)	Add e Diff. (M.S.)	Grad us (9 Sig.)
0	4 14 51		1017 29	38	2174	33	35 4		30
1	4 17 29	2 38	1016 91	39	2207	33	35 25	0 21	29
2	4 20 2	2 33	1016 52	39	2240	32	35 45	0 20	28
3	4 22 31	2 29	1016 13	40	2272	32	36 5	0 20	27
4	4 24 55	2 24	1015 73	41	2304	32	36 24	0 19	26
5	4 27 14	2 19	1015 32	41	2336	32	36 42	0 18	25
6	4 29 29	2 15	1014 91	42	2368	32	37 0	0 18	24
7	4 31 39	2 10	1014 49	42	2400	32	37 17	0 17	23
8	4 33 44	2 5	1014 07	42	2432	31	37 33	0 16	22
9	4 35 44	2 0	1013 65	43	2463	31	37 48	0 15	21
10	4 37 39	1 55	1013 22	43	2494	31	38 3	0 15	20
11	4 39 30	1 51	1012 79	44	2525	31	38 17	0 14	19
12	4 41 17	1 47	1012 35	44	2556	31	38 30	0 13	18

Grad us	Subt r. Pros th. Epic ycl. (G.M .S.)	Add. Diffe r. (M.S.)	Elon gat. à cent ro (Part ic.)	Sub. Diff.	Ecce ntric itas (Part ic.)	Ad. Diff.	Add e Vari atio (M.S.)	Add e Diff. (M.S.)	Grad us (9 Sig.)
13	4 42 59	1 42	1011 91	45	2587	30	38 42	0 13	17
14	4 44 35	1 36	1011 46	45	2617	30	38 55	0 12	16
15	4 46 5	1 30	1011 01	46	2647	30	39 7	0 12	15
16	4 47 30	1 25	1010 55	46	2677	30	39 18	0 11	14
17	4 48 50	1 20	1010 09	46	2707	30	39 28	0 10	13
18	4 50 6	1 16	1009 63	47	2737	29	39 37	0 9	12
19	4 51 16	1 10	1009 16	47	2766	29	39 45	0 8	11
20	4 52 21	1 5	1008 69	48	2795	29	39 53	0 8	10
21	4 53 21	1 0	1008 21	48	2824	29	40 0	0 7	9
22	4 54 16	0 55	1007 73	48	2853	28	40 6	0 6	8
23	4 55 5	0 49	1007 25	49	2881	28	40 12	0 6	7
24	4 55 49	0 44	1006 76	49	2909	28	40 17	0 5	6
25	4 56 28	0 39	1006 27	49	2937	28	40 21	0 4	5
26	4 57 1	0 33	1005 78	50	2965	28	40 25	0 4	4

Grad us	Subt r. Prosth. Epic ycl. (G.M .S.)	Add. Diffe r. (M.S.)	Elon gat. à cent ro (Part ic.)	Sub. Diff.	Ecce ntric itas (Part ic.)	Ad. Diff.	Add e Vari atio (M.S.)	Add e Diff. (M.S.)	Grad us (9 Sig.)
27	4 57 29	0 28	1005 28	50	2993	28	40 27	0 2	3
28	4 57 51	0 22	1004 78	50	3021	27	40 28	0 1	2
29	4 58 8	0 17	1004 28	50	3048	27	40 29	0 1	1
30	4 58 20	0 12	1003 78		3075		40 30	0 1	0
<i>foot</i>	Add e	Sub.		Ad.		Subt .	Subt .	Subt .	

Parsing notes: in the first (degraded) scan the Subtr. Prosth. column shows several long-s garbles (e.g. "4 20 2 2" for "4 20 2"; "4 H rr" for "4 24 55"); the clean second rendering of p. 246 supplies the readings above. Degree 0 carries no Add. Differ. entry.

3 Sig. (top-down) / 8 Sig. (bottom-up)

Grad us	Subt r. Prosth. Epic ycl. (G.M .S.)	Add. Diffe r. (M.S.)	Elon gat. à cent ro (Part ic.)	Sub. Diff.	Ecce ntric itas (Part ic.)	Ad. Diff.	Add e Vari atio (M.S.)	Add e Diff. (M.S.)	Grad us (8 Sig.)
0	4 58 20		1003 78	51	3075	27	40 30		30
1	4 58 26	0 6	1003 27	51	3102	26	40 29	0 1	29

Grad us	Subt r. Pros th. Epic ycl. (G.M .S.)	Add. Diffe r. (M.S.)	Elon gat. à cent ro (Part ic.)	Sub. Diff.	Ecce ntric itas (Part ic.)	Ad. Diff.	Add e Vari atio (M.S.)	Add e Diff. (M.S.)	Grad us (8 Sig.)
2	4 58 27	0 1	1002 76	51	3128	26	40 28	0 1	28
3	4 58 22	0 5	1002 25	51	3154	26	40 27	0 1	27
4	4 58 14	0 8	1001 74	51	3180	26	40 25	0 2	26
5	4 57 59	0 15	1001 23	51	3206	25	40 21	0 4	25
6	4 57 37	0 22	1000 72	51	3231	25	40 17	0 4	24
7	4 57 10	0 27	1000 21	52	3256	25	40 12	0 5	23
8	4 56 38	0 32	9996 9	52	3281	25	40 6	0 6	22
9	4 56 1	0 37	9991 7	52	3306	25	40 0	0 6	21
10	4 55 18	0 43	9986 5	52	3331	24	39 53	0 7	20
11	4 54 30	0 48	9981 3	52	3355	24	39 45	0 8	19
12	4 53 36	0 54	9976 1	52	3379	24	39 37	0 8	18
13	4 52 37	0 59	9970 9	52	3403	23	39 28	0 9	17
14	4 51 33	1 4	9965 7	52	3426	23	39 18	0 10	16
15	4 50 23	1 10	9960 5	52	3449	23	39 7	0 11	15

Grad us	Subt r. Pros th. Epic ycl. (G.M .S.)	Add. Diffe r. (M.S.)	Elon gat. à cent ro (Part ic.)	Sub. Diff.	Ecce ntric itas (Part ic.)	Ad. Diff.	Add e Vari atio (M.S.)	Add e Diff. (M.S.)	Grad us (8 Sig.)
16	4 49 7	1 16	9955 3	52	3472	23	38 55	0 12	14
17	4 47 46	1 21	9950 1	52	3495	23	38 43	0 12	13
18	4 46 21	1 25	9944 9	52	3518	22	38 30	0 13	12
19	4 44 50	1 31	9939 7	52	3540	22	38 17	0 13	11
20	4 43 13	1 37	9934 3	52	3562	22	38 3	0 14	10
21	4 41 31	1 42	9929 3	51	3584	21	37 48	0 15	9
22	4 39 43	1 48	9924 2	51	3605	21	37 33	0 15	8
23	4 37 51	1 52	9919 1	51	3626	21	37 17	0 16	7
24	4 35 54	1 57	9914 0	51	3647	20	37 0	0 17	6
25	4 33 51	2 3	9908 9	51	3667	20	36 42	0 18	5
26	4 31 42	2 9	9903 8	51	3687	20	36 24	0 18	4
27	4 29 29	2 13	9893 7	50	3707	20	36 5	0 19	3
28	4 27 11	2 18	9898 7	50	3727	19	35 45	0 20	2
29	4 24 48	2 23	9888 7	49	3746	19	35 25	0 20	1

Gradus	Subt r. Pros th. Epic ycl. (G.M .S.)	Add. Diffe r. (M.S.)	Elon gat. à cent ro (Part ic.)	Sub. Diff.	Ecce ntric itas (Part ic.)	Ad. Diff.	Add e Vari atio (M.S.)	Add e Diff. (M.S.)	Grad us (8 Sig.)
30	4 22 20	2 28	9888 3		3765		35 0	0 21	0
<i>foot</i>	Add e.	Add e.		Sub.	Subt .	Add.	Subt .	Add.	

4 Sig. (top-down) / 7 Sig. (bottom-up)

Grad us	Subt r. Pros th. Epic ycl. (G.M .S.)	Subt r. Diff. (M.S.)	Elon gat. à cent ro (Part ic.)	Subt. Diff.	Ecce ntric itas (Part ic.)	Add. Diff.	Add e Vari atio (M.S.)	Subt r. Diff. (M.S.)	Grad us (7 Sig.)
0	4 22 20		9883 8	49	3765	19	35 4		30
1	4 19 46	2 34	9878 9	49	3784	19	34 43	0 21	29
2	4 17 7	2 39	9874 0	49	3803	18	34 21	0 22	28
3	4 14 24	2 43	9869 1	48	3821	18	33 58	0 23	27
4	4 11 36	2 48	9864 3	48	3839	18	33 35	0 23	26
5	4 8 43	2 53	9859 5	48	3857	17	33 11	0 24	25
6	4 5 45	2 58	9854 7	47	3874	17	32 46	0 25	24
7	4 2 42	3 3	9850 0	47	3891	17	32 21	0 25	23
8	3 59 35	3 7	9845 3	46	3908	16	31 55	0 26	22
9	3 56 23	3 12	9840 7	46	3924	16	31 28	0 27	21
10	3 53 6	3 17	9836 1	45	3940	16	31 1	0 27	20
11	3 49 45	3 21	9831 6	44	3956	16	30 34	0 27	19
12	3 46 20	3 25	9827 2	44	3972	15	30 6	0 28	18
13	3 42 50	3 30	9822 8	43	3987	15	29 37	0 29	17

Grad us	Subt r. Pros th. Epic ycl. (G.M .S.)	Subt r. Diff. (M.S.)	Elon gat. à cent ro (Part ic.)	Subt. Diff.	Ecce ntric itas (Part ic.)	Add. Diff.	Add e Vari atio (M.S.)	Subt r. Diff. (M.S.)	Grad us (7 Sig.)
14	3 39 16	3 34	9818 5	43	4002	15	29 8	0 29	16
15	3 35 38	3 38	9814 2	42	4017	14	28 38	0 30	15
16	3 31 55	3 43	9810 0	41	4031	14	28 8	0 30	14
17	3 28 8	3 47	9805 9	41	4045	14	27 37	0 31	13
18	3 24 18	3 50	9801 8	40	4059	14	27 6	0 31	12
19	3 20 24	3 54	9797 8	40	4073	13	26 34	0 32	11
20	3 16 25	3 58	9793 8	39	4086	13	26 2	0 32	10
21	3 12 22	4 3	9789 9	38	4099	12	25 29	0 33	9
22	3 8 15	4 7	9786 1	37	4111	12	24 56	0 33	8
23	3 4 5	4 10	9782 4	37	4123	12	24 22	0 34	7
24	2 59 52	4 13	9778 7	36	4135	12	23 48	0 34	6
25	2 55 35	4 17	9775 1	35	4147	11	23 13	0 35	5
26	2 51 14	4 21	9771 6	34	4158	11	22 38	0 35	4
27	2 46 49	4 25	9768 2	33	4169	11	22 3	0 35	3

Gradus	Subtr. Prosth. Epicycl. (G.M. .S.)	Subtr. Diff. (M.S.)	Elongat. à centro (Part ic.)	Subtr. Diff.	Ecce ntric itas (Part ic.)	Add. Diff.	Add. e Variatio (M.S.)	Subtr. Diff. (M.S.)	Gradus (7 Sig.)
28	2 42 21	4 28	9764 9	33	4180	10	21 27	0 36	2
29	2 37 51	4 30	9761 6	32	4190	10	20 51	0 36	1
30	2 33 18	4 33	9758 4		4200		20 15	0 36	0
<i>foot</i>	Add. e.	Subtr. r.		Add.		Subtr. .	Subtr. r.	Add.	

Parsing notes: the first (degraded) scan badly garbles the left edge of this page (e.g. "300 H" for "3 8 15", "2 5T 3 5" for "2 55 35"); the clean second rendering of p. 248 supplies the readings above. Degree 0 has no Subtr. Diff. / Subtr. Diff. (variation) entry. Degree 12 Subtr. Diff. reads "3 26" in the first scan and is partly broken in the second; transcribed "3 26" with a [?]. The Subtr. Prosth. Epicycl. column reaches its maximum (~4 58') on the prior page (3 Sig.) and here decreases monotonically — confirming the polarity flip noted in the body header.

TABLE OF LUNAR PROSTHAPHAERESES [continued] — 5 Sig. / 6 Sig.

**** This is the reciprocal sign-block page of the *Tabula Prosthaphaereseon Resium Lunarium* (Table of the Lunar Prosthaphaereses of the Equation-Residuals). It is read top-to-bottom for the **5 Sig.** block (degrees 0→30 ascending in the rightmost "Gradus" column as printed top-to-bottom is actually 30→0; see below) and bottom-to-top for the **6 Sig.** block. The leftmost "Gradus" reference for 5 Sig. is given implicitly by the rightmost column, which runs 30 at top down to 0 at bottom. The column footers (Adde / Add. / Ad. / Sub. / Subtr. / Add.) give the addition/subtraction rule for the reciprocal **6 Sig.** block read from the bottom.

Columns: **Subtr. Prosth. Epicycl.** (Subtractive Epicyclic Prosthaphaeresis, °''') | **Add. Diff.** (Additive Difference, ''') | **Elongat. à centro** (Elongation from center,

particles) | **Sub. Diff.** (Subtractive Difference) | **Eccentricitas** (particles) | **Add. Diff.** (Additive Difference) | **Add. Variatio** (Additive Variation, ") | **Add. Diff.** (Additive Difference, ") | **Gradus** (rightmost).

Subtr

Prosthaphaeretic Epicycle cl. (G.M. S.)	Add. Diff. (M.S.)	Elongation at. à centro (Parti c.)	Sub. Diff.	Eccentricity tas (Parti c.)	Add. Diff.	Add. Variatio (M.S.)	Add. Diff. (M.S.)	Gradus
2 33 18		9758 4	31	4200	10	20 15		30
2 28 41	4 37	9755 3	30	4210	9	19 38	0 37	29
2 24 1	4 40	9752 3	29	4219	9	19 1	0 37	28
2 19 18	4 43	9749 4	28	4228	8	18 23	0 38	27
2 14 33	4 45	9746 6	27	4236	9	17 45	0 38	26
2 9 45	4 48	9743 9	26	4245	8	17 7	0 38	25
2 4 55	4 50	9741 3	25	4253	8	16 28	0 39	24
2 0 2	4 53	9738 8	24	4261	7	15 49	0 39	23
1 55 7	4 55	9736 4	23	4268	7	15 10	0 39	22
1 50 9	4 58	9734 1	21	4275	7	14 31	0 39	21
1 45 8	5 1	9732 0	21	4282	6	13 51	0 40	20
1 40 5	5 3	9729 9	20	4288	6	13 11	0 40	19
1 35 1	5 4	9727	19	4294	6	12 31	0 40	18

Subtr								
Prosth.		Elongat.		Ecce		Adde		
Epicycl.	Add.	to	Sub.	entrici	Add.	Varia	Add.	Grad
(G.M.S.)	(M.S.)	(Partic.)	Diff.	(Partic.)	Diff.	(M.S.)	(M.S.)	us
		9						
1 29 55	5 6	97260	18	4300	6	11 51	0 40	17
1 24 47	5 8	97242	17	4306	5	11 10	0 41	16
1 19 38	5 9	97225	16	4311	5	10 29	0 41	15
1 14 27	5 11	97209	15	4316	4	9 48	0 41	14
1 9 14	5 13	97194	14	4320	4	9 7	0 41	13
1 3 59	5 15	97180	13	4324	4	8 26	0 41	12
0 58 43	5 16	97167	12	4328	4	7 44	0 42	11
0 53 27	5 16	97155	11	4332	3	7 2	0 42	10
0 48 10	5 17	97144	10	4335	3	6 20	0 42	9
0 42 52	5 18	97134	8	4338	2	5 38	0 42	8
0 37 33	5 19	97126	6	4340	2	4 56	0 42	7
0 32 13	5 20	97120	5	4342	2	4 14	0 42	6
0 26 52	5 21	97115	4	4344	1	3 32	0 42	5
0 21	5 22	9711	3	4345	1	2 50	0 42	4

Subtr									
.									
Prost			Elong		Ecce		Adde		
h.			at. à		ntrici		Varia		
Epicy			centr		tas		tio		
cl.	Add.		(Parti	Sub.	(Parti	Add.	(M.S.)	Add.	Grad
(G.M.	Diff.		c.)	Diff.	c.)	Diff.	(M.S.)	(M.S.)	us
S.)	(M.S.)								
30			1						
0 16 8	5 22		9710	3	4346	1	2 8	0 42	3
			8						
0 10	5 22		9710	3	4347	1	1 26	0 42	2
46			5						
0 5 23	5 23		9710	2	4348	0	0 43	0 43	1
			2						
0 0 0	5 23		9710		4348	0	0 0	0 43	0
			0						

Column footers (for the **6 Sig.** block, read bottom-to-top): Adde | Add. | (Elongat.)
| Ad. | (Eccentr.) | Sub. | Subt. | Add.

6 Sig. (caption at foot of table). Catchword: De

Parsing notes: The "Elongat. à centro" and "Eccentricitas" columns are 5-digit particle values; a few digits in the source are faint (long-s/old-digit forms). Two cells where a difference figure was hard to resolve: the "Sub. Diff." at deg-21/deg-20 reads "21 / 21" repeated, and the "Eccentricitas" deg-21 reads 4275 (the apostrophe-like mark in the source over the difference "7" is ink, not a digit). The deg-17 Eccentricitas reads 4300; the adjacent batch handoff cited the same. Final two rows: deg-1 Variation 0 43, deg-0 Variation 0 0 with difference 0 43 — consistent with the column closing to zero.

THEORICORVM — Chapter 6

On Computing the True Longitude of the Moon from the Foregoing Tables

I.

First of all, for the given time — equated according to the method I set out above — determine both the mean longitude of the Sun and its true longitude reckoned from the mean equinox. Then, from the tables of the Moon's mean motions, extract the values appropriate to that same equated time in the customary manner: namely the mean longitude of the Moon from the Sun [*media longitudo Lunae à Sole*], the anomaly of the Moon [*anomalía Lunae*], and the motion in latitude [*motus latitudinis*], if that should be needed. Finally, add the mean longitude of the Sun to the Moon's mean motion from the Sun, so as to establish the longitude of the Moon from the mean vernal equinox.

[Margin: In the commentary on chapter 3 of this work.]

II.

Second, enter the table of prosthaphaereses with the Moon's anomaly, and extract the epicyclic prosthaphaeresis [*prosthaphaeresin Epicyclícam*] together with the elongation from center [*elongatione à centro*] — applying the correction by the proportional part if any minutes are attached to the degrees of anomaly. Add this prosthaphaeresis both to the Moon's longitude from the equinox and to the anomaly, as the headings of the columns require, or subtract it from them, so that both quantities come out equated.

III.

Third, subtract the true place of the Sun from the equated longitude of the Moon, and with the remainder doubled look up in the table of equations the eccentricity [*eccentricitatem*] together with the variation [*variatione*] — not neglecting, however, if needed, the proper proportional part — and set these aside separately.

IV.

Fourth: if that double distance of the luminaries [*duplex illa distantia luminarium*] with which you entered the table is less than a semicircle, halve its complement to the semicircle; or if it is greater than a semicircle, halve likewise

its excess over the semicircle. This will yield the angle of the second equation of the anomaly [angulus secundae aequationis anomaliae]. Before the semicircle of the double distance of the luminaries, add this angle to the equated anomaly; after the semicircle, subtract it from the same, so that the anomaly comes out equated a second time. And if this [angle] is less than a semicircle, bisect it; or if it is greater, bisect the complement of it to the whole circle. Then look up in the table of Tangents the tangent of this bisected angle — and this will be the third quantity found [inventum tertium].

[Margin: Chapter 4.]

V.

Fifth, add together the Moon's elongation from center and the eccentricity, halve the product, and you will have the first quantity found [inventum primum]. From this, with the eccentricity subtracted, the second quantity found [inventum secundum] remains. Multiply this by the tangent of the bisected arc — that is, by the third quantity found — and divide the resulting number by the first quantity found. (Or, more compactly, obtain it by the prosthaphaeresis of triangles, the fourth term.) The quotient will be a tangent, whose corresponding angle, subtracted from the bisected angle just mentioned, leaves the prosthaphaeresis of the eccentricity [eccentricitatis prosthaphaeresin]. This is to be subtracted before the semicircle of the anomaly last equated, but after the semicircle added to it.

Note this carefully, however: if the angle of the second equation is subtractive and greater than the anomaly itself, the anomaly is to be subtracted from it, and the tangent of what bisected remains is to be used as before; in that case the angle of the prosthaphaeresis of the eccentricity will be additive. This prosthaphaeresis, and the variation which you set aside, you are to add to the equated longitude of the Moon from the equinox — or subtract from it — according to the sign of each. You will thereby obtain the true place of the Moon from the mean equinox. When you have applied to this the inequality of the equinox [inaequalitatem aequinoctii] to be obtained as set out above, you will have the true place of the Moon in its own orbit, reckoned from the true vernal equinox.

FIRST EXAMPLE — Worked upon the Theory of the Moon Itself, by Means of the Doctrine of Triangles

In the year of Christ 1587, on the 17th day of the month of August, at hour 19, minute 25, p.m., the true place of the Moon's center was observed at Uraniborg

[Vraniburgi] with Tychonic instruments [Tycho Brahe] at Gemini $26^{\circ} 21\frac{1}{2}'$ — having here, of course, taken account of our correction in the fixed stars. The equation of the natural day [aequatio diei naturalis], with the meridian of Copenhagen [Hafniensis] also taken into account, adds 7 minutes. Therefore, with the completed time — namely 1586 years, the month of July, 16 days, hour 19, minute 32 — I enter the appropriate tables of the mean motions and transcribe them in order as given below.

	Sig.	Gr.	Mi.	Sec.
Anomaly of the equinoxes [Anomalia aequinoctiorum]	6	15	17	0
Whence its equation, additive			7	13
Longitude of the Sun from the mean equinox	5	5	42	16
Anomaly of the Sun	2	0	20	53
Longitude of the Moon from the Sun	9	25	12	6
Anomaly of the Moon	1	15	37	21
To the Sun's anomaly there corresponds a prosthapha	1	44	46	

	Sig.	Gr.	Mi.	Sec.
eresis, subtractive				
And	5	3	57	30
therefore the true place of the Sun from the mean equinox				

We shall now treat in order of the prosthaphaereses of the Moon by means of the following hypothesis.

The primary and genuine theory of the Moon, accommodated to the time and to the preceding mean motions.

[Margin: See chapter 2 below.]

Let the mean anomaly of the Moon ID in the appended diagram be 1 sign, 15°, 37', 21" — to which the [arc] that is equal in the first epicycle from the apogee (as was said earlier in the explanation of the hypothesis) is GK, [reckoned] up to the center of the second epicycle K. Let the same anomaly doubled — 3 signs, 1°, 14', 42" — be counted from H up to E, the place of the Moon in the second epicycle; and let the lines ED and KE be drawn.

Diagram. A geometric construction of the Tychonic twin-epicycle lunar model. A large circle (the Moon's deferent / eccentric orbit, radius DC = CD = 100000) is drawn with a vertical diameter. Its center region carries the small variation-circle construction at lower center, and the twin-epicycle assembly appears at upper left.

- N (top of upper-left assembly): outer point on the first epicycle
- K (upper left): center of the second epicycle; the first epicyclic arc GK runs up to it
- E (upper left, beside K): the place of the Moon in the second epicycle
- G (left of K): apogee reference on the first epicycle
- H (below K): point from which the doubled anomaly is counted to E
- D (below H): center of the first epicycle (carried on the deferent); endpoint of anomaly arc ID
- L (lower left, below D): point on the construction

- R, I, T (upper right, along the top): reference points / apsidal line markers (R and I near the top, T at upper right)
- F (center): point in the small variation-circle BFC, marking the motion of the orbit-center
- A (center, beside F): center of the small variation circle
- Q (center left of A): point on the small circle
- B (center, below F/A): the Earth (tellus); base of the small variation circle BFC

In the plane triangle DKE, two sides are given: DK, the radius of the first epicycle, 5800, and KE, the radius of the second epicycle, 2900 (in the units in which the radius of the Moon's orbit CD is 100000), together with the contained angle DKE, which is the doubled anomaly. Therefore, by the demonstrated properties of triangles, there result: the angle KDE, $26^{\circ} 18' 43''$, and the carried side DE, 6541. Then, in the second triangle DCE, the side DE just found, 6541, is given, and DC, the radius of the eccentric, 100000, together with [the angle...]

together with the contained angle CDE, $108^{\circ} 3' 56''$ — namely, the complement of the simple anomaly diminished by the angle KDE. Therefore by calculation the angle DCE becomes known, the epicyclic prosthaphaeresis [prosthaphaeresis epicyclica], $3^{\circ} 29' 16''$, together with the side of the Moon's elongation from center CE, 102217.

Furthermore, let the Sun's mean motion be added to the Moon's mean longitude from the Sun, so that the Moon's simple [longitude] from the mean vernal equinox is obtained: 3 signs, $0^{\circ} 54' 22''$. From this subtract the epicyclic prosthaphaeresis, $3^{\circ} 29' 16''$, and the equated longitude of the Moon remains: 2 signs, $17^{\circ} 25' 6''$. Let the same prosthaphaeresis be taken away from the anomaly, and this comes out equated: 1 sign, $12^{\circ} 8' 5''$ — which is contained by the angle ICE. Then from the equated longitude of the Moon subtract the true place of the Sun from the mean equinox, 5 signs, $3^{\circ} 57\frac{1}{2}'$, and there remains the distance of the luminaries [distantia luminarium]: 9 signs, $23^{\circ} 27' 36''$. The double of this — 7 signs, $16^{\circ} 55' 12''$ — measures, in the small circle BFC, the motion of the center of the Moon's orbit C about the Earth B, in the direction of the signs along FC, just as that too was set out above.

Now, so that the line CB may be had in the same little circle, let the radius AC be drawn. Since, then, in the isosceles triangle AB and AC, the individual sides are each given as 2174, together with the angle CAB of $133^{\circ} 4' 48''$ — which is the complement of the double longitude of the Moon from the Sun to the entire circle

— therefore the said line BC is given as 3988. And since the angle FBC is half of FAC, by Elements III.20, while this excess is that of the double distance of the luminaries beyond 6 signs — namely $46^{\circ} 55' 12''$ — its half is accordingly $23^{\circ} 27' 36''$, the measure of the said angle FBC, to which the angle ICR is also equal. By the subtraction of this angle, in this example, the Moon's anomaly is equated a second time, so that it becomes $18^{\circ} 40' 29''$ — which the angle RCE measures.

Therefore, in the last triangle ECB, from the two sides likewise given, CE 102217 and BC 3988, with the angle contained by them, ECB, $161^{\circ} 19' 31''$ — namely the complement of the Moon's anomaly last equated to a semicircle — the angle BEC is found: $0^{\circ} 41' 25''$. When this is added to the prior epicyclic prosthaphaeresis, there is made up $4^{\circ} 16' 41''$, subtractive.

There remains, in the last place, that libration of the Moon [*libratio illa Lunae*] to be worked out according to what was demonstrated above. The double longitude of the luminaries is 7 signs, $16^{\circ} [?] 55' 12''$ [recte: 16° , matching the value given just above]; hence the right sine of the excess beyond the semicircle — namely of the arc EC in the adjoined little circle — is 7304007, which is represented by the straight line FA. Therefore, as the radius 10000000 is to $40' 30''$, the maximum prosthaphaeresis of this [libration], so is CD [recte: FA] to the variation in the present case, $29' 35''$. Since this is of the same denomination as the remaining prosthaphaeresis — namely, subtractive — it too...

[...the variation] is likewise added to it; and from the first, second, and this third together is gathered the final lunar prosthaphaeresis just found: $4^{\circ} 40' 16''$. Subtracting this from the simple longitude of the Moon reckoned above from the mean equinox — 3 signs $0^{\circ} 54' 22''$ — there remain 2 signs $26^{\circ} 14' 6''$. When the equation of the equinox set out earlier is added to these, namely $7' 13''$, the true longitude of the Moon sought is here determined, for the given time, from the true vernal equinox (that is, from the beginning of Aries), as $26^{\circ} 21\frac{1}{2}'$. This differs only very slightly from the observed value, even when account is taken here of the Moon's own orbit.

Diagram. A small circle with two perpendicular diameters meeting at center A. The diagram illustrates the lunar variation/libration construction discussed in the worked example. Labeled points around and within the circle:

- F (top): upper point on the vertical diameter
- C (upper left, inside): point on the variation circle
- D (just below C, inside): point near the center on the vertical
- E (left, on horizontal diameter): the Moon's place

- B (left end of horizontal diameter): Earth's center
- A (center): the center of the construction
- H (right end of horizontal diameter): point opposite B
- K (bottom of vertical diameter): lower point

Now we shall fit this same example for working out by the table of lunar prosthaphaereses, and that in the order according to the rules set out above.

I.

The required quantities are taken in order from the synopsis of mean motions expounded above; for, the rest remaining unchanged, the single absolute prosthaphaeresis of the Moon will suffice in this comparison. From this the use of the table is shown, and the agreement of the table with the hypothesis becomes more evident.

II.

	Sig.	Gr.	Mi.	Sec.
To the simple anomaly of the Moon	1	15	37	21
There correspond s from the prosthapha eresis table		3	29	16

[This is] to be subtracted both from the longitude of the Moon from the equinox and from the anomaly.

III.

	Sig.	Gr.	Mi.	Sec.
Simple longitude of the Moon from the Sun	9	25	12	6
Simple	5	5	42	16

	Sig.	Gr.	Mi.	Sec.
longitude of the Sun				
Therefore simple longitude of the Moon from the equinox	3	0	54	22
Longitude of the Moon from the equinox, equated	2	27	25	6
Likewise the anomaly of the Moon, equated	1	12	8	5
Elongation of the Moon from the center			102217	

IV.

	Sig.	Gr.	Mi.	Sec.
Excess of the double longitude of the Moon from the Sun beyond the semicircle		46	55	12
Half of this, which is		23	27	36

	Sig.	Gr.	Mi.	Sec.
the final equation of the anomaly				
Therefore	0	18	40	29
the anomaly of the Moon, finally equated				
Half of this		9	20	15

Whose tangent, 1644280, is the third found quantity.

V.

Elongation of the Moon from the center	102217
Measure of the eccentricity	3988
Sum	106205

Half, which is the first found quantity [inventum I.]: 53102

Difference between the lesser side and the first found quantity, which is the
second found quantity [inventum II.]: 49114

Rule.

I.	III.	II.	
53102 ———	1644280 ———	49114	T. 1520793
	Gr.	Mi.	Sec.
There corresponds the arc	8	38	50
Half of the equated	9	20	15

	Gr.	Mi.	Sec.
anomaly			
The difference, •		41	25
which is the			
second			
prosthaphaere			
sis			
[subtractive]			
The third		29	35
prosthaphaere			
sis, which is			
the variation			
of the Moon			

Parsing notes: The "T." value 1520793 is the table-product (proportional fourth) for the Regula; it sits to the right of the I/III/II proportion line. In the lower block the leading-column value • for the "difference / second prosthaphaeresis" row is a degree-placeholder dot (0°), giving 0° 41' 25".

Since therefore all the prosthaphaereses of the Moon were here subtractive, they are rightly collected into one sum in this example in this manner:

	Gr.	Mi.	Sec.
from the	3	29	16
epicycle			
prosthapha	the second	41	25
eresis			
	the	29	35
	variation		
Sum	4	40	16

Now this prosthaphaeresis, since above it came out by the full resolution of the lunar hypothesis into triangles, makes plain that there is no difference between the triangular computation of the Moon's place and that by the prosthaphaeresis table — which was indeed what we set out to demonstrate in this example.

Example II.

This is added, so that the truth of this our lunar theory may be more fully demonstrated against the celestial phenomena themselves.

Among the observations which I had made here at Copenhagen [Hafniae] in these last years — also of the Moon, both by sufficient instruments and by frequent observations of its conjunctions with the more notable fixed stars in the zodiac, from which I have found the foregoing hypothesis, formerly established upon the Tyconic observations, to hold most correctly — I shall here bring forward as an example the notable conjunction of the upper horn of the Moon with Aldebaran, made visibly among us in the year of Christ 1608, on the 12th of February, at the evening hour 8, minute 43, when the Moon was near quadrature and at its greatest departure from the mean course. From this both of its prostaphaereses can be examined most excellently. This conjunction was seen three first minutes of time later at Wittenberg [Vitebergenses], namely at hour 8, minute 46, as the most distinguished and most lamented mathematician Mr. Melchior Jöstel [Melchior Jöstelius] of blessed memory afterwards wrote to me. Hence it is certainly clear, even from this, that the time has been most correctly assigned by us to this observation, or visible conjunction, because the difference of meridians, foreknown beforehand from eclipses, recurs here almost exactly. And because the Sun, at this time, was by its true motion in 3° of Pisces, the equation of the natural day to be added will be $7\frac{1}{2}'$. The mean time therefore was hour 8, minute $50\frac{3}{4}$, on the 12th day of February in the year of the running Epoch of the Savior 1608, or of the world (bissextile) 5572. To this time these motions from the foregoing tables correspond.

	Sig.	Gr.	Mi.	Sec.
Mean precession of the vernal equinox	2	17	3	20
Anomaly of the equinox	6	17	19	17
Therefore inequality of the precession of the equinox [additive]			8	9
Hence true precession	2	17	11	29

	Sig.	Gr.	Mi.	Sec.
of the equinox				

[Margin: See chapter 3.] [Margin: See chapter 4.]

		Sig.	Gr.	Mi.
Likewise	Longitude	Gemini	4	16 $\frac{1}{3}$
Aldebaran, true	Latitude	Scorpio	5	31

This same is the seen longitude, and latitude, of the upper horn [of the Moon].

[Margin: Cap. 5.]

	Sig.	Gr.	Mi.	Sec.
Furthermor e, simple longitude of the Sun	11	1	50	42
Prosthapha eresis of the Sun [additive]		1	43	58
Simple longitude of the Moon from the Sun	2	25	19	38
Anomaly of the Moon	8	18	8	20
Therefore prosthapha eresis of the Moon, from the epicycle [additive]		4	13	43

	Sig.	Gr.	Mi.	Sec.
Elongation from the center			99768 p.	

	Sig.	Gr.	Mi.	Sec.
Double longitude of the Moon from the Sun, true	5	26	58	46
To which corresponds the measure of the eccentricity			4346 p.	

	Sig.	Gr.	Mi.	Sec.
Variation of the center of the Moon [additive]			2	9
Anomaly of the Moon, first equated	8	23	2	3
Angle... complement of the ecliptic longitude of the Moon from		1	30	37

	Sig.	Gr.	Mi.	Sec.
the Sun, measured [dimicatus], which is the equation of the second anomaly [additive]				
Therefore anomaly of the Moon, finally equated	8	24	32	40
Compleme nt of the Moon's anomaly to the whole circle, bisected		47	43	40

To which the tangent, the third found quantity [inventum III.], corresponds:
11000525.

Inventum I.	52057
Inventum II.	47711

Rule.

I.	II.	III.	
52057 ———	11000525 —————	47711	T. 19082140

	Sig.	Gr.	Mi.	Sec.
There correspond		45	14	4

	Sig.	Gr.	Mi.	Sec.
s the arc				
The difference from the upper bisected [arc], which is the second prosthapheresis angle [additive]		2	29	36
Hence are collected into one sum all the prosthaphereses of the Moon — in that they are additive — and the aggregate is made [additive]		7	25	28
Distance of the simple Moon from the mean equinox	1	27	10	20
Prosthapheresis of the equinox, additive			8	9
Prosthapha		7	25	28

	Sig.	Gr.	Mi.	Sec.
ereses of the Moon, collected [additive]				
Therefore the true place of the Moon	Gemini	4	44	(approxima tely)

Now, since the seen longitude of the Moon appears from the place of Aldebaran to be in Gemini $4^{\circ} 16\frac{1}{3}'$, the parallax of its longitude at an altitude of about 39° — which around the said syzygy was found to have been $27\frac{1}{4}'$ — becomes apparent. And since account is taken of the reduction of the Moon to the ecliptic, by which we measure the star's place, there must still be added to the longitude of the Moon found in its own orbit $2' 38''$, so that thus the parallax of longitude grows beyond the half-degree, that is, beyond $30'$. That this agrees well enough with our supposition can be shown below, where the parallaxes of the Moon are to be treated.

[Margin: Cap. 8 of this [book].]

Likewise, that the seen latitude of the center of the Moon was about $5^{\circ} 47'$ northern [A], this too agrees — by reason not only of its distance from the Earth, but also of the necessary increase of its latitude around quadrature, which must be posited.

And within these bounds [bars] is everywhere contained the difference of the true longitude of the Moon from the mean; which, that it may be more easily investigated by a table derived from the following hypothesis, we shall strive with all our might, for the sake of the lovers of astronomy [Astrophili].

ON THE SECOND HYPOTHESIS OF THE MOON, afterwards discovered by us, which shows the equivalence of the prior one, but, resolved into tables of numbers, exhibits an easier practice in the computation of longitude.

Up to this point we have produced the genuine theory of the Moon from its own causes, and have confirmed it against the celestial phenomena by at least two examples here — though it was previously founded on many and indeed

sufficient observations, for variation of every kind — so that no one can doubt that, if one follows the prescribed tracks, one will represent the longitude of the Moon for any time exactly, in numbers, as it is in the heavens. But since

the complaint of certain people has time and again struck our ears — concerning the ψηφοφορίας [Greek: psēphophorias, "computation, reckoning"] (that is, the lengthy difficulty of calculation from the table above), also on account of the analysis of the single triangle necessarily annexed to the rules there. Although practiced astronomers dispatch this with easy effort — indeed those who know the doctrine of triangles perfectly, as is fitting, sometimes even take their own pleasure spontaneously in the whole compendious resolution of the hypothesis — beginners nevertheless perhaps, and others for whom the astronomical tables alone are thought sufficient for the investigation of a phenomenon (since they have judged otherwise), [are deterred]. And we, being equally ready to serve all by our labors and to demonstrate beforehand the most compendious path we know, are therefore prepared to give here another hypothesis of the Moon, but equivalent to the prior one. From it too we have afterwards constructed a whole table of prosthaphaereses, which, with all the ease that can be, will dispatch the appearances of the Moon's longitude. The syntax of this arises from the equivalences of hypotheses shown above in general terms, while in the same we use the eccentric-epicycle [Eccentrepicyclus]. For the lovers of astronomy and those moderately versed in the sidereal science, we shall here think it enough if such a secondary hypothesis of ours for the Moon — accommodated to the prior one by the example of what has gone before — both proves, by one and the same operation in the course of explanation, the equivalence I have spoken of, and demonstrates the structure of the following table from the geometric source of triangles.

[Margin: See chapter 2 of this [book].]

The secondary theory of the Moon.

On a straight line FM let two points A and B be conceived; and B being set as the center of the Earth, let a little circle KLC be drawn about A as center; and, C being set as center (according to the requirement of the foregoing prior example), let the primary eccentric orbit of the Moon FDM itself be described. Now, IC being drawn parallel to FA, it is clear that at I is to be understood the mean apogee of the eccentric orbit. Again, with the equant or epicycle HEN made about D as center, in it the Moon itself is now understood to be carried at E. For by the law of revolutions, when D is conjoined with I in the place of apogee, the Moon abides at H, the lowest part of the epicycle. And because, by an equal change, H and E come

to be with I and D, therefore the angles ICD and HDE are always supposed equal. Of these, the former, while it measures the simple anomaly of the Moon from the apogee I, the latter urges that same measure of the Moon's revolution onward in the order of the signs through the epicycle, the beginning being taken from the perigee H. Furthermore, AB being the mean eccentricity, since the center of the Moon's orbit is understood to move in the little circle KLC likewise in the order of the signs, at the double distance of the Moon from the Sun — therefore (as in the direct syzygies of the luminaries, that is, in true conjunction and opposition) the same center of the Moon's orbit dwells nearest to the Earth's center B, at K; but in the quadratures, at L, [farthest] from the

Diagram. A large geometric figure for the secondary lunar hypothesis, set on the vertical straight line FM. Outermost is the large primary eccentric orbit of the Moon, FDM. Within it, near the lower-left center, is a small circle KLC carrying the orbit-center, and a small epicycle HEN at upper left. Lines radiate from the lower center toward E (the Moon). Labeled points:

- F (top): upper end of the vertical line FM, on the large circle
- I (top, just left of F): point marking the mean apogee direction; IC drawn parallel to FA
- G (upper left): point on the large circle to the left of I/F
- N (upper left, on the small epicycle): one extreme of the epicycle HEN
- E (just below/right of N, on the epicycle): the Moon's place
- D (near E, center of the epicycle HEN)
- H (on the small epicycle, lower part): the perigee of the epicycle, lowest part
- L (lower center, on the little circle KLC): orbit-center position at quadratures
- A (lower center): center of the little circle KLC
- C (on the little circle, near A): center of the primary eccentric orbit
- K (below A, on the little circle): orbit-center position at syzygies, nearest B
- B (lower, on line FM below K): center of the Earth
- M (bottom): lower end of the vertical line FM, on the large circle

[Margin: See chapter 2 of this [book].]

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Theoricorum, Caput VI

[Continuation of the worked example for the secondary lunar hypothesis.]

at its most remote. Therefore, when the line B is drawn through C to G, BC in the present example will be the true eccentricity; C, moreover, will be the true position of the apogee, and the angle ABC (that is, the arc IG) the subtractive prosthaphaeresis of the anomaly. Whenever, then, the Moon stands in configuration at the true conjunction or opposition of the Sun, a single one of the two angles, gathered together, suffices for its appearances — namely angle EKD (to which ECD is always equal) together with KEB. This is the combined prosthaphaeresis that the first column of the table of prosthaphaereses presents, as will be explained more fully below.

For the rest, outside the aforesaid syzygies of the luminaries: since the eccentricity BK keeps increasing all the way to the quadratures — that is, until K, the center of the Moon's orbit, has reached L — the variation in the Moon's inequality therefore grows correspondingly larger as well. This increase, near the apogee and perigee, is to be assigned to the change in the Moon's distance from the Earth; but then, gradually, around the sides [of the orbit], it is to be assigned to the angle of greater elongation. All of this can be shown to the eye in the present hypothesis. And that this hypothesis differs not even in the slightest from the earlier one, so far as the outcome of the calculation is concerned, will soon be shown by the outcome alone in the proposed example — without those [demonstrations] which I judge it unnecessary to set out at length here.

Finally there remains that small anomaly which insinuates itself in its ordinary manner about the Moon on both sides of the cardinal syzygies. When the Moon is set midway on its course toward the quadratures, this becomes a maximum additive of $40\frac{1}{2}$ minutes, and just as much subtractive in the mean reception from quadrature. Since this, together with its cause, has been explained above, it will suffice merely to insert it into the following table of prosthaphaereses, from which it can then be drawn out for its own use.

This, then, has been a brief sketch of our secondary hypothesis, drawn so as to agree in every way with all the Moon's phenomena. We shall now test it by illustrating it geometrically with an example, and afterwards we shall hand down the structure of the following table of prosthaphaereses from the same source, together with its rules and use.

A worked example fitted to the secondary lunar hypothesis.

In the year of Christ 1587, on 17 August at 19 hours 25 minutes p.m., the true position of the Moon's center was observed at Uraniborg with the Tyconic instruments — as we set down before — at 26 degrees 21½ minutes of Gemini [□]. For at the time, equated as before, the mean longitude of the Sun from the mean vernal equinox is found to be 5 signs 5 degrees 42 minutes 16 seconds, and the anomaly of the Sun 2 signs 0 degrees 20 minutes 53 seconds; and on this account its subtractive prosthaphaeresis is 1 degree 44 minutes 46 seconds. Further, the simple longitude of the Moon from the Sun is 9 signs 25 degrees 12 minutes 6 seconds, with an anomaly of 1 sign 15 degrees 37 minutes 21 seconds.

This [anomaly] measures, in the lunar diagram given just above, the angle ICD (or the arc ID) — that is, the simple distance or remoteness of the Moon from the mean apogee I in its orbit. And since, by hypothesis, the angle HDE is equal to it, and CD, the radius of the Moon's orbit, is 100,000 (of which units DE is likewise 2,900), in the triangle CDE the two sides about the given angle D are given — namely [the angle is] 45 degrees 37 minutes 21 seconds. Whence, by what we prescribed in their place in the doctrine of triangles [trigonometry], the angle ECD first emerges as 1 degree 12 minutes 44 seconds, and likewise the side CE as 97,994 of those units of which CD, the radius, was (as I said) 100,000.

But since, at the true conjunction or opposition of the luminaries — when the center of the Moon's orbit shall be at K — the radius of the Moon's orbit is shown by the side KD just as it now is by CD, therefore, on account of these two equal sides, all the rest will be equal: that is, angle EKD equal to angle ECD, and side KE equal to side CE. So now in the triangle KEB, with KE known as 97,994, likewise KB as 5,800 (the minimum eccentricity of the Moon, which, as explained above, belongs to the conjunction and opposition of the luminaries), and besides the included angle EKB as 135 degrees 35 minutes 23 seconds (which is the complement of the angle ICE to a semicircle, that is, the equated anomaly of the Moon)...

[the included angle EKB] being subtracted from that simple supposed [angle], namely the angle FKE, the angle KEB is thereby brought to light as 2 degrees 16 minutes 32 seconds. And so this [angle KEB], together with the previously found EKD, composes the ecliptic prosthaphaeresis (so called by us below, because it pertains only to the conjunction and opposition of the luminaries — for the difference between the mean and true longitude of the Moon — and therefore also to eclipses): 3 degrees 29 minutes 16 seconds, entirely as it was found in the prior lunar hypothesis, in agreement with this same anomaly of the Moon.

Further, when this prosthaphaeresis of the Moon is set against the Sun's prosthaphaeresis found above, and the smaller is subtracted from the greater — since both are subtractive — there remains a residue of 1 degree 44 minutes 30 seconds, subtractive. When this is therefore taken away from the simple longitude of the Moon set down above, the remainder is the true ecliptic longitude of the Moon from the Sun: 9 signs 23 degrees 27 minutes 36 seconds. This, doubled, is found to be 7 signs 16 degrees 55 minutes 11 seconds, whose complement to a whole circle is the angle CAK, 133 degrees 4 minutes 48 seconds. Now this angle, together with the sides it encloses — BA 7,974 and AC 2,174 (in those parts, granted throughout, of which the radius of the orbit CD is 100,000, as was said above) — since it becomes known at all times from the revolution of the Moon's double distance from the Sun, gives likewise in this example the angle ABC, 9 degrees 31 minutes 49 seconds, the subtractive prosthaphaeresis of the anomaly; for the angle ICG (that is, the arc IG) is equal to it.

Whence the said anomaly is found, finally equated, to be 34 degrees 53 minutes 43 seconds, and the side BC, the true eccentricity, 9,590½. Lastly, in the triangle ECB, with the two sides ECB given — EC 97,994, likewise BC 9,590½ — and likewise the included angle ECB 145 degrees 7 minutes 12 seconds (which, as appears, is the complement of the equated anomaly, that is, of the angle GCE, to a semicircle), there is given also the angle CEB, 2 degrees 57 minutes 11 seconds.

Diagram. A large circle (the Moon's orbit) drawn about a center, illustrating the secondary lunar hypothesis worked example. Near the top of the circle stand three labeled points in a row, G — I — F, marking the apogeal region (I = mean apogee, F = the simple supposed apogee, G = the equated/true apogee position). At the upper left, a small epicyclic circle carries the labels N, D, E, H clustered together (D = center of the epicycle on the orbit, E = the Moon, with N and H on the small epicycle). Toward the lower-center interior of the great circle is a small circle (the little circle KLC about A) bearing the labels A, K, with C just above it — A being the mid-eccentricity point, K the syzygy position (nearest Earth), C the orbit-center for this configuration. Below this, at the bottom interior, is B (the Earth). Dotted construction lines run from B and from the interior points out to D/E and up toward the apogeal points G, I, F. At the very bottom of the great circle is the label M (the perigeal point opposite F).

- G, I, F (top, in a row): equated apogee (G), mean apogee (I), simple supposed apogee (F)
- N, D, E, H (upper left, on small epicycle): D = epicycle center on orbit; E = Moon; N, H = epicycle points

- C (interior, upper): true orbit-center for this configuration
- A (interior, on little circle): mid-eccentricity point
- K (interior, on little circle, below A): syzygy position (Moon's orbit-center nearest Earth)
- B (lower interior): Earth
- M (bottom of great circle): perigee

which, [added] to the angle ECD found at the beginning, 1 degree 12 minutes 44 seconds, constitutes the prosthaphaeresis of the Moon, 4 degrees 10 minutes 41 seconds, subtractive. To this the Moon's variation adds (as above) 29 minutes 36 seconds. And so the whole equation of the Moon — that is, the difference of the true longitude from the mean — is 4 degrees 40 minutes 17 seconds, to be subtracted.

But in order that the mean [longitude] from the true equinox may be had, let there be added [to the data] the Sun's [longitude]:

	Sig.	Gr.	Mi.	Sec.
The Moon's simple [longitude] from the Sun, namely	9	25	12	6
First, the simple longitude of the Sun from the mean equinox	5	5	42	16
Then the equinoctial equation			7	13
And there is compounded the mean longitude	3	1	1	35

	Sig.	Gr.	Mi.	Sec.
of the Moon from the true vernal equinox				
Finally, let the whole prosthapha eresis be subtracted		4	40	17
And there remains the longitude of the Moon in its proper orbit, □ [Gemini]		26	21	19

— as [it follows] from the theory above, exactly enough. Lastly the side BE, the distance of the Moon from the Earth, if it be required, is easily brought to light from the same final triangle ECB. And from this it may be recognized that there is found in this our secondary lunar hypothesis a complete equipollence [δεδομένοις — Greek: dedomenois, "(from the) givens"] with the prior one, which we have also proved true by many other examples.

There now follows the complete table of prosthaphaereses, with its explanation and use.

Tabula secunda Prosthaphaeresium Lunarium. [Second Table of Lunar Prosthaphaereses]

0. Sig. [Signs 0 at top / Signs 11 at bottom]

Column legend (left to right): **Gr. ano.** = degree of anomaly; **Subtrah. Prosth. Ecliptic. (G.M.S.)** = subtractive ecliptic prosthaphaeresis (deg./min./sec.); **Add. Diff. (M.S.)** = additive difference; **Subtr. Prosth. Æquan. (G.M.S.)** = subtractive prosthaphaeresis of the equant; **Add. Diff. (M.S.)**; **Subtrah. Excessus (G.M.S.)** =

subtractive excess; **Add. Diff. (M.S.)**; **Add. Equa. Anom. (G.M.)** = additive equation of the anomaly; **Scru. Prop. (M.S.)** = proportional minutes; **Add. Variatio (M.S.)** = additive variation; **Gra. ano.** (right-hand degree column).

Gra. ano.	Ecl. Pro sth. (Su btr.) G.M .S.	Diff. (Ad d) M.S.	Equ ant Pro sth. (Su btr.) G.M .S.	Diff. (Ad d) M.S.	Exc ess (Su btr.) G.M .S.	Diff. (Ad d) M.S.	Eq. Ano m. (Ad d) G.M .	Scr u. Pro p. M.S.	Vari atio (Ad d) M.S.	Gra. ano.
0	0 0 0	5 4	0 0 0	1 48	0 0 0	2 13	0 0	0 0	0 0	30
1	0 5 4	5 4	0 1 48	1 48	0 2 13	2 13	0 22	0 0	0 43	29
2	0 10 8	5 4	0 3 36	1 47	0 4 26	2 13	0 45	0 1	1 26	28
3	0 15 12	5 4	0 5 23	1 47	0 6 38	2 12	1 7	0 3	2 8	27
4	0 20 16	5 4	0 7 10	1 47	0 8 50	2 12	1 29	0 6	2 50	26
5	0 25 20	5 4	0 8 57	1 47	0 11 1	2 11	1 52	0 9	3 32	25
6	0 30 23	5 3	0 10 44	1 47	0 13 11	2 10	2 14	0 13	4 14	24
7	0 35 26	5 3	0 12 31	1 47	0 15 21	2 10	2 36	0 18	4 56	23
8	0 40 28	5 2	0 14 17	1 46	0 17 31	2 10	2 59	0 24	5 38	22
9	0 45 29	5 1	0 16 3	1 46	0 19 41	2 10	3 21	0 31	6 20	21
10	0 50 30	5 1	0 17 49	1 46	0 21 51	2 10	3 43	0 38	7 2	20
11	0 55 30	5 0	0 19 34	1 45	0 24 2	2 11	4 4	0 46	7 44	19
12	1 0	4 58	0 21	1 45	0 26	2 10	4 25	0 54	8 26	18

Gra. ano.	Ecl. Prosth. (Su btr.) G.M .S.		Equ ant Prosth. (Su btr.) G.M .S.		Exc ess (Su btr.) G.M .S.		Eq. Ano m. (Ad d) G.M .		Scr u. Pro p. M.S.	Vari atio (Ad d) M.S.	Gra. ano.
	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	
	28		19		12						
13	1 5 25	4 57	0 23 4	1 45	0 28 22	2 10	4 46	1 3	9 7	17	
14	1 10 21	4 56	0 24 48	1 44	0 30 32	2 10	5 7	1 13	9 48	16	
15	1 15 16	4 55	0 26 32	1 44	0 32 43	2 11	5 28	1 24	10 29	15	
16	1 20 10	4 54	0 28 15	1 43	0 34 54	2 11	5 49	1 36	11 10	14	
17	1 25 3	4 53	0 29 58	1 43	0 37 6	2 12	6 9	1 49	11 51	13	
18	1 29 55	4 52	0 31 40	1 42	0 39 17	2 11	6 29	2 2	12 31	12	
19	1 34 44	4 49	0 33 22	1 42	0 41 28	2 11	6 49	2 16	13 11	11	
20	1 39 32	4 48	0 35 3	1 41	0 43 39	2 11	7 9	2 30	13 51	10	
21	1 44 18	4 46	0 36 44	1 41	0 45 50	2 11	7 28	2 44	14 31	9	
22	1 49 3	4 45	0 38 24	1 40	0 48 0	2 10	7 47	2 59	15 10	8	
23	1 53 47	4 44	0 40 4	1 40	0 50 10	2 10	8 6	3 14	15 49	7	
24	1 58 29	4 42	0 41 43	1 39	0 52 18	2 8	8 24	3 29	16 28	6	
25	2 3 8	4 39	0 43 21	1 38	0 54 26	2 8	8 42	3 45	17 7	5	
26	2 7	4 36	0 44	1 36	0 56	2 8	9 0	4 2	17	4	

Gra. ano.	Ecl. Pro sth. (Su btr.) G.M .S.		Equ ant Pro sth. (Su btr.) G.M .S.		Exc ess (Su btr.) G.M .S.		Eq. Ano m. Scr u. Pro p. M.S.		Vari atio (Ad d) M.S.		Gra. ano.
	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	Diff. (Ad d) M.S.	
	44		57		34				45		
27	2 12	4 34	0 46	1 35	0 58	2 7	9 18	4 19	18	3	
	18		32		41				23		
28	2 16	4 32	0 48	1 33	1 0	2 7	9 35	4 37	19 1	2	
	50		5		48						
29	2 21	4 30	0 49	1 32	1 2	2 6	9 52	4 55	19	1	
	20		37		54				38		
30	2 25	4 27	0 51	1 31	1 5	2 6	10 8	5 14	20	0	
	47		8		0				15		
	Add	Sub	Add	Sub	Add	Sub	Sub		Sub		
	e	.	e	.	e	.	t.		t.		

11. Sig. [Sign 11, at bottom of page]

Parsing notes: This is a doubly-entered table — the left **Gra. ano.** column (0→30) reads with the **0. Sig.** heading at the top, while the right **Gra. ano.** column (30→0) reads with the **11. Sig.** heading at the bottom (the table is read upside-down for sign 11). The footer labels (**Add** / **Sub** / **Add** / **Sub** / **Add** / **Sub** / **Subt** / **Subt**.) give the operation to apply when reading from the bottom (sign-11 direction); the headers (**Subtrah.** / **Add.**) give the operation for the top (sign-0) direction. The three "Diff." columns are interleaved between the main value columns in the source. Several entries in the Diff. and seconds columns are tightly set; values were read at scan resolution — see uncertain readings below.

Tabula secunda prosthaphaeresium Lunarium. [Second Table of Lunar Prosthaphaereses]

1. Sig. [Sign 1 at top / Sign 10 at bottom]

Column legend identical to the previous page. The left **Gra. ano.** runs 0→30 (read with **1. Sig.** at top); the right **Gra. ano.** runs 30→0 (read with **10. Sig.** at bottom).

Gra. ano.	Ecl. Pro sth. (Su btr.) G.M .S.	Diff. (Ad d) M.S.	Equ ant Pro sth. (Su btr.) G.M .S.	Diff. (Ad d) M.S.	Exc ess (Su btr.) G.M .S.	Diff. (Ad d) M.S.	Eq. Ano m. (Ad d) G.M .	Scr u. Pro p. M.S.	Vari atio (Ad d) M.S.	Gra. ano.
0	2 25 47	4 25	0 51 8	1 31	1 5 0	2 4	10 8	5 14	20 15	30
1	2 30 12	4 22	0 52 39	1 30	1 7 4	2 2	10 24	5 33	20 51	29
2	2 34 34	4 20	0 54 9	1 29	1 9 6	1 59	10 39	5 53	21 27	28
3	2 38 54	4 17	0 55 38	1 28	1 11 5	1 59	10 54	6 14	22 3	27
4	2 43 11	4 14	0 57 6	1 27	1 13 4	1 58	11 9	6 36	22 38	26
5	2 47 25	4 12	0 58 33	1 26	1 15 2	1 57	11 23	6 59	23 1	25
6	2 51 37	4 9	0 59 59	1 25	1 16 59	1 56	11 37	7 23	23 48	24
7	2 55 46	4 6	1 1 24	1 24	1 18 55	1 55	11 50	7 48	24 22	23
8	2 59 52	4 2	1 2 48	1 23	1 20 50	1 55	12 3	8 14	24 56	22
9	3 3 54	4 2 [?]	1 4 11	1 21	1 22 45	1 55	12 16	8 41	25 29	21
10	3 7 53	3 59	1 5 32	1 21	1 24 39	1 54	12 29	9 8	26 2	20
11	3 11 40	3 56	1 6 52	1 20	1 26 32	1 53	12 42	9 35	26 34	19
12	3 15 42	3 53	1 8 11	1 19	1 28 24	1 52	12 54	10 1	27 6	18
13	3 19 31	3 49	1 9 28	1 17	1 30 15	1 51	13 6	10 27	27 39	17

Gra. ano.	Ecl. Pro sth. (Su btr.) G.M .S.	Diff. (Ad d) M.S.	Equ ant Pro sth. (Su btr.) G.M .S.	Diff. (Ad d) M.S.	Exc ess (Su btr.) G.M .S.	Diff. (Ad d) M.S.	Eq. Ano m. (Ad d) G.M .	Scr u. Pro p. M.S.	Vari atio (Ad d) M.S.	Gra. ano.
14	3 23 17	3 46	1 10 44	1 16	1 32 5	1 50	13 17	10 53	28 8	16
15	3 26 59	3 42	1 11 59	1 15	1 33 54	1 49	13 28	11 19	28 38	15
16	3 30 38	3 39	1 13 12	1 13	1 35 42	1 48	13 38	11 45	29 8	14
17	3 34 13	3 35	1 14 24	1 12	1 37 30	1 48	13 47	12 11	29 37	13
18	3 37 44	3 31	1 15 34	1 10	1 39 16	1 46	13 56	12 38	30 6	12
19	3 41 12	3 28	1 16 42	1 8	1 41 2	1 46	14 5	13 6 [?]	30 34	11
20	3 44 36	3 24	1 17 48	1 6	1 42 37	1 45	14 13	13 34	31 1	10
21	3 47 56	3 20	1 18 53	1 5	1 44 30	1 43	14 21	14 3	31 28	9
22	3 51 12	3 16	1 19 56	1 3	1 46 10	1 40	14 29	14 32	31 55	8
23	3 54 24	3 12	1 20 58	1 2	1 47 48	1 38	14 36	15 1	32 21	7
24	3 57 32	3 8	1 21 58	1 0	1 49 25	1 37	14 43	15 30	32 46	6
25	4 0 36	3 4	1 22 57	0 59	1 51 1	1 36	14 50	15 59	33 11	5
26	4 3 35	2 59	1 23 55	0 58	1 52 35	1 34	14 56	16 28	33 35	4
27	4 6 30	2 55	1 24 52	0 57	1 54 8	1 33	15 2	16 57	33 58	3

Gra. ano.	Ecl. Pro sth. (Su btr.) G.M .S.	Diff. (Ad d) M.S.	Equ ant Pro sth. (Su btr.) G.M .S.	Diff. (Ad d) M.S.	Exc ess (Su btr.) G.M .S.	Diff. (Ad d) M.S.	Eq. Ano m. (Ad d) G.M .	Scr u. Pro p. M.S.	Vari atio (Ad d) M.S.	Gra. ano.
28	4 9 21	2 51	1 25 48	0 56	1 55 40	1 32	15 8	17 26	34 21	2
29	4 12 8	2 47	1 26 42	0 54	1 57 10	1 30	15 13	17 56	34 43	1
30	4 14 51	2 43	1 27 35	0 53	1 58 38	1 28	15 18	18 27	35 4	0
	Add e.	Sub t.	Add e.	Sub t.	Add e.	Sub t.	Sub t.		Sub t.	

10. Sig. [Sign 10, at bottom of page]

Parsing notes: Same doubly-entered structure as the preceding page (1. Sig. read from top, 10. Sig. read from bottom). The "Excessus" main column and several Diff. columns are tightly set and partly ink-degraded at the gutter; a handful of seconds-place digits were read at scan-resolution floor (see uncertain readings). Note the Variatio column at row 3 (signed 22 38) versus row 4 (signed 22 33) appears non-monotone — flagged below.

Pars Altera, Liber Primus Theoricorum, Caput VI (continued)

Second Table of Lunar Prosthaphaereses (continued)

This batch comprises four consecutive full-page spreads of the **Second Table of Lunar Prosthaphaereses** [Tabula secunda Prosthaphaeresium Lunarium], covering the remaining sign-blocks: **2/9, 3/8, 4/7, and 5/6**. Each table is read two ways: top-down for the heading sign (e.g. "2 Sig.") and bottom-up for its reciprocal (e.g. "9 Sig."), with the column operation-labels (Add./Subtract) reversing at the foot of each page accordingly.

The columns (left to right) are:

- **Gradus anomaliae** — degree of anomaly (0–30), the argument.

- **Subtrahe Prosthaph. Ecliptica** [Subtract: ecliptic prosthaphaeresis] (°''') and its **Diff.** (difference, ''').
- **Subtrahe Prosthaph. aequata** [Subtract: equated prosthaphaeresis] (°''') and its **Diff.** (''').
- **Subtrahe Excessus** [Subtract: excess] (°''') and its **Diff.** (''').
- **Adde Aequata Anom.** [Add: equated anomaly] (°) — but see foot-row sense-reversals.
- **Scrupula proportionalia** [proportional minutes] (Scrup. prop.) (''').
- **Adde Variatio** [Add: variation] (''').
- reciprocal **Gradus anomaliae** (right edge).

Page 277 — "2 Sig." (top) / "9 Sig." (foot)

Heading (body): *Tabula secunda Prosthaphaeresium Lunarium.* — "Second Table of Lunar Prosthaphaereses." — **2 Sig.**

Gra d. ano.	Sub tr. Pro sth. Ecli pt. (°''')		Sub tr. Pro sth. aeq uat. (°''')		Sub tr. exc essu s (°' ') (''')		Add e Æq u. ano m. (°) (''')		Scr up. pro p. (' ') (''')		Gra d. ano.
	Diff. (''')		Diff. (''')		Diff. (''')						
0	4 14 51	2 38	1 27 35	0 51	1 58 38	1 26	15 18	18 27	35 4		30
1	4 17 29	2 33	1 28 26	0 49	2 0 4	1 24	15 23	18 57	35 25		29
2	4 20 2	2 29	1 29 15	0 47	2 1 28	1 22	15 27	19 28	35 45		28
3	4 22 31	2 24	1 30 2	0 46	2 2 50	1 20	15 30	19 59	36 5		27
4	4 24 55	2 19	1 30 48	0 43	2 4 10	1 18	15 33	20 30	36 24		26
5	4 27 14	2 15	1 31 31	0 41	2 5 28	1 17	15 36	21 0	36 45		25
6	4 29 29	2 10	1 32 12	0 39	2 6 45	1 15	15 39	21 30	37 0		24
7	4 31	2 5	1 32	0 37	2 8	1 14	15	22 0	37		23

Gra d. ano.	Sub tr. Pro sth. Ecli pt. (^{o'''})	Diff. (^{'''})	Sub tr. Pro sth. aeq uat. (^{o'''})	Diff. (^{'''})	Sub tr. exc essu s (^{o'} ")	Diff. (^{'''})	Add e Æq u. ano m. (^{o'})	Scr up. pro p. (' ")	Add e vari atio (^{'''})	Gra d. ano.
	39		51		0		42		17	
8	4 33 44	2 0	1 33 28	0 36	2 9 14	1 13	15 44	22 30	37 33	22
9	4 35 44	1 55	1 34 4	0 34	2 10 27	1 11	15 46	23 0	37 48	21
10	4 37 39	1 51	1 34 38	0 33	2 11 38	1 8	15 47	23 31	38 3	20
11	4 39 30	1 47	1 35 11	0 30	2 12 46	1 6	15 48	24 2	38 17	19
12	4 41 17	1 42	1 35 41	0 29	2 13 52	1 3	15 49	24 33	38 30	18
13	4 42 59	1 36	1 36 10	0 27	2 14 55	1 1	15 49	25 4	38 42	17
14	4 44 35	1 30	1 36 37	0 25	2 15 56	1 0	15 49	25 35	38 55	16
15	4 46 5	1 25	1 37 2	0 23	2 16 56	0 58	15 49	26 7	39 7	15
16	4 47 30	1 20	1 37 25	0 20	2 17 54	0 56	15 49	26 39	39 18	14
17	4 48 50	1 16	1 37 45	0 19	2 18 50	0 54	15 48	27 11	39 28	13
18	4 50 6	1 10	1 38 4	0 18	2 19 44	0 52	15 47	27 44	39 37	12
19	4 51 16	1 5	1 38 22	0 16	2 20 36	0 50	15 46	28 17	39 45	11
20	4 52 21	1 0	1 38 38	0 14	2 21 26	0 49	15 45	28 50	39 53	10
21	4 53 21	0 55	1 38 52	0 13	2 22 15	0 47	15 43	29 22	40 0	9

Gra d. ano.	Sub tr. Pro sth. Ecli pt. (^{o'''})	Diff. (^{'''})	Sub tr. Pro sth. aeq uat. (^{o'''})	Diff. (^{'''})	Sub tr. exc essu s (^{o'} ")	Diff. (^{'''})	Add e Æq u. ano m. (^{o'})	Scr up. pro p. (' ")	Add e vari atio (^{'''})	Gra d. ano.
22	4 54 16	0 49	1 39 4	0 12	2 23 2	0 45	15 41	29 54	40 6	8
23	4 55 5	0 44	1 39 16	0 11	2 23 47	0 42	15 39	30 26	40 12	7
24	4 55 49	0 39	1 39 27	0 8	2 24 29	0 39	15 36	30 57	40 17	6
25	4 56 28	0 33	1 39 35	0 3	2 25 8	0 36	15 33	31 28	40 21	5
26	4 57 1	0 28	1 39 38	0 3	2 25 44	0 34	15 30	31 59	40 25	4
27	4 57 29	0 22	1 39 41	0 1	2 26 18	0 31	15 27	32 30	40 27	3
28	4 57 51	0 17	1 39 43	[Su btr.]	2 26 49	0 29	15 23	33 1	40 28	2
29	4 58 8	0 12	1 39 42	0 1	2 27 18	0 26	15 19	33 32	40 29	1
30	4 58 20	[1 39 40]	1 39 40	0 2	2 27 44	—	15 15	34 2	40 30	0
	Add e	Sub.	Add e	Sub.	Add e	Sub.	Sub t.		Sub t.	

[Foot label: **9 Sig.**]

Parsing notes: The ^{o'} minor columns at deg-28/29 carry the table's standard turn-over markers (the "Subtr."/operation-row notations printed between the excessus and equated-anomaly columns); the equated-anomaly Diff. column terminates here. Row 5 variatio reads "36 25" but the smooth progression of the variatio column (...36 24, 37 0...) suggests the printed minor value is slightly garbled — flagged [?]. Row 29 excessus-Diff. printed "0 2" — flagged [?] (the Diff. column is closing out near the table's turning point).

Heading (body): *Tabula secunda prosthaphaeresium Lunarium.* — **3 Sig.**

Gra d. ano.	Sub tr. Pro sth. Ecli pt. (^{o'''})	Diff. (''')	Sub tr. Pro sth. aeq uat. (^{o'''})	Diff. (''')	Sub tr. exc essu s (°)	Diff. (''')	Add e Æq ua. Ano m. (°)	Scr u. pro p. (°)	Add e vari atio (''')	Gra d. ano.
0	4 58 20	[—]	1 39 40	[—]	2 27 44	0 25	15 15	34 2	40 30	30
1	4 58 26	0 6	1 39 38	0 2	2 28 9	0 22[?]	15 11	34 32	40 29	29
2	4 58 27[?]	0 1[?]	1 39 34	0 4	2 28 31	0 20	15 6	35 2	40 28	28
3	4 58 22	0 5	1 39 27	0 7	2 28 51	0 17	15 1	35 21	40 27	27
4	4 58 14[?]	0 8[?]	1 39 18	0 9	2 29 8	0 14	14 56	36 0	40 25	26
5	4 57 59	0 15	1 39 7	0 11	2 29 22	0 13	14 51	36 29	40 21	25
6	4 57 37	0 22	1 38 54	0 13	2 29 35	0 9	14 46	36 58	40 17	24
7	4 57 10[?]	0 27	1 38 39	0 15	2 29 44	0 9	14 41	37 27	40 12	23
8	4 56 38[?]	0 32	1 38 21[?]	0 18	2 29 50	0 6[?]	14 36	37 56	40 6	22
9	4 56 1[?]	0 37	1 38 0[?]	0 21	2 29 54	0 4	14 30	38 25	40 0	21
10	4 55	0 43	1 37	0 23	2 29	0 3	14	38	39	20

Gra d. ano.	Sub tr. Pro sth. Ecli pt. (^{o'''})	Diff. (^{'''})	Sub tr. Pro sth. aeq uat. (^{o'''})	Diff. (^{'''})	Sub tr. exc essu s (^{o'} ")	Diff. (^{'''})	Add e Æq ua. Ano m. (^{o'})	Scr u. pro p. (' ")	Add e vari atio (^{'''})	Gra d. ano.
	18[?]		37		57		24	54	53	
11	4 54 30[?]	0 48	1 37 12	0 25	2 30 0	[Su btra h.]	14 17	39 23[?]	39 45	19
12	4 53 36	0 54	1 36 46	0 26	2 29 59	0 1	14 10	39 50	39 37	18
13	4 52 37	0 59	1 36 20	0 26	2 29 58	0 1	14 3	40 18	39 28	17
14	4 51 33[?]	1 4	1 35 52	0 28	2 29 55	0 3	13 56	40 46	39 18	16
15	4 50 23	1 10	1 35 24	0 28	2 29 48	0 7	13 49	41 14	39 7	15
16	4 49 7	1 16	1 34 56	0 30	2 29 35	0 13	13 42	41 41	38 55	14
17	4 47 46	1 21	1 34 23	0 31	2 29 15	0 20	13 35	42 0[?]	38 43	13
18	4 46 21	1 25	1 33 52	0 31	2 28 48	0 27	13 27	42 35	38 30	12
19	4 44 50	1 31	1 33 20	0 32	2 28 16	0 32	13 19	43 2	38 17	11
20	4 43 13	1 37	1 32 44	0 36	2 27 39	0 37	13 11	43 29	38 3	10
21	4 41 34	1 42	1 32 6	0 38	2 26 57	0 42	13 3	43 55	37 48	9
22	4 39 43	1 48	1 31 28	0 38	2 26 10	0 47	12 55	44 21	37 33	8
23	4 37	1 52	1 30	0 40	2 25	0 50	12	44	37	7

Gra d. ano.	Sub tr. Pro sth. Ecli pt. (^{o'''})		Sub tr. Pro sth. aeq uat. (^{o'''})		Sub tr. exc essu s (^{o'} ")		Add e Æq ua. Ano m. (^{o'})		Scr u. pro p. (' ")	Add e vari atio (^{o'''})	Gra d. ano.
	Diff. (^{o'''})		Diff. (^{o'''})		Diff. (^{o'''})		Diff. (^{o'''})				
	55		48		20		47	47	17		
24	4 35	1 57	1 30	0 43	2 24	0 52	12	45	37 0	6	
	54[?]		5		28		38	12			
25	4 33	2 3	1 29	0 45	2 23	0 54	12	45	36	5	
	51		20		34		29	37	42		
26	4 31	2 9	1 28	0 46	2 22	0 55	12	46 2	36	4	
	42		34		39		20		24		
27	4 29	2 13	1 27	0 47	2 21	0 56	12	46	36 5	3	
	29		47		43		11	27			
28	4 27	2 18	1 26	0 49	2 20	0 58	12 2	46	35	2	
	11		48[?]		45			51	45		
29	4 24	2 23	1 26	0 52	2 19	0 59	11	47	35	1	
	48		6		46		53	15	25		
30	4 22	2 28	1 25	0	2 18	1 2	11	47	35	0	
	20		12	54[?]	45		44	39	4[?]		
	Add e		Add e		Add e	Sub t.	Sub t.		Sub t.		

[Foot label: **8 Sig.**]

Parsing notes: This page is more degraded than p. 277 — several Prosthaph. Ecliptica entries and their Diff. values (rows 2, 4, 7–11, 14, 24) carry double-scan garbling and are flagged [?]. Row 0 repeats the deg-30 turning values of "2 Sig." (4 58 20 / 1 39 40 / 2 27 44), as expected at the join between sign-blocks. The excessus column peaks near deg-11 (2 30 0) and the "Subtrah." sense-reversal marker is printed in that cell, confirming the maximum. Variatio and Scrup. prop. columns are clean and monotone.

Heading (body): *Tabula secunda Prosthaphaeresium Lunarium.* — **4 Sig.**

Gra d. ano.	Sub tr. Pro sth. Ecli pt. (^{o'''})	Diff. (^{'''})	Sub tr. Pro sth. aeq uat. (^{o'''})	Diff. (^{'''})	Sub tr. exc essu s (^{o'} ^{''})	Diff. (^{'''})	Add e Æq ua. Ano m. (^{o'})	Scr u. pro p. (['] ^{''})	Add e vari atio (^{'''})	Gra d. ano.
0	4 22 20	2 34	1 25 12	0 56	2 18 45	1 3	11 44	47 39	35 4	30
1	4 19 46	2 39	1 24 16	0 57	2 17 42	1 4	11 35	48 2	34 43	29
2	4 17 7	2 43	1 23 19	0 58	2 16 38	1 8	11 26	48 25	34 21	28
3	4 14 24	2 48	1 22 21	0 59	2 15 30	1 10	11 17	48 48	33 58	27
4	4 11 36	2 53	1 21 22	1 0	2 14 20	1 14	11 7	49 10	33 35	26
5	4 8 43	2 58	1 20 22	1 1	2 13 6	1 18	10 57	49 32	33 11	25
6	4 5 45	3 3	1 19 21	1 3	2 11 48	1 22	10 47	49 54	32 46	24
7	4 2 42	3 7	1 18 17	1 5	2 10 26	1 26	10 36	50 15	32 21	23
8	3 59 35	3 12	1 17 12	1 6	2 9 0	1 28	10 25	50 36	31 55	22
9	3 56 23	3 17	1 16 6	1 8	2 7 32	1 31	10 15	50 56	31 28	21
10	3 53 6	3 21	1 14 58	1 10	2 6 1	1 34	10 5	51 16	31 1	20
11	3 49 45	3 25	1 13 48	1 11	2 4 27	1 36	9 55	51 36	30 34	19
12	3 46 20	3 30	1 12 37	1 13	2 2 51	1 39	9 44	51 56	30 6	18

Gra d. ano.	Sub tr. Pro sth. Ecli pt. (^{'''})	Diff. (^{'''})	Sub tr. Pro sth. aeq uat. (^{'''})	Diff. (^{'''})	Sub tr. exc essu s (^{o'} ")	Diff. (^{'''})	Add e Æq ua. Ano m. (^{o'})	Scr u. pro p. (' ")	Add e vari atio (^{'''})	Gra d. ano.
13	3 42 50	3 34	1 11 25	1 14	2 1 12	1 42	9 33	52 15	29 37	17
14	3 39 16	3 38	1 10 12	1 15	1 59 30	1 45	9 22	52 34	29 8	16
15	3 35 38	3 43	1 8 58	1 16	1 57 45	1 48	9 11	52 53	28 38	15
16	3 31 55	3 47	1 7 44	1 16	1 55 57	1 51	9 0	53 12	28 8	14
17	3 28 8	3 50	1 6 29	1 16	1 54 6	1 55	8 49	53 30	27 37	13
18	3 24 18	3 54	1 5 13	1 17	1 52 11	1 55	8 38	53 48	27 6	12
19	3 20 24	3 58	1 3 57	1 17	1 50 13	1 58	8 27	54 5	26 34	11
20	3 16 25	4 3	1 2 40	1 19	1 48 12	2 1	8 16	54 22	26 2	10
21	3 12 22	4 7	1 1 21	1 21	1 46 8	2 4	8 5	54 38	25 29	9
22	3 8 15	4 10	1 0 0	1 21	1 44 0	2 8	7 53	54 54	24 56	8
23	3 4 5	4 13	0 58 38	1 22	1 41 49	2 11	7 42	55 9	24 22	7
24	2 59 52	4 17	0 57 14	1 24	1 39 36	2 13	7 29	55 24	23 48	6
25	2 55 35	4 21	0 55 49	1 25	1 37 21	2 15	7 17	55 39	23 13	5
26	2 51 14	4 25	0 54 24	1 25	1 35 4	2 17	7 5	55 54	22 38	4
27	2 46	4 28	0 52	1 26	1 32	2 19	6 53	56 8	22 3	3

Gra d. ano.	Sub tr. Pro sth. Ecli pt.		Sub tr. Pro sth. aeq uat.		Sub tr. exc essu s (° ")		Add e Æq ua. Ano m.		Scr u. pro p. (° ")		Add e vari atio (")		Gra d. ano.
	(°''')	(''')	(°''')	(''')	(°''')	(''')	(°')	(°')	(°')	(°')	(°''')	(°''')	
	49		58		45								
28	2 42	4 30	0 51	1 27	1 30	2 21	6 41	56	21	2			
	21		31		24			22	27				
29	2 37	4 33	0 50	1 28	1 28	2 23	6 29	56	20	1			
	51		3		1			35	51				
30	2 33	[—]	0 48	1 29	1 25	2 25	6 17	56	20	0			
	18		34		36			48	15				
	Add e	Add .	Add e	Add .	Add e	Add .	Sub t.		Sub t.				

[Foot label: 7 Sig.]

Parsing notes: This page is the cleanest of the four — all three "Subtrahe" columns are smoothly monotone and the foot operation-row reads "Adde / Add. / Adde / Add. / Adde / Add. / Subt. ... Subt." (the prosthaphaeresis and excessus columns all turn to "Add" for the ascending "7 Sig." read). No values flagged.

Page 280 — "5 Sig." (top) / "6 Sig." (foot)

Heading (body): *Tabula secunda prosthaphaeresium Lunarium. — 5 Sig.*

Gra d. ano.	Sub tr. Pro sth. ecli pti.		Sub tr. Pro sth. aeq uan		Sub tr. exc ess.		Add e æqu ano.		Scr u. pro p. (° ")		Add e vari atio (")		Gra dus ano m.
	(°''')	(''')	t. (° ")	Diff. (")	Diff. (")	Diff. (")	(°')	(°')	(°')	(°')	(°''')	(°''')	
0	2 33	[—]	0 48	1 29	1 25	2 27	6 17	56	20	30			
	18		34		36			48	15				
1	2 28	4 37	0 47	1 30	1 23	2 29	6 5	57 0	19	29			

Gra d. ano.	Sub tr. Pro sth. ecli pti. (^{o'''})	Diff. (''')	Sub tr. Pro sth. aeq uan t. (^{o'} ")	Diff. (''')	Sub tr. exc ess. (^{o'''})	Diff. (''')	Add e æqu ano. (^{o'})	Scr u. pro p. (['] ")	Add e vari atio (''')	Gra dus ano m.
	41		5		9				38	
2	2 24 1	4 40	0 45 35	1 30	1 20 40	2 32	5 53	57 12	19 1	28
3	2 19 18	4 43	0 44 5	1 31	1 18 8	2 34	5 41	57 23	18 23	27
4	2 14 33	4 45	0 42 34	1 31	1 15 34	2 37	5 29	57 34	17 45	26
5	2 9 45	4 48	0 41 3	1 32	1 12 57	2 38	5 17	57 45	17 7	25
6	2 4 55	4 50	0 39 31	1 33	1 10 19	2 40	5 5	57 55	16 28	24
7	2 0 5	4 53	0 37 58	1 34	1 7 39	2 43	4 53	58 5	15 49	23
8	1 55 7	4 55	0 36 24	1 35	1 4 56	2 44	4 41	58 14	15 10	22
9	1 50 9	4 55	0 34 49	1 36	1 2 12	2 46	4 28	58 23	14 31	21
10	1 45 8[?]	5 1	0 33 13	1 36	0 59 26	2 48	4 15	58 32	13 51	20
11	1 40 5[?]	5 3	0 31 37	1 37	0 56 38	2 50	4 3	58 40	13 11	19
12	1 35 1[?]	5 4	0 30 0	1 37	0 53 48	2 52	3 51	58 48	12 31	18
13	1 29 55	5 6	0 28 23	1 38	0 50 56	2 53	3 39	58 56	11 51	17
14	1 24 47	5 8	0 26 45	1 38	0 48 3	2 54	3 26	59 4	11 10	16
15	1 19	5 9	0 25	1 39	0 45	2 55	3 13	59	10	15

Gra d. ano.	Sub tr. Pro sth. ecli pti. (^{o'''})	Diff. (''')	Sub tr. Pro sth. aeq uan t. (^{o'} ")	Diff. (''')	Sub tr. exc ess. (^{o'''})	Diff. (''')	Add e æqu ano. (^{o'})	Scr u. pro p. (' ")	Add e vari atio (''')	Gra dus ano m.
	38		7		9			11	29	
16	1 14 27	5 11	0 23 28	1 39	0 42 14	2 57	3 0	59 18	9 43	14
17	1 9 14	5 13	0 21 49	1 39	0 39 17	2 58	2 47	59 25	9 7[?]	13
18	1 3 59	5 15	0 20 10	1 40	0 36 19	2 58	2 34	59 31	8 26	12
19	0 58 43	5 16	0 18 30	1 40	0 33 21	2 59	2 21	59 36	7 44	11
20	0 53 27	5 16	0 16 50	1 40	0 30 22	2 59	2 8	59 40	7 2	10
21	0 48 10	5 17	0 15 10	1 40	0 27 23	3 0	1 56	59 44	6 20	9
22	0 42 52	5 18	0 13 30	1 41	0 24 23	3 0	1 43	59 47	5 38	8
23	0 37 33	5 19	0 11 49	1 41	0 21 23	3 1	1 30	59 50	4 56	7
24	0 32 13	5 20	0 10 8	1 41	0 18 21	3 2	1 17	59 53	4 14	6
25	0 26 52	5 21	0 8 27	1 41	0 15 19	3 2	1 4	59 55	3 32	5
26	0 21 30	5 22	0 6 46	1 41	0 12 16	3 3	0 51	59 57	2 50	4
27	0 16 8	5 22	0 5 5	1 41	0 9 13	3 3	0 39	59 58	2 8	3
28	0 10 46	5 22	0 3 24	1 41	0 6 9	3 4	0 26	59 59	1 26	2
29	0 5	5 23	0 1	1 42	0 3	3 4	0 13	59	0 43	1

Gra d. ano.	Sub tr. Pro sth.		Sub tr. Pro sth.		Sub tr. Pro sth.		Add e æqu		Scr u. pro		Add e vari		Gra dus ano m.
	ecli pti. (^{o'''})	Diff. (^{'''})	t. (^{o'} ")	Diff. (^{'''})	ess. (^{o'''})	Diff. (^{'''})	ano. (^{o'})	p. (['] ")	atio (^{'''})				
	23		42		5				59				
30	0 0 0	5 23	0 0 0	1 42	0 0 0	3 5	0 0	60 0	0 0	0			
	Add e		Add e		Add e		Sub t.				Sub t.		

[Foot label: **6 Sig.**]

Parsing notes: At deg-30 ("5 Sig." foot, = "6 Sig." head) all three Subtrahe columns reach 0°0'0" and Scrup. prop. reaches 60'0" — the natural zero/maximum termination of the table, confirming the transcription. The early-row Prosthaph. ecliptica entries (rows 8–12) carry double-scan garbling in their seconds-fields and are flagged [?]. Row 17 variatio "9 7" flagged [?] (faint print). The "6 Sig." foot completes the full 0–6/6–12 sign range of the Second Lunar Prosthaphaeresis table.

The catchword "Expo-" at the foot of p. 280 signals that the next page (beyond this batch) begins an exposition — presumably the worked exposition/use of these tables, after which the lunar-latitude theory (nodes, argument of latitude, latitude table) is expected to follow within Caput VI.

Astronomia Danica — Pars Altera, Liber Primus Theoricorum, Caput VI (concl.) & Caput VII

Explanation of the structure of the preceding table of lunar prosthaphaereses

The first column lies next to the area [argument-field] of the preceding table. (The two outermost columns are not counted here — namely the degrees of commutation of the signs through the semicircle.) This first column is composed

of two angles added together, namely EKD and KEB. Now since for the true conjunction and opposition of the two luminaries — that is, for the true new moon and full moon — this single prosthaphaeresis of the Moon will suffice (and therefore also for eclipses, which are common to both luminaries), we have for that reason given it the more specific name *ecliptic* [prosthaphaeresis].

The second column contains only that one prosthaphaeresis which the angle DCE alone displays. This arises from the epicycle HEN, which we have added in place of the equant; for that reason it is called the prosthaphaeresis *of the equant*. And although such an angle, added to the other angle KEB, constitutes the prosthaphaeresis sufficient for the conjunction and opposition of the Moon with the Sun (as I said), nevertheless it had to be set out here separately as well. The reason is that, once the Moon's anomaly has been altered by the equation from the angle ICG and so on, here it fits only to that simple anomaly ID. Indeed, by hypothesis the angle HDE is always equal to the angle ICD — that is, to the mean anomaly of the Moon — and not to GCD, or rather not to the angle GCE, the anomaly finally equated. Therefore, in the assumption of the simple anomaly proceeding through individual degrees and so on: when the said prosthaphaeresis of the equant — that is, the angle EKD here — is subtracted from the whole ecliptic prosthaphaeresis, there remains by itself the angle KEB (which was one of these two components that existed before), the knowledge of which serves the steps that follow.

The third column is assigned to the excess [excessus] — namely that excess which lies between the shortest and longest eccentricity, that is, between the ecliptic syzygies of the luminaries and the quadratures. Its limits are K, A, L, as we showed above. Here too it arises for us from a triangle built upon the assumed simple anomaly: for when the eccentricity BK 5800 is increased by KL 4348 — that is, by the whole diameter of the small circle KLC — there will be that excess by which the angle LEB always surpasses the angle KEB at any distance of the Moon from apogee.

Finally, the fourth column contains the equation of the anomaly, which in the higher assumption is indicated by the angle ABC. Before the quadratures it is additive, but after them subtractive — as this too appears in the hypothesis itself. This [equation], moreover, is held to whole degrees of the Sun and to first minutes [scruples], both here on account of want of space and because it is nearly free of all perceptible error. For half of one first minute, which might perhaps be either in excess or in deficit in the anomaly, almost nowhere produces a deviation beyond 3 seconds through the longitude of the Moon. Accordingly we

judge that we should commend, in calculation, not everywhere that extreme ἀκρίβεια [Greek: ἀκρίβεια (akribēia, "exactness")], but rather a discreet [reasonable] precision.

Furthermore — so that here lovers of the stars [astronomers] might be relieved of the laborious resolution of triangles (which was indeed the very aim of this secondary theory we have devised) — we have bound the proportional minutes [scruples] attached to the fifth column, in order to discern the proportion of the aforementioned excess throughout the Moon's whole course outside the cardinal syzygies, following the practice of the ancients; and according to this practice their discovery too was easy. For at the contact of the eccentric and the epicycle, where the whole prosthaphaeresis is greatest, and where the angle CEB always surpasses the angle KEB by however much in the angle of appearance, beginning from K — that whole quantity too is comprehended under the name of *excess*. Therefore, since KL is found at the contact, that is, at the maximum excess of the prosthaphaeresis, to be 2 degrees 30 minutes, and since the whole excess is supposed to be made equal to a whole degree (that is, 60 minutes), then by the rule of arithmetical proportion a way is given both for assigning their proportional minutes to whatever other excesses are found near the contact, and afterwards, conversely, by the same rule for estimating the remaining excesses throughout the whole revolution of the lunar phenomenon.

Yet however ingenious this invention of the ancients may be, and however most useful it will seem for completing the calculation compendiously, nevertheless — to say how the matter stands — it rather often produces some discrepancy from the most exact analysis of triangles, and the discrepancy is the greater the greater the excess has been (as below in the case of Mars, Venus, and Mercury), or the more involved the equants are (as here in the case of the Moon).

The sixth and last column of the table, finally, is that variation [variatio] of the Moon which we set out clearly above together with its cause, and afterwards in an example as well. There now follow, therefore, the precepts of the lunar [calculation] itself, and their use thereafter in reiterating several of the preceding examples.

The method of computing the true longitude of the Moon from the new table of prosthaphaereses

I.

For the given and equated time, the mean motions of the luminaries are to be extracted in full from their appropriate tables, just as before, and written out in order — namely the simple [motion] of the Sun from the mean equinox, the anomaly of the Sun, the simple [motion] of the Moon from the Sun, and the anomaly of the Moon — having first set out (since this too is needed, in the last place) the inequality of the equinoxes.

II.

[Margin: The logistic of arithmetic at the end.]

Then, by means of the simple anomaly of each luminary, their prosthaphaereses are to be extracted individually: the Sun's indeed from its own table of prosthaphaeresis, but the Moon's only from the first column of this table, which we call the *ecliptic*; then likewise, by means of the same anomaly, [its prosthaphaeresis] from the second column, [namely] the prosthaphaeresis of the equant, or of the epicycle, which will be kept for its own use. Throughout, account must always be taken of the part of the proportion for the minutes adhering to the anomaly — let it suffice that I once delivered this in general elsewhere, but have now also given the reminder here.

III.

Thirdly, let the prosthaphaereses of the luminaries be compared with one another together with their signs [+/-]. If they are of the same denomination [sign], let them be subtracted from one another — the smaller, of course, from the greater; but if of different sign, let them be added together. Afterwards the remainder or the sum will be applied to the simple longitude of the Moon from the Sun, and that according to the sign of the prosthaphaeresis — considering at least in the remainder [the following]: if it [the larger] was that of the Sun prevailing, and indeed additive, let it be subtracted from the simple [longitude] of the Moon, and vice versa. For in the rest, as also in the sum, the addition and subtraction always follows the sign of the prosthaphaeresis of the Moon itself. This equated longitude of the Moon from the Sun, when doubled, shows that double true distance of the luminaries from one another which the hypothesis of the Moon requires.

IV.

With this double distance of the luminaries an entry is made into the said table of the Moon, and from its three last columns — namely the fourth, fifth, and sixth — there are excerpted in order, with their signs: first the equation of the anomaly of

the Moon; second the proportional minutes; third, finally, the variation of the center of the epicycle, which will be kept together with the proportional minutes. The equation of the anomaly, moreover, will be fitted to the simple anomaly of the Moon according to the sign of addition or subtraction, and thus the equated anomaly of the Moon arises.

V.

Furthermore, by means of the equated anomaly of the Moon, again the ecliptic prosthaphaeresis is extracted from the first column of the table, and that of the equant from the next [column], and the difference between them — which is what we seek here — presents itself at once, provided that no minutes of anomaly adhere; otherwise [it is found] compendiously, when you have sought only the difference of the first and second columns at the degree of the sign of anomaly offered, and likewise the difference of those same differences which are found set down in the table; and thus, through the remaining difference, you have hunted out the proportional part for the adhering minutes in the customary manner, and have duly fitted it, according to its sign, to the remaining ecliptic prosthaphaeresis — as will be plain in the example. To this, presently, the prosthaphaeresis of the equant

— kept earlier — must be added or subtracted according to its sign, but for the most part added. Then in the same way, by means of the same equated anomaly, the excess is sought from the third column, and presently is multiplied logistically into the proportional minutes kept earlier; and the proportional part emerging from this is always aggregated to the nearest prosthaphaeresis out of both remainders.

VI.

To this aggregated prosthaphaeresis, when the variation of the Moon — also kept earlier — has been duly applied according to its sign, there will at length stand established from this the whole prosthaphaeresis, that is, how much the true motion of the Moon is distant from the mean. Let this last prosthaphaeresis of the Moon therefore be fitted, according to its sign, first to the simple longitude of the Moon from the Sun; then let the simple longitude of the Sun from the mean equinox be added. To this, in the last place — when the equinoctial equation has been duly applied [Margin: On which see Cap. 4.] — is given the true longitude of the Moon from the true vernal equinox in its own orbit, which was what was proposed. These precepts are made clearer by the following examples.

FIRST EXAMPLE OF THE FORMER [the earlier worked examples]

	Sig.	Gr.	Mi.	Sec.	Differ entia Mi.	Sec.
Equati on of the preces sion of the equin oxes	0	0	7	13		
Simpl e [motio n] of the Sun	5	5	42	16		
I. Anom aly of the Sun	2	0	20	53		
Longit ude of the Moon from the Sun	9	25	12	6		
Anom aly of the Moon	1	15	37	21		
II. Prosth aph.		1	44	46	S.	

	Sig.	Gr.	Mi.	Sec.		Differ entia Mi.	Sec.
— of the Sun Prosth aph. — Moon' s eclipti c Prosth aph. — of the equan t Residu e of the eclipti c prosth aph. & the Sun		3	29	16	S.		
III.	7	16	15	12			
Theref ore doubl e [longit ude of the] Moon from							

	Sig.	Gr.	Mi.	Sec.		Differ entia Mi.	Sec.
the Sun, equat ed							
Equati on of the anom aly		9	32	7	S.		
IV. Propo rtiona l minut es			52	6			
Variat ion of the center			29	35	S.		
Anom aly of the Moon equat ed	1	6	5	14			
Anom aly equat ed, in signs & — Eclipt.		2	51	37		4	9
degre es, to		0	59	59		1	25

	Sig.	Gr.	Mi.	Sec.		Differ entia Mi.	Sec.
it corres ponds prosth aph. — Equan t							
Differ ence		1	51	38		2	44
Adher ing minut es			5	14			
Propo rtiona l part				14	Add.		
Prosth aphae resis — of the Eccent ric		1	51	52			
Prosth aphae resis — of the Equan t, kept		1	12	44			
Prosth aphae resis		3	4	36			

	Sig.	Gr.	Mi.	Sec.		Differ entia Mi.	Sec.
—							
Aggre gate							
Excess		1	17	9			
Propo rtiona l			52	16			
minut es							
Propo rtiona l part		1	7	9	Add.		
Whole		4	11	48			
first prosth aphae resis of the Moon							
Variat ion of the center of the epicyc le			29	35	Add.		
Absol ute		4	41	23	Sub.		
prosth aphae resis of the Moon							
Above		4	40	36			

	Sig.	Gr.	Mi.	Sec.		Differ entia Mi.	Sec.
, from both hypot hesis & table, it was [recte: "Super ius ex utraq; hypot hesi & tabula eadem erat"]							
The differe nce, theref ore, from the forme r [meth od] is at least				47	+		

on account of the cause I touched on above; nor will it be necessary in this example to carry the calculation further for the true longitude of the Moon, since everything else will agree with the foregoing.

Parsing notes: This computation worksheet is a single-column running tally rather than a true tabular grid; the rightmost "Differentia / Mi. / Sec." sub-

columns apply only to the block beginning at "Anomaly equated, in signs & degrees." Sign markers (S. = *Subtrahe*/subtractive; Add. = additive; Sub. = subtractive) are transcribed in the cell to the right of each value as printed. The "+" at the final difference row is the source's diacritic/cross mark on the residual (47 seconds).

[Margin: Lib. 1, Prog. [Progymnasmata] On the latitude of the Moon.]

SECOND EXAMPLE

[Reiterated] also from the latter [posterior] table of prosthaphaereses of the Moon, at least to take up the variation of the new table from the higher [earlier example].

	Sig.	Gr.	Mi.	Sec.	
Simple longitude of the Moon from the Sun	2	25	19	38	
Prosthaphaeresis of the Moon, Ecliptic		4	53	43	Add.
Prosthaphaeresis of the Moon, of the equant		1	39	50	Add.
Diff. of the prosthaphaereses of the Moon, Eclip., from the		3	9	45	Add.

	Sig.	Gr.	Mi.	Sec.	
Sun					
[26 / 1 a		1	43	58	a
—					
marginal					
cross-					
refs]					
Prosthap					
haeresis					
of the					
Sun					
Longitud	2	28	29	23	
e of the					
Moon,					
Ecliptic,					
from the					
Sun					
And the	5	26	58	46	
double of					
this					
There			29	0	Add.
correspo					
nds the					
Equation					
of the					
anomaly					
Proportio			59	58	
nal					
minutes					
Variation			2	9	Add.
of the					
center					
Anomaly	8	18	8	20	
— simple					
Anomaly	8	18	47	20	
—					
Moon's,					

	Sig.	Gr.	Mi.	Sec.	
equated					
There corresponds the Moon's prosthaphaeresis of the eccentricity		3	17	12	Add.
Prosthaphaeresis of the equant, kept		1	36	50	Add.
Excess		2	30	0	
Proportional part		2	29	55	Add.
Therefore, from these last four [quantities] to be aggregated here, there is made the absolute additive prosthaphaeresis of the Moon		7	26	6	
Above, it was the same		7	25	19	

	Sig.	Gr.	Mi.	Sec.	
Differenc e				47	+

Therefore this one too settles within a minute. Nor, if the calculation is duly carried out, will I judge that any greater deviation from the exact resolution of the hypothesis itself will occur anywhere. In the meantime, however, beginners in astronomy who are perhaps weary of the labor are opportunely relieved here, as regards the computation of the longitude of the Moon from the former [methods].

Parsing notes: This is again a running computation tally, not a grid. The marginal "26" and "1 a" appear to be cross-reference keys against the first example's corresponding rows; the bare "a" at the Sun's prosthaphaeresis row is a footnote/cross-reference marker, transcribed as printed. The "+" at the final difference is the source's cross mark on the 47-second residual.

CAPVT VII.

On the latitude of the Moon, and its true computation.

Since in the preceding chapter we have also at the same time provided for the perpetual restitution of the mean motion, or argument, of the latitude of the Moon, we shall here treat only of the Moon's recession from the ecliptic into latitude, together with the inequality of the nodes — that is, of the head and tail of the Dragon [caput et cauda Draconis], as they are called, which are the two points of intersection of the Moon's path with the ecliptic. And this we do from the *Progymnasmata* of Tycho Brahe, the heedless drowsiness of the ancients in this matter having been left aside.

"The astronomers of every age," says Tycho, "have long since established by examination that the Moon does not complete its course beneath the ecliptic, the Sun's path; so much so that it has been detected to maintain a wandering away from it, growing on this side and that to nearly five degrees, while the nodes — that is, the intersections with the ecliptic — do not, however, remain in the same places, but anticipate themselves backward [retrograde], such that they complete a period at 3 minutes 11 seconds each day, and within nearly 19 years. But hitherto all the practitioners have held it persuaded in vain that the limits of the Moon's maximum latitude remain perpetually equal to themselves, and reach precisely five degrees — and this chiefly on the authority of Ptolemy, whom al-Battānī [Albategnius] followed, and after him the Alphonsines, in whom too

Copernicus, as often elsewhere, acquiesced too carelessly. Moreover, that the nodes themselves maintain an equal and regular motion has hitherto been believed in vain. For in neither of these do our observations — made over several past years with the greatest diligence — agree up to now with the traditions of the ancients, although long received. For not only [do we find] the limits of the maximum latitude of others [recte: 'differently'] —

Astronomia Danica — Pars Altera, Liber Primus Theoricorum

CAPVT VII (continued) — On the latitude of the Moon, and its true computation

[Tycho, continuing his account of the Moon's variable latitude limits:] ...we find the limits to be not only different from what Ptolemy [Ptolemy] laid down, but also unequal among themselves. At the new and full moons, indeed, they correspond very nearly to Ptolemy's value: there we have established by observation that the Moon can depart from the ecliptic by $4^{\circ} 58' 30''$. But at the quadratures this departure increases by about a third of a degree, so that it then amounts to $5^{\circ} 17' 30''$ — as a careful and often-repeated examination has taught us, conducted attentively about each limit, the northern as much as the southern, and in both cases at the tropical points, so that allowance was everywhere made not only for parallax but also for refraction in the lower position.

Now if Ptolemy investigated the Moon's maximum latitude near the quadratures — as is probable — while it was near the tropic of Cancer [□] and close to the zenith, then that same latitude ought to have been 20' smaller than it is now. For owing to roughly that many additional minutes in the obliquity of the ecliptic at that time — being greater then — he could have made up about the third of a degree by which the Moon's maximum latitude found at Ptolemy's quadratures is exceeded by our value; since in this practice he made use only of the northern limit, established at the tropic of Cancer.

Likewise in the variation of the nodes we detect a not inconsiderable inequality, which nevertheless at the new and full moons, and also at the quadratures, alters the mean motion imperceptibly. It is therefore the less surprising that this difference was not noticed by earlier astronomers, who observed the Moon chiefly around those positions. But at the intermediate positions it can grow on either side by up to two degrees, less one quarter, so that the Moon's latitude

itself can on this account be increased or diminished by nearly a sixth of a degree near the nodes — that is, when the place is otherwise carefully attended to for the mean position there.

[Longomontanus resumes:] Having set out these matters concerning the Moon's latitude from Tycho [Tycho Brahe], which rest on the most certain observations — as may be seen in the four examples sufficient for the purpose in his work — and which our own experience in the rest abundantly confirms as a most luminous witness, we shall now add here the hypothesis of the Moon's latitude. [Margin: *Liber 1, Progymnasmata, p. 114 and following.*] By it the inequality both of the nodes and of the inclination of the Moon's orbit with respect to the ecliptic is most correctly handled and preserved. It is plainly similar to the variation of the equinoxes and of the ecliptic assigned above, since here too the pole of the lunar orb is taken to be mobile. For it is not possible to attach these inequalities to the hypothesis of the Moon's longitude, or the reverse — as Jacob Christmann [Jacob Christmann] dreamed, when by chance the calculation turned out right for him in a single example. For the period of the place of the lunar apogee is plainly incommensurable with the complete revolution of the nodes backward beneath the ecliptic: the former occurs within about 9 years, the latter within 19.

If from all this the astronomers have still not understood how that reformer, while he claims this theory of the Moon for his own under a counterfeit title, has wasted his labor (ἐν τῷ πιθηκίζειν [Greek: en tōi pithēkizein, "in playing the ape" — i.e. aping his betters], as the proverb runs) — then [Margin: *See the treatment of Christmann on the Theory of the Moon, in the year of our Lord 1611, printed pp. 25, 29, etc.*] let them institute a comparison of his own [results] with the outcome of Christmann's procedure in the lunar motions, especially the comparison of [eclipse predictions] beyond the eclipses, and they will confirm the experience of which we speak. That Christmann had certainly deserved something rather harsh from us, both on account of this deed and because, ignorant of astronomy, he did not blush to instruct anew, with his new instruments and a new method of observing the times, those very men who had grown grey in nearly the final elaboration of this art — by a preposterous handing-down of trigonometry, namely "from the sky," and borrowing his methods of calculation in multiplying and dividing besides, that is, taking shortcuts from others (as is sufficiently plain from our *Arithmetica* too). Yet now I would rather think the man's madness should be spared — a man otherwise renowned for learning — seeing that even his [trifles], skimmed freely by anyone, deserve to be held in contempt like a dog's barking. But while Tycho lived he spared him aptly enough, since he judged the man unworthy that he should lose any leisure in refuting him. For the Moon

herself once spoke, of old, in the hieroglyphic at Uraniborg depicted on the wall, when, running freely through her own orb full by night, she replied to the dog's barking with contempt: *I care nothing for trifles* [*Nil moror nugas*]. But enough of these matters, which are anyhow sufficiently refuted by the distinguished Lord David Origanus [David Origanus] — so that I may now return to expounding the hypothesis of the Moon's latitude.

Hypothesis of the inequality of the Moon's latitude and of the nodes

Diagram. A geometric construction illustrating the lunar-latitude hypothesis. About the center **B** (the pole of the ecliptic) a great circle is described in the plane **M L N**; a second great circle **M H N** (the mean path of the Moon under the zodiac) and a small circle **D G F E** (carrying the pole of the Moon's true orbit) are drawn about the pole **A**. The outer circle is the principal one; the lunar pole travels around the small circle **D G F E** as the luminaries' mutual distance revolves. Labeled points and arcs:

- **L** (top): northern extremity of the meridional arc through the poles
- **I** (just below **L**): intersection point on arc **L I**
- **R, H, S** (small cluster below **I**): vertices of the little inner triangle (**H** between **R** and **S**)
- **K** (below the **R/H/S** cluster): point marking the foot of arc **L K**
- **B** (center of construction): pole of the ecliptic
- **D** (just below **B**): position of the lunar pole at conjunction and opposition (minimum inclination, $4^{\circ} 58\frac{1}{2}'$, measured by arc **L I**)
- **A** (below **D**): pole about which circles **M H N** and **D G F E** are described
- **G** (left of **A**) and **E** (right of **A**): the lateral positions on the small circle (mid-courses, where the nodal inequality is greatest)
- **F** (below **A**): position of the lunar pole at the quadratures (maximum inclination, $5^{\circ} 17\frac{1}{2}'$, measured by arc **L K**)
- **P** (bottom): southern extremity of the principal circle
- **O** □ (outer left, upper): point **O** on the principal circle, at Aries
- **N** □ (outer left, middle): node **N** (cauda draconis / tail), toward Taurus
- **V** □ (outer left, lower): node **V** (cauda draconis / tail), toward Pisces
- **X** □ (outer right, upper): node **X** (caput draconis / head), at Cancer
- **M** □ (outer right, middle): node **M** (caput draconis / head), at Cancer

- [?] □ (outer right, lower): a second Cancer-region marker; letter unresolved at scan-DPI floor [?]

The node-letters are fixed by the page's own text: N and V lie on the cauda draconis (tail) on the left side, while M and X lie on the caput draconis (head) on the right side.

[One outer label (lower right) remains unresolved at scan-DPI — see QA note.]

[Margin: *See Chapter 4 of this [book].*] [Margin: *See the demonstration in T.B. [Tycho Brahe], Liber 1, Progymnasmata, p. 115.*]

Let a great circle be described about **B**, the pole of the ecliptic, in the plane of this [figure] M L N. Then, an arc of a great circle L I being drawn, let the distance B A be set at $5^{\circ} 8'$ in the sphere — which measures the mean latitude or inclination of the Moon with respect to the ecliptic. And about the pole **A** let both the great circle M H N and the smallest circle D G F E be drawn: the former displays the mean path of the Moon under the zodiac, the latter displays the pole of the Moon's true orbit according to the measure of its trace and magnitude, and this for the revolution of the twofold distance of the luminaries from each other.

Thus when the said pole is at **D** — which happens at the conjunction and opposition of the luminaries — the inclination of the Moon to the ecliptic is least, namely $4^{\circ} 58\frac{1}{2}'$, as is shown by the arc L I. But when the same pole, having rolled round through **G**, comes to be at **F** — which happens at the quadratures of those same luminaries — the Moon's inclination is then greatest, the one measured by the arc L K, $5^{\circ} 17\frac{1}{2}'$. In these positions no inequality of the nodes is felt, on account of the same continuous arc L B D F. But in the whole rest of the revolution of the pole of the Moon's orbit there is always some [inequality], greatest in the mid-courses around **G** and **E** — additive on the one side, subtractive on the other, as is sufficiently understood from the hypothesis itself.

The method of extracting each inequality depends on the doctrine of spherical triangles, in this way: in the triangle B A G, of which two sides are always given — B A $5^{\circ} 8'$ and A G $9\frac{1}{2}'$ — together with the angle B A G, to be estimated from the revolution of the true pole of the Moon's path to the twofold true distance of the luminaries (as suits the conjunction and opposition), the side B G is therefore first given, which determines the angle of the Moon's maximum latitude; then the angle A B G, by which the inequality of the nodes is shown at the ecliptic M X at the head [of the Dragon], and N V at the tail of the Dragon on the ecliptic. For the angle H B S is equal to the angle A B G.

In acquiring the inequality of the latitude conveniently, however, we use proportional minutes [scrupula proportionalia], in place of the obliquity of the ecliptic interchanged in the foregoing hypothesis of the equinoxes. For whereas there the proportional minutes were to be deduced, multiplied into the excess B G &c. beyond B D for any fixed longitude from the equinox arranged by us, here the same minutes arise from the excesses [discovered] by Tycho and us, to be multiplied into the difference of maximum and minimum latitude previously acquired for each degree (after the manner of the Sun's declination in the doctrine of the sphere), so that here too the sought latitude of the Moon may emerge. This interchange of the minutes, besides being agreeable in nature for learners to grasp, will also be useful in that these same minutes will serve the common use in working out the lunar parallaxes and the latitudes of the planets (of which later). These things briefly suffice for the exposition of the theory of the Moon's latitude, as also for the construction of the tables subjoined below; after which the method of computing the latitude and the motion of the nodes will follow in some example from Tycho's *Progymnasmata*.

Table of the Prosthaphaereses of the Nodes

Tabula Prosthaphæresium Nodorum [Table of the Prosthaphaereses of the Nodes]

[Margin: *Liber 1, Progymnasmata, p. 111 and following.*]

The column header reads **ADDE** [ADD] at the top (for argument signs 0, 1, 2 = 6, 7, 8 in the doubled-signs scheme) and **SVBTR.** [SUBTRACT] at the foot (for the complementary signs 11, 10, 9 = 5, 4, 3). Each sign-block gives: the argument degree (G.), the Prosthaphaeresis of the Nodes (Prosth. Nod. — degrees, minutes, seconds), and the proportional minutes (Scrup. Pro. — minutes, seconds). The rightmost degree column (0–30, descending) serves the bottom/right reading.

A rg . (0 / 6)	P r os th . N o d. (1 / 7)					P r os th . N o d. (2 / 8)					A rg . (1 / 9 et c.)					
	G	M	S	M	S	G	M	S	M	S	G	M	S	M	S	
0	0	0	0	0	0	1	3	2	1	2	1	3	3	4	2	3
							3	8	5	2		0	2	5	0	0
1	0	3	5	0	1	1	3	1	1	1	1	2	3	4	1	2
			0				5	2	6	8		8	4	6	3	9
2	0	7	3	0	4	1	3	4	1	1	1	2	3	4	5	2
			9				6	7	7	5		6	0	7		8
3	0	1	2	0	9	1	3	1	1	1	1	2	2	4	5	2
		1	7				8	2	8	3		4	1	7	4	7
4	0	1	1	0	1	1	3	3	1	1	1	2	6	4	4	2
		5	4		6		9	1	9	1		2		8	3	6
5	0	1	0	0	2	1	4	4	2	1	1	1	4	4	3	2
		9			6		0	2	0	0		9	5	9	1	5
6	0	2	4	0	4	1	4	4	2	9	1	1	1	5	1	2
		2	6		1		1	5	1			7	8	0	9	4
7	0	2	2	0	5	1	4	4	2	9	1	1	4	5	4	2
		6	9		6		2	4	2			4	6	1		3
8	0	3	9	1	1	1	4	3	2	1	1	1	9	5	4	2
		0			3		3	8	3	0		2		1	7	2
9	0	3	4	1	3	1	4	2	2	1	1	9	2	5	2	2
		3	7		2		4	9	4	1			7	2	8	1
1	0	3	2	1	5	1	4	8	2	1	1	6	4	5	8	2
0		7	3		3		5		5	3			1	3		0
1	0	4	5	2	1	1	4	3	2	1	1	3	4	5	4	1
1		0	6		6		5	4	6	6			9	3	6	9

A rg . (0 / 6)	P r os th . N o d. (1 / 7)					P r os th . N o d. (2 / 8)					A rg . (1 / 9 et c.)					
	G	M	S	M	S	G	M	S	M	S	G	M	S	M	S	G
1 2	0	4	2	2	4	1	4	5	2	1	1	0	5	5	2	1
		4	6		1		5	0	7	9			1	4	3	8
1 3	0	4	5	3	8	1	4	5	2	2	0	5	5	5	5	1
		7	2				5	6	8	2		7	0	4	9	7
1 4	0	5	1	3	3	1	4	5	2	2	0	5	4	5	3	1
		1	4		8		5	9	9	5		4	7	5	3	6
1 5	0	5	3	4	1	1	4	0	3	2	0	5	4	5	6	1
		4	2		0		6		0	8		1	2	6		5
1 6	0	5	4	4	4	1	4	5	3	3	0	4	3	5	3	1
		7	7		3		5	3	1	0		8	2	6	6	4
1 7	1	0	5	5	1	1	4	3	3	3	0	4	1	5	3	1
			6		8		5	6	2	3		5	8	7		3
1 8	1	4	0	5	5	1	4	1	3	3	0	4	0	5	2	1
					4		5	3	3	5		2		7	8	2
1 9	1	6	5	6	3	1	4	4	3	3	0	3	4	5	5	1
			9		2		4	1	4	7		8	0	7	2	1
2 0	1	9	5	7	1	1	4	0	3	3	0	3	1	5	1	1
			3		2		4		5	9		5	8	8	4	0
2 1	1	1	4	7	5	1	4	1	3	4	0	3	5	5	3	9
		2	2		4		3	0	6	1		1	3	8	5	
2 2	1	1	2	8	3	1	4	1	3	4	0	2	2	5	5	8
		5	5		8		2	4	7	2		8	6	8	3	
2 3	1	1	2	9	2	1	4	1	3	4	0	2	5	5	9	7
		8			4		1	0	8	3		4	7	9		

A rg . (0 / 6)	P r os th . N o d. (1 / 7)					P r os th . N o d. (2 / 8)					A rg . (1 / 9 et c.)				
	G	M	S	M	S	G	M	S	M	S	G	M	S	M	S
2 4	1	2 0	3	1 0	1 3	1	3 9	5 9	3 9	4 3	0	2 1	2 6	5 9	2 2
2 5	1	2 2	5 8	1 1	2	1	3 8	4 2	4 0	4 1	0	1 7	5 4	5 9	3 3
2 6	1	2 5	1 6	1 1	5	1	3 7	1 8	4 1	3 8	0	1 4	2 1	5 9	4 3
2 7	1	2 7	2 8	1 2	4	1	3 5	4 6	4 2	3 5	0	1 0	4 6	5 9	5 0
2 8	1	2 9	3 4	1 3	3	1	3 4	8	4 3	3 1	0	7	1 1	5 9	5 5
2 9	1	3 1	3 4	1 4	2 7	1	3 2	2 3	4 4	2 6	0	3	3 6	5 9	5 8
3 0	1	3 3	2 8	1 5	2 2	1	3 0	3 2	4 5	2 0	0	0	0	6 0	0 0
	(s ig n s 1 1 / 5)					(s ig n s 1 0 / 4)					(s ig n s 9 / 3)				

Bottom label: **SVBTR.** [SUBTRACT]. Foot signature: Z 4.

Parsing notes: This is a standard doubled-argument prosthaphaeresis table. The three sign-blocks head with 0/6, 1/7, 2/8 (top, ADD) and foot with 11/5, 10/4, 9/3 (bottom, SUBTR.); the left degree column (0–30) reads downward for the top headings, the right degree column (30–0) reads upward for the foot headings. The "Prosth. Nod." sub-columns are degrees/minutes/seconds; the "Scrup. Pro." sub-columns are minutes/seconds. p. 287 row 11, "Prosth. Nod." seconds cell rendered as "40 2" but is faint — read as 0° 40' 2" [?]; the smooth progression (0°37'23" → 0°40'2" → 0°44'26") supports 0°40'2".

Table of the Latitude of the Moon

Tabula Latitudinis Lunæ [Table of the Latitude of the Moon]

Argument-of-latitude table. Each of three sign-blocks gives, for the argument degree (0–30), the Latitude (Latitudo — degrees, minutes, seconds) and the Excess (Excessus — minutes, seconds). Top headings (read down, left degree column): 0 Bor. / 6 Mer., 1 Bor. / 7 Mer., 2 Bor. / 8 Mer. Bottom headings (read up, right degree column): 11 Mer. / 5 Bor., 10 Mer. / 4 Bor., 9 Mer. / 3 Bor.

A rg .	L at it u d o (0 B o r / 6 M er) G M S			L at it u d o (1 B o r / 7 M er) G M S			L at it u d o (2 B o r / 8 M er) G M S			E x c e s s u s M S			E x c e s s u s M S			A r g . (f o t) M S		
	0 0 0			2 2 6			9 2 4			1 2 1			2 1 2			3 2 3		
	0 5 1			2 3 3			4 4 4			2 5 1			5 1 3			2 3 2		
	3 0 3			6 3 6			5 5 5			0 9 6			6 5 9			9 5 9		
	0 1 2			2 3 3			4 2 4			2 2 1			4 1 4			2 4 2		
	0 5 0			8 8 0			3 8 6			5 8 5			8 5 8					

A rg .	L at it u d o (0 B o r/ 6 M er) G					L at it u d o (1 B o r/ 7 M er) G					L at it u d o (2 B o r/ 8 M er) G					E x ce ss u s M		E x ce ss u s M		A rg . (f o ot)	
	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S			
3	0	1	3	0	5	2	4	2	1	1	4	2	5	1	5	2	7	6			
4	0	2	4	1	1	2	4	4	1	3	4	2	1	1	4	2	6	7			
5	0	2	5	1	3	2	5	4	1	5	4	3	2	1	1	2	5	8			
6	0	3	9	1	5	2	5	1	1	7	4	3	3	1	2	2	4	9			
7	0	3	1	2	1	2	5	3	1	2	4	3	4	1	2	2	3	0			
8	0	4	2	2	3	3	3	3	1	3	4	3	4	1	3	2	2	1			
9	0	4	3	2	5	3	7	4	1	5	4	3	3	1	4	2	1	0			
10	0	5	4	3	1	3	1	4	1	1	4	4	2	1	5	2	0	9			
11	0	5	5	3	3	3	1	4	1	2	4	4	1	1	5	1	9	8			
12	1	1	5	3	5	3	1	3	1	4	4	4	5	1	5	1	8	7			

A rg .	L at it u d o (0 B o r/ 6 M er) G M S					L at it u d o (1 B o r/ 7 M er) G M S					L at it u d o (2 B o r/ 8 M er) G M S					E x ce ss u s M S		E x ce ss u s M S		A rg . (f o ot)	
	G	M	S	M	S	G	M	S	M	S	G	M	S	M	S	M	S	M	S	M	S
1 3	1	7	4	4	1 5	3	2 3	2 6	1 2	5 6	4	4 5	2 3	1 8	1 2	1	1	1	1	1	1
1 4	1	1 2	8	4	3 4	3	2 7	1 3	1 3	1 0	4	4 6	5 2	1 8	1 8	1	1	1	1	1	1
1 5	1	1 7	1 0	4	5 3	3	3 0	5 6	1 3	2 4	4	4 8	1 8	1 8	1 8	1	2 3	1	2	1	1
1 6	1	2 2	1 1	5	1 3	3	3 4	3 5	1 3	3 8	4	4 9	3 5	1 8	1 7	1	2 7	1	2	1	1
1 7	1	2 7	1 0	5	3 2	3	3 8	1 0	1 3	5 2	4	5 0	4 9	1 8	1 1	1	3 1	1	3	1	1
1 8	1	3 2	8	5	5 1	3	4 1	4 2	1 4	5	4	5 1	5 8	1 8	1 4	1	3 4	1	3	1	1
1 9	1	3 7	4	6	1 0	3	4 5	7	1 4	1 8	4	5 3	0	1 8	1 8	1	3 8	1	3	1	1
2 0	1	4 1	5 8	6	2 9	3	4 8	3 0	1 4	3 1	4	5 3	5 7	1 8	1 2	1	4 2	1	4	1	1
2 1	1	4 6	5 1	6	4 7	3	5 1	5 2	1 4	4 4	4	5 4	4 9	1 8	1 5	1	4 5	1	4	9	9
2 2	1	5 1	4 1	7	6	3	5 5	9	1 4	5 7	4	5 5	3 6	1 8	1 8	1	4 8	1	4	8	8

A rg .	L at it u d o (0 B o r/ 6 M er)					L at it u d o (1 B o r/ 7 M er)					L at it u d o (2 B o r/ 8 M er)					A rg . (f o ot)
	G	M	S	M	S	G	M	S	M	S	G	M	S	M	S	
2 3	1	5	3	7	2	3	5	1	1	9	4	5	1	1	5	7
		6	0		4		8	9	5			6	7	8	1	
2 4	2	1	1	7	4	4	1	2	1	2	4	5	5	1	5	6
			7		2			3	5	1		6	2	8	3	
2 5	2	6	1	8	0	4	4	2	1	3	4	5	2	1	5	5
								4	5	2		7	2	8	5	
2 6	2	1	4	8	1	4	7	2	1	4	4	5	4	1	5	4
		0	3		8			1	5	3		7	6	8	6	
2 7	2	1	2	8	3	4	1	1	1	5	4	5	5	1	5	3
		5	3		5		0	5	5	4		8		8	7	
2 8	2	2	0	8	5	4	1	5	1	5	4	5	1	1	5	2
		0			3		3		6			8	8	8	8	
2 9	2	2	3	9	1	4	1	4	1	1	4	5	2	1	5	1
		4	4		1		5	7	6	5		8	6	8	9	
3 0	2	2	6	9	2	4	1	2	1	2	4	5	3	1	0	0
		9			8		8	6	6	5		8	0	9		
	(1 1 M er / 5					(1 0 M er / 4					(9 M er / 3 B					

A rg .	L at it u d o (0 B o r / 6 M er) G M S					L at it u d o (1 B o r / 7 M er) G M S					L at it u d o (2 B o r / 8 M er) G M S				
	E x ce ss u s M S					E x ce ss u s M S					E x ce ss u s M S				
	B					B					o				
	o					o					r)				
	r)					r)									

Parsing notes: A standard argument-of-latitude table. Three sign-blocks: top headings 0 Bor./6 Mer., 1 Bor./7 Mer., 2 Bor./8 Mer. (left degree column descending); foot headings 11 Mer./5 Bor., 10 Mer./4 Bor., 9 Mer./3 Bor. (right degree column ascending). "Bor." = boreal (northern), "Mer." = meridional (southern). Each block: Latitudo (G./M./S.) and Excessus (M./S.). The seconds column of the first block is mostly absent in the original layout for the M./S. split of Excessus, but the printed pattern is degree-latitude then excess; transcribed per the cell grid as shown. Several cells in the photographed page have ink-faded seconds digits resolved against the smooth column progression. p. 288 first-block degree 24 latitude "2 1 17" rendered with a faint leading "2" — read 2° 1' 17" [?] (fits progression 1°56'30" → 2°1'17" → 2°6'1"). Third-block (2 Bor/8 Mer) latitude climbs smoothly toward the maximum 4°58'30" at argument 30, confirming the latitude limit established in the text.

Astronomia Danica — Pars Altera, Liber Primus Theoricorum, Caput VII (continued) and Caput VIII

[p. 273] Cap. 7. LIBER PRIMVS.

On deriving the Moon's true latitude from the tables.

For the proposed time, the same time for which the Moon's longitude has been computed, have ready the simple motion of latitude, the true distance of the luminaries, and the Moon's absolute prosthaphaeresis [prosthaphaeresis Lunae]. According to what the column headings indicate, add this prosthaphaeresis to the mean motion of latitude, or subtract it from it, in order to obtain the true motion. Next, using the true distance of the Moon and Sun, take from the earlier table the proportional minutes [scrupula proportionalia] that are to be kept aside, together with the prosthaphaeresis of the nodes [prosthaphaeresis nodorum]; and add this to, or subtract it from, the true motion of latitude — as the column headings require, whether additive or subtractive — so that the motion becomes fully equated. With this equated value, draw from the latter table the Moon's latitude together with its excess [excessus]. The proportional minutes, multiplied into the excess, yield the proportional part that is always to be added to the latitude, so that the true latitude appropriate to this inclination of the Moon's orbit and of the ecliptic comes out. Whether it is boreal or austral, the headings affixed at the top and the foot make plainly clear.

Example I.

In the year 1596, on the 4th of January, at 11 hours 43 minutes after midday, the apparent latitude of the Moon was observed as $4^{\circ} 31'$ north. To this a parallax of $28' 30''$ was added, establishing the Moon's true latitude as $4^{\circ} 59' 30''$. Now, for the proposed time, when equated, the mean motion of latitude is gathered as 3 signs $4^{\circ} 59' 6''$. From this the Moon's absolute prosthaphaeresis, $5^{\circ} 4' 19''$, when subtracted, leaves the true motion of latitude measured from the ascending node [♌] as 2 signs $29^{\circ} 54' 47''$. Next, with the true ecliptic distance of the luminaries — which is 5 signs $23^{\circ} 38'$ — I enter the earlier table and find proportional minutes of $0' 46''$ to be kept aside, and the prosthaphaeresis of the nodes, $24' 8''$, to be subtracted from the true motion of latitude in order to equate it; and the result will be 2 signs $29^{\circ} 30' 39''$ [recte for "69 gr." as printed]. With this I enter the latitude table, seeking the latitude and the excess, with the due correction made

by the proportional part; and I find a latitude of $4^{\circ} 58' 28''$, and an excess of $19' 0''$. This excess, multiplied logistically into the $0' 46''$ of proportional minutes kept aside, yields $15''$ to be added to the latitude — as was said, always to be added — so that the absolute latitude comes out as $4^{\circ} 58' 43''$ north. This differs from the observed value by only three-quarters of a minute, which is of no consequence.

On the method of investigating the nodes.

Now that the intricate computation of the Moon's latitude has been explained, it is worth the effort to inquire also into the nodes, where the Moon — being then entirely without latitude — crosses the ecliptic. [Margin: Lib. 1. Prog. pag. 116.] For the boreal node, then, to which the other is opposite, have ready at the given time these simple motions: the simple motion of the Sun from the equinox, the simple motion of the Moon from the Sun, and the simple motion of the Moon's latitude. Once these have been found, add the simple motion of the Sun to the simple motion of the Moon, so as to compose the simple longitude of the Moon from the vernal equinox. From this sum subtract again the simple motion of the Moon's latitude, and thus you will obtain the mean anticipation of the nodes — very nearly such as those who follow Ptolemy [Ptolemaeus], Alfonso [Alphonsus], and Copernicus commonly employ, persuaded (vainly) that no inequality underlies it here. But in order that this be limited in the proper manner, take out from the table of the prosthaphaeresis of the nodes the equation corresponding to the true distance of the luminaries, and add it to, or subtract it from, the mean motion of the nodes in the manner opposite to what the column headings themselves suggest; and thus you will obtain the true place of the node that carries the Moon toward the north — the node they call the head of the dragon [caput draconis]. To this, as was said, the other node, which they name the tail of the dragon [cauda draconis], is diametrically opposite. Both limits of the maximum latitudes, moreover, are removed from these by a quarter of a circle on either side, should one wish to ascertain them together.

[p. 274] THEORICORVM Cap. 7.

[Margin: Prog. 1. p. 3.]

Example II.

In the third example, in which Tycho [Tycho Brahe] engaged in investigating the Moon's latitude, the simple motion of the Sun is 11 signs $1^{\circ} 51' 1''$. To this is added the Moon's mean motion from the Sun, 4 signs $2^{\circ} 20' 59''$, establishing the simple motion of the Moon from the equinox as 3 signs $4^{\circ} 12' 0''$. From this the simple

motion of latitude, $54' 23''$, is taken away, leaving the mean anticipation of the nodes as 3 signs $3^{\circ} 17' 37''$. Then, with the distance of the luminaries 4 signs $4^{\circ} 13' 11''$, the prosthaphaeresis of the nodes is derived as $1^{\circ} 37' 36''$, here to be added to the mean motion of the nodes, so that the true place of the node is established as 3 signs $4^{\circ} 55' 13''$. Alternatively, if you have collected both the Moon's longitude and its latitude, subtract the equated motion of latitude — the one you used in extracting the latitude — from the Moon's true place, and the true place of the node will remain. Thus, in the same example, the Moon's true place is 3 signs $7^{\circ} 48' 45''$; from this, the equated motion of latitude, $2^{\circ} 53' 32''$, being taken away, there remains the true place of the ascending node $[\Omega]$, 3 signs $4^{\circ} 55' 13''$. This exceeds the Alfonsine and Copernican calculation by one and a half degrees, and, as the Prutenic Tables [Tabulae Prutenicae] give it, by 11 minutes more still — a difference which, in the very neighborhood of the Moon's nodes, is truly intolerable.

On the reduction of the Moon's place to the ecliptic.

Since the Moon's motion does not proceed beneath the ecliptic, but in some other circle inclined to it; and since the calculation yields that motion which takes place in the Moon's own orbit, and not the motion which instruments, through observation, present with respect to the ecliptic and its poles: therefore some adjustment is needed here, which is most easily carried out by means of an adjoined little table, in this manner.

[p. 275] Cap. 7. LIBER PRIMVS.

The construction of this table of reduction is as follows.

Diagram. A spherical triangle drawn as a narrow curvilinear figure. A long nearly-horizontal base runs from A (far left) to the right, with a short arc rising from the base; point C lies on the base, and B is raised above C. The vertices and points labeled are:

- A (far left vertex): the node, where the angle is given as $5^{\circ} 8'$.
- B (upper point, above C): apex on the Moon's orbit.
- C (on the base, below B): foot of the right angle on the ecliptic.
- D (upper right): terminus of the Moon's-orbit arc.
- E (lower right, on the base line): terminus of the ecliptic line.

In the spherical triangle ABC, since the angle at A is given as $5^{\circ} 8'$ — the mean latitude of the Moon, between the maximum and the minimum — and likewise the side AB, the Moon's true distance from the node, is given, with the angle at C

being a right angle: therefore the side AC on the ecliptic is given. This side AC is accordingly the difference between AB and AC, which yields the reduction that appears in the table.

Its use, after Tycho, is as follows.

When I wish to reduce the Moon's place, calculated from our tables in its own orbit, to the ecliptic, I enter the adjoined table with the true and equated motion of latitude, seeking in the usual way the sign at the top or the foot, and the degree by descending or ascending; and the minutes together with the seconds found in the common angle I add to, or subtract from, the place of the Moon found by calculation, so that I may obtain its true position with respect to the ecliptic. I do the contrary with these headings A and S [Add. and Subtr.] when the place of the Moon observed by instruments is to be reduced from the ecliptic and its poles to its own proper orbit. By this concise method, then, you will nowhere err from the truth by more than half a minute. But if you wish to pursue the highest precision, then in place of the angle $5^{\circ} 8'$ — which in the construction of this table we have employed as though invariable — use the true latitude found for the given time; and, according to the doctrine of spherical triangles, seek either the base from both sides, or the side of longitude from the base and the side of latitude, as the case requires.

Little table of the reduction of the Moon from its own orbit to the ecliptic, and vice versa.

Verus motus	1 / 7 (Subtr.) —			Verus motus
	0 / 6 — M. S.	M. S.	2 / 8 — M. S.	
0	0 0	6 6	6 5	30
1	0 15	6 12	5 57	29
2	0 30	6 18	5 48	28
3	0 45	6 24	5 39	27
4	0 59	6 29	5 30	26
5	1 13	6 35	5 21	25
6	1 27	6 40	5 12	24
7	1 42	6 44	5 1	23
8	1 56	6 47	4 51	22
9	2 10	6 51	4 40	21
10	2 24	6 54	4 29	20

Verus motus	0 / 6 — M. S.	1 / 7 (Subtr.) — M. S.	2 / 8 — M. S.	Verus motus
11	2 38	6 56	4 18	19
12	2 52	6 57	4 7	18
13	3 6	6 58	3 55	17
14	3 19	6 59	3 42	16
15	3 32	7 0	3 31	15
16	3 43	6 59	3 18	14
17	3 56	6 58	3 5	13
18	4 8	6 57	2 51	12
19	4 19	6 56	2 38	11
20	4 30	6 54	2 23	10
21	4 41	6 51	2 9	9
22	4 52	6 47	1 55	8
23	5 2	6 44	1 41	7
24	5 13	6 40	1 26	6
25	5 22	6 35	1 12	5
26	5 31	6 28	0 58	4
27	5 40	6 23	0 45	3
28	5 49	6 17	0 30	2
29	5 58	6 11	0 15	1
30	6 6	6 5	0 0	0
Latitudo	11 / 5	10 / 4 (Adde)	9 / 3	Latitudo

Parsing notes: The table is read top-down with the left "Verus motus" (true motion) column running 0–30 and the upper sign-headings 0/6, 1/7, 2/8; and bottom-up with the right "Verus motus" column running 30–0 and the lower sign-headings 11/5, 10/4, 9/3. The middle column block carries "Subtr." (subtractive) at the top and "Adde" (additive) at the foot, indicating the operation reverses depending on direction of entry. Each cell gives minutes and seconds (M. S.). The middle-column maximum 7' 0" occurs at argument 15, consistent with the reduction peaking a quarter-way along. The 22-row entry of the third block, 1' 55", is printed faintly but reads consistently with the smooth progression.

CAPVT VIII.

On the parallaxes of the Sun and Moon, both with respect to the circle of altitude and to the zodiac. Likewise on the refraction of the Sun, the Moon, and the stars.

[Margin: Lib. 2. Sphæ. cap. 10.]

So far we have treated the true motions of the stars and luminaries from their reconstructed hypotheses — that is, insofar as they regard the center of the Earth. Now we shall briefly append what deviation in these strikes the eye: partly by reason of the magnitude and bulging-out of the Earth, whence the parallaxes arise; partly by reason of the vapor diffused about it, on which the consideration of refraction depends. For although concerning both of these we taught generally in the doctrine of the sphere, insofar as they depend on observations, I judge that they must nonetheless be treated here too in particular. The variation in the Sun and Moon must be bound down in tables grounded on geometrical demonstrations — and this on account of the doctrine of eclipses, which follows shortly — now that the fixed distances of each luminary from the Earth, expressed in the Earth's semidiameters, have been given above. For since the inference of these distances stands in inverse proportion to the parallaxes in every phenomenon — so that a smaller distance of a body from the Earth causes greater parallaxes in the vertical circle, and conversely — therefore the first demonstration concerning these, for each luminary, will be carried out, as a beginning, in this manner.

Diagram. A quarter-circle (quadrant) construction illustrating parallax. From the center A (lower left) two lines are drawn meeting at a right angle at A: AD vertical (rising up the left side) and AF horizontal (running along the bottom to the right). Three concentric quadrant arcs are drawn from A: the largest, DF, the quadrant of the Sun's circle; within it, CG, the quadrant of the Moon's circle; and innermost, BK, the quadrant of the terrestrial globe. A line AHE is drawn from the Earth's center A, indicating the Sun at E and the Moon at H according to their true established motions. From the Earth's surface at B, lines are drawn to each luminary: BE to the Sun and BH to the Moon. Labeled points:

- A (lower-left vertex): center of the Earth.
- D (top of vertical line): terminus of the Sun's quadrant, vertical direction.

- F (right end of horizontal line): terminus of the Sun's quadrant, horizontal direction.
- C (on the vertical, below D): on the Moon's quadrant arc.
- G (on the horizontal, between K and F): on the Moon's quadrant arc.
- B (on the vertical, low, just above A): point on the Earth's surface (terrestrial-globe quadrant).
- K (on the horizontal, between A and G): on the terrestrial-globe quadrant.
- E (upper, on line AHE): the Sun in its true position.
- H (interior, on line AHE between B and the arcs): the Moon in its true position.

Let two lines be drawn from A, the center of the Earth, meeting at a right angle at A: AD and AF, of which the former is vertical, the latter horizontal. Then let a quadrant of a circle, DF, be described from the same A — a quadrant, I say, of the Sun's own circle; next that of the Moon, CG; and finally that of the terrestrial globe, BK. From here let the line AHE be drawn from the Earth's center A, marking the Sun at E and the Moon at H according to their true established motions. Then, when lines are likewise drawn to each luminary from the Earth's surface at B — namely BE to the Sun and BH to the Moon — the angle of the Sun's parallax will be BEA, and that of the Moon BHA. The proportionate magnitude of these depends on the proportion of the Earth's semidiameter AB to the lines AH and AE; for when this semidiameter vanishes — as in comparison with the semidiameter of the orbit of the fixed stars — every parallax angle too is exhausted.

The method of deriving the parallaxes in the circle of altitude — or rather, of constructing a table of the parallaxes of the Sun and Moon from triangles — is grasped by the following demonstration. In the plane triangle AEB, since two sides are given: AB, the Earth's semidiameter, as one part of those quantities of which AE, the Sun's distance from the Earth, defined by us as a mean of 1288 [semidiameters], is also one; [Margin: Cap. 6. lib. hujus.] together with the included angle BAE, which is measured by the Sun's true distance from the vertex DE — that is, the complement of its distance from the horizon — in the vertical circle FE: therefore the parallactic angle AEB also emerges. In exactly the same way, in the other phenomena, as with the Moon — its elongation AH being given from the hypothesis, in defined semidiameters of the Earth from the center, together with AB and the angle comprised, by hypothesis, in the vertical circle — the sought parallax angle AHB is investigated. For the rest, even though, according to the Sun's hypothesis, the difference from the mean to the maximum

and minimum is found to be 46 semidiameters of the Earth: nevertheless, because, on account of its more remote distance from the Earth, this inequality from the mean nowhere amounts to more than 6 seconds, therefore, with the mean elongation of the Sun fixed above at 1288 semidiameters...

In the case of the Moon, whose mean distance from the Earth I had set above at 56 whole Earth-radii — a figure perhaps the best established for me, drawn from many and most accurate observations — I believe we will proceed most correctly if we follow Tycho Brahe's earlier calculation of the Moon's parallaxes in the circle of altitude over a span of 4 whole diameters in both directions; that is, from 52 semidiameters at the nearest distance to a maximum of 60 semidiameters at the prior limit. From this it becomes easy to compute the Moon's individual distances from the Earth, as the requirements of the hypothesis demand — first in semidiameters and minutes, then, once these competent parallaxes have been supplied, from the values that follow. This is especially helpful for those who find it tedious to hunt down the Moon's distance from the Earth out of the hypothesis itself, by the resolution of a single triangle and reduction to semidiameters.

[Margin: See Tycho's *Progymnasmata* p. 123]

Now it is established from both parts of our lunar hypothesis that the limits of the quadratures lie more widely, both below and above, than those that occur at conjunction and opposition. Therefore, with the Moon's mean distance from the Earth set, according to repeated experience, at 56 Earth-radii, the remaining distances are given in this synopsis. For the Moon's distance from the Earth will be:

	Sign	Gr.	Mi.	
Maximum	♌ & ♍	57	38	Earth-radii
at	☐	60	4	
Minimum	♌ & ♍	54	23	
at	☐	51	57	

Therefore the difference between the first and second limit — that is, at quadrature [☐], and likewise at conjunction and opposition [♌ & ♍] at apogee and perigee — is 2 gr. 26 mi. Further, between the second and third — that is, the new or full Moon at apogee and perigee — the difference is 3 gr. 15 mi. of semidiameters. Finally, the greatest difference is 8 gr. 7 mi. of semidiameters.

Although I recall deriving a somewhat smaller value than this from the Tychonic observations — as Tycho himself in some places records — the difference is nevertheless not so great as to undermine our hypothesis; rather, it will be able to confirm the truth of that hypothesis far beyond the Copernican, and most of all beyond the Ptolemaic.

A general method of representing the Moon's distance from the Earth, in Earth-radii and parts thereof, for each of its positions according to the hypothesis.

Since the proportional minutes [scrupula proportionalia] attached to the earlier table of the Moon's latitude scarcely ever differ perceptibly from those to be applied on both sides in the present business, the procedure is as follows. First the Moon's true distance from the Sun will be given — what we above have everywhere called its position on the ecliptic. Then, by means of the same, the appropriate proportional minutes are taken, by which the difference between the first and second limit (or the third and fourth) — which is 2 gr. 26 min. of semidiameters — is always multiplied. There emerges a proportional part, which is continually to be added to the distance of the new or full Moon at apogee, namely 57 gr. 38 m. of semidiameters, and subtracted from that at perigee, namely 54 gr. 23 min. In this way the difference between these is sought.

Furthermore, the Moon's anomaly, equated by the ecliptic prosthaphaeresis, will be ready at hand. This is first reduced by a semicircle, then the remainder is halved, and with that residue we enter the same earlier lunar table, here too seeking the appropriate proportional minutes. These, multiplied into the difference previously found, yield the proportional part, which is always to be added to the Moon's least distance from the Earth as previously equated. Thus the Moon's true elongation from the Earth is conveniently obtained in Earth-radii and the minutes thereof.

Once the Moon's distance from the Earth has been found in Earth-radii by this third method, it is very easy to apply the appropriate parallaxes in the circle of altitude to any distance from the horizon. For the table contains the same parallaxes for each whole diameter from the Earth and each degree from the horizon; at least, due correction is thus made for the adhering proportional minutes. Everything, however, will be made clear by the following example.

Example.

Let the Moon's parallax in the circle of altitude be sought for the year 1608, the 12th day of February, hour 8, minute 43 — when, in the second example of the preceding section *on longitude*, from careful observation we reported the Moon conjoined by its upper horn with Aldebaran.

Now the data were then as follows:

	Sign	gr.	Mi.	Sec.
The Sun's distance from the Moon was	2	28½		
Equated anomaly	8	23	2	
Moon's true altitude above the horizon, roughly		39¼		
Proportion al minutes corresponding to the distance of the luminaries			59	56

This, multiplied into the difference (namely 2 gr. 26 min. of semidiameters), gives 2 gr. 26 mi. (nearly).

	Gr.	Mi.	
When this is added to the second limit, namely 57 gr. 38 of semidiameters, it [the	60	4	Earth-radii

	Gr.	Mi.	
maximum]			
becomes			
And when	51	57	
subtracted			
from the third			
limit, the			
Moon's			
distance from			
the Earth, 54			
gr. 23, here it			
[the minimum]			
becomes			
Therefore the	8	7	Earth-radii
difference			

[Then, for the proportional-minute correction:]

	Sig.	Gr.	Mi.	
Equated	8	23	2	
anomaly				
Semicircle	6	0	0	
to be				
subtracted				
Remainder	2	23	2	
Half	1	11	31	
To which			26	50 Sec.
correspond				
proportion				
al minutes				
Multiplied		8	7	Earth-radii
by the				
difference				
There		3	38	
result, to be				
added in				
Earth-radii				

	Sig.	Gr.	Mi.
Fourth limit, corrected		51	57
Therefore there are made up, in Earth-radii		55	35

[Margin: From the table of parallaxes following.]

These [values] establish the Moon's true distance from the Earth in these semidiameters, etc. To this distance of the Moon from the Earth, and to its altitude above the horizon — given as 39¼ gr. — there corresponds the Moon's parallax in the circle of altitude of 48 min. 7 sec. And following this example, all the rest is to be carried out generally outside a solar eclipse. In an eclipse, however, nothing is needed except the proportional minutes produced through the equated lunar anomaly. When these have been multiplied into the difference of the second and third limit (which is 3 gr. 15 min.), they yield the proportional part always to be added to the third limit, 54 gr. 23 min. This done, the parallax in the circle of altitude is likewise sought from the following table by means of the true distance of the Moon from the Earth thus found. We will add an example below in connection with a solar eclipse. For now we will append the table of parallaxes of the Sun and Moon in the circle of altitude; and after it we will briefly demonstrate the method by which these same parallaxes come to be distinguished, lengthwise and breadthwise, through the ecliptic.

Table of parallaxes of the Sun and Moon in the circle of altitude.

The table is read as follows. The leftmost column **G.** gives the altitude in degrees (0–30 on this first page). The next column gives the **Sun's mean parallax in the circle of altitude** (M. S. = minutes, seconds). The wide central block gives the **Moon's parallaxes in the circle of altitude from the horizon**, tabulated for each whole Earth-radius distance from 52 through 60 (column headers 52–60), each cell in M. S. (minutes, seconds). The rightmost columns repeat the altitude **G.** and give the **proportional minutes of longitude and latitude** (M. S.).

G.	Su n M. S.	52 (M. S.)	53 (M. S.)	54 (M. S.)	55 (M. S.)	56 (M. S.)	57 (M. S.)	58 (M. S.)	59 (M. S.)	60 (M. S.)	G.	Scr up. M. S.
0	2 40	66 6	64 51	63 39	62 30	61 23	60 20	59 17	58 16	57 18	0	0 0
1	2 40	66 6	64 51	63 39	62 29	61 23	60 19	59 17	58 16	57 18	1	0 3
2	2 40	66 5	64 50	63 38	62 28	61 22	60 18	59 16	58 15	57 17	2	0 5
3	2 40	66 4	64 50	63 38	62 28	61 21	60 17	59 15	58 14	57 16	3	0 8
4	2 39	66 1	64 47	63 35	62 25	61 18	60 15	59 12	58 11	57 13	4	0 11
5	2 39	65 57[?]	64 43	63 31	62 21	61 14[?]	60 11	59 8	58 7	57 9	5	0 13
6	2 39	65 52	64 38	63 26	62 16	61 10	60 7	59 3	58 3	57 5	6	0 15
7	2 38	65 46	64 32	63 20	62 10	61 4	60 1	58 57	57 58	57 0	7	0 18
8	2 38	65 38	64 24	63 12	62 3	60 57	59 54	58 50	57 51	56 53	8	0 21
9	2 37	65 29	64 17	63 3	61 54	60 48	59 46	58 42	57 43	56 44	9	0 23
10	2 37	65 19	64 5	62 53	61 45	60 39	59 37	58 33	57 34	56 35	10	0 25
11	2 36	65 8	63 54	62 42	61 34	60 28	59 27	58 23	57 24	56 25	11	0 27
12	2 36	64 55	63 41	62 30	61 22	60 16	59 16	58 12	57 13	56 14	12	0 28
13	2 35	64 41	63 28	62 17	61 9	60 3	59 3	57 59	57 1	56 2	13	0 30
14	2 35[?]	64 26	63 14	62 3	60 55	59 49	58 48	57 45	56 47	55 49	14	0 31

G.	Su n M. S.	52 (M. S.)	53 (M. S.)	54 (M. S.)	55 (M. S.)	56 (M. S.)	57 (M. S.)	58 (M. S.)	59 (M. S.)	60 (M. S.)	G.	Scr up. M. S.
15	2 34	64 10	62 57	61 47	60 39	59 34	58 32	57 30	56 32	55 35	15	0 32
16	2 34	63 53	62 40	61 31	60 22	59 18	58 16	57 14	56 16	55 20	16	0 33
17	2 33	63 35	62 22	61 13	60 4	59 1	57 59	56 57	56 0	55 4	17	0 33
18	2 32	63 15	62 3	60 54	59 45	58 42	57 40	56 40	55 43	54 47	18	0 32
19	2 31	62 54	61 43	60 34	59 27	58 22	57 21	56 21	55 24	54 29	19	0 32
20	2 31[?]	62 32	61 21	60 12	59 6	58 1	57 1	56 1	55 4[?]	54 10	20	0 31
21	2 30	62 8	60 58	59 49	58 44	57 40	56 40	55 40	54 44	53 50	21	0 30
22	2 29	61 44	60 34	59 26	58 21	57 18	56 18	55 18	54 22	53 28	22	0 28
23	2 28	61 19	60 9	59 2	57 57	56 54	55 54	54 54	53 59	53 5	23	0 26
24	2 27	60 52	59 42	58 36	57 31	56 29	55 29	54 29	53 34	52 40	24	0 23
25	2 26	60 24	59 15	58 9	57 5	56 3	55 4	54 4	53 9	52 15	25	0 21
26	2 25	59 55	58 47	57 41	56 38	55 36	54 38	53 39	52 44	51 50	26	0 18
27	2 24	59 25	58 17	57 12	56 10	55 8	54 11	53 13	52 18	51 25	27	0 15
28	2 23	58 54	57 46[?]	56 42	55 40	54 39	53 42	52 45	51 51[?]	50 59	28	0 11
29	2 21	58 22	57 14	56 11	55 9	54 9	53 12	52 16	51 23	50 32	29	0 7

	Su											Scr
	n	52	53	54	55	56	57	58	59	60		up.
	M.	(M.	(M.	(M.	(M.	(M.	(M.	(M.	(M.	(M.		M.
G.	S.	S.)	S.)	S.)	S.)	S.)	S.)	S.)	S.)	S.)	G.	S.
30	2	57	56	55	54	53	52	51	50	50	30	0 0
	19	48	42	39	37	38	42	47	54	5		

[Margin / column label: Proportional minutes of longitude and latitude.]

Parsing notes: This table is citation-grade and very dense; the Sun's mean parallax column is monotonically decreasing ($2'40'' \rightarrow 2'19''$), and each Moon-distance column should decrease monotonically down its column. A few cells are flagged [?] where ink-bleed or the double-scan garble made the seconds digit ambiguous: row 5 / col 52 (read "57", but partly obscured), row 5 / col 56 ("14"), row 14 / Sun ("35"), row 20 / Sun ("31") and row 20 / col 59 ("4"), row 28 / col 53 ("46") and row 28 / col 59 ("51"). Cross-checking against monotone progression, these readings are consistent with their neighbors.

Remainder of the table of parallaxes of the Sun and Moon.

This page continues the same table for altitudes (G.) 30 through 60. Column structure is identical to the previous page: altitude **G.**, the **Sun's parallax in the circle of altitude** (M.S.), then the Moon's parallaxes for whole Earth-radius distances **52–60** (each M.S.), then a repeat of **G.** and the **proportional minutes of longitude and latitude** (M.S.). (Row 30 repeats from the prior page as the overlap row.)

	Su											Scr
	n	52	53	54	55	56	57	58	59	60		up.
	M.	(M.	(M.	(M.	(M.	(M.	(M.	(M.	(M.	(M.		M.
G.	S.	S.)	S.)	S.)	S.)	S.)	S.)	S.)	S.)	S.)	G.	S.
30	2	57	56	55	54	53	52	51	50	50	30	0 0
	19	48	42	39	37	38	42	47	54	5		
31	2	57	56	55	54	53	52	51	50	49	30[	0
	17	13	9	16	4	6	10	16	24	35	?]	51
32	2	56	55	54	53	52	51	50	49	49	31	46[
	16	38	24	32	30	33	37	44	52	7		?]
33	2	56	54	53	52	51	51	50	49	48	32	40[
	14	2	58	57	55	59	4[?	10	19	34		?]

G.	Su n M. S.	52 (M. S.)	53 (M. S.)	54 (M. S.)	55 (M. S.)	56 (M. S.)	57 (M. S.)	58 (M. S.)	59 (M. S.)	60 (M. S.)	G.	Scr up. M. S.
							]					
34	2 12	55 25	54 21	53 21	52 20	51 24	50 30	49 35	48 48	48 0	33	32[?]
35	2 11	54 46	53 43	52 43	51 43	50 48	49 55	49 0	48 12	47 25	34	23[?]
36	2 9	54 4	53 1[?	52 0	51 1	50 3[?	49 18	48 23	47 37	46 49	35	15[?]
			]			]						
37	2 7	53 24	52 24	51 24	50 29	49 33	48 41	47 46	47 1	46 14	36	[?]
38	2 6	52 42	51 43	50 44	49 49	48 54	48 3	47 9	46 24	45 38	36	— 56[?]
39	2 5	52 0	51 1	50 3	49 8	48 15	47 24	46 31	45 46	45 0	37 45	—
40	2 3	51 17	50 18	49 21	48 27	47 35	46 44	45 53	45 8	44 22	38 34	—
41	2 1	50 32	49 34	48 38	47 45	46 53	46 3	45 14	44 28	43 43	39 22	—
42	1 59	49 46	48 49	47 54	47 2	46 10	45 21	44 34	43 47	43 4	40 9[?	—]
43	1 57	48 59	48 3	47 9	46 18	45 27	44 38	43 52	43 6	42 24	40 54[?]	—
44	1 55	48 11	47 16	46 23	45 32[?]	44 43	43 54	43 8	42 24	41 41	41 40[?]	—
45	1 53	47 23	46 29	45 36	44 46	43 58	43 12	42 28	41 44	41 0[?]	42 25	—
46	1	46	45	44	43	43	42	41	41	40	43	—

G.	Su n M. S.	52 (M. S.)	53 (M. S.)	54 (M. S.)	55 (M. S.)	56 (M. S.)	57 (M. S.)	58 (M. S.)	59 (M. S.)	60 (M. S.)	G.	Scr up. M. S.
	51	34	41	48	59	12	28	44	0[?]	18	10	
47	1 49	45 44	44 51	44 0	43 11	42 25	41 43	40 58	40 14	39 35	43 53	—
48	1 47	44 53	44 1[?]	43 11	42 23	41 38	40 57	40 9	39 24	38 44	44 35	—
49	1 45	44 1[?]	43 10	42 21	41 34	40 50	40 9	39 24[?]	38 37	37 58	45 17	—
50	1 43	43 8	42 18	41 30	40 44	40 1	39 20	38 37	37 49[?]	37 11[?]	45 58	—
51	1 41	42 14	41 26	40 39	39 54	39 11	38 30	37 49	37 4[?]	36 32[?]	46 38	—
52	1 38	41 20	40 33	39 47	39 3	38 20	37 40	37 0	36 23	35 45	47 17	—
53	1 36	40 25	39 39	38 54	38 11	37 28	36 49	36 10	35 34	34 57	47 55	—
54	1 34	39 29	38 43	38 0	37 18	36 37	35 58	35 20	34 44	34 9	48 32	—
55	1 32	38 32	37 47	37 5	36 24	35 45	35 7	34 29	33 54	33 20	49 9	—
56	1 29	37 34	36 50	36 10	35 30	34 52	34 15	33 38	33 4	32 30	49 44	—
57	1 27	36 36	35 52	35 14	34 35	33 57	33 23	32 47	32 5	31 40	50 19	—
58	1 25	35 37	34 54	34 17	33 39	33 2	32 29	31 55	31 21[?]	30 49	50 53	—
59	1	34	33	33	32	32	31	31	30	29	51	—

	Su											Scr
	n	52	53	54	55	56	57	58	59	60		up.
	M.	(M.	(M.	(M.	(M.	(M.	(M.	(M.	(M.	(M.		M.
G.	S.	S.)	S.)	S.)	S.)	S.)	S.)	S.)	S.)	S.)	G.	S.
	23	37	56	19	43	7	34	1	28	57	26	
60	1	33	32	32	31	31	30	30	29	29	51	—
	20	37	58	21	49	11	38	6	35	5	58	

Parsing notes: This page is the most degraded in the batch — the double-scan garble and ink-bleed make many seconds-digits and the Sun-parallax column hard to resolve, and the rightmost two columns (repeat-G. and proportional-minutes) are badly disordered in the scan: the printed layout shifts the proportional-minute values relative to the altitude rows, so the rightmost-column alignment is uncertain throughout and is transcribed only tentatively (marked – where I could not confidently assign a value to a row). The Sun's parallax column should continue its monotone decrease from 2'19" (G.30) down toward ~1' at G.60; the printed values for rows 54–58 (showing 1 34, 1 32, 1 29, 1 27, 1 25) appear to **break the monotone decrease** and are flagged as numerical impossibles / likely scan garble (see batch report). The Moon-parallax block (columns 52–60) is internally consistent and decreases monotonically both across each row and down each column, which I used as the primary cross-check for the seconds digits. **This whole table page**

Astronomia Danica — PARS ALTERA, LIBER PRIMVS THEORICORVM, Caput VIII (continued)

Remainder of the table of parallaxes of the Sun and Moon

of longitude and latitude. All Moon entries are minutes and seconds (M. S.); the Sun column is G. M. S.]

Heading (verbatim): *Residuum tabulæ parallaxium Solis & Lunæ.* — "Remainder of the table of parallaxes of the Sun and Moon."

Column headings (verbatim):

- *Parallaxis Solis media in circulo altitudinis.* — "Mean parallax of the Sun in the circle of altitude."

- *Parallaxes Lunæ competentes integris semidiametris terræ in circulo altitudinis ab horizonte.* — "Parallaxes of the Moon corresponding to whole Earth-radii, in the circle of altitude reckoned from the horizon."
- *Scrupula longitudinis & latitudinis.* — "Proportional minutes of longitude and latitude."

Alti tud e	Sun										Scr up.
(G. M.S .)	(G. M.S .)	52 (M. S.)	53 (M. S.)	54 (M. S.)	55 (M. S.)	56 (M. S.)	57 (M. S.)	58 (M. S.)	59 (M. S.)	60 (M. S.)	(M. S.)
60	1	33	32	32	31	31	30	30	29	29	51
	20	37	58	21	46	11	38	6	35	5	58
61	1	32	31	31	30	30	29	29	28	28	52
	18	36	58	22	48	14	42	11	41	12	28
62	1	31	30	30	29	29	28	28	27	27	52
	15	34	58	23	49	17	46	16	47	19	59
63	1	30	29	29	28	28	27	27	26	26	53
	13	31	57	23	50	19	49	20	52	25	28
64	1	29	28	28	27	27	26	26	25	25	53
	10	29	56	23	50	21	52	24	57	31	56
65	1 8	28	27	27	26	26	25	25	25	24	54
		26	54	22	50	22	54	27	1	36	22
66	1 5	27	26	26	25	25	24	24	24	23	54
		22	51	21	50	23	56	29	4	40	49
67	1 3	26	25	25	24	24	23	23	23	22	55
		18	48	19	50	23	57	32	7	44	14
68	1 0	25	24	24	23	23	22	22	22	21	55
		13	44	16	49	23	58	34	10	48	38
69	0	24	23	23	22	22	21	21	21	20	56
	58	8	40	13	48	22	59	36	13	52	0
70	0	23	22	22	21	21	20	20	20	19	56
	55	2	35	10	46	21	59	37	15	55	23
71	0	21	21	21	20	20	19	19	19	18	56
	53	56	30	6	43	20	59	38	17	58	43
72	0	20	20	20	19	19	18	18	18	18	57

Alti tud e	Sun	52	53	54	55	56	57	58	59	60	Scr up.
(G. M.S)	(G. M.S)	(M. S.)	(M. S.)	(M. S.)	(M. S.)	(M. S.)	(M. S.)	(M. S.)	(M. S.)	(M. S.)	(M. S.)
	50	49	25	2	40	19	59	39	19	0	3
73	0	19	19	18	18	18	17	17	17	17	57
	47	41	19	58	36	18	58	39	20	2	22
74	0	18	18	17	17	17	16	16	16	16	57
	44	32	13	54	32	16	56	38	20	4	40
75	0	17	17	16	16	16	15	15	15	15	57
	41	22	6	49	28	14	53	36	20	5	58
76	0	16	15	15	15	15	14	14	14	14	58
	38	15	59	43	24	11	50	35	20	7	13
77	0	15	14	14	14	14	13	13	13	13	58
	36	8	52	36	19	7	47	34	20	8	27
78	0	14	13	13	13	13	12	12	12	12	58
	33	1	45	29	14	2	43	33	20	9	41
79	0	12	12	12	12	11	11	11	11	11	58
	30	52	37	23	9	57	41	31	19	9	53
80	0	11	11	11	11	10	10	10	10	10	59
	28	43	29	16	4	52	39	29	18	8	5
81	0	10	10	10	9	9	9	9	9	9 7	59
	25	33	21	9	58	47	36	26	17		16
82	0	9	9	9 2	8	8	8	8	8	8 7	59
	22	23	13		53	42	33	24	15		25
83	0	8	8 5	7	7	7	7	7	7	7 7	59
	19	13		55	48	37	29	21	13		33
84	0	7 3	6	6	6	6	6	6	6	6 6	59
	16		57	48	43	32	25	18	10		40
85	0	5	5	5	5	5	5	5	5 8	5 5	59
	13	52	47	40	36	27	21	16			46
86	0	4	4	4	4	4	4	4	4 7	4 4	59
	16	42	37	32	29	22	17	13			51
87	0 9	3	3	3	3	3	3	3	3 6	3 3	59

Altitude		Sun									Scr
(G. M.S.)	(G. M.S.)	52 (M. S.)	53 (M. S.)	54 (M. S.)	55 (M. S.)	56 (M. S.)	57 (M. S.)	58 (M. S.)	59 (M. S.)	60 (M. S.)	up. (M. S.)
		32	28	24	21	17	13	10			55
88	0 6	2	2	2	2	2	2 8	2 6	2 3	2 2	59
		21	18	16	14	11					57
89	0 3	1	1	1 8	1 7	1 5	1 4	1 3	1 1	1 1	59
		10	10								59
90	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	60	
										0	

Parsing notes: The two PDF images (pp. 297, 298) are duplicate scans of the same leaf; p. 297 is poorly inked and doubled, so values were taken from the cleaner p. 298 image and cross-checked against p. 297 where legible. Row 85, column 52 prints as 6 52, which breaks the smooth descent of that column (84 → 7 3, 86 → 4 42); the neighboring columns at altitude 85 all read in the 5' range, so 6 52 is almost certainly a compositor error for ~5 52 — flagged below. Row 86, Sun column prints 0 16, breaking the monotone decrease of the Sun column (85 → 0 13, 87 → 0 9); it should read ~0 11 — flagged below. The "60 0" terminal entry of the proportional-minutes column at altitude 90 sits one cell to the right in the source rule-lines but unambiguously belongs to that final column.

On the distinction of parallaxes according to longitude and latitude, primarily with respect to the ecliptic

Section heading (verbatim): *De discrimine Parallaxium secundum longitudinem & latitudinem in primis respectu Eclipticæ.* — "On the distinction of parallaxes according to longitude and latitude, primarily with respect to the ecliptic."

[Margin: *Lib. Revol. c.* — i.e., Copernicus, *De Revolutionibus*, the cited chapter (number illegible in the margin).]

Such, then, was the method of working out the parallaxes of the Sun and Moon in the vertical circle, given the distance of each luminary from the Earth. Now I shall briefly touch on how these same parallaxes are to be distributed, in the most general and most concise way, lengthwise and breadthwise along the ecliptic. I am of course well aware how much labor Ptolemy, Copernicus,

[Erasmus] Reinhold [Reinholdus], Tycho Brahe, and most recently of all Johannes Kepler [Iohannes Kepplerus] have expended on this matter. Nevertheless, since — once the parallax in the vertical circle (which is often required for its own sake) is given — we can arrive at the resolution of that parallax with equal ease, but by a demonstration better suited to young students, I prefer to follow Copernicus and attend to two things here. Two things must first be sought out: the altitude of the phenomenon, and the angle that lies between the vertical circle and the ecliptic (or any circle of longitude). For once this alone is obtained, and the parallax in the vertical circle is known, the distribution of that parallax lengthwise and breadthwise is accomplished with almost no effort, as will be shown below. First, then, it must be shown how this same angle is found, for the present purpose, in any given revolution.

[Margin: *Vide 6 problem. lib. 2. Sphæricor.* — "See problem 6, book 2 of the *Spherics*."]

[Margin: *Per conversionem problematis primi c. 5. lib. 2. Sphær.* — "By the conversion of the first problem, chapter 5, book 2 of the *Spherics*."]

Diagram. A spherical-triangle construction on the celestial sphere illustrating the resolution of vertical-circle parallax into longitude and latitude components. The apex A is at the top; from it three "vertical" arcs descend to the horizon arc E F H at the base. The figure shows a small parallax triangle near the lower middle, with points L, D, B, G clustered together and K just below D.

- A (apex, top): the zenith of the place [*vertex loci*].
- D B C (an arc): some circle of longitude — the ecliptic or the Moon's orbit [*circulus aliquis longitudinis, ut ecliptica vel orbita lunæ*].
- E F H (base arc): the horizon [*horizon*].
- A C H (one of the three descending vertical arcs): the arc to the ninetieth degree (the nonagesimal), where no parallax of longitude occurs [*nonagesimi gradus, ubi nulla longitudinis parallaxis datur*].
- A B F (vertical arc through B): the meridian [*meridianus*].
- A D E (vertical arc through D): the vertical circle in which the phenomenon is here taken, for the example, to lie [*circulus verticalis, in quo phænomenon... haereat*].
- D K (small segment at D): the parallax in the vertical circle [*parallaxis in circulo verticali*].
- K L (small segment): therefore the latitude [component] [*latitudinis ergo est K L*].

- D L (small segment): and the longitude [component] [*longitudinis D L*].
- B, C, G (intermediate labeled points along and between the arcs): vertices of the construction triangles A B D, A B C, and A C D referred to in the text.

So in the present figure let A be the zenith of the place; D B C some circle of longitude, such as the ecliptic or the Moon's orbit; and E F H the horizon. Let three vertical arcs descend from A: A C H, the arc to the nonagesimal degree, where no parallax of longitude occurs; A B F, the meridian; and A D E, the vertical circle in which the phenomenon is here, for the sake of example, supposed to lie. Let the parallax of that phenomenon in the vertical circle be D K; then K L is its parallax of latitude and D L its parallax of longitude.

In the triangle A B D, then, in order to find the angle A D B, the data [*διδόμενα* [Greek: *didomena*, "the given quantities"]] must be sufficient. Given, by the doctrine of spherics, the declination of the phenomenon, together with the assumed elevation of the pole of the place, both A B — its distance from the zenith along the meridian — and A D — the complement of its altitude — will easily be found from the same doctrine. The third side, B D, is supplied by the distance of the star from the meridian itself. For when the right ascension M C is present (which is always given for the proposed time), together with the degree of the ecliptic that is then culminating, the side B D itself emerges directly from this datum and from the position of the phenomenon under the ecliptic. Hence, in the spherical triangle whose sides are all given, the sought angle at D is determined.

This has now been demonstrated somewhat more generally, for a Moon that also has latitude. But if the Moon is nearly devoid of any appreciable latitude — as happens in lunar eclipses — then it is better to take the angle that the meridian makes with the ecliptic from the table of the *primum mobile* [*tabula primi mobilis*], and afterwards to apply correctly the declination of the culminating degree. Finally, one numbers the arc from the said culminating point of the ecliptic to the position of the luminaries. In this way the two sides A B and B D, together with the angle A B D, are obtained. From these, by the resolution of triangle A B D, both the side A D — the complement of the luminaries' altitude — and the angle at D become known.

[Margin: *Per probl. 6. lib. 2. Sphaer. cap. 10.* — "By problem 6, book 2 of the *Spherics*, chapter 10."]

Indeed, once this same angle of intersection of the meridian with the ecliptic, and the side B A, are obtained, the side A C — the complement of the altitude of the nonagesimal degree — is at once given in the right triangle A B C (the right angle

being at C). From this, in turn, in the other right triangle A C D — since its base A D (namely the complement of the altitude) and the side A C are known, and the angle at C is right — the sought angle C D A is therefore determined. Which was to be demonstrated.

Example: of a Moon having latitude

Heading (verbatim, italic): *Exemplum. De luna latitudinem habente.* — "Example. Of a Moon having latitude."

The worked example is laid out as a column of named quantities with values in degrees (Gr.) and minutes (Mi.):

Quantity	Gr.	Mi.	Sign/note
In the year 1608, on 12 February, at 8h 43m, at Copenhagen [Hafniæ], the true place of ☾ was, in ☾ [Pisces]	3	43	☾ in ☾
Right ascension of the same	335	39	
Therefore, resolving the hours and minutes, the right ascension M.C. comes out as	106	24	
Whose culminating point in the ecliptic is	15	8	☾ [Cancer]
But the true place of the	4	44	☾ [Gemini]

Quantity	Gr.	Mi.	Sign/note
Moon in longitude was			
The difference, therefore, is the side B D	40	24	
Further, the declination of the culminating point (M.C.) is	22	40	B [boreal/north]
With respect to the node Ω , which was in	23	40	$\square$ [Leo]
There is subtracted from the declination of M.C.	3	14	
Therefore the true declination of the Moon's orbit at M.C.	19	26	B [boreal/north]
Which, when subtracted from the elevation of the pole at Copenhagen	55	43	
Leaves the side B A	36	17	
The side A D, complement of the Moon's altitude, is likewise	50	15	

Quantity	Gr.	Mi.	Sign/note
Therefore, in the aforesaid triangle A D B — since all three sides are given — the sought angle A D B comes out as	50	10	

Parsing notes: The zodiacal-sign symbols in the right-hand margin (♈, ♉, ♊, ♋, ♌) are reproduced from the source as glyphs read against the surrounding arithmetic; the "B" markers denote *borealis* (north) declination.

And it will suffice to have carried out all of this — including the sought angle — in whole degrees and first minutes only; for extreme precision contributes nothing toward varying the distribution of the parallax.

Now, with the angle A D B (that is, L D K) found — the angle that is always opposite the parallax of latitude, namely L K — its complement will be the angle L K D, which in this example is 39° 50'. This complement is likewise opposite the parallax of longitude L D. For in so small a triangle D L K, since the difference between its sides (which here are spherical) and rectilinear sides is imperceptible — as Copernicus too rightly pointed out, and as we have learned by experience in solving this triangle — therefore, given one angle besides the one at L (which is always right), the third angle is given as well.

[Margin: *Lib. 4. cap. 6.* — Copernicus, *De Revolutionibus* IV.6.]

And since the side opposite the right angle — by which the parallax in the vertical circle is comprehended — is reckoned as one degree, that is, 60', it is to this same standard that we have added, to the remaining angles in the foregoing table of parallaxes, the proportional minutes [*scrupula proportionalia*] occupying the last column. These were derived from the given right sines of the remaining angles, in that proportion in which 60' corresponds to the radius, that is, to the whole sine [*radio seu toti sinui*].

Therefore, whenever you wish to obtain the parallax of latitude, extract the proportional minutes that correspond to the angle A D B (or its opposite L D K). But when [you wish] the parallax of longitude, enter the table with the

complement of that angle — namely the angle at K — and look up the proportional minutes. Then, multiplying each by the parallax of altitude, the parallaxes of latitude emerge from the former and those of longitude from the latter, as in the present example:

Angle	Gr.	Mi.		Proportion al minutes (Mi. Sec.)
Latitude	50	10	correspond to	46 5
Longitude	39	50	correspond to	38 26

And since the parallax of the Moon's altitude in this example was earlier found to be 48' 7", therefore from the multiplication of the proportional minutes into that same value there results: the parallax of latitude, 36' 57"; and the parallax of longitude, 30' 49".

[Margin: *Cap. 6. exempl. plo 2. lib. jus.* [?] — apparent cross-reference to "Chapter 6, example..., book 2..." (partly garbled in the source margin).]

All these [results], in this observation as in the rest, agree most exactly with the restoration of the lunar hypothesis [*restitutioni hypotheseos lunaris*]...

The text above breaks at the foot of folio 283. The continuation on the next leaf (folio 284) opens:

the lunar [hypothesis]. I see that this prescribed method of distinguishing parallaxes is to be used most correctly whenever the Moon has a sufficiently appreciable latitude. In eclipses, however, I shall demonstrate another method below.

On the refractions of the stars, with their tables

Heading (verbatim, display caps): *DE REFRACTIONIBVS SIDERVVM cum suis tabulis.* — "On the refractions of the heavenly bodies, with their tables."

[Margin: *Lib. 2. Sphæ. cap. 11.* — "Book 2 of the *Spherics*, chapter 11."]

[Margin: *Legatur lib. 1. Prog. T. B. pag. 91. & seqq. de refractionibus.* — "Let book 1 of Tycho Brahe's *Progymnasmata* be read, page 91 and following, on refractions."]

Now that the first obstacle — which insinuates itself into observations through the complication of parallaxes — has been traced back to its causes, distinguished, and dealt with, a second one follows: entirely new, and not yet found among the [recorded] observations before Tycho. This obstacle arises from the refractions of the heavenly bodies, and these in turn from terrestrial vapors, which are condensed most of all near the horizon [*finitor*]; and it is there, accordingly, that the refractions are found to be greatest — just as this too has been demonstrated in general in our *Spherics*.

But truly, just as the doctrine of parallaxes affords a most beautiful contemplation — so much so that, given (besides the Earth's radius) either the distance of the phenomenon from the Earth or the angle of its displacement from its true motion, the other quantity is acquired with geometrical certainty — so the treatment of refractions has hitherto produced no certain demonstrations, on account of the highly variable constitution of the air in different places, such as the combination of geometrical reasoning with experience everywhere demands. We therefore rely here rather on experience, and we shall retain the refractions just as they were found at Uraniborg [Uraniburgi] and collected here and there throughout the first book of the *Progymnasmata* — leaving a deeper inquiry, gladly and without reluctance, to opticians and others, should they wish to learn any beyond what is inserted into the doctrine of spherics. A synopsis of the tables themselves for the Sun, Moon, and fixed stars therefore follows — that is, insofar as they here intrude in respect of the circle of altitude.

[Note: folio 284 is largely blank below this paragraph (the lower two-thirds of the page carry only ghost set-off from the facing leaf); the refraction tables themselves begin on the following leaf and will appear in the next batch. This trailing material from folio 284 is rendered here for sentence continuity; folio 284 falls within the next batch's page range and will be re-transcribed there as its primary owner.]

These refractions belong separately to the Sun, the Moon, and the fixed stars in the vertical circle of our hemisphere. They may perhaps vary by some small amount on account of the inequality of the air around sunrise or sunset, and likewise at different seasons of the year. But since among us this has not yet been sufficiently confirmed by experience in these matters, I gladly leave it aside in a question so doubtful, and here at least give this warning: since the heavenly bodies persist, as it were, proportionally in the growth and duration of their refraction according to the magnitude of their apparent disk, something must therefore be added to the refractions of the planets over and above those here

assigned to the fixed stars — and especially in the case of Venus, because her torch shines more brightly than the other five planets. As for how much is owed to each individual case, I leave that to the judgment of prudent astronomers, namely those who have charge of the celestial observations.

Table of refractions on the altitude circle at an elevation of the pole of 56°.

[Margin: See above, in the commentary, the Sun's refraction, which we have attributed to Venus, as observed from the Sun.]

G. from horizon	Sun M. S.	Moon M. S.	Stars M. S.
0	34 0	33 0	30 0
1	26 0	25 0	21 30
2	20 0	20 0	15 30
3	17 0	17 0	12 30
4	15 30	15 20	11 0
5	14 13	14 20	10 0
6	13 30	13 50	9 0
7	12 45	12 45	8 15
8	11 15	12 0	6 45
9	10 30	11 20	6 0
10	10 0	10 45	5 30
11	9 30	10 10	5 0
12	9 0	9 35	4 30
13	8 30	9 0	4 0
14	8 0	8 30	3 30
15	7 30	8 0	3 0
16	7 0	7 30	2 30
17	6 30	7 0	2 0
18	5 45	6 30	1 15
19	5 0	6 0	1 0
20	4 30	5 30	0 0
21	4 0	5 0	
22	3 30	4 35	

G. from horizon	Sun M. S.	Moon M. S.	Stars M. S.
23	3 10	4 10	
24	2 50	3 45	
25	2 30	3 20	
26	2 15	3 0	
27	2 0	2 40	
28	1 45	2 20	
29	1 35	2 0	
30	1 25	1 45	
31	1 15	1 30	
32	1 5	1 20	
33	0 55	1 10	
34	0 45	1 0	
35	0 35	0 50	
36	0 30	0 45	
37	0 25	0 40	
38	0 20	0 35	
39	0 15	0 30	
40	0 10	0 25	
41	0 9	0 20	
42	0 8	0 15	
43	0 7	0 10	
44	0 6	0 5	
45	0 5	0 0	

Parsing notes: The header row reads "G. ab hor." (Degrees from the horizon) over a column of integers 0–45; the three value columns are headed "Solis" (Sun), "Lunæ" (Moon), and "Stellar." (Stars), each subdivided into minutes (M.) and seconds (S.). The stars column terminates at altitude 20° (refraction 0 0); cells below that are blank in the source, as are the Sun and Moon columns left blank where the run continues only to 45°. At altitude 5° the Sun value reads "14 13" (the surrounding progression would suggest $\approx 14'$), and at altitude 0° the Sun reads "34 0" — both transcribed as printed; see Numerical impossibles.

CAPVT IX.

On the complete doctrine of eclipses of the Sun and Moon.

Preface, in which the usefulness and antiquity of eclipses, together with their general description and demonstration, are set out.

Our sole aim in this work is to pursue the causes — here as elsewhere — of these phenomena in the motions, the magnitudes, and finally in the nature of the light and darkness of the three principal world-bodies most affected with respect to one another, namely the Sun, the Moon, and the Earth. From these causes the eclipses of the luminaries, with their proper magnitudes, necessarily occur at fixed times according to the course of nature. Were it not for this aim, I would at once enrich this eclipse-doctrine, right at its very threshold, with the dissertations of [Margin: Pliny, *Natural History* II.10; Plutarch, *De Placitis Philosophorum* II; and elsewhere] Pliny [Plinius], Plutarch [Plutarch], and other philosophers. To read those authors I therefore send away the lovers of the stars — who in place of causes will often gather only the wonderment of the crowd and the follies of the ancient sophists. For since these men were not ashamed to philosophize without any knowledge of astronomy (which it ill befits a philosopher to lack), the result was that, reasoning in a way not sufficiently supported by experience, they slipped for the most part into inept and counterfeit fabrications of causes.

For us, who hope to demonstrate these things soundly, the end or usefulness of this doctrine presents itself first of all. It is not only the contemplation of the remaining causes in this matter, and the certitude — verified by us above — of an infallible astronomy and of the motion of the luminaries, so palpable that it strikes even the common understanding; it is also the certification of dates in the annals, in the verifying of which the eclipses of the Sun and Moon are most correctly used by chronologers as the best-known documents. Above all, moreover, there comes this notable advantage of the restored doctrine of eclipses: that from it, chiefly, the geographical tables concerning the longitudes of terrestrial places are refined and made whole again — to say nothing here of the foreknowing and most weighty significance attributed to eclipses, according to the astrologers, in harvests, politics, and the other conditions of the lower world.

Now in every age, from the first origin of the world, eclipses of the luminaries have occurred year by year, and indeed in roughly equal number (as I shall show below by way of a corollary). We may therefore rightly wonder why it was only at last — soon after the elapse of the one-and-a-third Olympiad, and before the birth of Romulus and Remus, that is, in the year 3198 after the beginning of the

world — that they first of all began to be recorded in the writings of historians, and even then not so frequently; or why their causes were not given before the ancient Babylonians in Syria, or even Thales of Miletus [Thaletem Milesium] in Greece. For in the science of the stars, the appearance of eclipses is, as [Margin: Flavius Josephus, *Jewish Antiquities* I.3] Josephus [Iosephus] reports, of all things the most ancient, and both is and was the best known. For my part, I judge that in those earliest times, besides the scarcity of practitioners, either antiquity itself begrudged us these most useful monuments of old, or the most lamentable fire stole them away from the most august library of Alexandria. For it is credible that — apart from the few things there that Ptolemy [Ptolemaeus] could transcribe for his own use from the observations of his astronomical predecessors — very many and most ancient records were foully consumed in the flames.

However that may be, even in this late age of the world the magnitude and dignity of these matters rightly stirs me to the Plinian exclamation [Margin: Pliny, *Natural History* II.12]: *Great men, equal to heaven in genius, interpreters of things, and capable of grasping the nature of the universe, discoverers of arguments by which they have vanquished gods and men alike! Who, beholding these things and the labors of the stars (since it has so pleased to call them so), does not pardon his own lot, born to mortal necessity?* Thus far Pliny.

Now the sum of what ought to be set forth in this doctrine, and what must thereafter be demonstrated by us with their proper anomalies, appears also in the Plinian text in these few words [Margin: *Natural History* II.10]: *It is manifest that the Sun is hidden by the intervention of the Moon, and that the Moon is hidden by the interposition of the Earth, and the favor returned — the same rays of the Sun being taken away by the Moon's interposing itself, and by the Earth from the Moon.* [Margin: See near the end of this book.] From these few words it is plain what the cause of the obscuration both of the Sun and of the Moon has been: in the case of the Sun, namely, as often as by the law of its motion the Moon thrusts itself between the Sun and our gaze.

For the light that is seen in the lunar body — otherwise opaque and devoid of light, but, like a mirror, fit for receiving the solar rays — flows entirely from the Sun, and in such a way that about half of the Moon is always illuminated by the Sun's rays (as is readily perceived in the various aspects of the Moon relative to the Sun). Hence it comes about that, at the visible conjunction of the luminaries beneath the ecliptic, the Moon — passing beneath the Sun from below and turning its own opaque face toward the Earth — takes away from our view just so much of the Sun as its visible magnitude permits along the straight line of sight.

Furthermore, the Moon's eclipse occurs "by the interposition of the Earth" (as Pliny puts it) because at the true opposition of the luminaries the Moon runs into the shadow that the Earth casts from the Sun toward the opposite quarter, shaped into the figure of a spinning-top or cone. For since the whole celestial sphere is forever bathed in the solar rays — except for that space over which the Earth's shadow extends — and since such a shadow, by its spread and length, far overtops the Moon's region as measured from the Earth (though it does not reach to the remaining stars), it follows that the Moon, occupying the point opposite the Sun beneath the ecliptic, passes through the said shadow, and therefore must for that while be deprived of the solar rays and of the light it enjoys from them. Pliny, then, aptly argues that "the favor is returned," in that the Earth here takes from the Moon the same solar rays that the Moon earlier took from the Earth by its own interposition — at the time, namely, when the Sun (which itself suffers nothing of the kind) is for the same reason hidden from mortals.

So much, then, is settled concerning the cause of eclipses. Now an account seems called for as to why these obscurations do not occur at every new moon in the Sun, nor at every full moon in the Moon; likewise why some are partial and some of them more universal, but in the Moon total — and indeed sometimes with a longer duration, whence lunar eclipses occur more often than solar ones in the same regions. And finally why lunar eclipses present themselves to be seen equally, or at the same instant, in all the same hemispheres, whereas solar eclipses, being rare and wandering, do not fall upon observers equally or at the same instant when those observers are far apart on the face of the Earth.

For understanding these matters, first and foremost there serves the knowledge of the deflection of the lunar orbit from the ecliptic, set forth generally before, in the doctrine of the Moon's latitude. For since such direct syzygies of the luminaries occur but rarely near the two intersections of the Moon's path with the ecliptic — which we call the nodes, or in common speech the Head and Tail of the Dragon — eclipses must therefore follow those same intermittent conditions of the revolutions. Further, from the same source is drawn the knowledge of both partial and universal obscurations. For the more nearly such syzygies, or conjunctions, of the luminaries are seen to be celebrated at the nodes, the greater and more universal are the eclipses, on account of a visible coupling nearer the node; and conversely they are the more partial, the more remote the said visible syzygies are from the nodes — so much so that if the quantity of the visible semidiameters of the Sun and Moon is exceeded by the apparent latitude in a solar eclipse, or [the quantity of the semidiameters] of the shadow and Moon by

the Moon's true latitude in a lunar eclipse, no obscuration on either side can be expected.

Furthermore, the greater bulk of the Earth's shadow at the Moon's passage makes the eclipses of the Moon itself appear more frequent. In the Sun, on the other hand, they are both rarer in the same tracts of the Earth and more wandering — both on account of the Sun's smaller apparent disk and on account of the entanglement of the parallaxes (concerning which I likewise treated above in the *Sphaerica*, and which will be treated further hereafter) [Margin: Book 2 of the *Sphaerica*, Chapter 10; likewise the preceding chapter of this book]. For to these the Moon, because of its smaller distance from the Earth, is more liable than any other heavenly body. But that all this may become clearer for beginners, to whom I leave the matter, it must be illustrated by the demonstrations that follow.

Sketch [Hypotyposis] of eclipses of the Sun and Moon.

Therefore, in the following diagram — which exhibits a general consideration of what has preceded — let two circles be drawn from A, the center of the Earth, beyond the Earth's own surface. The first of these is to be understood as forever lying in the same plane as the ecliptic, or path of the Sun; the other, IG, represents the Moon's orbit, deflected on either side of the ecliptic in latitude.

Hypothesis [Hypothesis] of eclipses of the Sun and Moon.

At the two places where the circles intersect each other — namely D, which is the Ascending Node [☿/♌ symbol: ♌], and C, the Descending Node [♍] — it is as was set out before concerning the Moon's latitude. But that it may be discerned in this single figure when eclipses occur in either luminary and when not, let the Sun's center be placed first at E; and when lines NQ and PQ have been drawn tangent to the Earth's globe, it is manifest that they meet and terminate at the point Q, and that the shadow projected by the Earth is produced thus far into the form of a cone or rounded pyramid. Accordingly, when the Moon in its course — which is roughly thirteen times swifter than that of the Sun and the shadow — passes through the said shadow at the node C during a full moon, then, as may be seen, it loses its light, because the Earth by its interposition impedes the influx of solar rays that would illuminate the Moon. And since the pyramidal shadow is far thicker, or broader, at the Moon's passage than the lunar diameter, it follows that when the middle of the obscuration has fallen on the node, not only a total eclipse of the Moon, but the same with a duration, must necessarily occur — for just so long, namely, until the Moon, by the benefit of its swifter motion, has extricated itself thence.

From this it further follows that the nearer the full moon is to the nodes, within the eclipse-limits, the greater the eclipse the Moon must suffer; and that this is gradually at last diminished, until the latitude of the Moon's limb, turned toward the Earth's shadow, has grown beyond the magnitude of the shadow's semidiameter. For in all the remaining tract — as at G, and so forth — no eclipse-bearing full moon is felt, the Moon's orbit bending away so far from the ecliptic that the semidiameter of the shadow (which by its cone always faces the ecliptic) in no way reaches the Moon.

Furthermore, since the projection of the shadow regards the Earth's center A as its midpoint, here account must be taken of the course of both luminaries as truly seen, not as apparent — for the former regards the center of the Earth alone, the latter merely the surface and the place of the observer.

So much for the eclipse of the Moon; the Sun's follows, whose cause we defined above, from Pliny, to have been the interposition of the Moon. Therefore the middle of such an eclipse occurs only at an apparent new moon — "apparent," I say, because here we measure the eclipse-conjunction of the luminaries not from the Earth's center, as before, but rather from the surface. Whence it comes about that

Diagram. A geometric construction illustrating the general theory of solar and lunar eclipses. At the upper right is a small circle labeled *Sol* (the Sun), within which lie the points P (top), E (center, the Sun's center placed there for the construction), and M; the point N is at the lower right of the Sun's disk. Two straight lines (the tangent rays NQ and PQ) descend from the Sun's disk toward the left, converging on a long narrow cone (the Earth's shadow) whose apex is at Q at the lower left. A further straight line labeled H runs out to the right past the Sun. In the middle-left of the figure are two large concentric circles centered on A (the Earth's center): the inner labeled *Via Lunæ* (the Moon's path / orbit) and *Ecliptica* (the ecliptic). Within and around this circle the lettered points of the construction appear: the Earth's center A; the node points; the small lunar disks shown at successive positions — one near the top (near Q within the orbit), the point K at the center of the shadow cone, the Moon shown crossing the shadow, and a lunar disk near C at the lower-left where the shadow cone narrows; and G to the right of center, marking the tract where no eclipse occurs.

- *Sol* (upper right): caption labeling the Sun's disk
- P (top of Sun's disk): tangent-line endpoint
- E (center of Sun's disk): the Sun's center, placed there for the construction
- M (within Sun's disk, below E): construction point

- N (lower right of Sun's disk): tangent-line endpoint
- H (right, beyond the Sun): labeled straight line
- Q (lower left, apex of the cone): point where the tangent lines NQ and PQ meet and the shadow terminates
- A (center of the concentric circles): the Earth's center
- *Via Lunæ* (upper-left arc of inner circle): "the Moon's path"
- *Ecliptica* (arc below *Via Lunæ*): "the ecliptic"
- K (center of the shadow cone, within the orbit): construction point at the Moon's passage through the shadow
- C (lower-left, where a lunar disk sits as the cone narrows): the Descending Node, the full-moon eclipse position
- G (right of center): position in the tract where no eclipse-bearing full moon is felt
- I (upper-left of the orbit, paired with G to name the orbit IG): orbit-endpoint label

The small lunar disks along the Moon's orbit are drawn with shaded/hatched faces to indicate their illuminated and shadowed portions.

than the true conjunction, which would be seen as if from the Earth's center. I gave a representation of this in the preceding figure too, in the region of the ascending node, where the lunar body D is set against the Sun from the Earth's center A in such a way that one understands the Sun to be entirely hidden on this side and that—that is, the span of the Sun within P and N along the solar diameter. This obscuration MN is defined, when referred to the Earth's surface B, as nearly the semidiameter MN, with the lines BM and BH drawn from B likewise touching the lunar body. Yet those observers who have the Sun at their zenith perceive the full eclipse of the Sun, the same that we earlier reckoned from the Earth's center. Consequently, because of the lunar parallax—by which the Moon's [parallax] exceeds the Sun's—the eclipses of the new moon are seen and occur for different inhabitants of the Earth in different ways, and they also set in at a different instant of time. And although, because the Moon's diameter is smaller than that of the Earth's shadow, these obscurations of the Sun do not happen as frequently at one single place on Earth, nevertheless the said parallaxes—by which the Moon's apparent position is variously displaced—distribute these obscurations among different inhabitants of the Earth in such a way that the number of solar eclipses easily exceeds that of lunar ones, as we shall demonstrate at the end of this doctrine by way of a coda. Solar eclipses, therefore, occur at least at an apparent new moon, and also on either side at each of the two

nodes, with the magnitude and form that the ratio of the semidiameters of the luminaries and the apparent latitude of the Moon require. For this reason a Moon situated beyond these limits—as at G and the like—never undergoes the eclipsing of the Sun. Moreover the duration of the eclipses, as well as their magnitude in particular obscurations both of the Moon and of the Sun, and likewise on which side these occur, depends wholly upon the motion and the latitude-condition of the Moon, and is to be estimated from these. And this is the first and general outline of eclipses. There follows the first and simple variation, which occurs both in the Moon and in the Sun.

On the simple variation in the magnitude or quantity of eclipses.

Heavenly bodies, like the Earth itself, even though they are spherical, are nevertheless perceived by our eyes as flat owing to their great distance—so much so that they can be measured only by their diametral magnitude, not by their surface. Optical theory furnishes us this visible form of theirs through the near-equality of the visual lines, on account of the greater distance of those bodies. [Margin: See *Optics* [ἄστρονομικά (astronomika); see note]] Indeed, from the same principle we recognize that these same bodies vary their visible magnitude according to their different distance from the eyes of observers, so that the more distant appear smaller to them, the nearer larger. Finally, those bodies that by their own light cast a shadow from smaller opaque objects onto an opposite surface, likewise produce that same shadow unequal in proportion to their unequal distance—so that at a greater distance the shadow is gradually enlarged from its root, while at a smaller distance it is diminished, and for the most part is cut short at its exit, since the margins of light lick the edges of the shadow more and more, all the way to the farthest immersion. This consideration must be applied above all to the assessment of the magnitude of eclipses, and set out for lovers of astronomy [Astrophiles] in the following diagram.

Diagram. A composite eclipse-geometry figure occupying the left column of the page. At the top is the solar disk, drawn through two superimposed circles representing the Sun at apogee and at perigee, with horizontal chords across them. Below, tangent lines descend on both sides to touch the terrestrial globe (a small circle near the bottom containing the Earth's center A), and then converge to a single shadow-apex P at the very bottom—the conical shadow of the Earth. Within the descending cone, a small Moon-circle is shown crossing at the level marked K/H, illustrating the Moon immersed in the shadow at apogee (K) and

perigee (H). The margin beside the figure cites the authorities for the construction.

- D, E (top, ends of the upper solar diameter — at apogee): the outermost points of the Sun's diameter when the Sun is at apogee (the higher position), from which lines DR and EQ are drawn tangent to the Earth.
- B (top center, between D and E): center/upper label of the apogee solar disk.
- F, G (just below D/E, ends of the lower solar diameter — at perigee): the outermost points of the Sun's diameter when the Sun is at perigee, from which lines FR and GQ are drawn tangent to the Earth.
- S, C, T (middle band, across the solar disks): interior construction points on the chords of the solar circles (C marks the center).
- R, Q (the two tangent points on the terrestrial globe, left and right): where the tangent lines touch the Earth's globe, drawn on the semidiameter AR.
- A (center of the small lower circle): the Earth's center.
- K (within the shadow-cone, upper of the small Moon-pair): the Moon at apogee, immersed in the shadow.
- H (within the shadow-cone, lower of the Moon-pair): the Moon at perigee, immersed in the shadow.
- P (at the very bottom, apex of the cone): the apex of the Earth's conical shadow.
- [Margin: *Ptolemy*, Book V, ch. 15 [Ptol. l. 5, c. 15]; *Copernicus*, Book IV, ch. 19 [Copern. lib. 4, cap. 19]]

In an eclipse, whether lunar or solar (as posited in the following figure), let a complete terrestrial globe be described about the Earth's center, on the semidiameter AR. Then, following the theories of the luminaries already set out, let the solar body first be represented as if at two centers, those most distant from one another with respect to the Sun—namely at apogee B and perigee C. Likewise let the Moon's distances be supposed at two places equidistant on either side from the Earth's center: above, at the new moon, near apogee M and perigee L; below, at the full moon, near the Moon's apogee K and perigee H. Now let straight lines be drawn—first from the outermost ends of the Sun's diameter, namely under the higher apogee D and E—which touch the terrestrial globe around R and Q, and therefore terminate in the apex P.

In the same way, when straight lines are drawn from the outermost ends of the diameter of the lowest Sun—namely, of the one situated at perigee—I mean the

lines FR and GQ, these likewise, touching the Earth, terminate at N, thereby revealing an unequal production of the shadow-cone at the distance PN, and besides leaving traces about each margin of the shadow, whereby that shadow is understood to be diminished whenever the Sun descends nearer toward the Earth, and vice versa. For the Sun's greater apparent size and presence contracts the shadow, as happens in other cases. Likewise the Moon at apogee presents on both sides under the ecliptic a smaller angle of vision than at perigee, as may be seen in this same adjoining figure in the lunar body immersed in the shadow at apogee K and at perigee H.

From these matters, thus generally sketched, follows that variation in the semidiameters of the Sun, Moon, and shadow which we shall later employ—according as the luminaries obtain their determined position in their hypotheses with respect to the Earth, which is to be learned thoroughly from the anomaly of each. The ancient astronomers reasoned geometrically from this symmetry of Sun, Moon, Earth, and shadow, concerning both the true distance of the luminaries from the Earth and the determined projection of the shadow from it—but in vain, namely on account of the refractions of the solar rays, which render this *πραγματεία* [Greek: *pragmateia*, "treatise" or "procedure"] void and introduce a certain anomaly or exception to what has been demonstrated; concerning which, therefore, we shall treat presently in what follows.

[Margin: *Ptolemy*, Book V, ch. 15 [Ptol. l. 5, c. 15]; *Copernicus*, Book IV, ch. 19 [Copern. lib. 4, cap. 19]]

ACCOUNT

On the anomaly of the foregoing, which is caused by the thickness of the air about the Earth, and hence by the refraction of the solar rays, in the eclipse both of the Moon and of the Sun.

The ancients' simple and uniform observations [Greek: *ἐπισκέψεις* (*episkepseis*, "examinations" or "observations")] concerning eclipses of the luminaries have thus been set out by us demonstratively. We shall now report what they either noticed less—being content with discovering the prior causes—or what in those places, namely Assyria and Egypt, where the ancient mathematicians once flourished (by whose industry the sublime knowledge of the stars has come down even to us), appeared less, owing to the purity of the air. For from observations made before, and still now, in Denmark, in nearer Germany, and in farther Norway—made most diligently by me and by others standing by—I have indeed detected a quite notable alteration in the magnitude of the eclipses in each

luminary. For when, from the year of the Lord 1589 up to the year 1597, together with six, seven, or sometimes more other students whom Lord Tycho Brahe [Tycho Brahe] then maintained in astronomical exercises, with the greatest diligence I observed all the eclipses visible in that interval—not by the pinhole-tube method alone (whose insufficiency we detected easily and early enough), but by various means through various instruments, especially the visual one, which deceives less in this business—I assuredly found here and there that the Moon's diameter at the eclipsing new moon undergoes, in these places, a remarkable diminutive change from that which appears at the full moon; so much so that, when the table to display this matter was being constructed, we did not hesitate to fit the difference of the Moon's diameter at new moon from that seen at full moon—and most of all at the eclipsing full moon—to that ratio which is of the number four to five,

—though it [the diameter] even still here somewhat exceeds the same. With this assessment the rest of the observations agree as closely as possible—those which I made a little later at Rostock by the banks of the Warnow, and near the shores of the Baltic Sea, and thereafter here in Denmark. In Norway, however, as is fitting, I observed an even greater diversity. For the solar eclipse of the year of the Lord 1601, on the 14th of December—which Johann Kepler [Johannes Kepler] adduced as observed at Prague in defense of his own opinion—I myself was observing 18 miles beyond Trondheim [Nidrosia], under an elevation of the pole of $64\frac{1}{2}^{\circ}$, one hour past noon, on the horizon, the Sun being sufficiently visible as to its upper limb on account of the parallaxes of the Moon and the remarkable refraction there (being in the vicinity of the horizon). Behold, afterward, when I had reached Bergen in Norway (whose pole is elevated $60\frac{2}{3}^{\circ}$), the bishop of that place, M. Andreas Foss [Anders Foss]—a theologian, mathematician, historian, botanist, and once a most outstanding man in languages and all the more polished literature—showed me a representation of the same, how, according to the account of the fishermen on the neighboring shore (who beheld the said eclipse with the greatest wonder), he had drawn it. In that same representation the Sun had so enclosed the lunar body within its compass, in the middle, that on every side toward the margin, with its light spread out, it stood out equally by about one and a half digits, as it were. And although the Moon was here near apogee, nevertheless in the reformed hypothesis it would not on that account have been seen with so diminished a diameter, had not a denser air and a kind of vapor of the Ocean intervened.

[Margin: See *Appendix, Progymnasmata* I, T. B. [Tycho Brahe]]

I shall give another proof of the present matter (should anyone still doubt) in the solar eclipse occurring in the year 1608, on the last of July—which, although that one too appeared at Wittenberg to be of about two digits, as the most distinguished and most longed-for Lord Melchior Jöstel [Melchior Jöstel] wrote to me, was nevertheless not seen at all at Copenhagen [Hafnia] under a sky most clear at the same time. For when, a little before, the most distinguished and most friendly man Johann Kepler, the Imperial Mathematician, had departed once his *Optics* was published, I was held by an even greater desire to demonstrate the truth through this eclipse as well. Therefore, in the tracking-down of this trace in the Sun—which, as I said, appeared not at all—I myself, most studious, applied five students of keenest eyesight. And although in this eclipse the parallax of latitude of the Moon (being lower in our vertical) was a little greater with us and would have subtracted somewhat of the obscuration, it was nevertheless not so great as to diminish that obscuration by even one digit, compared with the observation of the Wittenbergers—unless the intervention of this place's thicker air had withdrawn the whole of it from our eyes.

These and many similar observations the maritime places under a higher pole have furnished; but Bohemia furnished others, where in far purer air the greater defects in the Sun reveal themselves, all else being well considered. This I beheld with wonder in the year 1600, on the last of June, at Benátky in that same region, while staying with Lord Tycho in that place. For the eclipse that I expected from our earlier restitution of the Moon to be scarcely 4 digits, we saw to be greater by more than one and a half digits. And yet I could not then (as it seemed to me) subordinate so many observations of ours, made earlier here in Denmark, to one single marvel, nor charge these to the weakness of my former eyesight; but rather from that time I judged that effort should be devoted to bringing to light the causes of so great a variation—once it had become clearly established for me, from so many observations made in diverse places, that they depend on the unequal constitution of the air (which under a sky nearer the equator is purer, but toward the poles, more remote, is more turbid)—and so to render this eclipse-doctrine universal, as far as possible over the globe of the Earth, so far as experience and reason should permit. *For he denies first principles, who takes away experience in astronomy.* In the same way, in eclipses of the Moon, whether total or partial, I found a smaller thickness of the shadow in the Moon's transit under a higher pole and a thicker air.

But since it is credible—and as far as eyesight can attain it—that the magnitude, namely, of each luminary, at their mean distance from the Earth, is either very nearly equalized to the senses with one another, or that the Moon somewhat

exceeds the Sun (a test of which we have often made, and with various instruments): therefore I shall now, according to the capacity of my talent, explain in what follows—subject to the judgment of the opticians, should they be able to find better reasons—how the solar magnitude exceeds the lunar at the eclipsing new moon under an impurer sky, while at the full moon both a shorter transit of the shadow is gathered and the Moon from the same cause rises above its accustomed magnitude.

In eclipses, only two reasons, fairly closely related to each other, present themselves to us. Let me report both briefly—since it will be beside our purpose to be too much hindered, in the undertaken course of restoring the motions, by contemplations of this kind.

The former reason alleges a dilatation of the Sun's or the Moon's light in denser air, which reveals itself as soon as the opaque lunar body or the shadow is interposed—just as if the Sun, lingering near its rising or setting, were so concealed from view by some little island in the sea directly interposed between it and the eyes of sailors, that nevertheless on either side something of the solar body (which I have sometimes seen) remained visible. In this spectacle, it is wonderful how the Sun spreads itself out on either side by its rays, so much so that you would say the very σύστημα [Greek: systēma, "system" or "whole frame"] of the Sun is all but torn asunder and quite dissipated. But if the said island has insinuated itself between the eyes of the viewers and the Sun in no other way than to graze, by concealing, at least the outermost margin of the Sun on one side, here you will see the Sun gather itself together and as it were withdraw wholly into view—admitting, doubtless, a double refraction: one from the nearness of the horizon, the other from the nearness of the island at its side. When I have duly fitted these spectacles to a solar or even a lunar eclipse under a thicker sky, I detect no dissimilarity whatever—except that the Moon, in the Sun's obscuration, or the shadow, in the transit of the failing Moon, differ from the island by an immense disparity of distances; and further, that eclipses occur no more near the horizon than at any other position of the daily revolution. For the rest, two corollaries attach to this reason, which perhaps can scarcely be sufficiently approved by astronomers: the one, the visible dilatation of the solar body, etc., in the fuller eclipse; the other, the change of that same body relative to vision according to place, in a more partial obscuration of the Sun or Moon. Neither of these, however, can induce a diversification beyond 6 or 7 minutes, under the densest sky to which experience has thus far extended.

The second reason depends on the illumination of the dense air that closely surrounds the cone of shadow turned aside from the Moon. For it is credible that this shadow there (perhaps even on account of the reflection of the solar rays from the Earth) is rarer about its margins, and is radiated and diminished on every side by the dense, and therefore more strongly illuminated, air. And because the lunar obstacle, however small it may be, is represented to the eye by the phantom-image [idolum] of the shadow itself in the air, therefore, when this compression is made, less of the Sun is taken from us. Experience seems to dwell together with this reason: when in the evening, or even in the morning, the sky being more serene—especially in winter time—and the Moon being as little distant from the Sun as can be detected by sight, the Sun's rays so illumine the nearer semi-periphery of the Moon that this illumined part is perceived to stand out greatly above the remaining part of the opaque lunar body (which is discerned as if enclosed together in the form of a globe)—namely through the wider diffusion of light made in the denser air. And indeed this phantom-image of the dilatation of light can be either seen or exist only in the air. For if it really adhered to the Moon on account of certain circumstances [Greek: περιστάσεις (peristaseis, "surrounding conditions" or "circumstances")] causing a refraction of the Sun, then it would appear equally in all places on the Earth to which the Sun's obscurations reach, on account of the refractive lines descending directly from the Moon's extremities into the air and the eye—whence no variation whatever of solar eclipses in various climates would occur. Between the two reasons here recounted, therefore, either this latter—which seems to me the preferable—or the former is to be acknowledged as the true one; and no third is given. So much for the eclipse-anomaly of the Sun. That of the Moon follows.

Since above we showed the cause of the Moon's obscuration to be the terrestrial shadow projected from the Sun's opposing rays, accordingly, when these [rays] enter into air nearer the Earth, and therefore

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and therefore, the more they strike vapor-laden, dense air, the more they are refracted into the shadow, so as everywhere to shape it into a thinner and shorter point — and this indeed varies according to the differing quality of the air surrounding it at each separate place. Yet, in my judgment, the incidence of these refracted rays into the shadow can never be as great as the refraction of the visible solar body at the horizon, which strikes the eye. For the force of this kind

of refraction depends chiefly on the present visibility of the luminous body, as experience in optics attests. But the Sun, at night, when the Moon is most fully eclipsed, hems in the Earth with rays sent out at least obliquely, and casts the Earth's shadow toward the opposite side. It is therefore one thing for the visible solar body itself to be refracted at the boundary of the horizon, and another for the rays cast from the luminous visible body to illuminate the dense air, and so, refracted, to thin out the shadow by darting into it.

For often, in a clear sky, I am reminded by experience here — as if by a pleasant spectacle — of how the Sun, just on the point of setting, for the same reason hangs trembling for a little while above the horizon in a kind of balancing motion; then, after this struggle, as the rotation of the primum mobile urges it on instantly, with a momentary submersion as it were, it hides itself below the horizon — and yet with equal haste rises again. This happens to our sight by refraction from the Sun on account of its visible presence; I would believe that the same thing happens, in like manner, in the air, even when that body is hidden close at hand. The same lingering of the Moon as it is about to enter the Earth's shadow, and its haste to leave it, is likewise observed.

Since, then, optical reasoning supported by experience sets before the eyes well enough how visible objects present themselves by refraction in the same form by which they are in truth marked out and determined, the cause — when on several occasions an eclipse was observed in the Moon while both luminaries were visibly above the horizon, as in the observations of Pliny [Pliny the Elder], of Maestlin [Michael Maestlin], and indeed of my own — must therefore be sought nowhere else than in the Earth's shadow, just as I discussed this at greater length in the disputation on the diminution of eclipses. But what I had here noted from experience on this same point, I shall now briefly tie together with the present figure, so that it may be set before the eyes there more clearly.

Diagram. A geometric construction of the solar eclipse anomaly as seen from the Earth's surface. At the top is a large circle representing the Sun, hatched around its rim; inside it the letters **B** (center) flank-labeled with **C** at left and **D** at right, and **F** at the far left and right of the rim. Below the Sun, straight lines descend convergingly toward the Earth, defining the Moon's shadow cone. The small circle of the Moon (with **H** at top, **G** and **K** flanking, and **V** below) sits along the axis. Further down, **R** and **Q** mark the sides of a lens-shaped figure around point **A** (the Earth's center / observer's surface). The shadow narrows toward **L**, then to a point **M**, and finally to **P** at the bottom.

- **B** (center of upper circle): center of the Sun

- C (left of Sun): one extremity of the Sun's diameter
- D (right of Sun): the other extremity of the Sun's diameter
- F (far left and far right of Sun's rim): the visible illuminated circumference E F of the Sun's body remaining unobscured
- H (top of Moon's disk): the Moon's body in the solar eclipse
- G, K (flanking the Moon): edges of the lunar body grazed by the rays
- V (below the Moon): the eye / point of sight on the Earth's surface
- A (center of lens figure): center of the terrestrial globe / observer
- R, Q (left and right of A): sides of the shadow region near the Earth
- L: position of the Moon in the descending shadow
- M: the point to which the dense nearby air re-gathers the thinned shadow
- P (bottom): the true geometric apex of the shadow

Let **A** be the center of the terrestrial globe; **B** the Sun, **H** the Moon in a solar eclipse, **L** the Moon in its own [lunar eclipse]. But in order that we may first work out those things which, in a solar eclipse, are anomalous and do not present themselves equally in all places: let straight lines be drawn from the extremities of the Sun's diameter all the way to the Earth's surface, **C V** and **D V**, grazing the lunar body at **H** on either side. And although the greatest obscuration of the Sun is here brought about, nevertheless that same eclipse — even when total under a higher pole and in denser air — can never be so great that something visible of the Sun's own body does not remain, here represented by the bright circumference **E F**, which would appear while from those points too straight lines descend into the sight **V**, passing by the lunar body and therefore by the occultation to be induced by it.

So then, either the solar rays in denser air enlarge and dilate the visible circumference of the Sun, so that it presents itself to the view which experience requires; or the air close by, along the axis of the Moon's shadow cone **G V K**, on account of its density — by which it admits a notable illumination from the solar rays — acts upon the shadow in such a way that it everywhere feeds upon it with its stronger illumination...

Diagram. This is the same figure as on the facing page (p. 309), here printed in the left margin. A large hatched-rim circle (the Sun) at top labeled **B** at center with **C** and **D** within and **F** at the rim edges; below it the converging shadow-cone lines with the Moon's disk (**H**, **G**, **K**, **V**), the lens figure around **A** (with **R** and **Q** flanking), then narrowing through **L** and **M** to the apex **P**.

- B: center of the Sun

- C, D: extremities of the Sun's diameter
- F: ends of the visible bright circumference E F
- H, G, K: the Moon's body and its grazed edges
- V: the eye on the Earth's surface
- A: the Earth's center
- R, Q: sides of the shadow near the Earth
- L, M: positions along the thinned shadow
- P: the true apex of the shadow

feeds upon it as it were, and thins it out, and so, the phantom-image [idolum] of the lunar body imitating the residual shadow in the said air to the eye, takes away just as much from the Sun's due obscuration as has been fed away from the shadow by the more illuminated nearer air. (For any object whatever is perceived in that form and magnitude in which the rays from it flow into the eye through the transparent medium.)

Whatever the case may be, I have gathered from various places — and now assert it as a most certain matter of experience — namely that the solar eclipse is always smaller the nearer you incline toward our pole, and consequently the thicker the air through which you view it, provided everything else required for the change of parallaxes or of meridians has been most correctly handled. For this reason, below, for each particular climate, the proportion and ratio of this anomaly must be supplied by me. But whoever does not hesitate to refer this alteration to the discrimination of the sight, and to its defect in certain people — especially in these same, or in these particular places — will first convict the sight of a general defect in all those who live under a thicker sky, before he forces such an opinion of his upon us as the truth itself.

Furthermore, the denser air around the Earth likewise provides the cause, as I said above, of removing some portion of the terrestrial shadow into which the full and ecliptic Moon plunges. For whereas the straight lines **C R P** and **F Q P**, sent down from the Sun through the Earth, would really carry the shadow from **A** to **P**, the air surrounding the Earth, beyond **R** and **Q**, being denser and more illuminated, causes the solar rays to dart from it into the shadow, and everywhere thins it out and re-gathers it into a cone at **M**, in the form here seen. And so **M P** is the difference of this prolongation; and the gap on either side, in the Moon's transit at **L**, is sufficiently apparent — by which this shadow is shorter than the one that would have ended in the true point **P**. I have also found some variation of this under a denser sky (as reason demands), and likewise a greater apparent extent of the bright bit of the Moon in particular eclipses under the

same — though it is not so great as to contend with the Sun's apparent prominence.

Nor am I unaware that the Sun, on account of its true magnitude, which exceeds these bodies by many parasangs, illuminates a little more than half the combined system of the globes of the Moon and the Earth. But both this, and that other small matter which arises from the unequal motion of the nodes with respect to the Moon's latitude, and does not register well upon the senses, I judge must be omitted from the present consideration. For it is the mark of an astronomer to be willing to ignore certain insensible minutiae.

Meanwhile, however, we shall test this geometrically concerning the diminution of the terrestrial shadow, the Sun's mean distance from the Earth's center being set at 1288 semidiameters, and its visible diameter at 31 minutes, together with the magnitude of the shadow as observed in the Moon's transit here among us in Denmark, whose diameter extends to about $1^{\circ} 29\frac{1}{2}'$. For in vain did the ancients, as was said above, suppose that they could investigate the distance and mutual proportion of these three bodies — the Sun, the Moon, and the Earth — by means of eclipses, even granting in addition the Moon's distance from the Earth, since the anomaly which we have now expounded in the thinning of the shadow disturbs the reckoning of the dimension everywhere on Earth, though unequally.

Demon-[stration]

Demonstration of the true semidiameter of the Sun and the Moon, likewise of the true length and thickness of the terrestrial shadow in the Moon's transit — assuming, if it could be so, the purest air surrounding the horizon — and how much these differ from what is seen among us.

Diagram. A geometric construction (in the right margin) of the true shadow cone in purest air. At top, a large circle (the Sun) with **K** and **B** along its upper diameter and **G** at the left, **I** at the right. From the tangent points **G** and **I**, lines descend to graze the Earth's globe at **C** and **D**, converging far below to the shadow apex **H**. Midway down: **O** and **V** mark the Moon's true semidiameter position; lower, the small circle around **A** with **P** at left-of-center, **C** and **D** at its sides; and **E**, **R**, **T** marking the shadow's apparent breadth at the Moon's transit distance. The cone narrows to **H** at the bottom. Two marginal citations sit beside the figure: **Caput 8. hujus** ["Chapter 8 of this (book)"] and **Lib. 5. Almagest. cap. 15** [Ptolemy, *Almagest* V.15].

- G, I (Sun's diameter extremities): tangent points of the lines touching the Earth's globe

- K, B (along Sun's upper diameter): construction points within the Sun
- C, D (sides of Earth's globe): tangency of lines G and I to the terrestrial globe near its diameter
- O, V: true semidiameter of the Moon (full Moon in apogee)
- A (center of lower small circle): Earth's center
- P: foot of the perpendicular within triangle near A
- E, R, T: apparent breadth of the shadow at the Moon's transit distance (56 semidiameters)
- H (bottom apex): the true point of the terrestrial shadow cone

Let **A**, in the present adjacent figure, be the center of the terrestrial globe, and **B** the Sun at its mean distance from the Earth, which we set above at 1288 Earth-semidiameters — of which **A C** is one — this distance being measured by the line **A B**. Likewise, let the Moon's mean elongation from the Earth's center, on the opposite side, be the line **A R**.

Let lines descend from the points of contact at the extremities of the Sun's diameter **G** and **I**, tangent to the Earth's globe on either side near its diameter at **C** and **D** (for, as I said, these points are found within the contact scarcely perceptibly). They will therefore end at **H**, where the cone of the terrestrial shadow also runs out, to which point too the line **B A H** is continued. But since the proposal is first to ascertain the true semidiameters of the Sun and the Moon, and then the length of the shadow **A H**: let the line **A I** be drawn first; then, in the right triangle **A B I**, since the given quantities [Greek: διδόμενα (didomena, "the givens")] are **A B**, 1288 Earth-semidiameters, the distance of the Sun from the Earth; **B A I**, 15' 30", half the Sun's apparent angle; and finally the angle **A B I**, a right angle — therefore the side **B I** is found to be 5 51/100,0 [recte: $5^{51/1000}$] Earth-semidiameters, which measures the true semidiameter of the Sun, to which **B G** is equal.

In the same way, for the true semidiameter of the Moon: since in the right triangle **A O V** there are given **A O**, 57° 38' [recte: 57 38/?? — see note a] Earth-semidiameters — namely of the full Moon at apogee elongation, when also **O V**, the apparent semidiameter, is 16' — therefore the true semidiameter of the Moon is given as 16 1/12 [recte: $16^{1/12}$] of such parts of which **A D** is 60.

Furthermore, let the line **C K** be drawn parallel to **A B** along **B G**. Then, the one Earth-semidiameter **A C** being subtracted from **B G**, there remains **K G**, 4 51/1000 [recte: $4^{51/1000}$] semidiameters. But as this stands to **C K**, 1288 semidiameters, so does **B G** to **B H**; and that gives 1556 [recte: $1556^{1/2}$] semidiameters of the Earth

— from which, when **A B**, 1288 semidiameters, is subtracted, there remains the sought length of the shadow cone **A H**, 268 semidiameters of the Earth, very nearly, exactly so with Ptolemy [Ptolemy], when the Sun is itself at apogee.

Finally, for the apparent semidiameter of the shadow in the Moon's transit according to the mean elongation of the Moon from the Earth — namely 56 Earth-semidiameters — let the line **E F** be drawn through **E** parallel to **C D**. And since **R H** is given as 212 semidiameters by the difference of **A R** from **A H**: therefore again, as **A H**, 268 semidiameters, is to **A C**, one semidiameter, so is **R H**, 212 semidiameters, to **R E**, $47 \frac{81}{100}$ [recte: $47^{\frac{81}{100}}$] semidiameters — or, in resolved minutes, about $47' 28''$ nearly, of which **A C** is $60'$.

Then, the line **E A** being drawn: since in the right triangle **E R A** the given quantities [Greek: διδόμενα (didomena, "the givens")] are **A R**, 56 Earth-semidiameters, and **R E**, $47' 28''$, the semidiameter of one [Earth-semidiameter], together with the right angle enclosed at **R**: therefore the angle of apparent vision **R A E** is given as $48' 34''$. But among us, under a pole-elevation of about 56 degrees, that same angle, in very many and most careful observations of a lunar eclipse, has been found no greater than $44' 10''$ — to which measure we shall below show the Copernican [value] to come nearer, but the Ptolemaic [value] to exceed the same a little, so that you may see the quality of the air at the boundary of the horizon, in the various tracts of the Earth upon which the Sun's rays fall, thinning out the Earth's shadow, as was demonstrated above, to vary but little — unless perhaps

Diagram. The same figure as on the facing page (p. 311), here printed in the left margin: the Sun circle at top with **G, K, B, I**; tangent lines descending to **C** and **D** on the Earth; **O** and **V** marking the Moon's true semidiameter; the small circle around **A** with **P, C, D; E, R, T** below; and the shadow apex **H** at the bottom.

- **G, I**: extremities of the Sun's diameter (tangent points)
- **K, B**: construction points / center of the Sun
- **C, D**: tangency of the lines on the Earth's globe
- **O, V**: the Moon's true semidiameter position
- **A**: the Earth's center
- **P**: foot of the perpendicular near **A**
- **E, R, T**: apparent breadth of the shadow at the Moon's transit distance
- **H**: the true apex of the terrestrial shadow cone

unless perhaps in farthest Norway, Iceland, and Greenland, and the lands nearest the pole, the very thickest air may add something further — namely above $4\frac{1}{2}'$, by

which the angle $\mathbf{R A E}$ just now found roughly exceeds the same one observed among us, whose double, with respect to the whole diameter of the shadow, that is the angle $\mathbf{E A F}$, is $8\frac{1}{2}'$ [recte: $8\frac{2}{3}'$ — see note b].

From a careful consideration of these matters it becomes known that what we demonstrated above concerning the elongation of the Sun from the Earth, on the hypothesis of the Moon, agrees beautifully with the truth — without the mean of which the ancients could never arrive at the true distance of the Sun from the Earth, nor at the proportion of these three bodies, the Sun, the Moon, and the Earth, which is to be investigated from this — the true ratio of $\mathbf{E R}$ to $\mathbf{C A}$ being, of course, always hidden from them. But we, before we proceed further, shall presently acquire that same proportion, the opportunity being offered.

On the mutual proportion of the three world-bodies — Earth, Sun, and Moon.

Since the diameter of the Earth, as well as its whole volume, in any proposed measure becomes sufficiently known in geometry, accordingly the celestial bodies will be rightly commensurated to this proportion of the Earth — which we shall now do for the Sun and the Moon, both in respect of their symmetry with the Earth, and as set up between themselves. For astronomy promises not only knowledge of the motions, but also, as far as it can be done, of the magnitude of the celestial bodies.

Now since it is established from geometry that spheres are in the tripled ratio of their diameters [i.e., as the cubes of their diameters]: and the semidiameters of the luminaries have been acquired most nearly — the Sun's indeed $5\frac{51}{1000}$ [recte: $5\frac{51}{1000}$] minutes, the Moon's $16\frac{1}{12}$ [recte: $16\frac{1}{12}$] minutes, of which parts the Earth's is one — therefore, when these numbers are taken separately as cubes and compared with the cube of the one Earth-semidiameter, the Sun is found to be greater than the Earth 196 times — that is, taken twenty-eight times seven [$28 \times 7 = 196$] — while the Moon is smaller than the same Earth about $51\frac{4}{19}$ [recte: $51\frac{4}{19}$] times, or somewhat further; but, retaining $51\frac{11}{19}$ [recte: $51\frac{11}{19}$ — see note c], the Sun will be exactly ten thousand times greater than the Moon. And this is a sufficiently admirable mutual symmetry of these bodies, as it is now known, deduced from the heavens without any forcing.

DEMONSTRATION

Of the variation of the semidiameters of the Sun, the Moon, and the terrestrial shadow, together with the anomalies subjoined proportionally according to experience.

Having premised those things which concern the affections of the three world-bodies — namely the Sun, the Moon, and the Earth — with respect to their magnitudes and their uniform mutual distances in eclipses: now we shall further pursue, by geometric demonstrations and also by experience, by way of determination, what the variations in them have been, according to those things which we gave for contemplation in the preceding antepenultimate and penultimate figures — so that what the appearances of the semidiameters in eclipses have been may here be made known.

Astronomia Danica — Pars Altera, Liber Primus Theoricorum, Caput IX (continued)

I.

For the simple variations of the semidiameters indicated in the antepenultimate figure, let the last [figure] be taken up once more. From it, I wish first to examine the apparent magnitudes of the luminaries — that is, of the Sun and Moon — at apogee and perigee, and then those of the terrestrial shadow, on account of the variation of both the Sun and the Moon with respect to the Earth. Now, in accordance with the hypotheses of the luminaries set out above, and likewise with their determined mean distances from the Earth, we found the following [for the Sun and the new- or full-Moon]:

- Sun's distance from the Earth: at apogee **1334**, at perigee **1242** (in degrees [Grad.], i.e. Earth-radii [Semid. terrae]).
- Moon's (new or full) distance from the Earth: at apogee **57° 38'**, at perigee **54° 23'** (in degrees and minutes [Gr., Mi.] of Earth-radii).

Hence, in the figure set alongside, given the true semidiameter of the Sun $KI = 5\frac{7}{1000}$ in Earth-radii, $BA = 1334$ the distance of the Sun at apogee, and finally ABI a right angle, the angle BAI is obtained as **15' 0"** very nearly.

When the procedure is carried out in the same way with the Sun at perigee — only the line AB being changed, here put at 1242 — the apparent semidiameter of

the Sun comes out as $16\frac{1}{11}'$. The error in this variation of the semidiameter found in Tycho is therefore imperceptible, since he, from experience, puts this semidiameter of the Sun at apogee at $15'$ and at perigee at $16'$.

Similarly for the Moon: since its true semidiameter was found above to be $16\frac{1}{10}'$ of one Earth-radius very nearly — therefore, in the like right triangle AOV, from the given values $OV = 16\frac{1}{10}'$ (the semidiameter of one Earth-radius, perpetually), together with the following:

- AO at the Moon's apogee = $57^\circ 38'$, at perigee = $54^\circ 23'$ (in Earth-radii [Semid.]), with a right angle at O,

the apparent semidiameter of the Moon is given as:

- at apogee $16' 0''$, at perigee $17' 0''$, very nearly.

Therefore here in Tycho — through some error, I know not what, [arising] after my departure from Bohemia — a whole minute is in excess in the semidiameter of the Moon at perigee; although the same full Moon, when [measured] against the terrestrial shadow, even now by our own method of discovery exhibits a somewhat larger semidiameter, doubtless from one of those causes which I assigned above.

II.

Furthermore, as concerns the terrestrial shadow, and its first variation in this same transit of the Moon — the one it undergoes on account of the Sun's [distance] from the Earth — we shall assess it in this manner. Since the portion of the Sun's true semidiameter KG is established above as $4\frac{10007}{1000}$ degrees [parts], of which AC is one part: therefore, since CK — which is equal to AB — is given as **1334** at the Sun's apogee, it will hold that as KG is to KC, so AC is to AH. Hence in this proportion AH is found to be $277\frac{1}{2}$ Earth-radii. From these, with the mean distance of the Moon from the Earth, **56** Earth-radii, subtracted, there remains RH = $221\frac{1}{2}$ Earth-radii. Therefore, in order that RE may be obtained, it will hold that as HA is to AC, so HR is to RE = **7982** — namely, of those parts of which AC, one semidiameter of the Earth, was assumed as **10000** [parts], a measure we employ for the sake of brevity and greater precision. Since, then, in the right triangle ARE, AR = **560000** (the Moon's mean distance from the Earth) and RE = **7982** are given in the same ratio, together with a right angle at R, the angle of vision of the semidiameter of the shadow RAE — with the Sun at apogee — is given as $49' 0''$ of one degree very nearly. But with the Sun at perigee, the angle of vision RAE is found by the same method of derivation to be $48\frac{4}{5}'$. The difference is therefore

$\frac{4}{5}'$, or 56" — in complete agreement with the calculation of preceding astronomers.

III.

Diagram. A geometric figure illustrating the terrestrial shadow-cone and the determination of the angle of vision of the shadow's semidiameter. At the top is a large circle (the Sun) inscribed with points and chords; from it a long cone tapers downward to a vertex at the bottom (the shadow's tip). A smaller circle is shown crossing the cone partway down (the Moon's transit). Labeled points, from top to bottom:

- G (top left of the large circle): left extremity of the Sun's disk
- K, B (top, near the circle's upper edge): points on the Sun marking the true semidiameter chord
- I (top right of the large circle): right extremity of the Sun's disk
- O, V (middle, where the small circle crosses the cone): O on the cone axis/left side, V to its right — the Moon's position and semidiameter
- C, P, A, D (lower-middle, across the small circle / cone): C on the left side of the cone, P and A near the center (cone axis), D on the right side of the cone
- E, R, Γ (lower, across the narrowing cone): E on the left, R on the axis, Γ on the right
- H (bottom): the vertex (apex) of the shadow-cone

The remaining variation of the shadow is that due to the unequal advance of the Moon from the Earth, [introduced] by the hypothesis. For when the Moon is at apogee, it completes a shorter transit through the shadow than when at perigee, on account of the shape of the shadow itself, which extends into a cone with the Earth as its root, as was shown above. The difference here, however, comes out far smaller than the requirement of the ancients' lunar hypotheses demanded, on account of the equant of the Moon, rightly introduced into our own hypothesis. We track this [difference] at apogee and perigee as follows.

First, since it is established that the new or full Moon at apogee is distant from the Earth by $57^{\circ} 38'$ of Earth-radii: it will therefore hold that as $HA = 268$ Earth-radii (the mean length of the whole shadow that was found) is to $AC =$ one Earth-radius (which here, for the sake of pressing precision, is put at **60000** parts), so $HR = 210^{\circ} 22'$ of Earth-radii is to $RE = 47097$ — the first and third terms being resolved, that is, to the supposition of the mean. Hence, in the right triangle ARE, for the angle of vision RAE — since the two sides about the right angle at R are given

[δεδομένα, "given"] — that said angle RAE is given as **46' 49"**, namely the semidiameter of the true apparent shadow in the transit of the Moon when at apogee.

By precisely the same method of derivation, the semidiameter of the shadow which the Moon traverses when at perigee is found to be **50' 24"**, only the side HR being changed — [computed] for the distance of the perigee Moon from the Earth (when it is new or full) of **54° 23'** of Earth-radii. For HR is the difference between AR and AH at any supposed distance of the Moon from the Earth, and the semidiameter of the shadow at the mean distance of the Sun and Moon from the Earth is **48½'** from the foregoing inquiry. And we wish all these things to be reckoned for the true magnitude of the shadow, so that they may afterward be reduced to the apparent [magnitude] in accordance with experience, taken as a whole [καθ' ὅλα, "as a whole, in general"].

Concerning the precise determination of this [apparent magnitude], while I had been quite anxious about it — intending, as it seemed, to estimate the attenuation of the shadow arising from the inequality of the denser air around the horizons of various places — I find as yet almost nothing fixed, and [nothing] that undergoes a regular change. For when I inquire into the angle of vision of the shadow's semidiameter RAE from the mean distance of the Sun and Moon from the Earth, according to various astronomers, I find it to be:

- According to Ptolemy [Ptolemaeus]: **44⅔'** [44⅔]
- According to the Alphonsines [Alphonsini]: **41'**
- According to Copernicus [Copernicus]: **43¼'**
- According to Reinhold [Reinholdus]: **43¾'**
- According to Tycho [Tychonem]: **44½'**

[Margin: See Copernicus, book IV, chapter 19.]

Now, just as for Tycho and for us experience alone [serves] in this matter, so for the Alphonsines the not-sufficiently-accurate discovery of a certain Arab philosopher assists [them]. As for the ancients' measurement of the diameter of the shadow, this must be noted: if they measured it — as they doubtless did — by the passage of the Moon deep through the shadow, they always fell short of its true magnitude, on account of the swifter motion of the new and full Moon, first of all noticed by us, and computed from the Moon's own lingering in the shadow.

[Caput IX, continued — page 299]

Therefore, since the variation of the shadow's semidiameter in this same transit of the Moon — taken according to its mean distance from the Earth — was not, by the observations of various astronomers, perceptible to any great degree, it is credible that the vapid and gross air around the horizon (on account of which the attenuation of the shadow's diameter in the Moon's transit occurs at about $8\frac{1}{6}^\circ$ of Earth-radii, as we likewise hinted above) is not very unequal to itself — so that the Alexandrian refractions in the Sun, which I asserted above, may seem to be confirmed even from this. There now follows the table of the semidiameters of the Sun, the Moon, and the shadow, for any distance whatever from the Earth in eclipses.

Table of Semidiameters in Eclipses

[Tabula semidiametrorum in eclipsibus]

The table is read in two halves spanning the full page width. The left half gives, against the common anomaly of Sun and Moon [Anomalia ☉ & ☾] (in signs S. and degrees G.), the Sun's semidiameter [Solis] (minutes M., seconds S.) and the Moon's semidiameter [Lunae] (minutes M., seconds S.). The right half gives the shadow's semidiameter [Umbræ] (minutes M., seconds S.), the variation of the shadow [Variatio umbræ] (in seconds S.), and the corresponding anomaly [Anomalia ☉ & ☾] (degrees S. and signs G.) read in reverse. The two anomaly columns are complementary (left increasing 0s $0^\circ \rightarrow 6$ s 0° ; right decreasing $12^\circ \rightarrow 0^\circ$ with signs descending), so each printed row carries two argument-values.

Left half — Sun and Moon semidiameters vs. anomaly:

Anomaly ☉&☾ (S.)	Anomaly ☉&☾ (G.)	Sun M.	Sun S.	Moon M.	Moon S.
0	0	15	0	16	0
0	6	15	0	16	0
0	12	15	1	16	1
0	18	15	2	16	2
0	24	15	3	16	3
1	0	15	4	16	4
1	6	15	6	16	6
1	12	15	8	16	8

Anomaly ☉&☾ (S.)	Anomaly ☉&☾ (G.)	Sun M.	Sun S.	Moon M.	Moon S.
1	18	15	10	16	10
1	24	15	12	16	12
2	0	15	15	16	15
2	6	15	18	16	18
2	12	15	21	16	21
2	18	15	24	16	24
2	24	15	26	16	27
3	0	15	29	16	30
3	6	15	32	16	33
3	12	15	35	16	36
3	18	15	38	16	39
3	24	15	41	16	42
4	0	15	43	16	44
4	6	15	46	16	46
4	12	15	49	16	49
4	18	15	51	16	51
4	24	15	53	16	53
5	0	15	55	16	55
5	6	15	57	16	57
5	12	15	58	16	58
5	18	15	59	16	59
5	24	15	59	16	59
6	0	16	0	17	0

Right half — shadow semidiameter, variation of shadow, and (descending) anomaly:

Umbra M.	Umbra S.	Variatio umbrae (S.)	Anomaly ☉&☾ (S.)	Anomaly ☉&☾ (G.)
43	0	0	12	0
43	1	0	11	24
43	2	1	11	18

Umbra M.	Umbra S.	Variatio umbrae (S.)	Anomaly ⊙&ℳ (S.)	Anomaly ⊙&ℳ (G.)
43	6	2	11	12
43	9	3	11	6
43	14	4	11	0
43	19	6	10	18 [?]
43	25	9	10	18
43	33	12	10	12
43	41	15	10	6
43	49	18	10	0
43	58	21	9	24
44	9	24	9	18
44	20	27	9	12
44	31	30	9	6
44	43	34	9	0
44	55	37	8	24
45	7	40	8	18
45	19	43	8	12
45	31	46	8	6
45	44	49	8	0
45	56	51	7	24
46	7	52	7	18
46	17	53	7	12
46	27	54	7	6
46	36	55	7	0
46	44	55	6	24
46	50	56	6	18
46	55	56	6	12
46	58	56	6	6
47	0	56	6	0

Parsing notes: The table is degraded and the original numerals are partly broken; many digits required reconstruction from the smooth monotone progressions of

each column. The Sun and Moon semidiameter columns rise monotonically (Sun $15'0'' \rightarrow 16'0''$; Moon $16'0'' \rightarrow 17'0''$) and the shadow column rises $43'0'' \rightarrow 47'0''$ as the variation-of-shadow column rises $0'' \rightarrow 56''$. The two anomaly columns are complementary. In the right-half anomaly column the printed degree value at the row paired with shadow $43'19''$ reads "18" where a monotone descent from $11^\circ 0''$ to $10^\circ 18''$ to $10^\circ 12''$ would expect a value between; transcribed as printed and flagged [?] (likely a compositor repetition of the following row's "18"). Several cells on this scan are faint; values given are best readings against the column trends.

[New section, page 300]

On the Diminishing of the Moon's Semidiameter at the Ecliptic New-Moon and Its Enlarging at the Full-Moon — According to the Temper of the Air Successively Thickened in Different Places

[De lunae semidiametro in novilunio ecliptico diminuenda, plenilunio autem dilatanda, idque juxta aeris successive in diversis locis incrassati temperiem.]

We have expounded above at length, by optical reasoning but most of all relying on experience, how these phenomena in different places depend on the unequal temperament of the air. In accordance with that [experience], we here assign the decrease of the Moon's semidiameter at the ecliptic new-Moon, and its increase at the full-Moon — below and above the magnitude of its mean apparent size which the foregoing table comprises — and indeed both, for the various elevations of the world-pole [i.e. latitudes], always distributing, as it were, two-thirds of the semidiameter of the ecliptic new-Moon to the same full-Moon to be augmented; and this only for the Moon's semidiameter as it extends itself outside the shadow, which the phenomena of particular lunar eclipses, by our careful observation here, likewise demand for themselves.

No one ought to wonder at that sudden alteration which intervenes at the elevation of 50 and 56 degrees beyond the usual succession, since experience — to which we always appeal — teaches us the same. And why not? For in these maritime regions the thickness of the air, which is the cause of this appearance, suddenly bears down more heavily. Although this same [thickness] is packed more and more densely the nearer you go toward the pole, it nonetheless does not respond in a proportion answering to the successive change — a proportion that could produce an equal [variation] of the Moon's semidiameter. For

otherwise this same [air] would entirely exhaust every solar eclipse to be expected under a pole-elevation of 80 [degrees]. Since this would be incredible — and since experience in farther Norway and Iceland, which has hitherto become known to us, does not demand so great a successive change either — I have therefore judged that the individual values must here be adjusted to it as nearly as I could. And because the unequal apparent size of the luminaries at apogee and perigee introduces almost no perceptible change through these decrements and increments of the Moon, these [values] are therefore fitted to the Moon's semidiameter just as they come out, for the corrected pole-elevations, by the following little table.

Little table of the change in the Moon's semidiameter during eclipses [Tabella mutationis semidiametri lunæ in eclipsibus]

The header row gives the corrected pole-elevations [Elevat. poli] from 20 to 74 degrees. The body rows give: the amount to be subtracted from the Sun's semidiameter [Solis subt.]; the amount to be added to the Moon's semidiameter within the eclipse [Lunæ add.]; and the amount outside the eclipse [extra eclip.]. All values are in minutes (M) below the degree (G) column-heads.

Pol e- ele vat ion (°)	20	30	40	46	50	53	56	59	62	65	69	74
Su n's, to su btr act	0	10	30	40	50	2	3½	4½	5½	5½	6	6⅓
Mo on' s, to ad d (wi thi	0	6	20	30	34	1½	2½	3	3½	3½	4	4⅞

Pol
e-
ele
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ion

(°)	20	30	40	46	50	53	56	59	62	65	69	74

n
ecl
ips
e)

Parsing notes: The "Solis subt." and "Lunæ add." rows share a single bracketed left label (Semid. lunæ in eclips.). The first five columns (20–50°) read as small whole minutes (0, 10, 30, 40, 50 for the Sun; 0, 6, 20, 30, 34 for the Moon), while the remaining columns (53–74°) read as fractional minutes (2, 3½, 4½ ... etc.). This abrupt shift from whole tens of seconds to single-digit-plus-fraction minutes across the row is suspicious — the early columns appear to be in seconds and the later in minutes, or the table is mis-set; flagged as a table-reprocess candidate. The Moon's value at 74° reads 4½ [?]. The "extra eclips." sub-row label appears but its values coincide with the "Lunæ add." row as rendered; the two Moon sub-rows could not be cleanly separated at this resolution.

Table of the synodic revolution of the Moon to the Sun [Tabula synodicæ revolutionis lunæ ad solem]

Columns: month [Menses]; common-year offset [Cō = communis]; leap-year offset [Bis. = bissextilis]; Time [Tempus] of conjunction (Days, Hours, Minutes, Seconds); the Sun's anomaly [Anom. solis] (Signs, Degrees, Minutes, Seconds); the Moon's anomaly [Anom. lunæ] (S, G, M, S); the Moon's latitude [Latit. lunæ] (S, G, M, S).

Month	Brisbane						Sydney				Melbourne				Perth			
	C	B	D	H	M	S	S	G	M	S	S	G	M	S	S	G	M	S
January	31	31	29	12	44	3	0	29	6	21	0	25	49	0	1	0	40	14
February	59	60	59	1	28	6	1	28	12	41	1	21	38	0	2	1	20	28
March	90	91	88	14	12	9	2	27	19	2	2	17	27	1	3	2	0	42
April	120	122	118	2	56	13	3	26	25	23	3	13	16	1	4	2	40	56
May	151	155	154	15	40	16	4	25	31	43	4	9	5	1	5	3	21	9
June	181	181	181	4	2	1	5	2	3	4	5	4	5	1	6	4	1	2

Month	Sunom.						Monom.						Moonlat.					
	C ₀	Bis.	D	H	M	S	S	G	M	S	S	G	M	S	S	G	M	S
une	81	82	77		4	9		4	8				4					3
July	21 22	21 33	20 66	17	8	22	6	23	44	25	6	0	43	1	7	4	41	37
August	24 33	24 44	23 66	5	52	25	7	22	50	45	6	26	32	2	8	5	21	51
September	27 33	27 44	26 55	18	36	28	8	21	57	6	7	22	21	2	9	6	2	5
October	30 44	30 55	29 55	7	20	31	9	21	3	27	8	18	10	2	10	6	42	19

Month	Brisbane						Sydney						Melbourne					
	C	B	D	H	M	S	S	G	M	S	S	G	M	S	S	G	M	S
Nov	334	335	324	20	4	35	10	20	9	47	9	13	59	2	11	7	22	33
Dec	365	366	354	9	48	38	11	19	16	80	19	98	24	28	0	8	2	47
Half-m	—	—	14	18	22	2	04	13	31	11	92	22	54	30	65	15	20	7

[illegible]

Table of the Moon's hourly motion at conjunction and opposition,
together with the half-day before and after [Tabula motus horar. ☾ in
♌ & ♍, & simul dimidii diei ante & post]

Ar g. (G)	0: in ♂ M S	0: Ult ra M S	1: in M S	1: Ult ra M S	2: in M S	2: Ult ra M S	3: in M S	3: Ult ra M S	4: in M S	4: Ult ra M S	5: in M S	5: Ult ra M S	An o. (G)
0	27 43	27 12	28 4	27 34	29 1	28 46	30 24	30 42	31 51	32 53	32 58	34 45	30
3	27 44	27 12	28 8	27 39	29 9	28 57	30 33	30 55	31 58	33 5	33 7	34 53	27
6	27 45	27 13	28 13	27 45	29 17	29 8	30 41	31 7	32 6	33 18	33 7	35 0	24
9	27 45	27 14	28 18	27 51	29 25	29 19	30 50	31 20	32 14	33 30	33 11	35 8	21
12	27 47	27 15	28 23	27 58	29 33	29 29	30 58	31 33	32 21	33 41	33 14	35 14	18
15	27 48	27 16	28 29	28 5	29 41	29 41	31 7	31 46	32 28	33 54	33 17	35 18	15
18	27 51	27 19	28 35	28 12	29 49	29 52	31 17	32 0	32 35	34 5	33 20	35 24	12
21	27 53	27 22	28 41	28 20	29 58	30 4	31 26	32 15	32 41	34 17	33 21	35 29	9
24	27 57	27 26	28 47	28 29	30 6	30 17	31 34	32 28	32 47	34 26	33 23	35 32	6
27	28 0	27 30	28 54	28 38	30 15	30 29	31 43	32 41	32 53	34 36	33 24	35 34	3
30	28 4	27 34	29 1	28 46	30 24	30 42	31 51	32 53	32 58	34 45	33 24	35 37	0
(bl oc k in de x, fo ot)	11		10		9		8		7		6		

Parsing notes: This is a double-entry hourly-motion table; the six column-blocks at top (0–5) continue as blocks 11–10–9–8–7–6 read from the foot upward, and the right-edge "Ano." column counts 30→0 against the left-edge argument 0→30. The "in σ & σ° " / "Ultra" pairing inside each block follows the printed sub-heads. Several cells in blocks 4 and 5 were hard to resolve at this resolution (e.g. block 5 column entries around rows 6–9); the values given are best readings. Flagged as a table-reprocess candidate. Monotone progression of the leftmost-block "in σ & σ° " column (27 43 → 28 4) is smooth; block 5 "Ultra" rises 34 45 → 35 37 smoothly, supporting the readings.

The rules of computation [πραξις] for the eclipses of the Moon and Sun, in briefest form

[Greek: ψηφοφορίας (psēphophorias, "reckoning" or "calculation by counting-pebbles")] — the heading reads *Præcepta ψηφοφορίας, seu calculi eclipsium lunæ & solis, brevissima*, i.e. "The precepts of reckoning, that is, of the briefest computation of eclipses of the Moon and Sun."

Having now set out everything by which the eclipses of the luminaries, and the variations of those eclipses, can in any way be caused, I shall append the long-sought method of computing the failing of each luminary [deliquium]. I do this with the lightest possible labor, lest the difficulty that lovers of the stars [Astrophili] are otherwise forced to experience in this matter should wholly drive them away from the study of astronomy — the worthiest and most divine of all studies in philosophy.

The computation of eclipses turns chiefly on two things, both arising from the motion of the two luminaries. The first is the ecliptic syzygy itself, which, from the longitude of the luminaries beneath the ecliptic, supplies the indication of the time. The second is the representation of the semidiameters of the Sun, the Moon, and the shadow — together with the true latitude of the Moon in a lunar eclipse, and its apparent latitude in a solar obscuration — which bears on the magnitude and duration of the failing.

I.

[Margin: Judgment of a future failing — *Iudicium futuri deliquii*.]

As to the first point: since the motions, drawn from these restored theories and tables of the luminaries, are continuously arranged into ephemerides for times, and especially for future ones, I do not think it necessary to labor any further over the mean ecliptic syzygies. Rather, as soon as it has become known from the

motion of the Moon's latitude — which truly will lie within $12\frac{1}{4}^\circ$ from some node in the case of a lunar eclipse, and within $6\frac{1}{4}^\circ$ visibly in the case of a solar obscuration, if any is to occur — that an ecliptic full-Moon or new-Moon is at hand, the computation is to be directed to the true syzygy in either kind of eclipse. Then, once the reduction has been made by its proper table from the ephemerides here to the meridian of the place, the time of the true ecliptic syzygy becomes known.

[Margin: On this reduction see Cap. 3 at the end — *De hac reductione vide c. 3. in fine.*]

II.

[Margin: Investigation of the ecliptic syzygy — *Exploratio syzygiæ eclipticæ.*]

Next, for this time as offered and not yet equated, the Sun's true place from the equinox, and the Moon's from the Sun, are each to be sought from one's own restored tables. This is done for the sake of greater certainty, so that the motions in eclipses may be weighed most exactly. Here it must be noted that, in an ecliptic syzygy, only the prosthaphaeresis of the Moon — which belongs to the first column and to eclipses — is owed, and it is fitting to use it according to the simple offered anomaly of the Moon.

But if, without the continuous computation of motions that ephemerides comprise, anyone should wish to reckon eclipses for ages long past, let him consult the tables of the synodic revolution of the Sun and Moon, of Reinhold [Reinhold] and others. Or rather, let him himself investigate the motions of the luminaries at the beginning of the year in which he wishes to learn that such an eclipse occurred, and reduce them to synodic revolutions. Then let him augment these by monthly and half-revolutions for a time — by the benefit, that is, of the table which I have transferred here from Tycho's [Tycho Brahe] *Progymnasmata* and appended to the rest for this use — until he detects nodes near, as it were, the indicators of an eclipse. Afterward, let him correct the computation from these restored motions of the luminaries.

III.

[Margin: Finding the Moon's hourly motion before and after the eclipse — *Inventio motus horarii lunæ ante & post eclipsin.*]

Now, keeping the time previously supposed at the meridian of the place — which, according to our restoration, especially the lunar one, cannot lie far from the mean ecliptic syzygy — you may hunt down the Moon's hourly motion from the

Sun either by the table set out above, or else surely and compendiously in another way, as follows. For in reducing the time and the incidental quantities in either eclipse, it is better that the Moon's true motion be known for one hour both before and after the ecliptic syzygy. The Moon's mean hourly motion is therefore 30 min. 29 sec., and doubled it is 1° 0 min. 58 sec. To this corresponds, first, the equation of anomaly (since here, in so small an interval, the ratio of the equation can be [neglected]) of [continues on next page] 16 min. 30 sec.; likewise the variation of the Moon of 44 sec. Hence the mean anomaly of the Moon corresponding to one hour is 32 min. 40 sec. When the equation is added to this, there always remains the hourly motion of the equated anomaly, before and after the true syzygy, of 49 min. 10 sec. Therefore, if you always multiply the difference of the Moon's prosthaphaeresis of the first column — the difference which presents itself, at the sign and degree of the anomaly of the mean syzygy, just below it for the hourly motion before, and just above it for the hourly motion after the syzygy — into 49 min. 10 sec., there emerges the proportional part. This proportional part, namely 31 min. 13 sec., is to be fitted to the mean hourly motion of the Moon for either of the two hours, combined with the variation.

[Margin: Reduction of the lunar motions to the conjunction or opposition — *Reductio motuum lunarium ad coniunctionem aut oppositionem.*]

This is done according as the Moon has been placed, in the hypothesis, above or below its mean distance from the Earth — which is sufficiently understood from its own anomaly. For above, the proportional part is to be subtracted; below, it is to be added. And this way seems the easiest and safest. There is no reason for anyone to be anxious about deriving the proportional part from the excess and the proportional scruples, since here that excess can scarcely ever rise beyond 1½ [?] min. By these hourly motions immediately surrounding the middle of the failing, the Moon's reduction to the mean ecliptic is also made exactly, as will be clear in the examples.

IV.

[Margin: Estimation of the quantity of the failing — *Æstimatio quantitatis deliquii.*]

Once the hourly motions of the Moon and the true ecliptic syzygy have been found according to the foregoing, one must press on to the quantity of the failing and the time of its duration. Here the other thing comes into the practice that we said is required — concerning the Moon's latitude, and likewise the magnitude of

the semidiameters of the luminaries and of the shadow — and these can be sought from the tables above.

The quantity of a lunar eclipse, then, is to be judged from the Moon's own true latitude, together with the apparent semidiameters of the Moon and the shadow; but the magnitude of a solar failing is judged from the visible latitude of the Moon, together with the diameters of the Moon and of the Sun. All these are to be sought from the tables above: the semidiameters by the equated anomaly of the luminaries, the Moon's latitude from the first column of its table by the true motion of latitude. Now when the Moon's latitude is taken away from the sum of the two semidiameters, the maximum quantity of the failing remains. If you wish to have this same value in digits — of which the whole luminary is reckoned at 12 digits — then it will be: as the whole diameter of the obscured luminary is to 12, so the remaining quantity is to the fourth term, that is, to the digits of obscuration with their scruples.

It must be noted, moreover, that in a partial lunar failing, that small particle which we added above for the various pole-elevations must always be subtracted, on the deficient side, from the Moon's latitude at the middle of the eclipse. But in a solar defect, the Moon's semidiameter must always be diminished by that particle which we there left for the appropriate pole-elevation.

V.

[Margin: Knowledge of the duration of eclipses — *Cognitio durationis eclipseos.*]

Furthermore, to foreknow how long the defect will last, and likewise how much time is required for the Moon's incidence into, and tarrying through, the shadow in this total eclipse: for this purpose, then, let both the sum of the semidiameters and the Moon's latitude at the mean ecliptic syzygy be resolved into seconds. From the square of the sum of the semidiameters let the square of the Moon's latitude be subtracted, each squared separately, and there remains a square whose root shows the Moon's course in its own orbit from the beginning to the middle, or from the middle to the end — a course to be converted into time by the Moon's hourly motion.

In a total lunar eclipse, however — which is recognized when what remains from the difference of the semidiameters of the Moon and the shadow exceeds the Moon's latitude, after the Moon's semidiameter has first been increased by that particle which we added above for the various pole-elevations (though it will not be increased beyond this when, namely, a total failing of the Moon is foreknown) — therefore, since the beginning of the total obscuration, or again the beginning

of the Moon's first reappearance from the shadow reckoned from the middle (which time we call the half-tarrying in the shadow) is to be known: the said resolved difference of the semidiameters will be multiplied into itself, and from this number there will be subtracted (as before) [continues on next page]

[Margin: *Lib. 4. revolut. cap. 32. Arith. figurata cap. 3. Præcept. tabul. Rheinb. 52. & 63.* — Book 4 of the *Revolutions* [Copernicus, *De Revolutionibus* IV.32]; figurate arithmetic, ch. 3; precepts of the tables of Reinhold, 52 & 63.] the squared number of the latitude; for the root of the remainder will show the half-tarrying in the Moon's motion, likewise to be converted into time by the Moon's hourly motion. The demonstration of this is to be sought from Copernicus [Copernicus]; but the practice is compendiously aided by figurate arithmetic concerning square numbers, and so forth. The same can also be carried out more briefly by the tables of Reinhold [Reinhold], rightly accommodated.

VI.

[Margin: Knowledge of the true or apparent latitude of the Moon at the beginning and end of the eclipse — *Notitia veræ aut visæ latitudinis lunæ ad initium & finem eclips.*]

Finally, so that the true and apparent latitude of the Moon at the beginning and end of the eclipse may be established: let the Moon's motion of longitude found between the middle and the beginning be subtracted from the true or apparent motion of latitude at the middle, and from the remainder let the Moon's latitude at the beginning be sought by the first column only of the table of latitude above — the column assigned to the conjunction and opposition of the Moon with the Sun.

[Margin: This precept pertains chiefly to a lunar eclipse; another is given below on a solar eclipse — *Hoc quidem præceptum ad eclipsin lunæ maxime pertinet. Aliud vide infra de eclipsi solis.*]

And although the precession of the nodes in the meantime, in the foregoing reckoning, varies the latitude almost imperceptibly, it can nevertheless be taken into account — by adding, namely, $2\frac{1}{2}$ min. to the Moon's motion of latitude for each of the hours by which it exceeds the Moon's hourly motion of magnitude from the Sun, whether the Moon's motion of latitude is to be reduced backward or forward from the middle of the eclipse. For in the same way the Moon's motion of latitude is known at the end of the eclipse, and therefore the latitude itself as well — when the same motion of the Moon found from the middle to the

beginning is added to the true motion of latitude that agrees with the middle of the failing.

Thus far we have delivered the precepts in general concerning the defect of both the Sun and the Moon. While these suffice for the failing Moon, they by no means suffice for the Sun, on account of the entanglement of the lunar parallaxes — not unless these are first both found, and then applied to the ecliptic path according to longitude and latitude. For here it is not the true but the apparent conjunction of the luminaries that is required. And although above we treated of the parallaxes of the luminaries in general — both as to finding them in the circle of altitude, and as to resolving them according to longitude and latitude — nevertheless a more compendious method is offered in eclipses: for one must seek not only the Moon's distance from the Earth in semidiameters (and therefore the parallaxes in the vertical circle), but also the angle at the ecliptic, on which the resolving of the parallax chiefly depends — namely, when the visible Moon is deprived of almost any perceptible latitude, as happens in solar failings.

First, then, for the Moon's distance from the Earth, let there be taken at least the distance between the second and third limits, which is $3^{\circ} 15$ min. of Earth-semidiameters. Then, with the Moon's coequal anomaly diminished (as before) by a semicircle and the residue halved, the proportional scruples are sought from the table of the nodes (as above). When these have been multiplied into the difference of $3^{\circ} 15$ min., they leave a proportional part always to be added to the third limit — for instance, $54^{\circ} 23$ min. of semidiameters. Hence, with the corrected semidiameters of the Moon's distance from the Earth (as above), its parallax in the circle of altitude is sought, from which the Sun's parallax at the same altitude is also subtracted.

Now since the table of the parallaxes of the luminaries is computed for altitudes above the horizon, and here the Sun's altitude alone is supposed: if therefore anyone should wish to work out the Moon's ecliptic parallaxes in the circle of altitude more scrupulously than the ancients, let him add to the Moon's own altitude the parallaxes belonging to it at the supposed altitude of the Sun — or rather let him take account of [Greek: ἐν πλάτει (en platei, "in breadth," i.e. with respect to latitude)] in the same vertical. Thus he will obtain the Moon's true altitude above the horizon as well, and then the parallaxes precisely agreeing with it in the table.

[Margin: Method of investigating the Sun's altitude in a solar eclipse — *Ratio investigandi altitudinem solis in deliquio solari.*]

Furthermore, you will hunt down — most compendiously of all — the Sun's altitude above the horizon, and likewise the angle that the vertical circle makes with the ecliptic (on which, above too, the rest of the Moon's parallax is distributed into longitude and latitude), if you first, from the right ascension of the mid-heaven (which is given by the benefit of the supposed time), seek the culminating point of the ecliptic together with its declination. From this — as before, given the pole-elevation of the place and the conjoined place of the luminaries — two sides of the spherical triangle to be resolved immediately present themselves, together with the contained angle that the meridian makes with the ecliptic, in the table of the doctrine [of triangles] [continues on next page]

[Margin: *Lib. 2. Sphær. probl. 4. c. 1.* — Book 2 of the *Spherics*, problem 4, ch. 1.]

of the doctrine of triangles [doctrinæ sphæricæ], computed for each individual degree of the ecliptic. In resolving the said spherical triangle — for whose data it will everywhere suffice here to be given and worked through in whole degrees and minutes [in solis gradibus ac scrupulis primis] — the side opposite this given angle will be the complement of the altitude of the luminaries above the horizon, while the angle that looks toward the side lying on the meridian is the one that the vertical circle [verticalis] forms with the ecliptic. From this, thereafter, the Moon's parallax from the Sun is distributed along the circle of altitude into longitude and latitude as often as required, and is fittingly adjusted to the Moon's true motion so that the apparent place [visus] may be obtained.

Note, however, that although a general criterion concerning the ninetieth degree of the ecliptic [nonagesimus gradus] is drawn from this — namely that the signs of the Zodiac from ♈ to ♏ require the same point before their transit across the meridian, while afterward the remaining half of the Zodiac (that is, from ♏ to ♈) requires it after the meridian transit — yet from the character of the angle thus found one knows for certain whether the luminaries were in the eastern part of the ecliptic, where the parallax of longitude always increases the true motion of the phenomenon by its own quantity, or in the part to the west of the ninetieth degree, where the same parallax always diminishes the true motion by its own quantity. For only when that angle has been found:

				The luminaries are in the eastern part of the ecliptic
			If acute	
Before	the meridian	acute	(when the angle found is)	
After	the meridian	obtuse	If obtuse	The luminaries are in the western part of the ecliptic

(That is: an angle found acute **before** the meridian, or obtuse **after** it, places the luminaries in the eastern part; an obtuse angle **before** or acute **after** places them in the western part.)

[Margin: A criterion for which part or quadrant of the ecliptic the luminaries were in, with respect to the ninetieth degree from the horizon.]

This method of working out and distributing parallaxes seems to me an easy one. If some prefer to be led by other routes, Reinhold [Erasmus Reinhold], Kepler [Johannes Kepler], and others are at hand. And whom will the variety not sometimes delight, since several roads exist in the doctrine of the sphere for reaching those two things we desire? The demonstration of this method, together with its practice, will accompany the worked example of a solar eclipse below.

So much in general concerning the working-out and distribution of the Moon's parallax from the Sun at any given moment in a solar eclipse. Now we shall briefly and in order append what must specifically be carried through in such an eclipse.

1.

At the time of the true conjunction [vera synodus] of Sun and Moon, find the Moon's parallax of longitude from the Sun. Converted into time by means of the Moon's hourly motion, this is then applied to the time of true conjunction — to be subtracted if the luminaries are found in the part of the ecliptic east of the

ninetieth degree, and added in the western part. In this way the estimated apparent time of mid-eclipse emerges, which usually departs the less from the true time the smaller the parallax of longitude is on either side.

[Margin: Reduction of the true conjunction to the apparent.]

2.

So that the Moon's apparent motion from the Sun over the interval between the true and the apparent conjunction may be obtained, find once more — for the estimated time of apparent mid-eclipse — the Moon's parallax of longitude from the Sun. If this is smaller than the first (as in the eastern quadrant of the ecliptic), add it to the Moon's parallax of longitude found earlier; but if it is larger (as in the western part of the ecliptic), subtract it from that earlier value. What remains is the apparent motion of longitude corresponding to the interval assumed.

Then correct the estimated time of visible conjunction and reduce it precisely to its midpoint: multiply the interval of time found earlier by the earlier parallax of the Moon's longitude, and divide the product by the Moon's apparent motion found afterward. For the small change that meanwhile can occur in the time on account of the further slight variation of parallax is negligible. The Moon's parallax of longitude at the time of apparent conjunction must equal the true distance of the luminaries from each other, and conversely; and this ought to serve as a touchstone [δοκιμασία (dokimasia, "test" or "proof")] and rule for the visible conjunction's having been correctly determined.

[Margin: An exact working-out of the reduction.]

3.

Once the time of visible conjunction has been most correctly determined and (as seems convenient, before we proceed further) equalized, one seeks both the Moon's latitude — in minutes and seconds, by the Moon's true motion of latitude corresponding to the time of apparent conjunction — and the Moon's parallax of latitude from the Sun. This latter is always subtracted from a northern latitude of the Moon and added to a southern one (for us, that is, and for all who view the ecliptic below the zenith toward the south; for the rest it is the contrary). Thus results the Moon's apparent latitude at apparent mid-eclipse.

[Margin: Inquiry into the Moon's latitude at mid-eclipse, by the precept above, without the excess.]

4.

This apparent latitude of the Moon is then compared with the corrected and summed semidiameters of the luminaries. From this, by the fourth precept above, the magnitude of the solar obscuration [*quantitas deliquii solaris*] is also known; and by the fifth, the Moon's motion from the Sun to the beginning and end is derived as an equal quantity (the same here as the scruples of incidence [*scrupula incidentiæ*] and of mean duration). When this is resolved into time by the Moon's true hourly motion, it gives the estimated time of the beginning and end of the eclipse, once it has been duly applied to the apparent time at mid-eclipse.

[Margin: Knowledge of the magnitude of the solar obscuration. The Moon's equal motion from mid-eclipse to beginning and end. The estimated time of the beginning and end of the eclipse.]

5.

Further, for both the beginning and the end, investigate each of the Moon's parallaxes from the Sun — both of longitude and of latitude. Keeping the parallax of latitude, but converting that of longitude into time by the hourly motion, and applying it correctly to that estimated time (according, that is, to what we set before the eyes above in the general application of the parallax of longitude, where the one angle is to be found from whose character that parallax is distributed into longitude and latitude): then the time of incidence from beginning to mid-eclipse, and of emersion from mid-eclipse to end, and thus of the whole duration, becomes known — and that precisely enough, considering what the difference of the parallax of longitude between the estimated and the apparent beginning and end will introduce by thus varying the time.

[Margin: Knowledge of the time from mid-eclipse to beginning and end; likewise of the whole duration of the solar eclipse.]

6.

Finally, so that the Moon's apparent latitude at the beginning and end of the solar obscuration may be disclosed compendiously, one must enter the upper table of the Moon's latitude, from its beginning, with the mean motion of the Moon's latitude — corresponding, that is, to the intervals from mid-eclipse to beginning and from mid-eclipse to end. (This, as it seems to me, is here more correct to use than either the motion of longitude or the scruples of incidence, since even the inequality of the time-measure that falls in here is nearly compensated.) The latitude-scruples, without the excess, etc., are to be extracted thence and duly applied to the Moon's apparent latitude at mid-eclipse, according as its

disposition relative to one or the other of the nodes may be — taking account at the same time of the difference of the Moon's parallaxes of latitude from the Sun, both at the beginning and at the end, from that same mean apparent latitude (which is why we directed above that those values, once found, be kept for this use). And so too, through the just application of each difference from the mean, there emerges the Moon's apparent latitude at the beginning and end.

Nothing more then remains except the common delineation of each eclipse, of Moon and of Sun. Here, once the corrected semidiameters and likewise the Moon's latitude — with its designation north or south at the beginning and end of the eclipse — have been learned from the foregoing, let a common measure be laid out, extended in first minutes and their customary subdivisions along a straight line equal to one degree, or to the sum of the semidiameters; then, with the aid of compasses and ruler, the individual items are easily transferred onto the paper, and so the eclipse is set before the eyes as if in a single figure. Since this matter is plainly mechanical, it will conveniently be shown in its place below, among the individual worked examples.

[Margin: The Moon's apparent latitude at the beginning and end in a solar obscuration — how it is disclosed. A brief instruction in how eclipses are to be drawn out diagrammatically on paper.]

Example, in the eclipse of the Moon in the year of the Savior 1612, on the 4th day of the month of May, in the evening.

The precepts for computing eclipses given above must now be illustrated by examples, and any deficiency in them, if there be any, supplied. For both will rightly be expected and furnished from examples, which above all instruct beginners in this discipline. We adduce the example of the obscuration of each luminary from those eclipses that occurred in the year of the Lord 1612, in the month of May, in both luminaries, and which were observed here by us with what diligence could be managed. The first was of the Moon, whose mid-eclipse — according to the Ephemerides of the most distinguished gentleman Mr. David Origanus [David Origanus], and the restored motions of the luminaries therein — would have occurred at our meridian on 4 May, in the evening, at 10 h. 26 m., mean time. That this full moon would be eclipsed is sufficiently shown by the Moon's true motion of latitude, approaching the Dragon's Tail [cauda Draconis, the descending node] and distant from it by not even eight whole degrees.

For this time, then, the following motions were found from the tables above:

	Sig.	Gr.	Mi.	Sec.
Anomaly of the equinoxes [Anomalía æquinóctio rum]	6	17	45	6
Equation of the equinoxes, additive			8	20
Mean motion of the Sun	1	22	45	59
Anomaly of the Sun	10	16	59	0
Prosthapha eresis of the Sun, additive		1	21	35
Longitude of the Moon from the Sun	5	26	27	30
Anomaly of the Moon	8	18	16	14
Motion of the Moon's latitude	5	17	16	51
Ecliptic prosthapha eresis of the Moon, additive		4	53	51
Difference of the prosthapha		3	32	16

	Sig.	Gr.	Mi.	Sec.
ereses of Sun & Moon				
Which, when added to the longitude of the Moon from the Sun, yields that [longitude]	5	29	59	46
The Moon is therefore distant from true full moon, according to our calculation and the supposed time, at least		0	0	14

[Margin numerals beside the table: I., II., III.]

For the Moon's hourly motion before and after this eclipsed full moon — at the Moon's anomaly 8 sig. 18° &c.:

Difference of prosthaphaere sis for one degree		Mi.	Sec.
	below [infra]	0	59
	above [supra]	0	48
When this is		49	10

Difference of
prosthaphaere
sis for one
degree

Mi.

Sec.

multiplied
separately into
the anomaly

And when the proportional parts have been added to the Moon's hourly motion, taken as mean — 31 min. 13 sec. — there results the Moon's true hourly motion from the Sun, just before mid-eclipse, of 32 min. 1 sec., but just after it 31 min. 52 sec., with a slight difference intervening here.

Now since we found the difference in the Moon's motion appropriate to the supposed time, reckoned from the true opposition of the luminaries, to be 0 m. 14 sec., this, divided by the hourly motion of 32 m. 1 sec., shows that 26 sec. were wanting in the time. The true opposition of the luminaries, then, according to our restoration, occurred at 10 h. 26¼ m. [recte: h. 10 m. 26¼], on 4 May; to which there corresponds, from the values above reckoned from the equinox:

	Gr.	Mi.	Sec.	
true motion of the Sun	24	16		□ nearly
true motion of the Moon	14	16		□

	Sig.	Gr.	Mi.	Sec.
The Moon's equated anomaly [Anomalia Lunæ coæquata]	8	23	12	
The Moon's true motion of latitude	5	22	12	

[Margin: IV.]

From these are established the corrected semidiameters (the variation of the shadow being excepted):

	Mi.	Sec.	
of the Moon	16	34	
of the shadow	44	48	
Likewise the Moon's true latitude	40	56	North [Sept.]
Therefore the sum of the semidiameters of Moon & shadow	61	22	
Difference from the latitude	20	26	

[Margin: V.]

But if 2 min. 16 sec. be added to the Moon's latitude on account of the thicker air in this place — which is most convenient to do here, before the delineation of the following eclipse — the difference from the summed semidiameters will be 18 mi. 10 sec., which is the part actually deficient here, to which correspond eclipsed digits of 6 gr. [recte: digits] 35 mi.

Further, since this particular lunar eclipse is found only for the notice of the half-duration, the care will be — therefore the Moon's true latitude, namely 40 mi. 56 sec., resolved into second-scruples — that these are:

	Mi.	Sec.
Resolved sum of the semidia meters of Moon & shadow,	2456 (squared) →	to which the square correspo nds 6031936

	Mi.	Sec.
namely 61 min. 22 sec., is		
3822 (squared) →	the square correspo nds	13557124
Differenc e		7525188
Of this the root in second- minutes is		2743
Therefor e, reduced, they are	45	43

Parsing notes: This passage tabulates a Pythagorean-style computation. The Moon's latitude 40'56" reduced to seconds is squared ($2456'' \rightarrow 6\,031\,936$); the summed semidiameters 61'22" reduced to seconds is squared ($3682''$ — printed as "3822") to 13 557 124; their difference 7 525 188 has root 2743", which reduced gives 45' 43". The printed value "3822" for the side whose square is 13 557 124 appears to be a compositor transposition ($\sqrt{13557124} \approx 3682$), flagged below.

[Margin: See the Prutenic table, fol. 125 and following.]

The Prutenic table [tabula Prutenica] presents the same thing more compendiously, the things to be corrected having been corrected for the appended scruples; but while account is taken of the latitude increased above for □ [recte: the augmented latitude as above], they come to about 45¼ m. [45 m. 15 sec.].

	Mi.	Sec.
Now this motion of the Moon from mid-	45	30

	Mi.	Sec.
eclipse to beginning, or to end, is		
Converted into time by the hourly motion, it gives the time of incidence from mid-eclipse to:		
— beginning	h. 1 m. 25 s. 16	
— end	h. 1 m. 25 s. 40	

The whole duration of the eclipse is therefore h. 2 m. 50 s. 56.

Now from the time of true opposition given above, and the equation of time of $9\frac{1}{2}$ mi. to be added, the apparent time of the Moon's mid-obscuracion in our horizon is known to have been 10 h. $38\frac{2}{5}$ m. Hence the beginning is gathered as 9 h. $13\frac{1}{3}$ mi., and the end as 12 h. $4\frac{1}{3}$ mi.

[Margin: VI.]

Finally, when $49\frac{1}{2}$ mi. — namely the motion of incidence found, together with the summed motion of the nodes corresponding to the true motion of latitude at true opposition — are subtracted at the beginning, and the same amount added at the end, there results:

The Moon's
true motion
of latitude

at:	Sig.	Gr.	Mi.	Sec.
Beginning	5	21	23	4
End	5	23	1	28

To which
correspond
s the
Moon's
own

latitude at:	Mi.	Sec.	
Beginning	44	39	North [Sept.]
End	36	12	

Diagram. A delineation [typus] of the lunar eclipse. A large circle represents the Earth's shadow (the central disk is hatched/shaded to show the shadowed region, split by a vertical line into eastern and western halves, with stippling to suggest the surface of the Moon). Three smaller circles, each containing a stylized lunar "face," are drawn across the upper part: one at the left (the beginning, on the eastern side), one at the top center (mid-eclipse, the greatest darkening), and one at the right (the end, on the western side), illustrating the Moon's passage through the shadow from east to west. A horizontal line runs through the figure marked at the left and right ends.

- Left end of horizontal line: **Oriens** ☐ [East, with the symbol for Scorpio]
- Right end of horizontal line: **Occidens** ☐ [West, with the symbol for Leo]

The delineation [typus] of the lunar eclipse.

Of this lunar eclipse we were able to observe here only the beginning and the moment of greatest intensity, since the sky was rather overcast at the time. At the beginning, Regulus [Basiliscus] stood at an altitude of 35 degrees 52 minutes above the horizon toward the western quarter of the sky. From this the time is derived as 9 hours 14 minutes. And if, as is likely, we reckon that this darkening had set in a little earlier, no discrepancy will arise between the observation and the calculation.

Moreover, when we found Spica Virginis [Spica ☐] in the same quarter of the sky at an altitude of 23 degrees 57 minutes, and the time here ἐν πλάτει [Greek: en platei, "in breadth," i.e., roughly] was 10½ hours (which falls only very slightly short of the computed mid-point), 6¼ digits of the Moon — and no more — were obscured.

These were the observations I, the fourth observer, made of this eclipse from our observatory. From them it is clear that the eclipse agreed very closely with our own restitution.

An example in the solar eclipse which appeared in the morning on the 20th day of the same year and month.

The mid-point of this true ecliptic conjunction [synod], according to the ephemerides of D. David Origanus, reduced to our meridian, fell on the 19th of May at 23 hours 15 minutes. To this moment, then, by way of trial [δοκιμασίας — Greek: dokimasias, "testing, verification"], these motions are collected:

	Sig.	Gr.	Min.	Sec.
Anomaly of the equinoxes	6	17	45	21
Corresponding equation, to be added			8	22
Mean motion of the Sun from the mean equinox	2	8	4	39
Anomaly of the Sun	11	2	17	37
Prosthapheresis of the Sun, to be added			55	29
Mean motion of the Moon from the Sun	0	5	49	47
Anomaly of the Moon	3	11	13	31

	Sig.	Gr.	Min.	Sec.
Motion of the Moon's latitude	0	12	47	10
Ecliptic prosthapha eresis of the Moon, to be subtracted		4	54	18
Aggregate of the prosthapha ereses		5	49	47
Excess of the Moon's motion over the mean conjunction with the Sun		5	49	47

Therefore the assumed time agrees precisely here with the true conjunction.

Now the true place of the luminaries, reckoned from the true equinox, is gathered from these figures to be $9^{\circ} 8\frac{1}{2}'$ in Gemini [□].

I. For the parallax of the Moon's longitude from the Sun at the true synod:

	Gr.	Min.	
Right ascension of the Sun	67	27	
Time before noon, 45 minutes	11	15	to be subtracted
	Gr.	Min.	
Therefore the	56	12	

	Gr.	Min.	
right ascension of the mid- heaven [M.C.]			
The corresponding culminating point	28	27	♉ [Taurus]
Its declination is	19	53	N

The elevation of the pole is 55 degrees 43 minutes; when the declination is subtracted from this, there remains the side in the meridian of 34 degrees 16 minutes. Likewise the distance of the luminaries from the mid-heaven [M.C.] along the ecliptic is 2 degrees 48 minutes. Finally, the angle of intersection of the meridian and the ecliptic is 80 degrees 7 minutes.

Diagram. A spherical triangle ABC drawn in the left margin: a narrow curved triangle with vertex A at the top, B at the lower left, and C at the lower right, used for the parallax computation described in the adjacent text.

In the spherical triangle ABC here set out, then, from the givens [ἐκ δεδομένων — Greek: *ek dedomenōn*, "from the data"]:

Side / Angle	Gr.	Mi.	Meaning
AB	35	50	distance from Zenit[h] in the meridian
BC	10	42	distance of the luminaries in M.C.
ABC	77	10	angle of intersection
	Gr.	Mi.	
AC	34	50	complement of the Sun's altitude
ACB	87	45	angle of the

Side / Angle	Gr.	Mi.	Meaning
			parallax

Are found.

Since this [angle ACB] turns out to be acute, the luminaries have not yet passed beyond the 90th degree of the ecliptic; the angle of longitude is therefore 2 degrees 15 minutes, to which there correspond, from the table, proportional scruples of longitude of 2 minutes 21 seconds.

Now the distance of the Moon from the Earth in semidiameters is found to be 55 [units, with] 34 minutes of a semidiameter. Therefore the parallax of the Moon in the circle of altitude of 56 degrees is taken from the table as 35 minutes 8 seconds — for we reckon that the Moon's true altitude was very nearly this, once the addition for its latitude is made to the Sun's altitude. Likewise the Sun's parallax at that altitude is 1 minute 29 seconds. And the difference, which is the parallax of the Moon from the Sun in the circle of altitude, is 33 minutes 39 seconds. From these, by means of the proportional scruples, the parallax of the Moon's longitude from the Sun is elicited as 1 minute 19 seconds.

The Moon's true hourly motion just before and after the synod is 31 minutes 55 seconds. When the parallax of longitude, 1 minute 19 seconds, is divided by the Moon's hourly motion, there results the interval between the true and the apparent synod, 2 minutes 8 seconds, to be subtracted; and the estimated time of the apparent conjunction is obtained as 11 hours 13 minutes 52 seconds.

But since the parallax of the Moon's longitude from the Sun varies imperceptibly over so small an interval of time here, if then the equation of time [aequatio temporis] of 6 minutes 50 seconds appropriate to the Sun's place is added, the apparent time of the apparent synod is gathered as the 20th of May at 11 hours 20½ minutes before noon, very nearly.

The angle of latitude at the true conjunction was found earlier, from the complement of the longitude, to be 87 minutes 45 seconds [recte: 87 degrees 45 minutes — cf. angle ACB above]; and this same angle, on account of the brevity of the interval between the true and the visible synod, had remained practically unchanged in this example. To it, then, there correspond proportional scruples of 59 minutes 56 seconds, and therefore the parallax of the Moon's latitude from the Sun at the visible conjunction is found to be 33 minutes 36 seconds.

When this is subtracted from the Moon's true latitude — namely 40 minutes 50 seconds — it leaves the Moon's apparent latitude sought at the visible conjunction, 7 minutes 14 seconds N.

	Mi.	Sec.
Semidiameter of the Sun	15	4
Corrected semidiameter of the Moon at this place	13	13
Therefore the aggregate of the semidiameters	28	17
Apparent latitude of the Moon	7	14
Difference, which is the deficient part of the Sun toward the North	21	3
And in ecliptic digits	8 Part. 23 Min.	

[Margin: From the table of the Prutenic [Tables], p. 114 and following.]

Moreover, from the sum of the semidiameters of the luminaries and the apparent latitude of the Moon, there are given scruples of incidence of 27 minutes 10 seconds; and therefore the estimated time from the visible mid-conjunction to the beginning or to the end is 51 minutes 0 seconds.

			Min.
Therefore the estimated	beginning	H. 10	39½
	end	H. 12	11½

			Min.	Sec.	
The parallax of the	beginnin g	long.	9	17	Add.

		Min.	Sec.	
Moon				
from the				
Sun is				
given,				
accordin				
g to the				
method				
prescribe				
d, at the				
	latit.	37	22	
end	long.	5	6	Subt.
	latit.	32	36	

Since in this eclipse the parallaxes of longitude at the beginning and at the end subtract from the Moon's true motion: from the beginning to the mid-point, 7 minutes 58 seconds (which arise from subtracting the parallax of the mid-point from the parallax of the beginning, since both occurred while the luminaries lay in the eastern part of the ecliptic); and from the mid-point to the end, 6 minutes 25 seconds (by adding the parallax of the mid-point to the final parallax) — for this reason the intervals of time from the mid-point to the beginning and from the mid-point to the end are correctly found in this way: when the parallax of longitude of the several intervals has been divided by the Moon's true hourly motion, namely 31 minutes 55 seconds, the time emerging on each occasion is applied, by addition, to the time of incidence, that is, to 51 minutes 0 seconds. For thus I find, from the mid-point to the beginning, 1 hour 5 minutes 58 seconds, which is called the true time of incidence; likewise, from the mid-point to the end, 1 hour 3 minutes 4 seconds, which is called the time of emersion. These intervals, applied correctly to the equated mid-point (namely 11 hours 20 minutes 40 seconds), leave the beginning of this eclipse at 10 hours 14 minutes and the end at 12 hours 23 minutes.

Finally, so that the apparent latitude of the Moon may be had at the beginning of the eclipse, the mean motion of the Moon's latitude corresponding to the apparent time answers:

from the mid-			
point to the		Min.	Sec.
	beginning	36	22

from the mid-
point to the

	Min.	Sec.
end	34	44

From here, by the table of the Moon's latitude — and indeed by the latitude of the Moon appropriate to its starting-point reckoned from one of the nodes — the latitude in scruples is sought at each terminus; this is done by multiplying the adjusted motion of latitude by 5 minutes 13 seconds, which is the difference of one degree in the table, and so on.

And thus the change of the latitude itself emerges, from the apparent mid-point to the:

	Min.	Sec.	
beginning	3	10	Subt.
end	3	3	Add.

— since the visible synod took place with the Moon carried beyond the head of the Dragon [caput draconis, the ascending node] in the order of the signs, as may be perceived from the true motion of latitude at the mid-point of the eclipse.

Finally, so that the visible latitude of the Moon at the beginning and the end may be had precisely, the change of the parallax of the Moon's latitude at the beginning and the end, taken from the visible mean, must also be reckoned. This adds 3 minutes 46 seconds to the foregoing scruples of latitude at the beginning. Therefore the change of the apparent latitude from the mid-point to the beginning is 6 minutes 56 seconds, to be subtracted; and the Moon's apparent latitude at the beginning is only 0 minutes 18 seconds N. Reckoning similarly for the Moon's apparent latitude at the end, that comes out as 11 minutes 15 seconds N. So far the calculation; there follows the delineation of this solar eclipse.

[Margin: By the table of the Prutenic [Tables], p. 114 and following.]

Diagram. A delineation [typus] of the solar eclipse. A large outer circle is oriented by the four cardinal directions. Across the upper portion, three overlapping circles represent the Moon's disk in successive positions (beginning at the east, greatest obscuration at center, end at the west); the central position shows the Sun's disk as a stylized radiant face partly covered, with the uncovered remainder drawn as a strongly curved crescent (the "falcate" / sickle shape described in the text), serrated along the covered edge. At the top is a graduated scale bar marked **8 16 24 32** (digits of obscuration).

- Top: **Septent.** [North]
- Left: **Oriens** ☐ [East, with the symbol for Taurus]
- Right: **Occid.** ☐ [West, with the symbol for Leo]
- Bottom: **Merid.** [South]

The delineation [typus] of the solar eclipse.

The chief moments and phases of this solar eclipse I, the eighth observer, observed most carefully from our observatory, under a sky sufficiently clear throughout the whole duration.

We caught the visible beginning when the Sun was elevated a little above 51 degrees from the horizon, [measured] on the great geometric quadrant. The time deduced from this agrees well enough with the beginning derived from our calculation above. The end, however, was somewhat more drawn out by observation than by calculation. This must be attributed to the uncertainty of determining times near noon from the Sun's altitude — the cause of which I have set aside for treatment in the doctrine of the sphere [in spherical doctrine].

[Margin: Book 2 of the Sphere [Spherics], Cap. 5.]

As for the greatest obscuration of the Sun, which set in here at about half-past eleven, all of us, observing most diligently — both by the ocular view through a sheet of paper, with unanimous consensus, and by a new spyglass [spicillum] which transmits the rays of the Sun's body onto a little tablet with quite good distinctness — found it visually to be at most 8 digits; whereas the Copernican calculation supposes it at this place to be about $9\frac{3}{4}$ digits. Furthermore — and this is worth recording — the shining part of the Sun at mid-eclipse was seen by all of us here to be sharply sickle-shaped [falcate] on both sides, in the very manner in which you see it in the figure. So much so that from this, as from other solar eclipses observed in these places, it readily appears that the visible body of the Moon at the ecliptic new-moon is, on account of the denser constitution of the air with us, perceptibly thinned or diminished — quite appreciably, in the manner I prescribed above. It will be enough to have reported this here too, in good faith.

On the foursome of eclipses occurring each year in both luminaries — or thereabouts — from the beginning of the world, throughout the various tracts and regions of the whole Earth: a παράδοξον [Greek: paradoxon, "paradox," something contrary to expectation], ἀποδεικτικόν [Greek: apodeiktikon, "demonstrative," capable of proof] against the opinion of the common crowd.

Now that the courses of the first revolution and of the luminaries have at last, by the grace of God, been restored in this first Book of the Theorics, before I bring it to a close I have decided to append two matters by way of a coda [coronis]: the one concerning the nearly yearly number of eclipses, continued from the beginning of the world onward; the other concerning the spots that obtrude themselves on our sight in the body of the Moon, and their use, which these [discussions] will serve.

I. On the yearly number of eclipses.

Although those who understand the eclipse-doctrine set out above readily deduce and recognize the former point, you will nevertheless have difficulty persuading the common crowd of it — especially certain of those who, from a sacred place, as preachers, are led...

PARS ALTERA, LIBER PRIMVS THEORICORVM, Caput IX (coda continued)

[p. 313] Section I. On the yearly number of eclipses (continued)

[Margin: Joel 3, v. 15.] [Margin: Matth. 24, v. 29.] [Margin: Luke 21, v. [25].]

[those preachers who, from a sacred place,] are led to distort human effort, expended in these last ages of the world toward establishing the yearly tally of eclipses (which are entered as miracles or signs in certain Ephemerides), into a foretoken of the world's final day to come — just as those signs were foretold by the prophets and by our Savior himself.

[Margin: Pliny, bk. 2, ch. 12.]

I am, to be sure, not unaware that the failings of the luminaries — like the many changes, alterations, or destructions of other things in heaven and on earth — do at all times present a general token of the world's consummation. I cited this earlier from Pliny [Pliny the Elder], who fitted it elegantly to the mortal condition of the microcosm when he says: "Who, beholding the fixed labors of the stars, born to his own necessity, would not forgive himself for being mortal?"

Nevertheless, that the eclipses of the Sun and Moon — which occur necessarily according to the course of nature — are those obscurations of the luminaries of which Holy Scripture speaks, scarcely anyone who understands these matters would readily believe. But to inquire into that belongs to another place. Here, by contrast, the point to be shown is this: that no more failings of the luminaries appear over the whole earth in these last ages of the world (contrary to what those people maintain) than began at the very beginning of the world and have

continued through nearly every single revolution of the Sun. For I shall easily teach, by the demonstration that follows, that these yearly eclipses in each luminary, mentioned a little above, truly terminate in the number four.

Diagram. A large circle representing the ecliptic and the lunar circle, used to illustrate the eclipse limits at the nodes. Pole **A** lies at the center (marked with a dot), with the ecliptic **C D E F** described about it; pole **B** (also marked with a dot near the center) carries the lunar circle **C G E H**. The two circles intersect at **C** and **E** (the head and tail of the dragon — the nodes). Around the circumference, additional points mark the eclipse limits and parallax arcs.

- A (center): pole of the ecliptic
- B (center, near A): pole of the lunar circle
- C (upper right of rim): head of the dragon [caput draconis] — ascending node
- E (left rim): tail of the dragon [cauda draconis] — descending node
- D (upper rim): ecliptic point
- F (lower rim): ecliptic point
- G (top): point on lunar circle
- H (bottom): point on lunar circle
- Left side, top to bottom: S, L, N, O, M, V (lettered division points on the rim)
- Right side, top to bottom: S, K, R, ☾ (descending-node symbol), C, P, I, Q (lettered division points and node symbol on the rim)

With pole **A** placed in the sphere, let the ecliptic **C D E F** be described; and about pole **B**, the lunar circle **C G E H**. Thus **C** will be the head of the dragon and **E** the tail — the intersections at which the Moon's orbit, together with the ecliptic, gives rise (as I showed above) to natural eclipses of the luminaries alone. Now, since I here everywhere assume the mean distance of the luminaries from the Earth, the greatest eclipse limit [terminus eclipticus] of the Moon extends no farther than 12 degrees from either node, nor that of the Sun beyond 6½ degrees — as I expressed all these matters sufficiently with respect to the nodes in the foregoing diagram.

In the case of the Moon, then, since its limit is doubled (on account of the twofold intersection of the ecliptic by the Moon's orbit), the result is 48 degrees, which are to be retained. Next, since the tropical year, according to our restitution of 365 days, 5 hours, 48 minutes, 55 seconds, exhibits the Moon's mean course away from the Sun in a year beyond 12 synodic revolutions by 4 signs, 12 degrees, 35 minutes — to which are also added the 19 degrees, 20 minutes of the nodes'

yearly retrograde regression — therefore the whole yearly excess in the Moon's course away from the Sun comes to 5 signs, 1 degree, 55 minutes, beyond 12 synodic conjunctions (that is, beyond 12 meetings of the luminaries under the zodiac). Hence, working from these given quantities [Greek: διδομένοις (didomenois, "things given," the givens)], when we first compute logistically and concisely this matter for the Moon as it bears on our purpose, it will be as 12 half-revolutions, that is:

Sexag								
. 1	Sex. 2	Sex. 1	gr.	Mi.		gr.	Mi.	Sec.
36	ad 1.	14.	31.	55.	Sic	ad 1.	39.	23.
					48.			

So eclipses occurring yearly in the Moon, according to the mean courses of the Moon and of the nodes, scarcely extend (as you see) to $1\frac{2}{3}$ rounds of obscuration.

Similarly in the case of the Sun, the limit of this deficiency was established earlier at $6\frac{1}{2}$ degrees, as appears in the diagram at $\square$ and along arc **C R**, to which the parallax of the Moon from the Sun in latitude near the horizon adds nearly 58 minutes, 40 seconds, along arc **R S**, [making] 11 degrees, 20 seconds — since at some point on Earth, in the first revolution, the pole of the ecliptic [or] one or the other [node] grazes even the very zenith of the place, and adds such a parallax of the Moon's latitude from the Sun at the mean distance of each from the Earth, at least...

[p. 314] Section I (continued) — and end of Caput IX coda

leaves over. The limit, then, of a solar eclipse is reckoned over the whole earth at $17\frac{1}{2}$ degrees, which is likewise quadrupled (when the [computation] runs through both nodes as well; for solar eclipses occur at each node) — making 4 [arcs] of 6 sextants each. For the yearly number of solar deficiencies, then, the proportion will stand thus, [as 12 to 1]:

Sex	Sex.	Sex.			Sex.				
1	2.	1.	gr.	mi.	1.	gr.	o.	Mi.	Sec.
36.	ad 1.	14.	31.	55.	fic 1.	10	24.	55.	
					ad 2.				

Therefore this number of solar eclipses too is left a little short of $2\frac{1}{11}$. But when compounded in both luminaries — corresponding to the yearly revolution — it exceeds the number four by about $\frac{1}{11}$, which is a trifling amount, and rightly to

be left to the cause given above concerning the attenuation of eclipses on account of the thickness of the air in certain regions.

By this reckoning at least — that of the mean courses of the luminaries, and so forth — the yearly number of eclipses has been demonstrated. But when we turn our attention to their true motions and to the remaining circumstances of time, it may well happen that, with this number presumed, the eclipses in certain years (in alternate years, that is, or thereabouts) are either more or fewer; indeed, that in some of these there is even no obscuration at all, especially of the Moon. Yet so it nevertheless stands that, about every eighth or ninth-year revolution — within which the apogee of the Moon completes its whole period beneath the zodiac — that fourfold number of eclipses, which we elicit from the mean motions, necessarily returns full circle, as it were. This is sufficiently clear from the Ephemerides, above all from those of the most illustrious Mr. Origanus [David Origanus], who, with most studious and laborious diligence, had computed and described nearly all the deficiencies of both luminaries due to appear over the whole globe of the earth, year by year, for as long as his Ephemerides last — a thing accessible to know for anyone who loves the truth.

[Margin: On the most miraculous deficiency of the Sun in the time of the Lord's suffering I here inquire nothing.]

We conclude, therefore, briefly from these things that the parity in the number of eclipses has flourished in every age exceptionally — like the rest of natural things, by the ordination and conservation of the most wise Creator — though it is unknown to the crowd. And this is no wonder, since this very [computation] of the ancients, both of old and of more recent times, devised by industry, set against itself for certain, has dragged some into error concerning a plurality of eclipses toward the last age of the world; and if anyone imitates these, he will learn that the oracles of Christ in the Gospel are not here truthfully interpreted, since the circumstances thereof are not rightly estimated by inward understanding. This is what Mr. Tycho Brahe [Tycho Brahe] too sets forth in fuller words in his *Progymnasmata*.

[Margin: *Progymnasmata* bk. 1, p. 328.]

But as to the fact that in the same place I see it stated that six eclipses in the Sun and Moon occur yearly — that rests upon the old Ptolemaic [Ptolemy] supposition, and indeed upon the extreme stretching of the eclipse limits, which is neither true nor (as ours is) universal, [Tycho] hints; yet this in no way detracts from Tycho's intention in that passage, since he, like us, brings forward an equal

continuation of natural eclipses from the first beginning of things to the end. And this indeed had to be demonstrated, at least against those whom ignorance of these matters in some measure excuses.

[p. 314 → p. 315] Section II. On the spots on the Moon, and their use

I I.

De maculis in Luna, & ipsarum usu. — On the spots on the Moon, and their use.

In examining this subject we shall proceed thus: first, we shall briefly set out an induction toward demonstrating the causes of the spots on the Moon. Next we shall append an epilogism by which — given the elevation of the pole and the longitude of the place tolerably known — there is shown the time, or moment, in each daily revolution at which the Moon will be situated at the ninetieth degree of the ecliptic, reckoned from its rising or setting; whence one will judge of the visible disposition of the horns and of the spots, that is, of the whole face of the Moon, together with the boundary [finitor — the horizon] of the place.

Lastly, we shall add the use that this matter will furnish to mortals — and especially to those traversing the vastest Ocean — namely, that by its benefit they may rightly obtain the longitude of their place, just as [they do] by means of eclipses, now that the canons of the Moon and of the stars have been so restored that they are found conformable to a hair's breadth to the heavenly course of each.

That the Moon is a body by its own nature and essence in no way translucent, but opaque, many things together prove — among which the following especially present themselves.

First: the disappearance of the Moon at its conjunction with the Sun, to such a degree that no luminous traces of it remain in the sky. With which likeness, indeed, the poet of our age, Christianus [Christianus], elegantly and sweetly played upon the immortality of the pious deceased, when he sang thus:

[p. 315] (verse, continued)

The Moon, as though joined more closely to her brother, Seems to the foolish to have utterly perished; Yet she, who gazes from nearby on the lights of the Sun, Shines forth far more brightly. So the crowd think those have perished who seem to die; "Departed" I would sooner say than

"perished." They rather, having gained Christ the nearer, Obtain what they sought.

Second: the Moon, of whose visible opposition [to the Sun] she is the effective agent, displays the same to the eye in a solar eclipse, when a shadow is cast downward from the Moon in the manner of a cone — which shadow indeed reaches no body's surface except a luminous one. But as for the Moon's appearing to be endowed with a certain little light of its own in a total eclipse — by which, rather than by the solar [light] (in which we have sometimes also seen the [whole] body [of the Moon], as in the solar eclipse observed here on 23 September of the year 1605, with its face darkened) — penetrating the Earth's shadow, it presents itself somehow to be discerned by the eyes;

[Margin: See Job. Kepler [Johannes Kepler], generally in the *Optics*, and likewise M. Mästlin [Michael Maestlin], judging the same, *ibid.* p. 254.]

as it does also at the distance nearest the Sun, where in the evening just after the conjunction, and in the morning just before [it], by visible application to the Sun again, the Moon shows us her whole face, contained in one globe, against the sky now lit beneath — this is by no means to be attributed to her own native light, but partly to the rays of the Sun still lingering somewhat above the horizon in that uppermost region, and partly also to the diffused light of the remaining stars, to receiving which the Moon is equally fitted, like a mirror.

[Margin: Gen. 1, v. 16.] [Margin: 1 Cor. 15, v. 41.]

Nor will it offend anyone that in Scripture the Moon is called "the lesser light," and elsewhere in the same [Scripture] is compared with the Sun and stars as to a different glory — since Scripture (as is acknowledged) accommodates itself chiefly to appearance, as it were to our apprehension, a thing made manifest in a hundred other places. The Moon, then, which Scripture names the lesser between the two luminaries, is such not in itself and by its own nature, but solely by reason and proportion of appearance — being, for instance, the nearest of all to the Earth — whereas it would be surpassed by any smallest star among those called fixed.

[Margin: Job. ch. 2, [v.] 1.]

And what could be found, in the interpretation of this mystery — where the Sun [stands] for God of justice, or for Christ, and the Moon for the Church subject to Him — more apt than this: that just as that Church, from Him who enlightens every man coming into the world, [draws her light], so the Moon obtains from the

Sun the whole force and power of her light? But those who assert that a certain little light is left in man, whereby he might be able to do good for life even without the Spirit of Christ and total illumination — these, in resisting the testimonies of the sacred codex, fall most unhappily into the heresy of the Synergists and the like.

Accordingly, since from what has been adduced we gather that the Moon, lacking its own light, borrows everything from the Sun (to use Pliny's words) — which it displays with varied torch and face according to a fixed disposition with respect to the Sun and the Earth: inasmuch as in it we discern the boundary of waxing and waning, while

She is born dying, dies but now, freshly born wanders through all; Twice six [times] she takes upon herself her countenances in a year.

[Margin: On Galileo's [Galileo] little star.]

What, then, prevents the Moon herself from being a body opaque through and through, but on its surface illumined by the Sun's rays — when, reckoning perhaps rightly with Galileo of Florence [Galileo Galilei], we should posit around the Moon [an air] like that by which the Earth is surrounded, or thicker — thicker, I say, in that the lunar body is felt to abound more than the Earth's globe in copious moisture, and is as it were discerned [to be so]?

[Margin: Cael. Rhodig[inus] [Caelius Rhodiginus], bk. 1, ch. 3.]

That the Moon, then, with the ancients, is an ethereal earth, just as [is asserted] — so neither [should one] attribute to it an evaporation [perished?] from enclosed humors, which the solar rays from that part on which they fall illumine exceptionally. A similar [thing], without doubt, the lunar Menippus [of Lucian] would report from the appearance of our own globe of habitation, and would communicate it to us, if a trusty messenger between the worlds were not lacking. You see, then, as Galileo demonstrates [with his glass], the vapors [rising] from our [Moon] [into?] the lunar body round about, [render] the [ethereal] air [transparent], [and that is] why its visible limb is everywhere free of spots, while the rest of the face — chiefly the eastern and almost the uppermost [part] — teems in the manner of an orb [with such inequalities], because there [are] rough, earthy, [and] similar fields, which reflect the solar light more highly than they absorb it up to [their] surface. To these parts of the Moon, rather than to watery ones, I judge the spots that are seen in it can be attributed, along with the most excellent Kepler [Johannes Kepler] against Plutarch.

[p. 316] Section II (continued) — the lunar-map figure

[Margin: What the spots on the Moon are.]

Diagram. A schematic of the Moon's appearance at three positions on its diurnal arc across the sky, showing why the spots maintain a constant disposition relative to a line drawn from the pole of the ecliptic through the Moon's center. Three woodcut lunar disks are arranged in an arc across the top of the figure, each engraved with the dark surface markings (maculae) of the Moon's face, the markings rotated according to the Moon's orientation at that position. Two long straight lines descend from the eastern and western disks, converging downward to point **B** at the bottom; near their crossing is point **A**.

- "Oriens" (left margin): East
- "Occidens" (right margin): West
- C (top center disk): the Moon at the meridian / ninetieth degree
- D (left disk): the Moon near rising (east)
- E (right disk): the Moon near setting (west)
- A (center, above B): "Polus Verticis" — the pole of the vertex (zenith)
- B (bottom, apex where the lines converge): "Polus Eclipticæ" — the pole of the ecliptic

Accordingly, those interspersed spots in the Moon — which around full moon are most especially, and most conveniently, discerned in it by the benefit of the common viewing-tube [telescope] — are nothing other than the showing-through of the more solid and drier parts of the lunar fabric across the surrounding compass of vapors about it (which we earlier named the periphery [peristasis]), illumined by the solar rays. For these spots cannot, as some would have it, result from the form of the Earth, which the Moon might receive into itself like a mirror. For if they were formed by such an adventitious impression, they would surely thrust themselves upon our gaze differently in the various dispositions of the Moon at its rising or setting, and moreover on various horizons — the contrary of which our faithful observations have hitherto taught us, so that the said spots maintain a constant disposition with respect to a line drawn from the pole of the ecliptic through the center of the Moon. By this too it is shown that the Moon is non-rotatable about its own center, as may be seen in the present diagram. There, above pole **A** the horizon [is described], but above pole **B** the ecliptic, that is, the orbit of the Moon, is described. To this we have fitted the same face of the Moon in three places, just as it appears in its uniform spots — diligently observed by us several times, and indeed at different seasons of the

year, with the common spyglass [spicillum], which is believed to have been invented by the Belgians eight years ago.

By such observations of ours we have always noted that, when an arc is drawn from pole **B** of the ecliptic through the center of the Moon, the same disposition of the spots remains perpetually; but that from vertex **A** an inclination of each is seen, and the greatest [inclination] near the rising and setting of the ecliptic, until **B**, passing through **A** into one line **C**, halts the Moon at the ninetieth degree by its own indication — namely when a straight line let down from vertex **A** through the center **C** of the Moon to the horizon either touches each of the horns individually, or distinguishes the spots by that straight [line], in the manner here depicted; so that on this account the angle in the [triangle of the] sphere **A B D**, or **A B E**, which is discerned in the Moon's declination, measures the difference of this from the said ninetieth degree.

Furthermore, so that this part of astronomy may be sufficiently clear to beginners — namely, how it is recognized at what time the Moon, by calculation from a given pole-elevation and likewise from the longitude of the place, reaches the said ninetieth degree of the ecliptic measured from its rising, in each of the daily revolutions — I will here append, by the briefest of precepts, just one of the various methods that can be drawn from the doctrine of the sphere [doctrina sphaerica], as I promised.

For the noon of the day on which one afterward wishes to learn the time of the Moon's transit across the ninetieth degree of the ecliptic, first of all write out the restored motions of the Sun and Moon — that is, either from the *Ephemerides* of David Origanus [David Origanus], or according to the restitution to be established by the procedure set out in this book — and reduce them somewhat to the longitude of the place.

Then, from the table of houses computed for the elevation of the place in question, write out the time from noon corresponding to the Sun's place, seeking it under the heading of the tenth house. But to the Moon's place first add 90 degrees, or 3 signs; then, in the same way, under the heading of the first house, set out the time corresponding to the Moon's augmented motion.

Afterward subtract these two times from each other — namely the Sun's from the Moon's — and for the remainder one must seek the proportional daily motion of the Moon away from the Sun. This is done with the help of the daily motion of each body as presented in the ephemerides, which is added to the Moon's residual motion set down before; and again the time matching this is extracted.

From that time, subtracting last of all the amount previously assigned to the Sun's motion, what remains in the residue is the sought time of the Moon's transit across the ninetieth degree of the ecliptic. This is the briefest and quickest method, which we will now illustrate by example.

In the current year 1617, on the 4th of October — the day on which a full moon occurs — I wish to learn at what hour and minute the Moon climbs to the ninetieth degree of the ecliptic in the daily revolution measured from its rising; and this for our horizon of Copenhagen [Hafnia], where the pole is elevated 55 degrees 43 minutes.

For the noon of that same day, the motions of the luminaries, reduced from the *Ephemerides* of D. Origanus, are found to be:

	Position	Gr.	Min.
Sun	♎ (Libra)	22	6
Moon	♈ (Aries)	20	20

To which, adding 3 signs, the ascendant comes out in ♋ (Cancer) 20° 20'.

		H.	M.
To these there corresponds the time from	the tenth house [10 Domicilio]	13	18
	the first house [Prima Domo]	23	1
Difference		9	43

To this difference there correspond, out of the Moon's daily motion away from the Sun — namely 14 degrees 14 minutes — 5 degrees 46 seconds [recte: minutes] of motion. When this motion has been added to the Moon's residual position set down earlier, it amounts to 3 signs 26 degrees 6 minutes, that is, 26 degrees 6 minutes of ♋ (Cancer); and to this, finally, the time 23 hours 34 minutes corresponds from the first house under the elevation of 56 degrees. From this in turn, the Sun's time being subtracted, there results a remainder of 10 hours 16 minutes. And so on that day, 10 hours 16 minutes after noon, the Moon reaches the ninetieth degree of the ecliptic measured from its rising in our horizon — which was to be demonstrated.

It should be noted, however, that whenever the time corresponding to the Sun's motion appears to be in more hours than the Moon's, then 24 hours — that is, a

whole revolution of the heaven — must be added to the time corresponding to this latter motion, so that the subtraction can be made, as is commonly known.

Once the time has been found in this way at which the Moon would ascend to the ninetieth degree on any given day, I have many times applied to the Moon's face — and to the disposition of the spots upon it — that aforementioned optical instrument of ours [the telescope]; and this at different seasons of the year, and with the Moon in different positions above the horizon. In every case I have found her clothed with the same appearance of spots, and moreover I have found the same erection and disposition of those spots around the ninetieth degree, as shown beforehand. That this also befalls the Moon's horns in that location, the angle of her illumination by the Sun — drawn, that is, through the same line of the ecliptic — proves demonstratively. For when the arc of a great circle drawn from the pole of the ecliptic through the vertex of the place, and descending into the horizon, touches the ecliptic at the point of the ninetieth degree measured from the rising or setting, it always comes about that the said arc, or line, grazes the extremities of the Moon's horns, provided the Moon herself tarries in that place. Nor is there anything

perceptible by which the Moon's horns deflect from this said line, except when her recession produces latitude — chiefly at the greater distance of the Sun and Moon. Meanwhile, however, this variation reaches its maximum when the Moon, having advanced from one or other node through half a quadrant, or a little beyond, has acquired a latitude of about 4 degrees.

And so much for the indications by which the Moon's lingering around the ninetieth degree of the ecliptic — in every location — is recognized from the erected disposition of the horns and the spots. Now let us proceed further to the usefulness that this matter will have in the investigation of the unknown longitude of a place. This is the inquiry into the unknown longitude of a place by means of the Moon's motion in longitude, observed at that place. For since the Moon, among all the planets, possesses the swiftest periodic motion — so much so that she completes about $13\frac{1}{3}$ degrees each day — the difference in longitude of those same places is accordingly elicited from the proportion of the Moon's observed longitude at distant places of the Earth in particular: namely, places lying chiefly toward each other in the east-west direction (although this practical method applies to all places, provided they are situated under different meridians).

But since by the most accurate restitution of the Moon's course her longitude is granted as reckoned to the meridian of Hven [Hven], or rather to this our

meridian of Copenhagen, therefore — that meridian always being presupposed — there is gathered, from the difference of longitude given here but observed there, the thing proposed: namely, the difference in longitude (measured along the equator) of the unknown place from this known one. This holds whenever the time, too, suited to the place and to the observation to be made, has been sufficiently precisely known. For from the difference of this time from our own place — a difference which the difference in the Moon's motion shows by proportion through her daily motion — the sought difference of longitude results, once that interval of time has been resolved into degrees and minutes of the equator. This is no different from the way it is customarily reported for this purpose in the observation and comparison of a single eclipse of the Sun or Moon at various places. And so what is too obvious for those who understand — that is, for those somewhat versed in astronomy — will be explained by an example for beginners.

Let an example of this matter be taken for the 6th day of the month of October in the current year 1617, when the Moon was somewhere distant by observation from Aldebaran [Aldebaran] by 10 degrees, with respect to the rising of our horizon, while she was situated near the ninetieth degree there. Let that time be 11 hours past noon — which is known by the altitude of some fixed star under the given pole-elevation, or by some other method, according to what we handed down in the doctrine of the sphere concerning the investigation of time [Margin: Liber 2 Sphæricorum, Cap. 5], at the moment when the Moon there was situated near the ninetieth degree of the ecliptic, and the said distance of hers from Aldebaran was being taken.

Furthermore, since the longitude of Aldebaran, on this supposition, is for this year 4 degrees 26 minutes of $\square$ (Taurus) — while the Moon, at the supposed 11th hour of the day for our horizon, was in 27 degrees 20 minutes of $\square$ (Taurus) — the difference therefore given under this meridian of Copenhagen, in longitude, matching the 11th hour, is 7 degrees 6 minutes. But at the same hour, at the place of unknown longitude, it was 10 degrees 0 minutes by observation. The difference is therefore 2 degrees 54 minutes, which — measured by the Moon's daily motion, namely 14 degrees 41 minutes, in proportional time — gives 4 hours 45 minutes. By this it is signified that the place in which the observation was made is removed, with respect to meridians, by that many hours and scruples toward the east from this our place. And that same moment of time — namely $4\frac{3}{4}$ hours — resolved into parts of the equator, gives $71\frac{1}{4}$ degrees. Now since the longitude of the city of Copenhagen was $36\frac{1}{3}$ degrees, then, adding $71\frac{1}{4}$ degrees to this, the

longitude of the place that was sought amounts to 107 11/12 degrees [recte: 107 7/12° (= 107°35'); printed numerator "11" is a compositor error for "7"].

According to this example all other cases will be carried out, once it has been considered, in addition, toward which cardinal point the unknown place inclines from us — a thing easily known either by the Moon's differing motion or by some other conjecture. Now we shall first transcribe here, from their own canon, the more conspicuous stars that shine near the ecliptic and are most suited to this practical method, reduced to the end of the year 1620; then we shall display the construction of the instrument by which the Moon's distances from the said fixed stars can be most conveniently obtained; and finally, by what method these same distances are to be taken, so that they may represent the longitude-differences alone along the ecliptic, according to the hypothesis set out above.

A designation of fifteen of the more conspicuous stars near the ecliptic, according to longitude and latitude, complete for the year 1620, from which the Moon in her revolution may most conveniently be measured for ascertaining the longitudes of terrestrial places.

NAMES OF THE STARS Longi- tude Latit- ude								
Num.	S	(Sign)	(G.)	(M.)	(G.)	(M.)	(N/S)	Mag.
1	Lucida of Aries [Lucida Arietis]	♈ (Taurus)	2	22	9	57	Bor. (N)	3
2	Aldebaran	♈ (Gemini)	4	29	5	31	Auf. (S)	1
3	Lower horn ♈ (of Taurus)	♈ (Gemini)	19	28	2	14	Auf. (S)	3

Num.	NAMES OF THE STAR S	Longitude (Sign)	(G.)	(M.)	Latitude (G.)	(M.)	(N/S)	Mag.
	[Inferior conjunction]							
4	Procyon (Gemini)	♈ (Gemini)	25	38	0	13	Auf. (S)	3
	[Procyon]							
5	Lower head of Gemini (of Gemini)	♈ (Cancer)	18	0	6	38	Bor. (N)	2
	[Caput inferior]							
6	South Asellus (of Cancer)	♈ (Leo)	3	24	0	4	Auf. (S)	4
	[Asellus australis]							
7	Heart of Leo	♈ (Leo)	24	33	0	26	Bor. (N)	1

Num.	NAMES OF THE STARS	Longitude (Sign)	(G.)	(M.)	Latitude (G.)	(M.)	(N/S)	Mag.
	(Leo) [Cor Leoni s]							
8	First south ern of the wing □ (of Virgo) [Prim a austr alis alae]	□ (Virgo)	21	48	0	43	Bor. (N)	3
9	Spica □ (of Virgo) [Spic a]	□ (Libra)	18	32	1	59	Auf. (S)	1
10	South ern scale □ (of Libra) [Meri diona l. lanx]	□ (Scorpio)	9	47	0	26	Bor. (N)	2

Num.	NAMES OF THE STARS	Longitude (Sign)	(G.)	(M.)	Latitude (G.)	(M.)	(N/S)	Mag.
11	Northern bright star of the forehead (of Scorpio) [Septent. clara frontis]	♏ (Scorpio)	27	52	1	5	Bor. (N)	2
12	Preceding of the three in the head (of Sagittarius) [Trium praecedens in capite]	♐ (Sagittarius)	8	13	1	45	Bor. (N)	4
13	Lower	♏	28	47	4	41	Bor.	3

Num.	NAMES OF THE STARS	Longitude (Sign)	(G.)	(M.)	Latitude (G.)	(M.)	(N/S)	Mag.
	rhombus (of Capricorn) [Inferior cornu]	(Capricorn)					(N)	
14	Following in the tail (of Capricorn) [Sequens caudal]	(Aquarius)	18	16	2	29	Auf. (S)	3
15	Succeeding in the pouring-out of water (of Aquarius) [Succedens]	(Pisces)	6	20	0	20	Auf. (S)	4

NAMES OF THE STARS Longitude Latitude Sign Degrees Minutes Sign Degrees Minutes North/South Magnitude								
Num.	S	(Sign)	(G.)	(M.)	(G.)	(M.)	(N/S)	Mag.
	eden							
	s in							
	effusi							
	one							
	aqua							
	e]							

Parsing notes: The "Longitude" group has three sub-columns — the zodiacal sign (S.), degrees (G.), and minutes (M.); "Latitude" has degrees (G.) and minutes (M.), plus a Bor./Auf. (north/south) indicator that the source prints adjoined to the minutes. "Mag." is magnitude. "Bor." = *Borealis* (northern); "Auf." = *Australis* (southern). Star names: the sign-glyph in the name column indicates the constellation the star belongs to (e.g., "Inferior cornu ♉" = lower horn of Taurus). For row 5, the source minutes read "0"; row 1 latitude minutes "57" are clear.

I therefore thought these fixed stars should be transcribed here separately from their canon, so that they might be in view whenever zodiacal distances of the Moon were to be taken from them. Now we are about to describe the instrument by whose aid distances of this kind — namely, when the Moon lingers near the ninetieth degree of the ecliptic — are best to be drawn off. Having made a choice among the various kinds that can be devised, I see that that widely-used rod which sailors call the *Baculum Iacobi* [Jacob's Staff], with which they are also accustomed to investigate the pole's altitude on the open sea, is most accommodated to this practical purpose. For although we earlier rejected the use of this rod itself, when on a given occasion we treated of the sufficiency and reliability of instruments [Margin: Cap. 7, Lib. 2 Sphæricorum], nevertheless — because the aberration of the rod is smaller in such smaller distances, for whose measuring only its crosspiece [transversarium] suffices, and because nothing else can perhaps be applied by sailors in a turbid and surging ocean as conveniently — therefore let a rod of this kind be prepared, whose use can be useful to sailors besides; and whose crosspiece is in roughly a 3-to-2 [*sub-sesquidupla*, i.e. about 1½-fold] ratio with the index or staff, so that, taking this whole given rod as the radius (sine), the crosspiece alone subtends up to 22 degrees. Beyond that measure there is no need of anything else in distances of this kind, and therefore neither to shift the crosspiece from its fixed place here on the rod, nor to move it

nearer the eye. In such a proportion it suffices to prepare a rod 5 feet long and a crosspiece 2 feet long. But since I set out the rod's construction in the doctrine of the sphere, I therefore do not repeat it here [Margin: Cap. 7, Lib. 2 Sphæricorum]; I will only touch briefly on what concerns the completion of this particular matter.

Since the Moon's distance from a fixed star is to be taken solely along the zodiac [signifer], the disposition of the crosspiece is for that reason required to be uniform with the ecliptic, so that it may be made through the latitude of the fixed star equally with that of the Moon. Two transverse sliders must therefore be attached to the crosspiece of this instrument at right angles: one with its own sight [dioptra], through which the star's latitude is taken at the same time; the other, by which the Moon will be indicated, can be without a small wing [pennicidium], and this by hypothesis, since here directly

it falls into the horizon of the place, while it duly demonstrates the point of the ninetieth degree in which the Moon will be situated. Whether — the Moon being thirsty (i.e., not yet full) — the horns are taken, so that her center is reckoned; or the limb nearest the Sun, so that nonetheless, by the prosthaphaeresis [prosthaphaeresis] of the Moon's semidiameter — 16 minutes at most — the place of the center of the distances to be applied may arise: the transverse slider grazes it. The length of such sliders will be made on each side nearly equal to the length of half the crosspiece, while the degrees and minutes of the star's latitude are to be sought above and below. With these few remarks we shall be content for the explanation of this matter; and as for the example previously employed that remains, we shall represent it merely schematically, just as it would have been drawn off by a rod of this kind.

Diagram. A woodcut illustrating the use of Jacob's Staff to measure the Moon's distance from Aldebaran near the ninetieth degree of the ecliptic. A kneeling man holds the long staff (the index or *baculus*) up to his eye, with the crosspiece (*transversarium*) set on it at right angles. Dotted sight-lines run from the observer's eye along the staff and across the crosspiece to two celestial objects. An arc representing the ecliptic sweeps across the upper part of the figure from lower-left to upper-right.

- Luna (upper area, top of the arc): the Moon, shown as a partly-shaded disk, at the high (western) end near the top.
- Ecliptica (along the arc): label for the ecliptic, the great curving arc.
- Occidens (right end of the arc): "West" — the setting point.
- Oriens (lower-left end of the arc): "East" — the rising point.

- Aldebaran (mid-left, below the arc): the fixed star, shown as a small radiant point, toward which one of the dotted sight-lines runs.

Finis libri primi Theoricorum. [End of the First Book of the Theorica.]

LIBER SECVNDVS THEORICORVM

The Second Book of the Theorica

On the restored motions of the remaining five planets

CAPVT I

On the names of the five lesser planets, on the hypotheses [ὑποθέσεων, hypotheseōn, "hypotheses"] involved in the investigation of their motions, and on how — and from which observations — the restoration of those motions is to be undertaken and carried through by us.

In the preceding Book of the Theorica we set out a survey of the threefold structure of the world [ὑποτυπώσεως, hypotypōseōs, "outline, sketch"], gave a general delineation of the planetary hypotheses, and restored — fitted to every age — the motions of the fixed stars and likewise those of the Sun and Moon. In doing so we not only laid the foundation for renewing astronomy, but, relying on divine aid, also completed the better part of that renewal. Now, with that same heavenly favor smiling on our work, we undertake to restore the motions of the five wandering stars [the planets] by an equal effort, so far as we are able.

The nature and effects of these five planets — like those of the Sun and Moon — belong to a physiological [natural-philosophical] treatment. Therefore, in this place, before we proceed to their motions (whose doctrine is what properly belongs to astronomy), it may perhaps be necessary, for the sake of distinguishing them, that we report only their names, as they were called by the ancient Chaldeans, the Greeks, and Plato.

Saturn is called in Hebrew *Schabthai*, meaning "satiety" or "rest"; in Greek κρόνος [kronos]; but by Plato Φαίνων [Phainōn], that is, "the apparent one." For it does not, on account of its motion being the slowest of all, lie hidden beneath the Sun's rays as long as the others do.

Jupiter, again, is called in Hebrew *Tzedek*, that is, "justice," because it is believed to govern justice; in Greek Ζεύς [Zeus]; in Plato Φαέθων [Phaethōn], from its splendor.

Mars is called in Hebrew *Maadim*, that is, "redness like blood," both from its appearance and from its effect; by the Greeks and Ptolemy [Ptolemy] it is called Ἄρης [Arēs]; by Plato πυρόεις [pyroeis], "fiery," from fire.

Venus is called by the Hebrews *Nogah*, that is, "light"; by Plato Φωσφόρος [Phōsphoros], the morning star, and Ἑσπερος [Hesperos], the evening star; but by Ptolemy and the poets Ἀφροδίτη [Aphroditē], from the foam of the sea and from seed.

Mercury, finally, is called by the Hebrews *Choteb*, that is, "scribe," because it is believed to preside over scribes, the learned, and the ingenious; in Greek Ἑρμῆς [Hermēs], as it were "interpreter," and also Στίλβων [Stilbōn], from its sparkling and quivering light. But enough about these names.

Now, as to the construction of the hypotheses by which the appearances [φαινόμενα, phainomena, "phenomena"] of these planets are to be saved: although this is the place where one might be advised to choose, out of the threefold variety of those hypotheses set forth in the First Book and laid out in general outline, the one most agreeable to nature and truth — nevertheless we prefer to follow our original plan (which was also Tycho's [Tycho Brahe]), and to apply the motions of these planets in exactly the same way to the discoveries of all three most celebrated practitioners. We do this in such a way that the restoration of the superior planets, no less than that of the Moon, is everywhere adapted to the true motion of the Sun, and is thus first carried out separately with respect to longitude and latitude. For the inferior planets too, the estimation and judgment is made in its proper place from what the observations, compared with one another, conclude — namely, in the same way as these are subordinated to the Sun's mean motion in their own [motions].

For above all we hold that the astronomer must do this: that the celestial appearances [φαινόμενα, phainomena] be represented and accounted for, as accurately and concisely as possible, at any given time, by means of suitable hypotheses constructed on geometrical principles.

[Margin: Lib. 2. cap. Theor.]

We possess observations suited to this purpose. Some result from working out that synopsis of the restoration which we once labored to bring to a mean solar course at Uraniborg, during our apprenticeship under Tycho — relying then on the traces of the ancients, before the final lunar restoration, which was carried out in Bohemia. Others we acquired afterward by our own industry, here in this most famous Danish harbor and royal academy, and similarly resolved into

longitude and breadth by faithful computation of triangles. In all these matters I think it should be advised in advance that we by no means press — except where there is need — that extreme precision, too fastidious about the second-order minutes, which the observations themselves likewise reject; nor that we apply the little equation of time [aequatuncula temporis], demonstrated earlier; nor, again, that reduction by which the location of these planets is carried over from the ecliptic to their proper orbs, since it is of no great moment, and cannot be applied except by a very laborious restoration. This was the reason why the ancients too neglected it.

CAPVT II

On the restoration of the first inequality [aequalitas, here "inequality"] of Saturn according to longitude, fitted to our age.

Besides the simple revolution of the three superior planets in their orbs — by which their centers repeatedly look toward [the relevant point] — we showed generally in the preceding Book that they have a twofold anomaly: the first essential, namely attached to the centers of the orbs; the other accidental, arising from the Sun's course alone and from its configuration with the planet's position. Both of these adhere, as in the case of the Moon, so also to the motions of these three planets, though in a different manner. Now since the true motions are sought out from the mean (or simple) motions by means of fixed hypotheses or tables built from them — and yet the way of establishing these depends on the true motions, which themselves come into our perception and knowledge through certain observations of celestial appearances [φαινομένων, phainomenōn] — these true motions must accordingly be unraveled back into the mean ones, which will afterward be subjected to the perpetual laws of the hypotheses.

Now the three superior planets — Saturn, Jupiter, and Mars, of which we treat first in order — are, at their farthest nighttime brilliances (in which, with the Earth precisely interposed, they are opposed chiefly to the Sun according to its true course, as we now more correctly perceive), and thus are at the middle of their regressions [retrogradations]. At such times they are visible by their essential anomaly alone, and are therefore plainly free of the other, accidental or solar [anomaly]. Our first care, then, in each case will be to deduce these acronychal [acronychiis, "rising at nightfall, in opposition to the Sun"] positions correctly from observations, and to correct them this way and that chiefly according to their mean motions and eccentricities. For this task we judged the Prutenic Tables [tabulae Prutenicae] to be most suitable, inasmuch as they were

once geometrically recomputed and perfected with precision, from the Copernican [Copernicus] foundation, by that most distinguished man Erasmus Reinhold [Erasmus Reinhold].

With these tables, then, as the comparison of observations is faithfully carried out by us with repeated trials, and all things are tended with sufficient diligence — both what pertains to the appearance of the fixed stars, from which the positions of the planets are determined, and the Sun's course (restored by Lord Tycho for the nearest centuries, but by us in the preceding Book for all time) — we found the said Prutenic Tables, in the acronychal positions and first anomaly of Saturn at the year of our Savior 1600 completed (taken as the mean epoch of our restoration), to require correction in the following motions and anomalies in this way, since the reduction of these to our meridian is also omitted. Namely, that from the mean longitude there be subtracted

[from the mean longitude there be subtracted] 20 minutes at the apogee, 3 whole degrees; but at the prosthaphaeresis [προσθαφαίρεσι, prosthaphairesei, "prosthaphaeresis," the equation added or subtracted] of the eccentric there is everywhere added a fiftieth part — keeping here the same measure of the semidiameter of the equant, which is assigned to it as a third part of the whole eccentricity, according to Ptolemy and Copernicus.

When this restoration of the said tables has been carried out, the acronychal positions of Saturn come out as deduced from the computation, and — extended to the complete revolution of this star in the eccentric, together with the observed positions — they appear in the manner shown in the following synopsis. Into this synopsis we have, for the sake of brevity, condensed the requisite data of times and motions, which others are accustomed to explain at length.

Synopsis of fifteen positions of Saturn

Synopsis of fifteen positions of Saturn — both from observation in opposition with the true motion of the Sun, and from computation at the same moments, arising after the emendation of the tables has been applied.

No.	Year	Month	D.	H.M	Lon g. obs. (S. G. M.)	Lon git. med (S. G. M. S.)	Ano m. ecc. (S. G. M. S.)	Pra esc. aeq. (G. M. S.)	Lon g. sup. (S. G. M.)	Diff. (M.)
I	1582	Aug.	21	2 0	□ 7 26	10 □ 9 23 40	2 17 34 4	27 58 50	□ 7 22½	3½ —
II	1583	Sept.	3	1 0	□ 19 50	10 28 26 21	3 0 17 15	27 59 46	□ 19 48½	1½ —
III	1584	Sept.	15	6 ½	□ 2 34	11 11 7 41	3 12 57 58	28 0 40	□ 2 35¼	1¼ +
IV	1585	Sept.	28	19 ½	□ 15 39½	11 23 45 38	3 25 35 10	28 1 34	□ 15 39½	0¼ —
V	1586	Oct.	12	11 0	□ 29 2	0 6 25 47	4 8 14 47	28 2 27	□ 29 4 0	2 +
VI	1587	Oct.	26	9 0	□ 12 46	0 19 6 29 51	4 20 44 20	28 3 20	□ 12 46 0	0
VII	1588	Nov.	8	10 ½	□ 26 44	1 1 47 28	5 3 35 11	28 4 14	□ 26 44¼	½ +
VIII	1589	Nov.	22	10 ½	□ 10 53	1 14 28 50	5 16 15 55	28 5 8	□ 10 53¾	⅔ +
IX	1590	Dec.	6	20 ½	□ 25 9 24½	1 27 10 14	5 28 56 30	28 6 20	□ 25 9 23 0	1¼ —
X	159	Dec.	21	1 0	□ 9	2 9	6 11	28 6	□ 9	1¼

No.	Yea r	Mo n.	D.	H.M	Lon		Ano		Lon g. obs. (S. G. M.)	Diff. (M.)
					g. obs. (S. G. M.)	git. med (S. G. M.)	m. ecc. (S. G. M.)	Pra esc. (G. M.)	Lon g. sup. (S. G. M.)	
	1				24	51 20	37 3	58	23 0	—
XI	159 5	Ian.	30	21 0	□ 21 15½	3 17 53 36	7 19 37 30	28 9 40	□ 21 18½	2½ +
XII	160 8	Iuli.	9	3 0	□ 26 53	9 2 7 21	1 3 43 2	28 21 40	□ 26 57½	4½ +
XIII	160 9	Iuli.	21	13 0	□ 8 31	9 14 45 2	1 16 20 55	28 22 33	□ 8 31½	⅓ +
XIV	161 0	Aug .	2	22 ½	□ 20 10	9 27 22 37	1 28 51 57	28 23 30	□ 20 14½	4½ +
XV	161 1	Aug .	15	16 0	□ 2 12	10 10 1 0	2 11 35 38	28 24 30	□ 2 13½	1½ +

Parsing notes: This is a citation-grade table with eleven columns, several subdivided into Signs/Degrees/Minutes/Seconds (S. G. M. S.). Several cells are degraded by ink-bleed and the double-scan garble noted for this book; flagged values: row I "Longit. med." (10 □ 9 23 40), row II "Praesc. aeq." (27 59 4—), rows VIII–X (H.M and the □/□ longitude cells), row XII (the Longit.med./Anom.ecc./Praesc.aeq. block is misaligned in the source and partially illegible). The zodiac-sign abbreviations in the longitude columns I have rendered with their symbols; the "Long. sup." column repeats the sign of "Long. obs." The "Diff." column gives the observed-minus-computed difference in minutes, with a +/- sign. The whole table is a strong for clean cell-by-cell re-transcription. Row XII's anomaly/equation columns in particular appear column-slipped in the source rendering.

From such a comparison, since it is clear that the computed positions of Saturn nowhere depart from the observed ones — the difference rising, one way or the other, to [at most] 5 minutes — I therefore hope that the restoration of Saturn's first anomaly here, for our age, has been so satisfactorily accomplished by this method and by our industry, that I judge it will scarcely be of any moment to wish to repeat it for the sake of greater precision, or even to arrive at greater [precision] here or in what follows by another route, namely by computation of triangles. For that route — which proceeds by the assumption of three observed positions, after the manner of the ancients, by means of "problems" [πραγμάτεια, pragmateia, "treatise, method"] — is far more specialized, and therefore also more uncertain, and is wrapped in a labyrinth and in a great many windings and ambiguities, on account of the insinuation of the equant.

Now, before we come to assessing the second inequality of Saturn similarly from observations, the restoration of the first inequality already presented must be entrusted to the threefold hypothesis set forth earlier — but in such a way that both the old Ptolemaic geometry concerning the equant, and the remaining eccentricity of Copernicus, are conformed to the two little circles [circelli] on the circumference of the orb, in accordance with Tycho's restoration, as we showed in the previous Book. [Margin: See the preface to the reader in the *Theorica* books.] [Margin: Book 1, cap. 2.] For thus the phenomena will agree, in all three suppositions and for these superior planets, with one another in computation and in the tables, to a nicety.

The reformed Ptolemaic hypothesis for Saturn

The Ptolemaic hypothesis for Saturn, reformed and emended, upon which the form of motion of the remaining superior planets can be explained.

Diagram. A large geometric construction illustrating the reformed Ptolemaic hypothesis for Saturn. The Earth sits at center; a large circle (the deferent orb) is drawn around it, and a nested cluster of small circles (the major epicycle, the equant-circle, and the annual epicycle) is set at the upper-left of the deferent.

- A (center of the large circle): the Earth (*terra*)
- B (top of the large circle): point on the orb directly above A; the construction point where C lies when the planet is at the start of its revolution
- P (bottom of the large circle): point diametrically opposite B

- C (on the orb, upper left): center of the major epicycle E D; the simple revolution of Saturn through the whole zodiac is carried out on the orb B C P
- D (within the epicycle cluster): center of the minor epicycle (the equant) F H; semidiameter C D = 8721 parts (where the semidiameter of the orb A B or A C = 100000)
- E (on the major epicycle): point on epicycle E D; when C is at B, D is at E
- F, H (on the minor epicycle / equant F H): the equant circle, semidiameter D H = 2907 parts (a third part of C H)
- I, K, M, L (on the annual Ptolemaic epicycle about center H): epicycle I K M L; the planet is at M, always opposed to the Sun according to its true motion; semidiameter H M
- G (small label within the inner cluster, near D/F): construction point [?]

[Margin: See cap. 2, book 1, near the end.]

To continue the description of the construction: Let A be the Earth; B C P the orb upon which the simple revolution of Saturn is carried out through the whole zodiac. Again, about center C let the major epicycle E D be described, whose semidiameter C D, according to our restoration set out above, is reckoned at 8721 parts of the kind of which the semidiameter of the orb A B (or A C) is 100000. Finally, about center D let the minor epicycle — that is, the equant — F H be drawn, whose semidiameter D H, which is a third part of C H, comes from our same emendation to 2907 parts.

The proportion of the revolutions of these circles is as follows: the arc E D is commensurable with the arc B C under the same time; whence it follows that, when C is at B, D is at E. But the equant F H, relative to that same first epicycle E D, keeps a double measure of revolution; therefore, when C is at B, here H is at E, the angle F D H being always double the angle B A C (or D C E) throughout, as was set out in the previous Book.

Now upon these little circles and epicycles, although the first anomaly of Saturn and of the superior planets can be explained, nevertheless — so that the acronychal position of the planet may at the same time be shown in this Ptolemaic form — let there be added the annual Ptolemaic epicycle I K M L, described upon center H; and the planet will be at M, always opposed to the Sun according to the true motion of the Sun. The semidiameter H M of this epicycle will afterward likewise be emended from our observations, and everything that can pertain to determining the true position of Saturn at any given time will be

explained. For this turn it suffices to teach how we may obtain the prosthaphaeresis of the first anomaly — which befalls the planet when in opposition to the Sun — by computation of triangles from the foregoing supposition.

And so, in the plane triangle C D H: since these two sides are always given from what precedes — namely C D and D H, the former of which was 8721, the latter 2907 — together with the contained angle C D H (which by hypothesis is always double the angle B A C, or E C D), therefore from these the angle D C H is first given. This angle is here added to the angle E C D, together with the side C H. Then in the triangle A C H, since likewise two sides are given together with the contained angle — namely A C = 100000, which is the radius of the planet's orb; likewise the side C H just now found, together with the included angle A C H, which is the complement of the angle E C H to a semicircle — therefore here too, by the analysis of the present triangle A C H, there is found the angle of the whole prosthaphaeresis of the first inequality, C A H, to be subtracted here from the mean motion of Saturn at C, so that the true [planet] may be had at H (or M). For the line A I

running from A, grazes the Earth at H, which is set in the lowest part of the annual epicycle at M. Together with this prosthaphaeresis angle one usually also obtains the side AH, whose use will later serve, in connection with the prosthaphaeresis of the annual orb, for always tracking down at the same time the situation outside acronychal opposition. And this Ptolemaic supposition is, in itself, entirely alike for all three of the superior planets — except that a variation enters in the semidiameters of the individual epicycles. We will now compare the hypothesis of the more recent astronomers, and first that of Copernicus [Nicolaus Copernicus], with this Ptolemaic one, in a like configuration.

The hypothesis of the same configuration of Saturn and the remaining superior planets, in the Copernican form.

Diagram. A large circle represents the primary orb of revolution BC, drawn from the Sun A at the center of the universe (following Copernicus). The Sun A lies near the center of the great circle; B is at the top, P at the bottom. In the upper-left region sits a cluster of small circles: the first epicycle ED about center C, and beyond it the second epicycle FH, with the planet understood to be carried at H. Around A is drawn the annual orb of the Earth, IKML, a moderate circle with the labels I, K, M, L around it (M near the top toward H, A at its center). Lines run from the Sun A up through the epicyclic cluster.

- B (top of great circle): start-point of orb BC
- C (in the upper-left cluster): center of the first epicycle
- D (in the cluster): center of the second epicycle
- E (in the cluster): point on the first epicycle
- F, G, H (in the cluster): the second epicycle FH, planet at H
- A (center): the Sun, center of the universe and of the Earth's annual orb
- I, K, M, L (around the inner circle about A): the annual orb of the Earth; M is the position interposed between H and A
- P (bottom of great circle): point on the orb of revolution

Following Copernicus, let the primary orb of revolution BC of Saturn and the three superior planets be described from the Sun A at the center of the universe. With the center placed at C, let the first epicycle ED be drawn, and then the second one from D — namely FH, where the planet is understood to be carried around at H by a motion contrary to ED, as was explained above. Finally, from the same point A, that is from the Sun, let the annual orb of the Earth, IKML, be described.

Now when the Earth comes to be at M, exactly interposed between the place H of the planet and the apparent place of the Sun at A, the prosthaphaeresis of the first inequality arises through the computation of triangles from these same givens [Greek: δίδομένους (didomenois, "givens" — the given quantities)] (that is, with the Sun placed in the position of the Earth), the very ones demonstrated above in the Ptolemaic form. Nor does anything further enter on account of the second anomaly, except where the Earth M has migrated to some other place around its aforesaid annual orb. And since the semidiameter of the annual orb here — AM — is equal to the semidiameter of the upper Ptolemaic epicycle HM, the same phenomena are, with respect to appearances, elicited here in every way. Moreover, once the distance of the Sun from the Earth, AM, is given in a definite measure, in the same measure one can also obtain AC, the distance of Saturn's orb from the Sun, and so forth — which it was denied us to investigate by the earlier Ptolemaic method.

The hypothesis of the same configuration of Saturn etc., in the Tychonic form, and at acronychal opposition.

[Margin: Lib. 1, c. usu...] [reading uncertain — see QA]

Diagram. A large circle is the orb of Saturn BC, with B at the top and P at the bottom. In the upper-left region is the epicyclic cluster: the small circles built

from C and D, with points F, G, H and C marked. From the lower-central region a smaller circle (the Sun's orb AL) is drawn about center M with radius MA; A lies at the bottom of this inner circle, L to its upper left. Straight lines run from A up to the epicyclic cluster at H.

- B (top): start-point of orb BC
- C, D (in the cluster): centers from which the epicycles are made
- F, G, H (in the cluster): the epicycles; planet at H
- M (center of inner circle): center of the Sun's orb AL
- A (bottom of inner circle): the Sun; on line AH; carries the center of the planet's orb
- L (upper part of inner circle): point on the Sun's orb AL
- P (bottom of great circle): point on Saturn's orb

From the center of the true place of the Sun, let the orb of Saturn etc. be described as before, BC. Then let the epicycles be made from C and D, and let the angle CAH of the complete prosthaphaeresis of Saturn's first inequality be elicited, in every respect as above.

Afterward, on the straight line AH, about the center M and with radius MA, let the Sun's orb AL be drawn; in it, indeed, the center A of the planet's orb is understood to be carried along likewise in the order of the signs. Hence, when the Earth M has been placed directly between the Sun A (in its true motion) and the planet H — as in this present configuration — the whole second anomaly is entirely exhausted, because our line of sight from the Earth M coincides with the line drawn from A and extended to the planet's position H on the second epicycle. But in the other positions, outside acronychal opposition, the planet exhibits everywhere, from the annual orb, the same anomalies as the Ptolemaic and Copernican models do, as will also be shown further below.

[Margin: Cap. 9, lib. hujus.]

Now this Tychonic supposition, although it is plainly an inversion of the Copernican, nevertheless likewise implies epicycles in these three planets after the Ptolemaic manner — given the revolution of the Sun as the center of those same orbs — wider in proportion as the distances of the orbs from the Sun or the Earth are smaller, and vice versa, just as we hinted at in its proper place in the earlier book. [Margin: Lib. 1 Theor., c. 1, sub finem.] And thus far concerning the first inequality of Saturn etc.; the second follows.

CAPVT III.

On the restitution of the second, or annual, inequality in Saturn.

Since for the second inequality — to which these three superior planets are subject on account of the annual revolution of the Sun (or of the Earth, in Copernicus's view) — we have already prepared the hypotheses together in the preceding chapter, taking the occasion left to us in the old supposition: it therefore now seems to remain only that, in the present chapter, we should likewise investigate from several observations of Saturn the measure of the epicycle of this kind, by which we said this annual inequality is accounted for; after the semidiameters of the little circles in this same planet have, by the weighing of acronychal oppositions, been set out above in their own numbers.

Since, then, for this reason the whole theory of Saturn is recalled to the computation of triangles — a likeness that is nowhere abandoned in the two following [planets] — accordingly, if lovers of the stars [astrophiles] will once attend with due care to its application and execution, which governs and perfects this art, I promise that all the remaining matters that follow will be easier for them.

In the year of Christ our Saviour 1590, on February 8, at the 8th hour in the evening, the longitude of Saturn was found by us from observation at Hven [Hven (Tycho's island, Latin *Huena*)] to be at 7 deg. 32 min. of Gemini ♊, near its greatest annual digression in the order preceding the signs. For again, four days afterward — namely on February 12, around the same 8th hour — it was observed at 7 deg. 38½ min. of Gemini ♊, with a southern latitude of 1 deg. 30 min.

Now on February 8 at the 8th hour, the Sun by its true motion was at 0 deg. 0 min. 40 sec. of Pisces ♓ from the true equinox; but Saturn's mean longitude, corrected from the foregoing likewise from the true equinox, was 2 signs 15 deg. 10 min. 5 sec. Moreover, the same Saturn's anomaly of the eccentric — that is, its simple distance from the apogee — corrected at one and the same time from the acronychal correction, is given at this time as 5 signs 18 deg. 51 min. 40 sec. And the disposition of these motions, by the Ptolemaic theory of Saturn set out earlier, is as follows.

The theory of Saturn exhibiting both of its inequalities from the foregoing data, in the Ptolemaic form.

First, according to the given ratio of the anomaly of the eccentric, C is moved from B through Saturn's orb of revolution in the order of the signs, and (as before) the first epicycle ED is described about C, with an arc of equal measure in the order preceding the signs as BC is in the order following the signs. Hence, from the center D, let the second epicycle FGH be added, whose revolution from F through G in the consequent order therefore here ends at H, because by its own motion it completes in this little circle a double of the proposed anomaly H. Finally, about the center H we attach Saturn's annual orb in this Ptolemaic form, KML. And since we have proposed to inquire into the semidiameter of this annual orb HL in Saturn, we will accomplish this about the same point in this form by means of celestial observations, intending to accommodate the rest in the following order for the sake of greater understanding and delight — since indeed no difference at all intervenes in that triple supposition.

Diagram. A large circle with B at the top, A on the central vertical axis (somewhat below center), and P near the bottom. In the lower-left region is the epicyclic cluster of small circles, with labels L, G, M, D, H, C, K, P, F around and within it. Straight lines run from A down to the cluster.

- B (top): start-point of Saturn's orb of revolution
- A (on central axis): the Earth (Ptolemaic center of observation)
- C, D (in cluster): centers of the first and second epicycles
- E, D: first epicycle ED
- F, G, H (in cluster): second epicycle FGH; planet at H
- K, M, L (in cluster): the annual orb about H
- P (bottom): point on the orb of revolution

First, then, in the triangle CDH: since, as prescribed, the sides are given — namely CD 8721, likewise DH 2907 — together with the angle CDH, which here is the complement, to the whole circle, of the double of the given anomaly of Saturn, namely 22 deg. 16 min. 40 sec.: there is found, therefore, both the angle DCH, 10 deg. 22 min. 24 sec., and the side CH, 6120 parts. Moreover,

when the angle DCH thus found is subtracted from the simple anomaly, there remain 5 signs, 8 deg. 29 min. 16 sec., whose complement to the semicircle — namely 21 deg. 30 min. 44 sec. — measures the angle ACH. And so, in the same triangle ACH, since from there two sides are given, AC 100000 and CH 6120,

together with the said included angle ACH, the angle of Saturn's complete first prosthaphaeresis, CAH, is likewise sought; and it is found to be 1 deg. 21 min. 48 sec., together with the side AH 94330, in such parts as the radius of the orb AC is 100000. Now, when the prosthaphaeresis of the eccentric thus found is subtracted from Saturn's proposed mean longitude, there remain 2 signs, 13 deg. 48 min. 17 sec. If this longitude is again subtracted from the true place of the Sun, 11 signs 0 deg. 0 min. 40 sec., the remainder is 8 signs, 16 deg. 12 min. 23 sec., the anomaly of the annual orb, that is the arc KML. Accordingly, removing on each side 6 signs for the semicircle KM, there remain 2 signs, 16 deg. 12 min. 23 sec. — that is, in disjunct degrees, 76 deg. 12 min. 13 sec. — by which the angle AHL is measured.

Lastly, then, in the triangle AHL, for the sought HL, there are given: first the side AH 94330; second the angle AHL 76 deg. 12 min. 13 sec.; third the angle HAL 6 deg. 16 min. 17 sec., which results from the difference of the comparison of Saturn's true place in the eccentric H from the true equinox — which was 2 signs 13 deg. 48 min. 17 sec. — and of its place observed in the heavens, 2 signs 7 deg. 32 min. 0 sec. Having therefore premised here too the sufficient givens [Greek: διδομένοις (didomenois, "givens")], there comes forth, by the analysis of the said triangle AHL, that which we set out to seek from the beginning — namely the side HL, 103981 — which is the measure of the semidiameter of the annual orb in Saturn, arisen for us from observation through all three hypotheses. When this is compared with that which likewise comes out from the place of Saturn observed in the year 1591, in the month of March, on the 17th day of that month at the 7½th hour in the evening, at 22 deg. 24½ min. of Gemini ♊, a difference is found that is certainly tiny and almost none.

Moreover, so that the quantity of the semidiameter of this annual orb may be compared from the other side as well, we took up another observation of Saturn, made in the same year as the earlier one — namely 1590, in the month of September, on the 7th day, at the 12th hour of the following night, where Saturn was found at 28 deg. 6 min. of Gemini ♊ (with a southern latitude of 1 deg. 11 min.), near its greatest morning digression, that is, its earlier station; for the true place of the Sun at this time was at 24 deg. 28 min. 35 sec. of Virgo ♍. But Saturn's mean longitude from the true equinox was 2 signs 22 deg. 14 min. 40 sec., with an anomaly of the eccentric of 5 signs 25 deg. 55 min. 17 sec. With these supposed, the following Saturn hypothesis in the Ptolemaic form is accommodated, as above.

By a calculation carried out from the given quantities [Greek: διδομένοις (didomenois, "the given data")] in the manner described above, through the

doctrine of triangles, there results first the angle DCH of $4^{\circ} 2' 16''$; then the side CH of 5852; third, the prosthaphaeresis angle of the eccentric, CAH, of $0^{\circ} 30' 3''$, to be subtracted; and fourth, finally, the side AH of 94234, in the units where the radius of the orb AC is 100000. Next, when the true longitude of Saturn's eccentric with respect to the true equinox — namely $2s 21^{\circ} 44' 37''$ — is subtracted from the true place of the Sun, $5s 24^{\circ} 28' 35''$, there remain $3s 2^{\circ} 44'$, very nearly, for the anomaly of the annual orb, KHL; the complement of this to a semicircle is the angle AHL.

Likewise, this same true eccentric place of Saturn, $2s 21^{\circ} 44' 37''$, when subtracted from the observed place at $28^{\circ} 6' 0''$ of Gemini, leaves $6^{\circ} 21' 23''$ measuring the angle LAH, which shows Saturn's digression at L — that is, here in the annual orb, in the order of the signs. And so, in the triangle AHL, with its angles given together with the side AH of 94234 already obtained above, there emerges from this side the sought-for side HL of $10453\frac{1}{4}$. But the same side was previously found from the western side to be $10398\frac{1}{2}$. The difference, therefore, is 55 parts, half of which is $27\frac{1}{2}$; when this is added to the smaller value found, or subtracted from the larger, the semidiameter HL of Saturn's annual orb comes out, bounded at 10426, in the units where the radius of the orb AC is 100000 — and indeed so closely bounded that it does not yet fall short of the observation, or of Saturn's apparent position in the sky, by even one minute.

If, however, other observations are compared with this diameter as found — and hence with the complete restitution of Saturn down to our own age — or those that appear in Book I of the *Letters* of Tycho Brahe [Tycho Brahe], p. 56, they will agree beautifully with this emendation of ours and will confirm that we have accomplished the task. For the unequal distance of the Sun from the Earth introduces no variation into the apparent position of this star, so far as we can yet detect with certainty. Were it otherwise, a smaller distance of the Sun from the Earth would everywhere leave a smaller diameter of the annual orb HL as well, and conversely. For although the effect that the Sun's eccentricity would introduce, on account of the vast size of Saturn's orb, might be very small indeed, yet if this reasoning here carried any weight, a perceptible apparent difference would certainly result here and there — one that would sometimes extend beyond a fifth part of a degree.

CAPVT IV.

On the Establishment and Extension of Saturn's Mean Motions.

Since the establishment of the mean motions requires the longest possible interval of time between observations made in the heavens, we shall here scarcely produce anything worthy from any of Ptolemy's [Ptolemy] later assumptions — even granting (which is not the case) that his observations were of full reliability. Moreover, it is to be deplored that, neither from this most celebrated craftsman among the foremost, nor from the rest in the age before our Tycho, has anything been left so certain that it cannot be doubted in many ways. Were I to say that error crept into the Ptolemaic astrolabe — by which he indicates that he observed these planets — I should perhaps be saying it with no less right than I showed above in the cases of the Sun and Moon.

But had Ptolemy indicated his method of observing, and had he named the fixed stars individually against which his sightings were made for the distances, then the remedy for correcting the erroneous places of these stars would already have been so well prepared for us, through their restitution by Tycho Brahe — indeed, through our own eyes — that we should gladly and deservedly have devoted this effort to examining and correcting Ptolemy's own acronychal places. However that may be, we suppose his recorded places of the apogees in these planets — though they must first everywhere be reduced by us to the true place of the Sun at the acronychal positions. Then, retaining the epicycle semidiameters (which I taught above, in the case of the Sun, are not altered by time), let us turn for the correction of the individual mean longitudes to the observations of the ancients made outside the acronychal positions — the very observations from which Ptolemy himself once derived the periodic revolutions of these bodies. For at whatever moment these observations record some planet's conjunction with a fixed star — for which both the star's place and the planet's whole inequality are found from what precedes — surely, with respect to what we intend regarding the correction of this simple longitude, it will be granted that we reach it far more certainly and securely than from the Ptolemaic data.

Concerning the advancement of the apogee places in the order of the signs — which we shall first of all work out individually from the ancients' source — I judge that this point must not be passed over: that the opinion held by Ptolemy, the Alphonsines, and their successors down to N. Copernicus [Copernicus] was a consenting one, namely that this advancement was common with the precession

of the equinox itself, that is, with the progress of the fixed stars in the order of the signs. Now as for the motion of the apogee place in Saturn — since it is established from Ptolemy's acronychal observation, which was made in the 20th year of the Emperor Hadrian, that is, 136 of Christ, etc., that it was at 23° of Scorpio from the true equinox, to which scarcely half a degree is added — for the reduction of the same to the opposite of the Sun's true place (which addition is here easily gathered from the prosthaphaeresis of the Sun, $21'$ to be added, and from the anticipation of Saturn's motion in the meantime; likewise also from the addition of about $21'$ for the inequality of the precession of the equinox according to our establishment): for that reason it is gathered that, at Ptolemy's said time, the position of the apogee from the mean equinox — from which we measure the equality of the motions in longitude — was at $23^{\circ} 50'$ of Scorpio, provided that Ptolemy's acronychal observations were otherwise correctly determined.

But in our age — that is, in the completed year of Christ 1600 — through the acronychal restoration and the subtraction of the inequality of the equinox, the same apogee place in Saturn is found at $26^{\circ} 27'$ of Sagittarius. Accordingly, over the interval of 1464 Julian years, the apogee position of Saturn has advanced uniformly by $32^{\circ} 37'$. And this difference, when first reduced to the interval of years from the first bissextile of the world — likewise from the birth of Christ and from some one of these epochs — then, when its product has been distributed over the difference of years of the proposed age, and duly adjusted by multiplication of the same for the others following: there results the position of the apogee of this star: first, to the first bissextile of the world, $4s\ 22^{\circ} 29' 20''$; then, to the birth of the Savior of the world, $7s\ 20^{\circ} 48' 12''$.

Thereafter, from the same proportion, the motion of the same, corresponding to

		Gr.	Mi.	Sec.	Ter.
1000	years is	22	16	45	0
100		2	13	40	30
20		0	26	44	6
1		0	1	20	12

This motion, indeed, is reserved for future use.

Furthermore, for the restitution of the mean longitude in Saturn, let us take up again the observation of it made by the ancient Assyrians or Egyptians in the year 519 of Nabonassar, on the 14th day of the month Tybi among the Egyptians, at the 6th hour of the evening. [Margin: *Almagest* XI.7] This time was in the year 229

before the birth of Christ; but after the first bissextile of the world, in the year 3736, on the first day of March, at the 4th hour, 25' after midday, on our horizon of Copenhagen [Hafniensi]. For on that evening there was observed (as Ptolemy reports) the conjunction of Saturn with the star following beneath the southern shoulder of Virgo, than which it was more southerly by two digits.

Therefore, since the longitude of the said star from the true vernal equinox at this time is found, by our restitution, to be at $9^{\circ} 3'$, with a northern latitude of about $2^{\circ} 43'$, the longitude of Saturn at the same moment was likewise at $9^{\circ} 3'$ of Virgo, with a latitude of $2^{\circ} 38' N$. But the true motion of the Sun, from our restitution at the same time, is found at $7^{\circ} 31\frac{1}{2}'$ of Pisces, from the mean equinox; and from this same mean equinox Saturn's simple longitude is in any case given from the Prutenic Tables [tabulis Prutenicis] as $5s 2^{\circ} 24'$. Likewise the apogee place of Saturn from the same equinox, from the corrected values above, is $7s 15^{\circ} 22'$; this being subtracted from the longitude, there remains the anomaly of Saturn's eccentric, $9s 17^{\circ} 2'$. And since this not very much exceeds — by mounting — the mean elongation of the eccentric, therefore an error of one degree, had it been committed here, would introduce into Saturn's apparent position, and into its longitude (which we seek), only one and a third minutes.

These matters, then, being thus established, for the investigation of Saturn's first inequality let there be described, as before, with center A of Saturn's orb, the circle BPC; again, from B and C, the first epicycle, on whose circumference, the angle ECD being given from the hypothesis equal to the angle BAC, the point D is placed; from which, finally, let the equant circle FGH be formed. And because the angle in the same circle, FDH, must be double the angle BAC (or DCE) — this too from the hypothesis in this revolution — the point H is supposed to be the center of the annual orb in the Ptolemaic form, in all respects as above; but of that, later. For now, since our intention is first to seek the angle of the first inequality, CAH:

Diagram. A geometric construction for Saturn's first (eccentric) inequality. A large circle, the orb of Saturn centered at A (lower left), carries on its circumference points B (top) and P (bottom). Near the upper-right of the circumference a cluster of smaller circles is drawn: the first epicycle and the equant circle, with the annual orb. Lines run from A out to the cluster, meeting the points C, D, H, and the annual-orb points K, L, M, E.

- A (center, lower left): center of Saturn's orb BPC
- B (top of large circle): point on Saturn's orb
- P (bottom of large circle): point on Saturn's orb (diametrically opposite B)
- C (in the right-hand cluster): point on Saturn's orb; carries the first epicycle

- D (in the cluster): point on the first epicycle's circumference; center of the equant circle FGH
- E (cluster, right edge): epicycle point (angle ECD)
- F, G, H (cluster): points forming the equant circle; H is the supposed center of the annual orb
- K, L, M (cluster): points of the annual orb
- H, L (cluster, with K above): annual-orb points referenced in the triangles

Thus, as before, in the triangle CDH, since two sides are given — CD 8721 and DH 2907 (namely the semidiameters of the epicycles, from the Tychonic [Greek: omitted])

observations [Tychonic] and our own, corrected above, which we have supported with reasons earlier that they are by no means altered here) — together with the included angle CDH of $45^{\circ} 56'$, which presents itself when the eccentric anomaly BPC is subtracted from a whole circle and the remainder doubled: there is found, accordingly, first the angle DCH of $8^{\circ} 19' 28''$, and the side CH of 11246, in the units where AC is 100000. [Margin: *Theory* in the earlier book, in the *Commentary on the Sun and Moon*]

Next, let the angle DCH be added to the angle ECD, and there are gathered $81^{\circ} 17' 28''$ for the measure of the angle ECH. Accordingly, the complement of this to a semicircle — namely HCA — is $98^{\circ} 42' 32''$. In this triangle HAC, then, since two sides about the named angle at C are given — namely HC 11246 and CA the radius of the orb, that is 100000 — there is acquired, besides the side HA of 102261, also the angle CAH of the prosthaphaeresis of Saturn's first inequality, $6^{\circ} 14' 18''$. When this is added to Saturn's estimated simple longitude, $5s 2^{\circ} 24'$, there results indeed the true longitude of its eccentric from the mean equinox, $5s 8^{\circ} 38' 18''$. When the supposed place of the Sun is subtracted from this, there remain $5s 28^{\circ} 53'$ for the anomaly of the annual orb.

This anomaly we correct as follows, so that an imperceptible error may not creep into the disentangling of the given data [Greek: διδόμενα (didomena, "the given quantities")] of the annual inequality: because the inequality of the precession of the equinox in this age, according to our establishment, is found to be $6' 25''$ subtractive, therefore, these being added to the place of the fixed star, its longitude from the mean equinox is composed as $5s 9^{\circ} 9' 25''$. When about $8'$ are subtracted from these — because Saturn stood very nearly acronychal — we gather that, according to the first or eccentric inequality, it was elongated from the mean vernal equinox by $5s 9^{\circ} 1' 25''$. When the Sun's place, $11s 7^{\circ} 31' 20''$, is

subtracted from these, there remains the said anomaly of Saturn's annual orb, more correct than before, 5s 28° 30', very nearly.

Now from H let there be described Saturn's annual orb according to Ptolemy, KLM. And the planet being given at the point L in it, it is manifest, from the given complement of the anomaly to a semicircle, that the angle KHM is 1° 30'. By this angle, in the triangle LAH (given, together with the two included sides AH, previously found to be 102261, and HL 10426, the corrected semidiameter of the annual orb), there emerges the angle HAL of 10' 14". Because previously I supposed only 8', the angle LHA — the annual anomaly — was made too small by precisely two minutes. If account be taken of this here, and for those two minutes the adhering 14" be subtracted, the second inequality — that is, the angle LAH — comes out precisely 10'. When the prior inequality of 6° 14' 18" is joined to this, the whole [motion] of Saturn here is aggregated from the mean motion as 6° 24¼', very nearly; and from the mean

Diagram. The companion construction to fig-p331-1, drawn from the opposite (left) side. A large circle, Saturn's orb, is centered at A (left of center, low). Points B (top) and P (bottom) lie on its circumference. At the upper right a cluster of smaller circles — the first epicycle, the equant circle, and the annual orb — carries the lettered points. Lines run from A to the cluster.

- A (center, lower left): center of Saturn's orb
- B (top of large circle): point on Saturn's orb
- P (bottom of large circle): point on Saturn's orb
- C (right-hand cluster): point on Saturn's orb carrying the first epicycle
- D (cluster): point on the first epicycle; center of the equant circle
- E (cluster, right edge): epicycle point
- H, G (cluster, top): points of the equant circle; H supposed center of the annual orb
- K (cluster, upper right): annual-orb point
- L (cluster, left of the small circles): annual-orb point (planet position)
- M (cluster): annual-orb point

[Margin: *Theory* in the earlier book, in the *Commentary on the Sun and Moon, etc.*]

[is to be subtracted] from the value already defined above, namely 9° 9' 25" from the mean equinox. Once this is done, there finally emerges from that same equinox a mean longitude of Saturn of 2s 2° 45', in the year 229 before the birth of our Savior, on the first of March, at 4 hours 25 minutes past noon. This longitude

of Saturn, having thus been found and restored, when it is adjusted to noon on the first of January of the completed year 3736 from the first mundane bissextile — by the addition of $10^{\circ} 14' 40''$ (this being the motion of Saturn in longitude corresponding to the interval of time from the first of March, etc., to the end of the said mundane year 3736) — yields a simple longitude of Saturn from the mean equinox, at the end of the year just mentioned, of $5s 12^{\circ} 59' 40''$ [?].

In our own century, by contrast — namely in the mundane bissextile year 5564, that is, the completed year 1600 of Christ — we obtain, through our emendation made above, that same motion of Saturn's mean longitude as $6s 28^{\circ} 21' 26''$, likewise reckoned from the mean equinox. Therefore, over the interval of 1828 proposed Julian years, or 667,677 days, the motion of Saturn in longitude from the mean equinox amounts to $1s 15^{\circ} 21\frac{1}{4}'$, beyond 62 complete revolutions made in the meantime.

From this established proportion of time and of Saturn's motion in longitude, the epoch of that motion is derived — once the whole circles are discarded. First, at the beginning of the first mundane bissextile that we adopted above, at noon of the following first of January, it is found to be $5s 23^{\circ} 28' 44''$; but at the beginning of the year of our Savior it is found to be $2s 12^{\circ} 32' 51''$.

Furthermore, from this same proportion there emerges the corresponding motion in longitude:

		Sig.	Gr.	Mi.	Sec.	Ter.
1000		11	24	52	51	40
100	years	4	23	29	17	10
20		8	4	41	51	26
Commu		0	12	13	35	24
[tation]						
1						

All these values, like the foregoing ones derived for the motion of the apogee, are to be reserved for the canon of mean motions to be constructed later. And so much for the restoration of Saturn's longitude.

CAPVT V.

Concerning the restoration of the acronychal longitude, that is, of the orb of Jupiter, as it agrees with our own age.

Since in the foregoing restoration of Saturn we have also touched upon those matters that commonly concern the motions of the three superior planets — and have moreover added a description of the three hypotheses to which they may be applied without distinction — we may therefore be excused on grounds of brevity if we set out only those matters that serve the restoration of what follows. In the present case, this proves more difficult for us with Jupiter, inasmuch as we possess fewer observations of it. Indeed, after the restoration of that planet had once been worked out at Hven with respect to the mean course of the Sun, we sought nothing further here — until the acronychal appearances of subsequent years revealed to us a sufficiently difficult restoration in the region of the apogee and perigee, the longitude in nearly the same places of the eccentric gradually increasing beyond the earlier emendation. This was confirmed by the observations we afterward made here, and indeed carried still further: so much so that we hesitated in doubt whether to ascribe to the apogee of this star a certain anomaly contrary to the custom of the other planets (as once seemed the case to Tycho [Tycho Brahe]) — until we began to extend the first, or eccentric, inequality of Jupiter to the true motion of the Sun.

[the true motion] of the Sun. Here, indeed, this said anomaly around the apogee and perigee was rendered milder — so much so that, although it is not entirely removed, it nevertheless no longer appears so great that we should on this account suppose anything in the hypothesis of the Jovian star to be, in this respect, alien to that of the others; but rather we should remain within the mean limits of the restoration.

Now, after most diligently conducting an examination of the acronychal places of Jupiter that we have hitherto obtained from observations, comparing them against the Prutenic Tables [tabulae Prutenicae], and repeating this examination several times (as is fitting in this enterprise of restoration), I found that $4\frac{1}{2}'$ must be added to its simple longitude for this our century, and $40'$ to the apogee; likewise that $\frac{1}{4}$ part is everywhere to be added in the prosthaphaeresis of the eccentric. Once this is done, the observed and computed acronychal positions, with their requisites and distances, come out as follows.

Presentation of eight places of Jupiter, both from acronychal observation at the true opposition of the Sun, and from calculation issuing for the same times according to the aforementioned correction of the Prutenic Tables.

						Log	Lon					
	An	Me					g.	An				
	ni	ns.	D.	H.	M.	obs	Me	om	Pra	Lon		
						. (S.	d. (Se	alia	ec.	g.		
						G. (S.	x. °	(Se	aeq	sup		
						M.)	I II)	I II)	I II)	I)		Diff
												er.
I	158	Sep	6	20	10	□	5	2	27	23	5 $\frac{2}{3}$	
	3	te.				23	26	47	59	38	+	
						33	52	31	46	$\frac{1}{2}$		
							42	5				
II	158	Oct	13	7	20	□	0 0	3	28	22	0 $\frac{1}{4}$	
	4	ob.				22	19	20	0	0 $\frac{1}{4}$	+	
						0	12	57	44			
								34				
III	159	Apr	23	19	0	□	3	0	28	13	1 $\frac{1}{4}$	
	1	ilis				13	18	38	6	8 $\frac{3}{4}$	-	
						10	21	58	30			
							55	30				
IV	159	Sep	12	6	10	□	5	2	28	28	1 $\frac{1}{4}$	
	5	te.				28	31	52	10	54	-	
						56	30	11	30	$\frac{3}{4}$		
							55	7				
V	159	Oct	18	12	0	□ 5	4	3	28	5	5 -	
	6	ob.				40	56	25	11	35		
						0	23	32	25	0		
							[?]	23				
VI	160	Sep	17	11	10	□ 4	5	2	28	4	1 $\frac{1}{4}$	
	7	te.				10	36	56	20	8 $\frac{1}{2}$	-	
							8 3	32	50			
								7				
VII	161	Dec	30	14	40	□	1	4	28	19	2 $\frac{1}{8}$	
	0	e.				19	15	36	23	38	+	
						36	51	24	45	$\frac{2}{31}$	[?]	

						Lon					
						Log	g.	An			
						. obs	Me d.	om alia	Pra ec.	Lon g.	
						. (S.	(Se	(Se	aeq	sup	
						G.	x. °	x. °	. (°	. (°	Diff
An	Me					M.)	I II)	I II)	I II)	I)	er.
ni	ns.	D.	H.	M.							
						[?]	18	33			
VIII	161	Ma	1	22	0	□	2	5	28	21	2¼
	3	rt.				21	21	42	25	42	—
						45	41	14	40	¾	
							18	22			

Parsing notes: This citation-grade table is very densely set and several cells are smudged or column-ambiguous; many degree/minute/second sub-values are uncertain. The "Log. obs." column gives the observed zodiac-sign longitude (sign-glyph + degrees/minutes); the "Long. Med." (mean longitude) and "Anomalia" columns are sexagesimal sign-degree-minute-second; "Praec. aeq." is the precession/equation; "Long. sup." (computed superior longitude) and "Differ." (difference between observed and computed) are at right. — dense numerical table, multiple uncertain cells.

And yet, with a restoration of this kind so diligently carried out, the greatest difference here too in Jupiter — in either direction, between the observations we obtained at the acronychal places and the corrected tables — rarely exceeds 5 minutes; nor indeed will anyone who has considered the first observation together with the fourth and the last understand that any greater precision can be reached here.

I hope, moreover — indeed I am firmly persuaded — that the remaining observed acronychal positions of Jupiter, which are still preserved in the Tyconic treasury, will accord so well with this restoration that nowhere will a greater difference from the calculation appear. The earlier restoration, carried out with respect to the mean motion of the Sun, could in a way attest to this, were it too to be brought to light. It now remains for us to proportion the semidiameters of the epicycles of Jupiter's orb to this restoration.

Let there be recalled here the second supposition in Saturn...

Diagram. A geometric construction of Jupiter's orb in the Copernican form. A large outer circle (the orb of Jupiter) is drawn with B at the top and P at the

bottom, a vertical diameter B–P running through the figure. Near the upper-left interior is a small cluster of epicyclic circles around the eccentric center, and the planet's central point A lies just below this cluster, near the figure's center. Lines run from the epicyclic cluster through the inner small circle (the annual orb) to the lettered points.

- B (top): apogee point of the orb
- P (bottom): perigee point
- A (center): center of the annual orb / Earth's position
- G (cluster top), H (upper-right of G), D (epicycle center), F (lower-left), C (cluster bottom): centers and points of the eccentric and epicyclic circles. C–D–H is the epicycle-semidiameter chain (C D, D H) of the surrounding text; the construction also uses point E (angle D C E), present in the text but not separately resolvable in this figure at scan DPI [?].
- M, L (just above A): points on the annual orb
- K, I (right of A): points on the annual orb

[the second supposition], which we have intimated to be the Copernican one; upon which it also pleases us to explain the remaining phenomena of this Jovian star — although we have intimated above that the same constructions can be applied indifferently to any of the three superior planets. Since, therefore, the semidiameters of the epicycles on the circumference of the orb — namely C D and D H — take their measure in that proportion in which A C is 100,000 parts, according to our restoration (which earlier rested, in the tables of Reinhold [Reinhold] that we follow, upon the Copernican foundation) — so that C D is reckoned as 6870 parts and D H as a third part of the former, namely 2290 — it now follows, through the addition of $13\frac{3}{4}$ [?] (which make up the $\frac{1}{4}$ part of the maximum prosthaphaeresis of the eccentric), to enlarge the said semidiameters above the Copernican supposition to such an extent that C D becomes 7155 and D H its third part, namely 2385. For this augmentation expresses, as closely as possible, the said emendation in the supposition of the triangles — as we shall afterward show in the construction of the table, and immediately thereafter, when we are engaged in acquiring the measure of the semidiameter of the orb I K M L that is to be sought; which task indeed immediately follows upon this one that we have here imposed for the emendation of Jupiter's first inequality.

CAPVT VI.

Concerning the annual inequality that appears in Jupiter, to be examined and restored.

Among the observations by which we have explored, here and there, the measure of the semidiameter of Jupiter's annual orb, we adduce two — as above in the case of Saturn — by whose standard we measure this inequality of which we now treat in Jupiter. Of these, the earlier was made by us at Hven, in the year of our Lord 1593, on the 28th of September, at 8 hours in the evening, when Jupiter, at an altitude of about 9 degrees, was distant from the tip of the tail of Capricorn [$\square$] by $33^{\circ} 59'$, but from the right knee of Ophiuchus by $32^{\circ} 33'$, having at the same time an observed southern declination of $23^{\circ} 7'$. From this, by means of the verified places of the fixed stars and the reasoning of spherical triangles, the longitude of that same Jupiter — delimited and freed from refraction — was found to be $\square 13^{\circ} 56' [?]$, with a true southern latitude of $0^{\circ} 25'$.

For this time, the mean longitude of Jupiter from the true equinox was computed as $\square [10s] 0^{\circ} 20\frac{1}{2}' [?]$, and the place of the apogee as $6s 7^{\circ} 32\frac{1}{3}' [?]$, according to our aforementioned acronychal emendation. On this account, the anomaly of the eccentric is also found from these to be $3s 22^{\circ} 48\frac{1}{2}'$, and the prosthaphaeresis from the same restoration is $5^{\circ} 6\frac{1}{2}'$, to be subtracted. Hence the true or equated longitude of the eccentric comes out as $9s 25^{\circ} 13' 50''$. But the true place of the Sun at the same time was, for us, $6s 15^{\circ} 17' [?]$; and when the equated longitude of the eccentric is subtracted from this, there remain $8s 20^{\circ} 3' 10'' [?]$ for the annual anomaly, of which more later.

These matters are applied on the assumption of the hypothesis of Jupiter in the Copernican form, as in the following diagram. From the center of the universe — that is, the Sun, according to Copernicus [Copernicus] — let there first be described the orb of Jupiter B C; then, with the eccentric epicycles drawn from C and D, the planet will in this revolution be at H. But the side A H is given as 985,462; and in the triangle C A H, whose angles are given — after H C D has been found to be $10^{\circ} 55' [?]$ seconds, and subtracted from the angle D C E — the angle A C H becomes $78^{\circ} 7' 10'' [?]$. The prosthaphaeresis A C H, moreover, is $5^{\circ} 6' 40''$; whence A H C comes out as $96^{\circ} 45[?]' 40''$; and A C, the radius of the orb B C, is reckoned as 1,000,000.

Furthermore, let the annual orb M L K be described, homocentric with B C, and let the Earth be placed...

Diagram. A geometric construction of Jupiter's orb and annual orb in the Copernican form, paralleling fig-p334-1. A large outer circle (the orb of Jupiter, B C) has B at the top and P at the bottom on a vertical diameter. At center is point A with a small inner circle (the annual orb) bearing points K, I (upper right) and M (below A). At the left side is a cluster of small epicyclic circles labeled H and G (top), D, F, C (descending), and E (lower-left). (The earlier "B" reading here was spurious — B is the apogee at the top of the outer orb, not a cluster point.) A long line runs from the left-hand cluster across through the center to the right, intersecting the construction points.

- B (top): apogee of the orb
- P (bottom): perigee
- A (center): the planet's reference center
- K, I (upper right of A): points on the annual orb
- M (below A): point on the annual orb (Earth's position in this revolution)
- H, G (cluster top, set adjacent "H G"): construction points; the planet sits at H per the text (angle D C H / A C H)
- D (below H/G): epicyclic center (the D C H angle and the E D eccentric anomaly of the text)
- F (below D), C (cluster bottom): epicyclic points
- E (lower-left of the cluster, set apart): start point of the eccentric anomaly E D

[let the Earth be placed] according to Copernicus, on the circumference of that orb, at M; and let M A be sought. In the triangle A H M, therefore — since the angle H A M, the anomaly of the annual orb, was granted above to be $80^{\circ} 3' 10''$, while the angle of commutation A H M is found to be $11^{\circ} 17' 20''$, the observed place of Jupiter on the eccentric having been subtracted from the true — the angle A M H is thereby also given as $88^{\circ} 39' 30''$. And since the side H A in this same triangle was granted above to be 985,462, therefore from these the sought semidiameter of the annual orb A M comes out as 192,948, of the kind of which the radius A B is supposed to be 1,000,000.

The other, quite recent, observation was made by us here at Copenhagen [Hafnia], on Jupiter, in the year 1616, on the 10th day of the month of March, at 4 hours in the morning, just at the vernal equinox. At that time, by means of our sextant, Jupiter was distant from the Heart of [the Scorpion, π] by $23^{\circ} 46'$, but from the Bright Star of the Vulture [Lucida Vulturis = Vega/Lyra] by $39^{\circ} 44\frac{1}{4}'$; and this at an altitude of Jupiter of about 10 degrees.

From these, the true longitude of Jupiter was found to be $27^{\circ} 41'$ in [Taurus, $\rightarrow$] ♈ with a northern latitude of $0^{\circ} 24'$, refraction and parallax being eliminated in both cases. To this time, these restored motions correspond:

	Sig.	Gr.	Mi.	Sec.	
Mean longitude of Jupiter from the true equinox	8	21	52	0	
Anomaly of Jupiter's eccentric	2	13	56	30	
To which corresponds the restored prosthaphaeresis		5	10	10	Subt.
Therefore the true longitude of the eccentric is	8	16	41	50	
But the true longitude of the Sun	0	0	1	15	
From which, the true longitude of Jupiter's	3	13	19	25	

	Sig.	Gr.	Mi.	Sec.
eccentric				
being				
subtracte				
d, there				
remains				
the				
anomaly				
of annual				
commuta				
tion, just				
as we				
employ it				
in these				
planets				

With these granted, let the hypothesis of Jupiter be set up likewise in the Copernican form (as in the following diagram), in which everything is arranged as in the preceding one, with only the present variation of the revolutions taken into account. For — the rest being passed over as having been sufficiently and more than sufficiently impressed upon the reader so far — since the anomaly of the annual orb exceeds a quadrant by a little, the Earth will be located at M, and from there the planet is seen at H, namely by the line M H. But since our aim in the inquiry into the semidiameter A M of the annual orb in Jupiter is fixed here, so that it may be compared with the one found earlier from the other side — therefore let the first angle found, D C H, of $7^{\circ} 52' 0''$, be added here to the anomaly of the eccentric E D, and in the complement of the sum to a semicircle...

Caput VI (continued) — Jupiter's annual inequality

lies within the semicircle. Angle ACH is $98^{\circ} 11' 38''$. But angle CAH, the prosthaphaeresis, is $5^{\circ} 10' 10''$; therefore AHC is $76^{\circ} 38' 20''$. In triangle ACH, then, the angles are known, and together with the orbital radius AC = 1,000,000 the side AH is found to be 1,017,321.

With these things thus arranged and determined, triangle AHM likewise has its angles known, along with the side AH just now found as well. For when the equated longitude of Jupiter's eccentric, $8s\ 16^{\circ} 41' 50''$, is subtracted from the true observed longitude of Jupiter itself, $8s\ 27^{\circ} 41' 0''$, there remains for the annual commutation — that is, angle AHM — $10^{\circ} 59' 10''$. The angle HAM, however, is

recognized from the complement of LM to a semicircle, the arc by which the annual anomaly is measured, which we earlier set at $3^s 13^\circ 19' 25''$; accordingly the said angle HAM comes out as $76^\circ 40' 35''$. And therefore, by computation, the semidiameter of the annual orb AM is 194,033, which is what we sought.

But earlier the same quantity had been found to be 192,948. The difference, then, is 1,085. Half of this, 542, when added to the smaller value, yields as the limited semidiameter of Jupiter's annual orb — that is, the measure of the side AM — 193,490. Around this same measurement our other observations, on either side, fall very nearly into place: those we made at the morning hours of this year on 18 February and 8 March.

Nor indeed do I see how the Sun's unequal distance from the Earth — which is greatest at the opposite and fixed seasons of the year — could introduce an analogous inequality into the annual orb of this star. For even if only the ratio of half the Sun's eccentricity were taken into account here, the greatest annual commutation would appear smaller, with the Sun in perigee than with it in apogee, in this Jovial star by about $\frac{1}{2}^\circ$; and that certainly does not agree with observations. For when, among others, we consult the observation we made on 1610 January 26 in the evening and on the immediately following days — at least to the extent of a sighting fixed by threads — when Jupiter stood motionless, or stationary, in very nearly a straight line and at almost the midpoint between Aldebaran and the northern horn of Taurus (except that it had deflected a little eastward from that line, and was somewhat nearer to Aldebaran): we then reckon the longitude of this planet to be scarcely as much as the Prutenic Tables [tabulæ Prutenicæ] give for that time, namely $10\frac{3}{4}^\circ$ Gemini. That longitude certainly requires, even around the Sun's perigee, a semidiameter of Jupiter's annual orb still somewhat greater than the one found. But so much, up to this point, for the restoration of this star's longitude as suited to our own age. There follows a comparison with the ancients.

Caput VII — The restoration and periodic extension of Jupiter's apogee and longitude

CAPVT VII.

On the restoration and periodic extension of Jupiter's apogee and longitude.

Since we set out above, in the case of Saturn, the reasons why the correction of the mean motions in these planets is less safely derived from Ptolemy

[Ptolemæus] — especially that of the longitudes — we shall therefore for the present put forward only a few points that bear on the matter regarding this star.

First, for the correction of its apogee's position as found in Ptolemy, we proceed briefly as follows.

Ptolemy, in the first year of Antoninus Pius — that is, in the year 136 completed after the birth of Christ — found the point of Jupiter's apogee, or greatest elongation, in its eccentric at 5s 11° 0', measured from the true vernal equinox; and this by means of three acronychal positions of Jupiter directed opposite the mean course of the Sun. For this reason, even though we are compelled to retain that same measure of the apogee's position of necessity — because sufficient observations of the ancients are lacking through which we might discourse further on the matter — nevertheless, when we have regard both to the mean equinox and to the Sun's true motion, we find that about 3° must be added to it. For since the Sun's anomaly, around the point opposite Jupiter's apogee in Ptolemy's time, is found to be 9s 15° from the mean equinox, and therefore the Sun's prosthaphaeresis at that place is 2° 25' additive — but the combined daily motion of the Sun and Jupiter (ἐν πλάτει [Greek: en platei, "broadly," i.e. approximately], for precision is not required here), around the opposite point in that revolution of each, was 1° 8' — therefore, according to the said prosthaphaeresis of the Sun, there are elicited 1 day 19¾ hours by which Jupiter was set opposite the true position of the Sun earlier than it was opposite the mean position. To this interval there correspond, in the motion of Jupiter itself in the meantime, 14¾'; on account of which the apogee of this planet is drawn forward by 3 whole degrees, the distance between the true and mean equinox of Ptolemy's time being added in at the same time.

Accordingly the corrected position of Jupiter's apogee, in the year 136 completed after Christ, is, with Ptolemy, 5s 14° 0'. For we neither undertake nor even wish to derive it from Ptolemy more precisely, seeing that the positions of the fixed stars were uncertain to him. But in our own age — that is, in the year 1600 finished after Christ — we have that same apogee motion of Jupiter, restored above, at 6s 7° 39¼' from the true equinox, but 6s 7° 32' from the mean. In an interval, then, of 1464 Julian years, the position of Jupiter's apogee has advanced 23° 32' in the order of the signs. And these, drawn out proportionally through intervals of years, yield as follows:

	Gr. [°]	Mi. [']	Sec. ["]	Ter. ["]]
1000	16	4	28	51

		Gr. [°]	Mi. [']	Sec. ["]	Ter. ["]]
years	100	1	36	26	50
	20	0	19	17	23
	1	0	0	57	52

By continuing the same proportion we find the epoch of the apogee's motion in Jupiter at the beginning of the first bissextile year of the world to be 3s 8° 5' 38"; likewise at the beginning of Christ our Savior, 5s 11° 48' 50". All of which are to be preserved in the tables of mean motions.

With Jupiter's apogee thus restored, we come to its longitude — which is indeed of greater weight, inasmuch as any error it may contain is exposed more evidently by the passage of time. This is the reason why here too we by no means remain within the Ptolemaic observations, on account of the uncertainty already mentioned concerning them, but rather by restoration...

Caput VII (continued)

we draw the restoration of this planet to a certain conjunction with a fixed star observed by Timocharis [Timocharis] or some other, about 400 years before Ptolemy — provided that the position of such a coupling can still be precisely known in the heavens, the star's place being verified exactly through Tychonic diligence, and by our anticipation of the equinoxes being restored to its former state.

Ptolemy records that a conjunction of this kind, of Jupiter with the southern Ass [auster asinus] in the asterism of Cancer, took place in the year 45 according to the era of Dionysius, on day 10 of the month Virginion; but, reckoning from the death of Alexander, in the year 83, on day 18 of Epiphi according to the Egyptians, in the morning. [Margin: *Almagest* IX.3] For he writes that at that time the star of Jupiter had covered over the southern Ass. Now this time, when we compute Julian years backward, falls in the year 241 before the birth of Christ, and at the morning hour of 4 September — with us here, about the fourth hour of the same day, before completed noon. For which time I find the longitude of the southern Ass, according to our restoration, at 3s 7° 27' from the true equinox, with a southern latitude of 0° 25'; but the longitude from the mean equinox 3s 7° 33', exactly as Ptolemy presupposes the same value (which is most rare). Hence it is necessary that Jupiter then occupied the same position. Furthermore, I find the true position of the Sun at the same moment, even according to our restoration, at 5s 7° 9¼' from the mean equinox. Moreover, since its apogee position, restored

earlier and reduced down to here, falls at $7^{\circ} 50'$ Cancer, and the mean longitude is given, in whatever way, from Ptolemy's reasoning, at $22^{\circ} 59'$ Gemini from the mean equinox — therefore, with the former subtracted from the latter, there remains the anomaly of Jupiter's eccentric $9s\ 15^{\circ} 3'$, to which corresponds the restored prosthaphaeresis $5^{\circ} 12' 0''$. And when this is added to Jupiter's mean longitude, and the aggregate is taken away from the Sun's true position, there is left Jupiter's annual anomaly $2s\ 8^{\circ} 58\frac{1}{2}'$.

Diagram. With these things assumed, let the hypothesis of Jupiter be laid out in the Copernican form, agreeing with the earlier demonstrations in the same. The figure shows a large eccentric circle with the cardinal points B (top), P (bottom), and to the right a small epicyclic construction with points labeled H, G, F, E, C, and K, while A (the central reference point) and L lie within, and M is the Earth's position toward the lower part of the diameter.

- B (top of large circle): apogee direction
- P (bottom of large circle): perigee direction
- A (interior, on the central vertical axis): the Sun / reference center
- L (interior, left of center): point on the annual orb
- M (interior, below A on the axis): the Earth
- E (right side): point on the eccentric
- H, G (upper right of epicycle): planet positions
- F, C (center of epicycle group): deferent/epicycle centers
- K (lower, between center and epicycle): construction point

In this figure, since the angle HDC is given as $149^{\circ} 54'$ — namely, from the residue of the double anomaly to a full circle — together with the sides enclosing that same angle, CD and DH, which are the semidiameters of the epicycles restored above: therefore from these there emerges the angle DCH $7^{\circ} 23' 35''$. When this is subtracted from the anomaly of the eccentric, there remain $9s\ 7^{\circ} 33' 25''$, and on that account the angle HCA is $97^{\circ} 33' 25''$. But the angle CAH, the prosthaphaeresis of the eccentric, was $5^{\circ} 12' 0''$; therefore the angle CHA will be $77^{\circ} 14' 35''$. In the same triangle CHA the radius AC = 1,000,000 is also given, whereby the side AH is found to be 1,016,410.

Caput VII (continued)

Furthermore, in triangle HAM, since two sides are granted — HA = 1,016,410 and AM = 193,490, from our restoration above — together with the angle enclosed between them, HAM = $111^{\circ} 1' 50''$ (namely, the angle that lies in the complement

of arc LM to a semicircle, which measures the anomaly of the annual orb): accordingly, from these there is finally acquired the angle of annual commutation $AHM = 9^{\circ} 26' 0''$.

Diagram. This is the companion figure to the one on the preceding page: a large eccentric circle with B at top and P at bottom, the small epicyclic construction at upper right (labels H, G, D, F, C, E, K), the center A on the vertical axis, L to the left of center, and M below A toward the perigee.

- B (top of large circle): apogee direction
- P (bottom of large circle): perigee direction
- A (interior, on the vertical axis): the Sun / reference center
- L (interior, left of center): point on the annual orb
- M (interior, below A): the Earth
- E (right side of epicycle): point on the eccentric
- H, G (upper right of epicycle): planet positions
- D, F, C (center of epicycle group): deferent/epicycle centers
- K (lower, between center and epicycle): construction point

When this angle of annual commutation ($9^{\circ} 26' 0''$) is added to the equated longitude of Jupiter's eccentric — namely $2s\ 28^{\circ} 11'$ — there is composed the apparent longitude of Jupiter from the mean equinox: $[7^{\circ}] 37'$ Gemini. But it was observed at $7^{\circ} 33'$ Gemini. The difference, therefore, is found here to be only $4'$. After this difference has been taken away from the Ptolemaic mean longitude, that longitude remains, corrected to this time, at $2s\ 22^{\circ} 55'$. Nor indeed is there here, on account of so slight a difference between the assumed and the sought mean longitude of Jupiter — which redounds into both anomalies — anything of sensible error that befalls the determination of the mean longitude.

Now, since for the completion of the current year from the time mentioned, at which this observation was made, there are to be added $9^{\circ} 55' 30''$, which Jupiter would have completed in the meantime by its mean motion, there is composed its mean longitude from the mean equinox, at the noon next following the calends of January of the year 240 completed before Christ, as $3s\ 2^{\circ} 50\frac{1}{2}'$. But in our own age — that is, in the year 1600 likewise completed after the birth of Christ — we find the same corrected mean longitude of Jupiter from the mean equinox to be $5s\ 10^{\circ} 3\frac{1}{3}'$ [recte: the seconds/thirds digit is uncertain in the source]. In an interval, then, of 1840 Julian years, or 672,060 days, Jupiter had completed 155 whole revolutions and, in addition, $2s\ 7^{\circ} 48'$. From this proportion of time and motion given between themselves, the epoch of Jupiter's mean longitude at the beginning

of the first bissextile year of the world (whole circles being everywhere discarded) is found to be 2s 1° 16' 22"; likewise at Christ-in-the-flesh, 5s 29° 56' 46". Moreover, from the same proportion the following agree:

		Sig. [signs]	Gr. [°]	Min. [']	Sec. ["]	Tert. ["]]
	1000	4	2	56	5	13
years	100	5	6	17	36	31
	20	8	7	15	31	18
commo n	1	1	0	20	31	44

These are to be preserved for the motion of longitude by extending them through the Canon.

And so much, up to this point, for the restoration of Jupiter's longitude.

Astronomia Danica — Pars Altera, Liber Secundus Theoricorum

CHAPTER VIII.

On the restoration of the course of Mars, with respect to the first inequality, which appears all around its own orbit at opposition to the true place of the Sun.

In examining and restoring the acronychal [opposition] phenomena of Mars, we shall fit them to the Reinhold [Prutenic] tables by a method not entirely the same as for the other superior planets. For although the prosthaphaeresis — that is, the first inequality of Mars — comes out much closer to our wishes when the extreme nocturnal risings of that planet are referred to the true opposition of the Sun as well, nevertheless these same prosthaphaereses, when worked out all around by means of an eccentric orbit (or, what amounts to the same thing for us, a homocentric orbit equipped with two small circles), depart only slightly from the earlier Tychonic restoration in respect of distributing the prosthaphaereses around the orbit of Mars — provided we keep rather to the geometrical procedure that Copernicus [Copernicus] introduced. For here too it is not a third part of the whole eccentric inequality that is yielded to the semidiameter of the

second, or equant, small circle (as in the two superior planets), but only a fourth part.

For since the combined magnitude of both epicycles is found from observation to be 18,550 parts — where the radius of the orbit is 100,000 — in the proportion shown just above as closely as may be (a proportion that was indeed exactly fivefold in Ptolemy [Ptolemy]), it will follow, in the adjoined diagram and in Tycho's [Tycho Brahe] hypothesis (to which we promised above to apply the appearances of Mars), that the semidiameter of the greater epicycle CD is 14,840 and that of the smaller DH is 3,710. In all else the account given in the last figure of the second chapter is admitted throughout, with this one consideration: that here the Sun's orbit is cut in two places by the orbit of Mars as seen from the Earth M. From this it follows, on Tycho's hypothesis as well as on that of Copernicus, that Mars at opposition comes to be far nearer to the Earth than the Sun is.

[Margin: See the restoration of the acronychal [longitudes] in the Commentary of Johannes Kepler [Kepler] on the motion of Mars, pp. 54 and 55.]

Diagram. Caption: "The hypothesis of Mars in Tychonic form, representing its acronychal [opposition] situation." A large circle (the Sun's orbit) is drawn about the central point A near the bottom, with the vertical diameter running from B at the top down through the center to P at the bottom. Within the upper-left region a smaller circle (the orbit of Mars) is drawn about a center near M, and at its upper edge sits a compound epicycle construction — a small circle with the closely-grouped lettered points G, H, D, E, F, C marking the two nested epicycles (greater CD, smaller DH). Lines run from A up to the epicycle cluster.

- A (bottom center): the Earth / center of the Sun's orbit
- B (top): point on the Sun's orbit (upper end of the vertical diameter)
- P (bottom): lower end of the vertical diameter
- M (left of center): center of the orbit of Mars
- G, H, D, E, F, C (upper left, clustered): the nested-epicycle construction points (greater epicycle CD, smaller DH)

Furthermore, for the modern correction of the Prutenic tables, there are added to the mean longitude of Mars at the end of the year 1600 some $6\frac{2}{3}$ minutes; but to the apogee scarcely one and a third minutes [sesquiertium], so that at that time — leaving aside the precession of the equinoxes — it stands at 0 degrees 35 minutes of Leo. This same determination Johannes Kepler also retains in his very full commentary on Mars. From these data we shall continue the catalogue

inserted in that same commentary, adding two pairs of acronychal observations of our own, so that the required differences may be seen at a single glance, in the usual manner, in a small table.

A presentation of fourteen extreme risings of Mars, adapted to the true opposition of the Sun, together with the moments of time and the mean motions corresponding to these according to the restoration set forth above, and converted into true [motions], with the differences appended.

Nu m.	An n.	Me ns.	D.	H.	M.	Lon					Diff e. '
						Lon g. obs . °	Lon g. sim pl. °	An om alia °	Pra ec. aeq . °	Lon g. sup . °	
						Sig. . °	Sex . °	Sig. . °		Sig. . °	
I	158 0	No v.	18	1	30	□ 6 27 ½	0 27 54 35	4 27 11 10	27 57 50	□ 6 27 ½	0
II	158 2	Dec .	28	4	0	□ 16 54 ½	1 11 28 40	5 10 44 20	27 59 10	□ 16 55 ½	1 +
III	158 5	Ian.	30	19	15	□ 21 35 ½	1 52 10 10	5 51 25 30	28 1 5	□ 21 37	0½ +
IV	158 7	Ma rt.	6	7	20	□ 25 42	2 32 47 35	0 32 2 40	28 3 0	□ 25 42 ½	3½ +
V	158 9	Apr il.	14	6	20	□ 4 23	3 16 16 20	1 15 31 10	28 5 0	□ 4 26 ½	3½ +
VI	159 1	Iun ii	8	7	45	□ 26 42	4 7 40 5	2 6 54 23	28 6 45	□ 26 45	3½ +

Nu m.	An n.	Me ns.	D.	H.	M.	Lon		An om alia	Pra ec. aeq	Lon		Diff e. '
						Lon g. obs . Sig.	g. sim pl. □ Sex .''''			g. sup . □ Sig. '''	g. sup . □ Sig. '''	
VII	159 3	Au g.	25	17	30	□	5	3	28	□	0½	—
						12	11	11	8	12		
						15	49	3	40	14		
						20	20	½				
VIII	159 5	Oct o.	31	0	40	□	0 9	4 8	28	□	1½	—
						17	6	20	10	17		
						30	40	0	25	29		
						¾	½					
IX	159 7	Dec .	13	16	0	□ 2	0	4	28	□ 2	1 —	
						27	55	54	12	26		
						2	15	20				
						30	25					
X	160 0	Jun ii	18	14	0	□ 8	1	5	28	□ 8	0½	+
						37	36	35	14	37		
						24	37	5	½			
						45	10					
XI	160 2	Feb .	20	14	15	□	2	0	28	□	2 —	
						12	16	15	15	12		
						26	46	58	55	24		
						35	40					
XII	160 4	Ma rt.	28	16	20	□	2	0	28	□	0 —	
						18	58	57	17	18		
						36	45	57	50	36		
						½	20	0	½			
XIII	160 8	Iuli i	24	2	0	□	4	2	28	□	3½	+
						11	45	45	21	11		
						10	53	14	45	13		
						15	30	½				
XIV	161	Oct	8	16	50	□	5	3	28	□	3½	

Nu m.	An n.	Me ns.	D.	H.	M.	Lon					Diff e. '
						Lon g. obs . □ Sig.	Lon g. pl. □ Sex . °''' Sig.	An om alia □ Sig. °''' °'''	Pra ec. aeq . °''' Sig.	Lon g. sup . □ Sig. °'''	
	0	o.				25	48	47	23	25	—
						30	35	55	50	26	
							0	20		½	

Parsing notes: Source double-scan garble is heavy in the body-text overprint of this page; the clean table image at the page foot was used. The "Diffe." column gives small fractional minute differences with + / - signs. Sign-symbols read from the clean table: row VII "Long. obs." and "Long. sup." both show what appears to be ♓ (Pisces) but is more likely ♐ (Sagittarius) given the Anomalía progression — flagged [?]. Several seconds-digits in the "Long. simpl." and "Anomalía" columns are at the resolution floor; transcribed from the clean foot-of-page rendering.

In these fourteen acronychal positions of Mars, drawn from the heavens with the utmost diligence and collated against the corrected mean motions, the greatest difference very nearly everywhere settles within about 3 minutes either way — especially after I had subjected to calculation the previously-demonstrated measure of the epicycles' semidiameters. Nor indeed can I sufficiently understand how Copernicus, by his line of demonstration, did not hesitate to ascribe to an alteration of the Sun's eccentricity those 40 particles (out of the Ptolemaic 1500, where the orbit's radius was reckoned as 10,000) subtracted from the Sun's distance between the centers. For even supposing that any change had ever come about in the course of the centuries (which in fact there was none), it should then rather, by right and merit, have been attributed to the annual inequality of Mars, which proceeds from the Sun — and to examining and restoring this we devote the following chapter.

[Margin: *De Revolutionibus* V, ch. 16 and 19.]

CHAPTER IX.

On the annual [inequality] of Mars, to be explored and restored.

Just as the first inequality of Mars — that of its revolution in its own orbit — showed itself easy for us to handle (whether of the mean opposition of the Sun, as

formerly, or of the true, as now, once we had taken it up with respect to that opposition), so in its annual [inequality] several more anomalies insinuate themselves. Two departures from the pattern of the superior planets present themselves here chiefly in Mars. One is manifested in this planet alone (as it seems): namely, an equally twofold-unequal distance from the Earth through the course of its annual revolution. Hence it comes about that the semidiameter of the annual orb is here contracted and enlarged according to the Sun's approach to and recession from the Earth — and indeed so conformably in this star, once its phenomena have been directed to the true motion of the Sun (for earlier, while we arranged these same phenomena with respect to the mean place of the Sun, we ran here the hazard of a contradiction, the cause of which that notable change — not only of the apogee but also of the prosthaphaeresis — will explain to anyone who considers it carefully by way of comparison), that Johannes Kepler did not hesitate to assert, and indeed to demonstrate from observations as well,

that half the Sun's eccentricity will thus appear proportionally in the annual carrying-round of Mars through the Earth's (or the Sun's) orbit. From this he likewise judges it to come about that the Sun and each of the wandering stars leave only half of the whole of their own digression through the eccentric of the distance between the centers of the world and of the planet's orbit, while they grant the remaining half to the equation of apogee and perigee, and so forth. And so he reckons that, in the appearances of Mars alone, the very Sun has shown itself to be established as the center of all the phenomena of the world. This we shall embrace as most true only when Kepler has proved the same in the other [planets] as well, without forcing. That indeed we have so far been unable to keep silent about, neither in the superior planets Saturn and Jupiter, nor in the Moon, by way of their appearances, nor shall we hope to do so in what follows for Venus and Mercury — however earnestly we may be carried in that direction. Rather, I shall be inclined to believe that these wandering stars for the most part suffer these their own peculiar second inequalities, as certain symptoms, both from the solar rays and from other causes — the Moon especially, and Mars, as being by nature more closely akin to the Sun than the rest.

Concerning Mars in particular, this could be a special indication: namely, that it mimics the Sun's alteration from the Earth in the eccentric as well, since at the opposition of the Sun (which happens by a contrary law, as it were, of the Sun's apogee, Mars's perigee, and vice versa, and of the orbs of their revolutions) it exchanges its own prosthaphaereses. Whence perhaps it comes about that around the perigee of Mars's eccentric somewhat greater [prosthaphaereses] are required than in the other planets — the Sun, that is, then dwelling nearer to its

apogee and drawing Mars at opposition into a greater approach to the Earth; and conversely smaller ones around the apogee, with the Sun made lower [and farther] from the Earth. Although this conjecture will indeed correspond to the truth in every part only if (which, however, is not the case here) the planet's apogee were conjoined with the Sun's.

The other inequality regarding Mars's annual orb appears in this: that the semidiameter of the annual orb is always elicited as greater around the apogee of the eccentric than around the perigee, from the observations by triangular reasoning, and that the distance from the Earth would otherwise permit [a greater one] in the eccentric. This indeed betrays itself plainly enough not only in our former restoration to the mean [place] of the Sun, but in this new one as well — albeit with a far smaller difference of Mars's eccentric here at each of the apsides. Nor is there any reason why anyone should infer this inequality to be total from this, declining the Ptolemaic manner of equation [Greek:

ἀγεωμέτρητος (ageōmētrētos, "non-geometrical")], on the grounds that we leave a little more than half of the whole eccentricity in Mars to the difference of the apsides; since not even Mars's reduction to Ptolemy will so suffice but that we require here a still greater equation of the distances. Hence it had come about that Johannes Kepler, ascribing this discrepancy wholly to the eccentric (as it also seems), endeavored to devise other hypotheses of revolution in place of the customary eccentrics — namely [hypotheses] departing from the circle and emulating oval or elliptical figures; in which, too, the arcs of the revolutions do not correspond to wholly equal times, the Sun (the director of all motions) driving the planets disposed around it now more intensely, now more remissly.

[Margin: Copernicus, *De Revolutionibus* I, ch. 4. Tycho Brahe calls [it] an axiom of astronomy, in the *Progymnasmata*, p. 11.]

We, however, moved by the weightiest reasons — making the greatest account in astronomy of that Copernican [principle] that the motion of the heavenly bodies is uniform and circular, or composed of uniform circular [motions] — adhere to this and either trust it or hold it as a virtual axiom. Nor indeed can we, nor ought we, on account of such Martian [irregularities] or the anomalies of certain others, feel otherwise, until necessity has imposed it: that [the motions] be circular, that is simple, or — as is mostly the case — combined and composite, through which all such curved figurations, and indeed straight lines too, can be plainly described. The office of [astronomy] is fulfilled in regard to representing every kind of celestial phenomenon. Indeed, rather, granted that we do not set these up as material, we do not hesitate to affirm them to be real nonetheless, and of such

a kind as — armed with the force and power of the centers — accomplish their conversions in commensurable equal times; as these matters are found more fully discussed by us elsewhere.

[Margin: In the disputation published *On hypotheses, theses, etc.*]

If, therefore, we attribute to a libration — whether of the eccentric or of the annual orb of Mars — that which befalls it besides, in the matter of inequality at its apsides, we commit no offense against the received custom of astronomers; since it is agreed that Copernicus employed it both for the primum mobile and for saving the Mercurial phenomena. This we too did above in the case of the Moon.

These things being premised, we must approach the observations, so that the refutation [or, rather, the demonstration] of what we have just touched upon concerning the annual inequality may emerge more clearly. In examining the observations, we shall observe this method together with the foregoing: that we first resolve the hypothesis of Mars to a single observation for the investigation of the semidiameter of the annual orb, and one [observation] that can be had as close as possible to the mean place of the eccentric. Then we shall explore the rest, above and below, both in the apogee and perigee of Mars's eccentric and of the Sun's, by the very same reduction and method of calculation.

In the year 1594, on January 3, at hour 7 in the evening, the longitude of Mars was observed by us at Uraniborg [Uraniborg] to be 18 degrees 38 minutes of Aries □. To this time these corrected motions correspond:

	Sex. (Sig.)	Gr.	Mi.	Sec.
Mean longitude of the eccentric of Mars	0	48	23	30
Anomaly of the eccentric	4	19	42	20
Therefore the additive prosthapheresis		10	33	30
Whence the		58	57	0

	Sex. (Sig.)	Gr.	Mi.	Sec.
true longitude of the eccentric of Mars from the equinox				
But the true place of the Sun ♄		23	30	40
Therefore the anomaly of the annual orb	3	54	33	40

Parsing notes: In the "true place of the Sun" row the printed sign-symbol resembles ♄ (Saturn) but in context must be a zodiac sign for the Sun's place — likely ♑ Capricorn or ♒ Aquarius for an early-January solar longitude; the glyph is ambiguous at this resolution. Flagged [?]. The "Sex. (Sig.)" column is the zodiac-sign (30°-sign) count; degrees/minutes/seconds follow.

With these found, the hypothesis of Mars in Tychonic form is applied in the following manner.

Diagram. A large circle representing the Sun's orbit ALN is drawn about the center M (the Earth), with letters L on the left and N on the right of the horizontal diameter, R at the top, and P at the bottom of the vertical diameter. From point A (lower left, on the Sun's orbit) the orbit of Mars BPC is drawn; at its right edge a compound epicycle (first epicycle on C, second on D) appears as a small circle with closely-set points G, D, F, H, C, E. Lines run from A to H, from A to the epicycle cluster, and chords Q, K mark the triangle construction AMH used in the text.

- M (center): the Earth, center of the Sun's orbit ALN
- L (left): point on the Sun's orbit
- N (right): point on the Sun's orbit
- R (top): upper point on the Sun's orbit
- P (bottom): lower point on the Sun's orbit

- A (lower left): the point on the Sun's orbit from which the orbit of Mars BPC is drawn (Sun's true position from acronychal situation L through arc LA)
- B (upper): point on the orbit of Mars
- M (also marks center of Mars's orbit region)
- Q, K (lower center/right): triangle-construction points
- G, D, F, H, C, E (right, clustered): the nested-epicycle points (first epicycle on C, second on D)

Let ALN be the orbit of the Sun described upon the center of the Earth M; and upon A — distant from L by the arc LA, which the Sun by its true motion had completed from the acronychal situation at L — let the orbit of Mars BPC be described. Then [let] the first epicycle [be] upon C and the second upon D, according to that pattern of revolutions which has often been demonstrated above. Now since, for finding the semidiameter MA of the Sun's orbit in the annual appearances of Mars, the triangle AMH must here be worked upon: therefore, after the angle CAH has been established from the foregoing as sufficient for the restoration of the inequality of Mars's eccentric — namely 10 degrees 33 minutes 30 seconds — we first seek AH in this way.

The anomaly of the eccentric beyond the semicircle, namely the arc PC, to which KD is equal, is 79 degrees 42 minutes 20 seconds; accordingly its double, 159 degrees 24 minutes 40 seconds, [gives] the angle

measures the angle HDC by the law of revolutions. Now since the sides enclosing this same angle HDC were both given in the previous chapter — namely, the semidiameters of the epicycles and of the orb — and given not merely in themselves but also in a fixed ratio to one another (a fourfold ratio), the angle DCH is therefore disclosed most concisely (following what we demonstrated for triangles in the earlier volume) as $4^{\circ}5'20''$. [Margin: Prop. 29, lib. 1, Eucl. (Euclid, *Elements* I.29)] Adding this anomalous excess gives the angle ACH as $83^{\circ}47'40''$. Combining this with the prosthaphaeresis angle CAH, and subtracting the sum from a semicircle, leaves the angle CHA as $85^{\circ}38'50''$.

In triangle ACH, then, with the angles given and, in addition, the radius AC = 10000000, the side AH we seek is found to be 9969763. Once this side is acquired, all the angles in triangle AMH besides that side AH are known. For if, as required, the line LMN is drawn parallel to AH, the angle MAH equals the angle LMA. But the annual-orb anomaly MAH is given as $54^{\circ}33'40''$; and the angle at H — that of commutation, the difference between Mars's true eccentric position and his observed position — is gathered as $40^{\circ}19'0''$. Adding these two and subtracting the

sum from a semicircle yields the angle HMA = $85^{\circ}7'20''$. From these we again derive the orb semidiameter MA we sought, 64738, in the units where the radius AC is 100000 — this for when the Sun is near perigee while Mars is at about his mean distance on the eccentric.

For this example, treated as a paradigm for the remaining Mars observations that follow — taken near his apogee and perigee, and likewise the Sun's, for determining the difference in the semidiameter of Mars's orb — since we have already worked through the procedure, it will suffice in what follows simply to set before the reader the Tychonic observations themselves, drawn from Kepler's commentary, together with the chief data for investigating the said orb-semidiameter. [Margin: Comment. Iohan. Keppl. cap. 24 (Kepler's *Commentaries [on Mars]*, ch. 24)]

The observations are these:

Near the apogee of Mars's eccentric:

In the year 1600, March, day 6, hour $8\frac{1}{4}$, Mars was at $29^{\circ}18'$ Cancer ($\square$), at about his mean distance from the Earth.

In the year 1587, January, day 1, hour 7, minute 8, in the morning, Mars was at $1^{\circ}45'$ Cancer ($\square$), near the Sun's perigee.

Near the perigee of Mars's eccentric:

In the year 1591, September, day 26, hour $7\frac{1}{2}$, Mars at $18^{\circ}36'$ Capricorn ($\square$), at the Sun's mean distance from the Earth.

In the year 1589, November, day 1, hour $6\frac{1}{4}$, Mars at $20^{\circ}59[?]'$ Capricorn ($\square$), at a place midway between perigee and the Sun's mean elongation from the Earth.

The remaining chief quantities that come out of the computation are embraced by the following little table.

Near the apogee of Mars.

	True place of the eccentr ic ($S^{\circ} G'$ $M'' S'''$)	Distanc e from the center (Particl es)	True place of the Sun ($S^{\circ} G'$ M'')	Commu tation angle $\square$ ($G^{\circ} M'$ S'')	Orb anomal y ($S^{\circ} G'$ M'')	Resulti ng orb semidia meter
I	4 29 32	111297	11 26	30 14 5	6 27 $4\frac{1}{2}$	66586

	True place of the eccentr ic (S° G' M'' S''')	Distanc e from the center (Particl es)	True place of the Sun (S° G' M'')	Commu tation angle □ (G° M' S'')	Orb anomal y (S° G' M'')	Resulti ng orb semidia meter
	5	0	37 0			
II	4 27 17 20	111295 0	9 20 39½	33 47 15	4 23 22 0	65691

Near the perigee of Mars.

	True place of the eccentr ic (S° G' M'' S''')	Distanc e from the center (Particl es)	True place of the Sun (S° G' M'')	Commu tation angle □ (G° M' S'')	Orb anomal y (S° G' M'')	Resulti ng orb semidia meter
I	11 4 24 0	889149	6 12 45 ⅓	45 48 0	7 8 21 ⅔	64077
II	10 29 55 0	888800	7 19 14 0	38 55 40	8 19 19 0	63394

Parsing notes: The two "near apogee" rows; in row I the commutation column reads "30 14 5" but is printed split across the line ("II 26 37 0 | 30 14 | 5"); I have aligned it as G M S = 30°14'5". The "Distance from center" is in *particulae* (particles, i.e., units of 1/1,000,000 of the radius). Column header for the second column abbreviated as "Distan. □ a centro" (distance of the Sun from the center). The fractional sub-minute marks (½, ⅓, ⅔) are printed as small superscript fractions.

From these results is gathered the difference between the two semidiameters of the annual orb found at the apogee of Mars's eccentric, namely 895, and at its perigee, 683 — a difference to be attributed directly, in both cases, to the unequal distance of the Sun from the Earth. For as that distance decreases, the compass of the annual orb is contracted; and conversely, as it increases, the orb is dilated. Moreover, since the Sun here is, in both cases, displaced both from its mean elongation and from its lowest position, we therefore — by bringing in the Sun's true motion together with the place of its apogee — most conveniently and concisely arrive, from the given data, at a knowledge of the full inequality that

the Sun introduces into Mars's annual orb between its own apogee and perigee. We proceed thus:

From A describe the circle BCD, representing the Sun's annual epicycle, with its apogee marked at B and its perigee at D, and the midpoint of elongation at C and H, the circle having been bisected diametrically at right angles to the center by the lines BD and CH. Then the equated solar anomaly — which we here use rather than the mean, since it is both more correct and more concise, the Sun's true place and the place of its apogee being for the most part known beforehand — is applied to what we seek, adapted to the proportion of the right sines; and we readily obtain it. Thus, in the first example near the apogee of Mars, the Sun's place is $\square$ (Scorpio) $26^{\circ}37'$, the apogee $3^{\circ}5'39''$. Therefore the anomaly is 8 signs $20^{\circ}58'$ [$\square$ $20^{\circ}58'$]; whence, casting off six signs, the Sun is gathered as raised above its perigee in its eccentric by $80^{\circ}58'$.

Diagram. A circle BCD centered at A, representing the Sun's annual epicycle. The vertical diameter runs from B (top) through center A down to L (bottom); the horizontal diameter runs from H (left) through A to C (right). A point M lies on the horizontal line to the left of center A; below the horizontal diameter, parallel chords F–G (left, with G below H) and a point K near the bottom mark the constructed sines.

- B (top): apogee of the Sun's epicycle
- D (implied bottom / at L): perigee
- C (right): midpoint of elongation
- H (left): midpoint of elongation (opposite side)
- A (center): center of the epicycle
- M (mid-left, on horizontal): point defining line LM
- F (lower-middle): foot of the parallel-sine line FG / FA
- G (lower-left, below H): endpoint of chord
- L (bottom): perigee point of the diameter
- K (lower-right of L): point marking the constructed sine on the lower arc

These quantities are represented by the line FG, parallel to the semidiameter AH and below it, in which the arc DG is shown as the right sine; while the sine of the complement of this arc — namely $9^{\circ}2'$ — is 1570091, indicated by the line FA, by which the Sun's defect from its mean elongation is likewise measured. Given this, one proceeds similarly in the other observation near the apogee of Mars's eccentric, by means of the equated solar anomaly, which is found to be 6 signs $15^{\circ}11'$. For removing six signs here too gives the Sun's distance from its perigee in

its own circle DL as $15^{\circ}11'$, whose complement $74^{\circ}49'$ has the sine LM = 9650927. Subtracting from this the earlier value 1570091 leaves the line we sought, KF = 8080836. As this stands to the difference 895 emerging from observation, so the whole diameter BD = 20000000 stands to the quotient 2215 — which number indeed demonstrates the entire inequality that the Sun, dwelling at its apogee and perigee, brings to Mars's annual orb, taking the radius of Mars's eccentric as 100000.

In exactly the same way, at the perigee of Mars's eccentric, by means of the Sun's true anomalies and the difference there found between the two observations and inculcated above (683), we find the said inequality to be 2415. The difference from the prior value is therefore 200; half of which, subtracted from the greater or added to the lesser, yields here the same value as limited, 2315.

Furthermore, to fortify the measure of this inequality with other testimonies, we — relying on Kepler's notable industry — shall bring forward the two following observations. Among those he had elucidated from the Tyconic observations made on the nearest days on either side at Hven [Huenä], and which he had fitted as closely as possible to one and the same place of Mars's eccentric: the earlier, in the year 1585, on the 10th day of May, hour 6, minute 11, in the morning, set Mars at $26^{\circ}54\frac{1}{2}'$ Leo ($\square$); the later, in the year 1590, on the 31st day of December, hour 4, minute 44, likewise in the morning, places that same Mars at $9^{\circ}46\frac{1}{3}'$ Virgo ($\square$). Now at the earlier time the Sun's true place was at $28^{\circ}55\frac{1}{2}'$ Taurus ($\square$), Mars in his eccentric at $5^{\circ}22'$ Libra ($\square$); at the later time the Sun was at $19^{\circ}6\frac{1}{4}'$ Capricorn ($\square$), but Mars then at $6^{\circ}2'$ Libra ($\square$) — so that, on account of the two solar distances from one another, very nearly the maximum inequality arising from these would be suggested, while a minimal and almost imperceptible error would be left through Mars's eccentric. From the given data the same Kepler, for the earlier time

— the Sun approaching its apogee — found by his calculation the semidiameter of Mars's annual orb to be 62227; but for the later time — the Sun, namely, dwelling near perigee — the same to be 60291, supposing here too, deliberately, at such an eccentric anomaly, the distance of Mars from the center of the orb to be 100000. The sure measure of this, however — once our own restoration of the eccentric has been premised, since it stays very nearly around 109000 — means that if you multiply by 109000 at least the difference he left us here, 1936, and divide the sum by the radius, the true value 2110 results. In this we shall by no means err, so long as the difference alone is subjected to the computation for the semidiameter of Mars's annual orb.

Moreover, to extend this found difference to the full inequality that arises from the Sun's various distances from Mars, by the proportion of sines premised above — by the same method as the *πραγματεία* [Greek: *pragmateia*, "treatment, procedure"] instituted above (except that here the complements of the distance-arcs are added together) — the maximum inequality BD is drawn out as 2375. But this previously, by the limitation near the apogee and perigee of the eccentric, was 2315; the difference, then, is 60, and its half 30, whence this inequality, finally limited, comes out as 2345. For this we (so that it may be rendered closer to Kepler, even if it slightly exceeds half the measure of the Sun's eccentricity that we found above) shall posit 2350 as the maximum, in the units where the radius of Mars's eccentric orb is 100000. [Margin: Lib. priore Theor. cap. 5 (former book of the *Theorica*, ch. 5)]

So much, then, for the first proposed inequality of Mars's annual orb. The second follows, which concerns the eccentric of this same planet. This one, from the observations and differences which we earlier set forth as worked out near the apogee and perigee of Mars's eccentric, stands thus: when the semidiameter of the annual orb in Mars is considered everywhere at the Sun's mean distance from the Earth (the distance that resulted for these observations by our customary reduction), it appears at the apogee of his eccentric as $66787\frac{1}{2}$ [recte: the surrounding text and the symmetry with perigee suggest 667875 in the same particle-units; printed " $66787\frac{1}{2}$ "], at perigee as 642025, and therefore the mean as 654950, in the units where the radius of the eccentric is reckoned 1000000. This difference between the maximum and minimum — 25850 — when set in proportion with the other that flowed from the Sun, namely 23500 (for that one too must be increased by one digit to match this measure of the orb's radius), will stand as 11 to 10. Hence, with one given, the other is acquired without trouble.

For this reason, regarding the former — that of the Sun — we have at least drawn up the following table, framed by the method just above; for the benefit, namely, of those who from the hypothesis itself, by reasoning with triangles, are led to scrutinize the remaining phenomena of Mars more thoroughly along with us, as we shall then demonstrate by example. It is constructed thus: from each of the two anomalies, the semidiameter of Mars's annual orb comes out smallest of all — when both Mars and the Sun dwell at the perigees of their eccentrics — as 630275; but largest, at the apogees of both, as 679625, the difference of which is 49350, as is clear from comparison of the observations and of what we just premised. Half of this difference (the value to which, for the rest, it would have been permissible to arrive by limitation) would introduce an aberration of nearly two degrees into the phenomena of Mars in those places.

Now this twin inequality residing about Mars's annual orb, although it proceeds from a disparate cause, nevertheless — because it befalls only the annual revolution — we have, as it were, jointly appended both to the annual orb, to be saved by the librational reasoning through the diameter of the lunar theory shown above, rather than excusing the other, which flows from Mars's eccentric, by a libration of the axis in that same eccentric.

Besides these two inequalities of Mars's annual orb, a third is also found, which the inclination of this eccentric orb (like the others') to the ecliptic — that is, to the annual orb — causes (over and above the one that occurs by reduction), arising chiefly at the places of greatest inclination. But since this is of no great moment, and does not even here, where it is found greatest, extend so far as 4' — so long as we set and employ Mars's inclination as 1°50' in latitude — we therefore knowingly pass it by, both in the superior planets and in the body of Mars, lest we should seem to want to burden the theory of this planet with new and still more difficult labor.

And so much, then, for the modern restoration of Mars according to longitude.

A table showing the analogy of the Sun's distance from apogee, in Mars's annual orb, for each two degrees. [Tabula analogiam distantiae Solis ab apogaeo, in annuo orbe Martis, ad singulos binos gradus ostendens]

The table is laid out in six paired columns. The three columns across the top (read downward, with degree-arguments at the far left running 0, 2, 4, ... 60) are headed **0 Sexagene**, **1 Sexagene**, **2 Sexagene**; the three across the bottom (read upward, with degree-arguments at the far right) are headed **5 Sexagene**, **4 Sexagene**, **3 Sexagene**. Each block gives a value and its difference ("Diffe."). I transcribe the three top-headed blocks with their own degree argument, then the bottom-headed blocks (the "2 Sexagen." block shares the right-hand degree margin running 60 → 0).

0 Sexagene (degree argument at left)

Deg.	Value	Diffe.
0	23500	—
2	23493	7
4	23471	22
6	23436	35
8	23386	50

Deg.	Value	Diffe.
10	23321	65
12	23244	77
14	23151	93
16	23045	106
18	22925	120
20	22791	134
22	22643	148
24	22484	159
26	22311	173
28	22125	186
30	21927	198
32	21716	211
34	21491	225
36	21256	235
38	21010	246
40	20752	258
42	20482	270
44	20202	280
46	19912	290
48	19612	300
50	19303	309
52	18985	318
54	18657	328
56	18321	336
58	17977	344
60	17625 [printed "27625" — recte 17625; the column foot must equal the head of the "1 Sexagen." block]	352

(foot of block labeled **5 Sexagen.**)

1 Sexagene

Deg.	Value	Diffe.
0	17625	—
2	17266	359
4	16900	366
6	16529	371
8	16152	377
10	15770	382
12	15381	389
14	14989	392
16	14593	396
18	14199	400
20	13790	403
22	13386	404
24	12979	407
26	12570	409
28	12160	410
30	11750	410
32	11340	410
34	10930	410
36	10521	409
38	10114	407
40	9710	404
42	9307	403
44	8907	400
46	8511	396
48	8119	392
50	7730	389
52	7348	382
54	6971	377
56	6600	371
58	6234	366

Deg.	Value	Diffe.
60	5875	359

(foot of block labeled **4 Sexagen.**)

2 Sexagene (with the right-hand degree margin running 60 down to 0, which also serves the **3 Sexagen.** foot-block)

Deg. (left/top)	Value	Diffe.	Deg. (right margin)
0	5875	—	60
2	5523	352	58
4	5179	344	56
6	4843	336	54
8	4515	328	52
10	4197	318	50
12	3888	309	48
14	3588	300	46
16	3298	290	44
18	3018	280	42
20	3748 [printed "3748" — recte ~2748; breaks the monotone decrease 3018 → ... → 2490, and the difference column shows 270 between rows, consistent with 3018-270=2748]	270	40
22	2490	258	38
24	2244	246	36
26	2009	235	34

Deg. (left/top)	Value	Diffe.	Deg. (right margin)
28	1784	225	32
30	1573	211	30
32	1375	198	28
34	1189	186	26
36	1016	173	24
38	857	159	22
40	709	148	20
42	575	134	18
44	455	120	16
46	349	106	14
48	256	93	12
50	179	77	10
52	114	65	8
54	64	50	6
56	29	35	4
58	7	22	2
60	0	7	0

(foot of block labeled **3 Sexagen.**)

Parsing notes: This is a single continuous table of the function value vs. argument from 0° through (5×60)+60 = the full range, read in six 60-step blocks of 2° each. The top three blocks are read top-to-bottom (arguments at far left); the bottom three blocks (5, 4, 3 Sexagen.) are read bottom-to-top, sharing degree margins with the top blocks. The "Diffe." columns are first differences. The value 27625 at the foot of the "0 Sexagen." block is a clear compositor digit-error for 17625 (it must equal the head of the "1 Sexagen." block). The value 3748 at argument 20° in the "2 Sexagen." block breaks the column's smooth monotone decrease and is flagged below.

The use of the preceding table. [Vsus tabulae praecedentis]

Given the equated anomaly of both the Sun and Mars: first one must enter the table affixed to the Sun's anomaly, with the *sexagena*e (sixties) inscribed above

and below, and the whole and next-smaller degrees at either margin; according to which

the ratio of the Sun's distance from perigee is to be taken in the adjacent column, with the correction for the added minutes of anomaly (and so on, by the differences interpolated between each pair of degrees) carried out as needed. Then the corrected distance of the Sun is to be written out and kept aside separately. Next, in the same way, one enters the said table using the eccentric anomaly of Mars; the distance found in the column is written out and corrected. But this corrected value is at once to be multiplied by eleven, and the product, divided by ten, is added to the previously kept distance of the Sun. The aggregate of these two values, when added to the measure of the radius of Mars's orb at perigee—630275, fixed above (as will always be done)—forms the said radius of the orb for the correction of each new anomaly, just as was demonstrated and laid out above, and as will shortly be shown by an example.

[Margin: The fitting smaller radius of Mars's orb is 631000. See below under Cap. 12.]

An example showing the use of the preceding table, and thereby the entire analysis of Mars's theory in triangles.

In order that the truth of the hypothesis of Mars now restored by us may become more fully known—and at the same time how each of the triangles is assigned to the computation for obtaining the planet's true motion—we have selected one of our own observations to set down here as a model: namely, when Mars was in conjunction with the star in the northern knee and the uppermost part of Gemini [the star ζ Geminorum]. This was in the year 1613, on 25 March, at the 4th hour in the morning. For on the immediately preceding evening, around the 8th hour, Mars was distant from the same star toward the leading (westward) part of the signs by 10 minutes; whereas on the immediately following evening, around the same hour, Mars had passed the said star toward the trailing (eastward) part by 20 minutes—so that its daily motion was thus 30 minutes, which the Copernican calculation likewise yields almost exactly the same. From this we conclude that the conjunction of Mars and this star at the said time was, very nearly, an exact one. And since the longitude of the same star, reduced, was found to be at $4^{\circ} 31\frac{1}{2}'$ [of Cancer], with a northern latitude of $2^{\circ} 11'$, this was therefore likewise the place of Mars at that star.

For this time the following corrected motions are found:

	Sig.	Gr.	Mi.	Sec.
Mean longitude of Mars from the true equinox	4	7	34	0
Anomaly of Mars	11	8	27	40
True place of the Sun	0	14	31	20
True anomaly of the Sun	9	8	42	0

With these data given, the following hypothesis of Mars is fitted in the Tychonic form in this way.

Diagram. A geometric construction (continued onto the next page) of Mars's theory in Tychonic form, occupying the lower portion of the page and continuing at the top of the next page. A large circle centered on A represents Mars's eccentric orb (BP). A smaller circle centered on M, lower left, is the Sun's annual orb (LAN). Lines and the two epicycles (on C and D) carry the planet to its position H near the top. The accompanying prose (running across both pages) constructs it step by step:

- M (lower left, center of small circle): center of the Sun's annual orb
- L, A, N: the Sun's annual orb LAN (diameter LN passes through A)
- K: apogee of the Sun, shown by line MK erected, currently near 6° of Cancer
- A (center of large circle): center of Mars's orb, placed near 14° of Aries toward the leading part
- B, P: Mars's orb (apogee line BP drawn near the end of Leo and Aquarius)
- C: point removed from B toward the leading part (the prior eccentric epicycle is described upon C)
- D: point upon which the posterior epicycle is described
- H: position of Mars on the periphery of the second epicycle
- E, F, G: small construction points at the top near the epicycles

The construction text reads: First, describe upon M the Sun's annual orb LAN; and erect the line MK showing the Sun's apogee at K, which now dwells near 6° of Cancer. Then, placing A as center near 14° of Aries toward the leading part, describe Mars's orb BP, and draw the apogee line BP near the end of Leo and Aquarius. With C removed from B toward the leading part...

by 2° —namely at 8° and more of Leo—upon the same C let the prior epicycle of the eccentric be drawn with compasses, but upon D the posterior one. Thus, according to the law of revolutions and the things repeatedly stressed above, Mars's position will be at H on the periphery of the second epicycle. Afterward let M be joined to A, and from each of these let a line be drawn to the planet's position at H; and another line as well, namely the diameter LN of the annual orb, parallel to AH. All things will then be well expressed in this theory, which can show the true motion of Mars—even from the Earth at M through the annual orb LAK. Now the procedure for drawing out that same motion from the foregoing by the deduction of triangles is this.

First, in triangle CDH two sides—namely the radii of the epicycles, CD and DH—are given from the foregoing, together with the contained angle CDH of $43^\circ 4' 40''$ (since by the law of revolution that angle is reckoned as double the angle ECD). Thereby the angle DCH first becomes known as $11^\circ 48'$; then the side CH as 123880, in units where the radius AC is 1000000. But once the angle DCH is added to the arc ED, the angle ECH is formed as $33^\circ 20' 20''$, whose complement to the semicircle is the angle ACH, $146^\circ 39' 40''$. In this same triangle ACH, then, since likewise two sides containing the given angle at C are granted—namely AC, the radius of Mars's orb, 1000000, and CH, just acquired, 123880—there is therefore found from these, first, the angle of the eccentric prosthaphaeresis CAH, $3^\circ 31\frac{1}{2}'$, additive, so that the true or equated place of Mars on the eccentric becomes 4 signs $11^\circ 5\frac{1}{2}'$; then the side AH as 1105305 [?]. (The "0530" digits in the printed value 110·530·5 are imperfectly inked.)

With these matters dispatched, we arrive at extricating the prosthaphaeresis (or commutation) of the annual orb. Here, in triangle AHM, since the side AH has just been found, and likewise the angle HAM—which, on account of the parallels AH and LM, is equal to LMA, namely to the excess of the commutation anomaly of the annual orb beyond the semicircle, $63^\circ 25' 50''$. For since the true eccentric motion of Mars is subtracted from the true place of the Sun, there remains here the said anomaly, 8 signs $3^\circ 25' 50''$, which indeed measures the arc NLA. But NL is 6 signs; therefore what remains, 2 signs $3^\circ 25' 50''$, will be taken for the angle LMA, or HAM. Now with these two data granted in the aforementioned triangle AHM—

namely the side AH and the angle HAM—there remains the radius of the annual orb MA, to be equated from the foregoing and at the same time supposed, so that finally the angle MHA (or NMH) may be acquired. But to fore-determine the said MA, one proceeds thus.

First, with the Sun's equated anomaly 4 sixties $38^{\circ} 42'$, the [preceding] table...

Then entry is made into the same table with the equated anomaly of Mars, which in this example is given as 5 sixties $41^{\circ} 59'$ (with the simple eccentric prosthaphaeresis added to Mars's prefixed anomaly). The distance at 5 sixties 40° is found to be 22791, with an added difference of 134. Therefore for $1^{\circ} 59'$ the correction will be made by the rule of proportion in this way: As 120 is to 134 minutes, so 119 minutes are to 133 (nearly). These minutes, the excepted distances being also added here, make 22924—which number, when it is multiplied by 11 and the sum distributed by 10, yields the inequality of the radius of Mars's annual orb proceeding from the eccentric, 25216. This, combined with the other (the Sun's), which had previously been found and kept, makes a sum of 38743, to be added to the least quantity of the radius of the annual orb, 630275; and that radius will be, in this revolution of Mars's eccentric as well as the Sun's, equated to 669018, which in this example the side MA supplies. Therefore from the three data which we have just adduced separately, the angle MHA is acquired as $36^{\circ} 35\frac{1}{2}'$, to which HMN is equal, likewise on account of the parallels AH and MN.

But the place H on Mars's eccentric, [seen] from A—or N on the ecliptic and the Sun's orb, where, from the Earth at M, we detected the phenomenon of Mars in its acronychal [opposition] position—was shown to have been 4 signs $11^{\circ} 5\frac{1}{2}'$.

Therefore the angle MHA, just found, taken from the eccentric place of Mars in this example, leaves its longitude in the zodiac as $4^{\circ} 30\frac{3}{4}'$ of Cancer [recte: $4^{\circ} 30\frac{1}{2}'$ — see below], falling short of the observed value, according to our careful and earnest observation, by scarcely more than one minute, since that [observed value] was $4^{\circ} 31\frac{1}{2}' 20''$ [of Cancer].

CAPVT X.

On the restoration and extension of the periodic motions of the planet Mars.

As regards the periodic revolutions of Mars to be extended over all ages—whence both the apogee place and its mean longitude may be had at any time—we follow the method established in the foregoing.

For after, in the two preceding chapters, we had obtained the restoration, for our own age, of both the apogee's motion and that of the longitude, together with the radii of the epicycles and of the annual orb in Mars as well, we now have recourse to the observations of the ancients. And first we borrow from Ptolemy the determination of the apogee place and correct it; then we measure the emendation of the longitude by the ancient observation of Timocharis, as it seems, using the same fixed star that Ptolemy here employed. Once both of these are acquired together with the interval of time, we shall thereafter readily accomplish the extension of the mean motions.

Ptolemy, at the beginning of the reign of the Emperor Antoninus Pius (this was in the year 136 complete after the birth of Christ), from three acronychal observations directed to the mean opposition of the Sun, found the apogee of Mars at $25^{\circ} 30'$ of Cancer from the true equinox; which we correct in respect of the Sun's true motion as follows. Since the apogee place of the Sun in Ptolemy's age was in truth at 18° of Gemini, the prosthaphaeresis of the Sun in the place opposite the apogee of Mars was $1^{\circ} 30'$, additive.

[Margin: See Bk. 1, on the commutation of the Sun.]

The daily motion of the Sun and Mars combined, when acronychal Mars occupied the Ptolemaic apogee and the Sun the opposite point, is reckoned at $1^{\circ} 25'$. Whence the interval of time between the mean Severan place of the Sun [recte: between the Sun's mean place and] its true opposition to Mars comes out, from the data, as 25 hours 24 minutes—the Sun's prosthaphaeresis, namely $1^{\circ} 30'$, being multiplied by 24 hours, and the sum divided by the proposed daily motion of both Sun and Mars. From this we conclude that Mars's earlier arrival at the true opposition of the Sun was by a span of 25 hours 24 minutes; in which interval of time we reckon Mars's motion (from a daily $24^{\circ} 20'$) to have been 25 minutes—namely, to be added to the acronychal [position], or what is the same, to its mean motion at the apogee. To save this, the apogee place of Mars in Ptolemy's time had to be prolonged by $2^{\circ} 24'$ toward the trailing part, so that the acronychal position of Mars might look upon the true opposition of the Sun, as it should.

[Margin: Bk. 10, ch. 1. [Lib. 10. cap.] on the commutation of the motion of Mars □]

There is added, besides, the difference between the true and the mean equinox, 21 minutes, according to our restoration around Ptolemy's time, so that there are $2\frac{1}{2}^{\circ}$ to be added to the Ptolemaic apogee of Mars. It will therefore be—for the said year 136 complete after Christ, from the mean equinox—3 signs $28^{\circ} 15'$. But in our

own time, in the year 1600 complete of the world's Savior, we have detected the apogee of Mars at 4 signs 28° 42', likewise prolonged from the mean equinox toward the trailing part. The difference, therefore, of the motion of Mars's apogee from the same equinox over the interval of 1464 Julian years is found to be 30° 27'. According to this, by the method prescribed above, we have acquired the epoch of Mars's apogee place at the first bissextile [leap year] of the world's era, in our customary usage: 1 sign 2° 58' 21" from the mean equinox; but [from the era] of Christ incarnate, 3 signs 25° 29' 20".

Moreover, by the same proportion, the same motion of the apogee in:

Years		Gr.	Mi.	Sec.	Ter.
1000	correspo nds to	20	47	56	30
100	correspo nds to	2	4	47	39
20	correspo nds to	0	24	57	32
1	correspo nds to	0	1	14	52

—which indeed correspond to the tables of Mars's mean motions to be constructed below, and will be kept.

Parsing notes: the bracketed list is set with a brace grouping the four year-values {1000, 100, 20, 1} against the four time-value rows; "respondet" (corresponds) is the verb between the braces. Rendered as a table for clarity.

Furthermore, in order that we may also obtain in Mars the longitude corrected for the ancient age, we take up again an ancient observation, which Ptolemy reports to have been made in the year 13 according to Dionysius, on Capricorn 25, where: [Greek: ἑὸς ὁ τοῦ ἄρεως ἐδόκει προσβεβῆκεναι τῷ βορείῳ μετώπῳ τῷ τοῦ Σκορπίου (heōs ho tou Areōs edokei prosbebēkenai tō boreiō metōpō tō tou Skorpiou, "the morning [star] of Mars seemed to have drawn near to the northern forehead of the Scorpion")—that is, the morning star Mars appeared to be set close to the northern forehead of Scorpio. For I insert the Ptolemaic text itself here for this reason: that the bright star in the forehead of Scorpio—which is first in number in that asterism, and which Ptolemy had employed—Johannes Kepler called into doubt; and, departing from Ptolemy, in its place he chose the most northern star in the forehead of Scorpio, namely the fifth in number, which is of

the fourth magnitude and exceeds the bright star of second rank by $1^{\circ} 26\frac{1}{2}'$ in longitude and by $37'$ in latitude. I conjecture that Kepler did this, among other reasons, because he noticed that his own calculation, previously deduced through Ptolemy, agreed more correctly with that fifth star being substituted—at least in the longitude of Mars, though not likewise in latitude.

For the rest, I do not myself consider it sufficiently safe to depart from Ptolemy in the case of this star—both because the latitude of Mars (as Kepler himself admits, and as is later argued more evidently in connection with the emendation of latitude) is more correctly fitted to the bright star, and because the restoration of Mars (as of the other planets) to be derived through Ptolemy carries with it more error, for the causes explained above, so that I have judged these ancient conjunctions of the planets with stars the more to be safely chosen. And surely, when an observer names the northern forehead of Scorpio, it is credible that he pointed out this bright star [Greek: κατ' ἐξοχήν (kat' exochēn, "par excellence")], since not of all, but only of the three most splendid stars in the forehead of Scorpio, it is the most northern and the most splendid, holding also the most northern latitude among those same stars.

This star, therefore, we shall employ with Ptolemy, and we shall further attend to the time of the [Greek: συνθέσεως (syntheseōs, "conjunction"); — read here in the sense of conjunction] or conjunction of Mars with the same, which Ptolemy still more clearly sets forth, when he reports in express words in that same place that the same conjunction was made 52 years from the death of Alexander, but 476 years from Nabonassar, on the 21st day of the month Athyr among the Egyptians, in the morning. This time we have found to have fallen, in the current Julian year before the birth of Christ 272, on 18 January in the morning, around the 5th hour, at Copenhagen. To that moment the longitude of the said first star of Scorpio, by the Tychonic constitution and our precession of the equinox, is gathered to have stood, from the mean equinox, at $1^{\circ} 29' 40''$ of Scorpio, with a northern latitude of $1^{\circ} 16'$. Therefore the Ptolemaic text implies that Mars then occupied that same place very nearly—differing from us in this, that it sets the longitude of the said star at $2^{\circ} 15'$ of Scorpio with a latitude of $1^{\circ} 20'$ north, and consequently assigns Mars the same place. At this same time the true place of the Sun from the mean equinox, according to our computation, was 9 signs $24^{\circ} 38\frac{1}{2}'$, with its equated anomaly 7 signs $21^{\circ} 11'$. Then the apogee of Mars from the same mean equinox, according to...

Astronomia Danica — Pars Altera, Liber Secundus Theoricorum

Cap. 10. LIBER SECVNDVS.

we recover this same value, extended through Ptolemy according to the foregoing correction, as 3s 20°15'. Finally, since we still need to know in advance the mean longitude of Mars from the mean equinox — at least approximately — so that, taking it as our baseline, we may reason about both anomalies (eccentric and annual revolution) before arriving at the precise correction of that same longitude through the restored hypothesis, here we put more trust in Ptolemy's report than in the Prutenic Tables. We therefore place Mars's mean longitude at this date, reckoned from the true equinox, together with that report, at 4°12' Libra. When the 4½' of the inequality of the equinox are added to this, the longitude from the mean equinox becomes 6s 4°16½'.

When the apogeal motion of 3s 20°15' is subtracted from this longitude, the eccentric anomaly of Mars remains as 2s 14°1½'. This figure, diminished by 26½' according to the abridged prosthaphaeresis of Mars's eccentric — which falls below the Ptolemaic value as a result of our correction, when we reduced Mars's acronychal positions to the point opposite the Sun's true course — becomes 2s 13°35'. (For Ptolemy it is only 2s 12°47', since among other things he sets the apogeal motion here at 21°25'.) But since at this position a deviation of one degree in the eccentric anomaly scarcely yet produces an error of 4' in the eccentric's prosthaphaeresis, we may safely use this anomaly as we have corrected it — namely 2s 13°35' — in investigating the same quantity within our own Mars hypothesis. If anything in it still proves excessive or deficient, we shall correct it afterward.

On the basis of these established quantities, then, we shall now describe the following hypothesis of Mars in the Tychonic form, in our customary manner, and explain it still more fully for the benefit of students — assigning to both the Sun and Mars their due positions along their respective orbits, the positions from which the prosthaphaereses and the discrepancies from Mars's mean motion are to be drawn by calculation on either side.

Diagram. A geometric construction of the Tychonic Mars hypothesis. The text describes the construction step by step: With the center at M (the Earth), first describe the Sun's orbit A L O W. Draw the line O W, marking the position of the Sun's apogee at O and its perigee at W — which in that ancient age stood at about

3½° [of a sign, near Gemini/Sagittarius]. Then, placing the start of the sign Cancer (♋) at the top of this circle, and Capricorn (♑) at the opposite point, suppose the true position of the Sun at A — namely, removed from the sign Capricorn by 24°38' in the order of the signs (as found above). It is thus established that the Sun is distant from its perigee W by the arc W A, measured by the excess of the Sun's anomaly written above beyond a semicircle, namely 1s 21°11'. Accordingly, the perpendicular dropped from A to the axis O W falls at S, and W S measures the Sun's distance from perigee.

With these established, describe about A (the Sun's true position) the orbit of Mars B C P. Then erect to this orbit the mark of the sign Cancer (♋), drawn upward from A and parallel to M♋, so that within the planet's orbit there is also a representation of the ecliptic; and here let Mars's supposed anomaly, like every other individual element, be arranged in its due place according to the index of Mars's own apogee — which in this ancient age stood at 20°15' Cancer (♋), as was proven above.

Labeled points (read from the figure on both pages):

- M: the Earth, center of the construction
- A: the true position of the Sun (and center of Mars's orbit)
- O: the Sun's apogee
- W: the Sun's perigee
- L: point on the Sun's orbit (end of the annual-orb diameter L N)
- N: point on the Sun's orbit opposite L (the true eccentric position of Mars projected on the ecliptic)
- S: foot of the perpendicular from A to axis O W
- B: position of Mars's apogee on its orbit
- C: Mars's mean longitude on its orbit (foot of diameter B P at the eccentric)
- D: center of the second epicycle
- E: apogee of the first (prior) epicycle
- F: point on the lower epicycle (arc F H)
- H: the planet Mars (true position)
- P: Mars's perigee (foot of diameter B P)
- Q: foot of the perpendicular from H to the axis B P of Mars's orbit
- ♋ (Cancer), ♑ (Capricorn): zodiacal markers on the orbits

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[Continuation of the diagram description from p. 369:] ...let it be placed at B, the degrees and minutes occupying their position in the order of the signs, reckoned from the sign Cancer (♋) on Mars's orb. Then, the diameter B P being drawn as the index of the apsides of Mars's orb, let his mean longitude be supposed at C, removed from the apogee B in the order of the signs by the measure of the anomaly set down above, 2s 13°35'. Finally, about C let the first epicycle of Mars's orb be drawn, and about D the second, the distance of whose center from the apogee E of the prior epicycle equals the arc B C, while the distance of the planet at H along the arc F H of the upper epicycle E D is double it — in accordance with what has often been emphasized, from the law of revolutions, in the Tychonic hypothesis of the superior planets.

For the rest, I pursue every detail here in fuller demonstration, so that lovers of learning may both recognize the excellence of this hypothesis and learn at last to imitate it correctly and aptly in any other supposition of revolution — for it elegantly and καθ' ὑποτύπωσιν [Greek: kath' hypotypōsin, "in a sketch / by way of representation"] sets before the eye the very angles by which the true motions differ from the mean in each case. For the line from A to C is here the index of Mars's mean motion in his proper orbit, while A H is the index of the true motion. Since the diameter L N of the annual orb is set parallel to this line, the same line shows the true eccentric position of Mars from M to N upon the ecliptic. Lastly, the visual line running from M (the Earth) to the planet at H makes, together with A H, the angle of the annual commutation — namely M H A — to which the arc N Z on the ecliptic is always equal. Moreover, the angle H A M of itself also measures the Sun's distance from the point opposite Mars, derived from the Sun's supposed true position, since it is equal to the arc A L (that is, the angle L M A) from the same source, on account of the parallels A H and L N. Finally, once the perpendicular has been noted from H (the apparent position of Mars), extended to Q on the axis B P of Mars's orbit, the distance of Mars himself from his perigee P Q is also indicated — through which, as also through W S, the semidiameter of the annual orb M A is to be corrected in the case of this planet, as prescribed.

But let us return to working out the prosthaphaereses. With Mars's anomaly supposed, then — that is, the arc B C of 73°35' — to which the prosthaphaeresis of the eccentric C A H, and the side C H, are to be sought: first, in the triangle C D H, since from the given semidiameters of the epicycles — C D 14840 and D H 3710 — together with the contained angle C D H of 147°10' (namely, double the presupposed anomaly), there is first shown, by a compendious method, the angle

D C H of $6^{\circ}23\frac{1}{2}'$, to be added to the anomaly E D; and the angle E C H becomes $79^{\circ}58\frac{1}{2}'$. Then, in the same triangle C D H, the side C H is also given, by triangular calculation, as 18069.

Proceeding further down into the triangle C A H, we there repeat the two sides enclosing the given angle, likewise already granted: of these, A C, the radius of Mars's orbit, is 1000000; C H, just now found, is 18069; and finally the angle A C H is $100^{\circ}1\frac{1}{2}'$ —

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— namely, the supplement of the angle H C A. Therefore there is found, first, the angle of Mars's prosthaphaeresis C A H of $9^{\circ}47\frac{1}{2}'$, and then the side A H of 1046688.

Now, in order to possess here a moderately corrected longitude of Mars's eccentric, before we proceed to working out the prosthaphaeresis of the annual orb (which is comprised by the angle M H A), let us at the same time also correct the semidiameter M A of that orb — since in the former [the prosthaphaeresis of the annual orb] consists the measure of the angle H A M, to be known in advance together with the Sun's true position, while in the latter [the semidiameter M A] consist the two inequalities of Mars that we have recently discovered concerning his annual orb. On this account, first, with Mars's apogee presupposed as $3s\ 20^{\circ}15'$, let his anomaly of $2s\ 13^{\circ}35'$ be added; and the mean longitude is likewise compounded, from the mean equinox, as $6s\ 3^{\circ}50'$. When the prosthaphaeresis of the eccentric sought earlier, $9^{\circ}47\frac{1}{2}'$, is subtracted from this, there remain $5s\ 24^{\circ}2'45''$ for the true or equated longitude of Mars's eccentric. When in turn the Sun's position set down above, $9s\ 24^{\circ}38'10''$, is subtracted from this, there remain $4s\ 0^{\circ}36'$; and the complement of this arc to a semicircle — namely, about $1s\ 29^{\circ}24'$ — measures the angle A M L, that is, the Sun's distance from the point opposite Mars, and consequently also the angle H A M.

Further, in the correction of the orb's semidiameter M A: when, by means of the equated anomaly of the Sun, 3 sexagenes $51^{\circ}11'$, and likewise that of Mars's eccentric, 1 sexagene $3^{\circ}48'$ (the prosthaphaeresis of the eccentric being here too subtracted from its simple anomaly, and a reduction of the signs on both sides to sexagenes being made), each individual quantity has been found by the benefit of the table set out earlier — there is found the equation for W S, that is, the Sun's distance from the Earth, [as 4465], but for P Q, Mars's distance in the eccentric, as 18630. When these are added to each other, as was prescribed, and the sum is aggregated to the least semidiameter of the orb, 630275, there comes out the

corrected radius of the annual orb M A as 653370, in units such that the radius of the eccentric A C is 1000000. Accordingly, in this last triangle A H M, of the two found and given sides — namely A H 1046688 and A M 653370 — which also enclose the angle M A H given from the foregoing as $59^{\circ}24'$, we find at last the angle of the annual inequality M H A, here $38^{\circ}13\frac{1}{2}'$. When the prosthaphaeresis of the eccentric — namely $9^{\circ}47\frac{1}{2}'$ — is taken away from this, there remains, as the whole difference between Mars's mean and true longitude from the mean equinox, $28^{\circ}26'$, with [an adjustment] added.

Accordingly, when this is subtracted from the apparent star's — that is, Mars's — longitude established above likewise from the mean equinox, $7s\ 1^{\circ}29'40''$, there results the mean longitude of Mars from that same mean vernal equinox, $6s\ 3^{\circ}3\frac{1}{3}'$, which alone we were seeking here. We have just used this value as $6s\ 3^{\circ}50'$ while hunting for the prosthaphaeresis of the eccentric and the anomaly of the annual orb. According to the difference, therefore, which is about $46'$, when we examine each detail carefully, we find the prosthaphaeresis of Mars's eccentric to be $9^{\circ}44\frac{1}{4}'$, and therefore its true longitude $5s\ 23^{\circ}19'30''$, and consequently the anomaly of the annual orb $4s\ 1^{\circ}19\frac{1}{2}'$, to which corresponds a prosthaphaeresis of $38^{\circ}17\frac{1}{2}'$. When the eccentric's prosthaphaeresis of $9^{\circ}4[?]\frac{1}{2}'$ is again subtracted from this, there remain $28^{\circ}13\frac{1}{4}'$ to be subtracted from Mars's apparent longitude. This done, the mean longitude of Mars finally sought emerges as $6s\ 2^{\circ}56\frac{1}{2}'$ from the mean equinox — fairly precisely worked out, for the purposes of this business, for the aforementioned year 272 before Christ, the month of January, day 18, hour 17 before noon.

We thought it best to proceed by this route rather than the Ptolemaic one, so that we might directly investigate the mean longitude sought. But in order to reduce this same value to the completed year 272, let there be subtracted from Mars's established longitude of $6s\ 2^{\circ}56'30''$ the mean motion from the equinox corresponding to 16 whole days and 17 hours — the span by which the time of the observation fell later than noon of the next following first of January among us — namely $8^{\circ}45'22''$. This gives the mean longitude of Mars at the end of the year 272 before Christ, and indeed from the mean equinox: $5s\ 24^{\circ}11'8''$.

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[Margin: Liber superior, cap. 2, and cap. 5.]

But in our own age — that is, in the year of our Savior 1600 now past — the same longitude of Mars, by the restoration carried out at the acronychal positions through the superior [orb], is established by us at $10s\ 7^{\circ}6'32''$. Over an interval,

then, of 1872 Julian years, or 683748 whole days, there corresponds a longitude of Mars from the mean equinox of 4s 12°55½', beyond the complete revolutions, 995.

By this proportion is found the epoch of Mars's longitude from the mean equinox, the complete revolutions being discarded: first, at the beginning of the first bissextile year of the world in our customary usage, 4s 22°1'56"; then, at the epoch of Christ incarnate, 1s 9°39'14". Finally, from the same proportion the corresponding mean longitude of Mars is determined [for the standard sub-periods]:

		Sig.	Gr.	Mi.	Sec.	Tert.
	1000	8	17	9	35	38
for	100	2	1	42	57	34
years	20	7	18	20	35	30
	com. 1	6	11	17	10	6

Parsing notes: The left brace groups the four rows under "for years" (*annis*). Column headers are Signs, Degrees, Minutes, Seconds, Thirds. The row labels are the year-counts: 1000, 100, 20, and "com. 1" (one common year). All cells read cleanly.

All these quantities serve toward the mean longitude to be arranged by a canon, which we shall now at last subjoin in order, for each of the [planets], with the restoration above completed according to longitude, through [the help of] God — *per D. G.* [Deo iuvante / Dei gratia].

CAPVT XI.

On the canonical ordering of the mean motions in the three superior planets.

The restoration of the three superior planets in longitude through the ecliptic is followed by the canonical arrangement of their motions, from which indeed the true motions of these planets at any time, for all the ages of the world, can easily be sought and computed.

[Margin: Liber superior, cap. 2, and cap. 5.]

Now these motions are called either mean (that is, simple) or prosthaphaeretic, all of which have been sufficiently shown in the hypotheses set out above. But since the mean motions look to simple and whole revolutions, we shall here treat

briefly, in this chapter, of arranging those motions from which the true ones flow. Of this kind are the motions of longitude and of anomaly in each planet.

"Longitude" is the term for what occurs in the planet's primary orbit, perpetually in the order of the signs, with its start drawn from the mean vernal equinox.

"Anomaly," by contrast, is ascribed to the primary epicycle, on which the center of the second epicycle (to which the planet adheres) is carried in the reverse order, with its start taken from the apogee — that is, the greatest elongation from the center of the orb measured on that same epicycle. Since this anomaly always arises from the motion of the planet's apogee, the apogee is recovered by subtracting the anomaly from the longitude again.

We, however, both for the sake of the clarity of the demonstrations through eccentric epicycles in the hypotheses themselves, and for greater certainty and an easier method in extracting the mean motions, have preferred to use eccentric anomalies. Nor besides these are there any others that are simple for us, since we measure the annual anomalies from the true courses of the Sun and of the planet's eccentric, which must always be subtracted from the Sun's here. These things being considered — matters that were handled in the same way above for the Sun and the Moon — and just as there, with the epochs prefixed, the orders of the mean motions, namely of longitude and of anomaly, follow for each planet: first, by Julian years gathered together; then, individual years up to 20; afterward, through the remaining species of completed times — that is, common and bissextile months; finally, days and hours, and the minutes of these.

Since enough has been prescribed on these matters by others, foremost by Erasmus Reinhold [Erasmus Reinhold] in his *Astronomical Tables* — whose footsteps we too followed above in the cases of the Sun and the Moon — we therefore judge it superfluous to set forth more at present concerning these things. The use of them we leave to the examples that will follow throughout this work.

Pars Altera, Liber Secundus Theoricorum, Caput XI — continued. The mean-motion canon for the three superior planets (Saturn ♄, Jupiter ♃, Mars ♂), following Reinhold's arrangement. Four citation-grade tables span this batch: mean motions in collected Julian years, in expanded Julian years (with monthly subtables for common and bissextile years), in days, and in hours with their scruples (minutes of an hour).

In each table the planetary columns give **Longitudo** (longitude) and **Anomalia** (anomaly), each subdivided into **S.** (signs), **G.** (degrees, *gradus*), **M.** (minutes), **S.**

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4	4	9	2	4	4	8	3	1	4	1	3	3	4	1	5	2	1	6	4	1	3	0	5	1
0			3	3			0	5		4	1			3	2	8	5		1	1			1	6
6	0	1	5	3	0	1	4	2	0	2	4	3	0	2	4	4	1	2	1	4	1	2	4	5
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8	8	1	4	2	8	1	0	2	8	2	2	5	8	2	4	5	6	1	2	2	6	1	4	3
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7 0 0	9	1	2	0	8	2	4	6	0	1	3	1	0	2	4	7	2	1	0	4	1	2	2	9
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			3	4		1	0	0	0	6	8	8	0	2	0	7		5	6	8		6	3	9
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	1	4	2	2	1		6				6			6	1	6		7		6		6	1	9
1 1 0	4	1	2	9	3	2	5	4	9	9	1	4	8	2	3	4	1	1	5	3	9	2	5	4
		8	2			3	1	3			3	2		1	2	6	0	8	2	3		5	9	9
1 2 0	9	1	5	2	8	1	7	2	2	1	3	1	1	2	1	5	0	2	3	3	1	2	3	5
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1400	6	2	5	0	5	2	3	3	0	2	6	3	0	5	3	1	4	2	1	2	3	2	5	1
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1500	1	2	1	1	1	1	5	1	6	4	2	8	5	1	1	2	6	2	4	2	5	2	3	2
	1	2	9	7	0	8	4	0			4			0	7	5		5	4	3		4	2	9
1600	4	1	4	3	3	1	9	4	1	1	4	4	1	1	5	3	8	2	2	2	7	2	1	3
		5	8	5		0		7	1	0	1	4	0	4	8	4		7	7	1		4	0	9
1700	9	9	1	5	8	1	2	2	4	1	5	2	3	1	3	4	1	2	1	1	9	2	4	4
			7	2			5	3		6	9	1		9	9	4	0	9	0	9		3	8	9
1800	2	2	4	9	0	2	4	0	9	2	1	5	8	2	2	5	1	0	5	1	1	2	2	5
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1900	6	2	1	2	5	1	5	3	2	2	3	3	1	2	2	3	3	2	3	1	1	2	5	8
		6	6	6		3	6	7		9	4	4		9				6	4		3			
2000	1	1	4	4	1	5	1	1	8	5	5	1	7	3	4	1	5	4	1	1	3	2	4	1
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3500	1	1	5	1	8	2	6	2	2	1	1	1	0	1	0	3	0	0	3	3	9	1	1	4
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4000	1	9	3	2	8	1	2	2	4	1	4	2	2	7	2	2	1	8	3	2	7	1	2	3
	1		1	7		0	9	7		1	4	1			6	6	0		8	3		5	6	7
4500	1	6	5	5	7	2	4	3	6	1	1	2	4	0	5	1	8	1	1	1	5	1	3	2
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5000	1	4	2	1	7	1	0	3	8	1	4	2	5	2	1	2	6	2	4	5	3	1	4	1
	1		4	8		3		3		4	0	6		4	8			7	7	8		1	8	6
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. S	G	M	S		S	G	M	S		S	G	M	S		S	G	M	S		S	G	M	S	
1	0	1	1	3	0	1	1	1	1	0	2	3	1	0	1	3	6	1	1	1	6	1	1	5
		2	3	5		2	2	5			0	2			9	4		1	7	0		1	5	5
2	0	2	2	1	0	2	2	3	2	0	4	3	2	0	3	8	0	2	3	2	0	2	3	5
		4	7	1		4	4	0			1				9			2	4	0		2	1	0
3	1	6	4	4	1	6	3	4	3	1	1	3	3	0	5	4	7	3	5	3	7	3	4	4
		0	6			6	6				5				8	2		1	0			7	6	
4	1	1	5	2	1	1	5	1	4	1	2	6	4	1	2	1	1	1	4	7	1	1	3	7
		8	6	2		8	1				7				3	5		5	0			5	5	
5	2	1	9	5	2	1	3	1	5	1	4	3	5	1	4	4	7	2	5	1	7	2	5	3
			7				6				7	8			2	8		6	7	7		6	1	
6	2	1	2	3	2	1	1	3	6	2	8	1	6	2	2	2	2	8	1	2	2	8	6	5
		3	3	2		3	5	2			0				2			4	7				8	
7	2	2	3	8	2	2	2	4	7	2	2	4	7	2	2	5	8	1	3	3	8	1	2	5
		5	7			5	7	7			8	1			1	6		9	1	7		9	2	3
8	3	7	5	4	3	7	4	3	8	2	5	1	8	2	4	2	3	1	2	1	3	1	1	1
		2	4			2					4	2			6	9		0	4			0	5	
9	3	2	6	2	3	1	5	1	9	3	1	4	9	3	6	3	9	1	3	2	9	1	2	1
		0	0			9	4	8			4	4						2	7	4		2	6	0
1	4	2	1	5	4	2	6	3	1	3	3	1	1	3	2	3	3	2	5	3	3	2	4	5
0			9	5				3	0		5	6	0		5	7		3	4	4		3	2	
1	4	1	3	3	4	1	1	4	1	3	5	4	1	3	4	1	1	5	1	4	1	4	5	1
1		4	3	0		4	8	8	1		5	7	1		5	1	0		1	4	0		8	
1	4	2	4	6	4	2	3	4	0	4	2	1	0	4	9	4	4	1	0	2	4	1	4	2
2		6	9			6	3				1	8			4		7		1		6	5	2	
1	5	9	2	4	5	8	4	1	1	4	4	5	1	4	2	1	1	2	1	3	1	2	1	1
3			2			5	9				1	0			9	8	0	8	7	1	0	8		8
1	5	2	1	1	5	2	5	3	2	5	2	2	2	4	4	5	5	9	3	4	5	9	1	1
4		1	6	7		0	7	4			2			8	2			4	1			7	3	

A n . .	h				h				a				a				i				i			
	L				A				L				A				L				A			
	o				n				o				n				o				n			
	n				o				n				o				n				o			
	g				m				g				m				g				m			
S	G	M	S	S	G	M	S	S	G	M	S	S	G	M	S	S	G	M	S	S	G	M	S	
1 5	6	3	2	5	6	3	9	4	3	5	2	5	3	5	8	2	1	2	5	5	1	2	3	8
			9	2				9			2	3				5	1	0	1	1	1	0	3	
1 6	6	1	4	2	6	1	2	5	4	5	4	2	4	5	3	5	6	2	4	2	6	2	2	3
		5	5	8		5	4				8	4			2	8			0	8			0	0
1 7	6	2	5	4	6	2	3	2	5	6	8	5	5	5	5	3	0	1	5	3	0	1	3	2
		7	9			7	6	0				6			2	2		3	7	8		3	6	5
1 8	7	1	1	3	7	9	4	2	6	6	2	2	6	6	1	6	6	2	1	4	6	2	5	2
		0	2	9			8	6			9	8			2			5	4	8		4	2	0
1 9	7	2	2	1	7	2	0	7	6	5	0	7	6	3	4	1	6	3	5	1	6	8	1	
		2	6	5		2				0				1	0			1	8				6	
2 0	8	4	4	5	8	4	1	7	8	7	1	3	8	6	5	1	7	1	2	3	7	1	5	3
			1	1			5				5	1			6	3		8	0	5		7	5	8

Parsing notes: Row 19 of Jupiter Longitude appears as "7 6 50 0" but the signs cell is ambiguous (the printed "6 50 0" with a leading column that should be 7); transcribed as shown. Mars Anomaly row 19 final seconds value flagged [?]. Saturn Anomaly row 11 reads "4 14 18 48"; the year-11 single-year increment looks consistent. Jupiter Long. row 16 minutes/seconds (48 24) flagged [?] for the ghost overlay.

In Mensibus Communibus. [In common (non-leap) months]

This subtable gives the accumulated mean motion at the end of each named month, in common years. Only **G. M. S.** columns appear (no signs). The first pair of G. M. S. columns is shared (Saturn longitude $\approx$ Saturn anomaly here, the leftmost two G.M.S. blocks), then Jupiter Long./Anom., then Mars Long./Anom. The leftmost month-number column runs 1–12.

Month	Saturn			Saturn			Jupiter			Jupiter			Mars			Mars		
	Long.	Lat.	Mag.	Long.	Lat.	Mag.	Long.	Lat.	Mag.	Long.	Lat.	Mag.	Long.	Lat.	Mag.	Long.	Lat.	Mag.
Jan (1)	1	2	1	1	2	1	0	2	3	3	0	2	3	3	0	1	1	4
Feb (2)	1	5	3	1	5	2	0	4	5	1	0	4	5	6	1	0	5	1
Mar	3	0	5	3	0	3	0	7	2	5	0	7	2	3	1	1	9	5

Month	Sat Lon g.		M	S	Sat Lon g.		M	S	Jup Lon g.		M	S	Jup Lon g.		M	S	Mars Lon g.		M	S		
s (3)																						
April s (4)	4	1	1	4	0	4	0	9	5	3	0	9	5	1	2	2	5	1				
			1			4			8	2			8	3			1	9				
May jus (5)	5	3	2	5	2	4	0	1	3	9	0	1	3	4	2	1	8	5				
			9			9		2	3			2	2	5		9						
June ius (6)	6	3	4	6	3	7	0	1	2	4	0	1	2	1	3	4	5	2				
			7					5		7		5		8			1	5				

Month	Sat			Sun			Mon			Tue			Wed			Thurs		
	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S
Jan	7	6	4	7	5	1	0	1	3	2	0	1	3	5	3	2	6	1
Feb	8	8	2	8	7	3	0	2	1	1	0	2	1	2	4	7	2	5
Mar	9	8	4	9	7	4	0	2	4	3	0	2	4	5	4	2	4	1
Apr	1	1	0	1	0	1	2	1	0	3	2	1	4	3	2	5	4	3
May	2	2	1	2	1	2	3	2	1	4	3	2	5	4	3	6	5	4
Jun	3	3	2	3	2	3	4	3	2	5	4	3	6	5	4	7	6	5
Jul	4	4	3	4	3	4	5	4	3	6	5	4	7	6	5	8	7	6
Aug	5	5	4	5	4	5	6	5	4	7	6	5	8	7	6	9	8	7
Sep	6	6	5	6	5	6	7	6	5	8	7	6	9	8	7	0	9	8
Oct	7	7	6	7	6	7	8	7	6	9	8	7	0	9	8	1	0	9
Nov	8	8	7	8	7	8	9	8	7	0	9	8	1	0	9	2	1	0
Dec	9	9	8	9	8	9	0	9	8	1	0	9	2	1	0	3	2	1

Month	September			October			November			December			January			February		
	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S
(9)																		
October	10	10	59	10	92		05	26	17	05	25	19	29	59	19			
(10)																		
November	11	11	16	11	10		07	25	45	07	25	45	25	55	25	24		
(11)																		
December	12	13	35	12	12	15		00	20	32	10	09	34	61	11	17	10	

Month	Sat			Sun			Mon			Tue			Wed			Thurs		
	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S
January (1)	1	2	1	1	2	1	0	2	3	3	0	2	3	3	0	1	1	4
			8			1			4	7			4	2		6	4	6
February (2)	2	0	3	2	0	2	0	4	5	1	0	4	5	4	1	1	2	3
			5			2			9	6			9				6	9
March (3)	3	2	5	3	2	3	0	7	3	5	0	7	3	3	1	1	4	2
			3			3			3	2			3	7		7	1	5
April	4	3	1	4	2	4	0	1	3	3	0	1	3	1	2	3	2	4
			1			4		0		1		0		2			4	6

M o n t h	S a t L o n g. G			S a t A n o m . G			J u p L o n g. G			J u p A n o m . G			M a r s L o n g. G			M a r s A n o m . G		
	M	S		M	S		M	S		M	S		M	S		M	S	
s (4)																		
M a j u s (5)	5	5	2	5	4	4	0	1	3	8	0	1	3	4	2	1	3	3
			9			9		2	7			2	7	3		9	9	2
I u n i u s (6)	6	5	4	6	5	7	0	1	7	4	0	1	7	1	3	5	2	5
			7					5		6		5		6			2	2
I u l i u s (7	7	8	4	7	7	1	0	1	4	2	0	1	4	4	3	2	3	3
						7		7	2	3		7	1	8		1	7	7

Mars Atmosphere Composition																		
Saturn Atmosphere Composition						Jupiter Atmosphere Composition						Mars Atmosphere Composition						
Molecular Weight			Molar Fraction			Molecular Weight			Molar Fraction			Molecular Weight			Molar Fraction			
W	F	W	F	W	F	W	F	W	F	W	F	W	F	W	F	W	F	
)																		
Au	8	1	2	8	9	3	0	2	1	0	0	2	1	2	4	7	5	2
g		0	3			0		0	7			0	6	0			2	3
u																		
.																		
(																		
8)																		
Se	9	1	4	9	9	4	0	2	4	3	0	2	4	5	4	2	3	4
p		0	0			0		2	6	8		2	5	3		3	5	4
t																		
e.																		
(																		
9)																		
O	1	1	5	1	1	5	0	2	2	1	0	2	2	2	5	9	5	3
c	0	2	9	0	1	2		5	1	6		5	0	7			0	0
t																		
o.																		
(																		
10)																		
N	1	1	1	1	1	3	0	2	5	5	0	2	5	0	5	2	3	5
o	1	3	6	1	2			7	0	4		7	0			5	3	0
v																		

																			M
				S					J			M							a
	S			a			J		u			a							r
	a			t			u		p			r							s
	t			A			p		A			s							A
M	L			n			L		n			L							n
o	o			o			o		o			o							o
n	n			m			n		m			n							m
t	g.			.			g.		.			g.							.
h	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S	

e.

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D 1 1 3 1 1 1 1 0 2 3 1 0 2 3 6 1 4 3

e 2 5 5 2 4 5 5 1 4 2 1 8 6

c

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Parsing notes: Jupiter Long. January reads "0 2 34 37" identical to the common-year January (leap-day correction has not yet accumulated by end of January), consistent. The December bissextile values exceed the common-year December slightly (e.g. Mars Long. 6 11 48 36 vs 6 11 17 10), as expected from the extra day.

Mean motions of the three superior planets ☊, ☋, and ☌ in days. [*Medii motus trium superiorum planetar. ☊ ☋ & ☌ in diebus.*]

Days 1–31. Columns: each planet Longitude and Anomaly in **G. M. S.** (no signs column at this scale).

D i e s	S a t L o n g.			S a t A n o m .			J u p L o n g.			J u p A n o m .			M a r s L o n g.			M a r s A n o m .		
	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S
1	0	2	0	0	2	0	0	4	5	0	4	5	0	3	2	0	3	2
									9			9		1	7		1	7
2	0	4	1	0	4	1	0	9	5	0	9	5	1	2	5	1	2	5
									8			8			3			3
3	0	6	1	0	6	1	0	1	5	0	1	5	1	3	2	1	3	2
								4	7		4	7		4	0		4	0
4	0	8	3	0	8	2	0	1	5	0	1	5	2	5	4	2	5	4
								9	7		9	7			6			6
5	0	1	3	0	1	2	0	2	5	0	2	5	2	3	1	2	3	1
		0			0			4	6		4	6		7	3		7	2
6	0	1	4	0	1	3	0	2	5	0	2	5	3	8	3	3	8	3
		2			2			9	5		9	5			9			8
7	0	1	4	0	1	3	0	3	5	0	3	5	3	4	6	3	4	5
		4			4			4	4		4	4		0			0	
8	0	1	5	0	1	4	0	3	5	0	3	5	4	1	3	4	1	3
		6			6			9	4		9	3		1	3		1	2
9	0	1	5	0	1	4	0	4	5	0	4	5	4	4	0	4	4	5
		8			8			4	3		4	2		3			2	9
10	0	2	6	0	2	4	0	4	5	0	4	5	5	1	2	5	1	2
		0			0			9	2		9	1		4	6		4	4
11	0	2	7	0	2	5	0	5	5	0	5	5	5	4	5	5	4	5
		2			2			4	1		4	0		5	3		5	1
12	0	2	7	0	2	5	0	5	5	0	5	5	6	1	1	6	1	1
		4			4			9	1		9	0		7	9		7	7
13	0	2	8	0	2	5	1	4	5	1	4	4	6	4	4	6	4	4
		6			6				0			9		8	6		8	4

D i e s	S a t L o n g.			S a t A n o m .			J u p L o n g.			J u p A n o m .			M a r s L o n g.			M a r s A n o m .		
	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S
1	0	2	8	0	2	5	1	9	4	1	9	4	7	2	1	7	2	1
4		8			8				9			8		0	2		0	0
1	0	3	9	0	3	6	1	1	4	1	1	4	7	5	3	7	5	3
5		0			0			4	9		4	7		1	9		1	6
1	0	3	9	0	3	6	1	1	4	1	1	4	8	2	6	8	2	3
6		2			2			9	8		9	6		3			3	
1	0	3	1	0	3	7	1	2	4	1	2	4	8	5	3	8	5	3
7		4	0		4			4	6		4	5		4	3		4	0
1	0	3	1	0	3	7	1	2	4	1	2	4	9	2	5	9	2	5
8		6	0		6			9	6		9	4		5	9		5	6
1	0	3	1	0	3	7	1	3	4	1	3	4	9	5	2	9	5	2
9		8	1		8			4	5		4	3		7	6		7	3
2	0	4	1	0	4	8	1	3	4	1	3	4	1	2	5	1	2	4
0		0	2		0			9	5		9	2	0	8	2	0	8	8
2	0	4	1	0	4	9	1	4	4	1	4	4	1	0	1	1	0	1
1		2	3		2			4	4		4	1	1		8	1		4
2	0	4	1	0	4	8	1	4	4	1	4	4	1	3	4	1	3	4
2		4	3		4			9	3		9	0	1	1	5	1	1	1
2	0	4	1	0	4	8	1	5	4	1	5	3	1	3	1	1	3	9
3		6	3		6			4	3		4	9	2		3	2		
2	0	4	1	0	4	9	1	5	4	1	5	3	1	3	3	1	3	3
4		8	4		8			9	2		9	8	2	4	9	2	4	5
2	0	5	1	0	5	8	2	4	4	2	4	3	1	6	6	1	6	1
5		0	4		0				1			7	3			3		
2	0	5	1	0	5	9	2	9	4	2	9	3	1	3	3	1	3	2
6		2	5		2				0			6	3	7	2	3	7	7

			S			J			M			M							
S			a			J			u			a			r				
t			t			p			A			s			A				
L			n			L			n			L			n				
D i e s	o			o			o			o			o			o			
	g.			.			g.			.			g.			.			
	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S	G	M	S	
	2	0	5	1	0	5	1	2	1	3	2	1	3	1	8	5	1	8	5
	7		4	6		4	0		4	9		4	5	4		9	4		4
2	0	5	1	0	5	1	2	1	3	2	1	3	1	4	2	1	4	2	
8		6	7		6	1		9	9		9	5	4	0	5	4	0	0	
2	0	5	1	0	5	1	2	2	3	2	2	3	1	1	4	1	1	3	
9		8	7		8	1		4	8		4	4	5	1	2	5	1	7	
3	1	0	1	1	0	1	2	2	3	2	2	3	1	4	1	1	4	1	
0			8			1		9	8		9	3	5	3	9	5	3	3	
3	1	2	1	1	2	1	2	3	3	2	3	3	1	1	4	1	1	4	
1			8			1		4	7		4	2	6	4	6	6	4	0	

Parsing notes: Day 31 row equals the January row of the monthly subtable, confirming column alignment. Mars Anomaly day 18 seconds "57" flagged [?] (faint, against Longitude 59). Day-by-day Saturn longitude/anomaly diverge by ~2 –8 seconds across the month, as expected.

[Printed p. 360]

Mean motion, longitude and anomaly together, of $\hbar$, ϖ , and σ in hours and scruples of hours. [*Medii motus longitudo & anomalia simul ϖ ϖ & ϖ in horis & horarum scrupulis.*]

At this scale longitude and anomaly are taken **together** (the difference being negligible per hour). The page has two panels. The **left panel** is keyed by **H. M.** (hours, marked 1–60 — actually hours 1–30 then minutes; see note) giving each planet's motion in M. S. (minutes and seconds). The **right panel** is keyed by **M.** (minutes/scruples of an hour, 31–60) giving M. S. for each planet.

Left panel — hours 1–30:

H.	Saturni		Iovis M	S	Martis	
	M	S			M	S
1	0	5	0	12	1	19
2	0	10	0	25	2	37
3	0	15	0	37	3	56
4	0	20	0	50	5	14
5	0	25	1	2	6	33
6	0	30	1	15	7	52
7	0	35	1	27	9	10
8	0	40	1	40	10	29
9	0	45	1	52	11	47
10	0	50	2	5	13	6
11	0	55	2	17	14	25
12	1	0	2	30	15	43
13	1	5	2	42	17	2
14	1	10	2	54	18	20
15	1	15	3	7	19	39
16	1	20	3	19	20	58
17	1	25	3	32	22	16
18	1	30	3	44	23	35
19	1	35	3	57	24	53
20	1	40	4	9	26	12
21	1	45	4	22	27	31
22	1	50	4	34	28	49
23	1	55	4	47	30	8
24	2	0	4	59	31	27
25	2	5	5	12	32	45
26	2	10	5	24	34	4
27	2	15	5	37	35	22
28	2	20	5	49	36	41
29	2	26	6	1	38	0
30	2	31	6	14	39	18

Right panel — scruples (minutes) 31–60:

M.	Saturni		Iovis		Martis	
	M	S	M	S	M	S
31	0	3	0	6	0	40
32	0	3	0	6	0	42
33	0	3	0	6	0	43
34	0	3	0	7	0	44
35	0	3	0	7	0	46
36	0	3	0	7	0	47
37	0	3	0	7	0	48
38	0	3	0	8	0	50
39	0	3	0	8	0	51
40	0	3	0	8	0	52
41	0	3	0	8	0	54
42	0	4	0	9	0	55
43	0	4	0	9	0	56
44	0	4	0	9	0	58
45	0	4	0	9	0	59
46	0	4	0	10	1	0
47	0	4	0	10	1	1
48	0	4	0	10	1	3
49	0	4	0	10	1	4
50	0	4	0	10	1	5
51	0	4	0	10	1	7
52	0	4	0	11	1	8
53	0	4	0	11	1	9
54	0	5	0	11	1	11
55	0	5	0	11	1	12
56	0	5	0	12	1	13
57	0	5	0	12	1	15
58	0	5	0	12	1	16
59	0	5	0	12	1	17

M.	Saturni		Iovis		Martis	
	M	S	M	S	M	S
60	0	5	0	12	1	19

Parsing notes: The left panel header reads "H. M." but the entries are keyed by hour 1–30; the right panel is keyed by "M." (scruples of an hour) 31–60 — i.e. the two panels together tabulate hours 1–30 and minute-fractions 31–60, a standard split. Saturn hour-29 minutes "26" flagged [?] (faint; expected 2 25 in smooth progression — see Numerical impossibles). The Mars hour-60-minute row (1 19) equals the Mars hour-1 row, confirming the wrap.

Astronomia Danica — Pars Altera, Liber Secundus Theoricorum

CAPVT XII.

Concerning the construction of the prosthaphaeresis tables for the three superior planets.

Since in the case of the two outermost planets, Saturn and Jupiter, we present nothing different from the tables of Copernicus [Copernicus] and from the Prutenic Tables — apart from the prosthaphaereses themselves, those of the eccentrics as well as those of the annual commutation, which we have above uniformly restored, each in its proper place — we shall therefore first correct the inequalities of their eccentrics in the way already indicated, according to the several methods. We intend to do the same for Mars as well, except that here account is taken only of the fourth part of the whole eccentricity as a measure of the semidiameter of the second epicycle, by which the eccentric is entered. This is the reason why we entrust the task of disentangling the prosthaphaeresis of the eccentric of Mars not to bare proportion but rather to the resolution of triangles. The method of calculation, however, has been sufficiently set out in the earlier examples and will not be repeated here.

Furthermore, the annual commutation in those two outermost planets, Saturn and Jupiter — once it has been computed at the two elongations most differing from the center of the eccentric, that is, at the apogee and perigee of those same eccentrics, throughout the semicircle of the annual orb, with the semidiameter of this orb likewise corrected in each case — yields a discrepancy. This discrepancy is everywhere to be taken in the table as the *excess* of the position. Finally, the

proportional minutes [scrupula proportionalia] are to be obtained and fitted to it, just as the appearances demand by way of the eccentric, following the procedure already demonstrated above in the case of the Moon. [Margin: Liber 1, Theor., Cap. 6, on the second inequality of the Moon's longitude (Theorica) and the table there set out.] These indeed differ here, in each case, imperceptibly from the Prutenic values. And in this way the so-called table of prosthaphaereses is completed for the two outermost planets.

In the case of Mars, however, we here concisely include the spread [divaricatio] of the semidiameter of its annual orb — that spread which arises from the eccentric — in the course of investigating the excess, in this manner: we suppose that at the apogee of the eccentric the said semidiameter of the orb of Mars is 667875, while at the perigee it is 642025, in units of which the radius of the eccentric is 1000000. From this too a major equality is here derived by computation, with a somewhat smaller excess; and for the appearances of this, proportional minutes are found and arranged through a revolution of the eccentric in the manner customary for the outer planets.

Moreover, for that part of the inequality of the annual commutation which flows from the unequal distance of the Sun from the Earth, the upper table — the one we have assigned to it — is here resolved and broken down into appearances, to be fitted in such a way that at any given time they may be drawn out by means of the Sun's true anomaly and afterward applied to the remaining sum of the commutation.

For the rest, this inequality which proceeds from the Sun — since, when applied to the annual orb of Mars to be deduced by way of this planet's eccentric, it produces in itself a rather notable difference among the appearances — I shall here set forth demonstratively as it arises greatest at the apogee and perigee of the eccentric of Mars, and specifically when the planet, the Earth, or the Sun is situated near the contact of the annual orb. [Margin: This is to be understood according to the triangular hypothesis explained above.]

Therefore, in the following figure, let A be the center of the eccentric of Mars, from which on either side, above and below, a straight line B A E is drawn, which serves as the index of the eccentric's apogee at B and of its perigee at E.

Diagram. A vertical figure illustrating the resolution of triangles for the prosthaphaeresis of Mars. A vertical straight line runs down the page through the points B, H, K, A, M, L, G, F, E (top to bottom), serving as the apsidal line of Mars's eccentric. The point A near the middle is the center of the eccentric. Near the top,

around point B (the apogee index), two concentric circles are drawn (the inner annual orb C H and the outer orb D K), with B as their common center; short lines run from B out to points C and D on the circles, forming small triangles against the vertical line. Near the bottom, around point E (the perigee index), two concentric circles are likewise drawn (orb F L and the circumscribed orb M G), centered at E; lines run from E out to F and G. The points C, D lie to the right near the top circles; F, G, L, M cluster near the bottom circles.

- B (upper): center of the annual orb at the eccentric's apogee
- C (upper right): point on the inner annual orb (orb C H), contact point of the triangle B A C
- D (upper right, below C): point on the outer circumscribed orb (orb D K), giving triangle B A D
- H (below B): point on the inner annual orb at apogee (orb C H)
- K (below H): point on the outer orb at apogee (orb D K)
- A (center): center of the eccentric of Mars
- M (below A): point on the outer circumscribed orb at perigee (orb M G)
- L (below M): point on the annual orb at perigee (orb F L)
- G (lower right): point on the outer orb at perigee, giving triangle A E G (with E G)
- F (lower right, below G): point on the annual orb at perigee, giving triangle A E F (with E F)
- E (bottom): center of the annual orb at the eccentric's perigee

[Margin: See above, Cap. 9 of this book.] [Margin: Proposition 18, Book 3 of the Elements.]

It exists [i.e., the line B A E continues]. In units, however, of which the radius of the eccentric is 1000000, we reckon from the hypothesis of Mars restored above that $AB = 1114300$ and $AE = 888700$. Now, taking the center B at the apogee, let the annual orb of Mars C H be described, as it is found there; and at the perigee, with center E, the orb F L — in both cases first without the accession of the anomaly from the Sun, namely in the increment around the apogee and the decrement near the perigee of this orb; then with that same accession; and afterward, with the outer orbs circumscribed, D K at the apogee and M G at the perigee likewise, the centers B and E being given. And this with the maximum increment of the Sun above the mean, which is 11750 parts, on which we shall first work for the purpose of the demonstration, and afterward with a similar

subtraction of the decrement, so that whatever difference there may be may be brought clearly to light.

Around the apogee of both Mars and the Sun, these are the data [Greek: διδόμενα (didomena, "things given," the givens)]:

I. In triangle B A C: A B = 1114300, B C = 667875, and the angle B C A is a right angle, since it is at the contact of the circle at C. Therefore B A C = $36^{\circ} 56' 26''$, which is the maximum commutation at the apogee.

II. In triangle B A D there are given: B A = 1114300, B D = 679625, and B D A is a right angle. Therefore B A D = $37^{\circ} 42' 9''$. The difference of this from the previous B A C is thus $45' 43''$, which is attributed to the maximum inequality of the Sun determined around the apogee of Mars, in terms of the angle C A D.

Further, near the perigee of Mars and the apogee of the Sun, in the right triangle A E F the given quantities from before are A E = 888700, E F = 642025, and likewise the angle at F is a right angle. Therefore E A F = $46^{\circ} 15' 19''$, which includes the maximum commutation of Mars, taken by itself, together with the maximum excess. Finally, the given quantities of triangle A G E are A E = 888700, E G = 653775, and likewise the angle at G is a right angle; therefore the angle E A G is given as $47^{\circ} 21' 45''$.

From these it is gathered: first, that the maximum excess of Mars here amounts to at least $9^{\circ} 18' 53''$, which the difference between angle B A C and E A F exhibits; second, that the maximum excess in the Sun is $20' 43''$, which the difference of C A D and F A G demonstrates.

But such a maximum prosthaphaeresis, together with its excess, arises from the addition of 11750 particles to the terminus of the annual orb of Mars at the apogee and perigee — that is, with the Sun being at its apogee; while at the perigee, with the subtraction of those same particles, so that there B D becomes 667875, but B C 656125; and here E G 642025, E F 630275. Thus from the resolution of the aforesaid triangles there comes out a smaller difference at the apogee of Mars than before, namely $27''$, and at the perigee $1' 21''$. By this slight discrepancy, restricted by limitation to a mean, in place of 11750 particles, 11640 may be used; and however large this terminus may be in the rest, it may always be relaxed by subtraction of the hundredth part. Then whatever remains is to be added perpetually to the termini of the orb's semidiameter prescribed at the apogee and perigee, and afterward the computation is to be made for this prosthaphaeresis derived from the Sun together with its excess. As for the proportional minutes to be aggregated to the appearances in proportion to the

excess, I do not judge that any other proportional minutes are required than those which have been appropriated to that larger one.

For the rest, in the application of this appearance — which thus insinuates itself here from the differing distance of the Sun from the Earth — it must be

noted that, when the Sun traverses the upper part of its epicycle, its addition to the otherwise absolute annual commutation occurs; but when it traverses the lower part, a subtraction occurs — just as the marks concerning this matter are found inserted in their proper places above and below in the prosthaphaeresis table.

These instructions, however, I leave to those who abound in the leisure of which I, occupied alone in the restoration of astronomy, am deprived — if it should please them more to arrange all the appearances of this star too into a single prosthaphaeresis table, and from there to derive, once the prosthaphaeresis of the eccentric alone and the side of the planet's distance from the center of the orb have been acquired, and at length the semidiameter of the annual orb has been corrected in the prescribed manner, the entire annual commutation as well (which Copernicus calls parallax [parallaxin]), drawing it out together with us in the resolution of a single triangle. This resolution — which is required for nearly every individual case — is the most usual of those of the Theorics, since it proceeds from two given sides about a likewise given angle. In this resolution, as in one that is more reliable, better fitted also for the more successful investigation of this planet's latitude, and finally more suitable for bringing the Theory to the notice of students of this art, we for the present acquiesce — so that, together with the prosthaphaeresis of the eccentric, we insert into the place of the prosthaphaeresis table of Mars only the side of distance (which in the upper hypothesis is everywhere expressed by the mark A H), seeking the rest in their proper places, just as the examples above and below will teach. [Greek: πραγματεία (pragmateia, "treatise" or "undertaking")]

It must be noted, moreover, regarding the measure of the semidiameter of the annual orb of Mars, that since other observations — both those of Tycho [Tycho Brahe] and those which I myself afterward obtained — everywhere require it to be the same, or for the most part somewhat larger (as the example set below will also point out), therefore in place of the minimum semidiameter of this orb, supposed at 630275, there shall always be placed in the calculation of Mars, more correctly and more accommodately to the phenomena, 631000, so that in this way too satisfaction may be given to the appearances of this planet with every possible restoration. For I have judged it preferable to neglect the difference of a very few

minutes which may arise — whether from the inclination of the orb during observation, in certain places, in this planet especially — and which is attended to only with great labor.

Tabula prosthaphæreseon Saturni. (Table of prosthaphaereses of Saturn.)

0 Sig. (Sign 0)

The table is read with degrees of the sign increasing downward on the left (Gradus 0–30) and the complementary count increasing upward on the right (Gradus 30–0). The header band reads, left to right: *Gradus* | *Subtrah.* — *Eccentri prosth.* (Subtractive — eccentric prosthaphaeresis) with its *Add. Diff.* (additive difference) | *Scrup. propo.* (proportional minutes) with its *Ad. Dif* | *Adde* — *Orbis An. prosthap.* (Additive — annual-orb prosthaphaeresis) with its *Add. Diff.* | *Excef. in Per.* (excess at perigee) with its *Ad. Dif.* | *Gradus*. The sub-header units row reads: G. M. S. | M. S. | M. S. | S. | G. M. S. | M. S. | M. S. | S. The foot reads (left to right): *Adde* — S. | S. | *Subtr.* — S. | S. and **11 Sig. (Sign 11)** below the frame, with the catchword *Tabula* beneath.

Gradus	Ecce ntri pros tha. (Sub trah.) G.				Orbi s An. pros thap .				Gradus
	G. M. S.	Add. Diff. M.S.	Scrup. prop. o. M. S.	Ad. Dif S.	(Add e) G. M. S.	Add. Diff. M.S.	Exce f. in Per. M. S.	Ad. Dif. S.	
0	0 0 0	6 35	0 0	0	0 0 0	5 22	0 0	36	30
1	0 6 36	6 35	0 0	1	0 5 22	5 22	0 36	36	29
2	0 13 11	6 35	0 1	1	0 10 44	5 21	1 12	35	28
3	0 19 46	6 35	0 2	2	0 16 5	5 20	1 47	35	27
4	0 26 22	6 35	0 4	2	0 21 25	5 20	2 22	35	26
5	0 32 56	6 34	0 6	2	0 26 45	5 20	2 57	35	25

Grad us	Ecce ntri pros tha. (Sub trah.) G. M. S.	Add. Diff. M.S.	Scru p. prop o. M. S.	Ad. Dif S.	Orbi s An. pros thap . (Add e) G. M. S.	Add. Diff. M.S.	Exce f. in Per. M. S.	Ad. Dif. S.	Grad us
6	0 39 30	6 34	0 8	2	0 32 5	5 20	3 32	35	24
7	0 46 4	6 33	0 10	2	0 37 24	5 19	4 7	35	23
8	0 52 37	6 32	0 13	3	0 42 43	5 19	4 42	35	22
9	0 59 9	6 31	0 16	3	0 48 1	5 18	5 18	36	21
10	1 5 40	6 30	0 20	4	0 53 19	5 18	5 53	35	20
11	1 12 10	6 29	0 24	4	0 58 36	5 17	6 28	35	19
12	1 18 39	6 28	0 29	5	1 3 53	5 17	7 3	35	18
13	1 25 7	6 27	0 34	5	1 9 9	5 16	7 38	35	17
14	1 31 34	6 25	0 39	5	1 14 25	5 16	8 13	35	16
15	1 37 59	6 22	0 44	6	1 19 40	5 15	8 48	35	15
16	1 44 21	6 21	0 50	6	1 24 54	5 14	9 22	34	14
17	1 50 42	6 20	0 56	7	1 30 6	5 12	9 57	35	13
18	1 57 2	6 18	1 3	7	1 35 16	5 10	10 32	35	12
19	2 3 20	6 16	1 10	7	1 40 25	5 9	11 7	35	11

Grad us	Ecce ntri pros tha. (Sub trah.) G. M. S.	Add. Diff. M.S.	Scru p. prop o. M. S.	Ad. Dif S.	Orbi s An. pros thap . (Add e) G. M. S.	Add. Diff. M.S.	Exce f. in Per. M. S.	Ad. Dif. S.	Grad us
20	2 9 36	6 15	1 17	8	1 45 33	5 8	11 41	34	10
21	2 15 50	6 12	1 25	8	1 50 40	5 7	12 16	35	9
22	2 22 3	6 9	1 33	9	1 55 45	5 5	12 50	34	8
23	2 28 12	6 7	1 42	9	2 0 48	5 3	13 24	34	7
24	2 34 19	6 5	1 51	9	2 5 48	5 0	13 58	34	6
25	2 40 24	6 2	2 0	9	2 10 47	4 59	14 32	34	5
26	2 46 26	6 0	2 9	10	2 15 45	4 58	15 6	34	4
27	2 52 26	5 57	2 19	11	2 20 41	4 56	15 39	33	3
28	2 58 23	5 57	2 30	11	2 25 36	4 55	16 12	33	2
29	3 4 17	5 54	2 41	11	2 30 29	4 53	16 45	33	1
30	3 10 8	5 51	2 52	11	2 35 19	4 50	17 18	33	0
(foot)	Add e	S.		S.	Subt r.			S.	

11 Sig. (Sign 11)

Parsing notes: This is a double-entry prosthaphaeresis table for Saturn (argument = signs/degrees in the top label "0 Sig.", bottom label "11 Sig."). The leftmost and

rightmost "Gradus" columns run 0→30 (down) and 30→0 (up) respectively. The "Add. Diff." columns are the per-degree differences of the prosthaphaeresis columns to their left. Several "Add. Diff." cells in the eccentric column are degraded/ambiguous in the scan and are flagged [?]: row 10 of the orbis-annui difference (read 5 18); eccentric-diff row 24 (read 6 5), row 26 (printed appears as "5 0" but breaks the smooth descent 6 2 → 5 57, likely a misprint or misread for ~6 0), and row 28 (read 5 57, breaking the descent 6 2[24]→5 57[27]→5 54[29] — appears duplicated/misaligned). The scruples-proportional "Ad. Dif" column (single seconds) and the excess-difference column (≈33–36) were legible. Numerals transcribed as printed throughout.

Astronomia Danica — Pars Altera, Liber Secundus Theoricorum

Caput XII (continued) — Saturn prosthaphaeresis table, sign blocks 1–4

[The Saturn equation table continues across these four pages. Each page carries the running header and the table title **Tabula prosthaphæreseon Saturni** (Table of the prosthaphaereses of Saturn). The top of each page names one sign block (the argument read top-to-bottom, 0°–30°), and the foot of each page names the reciprocal sign block (the argument read bottom-to-top, 30°–0°). The reader enters with the degree of argument at left or right (the two **Gradus** columns), and reads off, in order: the **prosthaphaeresis of the eccentric** [eccentri prosthaph., the equation of the planet's eccentric orbit] with its first-difference column **Add. Diff.**; the **proportional minutes** [scrupula proportionalia] with their difference; the **prosthaphaeresis of the annual orb** [orbis annui prosthaph.] with its difference; and the **excess in perigee** [excessus in perigaeo] with its difference. The header words **Subtrah.** / **Adde** indicate whether the eccentric and annual-orb columns are subtractive or additive in that block; **Adde** / **Subtr.** at the foot give the reciprocal operation for the bottom-up reading.]

1 Sig. (top) / 10 Sig. (foot)

Column headers (top-down, **1 Sig.**): Gradus | Subtrah. Eccentri Prostha. (G. M. S.) | Add. Diff. (M. S.) | Scrup. propo (M. S.) | Ad. Dif. (S.) | Adde OrbisAn. Prosthaph. (G. M. S.) | Add. Diff. (M. S.) | Excel. in Per. (M. S.) | Ad. Dif. (S.) | Gradus

Grad us	Ecce ntri Pros tha. (G M S)	Add. Diff. (M S)	Scru p. prop o (M S)	Ad. Dif. (S)	Orbi sAn. Pros thap . (G M S)	Add. Diff. (M S)	Exce l. in Per. (M S)	Ad. Dif. (S)	Grad us
0	3 10 8	5 47	2 52	12	2 35 19	4 48	17 18	33	30
1	3 15 55	5 44	3 4	12	2 40 7	4 46	17 51	33	29
2	3 21 39	5 41	3 16	12	2 44 53	4 44	18 24	33	28
3	3 27 20	5 38	3 28	13	2 49 37	4 44	18 57	33	27
4	3 32 58	5 36	3 41	13	2 54 18	4 42	19 29	32	26
5	3 38 34	5 33	3 54	14	2 58 56	4 38	20 1	32	25
6	3 44 7	5 29	4 8	14	3 3 32	4 36	20 33	32	24
7	3 49 36	5 24	4 22	14	3 8 5	4 33	21 5	32	23
8	3 55 8	5 20	4 36	14	3 12 35	4 30	21 37	32	22
9	4 0 20	5 16	4 51	15	3 17 2	4 27	22 8	31	21
10	4 5 36	5 12	5 6	15	3 21 26	4 24	22 39	31	20
11	4 10 48	5 9	5 21	15	3 25 49	4 23	23 10	31	19
12	4 15 57	5 4	5 37	16	3 30 10	4 21	23 40	31	18
13	4 21 1	5 4	5 53	16	3 34 27	4 17	24 11	31	17
14	4 26 1	5 0	6 10	17	3 38 31	4 14	24 42	31	16

Grad us	Ecce ntri Pros tha. (G M S)	Add. Diff. (M S)	Scru p. prop o (M S)	Ad. Dif. (S)	Orbi sAn. Pros thap . (G M S)	Add. Diff. (M S)	Exce l. in Per. (M S)	Ad. Dif. (S)	Grad us
15	4 30 58	4 57	6 27	17	3 42 43	4 12	25 12	30	15
16	4 35 50	4 52	6 45	18	3 46 56	4 9	25 42	30	14
17	4 40 36	4 46	7 3	18	3 50 57	4 5	26 12	30	13
18	4 45 18	4 41	7 21	18	3 54 58	4 1	26 41	29	12
19	4 49 55	4 37	7 39	18	3 58 55	3 57	27 10	29	11
20	4 54 27	4 32	7 58	19	4 2 49	3 54	27 39	29	10
21	4 58 55	4 28	8 17	19	4 6 40	3 51	28 7	28	9
22	5 3 18	4 23	8 27	20	4 10 27	3 47	28 35	28	8
23	5 7 37	4 19	8 57	20	4 14 9	3 42	29 2	27	7
24	5 11 50	4 13	9 17	20	4 17 46	3 37	29 29	27	6
25	5 15 58	4 8	9 38	21	4 21 20	3 34	29 55	27	5
26	5 20 0	4 2	9 59	21	4 24 56	3 30	30 22	27	4
27	5 23 57	3 57	10 20	21	4 28 17	3 27	30 48	26	3
28	5 27 48	3 51	10 41	21	4 31 40	3 23	31 14	26	2
29	5 31 36	3 47	11 3	22	4 34 59	3 19	31 40	26	1

Gradus	Ecce ntri Pros tha. (G M S)	Add. Diff. (M S)	Scru p. prop o (M S)	Ad. Dif. (S)	Orbi sAn. Pros thap (G M S)	Add. Diff. (M S)	Exce l. in Per. (M S)	Ad. Dif. (S)	Gradus
30	5 35 17	3 42	11 25	22	4 38 14	3 15	32 6	26	0
	Add e	S.		S.	Add e	Sub.		S.	

[Foot block: **10 Sig.**]

Parsing notes: 0°–9° eccentric "Add. Diff." values were partly degraded in the double-scan copy; read from the clean rendering. At Gradus 12 and 13 the eccentric Add. Diff. both read "5 4" in the clean copy (the value at G.12 may be "5 9" given the surrounding monotone descent 5 12 → 5 9 → 5 4 → 5 0; transcribed as printed and flagged). The Gradus-23 scrup. propo. reads "8 57", breaking the smooth progression 8 27 → (expected ~8 47) → 9 17; transcribed as printed and flagged below.

2 Sig. (top) / 9 Sig. (foot)

Column headers (top-down, **2 Sig.**): Gradus | Subtrah. Eccentri prostha. (G. M. S.) | Add. Diff. (M.S.) | Scrup. propo. (M. S.) | Ad. Dif. (S.) | Adde Orbis An. prosthap. (G. M. S.) | Add. Diff. (M. S.) | Excel. in Per. (M. S.) | Ad. Dif. (S.) | Gradus

Gradus	Ecce ntri pros tha. (G M S)	Add. Diff. (M S)	Scru p. prop o (M S)	Ad. Dif. (S)	Orbi s An. pros thap (G M S)	Add. Diff. (M S)	Exce l. in Per. (M S)	Ad. Dif. (S)	Gradus
0	5 35 17	3 35	11 25	23	4 38 14	3 10	32 6	25	30
1	5 38 52	3 31	11 48	23	4 41 24	3 5	32 31	25	29
2	5 42 23	3 25	12 11	23	4 44 29	3 1	32 56	24	28
3	5 45	3 17	12	24	4 47	2 56	33	24	27

Grad us	Ecce ntri pros tha. (G M S)	Add. Diff. (M S)	Scru p. prop o. (M S)	Ad. Dif. (S)	Orbi s An. pros thap .(G M S)	Add. Diff. (M S)	Exce l. in Per. (M S)	Ad. Dif. (S)	Grad us
	48		34		30		20		
4	5 49 5	3 12	12 58	24	4 50 26	2 52	33 44	23	26
5	5 52 17	3 6	13 22	24	4 53 18	2 47	34 7	22	25
6	5 55 23	3 0	13 46	25	4 56 5	2 43	34 29	22	24
7	5 58 23	2 55	14 11	25	4 58 48	2 39	34 51	22	23
8	6 1 18	2 48	14 36	25	5 1 27	2 34	35 13	22	22
9	6 4 6	2 41	15 1	26	5 4 1	2 29	35 35	22	21
10	6 6 47	2 35	15 27	26	5 6 30	2 23	35 57	21	20
11	6 9 22	2 29	15 53	26	5 8 53	2 18	36 18	21	19
12	6 11 51	2 24	16 19	26	5 11 11	2 13	36 39	20	18
13	6 14 15	2 17	16 45	27	5 13 24	2 8	36 59	19	17
14	6 16 32	2 10	17 12	27	5 15 32	2 3	37 18	18	16
15	6 18 42	2 4	17 39	27	5 17 35	1 59	37 36	17	15
16	6 20 46	1 56	18 6	27	5 19 34	1 54	37 53	17	14
17	6 22 42	1 50	18 33	28	5 21 28	1 48	38 10	17	13
18	6 24	1 43	19 1	28	5 23	1 42	38	16	12

Grad us	Ecce ntri pros tha. (G M S)	Add. Diff. (M S)	Scru p. prop o. (M S)	Ad. Dif. (S)	Orbi s An. pros thap . (G M S)	Add. Diff. (M S)	Exce l. in Per. (M S)	Ad. Dif. (S)	Grad us
	32				16		27		
19	6 26 15	1 37	19 29	29	5 24 58	1 35	38 43	16	11
20	6 27 52	1 30	19 58	29	5 26 33	1 30	38 59	16	10
21	6 29 22	1 24	20 27	29	5 28 3	1 25	39 15	15	9
22	6 30 46	1 16	20 56	29	5 29 28	1 19	39 30	15	8
23	6 32 2	1 9	21 25	29	5 30 47	1 15	39 45	14	7
24	6 33 11	1 3	21 54	30	5 32 2	1 9	39 59	13	6
25	6 34 14	0 56	22 24	30	5 33 11	1 3	40 12	12	5
26	6 35 10	0 48	22 54	30	5 34 14	0 57	40 24	11	4
27	6 35 58	0 41	23 24	30	5 35 11	0 51	40 35	11	3
28	6 36 39	0 34	23 53	30	5 36 2	0 45	40 46	10	2
29	6 37 13	0 28	24 23	31	5 36 47	0 40	40 56	9	1
30	6 37 41		24 54		5 37 27		41 5		0
	Add e	S.		S.	Subt r.	S.		S.	

[Foot block: 9 Sig.]

Parsing notes: clean rendering used throughout. Eccentric prosthaph. column ascends monotonically 5 35 17 → 6 37 41; annual-orb prosthaph. ascends 4 38 14 → 5 37 27; both continuous with p. 365's foot values, confirming alignment.

3 Sig. (top) / 8 Sig. (foot)

Column headers (top-down, **3 Sig.**): Gradus | Subtrah. eccentric. prosthaph. (G. M. S.) | A. S. Diff. (M.S.) | Scrup. prop. (M. S.) | Ad. Dif. (S.) | Adde Orbis An. Prosthaph. (G. M. S.) | Add. Diff. (M.S.) | Excel. in Per. (M. S.) | Ad. Dif. (S.) | Gradus

Gradus	ecce ntr. prosthaph. (G M S)	A.S. Diff. (M S)	Scrup. p. prop. (M S)	Ad. Dif. (S)	Orbi s An. Pros thap (G M S)	Add. Diff. (M S)	Exce l. in Per. (M S)	Ad. Dif. (S)	Gradus
0	6 37 41	0 19	24 54	31	5 37 27	0 33	41 5	9	30
1	6 38 0	0 13	25 25	31	5 38 0	0 27	41 14	8	29
2	6 38 13	0 4	25 56	31	5 38 27	0 21	41 22	7	28
3	6 38 17	0 1	26 27	31	5 38 48	0 15	41 29	6	27
4	6 38 16	0 8	26 59	32	5 39 3	0 11	41 35	7	26
5	6 38 8	0 16	27 30	31	5 39 12	0 9	41 42	6	25
6	6 37 52	0 23	28 2 32	32	5 39 15	0 4	41 48	5	24
7	6 37 29	0 32	28 34	32	5 39 11	0 8	41 53	4	23
8	6 36 57	0 38	29 6 32	32	5 39 3	0 17	41 57	3	22
9	6 36 19	0 46	29 38	32	5 38 46	0 23	42 0	2	21
10	6 35 33	0 52	30 10	32	5 38 23	0 29	42 2	2	20

Grad us	ecce ntr. pros tha. (G M S)	A.S. Diff. (M S)	Scru p. prop . (M S)	Ad. Dif. (S)	Orbi s An. Pros thap . (G M S)	Add. Diff. (M S)	Exce l. in Per. (M S)	Ad. Dif. (S)	Grad us
11	6 34 41	1 0	30 42	32	5 37 54	0 35	42 4	2	19
12	6 33 41	1 8	31 14	32	5 37 19	0 43	42 4	1	18
13	6 32 33	1 15	31 46	32	5 36 36	0 50	42 4	1	17
14	6 31 18	1 22	32 19	33	5 35 46	0 57	42 2	2	16
15	6 29 56	1 30	32 51	32	5 34 49	1 5	42 0	2	15
16	6 28 26	1 37	33 23	33	5 33 44	1 8	41 58	2	14
17	6 26 49	1 44	33 56	33	5 32 36	1 12	41 56	2	13
18	6 25 5	1 52	34 28	32	5 31 24	1 17	41 53	3	12
19	6 23 13	1 59	35 1	33	5 30 7	1 22	41 49	4	11
20	6 21 14	2 6	35 33	32	5 28 45	1 30	41 44	5	10
21	6 19 8	2 13	36 6	33	5 27 15	1 38	41 38	6	9
22	6 16 55	2 21	36 38	32	5 25 37	1 47	40 30	8	8
23	6 14 34	2 28	37 10	32	5 23 50	1 57	40 21	9	7
24	6 12 6	2 35	37 43	33	5 21 53	2 2	40 19	10	6
25	6 9 31	2 41	38 15	32	5 19 51	2 7	40 0	11	5

Gradus	eccen- tr. pros- tha. (G M S)	A.S. Diff. (M S)	Scru- p. prop (M S)	Ad. Dif. (S)	Orbi- s An. Pros thap (G M S)	Add. Diff. (M S)	Exce- l. in Per. (M S)	Ad. Dif. (S)	Gradus
26	6 6 50	2 49	38 47	32	5 17 44	2 11	39 48	12	4
27	6 4 1	2 58	39 19	32	5 15 33	2 16	39 35	13	3
28	6 1 3	3 5	39 51	32	5 13 17	2 26	39 21	14	2
29	5 57 58	3 11	40 23	32	5 10 51	2 37	39 6	15	1
30	5 54 47		40 55	32	5 8 14		38 50	16	0
	Add e	S. A.		S.	Subt r.	S. A.		S.A.	

[Foot block: **8 Sig.**]

Parsing notes: At this sign block the eccentric prosthaph. column reaches its maximum (~6°38' near Gradus 3–4) and turns over to decrease, so the "A.S. Diff." column switches sense (the small values 0 19 → 0 1 → 0 8 around the turning point reflect this). Annual-orb prosthaph. likewise peaks near 5°39' (Gradus 6) and decreases. The "Excel. in Per." Ad. Dif. column reads "9 / 8 / 7 / 6 / 7 / 6 / 5 ..." near the top — the 6→7→6 wobble at Gradus 3–5 is transcribed as printed and flagged. At Gradus 22 the annual-orb "Excel. in Per." reads "40 30" where the descending neighbors are 41 38 (G.21) and 40 21 (G.23); this breaks the monotone descent and is flagged below.

4 Sig. (top) / 7 Sig. (foot)

Column headers (top-down, **4 Sig.**): Gradus | Subtra. Eccent. Prosthaph. (G. M. S.) | S. Diff. (M.S.) | Scrup. propo. (M. S.) | Ad. Dif. (S.) | Adde Orbis an. Prosthaph. (G. M. S.) | S. Diff. (M. S.) | Excel. Perig. (M. S.) | S. Dif. (S.) | Gradus

Gradus	Ecce nt. Pros tha. (G M S)	S. Diff. (M S)	Scru p. prop o. (M S)	Ad. Dif. (S)	Orbi s an. Pros th. (G M S)	S. Diff. (M S)	Exce l. Peri g. (M S)	S. Dif. (S)	Gradus
0	5 54 47		40 55		5 8 14		39 50		30
1	5 51 30	3 17	41 26	31	5 5 33	2 41	39 35	15	29
2	5 48 6	3 24	41 58	32	5 2 48	2 45	39 19	16	28
3	5 44 34	3 32	42 29	31	5 0 0	2 48	39 2	17	27
4	5 40 55	3 39	43 0	31	4 57 9	2 51	38 45	17	26
5	5 37 9	3 46	43 31	31	4 54 12	2 57	38 27	18	25
6	5 33 17	3 52	44 1	30	4 51 5	3 7	38 7	20	24
7	5 29 18	3 59	44 32	31	4 47 47	3 18	37 45	22	23
8	5 25 13	4 5	45 2	30	4 44 16	3 31	37 20	25	22
9	5 21 1	4 12	45 32	30	4 40 41	3 35	36 55	25	21
10	5 16 43	4 19	46 1	29	4 37 3	3 38	36 30	25	20
11	5 12 18	4 25	46 31	30	4 33 22	3 41	36 4	26	19
12	5 7 46	4 32	47 0	29	4 29 39	3 43	35 38	26	18
13	5 3 8	4 38	47 28	28	4 25 51	3 48	35 12	26	17
14	4 58 24	4 44	47 57	29	4 21 56	3 55	34 45	27	16

Gradus	Ecce nt. Pros tha. (G M S)	S. Diff. (M S)	Scru p. prop o. (M S)	Ad. Dif. (S)	Orbi s an. Pros th. (G M S)	S. Diff. (M S)	Exce l. Peri g. (M S)	S. Dif. (S)	Gradus
15	4 53 34	4 50	48 15	28	4 17 54	4 2	34 16	29	15
16	4 48 37	4 57	48 52	27	4 13 45	4 9	33 45	31	14
17	4 43 35	5 2	49 19	27	4 9 29	4 16	33 14	31	13
18	4 38 28	5 7	49 46	27	4 5 6	4 23	32 42	32	12
19	4 33 14	5 14	50 13	26	4 0 36	4 30	32 10	32	11
20	4 27 54	5 20	50 39	25	3 55 59	4 37	31 37	33	10
21	4 22 29	5 25	51 4	25	3 51 17	4 42	31 3	34	9
22	4 16 58	5 31	51 29	25	3 46 30	4 47	30 28	35	8
23	4 11 22	5 36	51 54	24	3 41 40	4 50	29 52	36	7
24	4 5 41	5 41	52 18	24	3 36 47	4 53	29 16	36	6
25	3 59 54	5 47	52 42	23	3 31 50	4 57	28 39	37	5
26	3 54 2	5 52	53 5	23	3 26 47	5 3	28 1	38	4
27	3 48 5	5 57	53 28	22	3 21 39	5 8	27 21	40	3
28	3 42 4	6 1	53 50	21	3 16 25	5 14	26 40	41	2
29	3 35 58	6 6	54 11	21	3 11 7	5 18	25 59	41	1

Grad us	Ecce nt. Pros tha. (G M S)				Orbi s an. Pros th. (G M S)				Grad us
	S. Diff. (M S)	Scru p. prop o. (M S)	Ad. Dif. (S)		S. Diff. (M S)	Exce l. Peri g. (M S)	S. Dif. (S)		
30	3 29 48	6 10	54 32	21	3 6 44	5 23 17	25 42		0
	Add e	A.		S.	Subt r.	A.	A.		

[Foot block: **7 Sig.**]

Parsing notes: clean rendering used. The two prosthaphaeresis columns now descend (eccentric 5 54 47 → 3 29 48; annual-orb 5 8 14 → 3 6 44), continuous with p. 367's foot values. The S. Diff. columns therefore run subtractive (header "S. Diff." / foot "A."). The "Excel. Perig." S. Dif. column climbs 15 → 42 as the excess descends from 39 50 to 25 17; smooth and monotone.

Pars Altera, Liber Secundus Theoricorum, Caput XII (continued)

This batch concludes the **Saturn** prosthaphaeresis table (sign blocks 5/6) and presents the opening four sign blocks of the **Jupiter** prosthaphaeresis table (blocks 0/11, 1/10, 2/9). Each table page carries two reciprocal sign blocks: the column headings at the top govern the top-down reading (the sign named at the top of the page), and the foot operation-row governs the bottom-up reading (the reciprocal sign named at the foot). Marginal operation labels (Subtrahe/Adde, Subtr./Adde, and the single-letter A./S. mode flags) are reproduced as printed.

Source p. 369 — Tabula Prosthaphæreseon Saturni (Saturn prosthaphaeresis table), sign block 5 (top) / 6 (foot)

Tabula Prosthaphæreseon Saturni — Table of the prosthaphaereses of Saturn.

Top heading: **5 Sig.** (Sign 5). Foot heading: **6 Sig.** (Sign 6).

Column structure (left block, read top-down for Sign 5):

- **Gradus** — degree argument

- **Eccentri. Prosthap.** under operation flag **Subtrahe** — eccentric prosthaphaeresis (the prosthaphaeresis of the epicycle)
- **Subtr. / Differ.** — its difference
- **Scrup. propo.** — scrupula proportionalia (proportional minutes)
- **Ad. Dif.** — difference of the proportional minutes
- (center) **Orbis An. Prosthap.** under operation flag **Adde** — prosthaphaeresis of the annual orb; **S. Diff.** — its difference
- **Excef. Perig.** — excess at perigee [Excessus Perigæi]; **S. Diff.** — its difference
- **Gradus** — degree argument (reciprocal count, read bottom-up for Sign 6)

Gradus (S.5)	Ecce ntr. Pros thap . (Sub trah e) G M S	Subt r. Diffe r. G M	Scru p. prop o. M S	Ad. Dif. S	Orbi s An. Pros thap . (Add e) G M S	S. Diff. M S	Exce f. Peri g. M S	S. Diff. M S	Gradus (S.6)
0	3 29 48	6 15	54 32	21	3 5 44	5 27 17	25 43		30
1	3 23 33	6 20	54 53	20	3 0 17	5 32 34	24 44		29
2	3 17 13	6 24	55 13	19	2 54 45	5 36 50	23 44		28
3	3 10 49	6 28	55 32	19	2 49 9	5 40	23 6 45		27
4	3 4 21	6 32	55 51	18	2 43 29	5 44 21	22 45		26
5	2 57 49	6 36	56 9	17	2 37 45	5 48 36	21 46		25
6	2 51 13	6 40	56 26	17	2 31 57	5 51 50	20 47		24
7	2 44 33	6 43	56 43	16	2 26 6	5 54	20 3 48		23
8	2 37	6 46	56	16	2 20	5 58	19 48		22

Grad us (S.5)	Ecce ntr. Pros thap . (Sub trah e) G M S	Subt r. Diffe r. G M	Scru p. prop o. M S	Ad. Dif. S	Orbi s An. Pros thap . (Add e) G M S	S. Diff. M S	Exce f. Peri g. M S	S. Diff. M S	Grad us (S.6)
	50		59		12		15		
9	2 31 4	6 50	57 15	15	2 14 14	6 2	18 27	49	21
10	2 24 14	6 53	57 30	15	2 8 12	6 5	17 38	49	20
11	2 17 21	6 56	57 45	14	2 2 7	6 8	16 49	50	19
12	2 10 25	6 59	57 59	13	1 55 59	6 11	15 59	50	18
13	2 3 26	7 1	58 12	12	1 49 48	6 13	15 9	51	17
14	1 56 25	7 3	58 24	12	1 43 35	6 16	14 18	51	16
15	1 49 22	7 5	58 36	11	1 37 19	6 18	13 27	52	15
16	1 42 17	7 8	58 47	10	1 31 1	6 20	12 35	52	14
17	1 35 9	7 11	58 57	9	1 24 41	6 22	11 43	53	13
18	1 27 58	7 14	59 6	9	1 18 19	6 25	10 50	53	12
19	1 20 44	7 16	59 15	8	1 11 54	6 27	9 57	53	11
20	1 13 28	7 17	59 23	7	1 5 27	6 28	9 4	54	10
21	1 6 11	7 18	59 30	6	0 58 59	6 30	8 10	54	9

Grad us (S.5)	Ecce ntr. Pros thap . (Sub trah e) G M S	Subt r. Diffe r. G M	Scru p. prop o. M S	Ad. Dif. S	Orbi s An. Pros thap . (Add e) G M S	S. Diff. M S	Exce f. Peri g. M S	S. Diff. M S	Grad us (S.6)
22	0 58 53	7 19	59 36	6	0 52 29	6 31	7 16	54	8
23	0 51 34	7 19	59 42	5	0 45 58	6 32	6 22	54	7
24	0 44 15	7 20	59 47	4	0 39 26	6 33	5 28	54	6
25	0 36 55	7 21	59 51	4	0 32 53	6 33	4 34	54	5
26	0 29 34	7 23	59 55	3	0 26 20	6 34	3 40	55	4
27	0 22 11	7 24	59 58	3	0 19 46	6 35	2 45	55	3
28	0 14 47	7 24	59 59	1	0 13 11	6 35	1 50	55	2
29	0 7 23	7 23	60 0	0	0 6 36	6 36	0 55	55	1
30	0 0 0	—	60 0	—	0 0 0	—	0 0	—	0

Foot operation-row (governs bottom-up reading for Sign 6): **Adde** | **A.** (eccentric prosthaphaeresis column) | **S.** (proportional-minutes difference column) | **Subtr.** | **A.** (annual-orb and excess columns).

Catchword (verso): **Tabula**

Parsing notes: Reciprocal layout confirmed — Eccentr. Prosthap. is *Subtrahe* reading down (Sign 5) and *Adde* reading up (Sign 6); Orbis An. Prosthap. is *Adde* down and *Subtr.* up. The "Differ." column entries are inter-row differences keyed between rows; transcribed against the printed value rows. Row 23 Eccentr. Differ.

printed "7 19" (repeating row 22's 7 19) — transcribed as printed (smooth progression, not flagged).

Source p. 370 — *Tabula prosthaphæreseon Iovis* (Jupiter prosthaphaeresis table), sign block 0 (top) / 11 (foot)

Tabula prosthaphæreseon Iovis — Table of the prosthaphaereses of Jupiter.

Top heading: **0 Sig.** (Sign 0). Foot heading: **11 Sig.** (Sign 11).

Column structure (left block, top-down for Sign 0):

- **Gradus**
- **Eccentr. Prosth.** under flag **Subtrah.** — eccentric prosthaphaeresis
- **A. Diff.** — difference (mode flag A)
- **Scru. prop.** — proportional minutes
- **Ad. Dif.** — its difference
- (center) **Orbis An. Prosthap.** under flag **Adde** — prosthaphaeresis of the annual orb; **Diff.**
- **Excef. Perig.** — excess at perigee; **Diff.**
- **Gradus** (reciprocal, bottom-up for Sign 11)

Gradus (S.0)	Ecce ntr. Prosth. (Subtrah.) G M S	A. Diff. M S	Scru. prop . M S	Ad. Dif. S	Orbis An. Prosthap. (Add) e) G M S	Diff. M S	Excef. Peri g. M S	Diff. S	Gradus (S.11)
0	0 0 0	5 28	0 0	0	0 0 0	9 22	0 0	46	30
1	0 5 28	5 28	0 1	1	0 9 22	9 22	0 46	46	29
2	0 10 56	5 28	0 2	1	0 18 44	9 22	1 32	45	28
3	0 16 24	5 27	0 3	1	0 28 6	9 22	2 17	45	27
4	0 21 51	5 27	0 4	2	0 37 28	9 22	3 2	46	26
5	0 27	5 26	0 6	2	0 46	9 21	3 48	45	25

Grad us (S.0)	Ecce ntr. Pros tha. (Sub trah.) G M S	A. Diff. M S	Scru. prop . M S	Ad. Dif. S	Orbi s An. Pros thap . (Add e) G M S	Diff. M S	Exce f. Peri g. M S	Diff. S	Grad us (S.11)
	18				49				
6	0 32 44	5 26	0 8	3	0 56 10	9 21	4 34	46	24
7	0 38 10	5 26	0 11	3	1 5 31	9 21	5 20	46	23
8	0 43 36	5 25	0 14	4	1 14 51	9 20	6 6	46	22
9	0 49 1	5 24	0 18	4	1 24 10	9 19	6 52	46	21
10	0 54 26	5 23	0 22	4	1 33 28	9 18	7 38	46	20
11	0 59 49	5 22	0 26	4	1 42 45	9 17	8 24	46	19
12	1 5 12	5 21	0 30	5	1 52 0	9 15	9 10	46	18
13	1 10 33	5 20	0 35	6	2 1 14	9 14	9 56	46	17
14	1 15 53	5 19	0 41	6	2 10 27	9 13	10 41	45	16
15	1 21 12	5 18	0 47	6	2 19 39	9 12	11 26	45	15
16	1 26 30	5 16	0 53	7	2 28 50	9 11	12 12	46	14
17	1 31 46	5 15	1 0	7	2 37 59	9 9	12 58	46	13
18	1 37 1	5 13	1 7	8	2 47 6	9 7	13 43	45	12
19	1 42	5 11	1 15	8	2 56	9 5	14	45	11

Grad us (S.0)	Ecce ntr. Pros tha. (Sub trah.) G M S	A. Diff. M S	Scru. prop . M S	Ad. Dif. S	Orbi s An. Pros thap . (Add e) G M S	Diff. M S	Exce f. Peri g. M S	Diff. S	Grad us (S.11)
	14				11		28		
20	1 47 25	5 10	1 23	9	3 5 14	9 3	15 13	45	10
21	1 52 35	5 8	1 32	9	3 14 15	9 1	15 58	45	9
22	1 57 43	5 6	1 41	9	3 23 14	8 59	16 43	45	8
23	2 2 49	5 3	1 50	10	3 32 11	8 57	17 28	45	7
24	2 7 52	5 1	2 0	10	3 41 6	8 55	18 13	45	6
25	2 12 53	4 59	2 10	10	3 49 58	8 52	18 58	45	5
26	2 17 52	4 57	2 20	11	3 58 48	8 50	19 43	45	4
27	2 22 49	4 55	2 31	11	4 7 35	8 47	20 28	45	3
28	2 27 44	4 53	2 42	12	4 16 19	8 44	21 13	45	2
29	2 32 37	4 52	2 54	12	4 25 0	8 41	21 58	44	1
30	2 37 29	—	3 6	—	4 33 38	8 38	22 42	—	0

Foot operation-row (governs bottom-up reading for Sign 11): **Adde** | **S.** (Scru. prop. difference) | (center) **Subtr.** | **S.** | **S.**

Catchword (verso): **Tabula**

Parsing notes: Row 4 Excef. Perig. printed "2 48" — see Numerical impossibles below (breaks the smooth ~0:46/step progression; rows 3→4→5 read 2 17 / 2 48 / 3 48, where a regular step would give ~3 02). Transcribed as printed and flagged. The Excef. Perig. Diff. column on this block is the running ~45–46 second step.

Source p. 371 — Tabula prosthaphæreseon Iovis (Jupiter prosthaphaeresis table), sign block 1 (top) / 10 (foot)

Tabula prosthaphæreseon Iovis — Table of the prosthaphaereses of Jupiter.

Top heading: **1 Sig.** (Sign 1). Foot heading: **10 Sig.** (Sign 10).

Margin note above the Gradus column on the degraded scan reads "Gradus 1q" (a stray ink/margin mark over the running header); the clean rendering shows the normal **Gradus** column head. Column structure as on p. 370:

- **Gradus**
- **Eccentri. proftha.** under **Subtrah.** — eccentric prosthaphaeresis; **Add. Dif.**
- **Scrup. propo.;** **Ad. Dif.**
- (center) **Orbis An. profthap.** under **Adde** — annual-orb prosthaphaeresis; **Add. Dif.**
- **Excef. in Per.** — excess at perigee; **Ad. Dif.**
- **Gradus** (reciprocal, bottom-up for Sign 10)

Gradus (S.1)	Ecce ntri. proft ha. (Sub trah.) G				Orbi s An. proft hap. (Add e) G				Gradus (S.10)
	Add. Diff. M S	Scru p. prop o. M S	Ad. Dif. S		Add. Diff. M S	Exce f. in Per. M S	Ad. Dif. S		
0	2 37 29	4 48	3 6	12	4 33 38	8 35 42	22	44	30
1	2 42 17	4 45	3 18	13	4 42 13	8 33 26	23	45	29
2	2 47 2	4 42	3 31	13	4 50 46	8 30 11	24	44	28
3	2 51	4 38	3 44	14	4 59	8 28	24	44	27

Gradus (S.1)	Ecce ntri. proft ha. (Sub trah.) G M S	Add. Diff. M S	Scru p. prop o. M S	Ad. Dif. S	Orbi s An. proft hap. (Add e) G M S	Add. Diff. M S	Exce f. in Per. M S	Ad. Dif. S	Grad us (S.10)
	44				16		55		
4	2 56 22	4 36	3 58	14	5 7 44	8 25	25 39	44	26
5	3 0 58	4 33	4 12	15	5 16 9	8 21	26 23	44	25
6	3 5 31	4 31	4 27	15	5 24 30	8 16	27 7	44	24
7	3 10 2	4 28	4 42	15	5 32 46	8 11	27 51	44	23
8	3 14 30	4 25	4 57	15	5 40 57	8 6	28 35	44	22
9	3 18 55	4 21	5 12	16	5 49 3	8 1	29 19	43	21
10	3 23 16	4 17	5 28	16	5 57 4	7 57	30 2	43	20
11	3 27 33	4 13	5 45	16	6 5 1	7 52	30 45	43	19
12	3 31 46	4 10	6 1	17	6 12 53	7 47	31 28	43	18
13	3 35 56	4 7	6 18	17	6 20 40	7 41	32 11	43	17
14	3 40 3	4 3	6 36	18	6 28 21	7 38	32 54	42	16
15	3 44 6	4 0	6 54	18	6 35 59	7 35	33 36	42	15
16	3 48 6	3 56	7 12	18	6 43 37	7 32	34 18	42	14
17	3 52	3 52	7 31	19	6 51	7 28	35 0	42	13

Grad us (S.1)	Ecce ntri. proft ha. (Sub trah.) G M S	Add. Diff. M S	Scru p. prop o. M S	Ad. Dif. S	Orbi s An. proft hap. (Add e) G M S	Add. Diff. M S	Exce f. in Per. M S	Ad. Dif. S	Grad us (S.10)
	2				9				
18	3 55 54	3 48	7 50	19	6 58 37	7 22	35 42	42	12
19	3 59 42	3 44	8 9	19	7 5 59	7 15	36 24	41	11
20	4 3 26	3 40	8 28	19	7 13 14	7 9	37 5	41	10
21	4 7 6	3 36	8 48	20	7 20 23	7 2	37 46	41	9
22	4 10 42	3 31	9 9	21	7 27 25	6 58	38 27	41	8
23	4 14 13	3 27	9 30	21	7 34 23	6 53	39 8	41	7
24	4 17 40	3 22	9 51	22	7 41 16	6 48	39 49	40	6
25	4 21 2	3 18	10 13	22	7 48 4	6 42	40 29	40	5
26	4 24 20	3 14	10 35	22	7 54 46	6 35	41 9	40	4
27	4 27 34	3 10	10 57	22	8 1 21	6 29	41 49	40	3
28	4 30 44	3 6	11 19	23	8 7 50	6 23	42 29	40	2
29	4 33 50	3 1	11 42	23	8 14 13	6 17	43 9	40	1
30	4 36 51	—	12 5	—	8 20 30	—	43 48	39	0

Foot operation-row (governs bottom-up reading for Sign 10): **Addē** | **S.** | (center)
Subtr. | **S.** | **S.**

Catchword (verso): **Tabula**

Parsing notes: Eccentr. and Orbis An. prosthaphaeresis values continue smoothly from p. 370 (Eccentr. 2 37 29 → 4 36 51; Orbis An. 4 33 38 → 8 20 30; Excef. in Per. 22 42 → 43 48). Block-boundary values match the foot of the prior page, confirming the chain.

Source p. 372 — **Tabula prosthaphæreseon Iovis** (Jupiter prosthaphaeresis table), sign block 2 (top) / 9 (foot)

Tabula prosthaphæreseon Iovis — Table of the prosthaphaereses of Jupiter.

Top heading: **2 Sig.** (Sign 2). Foot heading: **9 Sig.** (Sign 9).

Column structure as on p. 371:

- **Gradus**
- **Eccentri. proftha.** under **Subtrah.** — eccentric prosthaphaeresis; **A. Diff.**
- **Scrup. propo.;** **Dif.**
- (center) **Orbis An. profthap.** under **Addē** — annual-orb prosthaphaeresis; **Add. Diff.**
- **Excef. in Per.** — excess at perigee; **Dif.**
- **Gradus** (reciprocal, bottom-up for Sign 9)

Gradus (S.2)	Eccentri. proftha. (Subtrah.)		Scrup. propo.		Orbis An. profthap. (Addē)		Excef. in Per.		Gradus (S.9)
	G M S	A. Diff. M S	o. M S	Dif. S	e) G M S	Add. Diff. M S	G M S	Dif. S	
0	4 36 51	2 56	12 5	23	8 20 30	6 11	43 48	39	30
1	4 39 47	2 50	12 28	24	8 26 43	6 3	44 27	39	29
2	4 42 37	2 46	12 52	24	8 32 44	5 56	45 6	38	28

Grad us (S.2)	Ecce ntri. proft ha. (Sub trah.) G M S	A. Diff. M S	Scru p. prop o. M S	Dif. S	Orbi s An. proft hap. (Add e) G M S	Add. Diff. M S	Exce f. in Per. G M S	Dif. S	Grad us (S.9)
3	4 45 23	2 40	13 16	25	8 38 40	5 49	45 44	38	27
4	4 48 3	2 36	13 41	25	8 44 29	5 43	46 22	37	26
5	4 50 39	2 32	14 6	25	8 50 12	5 35	46 59	37	25
6	4 53 11	2 26	14 31	26	8 55 47	5 28	47 36	37	24
7	4 55 37	2 21	14 57	26	9 1 15	5 21	48 13	37	23
8	4 57 58	2 16	15 23	26	9 6 36	5 13	48 49	36	22
9	5 0 14	2 11	15 49	27	9 11 49	5 5	49 25	35	21
10	5 2 25	2 6	16 16	26	9 16 54	4 56	50 0	35	20
11	5 4 31	2 0	16 42	27	9 21 50	4 47	50 34	35	19
12	5 6 31	1 55	17 9	27	9 26 37	4 40	51 8	34	18
13	5 8 26	1 50	17 36	27	9 31 17	4 33	51 42	34	17
14	5 10 16	1 44	18 3	28	9 35 50	4 25	52 16	34	16
15	5 12 0	1 38	18 31	28	9 40 15	4 18	52 49	33	15
16	5 13 38	1 33	18 59	28	9 44 33	4 8	53 22	33	14

Grad us (S.2)	Ecce ntri. proft ha. (Sub trah.) G M S	A. Diff. M S	Scru p. prop o. M S	Dif. S	Orbi s An. proft hap. (Add e) G M S	Add. Diff. M S	Exce f. in Per. G M S	Dif. S	Grad us (S.9)
17	5 15 11	1 27	19 27	29	9 48 41	4 8	53 54	32	13
18	5 16 38	1 22	19 56	29	9 52 39	3 58	54 26	32	12
19	5 18 0	1 16	20 25	29	9 56 26	3 47	54 57	32	11
20	5 19 16	1 11	20 54	29	10 0 1	3 35	55 27	31	10
21	5 20 27	1 5	21 23	29	10 3 29	3 28	55 57	30	9
22	5 21 32	1 0	21 52	30	10 6 51	3 22	56 26	29	8
23	5 22 31	0 59	22 22	30	10 10 7	3 16	56 54	28	7
24	5 23 24	0 53	22 52	30	10 13 17	3 10	57 22	28	6
25	5 24 12	0 48	23 22	30	10 16 15	2 58	57 49	27	5
26	5 24 54	0 42	23 52	30	10 19 1	2 46	58 15	26	4
27	5 25 30	0 36	24 22	31	10 21 33	2 32	58 41	26	3
28	5 26 0	0 30	24 53	31	10 23 50	2 17	59 6	25	2

Grad us (S.2)	Ecce ntri. proft ha. (Sub trah.) G M S	A. Diff. M S	Scru p. prop o. M S	Dif. S	Orbi s An. proft hap. (Add e) G M S	Add. Diff. M S	Exce f. in Per. G M S	Dif. S	Grad us (S.9)
29	5 26 25	0 25	25 24	31	10 25 58	2 8	59 30	24	1
30	5 26 44	0 19	25 55	—	10 28 0	2 2	59 54	24	0

Foot operation-row (governs bottom-up reading for Sign 9): **Adde** | **S.** | (center)
Subtr. | **S.** | **S.**

Catchword (verso): **Tabula**

Parsing notes: Eccentr. and Orbis An. chains continue smoothly from p. 371 (Eccentr. 4 36 51 → 5 26 44; Orbis An. 8 20 30 → 10 28 0; Excef. in Per. 43 48 → 59 54). Row 17 Orbis An. Add. Diff. printed "4 8" (a repeat of row 16's "4 8") interrupts the otherwise monotone descent (row 16 = 4 08, row 18 = 3 58); transcribed as printed and flagged [?] — likely a compositor repeat for an expected ~4 03. Row 13 Excef. in Per. printed "51 42" is legible but the seconds glyph was faint; transcribed as printed with [?] on the cell.

PARS ALTERA, LIBER SECVNDVS THEORICORVM, Caput XII (continued) — Jupiter prosthaphaeresis table (sign blocks 3, 4, 5) finishing; then the Mars table of the prosthaphaeresis of the eccentric, with distance from the centre, opens (sign blocks 0/11, 1/10).

Cap. 12. LIBER SECVNDVS.

Table of the prosthaphaereses of Jupiter [Tabula Prosthaphæreseon Iovis].

3 Sig. (sign block 3; read with **8 Sig.** at the foot)

The two leftmost composite columns belong to the *Subtrahe* (Subtract) group; the right-hand composite columns belong to the *Adde* (Add) group, as marked.

Column headings: Degrees [Gradus] | eccentric prosthaphaeresis [Eccentri Prosthap.] | Difference [Differ.] | minutes of proportion [Scrup. propo.] | Add. Diff. [Ad. Dif.] | prosthaphaeresis of the annual orb [Orbis An. Prosthap.] | A. Diff. | excess at perigee [Excessus in perig.] | Diff. | Degrees [Gradus].

G.	Ecce ntr. Pros thap . (G M S)	Diffe r. (M S)	Scru p. prop o. (M S)	Ad. Dif. (S)	Orbi s An. Pros thap . (G M S)	A. Diff. (M S)	Exce ssus in peri g. (G M S)	Diff. (M S)	G.
0	5 26 44	0 11	25 55	31	10 28 0	1 26	0 59 54	23	30
1	5 26 55	0 3	26 26	31	10 29 56	1 42	1 0 17	22	29
2	5 26 58	0 1	26 57	31	10 31 38	1 30	1 0 39	21	28
3	5 26 59	0 0	27 28	31	10 33 8	1 19	1 1 0	20	27
4	5 26 59	0 3	27 59	32	10 34 27	1 8	1 1 20	19	26
5	5 26 56	0 16	28 31	31	10 35 35	0 58	1 1 39	18	25
6	5 26 40	0 26	29 2	32	10 36 33	0 48	1 1 57	17	24
7	5 26 14	0 36	29 34	32	10 37 21	0 36	1 2 14	16	23
8	5 25 38	0 39	30 6	32	10 37 57	0 25	1 2 30	15	22
9	5 24	0 43	30	32	10	0 11	1 2	15	21

G.	Ecce ntr. Pros thap . (G M S)	Diffe r. (M S)	Scru p. prop o. (M S)	Ad. Dif. (S)	Orbi s An. Pros thap . (G M S)	A. Diff. (M S)	Exce ssus in peri g. (G M S)	Diff. (M S)	G.
	59		38		37 22		45		
10	5 24 16	0 46	31 10	32	10 38 33	0 0	1 3 0	14	20
11	5 23 30	0 50	31 42	32	10 38 33	0 12	1 3 14	13	19
12	5 22 40	0 56	32 14	32	10 38 21	0 24	1 3 27	12	18
13	5 21 44	1 4	32 46	32	10 37 57	0 36	1 3 39	10	17
14	5 20 40	1 13	33 19	32	10 37 21	0 48	1 3 49	9	16
15	5 19 27	1 20	33 51	32	10 36 33	1 1	1 3 58	9	15
16	5 18 7	1 25	34 23	32	10 35 32	1 12	1 4 7	8	14
17	5 16 42	1 30	34 55	32	10 34 20	1 25	1 4 15	7	13
18	5 15 12	1 34	35 27	32	10 32 55	1 38	1 4 22	5	12
19	5 13 38	1 39	35 59	32	10 31	1 50	1 4	2	11

G.	Ecce ntr. Pros thap . (G M S)	Diffe r. (M S)	Scru p. prop o. (M S)	Ad. Dif. (S)	Orbi s An. Pros thap . (G M S)	A. Diff. (M S)	Exce ssus in peri g. (G M S)	Diff. (M S)	G.
					17		27		
20	5 11 59	1 45	36 31	32	10 29	2 4	1 4 29	0	10
					27				
21	5 10 14	1 53	37 3	32	10 27	2 16	1 4 29	1	9
					23				
22	5 8 21	2 0	37 35	32	10 25 7	2 29	1 4 28	2	8
23	5 6 21	2 6	38 7	32	10 22	2 42	1 4 26	3	7
					38				
24	5 4 15	2 11	38 39	31	10 19	2 55	1 4 23	4	6
					56				
25	5 2 4	2 16	39 10	32	10 17 1	3 9	1 4 19	5	5
26	4 59 48	2 22	39 42	31	10 13	3 21	1 4 14	7	4
					52				
27	4 57 26	2 28	40 13	31	10 10	3 35	1 4 7	9	3
					31				
28	4 54 58	2 34	40 44	31	10 6 56	3 48	1 3 58	11	2
29	4 52 24	2 39	41 15	31	10 3 8	4 3	1 3 47	12	1
30	4 49 45		41 46	31	10 59 5		1 3 45		0

Foot operation-row labels: **Adde** | (S.) | (S.) | | **Subtr.** | (S.) | | (S.)

8 Sig. (the same table read bottom-up for sign block 8)

THEORICORVM Cap. 12.

Table of the prosthaphaereses of Jupiter [Tabula prosthaphæreseon Iovis].

4 Sig. (sign block 4; read with **7 Sig.** at the foot)

Column headings as on the preceding page: Degrees [Gradus] | eccentric prosthaphaeresis [eccentr. prosth., *Subtrahe*] | A. Diff. | minutes of proportion [Scrup. prop.] | Add. Dif. | prosthaphaeresis of the annual orb [Orbis An. Prosthap., *Adde*] | Add. Diff. | excess at perigee [Excess. in perig.] | A. Diff. | Degrees [Gradus].

G.	Ecce ntr. pros tha. (G M S)	A. Diff. (M S)	Scru p. prop (M S)	Ad. Dif. (S)	Orbi s An. Pros thap (G M S)	Add. Diff. (M S)	Exce ss. in peri g. (G M S)	A. Diff. (M S)	G.
0	4 49 45	2 44	41 46	31	9 59 5	4 15	1 3 35	13	30
1	4 47 1	2 50	42 17	31	9 54 50	4 28	1 3 22	15	29
2	4 44 11	2 57	42 48	30	9 50 22	4 42	1 3 7	17	28
3	4 41 14	3 2	43 18	30	9 45 40	4 56	1 2 50	19	27
4	4 38 12	3 7	43 48	30	9 40 44	5 9	1 2 31	20	26
5	4 35 5	3 13	44 18	30	9 35 35	5 22	1 2 11	22	25
6	4 31 52	3 18	44 48	29	9 30 13	5 36	1 1 49	23	24
7	4 28 34	3 24	45 17	30	9 14 37	5 50	1 1 26	24	23
8	4 25 10	3 29	45 47	30	9 18 47	6 3	1 1 2	26	22

G.	Ecce ntr. pros tha. (G M S)	A. Diff. (M S)	Scru p. prop . (M S)	Ad. Dif. (S)	Orbi s An. Pros thap . (G M S)	Add. Diff. (M S)	Exce ss. in peri g. (G M S)	A. Diff. (M S)	G.
9	4 21 41	3 34	46 16	29	9 12 44	6 17	1 0 36	28	21
10	4 18 7	3 38	46 44	28	9 6 27	6 30	1 0 8	30	20
11	4 14 29	3 43	47 12	28	8 59 57	6 43	0 59 38	33	19
12	4 10 46	3 48	47 40	28	8 53 14	6 57	0 59 5	34	18
13	4 6 58	3 54	48 8	28	8 46 17	7 10	0 58 31	36	17
14	4 3 4	3 59	48 35	27	8 39 7	7 24	0 57 55	38	16
15	3 59 5	4 5	49 2	27	8 31 43	7 36	0 57 17	40	15
16	3 55 0	4 9	49 28	26	8 24 7	7 49	0 56 37	42	14
17	3 50 51	4 14	49 55	27	8 16 18	8 3	0 55 55	43	13
18	3 46 37	4 17	50 21	26	8 8 15	8 16	0 55 12	45	12
19	3 42 20	4 21	50 46	25	7 59 59	8 29	0 54 27	47	11
20	3 37 59	4 26	51 11	25	7 51 30	8 40	0 53 40	49	10
21	3 33 33	4 31	51 35	24	7 42 50	8 54	0 52 51	52	9
22	3 29 2	4 36	51 59	24	7 33 56	9 6	0 51 59	54	8
23	3 24 26	4 40	52 22	23	7 24 50	9 19	0 50 5	56	7

G.	Ecce ntr. pros tha. (G M S)	A. Diff. (M S)	Scru p. prop . (M S)	Ad. Dif. (S)	Orbi s An. Pros thap . (G M S)	Add. Diff. (M S)	Exce ss. in peri g. (G M S)	A. Diff. (M S)	G.
24	3 19 46	4 44	52 45	23	7 15 31	9 30	0 50 9	57	6
25	3 15 2	4 48	53 8	23	7 6 1	9 43	0 49 12	59	5
26	3 10 14	4 52	53 30	22	6 56 18	9 54	0 48 13	1 1	4
27	3 5 22	4 55	53 52	21	6 46 24	10 6	0 47 12	1 3	3
28	3 0 27	4 58	54 13	21	6 36 18	10 17	0 46 9	1 4	2
29	2 55 29	5 2	54 34	20	6 26 1	10 28	0 45 5	1 5	1
30	2 50 27		54 54		6 15 33		0 44 0		0

Foot operation-row labels: **Adde** | (S.) | (S.) | | **Subtr.** | (S.) | | (S.)

7 Sig. (the same table read bottom-up for sign block 7)

Cap. 12. LIBER SECVNDVS.

Table of the prosthaphaereses of Jupiter [Tabula prosthaphæreseon Iovis].

5 Sig. (sign block 5; read with **6 Sig.** at the foot)

Column headings: Degrees [Gradus] | eccentric prosthaphaeresis [Eccentri prosth., *Subtrah.*] | A. Diff. | minutes of proportion [Scrup. prop.] | Ad. Diff. | prosthaphaeresis of the annual orb [Orbis An. Prosth., *Adde*] | A. Diff. | excess at perigee [Excessus in Perig.] | A. Diff. | Degrees [Gradus].

G.	Ecce ntri pros tha. (G M S)	A. Diff. (M S)	Scru p. prop . (M S)	Ad. Diff. (S)	Orbi s An. Pros tha. (G M S)	A. Diff. (M S)	Exce ssus in Peri g. (G M S)	A. Diff. (M S)	G.
0	2 50 27	5 6	54 54	19	6 15 33	10 39	0 44 0	1 8	30
1	2 45 21	5 10	55 13	19	6 4 54	10 50	0 42 52	1 10	29
2	2 40 11	5 13	55 32	18	5 54 4	10 59	0 41 42	1 12	28
3	2 34 58	5 16	55 50	18	5 43 5	11 10	0 40 30	1 14	27
4	2 39 43	5 17	56 8	17	5 31 55	11 21	0 39 16	1 15	26
5	2 24 26	5 19	56 25	17	5 20 34	11 30	0 38 1	1 17	25
6	2 19 7	5 24	56 42	16	5 9 4	11 38	0 36 44	1 18	24
7	2 13 43	5 29	56 58	15	4 57 26	11 47	0 16 26	1 20	23
8	2 8 14	5 34	57 13	15	4 45 39	11 57	0 34 6	1 21	22
9	2 2 40	5 37	57 28	14	4 33 42	12 8	0 32 45	1 23	21
10	1 57 3	5 38	57 42	14	4 21 34	12 14	0 31 22	1 24	20
11	1 51 25	5 39	57 56	13	4 9 20	12 20	0 29 58	1 26	19
12	1 45 46	5 40	58 9	12	3 57 0	12 27	0 28 32	1 28	18
13	1 40 6	5 40	58 21	11	3 44 33	12 34	0 27 4	1 29	17
14	1 34 26	5 42	58 32	10	3 31 59	12 41	0 25 35	1 30	16

G.	Ecce ntri pros tha. (G M S)	A. Diff. (M S)	Scru p. prop . (M S)	Ad. Diff. (S)	Orbi s An. Pros tha. (G M S)	A. Diff. (M S)	Exce ssus in Peri g. (G M S)	A. Diff. (M S)	G.
15	1 28 44	5 44	58 42	10	3 19 18	12 48	0 24 5	1 31	15
16	1 23 0	5 46	58 52	9	3 6 30	12 55	0 22 34	1 32	14
17	1 17 14	5 48	59 1	9	2 53 35	13 1	0 21 2	1 33	13
18	1 11 26	5 51	59 10	8	2 40 34	13 5	0 19 29	1 34	12
19	1 5 35	5 54	59 18	8	2 27 29	13 10	0 17 55	1 35	11
20	0 59 41	5 56	59 26	7	2 14 19	13 14	0 16 20	1 36	10
21	0 53 45	5 56	59 33	7	2 1 5	13 17	0 14 44	1 36	9
22	0 47 49	5 57	59 40	6	1 47 48	13 20	0 13 8 [?]	1 37	8
23	0 41 52	5 57	59 46	5	1 34 28	13 23	0 11 31	1 37	7
24	0 35 55	5 57	59 51	4	1 21 5	13 27	0 9 54	1 38	6
25	0 29 57	5 58	59 55	3	1 7 38	13 30	0 8 16	1 38	5
26	0 22 59 [?]	5 58	59 58	2	0 54 8	13 31	0 6 38	1 39	4
27	0 18 0	5 59	60 0	0	0 40 37	13 32	0 4 59	1 39	3
28	0 12 1	5 59	60 0	0	0 27 5	13 32	0 3 20	1 40	2
29	0 6 1	6 0	60 0	0	0 13	13	0 1	1 40	1

G.	Ecce ntri pros tha. (G M S)		Scru p. prop (M S)		Ad. Diff. (S)		Orbi s An. Pros tha. (G M S)		Exce ssus in Peri g. (G M S)		G.
	A. Diff. (M S)						A. Diff. (M S)		A. Diff. (M S)		
						33	33	40			
30	0 0 0	6 1	60 0	0		0 0 0	13	0 0 0	1 40	0	
							33				

Foot operation-row label (left composite): **Adde** | (S.) | (S.) | | (S.) | (S.) | | (S.)

6 Sig. (the same table read bottom-up for sign block 6)

THEORICORVM Cap. 12.

Table of the prosthaphaeresis of the eccentric of Mars, with the distance from the centre [Tabula prosthaph. eccentric. Martis cum distantia à centro].

0 Sig. (sign block 0; read with **11 Sig.** at the foot — marked between the two halves of the page) and **1 Sig.** (sign block 1; read with **10 Sig.** at the foot).

This page presents two adjacent sign-block panels side by side. Each panel has columns: Degrees [Gradus] | eccentric prosthaphaeresis [Eccentr. Prosthaph., *Subtrah.*] | Add. Diff. [Ad. Diff.] | distance from the centre, in parts [Distāt. à centr., Partic.] | Add. Diff. [Ad. Dif.] | Degrees [Gradus]. The distances from the centre are tabulated in units (parts) of a scale where the mean distance ≈ 100000 ; values here run from ~ 111130 down through ~ 106775 .

Left panel — 0 Sig.

G.	Eccentr. Prosthaph. (G M S)		Ad. Diff. (M S)		Distāt. à centr. (Partic.)		Ad. Dif. (S)		G.
0	0 0 0		0 1 [?]		111130		1		30
1	0 10 1		10 1		111129		3		29
2	0 20 2		10 1		111126		7		28
3	0 30 2		10 0		111119		11		27
4	0 40 2		10 0		111108		17		26

G.	Eccentr. Prostha. (G M S)	Ad. Diff. (M S)	Distāt. à centr. (Partic.)	Ad. Dif. (S)	G.
5	0 50 1	9 59	111091	17 [?]	25
6	1 0 0	9 59	111672	19	24
7	1 9 59	9 59	111652	20	23
8	1 19 58	9 58	111631	21	22
9	1 29 56	9 56	111009	22	21
10	1 39 52	9 56 [?]	110986	23	20
11	1 49 46	9 54	110963	23	19
12	1 59 37	9 51	110939	24	18
13	2 9 25	9 48	110914	25	17
14	2 19 11	9 46	110888	26	16
15	2 28 55	9 44	110860	28	15
16	2 38 37	9 42	110824	36 [?]	14
17	2 48 16	9 39	110781	43	13
18	2 57 52	9 36	110732	49	12
19	3 7 26	9 33	110678	54	11
20	3 16 55	9 30	110621	57	10
21	3 26 22	9 27	110563	58	9
22	3 35 49	9 26	110504	59	8
23	3 45 13	9 24	110445	59	7
24	3 54 35	9 22	110385	60	6
25	4 3 56	9 21	110324	61	5
26	4 13 15	9 19	110263	61	4
27	4 22 31	9 16	110201	62	3
28	4 31 45	9 14	110139	62	2
29	4 40 57	9 12	110076	63	1
30	4 50 7	9 10	110013	63	0

Foot operation-row labels (left panel): **Adde** | | (S.) | (S.) |

11 Sig. (left panel read bottom-up).

Right panel — 1 Sig.

G.	Eccentr. prosth. (G M S)	Ad. Diff. (M S)	Distāt. à cent. (Partic.)	Ad. Dif. (S)	G.
0	4 50 7	9 6	110013	65	30
1	4 59 11	9 0	109948	70	29
2	5 8 14	8 54	109878	74	28
3	5 17 5	8 48	109804	77	27
4	5 25 53	8 42	109727	81	26
5	5 34 35	8 35	109646	85	25
6	5 43 11	8 30	109561	89	24
7	5 51 41	8 24	109472	92	23
8	6 0 5	8 18	109380	94	22
9	6 8 23	8 13	109286	96	21
10	6 16 36	8 7	109190	97	20
11	6 24 43	8 1	109093	100	19
12	6 32 44	7 55	108993	103	18
13	6 40 39	7 48	108890	105	17
14	6 48 27	7 43	108785	107	16
15	6 56 10	7 37	108678	111	15
16	7 3 47	7 30	108567	111 [?]	14
17	7 11 17	7 23	108453	114	13
18	7 18 40	7 17	108336	117	12
19	7 25 57	7 11	108267	119	11
20	7 33 18	7 5	108096	121	10
21	7 40 13	6 59	107973	123	9
22	7 47 12	6 53	107848	125	8
23	7 54 5	6 47	107720	128	7
24	8 0 52	6 41	107590	130	6
25	8 7 33	6 35	107459	131	5
26	8 14 8	6 29	107326	133	4
27	8 20 37	6 23	107191	135	3
28	8 27 0	6 15	107054	137	2

G.	Eccentr. prosth. (G M S)	Ad. Diff. (M S)	Distāt. à cent. (Partic.)	Ad. Dif. (S)	G.
29	8 33 15	6 6	106915	139	1
30	8 39 21		106775	140	0

Foot operation-row labels (right panel): **(S.)** | (S.) | | (S.)

10 Sig. (right panel read bottom-up).

PART TWO, BOOK TWO OF THE THEORICS — Caput XII (continued)

Mars eccentric-prosthaphaeresis table with distance-from-center (continued)

Table of the eccentric prosthaphaeresis of Mars with the distance from the center. [Tabula Prosth. Eccentr. Martis cum distantia à Centro.]

The table is presented in reciprocal sign-block panels: each page carries two side-by-side panels, with one sign-block read top-down from the head and its reciprocal read bottom-up from the foot. The "Subtract / Add" (Subtrahe / Adde) operation labels at head and foot govern the eccentric prosthaphaeresis column. Columns per panel: Gradus | Eccentric prosthaphaeresis (G. M. S.) | Diff. | Distance from center (parts) | Diff.

Source PDF p. 393 (printed p. 377) — panels "2 Sig." and "3 Sig."

Panel "2 Sig." (top: Subtract — Eccentric prosthaphaeresis; foot reciprocal "9 Sig.")

Gradus	Eccentric prosthaph. (G. M. S.)	Diff.	Distance from center (parts)	Diff.
0	8 39 22	5 57	106775	142
1	8 45 18	5 47	106633	144
2	8 51 5	5 38	106489	146
3	8 56 43	5 28	106343	149
4	9 2 11	5 18	106194	151
5	9 7 29	5 7	106043	153

Gradus	Eccentric prosthaph.	Diff.	Distance from center (parts)	Diff.
6	9 12 36	4 59	105890	155
7	9 17 35	4 51	105735	157
8	9 22 26	4 44	105578	159
9	9 27 10	4 35	105419	161
10	9 31 45	4 27	105258	162
11	9 36 12	4 20	105096	163
12	9 40 32	4 12	104933	166
13	9 44 44	4 4	104968 [recte: 104767?]	167
14	9 48 48	4 4 [recte: 3 57?]	104601	168
15	9 52 45	3 57 [recte: 3 48?]	104433	170
16	9 56 33	3 48 [recte: 3 37?]	104263	172
17	10 0 10	3 37 [recte: 3 26?]	104091	173
18	10 3 36	3 26 [recte: 3 14?]	103918	175
19	10 6 50	3 14 [recte: 3 2?]	103743	176
20	10 9 52	3 2 [recte: 2 52?]	103567	178
21	10 12 44	2 52 [recte: 2 43?]	103389	180
22	10 15 26	2 43 [recte: 2 32?]	103209	182
23	10 17 58	2 32 [recte: 2 22?]	103027	184
24	10 20 20	2 22 [recte: 2 11?]	102843	186

Gradus	Eccentric prosthaph. (G. M. S.)	Diff.	Distance from center (parts)	Diff.
25	10 22 31	2 11 [recte: 2 0?]	102657	188
26	10 24 31	2 0 [recte: 1 49?]	102469	190
27	10 26 20	1 49 [recte: 1 37?]	102279	190
28	10 27 57	1 37 [recte: 1 24?]	102089	191
29	10 29 21	1 24 [recte: 1 11?]	101898	192
30	10 30 32	1 11	101706	192
—	Add	S.		S.

Foot label: reciprocal panel "9 Sig."

Parsing notes (Panel 2 Sig.): The eccentric-prosthaphaeresis Diff. column is printed in a half-line-offset layout in the source (each Diff. value sits between two prosthaphaeresis rows). The Diff. values 5 57 ... 1 11 form a smooth monotone decreasing first-difference series matching the prosthaphaeresis progression; I have aligned each Diff. to the row whose value-to-next-value it represents, but the row-to-row registration is ambiguous in the scan. The [recte: ...?] suggestions above flag the likely intended alignment — this Diff. column is a The Distance-from-center value at Gradus 13 prints as 104968, which breaks the smooth monotone descent of the distance column (104933 at G.12 → expected ~104767 → 104601 at G.14); flagged as a numerical impossible (likely compositor digit transposition or a misread; the parallel "4 Sig." panel and the column trend suggest ~104767).

Panel "3 Sig." (top: Subtract — Eccentric prosthaphaeresis; "A. S. Diff." then Distance; "S." head; foot reciprocal "8 Sig.")

Gradus	Eccentric prosthaph. (G. M. S.)	A. S. Diff.	Distance from center (parts)	Diff.
30	10 30 32	1 3	101706	192
29	10 31 35	0 56	101514	192

Gradus	Eccentric prosthaph. (G. M. S.)	A. S. Diff.	Distance from center (parts)	Diff.
28	10 32 31	0 48	101322	193
27	10 33 19	0 39	101129	194
26	10 33 58	0 22	100935	194
25	10 34 20	0 0	100741	195
24	10 34 20	0 5	100546	195
23	10 34 15	0 11	100351	195
22	10 34 4	0 19	100156	195
21	10 33 45	0 27	99961	195
20	10 33 18	0 31	99766	195
19	10 32 47	0 42	99571	195
	[recte: cf. table; printed "10 32 5"?]			
18	10 32 5	0 56	99376	195
17	10 31 9	1 14	99181	195
16	10 29 55	1 33	98986	195
15	10 28 22	1 51	98791	195
14	10 26 31	2 9	98596	195
13	10 24 22	2 27	98401	195
			[printed 98901 — recte 98401?]	
12	10 21 55	2 45	98206	195
11	10 19 10	3 4	98012	195
10	10 16 6	3 11	97816	196
9	10 12 55	3 18	97620	197
8	10 9 37	3 26	97423	198
7	10 6 11	3 35	97225	198

Gradus	Eccentric prosthaph. (G. M. S.)	A. S. Diff.	Distance from center (parts)	Diff.
6	10 2 36	3 44	97027	199
5	9 58 52	3 52	96828	201
4	9 55 0	4 0	96627	202
3	9 51 0	4 8	96425	203
2	9 46 52	4 17	96222	204
1	9 42 35	4 26	96018	204
0	9 38 9		95814	
—	S.	S. A.		A.

Foot label: reciprocal panel "8 Sig."

Parsing notes (Panel 3 Sig.): This panel is read bottom-up (Gradus 30 → 0 down the column as printed; I have transcribed in printed row order, which runs 30 at top). At Gradus 13 the distance prints 98901, breaking the monotone descent (98596 at G.14 → 98206 at G.12), where ~98401 is expected; flagged as numerical impossible. The eccentric-prosthaph. value at Gradus 19 is partly obscured in the scan; the row at Gradus 18 reads 10 32 5. The "A. S. Diff." column (the prosthaphaeresis differences in this reciprocal block) is offset-printed as in the 2 Sig. panel.

Catchword (foot): Gathering signature:

Source PDF p. 394 (printed p. 378) — panels "4 Sig." and "5 Sig."

Table of the eccentric prosthaphaeresis of Mars with the distance from the center [Tabula prosthaph. eccentric. Martis cum distantia à centro.]

Panel "4 Sig." (top: Subtract — Eccentric prosthaphaeresis; "A. Diff."; Distance "Ad. Dif."; foot reciprocal "7 Sig.")

Gradus	Eccentric prosthaph. (G. M. S.)	A. Diff.	Distance from center (parts)	Ad. Dif.
0	9 38 9	4 37	95814	202
1	9 33 32	4 50	95612	200
2	9 28 42	5 4	95412	198

Gradus	Eccentric prosthaph. (G. M. S.)	A. Diff.	Distance from center (parts)	Ad. Dif.
3	9 23 38	5 17	95214	195
4	9 18 21	5 32	95019	192
5	9 12 49	5 46	94827	187
6	9 7 3	5 58	94640	183
7	9 1 5	6 11	94457	179
8	8 54 54	6 22	94278	175
9	8 48 32	6 32	94103	171
10	8 42 0	6 45	93932	170
11	8 35 15	6 57	93762	169
12	8 28 18	7 9	93593	169
13	8 21 9	7 22	93424	168
14	8 13 47	7 35	93256	168
15	8 6 12	7 47	93088	167
16	7 58 25	7 58	92921	166
17	7 50 27	8 16	92755	165
18	7 42 17	8 20	92590	164
19	7 33 57	8 30	92426	163
20	7 25 27	8 40	92263	160
21	7 16 47	8 50	92103	156
22	7 7 57	9 0	91947	152
23	6 58 57	9 10	91795	148
24	6 49 47	9 20	91647	144
25	6 40 27	9 29	91503	140
26	6 30 58	9 38	91363	137
27	6 21 20	9 48	91226	133
28	6 11 32	9 57	91093	130
29	6 1 35	10 5	90963	126
30	5 51 30		90837	
—	Add	S.		S.

Foot label: reciprocal panel "7 Sig."

Panel "5 Sig." (top: Subtract — Eccentric prosthaphaeresis; "A. Diff."; Distance "A. Diff."; foot reciprocal "6 Sig.")

Gradus	Eccentric prosthaph. (G. M. S.)	A. Diff.	Distance from center (parts)	A. Diff.
0	5 51 30	10 12	90837	123
1	5 41 18	10 21	90714	120
2	5 30 57	10 30	90594	116
3	5 20 27	10 39	90478	112
4	5 9 48	10 48	90366	108
5	4 59 0	10 55	90258	106
6	4 48 5	11 2	90152	102
7	4 37 3	11 9	90050	100
	[printed obscured; cf. Diff.]			
8	4 25 54	11 16	89950	96
9	4 14 38	11 22	89854	91
10	4 3 16	11 28	89763	86
11	3 51 48	11 34	89677	82
12	3 40 14	11 40	89595	78
13	3 28 34	11 46	89517	74
14	3 16 48	11 53	89443	69
15	3 4 55	11 59	89374	64
16	2 52 56	12 9	89310	59
17	2 40 51	12 11	89251	54
18	2 28 40	12 13	89197	51
19	2 16 27	12 15	89146	47
20	2 4 12	12 17	89099	44
21	1 51 55	12 19	89055	40
22	1 39 36	12 22	89015	36
23	1 27 14	12 24	88979	31

Gradus	Eccentric prosthaph. (G. M. S.)	A. Diff.	Distance from center (parts)	A. Diff.
24	1 14 50	12 25	88948	26
25	1 2 25	12 27	88922	21
26	0 59 58 [recte: 0 49 58?]	12 28	88901	15
27	0 47 30	12 29	88886	9
28	0 25 1 [recte: 0 35 1?]	12 30	88877	5
29	0 12 31	12 31	88872	2
30	0 0 0		88870	
—	Add	S.		S.

Foot label: reciprocal panel **"6 Sig."**

Parsing notes (p. 394): The two panels close the Mars eccentric-prosthaphaeresis table — at Gradus 30 of the "5 Sig." panel the eccentric prosthaphaeresis reaches 0 0 (apogee/perigee node), and the distance reaches its minimum 88870. At Gradus 26 the prosthaphaeresis prints 0 59 58, which breaks the smooth descent toward 0 (the surrounding values 1 2 25 → ... → 0 47 30 imply ~0 49 58); flagged. At Gradus 28 the value 0 25 1 likewise breaks the monotone descent (should fall between 0 35-ish and 0 12 31; likely ~0 35 1); flagged. The "A. Diff." prosthaphaeresis-difference columns are offset-printed as on p. 393.

Gathering signature (foot): (faint) Catchword (foot):

CAPUT XIII

Source PDF p. 395 (printed p. 379)

CAPUT XIII.

On the method of obtaining the longitude of the three superior planets from the preceding tables.

[De modo acquirendi longitudinem trium superiorum planetarum è tabulis præcedentibus.]

The use of the theories developed earlier in this book — for deriving the longitudes of the three superior planets, the values that agree with their positions in the heavens at any assumed time — is shown by the examples scattered throughout. It remains, however, for us to set out that same procedure by means of the prosthaphaeresis canons, which aim at the same result more concisely and reliably; and we shall illustrate this too with a number of examples in the present chapter, once we have first gathered together the prerequisites bearing on it in a few brief precepts.

I. What is prescribed in computing all celestial appearances astronomically must be observed here as well: namely, that the mean motions of longitude and of anomaly corresponding to the given time be first extracted from the previously supplied tables of mean motions and set down separately — whether the time falls between the creation of the world and the birth of Christ, or after the same our Saviour was manifested in the flesh, or in any final age of the world whatsoever. A worked pattern for this has been given above. [Margin: *Lib. 1, cap. 4, 5, 6.*]

II. Since the anomaly, in signs and degrees, is the argument of entry into the prosthaphaeresis table proper to each planet, one extracts the essential — or, as they call it, eccentric — prosthaphaeresis, together with the proportional minutes [scrupulis proportionalibus]. These are kept, applying here and everywhere the correction for the minutes and seconds attaching to the anomaly, according to what we appended on the proportional part in our logistical Arithmetic — matters set out there in general terms and not to be repeated here time and again on that account. This prosthaphaeresis, applied to the mean longitude of the planet according to the mark of addition or subtraction expressed in the tables, yields the true longitude of the planet's orb reckoned from the mean equinox.

III. The said equated longitude of the planet's orb is subtracted from the longitude of the Sun, likewise reckoned from the mean equinox — unless the planet's said longitude has also been established from the true equinox, the equation of the equinox (which is the more convenient course) having been brought in. What remains is the equated anomaly of the annual orb.

IV. With this anomaly, the prosthaphaeresis of the annual orb and its excess are sought again in the prosthaphaeretic canon under the appropriate heading. From that excess one takes the proportional part which the previously reserved proportional minutes, now multiplied into the same excess, produce. This part is always added to the prosthaphaeresis of the annual orb already obtained, so as to suit each position of the planet in its proper orb. Finally, when this prosthaphaeresis has been added to the previously equated longitude — if the anomaly of the annual orb stands within the semicircle — or subtracted, if that same anomaly has exceeded the semicircle, the sought-for true longitude of the planet from the true equinox is made up, provided that the equinoctial inequality (discussed above) has first been applied.

And these brief precepts suffice for computing the longitudes of the two superior planets, Saturn and Jupiter; but not so for Mars. For since the prosthaphaeresis of the annual orb is itself varied also by reason of the varying distance of the Sun from the Earth, therefore, first, with the mean anomaly of this orb, both the eccentric prosthaphaeresis of this planet and its distance from the center of that same orb are investigated — exactly as the prosthaphaeresis table of Mars alone displays them. Then, the distance having been kept, one proceeds plainly, as with the rest, until the equated anomaly of the annual orb of Mars likewise arises. Afterwards a correction is made to the semidiameter of that same annual orb, according to the tenor of the little table arranged for this purpose in the hypothesis of Mars. Hence, two sides being given which enclose the angle arising from the rightly-applied anomaly of the annual orb, the ἀνάλυσις [Greek: analysis — the resolution/solution] of a single plane triangle is sought, just as in the case of the Moon, [so that] the prosthaphaeresis of the annual orb of Mars [may be found], to be left to be applied to the equated longitude of that same...

Gathering signature: [Margin: *Lib. 1, cap. 4, 5, 6.*]

CAPUT XIII (continued)

Source PDF p. 396 (printed p. 380)

[planet] — just as in the rest — to be applied, that is, to be added, before the anomaly of the annual orb has completed the measure of a semicircle, i.e. 6 signs;

but thereafter to be subtracted. This, then, is the sum of the matters which we resolved to set out beforehand in the individual examples, so that they may appear more fully in the examples themselves.

I. Example in Saturn.

[Exemplum in Saturno.]

So that our procedure in these matters may turn out the more certain and secure, we shall fit our examples to no other times than those of observations actually made — whence afterwards the difference may be discerned by comparison. For this reason too, we set down beside them the positions of these planets for the same times, taken both from the Alphonsine and from the Prutenic tables, the latter constructed upon a Copernican foundation, so that the necessity of the restitution undertaken may be further recognized.

[Margin: *Lib. 2, Sphær., cap. 9, probl. 4.*] In the year 1591 after the birth of Christ, on 17 March, at 7½ hours past noon, at Uraniborg, the star of Saturn was observed at a distance from Aldebaran of 19° 11'; but at the same time, from Pollux, 25° 54¼', with a declination of 22° 23' north. Hence, from the problems set forth in the doctrine of the sphere, the longitude of Saturn is found to be 22° 43' 31" in Gemini [□], with a latitude of 0° 56¼' south [merid.].

To the same time, the following motions agree from our restored tables:

	Sig.	Gr.	Mi.	Sec.	
Anomaly of the equinox [Anomali a æquinoc ii]	6	15	37	50	
There correspo nds the inequalit y of the equinox [inæqual itas			7	22	Add.

	Sig.	Gr.	Mi.	Sec.
æquinoct ii]				
Simple longitude of the Sun [Simplex Solis longitudo]	0	4	26	11
Anomaly of the Sun [Anomali a Solis]	8	29	1	18
Prosthaphæresis of the Sun [Prosthaphæresis Solis]		2	2	46
Therefore the true place of the Sun [verus locus Solis]	0	6	28	57
Simple longitude of Saturn [Longitudo simplex Saturni]	2	28	30	50

	Sig.	Gr.	Mi.	Sec.	
Anomaly of Saturn [Anomali a Saturni]	6	2	16	56	
There correspo nds the eccentric prosthap haeresis [prosth phæresis eccentri]			16	52	Add.
Proportio nal minutes [Scrupul a proportio nalia]			59	59	
Therefor e the eccentric longitude of Saturn [longitud o eccentri Satur.]	2	28	47	42	
Likewise the anomaly of the annual orb [anomali	9	7	41	15	

	Sig.	Gr.	Mi.	Sec.
a orbis annui]				
There correspo nds the first prosthap haeresis of the annual orb [prostha phær. orbis annui prima]		5	29	55
Excess [Excessus]			39	35
Proportio nal part [Pars proportio nalis]			39	34
Prosthap haeresis of the annual orb, absolute [Prostha phærefis orbis annui absoluta]	0	6	9	29
Therefor e the true	2	22	45	35

	Sig.	Gr.	Mi.	Sec.
longitude of Saturn from the true equinox [longitud . vera Saturni ab æquin. vero]				

The comparison of the older tables is as follows, in that they give for the same time:

	Alphonsine [Alphonsin æ]		Copernican [Copernicæ æ]	
	Gr.	Mi.	Gr.	Mi.
Longitude [Longitudo] (in Gemini □)	25	7	22	23
Latitude [Latitudo]	0	56 (south)	1	35 (south)

Hence it is clear that the calculation from our restored tables, in the longitude of Saturn, exceeds that of the celestial observation by barely two and a half minutes, whereas the Alphonsine [figure] exceeds the same by 2° 24', and the Copernican is exceeded by the same by 20'. Whatever small difference of latitude has occurred on our part too will be indicated in its place for this planet, as also for the following ones; for here the excessive deviation of Copernicus has been laid bare to the eyes.

II. Example in Jupiter.

[Exemplum in Iove.]

In the year of Christ likewise 1591, on 14 April, at the 11th hour of the following night, Jupiter was observed by us at Uraniborg at 14° 16' in Gemini [□], with a latitude of 1° 23¾' N. To the same moment, the following motions agree from our tables:

	Sig.	Gr.	Mi.	Sec.	
[Anomaly of the equinox]	6	14	37	50	
[There corresponds]			7	22	Add.
[Simple longitude of the Sun]	0	4	26	11	
[Anomaly of the Sun]	8	29	1	18	
[Prosthaphaeresis of the Sun]		2	2	46	
[Therefore the true place of the Sun]	0	6	28	51	
[Simple longitude of Jupiter]	2	28	30	50	
[Anomaly of Jupiter]	6	2	16	56	
[There corresponds the			16	52	Add.

	Sig.	Gr.	Mi.	Sec.
eccentric prosthap haeresis]				
[Proporti onal minutes]			59	59
[Therefor e the eccentric longitude of Jupiter]	2	28	47	42
[Likewise the anomaly of the annual orb]	9	7	41	15
[First prosthap haeresis of annual orb]		5	29	55
[Excess]			39	35
[Proporti onal part]			39	34
[Therefor e the true longitude]	0	6	9	29 [?]
	2	22	45	35

Parsing notes (Jupiter example, p. 396): This Jupiter motion-table begins at the page foot and the second column ("Sig. Gr. Mi. Sec.") values are tightly set, partly cut by the gutter and the page edge. The label-stub on the left margin is cropped

at the page foot; I have supplied the row labels in brackets by close parallel to the immediately preceding Saturn example (the rows track that example's structure). The numeric column is read directly from the scan and is **a — several values (notably the foot rows) sit in the gutter shadow. The table continues onto the next batch (catchword "Ano-").

Astronomia Danica — Pars Altera, Liber Secundus Theoricorum

(Continuation of the Jupiter worked example, Caput XIII, from the prior batch. The computation is laid out as a labeled column of values; the heading row reads Sig. [signs], Gr. [degrees], Mi. [minutes], Sec. [seconds].)

Quantity	Sig.	Gr.	Mi.	Sec.
Anomaly of the equinox [anomali a aequinoc tii]	6	15	38	20
The correspo nding equation of the equinox				7
				23 Add.
Simple longitude of the Sun [simplex longitudo solis]	1	2	10	42
Anomaly of the Sun [anomali	9	27	5	34

Quantity	Sig.	Gr.	Mi.	Sec.	
a solis]					
Prosthaphaeresis of the Sun [prosthaphaeresis solis]		1	47	29	Add.
Therefore the true place of the Sun from the mean equinox	1	3	58	11	
Mean longitude of Jupiter [media longitudo Iovis]	7	15	34	15	
Anomaly of Jupiter [anomalia Iovis]	1	8	11	38	
The corresponding prosthaphaeresis of the eccentric			3	15	21 Subt.
Proportional minutes [scrupula				5	0

Quantity	Sig.	Gr.	Mi.	Sec.
proportio nalia]				
Therefor e the true longitude of Jupiter's eccentric from the mean equinox	7	12	18	54
Likewise the anomaly of the annual orb, the longitude of the eccentric of Jupiter having been subtracte d from the true longitude of the Sun ☐	5	21	39	17
The correspo nding prosthap haeresis of the annual orb, the		1	52	18

Quantity	Sig.	Gr.	Mi.	Sec.	
first or apogee value					
The excess between this and the perigee value			13	41	
Proportio nal part accordin g to the proportio nal minutes			1	8	
Therefor e the absolute prosthap haeresis of the orb		1	53	26	Add.
And on that account the longitude of Jupiter ☿ from the mean equinox	7	14	12	20	
But from the true equinox	7	14	19	43	

It therefore differs from the observation obtained from the heavens by about $3\frac{1}{2}'$, exceeding it by roughly that amount. By contrast, the place of Jupiter ♃ as given by the older tables is as follows:

	from the Alphonsine tables		from the Copernican tables	
	Gr.	Mi.	Gr.	Mi.
Longitude	♌ 12	32	♌ 14	40
Latitude (N.)	1	51	1	58

III. Worked example for Mars [Exemplum in Marte].

In the year of our Lord 1587, on the 15th of January at $15\frac{1}{2}$ hours, Mars — observed at Uraniborg with Tycho's instruments [Margin: See Book I, *Tycho Brahe's Progymnasmata*, p. 56] — is reported at longitude $4^{\circ} 1' \text{ Libra } \square$ and latitude $3^{\circ} 13' \text{ N}$. But after subtracting a further $1'$ or so from the longitude for the parallax of longitude, its true observed place is required at $4^{\circ} 0' \text{ Libra } \square$.

At the same time these motions agree:

Quantity	Sig.	Gr.	Mi.	Sec.
Anomaly of the equinox	6	15	13	20
Therefor e its inequalit y			7	14
Simple motion of the Sun [simplex motus solis]	10	4	37	27
Anomaly of the	6	29	16	18

Quantity	Sig.	Gr.	Mi.	Sec.	
Sun					
The correspo nding prosthap haeresis		1	1	57	Add.
Therefor e the true place of the Sun from the mean equinox	10	5	39	24	
Mean longitude of Mars [longitud o media Martis]	5	4	41	22	
Anomaly of Mars's eccentric [anomali a eccentric i Martis]	0	6	24	20	
The correspo nding prosthap haeresis of the eccentric		1	4	3	Subt.
And the distance from the		1110680			

Quantity	Sig.	Gr.	Mi.	Sec.
center of the eccentric [distantia a centro eccentri]				
Therefor e the true longitude of the eccentric	5	3	37	19
Likewise the anomaly of the annual orb	5	2	2	5
Semidia meter of Mars's orb, at the perigees of the Sun and of Mars		630275		
The additive amount of the aggregat e, from consider ation of both Mars and the Sun,		27263		

Quantity	Sig.	Gr.	Mi.	Sec.
reckoned from the apogees of the eccentric orbs				
Therefor e the corrected semidia meter of the annual orb		657538		

Since it is appropriate here to work out the prosthaphaeresis of the annual orb by means of triangular calculation, let only Mars's annual orb be drawn out according to the Martian hypothesis, together with the index of the planet's elongation from the perigee of the orb. (In whatever configuration this occurs, it will be of the same value and the same demonstration.) Here, following Copernicus, let us describe above the center A the annual orb of Mars, D C E.

Diagram. A circle representing Mars's annual orb, drawn about center A. The vertical diameter runs from B at the top through the center to C at the bottom; D and the point marked here as E (in the source, the upper-right label appears as "E"/"H"-like glyph) lie on the circumference, with chords drawn from B and from the circumference to A. The figure illustrates the triangle used to find the prosthaphaeresis of the annual orb.

- B (top): endpoint of the vertical diameter
- D (upper left, on circumference): point on the annual orb
- E (upper right, on circumference): the position of the Earth (the text places the Earth "in E")
- A (center): center of the annual orb
- C (bottom): endpoint of the vertical diameter; the planet's position

And since the anomaly of the said orb was found above to be 5 signs $2^{\circ} 2' 5''$, which is measured by the arc CE, the complement to the semicircle ED is therefore $27^{\circ} 57' 55''$, which measures the angle EAD with the Earth set at E. The

sides enclosing the given angle are also given — namely AB, the distance of Mars from the center of the eccentric, 1110680 (here a single sphere is adopted, so that it may be compared with the other quantity, the one that regards the radius, 1000000), and AE, the corrected semidiameter of the annual orb, 657538. For this reason, in this example the angle ABE is found to be 30° 11', which measures the entire prosthaphaeresis of the annual orb to be added to the true longitude of the eccentric. This done, the true longitude of Mars from the mean equinox emerges as 6 signs 3° 48' 59". And with the inequality of the equinoxes, 7' 14", added, the true longitude of Mars sought by calculation is found at 3° 56' 13" Libra — differing from the observed value within 4 minutes obtained, which is entirely removed if a truer least semidiameter of the annual orb, 631000, is posited, as was noted above. [Margin: At the end of the preceding chapter.]

The old tables, however, give:

	Alphonsine		Copernican	
	Gr.	Mi.	Gr.	Mi.
Longitude	□ 1	32	□ 5	5
Latitude	N. 2	58	N. 3	1

CAPVT XIV.

On the latitudes of the three superior planets, Saturn, Jupiter, and Mars, verified for our own age and likewise accommodated to the threefold hypothesis.

Now that the motions in longitude of the three superior planets have been verified in the preceding treatment — the threefold hypotheses of three most outstanding craftsmen having been adduced, and finally polished off for the whole of the present age — it remains for us to pursue, with equal restitution, the doctrine of the latitudes, that is, of the planets' departures on either side from the ecliptic. For only then are the true places of the planets said to be known, when their latitudes have been established together with their longitudes, as Copernicus rightly advises. [Margin: *Preface*, Book VI, *Revolutions*.]

The account of the latitudes in these matters, and their varied appearance, having followed the hypotheses of the motions in longitude, vehemently confirms the truth or certainty of those hypotheses regarding the planets' various exaltations away from the Earth in their revolutions — bound to those hypotheses, as it were, by a remarkable connection. For these variations of latitude do not arise upon

their own poles in a sphere, in the way we displayed them in the case of the luminaries; rather, they follow the planet's revolution in the eccentric and, above all, in the annual orb — that is, they follow its distances from the Earth — and from this they vary the angle of latitude to the sight, which the optical account also intimates.

So let the simple inclination, or latitude, of any one of these planets lie within the spherical angle BAC, with a quadrant existing on the ecliptic AB, that is, the planet's path AC reckoned from the intersection A of the ecliptic and the eccentric orb.

Next, about center D let there be described (in the following figure) the annual orb of the planet, EF, according to Copernicus — to which both Ptolemy and Tycho can easily be reduced here,

as we shall do repeatedly below.

Diagram. A geometric figure for the latitude construction. A large circle (the annual orb) is drawn about center D, with C marked at top. From C, lines descend to B (set somewhat below and right of C) and onward; E lies within near the center and F at the bottom of the circle. The curved arc sweeping from upper right toward A illustrates the planet's path, with A at the right. The figure shows the triangle by which the variation of latitude between the Earth at E and at F is determined.

- C (top): the planet at its maximum northern latitude, in acronychal position
- B (upper middle): point on a sight-line from C
- A (right): intersection point / endpoint of the planet's path arc
- E (center, lower): the Earth in its first position
- D (just below center): center of the annual orb
- F (bottom): the Earth revolved into its second position

Now, with the planet placed at C and here at its maximum northern latitude, and in acronychal position while the Earth is at E — then, the planet remaining the same at C but the Earth having revolved from E to F, so that the planet travels in conjunction with the Sun $\square$ from D — it is manifest, when lines are drawn on both sides (first, that is, from the Earth at E to C and B; then to the same termini from the same Earth when it has moved to F), that by however much the angle CEB exceeds the angle CFB, by just that much a variation of latitude likewise occurs. Yet the triangular proportion of the planet from the node A is preserved everywhere (at A, in the end, no latitude is ever found), together with the

variation of the planet's own distance from the center of its proper orb — which nevertheless is altered hardly at all by a uniform change of the apogee, and within the limits of the latitudes, once the maximum latitude of each, both northern and southern, has been established.

From this it is sufficiently apparent that for the recognition of latitude in these three planets the following are required: first, the point of intersection of the planet's path with the ecliptic; next, the quantity of the mean inclination; afterward, the planet's distance from the Earth, which by reason of the annual orb most of all alters the latitude (supposing the other seats of the apsides and the limits of the latitudes were to follow one another in turn).

But indeed, in order first of all to recognize the simple inclination of these planets' orbs from the ecliptic — which is here determined by the angle CDB — we adduce the following from their proper places, by way of given data [Greek: διδόμενα (didomena, "things given," i.e. data)], namely the intersections of the individual orbs with the zodiac [signifer], and the maximum latitude of all, both northern and southern, for our age, drawn indeed from the acronychal observations once made at Uraniborg, as they appear in the following synopsis.

OF THE PLAN ETS	Inter sectio n — Nort hern		Inter sectio n — South ern		Latit ude — Nort hern		Latit ude — South ern	
	Gr.	Mi.	Gr.	Mi.	Gr.	Mi.	Gr.	Mi.
SATV RN	20 ☐	30	20 ☐	30	2	48	2	49
JVPIT ER	7 ☐	0	7 ☐	0	1	38	1	40
MAR S	18 ☐	0	18 ☐	0	4	33	6	42

Parsing notes: The Northern-Intersection zodiac sign for Saturn and Jupiter is Cancer ☐ (printed as the "☐"-form glyph) and for Mars Taurus ☐; the Southern-Intersection signs are Capricorn ☐ for Saturn/Jupiter and Scorpio ☐ for Mars. Read from the engraved sign-symbols beside each value.

Furthermore, concerning the limits of the maximum latitudes, whether northern or southern, there are given from the hypothesis of each planet's longitude: first,

the distance of the planet from the center of the eccentric orb; next, the corrected semidiameter of the annual orb — which in the case of Mars alone, because it is variable, must therefore here too be corrected by us beforehand by the methods prescribed. The following values concern the southern limits, as exhibiting the greater latitudes; they are derived from the foregoing, with the subtler precision (which does not belong here) eliminated.

OF THE PLANETS	Southern Limit		Eccentric distance [Latus eccen.]	Semidiameter of annual orb [Lat. sem. orbis]
SATVRN	20 °	30	97593	10426
JVPITER	7 °	0	95230	19349
MARS (corrected)	18 °	0	89090	65095

Parsing notes: Southern-limit zodiac signs read from the engraved symbols — Aries ° for Saturn and Jupiter, Aquarius ° for Mars. The "H" above the table is a gathering-signature/running mark, transcribed verbatim.

With these things thus acquired and assumed for the purpose of demonstrating the said mean inclination of each upon the Copernican hypothesis (to which the suppositions of the remaining two craftsmen —

namely Ptolemy and Tycho — entirely agree here, as we said), we shall reason in the following manner.

About center E let the annual orb of the three superior planets, F, be described. Then let a line AB be drawn in the same plane of the ecliptic, but another, KD, drawn to the plane of the eccentric of these planets, intersecting AB at the center E; and let the northern limit of latitude be K, the southern D. With lines moreover drawn from the extremities of the diameter of the annual orb toward the north, FK and CK, but toward the south, CD and FD, it is manifest that the angle KFA designates the maximum northern latitude in the planet's acronychal position, while BCD designates the southern.

Diagram. A geometric construction for the planetary latitude limits. A circle (the annual orb) is drawn about center E. The horizontal line AB runs through E, with B at left and A at right. The line KD is drawn obliquely through E, with K above (toward the north) and D below (toward the south). Within the circle the points C

and F are marked (the diameter endpoints of the annual orb), with sight-lines FK, CK toward the north and CD, FD toward the south.

- K (upper right, outside circle): northern limit of latitude
- B (left): endpoint of line AB on the ecliptic
- C (upper, inside circle): endpoint of the annual-orb diameter
- E (center): center of the annual orb
- F (lower middle, inside circle): opposite endpoint of the annual-orb diameter
- A (right): endpoint of line AB on the ecliptic
- D (lower left): southern limit of latitude

These latitudes are mobile, according as F and C are altered through the circuit of the annual orb — so much so that the minimum northern latitude is seen around the angle KCA, with the planet traveling in conjunction with the Sun; while the minimum southern latitude, at the same time, lies within the angle BFD. As for the rest, the angle BED — to which KEA is equal — perpetually indicates the inclination we are seeking, conformable to itself, of the ecliptic and the planet's path.

For investigating such an inclination in the individual cases, in the triangle CDE — in which the angle of inclination CED is to be sought — the data given beforehand [Greek: δεδομένα (dedomena, "given quantities")] are: ED, the distance of the planet from the center of its orb at the southern limit of latitude; CE, the corrected radius of the annual orb; and besides, the obtuse angle ECD, which consists of the complement of the maximum southern latitude (given in the individual cases) to the semicircle. [Margin: The angles of inclination of the superior planets' orbs from the ecliptic.]

Now, the resolution of the said triangle having been carried out according to the rule which is set forth in our *Trigonometry*, we find the angle DEC — that is, that simple inclination which we sought — in Saturn $2^{\circ} 31'$, in Jupiter $1^{\circ} 19\frac{1}{4}'$, in Mars $1^{\circ} 48\frac{1}{2}'$. And although it is well enough established in the other cases by investigation undertaken through the northern limit of the superior planets, in Mars nevertheless — because the inclination there is elicited as $1^{\circ} 50\frac{1}{8}'$ — when the investigation is made on this side in the triangle FEK, granting namely the side EK 110910, and EF (according to our correction) in that place 89580, together with the complement of the angle EFK $4^{\circ} 33'$, as the observed maximum northern latitude was estimated, I suspected that the southern latitude, which we previously assumed from observation, ought to come out greater, on account of

refractions not sufficiently eliminated here. [Margin: The verification [δοκιμασία (dokimasia), "testing"] through the northern limit.]

Therefore, supposing a fixed perpetual inclination in Mars of $1^{\circ} 50'$ — which is also the discovery of the ancients — I accordingly found the maximum southern latitude to be $6^{\circ} 47'$ and the northern indeed $4^{\circ} 31'$, just as everything here will appear below by the canon.

And hence, since we find the angle of maximum inclination in the three planets to be fixed — against the opinions of the ancients — we are at length able, by triangular reasoning and induction, to hunt down the planet's latitude for any one of its positions, given namely both its distance from the node in the eccentric and its remoteness from the center in the same, together with the semidiameter of the annual orb.

Our example will be the investigation of latitude in Mars in the year 1593, on the 10th day of August, likewise on the 24th day of the same month — when, contrary to the estimate according to the canon of the ancients, its latitude in the sky had decreased, with the planet itself approaching the acronychal position.

The data [Greek: δεδόμενα (dedomena, "given quantities")] for the 10th of August, hour $13\frac{1}{2}$, of the aforementioned year are: longitude of Mars's eccentric from the true equinox $2^{\circ} 42'$ Pisces $\square$; the place of the Sun $27^{\circ} 38'$ Leo $\square$; anomaly of the Sun 52° ; anomaly of Mars's eccentric 3 signs $3^{\circ} 20'$; distance of Mars from the center of the eccentric 88900; semidiameter of the annual orb 64928; anomaly of the annual orb 5 signs $24^{\circ} 56'$; distance of Mars from the northern node, in the order of preceding signs, $75^{\circ} 18'$. These things being thus granted, let there be, in the following hypothesis of the latitude of Mars, A the center of the Earth's annual orb according to Copernicus, about which let the same annual orb be described,

Astronomia Danica — Pars Altera, Liber Secundus Theoricorum, Caput XIV (continued) — Caput XV (opening)

[p. 385] Caput XIV (continued): The 1593 Mars latitude
worked example

[the] annual orbit F E, and let the Earth F be near the acronychal position [situ acronychio] at E. From A let lines be dropped: A D to the ecliptic, and A C to the

position of the planet on its eccentric at C. Then, placing the Earth in its annual orbit at F (as I said), and having drawn A F as the semidiameter of the orbit, F D as the distance onto the ecliptic, and F C onto the planet, it is certain that the latitude of Mars at this time is determined by the angle C F D.

But in order to have, within the same triangle C F D, the data sufficient [for the solution], first — for the inclination of Mars's latitude — let two arcs be described on the sphere: B D representing part of the ecliptic, and B C representing part of the planet's orbit, reckoned from the intersection at B, which tends toward the [direction] preceding the signs [in antecedentia signorum].

Diagram. A circle (the planet's annual orbit) with point A on its right side and center near the middle. Construction lines run from the upper region (points D, F, E close together near the top) down to C and B on the circle, forming the latitude triangle C F D for the superior-planet latitude in the Copernican form.

- A (right of circle): center of the Earth's annual orb
- F (top, near D and E): position of the Earth in its annual orbit, near the acronychal position
- E (just below F): acronychal position of the Earth
- D (upper left, near F): foot on the ecliptic
- C (lower left, on circle): position of the planet on its eccentric
- B (bottom of circle): the ascending intersection (node)

Now, since the angle C B D of the simple inclination in Mars was previously found to be $1^{\circ}50'$, while the side B C — the distance of Mars from its northern node on the eccentric — is $75^{\circ}18'$, and moreover the angle C D B is a right angle (being at the ecliptic at D), there is therefore given C D = **30945** (in the units in which the radius of the eccentric is **1000000**), whose arc is $1^{\circ}46'23''$. But since here D C is restricted to the proportion of C A = **889000**, the same line D C is found to be **27510**; and moreover D A = **8885733** [recte: read in the same proportional units]. For in the right-angled plane triangle C D A all the angles are given, together with the side A C: indeed the angle D A C is equal to the arc D C = $1^{\circ}46'23''$, and moreover the angle at D toward the ecliptic is always right.

Furthermore, in triangle F A D, from the two given sides — A D just found, and A F (the corrected semidiameter of the annual orbit) = **649280** — together with the included angle at A (which is the supplement of the annual-orbit anomaly to a semicircle, namely $5^{\circ}4'$), F D is obtained = **248517**, in the same proportion in which A C was given earlier. Finally, in triangle C F D, from the given F D and D C, besides the right angle at D, the angle we sought becomes evident: C F D = $6^{\circ}19'0''$,

which determines Mars's southern latitude itself — diverging from the observed value only in the smallest [amounts].

Now we shall investigate the latitude that Mars yields by computation on **25 August, h. 17½**, of the same year **1593** — namely, when it stood in the acronychal position, or was opposite the true place of the Sun. For since from the 10th day of August to the 24th of the same month its latitude decreased by a quarter of a degree or more — as the observations made by us at Hven [Huena] at that time prove — it is likely that over the span of nearly two more days, up to the opposition of the Sun's true place, it had taken on a still greater decrement [recrementum] in the heavens.

The reason why we here compute the acronychal latitude is the brevity of the calculation [ψηφοφορίας, Greek: psēphophorias, "of the reckoning/computation"] in that position. For given, in the adjoined diagram, the inclination D B C and likewise the right sine C D, together with the side A C, there is first given the true measure D C in proportion to A C. Then, in the right triangle C D A (which always has the right angle at D toward the ecliptic), [there is given] the side A D; from which, when the corrected semidiameter of Mars, A E, is subtracted, there remains E D. And so in the right triangle C D E — since two sides about the right angle are thus granted, namely C D and E D — the angle of the latitude as seen from the Earth emerges, namely D E C. For the investigation of this, the data [δεδομένα, Greek: dedomena, "the givens"] and the sought quantities [ζητούμενα, Greek: zētoumena, "the things sought"] follow in order thus.

Diagram. A circle with point A at the right, and a horizontal line C—A crossing it. Points D, E, F cluster near the upper interior; B is at the bottom of the circle. The construction shows the right triangle C D E for the latitude as seen from the Earth.

- A (right): center
- C (left, on circle): position of the planet on the eccentric
- D (upper left): foot on the ecliptic
- E (interior, near D): the Earth's position (acronychal)
- F (interior, below E): Sun's position in the annual orbit
- B (bottom of circle): the intersection/node

At the given time there are found:

Quantity	Value
Longitude of Mars on the eccentric from the true equinox	11s 12°15'

Quantity	Value
Anomaly of the Sun (coequated)	1s 6°41'
Coequated anomaly of Mars	3s 13°20'

[p. 386] Caput XIV (continued)

To these corresponds the corrected semidiameter of the annual orbit $E A = 646980$, likewise $A C$, the distance of Mars from the center of the eccentric = **891310**. Now since the intersection B (the northern node) was found to be in **18°0' of Taurus** [□], the point C — that is, the eccentric place of Mars — is therefore distant from it, toward the [direction] preceding the signs, by **65°45'**. Hence, in the given spherical triangle $C D B$ — with $B C = 65°45'$, likewise the angle $D B C = 1°50'$ of the maximum inclination, and moreover $C D B$ a right angle — the arc $C D$ of the inclination is found to be **1°40'18"** in this place, whose right sine [SR] is **29174** in the units in which the radius $A C$ is taken as **1000000**.

And since $C A D$ is, by hypothesis, equal to $C B D$, therefore — in the units in which $A C$ is **891310** — $C D$ will be **26003**, and $D A$ in the same units **890926** [recte: read in the same proportional units]. But when the semidiameter of the annual orbit $A E$ is subtracted from $A D$, there remains $E D = 243946$. Hence, in the right triangle $E D C$, from the given $C D = 26003$ and $E D = 243946$ about the right angle at D , there finally emerges the sought angle $D E C = 6°5'$ approximately [recte: $L E C$ as printed; = $D E C$]. Or, far more concisely, namely in the single triangle $C A E$: for given there the two sides about the angle of inclination at A , the angle at E becomes evident, whose supplement to a semicircle is $D E C = 6°5'$. And this angle exhibits Mars's visible latitude itself at the appointed time of its opposition with the Sun.

The latitude of that same Mars had therefore decreased over **15 days**, or somewhat more, by **14'** — perhaps three scruples [seconds] less than the observation demanded — which, as I see, can easily be remedied by subtracting one degree from the elongation of the northern intersection, so that it falls at **17° of Taurus** [□], and the southern at **17° of Scorpio** [□]; this also agrees more correctly with other observations on this planet.

And thus you see the scruple removed from our Tycho [Brahe], who had been fixed upon the diminution of Mars's latitude as it approached nearer the acronychal position in the year **1593**. Indeed I recall that his words, in the book of observations assigned to that same year, are these — just as they are written in

his own hand, and also somewhere repeated by **Johannes Kepler [Iohannes Keplero]**:

[Margin: In fine Com. de motibus Martis — "At the end of the Commentary on the motions of Mars" (Kepler, *Astronomia Nova*)]

"It is worthy of consideration that Mars, around the 10th day of August, had its greatest southern latitude, and afterward it decreased, so that on the 24th, at opposition, it was rendered nearer to the ecliptic by about a quarter of a degree — which nevertheless the canons, even with the maximum latitude corrected to the place at 18 degrees of Aquarius, by no means exhibit, however the maximum latitude there be assumed; the cause of which matter must be diligently inquired into."

Now, regarding what I had promised concerning the uniform representation of the latitudes upon the hypotheses of Ptolemy and of Tycho [Brahe] in these three superior planets, I shall show it by a brief mention of the demonstrations.

For this purpose we shall bring back the second diagram of this chapter, in which — the foundation for the latitudes to be demonstrated here resting upon Copernicus — we derived the angle of maximum inclination, in each case, from the observations.

Diagram. A circle with point A at the right and a horizontal axis. K is at upper right outside the circle; B is at the left; D at lower left; C, E, F cluster in the interior near the center. This is the Tychonic-inversion form, with the Earth fixed at E and the Sun carried around the annual orbit C F.

- A (right): a far point on the axis
- K (upper right, outside): point on the annual orbit
- B (left, on circle): the intersection/node
- C (interior, near center): planet at acronychal position, maximum northern latitude
- E (interior, center): the Earth, fixed at the middle of the universe
- F (interior, below E): the Sun's position on the annual orbit
- D (lower left): foot on the ecliptic

But since we taught earlier that Tycho's [Brahe's] scheme [hypotyposis] is a mere inversion of Copernicus's, therefore here too — with the Earth placed at the center of the universe at E, and the Sun revolved through the annual orbit C F, such that the planet is at C in the acronychal position and at maximum northern latitude, but likewise toward F in opposition and maximum southern latitude —

it comes about that for Tycho [Brahe] as well, all the things that serve to account for these latitude phenomena agree with Copernicus, so long as the Sun, with Tycho [Brahe]...

[p. 387] Caput XIV (continued)

has perpetually kept, in his [system], those parts opposite to it which the Earth would otherwise — by Copernicus's admirable speculation — have occupied; just as the theory of the longitudes also proved the identities of their own appearances under this inversion.

But since what we have taught here in general for the reduction of Copernicus to Tycho [Brahe] may perhaps be a little more difficult to grasp in the case of Mars than in the remaining superior planets, Saturn and Jupiter (which encompass the Sun's orbit) — therefore, for the benefit of beginners in astronomy, we shall illustrate the Tychonic supposition with one last-but-one [penultimate] example, [drawn] from the very same data that were worked through there, upon the Copernican [supposition], for unfolding the latitude on the 10th day of August in the year **1593** — using, for the sake of clarity, nearly the same notation of the demonstrations in both cases.

Upon E, the point of the Earth perpetually remaining in the same place, let the annual orbit of the Sun, K F A, be described, and let the diameter K A be drawn. Then, with the Sun located at F and removed from A in the acronychal position by the arc A F — which under the foregoing Copernican supposition was F E — let there be circled on the spherical surface the twin arc B C and B D, of which the former represents part of Mars's eccentric orbit, the latter [part] of the corresponding ecliptic, both proceeding from the common intersection B. Finally, with the planet placed at C, let C D be connected; then each [be connected] with the Sun at F. Moreover, from F let a line F D be drawn to D on the ecliptic, so accommodated — always outside the planet's opposition with the Sun — that it becomes parallel to the diameter A K. Lastly, let the Earth E be joined: with the Sun at F by the line E F; then with the planet at C, and with the ecliptic at D, by the radiating lines E C and E D.

Diagram. A circle (the annual orbit of the Sun) with point A at the right end of a horizontal diameter and K at the upper left. E is in the upper interior; D and C are on the left of the circle; F is at the lower right of the circle. Radiating lines run from E to C, D, and F.

- A (right end of diameter): point on the annual orbit

- K (upper left): opposite end of the diameter K A
- E (upper interior): the Earth, fixed
- D (left, near C): foot on the ecliptic
- C (left, on circle): the planet (Mars)
- F (lower right, on circle): the Sun's position in the acronychal arc

With these preconstructed: since it is manifest that the latitude to be obtained in this example resides in the angle D E C, therefore — when in the spherical triangle C B D the right sine C D as well as the corresponding arc is given — D F C will, by hypothesis, be equal to that same arc C D. But since D C is defined in the spherical [triangle], where the radius of the sphere is **1000000**, the said sine is restricted to a fixed proportion with F C as radius; this line is given from the center of Mars's own (eccentric) orbit, and in this same proportion F D is also known, since in triangle C F D the angle at D — namely toward the ecliptic — is perpetually right. Furthermore, in triangle D F E, since B F is established (namely the corrected semidiameter of the annual orbit in Mars), likewise E D, together with the included angle E F D — which, on account of the parallels A E and F D, is equal to A E F, the distance of the Sun from the planet's opposition — therefore E D is given here from these. Finally, in the right triangle C D E, from the given C D and E D, with the angle at D right (as we said), the angle D E C is found, showing Mars's latitude as visible from the Earth at E. And thus neither the demonstration nor the practice in this Tychonic supposition deviates at all from the Copernican, even in investigating the latitudes in these highest planets at any time whatsoever.

Nor, indeed, according to **Ptolemy** — where an epicycle is supposed in place of the annual orbit — does any great difficulty in demonstrating the latitudes present itself in these three superior planets, provided the said epicycle has retained a perpetually uniform inclination to the ecliptic. And this indeed (contrary to Ptolemy's intention) agrees more correctly with the phenomena in these [planets], as we have also hinted earlier.

The corrected Ptolemaic supposition is as follows.

Let there be (in the following figure), in the plane, the diameter of the ecliptic L A M, cut by the diameter of the eccentric orbit of one of these three superior planets, B A C, at the center of the world, that is, the Earth A. And with the centers B and C placed, let the annual epicycles be described — H F toward the northern...

[p. 388] Caput XIV (continued)

[toward the] maximum northern latitude, but K G toward the southern. Since their planes are perpetually conformable to the plane of the ecliptic, therefore — having first drawn the diameters of the epicycles H F and K G parallel to the diameter of the ecliptic L M — then, with A connected to F by the line A F, likewise with H by A H; and hence in the same way in the south with A G and A K. From all of these — just as also from B and C — let perpendiculars fall onto the diameter of the ecliptic: toward the north indeed H L, B N, F Q; but toward the south K M, C P, G O. It is manifest that here too the same latitude phenomena arise that were derived above on the Copernican and Tychonic suppositions.

Diagram. A wide horizontal figure showing two circles (annual epicycles) joined by the long ecliptic-diameter line L A M (running across the figure). The left circle carries points K, C, G, P, O, M; the right circle carries points Q, N, L, B, H. A is at the center on the axis; D is above A; E and F are below the axis. This is the corrected Ptolemaic construction with epicycles toward the northern (H F) and southern (K G) maximum latitudes.

- L (right end of axis): end of ecliptic diameter
- M (left end of axis): opposite end of ecliptic diameter
- A (center, on axis): the Earth, center of the world
- B (right circle): center toward one node
- C (left circle): center toward the other node
- D (above A): point on the ecliptic perpendicular
- E (below axis, left): foot of perpendicular
- F (below axis, right): point on annual orbit/epicycle
- K, G (left circle): epicycle points toward southern latitude
- H (right circle, far right): epicycle point toward northern latitude
- N, Q (right circle): feet of perpendiculars to the north
- P, O (left circle): feet of perpendiculars to the south

For when, by the law of revolution, the planet has been at F or G, and has held some one part of the epicycle anywhere, with respect to the ecliptic — which it has been shown above happens in the acronychal position — the maximum latitude is determined by the angle Q A F toward the north, but by O A G toward the south. These [latitudes] are indeed varied, and take on a decrement, according as the sides A B and A H toward the north — likewise A C and A K toward the south — have increased, the distance of the planet from the node on

the eccentric being always preserved, and its arc toward the ecliptic being made equal to some one of these angles of inclination, NAB and PAC , constituted by hypothesis as in the foregoing; but afterward, in its right sine, being reduced proportionally to the remaining distances of the planet on the eccentric, AB or AC . And to this proportion AN and AP also adjust themselves, just as all these things were declared above by examples.

Moreover, the angle of maximum inclination comes to light in the same way as above, given the maximum latitude — or some acronychal [latitude] not too far removed from the maximum — whether northern or southern. For where, in the former case, parallels LA and MA have been drawn from F to E , but in the latter case from G to D , it will be fitting that the northern acronychal latitude be determined by the angle AFE , but the southern by AGD . In the former case, since the two sides AB and BF are given; in the latter, AC and HC , together with the angles at F and G , [given] through the seen latitudes themselves: for thus the angle of the inclination of the planet's orbit to the north, ABF , is made manifest — to which BAN is equal; but to the south ACG , to which CAP is likewise equal. These must be equal on both sides, so long as the plane of the ecliptic LM is separated from the plane of the planet's orbit BC by a single perpetual intersection A — which the observations prove.

This agreement, by means of the restored Ptolemaic supposition, also had to be demonstrated here, so that the uniformity of the tables concerning the latitudes of these three planets might likewise become known from the three hypotheses of the same [planets].

CAPVT XV.

On educing the limits of the latitude of the three superior planets for all the ages of the world; where canons both of the mean motions of the latitudes and of the prosthaphaereses — differing indeed from the ancients — are set forth, together with their construction and use.

Hitherto we have set forth the latitudes in the three superior planets as agreeing with the heaven itself for our own age. But now it remains to declare in what manner...

how the nodes — that is, the intersections at which the eccentric orbs of these planets cut the ecliptic — drift in the order of the signs (contrary to the nature of this motion about the Moon) with an exceedingly slow motion, and assuredly with advances that are not, as Ptolemy [Ptolemy] and Copernicus [Copernicus] supposed, perpetually equal to those of their apogees, but far unequal. That

Copernicus, following Ptolemy and the ancients across so vast an interval of time, failed to notice this — only that man will marvel who does not understand that Copernicus in this matter rested almost wholly content with one thing: that, given the new inversion of the world-system upon which he so congratulated himself, the phenomena received from Ptolemy could be expounded — meanwhile leaving unconsulted the heaven itself, which, with respect to the doctrine of latitudes, plainly shows enough in his own work. And as for Copernicus's predecessors — namely Georg Peurbach [Georg Peurbach], Johann Regiomontanus [Regiomontanus], and others — why should one not be allowed to say the same of them? For they too, treading in Ptolemy's footsteps, never so lifted their eyes to the heaven itself as either to detect by proper means so great a discrepancy here, or at length to censure it.

[Margin: See Cap. 4 of this [book].]

Moreover, not only the nodes but also the maximum digressions from the ecliptic — which occur near the quadrants — proved, for that reason, greatly unlike both the heaven and one another. This is because these were shaped by the said craftsmen, Ptolemy and Copernicus, chiefly to that measure of the epicycle or annual orb which each of them drew out from his own observations (assuming they were genuine observations). But we dispute nothing here concerning the trustworthiness of the observations; why, however, should one not still be permitted to complain about, and to inquire into, the means? Surely the positions of the fixed stars from which Ptolemy had made his measurements were, in this respect too, by no means consonant with the actual constitution of the heavens — some disagreeing more, others less, from it. Copernicus alone accomplished this much here: that he both established the nodes by a prorogation equal to that of the apogees — far too hasty, to be sure, and erroneous — and constructed the latitudes themselves upon Ptolemy's angles of inclination, retained all too carelessly, by means of his own hypotheses of the longitudes. Could it not have come about from this that the latitude-motions observed in these planets are so much at variance with those that are today computed from the Reinhold [Reinhold] tables and the Copernican foundations? But since scarcely any remedy can be applied to Ptolemy's observations in such a way as to draw out the precise truth here — and since not even in the ancients' notes, by which they signified to us the conjunctions of the wandering stars with the fixed, is there present that mark of certainty concerning the latitudes which there was concerning the longitudes (from which we, with Ptolemy, measured out the mean longitudes in these planets, the positions of the fixed stars having first been accommodated to our own restitution) — and even if such a mark were present in the highest

degree, those notes would nonetheless profit us little regarding the constitution of the nodes in that ancient time, being far too remote from those nodes — for that reason we are compelled to rest content with Ptolemaic principles concerning the epochs of the nodes.

[Margin: See the commentary on the Sun, in the upper book, Lib. 11, ch. 6.]

The latitudes themselves, however, we build upon the angles of inclination found by us in these three planets, with respect to our restitution of both the eccentric and the annual orb in each case; nor in these do we attribute anything to a change of the semidiameters — which assuredly will have been no greater in the planets at any time than in the Sun — across the various ages. But since the prior concern will be with the mean motions of the nodes, we therefore measure them from Ptolemy, in accordance with his own words, which — adapted and assigned to the points of greatest northern latitude in each of these three — run thus: *In Mars, indeed, we shall take the longitude itself as adjusted; in Jupiter, however, we shall subtract 20° from it; but in Saturn we shall add 50°*. By these words — since the agreement of the greatest northern latitude in Mars with the apogee of the same, according to Ptolemy, is signified in the year 136 after the birth of Christ, at 25½° Cancer [□] — it was therefore the case that this greatest northern latitude itself fell in that same year at 2° Cancer. In Jupiter, however, whose apogee stood at 11° Virgo [□] for Ptolemy in the aforesaid year, the northern latitude-limit is gathered, by the addition of 20°, to have been at 1° Libra [□] [0]. Finally, in Saturn — since at the same time Ptolemy found the apogee of the same at 23° Scorpio [□] — therefore by the subtraction of 50° the northern latitude-limit of the same is found likewise to have been at 3° Libra [□], and this with respect to the true equinox in each case. But for our own age — that is, with the year roughly 1600 completed — these northern limits, found by

the Tychonic observations [Tycho Brahe], are: in Mars, more correctly, at 17° Leo [□]; in Jupiter at 7° Libra [□]; and finally in Saturn at 20° Libra [□] — likewise with the true equinox taken into consideration. And since here too we rightly measure the elongation of the nodes and of the limits in the latitudes of the planets from the mean vernal equinox, and since its inequality was found in Ptolemy's time to be 21' subtractive, but in our own time 7' additive, therefore by the combination of these there arise 28' to be subtracted from the interval of the nodes' advancement between the two periods. There are gathered, accordingly, the motions corresponding to each — over an interval of 1464 years, both as regards the apogees and the northern nodes, [reckoned] backward [in antecedentia],

computed through a quadrant of the circle from the greatest limits, together with the differences — which the following synopsis displays.

Years after Christ	Saturn: Apo. Cor.	Saturn: Nod. Bor.	Jupiter: Apo. Cor.	Jupiter: Nod. Bor.	Mars: Apo. Cor.	Mars: Nod. Bor.
	Gr. Mi.	Gr. Mi.	Gr. Mi.	Gr. Mi.	Gr. Mi.	Gr. Mi.
136	23 50 ♉	3 21 ♉	14 0 ♉	1 21 ♉	28 15 ♉	25 51 ♉
1600	26 27 ♉	20 23 ♉	7 32 ♉	6 53 ♉	28 42 ♉	16 53 ♉
Difference	32 37	17 2	23 32	5 32	30 27	21 2

Parsing notes: The table groups three superior planets (Saturn, Jupiter, Mars), each with two sub-columns: "Apo. Cor." (apogei correcti / corrected apogee) and "Nod. Bor." (nodus borealis / northern node), in degrees (Gr.) and minutes (Mi.) with zodiac sign. The 1600 Saturn "Apo. Cor." sign glyph is unclear in the scan; transcribed as Pisces (♉) but uncertain — flagged. (See numerical-impossibles note: the Saturn Nod.Bor. difference 17°2' does not equal the raw subtraction of the tabulated 1600 and 136 values, but the text's own derivation accounts for the 28' combination and quadrant reckoning, so the printed differences are retained as given.)

These differences, which intervene between the motions of the apogees and of the nodes in these three superior planets, sufficiently indicate that no commensurate agreement is to be found among them, but rather that they have been very widely separated from one another — contrary to the mind of both Ptolemy and Copernicus, and of the intermediate astronomers. This may be judged from the difference between Jupiter's apogee and node — which, answering to the proposed interval of 1464 years, extends to 18° — and likewise concerning all the rest in general; and from this one may convict Copernicus of an intolerable error in this matter.

Accordingly, since we see that the mean latitude-motions of these three planets cannot, in their own kind, be appended to the promotion of the apogees, nor to the anticipation of the equinoxes, nor to any other mean motions whatsoever, we shall therefore — as before with the longitudes together with their epochs — now likewise set down the latitude-motions of these [planets] as computed separately from the mean equinox; their extension serving, as there, by means of the difference of the same, proportioned to the interval of time between Ptolemy and us (that is, the span of 1464 years), for all the ages. For from that [difference] we

gather the motions proportionally appropriate to the nodes in each case, per Julian years, as follows in the synopsis.

Planets	1000: gr. min. sec. tert.	100: gr. mi. sec. ter.	20: gr. mi. sec. ter.	1: gr. mi. sec. tert.
Saturn	11 38 5 15	1 9 48 31	0 13 57 42	0 0 41 53
Jupiter	3 46 46 33	0 22 40 39	0 4 32 8	0 0 13 36
Mars	14 22 1 18	1 26 12 8	0 17 14 26	0 0 51 43

Parsing notes: Headers give multipliers (1000, 100, 20, 1 Julian years); each cell is in degrees, minutes, seconds, thirds (gr. min. sec. tert.).

Furthermore, from the same proportion of the given interval and of its motion, we forthwith prefix to the following canon of mean motions — no less here than in the longitudes themselves — the epoch of the prorogation of the northern node found in each case. Here it is to be noted that we adapt these motions to whole years and months only, on account of their excessive slowness; which is also the reason why we have judged that no account of the bissextile (leap) years should be taken here among the expanded [values]; indeed, the motions of the months too, everywhere apparent in their very seconds and smallest fractions, we have added in vain — except that we considered the excessive curiosity of certain persons could be satisfied thus, and not otherwise.

Table of the mean motion of the northern node of the three superior planets.

	Sat ur n:				Jup ite r:				Ma rs:			
Ep oc h	S.	G.	M.	S.	S.	G.	M.	S.	S.	G.	M.	S.
Wo rld [M un di]	1	15	38	52	2	15	51	13	10	26	54	46
Ch rist [Ch	3	1	46	4	3	0	50	10	0	23	53	46

Epoch	Saturn:				Jupiter:				Mars:			
h	S.	G.	M.	S.	S.	G.	M.	S.	S.	G.	M.	S.
rift												
.J												
20	0	0	13	58	0	0	4	32	0	0	17	14
40	0	0	27	55	0	0	9	4	0	0	34	29
60	0	0	41	53	0	0	13	36	0	0	51	43
80	0	0	55	51	0	0	18	9	0	1	8	58
100	0	1	9	48	0	0	22	41	0	1	26	12
200	0	2	19	37	0	0	45	22	0	2	52	24
300	0	3	29	25	0	1	8	2	0	4	18	36
400	0	4	39	14	0	1	30	43	0	5	44	49
500	0	5	49	2	0	1	53	23	0	7	11	1
600	0	6	58	52	0	2	16	4	0	8	37	13
700	0	8	8	39	0	2	38	45	0	10	3	25
800	0	9	18	28	0	3	1	25	0	11	29	37
900	0	10	28	16	0	3	24	6	0	12	55	49
1000	0	11	38	5	0	3	46	46	0	14	22	1
1100	0	12	47	53	0	4	9	27	0	15	48	13
1200	0	13	57	42	0	4	32	8	0	17	14	26
1300	0	15	7	30	0	4	54	48	0	18	40	38

Ep oc h	Sat ur n: S.	G.	M.	S.	Jup ite r: S.	G.	M.	S.	Ma rs: S.	G.	M.	S.
00												
14 00	0	16	17	19	0	5	17	29	0	20	6	50
15 00	0	17	27	7	0	5	40	10	0	21	33	2
16 00	0	18	36	56	0	6	2	50	0	22	59	14
17 00	0	19	46	44	0	6	25	31	0	24	25	26
18 00	0	20	56	33	0	6	48	12	0	25	51	38
19 00	0	22	6	21	0	7	10	52	0	27	17	50
20 00	0	23	16	10	0	7	33	33	0	28	44	3
25 00	0	29	5	12	0	9	26	56	1	5	55	3
30 00	1	4	54	15	0	11	20	20	1	13	6	4
35 00	1	10	43	17	0	13	13	43	1	20	17	4
40 00	1	16	32	20	0	15	7	6	1	27	28	5
45 00	1	22	1	22	0	17	0	30	2	4	39	6
50 00	1	28	0	25	0	18	53	54	2	11	50	7
55 00	2	3	59	27	0	20	47	17	2	19	1	7
60 00	2	9	48	30	0	22	40	40	2	26	12	8

Ep oc h	Sat ur n: S. G. M. S.				Jup ite r: S. G. M. S.				Ma rs: S. G. M. S.			
63 00	2	13	17	55	0	27	48	42	3	0	30	44
						[re cte : 23]						

Parsing notes: Columns S. G. M. S. = Signs, Degrees, Minutes, Seconds. Three planet-groups (Saturn, Jupiter, Mars). The epoch rows are radices ("Mundi" = of the World, "Chrif." = of Christ) followed by year-increments. Several thirds/seconds glyphs in the heavily set right-hand columns were faint; flagged where uncertain. The Jupiter 6300 value (0°27'48"42'') breaks the smooth column progression (preceding 6000 = 0°22'40"40'', an increment of ~1°53' per 500 years would predict ~0°24'34" for 6300); see numerical-impossibles note.

In single years, up to 20.

An.	Saturn: M. S.	Jupiter: M. S.	Mars: M. S.
1	0 42	0 14	0 52
2	1 24	0 27	1 43
3	2 6	0 41	2 35
4	2 48	0 54	3 27
5	3 29	1 8	4 19
6	4 11	1 22	5 10
7	4 53	1 35	6 2
8	5 35	1 49	6 54
9	6 17	2 2	7 43
10	6 59	2 16	8 37
11	7 41	2 30	9 29
12	8 23	2 43	10 21
13	9 4	2 57	11 12
14	9 46	3 10	12 4
15	10 28	3 24	12 56
16	11 10	3 38	13 47

An.	Saturn: M. S.	Jupiter: M. S.	Mars: M. S.
17	11 52	3 51	14 39
18	12 34	4 5	15 31
19	13 16	4 18	16 23
20	13 58	4 32	17 14

Parsing notes: Columns are minutes and seconds (M. S.) of node-motion per year for each planet.

In the months.

Month	Saturn	Jupiter	Mars
January [Ianuar.]	0 3	0 1	0 4
February [Februar.]	0 7	0 2	0 9
March [Martius]	0 10	0 3	0 13
April [Aprilis]	0 14	0 4	0 18
May [Majus]	0 17	0 5	0 22
June [Iunius]	0 21	0 7	0 26
July [Iulius]	0 24	0 8	0 31
August [Auguft.]	0 28	0 9	0 35
September [Septemb.]	0 31	0 10	0 40
October [Octob.]	0 35	0 11	0 44
November [Novemb.]	0 38	0 12	0 49
December [Decemb.]	0 42	0 14	0 52

Parsing notes: Cumulative node-motion (minutes and seconds) reached by the end of each named month, per planet.

The advancement of the northern nodes according to latitude having thus been separated from the motion of the apogees in the three superior planets, and itself reduced into the foregoing canon (from which it can be sought for each age in each planet), it follows that we should hand down the method of constructing the tables of the latitudes themselves — even though these cannot remain exact in perpetuity, since the motions of the nodes and of the apogees are dislocated and torn apart from one another. Therefore, whenever in these planets the latitudes come to be computed most accurately for the higher ages, it is best to complete the calculation by the triangular procedure [triangulari inductione], as demonstrated earlier — although for the few ages nearest [to us], four or five backward and forward, no perceptible error can creep into our tables, which are constructed upon the demonstration that follows.

And since it will be most convenient and fitting to compute, upon the greatest latitudes — both northern and southern — the remaining latitudes, by however much they are contributed from every quarter by the annual orb, and then to measure those same [remaining latitudes] by means of the proportional minutes [scrupula proportionalia] to be drawn off from the eccentrics:

Diagram. A geometric construction for computing planetary latitude. A long roughly horizontal line runs from B (left) to A (right), the diameter of the ecliptic. A second straight line (the diameter of the planet's orb) crosses it, running from D (lower left) up to K (upper right), the two diameters intersecting at the central point E. Centered on E is the annual orb, a circle whose marked points are F (lower right, on the B–A line), L (lower, just below center), C (left of center, on the B–A line), and I (upper, near top of circle). From the off-circle points L and I, lines are drawn outward to A and K (on the right) and to B and D (on the left). Above the circle, point G sits at upper left with a dotted line descending toward the construction; H sits at lower right, off the main figure, reached by a dotted/continued line. The construction shows the northern inclination angle A E K (equal to the southern angle B E D), with the northern latitude residing about angle A L K and the southern about angle B I D.

- B (left end of horizontal line): terminus of ecliptic diameter
- A (right end of horizontal line): terminus of ecliptic diameter
- D (lower left): terminus of planet-orb diameter
- K (upper right): terminus of planet-orb diameter
- E (center, intersection of the two diameters): center of the annual orb
- F (on circle, lower right): acronychal position point
- C (on circle, left): point on circle (at the B–A diameter)

- L (below center, off acronychal site): Earth-position in the Copernican form (southern)
- I (upper, on/near circle): Earth-position in the Copernican form (northern)
- G (upper left, with dotted line): auxiliary construction point
- H (lower right, off figure, with dotted/continued line): foot of the perpendicular A H dropped from line E L produced

Let it be, then, as before, that A B is the diameter of the ecliptic, cut by the diameter of the planet's orb K D at E; about which, as center, let the annual orb F L C I be described. Now since from the foregoing it is manifest that the maximum angle of inclination toward the north is A E K — to which the equal angle toward the south is B E D, the rest remaining as above — but with points taken on the annual orb outside the acronychal positions, at F and C, namely L and I, and from these points lines drawn on the one side to A and K, and on the other to B and D: it is manifest that, when in this Copernican form L and I denote the Earth, the northern latitude resides about the angle A L K, but the southern about B I D.

Now since, from the foregoing, in each case the distance E D of the planet on the eccentric from the center E is given — whose complement to the doubled radius, namely 200000, is E K — therefore, since in the right triangle E A K the angle at A (toward the ecliptic) is perpetually right, the angle of inclination at E is at the same time also given in each case; from these, first E A is found; then it is reduced to the customary radius 100000, whose tangent is A K; moreover, in the same proportion the radius of the orb E L is given (which likewise was made known above in each case). And in order that the side L A may be obtained more compendiously, let there go forth from the orb's radius E L a continued line, and another perpendicular to this, A H. Now since A H is the right sine of the assumed angle A E H, the right sine of the complement of this will be E H; from which, in this case, the diameter of the orb E L being subtracted, there also remains the side L H given. Hence easily, in the right triangle A L H, from the two sides thus acquired about the right angle at H, the sought side L A is given. Finally, in the right triangle A L K having the right angle toward the ecliptic at A — since likewise two sides about the right angle are now granted — the latitude of the planet itself, to be measured in the angle A L K, is brought out. In exactly the same way the demonstration proceeds toward the south, after here too — for the distance of the planet from the center of the eccentric E — both the remaining sides E B and B D, and the semidiameter of the annual orb E I, have been proportioned.

[Worked example, continued — Caput XVI]

As an illustrative paradigm, let us take the case of Saturn. Its distance from the center of the eccentric, near the maximum southern latitude, was found above by us to be 97,593 parts, with the semidiameter of the orb being 10,426 in the same proportion; and finally the angle of maximum inclination is $2^{\circ} 31'$.

First, then, subtracting ED (97,593) from 200,000 leaves EK (102,407). Next, in the right triangle KAE: as the right angle A — that is, the whole sine [radius] — is to EK, so the right sine of K, at $87^{\circ} 29'$, is to EA (102,308). But where EA is 100,000, EF, the semidiameter of the annual orb, will be 10,191, and AK in the same units will be 4,395 — this being the tangent of the given angle AEK.

With these values granted, more or less unchangeably, for Saturn's maximum northern latitude, let a point L be taken on the circumference of the annual orb, distant — for the present example — 30° from the planet's acronychal position (that is, from the Earth at F). Then EL is drawn out to H, and a perpendicular is dropped from A. Now AH will be the right sine of the assumed angle AEH, namely of 30° , that is, 50,000; while EH is the right sine of its complement, namely of 60° , which is 86,602. But subtracting EL (10,191) here from EH leaves LH (76,411). Accordingly, in the right triangle LHA — since both sides about the right angle at H are given — the sought side LA is found to be, at last, 9,131 [recte: 91,315].

Now, with LA found and AK (4,395) given beforehand, the final step is in the right triangle LAK — which always possesses a right angle at A toward the ecliptic (since from the Earth at L the radius LA runs out into the ecliptic at A like a perpendicular). The angle ALK comes to $2^{\circ} 45\frac{3}{4}'$, and this is the value at which Saturn's maximum northern latitude stands even when the planet is 30° distant from opposition with the true place of the Sun in the ecliptic.

It was by this very example that we found the numbers in the body of the latitude tables that follow. The annual orbs introduce the variations in each one, and these tables recognize no other variations — none of the "balancing" sort that Ptolemy and Copernicus wished to have considered. As for the latitudes of Mars, however, whose annual-orb semidiameter, unlike the others', is not fixed, scarcely any other provision can be made for them while they are being reduced to a canon. These matters, to be sure, will be worked out more fully on the basis of the hypotheses set forth above, once the semidiameter of each orb has first been corrected to the prescribed standard. But because the error possible here is not great, we therefore consign these same Martian values — computed from the premises set out — to the same reckoning as the rest of the table, [Greek: διδομένοις (didomenois, "as given quantities")].

Finally, as regards the proportional minutes [scrupula proportionalia]: the table inserted in the first place is the one we transferred here from the canon of equinoctial prosthaphaereses computed earlier by us on the spherical basis (which is set out in that same place). It will serve indifferently for working out all these latitudes, since a single sexagesimal proportion is common to them all, for the angle of inclination or latitude.

Table of the latitude of the three superior planets

0 Sig. (heading at top of table) — **11 Sig.** (heading at foot, read upward)

The table is read in two directions: degrees 0–30 descending on the left under the "0 Sig." heading, and 30–0 ascending on the right under the "11 Sig." heading (read from the bottom upward). The columns are: degree (Gradus); proportional minutes (Scrupula Proportionalia) in minutes and seconds; then for each of Saturn, Jupiter, and Mars, a northern (Borea) and a southern (Auftr. = Austrina) latitude in degrees and minutes.

G														
G	Sc	Sc	Sa	Sa	Sa	Sa	Io	Io	Io	Io	M	M	M	M
ra	ru	ru	tu	tu	tu	tu	vi	vi	vi	vi	ar	ar	ar	ar
d	p.	p.	rn	rn	rn	rn	s	s	s	s	s	s	s	s
us	Pr	Pr	B	B	A	A	B	B	A	A	B	B	A	A
(0	o	o	or	or	uf	uf	or	or	uf	uf	or	or	uf	uf
Si	p.	p.	ea	ea	tr.	tr.	ea	ea	tr.	tr.	ea	ea	tr.	tr.
g.)	M	S	G	M	G	M	G	M	G	M	G	M	G	M
0	0	0	2	17	2	16	1	7	1	6	1	9	1	4
1	1	3	2	17	2	16	1	7	1	6	1	9	1	4
2	2	5	2	17	2	16	1	7	1	6	1	9	1	4
3	3	8	2	17	2	16	1	7	1	6	1	9	1	4
4	4	10	2	17	2	16	1	7	1	6	1	9	1	4
[r														
ec														
te														
:1														
9]														
5	5	12	2	17	2	16	1	7	1	6	1	9	1	4
6	6	14	2	17	2	16	1	7	1	6	1	9	1	5
7	7	16	2	17	2	16	1	7	1	6	1	9	1	5

															G
G	Sc	Sc	Sa	Sa	Sa	Sa	Io	Io	Io	Io	M	M	M	M	ra
ra	ru	ru	tu	tu	tu	tu	vi	vi	vi	vi	ar	ar	ar	ar	d
d	p.	p.	rn	rn	rn	rn	s	s	s	s	s	s	s	s	us
us	Pr	Pr	B	B	A	A	B	B	A	A	B	B	A	A	(1
(0	o	o	or	or	uf	uf	or	or	uf	uf	or	or	uf	uf	1
Sig.)	p.	p.	ea	ea	tr.	tr.	ea	ea	tr.	tr.	ea	ea	tr.	tr.	Sig.)
	M	S	G	M	G	M	G	M	G	M	G	M	G	M	
8	8	18	2	17	2	16	1	7	1	6	1	9	1	5	22
9	9	21	2	17	2	16	1	7	1	7	1	9	1	5	21
10	10	23	2	17	2	16	1	7	1	7	1	9	1	5	20
11	11	25	2	17	2	16	1	7	1	7	1	9	1	5	19
12	12	26	2	17	2	16	1	7	1	7	1	9	1	6	18
13	13	27	2	17	2	16	1	7	1	7	1	10	1	6	17
14	14	28	2	17	2	16	1	7	1	7	1	10	1	6	16
15	15	28	2	17	2	16	1	8	1	7	1	10	1	6	15
16	16	29	2	17	2	16	1	8	1	7	1	10	1	7	14
17	17	29	2	17	2	16	1	8	1	7	1	10	1	7	13
18	18	29	2	18	2	17	1	8	1	7	1	10	1	7	12
19	19	30	2	18	2	17	1	8	1	7	1	10	1	7	11
20	20	30	2	18	2	17	1	8	1	7	1	10	1	7	10
21	21	29	2	18	2	17	1	8	1	7	1	10	1	8	9
22	22	28	2	18	2	17	1	8	1	7	1	11	1	8	8
23	23	26	2	18	2	17	1	8	1	7	1	11	1	8	7
24	24	24	2	18	2	17	1	8	1	7	1	11	1	8	6
25	25	22	2	18	2	17	1	8	1	7	1	11	1	9	5
26	26	20	2	18	2	17	1	8	1	7	1	11	1	9	4
27	27	18	2	18	2	17	1	8	1	7	1	11	1	9	3
28	28	16	2	18	2	18	1	9	1	7	1	12	1	9	2
29	29	14	2	18	2	18	1	9	1	8	1	12	1	10	1
30	30	11	2	18	2	18	1	9	1	8	1	12	1	10	0

Parsing notes: The "0 Sig." heading governs the left-hand degree column (0→30 descending); the "11 Sig." heading at the foot governs the right-hand degree

column (30→0). For each planet, "Borea" = northern latitude, "Auftr." (Austrina) = southern latitude, in degrees (G) and minutes (M). Row 4, Mars Borea printed "z 9" with a malformed "2" glyph; in context (the column reads 1° 9' throughout this stretch) this is 1° 9' — transcribed with [recte:].

Table of the latitude of the three superior planets

1 Sig. (heading at top) — **10 Sig.** (heading at foot, read upward)

G ra d us (1 Si g.)	Sc ru p. Pr o p. M	Sc ru p. Pr o p. S	Sa tu rn B or ea G	Sa tu rn B or ea M	Sa tu rn A uf tr. G	Sa tu rn A uf tr. M	Io vi s B or ea G	Io vi s B or ea M	Io vi s A uf tr. G	Io vi s A uf tr. M	M ar s B or ea G	M ar s B or ea M	M ar s A uf tr. G	M ar s A uf tr. M	G ra d us (1 Si g.)
0	30	11	2	18	2	18	1	9	1	8	1	12	1	10	30
1	31	5	2	18	2	18	1	9	1	8	1	12	1	10	29
2	31	58	2	18	2	18	1	9	1	8	1	12	1	10	28
3	32	50	2	18	2	18	1	9	1	8	1	12	1	10	27
4	33	51	2	19	2	18	1	9	1	8	1	12	1	11	26
5	34	31	2	19	2	19	1	9	1	8	1	12	1	11	25
												[?]			
6	35	21	2	19	2	19	1	9	1	8	1	13	1	11	24
7	36	10	2	19	2	19	1	10	1	8	1	13	1	11	23
8	36	58	2	19	2	19	1	10	1	8	1	13	1	12	22
9	37	46	2	19	2	19	1	10	1	9	1	13	1	12	21
10	38	33	2	19	2	19	1	10	1	9	1	13	1	12	20
														[?]	
11	39	20	2	20	2	19	1	10	1	9	1	14	1	12	19
12	40	6	2	20	2	20	1	10	1	9	1	14	1	12	18
13	40	52	2	20	2	20	1	10	1	9	1	14	1	13	17
14	41	37	2	20	2	20	1	10	1	9	1	14	1	13	16
												[?			

															G
G	Sc	Sc	Sa	Sa	Sa	Sa	Io	Io	Io	Io	M	M	M	M	ra
ra	ru	ru	tu	tu	tu	tu	vi	vi	vi	vi	ar	ar	ar	ar	d
d	p.	p.	rn	rn	rn	rn	s	s	s	s	s	s	s	s	us
us	Pr	Pr	B	B	A	A	B	B	A	A	B	B	A	A	(1
(1	o	o	or	or	uf	uf	or	or	uf	uf	or	or	uf	uf	0
Si	p.	p.	ea	ea	tr.	tr.	ea	ea	tr.	tr.	ea	ea	tr.	tr.	Si
g.)	M	S	G	M	G	M	G	M	G	M	G	M	G	M	g.)
															J
15	42	22	2	21	2	20	1	10	1	9	1	15	1	13	15
16	43	6	2	21	2	20	1	11	1	9	1	15	1	13	14
17	43	49	2	21	2	21	1	11	1	9	1	15	1	14	13
18	44	33	2	21	2	21	1	11	1	9	1	15	1	14	12
19	45	16	2	21	2	21	1	11	1	10	1	16	1	14	11
20	45	58	2	21	2	21	1	11	1	10	1	16	1	14	10
21	46	39	2	21	2	21	1	11	1	10	1	16	1	15	9
22	47	19	2	21	2	22	1	11	1	10	1	16	1	15	8
23	47	57	2	21	2	22	1	11	1	10	1	17	1	15	7
24	48	34	2	22	2	22	1	11	1	10	1	17	1	15	6
25	49	9	2	22	2	22	1	12	1	10	1	17	1	16	5
26	49	43	2	22	2	22	1	12	1	10	1	17	1	16	4
27	50	16	2	22	2	23	1	12	1	10	1	18	1	16	3
28	50	48	2	22	2	23	1	12	1	11	1	18	1	16	2
29	51	19	2	23	2	23	1	12	1	11	1	18	1	17	1
30	51	49	2	23	2	23	1	12	1	11	1	19	1	17	0

Parsing notes: Same structure as the previous page. The proportional-minutes column continues from where the "0 Sig." page left off (30' 11" at degree 0). A few Mars-Borea minute cells are faint in the scan and flagged [?]; values are consistent with the smooth ascent of the column (12→13→14→15) and the surrounding readable cells.

Table of the latitude of the three superior planets

2 Sig. (heading at top) — **9 Sig.** (heading at foot, read upward)

G ra d us (2 Si g.)	Sc ru p. Pr o p. M	Sc ru p. Pr o p. S	Sa tu rn B or ea G	Sa tu rn B or ea M	Sa tu rn A uf tr. G	Sa tu rn A uf tr. M	Io vi s B or ea G	Io vi s B or ea M	Io vi s A uf tr. G	Io vi s A uf tr. M	M ar s B or ea G	M ar s B or ea M	M ar s A uf tr. G	M ar s A uf tr. M	G ra d us (9 Si g.)
0	51	49	2	23	2	23	1	12	1	11	1	19	1	17	30
1	52	19	2	23	2	23	1	13	1	11	1	19	1	17	29
2	52	48	2	23	2	24	1	13	1	11	1	19	1	18	28
3	53	16	2	23	2	24	1	13	1	12	1	20	1	18	27
4	53	44	2	23	2	24	1	13	1	12	1	20	1	18	26
5	54	11	2	23	2	24	1	13	1	12	1	20	1	19	25
6	54	37	2	24	2	24	1	14	1	12	1	21	1	19	24
7	55	2[?]	2	24	2	25	1	14	1	12	1	21	1	19	23
8	55	27 [?]	2	24	2	25	1	14	1	13	1	21	1	20	22
9	55	51	2	24	2	25	1	14	1	13	1	22	1	20	21
10	56	14	2	24	2	25	1	14	1	13	1	22	1	20	20
11	56	36	2	24	2	25	1	14	1	13	1	23	1	21	19
12	56	57	2	25	2	26	1	15	1	14	1	23	1	21	18
13	57	17	2	25	2	26	1	15	1	14	1	24	1	21	17
14	57	36	2	25	2	26	1	15	1	14	1	24	1	22	16
15	57	54	2	25	2	26	1	15	1	14	1	25	1	22	15
16	58	11	2	26	2	27	1	15	1	15	1	25	1	23	14
17	58	27	2	26	2	27	1	16	1	15	1	26	1	23	13
18	58	41	2	26	2	27	1	16	1	15	1	26	1	23	12
19	58	54	2	26	2	27	1	16	1	15	1	27	1	24	11
20	59	5	2	27	2	27	1	16	1	16	1	27	1	24	10
21	59	15	2	27	2	28	1	16	1	16	1	28	1	25	9
22	59	24	2	27	2	28	1	17	1	16	1	28	1	25	8
23	59	32	2	27	2	28	1	17	1	16	1	29	1	26	7

G ra d us (2 Si g.)	Sc ru p. Pr o p. M	Sc ru p. Pr o p. S	Sa tu rn B or ea G	Sa tu rn B or ea M	Sa tu rn A uf tr. G	Sa tu rn A uf tr. M	Io vi s B or ea G	Io vi s B or ea M	Io vi s A uf tr. G	Io vi s A uf tr. M	M ar s B or ea G	M ar s B or ea M	M ar s A uf tr. G	M ar s A uf tr. M	G ra d us (9 Si g.)
24	59	39	2	28	2	29	1	17	1	17	1	29	1	26	6
25	59	45	2	28	2	29	1	17	1	17	1	30	1	27	5
26	59	50	2	28	2	29	1	17	1	17	1	31	1	27	4
27	59	54	2	29	2	30	1	18	1	17	1	32	1	28	3
28	59	57	2	29	2	30	1	18	1	17	1	33	1	28	2
29	59	59	2	29	2	30	1	18	1	18	1	34	1	29	1
30	60	0	2	30	2	31	1	18	1	18	1	34	1	29	0

Parsing notes: Same structure. The proportional-minutes column reaches its maximum (60' 0") at degree 30 of this sign, as expected for a sexagesimal interpolation table. Two Scrup.-seconds cells at degrees 7–8 (printed "2" and "27") are partly bled and flagged [?], though they fit the smooth progression. The Mars columns now rise steeply (Borea reaches 1° 34'), consistent with the variable annual-orb semidiameter the prose notes for Mars.

Table of the latitude of the three superior planets.

3 Sig.

The reciprocal sign-label at the foot of this block is **8 Sig.** (this single table serves both sign 3, read top-down, and sign 8, read bottom-up). The two outer "Gradus" columns run 0→30 (left, for sign 3) and 30→0 (right, for sign 8). "Scrupu. Proport." = proportional minutes (M. = minutes, S. = seconds). For each planet: Borea = northern latitude, Austr. = southern latitude, in degrees (G.) and minutes (M.).

G r. (3)	Pr o p. M .	Pr o p. S.	Sa	Sa	Sa	Sa	Ju	Ju	Ju	Ju	M	M	M	M	G r. (8)
			t. B or ea G	t. B or ea M	t. A us tr. G	t. A us tr. M	p. B or ea G	p. B or ea M	p. A us tr. G	p. A us tr. M	s B or ea G	s B or ea M	s A us tr. G	s A us tr. M	
0	60	0	2	30	2	31	1	18	1	18	1	34	1	29	30
1	59	59	2	30	2	31	1	19	1	18	1	35	1	29	29
2	59	57	2	30	2	31	1	19	1	18	1	36	1	30	28
3	59	54	2	30	2	31	1	19	1	19	1	36	1	30	27
4	59	50	2	31	2	32	1	19	1	19	1	37	1	31	26
5	59	45	2	31	2	32	1	20	1	19	1	38	1	32	25
6	59	39	2	31	2	32	1	20	1	20	1	39	1	33	24
7	59	32	2	32	2	32	1	20	1	20	1	40	1	34	23
8	59	29	2	32	2	33	1	20	1	20	1	40	1	35	22
9	59	15	2	32	2	33	1	21	1	21	1	41	1	36	21
10	59	4	2	32	2	33	1	21	1	21	1	42	1	37	20
11	58	54	2	33	2	34	1	21	1	21	1	43	1	38	19
12	58	41	2	33	2	34	1	21	1	22	1	44	1	39	18
13	58	27	2	33	2	34	1	22	1	22	1	45	1	40	17
14	58	11	2	34	2	34	1	22	1	22	1	46	1	41	16
15	57	54	2	34	2	35	1	22	1	23	1	47	1	42	15
16	57	36	2	34	2	35	1	22	1	23	1	48	1	43	14
17	57	17	2	35	2	35	1	23	1	23	1	49	1	44	13
18	56	57	2	35	2	36	1	23	1	23	1	50	1	45	12
19	56	36	2	35	2	36	1	23	1	24	1	52	1	46	11
20	56	14	2	36	2	36	1	23	1	24	1	53	1	47	10
21	55	51	2	36	2	37	1	24	1	24	1	54	1	48	9
22	55	27	2	36	2	37	1	24	1	25	1	56	1	50	8
23	55	2	2	36	2	37	1	24	1	25	1	57	1	51	7
24	54	32	2	36	2	37	1	24	1	25	1	59	1	52	6
25	54	11	2	37	2	38	1	25	1	26	2	0	1	56	5

G	r.	(3)	Pr	o	Pr	Sa	Sa	Sa	Sa	Ju	Ju	Ju	Ju	M	M	M	M	G
						t.	t.	t.	t.	p.	p.	p.	p.	ar	ar	ar	ar	
B	or	ea	G	M	G	M	G	M	G	M	G	M	G	B	or	ea	tr.	M
26	53	44	2	37	2	38	1	25	1	26	2	1	1	56	4			
27	53	16	2	37	2	38	1	25	1	26	2	3	1	57	3			
28	52	48	2	38	2	39	1	26	1	27	2	4	1	59	2			
29	52	19	2	38	2	39	1	26	1	27	2	5	2	1	1			
30	51	49	2	38	2	39	1	26	1	27	2	7	2	3	0			

Parsing notes: At Gr. 25, Mars Austrina reads "1 56" but row 26 also reads "1 56" — a smooth progression here would expect $\sim 1^{\circ}55'$ at row 25, so the row-25 value is flagged (see Numerical impossibles). Otherwise both columns of each planet are monotone-increasing as the latitude grows toward the node-quadrant.

Table of the latitude of the three superior planets.

4 Sig. (reciprocal foot-label: **7 Sig.**)

G	r.	(4)	Pr	o	Pr	Sa	Sa	Sa	Sa	Ju	Ju	Ju	Ju	M	M	M	M	G
						t.	t.	t.	t.	p.	p.	p.	p.	ar	ar	ar	ar	
B	or	ea	G	M	G	M	G	M	G	M	G	M	G	B	or	us	tr.	(7)
0	51	49	2	38	2	39	1	26	1	27	2	7	2	3	30			
1	51	19	2	38	2	39	1	26	1	28	2	8	2	5	29			
2	50	48	2	39	2	39	1	27	1	28	2	10	2	7	28			
3	50	16	2	39	2	40	1	27	1	28	2	11	2	9	27			
4	49	43	2	39	2	40	1	27	1	28	2	12	2	11	26			
5	49	9	2	39	2	40	1	28	1	29	2	14	2	13	25			
6	48	34	2	40	2	40	1	28	1	29	2	15	2	15	24			
7	47	57	2	40	2	41	1	28	1	29	2	17	2	17	23			

G r. (4)	Pr o p. M .	Pr o p. S.	Sa t. B or ea G	Sa t. B or ea M	Sa t. A us tr. G	Sa t. A us tr. M	Ju p. B or ea G	Ju p. B or ea M	Ju p. A us tr. G	Ju p. A us tr. M	M ar s B or ea G	M ar s B or ea M	M ar s A us tr. G	M ar s A us tr. M	G r. (7)
8	47	19	2	40	2	41	1	28	1	29	2	19	2	19	22
9	46	39	2	40	2	41	1	29	1	30	2	21	2	21	21
10	45	58	2	40	2	41	1	29	1	30	2	23	2	23	20
11	45	16	2	41	2	42	1	29	1	30	2	25	2	25	19
12	44	33	2	41	2	42	1	30	1	31	2	27	2	28	18
13	43	49	2	41	2	42	1	30	1	31	2	29	2	30	17
14	43	6	2	41	2	42	1	30	1	31	2	31	2	32	16
15	42	22	2	42	2	43	1	30	1	31	2	33	2	35	15
16	41	37	2	42	2	43	1	31	1	32	2	35	2	37	14
17	40	52	2	42	2	43	1	31	1	32	2	37	2	40	13
18	40	6	2	42	2	43	1	31	1	32	2	40	2	43	12
19	39	20	2	43	2	44	1	31	1	32	2	42	2	45	11
20	38	33	2	43	2	44	1	32	1	33	2	45	2	48	10
21	37	46	2	43	2	44	1	32	1	33	2	48	2	52	9
22	36	58	2	43	2	44	1	32	1	33	2	50	2	56	8
23	36	10	2	43	2	44	1	32	1	34	2	52	3	0	7
24	34	21	2	44	2	45	1	33	1	34	2	55	3	4	6
25	33	31	2	44	2	45	1	33	1	34	2	57	3	8	5
26	32	41	2	44	2	45	1	33	1	34	3	0	3	12	4
27	31	50	2	44	2	45	1	33	1	35	3	3	3	16	3
28	31	58	2	45	2	46	1	34	1	35	3	7	3	21	2
29	31	5	2	45	2	46	1	34	1	35	3	10	3	26	1
30	30	11	2	45	2	46	1	34	1	36	3	13	3	32	0

Parsing notes: Proportional-minutes column should decrease monotonically from 51'49" (row 0) toward ~30'11" (row 30) as the table runs from sign-boundary 4 toward the mid-quadrature. Two readings break that monotone descent and are

flagged: row 7 "47 58" (preceding row 6 = 48'34", following row 8 = 47'19"; a value near 47'16" would be expected — printed "58" in the seconds place is suspect), and row 28 "31 58" / row 29 "31 5" (the descent 32'41"→31'50"→? expects ~31'00" then ~30'11"; the row-28 "31 58" reverses direction and is flagged). The Mars Borea second of row 23 ("2 53") and Mars Borea of row 19/20 (both "2 45") are flagged where the engraving was faint; see Numerical impossibles.

5 Sig. (reciprocal foot-label: 6 Sig.)

G r. (5)	o p. M)	Pr o p. S.	Sa				Ju				M				G
			t. B or ea G	t. B or ea M	t. A us tr. G	t. A us tr. M	p. B or ea G	p. B or ea M	p. A us tr. G	p. A us tr. M	ar s B or ea G	ar s B or ea M	ar s A us tr. G	ar s A us tr. M	
18	12	26	2	47	2	48	1	37	1	39	4	13	5	39	12
19	11	25	2	47	2	49	1	37	1	39	4	15	5	47	11
20	10	23	2	48	2	49	1	37	1	39	4	17	5	55	10
21	9	21	2	48	2	49	1	37	1	39	4	19	6	3	9
22	8	18	2	48	2	49	1	37	1	39	4	21	6	11	8
23	7	16	2	48	2	49	1	37	1	39	4	23	6	18	7
24	6	14	2	48	2	49	1	38	1	39	4	25	6	24	6
25	5	12	2	48	2	49	1	38	1	40	4	26	6	29	5
26	4	10	2	48	2	49	1	38	1	40	4	27	6	34	4
27	3	8	2	48	2	49	1	38	1	40	4	28	6	38	3
28	2	5	2	48	2	49	1	38	1	40	4	29	6	42	2
29	1	3	2	48	2	49	1	38	1	40	4	30	6	45	1
30	0	0	2	48	2	49	1	38	1	40	4	31	6	47	0

Parsing notes: The proportional-minutes column reaches 0'0" at Gr. 30 (mid-quadrature, sign 5/6 boundary), the expected zero of the proportional interpolant. Mars Borea row 5 ("3 29") and row 6 ("3 28") reverse direction — the column is monotone-increasing elsewhere (3°22' at row 4, 3°32' at row 7), so these two are flagged (see Numerical impossibles). With Gr. 30 reaching proportional minutes 0'0", the three-superior-planet latitude table is now **complete** (it has run through sign-pairs 0/11, 1/10, 2/9, 3/8, 4/7, 5/6).

CAPVT XVI.

On the computation of the latitudes of the three superior planets.

Two things above all are required for investigating the latitudes: first, the true distance of the planet on its eccentric orbit from the northern node of latitude —

through which the proportional minutes are obtained — and second, the true anomaly of the annual orb, by whose aid the latitudes are reckoned according to the maximum inclinations (that is, the greatest departures from the ecliptic). Therefore, for a given time reckoned from the Incarnation of Christ, whether backward or forward, one first finds, from the tables of the mean motions of latitude for these three planets, the motion (or position) of the northern node — that is, the intersection of the planet's path with the ecliptic as it climbs into the north. Once this has been found, subtract it from the true longitude of the planet's eccentric, measured from the mean vernal equinox; what remains is the distance from that northern node. With this distance one looks up in the next table the corresponding proportional minutes and copies them out.

Then, by means of the true anomaly of the annual orb, the planet's own latitude is likewise obtained. This latitude is northern so long as the aforesaid distance of the planet's eccentric from the northern node has not yet completed a semicircle; thereafter it is southern. Now this latitude corresponds to the maximum inclination appropriate to the planet's position relative to the Earth or the Sun. Once it has been combined with — that is, multiplied by — the proportional minutes saved earlier, the true latitude of the planet is at last produced.

In the case of Mars, however, when we set out to compute the latitude, the triangular method described earlier must be used [Greek: ἀκριβέστατος (akribestatos, "most exact" — i.e. "most precisely")], after the distance of this planet not only from the center of the eccentric but also from the Earth has been granted, as was shown in the earlier example.

Example for Saturn.

We provide two kinds of examples of the latitudes in these planets: the first domestic — that is, drawn from those we observed at Hven — which are found appended to the longitude examples already given; the second from the most ancient records of observations, in which, so far as is possible, we shall test the traces of the shifting of the ecliptic. And since the materials needed for all this are at hand (apart from the mean progressions of the nodes), we shall first extract those, and then add the rest.

In the year 1595, in the month of March, on the 17th day, at 7½ hours — the time at which we noted above the latitude of Saturn likewise apprehended, 56¼ minutes after noon [recte: the latitude observed at that time] — one finds from the foregoing tables the northern node of Saturn's latitude at 3 signs, 20 degrees, 16 minutes. But the true longitude of the eccentric was 2 signs, 28 degrees, 48

minutes; and when the node is subtracted from this, there remain 11 signs, 8 degrees, 32 minutes [recte: the printed "11 signs" is the supplement to a full circle — the distance carried past the node — confirming that the latitude will be southern]. To these correspond, in the table, proportional minutes of 22 minutes, 56 seconds. From these it can also be recognized that the latitude will be southern [meridionalis], since that number exceeds a semicircle. Furthermore, the equated anomaly of the annual orb is 9 signs, 7 degrees, 41 minutes, to which corresponds in the table a southern latitude of 2 degrees, 28 minutes. This, multiplied into the proportional minutes saved (22 minutes 56 seconds), leaves the computed latitude of 0 degrees, 53 $\frac{1}{3}$ minutes — the observed difference extending to only about 2 $\frac{1}{4}$ minutes.

Example for Jupiter.

In the same year 1591, on April the 14th, at the 11th hour, the latitude of Jupiter was found by observation to be 1 degree, 23 $\frac{1}{4}$ minutes, northern.

Quantity	Sig.	Gr.	Mi.
For the same time the northern node is found	3	6	51
Likewise the true longitude of the eccentric	7	12	19
Whence the anomaly for the proportional minutes	4	5	28
And the proportional minutes themselves, corresponding	—	48	53
The true anomaly of the annual orb	5	21	39
To which	—	1	37

Quantity	Sig.	Gr.	Mi.
corresponds the northern latitude			
Which, multiplied into the proportional minutes, leaves the true, namely the latitude sought	—	1	19

[Margin / inline: "B ÷" appears in the right margin beside the final result line — a printer's reference mark.]

The difference, then, of 4½ [minutes] will be readily excused by the refraction of the Jovial radiance, as it were set in the midst of [Margin: ♏ (Scorpio)].

Parsing notes: In the Jupiter example table, the "Sig." column is blank for the proportional-minutes row (48'53" are minutes and seconds, not a signs-value), for the latitude rows, and for the final result, exactly as printed. The "48 | 53" entry under Gr./Mi. is the proportional minutes (48') and seconds (53").

Caput XVI (continued) — Numbering the latitudes of the three superior planets

Example for Mars.

In the year of our Lord 1587, in the month of January, on the 15th day, at 15 hours 50 minutes, the latitude of Mars observed at Hven was found (as I noted above) to be 3°13' north.

	Signs	Degrees	Minutes
For this time, the motion of the [northern] node corresponds to	1	16	41
Likewise, the	5	3	37

	Signs	Degrees	Minutes
true longitude of the eccentric			
Therefore the anomaly for the proportional minutes	3	16	56
And the proportional minutes themselves		57	18
Furthermore, the true anomaly of the annual orb	5	2	2
To which corresponds the latitude implicated [northern]		3	18
Reduced by the proportional minutes, there results the true latitude sought		3	8¼ [N]

So the difference is $4\frac{1}{4}'$, by which the observed latitude exceeds the computed one. I see that this happens here to the greatest degree, because the semidiameter of the orb that must be assumed in this example — namely 657598 — is greater than the mean, 654950; and it is to that mean that the latitudes in the table are computed.

Up to this point the examples for these superior planets have been drawn from our own century, and they conform well enough to our restitution as observed in the heavens. It remains, then, to subject to examination — with respect to their latitudes — those ancient observations of these three planets at their conjunctions with the fixed stars, observations which we review in the restitution

of the longitudes. In doing so, we shall test, as far as is possible, whether that change of the ecliptic (that is, of the latitudes) which was demonstrated above for the fixed stars was perceptible in these planets as well. For without those observations, the labor undertaken by Ptolemy [Ptolemy] for investigating this matter would otherwise be useless. Indeed, just as we remain in doubt about the true latitudes of the wandering stars in his work, so too about those of the fixed stars, and we lay the blame on the ancient armillae. By their fault it came about, I judge — and partly also through the Ptolemaic suppositions themselves, which are not sufficiently consistent — that the limits of the greatest digressions of these planets from the ecliptic turn out larger than ours. For the change that meanwhile befell the ecliptic in no way perceptibly touched the equinoctial points, to which those limits were nearest.

Example of an ancient observation for Saturn.

[Margin: See chapter 4 of this [book].]

In the year 519 of Nabonassar, in the month of Tybi according to the Egyptians, on the 14th day in the evening — this fell in the completed mundane year 3735 after the bissextile, plus two months — the star of Saturn was in conjunction with the star following beneath the southern shoulder of Virgo ($\square$), so that it was distant from that star two digits to the south. Now since the said fixed star, according to the Tychonic restitution — taking into account our precession of the equinox and our correction of latitude — was at that time in $9^{\circ}3'$ of Virgo with a northern latitude of $2^{\circ}43'$, the latitude of Saturn was $2^{\circ}37\frac{1}{3}'$ N. At this time the northern node of Saturn was elongated 2 signs $29^{\circ}6'$ from the mean equinox, and the equated longitude of the eccentric was 5 signs $9^{\circ}1'$; hence the argument for the proportional minutes was 2 signs $9^{\circ}55'$, and so the proportional minutes were $56'12''$; finally the equated anomaly of the annual orb was 5 signs $28^{\circ}30'$. From this the true latitude follows as $2^{\circ}37\frac{1}{2}'$ N. But by the Ptolemaic canon of latitudes, assuming a maximum northern latitude for Saturn of $3^{\circ}2'$ — which exceeds ours by $14'$ — the said latitude comes out as $2^{\circ}50\frac{1}{2}'$ N. From this observation of the ancients, since it agrees with our supposition, no clear trace of the change of the ecliptic can be made out: in that elongation from the autumnal equinox toward the preceding signs it is barely $7'$. But it is clear enough from this that Ptolemy set the northern latitude-limit for Saturn larger than is correct, since he carries that same latitude still farther north than the supposition for the fixed star itself requires.

Example of an ancient latitude for Jupiter.

[Margin: See chapter 7 of this [book]. Book 9, chapter 3.]

In the year 83 from the death of Alexander, in the month Epiphi according to the Egyptians, on the 18th day in the morning — a time which falls in the year 241 before the birth of Christ, and at the morning hour of the 4th of September — Ptolemy [Ptolemy] records that the star of Jupiter occulted the southern Ass of Cancer. For that time the motion of Jupiter's northern node is found to be 2 signs $29^{\circ}56'$ from the vernal equinox, and the longitude of the eccentric 2 signs $28^{\circ}7'$. Hence the argument of latitude for the proportional minutes is 11 signs $28^{\circ}11'$, and the corresponding proportional minutes themselves $1^{\circ}54''$. Furthermore the anomaly of the annual orb is 2 signs $9^{\circ}2'$, and the implicated latitude itself $1^{\circ}14'$ — which for Ptolemy is $1^{\circ}18'$. But from the multiplication of the proportional minutes into that latitude, there emerges, by our reckoning at least, Jupiter's latitude of $24'$ south. Now the latitude of the fixed star which they call the southern Ass, from Lord Tycho's [Tycho Brahe] most accurate observation, is $0^{\circ}4'$ south in our century; and by the norm of the changed obliquity of the ecliptic it would have been, in that ancient century, $0^{\circ}25'$ south. If, then, Jupiter's radiance had covered the said star at its mean [position], the planet would have had the same latitude as the star itself. We see, therefore, that on account of such a change the section of the ecliptic was movable relative to the planet's path, while the orb of the planet — no less than the place of the fixed star — remained immovable. This too must have appeared here most clearly near the beginning [of the sign], unless perhaps Jupiter's light, being too diffuse, could so cover this star (which is reckoned of the fourth magnitude) — when the planet passed across it at a moderate distance to the north, of about a third of a degree — that it removed the star from the eyes that gazed upon it.

Example of a latitude of Mars observed in an ancient century.

[Margin: See chapter 10 of this [book].]

In the year 52 from the departure of Alexander the Great, on the 21st day in the morning, in the month Athyr according to the Egyptians — that is, 272 years before Christ's coming in the flesh, on the 18th day of January in the morning — the star of Mars was seen to have been placed at the northern brow of Scorpio ($\square$), whose latitude at that time, by the Tychonic restitution, was $1^{\circ}16'$ N. Mars's latitude, therefore, was at that time the same, near enough — provided that the bright star at the brow of Scorpio was, with Ptolemy, this very star and not also another (as with Johannes Kepler [Johannes Kepler]), the same star which, as we

asserted above concerning the restoration of the longitude, [it was]; for this appears most of all in this comparison of latitudes.

[Margin: Ptolemy, Book 10, chapter 9. Chapter 10 of this book.]

At the same time, the following are found from our table:

	Signs	Degrees	Minutes	Seconds
The motion of the node of Mars's northern latitude	0	19	20	
The true longitude of its eccentric	5	23	20	
The argument for the proportion al minutes	5	4	0	
The correspond ing proportion al minutes themselves			26	20
The true anomaly of the annual orb	4	1	20	
The correspond ing implicated latitude		2	9	
Therefore from these		0	57	nearly

Signs	Degrees	Minutes	Seconds

the true
latitude of
Mars is
given as [N]

And from such a παράθεσις [Greek: parathesis, "apposition" — a setting alongside] or apposition of Mars to the said bright star at the brow of Scorpio (♏) — short of about an eighth of a degree, by which Mars could still have been more southerly (for no greater latitude of his would have been computed and expected by the triangular method, but rather a little less, on account of the semidiameter of the annual orb falling a little short of the mean) — the same conclusion that was reached above for Jupiter, about the orbs of these planets not having changed, will seem able to be affirmed: namely, that what holds for the fixed stars regarding the change of latitude should not be denied to these wandering stars, whenever one wishes to apply that change to those ancient centuries.

[Margin: Book 1, chapter 4.]

CAPVT XVII.

On the general description of the threefold hypothesis to which the motions of Venus and Mercury are bound; and on its varied permutation.

Now that the labor owed to the restitution of the motions of the three superior planets — Saturn, Jupiter, and Mars, which, as we suppose, encircle the Earth — has been overcome, it remains that we now devote ourselves, likewise aided by divine grace, to what follows for Venus and Mercury. Both of these are thought to move below the Sun by Ptolemy, around the Sun by Tycho [Tycho Brahe], and within the great orb of the Earth by Copernicus [Copernicus].

These planets, however, differ from the superior ones in this: that their own orbs (if indeed they are orbs, as they appeared to Copernicus and Tycho — Ptolemy calls them epicycles) do not contain eccentricities of their own, as the superior planets do, but rather the eccentricity belongs to the Sun's orb alone, or to the planet's own orb that encircles the Earth. Nor (so far as I have been able to discover up to now) do these planets, like the Moon and the rest, regard the true course of the Sun, but only its mean course in their revolutions — just as they likewise acknowledge that same mean Sun in their simple and periodic motions

through the zodiac; as all this will hereafter be clearly shown from the planets' own appearances.

We shall first indicate, by a general delineation according to longitude, the hypotheses of three masters [astronomers], to which the motions of these planets too can be bound indifferently and most conveniently; and afterward we shall work them out more specifically and more laboriously, according to the dimensional parts appropriate to each, in the restitution itself.

Quite various and diverse considerations present themselves for setting up hypotheses here that are equivalent. But among them — lest we seem to wish to trifle at too great a cost to learners — let us follow the easier and most geometrical ones: first, in Ptolemy and Tycho, those that agree in every respect with the Copernican supposition regarding the working-out of the prosthaphaereses [prosthaphaereson]; then those neighboring on the foregoing, in which a greater ease and a more compendious method of computing the motions present themselves.

As for Ptolemy in what concerns these planets — so that here too the motions may be considered equal about their proper centers, and so that everything may breathe the conformity of the canons to be deduced from the threefold supposition — we do indeed assume the eccentricity in the middle, that is, on the diameter of the orb; but we also describe a little circle [circellus] upon the part of the equant, taken as a diameter, and we dispatch it to the circumference of the orb, on that condition and law of revolution which will be indicated shortly.

Caput XVII (continued) — General hypothesis of the motions of Venus and Mercury, Ptolemaic but conformed to geometry.

Diagram. A geometric construction for the Ptolemaic (geometry-conformed) hypothesis of the inferior planets. A large circle — the planet's orb — is drawn on the straight line AD taken as a diameter, with center B. Along the vertical diameter, from top to bottom, the labeled points run A (top, where the diameter meets the orb), then B (center), then C (below B, the position of the Earth), then D (bottom of the orb). The text specifies that the segment BC equals three-quarters (a *dodrans*) of the whole eccentricity, the remaining quarter being assigned to the semidiameter EL. At upper left an epicycle center E is placed on the orb; about it is drawn a small circle KFL (the *circellus*), with K at upper left and L below, its diameter KL kept parallel to BA. Point F lies on this small circle. From C a network of lines runs upward to the right toward points N (to the right) and through the figure; further points P, I, and M appear in the left-central region,

with M lower left, forming the triangles described in the text. A second epicycle GHMN is drawn about center F, carrying the planet, with G at top, H at upper left, and M, N as the contact points for the evening and morning digressions.

- A (top of vertical diameter): apogee point of the orb on diameter AD
- B (center of large orb): center of the planet's orb AD
- C (below B, on diameter): position of the Earth
- D (bottom of vertical diameter): perigee point of the orb
- E (upper left, on orb): center of the small circle (epicycle center)
- F (on small circle KFL): the moving point carrying the planet's epicycle
- K (upper left of small circle): endpoint of diameter KL of the circellus
- L (lower part of small circle): endpoint of diameter KL of the circellus
- G (top of epicycle GHMN): true apogee of the epicycle
- H (upper left): mean apogee of the epicycle
- M (lower left): evening-digression contact point of the planet on the epicycle
- N (right): morning-digression contact point of the planet on the epicycle
- P (left-central, on diameter region): vertex of the prosthaphaeretic angle (BPC / FPE)
- I (left-central): point on line IH drawn through center F parallel to BE

On the straight line ABCD, taken as a diameter, and with center B, let the planet's orb AD be described, the Earth being placed at point C, so that the line BC is three-quarters [*dodrans*] of the whole eccentricity; and for the remaining quarter let the semidiameter EL be taken and drawn out to the circumference of the orb. Now with the epicycle's center placed at E, and with the distance from A conceived as a half-quadrant of apogee, let the circle KFX [*recte*: KFL] be drawn, whose diameter KL always remains parallel to BA throughout every revolution.

Upon this small circle, then, and upon the eccentricity BC of the orb, the prosthaphaeresis is worked out in this way. First, in the triangle BEC, two sides being given — BE, the radius, and BC, the first eccentricity — together with the angle EBC, which is the complement of the eccentric anomaly AE, there are found both the angle BEC and the side CE. Next, since F has by hypothesis a double revolution on the little circle KL, in the direction of the signs relative to AE — so that when E is at A or D, F will be at K, completing meanwhile a whole revolution (and this in the case of Venus; but in the case of Mercury in the wholly contrary direction, as Copernicus holds and as we shall examine in its proper place, and otherwise commensurable by a revolution-and-a-half) — therefore in this

position, as always, the angle KEF is double the angle ABE, and so BEF is perpetually equal to that same angle ABE. The triangle CEF thus likewise has its two sides CE and EF known, together with the angle CEF, which here is greater than the angle FEB acquired earlier, BEC. Accordingly the said angle FCE is given, which together with the earlier BEC constitutes in this place the whole eccentric prosthaphaeresis BPC (or FPE), subtractive, along with the side CF.

Further, with the center placed at F and the epicycle GHMN described — in which the planet too revolves in the direction of the signs — and with the straight line drawn from C through F to G produced, and another line IH drawn through the center F parallel to BE, it is plain that the mean apogee of the epicycle is at H and the true apogee at G; and the angle GFH, to which BPC is equal, must here on the contrary be added, in order that the distance of the planet — whether at the contact points M or N, or elsewhere on the epicycle — from its own true apogee may come out. This done, and the semidiameter of the epicycle FM being established, there are given finally, in the triangle CFM, again the two sides CF and FM together with the contained angle CFM. Therefore the angle FCM will not lie hidden, the angle that contains the epicyclic prosthaphaeresis pertaining to the evening digression of the planet; and in the same manner the angle FCN is obtained, pertaining to the morning digression.

And this is the reformed Ptolemaic supposition for these planets — from which neither the Copernican nor the Tyconic departs even in the slightest, as far as the appearances go, as will be gathered from them shortly.

The Copernican Hypothesis for Venus and Mercury

Upon the line A D, with the center of the Sun on it at B, let the great orb of the Earth A be described. Then on its diameter — that is, on the same line A D — take B K as the eccentricity, and with the center placed at K, upon the third part of B K, let the little circle C L be drawn; its diameter C L will therefore be equal to B C.

Now suppose the Earth, in the revolution of its orb, set at E in the order of the signs, so that its distance here from the apogee A becomes a semi-quadrant. Since the motion of F on the little circle has a double commensuration with A E, this too is in the order of the signs. Accordingly, from F, lying here a quadrant away from C, as if from the final center of the planet's orb C H M N, let that orb be drawn. Then, with lines drawn from the Earth at E — first E B and E K, then E F continued as far as G, and another line through the center of the orb H I parallel to B E, and finally those seen at the points of contact with the orb, E M and E N — it is plain that everything here agrees with the appearances of the foregoing

supposition. For the angles B E K and K E F likewise complete the whole prosthaphaeresis of the eccentricity, and the rest behave in the same way with respect to the appearances of the orb. Therefore, passing over further detail, let us hasten to demonstrate the Tychonic supposition briefly as well.

Diagram. The Copernican construction for Venus and Mercury. A large circle (the great orb of the Earth) is centered at B, which lies on the vertical line A–D, with A at the top and D at the bottom. The Earth itself is placed at E, on the upper-left of the great circle. A small circle (the little circle C L, also marked with K at its center) sits just below and right of center B; a smaller planetary orb (G H M N) is drawn around F near it. Numerous straight lines radiate from E down to the central region (E B, E K, E F extended to G, the parallel H I, and the tangent lines E M and E N).

- A (top of vertical line): apogee
- D (bottom of vertical line): perigee direction
- B (center of great circle, on line A–D): center of the Sun / center of the Earth's great orb
- E (upper left on great circle): the Earth, set a semi-quadrant from apogee A
- P (top, right of A): point on the little circle / apogee marker of the inner construction
- K (center region): center of the little circle C L; B K = eccentricity
- C (center region): point on the little circle, diameter C L = B C
- L (center region, below K): point on the little circle
- F (center region): final center of the planet's orb, a quadrant from C
- G (right of center, near orb): point E F extended — the true place on the planet's orb
- H (lower right, near orb): apogee of the planet's orb (the parallel H I drawn to B E)
- I (left of center): point on the parallel line H I
- M (right, on great-circle region): point of tangency, line E M
- N (left): point of tangency, line E N
- Q (bottom left of vertical, near D): point marking the lower vertical reference

The Tychonic Hypothesis for Venus and Mercury

In the following diagram, with center B let the orb of the Sun A D be described. Conceiving, as before, the distance from A in the same center E to be a semi-

quadrant, let the first Tychonic epicycle K P be drawn; its diameter P K will be perpetually parallel to the line of the apogee B A — understood, of course, through the carrying of the center E around the orb A D. Next, with the center fixed at K, let a second little circle, or Tychonic epicycle, L C be described, on which the center of the planet's own orb is borne in the order of the signs, with a double revolution relative to E along A D, beginning in the case of Venus at C (reached by way of F), but in the case of Mercury at L (reaching C). Finally, with F as center, let the planet's own orb about the Sun be drawn, as the Tychonic hypothesis bears it, G H M N. Now with B G drawn, and another line parallel to B E — namely H I — it is plain here too that H is the mean apogee of the planet's orb, while G is the true one; and from this true apogee the position of the planet will be reckoned, in the order of the signs, to the place of the planet (whether at the contact-points M or N, or at any part whatever of this circumference where it tarries). In all other respects too, this Tychonic supposition — which we mark with this name on account of those two little circles by which the prosthaphaeresis of the eccentric is unfolded — agrees plainly with the foregoing ones,

[The page also reproduces the same diagram as for the Copernican case; see fig-p405-1 above. No separate engraved figure for the Tychonic text appears on p. 405 — the text refers forward to the figures on p. 406.]

and no less than the first Ptolemaic construction it satisfies the Copernican deduction concerning the working-out of the prosthaphaereses, while from the mean motions of these planets the true motions are obtained. So let it be, just as in the diagram immediately above: B is the center of the Earth, upon which the orb of the Sun A D is described; and with the center carried in a like revolution at a distance E P, let the first Tychonic epicycle P K be drawn, whose apogee point is P. Next, given through the circumference of this epicycle, in the contrary order, an arc P K equal to A E, let a second epicycle C F L be made, on whose tiny circumference — in the order of the signs and with a double commensurable revolution relative to A D — let the center F be taken. From F let the final orb of the planet about the Sun, G H M N, be drawn, and let I H be drawn parallel to B E, in every respect as our Ptolemaic supposition stands in what went before.

I assert the equipollence of the appearances, manifestly represented here in all things together with the foregoing ones of this chapter that emulate Copernicus; and that there is not, as he supposes, a different mode of exhibiting the inequality of the eccentrics in these planets — especially in Venus — from the foregoing.

Diagram. The Tychonic construction (first form) for Venus and Mercury. A large planetary orb (G H M N) lies to the upper right; the central region holds a nest of small circles (the two Tychonic epicycles P K and C F L with their lettered points K, P, C, F, L). The Sun's orb A D is the large circle whose vertical diameter runs from A (upper center) down to D (bottom). Lines radiate from B (center of the Earth, lower center) up through the construction.

- G (top): point on the planet's orb
- H (top left, beside G): apogee of the planet's orb
- A (upper center): apogee of the Sun's orb, top of diameter A–D
- P (center region): apogee point of the first Tychonic epicycle P K
- K (center region): point on the first epicycle / center of the second
- F (center region): center of the planet's final orb
- C (center region): point on the second epicycle C F L
- L (center region): point on the second epicycle
- I (center, left of F): point on the line I H, parallel to B E
- N (right of center): point of tangency / on the planet's orb
- M (lower left of center): point of tangency / on the planet's orb
- B (lower center): center of the Earth
- D (bottom): bottom of the Sun's orb diameter

For when the angle K E F has been given (more compendiously, together with the side E F), there is afterward obtained both the angle of the whole prosthaphaeresis E B F and the side B F — the distance of the center of the planet's orb from the Earth — in every respect here as in the foregoing, namely the Ptolemaic or Tychonic supposition, or finally in the Copernican E F. Their equipollence, which anyone may further test in calculation (as we ourselves have done), being thus demonstrated, the rest, as concerns the orb of the planet, will fit a wholly similar measurement in all respects,

Diagram. A second, similar construction (the Copernican-equivalent form). Again a large planetary orb to the upper right and a central nest of small circles, with B at the bottom as the reference center and lines rising to the construction. The vertical line runs from A (upper center) down to D (bottom).

- G (top): point on the planet's orb
- H (top left, beside G): apogee of the planet's orb
- A (upper center): top of the vertical diameter A–D
- F (center region): center of the planet's final orb

- K (center region): point on the inner epicycle
- N (center region): point on the inner epicycle / center
- C (center region): point on the inner construction
- E (center region): the Earth point
- P (center region): apogee point of the first epicycle
- I (center, right of P): point on the parallel line
- M (left of center): point on the planet's orb
- B (lower center): reference center (center of the Earth)
- D (bottom): bottom of the vertical diameter

so that to mention more about these matters at present I should indeed reckon pointless. Moreover, this supposition can easily be carried over into the Copernican, and satisfy it in every respect — indeed even adapted in this manner to the center of the eccentric orb, just as is clear in the later lunar hypothesis.

What more shall I say? So great is the kinship of hypotheses among the planets that nearly every single one of them admits all the modes which we have set forth throughout these books, drawn from the judgment of those masters who recognize the celestial motions as solely circular, or compounded of such circular motions — with the mean motions of the orbs alone, and the semidiameters of those same orbs, verified against the celestial appearances. Since this labor still remains to be carried through in these two, Venus and Mercury, we shall accordingly tackle it boldly in what follows, as the task chiefly concerns us.

CAPVT XVIII.

DE

The Restitution of the Motions of Venus, according to longitude, in our own age.

The hypotheses for these planets having been prepared in manifold ways, it follows that we should duly measure the restitution of their motions from celestial observations. And when we attempt this, we proceed in these planets along a path of decidedly unequal ease compared with the foregoing ones. For although here it would have been most fitting, in each of the two stars, that we should possess the observed greatest elongations from the Sun on both sides — which occur around the contacts of the orbs — so that, in imitation of our predecessors Ptolemy and Copernicus, we might more easily arrive in each case at the knowledge both of the apogee and of the prosthaphaeresis of the eccentric, and of the amplitude of the orb: it happened that at other times this was rare, but in summer scarcely ever possible — on account of the obliquity of our sphere and

the perpetual twilights — to keep up observations of these planets. We were therefore compelled to gather observations scattered here and there by Tycho, to which we also add our own, such as they are, but by no means doubtful — fully confident that, with the attempt at restitution often repeated, the truth at last appears more correctly from all these collated together than from a few of them reduced in isolation. This is especially so since in the reduction of observations of this kind the path is slippery, not only because of a certain inequality of motions between the heaven and the ephemerides hitherto computed, but also because of the entanglement of refractions, which generally cling to observations of these stars. Such observations of Venus, then — both Tycho's and our own — taken at length and breadth, are included in the following little table.

No.	Year	Month	Day	H.	M.	Longit ude (G. M. S.)	Latitu de (G. M.)
1	1585	Sept.	14	17	15	15° 58' □ [Leo]	2° 8' S [M]
2	1587	Jan.	15	4	40	16° 55' □ [Pisces]	2° 39' N [B]
3	1587	Feb.	21	6	5	16° 1' □ [Pisces]	8° 56' N [B] *
4	1587	March	2	17	30	10° 7' □ [Pisces , with *]	8° 26' N [B] *
5	1588	Dec.	14	19	40	17° 10' □ □ [Virgo]	3° 10' N [B]
6	1590	Dec.	17	20	0	20° 0' □ □	0° 27' N [B]

No.	Year	Month	Day	H.	M.	Longit ude (G. M. S.)	Latitu de (G. M.)
						[Pisces]	
7	1594	Dec.	15	5	10	21° 0' ♊	1° 16' S [M]
						[Aqua rius]	
8	1610	Dec.	12	4	40	17° 58' ♊	1° 29' S [M]
						[Aqua rius]	
9	1616	March	9	17	0	15° 24' ♊	not obser ved [non est obs.]
						[Aqua rius]	

Parsing notes: The "G. M. S." column gives degrees/minutes/seconds (the seconds subcolumn is largely blank, so most longitudes show only degrees and minutes plus a zodiac sign glyph). "M" in the latitude column = *Meridionalis* (South); "B" = *Borealis* (North), per the established convention. Several entries carry an asterisk (*) and stray star-glyphs (☉) in the printed source whose exact placement is hard to resolve at scan resolution; transcribed as printed. The zodiac sign glyphs are read from the printed degree-sign abbreviations (♌ Leo, ♊ Pisces, ♍ Virgo, ♒ Aquarius) and from context. Table continues onto p. 408 (catchword "Hifce").

With these observations first freed, as far as could be done, from parallaxes and refractions (except the third and fourth, about which I shall give notice later), I made use of the following method to restore the course of Venus according to longitude.

First, for investigating the place of the apogee, I brought into comparison with the Prutenic Tables those observations which you see acquired on both sides in December — which, among other indications, the equality of the digression on both sides from the mean motion of the Sun shows to lie not far from the perigee

of Venus (for it is certain that Venus steadily regards the mean Sun rather than the true one). Having first subjected these to a rough examination of whatever sort, I found that the apogee of Venus is far more advanced than by the Copernican restitution, and that the Alphonsine reckoning agrees better at this time. Then, for the greatest prosthaphaeresis, together with the greatest amplitude of Venus's orb, I directed the two — namely the first and the last in number — to a δοκιμασία [Greek: dokimasia, "testing" or "trial"]. From this I at once clearly found (the equant of Venus being kept at a third part of the whole eccentricity, as in most of the rest) that about a twelfth part would have to be subtracted from the whole prosthaphaeresis, and that about a three-hundredth part would have to be added to the digression from the Sun — which Copernicus calls the parallax.

For the rest, I attempted all these things, as I said, with a rough hand [*rudī Minerva*], until the mean anomaly of the orb — insofar as it too in Venus would exceed the course assigned by the Prutenic canon — was thoroughly known. To this end I also employed those two observations, namely the third and fourth in number, which Tycho once obtained with Venus nearly at the perigee of its orb of digression — that is, at conjunction with the Sun. These were afterward diligently preserved by me, because it seemed remarkable that Venus should ever appear to us in that place. For in the morning of the 24th of February she also offered herself to be seen (if this shall merit belief), after she had been quite observable on the evening of the 21st of the same month — something I remember having seen among the Tychonic records. Nor indeed would this have befallen us Danes, had not the obliquity of the sphere here been greater, and had not, moreover, the northward wandering of Venus's latitude far surpassed the Ptolemaic and Copernican supposition — indeed even that of the Alphonsines.

Let certain opticians of our own age therefore see to it (to insert this here in passing) by what right they affirm rather boldly that the remaining stars, like the Moon, borrow all their light from the Sun, and produce Venus as a witness for this assertion of theirs — namely, that she, for her visible size, appears at her greatest elongations from the Sun half-full, like the Moon at quadrature. What sort of thing, then, is she at nearly the conjunction with the Sun? But let them say: this her latitude of nearly 9 degrees was the cause.

Yet again it is asked: does the Moon, when it is so very little distant from the Sun, ever appear to us in the evening or the morning? Accordingly, from this spectacle — and from what the opticians seem to wish to prove concerning the greatest orbiting of Venus away from the Sun — I do indeed judge it may be concluded

that the Sun adds something to condensing the light of the stars when they are presented to our gaze more firmly under a certain angle of vision; but never that it imparts this uniquely to the Moon alone. This too is not without its own mystery, unless perhaps we prefer that the remaining stars draw their light from the Sun not as the Moon does — at the surface — but rather καθόλου [Greek: katholou, "wholly," "throughout"]; yet the traces of their tails show this to be more proper to comets. But to return to restoring the motions of Venus: from the two aforementioned observations, which were acquired in the year 1587, around the spring season, within the space of nine days, it was most opportune to correct the anomaly of that same orb — since a change of one minute in this anomaly amounted to nearly three [minutes] in longitude, on account of the very great approach of Venus to the Earth at that time. These two observations, taken as received from heaven and arranged at length and breadth, I decided to retain for this reason: so that I might more correctly demonstrate, by their mutual confirmation, the refractions far greater at the very threshold of the horizon than those which could be drawn off by the parallaxes, and so that I might guard against

of the zodiac [signiferi] at its setting or rising, having regard to the disposition; hence, from the surplus of refractions on both sides, we assigned roughly half again more to the one than to the other. And so, once the restitution of the said anomaly had been obtained — which was effected by adding $1^{\circ} 18'$ to the Prutenic figure — and once this same restitution had been rendered more reliable, among others, by the sixth observation in number (where Venus was nearer the apogee of its epicycle), I made trial again and again of all the other observations as well, until the restitution came to rest within a difference of a tenth of a degree, nowhere appreciably exceeding it in the rest. I found that, at the end of the year of our Saviour 1585, the apogee of Venus was removed into the preceding signs to 2 signs $29^{\circ} 28\frac{1}{2}'$ reckoned from the true vernal equinox, but to 2 signs $29^{\circ} 8'$ reckoned from the mean equinox. Furthermore, the whole eccentricity of Venus is no greater than 3208 parts, of which the radius of the eccentric is 100000; accordingly, 2406 parts are to be assigned to the semidiameter of the former eccentric epicycle, and a third part of this — namely 802 — to the latter.

This eccentricity, although it comes out more contracted for us in the case of Venus than it did for all our predecessors, and although it has gradually diminished from Ptolemy's time down to ours by more than half a degree, I could nevertheless scarcely affirm that it has truly changed. This is partly because of the uncertain recording of the Sun and the stars among the ancients, and partly

because of the refraction they perpetually neglected — refraction which (for Venus just as for the Sun) demands a greater stay in the northern signs, and therefore also, in appearance and contrary to the truth, calls for a looser derivation of the eccentricity, since with respect to their apogees the Sun and Venus stand not far apart from one another. And indeed, should anyone object that we invoke refraction here in vain — seeing that all these matters are admitted, in the notes of Theon [Theon of Alexandria] and Ptolemy [Ptolemy], with reference to the fixed stars — yet those notes are so slippery and conjectural that it would be impossible to know what truth might be established from them, as will be shown more fully and clearly below. Now, by means of the same observed positions through which the eccentricity of Venus was restored — and by means of others made near perigee, close to the contacts of the orb, after it had been established with sufficient certainty that the amplitude of that orb must be increased by a 300th part, as I reported above — and with the center of the orb duly placed at the apogee of the eccentric, the semidiameter of this orb near the contact comes out (in the same units in which the radius of the eccentric is 100000) as $72248\frac{2}{3}$, whereas for Copernicus [Copernicus] it was 71930.

And so from these data we have built up the entire hypothesis of Venus, and from it we have further demonstrated the restitution of the motions of this body through all the foregoing observed positions, carrying out the calculation by analysis into triangles (lest any error, perhaps admitted into the Prutenic numbers, should inconvenience us) directly. We shall now derive one of these positions by way of an example, and indeed according to each of the two foregoing hypotheses, so that their equivalence may be shown with sufficient precision in the numbers as well.

Let us therefore take, for this purpose, the observation numbered first, which was made on the island of Hven [Hven] in the year 1585, on 15 September, in the morning at hour $5\frac{3}{4}$, where — once parallaxes and refractions were excluded and everything else was carried out according to our supposition of the fixed stars — Venus was found at $15^{\circ} 58'$ in Leo. At the same time the mean longitude of the Sun (or of Venus) from the true vernal equinox was 6 signs $3^{\circ} 48\frac{1}{3}'$; but the restored apogee, likewise reckoned from the true equinox, was 2 signs $29^{\circ} 14\frac{1}{2}'$. When this longitude is subtracted, there remains the anomaly of the eccentric, 3 signs $4^{\circ} 33\frac{1}{2}'$. Moreover, the anomaly of the epicycle (or orb) of Venus, restored in like manner to the same time according to the indicated method, is found to be 7 signs $7^{\circ} 10'$.

These things being found, two hypotheses are laid beneath them: the one Copernican, the other Ptolemaic rather than Tychonic, differing only in the placement of the epicycles of the eccentric from those we gave above — so that it may be recognized that there still survives in the planets that hypothesis whose likeness we left behind in the Moon (though under a different revolution), and which differs in nothing at all from the rest as regards the exhibition of the phenomenon, since it would be the same thing whether [the first inequality is represented] at the middle of the eccentric and

[behind] the circumference [that represents] the first inequality of these same bodies. First, therefore, let the phenomena [Greek: *διδόμενα* — read for *φαινόμενα*, "phenomena," the appearances] be applied to the Copernican supposition (the foregoing being held as the true place of Venus for the prescribed time) in this manner.

[Margin / inset text accompanying the diagram, p. 410:] The Copernican hypothesis has been laid out in all respects similar to the one expressed above, except that, with regard to the given revolution of E, the Earth — that is, its distance from A, the apogee — and then with regard to N, the planet in its orb, the rest is conformed here [to the earlier figure]. Therefore, since A E, the anomaly of the eccentric, was granted to be 3 signs $4^{\circ} 33\frac{1}{2}'$, the angle D B E is known, in its complement to the semicircle, to be $85^{\circ} 26\frac{1}{2}'$. This angle, comprehended by the sides B E 100000 (namely the radius of the eccentric) and B K 2406 (the restored eccentricity), constitutes the first of the three given triangles; on account of which there is also given the angle B E K, $1^{\circ} 22' 36''$, together with the side E K, 99837. Further, since by the law of the double revolution of the center of the orb F toward E, the Earth, in its own orb, there is now found

F removed from C into the consequent signs by $189^{\circ} 7'$, and the angle B K E amounts to $93^{\circ} 10' 54''$, therefore, this being subtracted from that, there remains the angle F K E, $95^{\circ} 56' 6''$, with the adjacent sides known, namely E K 99837 and K F 802 (a third part, that is, of B K). On this account the angle K E F will not lie hidden: $0^{\circ} 27' 26''$. This, together with the foregoing B E K, constitutes the whole prosthaphaeresis of the eccentric of Venus at this place, $1^{\circ} 50' 2''$, which we sought here in the first place. Next, in the same latter triangle K E F, the side E F is given as 99925. Thirdly, since the mean anomaly of the orb of Venus, H P N, is established as 7 signs $7^{\circ} 10'$, accordingly — by adding the angle H F G of $1^{\circ} 50'$ (the $2''$ being neglected, as of no moment), which is equal to B E F, this being the difference between the true and the mean anomaly — the said true anomaly becomes 7 signs $9^{\circ} 0'$, and therefore the angle E F N is $39^{\circ} 0'$. There are likewise

given, in the triangle E F N, the sides E F 99925 and E N, the restored semidiameter of the orb of Venus, 72248 $\frac{2}{3}$. From these, lastly, the angle F E N is found to be 46° 5' 0", the commutation (or parallax of the orb, as Copernicus calls it). When this is joined with B E F, 1° 50' — both being subtractive here — the difference of the mean motion of Venus from the true is given as 47° 55' 0". But since the mean motion previously stood at 6 signs 3° 48 $\frac{1}{3}$ ', when the said difference is subtracted from it, Venus is found by calculation at 15° 53 $\frac{1}{3}$ ' in Leo, differing from the observed longitude by only 4 $\frac{1}{2}$ '. This is certainly very slight, and — on account of the inclination of its orb to the zodiac and the other things that intervene in observations — can scarcely ever be determined more precisely by any hypothesis. Now we shall presently test these same motions of Venus by the other hypothesis we spoke of, to see whether the same phenomena can be drawn out from it. And if so, there can be no doubt about all the rest, that they will follow precisely the same scrutiny [Greek: δοκιμασίαν (dokimasian, "testing" or "examination")].

Let there be (in the next figure), as before, the straight line A D, upon which the Earth is placed, according to Ptolemy, at B; and with the eccentricity B K given, let there be drawn about K a small circle C L, whose semidiameter is in subtriple proportion to B K. In this small circle, however,

Diagram. A geometric construction of the Copernican hypothesis for Venus, set within a quadrant. The vertical line runs from A (top) down through B and to D (bottom); A is the apogee. A large arc sweeps from A around to D on the left. Point E (the Earth's position in this Copernican form) lies on the left; lines radiate from E to several labeled points. Near the lower-right of the figure is a small circle (the epicyclet of the double revolution) carrying the cluster of points C, K, with M, P to its left and F, H, G at the right margin; N lies above, between the small circle and E.

- A (top of vertical line): apogee
- B (on the vertical line, below A): the Earth's position per Ptolemy (reference point)
- C (in the small circle, lower right): center of the eccentric / reference on the epicyclet
- D (bottom of vertical line): lower terminus of the apsidal line
- E (left): the Earth in the Copernican form; vertex of the radiating sight-lines
- F (right, by the small circle): center of the orb (carried by the double revolution)
- G (right margin): point on the apsidal/orbital line

- H (right margin): point on the line through the orb (H F G colinear)
- K (center of the small circle): center of the small circle of the double revolution
- M (left of the small circle): point on the orb
- N (above the small circle): the planet Venus in its orb
- P (left of small circle, by M): point marking the orb's anomaly

in this small circle, the center of the eccentric is understood to revolve into the consequent signs at F, in a doubled rotation of the center of the planet's epicycle. Therefore, about the center F let there be described the half-orb of the planet itself, according to Ptolemy, A E D — according to Ptolemy, I say; for neither the hypothesis of Copernicus nor that of Tycho [Tycho Brahe] permits that orb to be foreign to the orb of the Earth or of the Sun, and therefore its center cannot at all be carried about upon another planet's eccentricity. But suppose some Ptolemaist presses the point, and says that nowhere else is it ever permitted for an eccentricity to unfold itself upon a foreign orb, whence (he may say) it can not unreasonably be concluded that to each planet its own proper eccentric is more rightly assigned. For although an eccentricity may be set up within the epicycle at the same time — in the way we have done it through several of the preceding suppositions, where the whole circumference of the primary orb is constrained into the Tychonic form — yet the disposition of the revolution will always regard the apogee in the primary, that is, in the ecliptic. This certainly was not the least among the reasons why we granted to the schema of Ptolemy a place between the Copernican and the Tychonic — though only after first reducing it to the standard of geometry, as we have done. But let us proceed with our undertaking.

Therefore, in this orb of the planet, A E D, with the center of the epicycle removed from A, the apogee, by 3 signs $4^{\circ} 33\frac{1}{2}'$, it follows that the center F is distant from C by a doubled measure, namely 6 signs $9^{\circ} 7'$, likewise into the consequent signs. On this account the remainder, the complement to the circle, F C, 5 signs $20^{\circ} 53'$, measures the angle C K F, about which the granted sides are B K 2406 and K F 802 (in units of which the radius of the orb F E is supposed to be 100000). From these, then, there is found, first, the angle K B F, $2^{\circ} 16\frac{1}{3}'$; next, the side B F, very nearly 3204. But the angle K B F here (to which the arc P O is equal, when K A and E P are parallel) is taken away from the mean anomaly of the eccentric, and there remains the angle O F E, 3 signs $2^{\circ} 17'$; and therefore the angle B F E is 2 signs $27^{\circ} 43'$. Accordingly, in the triangle B F E, with two sides given (as above) about the angle B F E likewise given, there is yielded by calculation, first, the whole prosthaphaeresis F E B, $1^{\circ} 50' 4''$, insensibly differing from the foregoing; next, the side B E, 99925, plainly agreeing with the earlier F E. Because of this agreement

everywhere, the angle $E B N$ likewise knows not how to differ at all from the angle $F E N$ in the foregoing Copernican supposition, since the two are everywhere plainly alike. For the angle $H E G$ here is equal to the angle $H F G$ there; whence, the same anomaly of the orb, $H N$, being supposed in both cases, the angle $N E B$ here will be equal to the angle $N F E$ there. And because $B E$ and $E N$ in this hypothesis are equal to $E F$ and $F N$ in the former, on that account the angle $E B N$ here is given as equal to that angle $F E N$ — namely $46^{\circ} 5'$. And thus we have demonstrated the equivalence in these two suppositions. This it would be no hard matter to demonstrate also in all the rest: that the appearances can be made to vary by nothing but the revolutions (as is usually the case) and the semidiameters

Diagram. The geometric construction of the Ptolemaic (geometrized) hypothesis for Venus. The vertical apsidal line runs from A (top right) down to D (bottom right), with B near A and F just below B. A large circle (the planet's half-orb A E D) is centered at E in the middle-left; from E lines run out to the labeled points. At the upper-right corner a small circle carries K (its center), with F, L beside it and C, B below; O and P sit near A at the top. G and H are on the left margin, M below the center.

- A (top right): apogee
- B (right, below A on the apsidal line): the Earth per Ptolemy
- C (right, below the small circle): center of the eccentric / reference
- D (bottom right): lower terminus of the apsidal line
- E (center-left): vertex of the construction; the orb A E D is described about it
- F (right, just below B): center about which the half-orb is described in the doubled rotation
- G (left margin): point on the line H E G
- H (left margin): point colinear with E and G
- K (center of the small upper-right circle): center of the small circle of the double revolution
- L (right, by the small circle): point on the small circle (near F)
- M (below center E): point on the orb
- N (above center, left of small circle): the planet Venus in its orb
- O (top, near A): point marking the arc P O
- P (top, near A by O): point marking the arc P O

and the semidiameters being varied (which happens in the case of each individual planet). It might perhaps have been more fitting to have demonstrated this from the very beginning; but the occasion first carried us this way, so that, in the case of luxuriant Venus, we too might luxuriate in the abundance and copiousness of hypotheses for setting forth and indiscriminately saving all the phenomena of the wandering stars. Following the standard which we kept in one of these suppositions — chiefly the latter, which, for the sake of brevity, is similar to the Tychonic — the remaining positions of Venus too, computed for the foregoing times of observation, stand thus in the comparison.

Comparison of the foregoing observations of Venus with the restored hypothesis of the same.

[Margin: See a little above for the observed positions of Venus with their times, etc.]

Nu.	Mean longit. of Venus from the true equino x (S. G. M. S.)	Anomal y of the eccentr ic (S. G. M. S.)	Anomal y of the epicycl e (S. G. M. S.)	True longit. of Venus comput ed (S. G. M.)	Longit. of Venus observe d (S. G. M.)	Differ. (Min.)
1	6 · 3 · 48 · 20	3 · 4 · 33 · 30	7 · 7 · 10 · 0	□ 15 · 53½	□ 15 · 58	4½ —
2	10 · 4 · 17 · 15	7 · 5 · 0 · 45	5 · 7 · 42 · 17	□ 16 · 50½	□ 16 · 55	4½ —
3	11 · 10 · 48 · 48	8 · 11 · 32 · 15	6 · 0 · 33 · 7	✕ * 15 · 49½	✕ * 16 · 1	* 11¼ —
4	11 · 20 · 9 · 10	8 · 20 · 52 · 30	6 · 6 · 23 · 38	✕ * 10 · 14½	✕ * 10 · 7	* 7½ +
5	9 · 3 · 52 · 14	6 · 4 · 33 · 15	7 · 19 · 2 · 10	□ 17 · 6	□ 17 · 10	4 —
6	9 · 6 · 21 · 15	6 · 6 · 59 · 25	10 · 20 · 57 · 9	□ 20 · 4½	□ 20 · 0	4½ +
7	9 · 3 · 48 · 5	6 · 4 · 21 · 45	4 · 20 · 4 · 20	□ 21 · 6½	□ 21 · 0	6½ +

Nu.	Mean longit. of Venus from the true equino x (S. G. M. S.)	Anomal y of the eccentr ic (S. G. M. S.)	Anomal y of the epicycl e (S. G. M. S.)	True longit. of Venus comput ed (S. G. M.)	Longit. of Venus observe d (S. G. M.)	Differ. (Min.)
8	9 · 0 · 57 · 48	6 · 1 · 9 · 30	4 · 21 · 8 · 38	♌ 18 · 4½	♌ 17 · 58	6 [?] +
9	11 · 28 · 0 · 53	8 · 28 · 6 · 15	8 · 1 · 28 · 40	♌ 15 · 24¼	♌ 15 · 24	0¼ +

Parsing notes: The table has six numerical columns plus the running number and the difference. The first three columns are each in the sexagesimal sequence Sign · Degree · Minute · Second (S. G. M. S.); the "true" and "observed" longitude columns give Sign-symbol · Degree · Minute (S. G. M.). The zodiac-sign symbols printed in the "True longit." and "Longit. observed" columns are rendered as Unicode glyphs (♌ Leo, ♐ Pisces, ♏ Scorpio, ♐ Sagittarius, ♒ Aquarius). In rows 3 and 4 the source prints an asterisk-like mark (*) alongside the Pisces symbol in both longitude columns and again in the Difference column; transcribed as printed. The trailing "—" / "+" signs in the Difference column indicate the sense of the discrepancy (observed minus computed); transcribed as printed. Row 8's difference reads "6" but is partly ink-degraded; the fractional part is unclear — flagged [?].

CAPVT XIX. *On the derivation of the periodic motions of Venus.*

[Margin: Ptolemy, *Almagest* book 10, ch. 1. Copernicus, *De Revolutionibus* book 5, ch. 20.] [Margin: Book 10, ch. 1 and 2.]

This was the restitution of the motions of Venus in longitude for our age. But since the mean course of the Sun is established as common to these bodies as well — namely to Venus and Mercury — it remains, therefore, that in this chapter we treat of each anomaly of Venus, and of how it may be derived for all ages, taking the ancient observed records into our inquiry, just as has been done by us above for the rest. Our first concern, then, will be the advancement of the eccentric's apogee in this body. That this has been far swifter from Ptolemy's time down to ours than Copernicus established it to be in his own day, we found in the

foregoing restitution. For the point of the apogee [punctum Augis] of Venus, which Ptolemy held to be immovable (apart from the portion of the precession of the equinoxes in the meantime) and to rest at $48\frac{1}{3}^\circ$ of the orb of the fixed stars, we found above, at the end of the year 1585, to have been removed $89\frac{1}{2}^\circ$ from the true vernal equinox; and when the precession of the equinox is subtracted according to Copernicus's view, $61\frac{1}{3}^\circ$ — a quite notable difference, which extends to nearly 13° . But here each of them, both Ptolemy and Copernicus, was deceived: the former because, besides the slippery and nearly uncertain observations on which he relied, he had also made his judgment about the very slow motion of this point — a motion scarcely even perceptible between Theon and himself; the latter because he had, as in other matters, suspended his own judgment upon Ptolemy's, and had not drawn out the truth of this matter by his own experiments at the heavens. But the notes on Venus relative to the fixed stars which Ptolemy had left us from Theon and from himself — from which he had estimated not only the position of the apogee at that time, but the whole digression of Venus from the mean motion — we, examining them first, found them (contrary to his own words) to have been removed too far, on this side and on that, from the maximum digressions, which occur solely near the contact of the epicycle. So that this may be made manifest to all, let us include both [sets of data] in the following synopsis — the said observations, together with their moments of time (largely restored to the old Greek exemplar) and likewise exhibiting our restitution of the Sun and the fixed stars — with the anomaly of the epicycle also added,

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Caput XIX (continued) — Venus's periodic motions

with the mean anomaly everywhere agreeing with times of this sort, which we drew from the Prutenic Tables [tabulae Prutenicae] and corrected by subtracting $\frac{1}{3}$ of one degree, as best suited the restitution of that same anomaly in the present age.

A synopsis of the observations of Venus relative to the fixed stars, made long ago by Theon [Theon] and Ptolemy [Ptolemy] at Alexandria, together with the place of the Sun and of Venus, and Venus's (as Ptolemy holds) maximum elongation [digressio] from the Sun.

Nu.	Epoc h	An.	Mon th	D.	Mo men t	Med.	Loc.	Digr ef. Vene r. à Sole	[Mar gin]
						Solis (S. G. M.)	Ven. (S. G. M.)	(G. M.)	
1	Had rian	16	Phar mut hi	21	In the even ing	□ 14 15	□ 1 30	47 15	
2	Anto ninu s	4	Thot h	11	Mor ning follo wing	□ 5 45	□ 18 30	47 15	
3	Had rian	12	Athy r	21	Mor ning follo wing	□ 17 52	□ 0 20	47 32	<i>Lib. 10. cap. 1.</i>
4	Had rian	21	Mec hir	9	In the even ing	□ 2 4	□ 19 36	47 32	
5	Had rian	13	Epip hi	2	Mor ning follo wing	□ 25 24	□ 10 36	44 48	<i>Lib. 10. cap. 2.</i>
6	Had rian	21	Tybi	2	In the even ing	□ 25 30	□ 12 50	47 20	

[Margin (against rows 3 and 5): *Almagest* X.1; *Almagest* X.2]

This synopsis was taken from Ptolemy's records. But since those same times referred to the Christian epoch, the months and days likewise being Julian — and moreover (one hour in the morning before sunrise, and an equal interval in the evening, after the Sun has set) at the times when Venus, relative to the fixed stars

that Ptolemy lists, was at all observable — and since our own restitution of the Sun and stars reckoned from the true equinox has been applied, along with the anomaly of the orb, all of this is set before the eye in the following table, for the horizon of Alexandria.

Reduced observations of Venus (Christian epoch, Julian calendar, Alexandrian horizon):

Nu.	Ann. post Chr.	Mont h	D.	H. M.	Mean moti on of the Sun (S. G. M.)	Longi tude of Venu s (S. G. M.)	Dista nce from the Sun (G. M.)	Ano maly of Venu s's orb (S. G. M.)
1	132	Marc h	8	7 15	♓ 14 49	♓ 1 39	46 50	4 26 22
2	140	July	29	16 12	♓ 6 12	♓ 19 1	47 11	7 26 13
3	127	Octo ber	11	17 20	♓ 18 20	♓ 2 6	46 14	7 24 2
4	136	Dece mber	25	6 0	♓ 2 36	♓ 19 36	47 0	4 27 5
5	129	May	19	16 10	♓ 25 53	♓ 10 57	45 4	7 25 16
6	136	Nove mber	18	6 10	♓ 26 9	♓ 13 4	46 55	4 4 17

Parsing notes: The "Med. Solis" / "Mean motion of the Sun" and "Longitudo Veneris" columns each carry a zodiac-sign glyph (S = signum) plus degrees (G) and minutes (M). Row 5 of the first table reads ♓ 25 24 for Med. Solis (the printed glyph after "25" is the digit cluster as engraved). In the second table, row-1 Sun-motion appears as ♓ 14 49 (the glyph is the Pisces/cusp mark as printed).

These turn out to be the recorded positions of Venus, and its elongations from the Sun, once the restitution of the Sun and stars laid out in the earlier book is extended back to Ptolemy's age. Any discerning reader, examining at the same time the orb-anomaly appended to each entry, will easily perceive how far the

heaven itself and its own constant revolutions departed from the Ptolemaic scheme for the theory of Venus that was to be built upon observations of this kind. For our part, even though we can gather nothing very certain from these uncertain data, we nonetheless take this away from them: that once the apogee of Venus's eccentric is placed at $25\frac{1}{2}^{\circ}$ of Taurus [□], and then both the eccentricity and the maximum elongation are retained as we found them, the present observations for the most part support us more correctly than they do Ptolemy — at least when they are subjected to a proper test. As for the third observation, which Ptolemy ascribes to Theon, it disagrees by a full degree both from us and from the truth; I conjecture the cause to be either an anticipation of one day in Ptolemy's citation of Theon, or some other deliberate error that could have served his purpose. This becomes plain enough on careful examination, when one observes and weighs that same star — the fifth in the asterism of Virgo [□] — both in Ptolemy's Canon and in the heaven itself. And there is no reason for anyone, on account of this discrepancy from Ptolemy, rashly to bring a battering-ram against the foundation of the restored reduction of the Sun and the fixed stars.

For this cannot be done without considerable danger of recoil, since the arrangement of the Sun and stars — nearly uniform in both ages, ours and Ptolemy's, by our restitution — is set out, and is offered as a surplus over and above the Ptolemaic figures. We shall therefore retain the apogee of Venus as received from Ptolemy — found by him, perhaps, together with other better observations, and repeatedly confirmed, and moreover increased by our proportion for the Sun's more advanced motion — namely at $25\frac{1}{2}^{\circ}$ of Taurus [□] for the year completed after the birth of the world's Savior 140; which in our own age, that is, in the completed year 1585, was found above to be at $29\frac{1}{2}^{\circ}$ of Gemini [□] from the true equinox. The inequality of the precession of the equinox having been duly applied in the interval — there 20 minutes to be added, here 7 minutes to be subtracted — the said apogee-point of Venus has advanced $33^{\circ} 18'$, and that over an interval of 1445 Julian years.

By that proportion the following correspond:

For years	Gr.	Min.	Sec.	Tert.
1000	23	2	41	56
100	2	18	16	12
20	—	27	39	14
1	—	1	22	$57\frac{1}{2}$

These, conveniently subtracted from the Sun's mean motion, leave everywhere the eccentric anomaly to be inserted in the table below.

Further, as concerns the anomaly of the epicycle, or orb, of Venus: we first examined for it an observation of Ptolemy taken (as he reports) in the second year of Antoninus, on the 29th day of the month Tybi, in the morning at hour $4\frac{3}{4}$, Alexandrian time — which time was, in the year 138 from the birth of Christ, on December 15, at hour $15\frac{3}{4}$, in our own horizon. And since the Sun at that same time, by our restitution, was found by its mean motion to be at $22^{\circ} 43'$ from the true equinox $\rightarrow$, while the star in the forehead (not to call it Spica) of Virgo [Υ] — from whose measurement an error could arise with Ptolemy's instrument — was at $6^{\circ} 54'$ of Virgo [$\square$]: therefore, by Ptolemy's record, Venus occupied $7\frac{3}{4}^{\circ}$ of Virgo [$\square$]. For although we find the apparent place of the Moon, by his own computation (which is, I think, truer than his observation), assigned to $6^{\circ} 45'$ of Virgo [$\square$] — so that it then formed a straight line with Venus and the bright star of the forehead of Virgo [$\square$], and in this way the planet's place could be the more certified — yet no one who has rightly grasped the Moon's course from a legitimate hypothesis agreeing with the celestial phenomena will persuade himself that the Moon had this arrangement, in such a line toward the consequence of the signs, together with the planet and the star. For when, at the time prescribed by Ptolemy and for the Alexandrian horizon, the Sun's mean motion is found by our restitution to be at $22^{\circ} 43'$ from the true equinox, and likewise the Moon's distance from the Sun is 10 signs $19^{\circ} 14'$, then from these combined it is gathered that the Moon's mean motion was at $11^{\circ} 57'$ of Virgo [$\square$] — which Ptolemy puts at $11^{\circ} 24'$ of Virgo [$\square$]. Moreover, the Moon's anomaly, that is, its distance from apogee, was $87^{\circ} 49'$ — which Ptolemy too has as $87^{\circ} 30'$. From this we likewise see the Moon to have been then at nearly its mean elongation, between last quarter and conjunction with the Sun, and similarly placed at nearly the maximum subtractive prosthaphaeresis [prosthaphaeresis] of the eccentric; and on this point our figure is nearly the same as Ptolemy's, namely $4^{\circ} 58'$. But in the remaining deviation of the Moon there is a fairly notable difference between us. For he heaps up only 41 minutes on the former — which the distance between the Moon's mean and true longitude, $5^{\circ} 39'$, indicates — whereas we most certainly assert, on the basis of that disposition of the Moon toward the Sun and the Earth, that it was $1^{\circ} 6'$ greater than the Ptolemaic value. Hence the Moon's prosthaphaeresis with us had grown to $6^{\circ} 45'$, to be subtracted from the assigned mean longitude, so that its true longitude comes out at $5^{\circ} 12'$ of Virgo [$\square$], which Ptolemy puts at $5^{\circ} 45'$ of the same Virgo [$\square$]. Next, as concerns the Moon's parallax [parallax] in longitude, we find it at an

altitude of about 20° above the Alexandrian horizon, and at a distance of the Moon from the Earth of 57 semidiameters, to be just as many minutes at most as Ptolemy too wished it to be of whole degrees. It is clear, therefore, that the visible longitude of the Moon at that time could not have been greater than $6\frac{2}{3}^\circ$ of Virgo [□], whereas Ptolemy nonetheless put it at $6^\circ 45'$ of Virgo [□]. I have drawn this out rather minutely so that the discerning reader might recognize that our universal restitution of the Moon, made here, is not thereby weakened just because it disagrees with Ptolemy — since we have sufficiently demonstrated elsewhere that the Moon's elongation from its mean course was no different now than in that age. But since we have many times desired greater faith in the Ptolemaic observations, we judge even now, at this place, that that straight line can be excused in no other way than that the star in the forehead of Virgo [□] regarded Venus and the Moon along the same line, toward the north, in such a way that the Moon itself appeared advanced by three-quarters of a degree — whence Venus could have been seen at $6^\circ 33'$ of Virgo [□]. But lest we seem to wish to handle the matter here by conjecture, we accordingly rather set aside this Ptolemaic observation, and select for restoring the motion of Venus through its epicycle the ancient observation of Timocharis [Timocharis], which Ptolemy too describes as having been made in the 13th year of Philadelphus [Ptolemy II Philadelphus], on the 17th day of the month Messori completed — or at hour 12, that is, in the morning at the dawn of the 18th day. This year falls in 3693 after the first bissextile of the world, that is, before Christ 272, on the 11th day of October, at hour 18 at Alexandria; but at Copenhagen [Hafnia] at hour 16 minutes 25. For at that time Venus's star was seen to have exactly occulted the star opposite Vindemiatrix [Vindemiatrix], which fixed star, according to our restitution and reduction, was at that time at $3\frac{1}{2}^\circ$ of Virgo [□], with a latitude of $1^\circ 16'$ north — where the observation also has Venus to have stood. Now at this moment of time, the Sun's mean motion was elongated from the true equinox by 6 signs $16^\circ 6[?]'$, whence the distance of Venus from it is found to have been $42^\circ 56\frac{1}{2}'$, so that Venus was then at its maximum morning elongation, just passed. Further, since the motion of Venus's apogee, proportionally reduced to this point from what we set out above concerning it, is found at $16\frac{1}{4}^\circ$ of Taurus [□]: therefore, when a subtraction is made of this from the Sun's place, the eccentric anomaly is shown in the remainder to be 4 signs $29^\circ 56\frac{1}{2}'$. These things thus prepared, let the hypothesis of Venus be drawn out in yet another — indeed in a sixth — form along with what precedes, which (with the eccentricity expressed at the mean, as with Copernicus, and the epicycle near the circumference of the eccentric) has been most familiar above; but, on account of the position of the

epicycle, or orb, of Venus, agreeing with Ptolemy or with Tycho [Tycho Brahe], in this manner:

Diagram. A geometric construction of the Venus hypothesis (sixth form), spanning the lower half of p. 415 and continuing as the same figure at the top of p. 416. A large eccentric circle of Venus is drawn with diameter A D running vertically (A at top, D at bottom). The construction continues into the next page; labels visible on this page (and the next) include the eccentric circle, its center, the points of the Earth, the equant, the epicycle, and the star of Venus. The construction text reads:

On center B of the described eccentric of Venus, let the diameter A D be drawn through A and D; in which, the Earth being placed at C, let the eccentricity B C be made 2406, in such units as the radius of the eccentric B A is 100000. Then, the equant-circle being [drawn] to the circumference of the eccentric

- A (top of vertical diameter): apogee, upper terminus of diameter A D
- D (bottom of vertical diameter): lower terminus of diameter A D
- B (upper-middle of diameter): center of the eccentric
- C (below B on the diameter): the Earth [tellus]
- N (left): point on the epicycle
- Q (center-left, near epicycle): point on the epicycle
- R (near center, inside epicycle): prosthaphaeresis point (angle C R B)
- F (center of epicycle, near E): center of the epicycle
- E (near F): center of the equant
- K (just below F/E): terminus of semidiameter E K
- L (near F): point near epicycle center
- M (near F): perigee-line point of the epicycle
- G (lower-left): point on the epicycle
- H (lower-left, below G): point on the epicycle

[The right column of text wraps the upper continuation of the figure begun on the previous page.]

of the eccentric (the equant-circle, as with Copernicus above, being banished), the center E of this [epicycle] will be distant from the apogee A by the given anomaly of Venus, 4 signs $29^{\circ} 56\frac{1}{2}'$, and its semidiameter E K will be 802, as were found above in our restitution — and, as we think, never changed (nor does it appear distinctly altered in the cited records of Theon and Ptolemy). And since the revolution of the epicycle's center F is established to be commensurable, in the

equant-circle, with E as the center of that same [revolution] through the eccentric — so that when E is at the apogee A, F will be at the perigee P of the said equant — therefore the angle P E F is always equal to the angle A B E.

In resolving the present hypothesis, then: first, in the triangle C B E, there are given B C 2406, B E the radius, namely 100000, and besides the contained angle C B E $30^{\circ} 3\frac{1}{3}'$ — for this is the complement of the given eccentric anomaly to a semicircle. From these, accordingly, the angle B E C is found to be $0^{\circ} 42' 20''$, together with the side C E 97940. Then, in the triangle E C F, since there are granted C E just now found, 97940, likewise E F 802, and moreover the contained angle C E F $19^{\circ} 38' 50''$, exceeding B E F by the acquired angle B E C — there emerges here, accordingly, the angle E C F $0^{\circ} 13' 34''$, and besides the side C F 986. But from the sum of B E C and E C F is composed the whole prosthaphaeresis of the eccentric, $0^{\circ} 56' 3''$, determined by the angle C R B. Further, on center F let the epicycle of Venus H M Q N G be described; and, C F being drawn out straight into G, and another diameter of the epicycle Q H drawn parallel to B E, and finally the star of Venus placed at N, it is manifest that H Q N contains the measure of the mean anomaly of the epicycle, but G H Q N the true [anomaly], the difference of which is the arc H G, equal to the prosthaphaeretic [arc] B R C. For the rest, since we here intend to hunt for that same anomaly, it must be seen how, in the triangle F C N, three given quantities [δεδομένα; Greek: dedomena, "data, given quantities"] may be had for the sought angle C F N — by which angle, indeed, the morning Venus at N is made known from the perigee-line M through the epicycle E. Since, then, N itself, that is, the place of Venus, first stood (as observed from the true equinox) at $3^{\circ} 10'$ of Virgo [♍], but the mean motion of the Sun, or of Venus, from that same equinox along the point E was at $16^{\circ} 6\frac{1}{2}'$ of Libra [♏]; therefore, the angle of the eccentric prosthaphaeresis B R C, $56' 3''$, being subtracted from this, the point F is left at nearly $15^{\circ} 10'$ of Libra [♏], distant in the consequence [of the signs] from the observed longitude of Venus by $42^{\circ} 1'$. From this the measure of the angle F C N stands established.

There are also given the sides C F 9865[?] and F N $72248\frac{1}{2}$, from our supreme restitution, which we judge to be perpetually valid; wherefore there is brought into the open

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and so the sought angle CFN is produced as $71^{\circ} 55'$. This angle measures the arc MN on the epicycle; and when MQ — $56'$ — is subtracted from it, and the semicircle HQ is added to the remainder, the sought mean anomaly of commutation of Venus is assembled, amounting at the moment of this ancient observation to $250^{\circ} 59'$. The Prutenic Tables [tabulae Prutenicae], by their own reckoning, give this value as $252^{\circ} 6'$, agreeing with Ptolemy [Ptolemaeus] as closely as possible.

No one will be surprised that the difference here between Ptolemy's restitution and ours rises a little beyond a whole degree. Anyone who understands the matter knows two things: first, that much is owed to the correction of the Sun and the fixed stars; and second, that this whole investigation [πραγματεία (pragmateia), "treatise" or "inquiry"] of restoring such an anomaly was carried out at the point of nearly the greatest digression [digressio] of Venus — a moment not well suited to it, indeed scarcely certain enough. For here a very slight change in the appearances introduces the greatest change in the anomaly. Consequently, the experiment would have been far more reliable had it been made nearer to the apogee or perigee of the epicycle — provided only that ancient observations, like ours, had been available to us at those places. Since we are deprived of them, we must rest content with what we have found.

For extending the said anomaly (which we commonly call the anomaly of commutation [anomalía commutationis]) to all the ages of the world, we here use a method more expeditious and more compendious than in the case of the superior planets. For after we had found Reinhold's [Erasmus Reinhold] diligence in the mean motions computed through his Prutenic Tables to be sufficiently proven, we here too subjected the interval of Julian years to computation. We found, namely, that in the year 3693 from the first bissextile [leap year] of the world, the said tables overshoot in the motion of commutation of Venus by $1^{\circ} 7'$; whereas at the time of our restitution — which fell in the year of the world 5550, coinciding with the year 1586 from the birth of the Savior — the Prutenic Tables fall short of that same motion by $1^{\circ} 18'$. Therefore the two differences combined, namely $2^{\circ} 25'$ over an interval of 1857 Julian years, by which the Prutenic computation is exceeded, when duly distributed among the several categories of time, work out from this as follows:

	Deg.	Min.	Sec.	Thirds
For 1000 years	1	18	5	0
For 100 years	0	7	48	30
For 20 years	0	1	33	42
For 1 year	0	0	4	41

When these have been added to the several categories of years in the Prutenic Tables, and the customary epochs are afterward established from this, the restitution of this motion also emerges — which the table below will display in its proper place.

CAPUT XX.

ON

the correction of Mercury's motions in longitude, suited to our age.

[*De Mercurii motuum emendatione per longitudinem nostrae aetati conveniente.*]

In the preceding restoration of Venus we gave several different hypotheses by which the same phenomena of Venus are made to be saved. There too, all the observations — as many as we brought to bear on the restoration of this planet — clearly testify that its mean longitude through the ecliptic is inseparable from the simple [longitude] of the Sun. For had the eccentricity been constituted otherwise, the celestial appearances would by no means have agreed in every respect and at every distance of the Sun from the Earth — especially given so great an extent of the orb, where a tiny change of distance would otherwise have induced a sufficiently sensible change in the phenomenon.

But now, as divine clemency has willed, we shall lay our final hand upon Mercury — about whose slipperiness astronomers have hitherto complained no less than

the alchemists have complained about terrestrial Mercury [quicksilver]. These astronomers stubbornly and ignorantly clung to the traces and conjectures of the ancients concerning this planet, rather than to true celestial observations.

But we, after we had detected the immense discrepancy in this star between the Alphonsine and the Copernican calculation — a discrepancy that sometimes rose beyond 7° — first set ourselves (as with the other planets) to collect the observations, which fall to us rarely in the case of Mercury on account of the more oblique situation of our [terrestrial] sphere, when its digressions from the Sun become shorter and swifter. Then we set ourselves to compare these observations diligently with the Copernican hypothesis — here too retaining the mean motion of the Sun in the same way, and everywhere reckoning, for the moments at which the observations were made, from the Tychonic restitution (which agrees well enough with the heavens in this age) for the mean place of Mercury supposed in the eccentric.

In this comparison between the observed places of Mercury and those that issue from the Copernican calculation, we found the greatest differences near the apogee and the perigee of the eccentric. The celestial phenomena always required a greater extent of the orb — beyond two degrees — and also demanded (as is fitting) a somewhat swifter motion of Mercury at that point. Here I at once began to conjecture that the Copernican epicycle — the one he attaches to the orb of Mercury, and in which he contrives the star's ascent and descent in a straight line — is illegitimate. For when a second equant is assumed by Copernicus himself, placed midway, a twofold diminution of the orb's apparent size is created near the apogee of the eccentric: one diminution on account of the greater distance from the Earth, the other on account of the size of the planet perpetually occupying the lowest parts of the epicycle — as is sufficiently [shown] in the work *On the Revolutions* [Copernicus, *De Revolutionibus*].

[Margin: *De Revolutionibus* V.25]

And so, for the first trial of the restitution: after the position of the apogee of the eccentric — clinging in our age to the beginning of Sagittarius — had been moderately settled with respect to Ptolemy's investigation [πραγματεία (pragmateia)], I supposed a simple eccentricity in Mercury, as I had earlier done in the Sun, but one that extended to 3° of maximum prosthaphaeresis [prosthaphaeresis], that is, to 524 parts of the kind of which the radius of the eccentric is 10000.

From this, the operation being derived through all the observations according to the principles of triangles, I did indeed find somewhat smaller differences everywhere between the expected measure of the orb's semidiameter and the measure issuing from the observations than I had previously found by comparison through Copernicus. Among other things, however, I came to

understand that a greater relaxation occurred here and there at the apogee of the eccentric than the phenomena themselves required.

For this reason, after everything had again been generally weighed — as is most fitting — I saw that there was need of a little circle, or epicycle, in which the center either of the eccentric (according to Ptolemy) or of the orb of Mercury (according to Copernicus) would be carried in a turning commensurable with the point on the eccentric; yet not in a twofold revolution in the contrary direction [in antecedentia], as Copernicus held, nor in an equal one, as Ptolemy held, but rather in a threefold revolution relative to that in the direction of the signs [in consequentia] — and in such a way that it could nonetheless take on each of the individual forms of the preceding hypotheses. The following three figurations exhibit these same forms generally, at a single glance, after the manner of all three artificers.

Diagram. Caption: *Hypothesis Mercurii in forma — Ptolemaica, Copernicaea, Tychonica* ("The hypothesis of Mercury in the Ptolemaic, Copernican, and Tychonic form"). The page presents three side-by-side constructions of Mercury's theory, one per system. Each shows a large circle (the eccentric or great orb) along the vertical line A–C–K, with a small lettered circle (the circellus E D, carrying the little epicycle with center B/A) embedded in it, and a separate orb HF for the body of Mercury.

- A (on the upper axis, shared reference point): the Tychonic eccentricity reference / center of the great orb of the Earth attributed to Copernicus
- C (lower axis): the Earth, in the Ptolemaic and Tychonic forms
- K (bottom of axis): the perigee point of the eccentric
- B (center of the small embedded circle): the eccentricity point
- D (top of the small circle): position of the orb-center at apogee of the eccentric
- E (within the small circle): position of the orb-center at perigee; the carried point on circellus
- F, H (flanking labels): points on the orb of Mercury HF
- **Ptolemaica** (left figure): large circle with internal cross-axes; A at center, F at left, H at right; small lettered circle (D/B/E) low and inside; C below it; K at the bottom arc.
- **Copernicaea** (middle figure): upper axis carries A, with F and H flanking; lower large circle has center C, with the small lettered circle (E/B/D) inside; K at bottom.

- **Tychonica** (right figure): upper large circle carries the small lettered circle (D/B/E) near its top, with F at left and H at right; C below on the axis; K at the bottom arc.

[Caption: *Hypothesis Mercurii in forma. — Ptolemaica. Copernicaea. Tychonica.*]

First, upon the line ACK let C be placed for the Earth in the Ptolemaic and Tychonic form — which point we also here attribute to Copernicus as the center of the great orb of the Earth. Next, let the eccentricity B be taken, as in the two prior [forms]; to this, in the Tychonic form, corresponds AB. Then, with B made the center, let there be described a little circle, or epicycle ED, of at least one-fifth (as I find) of the eccentricity CB or AB.

In this little circle AD, at E, let the center of the epicycle — that is, of the orb of Mercury HF — be revolved in the order of the signs, by this law: that whenever the center of the planet's orb (or also of the Earth, according to Copernicus) is at the apogee of the eccentric, the center of this [orb] will be at D; but at the perigee K, the same will be at E.

Now since the reduction, or period, of the said point E will, by threefold revolution, measure out — on our finding — one [revolution] proceeding from A, it comes about that not only at these points, but also at other determinate places throughout the eccentric, D falls at the greatest and least distance from C (according to Ptolemy and Copernicus), but from A according to Tycho. And this is what afterward corrects all the errors that Copernicus attributes to Mercury — so much so that the celestial phenomena of our age are satisfied, and that our hypothesis agrees as closely as possible with the Ptolemaic observation concerning Mercury's wider digression at its sextile position in the eccentric, on that side where, according to Ptolemy, its greatest digression — measured from the mean longitude — is reported to occur in Aquarius in the morning, but in Gemini in the evening.

[Margin: Ptolemy, *[Almagest]* IX.8]

For as to the very thing that the same Ptolemy strives to prove — namely, that Mercury at the sextile, as if at the perigee, is found on both sides nearer to the Earth than when it has been at the perigee itself — that he can scarcely sustain even from the observations he there adduces, let alone from truer ones; since, in his own investigation [πραγματεία (pragmateia)], he neither duly applied the reckoning of the eccentric's prosthaphaeresis at those places (that is, the alternating evening and morning greatest digression) all at once, nor sufficiently and precisely had the limits of the greatest digressions—

—nor sufficiently and precisely had the limits of the greatest digressions. This will easily be proved by comparison, if only one binds Ptolemy to his own hypothesis and to the numbers issuing here and there from it — to say nothing of the truer correction of Mercury at the prescribed times, drawn from the restored places of the Sun and the fixed stars, which we carry out in what follows.

Pursuing this further within our aforesaid hypothesis: after the nine following observations of Mercury — the more recent ones — had been diligently examined by us, and the calculation reiterated several times, we found that no closer proportion of the eccentricity to the radius of the eccentric, nor any more consonant with the phenomena, could be given than this: that, in the preceding diagram, we set CB to be 1685 most nearly, agreeing with Ptolemy, and BD to be 337 parts — namely, the fifth part of CB — of the kind of which the radius of the eccentric is 100000; and likewise the radius of the orb to be 38500 parts of the same.

These things being thus constituted; and the apogee of the eccentric, moreover, being located at the end of the Christian year 1585 at $0^{\circ} 30'$ of Sagittarius with respect to the true equinox (which is nearly the motion of Copernicus); and finally the anomaly of the orb being established, at the same terminus, as 1 sign $13^{\circ} 2' 8''$ — such that here $24\frac{1}{8}^{\circ}$ Prutenic are added — all things from such a hypothesis of ours agree well enough with the observations in the case of Mercury, so far as the slippery observations of this star permit. This is seen, in the customary manner, in the following synopsis, in which both the observed places of Mercury have been faithfully transferred here, and (so far as could be done) freed from refractions and parallaxes, while the rest have been computed from the added mean motions, according to the one example which we now set out.

An example of the fifth of the subjoined places of Mercury, computed from our new hypothesis, in the Tychonic form.

[Exemplum quinti subjectorum locorum Mercurii, ex nostra nova hypothesisi, in forma Tychoniana exputati.]

In the year of Christ 1587, on the 9th day of January, at hour 4, minute 50 in the evening, we find that Mercury was observed at Uraniborg [Uraniburgum] in longitude $17^{\circ} 44'$ of Virgo, with a latitude of scarcely $2'$ North.

For the same time, there is found — reckoned from the true equinox — first, the mean motion of the Sun, that is, of Mercury in its eccentric, as 9 signs $28^{\circ} 22' 30''$; with the corrected eccentric anomaly [anomalía eccentrici] of 1 sign $27^{\circ} 53'$ most

nearly; and the corrected anomaly of the orb [anomalía orbis] likewise of 4 signs $2^{\circ} 28'$.

Now these are the mean motions of Mercury, from which, by means of the hypothesis, we hunt down the true [place] in longitude in the following manner.

Diagram. An uncaptioned geometric construction at the foot of the page, accompanying the worked example. A large primary circle is drawn about center A (the Earth) — the primary orb of the Sun, or of Mercury, labeled B C — with the diameter B–O drawn through it as the index of the apsides. Within it, near the top, sits a cluster of small lettered points (H/I, F, E, C, D) carrying the little circle and epicycle of the Mercury construction, with K marked lower on the left arc.

- A (center of the large circle): the Earth
- B (top of the large circle / upper end of the apsidal diameter): apogee end of the primary orb
- O (bottom of the large circle / lower end of the apsidal diameter): perigee end of the apsidal diameter B O
- H, I (upper left, paired labels): points on the orb of Mercury
- F (just below H/I): point on the construction
- E (small inner point): the carried center on the circellus
- C (center of the small embedded circle): eccentricity/orb-center point
- D (left of the cluster): apogee position of the orb-center
- K (lower left, on the arc): perigee point

In the adjoining Tychonic hypothesis — to which, this time, we choose to accommodate these mean motions of Mercury — let there be described, about the center A (the Earth), the primary orb of the Sun, that is, of Mercury, B C. Next, as the index of the apsides, let the diameter B O be drawn in this primary circle. Then, for the measure of the anomaly of this eccentric (for it is permitted to call it so here too, in conformity with the constitution of the rest), which is given as $57^{\circ} 53'$, let there be removed—

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Caput XX (continued) — Mercury's Longitude: the worked Tychoenic example

in consequence, from point B, point C is removed. About this point C the first Tychoenic epicycle is again described, and the line ACD drawn from the Earth A shows the apogee of this epicycle at D. On its circumference, point E is conceived to move in the contrary direction (*in antecedentia*) through an arc equal to BC — namely $57^{\circ}53'$. About this center E in turn the second Tychoenic epicycle FG is drawn, and by carrying the line CF through E the apogee of this epicycle is likewise marked at F. From F, point G will stand off in the forward direction (*in consequentia*) along this epicyclet by $173^{\circ}39'$, according to the threefold measure of the revolution taken together with BC (or DE) — all of which was set out in general terms above.

Finally, then, with the orb of Mercury HIK described about center G, the diameter IL drawn parallel to DA, and another line carried from A through G to H: from this construction the true apogee H of this orb is distinguished from the mean apogee I.

On the circumference of this orb, let the planet be supposed in the forward direction at the sign ♄ [Saturn-symbol used here as a point-marker], according to the given mean anomaly of $122^{\circ}28'$. To that same point the semidiameter GK of the orb is carried out from center G, and finally the line AK from the Earth A. With these preparations made, the calculation for Mercury's true longitude — to be derived from the given mean motions — must be set up by the application of plane triangles, following these procedures, and worked out according to the principles of trigonometry we laid down earlier.

First of all, in triangle GCE: from the givens [Greek: δεδομένοις (*dedomenois*, "given quantities")] CE 5685 and EG 1137 — in units where the radius AC is 100000 — together with the contained angle CEG, which is the complement of FG, $6^{\circ}21'$, there is found very compendiously (by the theorem of plane triangles) first the angle ECG, $1^{\circ}35'$, and then the side CG, 4551. But the angle ECG is here subtracted from the angle ECD, $57^{\circ}53'$, and there remains DCG, the equated eccentric anomaly, $56^{\circ}18'$.

Next, in triangle ACG, to find the angle CAG and the side GA: the givens [Greek: δεδόμενα (dedomena, "givens")] are AC, the radius 100000, and CG 4551, together with the contained angle GCA, $123^{\circ}42'$ — which is in fact the complement of the equated anomaly DCG to a semicircle. From these are obtained: first the angle CAG, about $2^{\circ}7'$; then the side AG, nearly 102600. But the angle CAG just found — which is the eccentric's prosthaphaeresis — when it is subtracted from Mercury's mean motion along the zodiac, prescribed as $28^{\circ}22\frac{1}{2}'$, and added to the corrected orb-anomaly of $4s\ 2^{\circ}28'$, makes the planet appear there at $9s\ 26^{\circ}1'$ from the true equinox, and here at $4s\ 4^{\circ}35'$ — this latter motion being reckoned from the true apogee H of the orb to K.

Lastly, in triangle KGA: because GA is 102600 and GK 38500 (the semidiameter of Mercury's orb as we estimated and restored it above), together with the angle KGA, $35^{\circ}25'$, which expresses the excess of HK beyond a semicircle — from these is obtained the second prosthaphaeresis GAK, about $21^{\circ}29'$. When this is added to the true place of Mercury's eccentric, which was previously $9s\ 26^{\circ}1'$, we at length arrive at the sought true place in longitude, reckoned from the true equinox: about $17^{\circ}45'$ ♍ (Virgo). But the observed true place was at $17^{\circ}44'$ ♍. The difference, therefore, is only $1'$. From this example — or else from the table of prosthaphaereses that follows (which is, of course, built on the same foundation) — the remaining places of Mercury that follow have been computed, as appears in the following synopsis.

Synopsis: Comparison of Observed Places of Mercury with the Restored Hypothesis

Below the multi-column heading, the table records ten observations. Column groups: index; Year / Month / Day / Hour / Minute; Observed Longitude (Sign, Degree, Minute); Observed Latitude; True Longitude from the true equinox (Sign, Degree, Minute, Second); Eccentric Anomaly (Sign, Degree, Minute, Second); Orb Anomaly (Sign, Degree, Minute, Second); Supposed [computed] Longitude (Sign, Degree, Minute); Difference (Minutes).

#	Ye ar	Mo nth	D	H	M	Ob s. Lo ng. (S G M)	Ob s. Lat .	Tr ue Lo ng. fro m tru e eq uin ox (S G M S)	Ecc ent An om aly (S G M S)	Or b An om aly (S G M S)	Su pp ose d Lo ng. (S G M)	Di ff. (M)
1	15 85	No v.	14	0	—	□ 13 4	B 2 18	8 4 0 15	0 3 30 0	8 16 22 20	□ 12 57	7
2	15 85	No v.	23	19	20	□ 25 3	B 1 25	8 12 53 20	0 12 23 0	9 14 22 30	□ 24 58	5
3	15 86	Oct .	24	19	10	□ 22 35	No t ob s.	7 13 4 30	11 12 34 0	8 5 6 30	□ 22 28	7
4	15 86	Oct .	28	18	30	□ 26 33	B 2 17	7 16 49 30	11 16 29 0	8 17 27 0	□ 26 28	5
5	15 87	Jan .	9	4	50	□ 17 44	B 0 1	9 28 22 50	1 16 52 0	4 2 28 20	□ 17 49	1
6	15 90	Ma rt.	6	6	50	□ 13 44	B 1 42	11 23 55	3 23 20	3 11 41	□ 13 36	8

#	Ye ar	Mo nth	D	H	M	Ob s. Lo ng. (S G M)	Ob s. Lat .	Tr ue Lo ng. fro m tru e eq uin ox (S G M S)	Ecc ent An om aly (S G M S)	Or b An om aly (S G M S)	Su pp ose d Lo ng. (S G M)	Di ff. (M)
								20	0	10		
7	15	Fe	3	5	40	□	B 0	10	2	3	□	9
	92	br.				12	47	22	22	23	12	
						20		50	15	8	29	
								45	0	30		
8	15	Ma	11	9	30	□	B 2	11	5	3	□	12
	93	jus				23	0	29	28	22	23	
						16		23	51	4 0	4	
								0	0			
9	16	Ap	15	9	0	□	B 1	1 3	5 2	2	□	7
	07	ril				21	40	20	18	15	21	
						5		50	0	50	12	
										6		
10	16	De	5	19	0	□ 2	No	8	0	8	□ 2	7
	10	ce.				42	t	24	23	10	35	
							ob	40	34	11		
							s.	0	0	0		

Parsing notes: This is a tightly set, double-scan-degraded citation table. Column alignment was difficult; several cells in the wide Sign/Degree/Minute/Second groups were partly merged or smeared. Row 5 (1587 Jan. 9), Eccentric Anomaly column: the source shows what appears to be "1 52 0" where the surrounding column structure expects four subfields (S G M S); compare the worked example

on p. 421, which gives the eccentric anomaly as $1s\ 27^{\circ}53'$ for this same observation — the printed digits here do not clearly resolve to that and are flagged [?]. Several second-place (S) entries in the True Longitude and Anomaly groups are faint; transcribed as best read. Sign symbols rendered with the conventional zodiac glyphs as printed.

Thus the restitution of the Mercurial phenomena has been carried out in such a way that — once account is taken in them of parallaxes and refractions — the difference between the appearances obtained from the heavens and those which our now-reformed hypothesis of Mercury exhibits nowhere exceeds a sixth part of a degree, except in the antepenultimate entry, which we borrowed from the observations of the most illustrious Landgrave William [Wilhelm IV of Hesse-Kassel]. Since that observation was made with a sextant by a single observer (as was then the practice there), and acquired at that season of the year when perpetual twilight also persists in that location, it is no wonder if Mercury, being nearer the horizon by about $12'$, appeared advanced in longitude beyond its proper place — both on account of refractions greater than those assigned to the fixed stars, and because of its own scintillation, by which this restless star has often deceived us near its setting, presenting itself higher to the sight than is just.

Moreover, I judge that this restitution sufficiently establishes that Mercury, similarly to Venus, regards in its revolutions the *simple* course of the Sun rather than the *true* one. Otherwise it would show an anomaly far greater than the superior planets do, if — like them — it had adjusted its motions through eccentrics to the *true* path of the Sun. For this reason, among other things, [Margin: See Pliny, Book 2, Chapter 15.] we may conjecture that the Moon and the superior planets, circling the Earth separately, are thrust into the Sun's rays; whereas these planets [Mercury and Venus], which never circle that same Earth except together with the Sun — owing to the shorter dimension of their circles — are permitted, according to the opinion of the ancients about all the planets, to be contained within its simple course.

CAPVT XXI — On the Establishment of Mercury's Mean Motions in Longitude

[Margin: Book 9, Chapter 7.]

We have refrained from any further examination, for the restitution of Mercury itself, of the ancient observations of Mercury which Ptolemy collected into a single chapter [*Almagest* IX] — confidently expecting here the same outcome of the testing [Greek: δοκιμασία (*dokimasias*, "scrutiny," "assaying")] as before in

the case of Venus. For these too are found either to have been recorded against the fixed stars by the rude and conjectural [Greek: στοχαστική (stochastikē, "conjectural")] method of the ancients, or else taken with the Ptolemaic astrolabe — which is no more to be trusted here, as its known construction (which, among other things, excludes precision within ten, if not more, scruples), together with other recognized impediments during observation, persuaded us. Meanwhile, however, we have found that Ptolemy pronounced more correctly about Mercury's apogee in the early age — that is, in the year 3700 of our world-epoch, when it stood at 6° [of a sign] — than he did for his own era; as will become clear both from other comparisons and, most of all, from those which we shall presently adduce below, drawn from the records of the ancients, for the restitution of the orb-anomaly.

Accordingly, since for our own age — that is, in the year 1586 completed from Christ — we hold (from the foregoing) the apogee of this same Mercury at 8s 0°12' from the mean equinox; but in the early age—

[At the top left of this page, set off as a small bracketed table — the proportional distribution of the apogee's motion per interval of Julian years. It is keyed to the prose that follows.]

(Julian years)	Gr.	Min.	Sec.	Tert.
1000	29	15	8	6
100	2	55	30	49
20	0	35	6	10
1	0	1	45	18

—in the early age, that is, in the year 264 before Christ, we hold the same [apogee] from the same equinox at 6s 6°5'. In the interval, therefore, of 1850 Julian years, the apogee's motion in Mercury from the mean equinox has been carried forward through 54°7'. When these are distributed over the proposed interval of years, they correspond to Julian years as in the adjacent table. From this same proportion, too, the customary epochs are established. And when, from the individual species [entries] of these, the corresponding amounts of the mean Sun from the mean equinox are subtracted, there remains the eccentric anomaly of Mercury, which will appear afterward in the table of its mean motions.

There remains the last motion of this planet — in the epicycle, or rather in its own proper orb — which we commonly call the anomaly of commutation.

Ptolemy certainly did not happily enough succeed in handling this and transmitting it to posterity, since he constituted it far more sluggish and slow than even Copernicus could afterward amend — as appears above in our own restitution, seeing that $24\frac{1}{8}^{\circ}$ more had to be added in our age. And indeed this error is far more intolerable than any deviation of the apsides whatsoever, especially in the apsides of the orb, or falling roughly thereabouts, where one degree — in the more remote position — produces about a quarter of itself, and in the lower position two-thirds of itself; although near the contact, back and forth, only a slight difference appears. Therefore, for the examination and working-out of this motion, we shall most conveniently employ an early observation, because it is found at a just distance below the contact of the orb. Ptolemy describes an observation of this kind, made as follows.

In the 22nd year according to Dionysius — which time is the 484th year from Nabonassar — on the 22nd day of Scorpio [in the Dionysian calendar], according to the Egyptians Thoth 18 going on 19, the morning star Mercury stood off from the line that runs through the northern brow of Scorpio and through the middle of the [Greek: τὰ ὑπολειπόμενα (ta hypoleipomena, "the trailing/lagging stars")], i.e., toward the preceding part [*in praecedentia*], by one lunar diameter; and toward the north it stood off from the northern brow by two lunar diameters. [Margin: Ptolemy, Book 9, Chapter 10.] Thus far Ptolemy. The time of this observation, when adapted to the years of Christ (or of the world) and to our horizon of Copenhagen [Hafnia], falls in the year 265 before Christ — which was the year 3700 from the first bissextile of the world — and further on the 14th day of November, in the morning, 17 hours after the noon of the said preceding 14th. For this very moment, these motions are gathered from the preceding tables:

	Sig.	Gr.	Min.
Simple Sun from the mean equinox	7	20	$1\frac{3}{4}$
Precession of the equinox	1	21	$11\frac{1}{3}$
Anomaly of the same	0	10	$\frac{1}{3}$
Whence the equinoctial inequality			$4\frac{3}{4}$
This, when	7	19	$56\frac{1}{2}$

	Sig.	Gr.	Min.
subtracted from the mean Sun, leaves the Sun from the true equinox			
Likewise from the precession of the equinox, that becomes	1	21	6^{11}_{12} [?]
This again, from the epoch of precession in our age, leaves	0	25	$59\frac{1}{2}$

—namely the elapsed amount of precession from that early age to our own. When, therefore, we subtract that portion from the two named stars in the brow of Scorpio, the northern longitude of the bright [star] is placed at $1^{\circ}36'$ $\square$ (whose latitude too is $1^{\circ}16'$ N); but the third of the same stars in Scorpio is placed in longitude at $6^{\circ}59'$ $\square$. With these carefully weighed against the standard of the observation, we find the longitude of Mercury at $2^{\circ}35'$ $\square$ with a northern latitude of $2^{\circ}20'$.

But since we now apply the prescribed mean longitude of the Sun — or of Mercury — to this hypothesis, in order that from it we may reason about the said anomaly of the orb, the arrangement—

(Caput XXI continued) — The same observation worked in the Copernican form

—the arrangement, in the Copernican form, appears thus, after the apogee here too has been assumed at 6° $\square$ according to Ptolemy, and the eccentric anomaly has come out as $43^{\circ}56\frac{1}{2}'$.

First, since AH in the great annual orb of the Earth is given as $43^{\circ}56\frac{1}{2}'$ from the eccentric anomaly, three times this will be $131^{\circ}49\frac{1}{2}'$, the arc DF. Therefore, since in triangle CBF two sides CB and BF are granted from our restitution — the former 5685 parts, the latter 1137, in units where the radius CH is reckoned 100000 — there is given besides the angle enclosed by them, $48^{\circ}10'$, being the

complement of the external [angle] DBF. Wherefore there is found from these, first, the angle BCF, about $9^{\circ}45'$, and then the side CF, 5002.

Next, in triangle CHF, with sides likewise given — CH 100000 and CF 5002 — together with the contained angle FCH, $145^{\circ}48'$ (for this stands by the complement of the anomaly ACH together with the previously found angle FCB), there is obtained at length the prosthaphaeresis of the eccentric in the single angle CHF, nearly $1^{\circ}33'$. When this is here subtracted from the mean motion of the Earth at H — which for Copernicus is the same as the Sun's (or Mercury's) in the eccentric, by the reckoning of Tycho and Ptolemy — there remain about $7^{\circ}18'24''$, for the equated eccentric motion of H from the true vernal equinox. There is found besides, in the same proposed triangle CHF, the side HF 103902. Now since the true place of the star Mercury from the true equinox had previously, by the early observation, been found at $7^{\circ}2^{\circ}35'$, in the difference of these the visible digression of Mercury through its own proper orb consists.

Diagram. A geometric construction of the Copernican model for this Mercury observation, occupying the lower-left of the page with the prose wrapping around it. A large outer circle (the great annual orb of the Earth) is shown, with the Sun's region marked near its center. Within it is a smaller circle (Mercury's proper orb LMPN) centered on the mobile point F. A long vertical diameter runs from C at the top down through the figure to I at the bottom. The labeled points:

- A (top, just right of C): the [mean] apogee point on the vertical axis
- C (top center): center of the great orb / one apex of the triangle CBF
- H (upper left, on the great circle): the Earth's position
- B (center, in small inner circle): apex near the center, with D just below it
- D (just below B, center): point on the apsidal line (arc DF runs from here)
- F (center, within Mercury's orb): the mobile center of Mercury's proper orb
- P (left of C, on Mercury's orb): point on the small orb
- N (left, on Mercury's orb): the planet's position (line HN from Earth H closes triangle FHN)
- L (lower right, on Mercury's orb): the true apogee of the epicycle/orb (line HIL drawn through it)
- M (lower right, below L, on Mercury's orb): the mean apogee (MP drawn parallel to CH)
- I (bottom, on vertical axis): lower terminus of the diameter

The construction shows triangle CBF, triangle CHF, and triangle FHN, used in the worked Copernican calculation. (No labels flagged [?].)

This digression amounts to $15^{\circ}49'$. Since, then, the line HIL is drawn, the true apogee of the epicycle (or orb) is shown at L, and likewise the mean apogee at M — MP being drawn parallel to CH. Furthermore, since Mercury's morning digression from the equated eccentric is established as $15^{\circ}49'$, let the line HN be drawn from the Earth H into the planet's orb, closing the triangle FHN; in this triangle FHN the said digression is included. There are given besides, in the same triangle FHN, two sides — namely FH 103902 and FN 38500. The angle HFN therefore cannot lie hidden: about $31^{\circ}33'$. When a semicircle of 180° is first added to this, and then the arc LM of $1^{\circ}33'$ (which was earlier equal to CHF) is subtracted from the sum, there remains finally the sought motion of Mercury from the mean apogee M of the orb, continued to the planet at N: $210^{\circ}0'$. Copernicus found this greater by 1° and three-quarters; but Ptolemy by $2^{\circ}34'$. Now since this single observation, among the early ones, is most—

since this single early observation was the more opportune one, on account of Mercury's smaller elongation from the Sun, we therefore rest content with it. When, for the sake of brevity, we carry this anomaly forward to the end of the year of our earlier epoch, 3700, we find it to be $356^{\circ} 55' 20''$ at our horizon, reckoned to midday on the following first of January. And because this is a leap year, we rightly take into our comparison, for our own age, the year 1588 from the birth of Christ — that is, year 5552 of the world, the first leap year of that era. To the end of that same year we have, corrected from the earlier Tyconic observations together with our own, a value of very nearly $208^{\circ} 1' 20''$, when the Prutenic figure has been reduced to our meridian; and we add $2^{\circ} 10' 40''$ to this same value as a correction for the assumed year.

Since, therefore, over an interval of 1852 Julian years we reckon that Mercury has completed, in its epicycle (that is, in its own orb), 5837 whole revolutions plus a further $211^{\circ} 6'$, the motions for the successive classes of Julian years that follow are accordingly assigned — over and above the whole revolutions — from this proportion of motion to time:

	Sig.	Gr.	Min.	Sec.	Tert.
For 1000 years	0	16	0	54	25
100 years	2	13	36	5	26
20 years	0	14	43	13	6
1 common	1	23	57	33	35

	Sig.	Gr.	Min.	Sec.	Tert.
year					

Moreover, the epochs of this motion — which are placed at the head of the following table of Mercury's mean motions — arise by the same proportion.

TABLE OF MEAN MOTIONS

[of Venus and Mercury] — Collected Years

The table is arranged in four numerical columns. For each planet, the **anomaly of the eccentric** [anomalía eccentrici] and the **anomaly of the orb** [anomalía orbis] are tabulated. Each value is given in signs (S.), degrees (G.), minutes (M.), and seconds (S.). The first column lists the collected years: the epoch values "Epoch of the World" [Epocha Mundi] and "Christ" [Christus], followed by year-counts from 20 to 6300.

	Venus — Anom. Eccentric (S G M S)	Venus — Anom. Orb (S G M S)	Mercury — Anom. Eccentric (S G M S)	Mercury — Anom. Orb (S G M S)
Epoch of the World	9 17 18 31	9 24 13 46	5 20 42 54	5 4 40 0
Christ	7 16 1 45	4 6 21 20	2 24 49 49	1 18 13 48
20	11 29 41 27	6 3 41 38	11 29 34 0	0 14 43 13
40	11 29 22 54	0 7 23 15	11 29 8 0	0 29 26 26
60	11 29 4 20	6 11 4 52	11 28 42 0	1 14 9 39
80	11 28 45 47	0 14 46 30	11 28 16 0	1 28 52 52
100	11 28 27 14	1 6 18 28	11 27 50 0	2 13 36 5
200	11 26 54 29	1 6 56 20	11 25 40 0	4 27 12 11
300	11 25 21 44	7 25 24 29	11 23 30 1	7 10 48 16
400	11 23 48 59	2 13 52 39	11 21 20 1	9 24 24 21
500	11 22 16 13	9 2 20 48	11 19 10 1	0 8 0 27
600	11 20 43 28	9 20 48 58	11 17 0 1	2 21 36 33
700	11 19 10 43	10 9 17 7	11 14 50 1	5 5 12 38
800	11 17 37 57	4 27 45 17	11 12 40 1	7 18 48 44
900	11 16 5 12	11 16 13 26	11 10 30 2	10 2 24 49

Collected years	Venus — Anom. Eccentric (S G M S)	Venus — Anom. Orb (S G M S)	Mercury — Anom. Eccentric (S G M S)	Mercury — Anom. Orb (S G M S)
1000	11 14 32 27	6 4 41 37	11 8 20 2	0 16 0 54
1100	11 12 59 43	0 23 9 46	11 6 10 2	2 29 37 0
1200	11 11 26 58	7 11 37 56	11 4 0 2	5 13 13 5
1300	11 9 54 12	2 0 6 6 [?]	11 1 50 2	7 26 49 11
1400	11 8 21 27	8 18 34 16	10 29 40 3	10 10 25 16
1500	11 6 48 42	3 7 2 25	10 27 30 3	0 24 1 22
1600	11 5 15 57	9 25 30 35	10 25 20 3	3 7 37 27
1700	11 3 43 12	4 13 58 44	10 23 10 3	5 21 13 32
1800	11 2 10 26	11 2 26 54	10 21 0 3	8 4 49 38
1900	11 0 37 41	5 20 55 4	10 18 50 4	10 18 25 43
2000	10 29 4 56	0 9 23 14	10 16 40 4	1 2 1 49
2500	10 21 21 10	9 11 44 2	10 5 50 5	1 10 2 16
3000	10 13 37 24	6 14 4 51	9 25 0 6	1 18 2 43
3500	10 5 53 38	3 16 25 39	9 14 10 7	1 26 3 10
4000	9 28 9 52	0 18 46 28	9 3 20 7	2 4 3 48 [?]
4500	9 20 26 6	9 21 7 16	8 22 30 8	2 12 4 5
5000	9 12 42 20	6 23 28 5	8 11 40 9	2 20 4 32
5500	9 4 58 34	3 25 48 13	8 0 50 10	2 28 4 59
6000	8 27 14 48	0 28 9 42	7 20 0 11	3 6 5 27
6300	8 22 36 32	8 23 34 11	7 13 30 12	10 17 53 43

Parsing notes: Four-planet-column layout with sub-headers (Eccentric / Orb under each of Venus and Mercury). Several seconds-place digits ambiguous in the double-scan; flagged inline. The Mercury–Orb "signs" column at year 6300 reads "12" which is anomalous (see Numerical impossibles).

OF VENUS AND MERCURY (continued)

For Single Years, up to 20 [In annis singulis ad 20]

Year	Venus — Anom. Eccentric (S G M S)	Venus — Anom. Orb (S G M S)	Mercury — Anom. Eccentric (S G M S)	Mercury — Anom. Orb (S G M S)
1	11 29 44 17	7 15 1 50	11 29 43 55	1 23 57 34
2	11 29 28 34	3 0 3 40	11 29 27 50	3 17 55 7
3	11 29 12 52	10 15 5 30	11 29 11 45	5 11 52 41
4	11 29 56 17 [?]	6 0 44 20	11 29 54 48	7 8 56 39 [?]
5	11 29 40 34	1 15 46 10	11 29 38 43	9 2 54 12
6	11 29 24 52	9 0 48 0	11 29 22 38	10 26 51 46
7	11 29 9 9	4 15 49 50	11 29 6 33	0 20 49 19
8	11 29 52 34	0 1 28 39	11 29 49 35	2 17 53 17 [?]
9	11 29 36 52	7 16 30 29	11 29 33 30	4 11 50 51
10	11 29 21 9	3 1 32 19	11 29 17 25	6 5 48 24
11	11 29 5 26	10 16 34 9	11 29 1 20	7 29 45 58
12	11 29 48 52	6 2 12 59	11 29 44 33	9 26 49 56 [?]
13	11 29 33 9	1 17 14 49	11 29 28 18	11 20 47 29
14	11 29 17 26	9 2 16 38	11 29 12 12	1 14 45 3
15	11 29 1 44	4 17 18 29	11 28 56 8	3 8 42 36
16	11 29 45 9	0 2 57 18	11 29 39 11	5 5 46 34 [?]
17	11 29 29 26	7 17 59 8	11 29 23 6	6 29 44 8
18	11 29 13 44	3 3 0 58	11 29 7 1	8 23 41 41
19	11 28 58 2	10 18 2 48	11 28 50 56	10 17 39 15
20	11 29 41 27	6 3 41 28	11 29 34 0	0 14 43

Parsing notes: Several values in the single-year table are internally inconsistent within their own column progression (e.g., Venus–Eccentric jumps to 11 29 56 at year 4, then back down at year 5; the same pattern recurs at years 8, 12, 16). These appear to be intrinsic to the source table — see Numerical impossibles.

Some seconds digits ambiguous in scan, flagged inline. Year 20 Mercury–Orb omits its seconds digit in the source (reads "0 14 43").

In Common Months [In mensibus communibus]

Month	Venus — Anom. Eccentric (S G M S)	Venus — Anom. Orb (S G M S)	Mercury — Anom. Eccentric (S G M S)	Mercury — Anom. Orb (S G M S)
January	1 0 33 11	0 19 6 43	1 0 33 9	3 6 18 31
February	1 28 8 57	1 6 22 30	1 28 8 53	6 3 17 51
March	2 28 42 9	1 25 29 13	2 28 42 3	9 9 36 22
April	3 28 16 11	2 13 58 58	3 28 16 3	0 12 48 30
May	4 28 49 23	3 3 5 41	4 28 49 13	3 19 7 1
June	5 28 23 25	3 21 35 6	5 28 23 14	6 22 19 10
July	6 28 56 37	4 10 42 10	6 28 56 23	9 28 37 41
August	7 29 29 48	4 29 48 53	7 29 29 32	1 4 56 13
September	8 29 3 52	5 18 18 38	8 29 3 33	4 8 8 20
October	9 29 37 3	6 7 25 22	9 29 36 43	7 14 26 52
November	10 29 12 6	6 25 55 6	10 29 10 44	10 17 38 59
December	11 29 44 17	7 15 1 50	11 29 43 55	1 23 57 32

In Leap-Year Months [In mensibus bissextilibus]

Month	Venus — Anom. Eccentric (S G M S)	Venus — Anom. Orb (S G M S)	Mercury — Anom. Eccentric (S G M S)	Mercury — Anom. Orb (S G M S)
January	1 0 33 11	0 19 6 43	1 0 33 9	3 6 18 31
February	1 29 8 6	1 6 59 29	1 29 8 1	6 6 24 15
March	2 29 41 17	1 26 6 12	2 29 41 11	9 12 42 46
April	3 29 15 20	2 14 35 57	2 29 15 11	0 15 54 54
May	4 29 48 31	3 3 42 41	4 29 48 21	3 22 13 26
June	5 29 22 34	3 22 12 26	5 29 22 22	6 25 25 34
July	6 29 55 45	4 11 19 8	6 29 55 31	10 1 44 5
August	8 0 28 47	5 0 25 54	8 0 28 40	1 8 2 37

Month	Venus — Anom. Eccentric (S G M S)	Venus — Anom. Orb (S G M S)	Mercury — Anom. Eccentric (S G M S)	Mercury — Anom. Orb (S G M S)
September	9 0 3 1	5 18 55 38	9 0 2 41	4 11 14 44
October	10 0 36 12	6 8 2 21	10 0 35 51	7 17 33 17
November	11 0 10 15	6 26 32 6	11 0 9 52	10 20 45 24
December	0 0 43 26	7 15 38 50	0 0 43 3	1 27 3 56

Parsing notes: April (leap-year) Mercury–Eccentric reads "2 29 15 11" in the source, where the surrounding rows (March "2 29 41 11", May "4 29 48 21") suggest the signs digit should be "3"; transcribed as printed and flagged. Both month tables follow the same four-column layout as the annual tables.

[Book p. 429] — Caput XXI (continued): Mean motions of Venus and Mercury

This opening (book p. 429) completes the **Table of the mean motions of Venus and Mercury** begun in the preceding pages — first the entries reckoned **in whole days**, then those **in hours and scruples of hours**. Both tables run on the same three motion-arguments established earlier: the *anomaly of the eccentric* (shared by Venus and Mercury), the *anomaly of the orb of Venus*, and the *anomaly of the orb of Mercury*. Two sub-tables sit side by side on this page.

Left sub-table — Anomaly of Venus and Mercury in whole days [in diebus simul]

Columns: day-count (1–31); anomaly of the eccentric of Venus and Mercury (G M S); anomaly of the orb of Venus (S G M S); anomaly of the orb of Mercury (S G M S).

Da y	Ecc . V & M — G M S			Orb Ve nus — S G M S				Orb Me rcu ry — S G M S			
	G	M	S	— S	G	M	S	— S	G	M	S
1	0	59	8	0	36	59	0	0	3	6	24
2	1	58	17	1	13	59	0	0	6	12	48

Da y	Ecc . V & M —			Orb Ve nus — S				Orb Me rcu ry — S			
	G	M	S	S	G	M	S	S	G	M	S
3	2	57	25	1	50	58	0	0	9	19	13
4	3	56	32	2	27	58	0	0	12	25	37
5	4	55	41	3	4	57	0	0	15	32	1
6	5	54	49	3	41	57	0	0	18	38	25
7	6	53	57	4	18	56	0	0	21	44	50
8	7	53	5	4	56	56	0	0	24	51	14
9	8	52	13	5	32	55	0	0	27	57	38
10	9	51	21	6	9	55	0	1	1	4	2
11	10	50	30	6	46	54	0	1	4	10	27
12	11	49	37	7	23	54	0	1	7	16	51
13	12	48	45	8	0	53	0	1	10	23	15
14	13	47	54	8	37	53	0	1	13	29	39
15	14	47	2	9	14	52	0	1	16	36	3
16	15	46	9	9	51	51	0	1	19	42	28
17	16	45	17	10	28	51	0	1	22	48	52
18	17	44	26	11	5	50	0	1	25	55	16
19	18	43	34	11	42	49	0	1	29	1	40
20	19	42	42	12	19	49	0	2	2	8	5
21	20	41	50	12	56	48	0	2	5	14	29
22	21	40	58	13	33	48	0	2	8	20	53
23	22	40	7	14	10	47	0	2	11	27	17
24	23	39	14	14	47	47	0	2	14	33	42
25	24	38	22	15	24	47	0	2	17	40	6
26	25	37	31	16	1	46	0	2	20	46	30
27	26	36	39	16	38	45	0	2	23	52	54
28	27	35	46	17	15	45	0	2	26	59	19
29	28	34	54	17	52	44	0	3	0	5	43

Da y	Ecc . V & M —			Orb Ve nus — S G M S				Orb Me rcu ry — S G M S			
	G	M	S	— S	G	M	S	— S	G	M	S
30	29	34	3	18	29	44	0	3	3	12	7
31	30	33	10	18	6[?]	43	0	3	6	18	31

Parsing notes: At day 31 the "orb of Venus" degree-entry reads 18 6 where the monotone progression of that column (...17 52, 18 29 at day 30) would lead one to expect roughly 19 6; the leading "1" of "19" may be a worn/dropped digit — flagged. The Venus-orb sign column is 0 throughout this page; the Mercury-orb sign column steps 0→1→2→3. Mercury-orb degree column at days 9→10 reads 27 57 38 → 1 1 4 2 (rollover of a sign), consistent.

Right sub-table — Anomaly of Venus and Mercury in hours and scruples of hours [in horis & scrupulis horarum]

This sub-table is laid out in two halves. The left half is keyed to **hours** (Hor. & Min. 1–24, then a few sexagesimal-second sub-rows); the right half is keyed to **minutes** (Min. 25–60... laid out as a second column block). The column heads are: hour/minute argument; anomaly of the eccentric of Venus & Mercury together (M S); anomaly of the orb of Venus (M S); anomaly of the orb of Mercury (G M S); then a minute-argument; anomaly of the eccentric of Venus & Mercury together (M S); anomaly of the orb of Venus (M S); anomaly of the orb of Mercury (M S).

H r	Ec							M in	Ec						
	c.		O		rb				c.		O		rb		
	V		V		rb				V		V		rb		
	&		e		M				&		e		M		
	M		n		er				M		n		er		
—	—		—		—			—	—		—		—		
M	S	M	S	G	M	S		M	S	M	S	M	S	M	S
1	2	28	1	32	0	7	46	31	1	16	0	48	4	1	
2	4	56	3	5	0	15	32	32	1	19	0	49	4	9	
3	7	24	4	37	0	23	18	33	1	21	0	51	4	16	
4	9	51	6	10	0	31	4	34	1	24	0	52	4	24	
5	12	19	7	42	0	38	50	35	1	26	0	54	4	32	

H r	Ec c. V & M —		O rb V e n —		O rb M er —		M in	Ec c. V & M —		O rb V e n —		O rb M er —		
	M	S	M	S	G	M		S	M	S	M	S	M	S
6	14	47	9	15	0	46	36	36	1	29	0	55	4	40
7	17	15	10	47	0	54	22	37	1	30	0	57	4	47
8	19	34	12	20	1	2	8	38	1	34	0	58	4	55
9	22	11	13	52	1	9	54	39	1	36	1	0	5	3
10	24	38	15	25	1	17	40	40	1	39	1	2	5	11
11	27	6	16	57	1	25	26	41	1	41	1	3	5	18
12	29	34	18	30	1	33	12	42	1	43	1	5	5	26
13	32	2	20	2	1	40	58	43	1	46	1	6	5	34
14	34	30	21	35	1	48	44	44	1	48	1	8	5	42
15	36	58	23	7	1	56	30	45	1	50	1	9	5	50
16	39	26	24	40	2	4	16	46	1	53	1	10	5	57
17	41	53	26	12	2	12	2	47	1	56	1	12	6	5
18	44	21	27	45	2	19	48	48	1	58	1	14	6	13
19	46	49	29	17	2	27	37	49	2	1	1	15	6	21
20	49	17	30	50	2	35	20	50	2	3	1	17	6	28
21	51	45	32	22	2	43	6	51	2	6	1	18	6	36
22	54	13	33	55	2	50	52	52	2	8	1	20	6	44
23	46	40	35	27	2	58	38	53	2	11	1	22	6	52
24	59	8	36	59	3	6	24	54	2	13	1	23	6	59
(s u b- ro w s, se c./ te								55	2	15	1	25	7	7

H r	Ec		O						Ec		O		O		
	c.		rb		O				c.		rb		O		
	V		V		rb				V		V		rb		
	&		e		M				&		e		M		
	M		n		er				M		n		er		
rt.)	—		—		—				M in	—		—		—	
	M	S	M	S	G	M	S	M		S	M	S	M	S	
25	(s		0	39	0	3	14	56	2	18	1	26	7	15	
	ec														
	.)														
	1														
	'2"														
26	1	4	0	40	0	3	22	57	2	20	1	28	7	23	
27	1	7	0	42	0	3	30	58	2	23	1	29	7	30	
28	1	9	0	43	0	3	37	59	2	25	1	41	7	38	
												[?			
												]			
29	1	11	0	44	0	3	45	60	2	28	1	42	7	46	
30	1	14	0	46	0	3	53								

Parsing notes: This right-hand table is the most layout-confused on the page; the double-scan garbles small superscript markers and the seconds/tertia sub-row labels. The lower block (rows 25–30 on the left half) carries small Sec./Tert. column heads indicating these are sexagesimal sub-divisions, not hours. Several cells are reconstructed from the smooth arithmetic progression of each column. Flagged cells: hour-22 eccentric "45 13" (the eccentric column should read close to 54' here given hour 21 = 51'45" and hour 23 = 56'40" on a smooth +2'28" step — the printed "45" looks like a compositor/scan error or a mis-set figure; transcribed as printed, flagged); minute-59 Venus-orb "1 41" (column elsewhere steps by ~1"–2", here jumps; the leading reading is uncertain). The whole right sub-table is a **table-reprocess candidate** for further given the cluster of uncertain small cells.

[Book p. 430] — Caput XXII

C A P V T XXII.

On the construction of the two prosthaphaeretic [prothaphæreticorum] tables for the two inferior planets, Venus and Mercury. [*De structura canonum prosthaphæreticorum duorum inferiorum Veneris et Mercurii.*]

The simple motions of these two planets, which were treated above, are similar in their constitution and use to those of the superior planets; so we lay down nothing further about them here. The same holds for the texture of the following table in the case of Venus, since — once the semidiameters have been supposed in accordance with the true celestial appearances — Venus's hypothesis has been resolved by us into numbers and into a table by the very same method Copernicus used [Greek: διδομένοις (didomenois, "given/granted")]. Indeed, we would not suppose that even here the proportional scruples [scrupula proportionalia] come out any differently from the Prutenic tables [Prutenicis].

As for the renewal of the theory of Mercury, it too departs little here from the common structure of the prosthaphaeretic table employed for the two superior planets and for Venus — provided only that the threefold analogy of the equating circlet [circellus aequans], commensurable with the revolution, with respect to the eccentric, is taken into account. For when this theory is restored into numbers, the proportional scruples are drawn out, in due proportion, from the distance of the center of the planet's orb (that is, the epicycle) from the Earth. Since the difference between the greatest and least distances — which the apsides, here too set opposite one another, also exhibit in this star, namely **11370** (in the units in which the radius of the eccentric is **100000**) — has been converted into **60**, and placed in the middle position of the rule of three [regula aurea, "golden rule"] (where the said difference 11370 will first occupy its place), while the third term is taken as any distance from the greatest, namely **106822**, by way of the anomaly of the eccentric reckoned beforehand and just now subsumed: from this there readily emerge the proportional scruples belonging to any degree, by way of the eccentric. We do the same in the remaining cases, once we have obtained the distances together with the equations of the eccentric, by the required operations, from the hypothesis of this star established above.

This method of constructing the proportional scruples — which we also applied above to the lunar appearances — I judge to be of all methods the most certain and the most correct, especially for recovering in turn the distances out of which

those scruples arise, since one sometimes has need of those very distances, namely for the more accurate estimation of parallaxes and latitudes.

As for the rest, and as concerns the appearances of the commutations [commutationum apparentias], nothing new is found in this star, since it will be sufficiently clear to those who understand that the parallax (or commutation) of the orb arises: at its greatest, from the given distance in the apogee of the eccentric, **106822**, with the radius of Mercury's orb **38500**; and the excess beyond this, similarly at the least distance in the perigee of the eccentric, **95452**, with the same orb-radius retained — and this for each single degree of the subsumed anomaly.

Let these few remarks on the construction of the following prosthaphaeretic tables for Venus and Mercury suffice; their use will be demonstrated afterward in the individual little precepts and examples.

[Book p. 431] — Table of prosthaphaereses of Venus (0 Sig. / 11 Sig.)

Table of prosthaphaereses of Venus [*Tabula prosthaphæreseon Veneris*]. Header **0 Sig.** at top (argument read downward, degrees 0–30), **11 Sig.** at foot (argument read upward via the right-hand degree column 30→0). The instructions in the body read **Adde** ("Add") for the left eccentric-prosthaphaeresis block at foot, and the column heads carry **Subtrah.** ("Subtract") / **Add.** ("Add") directions.

Columns (left to right): Degrees [Gradus]; Eccentric prosthaphaeresis, Subtract [Subtrah. Eccentr. Prostha.] (G M S); Difference [Diff.] (M S); Proportional scruples [Scru. prop.], Add [Add.] (M S); Diff. (S); Orb prosthaphaeresis [Orbis prosthap.], Add [Adde] (G M S); Diff. (M S); Excess in perigee [Excef. Perig.] (G M S); Diff. (M S); Degrees [Gradus].

G (down)		Ecc.Prostha. Subtr. G		M S		Diff M S		Scru.prop. Add M S		Dif											
S Orb prosthap. Adde G		M S		Diff M S		Excef.Perig. G		M S		Diff M S											
G (up)		---		---		---		---		0											
0	0	0	1	54	0	0	0	0	0	24	57	0	0	0	0	28	30		1	0	1
	54	1	54	0	0	0	0	24	57	24	57	0	0	28	0	28	29		2	0	3
	48	1	54	0	0	1	0	49	54	24	56	0	0	56	0	28	28		3	0	5
	42	1	54	0	1	1	1	14	50	24	55	0	1	24	0	28	27		4	0	7
	36	1	53	0	2	1	1	39	45	24	55	0	1	52	0	28	26		5	0	9
	29	1	53	0	3	2	2	4	40	24	55	0	2	20	0	28	25		6	0	11
	22	1	53	0	5	3	2	29	35	24	55	0	2	48	0	28	24		7	0	13

15 1 53 0 8 3 2 54 30 24 54 0 3 16 0 28 23 8 0 15
8 1 53 0 11 4 3 19 24 24 54 0 3 44 0 28 22 9 0 17
1 1 52 0 15 5 3 44 18 24 54 0 4 12 0 28 21 10 0
18 53 1 52 0 20 5 4 9 12 24 54 0 4 40 0 28 20 11 0
20 45 1 52 0 25 6 4 34 6 24 53 0 5 8 0 28 19 12 0
22 37 1 51 0 31 6 4 58 59 24 53 0 5 36 0 28 18 13
0 24 28 1 51 0 37 7 5 23 52 24 52 0 6 4 0 28 17
14 0 26 19 1 50 0 44 8 5 48 44 24 52 0 6 32 0 28 16
15 0 28 9 1 50 0 52 8 6 13 36 24 51 0 7 0 0 28 15
16 0 29 59 1 49 1 0 8 6 38 27 24 50 0 7 28 0 28
14 17 0 31 48 1 49 1 8 9 7 3 17 24 49 0 7 56 0 28
13 18 0 33 37 1 49 1 17 9 7 28 6 24 48 0 8 24 0 28
12 19 0 35 26 1 48 1 26 10 7 52 54 24 47 0 8 53 0
29 11 20 0 37 14 1 47 1 36 11 8 17 41 24 47 0 9 22
0 29 10 21 0 39 1 1 47 1 47 10 8 42 28 24 46 0 9
51 0 29 9 22 0 40 48 1 46 1 57 11 9 7 14 24 45 0
10 20 0 29 8 23 0 42 34 1 45 2 8 11 9 31 59 24 44
0 10 49 0 30 7 24 0 44 19 1 45 2 19 12 9 56 43 24
43 0 11 19 0 30 6 25 0 46 2 1 44 2 30 12 10 21 26
24 41 0 11 49 0 30 5 26 0 47 46 1 43 2 42 12 10 46
7 24 39 0 12 19 0 30 4 27 0 49 28 1 42 2 54 13 11
10 46 24 36 0 12 49 0 30 3 28 0 51 9 1 41 3 7 13
11 35 24 24 36 0 13 19 0 31 2 29 0 52 49 1 40 3 20
12 0 0 24 35 0 13 50 0 31 1 30 0 54 28 1 39 3 33
12 24 35 0 14 21 0 31 0

Foot directions: under the Eccentr. column **Adde** ("Add"); the foot row of each column carries **S.** (signs), and at very foot **11 Sig.** (so read upward for signs 11). The right-hand "Excef. Perig." Diff. column reads 0 28 throughout, stepping to 0 29, 0 30, 0 31 toward the bottom degrees.

Parsing notes: This is a multi-block sexagesimal table; columns were aligned against their smooth differences. Flagged cells: deg-12 orb-prosthap. seconds 4 58 59 (the orb column elsewhere steps ~24'55"; the "59" seconds reading is uncertain against the +24'53" difference shown); deg-15 eccentric seconds 0 28 9 (smooth column would give ~0 28 ?; "9" uncertain — at deg 15 the value should fall between deg 14 = 0 26 19 and deg 16 = 0 29 59, so ~0 28 09 is plausible but the units digit is faint); deg-22 proportional-scruples 2 57 (out of monotone order — neighbors read 1 47 and 2 8, so "2 57" appears to be a compositor transposition for ~1 57 or 1 58; transcribed as printed, flagged).

[Book p. 432] — Table of prosthaphaereses of Venus (1 Sig. / 10 Sig.)

Table of prosthaphaereses of Venus [*Tabula prosthaphæreseon Veneris*], continued. Header **1 Sig.** at top (degrees 0–30 downward), **10 Sig.** at foot (read upward via the right-hand degree column 30→0).

Columns (left to right): Degrees [Gradus]; Eccentric prosthaphaeresis, Subtract [Subtra. Eccent. prosth.] (G M S); Diff. (M S); Proportional scruples [Scru. Prop.] (M S); Diff. (S); Orb prosthaphaeresis, Add [Adde Orbis Prosthap.] (G M S); Add. Diff. (M S); Excess in perigee, Add [Add. Excessus in perig.] (G M S); Add. Diff. (M S); Degrees [Gradus].

G (down)		Ecc.prostha.		Subtr. G		M S		Diff M		S		Scru.Prop. M		S		Dif S			
Orb prosthap.		Adde G		M S		Add.Diff M		S		Excessus in perig.		G		M S					
Add.Diff M		S		G (up)															
	---		---		---		---		---		---		---		---		---		0
28		1		39		3		33		14		12		24		35		24	34
56		7		1		38		3		47		14		12		49		8	55
0		57		45		1		37		4		1		15		13		13	54
3		0		59		22		1		36		4		16		16		13	53
27				4		1		0		58		1		35		4		32	52
31		26				5		1		2		33		1		35		4	51
0		31		25				6		1		4		6		1		33	50
	0		31		24				7		1		5		38		1		32
58		0		31		23				8		1		7		9		1	31
18		29		0		31		22				9		1		8		39	30
0		19		0		0		31		21				10		1		10	29
18		0		19		31		0		31		20				11		1	28
	24		16		0		20		2		0		31		19				12
	29		24		14		0		20		34		0		32		18		
17		41		41		24		12		0		21		6		0		32	17
20		18		5		50		24		9		0		21		38		0	16
53		20		18		29		57		24		7		0		22		16	15
	8		13		21		18		54		2		24		5		0		22
	20		8		34		21		19		18		4		24		2		0
	1		18		8		55		22		19		42		3		23		59
	28		1		17		9		17		22		20		5		59		23
	23		43		1		15		9		39		22		20		29		53
21		1		24		56		1		13		10		1		23		20	53

9			22		1		26		8		1		12		10		24		23		21		17		32		23		48		0		26		2		0	
34		8			23		1		27		19		1		11		10		47		23		21		41		17		23		45		0		26			
36		0		34		7			24		1		28		29		1		10		11		10		23		22		4		59		23		42		0	
27		10		0		34		6			25		1		29		37		1		8		11		33		24		22		28		38		23		39	
		0		27		44		0		34		5			26		1		30		43		1		6		11		57		24		22		52		13	
		35		0		28		19		0		35		4			27		1		31		47		1		4		12		21		24		23		15	
		23		32		0		28		54		0		35		3			28		1		32		49		1		2		12		45		24		23	
		13		23		28		0		29		29		0		35		2			29		1		33		49		1		0		13		10		25	
		2		36		23		23		0		30		5		0		36		1			30		1		34		48		0		59		13		35	
		24		25		55		23		19		0		30		41		0		36		0																

Foot directions: left eccentric column **Adde**; foot row carries **S.** (signs); at very foot **10 Sig.** (read upward).

Parsing notes: Columns aligned against their smooth differences. Flagged cells: deg-8 orb-prosthaph. seconds 15 40 21 ("21" uncertain — the orb column near here reads ...15 1, 16 4 41; the seconds digit is faint); deg-14 orb-add-diff 24 9 ("9" faint; the Add.Diff column steps 24 33→24 5 monotonically, and ~24 09 fits). The eccentric-prosthaph. degree column rolls from 0° to 1° between deg 3 (0 59 22) and deg 4 (1 0 58), consistent. Several "4" glyphs in the scan are the long-form digit easily confused with the I/1 ligature; resolved by column arithmetic.

Table of the prosthaphaereses of Venus.

2 Sig. / 9 Sig.

This is a reciprocal sign-block page. The header "2 Sig." (sign 2) is read top-down with the left-hand Gradus column running 0→30; the footer "9 Sig." (sign 9) is read bottom-up with the right-hand Gradus column running 30→0.

Column structure (left to right): Gradus (left) | Eccentric prosthaphaeresis [eccentr. prosthaph.] — *Subtract* (G.M.S.) | Add. Diff. (M.S.) | Scruples proportional [scru. prop.] (M.S.) | Ad. Dif. (S.) | Orbit prosthaphaeresis [orbis prosthaph.] — *Add* (G.M.S.) | Ad. Diff. (M.S.) | Excess in perigee [excess. in peri.] (G.M.S.) | Ad. Diff. (M.S.) | Gradus (right).

Foot of columns (read at bottom): eccentric prosthaphaeresis = "Adde" (*Add*); scruples proportional / orbit prosthaphaeresis = "Subtr." (*Subtract*) [these are the directions for the reciprocal 9 Sig. block].

Grad .(L)	Ecce ntr. Pros tha. (Sub tr.) ° "	Diff. "	Scru. prop ."'	Dif. "	Orbi s pros thap . (Add e) °'''	Diff. '''	Exce ss. in peri. o'''	Diff. "	Grad .(R)
0	1 34 48	0 58	13 35	25	24 25 55	23 16	0 30 41	0 37	30
1	1 35 46	0 56	14 0	25	24 49 11	23 12	0 31 18	0 37	29
2	1 36 42	0 55	14 25	26	25 12 23	23 8	0 31 55	0 38	28
3	1 37 37	0 51	14 51	26	25 35 31	23 4	0 32 33	0 38	27
4	1 38 29	0 51	15 17	27	25 58 35	23 0	0 33 11	0 38	26
5	1 39 20	0 49	15 44	27	26 21 35	22 56	0 33 50	0 39	25
6	1 40 9	0 47	16 11	28	26 44 31	22 51	0 34 29	0 39	24
7	1 40 56	0 45	16 39	28	27 7 22	22 46	0 35 8	0 39	23
8	1 41 41	0 43	17 7	28	27 30 8	22 41	0 35 47	0 39	22
9	1 42 24	0 42	17 35	28	27 52 49	22 36	0 36 26	0 40	21
10	1 43 6	0 40	18 3	28	28 15	22	0 37	0 40	20

Grad .(L)	Ecce ntr. Pros tha. (Sub tr.) °	Diff. '''	Scru. prop .''''	Dif. "	Orbi s pros thap . (Add e) °'''	Diff. '''	Exce ss. in peri. o'''	Diff. "	Grad .(R)
					25	31	6		
11	1 43 46	0 38	18 31	29	28 37 56	22 25	0 37 46	0 41	19
12	1 44 24	0 36	19 0	29	29 0 21	22 29	0 38 27	0 42	18
13	1 45 0	0 34	19 29	29	29 22 41	22 14	0 39 9	0 42	17
14	1 45 34	0 32	19 58	29	29 44 55	22 8	0 39 51	0 43	16
15	1 46 6	0 30	20 27	30	30 7 3	22 2	0 40 34	0 43	15
16	1 46 36	0 29	20 57	29	30 29 5	21 55	0 41 17	0 44	14
17	1 47 5	0 27	21 26	30	30 51 0	21 49	0 42 1	0 44	13
18	1 47 32	0 24	21 56	30	31 12 49	21 43	0 42 45	0 45	12
19	1 47 56	0 22	22 26	30	31 34 32	21 37	0 43 30	0 46	11
20	1 48 18	0 21	22 55	30	31 56 9	21 30	0 44 16	0 46	10
21	1 48 39	0 19	23 25	31	32 17 39	21 22	0 45 2	0 46	9

Grad . (L)	Ecce ntr. Pros tha. (Sub tr.) ° "	Diff. "	Scru. prop ."	Dif. "	Orbi s pros thap . (Add e) °'''	Diff. "	Exce ss. in peri. o'''	Diff. "	Grad . (R)
22	1 48 58	0 17	23 56	30	32 39 1	21 14	0 45 48	0 47	8
23	1 49 15	0 14	24 26	31	33 0 15	21 5	0 46 35	0 47	7
24	1 49 29	0 13	24 57	31	33 21 20	20 57	0 47 22	0 48	6
25	1 49 42	0 11	25 28	31	33 42 17	20 49	0 48 10	0 49	5
26	1 49 53	0 8	25 59	31	34 3 6	20 40	0 48 59	0 49	4
27	1 50 1	0 6	26 30	31	34 23 46	20 32	0 49 48	0 50	3
28	1 50 7	0 5	27 1	31	34 44 18	20 24	0 50 38	0 51	2
29	1 50 12	0 3	27 32	31	35 4 42	20 15	0 51 29	0 52	1
30	1 50 15	—	28 13	—	35 24 57	—	0 52 21	—	0

Foot directions (for the reciprocal 9 Sig. block, read bottom-up): Eccentr. = "Adde" (S.); Scru. prop. = (S.); Orbis prosthap. = "Subtr." (S.); Excess. = (S.).

Table of the prosthaphaereses of Venus.

3 Sig. / 8 Sig.

Header "3 Sig." read top-down (left Gradus 0→30); footer "8 Sig." read bottom-up (right Gradus 30→0). Column directions: Eccentric prosthaphaeresis = *Subtract*; Diff. = *Add*; Orbit prosthaphaeresis = *Add*; foot = "Subtr." / "S. A."

This left-hand page (verso) is heavily ink-degraded in several columns; numerous cells are partly illegible. I transcribe what is legible and flag uncertain cells with [?]. This page is a strong

Grad .(L)	Ecce ntr. Pros thap . (Sub tr.) °		Scru p. prop o. °		Orbi s Pros thap h. (Add e) °		Exce ssus in peri g. °		Grad .(R)
	"	Diff. "	Ad. Dif. "	Ad. Dif. "	Diff. "	Diff. "	Diff. "	Diff. "	
0	1 50	0	28	32	35	20	0 52	0 53	30
	15[?]	1[?]	3[?]		24	5[?]	21		
					57				
1	1 50	0	28	32	35	19	0 53	0 53	29
	16	1[?]	35		45 2	54	14		
2	1 50	0	29 6	31	36 4	19	0 54	0 54	28
	15	4[?]			59	43	7		
3	1 50	0	29	32	36	19	0 55	0 55	27
	11	5[?]	38		24	32	1		
					39				
4	1 50	0	30 9	31	36	19	0 55	0 56	26
	6	8[?]			44	21	56		
					11				
5	1 49	0	30	32	37 3	19	0 56	0 57	25
	58	10[?]	41		32	10	52		
6	1 49	0 11	31	31	37	18	0 57	0 58	24
	48		21[?]		22	58	49		
					42				
7	1 49	0 13	31	32	37	18	0 58	0	23

Grad .(L)	Ecce ntr. Pros thap . (Sub tr.) ° "	Add. Diff. "	Scru p. prop o. '"	Ad. Dif. "	Orbi s Pros thap h. (Add e) °'''	Add. "	Exce ssus in peri g. °'''	Add. Diff. "	Grad .(R)
	37		44		41 40	46	47	58[?]	
8	1 49 24	0 16	32 15	31	38 0 26	18 33	0 59 45	0 59[?]	22
9	1 49 8	0 18	32 47	32	38 18 59	18 20	1 0 44	1 0	21
10	1 48 50	0 19	33 18	31	38 37 19	18 6	1 1 44	1 0[?]	20
11	1 48 31	0 21	33 50	32	38 55 25	17 52[?]	1 2 45	1 1	19
12	1 48 10	0 24	34 21	31	39 13 17	17 37	1 3 48	1 3	18
13	1 47 46	0 25	34 52	32	39 30 54	17 22	1 4 52	1 4	17
14	1 47 21	0 27	35 24	32	39 48 16	17 7	1 5 57	1 5	16
15	1 46 54	0 29	35 56	32	40 5 23	16 51	1 7 3	1 6	15
16	1 46 25	0 32	36 27	31	40 22 14	16 33	1 8 10	1 7[?]	14
17	1 45 53	0 34	36 59	32	40 38	16	1 9	1 8	13

Grad .(L)	Ecce ntr. Pros thap . (Sub tr.) ° "	Add. Diff. "	Scru p. prop o. '"	Ad. Dif. "	Orbi s Pros thap h. (Add e) °'''	Add. "	Exce ssus in peri g. °'''	Add. Diff. "	Grad .(R)
					47	14	18		
18	1 45 19	0 35	37 30	31	40 55 1	15 55	1 10 28	1 10	12
19	1 44 44	0 37	38 1	31	41 10 56	15 36	1 11 39	1 11	11
20	1 44 7	0 40	38 22[?]	31	41 26 32	15 16	1 12 52	1 13	10
21	1 43 27	0 41	39 3[?]	31	41 41 48	14 55[?]	1 14 6	1 14	9
22	1 42 46	0 43	39 44	31	41 56 43	14 34[?]	1 15 21	1 15	8
23	1 42 3	0 45	40 5[?]	31	42 11 17	14 12[?]	1 16 38	1 17	7
24	1 41 18	0 47	40 35[?]	30	42 25 29	13 50[?]	1 17 56	1 18	6
25	1 40 31	0 48	41 5[?]	30	42 39 19	13 26[?]	1 19 15	1 19	5
26	1 39 43	0 50	41 34[?]	29	42 52 45	13 0[?]	1 20 36	1 22[?]	4
27	1 38	0 53	42 4	30	43 5	12	1 21	1 23	3

Grad . (L)	Ecce ntr. Pros thap . (Sub tr.) °				Orbi s Pros thap h. (Add e) °				Grad . (R)
	"	Diff. "	Scru p. prop o. "	Ad. Dif. "	"	Diff. "	Exce ssus in peri g. °	Diff. "	
	53				45	32[?]	59		
28	1 38 0	0 54	42 33	29	43 18 17	12 3[?]	1 23 23	1 24	2
29	1 37 6	0 56	43 1	29	43 30 20	11 33	1 24 49	1 26	1
30	1 37 10	—	43 30	29	43 41 53	—	1 26 17	—	0

Foot directions (8 Sig. block, read bottom-up): Eccentr. = "Adde"; Diff. = "S. A."; Orbis Prosthaph. = "Subtr."; Excess. = "S."

Parsing notes: The verso (printed p. 434) is the most ink-degraded leaf of the batch. Several Diff. cells in the eccentric-prosthaphaeresis and orbit-prosthaphaeresis columns and several scruples-proportional values are partly bled and are flagged [?]; the main G.M.S. value columns are mostly legible. Numerous values were cross-checked against the smooth column progression. Row 30 eccentric prosthaphaeresis printed "1 37 10" which breaks the descending progression (29→1 37 6, then 30→1 37 10); flagged below as a numerical impossible / probable misread.

Table of the prosthaphaereses of Venus.

4 Sig. / 7 Sig.

Header "4 Sig." read top-down (left Gradus 0→30); footer "7 Sig." read bottom-up (right Gradus 30→0). Column directions: Eccentric prosthaphaeresis = *Subtract*; orbit prosthaphaeresis = *Add*; foot = "Adde" / "Ad." / "Subtr." / "S. A."

Grad .(L)	Ecce nt. Pros tha. (Sub tra.) °'''	Sub. Diff. '''	Scru. Prop .''''	Ad. Dif. ''	Orbi sAn. Pros thap . (Add e) °'''	Add. S. Diff. '''	Exce ss. in peri g. °'''	Ad. S. Diff. '''	Grad .(R)
0	1 36 10	0 58	43 30	28	43 41 53	11 2	1 26 17	1 30	30
1	1 35 12	0 59	43 58	29	43 52 55	10 33	1 27 47	1 32	29
2	1 34 13	1 1[?]	44 27	28	44 3 28	10 0	1 29 19	1 35	28
3	1 33 12	1 3	44 55	28	44 13 28	9 26	1 30 54	1 37	27
4	1 32 9	1 5	45 23	28	44 22 54	8 50	1 32 31	1 39	26
5	1 31 4	1 7	45 51	28	44 31 44	8 11	1 34 10	1 40	25
6	1 29 57	1 8	46 19	28	44 39 55	7 31	1 35 50	1 43	24
7	1 28 49	1 9	46 47	27	44 47 26	6 50	1 37 33	1 45	23
8	1 27 40	1 11	47 14	27	44 54 16	6 8	1 39 18	1 47	22
9	1 26 29	1 13	47 41	27	45 0 24	5 15	1 41 5	1 49	21
10	1 25	1 15	48 8	26	45 5	4 30	1 42	1 52	20

Grad . (L)	Ecce nt. Pros tha. (Sub tra.) °'''	Sub. Diff. '''	Scru. Prop . '''	Ad. Dif. "	Orbi sAn. Pros thap . (Add e) °'''	Add. S. Diff. '''	Exce ss. in peri g. °'''	Ad. S. Diff. '''	Grad . (R)
	15				39		54		
11	1 24 1	1 17	48 34	25	45 10 9	3 42	1 44 46	1 54	19
12	1 22 44	1 19	48 59	25	45 13 51	2 49	1 46 40	1 57	18
13	1 21 25	1 19	49 24	25	45 16 40	1 52	1 48 37	1 59	17
14	1 20 6	1 20	49 49	24	45 18 32	0 48	1 50 36	2 2[?]	16
15	1 18 46	1 22	50 13	24	45 19 20	0 13	1 52 38	2 4	15
16	1 17 24	1 24	50 37	24	45 19 7	1 27	1 54 42	2 7	14
17	1 16 0	1 26	51 1	23	45 17 40	2 17	1 56 49	2 9	13
18	1 14 34	1 27	51 24	23	45 15 23	3 40	1 58 58	2 12	12
19	1 13 7	1 28	51 47	22	45 11 43	4 50	2 1 10	2 15	11
20	1 11 39	1 29	52 9	22	45 6 45	6 20	2 3 25	2 17	10
21	1 10	1 30	52	22	45 0	7 46	2 5	2 18	9

Grad . (L)	Ecce nt. Pros tha. (Sub tra.) o'''	Sub. Diff. '''	Scru. Prop . '''	Ad. Dif. "	Orbi sAn. Pros thap . (Add e) o'''	Add. S. Diff. '''	Exce ss. in peri g. o'''	Ad. S. Diff. '''	Grad . (R)
	10		31		25		42		
22	1 8 40	1 32	52 53	22	44 52 39	9 19	2 8 0	2 20	8
23	1 7 8	1 33	53 15	21	44 43 20	10 55	2 10 20	2 22	7
24	1 5 35	1 35	53 36	20	44 32 25	12 39	2 12 42	2 24	6
25	1 4 0	1 36	53 56	20	44 19 46	14 28	2 15 6	2 26	5
26	1 2 24	1 37	54 16	19	44 5 18	16 23	2 17 32	2 27	4
27	1 0 47	1 38	54 35	19	43 48 55	18 26	2 19 56	2 27	3
28	0 59 9	1 39	54 54	18	43 30 29	20 38	2 22 26	2 26	2
29	0 57 30	1 40	55 12	18	43 9 51	22 55	2 24 52	2 24	1
30	0 55 50	—	55 30	—	42 46 56	—	2 27 16	—	0

Foot directions (7 Sig. block, read bottom-up): Eccentr. = "Adde"; Diff. = "Ad."; Orbis Prosthap. = "Subtr."; Excess. = "S. A."

Table of the prosthaphaereses of Venus.

5 Sig. / 6 Sig.

Header "5 Sig." read top-down (left Gradus 0→30); footer "6 Sig." read bottom-up (right Gradus 30→0). Column directions: Eccentric prosthaphaeresis = *Subtract*; orbit prosthaphaeresis = *Add*; Excess in perigee = *Subtract*; foot = "Adde" / "Ad." / "Subtr." / "Add." / "Add."

This recto (printed p. 436) is moderately ink-degraded; several cells are partly bled and flagged [?]. The clean lower rendering was used.

Grad .(L)	Ecce ntri Pros thap . (Sub trah e) °'''	Diffe .''''	Scru p. prop o. '''	Di. ''	Orbi s Pros thap . (Add) °'''	Diffe r. (Sub t.) °'''	Exce ss. in peri g. (Sub t.) °'''	Dif. '	Grad .(R)
0	0 55 50	1 41	55 30	17	42 46 56	—	2 27 16	2 23	30
1	0 54 9	1 43	55 47	16	42 21 33	0 25 23	2 29 39	2 21	29
2	0 52 26	1 44	56 3	16	41 53 34	0 27 59	2 32 0	2 19	28
3	0 50 42	1 45	56 19	15	41 22 49	0 30 45	2 34 19	2 13[?]	27
4	0 48 57	1 45	56 34	15	40 49 8	0 33 41	2 36 22	2 13	26
5	0 47 13	1 46	56 49	14	40 12 19	0 36 49	2 38 38	2 6	25
6	0 45 27	1 47	57 3	14	39 32	0 40 7	2 40 34	1 56	24

Grad .(L)	Ecce ntri Pros thap . (Sub trah e) °'''	Diffe .''''	Scru p. prop o.''''	Di. ''	Orbi s Pros thap . (Add) °'''	Diffe r. (Sub t.) °'''	Exce ss. in peri g. (Sub t.) °'''	Dif. ' ''	Grad .(R)
					12				
7	0 43 40	1 48	57 17	14	38 48	0 43 38	2 42 20	1 46	23
					34				
8	0 41 52	1 49	57 31	13	38 1 12	0 47 22	2 43 54	1 34	22
9	0 40 3	1 50	57 44	13	37 9 54	0 51 18	2 45 11	1 17	21
10	0 38 13	1 50	57 57	12	36 14	0 55 37	2 46 5[?]	0 54	20
					27				
11	0 36 23	1 51	58 9	12	35 14	0 59 53	2 46 42	0 37	19
					34				
12	0 34 22	1 51	58 21	12	34 10 4	1 4 30	2 46 49	0 7	18
13	0 32 41	1 52	58 33	11	33 0 42	1 9 22	2 46 24[?]	0 25	17
14	0 30 49	1 53	58 44	10	31 46	1 14 25	2 45 20	1 4	16
					17				
15	0 28 56	1 53	58 54	8	30 26	1 19 43	2 43 34	1 46[?]	15
					34				
16	0 27 3	1 54	59 2	7	29 1 25[?]	1 25 9	2 40 57[?]	2 37[?]	14
17	0 25 9	1 54	59 9	7	27 30	1 30 49	2 37 24[?]	3 33[?]	13

Grad .(L)	Ecce ntri Pros thap . (Sub trah e) °'''	Diffe .''''	Scru p. prop o. °'''	Di. °''	Orbi s Pros thap . (Add) °'''	Diffe r. (Sub t.) °'''	Exce ss. in peri g. (Sub t.) °'''	Dif. °' ''	Grad .(R)
					36				
18	0 23 15	1 55	59 16	7	25 54 3	1 36 33	2 32 48	4 36	12
19	0 21 20	1 55	59 23	6	24 11	1 42 25	2 27 5[?]	5 43	11
					38				
20	0 19 25	1 56	59 28	6	22 23	1 48 13	2 20 4	7 1	10
					25				
21	0 17 30	1 56	59 34	5[?]	20 29	1 54 4	2 11 49	8 15	9
					21				
22	0 15 34	1 56	59 39	5	18 29	1 59 36	2 2 3[?]	9 46	8
					45				
23	0 13 38	1 56	59 43	4	16 24	2 5 3	1 50 54	11 9	7
					42				
24	0 11 42	1 57	59 47	4	14 14	2 10 9	1 38 22	12 32	6
					33				
25	0 9 45	1 57	59 50	3	11 59	2 14 41	1 24 24	13 58	5
					52				
26	0 7 48	1 57	59 53	3	9 41 2	2 18 12	1 9 17	15 7	4
27	0 5 51	1 57	59 56	3	7 18 56	2 22 6	0 52 56	16 22	3

Grad .(L)	Ecce ntri Pros thap . (Sub trah e) °'''	Diffe . '''	Scru p. prop o. '''	Di. ''	Orbi s Pros thap . (Add) °'''	Diffe r. (Sub t.) °'''	Exce ss. in peri g. (Sub t.) °'''	Dif. '	Grad .(R)
28	0 3 54	1 57	59 58	2	4 54 4	2 24 52	0 35 40	17 8	2
29	0 1 57	1 57	59 59	1	2 27 31	2 26 33	0 18 4	17 44	1
30	0 0 0	—	60 0	1	0 0 0	2 27 31	0 0 0	18 4	0

Foot directions (6 Sig. block, read bottom-up): Eccentr. = "Adde"; Diff. = "Ad."; Orbis Prosthap. = "Subtr."; Differ. = "Add."; Excess. = "Add."

Parsing notes: At grad. 30 the eccentric prosthaphaeresis and orbit prosthaphaeresis both reach 0 0 0, the scruples-proportional column reaches its maximum 60 0, and the excess-in-perigee column reaches 0 0 0 — the table closes correctly at the perigee point (signs 5/6 boundary). The "Differ." (orbit-prosthaphaeresis difference) column begins blank at grad. 0 and is filled from grad. 1; this is the running first-difference. Several excess-in-perigee values in the lower-middle rows are ink-bled and flagged [?].

Astronomia Danica — Pars Altera, Liber Secundus Theoricorum, Caput XXII (cont.)

Table of the Prosthaphaereses of Mercury [Tabula prosthaphæreseon Mercurii]

0 Sig. (signs read top-down; the reciprocal **11 Sig.** read bottom-up)

Column headings: Degrees [Gradus] | Subtract [Subtrahe] — Eccentric prosthaphaeresis [Eccentr. prosthap.] | Add [Add.] Diff. | Add [Ad.] Proportional scruples [Scru. Prop.] | Diff. | Add [Adde] Orb prosthaphaeresis [Orbis Prosthap.]

| Add [Add.] Diff. | Add [Adde] Excess in perigee [Excessus in perig.] | Diff. |
Degrees [Gradus]

For the reciprocal block (11 Sig., read upward), the column operations shown at the foot are: Eccentric prosthaph. → **Add** [Adde]; Diff. → **Sub.**; Orb prosthaph. → **Subtract** [Subtr.]; Diff. → **Sub.**; Excess in perigee → **Add** [A.].

Gr.	Ecce ntr. prosthaph. (G M S)	Diff. (M S)	Scru. Prop. (M S)	Dif. (S)	Orbi s Pros thaph. (G M S)	Diff. (M S)	Exce ssus in peri g. (G M S)	Diff. (M S)	Gr.
0	0 0 0	1 54	0 0	0	0 0 0	15 54	0 0 0	1 20	30
1	0 1 54	1 53	0 1	1	0 15 54	15 54	0 1 20	1 20	29
2	0 3 47	1 54	0 2	1	0 31 48	15 53	0 2 40	1 21	28
3	0 5 41	1 54	0 3	3	0 47 41	15 53	0 4 1	1 21	27
4	0 7 35	1 54	0 6	4	1 3 34	15 52	0 5 22	1 21	26
5	0 9 29	1 54	0 10	7	1 19 26	15 52	0 6 43	1 21	25
6	0 11 23	1 54	0 17	7	1 35 18	15 51	0 8 4	1 21	24
7	0 13 18	1 55	0 24	8	1 51 9	15 50	0 9 25	1 21	23
8	0 15 13	1 55	0 32	10	2 6 59	15 50	0 10 46	1 22	22
9	0 17 9	1 57	0 42	11	2 22 49	15 50	0 12 8	1 21	21
10	0 19 6	1 57	0 53	11	2 38 40	15 50	0 13 30	1 21	20
11	0 21 4	1 59	1 4	12	2 54 30	15 49	0 14 51	1 21	19

Gr.	Ecce ntr. pros thap . (G M S)	Diff. (M S)	Scru. Prop . (M S)	Dif. (S)	Orbi s Pros thap . (G M S)	Diff. (M S)	Exce ssus in peri g. (G M S)	Diff. (M S)	Gr.
12	0 23 3	1 59	1 16	13	3 10 19	15 49	0 16 12	1 20	18
13	0 25 2	1 59	1 29	12	3 26 8	15 49	0 17 32	1 21	17
14	0 27 1	1 59	1 31	12	3 41 57	15 49	0 18 53	1 21	16
15	0 29 0	2 0	1 43	12	3 57 46	15 48	0 20 14	1 20	15
16	0 31 0	2 0	1 55	13	4 13 34	15 48	0 21 34	1 21	14
17	0 33 0	2 0	2 8	18	4 29 22	15 47	0 22 55	1 21	13
18	0 35 0	2 0	2 26	21	4 45 9	15 47	0 24 16	1 22	12
19	0 37 0	2 0	2 47	23	5 0 55	15 46	0 25 38	1 22	11
20	0 39 0	2 0	3 10	25	5 16 41	15 44	0 27 0	1 22	10
21	0 41 0	2 0	3 35	25	5 32 25	15 38	0 28 22	1 22	9
22	0 43 0	2 0	4 0	26	5 48 3	15 32	0 29 44	1 23	8
23	0 45 0	2 1	4 26	24	6 3 35	15 26	0 31 7	1 23	7
24	0 47 0	2 1	4 50	25	6 19 1	15 20	0 32 30	1 23	6
25	0 49 1	2 3	5 15	24	6 34 21	15 13	0 33 53	1 23	5
26	0 51 4	2 5	5 39	25	6 49 34	15 8	0 35 16	1 23	4

Gr.	Ecce ntr. prosthaph. (G M S)	Diff. (M S)	Scru. Prop. (M S)	Dif. (S)	Orbis Prosthaph. (G M S)	Diff. (M S)	Excessus in perigeo (G M S)	Diff. (M S)	Gr.
27	0 53 9	2 8	6 4	24	7 4 42	15 3	0 36 39	1 23	3
28	0 55 17	2 11	6 28	24	7 19 45	14 59	0 38 2	1 24	2
29	0 57 28	2 14	6 52	24	7 34 44	14 56	0 39 26	1 24	1
30	0 59 42		7 16		7 49 40		0 40 50		0
	Add e	Sub.		S.	Subtr. r.	Sub.		A.	

Parsing notes: The 0 Sig. table reads top-down for sign 0 (Cancer-relative start), bottom-up for sign 11. At Gr. 14 the proportional-scruples entry prints "1 31" where the smooth progression (1 29 ... 1 43) would expect ~1 36; flagged. The unlabeled "Diff." column between Scru. Prop. and Orbis Prosthaph. carries the small differences of the proportional-scruples column (header "Dif.", values in seconds).

Table of the Prosthaphaereses of Mercury [Tabula prosthaphæreseon Mercurii]

1 Sig. (read top-down; the reciprocal **10 Sig.** read bottom-up)

Column operations at foot (10 Sig., read upward): Eccentric prosthaph. → **Add** [Adde]; Diff. → **Sub.** [Subt.]; Orb prosthaph. → **Subtract** [Subtr.]; Excess in perigee → **S.**

Gr.	Ecce ntri pros thap . (G M S)	Diff. (M S)	Scru p. prop o. (M S)	Dif. (S)	Orbi s Pros thap h. (G M S)	Diffe . (M S)	Exce ssus in peri g. (G M S)	Diff. (M S)	Gr.
0	0 59 42	2 16	7 16	24	7 49 40	14 54	0 40 50	1 24	30
1	1 1 58	2 18	7 40	24	8 4 34	14 51	0 42 14	1 24	29
2	1 4 16	2 20	8 4	25	8 19 25	14 49	0 43 38	1 24	28
3	1 6 36	2 22	8 29	25	8 34 14	14 47	0 45 2	1 24	27
4	1 8 58	2 24	8 54	25	8 49 1	14 45	0 46 26	1 24	26
5	1 11 22	2 25	9 19	26	9 3 46	14 43	0 47 50	1 24	25
6	1 13 47	2 24	9 45	27	9 18 29	14 41	0 49 14	1 24	24
7	1 16 10	2 24	10 12	29	9 33 10	14 39	0 50 58	1 24	23
8	1 18 34	2 24	10 41	31	9 47 49	14 37	0 52 2	1 25	22
9	1 20 58	2 24	11 13	33	10 2 26	14 35	0 53 26	1 25	21
10	1 23 22	2 24	11 46	34	10 17 1	14 32	0 54 50	1 25	20
11	1 25 46	2 24	12 20	35	10 31 33	14 27	0 56 15	1 25	19
12	1 28 10	2 25	12 55	34	10 46 0	14 22	0 57 40	1 25	18
13	1 30 35	2 25	13 29	34	11 0 22	14 17	0 59 5	1 25	17
14	1 33	2 25	14 3	34	11	14	1 0	1 25	16

Gr.	Ecce ntri pros thap . (G M S)	Diff. (M S)	Scru p. prop o. (M S)	Dif. (S)	Orbi s Pros thap h. (G M S)	Diffe . (M S)	Exce ssus in peri g. (G M S)	Diff. (M S)	Gr.
	0				14 39	11	30		
15	1 35 25	2 25	14 37	34	11 28 50	14 5	1 1 55	1 26	15
16	1 37 50	2 25	15 11	34	11 42 55	13 58	1 3 21	1 26	14
17	1 40 15	2 25	15 45	35	11 56 53	13 51	1 4 47	1 26	13
18	1 42 40	2 25	16 20	35	12 10 44	13 44	1 6 13	1 26	12
19	1 45 5	2 26	16 55	36	12 24 28	13 36	1 7 39	1 26	11
20	1 47 31	2 26	17 31	36	12 38 4	13 30	1 9 5	1 27	10
21	1 49 57	2 26	18 7	36	12 51 35	13 26	1 10 32	1 27	9
22	1 52 23	2 26	18 43	37	13 5 1	13 20	1 11 59	1 27	8
23	1 54 49	2 26	19 20	36	13 18 21	13 14	1 13 26	1 27	7
24	1 57 15	2 27	19 56	37	13 31 35	13 9	1 14 53	1 27	6

Gr.	Ecce ntri pros thap . (G M S)	Diff. (M S)	Scru p. prop o. (M S)	Dif. (S)	Orbi s Pros thap h. (G M S)	Diffe . (M S)	Exce ssus in peri g. (G M S)	Diff. (M S)	Gr.
25	1 59 32	2 27	20 33	37	13 44 44	13 3	1 16 20	1 28	5
26	2 1 59	2 27	21 10	37	13 57 47	12 56	1 17 48	1 28	4
27	2 4 26	2 27	21 47	38	14 10 43	12 49	1 19 16	1 28	3
28	2 6 53	2 27	22 25	37	14 23 32	12 41	1 20 44	1 28	2
29	2 9 20	2 27	23 2	37	14 36 13	12 34	1 22 12	1 28	1
30	2 11 47		23 39		14 48 47		1 23 40		0
	Add e	Subt.		S.	Subt r.			S.	

Parsing notes: At Gr. 7 the excess-in-perigee prints "0 50 58", breaking the smooth progression (0 49 14 → 0 52 2) — interpolation expects ~0 50 38; flagged as possible compositor error or misread. At Gr. 8 the entry "0 52 2" likewise sits slightly off the expected progression; flagged. Gr. 25 eccentric "1 59 32" reads plausibly but the last second digit is ink-degraded; flagged.

Table of the Prosthaphaereses of Mercury [Tabula prosthaphæreseon Mercurii]

2 Sig. (read top-down; the reciprocal **9 Sig.** read bottom-up)

Column operations at foot (9 Sig., read upward): Eccentric prosthaph. → **Add** [Adde]; Scrup. → **S.**; Orb prosthaph. → **Subtract** [Subtr.]; Diff. → **Sub.**; Excess → **Sub.**

Gr.	Eccen- tr. pro- stha- p. (G M S)	Diff. (M S)	Scru- p. Prop- o. (M S)	Dif- (S)	Orbi- s Pros- thap- p. (G M S)	Diff. (M S)	Exce- ssus in peri- g. (G M S)	Diff. (S)	Gr.
0	2 11 47	2 28	23 39	37	14 48 47	12 33	1 23 40	1 29	30
1	2 14 15	2 28	24 16	38	15 1 20	12 22	1 25 9	1 29	29
2	2 16 43	2 28	24 54	37	15 13 42	12 12	1 26 38	1 29	28
3	2 19 11	2 28	25 31	38	15 25 54	12 1	1 28 7	1 29	27
4	2 21 39	2 28	26 9	37	15 37 55	11 50	1 29 36	1 29	26
5	2 24 7	2 28	26 46	37	15 49 45	11 39	1 31 7	1 30	25
6	2 26 35	2 28	27 23	37	16 1 24	11 28	1 32 37	1 30	24
7	2 29 3	2 28	28 0	37	16 12 52	11 18	1 34 7	1 30	23
8	2 31 31	2 28	28 37	37	16 24 10	11 8	1 35 37	1 30	22
9	2 33 59	2 29	29 14	37	16 35	10 58	1 37 7	1 30	21

Gr.	Ecce ntr. pros thap . (G M S)	Diff. (M S)	Scru p. Prop o. (M S)	Dif. (S)	Orbi s Pros thap . (G M S)	Diff. (M S)	Exce ssus in peri g. (G M S)	Diff. (S)	Gr.
					18				
10	2 36 28	2 29	29 51	37	16 46	10 49	1 38 37	1 31	20
					16				
11	2 38 57	2 27	30 28	37	16 57 5	10 41	1 40 8	1 31	19
12	2 41 24	2 27	31 4	36	17 7 46	10 33	1 41 39	1 31	18
13	2 43 51	2 23	31 41	37	17 18	10 25	1 43 10	1 31	17
					19				
14	2 46 14	2 19	32 17	36	17 28	10 18	1 44 41	1 31	16
					44				
15	2 48 33	2 14	32 53	36	17 39 2	10 9	1 46 12	1 30	15
16	2 50 47	2 13	33 29	36	17 49	10 0	1 47 42	1 30	14
					11				
17	2 53 0	2 10	34 4	35	17 59	9 52	1 49 12	1 30	13
					11				
18	2 55 10	2 5	34 38	34	18 9 3	9 45	1 50 42	1 30	12
19	2 57 15	2 3	35 11	33	18 18	9 36	1 52 12	1 30	11
					48				
20	2 59 18	2 0	35 44	33	18 28	9 26	1 53 40	1 31	10
					24				

Gr.	Ecce ntr. pros thap . (G M S)	Diff. (M S)	Scru p. Prop o. (M S)	Dif. (S)	Orbi s Pros thap . (G M S)	Diff. (M S)	Exce ssus in peri g. (G M S)	Diff. (S)	Gr.
21	3 1 18	1 56	36 17	33	18 37 47	9 6	1 55 11	1 31	9
22	3 3 14	1 52	36 50	33	18 46 53	8 49	1 56 42	1 30	8
23	3 5 6	1 44	37 24	33	18 55 42	8 32	1 58 12	1 30	7
24	3 6 50	1 40	37 57	33	19 4 14	8 15	1 59 13	1 31	6
25	3 8 30	1 38	38 30	33	19 12 29	7 58	2 1 14	1 31	5
26	3 10 8	1 33	39 3	33	19 20 27	7 42	2 2 45	1 31	4
27	3 11 41	1 27	39 37	34	19 28 9	7 25	2 4 16	1 30	3
28	3 13 8	1 22	40 11	34	19 35 34	7 8	2 5 46	1 31	2
29	3 14 30	1 17	40 44	33	19 42 42	6 50	2 7 17	1 30	1
30	3 15 47		41 16	32	19 49 32		2 8 47		0
	Add e		S.		Subt r.	Sub.		Sub.	

Parsing notes: At Gr. 25 the eccentric prosthaphaeresis prints "3 6 30", which is less than the Gr. 24 value "3 6 50" and breaks the otherwise monotonic-increasing column (expected ~3 8 8 by interpolation, with Diff. 1 38); flagged as a clear compositor error or misread of the degree/minute digits. At Gr. 5 the excess-in-perigee "1 31 7" and at Gr. 24 "1 59 13" are slightly ink-degraded; flagged.

Table of the Prosthaphaereses of Mercury [Tabula prosthaphæreseon Mercurii]

3 Sig. (read top-down; the reciprocal **8 Sig.** read bottom-up)

Column operations at foot (8 Sig., read upward): Eccentric prosthaph. → **Add** [Adde]; Diff. → **S. A.**; Orb prosthaph. → **Subtract** [Subt.]; Diff. → **Su.Ad** [Su. Ad.]; Excess → **S.**

Gr.	Eccentric Prosthaph. (G M S)	Diff. (M S)	Scrup. Prop. (M S)	Diff. (S)	Orbit Prosthaph. (G M S)	Diff. (M S)	Excess in peri. (G M S)	Diff. (M S)	Gr.
0	3 15 47	1 12	41 16		19 49 32	6 35	2 8 47	1 29	30
1	3 16 59	1 10	41 47	30	19 56 7	6 20	2 10 16	1 29	29
2	3 18 9	1 9	42 17	30	20 2 27	6 4	2 11 45	1 28	28
3	3 19 18	1 7	42 46	29	20 8 31	5 51	2 13 13	1 28	27
4	3 20 25	1 6	43 15	29	20 14 22	5 37	2 14 41	1 28	26
5	3 21 31	1 5	43 43	28	20 19 59	5 24	2 16 9	1 28	25
6	3 22 36	1 4	44 11	28	20 25	5 10	2 17 37	1 28	24

Gr.	Ecce ntri Pros thap . (G M S)	Diffe . (M S)	Scru p. Prop o. (M S)	Dif. (S)	Orbi s Pros thap h. (G M S)	Diffe . (M S)	Exce ssus in peri g. (G M S)	Diff. (M S)	Gr.
					23				
7	3 23 40	1 4	44 38	27	20 30 33	4 56	2 19 5	1 28	23
8	3 24 44	1 3	45 6	28	20 35 29	4 39	2 20 33	1 27	22
9	3 25 47	1 2	45 33	27	20 40 8	4 20	2 22 0	1 26	21
10	3 26 49	1 1	46 0	27	20 44 28	4 8	2 23 26	1 25	20
11	3 27 50	1 0	46 26	26	20 48 18	3 50	2 24 51	1 24	19
12	3 28 50	0 59	46 51	25	20 51 48	3 30	2 26 15	1 23	18
13	3 29 49	0 58	47 16	25	20 54 58	3 10	2 27 38	1 21	17
14	3 30 47	0 56	47 40	24	20 57 48	2 50	2 28 59	1 20	16
15	3 31 43	0 50	48 4	24	21 0 18	2 30	2 30 19	1 19	15
16	3 32 33	0 46	48 18 [?]	24	21 2 38	2 10	2 31 38	1 18	14
17	3 33	0 31	48	24	21 4	1 48	2 32	1 17	13

Gr.	Ecce ntri Pros thap . (G M S)	Diffe . (M S)	Scru p. Prop o. (M S)	Dif. (S)	Orbi s Pros thap h. (G M S)	Diffe . (M S)	Exce ssus in peri g. (G M S)	Diff. (M S)	Gr.
	19		52		26		56		
18	3 34 0	0 0	49 15	23	21 5 51	1 25	2 34 13	1 16	12
19	3 34 0	0 4	49 38	23	21 6 47	0 56	2 35 29	1 16	11
20	3 33 56	0 6	50 0	22	21 7 18	0 31	2 36 47	1 15	10
21	3 33 50	0 8	50 22	22	21 7 33	0 15	2 38 0	1 13	9
22	3 33 42	0 11	50 43	21	21 7 33	0 10	2 39 13	1 10	8
23	3 33 31	0 18	51 3	20	21 7 23	0 20	2 40 23	1 8	7
24	3 33 13	0 22	51 22	19	21 7 3	0 25	2 41 31	1 5	6
25	3 32 51	0 26	51 41	19	21 6 3	1 0	2 42 36	1 2	5
26	3 32 25	0 30	52 0	19	21 4 38	1 25	2 43 38	0 59	4
27	3 31 55	0 34	52 18	18	21 2 48	1 50	2 44 37	0 56	3
28	3 31 25	0 40	52 36	18	21 0 28	2 20	2 45 33	0 53	2
29	3 30 45	0 40	52 53	17	20 57 38	2 50	2 46 26	0 50	1
30	3 30 0		53 10	17	20 54 21	3 17	2 47 16		0
	Add	S. A.		S.	Subt.	Su.A		S.	

	Ecce				Orbi			Exce	
	ntri			Scru	s			ssus	
	Pros			p.	Pros			in	
	thap	Diffe		Prop	thap	Diffe		peri	Diff.
	. (G	. (M		o. (M	h. (G	. (M		g. (G	(M
Gr.	M S)	S)		S)	M S)	S)		M S)	S)
									Gr.
	e					d			

Parsing notes: The 3 Sig. eccentric column reaches its maximum ($\sim 3^{\circ}34'$) around Gr. 18–19 and turns over (decreasing thereafter), so the Diff. column switches sense — printed foot is "S. A." (subtract/add). The orb-prosthaphaeresis column likewise reaches maximum ($\sim 21^{\circ}7'$) around Gr. 20–22 then decreases, foot "Su.Ad". Several cells in the turnover region of the orb column (Gr. 3, 25, 26, 28) and its Diff. are ink-degraded and read inconsistently against the smooth turnover; flagged. The orb-prosthap. Gr. 30 value "20 54 21" continues the descent into the next block (4 Sig.); flagged for cross-batch verification. The left margin of this page (verso of the bound leaf) is partly cropped/degraded — the left-most repeated fragment column was the Excessus/Diff. tail of the facing 3 Sig. table; transcription above is from the clean recto rendering.

Astronomia Danica — Pars Altera, Liber Secundus Theoricorum

Caput XXII (continued) — Table of the prosthaphaereses of Mercury

Table of the prosthaphaereses of Mercury — sign block 4 Sig. (top, descending) / 7 Sig. (bottom, ascending)

The column-head titles for the 4 Sig. reading (top-down) are: Eccentric prosthaphaeresis = **Subtract**; Orb prosthaphaeresis = **Add**; the reciprocal 7 Sig. block reads bottom-up with the foot-titles (Eccentric = **Add**; Orb = **Subtract**).

Grad us (4 Sig.)	Ecce nt. pros th. (° ")	Diff. (")	Scru p. prop . (")	Dif. (")	Orb pros thap . (°")	Diff. (")	Exce ff. in peri g. (° ")	Diff. (")	Grad us (7 Sig.)
0	3 30 0	0 50	53 10	17	20 50 38	4 15	2 47 16	0 48	30
1	3 29 10	0 54	53 27	16	20 46 23	4 44	2 48 4	0 45	29
2	3 28 16	1 0	53 43	16	20 41 39	5 13	2 48 49	0 41	28
3	3 27 16	1 6	53 59	15	20 36 26	5 41	2 49 30	0 38	27
4	3 26 10	1 14	54 14	15	20 30 45	6 9	2 50 8	0 34	26
5	3 24 56	1 22	54 29	14	20 24 36	6 39	2 50 42	0 30	25
6	3 23 34	1 32	54 43	14	20 17 57	7 8	2 51 12	0 25	24
7	3 22 2	1 43	54 57	14	20 10 49	7 40	2 51 37	0 22	23
8	3 20 19	1 55	55 11	13	20 3 9	8 16	2 51 59	0 19	22
9	3 18 24	2 6	55 24	13	19 54 53	8 55	2 52 18	0 18	21
10	3 16 18	2 18	55 37	13	19 45	9 25	2 52 36	0 16	20

Grad us (4 Sig.)	Ecce nt. pros th. (° ")	Diff. (")	Scru p. prop . (")	Dif. (")	Orb pros thap . (°")	Diff. (")	Exce ff. in peri g. (° ")	Diff. (")	Grad us (7 Sig.)
					58				
11	3 14 0	2 30	55 49	12	19 36 33	9 56	2 52 54	0 6 [?]	19
12	3 11 30	2 37	56 1	12	19 26 37	10 30	2 53 0	0 7 [?]	18
13	3 8 53	2 45	56 13	11	19 16 7	11 6	2 52 53 [?]	0 12 [?]	17
14	3 6 8	2 53	56 24	11	19 5 1	11 40	2 52 41 [?]	0 19 [?]	16
15	3 3 15	2 57	56 35	10	18 53 21	12 22	2 52 22	0 25	15
16	3 0 18	3 0	56 45	10	18 40 59	13 8	2 51 57	0 31	14
17	2 57 18	3 4	56 55	10	18 27 51	13 56	2 51 26	0 39	13
18	2 54 14	3 5	57 5	10	18 13 55	14 18	2 50 47	0 44	12
19	2 51 9	3 6	57 15	9	17 59 37	14 45	2 50 3	0 52	11
20	2 48 3	3 7	57 24	8	17 44 52	15 30	2 49 11	1 3	10

Grad us (4 Sig.)	Ecce nt. pros th. (° ")	Diff. (")	Scru p. prop . (")	Dif. (")	Orb pros thap . (°")	Diff. (")	Exce ff. in peri g. (° ")	Diff. (")	Grad us (7 Sig.)
21	2 44 56	3 8	57 32	9 [?]	17 29 22	16 0	2 48 8	1 12	9
22	2 41 48	3 9	57 41	8	17 13 22	16 34	2 46 56	1 18	8
23	2 38 39	3 12	57 49	8	16 56 48	17 6	2 45 38	1 24	7
24	2 35 29	3 14	57 57	8	16 39 42	17 46	2 44 14	1 31	6
25	2 32 17	3 16	58 5	8	16 21 36	18 31	2 42 43	1 38	5
26	2 29 3	3 19	58 13	8	16 3 25	19 25	2 41 5	1 50	4
27	2 25 47	3 22	58 21	8	15 44 0	20 15	2 39 15	2 6	3
28	2 22 28	3 26	58 29	7	15 23 45	21 5	2 37 9	2 22	2
29	2 19 6	3 26 [?]	58 36	6	15 2 40	21 56	2 35 47	2 34	1
30	2 15 40	—	58 42	—	14 40 44	—	2 33 13	—	0

Foot-titles (for the 7 Sig. block, read bottom-up): Eccent. prosth. = **Add**; Scrup. prop. = **Ad.**; Orb prosthap. = **Subtr.**; Exceff. = **S. A.**

Parsing notes: At Gr 11–14 of the 7-Sig "Exceff. in perig." difference column, the printed digits are ink-degraded (values read 0 6, 0 7, 0 12, 0 19); these break the

otherwise smooth small-difference run near the excess maximum and are flagged. The Exceff. main-column entries at Gr 13–14 (2 52 53, 2 52 41) appear to dip slightly below the Gr 12 peak (2 53 0); the digits are faint — flagged as possible misreads.

Table of the prosthaphaereses of Mercury — sign block 5 Sig. (top, descending) / 6 Sig. (bottom, ascending)

Column-head titles for the 5 Sig. reading (top-down): Eccentric prosthaphaeresis = **Subtract**; Orb prosthaphaeresis = **Subtract**; the reciprocal 6 Sig. block reads bottom-up.

Grad us (5 Sig.)	Ecce ntri pros thap h. (°°°)	Diff. (°°)	Scru p. prop o. (°°)	Dif. (°)	Orb pros thap h. (°° °)	Diffe r. (°°°)	Exce ffus in peri g. (°° °)	Diff. (°°)	Grad us (6 Sig.)
0	2 15 40	3 30	58 42	5	14 40 44	22 34	2 33 13	2 36	30
1	2 12 10	3 34	58 47	5	14 18 12	23 2	2 30 37	2 40	29
2	2 8 36	3 38	58 52	4	13 55 10	23 37	2 27 57	2 44	28
3	2 4 58	3 42	58 56	4	13 31 33	24 2	2 25 13	2 48	27
4	2 1 16	3 46	59 0	5 [?]	13 7 31	24 24	2 22 25	2 51 [?]	26
5	1 57 30	3 50 [?]	59 5	4	12 43 7	24 44	2 19 24	2 54	25
6	1 53 40	3 56	59 9	5 [?]	12 18 23	25 19	2 16 30	3 4	24
7	1 49	4 1	59	4	11	25	2 13	3 15	23

Grad us (5 Sig.)	Ecce ntri pros thap . (°''')	Diff. (''')	Scru p. prop o. (''')	Dif. (''')	Orb pros thap h. (°')	Diffe . (''')	Exce ffus in peri g. (°')	Diff. (''')	Grad us (6 Sig.)
	44		14		53 4	54	26 [?]		
8	1 45 43	4 1	59 18	5 [?]	11 27 10	26 32	2 10 11	3 31	22
9	1 41 38	4 7	59 23	4	11 0 38	27 13	2 6 40	3 47	21
10	1 37 25	4 13	59 27	4	10 33 25	27 54	2 2 53	4 10	20
11	1 33 5 [?]	4 20	59 31	4	10 5 31	28 35	1 58 43	4 40	19
12	1 28 38	4 27	59 35	3 [?]	9 36 56	29 11	1 54 3	5 10	18
13	1 24 3	4 35	59 38	3	9 7 45	29 41	1 48 53	5 40	17
14	1 19 20	4 43	59 41	3	8 38 4	30 6	1 43 13	6 5	16
15	1 14 31	4 49	59 43	3	8 7 58	30 26	1 37 8	6 15 [?]	15
16	1 9 38	4 53 [?]	59 46	2	7 37 32	30 46	1 30 53	6 20 [?]	14
17	1 4 43	4 55 [?]	59 48	2	7 6 46	31 7	1 24 33	6 23	13
18	0 59 47 [?]	4 56 [?]	59 50	2	6 35 39	31 27	1 18 10	6 24	12
19	0 54 50	4 57 [?]	59 52	2	6 4 12	31 48	1 11 46	6 25 [?]	11
20	0 49	4 58	59	2	5 32	32	1 5	6 27	10

Grad us (5 Sig.)	Ecce ntri pros thap . (°''')	Diff. (''')	Scru p. prop o. (''')	Dif. ('')	Orb pros thap h. (° ')	Diffe . (''')	Exce ffus in peri g. (° ')	Diff. (''')	Grad us (6 Sig.)
	52		54		24	17	21		
21	0 44	4 58	59	1	5 0 7	32	0 58	6 29	9
	54		56			36	54		
22	0 39	4 59	59	1	4 27	32	0 52	6 30	8
	56		57		31	56	25		
						[?]			
23	0 34	4 59	59	1	3 54	33	0 45	6 31	7
	57		58		35	10	55		
24	0 29	4 59	59	0	3 21	33	0 39	6 32	6
	58		59		25	20	24		
25	0 24	5 0	59	0	2 48	33	0 32	6 33	5
	59	[?]	59		5	30	52		
26	0 20	5 0	59	0	2 14	33	0 26	6 34	4
	0		59		35	35	19		
27	0 15	5 0	59	0	1 41	33	0 19	6 34	3
	0		59		0	35	45		
28	0 10	5 0	60 0	0	1 7	33	0 13	6 35	2
	0				21	49	11		
29	0 5 0	5 0	60 0	0	0 33	33	0 6	6 36	1
					41	41	36		
						[?]			
30	0 0 0	—	60 0	—	0 0 0	—	0 0 0	—	0

Foot-titles (for the 6 Sig. block, read bottom-up): Eccentri prosthaph. = **Add**; Scrup. propo. = **S.**; Orb prosthaph. = **Subtr.**; Exceffus = **A.**

Parsing notes: This page is the most ink-degraded of the batch; many of the small Diff. column entries (single- and double-digit minutes/seconds) are faint. Where a digit was uncertain I flagged it [?] but transcribed my best reading; the diff-columns are smooth monotone progressions, which constrains the readings. The "Orbis Prosthaph." column descends smoothly to 0 0 0 at Gr 30, and the "Exceffus

in perig." column likewise to 0 0 0 — consistent with the perigee endpoint. The Gr 14 5 Sig. eccentric reads 1 19 20; the Gr 11 value (1 33 5) and Gr 10 value (1 37 31) are faint in the units place — flagged.

CAPVT XXIII

On the method of computing the true longitudes of Venus and Mercury by the foregoing canons.

The procedure for computing the phenomena of these stars with respect to longitude is for that reason somewhat easier than for the others, and it scarcely differs at all from the Prutenic method [Prutenica], since it everywhere takes the Sun's mean motion as its basis.

First, for the required time, the Sun's mean motion is extracted from the solar table inserted in the previous book — and that, as the table gives it, reckoned with respect to the mean vernal equinox. When the inequality [of precession] is applied to this, the mean elongation of these planets from the true equinox is established within this motion of the Sun.

Second, in the same way, for the same time the mean motions of the anomalies of each of these planets are worked out and copied off, as contained in the tables just set out.

Third, by means of the eccentric anomaly of each, its prosthaphaeresis is sought, together with the proportional scruples [scrupula proportionalia] adjoined to it. When this prosthaphaeresis is applied to the mean eccentric motion according to the titles inscribed on the canon, the planet's true motion in the eccentric, reckoned from the true equinox, is produced. By the contrary operation, the same prosthaphaeresis is to be applied to the anomaly of commutation [anomalía commutationis], so that this too is rendered equated.

Fourth, by means of the anomaly of commutation thus equated, the equation of the orb [aequatio orbis] of each is investigated in the table of prosthaphaereses — the quantity that Copernicus [Copernicus] calls the parallax [parallaxis] — together with the excess [excessus], of which, according to the scruples that have been kept, a proportional part is taken. This part is always to be added to the equation, so that the equation may be discerned for every position of the orb's center in the eccentric. Afterwards, according to the titles for subtraction or addition, it is to be applied to the planet's true elongation in the eccentric. When this has been done, the sought longitude of the planet from the true equinox will be established — which was what was proposed.

A worked example for both, namely Venus and Mercury together.

In the year 1607, in the month of April, on the 14th day, in the evening at the 8½th hour [hora 8½] — when Venus stood above our horizon by about 2°, and Mercury above her in an almost straight line, tending toward Capella [Capella], from which, however, it was deflecting fairly perceptibly toward the north — I observed here, by means of the Astronomical radius [radius Astronomicus], at least the distance of these planets from one another to be 2½°. Now at that time Venus's latitude was some southerly value, about 10 minutes (at most), as will be shown afterward; while Mercury's was about 1½° north [Septen.], as I investigated more precisely on the following days through other observations. With these matters noted beforehand in the heavens, we must now see what arrangement with respect to longitude the calculus of the reformed hypotheses assigns to these two planets relative to one another.

First, then, for the assumed time the longitude of the Sun from the mean equinox comes out commonly to 1 sign 2° 11' 41". When the inequality of precession is added to this — namely 8' 9" — the mean elongation of both planets from the true equinox becomes 1 sign 2° 19' 50". Then, for the same time the following are found:

			Sig.	Gr.	Mi.	Sec.
anomal y in	Venus	Eccentr ic	10	1	34	16
		Orb	1	6	23	7
	Mercur y	Eccentr ic	5	1	24	5
		Orb	2	12	48	11

Furthermore, from the table of prosthaphaereses for each there is given:

	Gr.	Mi.	Sec.	
prosthapha eresis of the eccentric in Venus	1	33	15	add.
with proportion		12	56	

	Gr.	Mi.	Sec.	
al scruples				
likewise in Mercury	2	10	46	ablat.
with proportion al scruples		58	48	

When these prosthaphaereses have been duly applied according to their titles, there results:

in			Sig.	Gr.	Mi.	Sec.
	Venus	True longitude of the eccentric	1	3	53	5
		Equated anomaly	1	4	49	52
	Mercury	True longitude of the eccentric	1	0	9	4
		Equated anomaly of the orb	2	14	58	57

Further, since the equated orb anomaly of each is prescribed, the equation of the orb is likewise found from the tables set out above:

In		Gr.	Mi.	Sec.		(pars propo rt.) Mi.	Sec.
	Venus	14	23	1	add. with the excess	16	25
	(hence the propo rtiona l part)					3	27
	Mercu ry	17	38	53	add. with the excess	(Gr.) 1 (Mi.) 33 (Sec.) 18	
	(when ce the propo rtiona l part)					1 44	10

Parsing notes: This is a two-tier nested brace layout in the source. The left brace groups Venus and Mercury; the right-hand small columns give the proportional part taken from the excess. For Venus the equation of the orb is 14° 23' 1" "add. with the excess," and the proportional part is 16' 25" (with a further 3' 27"). For Mercury the equation of the orb is 17° 38' 53" "add. with the excess," with proportional parts 1° 33' 18" and 1° 44' 10" given to the right. The exact pairing of the two right-hand sub-values to "Pars proport." / "unde pars proportion." is as the source brackets them; the column alignment is loose in the print.

Since, however, in the apogee the proportional parts are added to the orb equations, and these too — as the titles direct — to the eccentric elongations from the true equinox obtained earlier, there finally emerges the sought longitude of Venus: 18° 19' 33" of Taurus [□]. Similarly, too, the sought longitude of Mercury: 19° 32' 7" of Taurus [□]. So it appears that Mercury, by our calculation, followed behind Venus as regards longitude alone by about 1° [recte: 1¼°? — the text reads "quasi 1½ gra." but the figures give a ~1°12' separation]; the remaining part of

their distance, which had been about equal to this, was made up by the withdrawal of the two into latitudes of opposite directions.

And so much, then, for the restitution of these planets too — Venus and Mercury — through their longitudes.

CAPVT XXIV

On the demonstration of the latitudes of Venus and Mercury, agreeing with the celestial appearances, together with their canon.

The latitudes of these planets, rightly considered, confirm the hypotheses of their longitudes no less than do those of the three superior planets; indeed there is great agreement between these and the superior ones, provided we weigh the matter correctly. For just as there the inclination of the eccentric orbs to the ecliptic is perceptible on both sides — so much so that they are not varied except at the common sections [communes sectiones] of these circles over a fairly long stretch of time — in the same way the latitudes of these planets behave likewise, so that their epicycles, or rather their orbs, are found tilted toward the ecliptic with a perceptible inclination on both sides, in such a way indeed that only the common segments are altered in these with the passage of time.

As for the segments of circles of this kind, which in the superior planets were by no means bound uniformly to the motions of the apsides — contrary to what was the opinion of the ancients, and especially the Ptolemaic [Ptolemaica] opinion, which Copernicus followed — although in Mercury they are understood to pass through its apsides (which at the beginning of our century we showed to lie in [the beginning of] Pisces [♓] and Gemini [♊ recte ♊]) [?]: nevertheless in Venus these same nodal points [sectionum puncta] of the apsides — which were found near the end of Gemini [♊] and Pisces [♓] — run ahead by 16 degrees, so that in this century they fall at about 13° of Gemini [♊], as the celestial observations have taught us.

But to the said common sections of the ecliptic and of the planet's orb we refer a stable and invariant inclination of latitude, in every [position] through the eccentric or...

— that is, by the carrying-round of the center of the planet's orb, with Ptolemy [Ptolemy] and Tycho [Tycho Brahe], or of the earth itself, with Copernicus [Copernicus]. The motion is indeed of the same kind: from the first departures from the ecliptic's common section of the orb, into the north for Venus, but into the south for Mercury — and so it is everywhere reversed by turns according to

the relation of the section. And because the visible angles of inclination of these planets too are regularly altered with their distances from the earth, it cannot be that there is some eccentric, distinct from the plane of the ecliptic, to which this orb (like the epicycle in the case of the Moon) will conform itself uniformly. For that reason it is by now consonant with the truth that these two stars acknowledge the supposition of Copernicus, or indeed of Tycho, in which the orbs themselves are substituted for the Ptolemaic epicycles.

Furthermore, the observations decidedly reject the two kinds of variation that Ptolemy devised concerning their latitudes. The first is the one by which he imagined that the angles of declination and of reflexion (which are in fact the same thing) are increased at the contact of the epicycle — that is, at the planet's greatest digression to the sides. Into this error the apparent latitudes of Venus around the said places chiefly led Ptolemy, after he had made the perpetual angle of inclination one whole degree smaller than the angle that results from observations taken near the perigee of the epicycle and the quadrants of the eccentric — such observations as could scarcely ever have been obtained by Ptolemy in a less oblique sphere. Accordingly, what was thus left out through ignorance he chose to repair by some makeshift or fictitious addition, but in a manner — as in the cases above — that ran almost contrary [to the truth]. And yet all the latitude of these five planets in general, and its variation, depends on the recession of the center of the planet's orb from the said common section (where the angle of perpetual inclination resides, to be ascertained before everything else in each case); likewise on the motion of the planet in the epicycle or its proper orb; and no less on its distance from the earth as required by the demands of longitude under the hypothesis. The other variation of the latitudes in these planets, similarly, which they call the deviation [*deviationem*], appears most strongly near the apsides of the eccentric — namely the deviation that in Venus would extend northward to a sextant of a degree [a sixth, i.e. 10'], but in Mercury, on the contrary, southward, with a perpetual maximum declination reaching a dodrans [three-quarters, i.e. 45']. We shall truly call this a deviation or Ptolemaic error, since neither is the cause of it anywhere manifest in the hypothesis of longitude, nor do we yet detect any sure and firm traces of it in our own — or even in the ancients' — observations, once they have been duly examined and restored. For neither does that variation which is produced (below the celestial appearances) in connection with the latitudes of Mercury at apogee and perigee, and which will be excused by the reformed hypothesis of longitude, fit here. Hence celestial experience itself, rather than commonplace imitations, has furnished us the occasion of cleansing our part of the Augean stable, and of here too restoring the suppositions of the latitudes to their native simplicity.

Comparing the observations with one another, we found the angle of the mean inclination in Venus to be $3\frac{1}{2}$ degrees — by a whole degree, that is, greater than under the Ptolemaic and Copernican supposition — but in Mercury $5\frac{3}{4}$ degrees.

With these things presupposed, and the rest that we have just pressed home as required for the investigation of the latitudes, we shall complete the recapitulation [epilogismum] of those latitudes — far more consonant with the heavens — with almost uniform ease. For this, in each of the two planets, a general diagram drawn in the received forms must be set out beforehand and briefly explained, so that it may lie open, as it were at a single glance, to the reader's eyes, into what latitude each of these planets passes from the ecliptic at any time throughout the whole eccentric — in the Ptolemaic and Tychonic form together, that is, since in either case the epicycle or orb of the planet stands upon the eccentric obliquely with respect to the plane of the ecliptic.

Schema of the latitudes of Venus and Mercury in the Ptolemaic and Tychonic form.

Diagram. A geometric construction showing the latitude theory of Venus and Mercury in the Ptolemaic/Tychonic form. At the center is point **A**, the center of the eccentric. Around it a large eccentric circle is described, with eight epicycle positions marked at points **B, C, D, E, F, G, H, I** around its circumference. Two perpendicular diameters **B F** (running top–bottom, vertical) and **D H** (running left–right, horizontal) divide the figure. The left side is labeled **Orient**. (East) and the right side **Occident**. (West). At each of the eight positions a small inclined epicycle is drawn, each carrying the labels **R, S** (the points where the epicycle's section corresponds to **B F**) and **K, L** (the points answering to **H D**), together with **M, N** (the endpoints of the diameter drawn through the epicycle's center where the common intersection with **L K** occurs). The inclination angle **L B N** (= **K B M**) at each position gives the measure of the mean inclination.

Labels around the central eccentric, reading clockwise from top: **B** (top), **C** (upper-left), **D** (left), **E** (lower-left), **F** (bottom), **G** (lower-right), **H** (right), **I** (upper-right). Center: **A**.

Each peripheral epicycle is lettered with **R** (top), **K** (right), **S** (bottom), **L** (left), and the inclination diameter endpoints **M, N**.

Beneath the diagram the text continues:

Upon center **A** let the eccentric **B D F** &c. be described, and with the diameters **B F** and **D H** drawn at right angles, let there be supposed, at the places **B C D E F G H**

I, the epicycle or orb of Venus, or also of Mercury — the eccentricity, of course, that accounts for their inequality in longitude being omitted here for the sake of brevity and convenience. Further, this epicycle, in any position whatever throughout the eccentric, takes sections similar to those of the eccentric, and indeed plainly parallel to them — so that **R S** answers to **B F**, and **K L** looks toward **H D**, and this in every carrying-round of the epicycle through the eccentric.

But since the orb or epicycle of the planet is perceived to be inclined to the ecliptic, let the diameter **M N** finally be drawn everywhere through the center of the epicycle, where the common intersection occurs with **L K**, so that the angle **L B N** (or **K B M**) may hold the measure of the mean inclination in each of these planets — which in Venus we have asserted above to be $3\frac{1}{2}$ degrees, and in Mercury $5\frac{3}{4}$, found by us from the heavens, as we shall also make plainer below.

When the schema of the latitudes of these planets is set up in this way in the form, whether Ptolemaic or Tychonic, the denominations and varieties of the latitudes are sufficiently in view — varieties that follow the planet's revolution, both through the eccentric and through the epicycle, that is, its proper orb. Thus we, for example, beginning the consideration of the latitudes in Venus from the intersection **B**, which precedes the apogee by 16 degrees (as was said before), pronounce the whole semicircle **R N S**, which faces the rising, to be of northern latitude perpetually and in every revolution; but the remaining semicircle **S M R** to be of southern. The turns change throughout, however, so that the hemisphere of the epicycle toward the rising completes the southern latitude, and the other, toward the setting, the northern; while the crossings occur at **R** and **S**, the line of which is perpetually parallel, in these planets, to the line of the apogee **B F**.

So much for the general indication of the latitudes in these two planets according to the Ptolemaic or Tychonic form. The Copernican follows, which — being still of the same equivalence — is easier, since the orb of the planet invariably occupies the middle, in this manner.

Diagram of the latitudes of Venus and Mercury in the Copernican form.

Diagram. A geometric construction (upper-right inset) for the latitude theory in the Copernican form. Upon center **A** the great eccentric, that is, the great orb of the earth, **B D** is described as a large circle (with **B** at top, **D** at bottom). Within it a smaller circle, the homocentric inclined orb of the planet, is set, carrying the points **K** (upper) and **C** (lower) on the line of common intersection. The line of common intersection **B D** runs vertically through the center. The line **E F** runs

through the center at right angles to **B D** (with **F** at left, **E** at right), showing the plane of the ecliptic, i.e. of the earth's great orb. A further line **G H** is drawn through center **A** (with **G** upper-right, **H** lower-left), making with **E F** on either side the angle **H A F** (or **G A E**) equal to the perpetual inclination of the planet's orb to the plane of the eccentric (the great orb of the earth).

Labels: **B** (top), **K** (inner, upper), **G** (right of center, upper), **A** (center), **F** (left), **E** (right), **H** (lower-left), **C** (inner, lower), **D** (bottom).

The text reads:

Upon center **A** of the eccentric (as before), let the eccentric itself — that is, the great orb of the earth, **B D** — be described; and upon it let the planet's homocentric inclined orb **K C** be imposed, for convenience' sake — although it would be more correct to fix the center of this orb at the movable terminus of the eccentricity, since this has some weight in the Mercurial latitudes, even though in Venus it imports almost nothing perceptible, on account both of the eccentricity and of the orb's inclination, which is almost twice smaller than in Mercury. Let the line of common intersection **B D** be drawn, which in Mercury is at the apogee, but in Venus anticipates the apogee by 16 degrees. This passes through the common segments **K** and **C**; and let **E F** be set up through the center and at right angles [to it], showing the plane of the ecliptic, that is, of the great orb of the earth. Finally let **G H** be drawn across through the center **A**, which here makes on either side, with **E F**, the angle **H A F** (or **G A E**) equal to the perpetual inclination of the planet's orb from the plane of the eccentric, or great orb of the earth.

These things being thus established — and with the contrary disposition of the latitudes in Venus and Mercury (as shown above) being considered — in whatever situation of the earth, throughout its own circle **B D**, the latitude of Venus, as it traverses the eastern semicircle of its orb **K H C**, will always be northern. But in the other semicircle **C G K** all things behave in the contrary manner, so that the latitude of Venus through that becomes southern, and that of Mercury, on the contrary, northern. Nor are more things required for ascertaining the state of the latitudes in these planets in general. Indeed — what we hoped for in the outcome — they differ in no respect from the foregoing, since in both cases we contemplate a sure and stable digression of the orbs into latitude with respect to the ecliptic, and measure the rest, which touch upon the distances, from the longitude hypothesis of each. How these now come about, and how the latitude of each of these planets at any time may be obtained by a uniform application of plane triangles, we shall make plain in the appended example.

Let us take that example from the first observation of Venus among us, where we found its true southern latitude, with the parallax removed, to be 2 degrees 8 minutes — namely near the end of the 16th degree [of Leo], which fell in the year 1585, on 15 September in the morning, at hour 5¾, as was noted above. [Margin: Cap. 18, h[oc] in[...].]

Now the coequated anomaly of the eccentric was:

	Sig.	Gr.	Mt.
The coequated anomaly of the eccentric was	3	2	43
But the orb of the epicycle, likewise coequated	7	9	0

— although it would have been the same had one applied these simple [values] on either side.

Further, the distance of Venus from the earth is 63120, of the kind in which the radius of the eccentric is 100000.

These things being given, and at the same time granting that the northern node in Venus anticipates the apogee by 16 degrees, the schema of the latitude of Venus is supposed in the Ptolemaic or Tyconic form, as before.

Diagram. A geometric construction (upper-right) accompanying the worked latitude example for Venus. A large circle (the eccentric) is drawn with **B** at top and **D** at bottom on a vertical diameter; center near **A** (right of middle). A second, dashed circle (the epicycle/orb of Venus) is drawn to the left, centered at **C**. Points and lines shown: **B** (top), **D** (bottom), **A** (center-right), **C** (center of the smaller circle), **R** and **P** (upper-left, on the small circle and its diameter), **M** (between, near the chord), **N** and **F** (right of center, on the horizontal line **E...F**), **E** (upper-left, on the small circle), **H** (left), **K** (lower portion of the small circle). The construction triangles **L M E** and **A M L** of the text are formed among these points (the engraving renders the labels small; **L** lies near the lower-left/center where the dashed circle meets the lines).

In the left margin, the text begins:

For the arc of the eccentric **B C** is composed by adding to the anomaly of the eccentric the anticipation of the node, namely 16 degrees; and it becomes 3 signs 18 degrees 43 minutes. Equal to this is [the arc] in the orb, **E H**, since **B D** and **E K** are parallel. But with Venus tarrying in **M**, the arc **H M** is 7 signs 9 degrees. And so, **E H** and **H M** being added, **E H M** is gathered as 10 signs 27 degrees 43 minutes.

But since **N C** is perpendicular to **E K**, the angle **N C M** — that is, the arc **N M** — is likewise established as 57 degrees 43 minutes, the dodrans [three quarters] of the circle being subtracted, that is, from the given arc **E H M** alone. And since Venus here holds the western part of its epicycle with respect to the line of intersection **E K**, therefore, according to the things shown above, its latitude is pronounced to be southern. This is to be elicited in the following manner, and compared with the observed [latitude].

First, in order to have the side **M L** in the triangle **L M E**, that side must be sought first in proportion with the right sine **M P** — namely the sine of the complement of the angle **M C N**. For as the radius 100000 is to the said sine, which in this example is 53411, so is the perpetual angle of inclination of Venus's orb, demonstrated before as 3 degrees 30 minutes, **L E M**, to the said **M L** 3261 — which, offered more swiftly [Greek: κατὰ προσθαφάρεσιν (kata prosthaphairesin, "by [the method of] addition-and-subtraction")], and a second time proportioned with the radius of Venus's orb 72248, comes out as 2356.

Finally, in the triangle **A M L**, from the given **M A** (the distance of Venus from the earth) 63120, the side **M L** just found 2356, and the angle at **L** (which, as it were, is always toward the ecliptic) being right, there is found, by way of the angle **M A L**, the sought latitude of Venus, 2 degrees 8 minutes — which agrees plainly with the observation. The Alphonsine calculation [calculus Alphonsinus] exhibits this found and observed latitude of Venus quite competently in this place; whereas Copernicus falls short of it by half a degree.

So much concerning Venus. But concerning the other, additional small variation observed in connection with the apogee and perigee of the eccentric of Mercury, the celestial observations that have been inserted in the synopsis above inform us. For in the year of the Lord 1585, on 15 November, in the morning at hour 7, around the greatest digression of Mercury from the Sun — namely, with the corrected anomaly of the orb at 8 signs 16 degrees 22 minutes — the true observed latitude of Mercury at Uraniborg was 2 degrees 18 minutes north [Boreal]; and this near the apogee of the eccentric, or at least $3\frac{1}{2}$ degrees from it in consequence [eastward].

Further, in the year of the Lord 1593, I find that Mercury was likewise observed near the perigee, and at its greatest evening digression, at Kassel by the Landgrave's [astronomers], for several continuous days, around the middle of May — and this not without wonder, since the perpetual twilight, even of that place, lasts then, though perhaps not so conspicuously as among us.

But among these observations, when [we take] that one which was had on 11 May in the evening at hour $9\frac{1}{2}$,

(continuing from the Mercury latitude worked example) ...it was; and once I have assumed the latitude of Mercury that this observation of theirs yields, namely $2^{\circ}4\frac{1}{2}'$, and have in any case freed myself of the parallaxes that exceed the refractions in the contrary direction, I do not find that this same latitude was in fact larger than 2° . Now at that time the eccentric anomaly of Mercury was 5 signs 29° nearly — that is, 1° short of perigee; and the anomaly of commutation was about 3 signs 22° , as these corrected motions too are more precisely seen above. Therefore Mercury was then likewise near its greatest digression [digressio] from the Sun.

Between these two observed latitudes there is a difference of 18 minutes, and this seems plainly contrary to the longitude hypothesis: for the angle of the planetary orb's departure from the ecliptic appears larger at apogee than at perigee, whereas by optical reasoning it ought to be smaller. For this reason neither the Ptolemaic deviation, nor the Sun's greater distance from the Earth at the time when the planet was at the perigee of the eccentric (and conversely), can save the appearance. The former produces those leaps of latitude (of which Copernicus [Copernicus] complains) suddenly into opposite directions, and moreover shows here, near apogee, an appearance that contradicts it; while the latter at the same time leaves Mercury's digressions in longitude shorter in its orb around perigee — which likewise runs counter to the appearance in longitude.

On this account, what Copernicus supposed belonged to his longitude hypothesis for Mercury, for representing the Ptolemaic deviation — namely the epicyclet he had devised — that same thing we, with better right, assign to our small circle [circellus], the equant being granted: that is, it is posited as inclined toward the north at apogee, and equally far toward the south at perigee. For since the restitution [of this small circle] is made at one and a half times the rate, relative to the half-measure of the eccentric, it consequently takes up the contrary direction at the contrary apsides of the eccentric. Now this inclination, on both sides close to the latitude, is felt to amount to about half the appearance in longitude — namely about $15'$ near apogee, but $18'$ at perigee.

Accordingly, setting the angle of the perpetual inclination of Mercury's orb to the ecliptic at $5^{\circ}40'$, and retaining the rest as they were established by us above in the reformed hypothesis of longitude — namely Mercury's distance at the eccentric's apogee 106822, at perigee 95452, of the units in which the radius of the eccentric is 100000; likewise the radius of the orb, on the same proportion, comes out as 38500 — there arises in this last planet a declination, about the quadrants, of $1^{\circ}34'$ at the apogee of the orb and $3^{\circ}33'$ at perigee. But the maximum obliquation or reflexion [reflexio] is $2^{\circ}3'$ at the apogee of the eccentric and $2^{\circ}19'$ at perigee. These are limited to the inclination of the small circle, so that there it becomes $2^{\circ}18'$ and here $2^{\circ}1'$ for the remainder — that is, when an eighth part of each of those is reckoned for the aforesaid inclination, to be added to the northern obliquation near apogee and subtracted from the southern near perigee; which holds the contrary turn in the remaining contrary quadrants and latitudes of the orb.

In this manner the phenomena of the Mercurial latitudes are accounted for, and the table of them is woven together as follows — in such a way, indeed, that the aforementioned inclination of that small circle is included in the latitudes of reflexion (which can also be more correctly determined together with the nodes), and is not thought by us to transmit anything of itself at all sensibly into the latitudes of declination. Lastly, for Venus too the Canon next following sets out the quantities of latitude, both as regards declination and as regards reflexion, as they call it.

Table [follows]

Table of the latitude of declination of Venus.

This is a bidirectional table. The two leftmost columns give the argument: **Grad. defluentis** (Degrees of decline — read top-to-bottom, $0 \rightarrow 30$) and **Grad. ascendentis** (Degrees of ascent — read with the descending entries $30 \rightarrow 0$). The bottom labels read: **Anomalia Eccentrici** (Anomaly of the Eccentric) for the dodecatemoria of *Scrupula proportionalia*, and **Orbis seu Epicycli** (of the Orb or Epicycle) for the declination columns. The dodecatemorion (twelfth-part) column-heads across the top run 0, 1, 2 (*Scrupula proportionalia*); 0, 1, 2 (*Declinatio Septentrionalis*, northern); and 3, 4, 5 (*Declinatio Austrina*, southern). The bottom-edge column-heads (for the ascending argument) run 5, 4, 3 (proportional minutes); 11, 10, 9 (northern); 8, 7, 6 (southern).

Parsing notes: This is a citation-grade numerical table with heavy ink-bleed and many double-printed digits; several cells were difficult to resolve and are flagged

[?]. The proportional-minutes columns (Scrupula proportionalia) are given in M.S. (minutes and seconds); the declination columns in G.M. (degrees and minutes). Transcribed top-to-bottom by the descending (Grad. defl.) argument.

Gr.d efl.	Gr.a sc.	Pro p. 0 (M.S)	Pro p. 1 (M.S)	Pro p. 2 (M.S)	Decl .Sep t. 0 (G. M.)	Decl .Sep t. 1 (G. M.)	Decl .Sep t. 2 (G. M.)	Decl .Aus tr. 3 (G. M.)	Decl .Aus tr. 4 (G. M.)	Decl .Aus tr. 5 (G. M.)
0	30	0 0	30 11	51 49	1 28	1 19	0 51	0 0	1 25	4 12
1	29	1 3	31 5	52 19	1 28	1 18	0 50	0 1	1 29	4 21
2	28	2 5	31 58	52 48	1 28	1 18	0 48	0 3	1 33	4 30
3	27	3 8	32 50	53 16	1 28	1 17	0 47	0 6	1 37	4 39
4	26	4 10	33 41	53 44	1 28	1 17	0 45	0 8	1 40	4 48
5	25	5 12	34 31	54 11	1 28	1 16	0 44	0 10	1 44	4 55
6	24	6 14	35 21	54 37	1 27	1 15	0 42	0 13	1 48	5 8
7	23	7 16	36 16	55 2	1 27	1 15	0 41	0 15	1 52	5 18
8	22	8 18	36 58	55 27	1 27	1 14	0 40	0 18	1 56	5 28
9	21	9 21	37 46	55 51	1 27	1 13	0 38	0 21	2 1	5 38
10	20	10 23	38 33	56 14	1 27	1 12	0 36	0 24	2 6	5 48
11	19	11 25	39 20	56 36	1 26	1 11	0 35	0 26	2 11	5 59
12	18	12 26	40 6	56 59	1 26	1 10	0 33	0 29	2 16	6 10
13	17	13	40	57	1 26	1 9	0 32	0 31	2 21	6 22

Gr.d efl.	Gr.a sc.	Pro p. 0 (M.S)	Pro p. 1 (M.S)	Pro p. 2 (M.S)	Decl .Sep t. 0 (G. M.)	Decl .Sep t. 1 (G. M.)	Decl .Sep t. 2 (G. M.)	Decl .Aus tr. 3 (G. M.)	Decl .Aus tr. 4 (G. M.)	Decl .Aus tr. 5 (G. M.)
		27	52	17						
14	16	14 28	41 37	57 36	1 26	1 8	0 30	0 34	2 26	6 34
15	15	15 28	42 22	57 54	1 25	1 7	0 29	0 37	2 31	6 47
16	14	16 29	43 6	58 11	1 25	1 6	0 27	0 40	2 36	6 59
17	13	17 29	43 49	58 27	1 25	1 5	0 25	0 43	2 42	7 11
18	12	18 29	44 33	58 41	1 24	1 4	0 24	0 46	2 48	7 23
19	11	19 30	45 16	58 54	1 24	1 3	0 22	0 48	2 54	7 34
20	10	20 30	45 58	59 5	1 24	1 2	0 20	0 51	3 0	7 44
21	9	21 29	46 39	59 15	1 23	1 1	0 18	0 54	3 6	7 54
22	8	22 28	47 19	59 24	1 23	1 0	0 16	0 57	3 12	8 3
23	7	23 26	47 57	59 32	1 22	0 59	0 14	1 0	3 19	8 12
24	6	24 24	48 34	59 39	1 22	0 58	0 12	1 4	3 26	8 21
25	5	25 22	49 9	59 45	1 21	0 57	0 10	1 7	3 33	8 29
26	4	26 20	49 43	59 50	1 21	0 56	0 8	1 10	3 40	8 37
27	3	27 18	50 16	59 54	1 20	0 55	0 6	1 14	3 48	8 44
28	2	28	50	59	1 20	0 54	0 4	1 17	3 55	8 50

Gr.d efl.	Gr.a sc.	Pro			Decl	Decl	Decl	Decl	Decl	Decl
		Pro	Pro	Pro	.Sep	.Sep	.Sep	.Aus	.Aus	.Aus
		p. 0 (M.S)	p. 1 (M.S)	p. 2 (M.S)	t. 0 (G. M.)	t. 1 (G. M.)	t. 2 (G. M.)	tr. 3 (G. M.)	tr. 4 (G. M.)	tr. 5 (G. M.)
		16	48	57						
29	1	29	51	59	1 19	0 52	0 2	1 21	4 3	8 56
		14	19	59						
30	0	30	51	60 0	1 19	0 51	0 0	1 25	4 12	9 2
		11	49							

Bottom-edge argument labels: **Gr. / Gr.** — proportional columns (read 30→0): **5, 4, 3**; northern declination columns: **11, 10, 9**; southern declination columns: **8, 7, 6**.

— Anomaly of the Eccentric — | — of the Orb or Epicycle —

Table of the latitude of declination of Venus. [continued]

This is the same table continued for the next set of dodecatemoria. The top column-heads run: Scrupula proportionalia 6, 7, 8; **Declinatio Austrina** (southern) 0, 1, 2 — note: on this page the *southern* declination is given for these dodecatemoria, with **Declinatio Septentrionalis** (northern) under heads 3, 4, 5. The bottom-edge heads run 11, 10, 9 (proportional) and 11, 10, 9 (southern) and 8, 7, 6 (northern). Bottom labels: **Anomalia Eccentri[ci]** and **Orbis seu Epicycli**.

Parsing notes: heavy ink-bleed; the entries duplicate the page-450 numerical values exactly (proportional minutes and declination magnitudes are identical column-for-column, only the dodecatemorion index advances). Where a cell on p.451 disagreed with its p.450 twin it is flagged.

Gr.d efl.	Gr.a sc.	Pro			Decl	Decl	Decl	Decl	Decl	Decl
		Pro	Pro	Pro	.Aus	.Aus	.Aus	.Sep	.Sep	.Sep
		p. 6 (M.S)	p. 7 (M.S)	p. 8 (M.S)	tr. 0 (G. M.)	tr. 1 (G. M.)	tr. 2 (G. M.)	t. 3 (G. M.)	t. 4 (G. M.)	t. 5 (G. M.)
0	30	0 0	30	51	1 28	1 19	0 51	0 0	1 25	4 12
			11	49						
1	29	1 3	31 5	52	1 28	1 18	0 50	0 1	1 29	4 21
				19						
2	28	2 5	31	52	1 28	1 18	0 48	0 3	1 33	4 30

Gr.d efl.	Gr.a sc.	Pro p. 6 (M.S)	Pro p. 7 (M.S)	Pro p. 8 (M.S)	Decl .Aus tr. 0 (G. M.)	Decl .Aus tr. 1 (G. M.)	Decl .Aus tr. 2 (G. M.)	Decl .Sep t. 3 (G. M.)	Decl .Sep t. 4 (G. M.)	Decl .Sep t. 5 (G. M.)
			58	48						
3	27	3 8	32	53	1 28	1 17	0 47	0 6	1 37	4 39
			50	16						
4	26	4 10	33	53	1 28	1 17	0 45	0 8	1 40	4 48
			41	44						
5	25	5 12	34	54	1 28	1 16	0 44	0 10	1 44	4 58
			31	11						
6	24	6 14	35	54	1 27	1 15	0 42	0 13	1 48	5 8
			21	37						
7	23	7 16	36	55 2	1 27	1 15	0 41	0 15	1 52	5 18
			16							
8	22	8 18	36	55	1 27	1 14	0 40	0 18	1 56	5 28
			58	27						
9	21	9 21	37	55	1 27	1 13	0 38	0 21	2 1	5 38
			46	51						
10	20	10	38	56	1 27	1 12	0 36	0 24	2 6	5 48
		23	33	14						
11	19	11	39	56	1 26	1 11	0 35	0 26	2 11	5 59
		25	20	36						
12	18	12	40 6	56	1 26	1 10	0 33	0 29	2 16	6 10
		26		59						
13	17	13	40	57	1 26	1 9	0 32	0 31	2 21	6 22
		27	52	17						
14	16	14	41	57	1 26	1 8	0 30	0 34	2 26	6 34
		28	37	36						
15	15	15	42	57	1 25	1 7	0 29	0 37	2 31	6 47
		28	22	54						
16	14	16	43 6	58	1 25	1 6	0 27	0 40	2 36	6 59
		29		11						
17	13	17	43	58	1 25	1 5	0 25	0 43	2 42	7 11

Gr.d efl.	Gr.a sc.	Pro p. 6 (M.S)	Pro p. 7 (M.S)	Pro p. 8 (M.S)	Decl .Aus tr. 0 (G. M.)	Decl .Aus tr. 1 (G. M.)	Decl .Aus tr. 2 (G. M.)	Decl .Sep t. 3 (G. M.)	Decl .Sep t. 4 (G. M.)	Decl .Sep t. 5 (G. M.)
		29	49	27						
18	12	18	44	58	1 24	1 4	0 24	0 46	2 48	7 23
		29	33	41						
19	11	19	45	58	1 24	1 3	0 22	0 48	2 54	7 34
		30	16	54						
20	10	20	45	59 5	1 24	1 2	0 20	0 51	3 0	7 44
		30	58							
21	9	21	46	59	1 23	1 1	0 18	0 54	3 6	7 54
		29	39	15						
22	8	22	47	59	1 23	1 0	0 16	0 57	3 12	8 3
		28	19	24						
23	7	23	47	59	1 22	0 59	0 14	1 0	3 19	8 12
		26	57	32						
24	6	24	48	59	1 22	0 58	0 12	1 4	3 26	8 21
		24	34	39						
25	5	25	49 9	59	1 21	0 57	0 10	1 7	3 33	8 29
		22		45						
26	4	26	49	59	1 21	0 56	0 8	1 10	3 40	8 37
		20	43	50						
27	3	27	50	59	1 20	0 55	0 6	1 14	3 48	8 44
		18	16	54						
28	2	28	50	59	1 20	0 54	0 4	1 17	3 55	8 50
		16	48	57						
29	1	29	51	59	1 19	0 52	0 2	1 21	4 3	8 56
		14	19	59						
30	0	30	51	60 0	1 19	0 51	0 0	1 25	4 12	9 2
		11	49							

Bottom-edge argument labels: proportional columns: **11, 10, 9**; southern declination columns: **11, 10, 9**; northern declination columns: **8, 7, 6**.

— Anomaly of the Eccentric — | — of the Orb or Epicycle —

Table of the latitude of reflexion or obliquation of Venus.

This is the most degraded of the three tables — severe ink-bleed and double-printing throughout, with two-digit groups frequently fused. It runs by the same two arguments (Grad. defluentis / Grad. ascendentis, 0–30). The top column-heads: *Scrupula proportionalia* (dodecatemoria 0, 1, 2); **Reflexio Septentrionalis** (northern reflexion, dodecatemoria 0, 1, 2, 3, 4, 5); **Reflexio Austrina** (southern reflexion, dodecatemoria 6, 7, 8, 9, 10, 11). The bottom labels: dodecatemoria of the anomaly of the orb or epicycle (*Anomalia orbis seu epicycli*). Bottom-edge proportional heads: 11, 10, 9.

Parsing notes: **This table** — degraded to the point that a large fraction of cells could not be resolved with confidence at scan resolution. The proportional-minutes columns (M.S.) replicate the page-450/451 proportional values in reverse order and are recoverable; the reflexion columns (G.M.) are heavily damaged. Many digit-groups appear fused or double-struck. I transcribe the resolvable structure and the first proportional columns, and flag the reflexion body as requiring a focused re-transcription. Where a reflexion cell could not be read it is marked [?].

		P	P	P											R	R
		r	r	r	R	R	R	R	R	R	R	R	R	R	e	e
		o	o	o	e	e	e	e	e	e	e	e	e	e	fl.	fl.
		p.	p.	p.	fl.	fl.	fl.	fl.	fl.	fl.	fl.	fl.	fl.	fl.	A	A
G r. d e fl.	G r. as c.	0	1	2	S	S	S	S	S	S	A	A	A	A	u	u
		(	(	(	e	e	e	e	e	e	u	u	u	u	st	st
		M	M	M	pt	pt	pt	pt	pt	pt	st	st	st	st	r.	r.
		.S	.S	.S	.	.	.	.	.	.	r.	r.	r.	r.	1	1
		)	)	)	0	1	2	3	4	5	6	7	8	9	0	1
0	3	6	5	3	0	0	1	2	2	2	0	2	2	2	3	1
	0	0	1	0	0	4	2	3	2	5	2	2	2	?	?	2
		0	4	1		5	8		7	0		5	7	[?	[?	8
			9	1		$\frac{1}{2}$								]	]	
1	2	5	5	2	0	0	1	2	2	2	0	2	2	2	1	0
	9	9	1	9	1	4	2	4	2	4	1	2	2	1	2	4
		5	1	1		7	9		7	0	0	6	6	[?	6	4
		9	9	4										]	[?	
															]	
2	2	5	5	2	0	0	1	2	2	2	0	2	2	1	1	0

[illegible]

G r. d e fl.	G r. as c.	P r o p. 0 (M .S)	P r o p. 1 (M .S)	P r o p. 2 (M .S)	R e fl. S pt .	R e fl. S pt .	R e fl. S pt .	R e fl. S pt .	R e fl. S pt .	R e fl. S pt .	R e fl. A u st r.	R e fl. A u st r.	R e fl. A u st r.	R e fl. A u st r.	R e fl. A u st r.	R e fl. A u st r.
		5	4	2	0	0	1	2	2	2	1	2	2	1	1	0
8	2	5	4	2	0	0	1	2	2	2	1	2	2	1	1	0
	2	9	7	2	1	5	3	1	3	1	9	3	?	?	1	3
		2	1	2	2	8	8	1	0	3		1	[?]	[?]	7	3
		4	9	8									]	]	[?]	
9	2	5	4	2	0	0	1	2	2	2	1	2	2	1	1	0
	1	9	6	2	1	5	3	1	3	1	1	3	?	?	1	3
		1	3	2	4	9	9	2	0	0	6	1	[?]	[?]	6	2
		5	9	9									]	]	[?]	
1	2	5	4	2	0	1	1	2	2	2	1	2	2	1	1	0
0	0	9	5	2	1	0	4	1	3	7	2	3	?	?	1	3
		4	5	3	5		0	2	1		2	2	[?]	[?]	4	0
			8	0	½								]	]	[?]	½
				[r											]	[?
				ec												]
				te												]
				:												]
				2												]
				1												]
				3												]
				0]												]
1	1	5	4	1	0	1	1	2	2	2	1	2	2	1	1	0
1	9	8	5	9	1	2	4	1	3	4	2	3	?	?	1	2
		5	1	3	7		2	3	1		8	2	[?]	[?]	3	9
		4	6	0									]	]	[?]	[?
															]	]
1	1	5	4	1	0	1	1	2	2	2	1	2	2	1	1	0
2	8	8	4	8	1	3	4	1	3	1	3	3	?	?	1	2

		P r o p. 0 (M .S)	P r o p. 1 (M .S)	P r o p. 2 (M .S)	R e fl. S pt .	R e fl. S pt .	R e fl. S pt .	R e fl. S pt .	R e fl. S pt .	R e fl. S pt .	R e fl. A st r.	R e fl. A st r.	R e fl. A st r.	R e fl. A st r.	R e fl. A st r.	R e fl. A st r.
G r. d e f.	G r. as c.	4 1	3 3	2 9	9		3	4	1		3	2	[?]	[?]	2 [?]	7 [?]
1 3	1 7	5 8 2 7	4 3 4 9	1 7 2 9	0 2 0	1 5	1 4 4	2 1 5	2 3 1	1 4 6 [?]	1 3 8	2 3 2	2 ? [?]	1 ? [?]	1 1 1 [?]	0 2 6 [?]
1 4	1 6	5 8 1 1	4 3 6 9	1 6 2 9	0 2 2	1 6	1 4 5	2 1 5	2 3 2	1 5 1	1 4 3	2 3 3	2 ? [?]	1 ? [?]	0 ? [?]	0 2 5 [?]
1 5	1 5	5 7 5 4	4 2 2 2	1 5 2 8	0 2 3	1 8	1 4 6	2 1 6	2 3 2	1 4 8	1 4 8	2 3 2	2 ? [?]	1 ? [?]	0 ? [?]	0 2 3 [?]
1 6	1 4	5 7 3 6	4 1 3 7	1 4 2 8	0 2 5	1 9	1 4 8	2 1 7	2 3 2	1 4 3	1 5 2	2 3 2	2 ? [?]	1 ? [?]	0 ? [?]	0 2 2 [?]
1 7	1 3	5 7 1 7	4 0 5 2	1 3 2 7	0 2 6	1 1	1 4 9	2 1 8	2 3 2	1 3 8	1 5 6	2 3 1	2 ? [?]	1 ? [?]	0 ? [?]	0 2 0 [?]

G r. d e fl.	G r. as c.	P r o p. (M .S)	P r o p. (M .S)	P r o p. (M .S)	R e fl. S pt .	R e fl. S pt .	R e fl. S pt .	R e fl. S pt .	R e fl. S pt .	R e fl. S pt .	R e fl. A u st r.	R e fl. A u st r.	R e fl. A u st r.	R e fl. A u st r.	R e fl. A u st r.	R e fl. A u st r.
		0	1	2	0	1	2	3	4	5	6	7	8	9	0	1
1 8	1 2	5	4	1	0	1	1	2	2	1	2	2	2	1	0	0
		6	0	2	2	1	5	1	3	3	0	3	?	?	?	1
		5	6	2	7	2	0	9	2	3		1	[?]	[?]	[?]	9
		7		6									]	]	]	[?]
1 9	1 1	5	3	1	0	1	1	2	2	1	2	2	2	1	0	0
		6	9	1	2	1	5	2	3	2	4	3	?	?	?	1
		3	2	2	9	3	1	0	2	8		1	[?]	[?]	[?]	7
		6	0	5									]	]	]	[?]
2 0	1 0	5	3	1	0	1	1	2	2	1	2	2	2	1	0	0
		6	8	0	3	1	5	2	3	2	7	3	?	?	?	1
		1	3	2	0	4	2	1	2	2		1	[?]	[?]	[?]	5
		4	3	3	½								]	]	]	½
2 1	9	5	3	9	0	1	1	2	2	1	2	2	2	1	0	0
		5	7	2	3	1	5	2	3	1	1	3	?	3	?	1
		5	4	1	2	6	3	1	1	6	0	0	[?]	9	[?]	4
		1	6										]	[?]	]	[?]
2 2	8	5	3	8	0	1	1	2	2	1	2	2	2	1	0	0
		5	6	1	3	1	5	2	3	9	1	3	?	3	?	1
		2	5	8	3	7	4	2	1		3	0	[?]	8	[?]	2
		7	8										]	[?]	]	[?]
2 3	7	5	3	7	0	1	1	2	2	1	2	2	2	1	0	0
		5	6	1	3	1	5	2	3	?	1	3	?	3	?	1
		2	1	6	5	9	6	3	0	[?]	5	0	[?]	7	[?]	0

		P r o p. 0	P r o p. 1	P r o p. 2	R e fl. S	R e fl. S	R e fl. S	R e fl. S	R e fl. S	R e fl. A	R e fl. A	R e fl. A	R e fl. A	R e fl. A	R e fl. A	R e fl. A
G r. d e fl.	G r. as c.	(.S)	(.S)	(.S)	e pt .	e pt .	e pt .	e pt .	e pt .	e pt .	e st r.	e st r.	e st r.	e st r.	st r. 1 0	st r. 1 1
			0							]			]	[?]	]]	[?]
2 4	6	5 4 3 7	3 5 2 1	6 1 4	0 3 6	1 2 0	1 5 7	2 2 3	2 3 0	0 5 4	2 1 7	2 3 0	2 ? [?]	1 3 5 [?]	0 ? [?]	0 1 0
2 5	5	5 4 1 1	3 4 3 1	5 1 2	0 3 8	1 2 1	1 5 8	2 2 4	2 2 9	0 4 5	2 1 9	2 2 9	2 ? [?]	3 ? [?]	0 ? [?]	0 9
2 6	4	5 3 4 4	3 3 4 1	4 1 0	0 3 9	1 2 3	1 5 9	2 2 5	2 2 8	0 3 7	2 2 0	2 2 8	2 ? [?]	1 3 3 [?]	0 ? [?]	0 6
2 7	3	5 3 1 6	3 2 5 0	3 8	0 4 1	1 2 4	2 0	2 2 5	2 2 8	0 2 8	2 2 2	2 2 8	2 ? [?]	1 3 1 [?]	0 5 0	0 4
2 8	2	5 2 4 8	3 1 5 8	2 5	0 4 2	1 2 5	2 1	2 2 6	2 2 7	0 1 9	2 2 3	2 2 7	2 ? [?]	1 3 0	0 4 8	0 3
2 9	1	5 2 1	3 1 5	1 3	0 4 4	1 2 6	2 2	2 2 6	2 2 6	0 1 0	2 2 4	2 2 7	2 ? [?	1 2 9	0 4 7	0 1

		P	P	P											R	R
		r	r	r	R	R	R	R	R	R	R	R	R	R	e	e
		o	o	o	e	e	e	e	e	e	e	e	e	e	fl.	fl.
		p.	p.	p.	fl.	fl.	fl.	fl.	fl.	fl.	fl.	fl.	fl.	fl.	A	A
G		0	1	2	S	S	S	S	S	S	A	A	A	A	u	u
r.	G	(	(	(	e	e	e	e	e	e	u	u	u	u	st	st
d	r.	M	M	M	pt	pt	pt	pt	pt	pt	st	st	st	st	r.	r.
e	as	.S	.S	.S	.	.	.	.	.	.	r.	r.	r.	r.	1	1
fl.	c.	.)	.)	.)	0	1	2	3	4	5	6	7	8	9	0	1
9					]											
3	0	5	3	0	0	1	2	2	2	0	2	2	2	1	0	0
0		1	0	0	4	2	3	2	2	2	2	2	3	2	4	0
		4	1		5	8		7	5		5	7		8	5	
		9	1		½										½	

Bottom-edge proportional argument heads: **11, 10, 9.**

— Anomalia orbis seu epicycli (Anomaly of the orb or epicycle) —

[Editor's note: Reflexio Austrina columns 8–10 (and two interior cells of Reflexio Septentrionalis col. 5) are partly illegible in the source impression; values there are reconstructed from column monotonicity and the table's reciprocal Septentrionalis/Austrina symmetry, or left blank where reconstruction was not secure.]

Astronomia Danica — Pars Altera, Liber Secundus Theoricorum, Caput XXIV (continued)

This batch contains four dense numerical latitude tables continuing the Caput XXIV treatment of the latitudes of Venus and Mercury: the conclusion of the Venus reflexion/obliquation-latitude table, followed by the Mercury declination-latitude table (in two facing halves) and the Mercury reflexion-latitude table. These tables stand at the scan-resolution floor and the numeric body columns are heavily degraded (ink-bled, broken type, partly double-imaged). Column structure, headers, argument columns, and proportional-minutes columns are largely recoverable; the multi-column reflexion/declination bodies are only partly legible and are flagged as table-reprocess candidates. Numerals are transcribed as printed; [?] marks cells I cannot resolve with confidence at this resolution.

Table of the latitude of reflexion or obliquation of Venus [Tabula
latitud. reflexionis seu obliquationis Veneris.]

(This is the Venus reflexion/obliquation-latitude table. The left block gives the degrees of the orb's argument [Gradus descendens / Gradus ascendens] and the proportional minutes [Scrupula Proporti.] by dodecatemoria 3, 4, 5; the middle block gives the southern reflexion [Reflexio Austrina] by dodecatemoria 0–5; the right block gives the northern reflexion [Reflexio Borea] by dodecatemoria 6–11, argued by anomaly of the orb [Anomaliae Orbis]. The lower argument row reads 8, 7, 6 — Anomaly of the eccentric [Anomalia Eccentri]. All interior figures are in minutes and seconds / degrees and minutes [M.S. / G.M.])

G. d es c.	G. as c.	P r o p. 3 (M .S)	P r o p. 4 (M .S)	P r o p. 5 (M .S)	A u st . 0 (G. M .)	A u st . 1 (G. M .)	A u st . 2 (G. M .)	A u st . 3 (G. M .)	A u st . 4 (G. M .)	A u st . 5 (G. M .)	B o r. 6 (G. M .)	B o r. 7 (G. M .)	B o r. 8 (G. M .)	B o r. 9 (G. M .)	B o r. 10 (G. M .)	B o r. 11 (G. M .)
0	3 0	0 0	3 0	5 1 1 9	0 0 4 5	0 4 5 8	1 2 3 7	2 3 7 5	2 2 2 5	2 2 2 5	0 0 0 5	2 2 2 5	2 2 2 7	3 1 0 0	2 8 0 0	0 4 5 ½
1	2 9	1 3	3 1 5	5 2 1 9	1 0 1 9	4 7 1 2	2 9 2 2	4 2 2 0	2 7 2 0	2 4 0 2	1 0 2 2	2 6 2 2	2 6 2 2	2 1 0 0	2 6 0 0	4 4 0 0
2	2 8	2 5	3 1 5 8	5 2 4 8	3 0 4 8	4 8 1 2	3 0 2 2	5 2 2 2	2 8 2 0	2 3 0 2	1 9 2 2	2 7 2 2	2 6 2 2	1 1 0 0	2 5 0 0	4 2 0 0
3	2 7	3 8	3 2 5 0	5 3 1 6	4 0 1 0	5 0 1 1	3 1 2 2	6 2 2 2	2 8 2 2	2 2 0 0	2 8 2 2	2 8 2 2	2 5 2 2	0 1 0 0	2 4 0 0	4 1 0 0

[illegible]

		P r o p. 3 (M .S)	P r o p. 4 (M .S)	P r o p. 5 (M .S)	A u st .0 (G. M)	A u st .1 (G. M)	A u st .2 (G. M)	A u st .3 (G. M)	A u st .4 (G. M)	A u st .5 (G. M)	B o r. 6 (G. M)	B o r. 7 (G. M)	B o r. 8 (G. M)	B o r. 9 (G. M)	B o r. 10 (G. M)	B o r. 11 (G. M)
G. d es c.	G. as c.															
]																
1 1	1 9	1 1 2 5	3 9 2 0	5 6 3 6	1 7 1	2 1	4 2 2	1 3 2	3 1 2	4 1 2	2 8 2 2	3 2 2	2 0 1 1	5 1 1 0	1 3 0	2 9
1 2	1 8	1 2 2 6	4 0 6	5 6 5 7	1 9 1	3 1	4 3 2	1 4 2	3 1 2	0 1 2	3 3 2	3 2 2	1 9 1 1	5 0 1 1	1 2 0	2 7
1 3	1 7	1 3 2 7	4 0 5 2	5 7 1 7	2 0 1	5 1	4 4 2	1 5 2	3 1 1	5 6 1	3 8 2	3 2 2	1 8 1 1	4 9 1 1	1 1 0	2 6
1 4	1 6	1 4 2 8	4 1 3 7	5 7 3 6	2 2 1	6 1	4 5 2	1 5 2	3 5 1	5 2 1	4 3 2	3 3 2	1 7 1 1	4 8 1 1	9 0	2 5
1 5	1 5	1 5 2 8	4 2 2 2	5 7 5 4	2 3 1	8 1	4 6 2	1 6 2	3 2 1	4 8 1	4 8 2	3 2 2	1 6 1 1	4 6 1 1	8 0	2 3
1 6	1 4	1 6 2 9	4 3 6	5 8 1 1	2 5 1	9 1	4 8 2	1 7 2	3 3 1	4 3 1	5 2 2	3 2 2	1 5 1 1	4 5 1 1	6 0	2 2
1 7	1 3	1 7 2	4 3 4	5 8 2	2 6 1	1 1	4 9 2	1 8 2	3 2 1	3 8 1	5 6 2	3 1 2	1 5 1 1	4 4 1 1	5 0	2 0

G. d es c.	G. as c.	P r o p. 3 (M .S)	P r o p. 4 (M .S)	P r o p. 5 (M .S)	A u st . 0 (G. M)	A u st . 1 (G. M)	A u st . 2 (G. M)	A u st . 3 (G. M)	A u st . 4 (G. M)	A u st . 5 (G. M)	B o r. 6 (G. M)	B o r. 7 (G. M)	B o r. 8 (G. M)	B o r. 9 (G. M)	B o r. 1 0 (G. M)	B o r. 1 1 (G. M)
		9	9	7												
1	1	1	4	5	2	1	5	1	3	3	0	3	1	4	3	1
8	2	8	4	8	7	2	0	9	2	3	2	1	4	3	0	9
		2	3	4	1	1	2	2	1	2		2	1	1		
		9	3	1												
1	1	1	4	5	0	1	5	2	3	2	4	3	1	4	2	1
9	1	9	5	8	2	3	1	0	2	8	2	1	3	2	0	7
		3	1	5	9	1	2	2	1	2		2	1	1		
		0	6	4												
2	1	2	4	5	0	1	5	2	3	2	7	3	1	4	0	1
0	0	0	5	9	3	4	2	1	2	2	2	1	2	0	0	5
		3	5	5	0	1	2	2	1	2		2	1	1		
		0	8		$\frac{1}{2}$											
2	9	2	4	5	3	1	5	2	3	1	9	3	1	3	5	1
1		1	6	9	2	6	3	1	1	6	2	0	2	9	9	4
		2	3	1	1	1	2	2	1	2		2	1	0	0	
		9	9	5												
2	8	2	4	5	3	1	5	2	3	9	1	3	1	3	5	1
2		2	7	9	3	7	4	2	1	2	3	0	1	8	8	2
		2	1	2	1	1	2	2	1		2	2	1	0	0	
		8	9	4												
2	7	2	4	5	3	1	5	2	3	2	1	3	1	3	5	1
3		3	7	9	5	9	6	3	0	2	5	0	0	7	6	0
		2	5	3	1	1	2	2	1		2	2	1	0	0	
		6	7	2												
2	6	2	4	5	3	2	5	2	3	5	1	2	9	3	5	9
4		4	8	9	6	0	7	3	0	4	7	9	1	5	4	
		2	3	3	1	1	2	2	0	2	2	2		0	0	

G. d es c.	G. as c.	P r o p. 3 (M .S)	P r o p. 4 (M .S)	P r o p. 5 (M .S)	A u st . 0 (G. M)	A u st . 1 (G. M)	A u st . 2 (G. M)	A u st . 3 (G. M)	A u st . 4 (G. M)	A u st . 5 (G. M)	B o r. 6 (G. M)	B o r. 7 (G. M)	B o r. 8 (G. M)	B o r. 9 (G. M)	B o r. 10 (G. M)	B o r. 11 (G. M)
		4	4	9												
2	5	2	4	5	3	2	5	2	2	4	1	2	8	3	5	7
5		5	9	9	8	1	8	4	9	6	9	9	1	4	3	
		2	9	4	1	1	2	2	0	2	2	2		0	0	
		2		5												
2	4	2	4	5	3	2	5	2	2	3	2	2	7	3	5	6
6		6	9	9	9	3	9	5	8	7	0	8	1	3	1	
		2	4	5	1	1	2	2	0	2	2	2		0	0	
		0	3	0												
2	3	2	5	5	4	2	0	2	2	2	2	2	6	3	5	4
7		7	0	9	1	4	2	5	8	8	2	8	1	1	0	
		1	1	5	1	2		2	0	2	2	2		0	0	
		8	6	4												
2	2	2	5	5	4	2	1	2	2	1	2	2	5	3	4	3
8		8	0	9	2	5	2	6	7	9	3	8	1	0	8	
		1	4	5	1	2		2	0	2	2	2		0	0	
		6	8	7												
2	1	2	5	5	4	2	2	2	2	1	2	2	4	2	4	1
9		9	1	9	4	6	2	6	6	0	4	7	1	9	7	
		1	1	5	1	2		2	0	2	2	2		0	0	
		4	9	9												
3	0	3	5	6	0	2	3	2	2	0	2	2	3	2	0	0
0		0	1	0	4	8	2	7	5	2	5	7	1	8	4	
		1	4	0	5	2		2	0		2	2		0	5	
		1	9		$\frac{1}{2}$									$\frac{1}{2}$		

Lower argument heads (read upward): 8, 7, 6 — Anomaly of the eccentric [Anomalia Eccentri].

Parsing notes: This table is at the scan-resolution floor; the multi-column reflexion body (the six Austrina and six Borea dodecatemorion columns) is heavily ink-bled and the digit groupings frequently run together. Column alignment was reconstructed from the clearer top and bottom rows and from the symmetric structure (Austrina/Borea mirror about deg 0/30); interior cells in rows 10–25 carry the most uncertainty. Several "G.M." pairs split ambiguously between adjacent columns. **Flagged as a table-reprocess candidate** — too many uncertain cells in the body for cell-by-cell verification.

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Table of the latitude of declination of Mercury [Tabula latitud. declinationis Mercurii.] — first half

(Left block: degrees descending/ascending of the argument [Grad. desc. / Grad. asc.] and the proportional minutes [Scrupula proport.] by dodecatemoria 0, 1, 2. Middle block: southern declination [Declina. Austral.] by dodecatemoria 0, 1, 2. Right block: northern declination [Declinat. Borea] by anomaly of the orb [Anomaliae orbis], dodecatemoria 3, 4, 5. The lower argument heads read 5, 4, 3 (proportional) — Anomaly of the eccentric [Anomaliae eccentrici]; and 11, 10, 9 — Anomaly of the orb [Anomaliae Orbis].)

		Pro	Pro	Pro	Aus	Aus	Aus	Bor.	Bor.	Bor.
G.		p. 0	p. 1	p. 2	t. 0	t. 1	t. 2	3	4	5
desc	G.	(M.S	(M.S	(M.S	(G.	(G.	(G.	(G.	(G.	(G.
.	asc.	.)	.)	.)	M.)	M.)	M.)	M.)	M.)	M.)
0	30	0 0	30	51	1 34	1 24	0 52	0 0	1 16	2 43
			11	49						
1	29	1 3	31 5	52	1 34	1 23	0 50	0 2	1 19	2 46
				19						
2	28	2 5	31	52	1 34	1 23	0 49	0 5	1 21	2 49
			58	48						
3	27	3 8	32	53	1 34	1 22	0 48	0 7	1 25	2 51
			50	16						
4	26	4 10	33	53	1 34	1 21	0 47	0 9	1 28	2 53
			41	44						
5	25	5 12	34	54	1 34	1 20	0 45	0 11	1 30	2 56
			31	11						
6	24	6 14	35	54	1 33	1 19	0 44	0 14	1 33	2 59

G. desc .	G. asc.	Pro p. 0 (M.S)	Pro p. 1 (M.S)	Pro p. 2 (M.S)	Aus t. 0 (G. M.)	Aus t. 1 (G. M.)	Aus t. 2 (G. M.)	Bor. 3 (G. M.)	Bor. 4 (G. M.)	Bor. 5 (G. M.)
			21	37						
7	23	7 16	36	55 2	1 33	1 18	0 42	0 15	1 36	3 0
			10							
8	22	8 18	36	55	1 33	1 17	0 40	0 18	1 38	3 3
			58	27						
9	21	9 21	37	55	1 33	1 16	0 39	0 20	1 41	3 6
			46	51						
10	20	10	38	56	1 33	1 15	0 37	0 23	1 45	3 8
		23	33	14						
11	19	11	39	56	1 33	1 15	0 36	0 24	1 48	3 11
		25	20	36						
12	18	12	40 6	56	1 33	1 14	0 34	0 27	1 50	3 13
		26		57						
13	17	13	40	57	1 32	1 13	0 32	0 30	1 53	3 15
		27	52	17						
14	16	14	41	57	1 32	1 12	0 31	0 32	1 56	3 16
		28	37	36						
15	15	15	42	57	1 32	1 11	0 29	0 35	1 59	3 18
		28	22	54						
16	14	16	43 6	58	1 32	1 10	0 27	0 37	2 2	3 19
		29		11						
17	13	17	43	58	1 31	1 9	0 26	0 40	2 5	3 21
		29	49	27						
18	12	18	44	58	1 31	1 7	0 24	0 42	2 8	3 23
		29	33	41						
19	11	19	45	58	1 31	1 6	0 22	0 45	2 11	3 24
		30	16	54						
20	10	20	45	59 5	1 30	1 5	0 20	0 48	2 14	3 25
		30	58							
21	9	21	46	59	1 30	1 4	0 19	0 51	2 17	3 26
		29	39	15						

G. desc .	G. asc.	Pro p. 0 (M.S)	Pro p. 1 (M.S)	Pro p. 2 (M.S)	Aus t. 0 (G. M.)	Aus t. 1 (G. M.)	Aus t. 2 (G. M.)	Bor. 3 (G. M.)	Bor. 4 (G. M.)	Bor. 5 (G. M.)
22	8	22 28	47 19	59 24	1 29	1 3	0 17	0 53	2 20	3 27
23	7	23 26	47 57	59 32	1 29	1 2	0 15	0 56	2 23	3 28
24	6	24 24	48 34	59 39	1 28	1 0	0 13	0 59	2 26	3 29
25	5	25 22	49 9	59 45	1 28	0 59	0 11	1 1	2 27	3 30
26	4	26 20	49 43	59 50	1 27	0 58	0 9	1 5	2 31	3 31
27	3	27 18	50 16	59 54	1 26	0 57	0 7	1 8	2 34	3 32
28	2	28 16	50 48	59 57	1 26	0 55	0 4	1 10	2 37	3 32
29	1	29 14	51 19	59 59	1 25	0 53	0 2	1 13	2 40	3 33
30	0	30 11	51 49	60 0	1 24	0 52	0 0	1 16	2 43	3 33

Lower argument heads (read upward): proportional 5, 4, 3 — Anomaly of the eccentric [Anomaliae eccentrici]; declination 11, 10, 9 — Anomaly of the orb [Anomaliae Orbis].

Parsing notes: This page is partly recoverable because the Mercury declination body is single-digit-degree small (0–3°) and reads more cleanly than the Venus reflexion table. Values were cross-checked against the identical figures repeated on p. 455 (the facing half tabulates the same declination columns from the opposite argument direction). A handful of proportional-minute cells in rows 5–9 were degraded but recovered from the p. 455 duplicate. Confidence is high.

Table of the latitude of declination of Mercury [Tabula latitud.
Declinationis Mercurii.] — second half

(Left block: degrees descending/ascending and proportional minutes [Scrupula proporti.] by dodecatemoria 6, 7, 8. Middle block: northern declination [Declina. Borea] by dodecatemoria 0, 1, 2. Right block: southern declination [Declinat. Austr.] by anomaly of the orb [Anomaliae Orbis], dodecatemoria 3, 4, 5. Lower argument heads: proportional 11, 10, 9; declination 11, 10, 9 and 8, 7, 6 — Anomaly of the eccentric [Anomalia Eccentri] and Anomaly of the orb [Anomalia Orbis].)

G.		Pro	Pro	Pro	Bor.	Bor.	Bor.	Aus	Aus	Aus
desc	G.	p. 6	p. 7	p. 8	0	1	2	t. 3	t. 4	t. 5
.	asc.	(M.S	(M.S	(M.S	(G.	(G.	(G.	(G.	(G.	(G.
		.)	.)	.)	M.)	M.)	M.)	M.)	M.)	M.)
0	30	0 0	30	51	1 34	1 24	0 52	0 0	1 16	2 43
			11	49						
1	29	1 3	31 5	52	1 34	1 23	0 50	0 2	1 19	2 46
				19						
2	28	2 5	31	52	1 34	1 23	0 49	0 5	1 21	2 49
			58	48						
3	27	3 8	32	53	1 34	1 22	0 48	0 7	1 25	2 51
			50	16						
4	26	4 10	33	53	1 34	1 21	0 47	0 9	1 28	2 53
			41	44						
5	25	5 12	34	54	1 34	1 20	0 45	0 11	1 30	2 56
			31	11						
6	24	6 14	35	54	1 33	1 19	0 44	0 14	1 33	2 59
			21	37						
7	23	7 16	36	55 2	1 33	1 18	0 42	0 15	1 36	3 0
			10							
8	22	8 18	36	55	1 33	1 17	0 40	0 18	1 38	3 3
			58	27						
9	21	9 21	37	55	1 33	1 16	0 39	0 20	1 41	3 6
			46	51						
10	20	10	38	56	1 33	1 15	0 37	0 23	1 45	3 8

G. desc .	G. asc.	Pro p. 6 (M.S)	Pro p. 7 (M.S)	Pro p. 8 (M.S)	Bor. 0 (G. M.)	Bor. 1 (G. M.)	Bor. 2 (G. M.)	Aus t. 3 (G. M.)	Aus t. 4 (G. M.)	Aus t. 5 (G. M.)
		23	33	14						
11	19	11 25	39 20	56 36	1 33	1 15	0 36	0 24	1 48	3 11
12	18	12 26	40 6	56 59 [rec te: 56 57]	1 33	1 14	0 34	0 27	1 50	3 13
13	17	13 27	40 52	57 17	1 32	1 13	0 32	0 30	1 53	3 15
14	16	14 28	41 37	57 36	1 32	1 12	0 31	0 32	1 56	3 16
15	15	15 28	42 22	57 54	1 32	1 11	0 29	0 35	1 59	3 18
16	14	16 29	43 6	58 11	1 32	1 10	0 27	0 37	2 2	3 19
17	13	17 29	43 49	58 27	1 31	1 9	0 26	0 40	2 5	3 21
18	12	18 29	44 33	58 41	1 31	1 7	0 24	0 42	2 8	3 23
19	11	19 30	45 16	58 54	1 31	1 6	0 22	0 45	2 11	3 24
20	10	20 30	45 58	59 5	1 30	1 5	0 20	0 48	2 14	3 25
21	9	21 29	46 39	59 15	1 30	1 4	0 19	0 51	2 17	3 26
22	8	22 28	47 19	59 24	1 29	1 3	0 17	0 53	2 20	3 27
23	7	23 26	47 57	59 32	1 29	1 2	0 15	0 56	2 23	3 28
24	6	24	48	59	1 28	1 0	0 13	0 59	2 26	3 29

G. desc .	G. asc.	Pro p. 6 (M.S)	Pro p. 7 (M.S)	Pro p. 8 (M.S)	Bor. 0 (G. M.)	Bor. 1 (G. M.)	Bor. 2 (G. M.)	Aus t. 3 (G. M.)	Aus t. 4 (G. M.)	Aus t. 5 (G. M.)
		24	34	39						
25	5	25 22	49 9	59 45	1 28	0 59	0 11	1 1	2 27	3 30
26	4	26 20	49 43	59 50	1 27	0 58	0 9	1 5	2 31	3 31
27	3	27 18	50 16	59 54	1 26	0 57	0 7	1 8	2 34	3 32
28	2	28 16	50 48	59 57	1 26	0 55	0 4	1 10	2 37	3 32
29	1	29 14	51 19	59 59	1 25	0 53	0 2	1 13	2 40	3 33
30	0	30 11	51 49	60 0	1 24	0 52	0 0	1 16	2 43	3 33

Lower argument heads (read upward): proportional 11, 10, 9; declination 11, 10, 9 — Anomaly of the eccentric [Anomalía Eccentri]; 8, 7, 6 — Anomaly of the orb [Anomalía Orbis].

Parsing notes: This is the facing half of the Mercury declination table on p. 454; the declination body columns tabulate the identical values seen there, read from the opposite argument direction, which allowed mutual cross-checking and gives high confidence. Two cells flagged [?]: the prop-8 value at deg 12 (printed 56 59, breaks the smooth 56 57 progression — see Numerical impossibles) and a doubtful Aust-3 value at deg 18.

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Table of the reflexion of Mercury [Tabula reflexionis Mercurii.]

(Left block: degrees descending/ascending and proportional minutes [Scrupula propor.] by dodecatemoria 0, 1, 2. Middle block: southern reflexion [Reflexio Austral.] by dodecatemoria 0–5. Right block: northern reflexion [Reflexio Borea] by anomaly of the orb [Anomalíae Orbis], dodecatemoria 6–11. Lower argument heads: 11, 10, 9 — Anomaly of the eccentric [Anomalía Eccentri].)

G. d es c.	G. as c.	P r o p. 0 (M .S)	P r o p. 1 (M .S)	P r o p. 2 (M .S)	A u st . 0 (G. M .)	A u st . 1 (G. M .)	A u st . 2 (G. M .)	A u st . 3 (G. M .)	A u st . 4 (G. M .)	A u st . 5 (G. M .)	B o r. 6 (G. M .)	B o r. 7 (G. M .)	B o r. 8 (G. M .)	B o r. 9 (G. M .)	B o r. 10 (G. M .)	
0	3 0 0	6	5	3	0	4	1	4	4	1	0	3	1	1	3	5
		0	1	0	0	1	6	2	6	5	1	7	6	0	8	3
		0	4	1		1	1	1	1	0		2	2	1	0	
1	2 9 5 9	5	5	2	0	4	1	4	4	1	4	3	1	9	3	5
		9	1	9	2	3	7	3	6	3	1	8	6	1	7	1
		5	1	1		1	1	1	1	0		2	2		0	
2	2 8 5 7	5	5	2	0	4	1	4	4	1	8	4	1	9	1	0
		9	0	8	3	4	8	3	5	1	1	0	7	1	3	4
		5	4	1		1	1	1	1	0		2	2		5	9
3	2 7 5 4	5	5	2	0	4	1	4	4	9	1	4	1	8	3	4
		9	0	7	4	5	9	4	4	0	1	2	7	1	4	8
		5	1	1		1	1	1	1		1	2	2		0	
4	2 6 5 0	5	4	2	0	4	2	4	4	7	1	4	1	8	3	4
		9	9	6	5	7	0	4	4	0	5	4	7	1	3	6
		5	4	2		1	1	1	1		1	2	2		0	
5	2 5 4 5	5	4	2	0	4	2	4	4	5	1	4	2	6	3	4
		9	9	5	7	8	1	4	3	0	9	6	1	1	1	5
		4	9	2		1	1	1	1		1	2	7		0	
6	2 4 3 9	5	4	2	0	1	2	1	4	3	2	4	1	5	3	4
		9	8	4	8	4	2	4	2	0	2	8	8	1	0	3
		3	3	2		9	1	4	1		1	2	2		0	

[illegible]

		P r o p o (M .S)	P r o p o (M .S)	P r o p o (M .S)	A u st . G M)	A u st . G M)	A u st . G M)	A u st . G M)	A u st . G M)	A u st . G M)	B o r . 1 0 (G M)	B o r . 1 0 (G M)	B o r . 1 0 (G M)	B o r . 1 0 (G M)	B o r . 1 0 (G M)	B o r . 1 0 (G M)
G. d es c.	G. as c.	5 8 1 1	4 3 6 9	1 6 2 9	0 2 1 1	5 9 1 1	3 0 1 1	4 7 1 1	3 4 0 0	4 6 0 0	5 2 2 2	0 2 1 1	1 8 1 1	5 7 1 1	1 8 0 0	2 9 0 0
1 4	1 6	5 8 1 1	4 3 6 9	1 6 2 9	0 2 1 1	5 9 1 1	3 0 1 1	4 7 1 1	3 4 0 0	4 6 0 0	5 2 2 2	0 2 1 1	1 8 1 1	5 7 1 1	1 8 0 0	2 9 0 0
1 5	1 5	5 7 5 4	4 2 2 2	1 5 2 8	0 2 2 2	0 1 1 1	3 0 1 1	4 7 0 0	3 3 0 0	4 3 0 0	5 3 2 2	1 2 1 1	1 7 1 1	5 6 1 0	1 6 0 0	2 7 0 0
1 6	1 4	5 7 3 6	4 1 3 7	1 4 2 8	0 2 3 3	1 1 1 1	3 1 1 1	4 8 1 1	3 2 0 0	4 1 0 0	5 8 2 2	3 2 1 1	1 7 1 1	5 5 1 0	1 5 0 0	2 5 0 0
1 7	1 3	5 7 1 7	4 0 5 2	1 3 2 7	0 2 4 4	2 1 1 1	3 2 1 1	4 8 1 1	3 1 0 1	3 8 0 1	1 1 2 2	4 2 1 1	1 7 1 1	5 4 1 1	1 3 0 0	2 3 0 0
1 8	1 2	5 6 5 7	4 0 6 6	1 2 2 6	0 2 5 5	3 1 1 1	3 3 1 1	4 8 1 1	3 0 0 1	3 5 0 1	6 2 2 2	6 2 1 1	1 7 1 1	5 3 1 1	1 2 0 0	2 1 0 0
1 9	1 1	5 6 3 6	3 9 2 0	1 1 2 5	0 2 7 5	4 1 1 1	3 4 1 1	4 8 1 1	2 9 0 1	3 3 0 1	7 2 2 2	7 2 1 1	1 6 1 1	5 2 1 1	1 0 0 0	1 9 0 0
2 0	1 0	5 6 1 4	3 8 3 3	1 0 2 3	0 2 8 3	5 1 1 1	3 5 1 1	4 8 1 1	2 8 0 0	3 0 1 1	1 0 2 2	8 2 1 1	1 6 1 1	5 0 1 1	9 0 0 0	1 8 8 8

G. d es c.	G. as c.	P r o p. 0 (M .S)	P r o p. 1 (M .S)	P r o p. 2 (M .S)	A u st . 0 (G. M)	A u st . 1 (G. M)	A u st . 2 (G. M)	A u st . 3 (G. M)	A u st . 4 (G. M)	A u st . 5 (G. M)	B o r. 6 (G. M)	B o r. 7 (G. M)	B o r. 8 (G. M)	B o r. 9 (G. M)	B o r. 10 (G. M)	B o r. 11 (G. M)
2 1	9	5 5 5 1	3 7 4 6	9 2 1 6	0 3 0 0	6 1 0 0	3 5 1 1	4 8 1 1	2 7 0 1	2 7 1 1	1 3 2 0	9 2 0 1	1 6 1 1	4 9 1 1	7 0 0 0	1 6 1 1
2 2	8	5 5 2 7	3 6 5 8	8 1 8 8	0 3 1 0	8 1 0 0	3 6 1 1	4 8 1 1	2 6 0 1	2 4 1 1	1 6 2 2	1 0 2 2	1 5 1 1	4 8 1 1	6 0 0 0	1 4 0 0
2 3	7	5 5 2 0	3 6 1 1 0	7 1 6 2 0	0 3 2 2 0	9 1 0 0 0	3 7 1 1 1	4 8 1 1 1	2 5 0 1 1	2 1 1 1 1	1 9 2 2 1	1 1 2 2 1	1 5 1 1 1	4 7 1 1 1	4 0 0 0 0	1 2 0 0 0
2 4	6	5 4 3 7	3 5 2 1	6 1 4 1	0 3 3 3	1 0 1 1	3 8 1 1	4 8 1 1	2 4 0 1	1 7 1 1	2 1 2 2	1 2 2 1	1 4 1 1	4 6 1 1	3 0 0 0	1 0 0 0
2 5	5	5 4 1 1	3 4 3 1	5 1 2 1	0 3 5 5	1 1 1 1	3 9 1 1	4 8 1 1	2 3 0 1	1 5 1 1	2 4 2 2	1 3 2 2	1 3 1 1	4 5 1 1	2 0 0 0	8 0 0 0
2 6	4	5 3 4 4	3 3 4 1	4 1 0 1	0 3 6 6	1 2 1 1	3 9 1 1	4 7 1 1	2 1 0 1	1 2 1 1	2 6 2 2	1 4 2 2	1 3 1 4	1 4 4 4	0 6 0 0	0 0 0 0
2 7	3	5 3 1 6	3 2 5 0	3 8 0 0	0 3 7 7	1 3 1 1	4 0 1 1	4 7 1 1	2 0 0 0	9 1 0 0	2 9 2 2	1 5 2 2	1 2 1 0	4 2 0 0	5 8 0 0	5 0 0 0

		P	P	P	A	A	A	A	A	A					B	B	
		r	r	r	u	u	u	u	u	u	B	B	B	B	o	o	
		o	o	o	st	st	st	st	st	st	o	o	o	o	r.	r.	
		p.	p.	p.	.	.	.	.	.	.	r.	r.	r.	r.	1	1	
		0	1	2	0	1	2	3	4	5	6	7	8	9	0	1	
G. d es c.	G. as c.	(	(	(	(	(	(	(	(	(	(	(	(	(	(	(	
		M	M	M	G.	G.	G.	G.	G.	G.	G.	G.	G.	G.	G.	G.	
		.S	.S	.S	M	M	M	M	M	M	M	M	M	M	M	M	M
		)	)	)	)	)	)	)	)	)	)	)	)	)	)	)	)
2	2	5	3	2	0	1	4	4	1	6	3	1	1	4	5	3	
8		2	1	5	3	4	1	7	8	1	1	5	2	1	6		
		4	5		8	1	1	1	0		2	2	1	0	0		
		8	8														
2	1	5	3	1	0	1	4	4	1	3	3	1	1	4	5	2	
9		2	1	3	3	5	1	6	7	1	4	6	0	0	5		
		1	5		9	1	1	1	0		2	2	1	0	0		
		9															
3	0	5	3	0	0	1	4	4	1	0	3	1	1	3	5	0	
0		1	0	0	4	6	2	6	5	1	7	6	0	8	3		
		4	1		1	1	1	1	0		2	2	1	0	0		
		9	1														

Lower argument heads (read upward): 11, 10, 9 — Anomaly of the eccentric [Anomalia Eccentri].

Parsing notes: The Mercury reflexion table is the most degraded of the four — heavy ink-bleed and broken type throughout the body, and the column-pair boundaries (each cell holds a "G.M." reflexion pair) repeatedly split ambiguously between adjacent columns. The argument columns (Grad. desc./asc.) and proportional-minutes columns are confidently read; the southern and northern reflexion bodies are only partly recoverable. Multiple interior [?] flags remain. **Flagged as a table-reprocess candidate.**

Astronomia Danica — Pars Altera, Liber Secundus Theoricorum

Caput XXIV (conclusion) — Table of the Reflexion of Mercury

Table of the Reflexion of Mercury [Tabula Reflexionis Mercurii].

The table gives the reflexion (obliquation) latitude of Mercury. The two leftmost columns are the degrees of the argument, running 0–30 downward on the left (Gra. desc., degrees descending) and 30–0 upward on the right (Gra. asc., degrees ascending). The next block, headed **Proportional Minutes / Twelfth-parts of the Eccentric [Scrupula Proporti. — Dodecat. Eccent.]**, carries the column heads 3 4 5 (and at the foot 8 7 6), each entry in minutes and seconds (M. S.). The right-hand blocks give the **Northern Reflexion [Reflexio Borea]** and the **Southern Reflexion [Reflexio Austrina]**, subdivided into **Twelfth-parts [Dodecatemoria]** and **Anomaly of the Orb [Anomaliae Orbis]**, with column heads running 0 1 2 3 4 5 6 7 8 9 10 11 across the top; each cell is given in degrees and minutes (G.M.). The bottom argument heads for the right portion read 8 7 6, and the foot of the table is labelled **Anomaly of the Eccentric [Anomalia Eccentri]**.

Parsing notes: this page is the heavily inked double-scan region flagged in the prior batch's handoff. The argument columns and the proportional-minutes block are confident; several cells in the dense reflexion blocks (especially the high-anomaly columns 0–5 and the degree/minute split in the Borea section) suffered ink-bleed and run-together digits. The transcription below preserves the printed structure; cells whose digit grouping could not be resolved with confidence are flagged [?]. Recommend table-reprocess at further review.

G ra . d es c.	G ra . as c.	P r o p. (M .S)	P r o p. (M .S)	P r o p. (M .S)	B o re a 0 (G. M)	B o re a 1 (G. M)	B o re a 2 (G. M)	B o re a 3 (G. M)	B o re a 4 (G. M)	B o re a 5 (G. M)	A u st r. 6 (G. M)	A u st r. 7 (G. M)	A u st r. 8 (G. M)	A u st r. 9 (G. M)	A u st r. 1 (G. M)	A u st r. 1 (G. M)
		0	0	1	4	0	1	1	2	1	0	1	2	2	1	0
	0	0	3	1	9	4	2	5	0	2	0	5	3	2	4	5
			0	5	0	6	4	4		3		3	5	6	8	6
				1												
1	2	1	3	5	0	0	1	1	2	1	0	1	2	2	1	0
	9	3	1	2	2	4	2	5	0	2	5	5	3	2	4	5
			5	1		7	5	5		6		5	5	5	6	6
				9												
2	2	2	3	5	0	0	1	1	1	1	0	1	2	2	1	0
	8	5	1	2	3	4	2	5	5	2	9	5	3	2	4	5
			5	4		9	7	5	9	4		7	5	4	5	5
			8	8												
3	2	3	3	5	0	0	1	1	1	1	0	1	2	2	1	0
	7	8	2	3	4	5	2	5	5	2	1	5	3	2	4	5
			5	1		0	8	6	9	1	3	9	5	3	3	2
			0	6												
4	2	4	3	5	0	0	1	1	1	1	0	2	2	2	1	0
	6	1	3	3	5	5	2	5	5	1	1	2	3	2	4	5
		0	4	4		1	9	6	9	9	7		5	2	2	1
			1	4												
5	2	5	3	5	0	0	1	1	1	1	0	2	2	2	1	0
	5	1	4	4	7	5	3	5	5	1	2	4	3	2	4	4
		2	3	1		3	0	7	8	7	1		6	1	0	9
			1	1												
6	2	6	3	5	0	0	1	1	1	1	0	2	2	2	1	0
	4	1	5	4	9	5	3	5	5	1	2	6	3	2	3	4
		4	2	3		4	1	7	7	5	5		6	0	9	7

G ra . d es c.	G ra . as c.	P r o p. (M .S)	P r o p. (M .S)	P r o p. (M .S)	B o re a 0 (G. M)	B o re a 1 (G. M)	B o re a 2 (G. M)	B o re a 3 (G. M)	B o re a 4 (G. M)	B o re a 5 (G. M)	A u st r. 6 (G. M)	A u st r. 7 (G. M)	A u st r. 8 (G. M)	A u st r. 9 (G. M)	A u st r. 10 (G. M)	A u st r. 11 (G. M)
		7	7													
7	2	7	3	5	0	0	1	1	1	1	0	2	2	2	1	0
	3	1	6	5	1	5	3	5	5	1	3	8	3	1	3	4
		6	1	2	1	5	3	7	7	2	0		6	9	7	5
		0														
8	2	8	3	5	0	0	1	1	1	1	0	2	2	2	1	0
	2	1	6	5	1	5	3	5	5	9	3	1	3	1	3	4
		8	5	2	2	7	4	7	6		4	0	6	8	5	3
		8	7													
9	2	9	3	5	0	0	1	1	1	1	0	2	2	2	1	0
	1	2	7	5	1	5	3	5	5	6	3	1	3	1	3	4
		1	4	5	4	8	5	8	5		8	2	6	7	3	1
		6	1													
1	2	1	3	5	0	0	1	1	1	1	0	2	2	2	1	0
0	0	0	8	6	1	5	3	5	5	4	4	1	3	1	3	3
		2	3	1	5	9	6	8	5		2	4	6	6	2	9
		3	3	4												
1	1	1	3	5	0	1	1	1	1	1	0	2	2	2	1	0
1	9	1	9	6	1	0	3	5	5	2	4	1	3	1	3	3
		2	2	3	7		7	9	4		6	6	6	4	0	7
		5	0	6												
1	1	1	4	5	0	1	1	1	1	1	0	2	2	2	1	0
2	8	2	0	6	1	1	3	5	5	0	5	1	3	1	2	3
		2	6	5	8		0	9	3		1	7	6	3	8	5
		6		7												
1	1	1	4	5	0	1	1	2	1	0	0	2	2	2	1	0
3	7	3	0	7	2	3	4	0	5	5	5	1	3	1	2	3

G ra . d es c.	G ra . as c.	P r o p. 3 (M .S)	P r o p. 4 (M .S)	P r o p. 5 (M .S)	B o re a 0 (G. M)	B o re a 1 (G. M)	B o re a 2 (G. M)	B o re a 3 (G. M)	B o re a 4 (G. M)	B o re a 5 (G. M)	A u st r. 6 (G. M)	A u st r. 7 (G. M)	A u st r. 8 (G. M)	A u st r. 9 (G. M)	A u st r. 0 (G. M)	A u st r. 1 (G. M)
		2 7	5 2	1 7	0	0	0	2	7	5	8	5	2	7	3	
1 4	1 6	1 4 2 8	4 1 3 7	5 7 3 6	0 2 1	1 5	1 4 1	2 0	1 5 1 3	0 5 3	0 5 9	2 1 9	2 3 5	2 1 1	1 2 5	0 3 1
1 5	1 5	1 5 2 8	4 2 2 2	5 7 5 4	0 2 3	1 6	1 4 2	2 0	1 5 0	0 5 0	1 3	2 2 1	2 3 5	2 1 0	1 2 3	0 2 9
1 6	1 4	1 6 2 9	4 3 6 1	5 8 1 1	0 2 4	1 7	1 4 3	2 0	1 4 9	0 4 7	1 7	2 2 3	2 3 4	2 9	1 2 2	0 2 7
1 7	1 3	1 7 2 9	4 3 4 9	5 8 2 7	0 2 6	1 9	1 4 4	2 0	1 4 7	0 4 4	1 1 1	2 2 4	2 3 4	2 8	1 2 0	0 2 4
1 8	1 2	1 8 2 9	4 4 3 3	5 8 4 1	0 2 7	1 0	1 4 5	2 1	1 4 7	0 4 0	1 1 6	2 2 6	2 3 3	2 6	1 1 8	0 2 2
1 9	1 1	1 9 3 0	4 5 1 6	5 8 5 4	0 2 9	1 1	1 4 5	2 1	1 4 6	0 3 7	1 1 8	2 2 7	2 3 3	2 5	1 1 6	0 2 0
2	1	2	4	5	0	1	1	2	1	0	1	2	2	2	1	0

[illegible]

		P	P	P	B	B	B	B	B	B	A	A	A	A	A	A
		r	r	r	o	o	o	o	o	o	u	u	u	u	u	u
		o	o	o	re	re	re	re	re	re	st	st	st	st	st	st
		p.	p.	p.	a	a	a	a	a	a	r.	r.	r.	r.	r.	r.
G	ra	3	4	5	0	1	2	3	4	5	6	7	8	9	0	1
.	ra	(	(	(	(	(	(	(	(	(	(	(	(	(	(	(
d	.	M	M	M	G.	G.	G.	G.	G.	G.	G.	G.	G.	G.	G.	G.
es	as	.S	.S	.S	M	M	M	M	M	M	M	M	M	M	M	M
c.	c.	)	)	)	)	)	)	)	)	)	)	)	)	)	)	)
2	3	2	5	5	0	1	1	2	1	0	1	2	2	1	1	0
7		7	0	9	4	2	5	0	3	1	4	3	2	5	3	5
		1	1	5	2	0	2		3	0	4	3	8	2		
		8	6	4												
2	2	2	5	5	0	1	1	2	1	0	1	2	2	1	1	0
8		8	0	9	4	2	5	0	3	7	4	3	2	5	2	3
		1	4	5	3	1	3		1		7	4	7	1		
		6	8	7												
2	1	2	5	5	0	1	1	2	1	0	1	2	2	1	1	0
9		9	1	9	4	2	5	0	2	4	5	3	2	4	0	2
		1	1	5	5	2	4		0		0	5	6	9		
		4	9	9												
3	0	3	5	6	0	1	1	2	1	0	1	2	2	1	0	0
0		0	1	0	4	2	5	0	2	1	5	3	2	4	5	0
		1	4	0	6	4	4		3		3	5	6	8	6	
		1	9													

[Foot of table, proportional-minutes block: 8 7 6 — Anomaly of the Eccentric (Anomalia Eccentri).]

[Foot of right portion: 8 7 6 — read upward against the ascending-degree column.]

CAPVT XXV.

On computing the latitudes of Venus and Mercury from the preceding tables [De latitudinum Veneris ac Mercurii è tabulis praecedentibus computatione].

[Margin, p. 458: A precept for computing latitude in Venus and Mercury — Cap. 19 of this book. (Praeceptum latitudinem in Venere & Mercurio supputandi, Cap. 19 hujus.)]

We have constructed these canons for the latitudes of Venus and Mercury throughout on the pattern of the Prutenic Tables, both for greater ease of use and to avoid error in this matter. We therefore depart hardly at all from Reinhold's [Erasmus Reinhold] precept on the representation of these latitudes — even though, in fact, an enormous discrepancy is sometimes found in the phenomena themselves, indeed one so great that it often reaches a full three degrees, chiefly in the case of Venus, as the reformed tables show when compared with the old. The latitude, then, although in itself uniform — as its computation by triangles demonstrated above — we have nonetheless distributed, after the fashion of the ancients (such as it is), so that it can be the better comprised in tables and adapted to use by means of the proportional minutes. We have divided it into the **declination**, which is reckoned only at the actual quadrants of the eccentric, and the **reflexion or obliquation** [**reflexionem seu obliquationem**], which likewise is computed only at each apse. These two kinds we here track everywhere blended together at the intermediate places; the **deviation** (as they call it) we have neglected, for the reason mentioned above.

The sum of the precept, then, is this. By the equated anomaly of the eccentric [anomalía coaequata eccentrici] — after, in the case of Venus, the 16 degrees that belong to our age have been added to it — extract the proportional minutes. By the equated anomaly of commutation [anomalía coaequata commutationis] extract the latitude, with this caution: from whichever part of the canon yields the proportional minutes, from that same part you are also to take both the declination-latitude and the reflexion-latitude. The headings indicate what kind of latitude it is.

Next, applying the proportional minutes, take from each of the latitudes the parts that correspond to their own latitudes. Finally, if both — namely the declination and the reflexion, thus corrected — are of one and the same affection [i.e., both north, or both south], the latitude sought is the sum of them; if not, subtracting the smaller kind from the larger leaves a difference, which is to be reported as

the latitude sought, and which will keep the name (north or south) of whichever component is the excess or remainder.

Example [Exemplum].

We shall here exemplify this general precept for the latitude of Venus only and chiefly by comparing it with an ancient observation, so that it may be established to some degree that the motion of its nodes does not stick perpetually at the apses, but in a manner follows a motion commensurable with them.

For this purpose I think we should use that observation of Timocharis [Timocharis] which we employed for correcting the longitude, since the data required for it were ready to hand there. This took place, as we reported above from Ptolemy [Ptolemy], in the 13th year of Philadelphus [Ptolemy II Philadelphus], at the beginning of the 18th day of the month Messori, in the morning; where, among other things, we found the latitude of Venus, at its conjunction with the fixed star in the southern wing — namely the star opposite Vindemiatrix [in Virgo, ♍] — to be 1 degree 16 minutes north.

At the same time the equated anomaly of the eccentric was 4 signs 29 degrees 0 minutes [4 sig. 29 g. 0 m.], and that of the orb, once restored, the equated anomaly of commutation was 8 signs 11 degrees 55 minutes [8 sig. 11 g. 55 m.]. Therefore, in the first table of Venus — the one inscribed "of declination" — first, opposite the proposed anomaly of the eccentric, the proportional minutes are found to be 31 minutes 5 seconds; and second, opposite the anomaly of the orb, the southern declination itself is 0 degrees 46 minutes, whence the proportional part is deduced as 23 minutes 50 seconds, to be kept. Then in the same way entry is made into the following table of Venus, which exhibits the reflexion, and likewise, with 4 signs 23 degrees 0 minutes of the equated anomaly of the eccentric, there are found in the area

— in the area, the proportional minutes 52 minutes 19 seconds; and with the anomaly of the orb likewise 8 signs 11 degrees 55 minutes, the northern obliquation is taken as 2 degrees 19 minutes, whence, again by the foregoing proportional minutes, the proportional part is elicited as 1 degree 48 minutes 53 seconds. Since this is greater than the declination and, moreover, of a different affection, that [declination] subtracted from this [obliquation] leaves the sought northern latitude of Venus as 1 degree 25 minutes 3 seconds. This exceeds the observed value by only 9 minutes, if we suppose the orb of the planet to have been unalterable with respect to the change of the ecliptic; but otherwise it

agrees entirely. For the star in question, in the wing [of Virgo], holds a latitude of 1 degree 25 minutes north in our own age.

Yet the ancient observations are not to be so trusted, nor is such precision to be granted to them, that anything should be concluded precisely from this case — contrary to what was done above concerning the latitudes of the three superior planets. Rather, the point — for which we introduced this example, as we hinted before — is just this: that we may understand that the northern section of Venus's latitude in that age clung as closely as possible to its own apogee, or preceded it a little in the direction of the signs. Whereas now, in our own age — as we have said, and as all our modern observations prove — it has fallen back at least 16 degrees behind that same apogee. Around the time of Christ's birth, therefore, since we gather that the point of the said intersection lingered near the apogee, and since it must accordingly be drawn back or forward to the intermediate times, reasoning establishes that to roughly every hundred years one degree corresponds — by which the motion of the ecliptic section and of the orb of Venus will be reckoned slower than that of this apogee. In Mercury, however, the section of the southern node accompanies its apogee unceasingly and without dislocation, just as our observations sufficiently support Ptolemy's view in this respect.

And here, by the grace of God, we set an end to the **Restoration of Astronomy [Astronomiae Redintegratio]** — wherein, according to our plan, we have corrected the hypotheses and Tables that are to exhibit the celestial phenomena against the infallible observations that we were able to obtain, and so indeed that we would not wish these persistent labors of ours — which we hope will profit all honest people and lovers of these disciplines — to be judged by anyone (that is, to be derided or perversely despised) except by one who sets himself to make a proper comparison of them with the celestial appearances. Let such a one rather pronounce from these a just verdict on the matter, or, by at last setting his own hand to the table, himself perceive how difficult this task has been.

As for the **station, retrogradation, and likewise the appearing and occultation of the planets [statione, retrocessione, item apparitione & occultatione planetarum]**, we judge there is no need at all of a separate treatment after the manner of Ptolemy or Copernicus, since the recognition of these falls out better within the computation of the planets' true motions — as Copernicus rightly judged of old — while the science of them is most rightly drawn from Book 2, Chapter 4 of our doctrine of the sphere [doctrinae nostrae sphaericae lib. 2 cap. 4], which clearly handles the rising and setting of the stars.

To God alone be glory [Soli Deo Gloria].

THE END [FINIS].

[This concludes Liber Secundus Theoricorum and Pars Altera.]

[Appendix title]

APPENDIX TO THE *ASTRONOMIA DANICA*

On the Adventitious Phenomena of the Heavens [De
Asscitiis Coeli Phaenomenis]

Namely, On New Stars and Comets

By Christian Severini's son [F. = filius] Longomontanus [Christen Sørensen Longomontanus].

Diagram. The printer's device of Johannes and Cornelius Blaeu. At the center stands an armillary sphere; to its left is the winged god Mercury holding a caduceus, and to its right a standing figure (a "wild man" / Hercules-type holding a club). Below, a banner carries the motto:

- Banner (center): "INDEFESSVS AGENDO" [Tireless in acting]

Amsterdam: At the press of Johannes and Cornelius Blaeu [Apud Ioannem & Cornelium Blaeu]. In the year 1640.

[Dedicatory poem]

To the most distinguished and excellent gentleman, Dr. Christian S. Longomontanus, Royal Professor of Mathematics in the University of Copenhagen [Academia Hafniensis], on the publication of his new work on comets.

Far beneath your spirit, great man, was the care of these prostrate lands,
or of whatever this slippery globe contains. Your one concern was to
leave behind mortal cares, and with lofty mind to scale the dwellings of
heaven. You enter the venerable palaces of the sky; with shrewd mind
you mark down the hidden causes of things. You disclose what a comet,
that hostile torch, portends, and what God himself intends by these
sacred fires. For new portents are not seen in the sky in vain: they are
sure tokens of impending evil. Those who have learned it by experience

believe it — those whom the hostile force of an angry hand has thus far struck with a hard lash. But not yet is there an end to punishment, because not yet is there an end to crimes. Take away the crime, and no stars do harm.

Caspar Erasmus Brochmand [Caspar Erasmus Brochmand], Professor of Sacred Letters in the University of Copenhagen.

[Dedicatory epistle, p. 1]

TO THE MOST SERENE AND MOST EXALTED PRINCE, LORD CHRISTIAN IV [Christian IV of Denmark], ELECTED KING OF DENMARK AND NORWAY, HIS OWN MOST GRACIOUS LORD.

There was good reason, most exalted Prince, that I should recently dedicate the *Astronomia Danica* — worked out so far as the powers of my mind and body have hitherto allowed — to the most serene and mighty King of Denmark, your father, that renowned father of his country, with all the submission of spirit that is fitting. For nearly the same reason I am now moved to inscribe and present its appendix, on the new phenomena of the heavens, humbly to Your Serene Highness. For God, the Best and Greatest, has in these times shown such clemency toward us that not only do these ancestral kingdoms begin to rise daily higher and to flourish under so great a king, but — what must be reckoned of the highest importance — a sound confession concerning the Triune God and the mystery of our salvation in Christ Jesus is kept with one accord, while in the meantime no honorable arts or disciplines are neglected. For my own part, to the extent of my slender powers, once I had been appointed to this Royal University, I judged it a wrong if the work formerly undertaken at great expense and brought down from heaven in our homeland for the restoration of astronomy, or the things that God meanwhile chose to make shine forth in the sky as a warning to mortals, should be suppressed and perish utterly — especially since I had come to know that whatever industry of mine devoted to bringing these matters to light would be welcome to the gracious will of my superiors. Among other tasks, I directed my zeal to this in particular: to record carefully that fateful comet which, three years ago now, blazed threateningly enough in the sky. And not in vain — for from the time of its rising, and thereafter, the Roman Empire was so shaken and thrown into disorder that even now its own safety is imperiled, and any solid restoration is called into doubt, unless the merciful God himself should as it were interpose directly and bring timely aid to its exhausted affairs.

But although, while it lasted, I omitted here neither occasion nor any effort to grasp the motion and position of this tailed star as well — which my observations, when compared with those of others, will, I hope, sufficiently prove — yet very many causes of delay pressed in upon me, which too long suspended the prompt rendering of the duty I owed in its description. Among these was that deadly plague which, sent by divine will at the same time as a punishment for our offenses, began to rage at the rising of this comet and lasted a year and a half; on account of it this University was for about three-quarters of a year suspended and dispersed. But lest I bring this offspring on this single comet to birth too late, amid the writings of others recently rushed into print, I judged it worth my while to make the material — so specialized when on a single comet — richer, and to undertake the publication of a general treatise on new celestial phenomena, derived as far as I could through their causes as through first principles; and to append to it, as examples, the phenomena and such astrological judgments as I could give of the two most recent comets — so that in this way at last I might return to discharging humbly, as I had vowed, the duty owed. This treatise, although I confess it to be far smaller than befits so great a Prince — who, like a new Sun, spreads himself in rays and beams of ancestral, fatherly, and wholly heroic virtue and clemency to all —

[Dedicatory epistle, p. 2]

— so that the hope of these kingdoms may, with the good God's help, be uniquely secured in Your Highness for the future: nevertheless, since among the languages, arts, and sciences in which Your Highness is exceptionally accomplished, you also pursue with singular skill the exercises of mathematical subjects, there can be no doubt that that lofty mind of Your Highness will also take much delight, as occasion offers, in things celestial and more sublime — which Astronomy, the queen of the disciplines, embraces and shares with many other matters — and especially in duly investigating the nature of new phenomena, above all of comets. For this study, which calls the noble mind away from earthly dregs and restores it, as into its own inspired place, to the admiration — indeed the knowledge — of so harmonious an order and motion of the celestial bodies, has always been sought after by kings, by princes, and by men in public affairs, as the examples among the most serene and most praiseworthy kings of the Danes remain to this present day. Press on, then, in virtue, most exalted Prince, and henceforth emulate your great-grandfather and your father's grandfather not only in piety and clemency, but also in the skill in and delight in this sublime science — and emulate your father besides in his effective desire to advance this commonwealth. No memory will ever blot out

their virtues and good deeds toward these kingdoms. For (as our Saxo [Saxo Grammaticus], in his own time no less a poet than a most elegant historian, sang):

Glory follows the dead, and Fame outliving the ashes of each shall survive; nor does it sink into any age — the virtue that one has accomplished, perfected, in his own time.

But lest this glory should prove vain, those Heroes so devoted their effort to piety toward God and clemency toward their subjects that they held Him above all favorable, and these bound fast to themselves — Him, I say, the δημιουργόν [Greek: dēmiourgon, "Maker" / Creator], on whom all the safeguards and salutary counsels for the protection and enlargement of the commonwealth depend. True indeed is what Plato somewhere says: "Where not God, but some mortal man, reigns, there is no escape from calamities and evils." This I think must henceforth be religiously observed, lest — if either other evils, or those which these latest comets threaten (and not in vain) against the world, should reach us too, by divine will and (alas) by our own deserts — we should once more have God on our side, who alone knows how to bend the fates and to pour down from on high, in place of a portentous omen, a blessing — intimately, and most graciously wills to do so. To Him we devoutly commend Your Serene Highness, together with your most illustrious brothers, that piety and the other ancestral virtues, in whose footsteps you so beautifully tread by divine grace, may extend themselves into the most certain supports of these kingdoms, may continue, and may be happily propagated to a distant posterity, if any is to be. At Copenhagen [Hafniae], on the day before All Saints, in the year of Christ our Redeemer's redemption 1621.

Your Serene Highness's

most humble [servant],

Christian Severini's son [Christen Sørensen] Longomontanus.

PROOEMIUM [Preface]

All the luminous bodies presented to our eyes in the theater of the heavens are the subjects of astronomical consideration — whether, like the fixed stars, they hang motionless, or whether they detach themselves from those stars by motions of whatever kind. It is therefore only fitting that something concerning the adventitious phenomena of the heavens [ascititiis coeli phaenomenis] should also be appended, now that our *Astronomia* proper is complete. For although these phenomena have not been subject to the eternal laws of motion in the way

the other bodies coeval with the world are — being, as it were, less durable — nevertheless their investigation must by no means be neglected. As much as can be obtained by external aids, namely by properly conducted observations, and as much as can be drawn out, here and there, by skillful reasoning toward what is nearest the truth, ought to be pursued. This is what the goal — and as it were the fruit — of so illustrious a study, one most worthy of a mind well endowed, recommends. By it, at the same time, the immense longing of our souls, and the near-utmost concern of those given over to it, is drawn — in fear and reverence of the Divine Power [Numen] — by the appearance of new portents in the heavens.

For just as the heavenly bodies are seen to be present for the general governance of the world, according to God who created them in the beginning — not only when they mark out the seasons, but also when, in producing whatever happens in these lower regions, they operate according to the course of nature, while other events they signify in advance — so these new phenomena, whenever they blaze forth in the heavens, undoubtedly portend something beyond the customary law of nature. By the singular providence and dispensation of the Divine Power within this earthly economy, they intimate, before it has come to pass, and they exhort men, the pious as well as the impious, to repentance. For signs of this kind, and above all the comet, are generally of ill omen — a thing to which the history of every age bears witness, so that from it the following maxim [apotelesma] has been constructed:

οὐδεὶς κομήτης ὅστις οὐ κακὰ φέρει [Greek: "no comet that does not bring evils"]

Never has a comet been seen to blaze without bringing harm. [Impune nunquam visus fulgere cometes.]

The example of Hipparchus of Rhodes [Hipparchus], that ancient and distinguished astronomer — but above all of our own Danish Atlas, Tycho [Tycho Brahe] — will likewise move posterity not to pass over such new celestial phenomena in silence, but rather, on their occasion, to cultivate the art of astronomy all the more intensively. For both of these men observed two new stars apiece, blazing in the heavens in their respective ages, and they set them in canonical order, in relation to the ecliptic, along with virtually all the rest of the fixed stars. Indeed, in the matter of observing comets as well, and of duly assigning them their place among the celestial bodies, our Tycho in the earliest period devoted no less care — if not greater — than he did to the planets and the other bodies coeval with the heavens. Accordingly, since new stars of this kind

[Greek: τερατώδεις (teratōdeis, "monstrous" or "prodigious")] do not cease to be seen in the heavens in our own times as well, but appear more frequently as the otherwise enfeebled world nears its end — which without doubt is shortly at hand — and since by these (clearly) divine providence wishes pious minds to be warned of the most disturbed state of the world about to come, which is everywhere indicated in Scripture, I too, professing the mathematical arts, and especially the astronomical, in this Royal Academy, have rightly been roused by some concern for these matters. As it once impelled me to observe, so now it has impelled me to write.

[Margin: Eccles. 38, also 39; Apoc. 20]

And although this comes about, for various reasons, later than one might have wished — after the last comet, which was terrible enough, had already been received by many distinguished men too, even immediately upon its first appearance, and described, and reported by most as bringing dire and ominous tidings, just as the present state of affairs seemed to demand — nevertheless, because I have not found observations of it by others so reliable and precise, nor its causes and effects so thoroughly debated by those same men (even amid so great a diversity of judgments) that I could not trust my own labor (belated though it be [postliminium]) would find a place among the discerning: for this reason, in order that I might more rightly serve the benefit of mathematical youth, to whom I had devoted my chief effort, it seemed good to me first to compose a general treatise on new phenomena of this kind, through their causes and effects, after first collecting the opinions of the ancients and the moderns, recalling them to the anvil of reason, and adjudicating them; and then to set out, from my own observations, the appearances, motions, and positions of the two bodies that presented themselves in the form of comets, moving in contrary directions as if struggling against each other — namely those that blazed forth from the heavens in the years 1607 and 1608 since the birth of Christ — and to attach to them their significations, such as they are. This I did both on account of these perilous times, of which these comets were without doubt the signs — or whose [perils] (which is to be feared) had not yet fully arrived; and also so as not to leave the youth destitute hereafter of examples concerning the observation of comets, the working-out of parallaxes [parallaxium enucleationem], and finally the prediction of effects of whatever kind, conducted duly and astrologically — that is, according to the most rational method.

the rest. What I attempt with such slender powers as I recognize myself to possess, by the strength of my own talent, and aided here by the divine Power, I

have applied so that the material I have undertaken may be directed and decided according to those principles which I too judge to be most likely true in nature. I see that I must beg pardon for myself from those very men who otherwise cherish opinions widely differing among themselves concerning the new celestial phenomena. Yet those opinions must, before all else, be tested against true and competent astronomical observations, which fall under the senses, and which are afterward — by the help of geometry and arithmetic — subjected to higher reasoning. For these [observations] alone supply the steps by which to enter the threshold of so lofty a philosophy, and indeed they do so most rightly of all, in preference to the many speculations of the ancients and moderns alike.

Contents of the chapters of this treatise.

Cap. I. On the kinds [species] of new phenomena.

Cap. II. On the various opinions, both of the ancients and of the moderns, concerning the origin and definition of new phenomena, and especially of comets; how their causes are briefly handed down by those authors, and reduced under definite genera.

Cap. III. On the assessment of the chief opinions, both ancient and modern, concerning comets — what in them we judge should be rejected, and what retained.

Cap. IV. On the efficient cause of the origin and generation of new celestial phenomena.

Cap. V. On the material cause of new celestial phenomena.

Cap. VI. On the formal cause of new celestial phenomena.

Cap. VII. On the final cause of new phenomena, and on their description.

Cap. VIII. On the judging of the effects of new celestial phenomena, and especially of comets.

Cap. IX. On the determination of the course of the comet of the year 1607 from my own observations; and on its parallaxes [being] so slight that from them we conclude it lay within the celestial region, far above the Moon.

Cap. X. On the phenomena, parallaxes, distance from the Earth, and magnitude of the comet of the year 1618, gathered from my own observations; with the addition of a report of this same comet observed elsewhere in Persia.

Cap. XI. On the effects or significations of these two comets, to be adjudicated separately and briefly according to the common decrees of the astrologers.

Cap. XII. On the common character of the significations of the two aforementioned comets, drawn from the same efficient causes; on their mutual contrariety; likewise, on the nature of the signs or asterisms through which they passed; and further, on two others that arose at nearly the same time. From all of which, the detestable heedlessness of mankind in these final times is, before all else, exposed.

CH. S. LONGOMONTANUS, ON THE NEW CELESTIAL PHENOMENA, APPENDIX.

CAP. I.

On the kinds of new phenomena.

The method I have undertaken of weaving together this treatise departs somewhat from the natural method, which is commonly begun from the definition of a thing. For what we chiefly inquire here — namely, of what nature, in the end, such new celestial phenomena have been — will first be transmitted through external criteria, and will be accurately judged by various circumstances drawn from these, chiefly by the testimony of the eye and by the historical record. The nature of the phenomenon, which is to be recognized through its causes by reasoning that follows afterward, can then be taught or expounded tolerably well. For a new phenomenon can be called such only in general — as something that, in the manner of a fixed star or a comet, arises anew above us, whether in the air or rather in the celestial theater, since it was certainly not seen before. Yet until this description has been made (imperfect though it be), the opinions of the ancients and moderns, which will be recited in the following chapter, will plainly demonstrate the matter.

Therefore, beginning from the visual criterion, the kinds of those phenomena which appear in this way — beyond what is customary in the heavens, and which from time to time seem to appear there — we shall here review first of all. For this, even Pliny [Pliny] alone would suffice us, provided he had not extended the subject matter, in imitation of the Greeks (from whom the doctrine of comets was, as others hold, first transmitted to the Latins and originally derived), too far into the subdivision of such kinds. For he enumerates twelve kinds of comets in *Natural History* II.25 (*N.H.* 2.25), which it is better to write out in Greek

characters, so that their etymology [Greek: αἰτιολογία (aitiologia, "explanation of causes")] may be more rightly grasped, and with what things they were conformable:

- Κομήτης [Greek: komētēs, "comet"]: a hairy or curled star.
- Πωγωνίας [Greek: pōgōnias, "bearded"]: bearded, from whose lower part there is sent forth a beard, as it were, in the shape of a long beard.
- Ἀκοντίας [Greek: akontias, "javelin-shaped"]: a tuft, from the form of a javelin.
- Ξιφίας [Greek: xiphias, "sword-shaped"]: from the form of a sword — or rather almost without rays, except for sparse ones at the edge, of an amber color.
- Δισκεύς [Greek: diskeus, "disc-shaped"]: like a cask, in its concave [interior] of smoky light.
- Πιθήτης [Greek: pithētēs, "jar-shaped"]: it has the appearance of a horn. [recte: this label appears displaced; the description "has the appearance of a horn" corresponds to Κεραιοειδής below]
- Κεραιοειδής [Greek: keratoeidēs, "horn-shaped"]: a star [?]. — [the descriptions "which represent burning faces" and "like a cask in its smoky concave light" are entangled in the source typesetting here; see Parsing notes below]
- Λαμπαδίας [Greek: lampadias, "torch-shaped"]: stars which represent burning faces.
- Ἴππεύς [Greek: hippeus, "horse-shaped"]: representing horses' manes, of swiftest motion, and going around in a circle about themselves.
- Ἀργυροκόμης [Greek: argyrokomēs, "silver-haired"]: shining so brightly with silvery hair that one can scarcely look upon it, and showing in itself, in a human likeness, the image of God.
- Λογχίτης [Greek: lonchitēs, "lance-shaped"]: in the form of a spear-point.

There also arise [comets] with the appearance of shaggy goats and surrounded by some tuft; at the same time the image of the tuft, changed into a spear-point, occurred on the 108th Olympiad [Olymp. 108]. — Thus far Pliny.

Now then, although Pliny establishes these distinctions of comets in imitation of the Greeks, as I have said, [I nevertheless subscribe] to the opinion of Seneca, *Natural Questions* VII.11 (*Nat. Quaest.* 7.11), where he discourses on comets thus: "Whether there are some differences among them, or whether there are not, it is necessary that they come about by the same principle, since they are comets." I

willingly subscribe to this, and therefore add that remark of his about the Plinian distinctions — namely, that a great part of them, or even their visual appearance alone, is expressed by the [relative] position between the Sun and the comet, as will be shown more rightly below, once the material proper to comets has been supplied. But concerning the Plinian forms of comets, such as they are: certainly those that have been seen with our own eyes have been either tufted [comati], like the one of 1585, or tailed [caudati], like all the rest — except for the one which the distinguished Johannes Kepler [Kepler], in the year 1618, reported to have been seen first among the three that were visible in the heavens in that same year, detected in the north. Its tail, like that of the others, was turned diametrically away from the Sun, and appeared in the manner of household brooms, so that on this account it might seem it could be referred to the second kind, the Πωγωνίας [Greek: pōgōnias, "bearded"] mentioned above. — Not to dispute about that form which the same Kepler mentioned, [said] first by others, then by himself, to have been observed near the rising of the last comet, and a few days before it, having the appearance of a Ceratias or rather of a beam [trabis] (concerning which Seneca speaks in *Natural Questions* VII.5), and stretched out lengthwise away from the Sun.

But we, treating of new phenomena in general, see that that distinction of them can rightly be drawn which Epigenes [Epigenes] attributes to comets alone — and indeed in another sense — which is read in Seneca, *On Natural Matters* VII.6, in these words: "There are," says Epigenes, "two kinds of comets: some pour out fire on every side and do not change their place; others gather wandering fire into one direction in the manner of a tuft, and pass through the stars." Of these kinds, I say — although Epigenes maintains that the former is born from earthly matter and afterward raised aloft by the force of wind [spiritus] (an opinion of his which Seneca, having mentioned it at greater length, refutes below) — nevertheless, in granting that the same comet is, with respect to the fixed stars, motionless (not, certainly, with respect to the primary motion), while the other does pass through the stars, he does not obscurely hint at what otherwise common observation can grasp and attest, both in new stars and in comets. This [observation], without distinct observations and parallaxes worked out here and there — of which it [common observation] is deprived — can establish nothing certain about phenomena appearing above us by night, nor distinguish their elevations [sublimationes] from the face of the Earth. Therefore, what Epigenes there calls unmoving, but low, we too declare to be motionless among the stars — yet of all things that are seen newly in the heavens, the most sublime. On this account, from this we have established the first genus, called the new star [stellam novam]. But from the other — what appears mobile among the stars, and

stretches out wandering fire in the manner of a tuft — we make a genus of comets, namely of those wandering about in the lower, or planetary, heaven, and appearing sometimes in various forms. The causes of this opinion will be rendered in what follows. And so much for the kinds of new phenomena, since they are the first of all to fall under our senses, and therefore have also been assigned the first place in this treatise.

CAP. II.

On the various opinions, both of the ancients and of the moderns, concerning the origin and definition of new phenomena — and especially of comets; how their causes are briefly handed down by those authors, and reduced under definite genera.

If the new stars [sidera] of which we are treating must be referred to causes in nature, and not rather (as some would have it) be reckoned outright among miracles, then surely no matter in all of natural philosophy [Physiologia] seems to me — whether it be found in the lower elements, or above in the heaven itself, where nature in her most excellent state flourishes — to have given rise to more mutually discrepant opinions among the philosophers than the generation and origin of comets, when it must be explained through their causes. (I pass over the new stars, whose appearances were either never detected by the ancients, in those uncultivated ages, or never handed down in writing to posterity. For the one that Pliny, in II.26 (lib. 2, cap. 26), reports to have been observed by Hipparchus of Rhodes — because it was not without a peculiar motion, it should rather (if otherwise credit is to be given to Pliny) be counted among the comets, appearing in opposition between the Sun and the Earth.)

But to return to comets, it will be pertinent to my purpose first to set out briefly in this chapter the chief opinions, both ancient and modern, concerning their nature and causes, and then to distinguish them into definite genera. As for the opinions of the ancients on these matters, Plutarch [Plutarch], in *On the Opinions of the Philosophers* III.2 (lib. 3, cap. 2), collected them with marvelous brevity and sets them out in this manner:

Some disciples of Pythagoras [say] that a comet is one of those [bodies] which are not seen perpetually, but at a fixed time return in a recurring orbit and become visible. Others [hold it to be] a reflected likeness of our vision back from the Sun, in the manner of a mirror-image —
Anaxagoras, Democritus: a conjunction of two or more stars mutually

illuminating one another. *Aristotle*: an ignited concretion of dry exhalation. *Strabo*: the light of a star densely surrounded by cloud, as happens in torches. *Bontius Heraclides*: a lofty cloud illuminated from on high. *Epigenes*: an earthy vapor [spiritus], seeking the upper regions, becoming inflamed. *Boethus*: a species of milky air. *Diogenes*: that comets are stars. *Anaxagoras* [again]: glittering [particles] which, as they are called, fall from the aether in the manner of sparks, and are therefore at once extinguished. *Metrodorus*: a violent compression sparking against a cloud. *Xenophanes*: that all such things are aggregations or motions of ignited clouds.

Thus far Plutarch has reviewed most of the ancients' opinions concerning the causes and generation of comets — which it is likely he himself took from others, but especially from Seneca, who in *Natural Questions* VII (lib. 7) disputes more fully about these same matters, and refutes these same opinions, yet in such a way that he renders even himself uncertain about the generation of comets, except that he rightly judges them to come about in the heavens.

When we turn from these ancients to the moderns, we shall bring into the open the assessments — not of all, indeed, but of the chief mathematicians and natural philosophers [Physiologi] — concerning the causes of comets, and, emulating Plutarch, confine ourselves to a brief working-out. For one can scarcely easily take in the multitude of those opinions which were brought forward by so many writers concerning the last comet that shone forth. Cardano [Cardano] and our Tycho [Tycho Brahe] hold that a comet is celestial matter, generated anew, aggregated into a globe, and suddenly illuminated by the Sun [Greek: καθ' ὅλα (kath' hola, "wholly" or "as a whole")]. Julius Scaliger [Scaliger], on the contrary, contends that the same is produced from terrestrial exhalation, raised by the force of the sidereal [bodies] into the supreme aerial region, and is not kindled but only illuminated by the rays of the Sun. To this opinion Rothmann [Rothmann] assents in part, under this condition: that, once he had understood the heaven to be sufficiently penetrable, he did not doubt that a terrestrial exhalation could climb into the aethereal region, and there for the first time a comet be generated, and be illuminated only by the Sun's rays (as Cardano, Tycho, and Scaliger [hold]).

Kepler [holds] that comets are generated in the vast aether, like fish in the Ocean — and just as these [fish], or birds in the air, [move about], so comets traverse the whole universe in a rectilinear course. Whence he supposes it comes about that, should they by chance at some time fall upon the Earth, they would be pierced

through by it as by something more solid. Nor can anything else, among other things, come about by Kepler's hypothesis — that rectilinear cometary trajectory. Ambrosius Rhodius [Ambrosius Rhodius] [holds] that comets are celestial bodies, luminous indeed, but outside the order of nature. Willebrord Snel [Snellius] discourses that comets are [Greek: κατακάύματα (katakaumata, "burnt-out things" or "conflagrations")] or burnt-up exhalations, emitted from the Sun, which therefore, like the Sun, will glow with a coal-like burning. Finally D. Habrecht [Habrecht] defines a comet to be a new star, generated from celestial matter, and sometimes existing in the supreme air, but for the most part in the aether itself — like L. Regiomontanus [Regiomontanus], P. Apian [Peter Apian], Andreas Craig [Andreas Craig], Scot, and others, otherwise excellent men. I shall not touch upon the natural philosophers, who...

Astronomia Danica — Appendix, *De novis coeli phænomenis*

[Appendix p. 3 / PDF p. 485 — end of Cap. II]

[they held that] no [account] of the generation of comets should be admitted except Aristotle's [Aristotle] opinion, lest perhaps they should seem to want to sprinkle and defile the heavens with new impressions; rather, they held that on this matter, as on others, the hypothesis of the single philosopher of Stagira [Aristotle] was to be followed.

This collection of opinions, ancient and recent, on the causes of comets, however varied it was, nonetheless falls under three heads — namely, when one considers the diversity of *matter*, of *appearance*, and of *place*. For, first, with respect to matter, a comet is produced as arising either from a celestial or from a terrestrial source. Next, as to the form of its appearance — which will not differ from the truth — a comet either blazes kindled like a fire, or at least appears illuminated by the stars, and above all by the Sun. Finally, with respect to place, comets will be situated either in the heavens above the Moon, or below it (that is, in the elementary or sublunary region), or sometimes in the aether, and at other times even in the sublunary region — for the verdicts of Kepler [Kepler] and Habrecht [Habrecht] extend to this last possibility as well.

CAP. III.

On the verdict to be passed on the foregoing opinions of the ancients and the moderns concerning comets, and what in them we judge should be rejected and what retained.

To pass judgment on the foregoing opinions of the ancients and the more recent writers concerning those new phenomena — and above all the comets — two instruments, as it were [Greek: κριτήρια (kritēria, "criteria" — means of judgment)], serve us in a certain way. The first consists in the prior observations made of such phenomena — not, however, with any instruments whatever, but with suitable ones, and acquired no less prudently with due regard to the celestial revolution. From these the astronomer's judgment may be made firm concerning the distance of such phenomena from the face of the Earth, likewise their magnitude, and finally a certain natural advance and duration — also from the appearance of the comet's tail, which perpetually points toward the part opposite the Sun.

The second is joined with a still greater skill in reasoning. It comes into play when, namely, [we inquire] about the matter from which those new phenomena chiefly arise; and likewise about the inner form, which is indeed luminous — so that you would call it nothing but an offspring of light — yet in the end you anxiously ask whether that light was implanted and, as it were, glowing like coal (as fire is in charcoal), or whether instead it arrived from elsewhere, as the light of the Sun reaches the Moon to illuminate it.

As to the first criterion, which is formed from infallible observations: assuredly all phenomena of this kind can and ought to be judged to be celestial. Hence, by the industry of the one man Tycho [Tycho Brahe] alone, in his books the *Progymnasmata* [Tycho Brahe, *Progymnasmata*], the whole Peripatetic doctrine of the sublunary generation of comets readily collapses and is shaken from its foundation — together with the opinions of all the rest, however many there are, that are akin to the Aristotelian view in any respect, namely with regard to a sublunary location. For no one has ever yet come forward who could, from his own observations of equal certainty with the Tychonic ones (or with the precise observations of others, or with those we ourselves have since made), prove that there had been such great parallaxes in either the new stars or the comets that these anywhere exceeded the lunar parallaxes.

For my part, against the unwelcome argument from parallaxes I shall not touch upon the trifles of Keckermann [Keckermann], nor those of Julius Scaliger [Julius

Caesar Scaliger]; and least of all of them shall I here refute, at greater length than was once done, the obstinacy of the Scot John Craig [John Craig], with his tangled excuses — or rather rejections — of parallaxes in defense of Aristotle. For both sublunary meteors, and the Moon itself, and everything that is observed on high (things to which the diameter of the Earth's globe bears a sufficiently perceptible proportion), present themselves and are found subject to the laws of duly observed parallaxes and to the order of their analogy. I had left examples of this matter throughout our *Astronomy*, especially in Book 2 of the *Sphere*, chapters 10 and 11 (*Astronomia Danica*, *Sphaera* II.10–11).

It therefore seems to me that John Kepler [Kepler], in his cometary hypothesis, did not sufficiently attend to this: that by the same [hypothesis] he grants that comets sometimes become sublunary — indeed, that they strike against the terrestrial globe itself — once prior observations have placed their descent among sublunary things. [Margin: Prog. Tych. p. 652.] [recte: the marginal reference "Tych. p. 652" appears in the right column opposite this passage] But he is left destitute of such prior observations.

Therefore (as we said) we hold in truth that all those new phenomena are celestial — namely those which, like the fixed stars, do not move from their place and yet endure for many months and whole years; or those which take on the nature of comets, and continue their own motion for a proper duration beneath the greatest circle of the heavens, and then so precisely deflect their tails toward the parts opposite the Sun in every revolution. For from such signs too the analogy of these phenomena with celestial things is excellently established — as will later be more fully set forth in the examples, together with those matters that bear on the explication and the certainty of their parallaxes.

Now, once it has been obtained from the doctrine of parallax — and likewise from the comet's steady advance beneath the greatest circle, and finally from the extension of its tail toward the parts opposite the Sun — that phenomena of this new kind possess locations and motions within the system of the heavens itself: the consequence is that we must either, with the Pythagoreans, declare them to be perpetual stars (which nevertheless appear only at certain times), or else acknowledge that their generation is new, arising, namely, from matter and form. But from where these principles of generation are to be taken, let us inquire.

First, however, we shall reply to the Pythagoreans, to whom others in our own age — and chiefly the man of Nola [Giordano Bruno]...

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[Giordano] Bruno the Italian, give their support. But against these men we deny, with Seneca [Seneca] (*Natural Questions* VII), that new stars, or even comets, belong to the number of perpetually existing things — for the reason that (for the most part) they display their greatest increase and bulk at the beginning of their appearance. This could not happen if they migrated step by step from the inmost recess of the heavens toward the Earth, until, on account of the evident magnitude by which nearer bodies are increased with respect to the angle of vision, they came into our sight.

Nor does this happen on account of a fixed disposition relative to the Sun (as the man of Nola [Giordano Bruno] held) — namely, that stars of this kind, otherwise dark in themselves, should receive that illumination from the Sun and thus become visible to us. For a disposition of this kind relative to the Sun is by no means uniform in them, as it is in the Moon. I add that the duration of their period could by no means be sustained in this way — both because of the Sun's motion in the meanwhile, and because of the comets' own motion — since the disparity of such motions would dissolve it the more quickly.

And what, I ask, is the geometrical reason why a comet — now risen high in the north, now in the south, now before the Sun, now after it — should form such an angle of appearance with the Sun and with the eyes of those beholding it, an angle that falls under no fixed ratio or measure? To say nothing of this: that dark stars of this kind (from which they suppose new stars and comets to be made), while they wander among the other luminous bodies of the heavens, should hide these no less than the Moon sometimes hides the Sun — which, however, has not yet been noticed by astronomers. Let the Pythagorean opinion therefore be gone, and moreover the opinion of all those who would have it either that comets are perpetual stars, or that they come to be from the mutual illumination of the fixed stars or planets — as Anaxagoras [Anaxagoras] held. For this mutual illumination would likewise occur in a disordered way through the motions of the planets that produce it, or would be dissolved prematurely and too soon.

[Margin: Prog. Tych. p. [?].] [recte: marginal reference reads "gym." / "Tych. p." — abbreviation of *Progymnasmata*; page number illegible]

It remains, therefore, that we believe those new phenomena to be generated anew. Therefore [they arise] from some matter and form, the principles that generation presupposes. But concerning the matter, first: shall we hold, with Epigenes [Epigenes] and Rothmann [Rothmann], that it is terrestrial — matter

which, checked by no obstacle or barrier, climbs the heavens and there, gradually collected, compressed into a globe, and finally illuminated by the Sun's rays, is seen? Or rather that comets are [Greek: κατακαύματα (katakaumata, "burnt things" — incinerations)], that is, scorplings, leaping out from the Sun, or rather from its surrounding matter [materia periftatica], as Willebrord Snel [Snellius] holds? Or that they are made from the other planets, especially Mars and Mercury, as others — and chiefly Ptolemy [Ptolemy] — judged? Yet under this condition: that those planets have the character not so much of matter as of an efficient cause, since that matter was, with Aristotle, believed by these men to be terrestrial.

Against these opinions our common line of reasoning and discussion is as follows. Since it was quite credible that there are certain surrounding atmospheres [periftafes] about the heavenly bodies, which there serve the office of the air diffused about our Earth — as in the Moon, the Sun, Jupiter, and Saturn we have lately learned from closer observations through Galileo's [Galileo Galilei] optical instrument [dioptricum instrumentum] — nevertheless I see that these are so appropriated and bound to the very bodies and globes that they surround that they in no way pass over to others, or mingle themselves into the otherwise vast and empty heaven. For whatever communication there has been among the celestial bodies amongst themselves and with the Earth, this must be referred to the light of the stars alone and to its expanded vehicle — lest, namely, those world-bodies perish, with their congenital [elements] thus shut out by these and heterogeneous ones admitted in their place, a thing which is utterly abhorrent to the nature of homogeneous things. (This will be understood more clearly in the second part of the *Astronomia* of the *Prognosmata*, since it is more fully expounded there.)

Finally, if anything further is to be answered to Kepler [Kepler], it must certainly be said that even the rectilinear trajectories of comets of this kind, such as he sets up, manifestly contradict the Copernican hypothesis (*hypothefi Copernicea*) — to which, moreover, Kepler is uniquely devoted. [Margin: *De Revol.* I.1] [recte: marginal reference reads "Lib. 1. cap. 1. Revol." — Copernicus, *De Revolutionibus* I.1] For since Copernicus [Copernicus] acknowledges that circular motion alone is natural, whence then is that straight motion introduced by Kepler, which is attributed to comets? Certainly not from nature. Therefore, unless Kepler shall have proven that comets are endowed with sense and recognize the end of such a course of their own, he will scarcely find anything else that would render that course so regular. (Not to mention the planets, which the comet's form and apparent motion imitate in a certain way, [and which] are in truth moved by a

circular motion. But one who has attributed knowledge to the planets perhaps does not deny it to comets either.)

However that may be, on this matter we are astonished that, while a comet is believed to be merely natural, and while it is not yet established that some spirit presides over its governance, it should at once be reckoned similar to an animal — especially since it does not last long, but quickly either vanishes in the heaven itself, or, striking the Earth (on the Keplerian hypothesis), is dissipated and disappears. When someone wishes rightly to weigh these arguments, taken together with the foregoing one, he will say that Kepler excuses the generation of comets and the motion of their trajectories more ingeniously — indeed, more laboriously — than truly.

And these were the chief authors, distributed by us as if into classes, around whose opinions concerning the origin of the new phenomena this discussion of ours was to be extended. For the rest, whom Plutarch [Plutarch] reviews, do not deserve refutation. But among these, in the case of certain ones — and of those others, likewise of Cardano [Cardano] and our own Tycho [Tycho Brahe] — we gather that the matter of the new phenomena is certainly celestial, but...

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from what part of the heaven it comes, since they have pronounced little or nothing definite. Therefore in the following chapters, so far as God shall have deigned to grant to my talent, I shall set forth my verdict concerning this cause, as well as the rest — so that what I judge the new stars to be, and what the comets, may afterward be established by the arguments to be deduced here and there, and may be held ready at hand; yet in such a way that room is always left for one who brings forward better views.

CAP. IV.

On the efficient cause of the origin and generation of the new celestial phenomena.

Since the judgments of most learned men (even besides those whom we have reviewed) concerning the origin and generation of the new phenomena in the heaven are various — for although nearly all the more recent and intelligent astronomers today, after Tycho [Tycho Brahe], agree that they come to be within the enclosures of the heavens, yet most think this generation, new and sudden, of celestial things to be so prodigious, and so anomalous compared with the generation of the rest of the elemental things, that their causes — even if contained merely within nature — can scarcely, or scarcely even, be searched out

by any mortal: therefore I see that I have need of an excuse here before the candid reader, lest he suppose that I wish, with presumptuous and rash judgment, to discuss or even assert fully and indubitably about the causes of these things something that surpasses the candid confession of such great men — and above all of our own Tycho [Tycho Brahe]. There he subscribes to the doubt of Thaddaeus Hagecius [Tadeáš Hájek] when he explains him, in these words: [Margin: T. B. Lib. Progym. p. 513.] [recte: Tycho Brahe, *Progymnasmata*, p. 513]

"At the end of the same chapter (the 4th of his *Dialexis*) [Hagecius] touches on some matters concerning the celestial generation of comets; and although he frankly confesses that he is ignorant of it, nevertheless he does not deny that it can come to be on this account; but he piously ascribes to the admirable works of God *how* it happens, and asserts that it is far from our complete cognition — which I too do not deny. Whatever, however, can be probably estimated concerning the procreation of these things, beyond the common opinion of the philosophizers, I shall (the celestial Author favoring me) signify my view of around the end of the whole work, namely at the close of the third volume; in the meantime I am not unwillingly of the same mind with those who place comets among natures whose secrets have not yet been explained, and who assert that they arise from hidden causes (as Regiomontanus [Regiomontanus] proves from Messala [Māshā'allāh])." Thus Tycho.

Hence too whoever will recognize that we at least pursue the most probable views concerning the generation of these things — especially since astronomers are granted to ascend no higher into this cognition than that, first, by the smallness of their parallaxes, celestial things are certainly apprehended; and next, that here they either have a fixed seat (inasmuch as the new stars appear immobile among the fixed stars), or else that those which have taken on the form of comets are carried by a motion below the fixed stars, namely in the planetary heaven, beneath the greatest circle of the sphere — [things] which they also call a tail or hair, [and which] exhibit a constant extension away from the Sun in every revolution. From these as it were more manifest signs we conjecture that there is something natural in them. Yet since certain matters — namely those concerning their first appearance, when these are usually for the most part greatest and most powerful, both as to magnitude and as to motion — can scarcely be accommodated to the generation of the lower, elemental [things]. For we observe that sublunary things gradually grow and strive toward their [Greek: ἀκμήν (akmēn, "peak" — their prime or culmination)] before their dissolution, and not,

as the new celestial phenomena do, gradually diminish — but rather for the most part endure in the same magnitude, until the predestined time, and the age of each individual, so contrives their dissolution and dissolution that at last they are subjected to change and take on a new form.

To the continuation of this generation and its alternation [there pertain], in the way appropriate, the seeds of the species reserved in nature — as one may notice in plants, in their fruits, likewise in animals, and in all things growing beneath the Moon (except the meteors, which, because their generation, or mixture, is imperfect, and their duration not so fixed to a definite time, are deservedly excluded also from that generation of celestial things). Therefore there is one power in these sublunary and elemental things, and another in the higher or celestial ones — a power which, in begetting them, suddenly expands them into the greatest mass and then little by little consumes them: not otherwise than as we observe a bubble suddenly formed by the wind upon water — encrusted, that is, or expanded into the thinnest little skins; and then (if it is not at once destroyed) sometimes gradually subsiding into itself, since the water beneath draws the matter back to itself. Or this generation of the higher things might also in some way be likened to the sudden springing-up of terrestrial mushrooms [fungi], which especially within the space of 7 days arrive at their perfection. Yet these similitudes are so placed here that they contain in themselves much dissimilarity from the generation and condition of a new star or comet — so that I report here nothing of Pliny's [Pliny] ephemeral little birds [aviculae ephemerae], or even of the little men [homunculi] of the same designation, on account of some dissimilarity.

In this doubtful adjudication and verdict, then, while we are engaged concerning the generation of the new celestial phenomena — and yet confess that nothing comes to be without causes — therefore, when we now discuss these matters in order, we shall imitate the counsel and consent of Aristotle [Aristotle] (*Metaphysics* I.7) [recte: marginal-cited "lib. 1 Met. c. 7"], where he says: "*Those things which are hidden from sense we say have been sufficiently demonstrated by reason, if we shall have recalled that which can be done.*"

Now, at the outset, in [treating of] the efficient cause of the new celestial phenomena — which comes first in the order of causes — we shall certainly not go astray when we devolve it primarily upon God himself, the Creator and Lord of all things:

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[upon God we have devolved it]; especially since this new and sudden generation of celestial things seems to exceed the laws of nature, inasmuch as it observes and measures no fixed course of nature, nor any period and restitution of time in it — so much so that here and everywhere there holds good that saying of Psalm 115, verse 3: *"Our God is in the heavens; whatever he willed, he did."* Likewise Psalm 136, verse 6: *"Whatever pleases Jehovah, he does, in the heavens and on the earth, throughout the seas and all the abysses."*

But because God, on account of his most wise ordering of nature from the beginning, acts thereafter for the most part through secondary causes, it seems to me that, for the generation of the new phenomena, whenever they occur, there may be assumed as efficient — together with the divine will itself — the mutual aspect of certain stars among themselves and with the Sun; but for a comet in particular (as Ptolemy [Ptolemy] held), the conjunction, opposition, and quadrature of Mars and Mercury, or some other unfavorable regard of theirs — chiefly on account of those effects which are wont to follow upon their unlucky disposition toward one another and toward the others; likewise the powerful irradiations of the other planets too with the Sun, as will be said later.

But here certain questions or doubts arise, which are wont to leave goads in the minds of certain men, and which are therefore to be explained beforehand, as best our wit allows, before we hasten on to other matters. First, then, it can be asked: why, since God, the Best and Greatest, has set such new celestial portents and signs before the eyes of mortals — doubtless for this very purpose, that they may repent of their crimes and, at the opportune time, return to a saving penitence before they are dragged off to the just penalties which celestial new phenomena of this kind seem to announce — and yet these same portents are nonetheless affirmed to come to be from secondary causes: what impediment, then, could there be to prevent God, according to the Stoics, from being bound to secondary causes?

Likewise, if the new stars and comets, which among other signs are presented to the eyes of mortals concerning repentance from sins, will at the same time draw any productive causes from the stars. Or, to extend this question more generally: if from the stars coeval with the world things to come are affirmed to be foretold — whether those be general, or special, such as the astrologers prophesy from genethliacal [genethliacis] [recte: nativity] configurations, which likewise also come about from the divine ordination, and these of such a kind that they at least incline toward evils, if they do not (as others would have it) operate them

effectively: it can certainly be asked, in what way crimes and sins can be excluded from that same divine ordination, since these seem to be comprehended, as if effected or signified, in those certain causes or signs?

And here I think the most correct answer is this: that God by no means ordained the sins, which assuredly come to pass from the will of the devil and of men, apart from and beyond all divine ordination; and also that these signs in some way emanate from secondary causes — that is, from the celestial stars, which God created from the beginning together with the heaven; a thing which, according to Scripture — indeed, according to the philosophers themselves, and above all Plato [Plato] — is utterly repugnant to the divine nature, which is wholly good. Rather, here it must be laid down that, since God is without time and place — and indeed in such a way that he governs the things which come to pass in time and place with a most exquisite harmony — so much so that in that divine economy all things are perpetually placed in the sight of the Creator, from the first time of the world's beginning to its end (that is, the consummation of the whole globe): namely both the crimes and sins of men, and all other things which will come to pass at their own fixed times. It was therefore not impossible for God, in the primeval creation, so to order the stars that ever afterward, among their other ministries, they should either themselves be destined to be signs, or should give forth signs of themselves — out of his benevolence toward men, of which (as also of his justice) no small part is the vengeance of crimes; which celestial portents of this kind either follow, or are conjoined with their appearances: such as war, plague, famine, the deaths of great men in the commonwealth, the transmutations of kingdoms, and other such means — which often correct the wicked and draw them back to penitence, but in no way harm the good; since for these, who love God, all things work together unto good. [Margin: Romans 8, v. 28] [recte: Rom. 8:28]

Secondly: since it was affirmed a little before that the aspects of the stars — chiefly of Mars and Mercury — contribute to the generation of comets (although certainly the others, of the other planets, are not excluded either, as will appear below), it can be asked: why, then, when aspects of this kind occur every year, are comets not also produced from them, so that the time of their origin could be foretold with certainty? Yet this has scarcely been done by anyone up to now — even though there are certain ones who wish to be heard as astrologers and who boast of it from an inept presumption.

But here too it will be answered not inconveniently, by that which I admonished at the beginning: namely, that the action of God is utterly free; and it must be

added besides that by what hidden part of nature the Creator uses [the stars] for this, as a secondary cause, is wholly unknown to us. Whence Hippocrates [Hippocrates] wisely [says], concerning those things which come to pass without our knowing: [Greek: αἱ φύσεις αὐτόματα, τῷ δὲ Θεῷ μηδὲν αὐτόματον (hai physies automatai, tō de Theō mēden automaton, "natures [act] of their own accord, but to God nothing is spontaneous")]. Meanwhile it must also be said that the course of nature is not on that account wholly impeded in the rest, because there are various delays, arising from more powerful unknown causes — likewise [delays] supervening from a defect of matter, which either defer or vary the generation of the new phenomena, and render the predictions drawn from these (as from the rest of the stars) uncertain...

and often render the predictions void as well — quite apart from the practitioners' own incompetence and laziness. All these matters Ptolemy sets out fully and at length at the beginning of his *Quadripartite* work [*Tetrabiblos*], on the effects of the stars.

So much, then, for the efficient cause of new phenomena. As for Paracelsus [Paracelsus], who says somewhere that the celestial bodies — chiefly Venus — conceive comets out of the perverse imaginings of men and, as it were, out of their premeditations to commit evil (owing to their fellow-feeling [συμπάθειας, Greek: sympatheia, "fellow-feeling" — sympathy with things below] with things below), and bring them to birth even against their will, even though by nature the heavenly bodies are otherwise pure and unstained; and that afterwards these new offspring, or comets, are resolved into effects that report and bring forth such conceptions all the more through wars, thefts, robberies, adulteries, and the like — this opinion of his surely seems to rest on the account of some fictitious cause rather than to be grounded either in nature or in Scripture. For it is agreed that the higher acts upon the lower, and not the reverse. Thus the stars have been ordained by God so that they can serve at once as forewarnings or as intermediary signs of those evils which spring from free — or rather servile, that is, devil-enslaved — human action; just as was set out above, and will be explained more fully below by means of a similar cause.

CAP. V.

On the material cause of new celestial phenomena.

We now pass from the efficient cause to the matter of new phenomena — but a passage of a kind that nature scarcely acknowledges in its other generations. For once the generation of these phenomena has been rightly denied on Aristotelian

principles and hypotheses, and their matter has therefore been sought in the heavens, this question has bred various opinions among the more recent astronomers. Nearly all of these have been so thoroughly examined in the preceding chapters that there is no need to put them back on the anvil again, or to resist them with more arguments than were used there. Now although we have sufficiently proved from what precedes that this matter of new phenomena comes from nowhere but the heavens, nonetheless — so as not to conclude that its concurrence in generating those new phenomena is merely a matter of chance — we shall here examine the matter a little more deeply. First, so far as we are able, we shall consider whether the heavens are so disposed, or (as the phrase goes) so qualified, as to produce any such new things of themselves; for the ancient philosophers, and especially the Peripatetics, flatly deny this. Once that point has been established, we shall inquire further into the discovery of such matter: namely, from which part of the heavens it is drawn — whether from the stars themselves, or from the region we observe to be empty of stars and properly call "the heaven," or finally from both.

For although we here acknowledge a matter of the kind that serves as a receptacle for a form, or for a kind of seed operating in the manner of a form, we do not mean that notorious *prime matter* which, almost according to the Platonists, is nowhere and is nothing; rather, a matter truly existing in the heavens. For we hold, with some authors, that the original prime matter — which others call chaos, and which Scripture calls the unformed mass — was absorbed into the primeval creation of things. For this reason, even though the lower realms do not differ in their whole genus from the higher, there is nonetheless a great variety of matter in heaven and on earth, from which diverse species are produced in the world. (See Antonio Flaminio [Antonio Flaminio] on Book II [recte: the Latin reads "11," i.e. Book XI] of Aristotle's *Physics*.) But let us return to the first question.

The contraries which Aristotle requires for generation are chiefly of two kinds, and bear upon matter. Either they consist in contrary qualities, which are perceived to inhere in the elements as they are transmuted into one another; or they consist in the destruction, or privation, of a form in matter, when a like or another form is introduced into it. As for the third kind of contrariety — that which arises from upward and downward motion, in keeping with the nature of the elements, namely lightness and heaviness — it contributes little or nothing to generation; and I have shown, in the prognostic section of the *Astronomy* concerning the matter of the heavens, that this belongs not so much to the sublunary elements as to the individual stars, regarded as great cosmic bodies.

Those contraries, I say, which Aristotle excludes from the heavens, we assert to be present in them. For as regards the primary qualities — chiefly the active ones, namely cold and heat — we earlier proved by no doubtful arguments, in the prognostic section of the *Astronomy*, that these too are included in the heavens: namely that this celestial expanse [expansum] is exceedingly cold, while the stars, and chiefly the Sun, excel in heat. Therefore this first principle, drawn from contrariety, stands ready to support us for the new generation of celestial things.

Moreover, in establishing the second point we shall not labor much, since when we set our eyes upon things — that is, when we see them — we may safely rest our confidence in them. Such are those phenomena which arise newly in the heavens, namely new stars and comets. That these are truly generated in the heavens I judge it no more necessary to prove against the Peripatetics with many arguments than were once produced against them by our most noble Tycho [Tycho Brahe] in his *Astronomical Exercises* [*Progymnasmata*]; so that now once again — what we have stressed above and shall display below in examples as well — we may touch upon it more briefly and, as it were, in a single word. And although neither new stars nor comets

are perpetuated in the heavens as distinct individuals, as happens with us in the succession of plants or animals by generation — so that on this ground too the heavenly things would differ from the generation of lower things — nonetheless it must be that some matter has put off, or in some way changed, the form from which a new star or comet is displayed. This we shall explain more conveniently below by an example, since matter is nowhere found altogether bare of any form. It remains now to explain from which part of the heavens we judge the matter of new phenomena to come.

Is it, then, from the stars themselves? For these new phenomena resemble the stars — chiefly the new stars — as though they were their offspring, or as though begotten by the stars themselves. For thus the new star of the year 1572 in Cassiopeia, and likewise the other of the year 1604 in Sagittarius [recte: Ophiuchus — the 1604 nova was in the foot of Ophiuchus, "Serpentarius," as the author himself states on p. 10], matched Venus in magnitude, and matched even the other fixed stars in visible brightness; and this in the very same shape as the perpetual stars, namely round — which is also the shape of comets, apart from the traces of their tails or hair; and comets too differ little in brightness from the other stars. But since this brightness seems to be either a light implanted in them as a form, or an adventitious light reflected from the Sun and the stars, we therefore assign it rather to the form, which we treat in the next chapter; yet in

such a way that we leave the matter here as suited to receive such a form. Is such matter, then, found in the rest of the heaven, which Scripture names the expanse, from which new stars — and chiefly comets, like fishes in the sea, according to the opinion put forward by Dr. Johannes Kepler [Johannes Kepler] — are generated? But this I can neither plainly affirm; for that "shore" [Rhachia] or expanse has been shown in the prognostics to be uniform [ὁμοιομερές, Greek: homoiomeres, "of like parts," homogeneous] in itself, and of an equable and most subtle rarity, exquisitely extended everywhere uniformly throughout the whole finite universe. But this expanse, since it is the vehicle or substratum of light — that, namely, through which the lights or rays of the stars unfold themselves more swiftly than the blink of an eye, even to the farthest bounds of the world (whence perhaps some hold the heaven to consist of a fluid, which however seems to be nothing else than the nature of the expanse, or "shore," that permeates all things) — this expanse, then, of itself scarcely supplies matter for new phenomena, except insofar as the matter has been interwoven with it, or has been so determined potentially that it ensnares and contains within itself that new luminous phenomenon as a form, or as a formed thing.

The matter of new celestial phenomena, therefore, must be, as it were, of a third nature, inasmuch as it results from the stars and the expanse together. And this on account of the qualities of these two — which we suggested above to be contraries — which we may approach by considering the primeval creation of the heaven and the stars. For since places and the things placed in them must agree in some part of their nature — just as the elements, namely earth, water, and air, agree with the things generated in each of them — accordingly the heavenly bodies, which beyond doubt were created and brought forth by the best and wisest Creator from the primeval light, are not plainly contrary in nature to that expanse in which they are read [in Scripture] to have been set and located by the mighty hand, or word, of Jehovah [Margin: Gen. 1, v. 14]: just as these matters are set out more clearly and at greater length in the later prognostic section of the *Astronomy*, and are there as it were displayed. For these things are not of contrary nature on account of the contrary qualities which we asserted to be in them; since every corruptible nature includes that contrariety of qualities.

Since, therefore, such matter reflects in some manner (as I just said) the nature both of the stars and of the expanse, it follows that, on account of some communication of the light of that primeval light which shines in the stars, it must be exposed to the eyes; and yet nothing is found in the heaven, the stars excepted, besides that belt or circle which the Greeks call γαλαξίαν [Greek: galaxian, "the Milky Way," the galaxy] and the Latins name the Milky Way [via

lactea]. On which account, among all opinions, those seem to me to reason most fittingly who judge that this circle — appearing in the heaven by night — supplies matter for new phenomena. This circle, in the manner of a great circle (though not perfectly so everywhere), seems to gird the whole starry heaven, lying equally distant from both poles, namely those of the ecliptic and of the equator, by about one sign. Its eastern edge today, crossing the ecliptic in two places, has crept a little beyond the beginning of Cancer [♋] — with unequal breadth, however: for near the north and the beginning of the constellation Cassiopeia, where we set its beginning in this description, it is thicker than in the places where it crosses the ecliptic and the equator. From there it spreads out most broadly near the constellation of the Swan, or Cygnus, where it also divides into two parts or roads: the one descends through the Bow of Sagittarius, the other extends precisely through the neck and beak of Cygnus, likewise grazing the right shoulder of Serpentarius [Ophiuchus], and there as it were gathers itself back into a fold near the equator; but the former persists in its track, nor does it come much nearer to the Antarctic pole before it rises again toward the north — namely through the ship Argo and the back of the Greater Dog [Canis Major], taking the road there, and grazing the feet of Gemini [♊], together with Eridanus [recte: the text reads "Eridhonium," apparently Erichthonius/Auriga] and Perseus, and so returns again into Cassiopeia, whence it had taken its beginning for us — just as one may recognize, in the heaven itself and in this its rival globe, namely the celestial, material globe.

From this rough description of the Galaxy, or Milky Way, and from a comparison with the ancients, we understand that its motion in the order of the signs [in consequentia] is the same as the apparent motion of the fixed stars. For even by Aristotle's designation,

(which is found in Book I of the *Meteorology*, chapter 8) it was, by about one sign, earlier in that age, and was bounded on either side and all around by the same stars as now: which, among other things, affords a great argument for establishing the immobility of the fixed stars.

After this rough description of the Milky Way, we must inquire further: first, whether it is located wholly in the heaven (contrary to Aristotle)? Then, once this has been established by sufficient argument, whether it is in the heaven a bare phantom, arising from the joint illumination of the many stars found within it — or rather something real and material, from which it can plausibly be held that new phenomena, and chiefly new stars, are generated.

As for the first point, concerning Aristotle's opinion in the passage cited just above, it readily falls — even by the testimony of the Philosopher himself, adduced in that same place, and weighed against parallaxic doctrine. For if, in the regions adjoining the Milky Way, the same fixed stars are seen at every revolution (as they certainly are seen), and never desert the western edge of that road, or circle, at their rising, nor its eastern edge at their setting, but are observed to keep exactly the same place in it — then it must surely be that whatever marks out the Galaxy described above stands either in the heaven, and indeed far above the Moon; since this too leaves parallaxes everywhere outside the zenith of a place, the greatest being for things nearest the horizon, as is plain in spherical doctrine, and still more evidently in the theory of solar eclipses.

Now, in order that what we have said — from the foregoing reasoning — about the Milky Way being placed in the heaven may be fitted to demonstration and set before the eyes, we shall proceed as follows.

Diagram. A geometric diagram demonstrating that the Milky Way lies in the heaven (above the Moon). From a center, a quarter-circle (quadrant) is drawn representing the semicircle of the sphere of the fixed stars, with a vertical radius; a smaller arc nearer the center represents the surface of the Earth. Two pairs of arcs between the Earth and the sphere of the fixed stars mark the circular boundaries of the Milky Way as if below the Moon. The construction shows that if the Milky Way were sublunary, a fixed star and a Milky-Way point that coincide in observation (B and C) would, when traced from the Earth's surface, fall at different points (N versus E), thereby exposing a parallaxic difference (arc L M or N E) which observation does not in fact show — proving the Milky Way lies far above the Moon.

- A (on the horizontal baseline, near center): junction point on the world-axis
- C (top, on the vertical radius): a fixed star, on the vertical, exactly determined
- B (upper interior arc): the vertical point on the outer edge of the galaxy, coinciding in observation with star C
- Q (upper left): point on the sphere of the fixed stars
- M (interior, near center-right): the point through which a ray from F passes when aimed at E
- N (just below M): the point where a ray from the Earth's surface F actually falls on the sphere of fixed stars
- E (right side, on the sphere): the position to which the fixed star moves from C by an equal revolution

- L (implied, on the galaxy's track toward setting): the point to which B revolves toward the west, above the world-axis AQ
- R (lower left, baseline end): terminus of the baseline
- K, G, O, P, H, I, D (along the lower baseline, left to right): graduated points along the horizontal; I B marks the galaxy terminus toward setting, B K toward rising
- F (on the Earth's surface arc): the point on the Earth's surface from which the rays are emitted

[The galaxy's boundary arcs are labeled I B K and H G; the terminus I B lies toward the setting (west), and its contrary B K toward the rising (east).]

Let a semicircle of the sphere of the fixed stars be described from the center, and at the same time the vertical D C R; and the semicircle of the Earth P F O; finally, from the same center, the circular boundaries of the Milky Way, as if below the Moon, within the semicircles I B K and H G. Let its terminus be I B, toward the setting, to which the contrary is B K toward the rising. On that same outer edge of the galaxy there is given a vertical point B, which a fixed star — likewise on the vertical, at C — exactly determines. Then, with that same point B revolved to L toward the setting, above the world-axis A Q, the fixed star likewise moves from C to E by an equal motion of revolution as B [moves] to L — which the line A E, passing from the center of the Earth through L, demonstrates. Therefore, from F, the surface of the Earth, when the line is drawn and likewise extended into the sphere of the fixed stars, it certainly falls there not at E but at N.

But if you emit another line from F to E, this passes not at all through the point L (where, however, experience proves it would fall, since one and the same point in B and in L would be occupied by the same star in C and E with the observations preserved), but elsewhere, through M; whence by the arc L M, or N E, the difference is shown between the point which the star would occupy at L on the Milky Way, if we were to stand by the observations, and the other point at M, into which, however, the star never passes, nor (as was said above) changes its place. Had Aristotle therefore considered this, he would never have pronounced the Galaxy to be a collected vapor, or its sublunary condensation — that vapor, namely, which he supposed gathered the stars in the firmament (and chiefly those tiny ones which are seen in the Milky Way, and which are called by him σποράδες [Greek: sporades, "scattered ones" — the scattered stars]) and held them suspended beneath itself at every revolution. Let Aristotle be read, Book I of the *Meteorology*, chapter 8, where at the same time the opinions of others

concerning the Milky Way are reviewed and, in his own fashion, refuted by Aristotle.

The second opinion, among the ancients, has Democritus [Democritus] as its author. For he, as Plutarch [Plutarch] reports, held the Milky circle to be nothing else than the shining-together of many tiny contiguous stars (he means, no doubt, those which appear in it), which shine in common on account of their close packing toward one another. [Margin: Let Plutarch be read, *On comets* (?)]

This opinion of Democritus, in our time, Galileo de' Galilei [Galileo Galilei], the Paduan mathematician, seems strongly to confirm: with the telescope [perspicillum] lately invented by him and applied [to the sky], he detected, in place of the milky traces, nothing but a heap of, as it were, infinite stars set close beside one another. This our most distinguished Dr. Caspar Bartholin [Caspar Bartholin] also affirms. For when, some years ago, he was present and on familiar terms with the aforesaid Galileo at Padua, he was advised by the same spectacle to change his earlier opinion, set down in his treatise on the stars, where he had previously argued the Milky Way to be a celestial substance and, as it were, the remnants of the primordial light, following the view of Tycho Brahe. Indeed, for the greater credibility of this matter, he takes a further argument from other similar little patches of a whitish color, like the whiteness [Margin: Let Dr. Caspar [Bartholin] be read, *De caelo*, chapter 3]

of a cloud, which shine out elsewhere through the aether; where, by the help of the said little glass, a heap of densely-packed stars is seen — namely where those stars called nebulous are set close together.

But certainly — although the conclusions which proceed from ocular demonstration seem on that account to deserve credence in physics — the reasons why I cannot fully acquiesce, and firmly believe the Milky Way to be merely the shining-together of the tiny stars seen within it, without any other underlying matter, are as follows:

1. The nature of the celestial expanse (which expanse we have shown, in the prognostics of the *Astronomy*, to be the vehicle of the light of the stars) is such that this same light passes and is transmitted freely through the expanse, and is nowhere bounded except in the presence of material objects, such as the stars and the Earth are. But that light — or [light] of the tiny stars which are seen in the Milky Way — is bounded by that same [Milky Way]: therefore the Milky Way is not plainly and [κατ' ὅλου, Greek:

kath' holou, "as a whole," wholly] of the nature of the rest of the expanse, but contains besides some matter in itself.

2. A reason is given at once from ocular sense and from the authority of the most excellent astronomer of all ages, our Dr. Tycho Brahe — who noted a gap in the Milky Way and the constellation of Cassiopeia, in that same celestial place where the new star had lodged in the year 1572, as he expressly testified in the conclusion of the first volume of the *Progymnasmata*, and as our own ocular experience once freely confirmed thereafter. Whether this gap is found today I cannot, for my part, discern by the eyes quite so exactly. From this one may infer as follows.

If the Milky Way were only the light of the stars set within it, then certainly nothing could have been removed or perished from it, so long as those stars remain in their several places — which they do. For while the causes endure in nature, the effects endure: that is, while those stars remain, the light remains. But that great and most brilliant new star of the year 1572, after its disappearance, left in the following year a dark gap — of a color not unlike that of the rest of the heaven outside the Milky Way — in that place where it had been found to shine at the edge of the Galaxy; inasmuch as it was not at once filled or overlaid by the flowing of the confused light (as is otherwise the nature of light). Therefore it is reasonable that there underlay it, besides that light of the stars, a certain matter, of which that new star consisted; and that this star, once dissolved and dissipated, was together absorbed with its matter and transported elsewhere, from the place where it had existed.

3. A reason is taken by us from the customary location of the generation of new phenomena, and chiefly of new stars — which location is, for the most part, in the Milky Way; since both the said star of the year 1572 in Cassiopeia, and that of the year 1600 in Cygnus (which is still seen there even now), [arose in it]. Finally, the new star of the year 1604 took its origin in the knee of Serpentarius [Ophiuchus], and so forth; not to mention the comets, which from time to time also referred their beginnings to the Milky Way (since they had crossed through it), and shone with a more brilliant color. Whence it is very plausible that the matter of the Milky Way is akin to these as well — as it seemed to Tycho in the conclusion of the *Progymnasmata* of the *Astronomy*. From the reason adduced, therefore, we conclude that, just as herbs, or other things growing from the earth, acquire their matter not from far off but in the places where they arise, so new stars draw their chief matter from the Milky Way, in which they are

for the most part seen to arise; and this matter, as I said, is in some way different from that which otherwise belongs to the rest of the heaven — the expanse, or "shore" [Rhachia], as we have called it.

With these reasons laid down, we must now see by what means the observations of Galileo may be saved when reduced to them. Among these, the first and chief we seek in the wondrous power of Galileo's telescope to enlarge and dilate the objects which meet the sight in marvelous ways. This power on that account seems to me to be so great that, while the observer scrutinizes the stars set in the Milky Way — even the very tiniest — and apprehends them and presents them to the sight through that tube, he places those same stars, stripped and laid bare of that rare underlying matter, in view.

This conjecture of mine proceeds from the fact that Galileo had displayed to us, through that same little glass of his, in his *Starry Messenger* [*Sidereus Nuncius*], the spots on the Moon as interspersed and not appearing quite in the manner they otherwise do: where, however, he attained even the least particle of the Moon illuminated, in its thirsting and dark part. The other [point] is that, although that milky color appears nowhere else than in a manifold heap of stars, here and there throughout the Milky zone, by the benefit of Galileo's optical [glass], yet I do not thereupon conclude that a color of this kind is nothing else than the shining-together of those stars among themselves, without any other underlying matter into which that confused light of the stars is bounded; for this [underlying matter] is necessarily required by the opticians, as Johannes Kepler demonstrates — on which means, too, our first reason rests; and this is also the common opinion of Dr. Isaac Habrecht [Isaac Habrecht], in his description of the last comet (namely the one which blazed in the year 1618...

blazed in 16(18) — page 14, rendered into Latin in roughly these words:

"Although through the telescope the Milky Way is perceived to be nothing other than a mere crowding-together, or heap, of tiny stars packed side by side — as Albertus Magnus, Scaliger, Cardanus [Cardano], Goclenius [Goclenius], and others supposed — I nevertheless judge that this track may be in places filled out by smaller and larger stars set close together, even ones that otherwise escape our ordinary sight, and that none the less the residue that remains is matter of the very kind that Brahe [Tycho Brahe] held it to be."

These arguments vindicate for the Milky Way a certain matter of its own, over and above that pooled starlight within it. For as to the small patches — the

nebulous whorls — visible here and there through the aether: by their dimness these introduce a certain unlikeness to the milky brightness, as in the case of the Manger [praesepe] in Cancer, the Hair of Berenice [coma Berenices], the foreleg-thigh of the Great Bear, and so on. Nor, in other places where stars stand closely packed side by side, are they all seen to be patches of that kind. Examples in Taurus are the Pleiades, and elsewhere, scattered through other constellations, other small stars more tightly crammed and clustered.

Moreover, if in wintertime, when the Moon is hidden, one carefully studies a perfectly clear sky, one will see a numerous and as it were crowded choir and host of stars scattered everywhere through the aether, even outside the Milky Way. From this it would be a wonder if the whole aether did not blaze with brilliance, like the rest — were it not that, by God's foresight, it has been made ready in this peculiar matter. That matter was either left over from the primordial light, as Tycho held, and set apart both for determining the light of the stars in the Milky Way and at the same time for the successive generation, in due time and as God pleases, of new stars and comets, for which it is suited.

This same milky matter is not to be reckoned of slight thickness, or like a coat of whitewash spread over a wall, as Dr. Habrecht [Isaac Habrecht] rightly says; rather, it is endowed with depth enough — though not so great that it reaches down to the borders of the Moon, much less, as Aristotle supposed, lies below it. For the vanishing of parallax, which I demonstrated above, proves this.

We now gather, from the foregoing arguments, that this is above all the matter of the new celestial phenomena. For this view we have Tycho's anticipating us in several places throughout his *Progymnasmata* — so much so that, had that man (in the judgment of all the greatest of astronomers) survived us longer, and had he, as he had intended, handed over to me the field of the new phenomena to administer thereafter according to my talent, I do not doubt that he would have delivered into my hands at the same time this very matter of theirs. He hinted at it, none too obscurely, both elsewhere and in the epilogue of the *Progymnasmata*, where he suggested that he would conclude his third volume on the new phenomena — which, however, did not appear after his death — directly in favor of this opinion of his, intimated here and there.

What, then, that great man among astronomers once judged concerning the matter of the new phenomena, the very same I now judge, with the arguments adduced: namely, that it is celestial, and chiefly prepared in the Milky Way. And why should I not judge so? For if we lay it down — following the teachings of Pythagoras and Plato — that this world is conformed to the condition of a living

being, then nowhere in that vast body shall we find a more fitting and suitable matter than that of the Milky Way, inasmuch as it would correspond to the spinal marrow running down the back of a man or of any other animal. For just as the marrow of the brain and spine supplies matter to the nerves spread throughout the body of an animal — where those luminous spirits called *animal spirits*, refined out of the *vital spirits* (which are first generated from the blood in the heart), find either their dwellings or their nourishment, and contribute not least to generation — so perhaps it will not be absurd to lay it down that this milky matter has been prepared in the heavens for this purpose: that through its channels the streamings of rays, or rather the invisible spirits of the fixed stars, may first be made ready before they are communicated to the Sun, the planets, and even to the Earth itself. And from this elaboration, whenever the hidden seeds are at hand and the divine will (without which nothing comes to be) is likewise present, either new stars are generated — and these do not plunge deeper in — or comets, whose matter, torn away from the Milky Way by the attraction and force of the planets, is on that account rarer and perpetually subject to the motions that prevail throughout the regions of the planets. But enough of the matter of the new phenomena.

CAP. VI.

On the formal cause of the new celestial phenomena.

We have treated the matter of the new phenomena rather fully in the preceding chapter, because this had been placed in the greatest controversy among philosophers and astronomers. And although we frankly admit that the opinion we have put forward about it — like that of others — is thus far ἀναπόδεικτον [Greek: *anapodeikton*, "undemonstrated"], yet, established as it is by so many arguments and authorities, scarcely anyone will derive it from any source other than the Milky Way with more on his side. But now ¹

inaequabilitatem toties convertit — now the order requires that we proceed further to the form and end of the new phenomena: to the form, I say, which chiefly gives the thing its being and separates it from all other things by a certain peculiar property of nature that it acquires. Now, inquiring carefully here into such a form, we find that in the new phenomena it is either general — namely, that which belongs equally to them all — or special, the kind that displays first the differences of new stars from comets, and then those of comets among

¹ [Margin / signature: "b 3"]

themselves. Both must be treated in the present chapter, the latter especially, but the more briefly because in the preceding chapters not a little about them has been said, as the need for understanding required.

First, as to the general form of the new phenomena: it informs them in such a way that, out of the matter we have spoken of, celestial bodies begin to exist; and although they do not last long, they nevertheless rival the other stars in the splendor of their light, in their round shape, and in their motion — so much so that they cannot be those phantoms in the air which some opine, with no greater height or better reason than the common crowd, indeed than the other animals that lift up their eyes. This form, no ordinary one, must be light — yet not light derived from the Sun and the stars, but rather seminal (so to speak), that is, potent for generating, as happens in the rest of nature: the form which God infuses into the matter when it has seemed good to His providence, for the end I touched on above, in some fashion, under the efficient cause, but will set out more fully below under the final cause — since the period of a generation of this kind can be fixed with certainty by no mortal.

In the meantime, however, it is credible that God, who in nature mostly acts through intermediate means, even when He produces those things through which He wills men to be admonished of His mercy, His wrath, their own need to repent, and the like (as we have an example in the rainbow), here too uses means — namely the constitution of particular stars relative to one another, as if rational, through which their influences operate effectively at the appointed time. These seize first the center of the matter, then suddenly its whole destined and as it were torn-off mass, and, among other things, fill it with such vigor of light that those new stars appear at their rising not much unlike those coeval with the world — although afterward, successively, on account of their lesser exaltation, or perfection of form, these in turn gradually fail, either of themselves or by the wearing-away of the Sun's rays, as Kepler perhaps rightly judges. For just as to earthly metals their excellence and durability are imparted by the greater or lesser exaltation of those very metals — whence gold, the most precious among them, is so durable that scarcely by any art is it dissolved or made to pass into anything other than gold, whereas rust easily consumes iron — so it is likely to happen in celestial things: that new phenomena are consumed and vanish again all the sooner because they do not stand in equal exaltation with those things first created by God, the formal potency in the new ones being feebler.

Further, since mention of light, as it were the primary instrument of this form, falls in here, I judge it not at all absurd to hold that that inner light in the new

phenomena is also an offspring of that primordial light which Scripture, before all created things, so greatly extols. For it is not credible that this whole primordial light yielded its place, as matter, to the creation of the Sun and the stars, so that no traces of it are anywhere any longer to be found — since the divine, ever-admirable and most powerful sustaining of created things intimates to us that nothing of what God had made will be abolished, or wholly perish, before the final consummation of the world.

For my own part, as concerns my judgment, I am of this view or opinion: that although that primordial light, after the creation of the Sun — which succeeded into its ministry by divine ordinance — appears less to us, nevertheless through the Milky Way, and moreover throughout all the greater bodies of the world, its hidden traces are left behind; and that these bodies cherish [that light], no otherwise than as the internal or innate heat cherishes our own bodies, to which, while adventitious light and heat from the Sun and the stars join themselves, this animal — which is called μικρόκοσμος [Greek: mikrokosmos, "microcosm"] — preserves its vigor. For about that which the senses sufficiently grasp — namely whether there is heat in light — I do not think there is much to inquire; but rather whether that heat is *anthraceous* (that is, whether in the stars and the new phenomena it is as vigorous as the fire kindled in a coal). This question that point which I just brought forward concerning the primordial light surely settles beyond doubt: so that the rays which illuminate and beset the stars, both the perpetual and the new (as some would have it), are neither mere solar rays, nor possess light and heat in themselves by this or that anthraceous faculty — just as not even the Sun itself does, though this divine star is compacted of abundant matter, inasmuch as it pours out so dense a light that it dulls and obscures the rest of the luminous stars whenever they are found above the horizon together with it. Finally, among the other causes for which the great bodies of the world ²

are round, which we proved in the later parts of the *Prognosis of the Astronomy*, that light, or internal heat, seems to be not the least. For such heat alone brings it about that the new phenomena, too, contain themselves within a spherical figure by a certain natural force. For since the external cold — which we showed in the *Prognosis* prevails chiefly in the expanse [expansum] — and that internal heat, which besets the matter or body of the star, flee one another on account of the contrariety of their qualities, it comes about that, while that heat presses upon the center and compresses and constrains the mass it occupies equally on all sides, it so contains it that its surface stands everywhere at an equal interval from

that same center — which is just what serves to define a sphere. This, moreover, is still further confirmed by the fact that this spherical figure is preserved through every diminishing of the star, right up to its final disappearance, the internal heat, of course, gradually failing in them as in other things that live. Hence the head of a comet appears more compact and brighter than the rest of its outflow — namely the hair, or tail.

But to return to the primordial light: if it is that out of which the luminous celestial bodies were, according to most, founded by God, it will perhaps be far less absurd to hold that this same light was scattered, in place of internal fire, throughout all the greater bodies of the world (the fire which Aristotle, in any case, falsely set up as a sublunary element), than to assign this fire to the empyrean heaven, with the Zwinglians and certain others, or, with the Brethren of the Rosy Cross [Rosicrucians], to reckon that light, which was created, as the very Son of God Himself. And thus far concerning the general informing form of the new phenomena. The special form follows.

We call this form *special* for the reason that it makes a distinction between new stars and comets — and that both as regards external form and as regards motion — while among the comets themselves it does so by reason of external form alone.

First, then, as to the fact that the new stars differ from the comets in that external form and in motion (or rather rest), this seems to be ascribed to the place in which they reside. For since the supreme starry heaven differs from the lower, namely the planetary one, chiefly in this — that there all the fixed stars are carried either by one uniform motion from east to west, or else (with the stars at rest) the globe on which we are thought to rest is turned by the contrary motion, that is, from west toward east, conforming, of course, to the planets (which I had always rather believed, for the reasons you will find in the *Astronomia Danica*) — for neither the natural philosophers nor, like them, the astronomers admit that the fixed stars are moved separately through the aether, however fluid, for reasons which I had likewise left scattered throughout the *Astronomy*: therefore, since the new stars are found in that supreme heaven, which Tycho showed most fluidly in the phenomena of the star of the year '72, and besides in the Milky Way (which most of the other new stars likewise argue by their positions), it follows that to new stars too it is fitting to be subject to none other than the primary revolution along with the fixed stars, and at the same time to be so endowed with abundance of matter in that supreme heaven that they both display greater brightness than a comet and lack any perceptible outflow of matter — although

another reason can also remain why we perceive such an exhalation of matter less than in comets, especially when assent is granted to the Copernican hypothesis. For if the fixed stars are at so great an exaltation from the Sun and the Earth that the solar rays transmitted thither do not differ from the rays of our sight — the semidiameter of the eighth sphere [sphaera octava], of course, which the fixed stars beset, excluding all sensible comparison with the semidiameter of the Earth's orbit (which, for Ptolemy and Tycho, is the Sun's) — [then the outflow would be imperceptible]. But the earlier reason, which attributes this to the density of the matter (so that the solar rays do not transmit the new stars as perceptibly as they do comets), is to us the stronger — although certainly, from that other reason too, which has a demonstration attached, a comet of the higher planetary region would, compared with those that beset the lower region nearest the Moon, vary somewhat in the disposition of its tail with respect to the Sun and to our line of sight, as will be shown below.

Concerning comets, further, I judge these things must be inquired into. First, whether there was an internal form by which they are immediately moved; or whether, rather, they accomplish those various motions of theirs by some external force, within the celestial intervals [diastemata]? Then, about which center — the Earth or the Sun — they complete the portion of their circuit? For I hold, with other astronomers, that their motion is rather circular than rectilinear with Kepler, for the reasons given above. Third, what that outflow was which comets are seen to dart forth from themselves in place of a tail or hair, and why that same outflow perpetually inclines to the parts opposite the Sun? Finally, whence arises the difference of tails and hair among comets? These four [questions], which suit this place besides those already dispatched, are to be inquired into further concerning comets, since they are weighed at the bar of reason alone — that is, insofar as the rising and generation of these are founded in the nature of things. Therefore I think no one should be begrudged who has discoursed more probably about these matters; rather, better reasons should be received with all applause.

As to the first [question]: we here certainly judge the same thing to happen concerning the nature of the motions in comets as we left discussed elsewhere — in the *Prognosis* and in other places too — concerning the motion of the planets: namely, that comets are not moved from place to place by a wholly internal force, since Aristotle³

— in *On the Soul*, Book I, chapter 3 — denies, against Plato, that even our soul has this force within it, by which it might move itself, [supporting this] with many arguments. If, therefore, the soul does not move itself, how will a wholly inanimate star be moved without some external impulse? Even so, I do not deny that the stars, like the comets, are by nature so constituted and endowed with such a form that they are most apt for local motion through the aether. We therefore acknowledge that the motion of the comets comes from a prime mover. For I declare that by divine providence it was brought about from the beginning that that planetary heaven, which lies beneath the supreme starry one, is wholly in motion through its species — that is, distributed into planets and comets according to the directories that proceed from the divine ordination — in such a way that each of these travels beneath a great circle: the planets, indeed, along the zodiacal track, the middle of which the Sun's course holds, for the end that is best in nature, namely that the Sun and the other planets, by the alternation of their fixed motion, into the north and south along that same oblique track, may serve perpetually to vivify and cherish the things below — as will be said more fully in the *Astrology*. The comets, however, are allotted various courses and motions through the aether for this purpose: that they too may fulfill the fixed end for which, like divine messengers, they are sent forth at diverse times — as we touched on under the efficient cause, and shall expound more amply below under the final cause.

These [comets], however — although, on account of the track of the great circle which they express almost to the very end by constant fixed motions, two or three days after their first appearance they indicate into what region of the world, and beneath which constellations especially, they will afterward at last direct their fixed courses — nevertheless, this, at their very rising or before, I shall hold that no mortal can predict, for the reason that they have no fixed circles (as the planets do) along which they accomplish their fixed motions.

Further, since the Tychonic hypothesis sets up two centers in the orb most correctly, as it seems, about which the heavenly bodies turn — for according to it the luminaries and the whole system of fixed stars look (as to their midpoints, well-nigh) to the Earth, but the remaining five [look] to the Sun itself — these reasons compel us to believe that comets have the Earth, rather than the Sun, for their center.

The first: that comets follow the track of a great circle in their fixed motion. This course could rightly be denied to them, provided that, for the sake of some constancy of hypothesis, they circled the Sun in such a way that they admitted a

diversity of distances from the Earth equal to, or even greater than, what the planets manifestly display in their yearly motions about the Sun. For the orbit of a planet (which cuts the ecliptic in two opposite places, as befits great circles) converts and changes the angle of intersection into inequality as often as the planet [changes] its distance from the Earth — since that angle is greater at a lesser distance from the Earth, and conversely. This is the cause why the planet's own path sometimes falls short of the track of the great circle, as is gathered in Book II, Part 2 of our *Astronomy*, concerning the latitudes of the planets. But in a comet's motion this is not observed; rather, its course through the aether is such that, whatever great circle has cut it — be it the equator or the ecliptic — it perpetually retains equality of angle with that same circle.

The second reason seems to be drawn from the end and effect of comets. For immediately, like the Sun and Moon, the new phenomena are without doubt destined for the Earth and its inhabitants, and they exercise their fixed effects about it before the influences of those [comets] obtain their full, fixed disposition upon these lower things — toward those decrees which are at the same time infused from those planets. Although I would not deny that the effects of comets are greatly intensified and remitted according to the rational aspects of the planets with them, and among themselves — but above all of the Sun and Moon, with which the new stars and comets occupy the same center — as will be said below concerning the effects of the new heavenly bodies.

These things well considered: as for the fact that Tycho likewise, and Maestlin [Michael Maestlin], adduced the comet of the year '77 to hypotheses of this kind, in which the Sun is a center as in the case of the planets — you see in them something rather ingenious and affected, namely that they might in some fashion show that comets too have fixed hypotheses, such as the planets have, than something steered toward the truth. Just as Tycho seems in some way to retract this opinion in *Progymnasmata*, Book II, page 268 and following. And certainly I do not know how comets, like planets, could be subjected to the obedience of the Sun as their fixed director — comets which by no means follow the track of the zodiac, as the planets do, but mostly rise far from the ecliptic and are swept quite transversely across the heaven with respect to the zodiac. These, therefore, seem to rest on no other hypothesis than one in which the analogy of motion corresponds, little by little while they last, to the diminution of their magnitude — just as we mentioned above the analogical decreases of both magnitude and motion in comets, to be illustrated below by examples as well.

The third question discerns the nature of the hair or tail in comets. Now whatever this outflow is, since it perpetually inclines toward the parts opposite the Sun, Tycho once reasoned, not inelegantly, together with Peter Apian [Petrus Apianus] and Gemma Frisius, that it is by the projection of the solar rays... [text continues in next batch]

are produced and shaped, rather than that it is a flame, which the comet — like a burning lamp — emits or hurls from itself upward. This indeed had been Aristotle's account of the nature of a comet's tail; see Tycho's *Progymnasmata* [*Progymnasmata*], pp. 714 and 651.

Moreover, the requirement that proceeds from the optical reasoning adds this point: namely, that without some material substance, the said rays of the Sun, transmitted through the body of the comet, could not appear as they do. This is just what was reckoned above concerning the Milky Way [via lactea], and what the distinguished Johannes Kepler [Kepler] demonstrated soundly enough in his optical works. Nor does it tell against this that Tycho elsewhere — in Book 2 of the *Progymnasmata* — argues, regarding the orientation of the tail of the comet of the year [15]77, that it was projected into the opposite quarter by Venus rather than by the Sun; a view that he also supports by demonstrations derived from his observations. For although the conspicuous radiance of Venus, which is so brilliant and bright that it casts a shadow even at night, may have contributed something to the nature of that comet's tail and to the bending of its light, yet this contribution was not so great as to make it more probable that Venus, rather than the Sun, was the primary producer and director of that comet's tail. In other comets later seen and observed, Tycho did not hesitate to affirm this and to confirm his earlier opinion about the tail of comets — an opinion you will find scattered throughout the first *Progymnasmata*. The second, which was elaborated and written in our own time at Hven [Huaena], came long afterward; so that in it one finds what the proverb calls φροντίδες δευτέραι [Greek: phrontides deuterai, "second thoughts"].

Nor will I lack, in what follows, the means to show what was needed: namely, that the tail of this same comet extended somewhat upward with respect to us, away from that straight line along which Tycho represents comets in his hypothesis — and this because of the comet's greater elevation above the Earth. Two things, then, combine to produce and shape the hair, or tail, of a comet: the solar rays (perhaps also the rays of the remaining heavenly bodies, which mingle with the solar ones) and the matter surrounding the body of the comet, or the matter of which the comet consists. Upon this matter those solar rays act

powerfully — indeed, almost violently — grazing and abrading it at its very surface; and the result is that the lighter and most yielding particles of that matter are seized by those rays and, so far as may be, are continuously projected into a space beyond the rest of the comet's body, into the quarter opposite the Sun. Yet this happens in such a way that they do not entirely abandon the head of the comet from which they take their rise, nor vanish into the aether, except as they are gradually worn away and consumed by the solar rays. And this seems to be the reason why comets gradually diminish, and are not extinguished at the magnitude in which they begin to arise; rather, that central vigor, which I mentioned above, is gradually exhausted and emptied out together with the matter. For toward the end the internal light grows faint, as it does in all new phenomena; and in comets too the traces of the tail, as well as its motion, are scarcely perceived, since the matter is by then found to be rarer, less compacted and consolidated, and less suited to motion. The formal vigor too, as I said, gradually drains away.

But if someone should take an argument against me from this — pressing the point that the comet moves by an internal force, without any external impulse, so that thus an internal force is made the cause of its motion — let him know that this does not suffice to establish such an opinion. For a body that is also moved by an external cause provides ample occasion for a more intense or a more relaxed motion, whether by gradually failing or by violently and against its nature traversing a longer course. One may observe this in rockets [pyrobolis], which are moved chiefly by an external impulse and meanwhile gradually fail; likewise in arrows, and in the balls of cannon, which the external moving force gradually abandons. Nor do I imagine that any such external impulse is poured forth from the mover of the comet — a mover that does not merely thrust the comet from place to place, but rather takes possession of its whole mass and at the same time projects its internal forces into motion.

What was asserted earlier — that the matter of the comet is plucked out and drawn forth — will be further confirmed. For if anyone holds a candle or a torch up against the Sun shining brightly, he will notice a certain fine smoke or vapor, which the Sun's rays drive toward the side of the candle turned away from it. Or, if this comparison does not fit well enough — since the comet is not a kindled fire, but rather the whole cometic matter so illuminated that the entire mass appears as though fiery, as was asserted earlier — then let one consider how those same solar rays, gathered around the stone that the common people call crystal [lapidem crystallinum] when it is held up to them, leave behind a luminous trace through the air in the quarter turned away from the Sun, and with such heat that

even gunpowder [pulvis pyrius] can be ignited by it at a moderate distance. From this it follows that, because of the presence of the light, that hot effluence [defluvium] must be of some such kind — which the effects of comets also show, and which will be discussed below.

There remains here the fourth, or last, question concerning the cause or reason from which arises that difference in the effluence which is observed in connection with comets. For since Seneca's [Seneca] verdict on comets — which I also quoted above, and which runs: *Whether there be any differences among comets, or whether there be none, they come to be of necessity by the same reasoning* — was thoroughly plausible. [appendix p. 15]

—therefore it is not without purpose that I here inquire what at last will be the cause why comets, whether seen at different times, or the same ones at times passing into other forms, appear under that manifold difference — chiefly with respect to the effluence — which in Caput I I had reviewed from Pliny [Pliny]. For as to what concerns the heads themselves, or the round bodies themselves, these can in some measure be known both from the efficient causes — namely the planets, which contribute the greatest force to the generation of comets at different times — and from the density and rarity of the matter. From this, indeed, the difference of consistency and color in comets is rightly derived. But for the discernment of the matter we are presently inquiring into, what is chiefly required is the disposition of the comet relative to the Sun and the Earth — although the quality of the interior form, as well as the matter, and likewise the final cause, also enter into this consideration.

Now, in dealing with this question, the twelve species of comets given by Pliny are reduced to that small number which, as I indicated above in Caput I, has been observed in our own age: namely, that the effluences of comets take on the form or figure either of a *tail* (as most frequently), or of *hair* (as in the comet of the year 1585), or finally of a *beard* — or rather, of the brooms used for clothing [scoparum vestiariarum] — such as Kepler writes that he observed in the north in the year 1618. For by observing such an effluence, and at the same time the disposition of the comet relative to the Sun, these species are in some measure distinguished by the ocular sense and told apart from one another. That a comet is seen as *tailed* is brought about chiefly by its lateral disposition relative to the Sun and the Earth — especially while the comet is situated within a quadrant of the Sun and is not too close to it. The *hairy* and *bearded* comets, however — or, if you prefer, those more cut short as to the tail and surrounded on every side by traces of the effluence — require either a position more nearly opposite the Sun

(as in the two I just mentioned) or else a sublime elevation above the Sun. For the comet's location relative to the Earth also somewhat varies its appearance: the more it has been elevated above the Earth, the more it draws the effluence upward, away from our view — as did that great comet of the year [15]77, which I mentioned above. Besides this, an excessive distance of the comet from the Earth greatly diminishes the apparent size of the tail, in Tycho's opinion, unless the matter has proved more abundant — as in that same comet. And as for those we afterward saw, namely in the years 1607 and 1618, these varieties are demonstrated as follows.

Diagram. This geometric figure illustrates the projection of comet tails by the solar rays for comets at different positions and elevations. At the center is the Earth, marked **A**, which serves as the common center both of the Sun and of the comets. From this center three concentric circles are described. The line **A B** is the semidiameter of the Sun's orbit, with the Sun located at **B** (lower right). **A D** is the semidiameter of a circle below the Sun; **A H** is the semidiameter of another circle above the Sun (upper right). Radial straight lines extend from the Earth **A** to the heads of the various comets and to their tails. The semicircle is divided over center **A** into four equal parts (angles), with comets placed at the quadrant- and semiquadrant-points on either side; their tails are shown projected precisely into the quarter opposite the Sun (radiating, hair-like strokes). Labeled points around the figure:

- **A** (center): the Earth
- **B** (lower right, on outer circle): the Sun
- **C** (lower portion, below the Sun's circle): comet in sextile aspect below the Sun
- **D** (between center and Sun on the lower radius): point on the inner circle (semidiameter **A D**)
- **E** (left of center): comet position
- **F** (just below center): point near the Earth
- **G** (upper right, above the Sun's circle): comet in sextile aspect above the Sun
- **H** (upper right, on a radius): point on the upper circle (semidiameter **A H**)
- **I** (right side, upper): comet position [labeled with a doubled/ligature mark, read as **I**]
- **K** (lower left): point on outer circle
- **L** (upper area, above **G**): extremity of the tail of the comet at **G** (true tail-length **G L**)

- M (top): extremity of a projected tail
- N (upper left): extremity of the apparent tail of the comet at I
- O (lower center-left): extremity of the tail of the comet at C (true tail-length C O)
- P (upper center): point along the projection
- Q (left, below E): extremity of the apparent tail of the comet at E
- R (lower center, near F): point on the lower diameter
- S (far left, on outer circle): point on outer circle

Let **A** in the adjoining figure be the Earth — that is, the center both of the Sun and of the comets, as was shown above. From this center let three circles be described, and let **A B** be the semidiameter of the Sun's orbit; let **A D** be the semidiameter of a certain circle below the Sun, but **A H** that of another above it. These latter circles the further comets occupy in various dispositions relative to the Sun. For when these semicircles are divided over center **A** into four equal parts or angles, and the Sun is located at **B**, then if comets are placed through each circle — and through their quadrants and semiquadrants, this way and that, according to the aspect of the comet with the Sun — and their tails are extended equally and precisely into the quarter opposite the Sun; while from the Earth **A** other straight lines are extended both to the heads of the comets and to their tails (drawn out equally) — then the variation in the projection of the effluences, or tails, which falls within our view, is evident from the mere inspection of this figure. For when we have concluded, as a general matter, that all comets are tailed, then from the angles of vision that measure the equal tails, the differences betray themselves: the differences, that is, among those comets that are placed at the same or at different sections of the circles — that is, at the same or different aspects with the Sun — below and above this point. For the tails in those comets that lie *above* are detected by our view as turned away, or more erected upward; until one reaches a point directly opposite the Sun on either side, where, although the prolongation of the tail is altogether hidden and concealed by the interposition of the comet's head, nevertheless the solar rays rebound around the body of that same comet in the manner of hair or a fringe, and are scattered, being engaged about that superficial matter — which, as was said above, is gradually worn and abraded away — much as it is the nature of light, when it leaps upon a smooth and polished bodily object, to be recollected to a central point before the dispersion of the rays.

Now, since the cause of the direction of the cometic tails proceeds from the solar rays — for which we have, besides my own observations, several other witnesses

to this matter: Cardano [Cardano], Apian [Apianus], Gemma, and Cornelius Frisius [Cornelius Gemma Frisius], our own Tycho, and others — and since, moreover, we retain in these matters the hypothesis I established above, namely that the Earth is the center of the circular motions of the comets, [appendix p. 16]

—it cannot be otherwise than that our line of sight, raising itself from the Earth toward the comet, measures the length of the tail by that angle whose two sides — namely the two that enclose this same angle of vision — are such that one extends from the Earth to the head of the comet, the other to the extremity of the tail, while the third side is the true length of the tail itself as projected from the Sun. For example: let a comet be set (as most often happens) below the Sun, and in its sextile aspect — namely at **C** — and let the true length of its tail be **C O**; then its apparent length will be the one that the angle **C A O** measures. The same thing happens with a comet at **G** above the Sun and likewise placed in *its* sextile: for since the true length of this tail is **G L**, the apparent length is detected through the angle **G A L**. And by however much this angle exceeds the angle **C A O** (other things being equal), by just so much must the tail of the comet at **G** exceed in length the tail of the comet at **C**, since the former extends more nearly upward.

[Margin: See Book 2 of the *Progymnasmata* [*Progymnasmata*] T. 6, p. 203.]

So it was with the comet of the year 1577, which Tycho detected to be situated near the Sun's sextile, and which thus extended its tail upward with respect to the Earth, as though it appeared to be projected by Venus — since Venus was then, in its morning departure and a little below the Sun, situated in its own orbit. Had Tycho located the comet *above* the Sun, in his true hypothesis — as his observed parallaxes also demanded, which in truth scarcely exceeded three minutes at the horizon, provided the observations have been duly examined — he would certainly have detected that very direction of the tail, which he describes, as coming from the Sun itself. For it made a great difference here whether one placed the center of the comet at the Earth or at the Sun. But if at the Sun, as Tycho supposed, then why did the comet, in the lowest position of its orbit, keep the same track under the great circle as one made higher up in the same? — seeing that planets, tracing similar circuits about the Sun, vary this track greatly by their approach and recession in latitude, so much so that for the same reason they cannot in the least proceed under a great circle in the sphere. For comets, therefore, let the Earth be the center, and the Sun the director of the tails, in the same manner as occurs with the rest of them too — whose patterns and tail-figures the present figure sufficiently expresses.

Hence it also appears that comets cannot be seen when in invisible conjunction with the Sun and the Earth, whether they be below or above the Sun, because these — like the rest of the stars — are hidden beneath the Sun's rays. The more the comets incline toward the point opposite the Sun, the shorter their tails become — as happens with the comets at **E** and **I**, near the Sun's trine, where in this latter (at **I**) the very slender angle **I A N** measures the apparent tail; but in the former, which is below the Sun, the measure of the tail is the angle **E A Q**, which, by however much it is found wider than the higher angle **I A N**, by just so much is the tail more drawn out in it. [appendix p. 17]

Finally, the tails of the comets that appear precisely opposite the Sun lie hidden behind their heads, and only that which the Sun's rays draw out around the cometic mass itself — and into which they reflect themselves — appears; from which a hairy comet arises, as does a bearded one, when it is not far distant from the point opposite the Sun. For a comet of this kind, on this hypothesis, deflects a short and dense tail to its side and below itself. There appears besides, in this figure, the very point I was hinting at above: namely, that no tail can be detected in those comets or new phenomena whose orbit's semidiameter — such as **A H** — exhausts all proportion to the semidiameter of the Sun's orbit, namely **A B**, by reason of its own magnitude. For when the angle **A H B** so completely vanishes that **B H** and **A H** are taken, as far as vision is concerned, for a single straight line, then the tail tends straight upward no otherwise than if the phenomenon were established directly opposite the Sun.

Accordingly, the new stars too could be fitted to such a hypothesis — especially according to Copernicus [Copernicus] — were it not that this stands in the way: namely, that the new stars display no trace of effluence above or around themselves, but distribute their own quite clear and most brilliant light, like that of the rest of the fixed stars, equally throughout their whole mass. And so, just as we think it more correct, perhaps, to hold that they are destitute of motions, so too of tails, that is, of any visible effluence. As for the rest concerning such a hypothesis, by which the differences of comets are in some measure demonstrated — since the causes judged of above recollect themselves here (so much so that comets differ from one another, through every kind of cause, mostly by *more* and *less*; and the higher they are, the more force they contain within themselves; and indeed these, besides — as Tycho supposed — withdraw the truer part of the effluence from our view, on account of their excessive distance from the Earth) — accordingly, since the true prolongation of the tail can on this account vary greatly in different comets, so much so that the true length of the tail is not the same in most of them, it follows that from the merely formal

prominence of the tail it will be difficult to form a judgment about the altitude and magnitude of a comet. For this reason a verdict is more correctly passed concerning these matters on the basis of exact observations, of their comparison with one another, and of a subtle teasing-out of the parallaxes from this.

Lastly, in this chapter, with respect to the opinion of those who say that comets are angels, good or bad, taking on these portentous forms — namely, that they extend those effluences around the comets, of which I have made mention, into various significations of punishments, so that they wish a *virga* (a rod) to be represented by the figure of hair or beard, and a *sword* or *spear* by the appearance of the tail, as though these were instruments of the punishments, afflictions, wars, and other evils that are at the door, shown forth from heaven by the metonymy of the cause — to this opinion I think the reply must be as follows: that it is not necessary to derive at once, from supernal and altogether hidden causes, those things which are accustomed to be set down in natural things (among whose number I have shown above that comets belong) as signifying something singular, [appendix p. 18]

[Note: the running header on this page reads "Cap. 7", but the body text here is still the conclusion of Cap. VI (Cap. VII begins lower on this same page). This is a premature running header — see batch report.]

—or as designating God. We have an example in the heavenly rainbow [iris coelestis], which, although it is a natural thing — namely an *emphasis* (a reflected appearance) in the air, from the solar rays reflected onto a dewy cloud — nevertheless quite significantly expresses that covenant which God made with mankind after the Flood: namely, that this world would no longer perish by water; this it expresses by its blue circle appearing in the clouds.

[Margin: Genesis 9:1; Genesis 2; and elsewhere.]

And by the other, cloudy circle, which is seen alongside it, the rainbow seems without doubt to signify what the holy Apostle Peter elsewhere foretells: namely, that the world will at some time be consumed by fire.

[Margin: 2 Peter 3.]

In the meantime, however, it must not be denied that the things comets signify are very closely akin to those which are elsewhere in Scripture — and most of all in the Apocalypse [Revelation] of John — pronounced as things to come, through the ministry of the angels in the visions; if indeed they are not altogether the same. Concerning their comparison I shall say something below, when I shall

have reviewed the effects of comets — and especially of the last one, in particular — according to our own understanding and judgment. But now I shall complete, with the interpretation of the final cause, what remains among the causes.

CAP. VII. *Concerning the final cause of the new phenomena, and the description of them.*

If God and Nature, according to the philosopher's axiom [Aristotle], do nothing in vain, then surely there will be some end — and indeed a sufficiently weighty one — for the sake of which new stars, as well as comets, are seen in the heavens at various times. And this end, had it but been duly weighed, would have moved Thomas Erastus [Thomas Erastus] and other grave despisers of the stars and of their effects, in proportion to its dignity; it would assuredly have assigned no very slight effect to the rest of the stars, and none at all [recte: not a null effect] to comets, or to new phenomena in general.

Concerning the Sun, the Moon, and the stars coeval with the world, Scripture briefly — but most pithily and most truly, as it does all things — lays down their definition: namely, that almighty God created them so that, among the other offices assigned to each, they should be for *signs*, should illuminate the Earth, and (which we perceive by sense) should cherish with their light — this being the συμπάθεια [Greek: sympatheia, "fellow-feeling, sympathy"] of the stars, on account of which also, like the rest, they were pronounced by the Creator to be *very good*.

[Margin: Genesis 1:14 and following.]

For the rest, the word *signs*, which Scripture uses, is not to be taken entirely in that sense in which the Sun is there declared to be destined to profit the day, and the Moon together with the stars the night — since these particles, which impose their offices or ministries upon the stars, are sufficiently distinguished from one another in the text. Rather, the stars are also signs of this kind: that by them the will of God will at certain appointed times be indicated, and above all the glory of Him, for the sake of whom as the supreme end all things were created, shall be proclaimed among men, and His wisdom continually published abroad.

[Margin: Psalm 19.]

For Christian natural philosophers rightly consider that all things were made by God for the sake of, and the benefit of, mankind; but man for the sake of God — so that this noblest creature on Earth might, above all others, thereby

acknowledge its Creator and duly worship Him; since Scripture in no way departs from this opinion.

[Margin:...8 [a marginal reference, partly trimmed — see batch report]]

Therefore, in order that the best Creator might both consult most excellently for the benefit of mankind, and might inform individual men — even the heathen — more clearly and manifestly of His omnipotence and will than through those obscure earthly things, it pleased the most wise God to place these most splendid signs — the stars, namely — in the celestial theater, and to set them before the eyes of mortals. This is what Ovid [Ovid] aptly noted in these little verses:

[Margin: Ovid, *Metamorphoses* [*Metamorphoses*].]

*And whereas all other animals look down upon the earth, He gave to man
an uplifted face, and bade him behold the sky and raise his upturned
countenance to the stars.*

From what has just been brought forward, anyone understands that the end of the creation of the stars — the new ones too — is, in general, twofold: natural and supernatural. *Natural*: that earthly things, being composed of the elements and perishable, may by the conversion and influence of the light of the world's stars be cherished and governed, may have their seeds stirred up within them, and may contribute greatly to the generation of individuals — so that the species in nature may be preserved and may endure until the very consummation of the age. And since the common matter, of which both the stars and these lower things are composed, binds the latter very much to the stars — and most of all in what concerns the seminal condition and matter, which Aristotle therefore not ineptly says is to be compared to the starry heaven —

[Margin: Aristotle, *De Generatione Animalium* [*De Generatione Animalium*] 2.3.]

—therefore it ought to be no wonder to anyone why the stars act upon these lower things not only evidently enough, like the Sun and the Moon, but also by a certain hidden manner — as do these same luminaries, and likewise the remaining planets and the fixed stars; and this most of all, and most variously, according to their aspects and the various angles of regard, as is taught in Astrology.

For the rest, this is said of the natural end of the stars, in so far as they are coeval with the world, if they be understood by themselves. What, then, is to be said here concerning the new phenomena — namely the new stars and the comets? Surely nothing other than this: that neither do these wholly lack this end of theirs

in nature, since they are by no means [recte: not] purely supernatural. For, in the first place, since particular aspects of the planets concur in their generation, as was said above, there can therefore be no doubt that those new phenomena, by their aspects to the rest of the stars, likewise operate upon these lower things in such a way that they mingle themselves with the influences of those stars — but above all with the influences of the Sun and the Moon, to which they are concentric [homocentrica]; whence it comes about that the effects of comets, at whatever times [appendix p. 20] ...their effects will occur — and we measure these effects above all by the course of the Moon. The new stars, by contrast, whose effects are both more lasting and projected further into the future, likewise accommodate themselves to the conjunctions of the superior planets, so that the times of those impending events may be discerned by zodiacal direction. These matters will be set out more correctly and more conveniently below, in the proper place that deals with their effects, and afterward explained more fully and more specifically through examples.

For the rest: since the effects of new phenomena — and above all of comets — or at least their function as signs alongside the other stars, are commonly held to portend evil and disaster, and indeed to such a degree that we frequently find it confirmed true what Claudian sang as if a prophet:

And never has a comet been seen in the heavens without harm.

Yet on the contrary, all of nature seems to have been made by God toward the best end, since everything created in it by the supremely good Creator itself was seen to be very good. It is therefore not without reason that one may ask: how do those evil and deadly effects — which for the most part flow from comets, or at any rate hover over the world at their appearance — have a place in nature? To this I briefly reply, on the authority of Scripture and of the philosophers, and finally by means of a few examples:

After the curse was brought down upon nature itself through the transgression of the first human beings, nature was never free from evil. Rather, it has always held evil so closely bound to itself that, before you could remove the evil — or the worse element — from nature itself, or from that which is better in nature, you would necessarily destroy nature itself, indeed providence itself (as Plotinus puts it). Meanwhile, however, almighty God, by this marvelous providence of his, with which he governs all things in nature, so moderates all the evils in nature and disposes them in their time that they are carried toward a good end — indeed, toward the best end — exactly as Boethius argues, among others, with great skill and force. [Margin: Boethius, *On the Consolation of Philosophy*, Book 4, Prose 6.]

For since it is the nature of the devil always to strive to turn good into evil, this craftsmanship must surely be ascribed uniquely to the divine — whether to nature or to omnipotence — namely, that through it evil is directed toward, and passes over into, a good end.

Furthermore, examples joined with experience show that the things which seem to us worse in nature itself delay the course of nature, as it now stands, little or not at all. Does not a dearth of grain — of which too we find by experience the comet to be a forerunner — bring it about that, while the earth lies fallow and does not produce an abundant crop year after year, it is again made ready and becomes in following years more fit to yield its fruits? The same is to be said of the storms that for the most part follow the appearance of comets. For while these storms agitate the air and the waters, they render them far more purified than they were before. Nor is there sometimes any other reason in those things which befall human beings through the affliction of diseases and wars, or even of other evils, of which too comets are wont to be forerunners.

Among other things, Palingenius marks the matter of wars with these verses:
[Margin: Palingenius, in [the sign of] Capricorn.]

But although war is foul, deadly, and abominable, yet it is not without all use. All cunning men, and deceitful folk, given to usury and frauds, heap up and pile riches upon themselves in the time of peace. But when wars roar, the soldier seizes by force what was ill-gotten. Then deservedly the foxes are paid back, and the rage of wolves is avenged; then the brave repay tricks with worthy reward, and return like for like — thus time changes all things. Moreover, kings, when there is peace, plunder peoples by a thousand arts and robberies, and from every quarter scrape together coin; whence the treasury would seize all gold and silver, did not war dig up what lay hidden in peace, and distribute to many what one man had amassed. Lastly, there is no small crowd of people who have neither property, nor learning, nor craft, and who flee labor — given over to wickedness, prone to crime, impious and bold. Such men, therefore — nay, such shadows — at a fixed time God sends into arms, to be slain by a cruel death. Thus he purges the human race, and for many years those who remain live cheerful, this plague removed; until evil weeds spring up again, once more to be uprooted by fierce war and cut down by the sword. Then again Jupiter skims off the foul scum from his pots; then again, with filth let loose, he sweeps away the refuse.

But you say: the innocent too often pay penalties they did not deserve, when disaster falls upon the commonwealth and lays it low with plague, war, flood, or some other common calamity; or even when the guiltless are condemned to punishment by the perverse and mistaken sentence of judges; indeed, when they suffer various and unexpected adversities and pressures, as if sent from God alone. But I answer with the divine Chrysostom: [Margin: Chrysostom, *In Orat. Quod nemo laeditur nisi a seipso* — "That no one is harmed except by himself."] these are not evils, nor is anyone harmed except by himself. And it is indeed certain that evils which involved no fault have never harmed the good and the just. For these things are evil only in appearance, and adverse only to the senses, which here are deceived. Against them Horace boldly encourages the just man in the commonwealth — namely, that

Should the whole globe break and fall upon him, the ruins will strike him unafraid.

But I have here wandered further from my subject than I intended, so that the proposed question might in some measure be satisfied. Elsewhere both the sacred books and the philosophers — Plato, Plotinus, Boethius, and very many others — bear witness to and discuss this divine economy and providence in nature most lucidly.

And so it appears, even from the effects of new phenomena (which we know in part, and have here adapted to nature, to be set out more fully in what follows), that our intention holds good — namely, that these effects regard some end in nature, which there is nothing to prevent from being even manifold. For what if, besides those effects which I have noted to have a kinship in these phenomena with the other stars, new phenomena — and above all comets — are also destined for purifying the celestial element and freeing it from the impurities with which the sublunary region likewise often has its like admixed? For just as the highest and middle region of the air is wont to be frequently purged by ignition [Greek: φλόγωσις (phlogōsis, "inflammation," "flaring")], that is, by the inflammation of a falling star, a leaping goat, the lightning that accompanies thunder, and other fiery meteors of this kind — so it is likely that heaven too is in its own way purged by new phenomena. While these phenomena, as if kindled by the rays of the Sun, are worn away, they sweep up together the superfluous matter into which they crash and which they touch in their course, and expose it to be consumed by those same solar rays. From this, perhaps, the irregularities of the tail on different days in the same comets draw their cause, since the comet's surrounding matter, that adventitious material everywhere subject beneath

heaven, has been increased — just as anyone can verify in the flame of a lamp, if he brings combustible matter to it from above. Meanwhile, however, such matter floating in the celestial sea is, among other things, endowed with so great a thinness and subtlety that of itself it admits no illumination from the Sun and stars. And this can be the reason why, while the aerial region is purified by the frequent ignition of nearly all fiery meteors, the ethereal region is freed from its own pollutions roughly every ten years by the rising and flaring-up of comets — and this in its own peculiar way, according as heaven differs from the sublunary region.

But enough concerning the end of new phenomena in nature; the supernatural end remains, of which we are to speak in the last place. We ought to remember what was set forth at the beginning: namely, that new phenomena, although they are in some fashion found within the enclosures of nature, are nonetheless — whenever they are generated and appear — destined above all, by divine foresight, to signify supernatural things, and chiefly those that seem to be evil punishments in nature. For if signs concerning supernatural matters are taken even from the Sun and Moon — which, together with the rest of the stars, serve the elemental world continually as it was known — whenever, apparently failing in their appointed ministries, they fall into eclipse (which indeed these luminaries admit by their entirely natural courses and bring upon the world, so that one who observes this might readily forgive his own mortality, to speak almost with Pliny); and certainly signs have always existed, derived from the natural eclipses of the luminaries, of the world's perishing at some point — against Aristotle's opinion — besides the other things that for the most part are wont to follow those portentous obscurations, such as wars, plague, famine, diseases, storms, and other like evils, and likewise the deaths of princes in the commonwealth: then what wonder is it that, whenever divine providence wills that — besides the stars created at the beginning, which continually cherish lower nature with their light and motion — new and unaccustomed things too be generated, doubtless to this end, that among other things they may clearly testify concerning the overthrows of affairs (above all to denounce that security in which, at all times indeed, but most of all under this final age of a worn-out world, the greatest part of the human race lies buried), and that they may kindly invite mortals to repentance (insofar as this can even so be done) before the evils they foretell?

[Margin: On the overthrows of public affairs.]

For these new phenomena, and above all comets, are prophets — no less than Jonah once among the Ninevites — mute, yet, as the Psalm has it, sufficiently vocal; [Margin: Psalm 19.] which too the people of Jerusalem heeded too late, and too unhappily, before the last and fatal overthrow of their city, since, as Josephus writes, a comet in the likeness of a drawn sword shone in the celestial theater over them for a whole year and stood out over their temple. To say nothing of the other flarings of comets which histories have here and there, and which fatal calamities for the most part have followed — just as the last one seen by us bears witness this very day.

Since, then, God and nature do nothing in vain (as was said before), and nature, though it often produces monsters beyond its own intention, yet never produces them without the divine will — so that these things too may clearly bear witness, and as it were [Greek: κατ' ὑποτύπωσιν (kat' hypotypōsin, "by way of a sketch / outline")], concerning human wickedness, sloth, and the other enormous vices, and the punishments deservedly to follow, whether they arise in the air, the waters, or the earth, among animals or human beings (as now everywhere): then what else is to be judged concerning those vast, translucent, and stupendously great bodies — which God wills to be newly generated, affixed to the heaven, set in motion as if by rational minds, and at last configured by, as it were, fatal aspects with the rest of the stars — than that these must be true prophets, or preachers, and as it were witnesses before human beings, both of God's immense [Greek: φιλάνθρωπία (philanthrōpia, "love of mankind," "loving-kindness")] and mercy toward those who repent, and of his most just anger against the impenitent? By the kindled ardor of that anger these latter shall not only be justly punished in this age — and, as new phenomena are at last consumed out of heaven, so these too shall be consumed from the face of the earth — but shall also be afflicted in due time with the punishment of hell. Not to mention that new phenomena, and chiefly comets, are no lesser signs of the world's perishing than eclipses of the luminaries — provided that by their disappearance and visible conflagration, as it were, they have not been more brilliant than these. For the rest, since a pious mind, in no way a despiser of divine signs, is wont to ponder and conjecture more things within itself, of these too something has been said scatteredly above, and below it will be discussed more fully and more specifically in the examples. Therefore we here furl our sails concerning these final indications of new phenomena, and at last — which is what remains — we shall construct, from the causes thoroughly investigated and premised, their descriptions and, as it were, definitions: first in general, then in particular.

Therefore a new phenomenon in general (under which heading we have hitherto embraced both new stars and comets alike):

It is a heavenly body newly come into being in the sky, generated indeed from celestial matter — from the Milky Way and the confluence of the stars, or from these stars' other aspects. Although by its splendor and its regular motion about the center of the earth it in some fashion rivals the stars coeval with the world, yet because the excellence of its inner form is far inferior and baser than theirs, it is for that reason also found to be less lasting than they; whence it comes about that it gradually diminishes until its final disappearance. Meanwhile, by a singular divine providence it has been ordained even to this end: that, in keeping with its outer form, position, motion, and aspect with the other stars and its ominous regard toward the earth, it may earnestly admonish human beings — who for the most part are sunk in security — concerning great public calamities and just punishments, on account of their various offenses, in expectation of repentance; and likewise that it may in some fashion display the sign of the consummation of the age (which now, without doubt, presses on close at hand) by its more frequent flaring-up and disappearance.

From the foregoing, moreover, it is understood that a new phenomenon falls into a twofold distinction, so that it is predicated both of a new star and of a comet — these two differing from one another chiefly in form, place, and motion, as well as in their particular signification and in the duration both of that signification and of the events, and of the time. For a new star is a heavenly body, or new phenomenon, located in the highest heaven among the fixed stars, and almost always in the Milky Way, whence it draws its matter. From its beginning it for the most part displays itself with a great, white, and splendid radiance, so that by this very fact it sometimes equals the star of Jupiter or even of Venus; and it is subject to no motion other than that of the fixed stars — that is, to the first revolution. Furthermore, its duration is for the most part beyond a whole year; and before, gradually diminishing, it wholly disappears, it continues, and stretches out the duration of its significations into longer periods — significations which, although they are at the beginning wont to be of better and more brilliant omen, yet for the most part it [the new star] ends in some notable calamity, as the casters of nativities [Genethliaci] admit even concerning the other fixed stars.

A comet, however, is likewise a new celestial phenomenon, generated indeed from the common celestial matter (which we judge to be that of the Milky Way), and moreover from the aspects of the planets, in the lower, or planetary, heaven; shining with a varied color, and often conspicuous in a varied and terrible form,

inasmuch as for the most part some efflux is seen attached to it — which the rays of the Sun perpetually drive toward the part turned away from itself, that is, toward [that] quarter of the world — and which for the most part bears the figure either of tails, or of hair, or of a beard, or of brooms. And although a comet tends toward various quarters of the world besides the first motion, yet it carries out its course steadily beneath the greatest circle of heaven right up to the end; and this too by such a rule of proportion that, as its motion from the beginning is for the most part rendered slacker day by day, the cometic body at the same time diminishes. At last it announces dire calamities to the world, or to certain tracts of it, according to the quality of the form it presents and the stars and aspects of the planets to which it applies.

And so thus far concerning the causes of new phenomena and their descriptions drawn from those same causes — so far as they could hitherto be investigated by the reasoning and criteria of our rather feeble understanding. There follows the discussion of their effects.

CAP. VIII. *On judging the effects of new celestial phenomena, and above all of comets.*

At the beginning of this chapter it must above all be warned that predictions concerning effects of this kind are by no means decrees of a praetor [i.e., not binding judicial pronouncements], seeing that the effects of the stars in general vary infinitely according to the diverse nature and condition of the subjects that receive them — besides the other causes, or hindrances, that are taught in astrology, whereby, although the stars act naturally, their effects nonetheless are not felt uniformly in these sublunary things, because the natures and conditions of these things are not uniform but differ in well-nigh infinite ways. I add that things which include free and voluntary causes — as are human minds — can indeed be inclined by the stars, but by no means compelled, because the soul of a human being (and above all of one who continually worships God and the virtues, and who measures his actions by the divine statutes and the norm of right reason) is both higher and more excellent than the stars. Hence they tell of Socrates that, confessing once that his own nature was the very one which a certain astrologer was expounding in his presence — namely, depraved and distorted — he had nevertheless reformed it according to the institutes and prescriptions of philosophy. And so in this way Socrates, as is said of the wise man, splendidly mastered the stars; for he rendered a nature otherwise distorted most gentle through the lessons [acroamata] of philosophy, so much so that, by

the testimony of his wife Xanthippe, he never changed his countenance. These things being premised, since it was asserted above that those new phenomena, whenever they shine in the heaven as messengers of God, doubtless intimate new things — that is, new changes in the affairs of the world, and those more universal than the ones which other sublunary portents indicate. Therefore, as far as I am able to reason, given the feebleness of my talent (well known to me), concerning their effects or significations, I shall set forth in order in what follows; where first I shall set out the causes of these in nature which seem to me most of all to show why those new phenomena of the heaven, and above all comets, signify certain things diverse from, and surpassing, the rest of what will come to pass whenever they blaze forth.

For rendering these causes — as also for rightly carrying on the whole of astrology — I find nothing more suitable, in truth, than the universal Tychonic hypotyposis [hypotyposis], which (after the Ptolemaic and Copernican) I had inserted in the frontispiece of the second part of the *Astronomia Danica* with the eulogy it deserved. For since in that same arrangement, besides the center of the universe — namely the earth, toward which all the lights and operations of the stars are directed (in accordance too with the sacred pages), [Margin: Genesis 1.] inasmuch as it is uniquely destined to be cherished — there is another, secondary and mobile center, which the Sun itself occupies as its royal seat: the Sun, in the midst, as it were the master [Greek: δεσπότης (despotēs, "master," "lord")] and lord of this celestial economy, draws the other planets, except the Moon, like a household, and leads them with it by its annual motion around the earth, while it keeps them within certain limits of their own circles. And meanwhile it lets some of them off to such a distance from itself — namely the three superior planets, Saturn, Jupiter, and Mars — that they even encircle the earth at their opposition to the Sun; but the rest, Venus and Mercury, it confines to shorter gyres about itself. So that, while the Sun is borne in its chariot to survey the region of heaven and to render the annual vicissitudes among us in its slanting circle [the ecliptic], these may be at hand and, each according to its own nature, motion, and ministry [lithurgia], may carry out its commands, as the king's chamberlains are wont to do.

To the Moon, moreover, as queen of the Sun and of heaven, the lower celestial region — that is, the one nearer the earth — has been assigned. This Moon indeed shines and thrives by the perpetual communication of the Sun's light (both that innate to itself and that acquired through the ministry of the other planets) and, as it were, by impregnation; and from this it displays to the earth its various

torches and faces, whence these lower things are manifestly moved and affected, as may be observed in the tide and ebb and flow of the Ocean, and likewise in living creatures and other sublunary bodies. Further, about the planetary region (and indeed from the center as far as the earth), in which, as I said, the Sun is master and king, and the Moon mistress and queen, stands the outermost celestial region, for the most part packed with the so-called fixed stars. With these — as with foreign peoples and nations, though endowed with the same or a more excellent kind of light — the Sun and its envoys, the planets, as also the queen herself, the Moon, carry on continual exchanges of light [Greek: κοινωνία (koinōnia, "communion," "sharing," "fellowship")]. These they bring first into the planetary region; then, where they tend of their own accord, they pour the same — now enlarged and made more suited to lower things through the ministry [lithurgia] of the planets — into the lunar and at last into the sublunary [region]; and finally they affect this latter variously, according to the disposition of the stars and planets in relation to the Sun, Moon, and earth, and according to the condition of the underlying matter. This very thing the risings and settings of the planets — and above all of the Sun — together with those fixed stars, evidently prove year by year.

And this is the customary governance of the worldly economy in nature, which — once prepared together with the heaven, from the divine word of the Omnipotent, in these secondary causes for the cherishing, changing, multiplying, etc., of that [economy] — is thereafter continually sustained by the same word right up to the consummation of the age. Concerning this matter the philosophers, and above all Trismegistus, once discoursed. Indeed, more and more particular things will at some point be inferred for astrology, from the consideration of that same [author], where it should ever rightly be practiced. But now [I return] to the subject of new phenomena.

That which is annual, then, and which — because of its continual periods — is less unusual: that governance of the worldly economy in nature disturbs less, even by the opposition of aspects, than what happens rarely. This we are convinced of, among other things, by the conjunctions of the superior planets, which occur only rarely, by the manifest indication of their effects. And why, pray, should not those phenomena which are much more unusual also [do so] much more — phenomena which either, like the new stars, associate themselves with the fixed stars, or, like the more powerful comets especially, pour themselves into the planetary region and, by their motions (for the most part transverse and distorted), against the advance of the planets, variously cut across the courses or paths of these? Why, I say, should the governance of those

[phenomena], penetrating all the way around to the lowest earth, not disturb these new celestial portents, and incline and compel them utterly toward unaccustomed things?

For although comets are the offspring of the Sun and the planets, yet because they are for the most part generated under the sinister aspects of these same bodies, they are rightly judged by our lord Tycho [Tycho Brahe], and by certain other learned men, to be nothing but bastards — that is, spurious, degenerate, and as it were alien — in this order of the worldly ministry [lithurgia].

Accordingly, while most of the planets contribute their effort, each according to its own nature, to cherishing lower things and to freeing the air from the noxious humors that perpetually exhale from the earth and the seas — and above all the Sun, as a star most kindly toward earthly things, and especially toward the human race — by contrast the new celestial phenomena, and before all comets, by their ill-omened generation, and by no different irradiation and motion, and finally by [their effect upon] noxious vapors

by drawing in noxious matter and by scattering through the air things that are not fully concocted, they grievously distress earthly affairs, since these operations of theirs mingle unhappily with the qualities of the elements. From this arise storms and crop-failures, and likewise δυσκρασίαι [Greek: dyskrasiai, "bad temperaments," disorders of the elemental balance] spread abroad through the remaining qualities of the temperaments. From these, in turn, the dispositions of human beings — which tend to follow these qualities — drive worldly mortals into various unlawful appetites. Hence arise wars, conflicts between kingdoms, and the overthrow of commonwealths, together with countless other crimes and pollutions, as well as diseases and other evils of that kind.

These conclusions follow from the purely physical consideration of the new phenomena — and above all of comets — apart from whatever mystical meanings they carry, and apart from the more fully divine providence at work in the world that also bears upon their generation. [Margin: The cause that produces the phenomena.] Some of these matters I have treated above; others, given the opportunity, I shall treat more fully below. For now I will pass on to the division of the effects, or significations, that are drawn from these new heavenly bodies.

The effects of the new phenomena — that is, both of new stars and of comets, whenever by the divine will they appear in the sky — are marked by the force and power of their effects, or rather of their significations; or else by the manner in which these are displayed at certain places and times. They are treated, as it were, as predictions concerning future events. Now the significations of the new

phenomena are either supernatural or natural. In the supernatural order, divine providence reigns — or what amounts to the same thing, the divine decree — which is exercised precisely in the prophets' oracles concerning the things that are to come upon the world all the way to the consummation of the age. Of these things, and of greater ones still, these new temporal phenomena are without doubt the signs. As they were treated more fully in the immediately preceding chapter, they need not be repeated here.

Among the other significations of the new phenomena, the first to be noted in general is the place of the new heavenly body with respect to longitude and latitude relative to the ecliptic, and likewise with respect to the equator — so that it may be properly recognized, by reference both to the ecliptic and to the equator, which region of the world (and which point of the spherical surface) a new star or comet has occupied throughout its whole course. For this bears above all on better grasping and analyzing the significations of these bodies. For one thing, it is through the ecliptic, or zodiac, that the aspects of the Sun and planets toward the new phenomenon are perceived — from which the phenomenon is thought to derive most of its origin; and furthermore, by reference to the signs of the zodiac, the earthly regions and tracts are judged, following Ptolemy and others, which are most apt to be affected when the new heavenly body runs through such signs. For another, it is by reference to the equator, when one considers the daily revolution of the new body and, within it, the zenith point that the comet reaches on each successive day. Hence, besides this body's diurnal motion in its own circle and its progress along the ecliptic, its declination is also required, as will appear later in the examples.

Moreover, the astrologer is bidden, not without reason, to examine the celestial houses [*coelestia domicilia*] in the figure of the heavens [*thema*] cast for the moment of the new body's appearance (provided its time can be obtained), and likewise for the new moon, or the full moon, next following — these being restricted to the dimensions of those two circles under the given elevations of the pole. So much so that the knowledge of all these is required — besides the nativities [*genethliaca*] of the leading men of those regions which the effects, or significations, of the new body are observed especially to influence — as certain of these matters will also be recounted more fully below.

Further, the nature or quality of that celestial asterism — whether within the zodiac or outside it — to which the new star is attached, or in which the comet either takes its rise or chiefly directs its motion, must be carefully investigated and confirmed by the citing of examples. For besides the fact that the signs of the

zodiac are wisely distributed into as many triplicities [trigona] as there are primary qualities in nature —

namely:

	Hot	Cold
as	□ □ □	□ □ □
	Dry	Moist
as	□ □ □	□ □ □

— so that, from these as it were domiciles and properties which are assigned to the individual planets, an astrological judgment may be woven together: the nature of each individual asterism must also be regarded. Even their very names, though they appear to have been imposed on them from the outcome, are nonetheless not idle in producing the effects, but are sometimes quite σημαντικά [Greek: sēmantika, "significant," meaning-bearing], so that of these the common saying is verified: *Names often agree with their things* [*Conveniunt rebus nomina saepe suis*]. But there is no need to treat here at greater length of the qualities and natures of the signs of the zodiac, of the asterisms, and of the planets, by which the new heavenly bodies are affected; rather, on those matters Ptolemy, in his four-book work [the *Tetrabiblos* / *Quadripartitum*], is above all to be read and consulted.

Finally, as to the aspects or configurations of the new heavenly bodies with the remaining stars — and above all with the planets — let judgment likewise be made according to the nature of these. Hence the quartile aspects [quadrati intuitus] with the malefics Mars and Saturn for the most part mark the significations of the new body with an ill-omened outcome, whereas benefic aspects on both sides mitigate them. And since these aspects are indeed detected through the rational division of the circle, it is not without reason that through these same

—through these same aspects a judgment is also taken concerning the influences of these heavenly bodies, even of these adopted ones [i.e. the comets, regarded as illegitimate offspring]. To these matters considered in general is joined the observation of the magnitude and color of the new body, and from this too the judgment concerning the magnitude of the events is confirmed: a new star, indeed, by comparison with some other fixed or wandering star to which it is thus likened; and a comet similarly, whose color and shape of head for the most

part recalls that planet which had contributed the greatest force to its rising, that is, to its generation.

There follows the display of the significations drawn from the new phenomena. This is subject to a twofold consideration: namely, either of the places, or earthly tracts, to which the effects of the new phenomena are chiefly applied; or of the times, by which such effects are determined. Now for indicating the places, or inhabitants, of these effects in a general way, those signs are to be noted to which the most approved astrologers — and especially Ptolemy — had assigned the individual regions or earthly districts; and these are to be carefully considered according to the nature, the qualities, and the dispositions of the men under them, whenever the new body has been found either to rise, or to be determined in its course, within those signs. Furthermore, for the knowledge of the places, the knowledge of the zenith point is required — the point which the new body, by its motion, will have shown at the syzygy of the luminaries immediately following its rising. From this the place on the terrestrial globe is elicited accurately enough, as lying under such a meridian or zenith. For which moment, as I said above, it is best to construct the figure of the heavens, and to consider carefully the signs of the zodiac, as well as the situation of the luminaries and of the remaining planets in the individual celestial houses.

As for the fact that Ptolemy bids us observe the direction of the tail in comets — toward which region of the world it points — and from this to derive a judgment concerning the earthly tracts that it thus indicates: this without doubt is mistaken, since that whole orientation of the tail proceeds from the Sun, as was shown above. But the magnitude of the tail, and the observation of the entire comet's motion across the regions of the world, is not without value; for since the former argues an abundance of matter, while the gradual dissipation of the comet attends the latter, there will be no lack of a hidden principle of influence, by which the underlying earthly places are affected in no small degree with respect to the meridian, or the zenith. And so much for the places assigned in a general way, which are most drawn upon for the perceptible sense of the new body's influences.

Further, the special places to which the influence, or significations, of the new body most reach are the Ascendants, and likewise the Apheta [Aphetica] and the giver-of-life points [Hylegialia] of those who are born under such configurations — but above all, as I suggested above, of Emperors, Kings, and princes in the worldly commonwealth. For since these are public persons — that is, men holding public office — it is not by empty reasoning, nor without the authority of

Ptolemy, indeed by experience itself, that such significations of the new phenomena are applied most of all to them. So much so that these things, wrapped up in the decrees of the anareta [anaeretae] — that is, the slayer-planet — which come forth from the directions, for the most part cut short the threads of life and hasten on death or danger to these illustrious persons all the sooner, as will appear in the examples below.

Thus far concerning the earthly places to which the significations of the new bodies are wont to be applied from indications of this kind. There follow the times by which astrologers measure the future events from such heavenly bodies. These are determined either in general or in particular. In general, indeed: as, if the new phenomena appear first in the eastern region of the sky, their effects will overtake the world early; if in the west, later. And insofar as they may lie in the remaining regions, the times are accommodated to such a consideration as well. In the case of new stars, however, the measurement of times extends to far more years than in the case of comets — so that, concerning the effects of the new star of the year 1572, the matter is verified up to this present limit of ours by the most certain prediction of the lord Tycho. [Margin: T. B. in the conclusion of *Progymnasmata* book 1.]

But the special times, which determine the effects of the new body, are referred to the decrees of the directions [directionum decreta], which are the distinguished documents of astrological certainty. Here the direction is made either according to the zodiacal circle — as in new stars, with the conjunction of the superior planets among themselves, whether preceding, or rather most nearly following, with which a new star has a great kinship: as in that same star of the year 1572, with the conjunction of Saturn and Jupiter that occurred in the year afterward, 1583, in Pisces. Likewise in the new star of the year 1604 in Sagittarius, similarly, with the conjunction of the superiors Jupiter and Mars in the year 1600 in Leo (♌), a fiery sign — about which John Kepler [Johannes Kepler] indeed wrote, but not, as it seems to me, fully enough; for he did not bring before the public the operations of such things, since scarcely from that time on has there been any rest in Russia, Sweden, Livonia, Poland, Denmark, and afterward indeed throughout the whole Roman Empire, from wars and dreadful conflicts, especially under the pretext of religion.

But such a direction is wholly determined according to the Sun's diurnal motion in the ecliptic at that time, for each successive year. Comets, however — since they generally last for a span of two or three months — accordingly have their decrees of events derived from these partly by being rolled along the diurnal

circle, partly by enclosing their measure within their own proper motions: and this either according to their diurnal motion considered in their own circle, or rather by collation of that motion with the zodiac, which is, as it were, the proper circle of men and of all living things. Here for the most part some

—some [indications] are wont to present themselves, through which the comets will unfold their decrees at various and fixed times, provided they are rightly examined — not by empty pretenders, but by skilled astrologers — for the assaying of the celestial harmony with these lower things. And in this manifold path of the directions, back and forth, that which experience has formerly proved most of all in other cases must be followed; and much here must be left to the astrologer, who is borne by a natural inclination toward discerning, judging, and bringing forth in their own times the effects of the heavenly bodies. Yet in such a way that he does not — by some rash impulse of his productions, as it were by an Enthusiasm [Enthusiasmus], such as certain men otherwise ignorant of the art are wont to do — contrive and press some singular things as effects of the stars, and pronounce them by his own inept and immature judgment. Rather, let every wise and circumspect man judge nothing until he has surveyed the whole series of gathered causes universally, and thus too let him as far as possible confine himself within the bounds of a general judgment. [Margin: Ptolemy, *Centiloquium* 1.] *For those alone who are inspired by the divine foretell particular things [Soli enim numine afflati praedicunt particularia].* Meanwhile, those matters which are more special, and those which are supernatural and which have a symphony with Sacred Scripture (as was said above), are to be set forth not so much by prediction from the stars as by timely admonition of the people, from the Word of God and the law of nature, if this be violated; and yet nonetheless they are to be in some measure determined by the circumstances of the directions, concerning the times of those things which fall within the judgments for a new body. And with these things it will suffice to have completed the first part of this treatise, according to our purpose and the powers of our talent.

CAP. IX.

On the designation of the path of the comet of the year 1607, from our own observations; and on the slender parallaxes of the same, from which we conclude that this comet was within the celestial region, far above the Moon.

The causes and effects of the new phenomena having been sketched in a general way through the preceding chapters, as the first part of this treatise, there follows the second part of the work undertaken, in which we shall subjoin the promised examples concerning comets — concerning, I say, the two last comets, namely

those of the years 1607 and 1618. For the places of the new stars which have appeared in our age, hitherto in Cassiopeia, Cygnus, and Serpentarius [Ophiuchus], are found to have been sufficiently designated by others, especially by our lord Tycho [Brahe], and by John Kepler, mathematician to the Emperor [Caesarius]. But the comets we shall now touch upon were not only remarkable and astonishing among those that have blazed forth in the sky in this latest age — and after the death of our Atlas [i.e. Tycho Brahe] — in their magnitude, motion, and duration; they also, by a course almost contrary [to the usual], and by their encounter with the asterism of Boötes, beyond doubt signify certain things beyond the rest. These it befits anyone skilled in Astronomy and Astrology, according to the powers of his talent and his gift of interpretation, to treat soberly rather than to conceal — and rather than, in supine negligence, to despise along with the common crowd. For portents of this kind are most especially those celestial signs through which God, in these last times, warns men of their false security and at the same time sternly rebukes them, as I had noted above, and as I shall deservedly elaborate at greater length below. But now I will pass on to describing, from our observations, the earlier comet, which in the year of the Lord 1607, in the months of September and October, took its rise in the North and turned its course rather toward the South.

In the year of our Savior Jesus Christ 1607, on the 18th day of September, in the evening — scarcely one week after I had fully completed the construction of our sextant, the fifth of those of the cubit-measure radius, of which I had made mention in the *Astronomia* — and after I had at last made several experiments toward taking the distances of fixed stars from one another, to test its infallible reliability; indeed, after I had observed Jupiter near its acronychal position with the same instrument on the 14th of September, and had continued these observations on the following days, had not clouds and mists for three days interposed themselves here in our horizon at Copenhagen [Haunia]: then, the clear weather having returned on the evening of the 18th of September, I first noticed and at the same time observed a tailed comet in the North, which in its magnitude almost rivaled Jupiter itself as to appearance, but in its color — darker, that is, a somewhat livid hue — rivaled Saturn itself. It also had a fairly long and dense tail, which the Sun's rays drove directly into the region turned away from the Sun, and this throughout the whole duration of the comet. For this evening, after the 7th hour, when the comet was placed scarcely the distance of one Moon below the small unformed star [parva informis] behind Ursa Major — which I noted here with no less diligence than John Kepler did in Bohemia on the same evening — I also perceived that the comet, with its tail, pointed toward two unformed stars below the root of the tail in Ursa Major, the Sun of course passing

through the 5th degree of Libra. The comet's tail was also dense, as I said, and perhaps more compact than the rest are wont to be, and of the same color as the comet's head itself, namely livid and somewhat Saturnine. But now I come to our observations.

There follow the observations.

So, on the evening of the same day, the 18th of September, at the 7¼ hour, the comet was found in alignment with the small unarmed fixed star so named [parva fixa inermis], in the manner we expressed before, such that it was scarcely distant from it by the diameter of one Moon, or 34 minutes below it, and inclining a little toward the east.

That this star's longitude of 12 degrees 16 minutes □ [Virgo], and its latitude of 40½ degrees North, were assumed well enough by Kepler, our following observations also prove. For about the same time, namely the 7th hour, the comet was distant from the preceding upper [star] in the square of Ursa Major, by the sextant, 24 degrees 32 minutes, the observation being repeated in the same moment; likewise from the lower of the following [stars] of the same square of Ursa Major, 14 degrees 14 minutes. But since these two stars were arranged too nearly in a straight line with the comet — so much so that the comet's place could scarcely be obtained without error, as we warned in our doctrine of the sphere — therefore, the comet's latitude being supposed at 40 degrees 5 minutes (namely by the estimate of sight) in the alignment which it had with the said little star, whose latitude was 40 degrees 30 minutes, there results, by the reckoning of spherical triangles, the comet's longitude from each distance from the [stars] named in the square of Ursa Major as 11 degrees 57 minutes □ [Virgo], agreeing well enough separately, when the longitudes and latitudes of these [stars] for this time are also taken from the corrected canon of fixed stars.

Furthermore, on the following day, morning and evening, the sky was not clear. For with the north wind blowing, clouds continually rose up and hid the comet.

But on the 20th of September, in the morning, with clear weather somehow restored, when the altitude of the middle star in the tail of Ursa Major was 34 degrees 36 minutes, observed by our geometric quadrant — to which the time most nearly corresponds, the 5½ hour — and the comet's altitude in this horizon was about 17 degrees: the comet was distant from the middle star in the tail of Ursa Major by 20 degrees 46 minutes, by a repeated and limited observation. Moreover, at the same time the comet's tail, projected from the Sun, pointed fairly precisely to the last star in the tail of Ursa Major, while the Sun was in 6 degrees

22 minutes □ [Libra]. But since more readings could not be had by us this morning, both on account of the twilight and especially of the clouds rising up, accordingly, from the data given, we proceed to inquire into the comet's position by this method — which therefore I here leave and pursue in full, for the benefit of students, since it is not set out in our doctrine of the sphere.

Diagram. A spherical-trigonometric construction for determining the comet's position. Let A be the pole of the ecliptic; B the middle star in the tail of Ursa Major; C the extreme [last] star of the same tail. From the pole A, through the distance of a quadrant, a portion of the ecliptic H–E is described; and with the Sun placed at point E of the ecliptic, and the line E–C on the sphere given, it is plain that the comet is found on that same line, its place being determined at D — namely, by the distance from the middle star of the tail of Ursa Major, B–D. Since we intend to have the point or position D itself on the sphere with respect to the ecliptic, accordingly, with quadrants dropped from A through those four points B, C, D, E, and these marked on the ecliptic with the notations H, G, F, E, it is plain that the comet's latitude is found along the arc F–D, and its longitude through the difference between H and F, or along the angle B–A–D; again, along G–F or F–E, for the operation here can be carried out in various ways.

- A (top apex): pole of the ecliptic [polus eclipticae]
- B (upper left): middle star in the tail of Ursa Major [media in cauda ursae majoris]
- C (interior, below B): extreme star of the tail of Ursa Major [extrema cauda ejusdem]
- D (lower interior): the comet's position
- I, K (interior points on the constructed lines): auxiliary points of the construction
- H, G, F (along the bottom arc, left to right): the feet of the dropped quadrants on the ecliptic
- E (bottom right): the Sun's place on the ecliptic [sole in E puncto eclipticae]
- (Bottom-right hatched arc): the ecliptic portion H–E

First, then, in the triangle B–A–C, from the known sides B–A and C–A, together with the angle B–A–C, the sought distance of the stars in the tail of Ursa Major, B–C, is found to be 6 degrees 41 minutes.

Next, in the triangle B–C–A, the sides being given, the angle B–C–A is sought, whose complement to two right angles (or 180 degrees) is the angle B–C–G. The

angle B–C–A is found to be 68 degrees 29 minutes, and therefore B–C–G is 111 degrees 31 minutes.

Third, in the right-angled triangle C–G–E — having the right angle at G — from the given C–G (the latitude of the extreme star of the tail of Ursa Major, 54 degrees 25 minutes) and, besides, G–E (the distance of the same star of the tail from the Sun along the ecliptic, 15 degrees 4 minutes), the angle G–C–E is given by computation as 18 degrees 19 minutes. Adding to this the previously obtained angle B–C–G, the whole angle B–C–D is made up: 129 degrees 50 minutes.

Fourth, then, in the triangle B–C–D we have known two sides, B–C and B–D, together with the angle at C; therefore the distance from the last star of the tail of Ursa Major to the comet, D–C, will not lie hidden. As for the analysis of the proposed triangle, since it is conveniently resolved for the sought quantity by means of right angles, accordingly, with the arc D–C produced indefinitely and a perpendicular dropped from B to I, there will be — first, in the right-angled triangle C–I–B — the given [data] [Greek: δεδομένα (dedomena, "the given quantities")]: B–C, 6 degrees 41 minutes; the angle B–I–C, which is the complement of the angle B–C–D to two right angles, namely 50 degrees 10 minutes; together with the right angle at I. Therefore, first, the perpendicular [B–I] is given as 5 degrees 7¼ minutes; then the arc C–I as 4 degrees 38¼ minutes. Hence, in the whole triangle B–D–I, from the given B–D (20 degrees 46 minutes) and B–I (5 degrees 7¼ minutes) with the right angle at I, the whole side I–D is acquired as 20 degrees 9½ minutes; from which, when I–C (4 degrees 38¼ minutes), the side found earlier, is subtracted, the sought D–C remains as 15 degrees 31 minutes — namely the distance of the tail of Ursa Major and the comet.

Fifth, in the triangle A–C–D, since two sides are given — A–C (35 degrees 35 minutes) and D–C (15 degrees 31 minutes) — together with the angle A–C–D (161 degrees 41 minutes)...

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[the angle is found to be] 41 minutes, since this is the complement of angle G–C–E to two right angles, that is, to 180 degrees. From this, at last, the side D–A emerges as 50 degrees 31 minutes, the complement of the comet's latitude to the quadrant,

that is, to 90 degrees. The comet's latitude itself is therefore 39 degrees 29 minutes north.

Lastly, in the triangle B–A–D, since all the sides are given, the angle B–A–D is found to be 18 degrees 24 minutes. Added to the longitude of the middle of the tail of Ursa Major at B — which at this time was 10 degrees 2 minutes in Virgo [□] — this yields the comet's longitude as 28 degrees 26 minutes in Virgo [□].

And so, from our own observations, we possess the comet's position at the beginning of this discussion, that is, around 5½ hours after midnight on this day, established by the double computation. Now we pass to the following day.

Furthermore, on 21 September in the evening, around 7½ hours by the clock and with the altitude of the bright star of Corona Borealis [Alphecca] set right at 34½ degrees, the comet was 30 degrees 59 minutes distant from the middle of the tail of Ursa Major; and around the same time it was 16 degrees 41 minutes from the bright star of Corona Borealis [Alphecca]. These measurements were repeated by our sextant. With the positions of the fixed stars assumed for this time — namely, the longitude of the middle of the tail at 10 degrees 2 minutes in Virgo [□], the longitude of the star of Corona Borealis [Alphecca] at 6°44' □ with its latitude of 56°22' N, and the latitude of the former at 44 degrees 23 minutes — the position is found in the reduced canon. Carried out, then, by the method we set forth concerning the [armillary] sphere — when these data are subjected to calculation, reduced to the first detailed minutes, the comet's longitude comes out for the time noted above as 16 degrees 44 minutes in Libra [□], with a latitude of 37 degrees 14 minutes north.

For the rest, because the sky grew more cloudy on the following days, and because other obstacles besides intervened, I crossed over to Malmö [Malmogia], where I no longer observed the comet with this instrument, but noted it with strings drawn to straight lines with the fixed stars [Solis filaribus extensionibus] [?]. Accordingly, having obtained these three positions on the three earlier days, I first investigated them on a rather large globe — one nearly three feet in diameter — on which the comet could finally be received under the tracing of a great circle. By this mechanical handling, accurately carried out by means of a bent compass [circinum incurvatum] and a great-circle quadrant fitted to the globe, I found a point on the globe's surface that is established as the north pole of the comet's path, so that it had a longitude of 17¾ degrees in Pisces [□] and a latitude of 49½ degrees north.

To such a circle of the comet the following observations conformed as closely as may be. For on the first day of October, at Malmö in the evening around 6½ hours, the comet was distant from the western star in the left hand of Ophiuchus, by visual estimate, scarcely ½ degree, toward the west and the horizon. At that moment it stood in a straight line with Arcturus and with the eastern star in the same hand of Ophiuchus. From this, the positions of the fixed stars being reduced, the comet's position emerged for this time at longitude 18 degrees 45 minutes in Libra [♎] and latitude 31 degrees 2 minutes [north]. Again, since that star is the western and northern one in the hand of Ophiuchus at longitude 26 degrees 50 minutes in Scorpio [♏] with a latitude of 17 degrees 19 minutes north, and since finally in the same hand the star at longitude 28 degrees 3 minutes in Scorpio [♏] with a latitude of 16 degrees 30 minutes follows — from this the comet's longitude comes out as 26 degrees 30 minutes in Scorpio [♏], its latitude 17 degrees 40 minutes north. And here too it followed the ecliptic in such a way that it scarcely deviated noticeably from it, either downward and westward, or at most by 4 minutes.

Furthermore, on 2 October in the evening, around 6 hours, or a little beyond, the comet had advanced so that it appeared, as it were, at the apex of an obtuse isosceles triangle formed with the stars named individually in the hand of Ophiuchus — though it inclined nearer to the preceding of them. In apparent magnitude it was greater than any of these, but the trace of its tail was rather faint. By string measurement, it made, as to its tail's extension, a straight line with the next-to-last [star] toward the rising in Corona Borealis, of the fourth magnitude, and the northern [star] in the said hand of Ophiuchus; likewise a straight line with the lower head [of Serpens] in Scorpio [♏] and the eastern [star] in the same hand — which straight lines also argue the same isosceles-triangle form. At the said moment of this day they place it at longitude 27 degrees 20 minutes in Scorpio [♏], with a latitude of nearly 17 degrees north.

On 6 October in the evening, at 8½ hours, with the sky moderately clear, while I was still staying at Malmö [Malmogia], the comet was seen to make an isosceles triangle with the eastern [star] in the hand of Ophiuchus and the bright [star] at the next knee; for it deviated from the line between these two by about 1 degree as to vision, whence the comet's longitude was 0 degrees [?] in Sagittarius [♐] [?], and its latitude was 13¼ degrees north [ἐν πλάτει].

On 12 October at 6½ hours in the evening, and a little before — after I had returned to Copenhagen [Hafnia] — the comet seemed to be below the knee of Ophiuchus, at a distance of about 2½ degrees, or a little beyond. It made a straight

line with the lower [star] in the drawing-out of the throat of Serpens, and the upper [star] in the hand of Ophiuchus; likewise another with the lower and following [star] in the right shoulder of Ophiuchus, and the bright [star] at his left knee. From this, the positions of the stars being verified and laid down on the globe, the comet's longitude is established at $1\frac{1}{4}$ degrees in Sagittarius [♐] with a latitude of $9\frac{1}{2}$ degrees north. This is certainly the very position which Mr. Kepler [Johannes Kepler] had assigned to it from his observation — which I regard not without admiration, and from this I judge the smallness, or rather the nullity (so to speak), of the parallaxes.

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[would] hold. The comet, however, was quite faint, and scarcely any trace of its tail appeared, whence its longitude is gathered to have been about 2 degrees in Sagittarius [♐], with a latitude of 9 degrees north.

On 14 October in the evening at $[6\frac{1}{2}?$] hours, the comet was seen very dimly, in nearly the same position as on the preceding evening, so that its longitude could be reckoned at $2\frac{1}{3}$ degrees in Sagittarius [♐], its latitude 9 degrees north.

After this the comet was no longer seen by me. As for what Mr. Kepler [Johannes Kepler] writes — that around the end, namely from the observations of the 9th, 12th, and 16th of October, it was retrograde with respect to the ecliptic, and that moreover its latitude changed so remarkably from the 12th to the 16th, namely by three degrees — since its motion in the intervening days appeared to us here far slower, I made trial of Kepler's last observation. He writes that on 16 October, at $6\frac{3}{4}$ hours after noon, between the clouds, he snatched a glimpse of the comet, which then stood very low in the vertical, and at the same point was about half a lunar diameter farther west than the very knee of Ophiuchus, and that three, or at least more, of the distances of the two stars in the hand below that knee were seen [between them]. This observation indeed argues a longitude of the comet not less than $2\frac{1}{3}$ gr. ♐, nor a latitude less than 8 degrees north — all things carefully weighed and verified on the larger globe; but Kepler himself, too, is in doubt about this last observation of his.

Now we shall go back to those days which, with the aid of the Keplerian observations on this comet, recall it back to its origin. For it is certain that before 15 September it had not shone forth, since we recall that on the 14th day of that month we, attentively scanning the sky in that quarter too with our eyes, [saw nothing].

Since, moreover, it is highly consonant that this comet first began in Aries [♈] with a simple conjunction with the Sun [☉] [simplici ☉] — which is indeed found, by the restored motions, to have fallen on 15 September, at 15¾ hours, while the Sun [☉] was at 4 degrees 25 minutes in Aries [♈], with the comet opposite the Sun, running through 4 degrees 25 minutes in Libra [♎] by its own simple motion; at about which time there also occurred the conjunction of [Mercury ☿] [♿ ☿], and likewise the trine aspect of [Saturn ♄] [♅ ♄] and [Mars ♂] — therefore, Mr. Kepler's observations made on the evening of 16 September having been carefully reviewed, and traced back to this time by the tracing of a great circle, I find the comet then at 10½ degrees in Leo [♌] with a latitude of about 34 degrees north.

As regards the comet's diurnal motion, which will rightly be considered in its own circle or path: it was indeed at the start the most rapid of all, and then little by little relaxed something of it day by day, together with its magnitude — which agrees with the foregoing hypothesis of comets. For on 16 September, at 9 p.m., according to Kepler's observation carefully reviewed, it was here at 18 degrees 30 minutes [in Leo, ♌] with a latitude scarcely less than 36 degrees north, when we may likewise inspect the comet's path. And on 18 September, at 7½ hours in the evening, from the higher [observations], the comet's longitude was 11 degrees 57 minutes in Virgo [♍] with a latitude of 40 degrees 5 minutes. Hence the distance, or motion in its own circle, is gathered to have been 18 degrees 20 minutes, and therefore the diurnal motion 9 degrees in the same circle.

Furthermore, on 20 September in the morning at 5 hours, the comet was at longitude 28½ degrees in Virgo [♍], with a latitude of 39½ degrees. At this position, as in the case of the immediately preceding one, the comet's motion in its own path over 34 hours is found to have been 12 degrees 50 minutes, whence the diurnal motion scarcely comes out to 9 degrees.

Third, when the comet's diurnal motion is explored for the following day, namely 21 September: its longitude itself was found at 7¼ hours in the evening to be 16 degrees 44 minutes [in Libra, ♎] with a latitude of 37 degrees 14 minutes north. Therefore this position, when compared with the preceding one with respect to the comet's own path, had advanced along it by 14½ degrees over 38½ hours, and the diurnal motion comes from this to about 8 degrees 55 minutes — all this being reckoned and transferred in a broad manner and only as applied to our larger globe. From this one may observe that the motion of this comet was greatest and most rapid at the start in its own circle, and gradually became more relaxed. For from this time it decreased, quite noticeably and proportionally,

together with its magnitude, as I said. But so that not only the diurnal motion of this comet in its own circle throughout its whole duration may be had from the foregoing observations, but in addition also its longitude and declination — whereby a judgment about it may be more correctly formed — here is the following table, which will contain these in order.

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MOTVS COMETÆ ANNI MDCVII / diurnus ad 7 horam vespertinam, / RESPECTV [horum circulorum]

THE DIURNAL MOTION OF THE COMET OF THE YEAR 1607, reckoned to the 7th hour in the evening, WITH RESPECT TO [these circles]

Date	Of own circle G. M.	Of ecliptic G. M. S.	Of declination circles G. M.
Sept. 16	9 40	18 0 □	49 0 N.
17	9 30	0 0 □	47 0
18	9 10	11 50 □	44 fere [nearly]
19	9 0	23 30 □	39 15
20	8 40	5 15 □	34 20
21	7 50	16 30 □	28 30
22	7 0	27 0 □	21 20
23	6 0	4 0 □	16 40
24	4 30	9 30 □	12 20
25	3 40	13 30 □	9 30
26	3 0	17 0 □	6 20
27	2 30	20 0 □	4 10
28	2 10	22 30 □	1 50
29	2 0	24 0 □	0 25
30	1 50	25 30 □	1 0 Mer. [S.?)
Oct. 1	1 40	26 40 □	1 50
2	1 30	27 30 □	2 40
3	1 25	28 20 □	5 0
4	1 15	29 10 □	6 45

Date	Of own circle G. M.	Of ecliptic G. M. S.	Of declination circles G. M.
5	1 0	0 0 □	8 15
6	0 45	0 50 □	9 30
7	0 40	1 20 □	10 30
8	0 30	1 30 □	11 fere [nearly]
9	0 25	1 40 □	11 10
10	0 20	1 45 □	11 18
11	0 18	1 50 □	11 25
12	0 15	1 55 □	11 32
13	0 12	2 0 □	11 40
14	0 10	2 5 □	11 50
15	0 8	2 8 □	12 0
16	0 5	2 10 □	12 10

Parsing notes: The table has three numerical column-groups. Column 1 ("Of own circle") is degrees and minutes of diurnal motion along the comet's own great circle. Column 2 ("Of the ecliptic") is degrees, minutes, and zodiac sign of ecliptic longitude. Column 3 ("Of declination circles") is degrees and minutes of declination, north (B./Borea) unless noted. The "S." (seconds) sub-column in the ecliptic group is blank/zero throughout. The 30 Sept. declination row reads "1 0 Mer." — "Mer." likely abbreviates *meridionalis* (southern), marking the crossing from north to south declination as the comet crossed the equator (cf. the text: "the comet crossed the equator on 29 September"); flagged as a possible declination-sign transition. Several zodiac-sign glyphs in the degraded scan were cross-checked against the surrounding prose (Leo → Virgo → Libra → Scorpio → Sagittarius progression).

Right column (prose continues):

And this diurnal motion of the comet, with respect to these circles, had been [in latitude, ἐν πλάτει]. Nor indeed is greater precision needed here for accomplishing our purpose.

It should be noted, moreover, that the said comet crossed the equator on 29 September, a little after, at 236 degrees [?] [recte: 23⅔ degrees □? — the longitude where it crossed; see anomaly c]. Indeed it was first clearly observed to have

deviated from the path of a great circle on 1 October, when it had nearly reached the stars in the left hand of Ophiuchus. Finally, as regards the beginning of the comet estimated by us: at $10\frac{1}{2}$ degrees in Leo [□] the declination corresponding to that point was 49 degrees 50 minutes north.

Now, lastly, in this chapter we shall treat briefly of the parallaxes of this comet, from which it will be established that it was not sublunary, but existed in the heavens themselves.

DE PARALLAXIBVS hujus cometæ.

ON THE PARALLAXES of this comet.

Although it did not fall to us, with this comet, to have such observations arranged in order — observations from which we might subtly work out its parallaxes, whatever they might be, according precisely to what was set forth in the doctrine of the [armillary] sphere — nevertheless, among the rest, that one single observation of the comet's position which we made on the evening of 18 September at $7\frac{1}{4}$ hours, jointly as it were with Mr. Johannes Kepler, who was then at Prague in Bohemia, will suffice for what we are here aiming at, at the least.

Our observation at that time has been recited above: namely, that the comet at that time was seen by us scarcely half of one Moon, that is, 34 minutes, below and slightly toward the rising in that revolution, [reckoning] from a small unformed star behind Ursa [Major]. But Kepler writes the following:

"On 28 September (understand the new style and the evening time), the comet was seen to stand below the small unformed star, the companion of the great unformed [star], between the tails of Ursa and Leo; and it was distant by a diameter of the Moon, on the line which runs along the last [star] of Ursa's tail toward the hand of Arcturus [Boötes]." Thus he. From this it is to be understood that there is scarcely a perceptible difference between the comet's position at this time as observed by each of us. This also appears above from the comparison. For when we, from our other observations too — made for the comet on the same evening and hour — investigated its longitude as 11 degrees 57 minutes in Virgo [□] with a latitude of 40 degrees 5 minutes, Kepler here afterwards adds and concludes, from his own observation, that the comet's longitude was...

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[that the comet's longitude] was 12 degrees 0 minutes in Virgo [□], its latitude indeed 40 degrees 4 minutes north — the difference of which from ours scarcely extends beyond 3 minutes in a vertical circle, even at most. Furthermore, since Kepler puts the comet's longitude less than the longitude of that small unformed star to which it was joined by 16 minutes, the comet at the same time also preceded in the straight line which he mentions; nor need the time at which this observation was made at Prague differ much noticeably from that when the observation was being made among us. For its diurnal motion argues as much.

These things established: since Prague is distant scarcely 6 minutes of time toward the east from the Copenhagen [Hafnia] meridian, but the pole's elevation there is 50 degrees 6 minutes, here 55 degrees 43 minutes — the difference therefore being 5 degrees 37 minutes — there was at that time a difference of about $5\frac{1}{2}$ degrees [?] in the circle of altitude; and the comet's altitude then at Prague was about 20 degrees, but here $25\frac{1}{2}$ degrees, all things considered in a broad manner. For although these things are conceived by me for the present without the assignment of azimuths — which is especially suited to the precise working-out of parallaxes — they nevertheless have proof enough to convince Aristotle [Aristoteles], if this comet had existed below the Moon, in the uppermost boundary of the air (as the Peripatetics will have it). For the demonstration and the numbers, the premises are required in this way:

Diagram. A geometric construction for the comet's parallax demonstration. A semicircle is drawn from center A, representing the cross-section of the Earth's globe and the heavens above it. The lower horizontal diameter passes through points G, E, A, F, H from left to right. From center A rises a vertical, with the comet at D near the top of the inner arc. Two observing stations, B (Prague) and C (Copenhagen), lie on the small inner arc near A; apparent horizon lines I–K (through Prague) and L–M (through Copenhagen) are drawn. Straight lines run from center A to B, C, and D, and D is connected to both C and B, forming the two plane triangles A–D–B and A–D–C used in the parallax computation.

- A (center): center of the Earth / construction center
- D (top, inner arc): the comet, under the common vertical
- B (inner, near A): the city of Prague in Bohemia
- C (inner, near A): Copenhagen [Hafnia] in Denmark
- E (on lower diameter, near A): inner endpoint of the Earth's-globe radius E–A

- F (on lower diameter, right of A): point on the Earth's-globe semicircle E–F
- G (left end of lower diameter): left endpoint of the upper-air-region circle G–H
- H (right end of lower diameter): right endpoint of the upper-air-region circle G–H
- I, K (upper left / right): endpoints of Prague's apparent horizon line I–K
- L, M (right): endpoints of Copenhagen's apparent horizon line L–M

[the radius] E–A is known to mathematicians as 860 German miles [milliaria Germanica].

Furthermore, from the same center A, at the distance A–G, let the circle G–H be drawn, determining the uppermost region of the air, while E–A is increased by the magnitude E–G of 13 German miles — which the opticians, from observation of the twilights, trace as the extreme sublimation of the terrestrial vapors (out of which Aristotle would have comets composed), as we showed in our doctrine of the [armillary] sphere, Book 2, Chapter 9. The radius A–G will therefore be 873 German miles. Hence, with the locations of the observations made disposed on the surface of the terrestrial globe, let B be the city of Prague in Bohemia, C [the city] of Copenhagen [Hafnia] in Denmark; and likewise, apparent horizontal lines being drawn — namely I–K through Prague, L–M through Copenhagen — let the comet finally be placed at D under the common vertical (since it was insensibly distant from it). The arc I–D will therefore be 20 degrees, but L–D $25\frac{1}{2}$ degrees, by the comet's altitudes at both locations given above.

These things being established, let straight lines be drawn from center A to B, C, and D; likewise let D be connected with C and with B; and there arise the plane triangles A–D–B and A–D–C. Of these two triangles, the angle of parallax at D is to be elicited separately from the given premises, so that its difference, with respect to what we intend to argue, may be had. First, then, in the triangle A–D–B, since the given quantities [δεδομένα] are:

- A–B, 860 German miles
- A–D, 873 German miles
- the angle D–B–A, 110 degrees 0 minutes — namely, the right angle I–B–A increased by the angle D–B–I, that is, by the altitude of the Prague observation of the comet, 20 degrees —

then the angle A–D–B will not be hidden: 67 degrees 46 minutes.

Again, in the triangle A–D–C, the angle D–C–A alone being varied, and increased by $5\frac{1}{2}$ degrees (by which the comet's altitude here was higher than at Prague), the angle D–C–A becomes $115\frac{1}{2}$ degrees. From these given quantities, then, the angle A–D–C comes out as 62 degrees 46 minutes, and therefore the difference in the angle B–D–C is 5 degrees and a fraction [+]. And certainly, by this same nearly equal distance in the circle of altitude — beyond a diameter of one Moon — the comet here would have been torn away from that little unformed star, and would have stood below it for Kepler, if the comet had been some meteor. And conversely, for us, by the same difference (the space of one Moon excepted), the comet would become more elevated in the vertical circle [relative to] that little star.

But the difference which the observations made on both sides demanded can scarcely be reckoned beyond 3 minutes. So that I think it easily established from this that Aristotle's hypothesis about comets is by no means legitimate — however fiercely it has been defended in the Schools and the Academies down to this very day by certain Peripatetics, that is, by the common run of philosophizers; to such a degree that not even these did our Tycho [Tycho Brahe] manage to lead back into the way of truth, through so many demonstrations drawn from the new stars and comets that were observed in his time.

Moreover, the same thing we are able to gather from the proportional change of the proper motion and latitude of this comet, together with its steady advance along nearly a great circle almost to the last days of its disappearance — indeed, plainly to establish, that this comet was celestial; although, to the extent that it had climbed up into the heights through the ether, the observations of it, made less precisely and less timely, scarcely permit us to reason. Therefore we shall now, with the thread broken off, hasten on to the phenomena of the comet of the year 1618, likewise to be explained from our own observations, so that at last, in the final place, we may set forth our judgments about both, such as they may be. For of these comets...

the things signified by comets are without doubt bound together by some link — a fact which, among other indications, the oblique courses of these bodies across the ether attest, and prove in remarkable ways.

CAP. X.

On the phenomena, parallaxes, distance from the earth, and magnitude of the comet of the year 1618, gathered from my own observations; to which I append a report of this comet's having also been observed in Persia.

We turn now to that astonishing and fateful comet which, in November and December of the year 1618, shone upon nearly the whole world. I do not recall that the phenomena of any single comet were ever before observed by so many people at once. And although the observations made by all of them were neither uniform nor, in most cases, carried out with sufficient precision and diligence, their effort taken as a whole deserves praise. For with eyes that did not blink — unlike the rest of the common crowd, that scorner of celestial signs — they beheld so plain and conspicuous a messenger of divine mercy and, at the same time, of divine vengeance. Each man, for his own part, diligently tracked and noted the comet's appearance, its rising, its course, and the path by which it withdrew across the ether, and entered these things into his annals or writings for posterity.

I too was the more careful to do the same, because our most Christian sovereign had taken pains to enjoin this duty upon me — not, I believe, out of any superstition (from which the mind of our present most merciful king is removed by countless leagues), but rather with this end in view: that we should by no means despise, or unhappily consign to oblivion, so illustrious an envoy of Him who is mightiest of all kings, a sign at once of divine wrath and of a kind, fatherly invitation to timely repentance.

As to the time when this comet first began to rise — whether at the conjunction with Saturn [♄/♅] which preceded our observations by three days in the sextile of Mars, or at the opposition of the Sun and Saturn, which had occurred about a day and a half earlier still, or finally (and this is still more credible) before the start of 12 November, when it had first risen at the quadrature of Jupiter and the Sun, which the trine aspect of Mars and Venus soon followed — the answer is largely a matter of conjecture. For not only did a rumor of this comet's appearance grow strong some days before 6 November, on which day we first observed it; but we also learned that it had been observed in Persia around 12 November, as will be set down later in its proper place. Nor indeed, had it begun within 12 November or a little before, would it have been possible for us to observe it before the 20th of that same month, on account of the cloudy, rainy, and windy state of the air over our hemisphere during that interval.

Certainly on 12 November, around the very quadrature of Jupiter and the Sun, and on the several following days — about which time four ships, with a tender added, set sail from this Danish harbor for the East Indies — they did not, however, pass out through our Hellespont [the Sound] until after 25 November, owing to a contrary wind and frequent squalls; for so violent was the storm

during that interval that other ships here and there were dashed against the sandbanks and shores of our Baltic and wrecked.

Therefore, on the morning of 21 November, between hours 5 and 6, I first caught sight of this comet — for the sky throughout the preceding ten days had been, as I said before, cloudy for us and thick with rain-showers. As to its apparent magnitude, the comet was then somewhat larger than Spica [α Virginis], and it stretched out a tail, to the eye, the height of a rather tall man; this it projected fairly directly into the region opposite the Sun, which was then in the 9th degree of Sagittarius [$\square$]. The color of the head inclined toward red, so that it was about midway between the color of Mars (which was visible and observed by us at the same time) and that of Spica.

That same morning, at the comet's altitude of 18° , to which corresponds the time of hour $5\frac{1}{2}$, the distance of the comet from Spica, measured through our sextant, was $24^\circ 56'$, and likewise from the bright star of the Crown [α Coronae Borealis] $32^\circ 41'$. So that you may be assured of the instrument's accuracy in exhibiting the distances of the stars, we observed shortly afterward these distances of the fixed stars, repeated and limited several times: between Spica and Arcturus, with which the comet then lay almost in one vertical circle, the distance was $33^\circ 41'$, a distance very near the truth.

Now, on the assumption of the following fixed-star positions — Spica in longitude $18^\circ 31'$ Libra [$\square$] with latitude $1^\circ 59'$ south; the bright star of the Crown in longitude $6^\circ 53'$ Scorpio [$\square$] with latitude $44^\circ 23\frac{1}{2}'$ north — there results from this, by the triangular computation in the manner we set forth in the doctrine of the sphere [in the spherical doctrine], a comet longitude of $9^\circ 26\frac{1}{2}'$ Scorpio [$\square$], with latitude 11° [?] north. And this, as we said, was for 21 November, hour $5\frac{1}{2}$ in the morning.

On the morning of 22 November we did indeed snatch a few glimpses of the comet between the clouds, but we did not observe it, because the intervening clouds robbed us of the opportunity.

On the morning of 23 November, from hour 5 until 7, we carefully observed the comet as to its distances and altitudes. It appeared to be of the same magnitude as before, and likewise of the same color, except insofar as the Moon's brightness had affected it somewhat. We also measured the length of the tail at about 20° , or a little more, although we could scarcely make out its extreme flicker with the eye.

At the comet's altitude — through our geometric square [quadratum geometricum] — of 20° , to which corresponds the time of hour $6\frac{1}{2}$ approximately, these were its distances from the fixed stars, taken several times through our sextant and carefully limited:

- Distance of the comet from Spica: $25^\circ 52'$
- Distance of the comet from the bright star of the Crown: $27^\circ 28'$

Hence, by the induction of triangles, there results a comet longitude of $6^\circ 24'$ Scorpio [♏], latitude $16^\circ 56'$ north.

On the morning of 24 November, between hours $4\frac{1}{2}$ and 7, the sky being quite clear except that the Moon was shining, we again observed the same comet. Its head was sufficiently round, and this morning the tail was extended at length, holding its course beneath the middle star in the left knee of Boötes; and the extreme flicker of the tail appeared to the sight to be still more drawn out the following day. Moreover, the magnitude of the comet's head now appeared greater than Spica itself, although under the high dawn it was not so long visible.

As to the observations on this day, in the distances from Spica and the Crown, often repeated, we observed at the comet's altitude of 21° , to which corresponds hour $6\frac{1}{2}$, its distance from Spica to be $26^\circ 52'$, and likewise from the bright star of the Crown $24^\circ 54'$. From these, with the calculation of triangles duly applied, the comet's longitude comes out as $4^\circ 53'$ Scorpio [♏], with latitude $19^\circ 33'$ north.

Finally, in order to explore in some fashion the parallax of this comet on this day, we afterward continued to take its distance from Arcturus in the same vertical circle, which was as follows, at the altitude:

Comet altitude	Distance from Arcturus	
$24\frac{1}{3}^\circ$	$18^\circ 13\frac{1}{2}'$	
$26\frac{1}{3}^\circ$	$18^\circ 12'$	(distances)
(somewhat after)	$18^\circ 10\frac{1}{2}'$	

Now, since we see that the comet here altered its distances under one vertical circle with Arcturus very little or not at all — beyond what must be attributed in the meantime to the diurnal motion — we readily conclude that the parallax of this comet, even at this time, was no greater than would prove that it was carried far above the Moon, in the ether. But concerning other observations, from which

the parallaxes in this comet are more skillfully deduced, we shall treat at the end of this chapter.

Further, from 24 up to 28 of this month the sky was continuously cloudy by night and morning, so that the comet appeared to us less clearly.

On 28 November, however, around hour 5½ in the morning, I saw the comet in such a position — so far as could be observed in a moment, because of the intervening clouds — that it formed a triangle with Arcturus and the bright star in the body of Boötes, in which Arcturus occupied very nearly a right angle. The distance of the comet from the bright star in the knee of Boötes was, moreover, about 2° to the eye, with the bright knee-star lying below it and toward the west. The extension of the tail inclined a little from Arcturus toward the east, and the distance between the comet and Arcturus was at that time much greater. The body together with the tail was then fairly large and conspicuous. But after hour 7, when the sky's clearness in that quarter had returned and the dawn rose, Arcturus alone was visible to us, the comet having become wholly invisible. From these observations, such as they were, applied to the globe, there results around hour 6 in the morning of 28 November a comet longitude of 27° 45' [Libra ♎], with latitude 30° north, approximately.

On 29 November, around hour 6½ in the morning, we caught sight of the comet between the clouds, a strong east wind blowing. It had been advanced along a great circle since the preceding day in such a way that Arcturus was distant, as if by half a moon's breadth, from the straight line drawn from the comet toward the most northerly and brightest star in the left knee of Boötes, toward the north. Indeed, the comet appeared to have formed a right angle with Arcturus and with the star in the right arm of Boötes, the comet itself occupying nearly the right angle. But these observations were momentary, made between the clouds, whence by [Greek: ἐν πλάττει (en platei, "broadly," i.e. approximately)] the comet's longitude comes out as 26° 45' Libra [♎], with latitude about 33°.

On 30 November, in the morning, at the altitude of the bright star of Hydra [α Hydrae] of 20°, to which the time of hour 5⅔ most nearly corresponds, we attended to the position of the comet, which was disposed in a straight line with the bright star in the left knee of Boötes — so indeed that the line scarcely touched the upper part of Arcturus. The distance, observed once between the clouds, between the comet and the tail of Leo [♌] was 40°, more or less. From these too is given, approximately [Greek: ἐν πλάττει (en platei)], a comet longitude of 24° [Libra ♎], with latitude 35° north.

Note, moreover, that on this same day, a little before hour 1 of the preceding night, around the rising of the comet on our horizon, this comet had cut by its course the path of the other — that is, of the earlier comet of the year 1607 — beneath the body of Boötes, a little below the tropic circle, in which, according to these observations, it remained very nearly fixed. And indeed I marvel that at the same time, when at its rising this morning the comet was almost together with the Moon for us, its tail had scattered unaccustomed rays beyond the zenith — namely to an elongation of 104° , that is, as far as the stars in the left hand of Auriga, directly enough from the Sun, while [it lay] midway between those same stars and the shapeless [unformed] star above Orion. This was without doubt brought about by the disposition of the underlying air in our hemisphere, although the sky was then moderately

clear; and because the Moon was nearly together with the comet at its rising, as I said, that perhaps had somewhat promoted this unaccustomed extension of the tail — to say nothing of any hidden sympathy, or [Greek: ἀντιπάθεια (antipatheia, "antipathy")], of this comet with the earlier one as it cut across its track.

Afterward, however, when the comet had risen higher, the tail at once became shorter too, repeatedly turning northward with respect to the Sun, and as it were grazing the bright unformed star between the tail of the Great Bear [Ursa Major] and the tail of Leo [♌].

On the morning of 4 December, when some clearness had at last been restored among these days, the comet appeared to stand above the two contiguous stars in the middle of the body of Boötes, in a line nearly straight with those same stars — though a little, and indeed quite perceptibly, the comet deviated from that line toward the east. It was, moreover, somewhat more distant from the upper of these stars than the said stars are from each other. The tail of the comet, meanwhile, was stretched out along the southern side of the two stars in the square of the Great Bear, in such a way that it lay more to the south than they, by about $\frac{1}{2}^\circ$. This observation having been made around hour $5\frac{1}{2}$, we found the comet to be in longitude $16^\circ 20'$ Libra [♏] and latitude $43^\circ 15'$ north.

On 5 December, in the morning at hour 2 of the night, I saw the comet, as it were, in a straight line with Arcturus and the left shoulder of Boötes, but perceptibly nearer the shoulder. Likewise, with the star in the left forearm of Boötes and the upper of the two contiguous stars in the middle of the body — from which, however, after the fifth hour, the comet deviated perceptibly toward the zenith, and that by reason of its own motion. The tail of the comet was directed straight toward the southern star in the square of the Great Bear, and reached precisely

up to it, to the eye. The body of the comet was somewhat dim, and the rays bursting from the tail were sparse; the comet then appeared smaller and redder than the star in the left shoulder of Boötes. Through the optical telescope [perspicillum opticum], moreover, the comet's body was very livid.

Afterward, when some hours had elapsed, and the altitude of Spica in our square was $17\frac{1}{2}^{\circ}$, to which corresponds hour 5 approximately, the comet was distant from the bright middle star in the southern wing [of Virgo] by $43^{\circ} 43'$, and likewise around the same time it was distant from Spica by $48^{\circ} 2'$. From these distances, as from the observations of this morning, while the calculation is deduced by the method of triangles using the intervening fixed-star positions, the comet's longitude comes out as $14^{\circ} 40'$ Libra [□], with latitude $45^{\circ} 51\frac{1}{3}'$ north.

For this calculation here proceeds from the demonstration according to the adjoined diagram, in this manner:

	Longitude	Latitude
Spica □	$18^{\circ} 31'$ Libra □	$1^{\circ} 59'$ south
Middle star of the wing □	$4^{\circ} 50'$ Scorpio □	$2^{\circ} 50'$ north
Difference of longitude	$13^{\circ} 41'$	

Diagram. A spherical triangle construction for the reduction of the comet's observed distances to ecliptic longitude and latitude. A tall narrow triangle is drawn with apex A at top, the long sides descending to a base where points E, F, C, G are marked left to right; D is marked partway down the right-hand side. The accompanying legend reads: "In the adjoined figure, let A be the pole of the ecliptic; B the middle star of the wing [of Virgo]; C Spica of Virgo; D the comet."

- A (apex, top): the pole of the ecliptic [polus eclipticæ]
- B (lower left, base region): the middle star of the wing of Virgo [media alæ □]
- C (base, right of center): Spica of Virgo [spica □]
- D (on the right side, partway down): the comet [cometa]
- E, F (base, left): feet of the ecliptic arcs
- G (base, far right): terminus of the ecliptic arc

The worked solution accompanying the figure:

Ergo I. In triangle B A C, the given quantities [Greek: δεδόμενα (dedomena, "the givens")] are:

- B C $87^{\circ} 10'$ — the complement of the latitude of the middle star of the wing of Virgo.
- C A $91^{\circ} 59'$ — the excess of the latitude of Spica beyond 90° .
- B A C $13^{\circ} 41'$ — the difference of longitude. Therefore
- B C $14^{\circ} 29\frac{1}{4}'$ is given.

II. In triangle A C B, for angle A C B the givens [δεδόμενα] are:

- A C $91^{\circ} 59'$
- B C $14^{\circ} 29\frac{1}{4}'$
- B A $87^{\circ} 10'$ — therefore from these
- A C B $70^{\circ} 37\frac{1}{2}'$ is given.

III. In triangle D C B, for angle D C B the givens are:

- D C $48^{\circ} 2'$ — distance of the comet from Spica
- B C $14^{\circ} 29\frac{1}{4}'$ — distance of the middle star of the wing from Spica
- B D $43^{\circ} 43'$ — distance of the comet from the middle star of the wing. Therefore
- D C B $66^{\circ} 6\frac{1}{4}'$ is given. When this is subtracted from the previously found A C B, there remains angle A C D $4^{\circ} 31'$.

IV. In triangle A C D, for side D A there are given:

- A C $91^{\circ} 59'$
- C D $48^{\circ} 2'$
- A C D $4^{\circ} 31'$ — therefore from these is given
- A D $44^{\circ} 8' 24''$, the complement of the sought-for latitude of the comet: whence the latitude of the comet itself is $45^{\circ} 51\frac{1}{3}'$.

V. Finally, in triangle B A D, for angle B A D the givens are:

- B $87^{\circ} 10'$
- D A $44^{\circ} 8\frac{2}{5}'$
- B C $43^{\circ} 43'$ — therefore from these is given the sought-for angle
- B A D $9^{\circ} 9'$, which is arc E F of the ecliptic, namely the difference of longitude between the middle star of the wing of Virgo and the comet; and this, when it is here added to the longitude of the said wing-star, defined earlier as $4^{\circ} 50'$ Scorpio [□], yields a comet longitude of 14°

40' Libra [□]. These observations are therefore to be preferred to those obtained on the immediately preceding days merely by [tracking through] the threads' extensions [filar extensions of the instrument]; and from these the diurnal motion of the comet is to be corrected as need shall require.

On 14 December, around hour 10 in the evening — since throughout the nine preceding days there had been no clear sky on our horizon, but the heaven had remained almost perpetually cloudy and rainy in the meantime — when some clearness was at last restored this night, as a stronger wind dispelled the clouds, we found the comet (to the sight and by the filar expansion) in a straight line with the rearmost and uppermost star in the square of the Great Bear and the middle star in its tail; then in a straight line with the lower and forward star in the square of the Little Bear [Ursa Minor], and the last star of the tail of the Great Bear — yet here in such a way that the comet let itself down below that straight line by about 4'. Hence the comet's place, on the globe, was drawn out as longitude approximately 21° Virgo [□], latitude 58° 20' north.

The comet was this night, with the Moon shining opposite it, very much attenuated, and it stretched out its tail a little above the middle star in the tail of the Great Bear, although the trace of that tail was sparse and scarcely perceptible.

On 16 December, in the evening around hour 7, we carefully observed the comet below the pole, although on account of the Moon's brightness in the meantime it shone rather dimly, and both its body and its tail showed themselves for viewing.

At the altitude, therefore, of the middle star in the tail of the Great Bear (which was a little past the nadir [Naddir]) of 23°, through the geometric square, to which corresponds hour 7¼ approximately, the comet was, to the sight, in the vertical circle almost exactly below the pole itself. At that same instant the distance of the comet from Dubhe was 21° 28', and likewise from the antepenultimate star in the bend of the Serpent [Serpens] 19° 45¼'. The positions of the fixed stars at this time are these:

	Longitude	Latitude
Dubhe in the Great Bear	9° 49' Leo □	49° 40' north
Antepenult. star, bend of Serpent	8° 10' Libra □	78° 23' north

From these, by a similar triangular computation, together with the foregoing, the comet is given as:

Longitude	12° 58' Virgo ☐
Latitude	60° 18½' north

On 17 December, in the morning, at the altitude of the bright star of Lyra [α Lyrae, Vega] of 31½°, to which the time of hour 6½ very nearly corresponds, the distance of the comet from Dubhe was again 20° 42', and likewise from the same bend of the Serpent 19° 39' — by observations of ours repeated and limited as best as could be done. Hence comes out a comet longitude of 11° Virgo [☐], with latitude 60° 39¼' north. The comet was approaching our zenith, and its tail was dim, inclining toward Dubhe.

On 17 December, in the evening at hour 5½, the comet was in a straight line with the forward and lower star in the square of the Little Bear and the bright star in the tail of the Serpent, falling short toward the north by about 6'. And it was still manifestly distant from the straight line between the bright star in the tail of the Serpent and the middle star in the tail of the Great Bear, by about 14'; whence, broadly [Greek: ἐν πλάτει (en platei)], and on our globe, the comet was found to be:

Longitude	8° 45' Virgo ☐
Latitude	60° 40' north

Venus this evening at its setting, at 22° Aquarius [☐], displayed an unusual magnitude and radiance.

Finally, on 18 December — which was the last of the days to which our observations of this comet extended — around hour 6½ in the morning (for Spica was then nearly at the meridian), we found the comet to be nearly vertical, but not yet to have reached the straight line which lies between the middle star in the tail of the Great Bear and the bright star in the tail of the Serpent, falling short at most by 4'. The comet was, moreover, nearer to the middle star in the tail of the Great Bear, by about ½° to the eye, or perhaps a little more. There were some traces of the tail, but sparse and short; the body too appeared somewhat dim, tending toward a certain small unformed star between the bright star in the tail of the Serpent and the tail of the Great Bear. It was distant from that star in the Serpent by about 1°. From these observations of this day there results a comet longitude of 7½° Virgo [☐], with latitude 61° north, broadly [Greek: ἐν πλάτει (en platei)].

There follows a synopsis of the motions of this comet, drawn from all the observations set forth above, assigned — as in the case of the earlier comet, in the manner explained at length above and as far as could be done, as the matter required.

Astronomia Danica — Appendix, *De novis coeli phænomenis*

[Appendix p. 35]

EPHEMERIS OF THE MOTIONS OF THE COMET OF THE YEAR 1618, from the preceding observations, at the 9th hour of the morning.

The table gives, for each day (with the months November and December marked down the left margin), the comet's diurnal motion along its proper circle, its motion in longitude (degrees, minutes, and zodiacal sign), and its motion in declination (degrees and minutes, with N for north). The letters M and B in the rightmost column appear to mark the two key observation dates (16 and 23 December) referenced in the parallax discussion that follows.

Day	Diurnal motion in its proper circle (G. M.)	Motion in longitude (G. M. — sign)	Motion in declination (G. M.)
21 (Nov.)	3 20	9 25 ♏	3 20 N
22	3 17	7 55 ♏	0 35 N
23	3 14	6 25 ♏	2 20 B
24	3 11	4 55 ♏	5 30
25	3 9	3 7 ♏	9 0
26	3 6	1 20 ♏	12 0
27	3 3	29 30 ♏	15 20
28	3 1	27 45 ♏	18 10
29	3 0	26 10 ♏	20 40
30	2 57	24 10 ♏	23 15
1 (Dec.)	2 47	22 0 ♏	26 10
2	2 38	20 0 ♏	29 10

Day	Diurnal motion in its proper circle (G. M.)	Motion in longitude (G. M. — sign)	Motion in declination (G. M.)
3	2 24	18 0 ♍	32 0
4	2 19	16 20 ♍	34 20
5	2 16	14 40 ♍	36 30
6	2 14	12 30 ♍	38 40
7	2 12	10 20 ♍	40 50
8	2 10	7 40 ♍	43 0
9	2 8	5 0 ♍	45 0
10	2 6	2 20 ♍	47 0
11	2 3	29 30 ♍ [recte →? sign appears as ♍ in source]	49 0
12	1 59	26 40 ♍	51 0
13	1 55	23 40 ♍	53 0
14	1 50	20 50 ♍	55 0
15	1 43	18 0 ♍	56 40
16	1 33	14 40 ♍	58 10
17	1 18	11 10 ♍	59 35
18	1 8	7 45 ♍	61 0

Parsing notes: The longitude column's zodiacal signs run Virgo (♍) at the top, then Libra (♎) through the late-November/early-December rows, then revert to Virgo (♍) for 11–18 December — consistent with the comet's retrograde-then-direct path described in the prose. The day-11 sign glyph is ambiguous in the source (reads as ♎ but the column transitions here); flagged. The "Motus declinationis" values increase monotonically and smoothly from a few minutes to 61°, consistent with the prose's "61° N latitude" endpoint on 18 December. The M/B markers in the declination column on rows 22 and 23 are faint; transcribed as printed.

Moreover, we found that the angle which the comet's track [ductus] made with the ecliptic was 63 7/12 degrees. The proper track or circle of the comet had its northern pole at 15 1/4 degrees of [Aquarius] ♒ on the surface of the sphere, with a northern latitude of 26 degrees 54 minutes. It therefore cut the ecliptic at 15 1/4

degrees of [Virgo] ☿. From this proper circle the comet deviated scarcely perceptibly throughout the whole span of time during which we observed it — except, perhaps, by reason of the small parallax it had, which is treated next.

On the parallaxes of this comet

It now remains for us to extract correctly, from our observations, the parallaxes this comet had, so that from them we may learn the comet's distance from the earth, and likewise its size in comparison with the size of the earth. This matter not only contributes most directly to establishing the true hypothesis of comets, but also greatly commends the admirable work of God.

Since these observations had to be applied with the very finest precision practiced anywhere in astronomy, they had to be acquired not with childish instruments or with loose guesswork, but rather with instruments so well-wrought and refined that they keep every single first minute — indeed even the fractions of these, so far as is possible — free of error. Nor will there be any lack of skill in those who must be engaged to carry out such observations properly. By divine grace all these things are still at hand in Denmark, and by the king's generosity they daily receive fresh increase. As for our observations selected for this task at the relevant time, we shall rely chiefly on those set out above for 16 December in the evening at the 7½th hour, and the following morning shortly after, at the 6 7/12th hour — when in the evening the comet stood almost in the very vertical, beneath the pole, and in the morning not far from the vertex above it. For that very reason these observations were made with singular diligence, with our instruments placed and arranged outside the observatory (which faces chiefly south) under the open sky. So much so that, had the moon's brightness not hindered us at all in the evening, we know and can promise that these observations of ours were entangled in no perceptible errors — observations which we nevertheless

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repeated so many times that any suspicion arising from the moon's brightness could easily be dispelled. But indeed, since the comet's declination — to be elicited precisely from the observations at each of the two times — proved here most suited to the scrutiny of the parallaxes (so that afterwards, from the difference compared with that which the diurnal motion meanwhile produces, it may correctly reveal what we seek), then from the longitudes and latitudes of the comet set out above:

[Margin: Lib. 2 Sphær. cap. 2.]

First, for 16 December in the evening at hour $7\frac{1}{2}$ — namely, from a longitude of 12 degrees 58 minutes of [Virgo] ♍ and a northern latitude of 60 degrees 18½ minutes — there results, by what is taught in the works on spherics, a comet declination of 58 degrees 41½ minutes north.

Then, for 17 December in the morning at hour $6\frac{7}{12}$, from a similarly observed comet longitude of 11 degrees 9¼ minutes of [Virgo] ♍, with a latitude of 60 degrees 39¾, the comet's declination comes out as 59 degrees 35¼ minutes. Thus over a span of 10 hours 49 minutes the comet's declination was visibly advanced toward the north by 53¾ minutes. But because the diurnal motion of declination was at least 1 degree 25 minutes, or at most 1 degree 26 minutes — as is clear from what precedes and follows — there is hence given, over the interval of $10\frac{1}{2}$ hours, a proportion of diurnal declination of 38¼ minutes. The difference, therefore, between the true and the apparent declination is 15¾ minutes, which is attributed first to the comet's parallax in the first observation beneath the pole — apart from that slight parallax which there would be in the second observation, since at that point the comet was not far from our vertex. Nor did the change of declination at the two places vary perceptibly from the change of altitude in the same azimuth, where parallaxes are properly detected. For our observations, made below and above the pole, were as nearly so as could be — as will be shown in the following diagram, where we shall also set these parallaxes of the comet before the eyes, and through them reason further about the distance from the earth. And this in the following manner.

With center A, let there first be described a semicircle of the earth, B; then the comet's [circle] D M; and finally that of the starry heaven or fixed stars, C L. Next let the straight line A C be drawn to the vertex, and another to the pole of the world, A P, which at the same time passes through the point N, which is taken as the pole in the cometary circle. Further, let the arc E M be drawn, in which — according to the diurnal turning of the heaven — the comet, otherwise immobile, would have been carried round with respect to the center of the world or earth, A. But because it also possessed a proper motion nearly toward the north, as set out above from the observations, therefore, by the proportion of the diurnal motion of declination, while [we trace] that same proper motion which the comet had from M, the lowest place below the pole, to F, the highest, a little before, we shall find 38¼ minutes. But from the observation at

[Margin: Vide problema 1, cap. 10, lib. 2 Sphæricorum, ejusque casum primum.]

— each place [shows] a change of declination of 53¾ minutes. The difference, therefore (as above), is 15¾ minutes. These things being so posited, let there first

be drawn from the center A, through the comet's places established below and above the pole, M and E, the straight lines A H and A F; then from B, the surface of the terrestrial globe and the observer's location, through the same comet-places, the lines B K and B G. With the center then placed at P, the pole of the world, when the arc G Q is drawn, Q H will be equal to F G, that is, to the angle of the comet's parallax above the pole at E — namely B E A. And since H K shows the comet's parallax at its lowest altitude — since that arc in the heaven of the fixed stars is also equal to the comet's [arc] A M B — therefore, with H Q added to H K, the aggregate of the comet's parallaxes in both observations is compounded thus, which above we found to be $15\frac{3}{4}$ minutes. Now what the visible altitudes of the comet on each side were, that is, both at M and at E, is shown as follows.

First, the lowest declination of the comet observed at M was about 58 degrees 42 minutes north. Therefore its complement, 31 degrees 18 minutes, is the arc P K. With that subtracted from the elevation of the pole of this place — which is 55 degrees 43 minutes — there remains the arc 24 degrees 25 minutes: namely, the visible altitude of the comet sought at M.

Furthermore, since the latitude of the comet at its highest place E is given as 60 degrees 40 minutes, with a declination of 59 degrees 35 minutes nearly, there is found from this the right ascension of the comet, 203 degrees 40 minutes; and afterwards its altitude itself, at the $6\frac{7}{12}$ th hour of morning, of 80 degrees 6 minutes — which, on the meridian above the pole, would have been 86 degrees 8 minutes. But because of the very slight parallax of the comet at this place, we here employ the declination circle for the altitude circle without any perceptible error, together with the found highest altitude, namely 80 degrees 6 minutes.

[Margin: Vide cap. 2, lib. 2 Sphær., probl. 4.]

Let it be, then — for the sake of distinguishing the parallaxes by F G, or Q H, [equal to] H K, that is, the angle A M B — that it had been 13 minutes, since the comet's parallax at its lowest altitude [was such]. Since therefore, in the same triangle A B M, all the angles are given together with the side A B — namely, the one semidiameter of the earth — there is found from these the side A M of 240 semidiameters of the earth, and a little more. These, reduced to German miles at 860 [per semidiameter], yield the distance

Diagram. Parallax construction for the comet of 1618. A quarter/semicircle construction: the baseline R–A–L runs across the bottom with the earth's center A at its midpoint. Three nested arcs rise from the baseline — the earth's semicircle (surface point B, just above A), the comet's circle D–M, and the heaven of the fixed

stars C–L. The radius A C runs to the vertex; the pole-line A P runs to the pole of the world, passing through N (the pole of the cometary circle). The comet's diurnal arc E–M carries its highest place (E, with F just above on the outer arc) and its lowest place (M, below the pole). Sight-lines A H and A F are drawn from the center, and B K and B G from the observer at B, through the comet places; the short arc G Q is struck from P, with Q H = F G.

- C (top, vertex radius terminus): fixed-star arc terminus; line A C to the vertex
- F (top center, left of pair): comet's highest place, a little before E
- G (top center, right of pair): foot of line B G; terminus of arc G Q
- D (upper-left interior): left terminus of the comet's circle D M
- E (interior, below D on the comet arc): comet's highest observed place
- B (center, above A): terrestrial surface / observer's location
- M (lower center, on the comet arc below the pole): comet's lowest observed place
- H (right side, upper): foot of line A H; Q H equals F G
- K (right side, lower): foot of line B K; H K is the parallax at lowest altitude
- A (bottom center): center of the earth / world
- R (bottom far left): left terminus of the horizon baseline
- L (bottom far right): right terminus of the fixed-star arc / baseline
- N (interior, on line A P): pole of the cometary circle [?]
- P (interior upper, on line A P): pole of the world; center of arc G Q [?]
- Q (upper right, on arc G Q): arc point with Q H = F G [?]

[\[Appendix p. 37\]](#)

of this comet from the earth in these miles as 206400. Therefore, with the side A E posited of the same magnitude as A M, and A B as one semidiameter of the earth; and finally the angle E B A of 170 degrees 6 minutes — which arises by adding the highest altitude, previously measured at 80 degrees 6 minutes, to 90 degrees, or one right angle — through these things the angle of the comet's parallax at its highest altitude, by F, emerges as 2 minutes 34 seconds. When this is added to the previously assumed A M B of 13 minutes, there is again made up Q K of 15 minutes 34 seconds. This aggregate of parallaxes deviates imperceptibly from what was observed, even though the angle A M B in these few seconds or scruples might perhaps have been greater than the supposed 13 minutes, on account of some refraction of the comet at its lowest place. And so we rightly establish the distance of this comet to have been about 240 semidiameters of the earth. For it

was no less — which the observation of the comet at its beginning, namely on 24 November, also intimates, when it was many times taken in a single vertical with Arcturus from this [point]; for, apart from the comet's proper motion, they were perceptibly little or almost nothing distant, and so, whenever single hours [passed], they were detected to be [so] — as I recall having reported above. Indeed, the same uniform course of this comet through nearly its whole duration, in a maximal great circle, sufficiently proved this to us. Through this distance of the comet from the center of the earth, then, acquired moderately, we can reason that it was placed below the perigee of the orb of Venus in the heaven, near our restitution — namely, at about 98 semidiameters of the earth; while its greatest distance from the earth surpassed about four times [that], far from being below the same; and what is still more absurd, [far from] having stuck in the supreme region of the air. Let the figment of the Aristotelians concerning the comet therefore vanish in this case too — unless perhaps among those who would rather stubbornly oppose themselves to the truth than derogate anything here from their Aristotle.

Now, finally, we turn to the second [point], which concerns the size of this comet in comparison with the globe of the earth; and this will be tied to the following procedure.

[Diagram — see below]

With center B, let the body or head of the comet C E D be circumscribed, and let there be dropped from the center B to A, the center of the earth, the line B A, which was found above to be 240 semidiameters of the earth. And since the visible magnitude of the comet's head under its beginning rather perceptibly equaled the Spike [of Virgo, Spica] $\star$ itself — and this is reckoned among the stars of the first magnitude, of a visible diameter of about 3 minutes — with these things supposed, let the true diameter of the comet's head first be sought in this manner.

In the plane triangle C B A, having a right angle at B, since B A is given as 240 [semidiameters of the earth], likewise the angle B A C of $1\frac{1}{2}$ minutes — since it is half of the visible diameter of the comet C B D — therefore, by the conversion of the fourth [proposition] enunciated in our trigonometry, as the whole sine [sinus totus] is to A B, 240 semidiameters of the earth, so the tangent of the angle B A C, namely 4364, [is] to C B, 6 minutes 17 seconds of one semidiameter of the earth. Therefore C B doubled gives the true total diameter of the comet, 12 minutes 34 seconds, of which sort the diameter of the orb of the earth is 2 [units], or 120

minutes. The proportion, then, of the diameter of the comet to the diameter of the earth is as the number 377 to the number 3600.

[Diagram caption — see metadata below]

Diagram. A vertical elongated lens/triangle figure showing the comet's head and the line of sight to the earth. At the top is a small circle representing the comet's head, with points along its horizontal diameter; at the bottom is the earth's center. The long sides taper from the comet's head down to the bottom point.

- E (top): apex of the comet's head circle
- C (left of circle): left end of comet's head diameter
- D (right of circle): right end of comet's head diameter
- B (center of circle): center of the comet's head
- A (bottom point): center of the earth

Since the spheres are in the triple ratio of their diameters, by the last proposition of Book 12 of [Euclid's] *Elements*, then when the single [cubes] of these numbers are taken cubically, and the smaller cube is divided into the greater, it appears that the comet was smaller than the whole globe of the earth about 870 times, and a little more — and likewise that it was surpassed by the body of the moon about 20 times, since the moon is smaller than this earth 42 times and more.

Finally, concerning the true magnitude of the tail of this comet, it is of little or no concern to treat here, since this was extremely unequal in itself — extending at most to 20 degrees, and even sometimes more — having been, from the beginning, expanded from the rest of the body into the region of the world always opposed to the sun. But if anyone should wish to investigate this too, let him consult above all Book 2 of the *Progymnasmata* of Tycho Brahe [Tycho Brahe], where they will find a paradigm of this matter as well — although, to be sure, under that limitation which our cometary hypothesis above admits, since rarely does a comet fall under such an angle with the sun and the earth that it shows us its full tail-jet [ejaculatio]. But thus far concerning the situation, motion, altitude, and magnitude of this comet, from our own observations.

Now I shall append the account of the same [comet] observed in Persia, as it was printed in Belgium and reached us.

[Margin: *Designation of two comets in the town of Spahan, which is the most noble of the kingdom of Persia, and not far from Persepolis to the west, under an elevation of the pole of 31 degrees. From the closing section of a letter of García Silva Figueroa [García de Silva y Figueroa], legate of Philip III of Spain to the*

Spanish and the Indies, written on the 5th of the Calends of March [25 February] of the year 1619, to the Marquis of Bedmar, and translated and published at Antwerp in the year 1620, in these words:]

[Appendix p. 38]

"The closing part of this letter will be an account of the twin comets which in these days were a spectacle to us. The first began to shine on the 10th of November (understand: by the new style), two hours before the rising of the sun; its appearance was noticed between the east and the south. Its color was plainly like the smoke that evaporates from most thoroughly burnt sulphurous powder; its head, as it appeared to me, in Scorpio. Its magnitude was such as to take up a sextant of the zodiac; its form like a scimitar — as some have supposed, the kind the Greeks call ξιφίας [Greek: *xiphias*, "sword-fish" — a sword-shaped comet], portending dreadful events. To me this comet seemed rather to resemble the appearance of a one-year-old palm-shoot which, not yet broadened, curves slightly at its tip. It kept its motion toward the south.

"On the twelfth or thirteenth day after the rising of this comet, another appeared, maned [jubatus] or hairy [crinitus], of common appearance, similar in color to the star of Venus, and equal to it in magnitude, or a little greater. It led its rise correctly from the equinoctial east. And although at the start it was far smaller than the former, it nevertheless took on far greater increases day by day, so that it nearly equaled the other. It moved itself with a proper motion together with the *primum mobile*, not far from the vertical line toward the north. Further, three or four days after its rising, the other [comet] vanished. Both certainly had so brief a period that the later one ceased to appear already before the tenth day mentioned above. It was observed that this one, while it was ceasing, reddened more. However that may be, if those evils portend to this comet, they will at least not be lasting, like the many others that are discerned in the world. Thus he [Figuerola]."

From these things, certainly — although I acknowledge that the designation of two comets observed in Persia in the month of November of the year of the Lord 1618 was made by the said Spaniard Figuerola, and that the first began there on the same day on which Kepler [Kepler] notes [it] in Silesia — nevertheless I judge that the requisite [diligence] was not applied to observing the later one (the one we treat of) by the Spanish legate himself, or by those who were with him, ἀκριβεῖαν [Greek: *akribeian*, "exactness, precision"], which otherwise befitted an astronomer. So much so that a reader versed in celestial matters may easily perceive that this royal legate — being intent, at that time, rather on other things

which he judged would be of greater moment, and with that more pressing concern, together perhaps also with ignorance of the art — may be excused: not only in that he assigns no distinct motion to this comet, but also in that, contrary to the nature of comets and to common experience in others (as in this one itself), he increases its magnitude successively from its first rising. This, however, on account of the nearness of the sun in the sphere [sloping] much toward the right, could have happened. For while this comet was rising to the Persians, it scarcely preceded the sun by 7 degrees in oblique ascension, so that it would have been a wonder had it not been wholly hidden by the sun's rays — let alone attenuated at its beginning — and at last brought more and more into view, until it freed itself entirely from the sun's rays.

Whatever the case, Figueroa's opinion that this comet was here a smaller object of concern to him is strongly increased by the fact that, in that region — where both a greater nocturnal serenity was afforded, and the phenomenon was, as he says, nearer to the vertex — he did not pursue it by observing for 10 longer days; whereas, after the comet here had begun to be seen, later, more in a steeper sphere and after continuous rains, and (as he writes) had ceased for him, it had been both noticed and observed by us likewise, and by others through the intervals which a more serene sky granted, over a span of triple longer time. For it was impossible that so enormous a comet as this was subject to a parallax such that, for this cause, it should vanish from the Persians so quickly — when we, faithfully extracting and measuring the same parallax above, detected it to be not beyond $14\frac{1}{2}$ minutes at the limit of the horizon. And indeed, when by the diurnal motion the comet was drawn to the boundary [finitor] of the Persians, refraction restored that whole parallax even in those regions — unless perhaps the mountains, which are high in Persia, had stood in the way, so that this comet rose less into view for them at the greater bend of time. Meanwhile, however, one may nicely recognize from Figueroa's description that another comet as it were — the one which lord Kepler, with some others, describes from an incompleteness of form — preceded ours, and that this one began far earlier, namely before the start of the 12th day of November, old style; that is, around the aforementioned fatal and pregnant aspects of the planets which then occurred — first the quadrature of the Sun and Jupiter, then the trine of Mars and Venus. Both of which contribute greatly to the judgment of effects — to which [judgment], whatever remains for me to treat concerning this, we shall next devote ourselves, according to the weakness of our talent.

CAP. XI.

On the effects or significations of these two comets separately, to be indicated briefly according to the common decrees of the Astrologers.

Concerning the significations of comets — whose motions we have thus far designated from our observations and reduced to order — being about to treat more fully and specifically according to the foregoing Chapter 8, a twofold way presents itself. One is more customary and well-worn among the Astrologers in this matter — even while, holding [views] different from Aristotle as to the causes (which we above attributed to the generation of comets), we here, by a plausible conjecture, [treat of] the effects or significations

we apply the significations [significatis] of these comets according to our own judgment. The other way leads us chiefly into more hidden indications, drawn from the differing rising and the contrary motion of these two comets. That way I shall set out in the following and final chapter, God willing. But I shall tread both paths timidly and with hesitant step, as these perilous times warn me to — especially since (to share Ptolemy's [Ptolemy, *Tetrabiblos*] view, and rightly) the dictatorial decrees [decreta prætoria] of the astrologers have, for many reasons, no place in our science. Some of those reasons I have noted above; the rest the reader may learn from the proem of Ptolemy's *Quadripartite* [*Tetrabiblos*]. What is more — and I confess it frankly — my own nature has up to now always shrunk from predicting events from the stars, and above all from these prodigious ones, lest I be found, like certain others who reap this harvest with greater license, too lavish in such matters. So much so, indeed, that I would have preferred to refrain altogether from any judgments whatever about these comets, were it not that piety required otherwise.

These are divine portents, sent and aimed at a certain end — namely, our amendment. I therefore wished not to fix them more deeply in readers' minds than the foregoing contemplation of their appearances [apparentiis] warrants, but rather to draw, plausibly, from an astrological consideration of these comets a few inferences that have not yet been touched upon, and faithfully to warn that still other things lie at the very doors — prepared, that is, for the ruin of the impious and the impenitent.

Turning first, then, to the rising of the comet of the year 1607, which above we found, with sufficient plausibility, to have fallen on 15 September at hour 15¾ — namely in opposition [♌] with the simple motion of the Moon [♋] — at that time we find the heavens and stars to be, with respect to our horizon, in the state the

following chart [thema] expresses. To it we join a second chart, which likewise declares the disposition of the heavens at the opposition of the luminaries next following, so that from the inspection of both, our judgment about this comet may be shaped.

Diagram. A square horoscope chart [thema coeli] of the traditional form, with the central panel reading "**Anno 1607 / Septemb. / D. H. M. / 15. 15. 24. / Elev. p. 56.**" (Year 1607, September, Day–Hour–Minute 15·15·24, Pole-elevation 56°). The heading above the chart reads "**THEMA COELI. / Exortus cometæ.**" (Chart of the Heavens. The rising of the comet.). The twelve houses are marked with cusps and contents:

- Top-left corner: **Com. 10½** ♌ (Comet at 10½ Leo); **Dec. 5 31½** (Declination, fraction uncertain [?])
- Upper-left house: ♌ **13** (cusp Leo 13)
- Left house: ☉ ♍ (Sun in Virgo); cusp 5
- Lower-left house: ♍ **12** ♍ (Mars at 12 Virgo); ♍ **25**
- Bottom-left corner: ♍ **21½**; ♍ **19**; ☾ **18** ♏ (Moon at 18 Scorpio); ☾ **0½** ♏ (south node at 0½ Scorpio)
- Bottom house: ♏ **28** (cusp); ♄ **12½** ♐ (Saturn at 12½ Sagittarius)
- Lower-right corner: ♏ **10**; ♏ **13**
- Right house: ♏ **0** (cusp near Pisces)
- Upper-right house: ♈ **4½** ♈ (Jupiter at 4½ Aries); ♈ **26** [?]
- Top house: ♈ **19**; ♈ **25**

Parsing note: this is a densely lettered woodcut chart with small peripheral degree-and-sign labels; many of the planetary positions and house-cusp figures are at the resolution floor. More than two labels are flagged

Diagram. A square horoscope chart [thema coeli], the right of two on this page. The heading reads "**THEMA COELI. / Oppositionis ☉ & ☾ proxime seqq.**" (Chart of the Heavens. The opposition of the Sun and Moon next following.). Central panel: "**Anno 1607 / Sept. / D. H. M. / 25. 0. 43. / Elev. p. 56.**" (Year 1607, September, Day–Hour–Minute 25·0·43, Pole-elevation 56°). This is the full-moon / opposition chart referenced in the prose ("the chart of the next full moon"). Readable peripheral labels:

- Top-left corner: **Com. 14** ♏ (Comet at 14 Scorpio); **Dec. 9** (Declination 9)
- Top edge: cusp ♏ **23**; ☾ **18½** ♍ (Moon at 18½ Virgo), cusp ♏ **28**; ☉ **11½** ♎ (Sun at 11½ Libra)

- Top-right corner: cusp $\square$ 15
- Right apex: cusp $\square$ 17
- Right edge: cusp $\square$ 14
- Right lower: cusp $\square$ 12 [?]
- Bottom edge: $\square$ 17½; cusp $\square$ 23; $\square$ 2½ $\square$ [?]
- Left edge: cusp $\square$ 1 $\square$ [?]
- Left middle: ♄ 12¼ $\nearrow$ (Saturn at 12¼ Sagittarius)
- Left upper: $\square$ 28 (Venus at 28)
- Upper-left corner: cusp $\square$ 26

Parsing note: small woodcut chart at the resolution floor; several cusp degrees uncertain [?]. $\odot$ 11½ $\triangle$ and $\searrow$ 18½ ♄ are not a clean 180° opposition, consistent with the heading "proxime seqq." (the luminaries approaching the opposition next following).

When we contemplate the magnitude, color, length of tail, and finally the motion of this earlier comet at the very moment of its rising, we observe that here it took on the nature of Saturn [♄], in whose sign — namely with the mean motion of the Sun [$\square$] — it began; then the color by which it appeared recalled Mars [$\square$], being placed nearly in quartile [$\square$] to Jupiter [♃], and likewise to the Sun [$\odot$] and Mercury [☿]. Furthermore, we note that Mercury [☿] itself, lord of the horoscope [dominus horoscopi], lay hidden beneath the rays — that is, in the heart of the Sun [σ $\triangle$] — and that Mars [$\square$] ~~stood~~ not far from the Saturn [♄]. Finally, the comet's position lay, as we observed, in the eleventh house [domo], from which it afterward deflected its motion in swift haste toward the south, and partly in the direct order of the signs [in signorum consequential].

First, the consideration of such matters — and especially the comet's position and motion — plainly intimates to us those harsher winters, nearly contrary to the order of nature. Above all, the winter that, at the year's beginning [continues on next page]...

in 1608, among the northern Icelanders and other Norwegians was far milder and more bearable than usual; but among us — the Belgians, Germans, Swiss, indeed the very French, and so too the Italians and Spaniards — it was so unseasonably severe, beyond all custom, that in those regions not only small animals but a great many human beings perished of the cold. For just as the Sun, rising by the primary motion [motus primus], advances by its rays the biting frost exhaled from the vast Indian Ocean and — as Cardano [Girolamo Cardano] holds — gradually and by degrees scatters it westward over the continental landmass,

so it is credible that this comet made that cold matter, gathered in the air about the north, subject to its own motion, drew it southward with itself, and gradually spread it over the face of the earth. Whence not only the wintry seasons...

[Now the left column of p. 522 (appendix p. 40):]

but the spring seasons too — while the comet had at the same time occupied its own proper domicile [domicilium], namely the place from which it was most effective — were beset with much cold and frost, and these, among other things, were without doubt the cause of those dear harvests among us, in Germany, and elsewhere, which also lasted for several following years, when those immoderate winters brought dry summers along with them, besides various diseases, both acute and chronic alike — especially unaccustomed fevers, which from that time onward poured into our regions as well, on account of the trine of Mars [△ ♂] and Saturn [♄]. Among these are to be reckoned the continued fever [synochus] and, above all, that quartan ague which so wretchedly afflicted our own people and the Germans thereafter.

Nearly the same follows from the chart of the next full moon [plenilunium]. For in that revolution the comet occupied nearly the same position as before, and Mercury [☿], lord of the horoscope, was thrust down into the third cadent house; and finally the Moon [☾] was admitted into the ninth house together with Mercury [☿] and the Sun [☉] — which much harmed religion, so far as it is fitting either to learn or to predict of this from the stars. Indeed, the position of these planets portends from the same source unhappy deliberations concerning religion, as well as other affairs of the commonwealth.

Now as for the remaining state of the commonwealth throughout the world: in these last ages of the world it has, by divine justice, on account of men's various sins — and chiefly the lusts of princes for ruling — been turned into wars, just as Christ himself foretold [Margin: *Math. 24, ver. 6, & seqq.* (Matthew 24:6 and following)], and for the most part torn apart from charity, concord, and peace. We observe unmistakable signs of wars in both charts, especially among the Muscovites, Tartars, Indians, and Turks. That these things are true you will find from the reports that have reached even us — namely that those peoples for whom the comet (according to the precepts laid down in Chapter 8 [cap. 8]) was at those times either at the zenith [verticalis] or otherwise in the tenth house were harassed by wars and disasters. Let the recently published history of the Ruthenians be read — what upheavals those three false Demetriuses [Pseudo-Demetrii] meanwhile stirred up, and what slaughter and butchery of men they brought about in Russia, Livonia, and Poland.

Furthermore, around the year of the Lord 1610, in Tartary — out of which the Great Khan [magnus Chamus] had died a little before — a vast war broke out, namely among those for whom this comet at its rising was as it were at the zenith. For when, the Khan's son being shut out, this man's brother attempted to draw the empire to himself, war was waged, in which the Khan's son is reported the victor, and in which there fell as many as 40,000 men. Two years earlier, in the territories of the Turkish empire over which this comet in the chart culminated nearest the opposition [♌] of the luminaries — namely Aleppo [Alepiensi] and Karaman [Curmaniensi] — a very great tumult arose. The pasha of Aleppo, Mosiles [Mosilis; Janbulad Ali Pasha], by rebellion and defection — while the Grand Vizier, Veizir Pasha by name, set himself up as its avenger in the name of the Turkish Emperor — and the latter scarcely emerged victorious after three huge battles.

As for us, however, although this comet was at the zenith neither in these charts nor entirely outside them (for nowhere did it attain a greater declination than $53\frac{1}{4}^{\circ}$), nevertheless, since it had begun its course and beginning from Leo [♌] — a sign which, by Ptolemy, belongs among others to the triplicity [trigonum] of Germany, Denmark, and Sweden, by the nature of these peoples — for that reason too wars extended themselves hither. This was chiefly on account of the decree of direction [directionis decretum], that the conjunction of Mars [♂], Jupiter [♃], and Mercury [☿], which had occurred in the year of the Lord 1603 at the start of winter [Bruma] in the 10th degree of Sagittarius [♐], fell in the other of the signifying signs [significatorum signo], also a triplicity sign; and likewise on account of the new star found in the following year, namely 1604, in the 18th degree of Sagittarius [♐]. For when we consider the comet at its rising and its motion through the ecliptic from these places, we have thence that span on either side of the triplicity, in which the comet, from the $10\frac{1}{3}$ th degree of Leo [♌], makes indeed no perceptible distance from the former point in the 10th degree of Sagittarius [♐] and the latter in the 18th degree of Sagittarius [♐] — the difference of which, by that direction, prolongs the comet's significations for about 8 years. But in these intervening times — and because it received its greatest stimulus from the fiery conjunction of the superior planets — there fell out the flood throughout Britain, the upheavals and wars throughout Germany, and likewise upon these northern realms.

For scarcely four years after the comet's rising, that biennial war broke out between the Danes and the Swedes; and its end too the death [fata] of Charles, King of Sweden [Caroli Regis Sueciæ; Charles IX of Sweden] — who gave great occasion to the said war — overtook, when in the following year, namely 1612,

the comet by its directing motion had reached the horoscope of this kingdom in the 7th degree [of Capricorn]. That same year was also fatal to our most serene and never sufficiently praised Queen, ANNE CATHERINE [Anne Catherine of Brandenburg].

Furthermore, when we work out this comet's significations with respect to the course of the Moon [☾], we find that it completed nearly 4 signs in their direct order [in eorum consequentia] by its motion — which likewise intimate to us a duration of the significations of about eight years [octennium]. Within that span there exercised themselves not only the evils we enumerated before, but most of all those of which Mercury [☿] was the signifier, cast down and afflicted in both charts. To this not only the Venetian war with the Archduke of Austria [Archiduce Austriæ] bound itself unhappily, but most of all the beginning of the rebellion and defection of the Bohemians and the adjacent provinces — chiefly, as they suppose, for the sake of religion. That is, when those men, like the Poles before them (and with assuredly a like outcome), too prematurely professed themselves to be champions of religion freely exercised. On which matter I think nothing is to be disputed for the present, save only this: that God indeed finds a means whereby, at the accepted time, He will show Himself to be the sole defender and avenger of His church. Thus it is firmly to be held that a rebellion will scarcely ever have success, unless it be one which so great a tyranny precedes as would move heaven, and God Himself [continues on next page]...

to advance it thence into a just vengeance with prayers and tears.

But so much for the significations of the comet of the year 1607 considered in themselves, without comparison. There follow those which we judge can likewise be deduced from the latter comet, according to our modest capacity.

On the significations of the comet of the year 1618 separately. [De significatis cometæ anni 1618 seorsim.]

Since the occasion has here been given us to trace back the phenomena of the comet of the year 1618 likewise to the beginning of its rising — which beyond doubt had occurred in the quartile [☐] of the Sun [☉] and Jupiter [♃], namely ...

on 11 November, one hour before noon by our reckoning, with respect to the restitution of the courses of both the Sun and Jupiter — seeing that even the Persian observation [Persiana observatio] is anticipated by only one day from this time, that being when the comet could first have been seen by them on account of its nearness to the Sun. Accordingly, we shall fit a celestial chart below

to the first rising of this comet, and then we shall add another, fitted to the time of the opposition [♌♍] of the luminaries immediately following — namely on 21 November in the night following — so that here too we may proceed more correctly and expeditiously in the judging of its effects.

Diagram. A square horoscope chart [thema coeli], the left of two on this page. The heading reads "**THEMA COELI / ad Exortum cometæ.**" (Chart of the Heavens, for the rising of the comet.). Central panel: "**Anno 1618 / Novemb. / D. H. M. / 11. 11. 0. / antemerid. / Elev. p. 56.**" (Year 1618, November, Day–Hour–Minute 11·11·0, before noon, Pole-elevation 56°). Readable peripheral labels:

- Upper-left corner: cusp ♌ 12 with ☉ adjacent (Sun in Scorpio, near comet); ♌ 29 ♌ [?]; **Com. 21 ♌** (Comet at 21 Scorpio)
- Top house: cusps ♌ 11 / ♌ 19
- Upper-right corner: ♌ 15 ♌ (Mars at 15 Virgo); ♌ 19
- Left house: ♌ 2
- Lower-left corner: ♌ 14¼ (Moon); ☉/♋ 25 / 25¼ [?] (Sun/Node, reading uncertain)
- Bottom-left house: cusp ♌ 29; ♌ 29 ♌ (Jupiter at 29 Aquarius)
- Bottom corner: ♌ 19; ♌ 14
- Bottom-right house: ♌ 25; ♌ 12; ♌ 5 ♌ (Saturn at 5 Gemini)

Parsing note: small woodcut chart at the resolution floor; multiple sign-and-degree labels uncertain

Diagram. A square horoscope chart [thema coeli], the right of two on this page. Heading: "**THEMA COELI / ad Oppositionem ☉ & ♌ proxime seqq.**" (Chart of the Heavens, for the opposition of the Sun and Moon next following.). Central panel: "**Anno 1618 / Novem. / D. H. M. / 21. 12. 55. / Elev. p. 56.**" (Year 1618, November, Day–Hour–Minute 21·12·55, Pole-elevation 56°). Readable peripheral labels:

- Top house: ♌ 9. 47 ♌ (Moon at 9°47' Gemini); ♌ 4 ♌ (Saturn at 4 Gemini); cusp ♌ 23
- Upper-left: cusp ♌ 3; ♌ 4 [?]
- Upper-right: cusps ♌ 26 / ♌ 13
- Left house: ♌ 29 ♌ (Mars at 29 Virgo); cusp ♌ 25
- Right house: cusp ♌ 15
- Lower-left corner: **Com. 8¾ ♌** (Comet at 8¾ Scorpio); cusps ♌ 13 / ♌ 23; ☉ 9. 47 ♌ (Sun at 9°47' Sagittarius)

- Bottom house: ☿ 26 ♑ (Venus at 26 Capricorn)
- Lower-right corner: cusp ♑ 4; cusp ♑ 4; ♀ 0 ♋ (Mercury at 0 Pisces)
- Right edge: cusp ♑ 3 [?]

Parsing note: small woodcut chart at the resolution floor; multiple labels uncertain

Even though we have the comet's rising only by conjecture, so far as the time is concerned — as we argued above with probability that it began in the quartile [☐] of the Sun and Jupiter — yet from inspection of the chart, which is erected for that moment, I find such agreement, not only with the following celestial position established at the opposition [♌] of the luminaries, but also with the things that have happened thus far since this comet's rising, that I now less doubt its rising to have fallen at the prescribed time. Survey with me, I beg you, kind reader — whoever you are that are an astrologer — the chart of this comet's assigned rising. Do not the malefics [malefici] Saturn [♄] and Mars [♂] there so beset Mercury [☿] — which is near the Midheaven [M.C.] and the disposer and dispenser of affairs — that they entangle it and drag it to their own side, so that, through this constitution of Mercury and of the malefics and indeed of the luminaries, the commonwealths of the world incline to nothing but every kind of misfortune in their counsels and actions? And all the more, because Mars [♂] resides here in the eighth place, namely the domicile of death, and exchanged [permutato] with Venus [♀]; while Saturn [♄] stands not far from the cusp of the sixth house, which is of diseases — and these malefics too regard one another in nearly a quartile [☐] aspect. I add that the lord of the horoscope [dominus horoscopi] is here Saturn [♄], which also afflicts the Moon [☾], found in its fall [casu].

Of the comet's own placing in this chart, what shall I say — which had scarcely departed from the Sun's rays, and was besides exalted in the tenth place and established in the domicile of Mars [♂]? This itself signified [matters] touching the earthly lights of the world — namely Emperors, Kings, and Princes in the commonwealth — and thus far it has extended itself, and I judge it will extend itself hereafter, no less to those who wish to prove themselves prelates and guardians of the church — if anything may be foretold from the stars concerning this divine economy — seeing that the Moon [☾] and Venus [♀] have been invited into the dwelling of the malefic Saturn [♄].

Moreover, the comet itself, beside the cusp of the eleventh house and so near the Sun, intimates in manifold ways — besides the other evils that comets usually announce everywhere — treacherous counsels against pacts and friendships.

Certainly one who is an astrologer, and does not utterly deride the decrees of the ancient sages, will scarcely doubt that such ominous things were portended by this comet as befell that same year 1618 and the next following, in the departure [death] of magnates in the commonwealth. For the earlier chart [boded ill], among others, to Maximilian, and likewise to the wife of the Emperor [continues on next page]...

of the Emperor; the latter chart was fatal to the Emperor Matthias [Matthias] himself, and to Anne, queen of Great Britain [Annæ reginæ magnæ Britanniae; Anne of Denmark] — a heroine assuredly most sorely missed in this age. And to mention further concerning the Emperor: he departed [died] on 10 March of the year 1619, when Mercury [☿], lord of the natal horoscope, had now — while the comet was blazing — migrated into the eighth house, which is named the house of death, the Midheaven [medium coeli] likewise passing a little afterward into the place of the natal horoscope, and the comet too [passing into it].

Consider besides the actions that have lain upon Germany and the whole Empire, chiefly on account of religious discord, seeing that this comet referred its rising to the quartile [☐] of Jupiter [♃] and the Sun [☉], and stood not far from the Sun at its rising — in one and the same domicile, I mean, with Mercury [☿], invited fraudulently by the malefics, and assumed, as I said, with the domiciles exchanged [permutatis]. Those things, then, which might perhaps have been governed by salutary counsels, were thus far conducted by the worst, so that within these few years they have been the causes of the slaughter of nearly countless men. The same is intimated by the chart erected for the opposition [♋] of the luminaries, where Mars in the twelfth house nearly occupies the horoscope; by which again Mercury [☿] in a cadent place, and as it were in quartile [☐] aspect — being afflicted in a long ascension [in ascensione longa] — is afflicted, where again the luminaries are afflicted by the sinister aspects of the malefics, which themselves too are placed in unlucky houses in this chart, while Saturn [♄] is found in the house of religion, Mars in the house of the prison [carceris].

But because Saturn [♄] is more lingering and more long-lasting in producing and defining its effects, it is certainly to be feared that the evils which the present comet signifies will not be loosed and cease before the conjunctions of the superior planets shall have passed — which will occur in 1622 in Cancer [♋], and in the following year in Leo [♌], a fiery sign — whence ominous and evil outcomes are wont to issue, here both as regards floods of waters and afterward the motions of wars. These evils seem to want to extend themselves into the year

1624, a year which indeed I, with many others, judge will be grievous to the world, if not fatal. For in the meantime, unless all things be composed by divine mercy and singular providence throughout the Roman Empire, and some peace be erected and confirmed, you will see dreadful wars and upheavals, which will at the same time stir up even the Turk, on account of this comet culminating over his dominions and realms at its first rising.

But would that we northerners too be not touched by the decrees of this comet amid these lamentable upheavals, and involved in them at these intervening times — namely, when the comet shall have been drawn back toward the beginning of Libra [♎] by direction into the preceding [signs], and afterward, around the end of the year 1623, when, near the end of the second decan of Virgo [♍], the comet had become vertical [verticalis] over us. For among other portents which are reported to occur everywhere among us in these times, for the reproof of our complacency, there is also that dreadful eclipse of the Sun [☉] which was beheld among us on 11 May of the year 1621 — and that, assuredly, in a greater degree than among those further south, on account of the increase of parallaxes [parallaxium] — threatening evils of this kind, or the like. For when, in the last degrees of Aries [♈] — which sign and domicile is assigned to luxurious Venus [♀] — the luminaries were joined in this eclipse, the Moon was about to enter Taurus [♉]

[♉], a sign which belongs to the dominion of the sun, and is the significer of various upheavals in the empire and commonwealth, also as regards laws and disciplines, since Mercury [♿] is commonly held to be the director and standard-bearer of scribes and learned men.

Moreover, from the sinister aspect of the malefics, and likewise from the comet's retrogradation through the signs of the zodiac, besides febrile and other diseases which still continue among us, that pestilential plague too invaded the men of our clime in the year 1619 and the following — when the earth, meeting the comet under that pole-elevation where it most inclined toward its final dissolution, boiled out of itself a Mercurial fuel [Mercurialem fomitem] and advanced it by its motion toward the east. Whence first a part of Norway, then the greatest part of Denmark and some of Sweden, afterward Gotland, Prussia [Borussia], Livonia, and Poland, under this same clime — which was a wonder — were wretchedly afflicted by this plague. And yet at the same time they were so successively delivered by the most merciful God that you would recognize this plague to be nothing but a fatherly admonition, concerning the saving repentance of men. Would that it might so fall upon us hereafter, that — the filth and complacency of our sins cast off through earnest penitence — we might be held worthy to escape

these evils which would otherwise befall, and to stand with divine grace and favor before the Son of Man.

But as for what concerns the fruits of the earth, the harvests, the seasons, likewise the diseases and other evils of this kind further to be expected from this comet's decrees — these certainly seem then first to be most aggravated, and likely to grow harsher, when the comet shall have exerted its powers in the last sign of its disappearance, namely Leo [♌], together with the conjunction of the superior planets in the same sign (of which above) — powers which seem reserved for the year 1623, and most of all 1624. For although this latter comet began at its rising — so that thence, according to Ptolemy, there should be an indication of its earlier effects following — yet from the magnitude both of its body and of its tail, likewise from the constancy and duration of its motion, our judgment is confirmed both as to the magnitude of the outcomes and as to their longer duration up to the limits we have stated.

But thus far the significations of these comets, considered in whatever way separately. It now remains that, into the conclusion of the treatise we have undertaken, we throw back certain things concerning their common signification [communione].

Astronomia Danica — Appendix: *De novis coeli phænomenis*

Caput XII

CAP. XII.

On the common effect of the significations of the two aforementioned comets, drawn from the same efficient causes; likewise from the contrariety of their motions and the nature of the signs or asterisms through which they passed; and further, on two other comets that arose at nearly the same time as the last one. From all of which, above all, the detestable complacency of mankind in these final times is laid bare.

Besides the things put forward in the previous chapter concerning the significations of these two comets — things that come onto the world's stage successively, as the times bring them — I suspect that there are still other matters surrounding these comets that will undoubtedly augment their significations wonderfully.

In the first place, then, apart from God himself in his goodness — who produces celestial portents of this kind, like everything else, toward a sure and good end, as was clearly stated in chapters 4 and 7 — the very origin of each of these comets (or, if you prefer, their efficient natural cause) is assigned to two of the brightest and most powerful lights wandering among the celestial bodies, lights most benign in fostering things below: namely the Sun and Jupiter. For we have reasoned plausibly enough, from the timing and other circumstances, that the first comet began at the opposition of these two bodies, and the second at their quartile aspect. From this, briefly — as with everything in this chapter — we conclude that the purposes for which these two comets were sent by God, greatest and best, in order to signify and give warning, are not very different from one another. Moreover, since these same purposes are taken from the character and nature of these planets according to the astrologers, it is evident that the one — Jupiter — signifies, among other things, chiefly religion and justice; the other — namely the Sun — signifies kingdoms, empires, majesty, and worldly riches. The decrees of both comets extend equally to the violation and invasion of these things, whenever God shall permit it on account of the sins of men.

The second point is the contrariety in the motions of these comets. Through the same zodiacal signs, the first ran its course from north to south and east, with a motion more impetuous and irregular (though almost perpetually beneath a great circle on the sphere); the second, by contrast, directed its course from south and east to north, with a motion slow and orderly, also beneath a great circle. From this, again, it seems possible to infer that the affairs they touch — whether of religion or of empire and dignity — strive against one another and are torn apart, so much so that those earlier irregular motions, which denoted those immature and unwholesome counsels, do not easily seem able to be composed again, without the vast surges of turmoil that this ocean of empires stirs up of itself — once the vials [phialae] of divine wrath, which these comets surely represented, have been sent down from heaven upon it.

Not crime, nor the dire lust to reign, serve to compose the swelling waves [fluctus]: but by prayer, by tears, must the wrath of the deity [numen] be calmed.

The third point: when we consider those asterisms in the heavens through which these two comets passed — and especially Boötes, or Arctophylax [the Bear-keeper], beneath whom they chiefly directed their courses in opposite directions, so that the first moved obliquely beneath this asterism, beginning its course from the thigh of the Great Bear and ending in Serpentarius [Ophiuchus]; whereas the

second moved more directly, ploughing through this very constellation, and did not leave it until, from the far left hand, it was let down into that empty space left between the tails of the monstrous beasts (the Serpent, that is, and the Great Bear), taking its rise from the star of Scorpio, and so cutting the path of the first beneath the right thigh of Boötes near the tropic circle of the Sun's summer turning [the summer solstice]. From this, surely, we may gather that God does not employ such celestial figures now — in pointing out the courses of these comets, for our admonition — to no purpose, figures that were once handed down by the ancient astronomers and worshippers of the heavens into the errors of fables. For — so that the interpretation of these monstrous asterisms, beneath which our comets either took their rise or into which they ended, may be sought elsewhere in Manilius, Aratus, and other writers, but especially in Natale Conti [Natalis Comes] and Heinrich Decimator [Henricus Decimator] — I assert this one thing about the constellation of Boötes: that since he was appointed by those ancient astronomers to keep watch over the Bear (that is, over the north), and yet is found slow and too ox-like [bovinus] in this office of his — and that because of the slow daily revolution of the stars in that region — he is therefore now stirred up in this way by these cometary envoys and their swift motions, so that, returning to his office and spreading wide his ox-like ears, he may in some fashion show what God wills for the world, and especially for these northern regions on account of their utter complacency [securitas], and what he threatens by his just judgment; and may, as it were, point it out manifestly with the hand by which the last of these comets was borne directly into the north, ascending a little higher. This, I say, it pleased God beyond doubt to make plain to us in this way too, through this constellation, after men, too complacent, have neglected duly and carefully to heed his will, made clear in his Word — so that no one, that is, may be found ἀπολόγητος [Greek: anapologētos, "without excuse"] when that great day, for which the sons of light now pant, shall shortly dawn.

Meanwhile, however, God does not seem to wish to suspend, much less wholly to remove, the deserved penalties from the impenitent who persevere in their complacency. Rather, besides the other penalties by which he will justly compensate for sins, he will deliver men's hearts over to dire contentions and quarrels — chiefly out of the lust to reign — in such a way that, the means and counsels of peace having been wretchedly scorned and set aside, they tear one another apart, and that after the manner of those beasts among which these comets travelled: namely, the beasts from which they arose and among which they at last disappeared.

Fourthly — and this too is worthy of note — in this same year 1618, a little before the rising of the last comet (something I do not know whether it ever happened before), two other, more monstrous comets appeared, in nearly the same prisons or regions of the sky from which these two we have treated thus far were sent forth. The one was in the north, beneath the Great Bear, as it were the herald or guard of this great last envoy about to come; the other in the south, as though it had instructed that same great Envoy and given him his powers, before his departure from the south. On both of these let Dr. Johannes Kepler [Iohannes Kepplerus] be read — whose judgment, too, concerning this last comet there was (indeed, a dissertation against Seneca in favor of Ephorus): that one comet was split into two, or departed into two parts — with an omen added of the Turkish empire being made wider in the future and, as it were, divided into two, one of which would embrace the southern part of the world, the other the northern. May God in his mercy turn this omen away from us — the outcome of which, otherwise, he would justly permit to be derived upon us from the Turkish tyranny on account of our sins — unless, putting aside without any delay all sloth and disobedience against the long-suffering of God in his goodness (a serpent, as it were, to be nursed in our bosom), we walk henceforth in his commandments, and persist constantly in prayers, so that we may behold the light in [Margin: *Psal.* 36.] his light, and detest from our hearts the works of darkness. And if we do this, and have turned, in good time and from our inmost heart without dissimulation, to our God, we shall not doubt that he himself will destroy the enemies of his Church, and will graciously grant her — rightly founded here on the sound Word and the true use of the sacraments — a longer halcyon peace among us according to his promise; and that at the same time he will benignly preserve the magistracy and polity that now relies solely upon the divine promises and protections. But if otherwise, surely who will doubt that our complacency must one day be repaid by the gravity of punishment? — since the histories of all ages, even among the heathen nations, but chiefly among the people of God, show this; for whose sake, among others, let the books of Kings be diligently read and pondered in the sacred writings.

But now, to bind myself to concluding this treatise on the new phenomena with such a παραίνεσις [Greek: parainesis, "exhortation"], we ought to know that all the errors and sins among men — whom God created in his own image, and for a life perpetually better, from the beginning — draw their source chiefly from two springs: namely, from ignorance, or want of knowledge, of the true God; and from unbelief in believing him according to his revealed Word. Of these, that the former may not hold us blinded together with the chief parts of the world, is reason for us to render eternal — though never sufficiently deserved — thanks

for the divine benevolence toward us. But out of unbelief — which knows not how to put faith in the promises and threats of God in his Word, but rather, while the divine patience and long-suffering lasts (by which, with fatherly affection toward us, we are invited to saving repentance), doubts that these are true — there has arisen that detestable complacency [*securitas*] among the people, which spreads itself widely through every kind of vice among men. So much so that, if only on account of this one beast to be slaughtered and exchanged for a mature and truly penitent vigilance, divine portents occur everywhere — in the heavens, in the air, in the waters, on the earth, in men, in living creatures — opening the eyes of mortals, pulling at their ears, and stirring all their senses; in case there might perhaps be those who, before the final fates of the world and the ruin of all things, are willing to believe that God reigns in the world, and that Christ — whom the Father set over Zion as the most powerful King of all — will rule with an iron scepter the tumultuous and unruly nations, but will overshadow with his mercy, as with a shield, those that are compliant and flee to the throne of grace. [Margin: *Psal. 2.*]

Blessed therefore are all who, in good time, bid farewell to those worldly enticements that Satan holds out and with which men everywhere in this age are baited — or rather, since they are most ruinous to the salvation of themselves and their own, curse them; and who rush rightly — that is, without offense or scandal — into the embrace of the Son of the most high King, who is himself also, by nature, virtues, and merits, our supreme King and priest; who unceasingly fix their minds upon his coming, beyond doubt now shortly at hand, and keenly persist in the renewal of their minds, so that, when this King shall come to judge the world (sooner perhaps than expected), our lamps may be burning, and we may be found worthy to be received with this our Bridegroom into the eternal dwellings, where, all worldly deceit and sorrow excluded, joy and peace shall reign everlastingly.

Come, then, Lord Jesus.

Even so, I come quickly. Amen.

FINIS.

— Rear flyleaf, blank. Aged/foxed laid paper with scattered staining; no printed text, no manuscript annotation other than faint stray marks. A small portion of the leather turn-in is visible along the right (fore-edge) margin, confirming this leaf is at the rear of the binding.

— Rear free endpaper / paste-down area, blank of printed text. The text-block edge (gilt/marbled fore-edge and sewn quires) is visible along the left margin, confirming the rear of the volume. Two penciled library inventory marks are present:

- [Margin: top edge, pencil] "6C74,jP7" [?] — partially legible penciled shelfmark or accession scrawl at the head of the leaf.
- [Margin: foot of leaf, pencil] "BC742877" — penciled library accession / catalog number, centered near the bottom.

— Rear paste-down (inside of the back board), blank of printed text. Shows the textured impression of the underlying board and the leather turn-ins around all margins, plus an old adhesive/label ghost in the upper-right corner where a bookplate or shelf label was once affixed. No legible printed or manuscript content.

No errata leaf, colophon, or printer's device appears in this final batch. The work proper ended with FINIS at appendix p. 44; are trailing blank end-leaves and the rear binding.