

**Opera Mathematica,
Tomus Tertius —
complectens
Commentarium in
Sphaeram Ioannis de
Sacro Bosco, &
Astrolabium**

*[Mathematical Works, Volume Three —
Commentary on the Sphere of Sacrobosco, and the
Astrolabe]*

(1611)

by Christoph Clavius (1538–1612), S.J. — commentary on the *De sphaera* of Johannes de Sacrobosco (c. 1195–c. 1256)

Translation by Scott Weisman



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Translator Version 2

Upright reason dictates that the recipients of the good, of whatever type and level of beneficence it may be, must show gratitude and blessing to the beneficent person in every way possible, commensurate with the value of the beneficence. And one who has benefited all the people of the world — for example, one who invented a new instrument for the good of the world, or a good book — it is fitting for every discerning person, out of the obligation of love of fellow beings, to at least purchase it, so that the man will profit and his heart will be encouraged thereby to invent yet more good instruments in the world. And similarly, all other wise-hearted people will likewise strive and exert themselves to invent good things and instruments needed for the repair of the world and its perfection.

And therefore, whoever says: "What do I need this new instrument for?" — he does not act well towards the world. For if not for the man who invented it, where would you be? And what would your city do? And more than this, if the man did not exert himself, what would the world do?

— R. Pinchas Eliyahu Hurwitz, *Sefer HaBris HaShalem* (1797)

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A note on the work

Tomus III holds Clavius's astronomy. The first and larger part is his commentary on Sacrobosco's *De sphaera* — the most influential Renaissance astronomy textbook — reprinting Sacrobosco's brief text and surrounding it with a vast commentary on spherical astronomy: the celestial and terrestrial spheres, the circles of the sphere, risings and settings, the climes, the motions of the Sun and planets, eclipses, and the computus. The second part, the *Astrolabium*, is a comprehensive treatise on the construction and use of the astrolabe, with its projection geometry and tables. Both are richly illustrated with woodcut diagrams.

A note on the translation process

This translation was produced with the [translation-pipeline](#). See the linked repository for details on the process.

This text is the product of a highly refined automated translation process. It has undergone only cursory human review and should not be treated as an authoritative or scholarly rendering. It is offered solely as a reference and reading aid — a way to make an otherwise inaccessible work approachable — and any passage of real consequence should be checked against the original source.

Acknowledgements

Translated with the assistance of Claude. The translator thanks the [Anthropic](#) team for making this work possible.

Opera Mathematica, Tomus III — Front Matter

Binding and endpapers

This opening covers the volume's binding cover and its front endpapers; no printed text of the work appears yet.

Upper cover. Blind-tooled leather binding with a densely worked ornamental panel design: an outer floral roll border enclosing concentric rectangular frames of scrollwork and foliate tooling, with a central oval cartouche containing an armorial device (a quartered shield). The tooling is worn and no lettering is legible.

Front flyleaf. Blank leaf carrying two handwritten notations in the upper portion:

- [Margin: Upper left, ink shelfmark:] "2 Math. u. 10 / 3" [?] — an early library pressmark (partly written as a fraction, "2 Math. u. 10" over "3").
- [Margin: Center, ink:] "Clavius" — the author's name, written as an identifying note.

The remainder of the leaf is blank, with staining, show-through, and edge damage.

Front paste-down / endpaper. Blank leaf showing faint mirror-image offset (show-through) of the printed title page from the facing recto — a few reversed capitals are visible but not reliably transcribable. Applied at the lower center is the modern Bayerische Staatsbibliothek barcode label:

- "<36613880450015" (printed twice)
- "Bayer. Staatsbibliothek"
- A red inked shelf/serial number "5" [?] beside the barcode, and a pencilled "33" below.

Blank front endpaper. No printed text. A handwritten shelfmark appears at the top in ink: "2 Math. u. 10 - 3" [the Bavarian State Library call-number "2 Math. u. 10-3"]. The lower part of the leaf is blank, with a faint show-through of the woodcut on the following title page.

CHRISTOPHORI CLAVII BAMBERGENSIS E SOCIETATE IESV OPERVM MATHEMATICORVM TOMVS TERTIVS

The Mathematical Works of Christoph Clavius [Christoph Clavius] of Bamberg, of the Society of Jesus: Volume Three.

Comprising the *Commentary on the Sphere of Johannes de Sacro Bosco* [Sacrobosco], and the *Astrolabe*.

Diagram. The title-page device is a woodcut armillary sphere shown in oblique perspective, seen slightly from the front and above. A graduated meridian ring encircles the whole, marked off in tens of degrees; running down each quadrant the numbers read 10, 20, 30, 40, 50, 60, 70, 80, 90, then back 80, 70, 60, 50, 40, 30, 20, 10 toward the opposite pole. Within it are set the principal circles of the sphere — the two polar (Arctic and Antarctic) circles, the two tropics, the equator (equinoctial), the horizon, and the oblique zodiac band bearing its signs — with the earth shown as a small globe at the centre. The poles are marked by small star-shaped figures at upper left and lower right.

- POLVS SEPTENTRIONALIS (upper left, at a star): North Pole
- [meridian ring, both quadrants]: 10 · 20 · 30 · 40 · 50 · 60 · 70 · 80 · 90 · 80 · 70 · 60 · 50 · 40 · 30 · 20 · 10 (degree graduation)
- ARCTICVS (upper left interior): the Arctic Circle [Circulus Arcticus]
- TROPICVS ÆSTIVVS (upper interior, printed "TROPICVS... ESTIVVS"): the Summer Tropic (Tropic of Cancer)
- ÆQVINOCTIALIS (mid-right interior, printed "EQVINOCTIALIS"): the Equinoctial (celestial equator)
- ORIZON (across the middle): the Horizon
- ARIES ♈ · TAVRVS ♉ · GEMINI ♊ (on the zodiac band, upper right, with degree subdivisions 10 · 20 · 30): the zodiac signs Aries, Taurus, Gemini
- AQVARIVS ♒ · PISCES ♓ (on the zodiac band, lower interior, with degree subdivisions 10 · 20 · 30): the zodiac signs Aquarius, Pisces
- TROPICVS HIEMALIS (lower interior): the Winter Tropic (Tropic of Capricorn)
- ANTARCTICVS (lower interior): the Antarctic Circle [Circulus Antarcticus]
- POLVS AVSTRALIS (lower right, at a star): South Pole

MOGVNTIÆ, — At Mainz [Moguntia]:

At the expense of Antonius Hierat [Anton Hierat]; printed by Reinhard Eltz [Reinhardus Eltz].

Cum gratia & privilegio sacræ Cæsar. Maiest. — With the grace and privilege of His Sacred Imperial Majesty.

ANNO M. DC. XI. — In the year 1611.

[Handwritten ownership marks on the title page: a large pen flourish (a signature) across the centre, over the printer's device [?]; a cursive signature to the right of "Complectens," reading approximately "de Berennis" [?]; and, below the woodcut above "MOGVNTIÆ," a monastic ownership inscription reading approximately "Monast[erii]... senis" [?]. All are illegible manuscript hands.]

Blank verso of the title leaf. No printed text of its own; the whole title-page setting shows through in mirror image. The leaf carries the ownership stamp of the Bavarian State Library:

[Ownership stamp: **BIBLIOTHECA REGIA MONACENSIS** — Royal Library of Munich]

A handwritten cursive signature also appears on this side, illegible [?].

Dedicatory Epistle

TO THE MOST REVEREND AND MOST ILLUSTRIOUS PRINCE AND LORD, LORD JOHANN GOTTFRIED [Johann Gottfried von Aschhausen], Bishop of Bamberg [Episcopo Bambergensi], and Provost of the Metropolitan Church of Würzburg [Ecclesiæ Metropolitanæ Herbipolensis Præposito], etc., his own Most Gracious Lord: CHRISTOPHORUS CLAVIUS [Christoph Clavius] OF BAMBERG, OF THE SOCIETY OF JESUS.

I have been especially glad, Most Reverend and Most Illustrious Prince, that at the very time when this third volume of my mathematical studies, brought to completion by my later revisions, was pressing to be released by me and made public, you — not long after you had taken up the governance of the Church of Bamberg — set all your affairs at home swiftly in order and, in the harshest season of the year and by arduous journeys, went off to Carinthia [Carinthia]. Your aim was to carry out the duty of a diligent shepherd: to visit kindly, and see for yourself, your sheep that lay far removed from the fold, and to learn what

dangers they were exposed to, or what hardships oppressed them, and with what help of yours above all they stood in need.

For when I came to understand the extraordinary devotion of those who are subject to your rule — how they first awaited you in your absence, then welcomed you on your arrival, honored you in your presence with every kind of service, as was fitting for subjects to do, and at last, when you departed, accompanied you with no gifts so much as with sighs and tears — I was very glad that I could take part in it, and could join my own reverence toward you to their acts of homage. The service they once rendered to the Emperor Henry [Henry II], who first founded the Church of Bamberg, when he was passing through those regions on his way into Lombardy, they now rendered to you with as much greater zeal as their reason was the better one. [Margin: Baronius [Cesare Baronio], *Annales Ecclesiastici*, vol. II.] Him, under arms, they had perhaps feared; you, in the civilian's gown and in no way to be feared, more renowned for the fame of your virtues than he for the terror of war, they could only love. Him, the most powerful monarch of Germany, they deservedly revered; but you — their lord, their bishop, and their most kindly shepherd, and (to come nearer to their own feeling, and indeed to the truth of the matter) their most beloved father — they were bound to embrace with their whole heart.

As for me, I am inclined to think that my own reasons for love and reverence are weightier still — the very ones I already recounted earlier, in the first volume. Receive, therefore, Most Reverend and Most Illustrious Prince, this my third gift, which I offer to you on your return to your homeland in the same spirit in which the Carinthians offered you their loyalty and homage as you came to them, and the people of Bamberg rejoiced with you over your safe journey as you returned — rejoicing no more on your account than on their own.

Farewell, both for your own sake and for that of the orthodox religion. Rome, in the year of the Lord 1611.

In Sphæram Ioannis de Sacro Bosco Commentarius

**A COMMENTARY BY CHRISTOPHORUS CLAVIUS [Christoph Clavius] OF
BAMBERG, OF THE SOCIETY OF JESUS, ON THE SPHERE OF JOANNES DE
SACRO BOSCO [Sacrobosco].**

[The remainder of this half-title leaf is otherwise blank; heavy show-through from the verso (an index/preface *Ad Lectorem*) is visible but belongs to the following page.]

To the Reader

AD LECTOREM. [To the Reader]

So that greater profit might be drawn from our Commentary on the *Sphere*, we have added — for the benefit of students, over and above the Author's own exposition, along with much else — these items in particular, which follow.

I.

A most useful disputation on the fourfold motion of the eighth sphere, according to the periods discovered by Nicolaus Copernicus; in which the emptiness of the motion of trepidation [motus trepidationis] is refuted by the most compelling arguments, and an eleventh heaven, the *Primum Mobile*, is established. Page 33.

II.

By what technique the declinations of the stars are to be computed by means of sines, once their longitudes and latitudes are known. Page 99.

III.

The history and my own view concerning the new star [*stella nova*] that appeared in Cassiopeia in the year of the Lord 1572; and concerning two others, likewise new, one of which was seen in Cygnus in the year 1600, and the other in Sagittarius near the ecliptic in the year 1604. Page 103.

IV.

By what means the distances of the heavens, their thickness, and their circumference may be investigated, together with the magnitudes of the stars. Page 117.

V.

A digression on the number of the sand, drawn from Archimedes. Page 120.

VI.

By what method the declination of any given point is to be investigated from the doctrine of sines. Page 149.

VII.

By what device the place of the Sun in the zodiac, and its entry into the twelve signs, may be found from memory, more or less. Page 158.

VIII.

By what means the interval between any two cities whose longitude and latitude are known may be found, both geometrically by lines and from the sines. Page 177.

IX.

Likewise, how the latitudes of rising and of setting [Latitudines ortivae atque occiduae] are to be computed by sines. Page 182.

X.

Next, by what way the right and oblique ascensions of all points, or of the arcs of the ecliptic, are found from the doctrine of sines. Pages 198 and 203.

XI.

Again, in what manner the lengths of the days and nights throughout the whole year, in every clime, may be inquired into by sines. Page 238.

XII.

Besides these, tables most necessary to astronomical work: namely, a table of all the fixed stars, with the longitude, latitude, magnitude, and position within the constellations of each. In which matter we have followed the order of all the asterisms, or constellations, as they were observed by the most ancient astronomers. Page 76.

A table for reducing degrees to hours, and the reverse. Page 128.

A table for converting degrees, minutes, seconds, thirds, and so on of the equator into minutes, seconds, thirds, and so on of days, and the reverse. Pages 130 and 131.

A table of the declinations of the points of the ecliptic, far fuller than any published by others, inasmuch as it proceeds by five-minute steps. Page 150.

A table of right ascensions. Page 199.

A table of ascensional differences for all elevations of the pole. Page 203.

A table of oblique ascensions for various altitudes of the pole. Page 209.

A table of semidiurnal arcs computed for all elevations of the pole. Page 288.

XIII.

A geometric digression on the twilights [de Crepusculis]. Page 256.

XIV.

A demonstration of why the more northerly climes are the narrower. Page 286.

XV.

A most useful disputation on eccentric orbs and epicycles, against certain philosophers. Page 290.

XVI.

Finally, the *Theorics* of the planets arranged into tables. Page 308.

[The lower half of this page carries a large, faint mirror-image ghost of a set of concentric circles — offset/show-through from the woodcut cosmological figure printed on the facing page; not an independent figure.]

Diagram. A new figure and series of the twelve heavens — a full-page geocentric woodcut of concentric spheres. The heading above reads: "A NEW FIGURE AND SERIES OF THE TWELVE HEAVENS; being one that until nearly the present day was unknown, but is explained in this Commentary. Page 42 and following." The circles are labelled, reading from the outermost inward, with the seven planetary spheres each flanked on both sides by the planet's astronomical symbol. Small star-figures (asterisks) are scattered through the firmament and the outer rings; at the very center is a small woodcut of the terrestrial globe (the elemental Earth).

- Outermost ring: *Caelum Empyreum* (the Empyrean Heaven)
- *Primum mobile* (the First Movable)
- *Decimum caelum* (the Tenth Heaven)

- *Nonum coelum* (the Ninth Heaven)
- *Firmamentum* (the Firmament)
- ♄ *Saturnus* ♄ (Saturn)
- ♃ *Iuppiter* ♃ (Jupiter)
- ♂ *Mars* ♂ (Mars)
- ☉ *Sol* ☉ (the Sun)
- ♀ *Venus* ♀ (Venus)
- ☿ *Mercurius* ☿ (Mercury)
- ☾ *Luna* ☾ (the Moon)
- Center: the terrestrial globe (Earth)

Christoph Clavius of Bamberg, of the Society of Jesus, on the Sphere of John of Sacrobosco

Preface

In various ways — as Proclus Diadochus [Proclus] reports in the Commentary he wrote on the first book of Euclid — the ancient philosophers divided up the mathematical disciplines. Among all their divisions, the one ascribed to the Pythagoreans, and followed by Plato, Aristotle, Boethius, and other philosophers of the greatest weight, both ancient and more recent, has always been the most celebrated. By it all the mathematical disciplines are distributed into four principal kinds, namely Arithmetic, Geometry, Music, and Astronomy; and this was not done without reason. For since all the mathematical sciences are concerned with quantity, and quantity is twofold — discrete and continuous — and since, again, discrete quantity can be considered either absolutely and in itself, or in comparison with another, and likewise continuous quantity falls under our understanding either as immobile or as mobile, it was entirely right that four principal mathematical disciplines should be established, treating of the twofold quantity under the twofold consideration of each. Such are the four just enumerated.

Arithmetic, then, deals with discrete quantity — that is, with number considered absolutely and in itself — investigating and setting out at length and with care its properties and the whole force of numbers, together with the art of counting. Geometry next treats of continuous immobile quantity, and teaches how to measure the magnitudes of the earth and of other things. Music, for its part, deals with discrete quantity — that is, number — considered in comparison with another, and studies the concord and harmony of sounds. Astronomy, finally,

undertakes the discussion of continuous quantity, of mobile magnitude, and scrutinizes the heavens and the unchanging motions of the stars.

[Margin: The division of the mathematical disciplines.]

Of these four mathematical disciplines — from which, indeed, all the others that deal with quantity in any way flow and are propagated — Astronomy is the most far-reaching, and for that reason at once the most worthy and the most delightful. On account of the multitude of things it considers, it has always been held in such esteem that the ancient philosophers devoted more study, labor, and diligence to no other science than to this one Astronomy. But since, as time went on, very many shrank back from this excellent discipline, daunted by the length of its books and the difficulty of the subject, so that it nearly collapsed, therefore John of Sacrobosco, an Englishman by birth, an outstanding philosopher and mathematician of his day, who flourished around the year of the Lord 1232, wishing to remedy this evil, composed for the common benefit of students — drawing on the most approved and choicest astronomers, Ptolemy, Alfraganus [al-Farghānī], al-Battānī [Albategnius], and very many others — a certain small compendium of the whole of Astronomy, to serve as a kind of introduction to the weightier writers on Astronomy, with all the care he could; and this work has indeed always held a great name in all the schools right up to our own times.

This little work, therefore, we have thought good to illustrate, for the benefit of students, with fuller annotations. In them we have tried, so far as possible, first to explain the author's meaning in a plain account; then to supply what he himself seems to have omitted; and to clarify at greater length what he taught too briefly — always keeping to the footsteps of the ancient astronomers, while adding besides a number of observations of more recent writers, so that what has been obscurely stated by others concerning the motions of the heavens and the form of the whole universe may be more perfectly understood.

But before we approach the author himself to interpret him, it will be worthwhile to say a few things first about Astronomy in general, so that students may come to this science with a better-prepared and more eager mind. These we shall divide into four chapters. In the first we shall briefly teach who were the first inventors of this discipline, and who were its most eminent authorities. In the second we shall make clear what parts this whole science of Astronomy comprises. In the third we shall discuss the excellence and dignity of Astronomy. And in the fourth, finally, we shall bring forward some remarks about its usefulness and necessity.

[Margin: When John of Sacrobosco flourished, and why he wrote this compendium of Astronomy.]

Commentary on the First Chapter of the Sphere

On the Inventors of Astronomy

[Margin: Who were the first inventors of Astronomy.]

No one ought to doubt that the first inventors of Astronomy were the forefathers and propagators of the human race — I mean Adam, Noah, Abraham, and others of this kind — from whom, as the most ancient histories testify, all the other honorable disciplines also drew their origin. For as Josephus writes in the *Antiquities of the Jews* I.4, when Adam had foretold to his sons the destruction of all things — one coming by the force of fire, the other by the might and mass of waters — they, fearing that the science of celestial things which they had first discovered might slip away from among men, or might die out before it came to their knowledge, made two columns, one of brick and the other of stone, and inscribed on both the things they had discovered, so that if the one built of brick should be destroyed by the rains, the stone one, remaining, might furnish men with the writings concerning celestial matters. This stone column, Josephus reports, survives to this day in Syria; and if the stone column were consumed by fire, the brick one would remain unharmed and would present the science of the stars to mortals. The same Josephus, in chapter 8 of the same book, affirms that those ancient fathers were granted so long a life precisely so that they might have leisure for matters astronomical and geometrical; and his very words are these:

[Margin: The two columns, on which the sons of Adam inscribed the sciences lest they perish in the flood, and what they were.]

"Let no one, comparing modern life and the brevity of the years in which we now live with those of the ancients, suppose false what is said of them; nor, because life is now not prolonged to so great a length, let him believe that they in no way reached that length of life. For they, being religious and made by God himself, and having more suitable nourishment provided for a longer span of time, rightly lived through the cycles of so many years. Moreover, on account of the virtues and glorious benefits which they continually investigated — that is, Astrology and Geometry — God granted them ampler spaces of life, which they could not have learned thoroughly unless they lived six hundred years. For it is through so many years' courses that the Great Year is completed."

[Margin: Why God prolonged so long a life for the first parents, according to Josephus's opinion.]

Again, in the same book he names him [Abraham] as experienced in celestial matters; and in chapter 16 he testifies that Abraham was the first to instruct the Egyptians in Arithmetic and the science of the stars. For thus he writes of him: "Arithmetic too he imparted to them (that is, to the Egyptians), and he himself conveyed to them what pertains to Astrology. For before Abraham's coming into Egypt, the Egyptians were entirely ignorant of these things." For these things are known to have passed from the Chaldeans to the Greeks.

It is established, therefore, that Astronomy existed even at the very beginning of the world, so that with the fullest right it may contend with all the other arts and disciplines in point of antiquity, since we read of none more ancient. Hence it comes about that those authors who are read of in the histories as having been the first inventors of Astronomy should be judged rather to have illustrated it — a science already discovered long ago, indeed sprung from the very beginnings of the world — and to have enriched it with new additions, than to have first discovered it; and that they are called the first authors of this discipline only in that sense.

[Margin: Who are said by writers to be the first inventors of Astronomy.]

As to whom this invention — or rather amplification — of Astronomy is chiefly to be ascribed, there has always been great controversy among authors, and the case is still before the judge. For some attribute it to the Egyptians, some to the Assyrians, some to the Babylonians; while others assert that it was first discovered by the Ethiopians, on the ground that, living under the equinoctial circle, they always enjoy a most serene sky, from which they can easily observe the courses of the stars. These authors, however, do not deny that the Egyptians afterward rendered it more perspicuous and more illustrious. Nor, indeed, are there wanting those who make Atlas the first inventor of this discipline, and who would have it that from this arose that famous fable — namely, that he bore the heaven on his shoulders — because he was the first to take care, by the vigor of his mind and his skill, to hand on to men the courses of the Sun and Moon and the revolutions and principles of all the stars. Concerning this Diodorus Siculus writes thus in Book 4: "They say that Atlas was most skilled in Astrology, and was the first among men to dispute about the sphere; from which circumstance he was thought to bear the heaven on his shoulders, the invention of the sphere giving occasion to the fables." Concerning the same, St. Augustine [Augustine] in Book 18 of the *City of God* [*De Civitate Dei* XVIII] says thus: "Atlas is said to have

been a great astrologer, from which the fable took its occasion, so that he was feigned to bear the heaven." Eusebius of Caesarea, too, in his *Preparation for the Gospel*, would have Enoch and Atlas to be one and the same man; but from the histories it is established that Atlas is younger by at least seven hundred years. Caelius Rhodiginus [Lodovico Ricchieri], in Book 18 of his *Ancient Readings* [*Lectiones Antiquae* XVIII], thinks that Astronomy was first discovered by the Sidonians on account of the practice of navigation. For just as the Egyptians laid the first foundations of Geometry on account of the measuring of fields, which they could not do without Geometry, and the Phoenicians, on account of their frequent trade and commerce, are thought to have handed down the first rudiments of Arithmetic, so too the Sidonians, on account of the constant navigation they practiced, are believed to have first discovered Astronomy, since the practice of navigation cannot in the least subsist without this science. This, however, was afterward increased in a wonderful manner by the Chaldeans, Persians, Indians, Egyptians, Greeks, and very many Arabs.

[Margin: Various authors who flourished in Astronomy.]

Whatever, then, may finally be certain and clear about the first inventors of Astronomy, many distinguished authors excelled in it, of whom I shall review only the more eminent. In the first place there flourished in it Atlas, the brother of Prometheus, king of Mauretania, born in Egypt, who handed it on to Hercules; and Hercules is said to have made such progress in this discipline that, on account of the knowledge of celestial matters with which he was endowed, he is reported to have borne on his shoulders the heaven taken over from Atlas, and to have attained great glory — as the histories testify — because he was the first to bring the sphere of the stars into Greece. After these, very many distinguished astrologers followed in succession, such as Anaximander of Miletus, Thales of Miletus, Pythagoras of Samos, Eudoxus of Cnidus (in Plato's time, a pupil of the Egyptians and Chaldeans), Callippus, Archytas of Tarentum, Euclid of Megara, Aratus of Soli, Timocharis of Alexandria, Abrachis — who by another name is called Hipparchus, although very many think Abrachis different from Hipparchus — Eratosthenes of Athens, Archimedes of Syracuse, Sosigenes, Julius Caesar (who by the work of Sosigenes adapted the year to the Sun's course), Andromachus of Crete, who is said to be the inventor of the Theorics, Proclus Diadochus, Menelaus the Roman (also called Mileus the Geometer), Theodosius of Tripoli, author of the three books on spherical elements, Ptolemy the most skilled of all, Theon of Alexandria, Pappus of Alexandria, Albumasar [Abū Ma'shar], Almeon the Arab, Abraham Avenezra [Abraham ibn Ezra], al-Battānī [Albategnius], Thabit [Thābit ibn Qurra], the discoverer of the motion of

trepidation in the eighth sphere, who flourished 1140 years after Ptolemy, Haly [‘Alī], Geber of Seville [Jābir ibn Aflāḥ], Alfraganus [al-Farghānī], Alphonsus king of Spain (in the year of the Lord 1250), from whom the Alphonsine Tables took their name, George Peurbach [Georg Peuerbach], John of Königsberg [Regiomontanus], John Werner of Nuremberg, John Bianchini of Ferrara (who also composed astronomical tables), Johannes Stöffler [Stoflerinus], Nicolaus Copernicus, Francesco Maurolico the Sicilian abbot, Pedro Nunes of Alcácer [Petrus Nonius Salaciensis], the Portuguese, and Giovanni Antonio Magini of Padua, and others almost without number.

On the Parts of Astronomy

[Margin: What Astronomy is.]

In order that we may more correctly gather what parts Astronomy comprises under itself, we shall not inappropriately take our beginning from the explanation of the name. This science concerning celestial things, then, which is called Astronomy, according to the meaning of the name and its etymology seems to signify nothing else than the reckoning and law of the stars, so that Astronomy is the same as the science of the stars. For it treats of the motions of the stars, and of the fixed and perpetual alternations and laws of those motions; of the order of the stars and heavens, their site and position, rising and setting, multitude and magnitude, distance from the earth and from one another, their mutual conjunction, eclipses, and other things of this kind. This is by others usually called Astrology. For in our age Astronomy and Astrology are used almost interchangeably for the same science; and therefore we too shall use these names without distinction in these our commentaries, although some mathematicians would have some distinction established between these terms — that Astronomy should signify that doctrine which considers the motions of the heavens and stars, while Astrology should be applied to that art which, from the revolutions of the heavens and the conjunctions or oppositions of the stars, predicts future events and certain presentiments and significations bearing on the safeguarding of health and of the household.

[Margin: Astronomy and Astrology — in what way they differ from each other.]

Astronomy is divided into Theoretical — that is, contemplative — and Practical, that is, working and active. Theoretical astronomy considers the whole machine of the world as it is in itself, describing the constitution of the world and dividing the whole framework of the world into the ethereal and the elementary region. Then it investigates the number, magnitude, and motion of all the celestial bodies,

and speculates on the risings and settings of all the stars and planets. In like manner it considers the figures and images of all the constellations and signs, and teaches how to compute by calculation of numbers the true places both of the fixed stars and of the wandering ones, which they call Planets. Likewise it most diligently inquires into the progressions, stations, retrogradations, conjunctions, and oppositions of the planets, together with the eclipses of the luminaries — namely the Sun and Moon — and other such things almost infinite in number. And this Astronomy is expounded partly in the *Almagest*, or Great Construction of Ptolemy, or also in the *Epitome* of Regiomontanus, in the astronomical work of al-Battānī [Albategnius], in the little work of al-Farghānī [Alfraganus], in the *Theorics of the Planets* of George Peurbach, in the *Celestial Revolutions* of Nicolaus Copernicus [*De Revolutionibus*], and in the volumes of almost countless other authors; partly by very many instruments devised for this purpose by astronomers with the greatest industry, to set the celestial motions before our eyes — such as the common Astrolabe, or planisphere of Ptolemy, the Astrolabe of Gemma Frisius, the Catholicon or universal instrument, the universal Planisphere of Johannes de Royas, the Astronomical Ring, the Quadrant, the Torquetum, the Astronomical Radius [cross-staff], and other things of the kind; and partly, finally, Theoretical Astronomy is taught in that part which is usually called tabular, because through numbers arranged into tables astronomers scrutinize the motions of the heavens — such as the tables of Alphonsus king of Spain [Alphonsine Tables], of Regiomontanus, of Bianchini of Ferrara, and of Nicolaus Copernicus (which tables are usually called Prutenic [Prutenicae]), and of many others.

[Margin: The division of Astronomy into Theoretical and Practical.]

Practical Astronomy, however — which others call Judiciary, or Prognostic, that is, Divinatory — accommodates all these things to the use of human life. For it contemplates the complexions and natures both of the signs and constellations, and also of the Planets and the other stars, and explains which signs are hot, which cold, which temperate, which masculine, which feminine, and other such things. Again, from the motions of the orbs and stars it predicts future events in these lower regions. But since many have rashly and wrongly dared to add many things to this part of Astronomy, and have so amplified this prognostic part that it has now become an altogether superstitious and hateful matter, and is deservedly held suspect by the Church, and wonderfully condemned by St. Augustine in his books *On Christian Doctrine* [*De Doctrina Christiana*], therefore I judge that nothing at all should be said of it by us — except that it is utterly overthrown by John Pico della Mirandola [Giovanni Pico della Mirandola] in his

twelve books written against the astrologers; by Francesco Pico, his nephew, in his books *On Foreknowledge* [*De Praenotione*]; by Antonio Bernardi della Mirandola, Bishop of Caserta, in books 22, 23, and 24 of the *Monomachia*; by Michael Medina in Book 2 *On the Right Faith in God*, chapter 1; and by Julius Syrenus in his books *On Fate*.

[Margin: Judiciary Astrology is a superstitious matter.]

On the Excellence of Astronomy

Since the nobility of any science, on the authority of Aristotle, ought to be derived from two things — namely, from the excellence of the subject with which it deals, and from the certainty of the demonstrations by which it confirms what it considers (for he says that that science is the more excellent and noble which either deals with more excellent things or is more certain) — how great the dignity and excellence of Astronomy is can be recognized without obscurity from both heads. For if we look to the subject or matter of Astronomy, it must be granted, and we are compelled to confess, that almost the supreme place among all the other human disciplines, or those acquired by the natural light, is to be conceded to it. For this science treats of the celestial bodies, which are the most noble of all, for many reasons. First, because, according to the philosophers, they are ungenerable and incorruptible, free from all corrupting alteration, and, in short, unchangeable by any motion that in any way varies their substance — which is not the case with the other bodies of which the natural philosopher disputes. For although the elements, as Aristotle holds together with the philosophers, cannot as wholes be generated or corrupted, yet in respect of their parts they are subject to generation and corruption. Second, because the celestial bodies are the cause of all these lower things, as Aristotle holds in *Meteorologica* I, where he says that the lower world must necessarily be continuous with the higher motions, so that all virtue may be derived from them. Likewise in *Physics* VIII he asserts that all things are produced by means of the motion of the heaven, and on that account he did not hesitate to call the celestial motion the life of all beings. Again, in *On the Heavens* II [*De Caelo* II] he affirms that the heaven acts upon these lower things by means of light and motion. Finally, in *On Generation and Corruption* II he testifies that, on account of the motion of the Sun and of the other planets in the oblique circle — that is, in the Zodiac — generations and corruptions come about in these lower regions; and he affirms the same in very many other places, in which almost the whole assembly of philosophers agrees with him. Third, because the celestial bodies are nearer to the most noble and first being, namely to the glorious God; indeed, according to Averroes, the

celestial body is the mediator and bond of the higher things with the lower, and the place of things eternal and divine. For all philosophers and nations, however barbarous, place God in the heaven as in his proper seat. For although God is not bound to this or that place, but exists everywhere (which belongs to no other things), yet he is placed in the heaven as in the more noble part of the world, where he most manifests his omnipotence and goodness, as the Theologians assert. Fourth and last, because among all other bodies the celestial bodies possess the most noble place, namely the supreme; and the higher bodies are, the more noble they must be esteemed, as all philosophers confess. For just as the earth is the lowest of all the elements in site and place, so too it is the last in dignity; to it in nobility succeeds water, because it occupies a higher place; then follows air, because by its lightness it transcends water; and lastly fire obtains the primacy among all the elements, since it is set above them all. There accrues also to the dignity of the celestial bodies the fact that they possess most noble accidents — namely both motion and circular figure (as we shall show in its own place), light, and other things of this kind — so that not undeservedly does Aristotle seem to have called these bodies divine.

[Margin: The excellence of Astronomy from its subject.]

[Margin: The celestial bodies are the most noble of all.]

But if we consider the mode of demonstrating which Astronomy employs, no one will deny that all the natural disciplines are far surpassed by this science. For to confirm the matters of which it treats, it employs the most effective demonstrations — namely the Geometrical and Arithmetical — which, in the opinion of all philosophers, obtain the first degree of certainty. Wherefore not without reason...

[Margin: The dignity of Astronomy from its mode of demonstrating.]

(Continuing from the preceding page, on the nobility of astronomy from its mode of demonstration.)...on both counts — namely by the nobility of its subject and by the certainty of its demonstration — Ptolemy, at the beginning of the *Almagest*, held astronomy to be simply the first among all the other sciences. For he says that natural philosophy and metaphysics, if we consider their mode of demonstration, ought rather to be called conjectures than sciences, because of the multitude and disagreement of opinions about them.

DE VTILITATE ASTRONOMIAE.

(On the Usefulness of Astronomy)

[Margin: Astronomy is useful for theology.]

How great is the usefulness — indeed the necessity — of this most excellent science can scarcely be explained. For astronomy seems in some way to prepare the road to all the disciplines and to point out a safe approach to them. It is above all of the greatest service to sacred theology. For from the contemplation of the celestial orbs and of their motions, which always keep themselves in the same manner and unchangeably, the greatness and excellence of their Creator is made known. Hence Ptolemy, at the beginning of the *Almagest* and following the tradition of the Arabs, not without reason declared this one science to be the road and pathway to the knowledge of God most high.

From this opinion St. Paul does not differ, where he says (Romans 1) that "the invisible things of God are clearly seen, being understood from the creation of the world by the things that are made," and so on. In that passage, when he speaks of all created things, he seems most of all to have understood the celestial bodies. For these, by their beauty, their magnitude and multitude, by the wondrous variety of their motions and influences, and by their perpetual stability, commend in a marvellous way the goodness, wisdom, and providence of the glorious God, and most powerfully draw us to the knowledge, love, and admiration of Him. This the royal prophet David admirably testifies when he says: "The heavens declare the glory of God, and the firmament proclaims the work of his hands." And again: "For I will behold thy heavens, the works of thy fingers, the moon and the stars which thou hast founded." This opinion is supported by what is written in the book of Wisdom, chapter 13, where it reads thus concerning the celestial bodies: "Let those who, delighted by their beauty, supposed them to be gods, know how much more beautiful is the Lord of them; for by the greatness of the beauty and of the creature the creator of them may be seen so as to be known." From this it has come about that astronomy, which treats of these most excellent bodies, is by many called natural theology.

[Margin: Astronomy is useful for metaphysics, physics, medicine, poetry, and navigation.]

This discipline also serves metaphysics, since on the authority of the astronomer Aristotle, in Book 12 of the *Metaphysics*, gathered the number of the intelligences from the number of the orbs. By the same reasoning, from the motions of the

celestial orbs the power and substance of the intelligences that move them can best be investigated and grasped.

Nor does this science contribute a little to natural philosophy, since the philosopher borrows much from the discoveries and demonstrations of astronomy, as may be seen in the second book of *On the Heavens* and in other books of Aristotle. Moreover, from the unchanging celestial motion Aristotle, in the eighth book of the *Physics*, tracked down the first mover, eternal and free from all change.

To medicine, too, astronomy is so conducive that Galen, the prince of physicians, warns the sick not to entrust themselves to the hands of doctors ignorant of astrology. For, he says, medicines administered at unsuitable times profit little or nothing; indeed, they often do harm. But these seasons can be known only from the motions of the planets, which belong to the astronomer.

And what would the poets produce if they were entirely deprived of this splendid discipline? For what in their poems or writings is fine or excellent that is not filled with the motions of the stars, the rising and setting of the signs, and the constellations? Add to this that no one will understand the poems of the ancients unless he has first been thoroughly versed in the study of astronomy.

The art of navigation, too — so useful and necessary to the human race — can by no means worthily guard its own bounds without the aid of astronomy, as all writers on the nautical art frankly admit.

[Margin: Astronomy is necessary for churchmen.]

Astronomy is furthermore quite indispensable to men set in ecclesiastical dignity — as St. Augustine testifies — for calculating more exactly the conjunctions and oppositions of the two luminaries in relation to the movable feasts, and other matters of that kind that concern the honor and standing of the Church. It was through neglect of this astronomy that, right up to the year of our Lord 1583, we for the most part strayed so far from the true observance of the sacred Easter and of the other movable festivals that the Jews, the Turks, and other nations reproached us marvellously for our ignorance. This indeed the most numerous and weighty mathematicians very often, and very bitterly, lamented; although it is said that at an earlier time Nicholas V, Leo X [Popes Nicholas V and Leo X], and many other supreme pontiffs wished to apply a remedy, if only there had then been available to them an abundance of outstanding and excellent astronomers to whom they could safely have entrusted the task of correcting the calendar. Astronomy has, moreover, among its other almost infinite uses, this notable one

as well: that it demonstrates the fixed limits of the year and the true description of its parts, once the equinoxes and solstices have been carefully noted; it defines the measured intervals, measures out and distinguishes most accurately the alternations of days and nights, their intervals, and their lengths. But by divine goodness and providence it has at last come about that in our own times Gregory XIII [Pope Gregory XIII], best and greatest pontiff, put the final hand to the correction of the Roman calendar and brought the equinoxes and solstices back to the times of the Council of Nicaea. Hence it is that henceforth we shall always celebrate the most holy Easter, together with the other movable feasts, rightly and in accordance with the decrees of the holy Fathers and of the Roman pontiffs. In this matter I too, by command of the same supreme pontiff, expended no small effort and labor; and toward the explanation of the new calendar I published, by order of Clement VIII [Pope Clement VIII], supreme pontiff, a fairly large volume, in the year 1603.

[Margin: Astronomy is useful for cosmography.]

Astronomy is, besides, as it were the fountain and origin of cosmography, since without the aid of this science the description of the terrestrial globe, the doctrine of place and interval, the designation of regions, and other matters of this kind — which bring a wonderful adornment together with usefulness to all commonwealths — can in no way be perfectly obtained.

[Margin: Astronomy is useful for the governance of the state.]

I pass over the fact that this science is in the highest degree necessary to the administration of the commonwealth, to agriculture, to the waging of wars, and the like. Of this history sets before us many examples. Sulpicius [Gaius Sulpicius Gallus], on account of his knowledge of the lunar eclipse — which is taught only in astronomy — is reported to have freed a whole army from immense fear. Historians relate that Pericles [Pericles of Athens] did the same, as did Dion the Sicilian [Dion of Syracuse]. But Nicias, the general of the Athenians, through ignorance of this matter, was struck with fear and did not dare to lead his fleet out of harbor — to no small loss and injury of the Athenian commonwealth.

Nor should it be passed over that not so many years ago — as Juan de Rojas [Juan de Rojas Sarmiento] reports in the letter to the Emperor Charles V [Charles V] which he prefixed to his commentary on the universal planisphere — Columbus [Christopher Columbus], leader of the army of Ferdinand king of the Spains, in those earlier years in which the new world of the West Indies was discovered and explored, rescued his whole Christian army near the island of Jamaica from utter

want by the aid of this divine discipline. For when the entire Spanish army had been placed in the last extremity of life, and the commander could by no means obtain provisions from the people of Jamaica — for in this way the barbarians hoped easily to overcome the Christian army by hunger and by arms — it was announced to the rulers of the Jamaicans that, unless they supplied to him and to all his men the necessities for sustenance, very great and utmost evils threatened them: in proof of which they would, not long after, see the moon darkened, which he himself, being eminently versed in astronomy,

already knew was about to fail. At first the barbarians scorned the commands and threats of the Christian commander; but when, at the appointed time, they saw the moon gradually failing, and did not understand the cause of it, then for the first time giving credence to his words, they supplied the Christians with provisions in abundance, and, throwing themselves at the feet of the commander and of the rest of the soldiers, they earnestly begged forgiveness. I say nothing of many other similar examples — so that Ptolemy would seem not without reason to have declared that a good astrologer prevents much evil, and that a wise astronomer can procure much good for men.

[Margin: Among the ancients astronomy was held in great esteem.]

To all these praises this is added: that this science of celestial things — astronomy, that is — has always been held in great esteem. Thales of Miletus [Thales] took such delight in this art that he philosophized in complete poverty and seemed to have no care whatever for his household affairs. When, as often happens, he was mocked by idle men as though forgetful of his own advantage, he — having learned from astrology of the marvellous fertility of that coming year — is said to have bought up beforehand all the olives in the Milesian countryside before they had begun to blossom, and to have come out extremely rich. In this way he wished to show the Milesians that a prudent and wise man can make money if he chooses.

It should not be passed over in silence that among the Egyptians no priests and no pontiffs were appointed unless they were mathematicians (for so they called astrologers, by *antonomasia*); that among the Lacedaemonians none sat beside the kings unless they were mathematicians; that among the Persians none were saluted as kings unless they were mathematicians. Indeed, Alexander the Great [Alexander] is reported to have written thus (though this be far from a Christian): "O most merciful king, neither rise, nor sit, nor take food or drink, nor do absolutely anything without the counsel of a skilled mathematician, if it can possibly be done."

By this discipline, we read, Dionysius the Areopagite [Dionysius the Areopagite] recognized that the eclipse of the sun which occurred at the full moon — which could not then happen by the powers of nature — announced the Passion of the Lord, when he cried out: "Either the God of nature suffers, or the fabric of the world is being dissolved." A little later, by the preaching of the Apostle Paul, he was converted to the faith of Christ. This discipline, if we believe Josephus, Abraham first handed on to the Egyptian priests; in it Moses, that eminent leader of the people of God, excelled, as St. Stephen testifies in the Acts of the Apostles, saying that he was instructed in all the wisdom of the Egyptians — which indeed consisted chiefly in astronomy.

[Margin: Kings and emperors have always taken delight in astronomy.]

To all these praises add this: that there is no profession in which the greatest kings and emperors have taken more delight than in astronomy — as also in our own age. For this astronomical study was pursued with the greatest care by Julius Caesar [Julius Caesar], emperor of the Romans, who, as the histories relate, brought back with him from Egypt Sosigenes, a distinguished and skilled mathematician, whose services he used greatly in the ordering of the year and the course of the sun; and from that time the mathematical arts began to be more diligently cultivated in Italy. This Caesar took such delight in this study that of himself he said, in Lucan [Lucan]:

.....media inter praelia semper Stellarum, cœlique plagis, superisque
vacauit.

"...ever amid the midst of battle I have had leisure for the stars, for the tracts of heaven, and for the gods above."

Following him, the Emperor Hadrian [Hadrian] was so versed in the motions of the stars that, as the histories relate, he used to write out for himself each year his own prognostication.

What shall I say of Alfonso, king of the Spains [Alfonso X of Castile]? He was so learned in the science of the stars that he composed that notable work, the Astronomical Tables.

I pass over, among the more recent, the Emperor Charles V, ever Augustus, and Ferdinand his brother [Ferdinand I], who in a marvellous way found refreshment in these studies and in astronomical instruments; whose example was followed by Philip, king of the Spains [Philip II], the Emperor Maximilian

[Maximilian], Emmanuel Philibert, Duke of Savoy [Emmanuel Philibert], and very many others who are still living today.

[Margin: From no science is greater pleasure derived than from astronomy.]

To this it is further added that from no other human science is so much pleasure and delight taken as from astronomy. For what can be more agreeable, more charming, more sweet, and finally more delightful than to survey with the eyes that most lovely array of so many and so great lights? That there is nothing in this life which more delights the mind of man, very many weighty authors affirm — so that it ought not to seem so strange why some have spent twelve whole years, some forty, some more or fewer, on the mountaintops under the open sky, for the sake of observing the stars. Indeed, Plato [Plato] did not scruple to assert that eyes have been granted to us by God for the sake of astronomy alone. To this Ovid [Ovid], the most ingenious of poets, seems to allude when he sings thus:

Finxit in effigiem moderantum cuncta Deorum, Pronaque cum spectent
animalia cætera terram, Os homini sublime dedit, cœlumque videre
Iussit, & erectos ad sidera tollere vultus.

"He shaped [man] into the likeness of the gods who govern all things; and whereas the other animals look bent down toward the earth, he gave to man an uplifted countenance, and bade him look upon the sky and raise his upright face to the stars."

[Margin: And in another place.]

Felices animæ, quibus hæc cognoscere primum, Inque domos superas
scandere, cura fuit.

"Happy the souls whose first care it was to gain this knowledge, and to climb up to the mansions on high."

[Margin: And a little after.]

Admouere oculis distantia sidera nostris, Aetheraque ingenio supposuere
suo.

Sic petitur cœlum, non vt ferat Ossan Olympus, Summaque Peliacus
sidera tangat apex.

"They brought the distant stars near to our eyes, and subjected the heavens to their genius. Thus is heaven attained — not that Olympus should bear up Ossa, and the peak of Pelion touch the topmost stars."

For in this most beautiful art those things are made clear than which nothing greater or more beautiful can be conceived. By it the powers of our mind are seized and drawn away from the things of this earthly world, which never remain in the same state, to those things which lie subject to no corruption. In it, with contempt for the narrow confines of this earthly speck, the mind wanders with wondrous sweetness and delight through the spacious air, among the golden Suns, the silvery and changing Moons, and the shining stars. Let these few things, out of many that could be brought forward concerning the praises and usefulness of this excellent discipline, suffice to have been said. Now let us proceed to the exposition of the author of the *Sphere*.

PROOEMIUM IOANNIS DE SACRO BOSCO.

(The Prooemium of Johannes de Sacrobosco)

(Sacrobosco's base text — the lemma:)

[Margin: The order the author keeps in the treatment of the sphere.]

We divide the treatise on the sphere into four chapters. First we shall speak of the composition of the sphere: what the sphere is, what its center is, what the axis of the sphere is, what the pole of the world is, how many spheres there are, and what the shape of the world is.

In the second, of the circles from which the material sphere is composed, and from which the supercelestial sphere — which is represented by it — is understood to be composed.

In the third, of the rising and setting of the signs, of the diversity of days and nights, and of the division of the climes.

In the fourth, of the circles and motions of the planets, and of the causes of eclipses.

COMMENTARIUS.

(Commentary)

[Margin: The title of this work.]

This little book is entitled *On the Sphere* — that is, on a certain globular or round figure containing various and diverse circles, which is customarily called the *material sphere*, devised with marvellous artifice for this purpose: that we might be able to have some knowledge of celestial things. For since it is not in our power to ascend to the heavens whenever we please, so that we might there perceive by sight the degrees and circles once we have considered them, and revolve them from wherever and to wherever we wish; and since, again, the span of a man's life does not suffice to wait for all the things that are to be in the heaven, nor can any man, while he lives, behold all the things that are present; and further, since it is now day here and night there, and to some the Sun (or any other star you please) is now rising while to others it is setting, and some dwell under the oblique sphere and others under the right sphere; and since, finally, no one can dwell in all places at the same time — all of which, nevertheless, are required if we are to have any knowledge of the things that are in that celestial region — for this reason the craftsmen of this discipline, endowed with wondrous learning, with great industry and the highest ingenuity devised some material instrument which might set before our eyes all those things which we imagine in the heaven and desire to know. This instrument, then, is called the material sphere, about which the author here has entitled his little book — not that he wishes, as if by his own set purpose, to discourse about this sphere; for his principal intention in this book is to treat of that celestial sphere, for whose sake this material one was devised. But since, as we have said, knowledge of the things that appear in the heaven cannot at all be acquired without the use of the material sphere, he therefore entitled his little book *On this Sphere*, yet in such a way that all the things that are said about this sphere are to be referred to that celestial one.

[Margin: Why the material sphere was devised by astronomy. It is chiefly the celestial sphere that is treated in this book. The intention of the author. The subject of astronomy and of his book, and what it is.]

The whole aim of the author, then, is set in this: that through the material sphere he may make clear to us the constitution and figure of the whole world, and may teach how the celestial bodies are moved, by what reasoning the stars and signs rise and set, and finally what follows from this rising with respect to days and nights in the various climes — so that this treatise is almost a compendium of the whole of astronomy. Wherefore not inappropriately we lay down the same subject for this little book as for the whole of astronomy, namely the *movable celestial body about the center*. For according to the principles of the philosophers, the subject of any book ought to have three conditions. First, that the parts of the

subject and the properties of that which is called the subject be set forth in that book. Second, that all the things which are treated in that treatise be referred to the subject itself. Third, that that which is constituted the subject of that book distinguish the book — or that science — from all others. All these conditions belong to the movable celestial body about the center with respect to this little book. For in it the subject parts of the movable celestial body are investigated — namely how many particular ones there are in number — and its properties are most diligently explained, such as motion, position, figure, quantity, and the like. Next, all the things treated here are considered by attribution to the movable celestial body about the center, as that earth and water make up a round body, that the earth is situated immovable at the center of the world and is a point with respect to the firmament, and other things of that kind; for the account of the things that appear in the celestial bodies could not be assigned without this cause. And this was the reason why Ptolemy in the *Almagest*, and our own author, and Alphraganus [al-Farghānī], and all the other astronomers said much about the four elements, and especially about the earth — indeed, so that they might the more easily explain the celestial motions, which take place about the earth as about a center. Lastly, through the movable celestial body about the center this little book is distinguished from all other disciplines. For although Aristotle too treats of the heaven in the book *On the Heavens*, yet he does so on a different principle than the astronomer does. For the philosopher chiefly endeavors to investigate the nature and substance of the heaven, and whatever he asserts about the motion of the heaven in particular, all of it he begs from the astronomers; whereas the astronomer treats of the same celestial body on this precise principle: insofar as it is movable about the center of the universe — that is, so that he may assign the periods and varieties of all the motions, always understanding only local motion. For the celestial bodies, according to the philosophers, do not admit of other motions, or of alteration and the like: for they do not admit — according to the philosophers — of corruption, augmentation, diminution, generation, and corruption.

[Margin: What is contained in each of the chapters of this book.]

In this Prooemium, therefore, the author makes clear to us his intention and his manner of proceeding, dividing the whole treatise into four parts. In the first of these he declares that he will set forth the parts of the sphere and what the shape of the world is — which is indeed most worthy to be known. For how will it not be most delightful, as well as most useful, to know in what manner the fabric of this world, by which we are covered and in which we continually pass our life, is constructed and arranged? In the second he promises that he will speak of the

circles of the sphere. In the third and fourth he declares that he will discuss the motions of the stars — that is, the rising and setting of the signs and constellations. But since the treatment of the celestial motions can be twofold — one which inquires into and unfolds the *first motion*, which is proper and peculiar to the *primum mobile*, from east to west, and carries all the other orbs along with it in the space of twenty-four hours; the other which considers and explains the *second motion*, which is peculiar and proper to the other heavens below the first —

the *primum mobile*, and it proceeds from west to east. For the individual lower orbs strain, in a way, against that first motion—each of them also by its own proper motions—the first motion by which they are dragged from east to west. Therefore our author, wishing to touch briefly on both treatments, in the third chapter deals with that first motion and with everything that happens on its account in various regions: namely, the rising and setting of the signs, which are perpetually carried by the *primum mobile* from east to west; likewise the variation of days and nights, which differs from place to place because of the differing rising and setting of the signs; and finally the climes, in which such variation is found. In the fourth chapter he discusses the circles, orbs, and motions of the planets, the causes of eclipses of the Sun and Moon, and those things that come about by reason of the second motion. And so, in a kind of compendium, he seems in this little book to have embraced the whole science of celestial things.

CAPVT PRIMVM. [Chapter One]

The sphere, then, is described by Euclid as follows. A sphere is the passage of the circumference of a half-circle which, its diameter held fixed, is carried round until it returns to its own place. That is, a sphere is a round, solid figure of the kind that is described by the arc of a semicircle carried around.

[Margin: Definition of the sphere.]

COMMENTARIVS.

This first chapter contains the principles and foundations of the whole of Astronomy, on which Ptolemy [Ptolemy] too discourses most learnedly in the first Book of his *Great Composition* [*Almagest* I]. It may most conveniently be divided into four principal parts. The first part contains five definitions: two of the sphere, the third of the center of the sphere, the fourth of the axis of the world, and the fifth of the poles of the world.

[Margin: What is treated concerning the sphere in the first chapter.]

In the second part are contained certain divisions of the sphere. In the third, the author explains what the shape of the world is. In the fourth, finally, he demonstrates certain conclusions about the celestial and elemental regions.

Now, so that the two definitions of the sphere may be understood, it must be observed that among Mathematicians only three kinds of quantity are found. Under the first are contained all lines, whose extremities are points. Under the second are included all surfaces, which are bounded by lines. The third kind, finally, comprises bodies, or solids, whose boundaries are surfaces.

[Margin: There are only three kinds of quantity.]

A line is length without breadth, having only one dimension, along which it is divided lengthwise. A surface is breadth without depth, admitting only two dimensions, one according to length, the other according to breadth. A body, finally, or solid, is a magnitude admitting three dimensions: namely length, breadth, and thickness or depth.

[Margin: What a line is. — What a surface is. — What a body is.]

And no other magnitude or quantity, besides these three, is considered by the Mathematician, because no more can be given—since not even more than the three aforesaid dimensions can be found. Although Aristotle [Aristotle], at the beginning of his books *On the Heavens* [*De Caelo*], tries to confirm this by many probable arguments, the Mathematicians nonetheless demonstrate the same point most clearly by a single proof, which I have chosen to set down here, since it is found well explained by few.

It must be known, then, that all things are measured by Mathematicians by the perpendicular line, so that any magnitude is said to be as long as is the perpendicular drawn from one extremity of the figure to the other. Thus, in the parallelogram ABCD here proposed, the length will be the perpendicular line LM, drawn from point L on side AD to the opposite side BC (produced), or the perpendicular AF. In the same way they say that the breadth of any quantity is as great as the perpendicular drawn from one side to the other: thus the breadth of the proposed parallelogram will be the perpendicular BE, extended from side AB to side DC (produced). Depth, finally—that is, thickness or height of any body whatever—is judged to be as great as the perpendicular drawn from one part to the other. For which reason Euclid, most beautifully, at the beginning of the sixth

book, in defining the height of any figure, said that it is the perpendicular line drawn from the vertex to the base.

[Margin: Mathematicians measure everything by the perpendicular line.]

Diagram. A parallelogram construction illustrating measurement by perpendiculars, drawn with several internal and produced lines. The four corners of the parallelogram are labelled A (top left), B (top right), C (bottom right), and D (bottom left); several perpendiculars and their feet are marked within and below it. Labels read on the figure:

- A (top left corner)
- G (on the top side, between A and B)
- B (top right corner)
- L (on the left side / near AD)
- M (interior, foot of the perpendicular LM)
- F (interior toward the right side)
- D (bottom left corner)
- K (along the bottom, right of D) [?]
- I (along the bottom, right of K)
- C (bottom, center-right corner)
- E (bottom right)
- [?] (a further label set below C, near the base — possibly H) [?]

The perpendiculars discussed in the text are LM (from L on AD), AF, BE (from AB to DC), and GH, where GBEH forms a parallelogram; the non-perpendicular lines GI, GK are also referenced.

The reason why Mathematicians measure everything by the perpendicular line is the one Ptolemy adduces in the little book he wrote *On the Analemma*, and which Simplicius took from a book of the same Ptolemy *On Dimension*: namely, because the measure of anything ought to be settled and determinate, not indefinite. Now, among all the straight lines by which any measurement is taken, only the perpendicular line is of certain and determinate length; all the others are indeterminate. Thus, in the parallelogram above, the perpendicular line BE, by which we took the breadth of the figure, is—among all the lines that can be drawn from side AB to side DC, whether produced further or not—the only one of settled and invariable quantity. For from whatever point of side AB you draw a perpendicular to side DC, this will have exactly the same length as the perpendicular BE—such as is the perpendicular GH. For since GBEH is a

parallelogram (as is plainly established from the first book of Euclid), the opposite sides BE, GH will be equal; and so of the rest.

[Margin: Why Mathematicians measure everything by the perpendicular line. — a. Euclid I.34.]

This by no means happens with the other lines, which are not perpendicular. For from any point of side AB, to side DC, innumerable non-perpendicular lines can be drawn, of which one is greater than another, and the perpendicular drawn from the same point is smaller than all of them—as is evident in the lines GH, GI, GK. Since this is so, it is not without great design—nay, with Nature herself as guide—that the measures of quantities are taken by perpendicular lines, which alone are determinate and invariable, and not by the others, which can be drawn in infinite ways, now shorter, now longer. Just as, not only among Mathematicians but also among ordinary people, distances and the intervals of journeys are reckoned along straight lines, which are the shortest,

[Margin: b. Euclid I.19.]

and not along curved ones, which can be varied in six hundred ways. Thus the space lying between the two points A and B is defined to be only as great as the straight line ACB, and not as great as the circular line ADB, or AEB, or AFB; for these are not of the same length, but one is greater than another, whereas the straight line is always the same, and the shortest of all that can be drawn from point A to point B.

Diagram. A figure showing a straight base line with a series of nested arcs above it, illustrating that the straight line between two points is shorter than any arc joining them. Labels read on the figure:

- A (left end of the base)
- C (midpoint of the straight base line)
- B (right end of the base)
- D (lowest arc, ADB)
- E (middle arc, AEB)
- F (highest arc, AFB)

The straight line ACB is the shortest path from A to B; the successive circular arcs ADB, AEB, AFB are longer and unequal.

This, then, being shown—namely, that all things are measured by the perpendicular line—it will easily be demonstrated that there are, from the nature

of the thing, only three dimensions in every corporeal thing: one according to length, another according to breadth, and the third according to depth.

[Margin: Why there are only three dimensions.]

The reason for this is that at any point taken within a body, only three perpendicular lines can be set up—such that each of them is at right angles to the other two—and no more; of which any two, taken in any way, will lie in one and the same surface, while the remaining one lies in another, different surface. By one of these lines, then, the length of the body is measured, by another its breadth, and by the third its height or depth. From these things it is established why a body has only three dimensions. Hence some are not unfitting when they define a body thus: A body, or solid, is a magnitude in which three straight lines, intersecting one another at right angles, can be drawn through one and the same point; whereas in a surface only two can be drawn.

That at any point three lines can be drawn, each perpendicular to the other two, we shall demonstrate as follows. In the figure above, where the two straight lines AB, BE cut one another at right angles at B: if from B a straight line is understood to be raised at right angles to the plane in which those straight lines lie (for two intersecting straight lines are always in one plane), this line will be perpendicular to both AB and BE, by the third definition of the eleventh book of Euclid; and consequently each of them in turn will be perpendicular to it. From which it follows that any one of them is perpendicular to the other two.

[Margin: a. Euclid XI.2. — b. Euclid XI.12.]

That no other line, however, can be perpendicular to these three, we shall make clear in this way. Let there be drawn, if it can be done, a fourth line from B perpendicular to the straight lines AB, BE; this will necessarily be at right angles to the plane in which the straight lines AB, BE lie. Since, then, the third line too has been raised at right angles to the same plane, there will be drawn from point B two straight lines perpendicular to the same plane on the same side—which cannot happen.

[Margin: c. Euclid XI.4. — d. Euclid XI.14.]

These things being rightly understood, the two definitions of the sphere will easily be grasped. For the first definition runs thus (which the author testifies he took from Euclid): *A sphere is the passage of the circumference of a half-circle which, its diameter held fixed, is carried round until it returns to its own place—*

that is, as the author himself explains: *A sphere is a round, or solid, figure of the kind that is described by the arc of a semicircle carried around.*

[Margin: Explanation of the preceding definition of the sphere. — The definition of the sphere handed down by Euclid.]

For the sphere is not the passage or revolution itself, but is produced from such a passage or revolution—so that this predication, "a sphere is a passage," is causal, and by no means formal. For the sense is that a sphere is a solid of the kind understood to be circumscribed by the arc of a semicircle in one complete revolution, its diameter remaining immovable and fixed; and that Solid is understood to be circumscribed which is continuously touched by the arc as it is carried around. Thus, if one takes clay, or any other workable material, into which some diameter is inserted to serve as the material's thickness, and the circumference of the semicircle, applied at both ends of this diameter, is carried around until it returns to the place from which it began to move, all unevenness of the clay will be removed, and a spherical or round figure will be produced. Such a round body, then, described by the circumference of the semicircle, is called a sphere.

But someone will say: since the circumference of a semicircle is a certain curved line devoid of all breadth, and since—by the consensus of all Mathematicians—nothing but a surface is produced by the imaginary drawing or motion of any line, how can it be that a sphere, which is a solid thing (as the author himself asserted in explaining his definition, and as will presently be added again from Theodosius), is generated from the drawing, revolution, or turning-round of the circumference of a semicircle? For from such a carrying-round only the outermost surface of the sphere is produced.

[Margin: A difficulty against the author's preceding definition. — Solution of the difficulty.]

To this it must be answered that this definition of Euclid has not been faithfully reported by the author. For Euclid, in Book XI, definition 14, did not say that a sphere is produced from the turning of the *circumference* of a semicircle about the diameter, but from the drawing and revolution of the *whole* semicircle—which, indeed, is agreed to be a surface. Wherefore, just as from the revolution of a finite straight line about one fixed end a circle is described (so that the line itself makes the surface, while the other end—the point—marks out the circumference), so likewise from the turning-round of the surface of the semicircle the solidity of the sphere will be produced, and from the revolution of

the semicircumference the outermost round surface; and in this way the perfect spherical body arises.

[Margin: Another definition of the sphere handed down by Theodosius.]

A sphere is also described by Theodosius thus: A sphere is a certain solid contained by a single surface, in the middle of which is a point, from which all lines drawn to the circumference are equal.

COMMENTARIVS.

This is the second definition of the sphere, taken from Theodosius, *On the Elements of Spherics*; in it three parts are contained. The first is "solid"—that is, a body—and it is set down to distinguish it from plane figures, of which sort are the circle, the square, and so on. The second, "contained by a single surface," is added to exclude solid figures comprised by several surfaces, such as a cartwheel, a millstone, a pyramid, a cube, and so on. But since surface is of two kinds—one plane, which on every side can be measured exactly by a straight line, such as the surface of a well-leveled wall, or of a board, or of a well-stretched sheet of paper; the other curved, which cannot be measured on every side by a straight line, and this is either concave, as is the inner surface of some water-jar, or convex, of which sort is the outer surface of a water-jar or of a ball—the sphere is contained by a curved surface, and that a convex and single one. The third part, finally, is "in the middle of which," etc., and it is joined on to distinguish it from very many solids that are indeed contained by a single surface, but in which nevertheless no such point can be assigned: such as an egg-shaped body, a lentil-shaped body, and others of this kind.

[Margin: Explanation of the definition of the sphere handed down by Theodosius.]

But if we compare this definition with the former, we shall find that the former furnishes us with the manner and technique of fabricating a sphere, whereas this one explains the substance of a sphere already fabricated; and for that reason the former should rather be called a description, and this one a definition. This definition of Theodosius, taken from Plato's *Timaeus* [Tymæo], Cicero elegantly expressed in his book *On the Universe*, speaking of the world in these words: *Therefore it was made globe-shaped—what the Greeks call σφαιροειδές* [Greek: σφαιροειδές (sphairoeides, "spherical, sphere-shaped")]—*every extremity of which is reached by equal radii from the center.*

[Margin: Comparison of the two definitions of the sphere with each other.]

For this definition also fits the whole world; since the world is a solid sphere, there being no vacuum in it, but all things being filled with bodies from the convexity of the world right down to its center, as Aristotle proves in the fourth book of the *Physics* [Aristotle, *Physics* IV].

But if we look into the matter more carefully, both of the aforesaid definitions of the sphere can be fitted rather to any globe or ball than to that sphere about which the author wrote his little book, and about which our discussion is chiefly to be. Therefore we shall adduce another description, in this manner: The sphere (of which we are to treat) is a certain round instrument, in which various circles, or rings, are contained, by which the motions of the heavens and the situation of the whole world are most conveniently explained. Such, indeed, is the instrument they call a *material sphere*.

[Margin: Description of the material sphere, which is treated here.]

Who were the first and most excellent inventors of this instrument is not sufficiently established. For some think that Atlas first devised the sphere, and that it was afterward carried into Greece by Hercules, as Pliny reports. Some, however, as the same author testifies, make Anaximander of Miletus the first inventor. Diogenes Laertius ascribes this invention to Musaeus. Others, finally, name yet other inventors, among whom is also counted Archytas of Tarentum, a writer of no small distinction. Cicero, however, and Maternus testify that Archimedes of Syracuse, that most subtle Mathematician, was the first inventor of the instrumental sphere, which would represent to the life that celestial sphere.

[Margin: Who are said to be the first inventors of the material sphere.]

For, in order to set before our eyes the composition, arrangement, and motions of the heavens, he fashioned (they say) a certain glass sphere, wholly transparent, with such craftsmanship that in it the globes of the planets—especially of the Sun and Moon—moved by their own motions into the different regions of the world, exactly as they move in the heaven itself. So perfectly and precisely did this glass sphere, built by Archimedes with the utmost care and art, imitate the celestial sphere. Of this sphere the poet Claudian wrote a most elegant epigram, which I have chosen to set down here:

[Margin: The wonderful sphere of Archimedes.]

When Jupiter beheld the heavens in a little glass, he laughed, and to the gods above spoke thus: Has the power of mortal effort advanced this far?

Now in a fragile globe my labor is made sport of. The laws of the sky, the constancy of things, and the ordinances of the gods— behold, the old man of Syracuse has transferred them by his art. An enclosed spirit serves the various stars, and drives the living work by fixed motions. A counterfeit Zodiac runs through its own year, and a feigned Cynthia [the Moon] returns with each new month. And now bold industry, rolling its own world, rejoices, and governs the stars with human mind.

And that point is called the center of the sphere. A straight line passing through the center of the sphere, touching its ends to the circumference on either side, about which the sphere revolves, is called the axis of the sphere. And the two points bounding the axis are called the poles of the sphere.

[Margin: What the center, axis, and poles of the sphere are.]

COMMENTARIVS.

Here, by the three remaining definitions, he declares what the center is, what the axis is, and finally what the poles of the sphere are; all of which are clear in the author.

The center of the sphere Euclid describes thus in Book XI: The center of the sphere is the same as that of the semicircle, from whose revolution the sphere is understood to be produced.

[Margin: What the center and axis of the sphere are, according to Euclid.]

The axis he defines thus, in the place cited: The axis of the sphere is that motionless line about which the semicircle (from whose turning, of course, the sphere is made) is turned. Proclus Diadochus, however, defines it thus: The axis of the world—which we have already said to be a sphere—is called its diameter, about which it revolves. From all these definitions it is evident that not every line which, passing through the center of the sphere, touches its ends to the circumference on either side, is called an axis (although it is called a diameter), unless the sphere revolves about it. For the diameter embraces far more than the axis, since the axis is something inferior, and the diameter something superior: every axis, indeed, is a diameter, but not conversely. For in the celestial sphere only those diameters can be called axes about which some motion takes place—and these are few—and the principal axis is the one that stretches from the North through the middle of the earth toward the South. Yet innumerable diameters can be assigned—all the lines, that is, passing through the center of the sphere;

may, even plane figures have diameters, such as the circle and so on, but not an axis.

[Margin: The distinction between the diameter and the axis of the sphere.]

For an axis is found only in solid bodies. Yet any diameter can also be called an axis, because a sphere can be revolved about it—just as about the axis of the world—although in fact it does not itself move. So too, among Geometers and Astronomers, every circle in the sphere is said to have its own proper axis, about which it would move circularly and uniformly, if it had to move—although in actual fact it does not move. An axis of this kind is a diameter of the sphere drawn through the center of the circle and standing at right angles to the plane of that same circle. And that diameter about which the heaven, or sphere, is turned is called the axis, by a likeness taken from the wooden axle upon which the wheel of some cart is turned; and this name is derived from "acting"—that is, from moving—because, of course, the world is turned about it without ceasing. This axis the poet Manilius elegantly depicted for us in these verses:

A slender axis is drawn through the cold air, and holds the world
balanced upon its opposite pivots; the starry sphere revolves about its
middle, and, itself unmoved, wheels its eternal courses...

The ancients also imagined that heaven and earth are supported by an axis.
Hence Cicero says:

the earth, which is held up by the axis that runs through it. Lucan alludes to this when he points out to Caesar his seat in heaven, writing:

If you should press upon a single part of the boundless sky, the axis will
feel the burden — the weight of the balanced heavens.

[Margin: The poles of the world.]

On the poles. Now there are two poles of the world — that is, the two points that terminate the axis. The one that is always visible to us here, living in Europe, and stands plainly in view, is called the northern pole (*Borealis*, *Boreus*, *Septentrionalis*, or *Aquilonius*). The astronomers, however, call it the Arctic pole — that is, the "Bear pole" — from a certain notable constellation which in Greek is called ἄρκτοι [Greek: arktoi, "bears"], in Latin *ursa* ("the Bear"), and which revolves perpetually around this pole. Most nations also call it "North"; the Italians call it *Tramontana*. The other pole is called the southern pole (*Australis*, *Austrinus*, *Meridionalis*, or *Notius*). The astronomers call it the Antarctic, because it is diametrically opposite the Arctic pole. This one is never visible to us: it

always lies hidden below our horizon by just as much as the other rises above that same horizon — here at Rome, for instance, by roughly 42 degrees. Virgil describes both poles beautifully when he says:

This vertex is always high above us; but the other, beneath our feet,
black Styx and the deep shades behold.

[Margin: Why the pole is called the "sea-star."]

Sailors call each pole a "sea-star" (*stella maris*) or "sailors' star" — not because the poles themselves are stars, but because near them are certain stars so close by that they are scarcely seen to move (though near the Antarctic pole no notable star has been found that is less than 30 degrees from the pole itself). Of these, the one nearest the Arctic pole is at the tip of the tail of the Lesser Bear; the one observed to be nearer the Antarctic pole is set in the left foot of the Centaur. And since sailors, keeping their eyes on these stars, steer their courses across the open sea by them, each has customarily been called the "sea-star" or "sailors' star."

[Margin: Why they are called "poles."]

The poles take their name from the Greek verb πωλέω [Greek: pōlēō, "to turn" or "to revolve"], which means "to turn" or "to spin around"; for the whole machinery of the world revolves ceaselessly about these two points. Moreover, some call these two points the "vertices" or "hinges" (*cardines*) of the world. For just as a door turns on its hinges, so too the entire structure of the world turns about these said points, which alone are motionless.

DIVISION OF THE SPHERE OF THE WORLD

[Margin: Division of the sphere according to substance.]

[Lemma] The sphere of the world, moreover, is divided in two ways: according to substance, and according to accident. According to substance it is divided into nine spheres — namely, the ninth sphere, which is called the first motion or the primum mobile; and the sphere of the fixed stars, which is called the firmament; and the seven spheres of the seven planets, of which some are larger and some smaller, according as they approach nearer to, or recede farther from, the firmament. Hence among these spheres the sphere of Saturn is the largest, while the sphere of the Moon is the smallest.**

COMMENTARIUS

This is the second part of this chapter, in which two divisions of the sphere of the world are set forth: one according to substance, the other according to accident. According to substance, the author divides the sphere of the world into nine spheres.

[Margin: "Sphere" is here taken to mean the celestial sphere.]

In this division, "sphere" is not taken in the sense that embraces all the bodies that compose the entire universe — that is, the heavens together with the elements. For on that reckoning there would be more than nine spheres, as will soon be evident when we discuss the number of the heavens and the elements and their order. Rather, it is taken to mean the celestial sphere, which indeed consists of, or is bounded by, two surfaces: a convex outer surface and a concave inner one — and this is properly called an *orb*.

[Margin: The difference between an orb and a sphere.]

For an orb differs from a sphere in this: a sphere is solid all the way to its center and is enclosed by only a single surface, namely the convex outer one; an orb is not like this, but is bounded by two surfaces, one outer and one inner — such as all the heavens are.

[Margin: "Sphere," or celestial orb, is taken in two ways.]

But since "sphere," or celestial orb, can be taken in two ways: in one way, for any orb distinct from another, whether it be concentric with the world or eccentric — that is, whether it has the same center as the world or a different one. In this sense each planet is said to contain its own orbs, the treatment and study of which belong to the *Theories of the Planets*, although our author too attempts to touch on them very briefly in chapter 4. In the other way, the celestial sphere is taken for a total orb, distinct from the others, which is everywhere equidistant from the center of the world and is concentric with the world both on its convex and on its concave side; and it is sometimes conceived as made up of several partial orbs that are arranged to produce the motion of a planet. In this sense each planet is said to have its own proper and peculiar orb, containing other partial orbs, some concentric, some eccentric.

[Margin: In what sense "sphere" is taken in this division.]

as is clear in the *Theories of the Planets*. It is therefore in this latter sense that "sphere" is taken in the present division — namely, for the entire celestial orb

which contains (if we are speaking of the heavens of the planets) several other partial orbs arranged to produce the planet's motion, whether these be concentric or eccentric. And so the author divides the sphere, taken in this sense, into nine spheres: namely, the ninth sphere, which is called the first motion or *primum mobile*; the sphere of the fixed stars, which is called the firmament; and the seven spheres of the seven planets — that is, the sphere of Saturn, of Jupiter, of Mars, of the Sun, of Venus, of Mercury, and of the Moon. This division, however, we shall examine a little later, since the more recent astronomers posit more celestial spheres.

[Margin: The celestial orbs are contiguous with one another.]

Now all the celestial orbs are entirely contiguous and immediately adjacent to one another, so that the higher always encloses the lower, and there is nothing intermediate between any one and the next — just as we see in the coats of an onion, where the outer everywhere surrounds the one beneath it. This we shall demonstrate to be so when we discuss the order of the heavens. Therefore, since every containing body is, in its circumference, equal to the body it contains, the author rightly adds that some of the celestial spheres are larger and some smaller, according as they approach nearer to, or recede farther from, the firmament. For on this reckoning the ninth sphere will be the largest of all.

[Margin: Why the ninth sphere is called the sphere of the first motion, or first mover.]

Then the firmament will be larger than the sphere of Saturn, which comes next after it, and so on in turn, until we come down to the sphere of the Moon, which is the lowest. For this is the smallest of all the spheres.

The ninth sphere is called by the author, and by the other astronomers, the first motion or *primum mobile*, because, as they suppose, no other movable heaven exists above it, and by its own extremely swift motion — as we shall explain in its proper place — it

carries off with it all the lower spheres it surrounds, from east to west, in the space of twenty-four hours. Yet, although the author here holds the ninth sphere to be the highest and the *primum mobile*, we can without any difficulty call it both the first sphere and the ninth (or last). The first in the order of nature, because it is nearer to the First Being — in which sense the sphere of the Moon is last, since it is farthest from the First Being. But the ninth, or last, with respect to us, because it is the more remote from us — in which sense the sphere of the Moon, being nearer to us, will be called the first. Yet the astronomers have not

customarily called it the "last motion" or "last mover," but only the "first motion" or *primum mobile*, on account of the dignity and pre-eminence it has in carrying the lower spheres around with it by its own proper motion — in which respect it seems to hold the primacy.

[Margin: Why the eighth sphere is called the firmament and the sphere of the fixed stars.]

The author, with the astronomers, also calls the sphere that is eighth with respect to us the "firmament" and the "sphere of the fixed stars." The firmament, because just as a rampart, wall, and city walls set at the outermost boundaries encircle, fortify, and secure a city, so too the eighth sphere, which is called the firmament — and which all antiquity held to be the highest and outermost heaven — secures, contains, encompasses, and, as it were, fortifies not only all the remaining lower spheres but also everything whatsoever that exists in the whole universe. Or else it is called the firmament because it holds the stars fixed more firmly, as will presently be said.

[Margin: Why the stars of the firmament are called "fixed."]

He calls it the "sphere of the fixed stars" because it carries, conveys, and contains all the fixed stars. Now these stars are not called "fixed" because they do not move, or because they remain wholly stationary; for that is false, since experience has shown most clearly that they do move, as will be said in its proper place. Nor again are they called "fixed" in the sense that they move only with the motion of the orb in which they are set; for on that reckoning the planets too would have to be called "fixed," since they are carried around only by the motion of the orbs in which they exist, as we shall later show. Rather, they are called "fixed" because they always keep the same position, order, and distance among themselves — as the observations both of the ancient astronomers (Ptolemy [Ptolemy], al-Battānī [al-Battānī], and the rest) and of the more recent ones make most evident to us. For the stars of that brilliant constellation called Orion always keep the same position, order, and distance among themselves: the three stars that make up Orion's belt, for instance, perpetually form what is almost a straight line. The same has been observed in the stars of the Greater and Lesser Bear, and finally of the other constellations. On this matter, read Ptolemy in Book 7 of the *Almagest*, and Regiomontanus [Regiomontanus] in his epitome of the same Book, where very many observations of the stars are brought forward, from which it is clearly gathered that the stars of the firmament always keep the same order and position among themselves. For this same reason the eighth sphere was called by the Greeks ἀπλανής [Greek: aplanēs, "not wandering," "fixed"], as if to say "not

straying" and "not erring," because all the stars fixed in it proceed without any wandering or intermingling.

[Margin: Why the sphere of a planet is so called.]

Finally, the remaining seven spheres, each of which contains a single star, are called the "spheres of the planets," because they carry stars — or heavenly bodies — that are called planets, that is, "wandering stars" or "wanderers" (*Errones*). This is not because they stray about the heaven in the sense of being carried by no ordered, certain, and determinate motion — for on that reckoning there could be no science of them, which is not true, since they have fixed periods of their motions. Rather, they are called "wandering stars" because they neither keep always the same distance among themselves, nor keep the same order with respect to the fixed stars of the eighth orb. This we perceive more clearly than daylight, day by day, in the Sun and Moon. For at one time these two planets are wholly joined together, as happens at new moon; at another the one is opposite the other, and the one recedes as far as possible from the other, as happens at full moon; at another they are seen nearer to, or farther from, each other. Again, at one time they appear near a given fixed star of the eighth orb, or firmament, and at another near a different one; and exactly the same has been observed in the rest of the planets. For now they are seen to advance in a direct course, now to go backward and press in the contrary direction; now to be hidden and to lie concealed on account of their nearness to the Sun; then, when the Sun withdraws from them, or they from the Sun, to come forth again into the light, and to open and reveal themselves; now to go before the Sun, now to follow it; now to be hurried along, as it were, with the swiftest course; now, on the contrary, to slow down so much that they are not even thought to move, but to stand still in exactly the same place of the Zodiac; now, finally, to run off toward the north, now toward the south. On this matter more is set forth in the *Theories of the Planets*. For this reason, then, the stars appear to stray about the heaven in such a way that they seem to be driven by a kind of chance and fate; and this is why they are rightly called "planets" by the astronomers.

[Margin: Division of the sphere according to accident.]

[Lemma] According to accident, moreover, it is divided into the right sphere and the oblique sphere. Those are said to have a right sphere who dwell under the equator, if anyone could dwell there. And it is called "right" for them because neither of the poles is raised higher than the other for them; or because their horizon cuts the equator, and is cut by it, at right spherical angles. But those are said to have an oblique sphere who dwell on this side of the equator, or beyond it. For them, one of

the poles is always raised above the horizon while the other is always depressed; or because their artificial horizon cuts the equator, and is cut by it, at unequal and oblique angles.

COMMENTARIUS

He now divides the sphere according to accident into the right sphere and the oblique sphere. But since what is said in this division, and what follows next, cannot be understood unless certain circles of the sphere are first known — circles frequently mentioned in what follows — I think it worthwhile to explain the circles of the sphere briefly and in general, intending to discuss them more fully, along with their functions and names, in chapter 2, where the author himself treats of them. For now I shall only set out the names of the circles in a rough and ready way.

ON THE CIRCLES OF THE SPHERE

[Margin: The ten circles of the sphere. What a great circle and a lesser circle of the sphere are.]

The circles of the sphere are 10, whose names are these: the Equator (*Aequinoctialis*), the Zodiac, the Colure of the Solstices, the Colure of the Equinoxes, the Meridian, the Horizon, the Tropic of Cancer, the Tropic of Capricorn, the Arctic Circle, and the Antarctic Circle. The first six are called "greater," or "greatest" (great circles); the latter four are "lesser," or "not greatest." A great circle is one that has the same center as the sphere and divides

the sphere itself into two equal hemispheres. A lesser circle, on the other hand, is one that has a center different from the center of the sphere and divides the sphere into two unequal segments. For the rest, every circle

[Margin: What the pole of a circle in the sphere is.]

of the sphere, whether greater or lesser, is said to have two poles, about which, if it were to move, it would be carried uniformly. Indeed, it is from the poles themselves that all circles are described on the surface of the sphere. For the pole of any circle of the sphere is that point on the convex surface of the sphere from which all straight lines drawn to the circumference of the circle are equal. For since the circumference is described from the pole of the circle, it necessarily follows that the pole is equally distant from all points of that circumference.

[Margin: The Equator.]

The **Equator** is that great circle in the sphere which is described from the poles of the world, and which is equally distant, in all its parts, from each pole of the world.

[Margin: The Zodiac.]

The **Zodiac** is likewise a great circle, described from poles that are distant from the poles of the world by a fourth part plus a ninetieth of one quadrant — that is, by 47 of the 180 parts into which the quadrant is understood to be divided. It cuts the equator, and is cut by it in turn, into two halves, but obliquely; so that the Zodiac is inclined to the equator, one half slanting toward the north and the other toward the south. The midpoint of each half recedes from the equator by just as much as the poles of the Zodiac recede from the poles of the world — a distance that amounts to $23\frac{1}{2}$ degrees. And it calls a "degree" one part of any circle divided into 360 parts; for the astronomers divide every circle into that many parts. Now in the Zodiac four principal points are considered, of which two are called equinoctial and two solstitial. The equinoctial points are those at which the Zodiac cuts the equator; the two solstitial points are those which, as we said, are farthest removed from the equator. Again, of the equinoctial points, the one that lies to the right of the Arctic pole (assuming, that is, that the half of the Zodiac which slants toward the north is placed in the upper hemisphere) — or that is placed in the west — is called the vernal point, and is the beginning of Aries. The other, which lies to the left of the same pole (the sphere keeping the same position), or is placed in the east, is called the autumnal point, and is the beginning of Libra. Or, if you prefer: that point of the Zodiac belongs to the vernal equinox which is the beginning of the semicircle slanting toward the Arctic pole, proceeding from west to east; while the end-point of that same semicircle — that is, that point of the Zodiac — belongs to the autumnal equinox, which is the beginning of the other semicircle, slanting toward the Antarctic pole, likewise proceeding from west to east. Of the solstitial points, too, the one that recedes from the equator toward the north is called the summer point, and is the beginning of Cancer; the other, which withdraws toward the south, is called the winter point, and is the beginning of Capricorn. And these four points are to be carefully noted, so that the other circles of the sphere may be understood.

[Margin: The Colure of the Solstices.]

The **Colure of the Solstices** is that circle which passes through the poles of the world, the poles of the Zodiac, and the solstitial points.

[Margin: The Colure of the Equinoxes.]

The **Colure of the Equinoxes** is that circle which is drawn through the poles of the world and the equinoctial points, but not through the poles of the Zodiac.

Diagram. A woodcut armillary sphere showing the principal circles of the sphere set within a graduated meridian ring. The outer ring (the meridian) is graduated in degrees, marked in tens from 10 to 90 and back down (10, 20, 30, 40, 50, 60, 70, 80, 90, 80, 70...) around each quadrant. Four cardinal reference letters sit at the ends of the vertical and horizontal axes: **B** at top, **D** at bottom, **A** at the left. [A corresponding letter at the right (expected **C**) is not legibly resolved — [?].] The inner rings run from the upper-left pole to the lower-right pole, tilted, and carry the following labels:

- POLVS SEPTENTRIONALIS (upper left): "North Pole"
- ARCTICVS: the Arctic Circle
- TROPICVS AESTIVVS (upper): "Summer Tropic" (Tropic of Cancer)
- AEQVINOCTIALIS: the Equator
- ORIZON (horizontal band across the middle): "Horizon"
- TROPICVS HIEMALIS (lower): "Winter Tropic" (Tropic of Capricorn)
- ANTARCTICVS: the Antarctic Circle
- POLVS AVSTRALIS (lower right): "South Pole"

[Margin: The Meridian. The vertex of the place, or Zenith. The Nadir of the place. The Horizon.]

The **Meridian** is that circle which is drawn through the poles of the world and the vertex of the place, and which stands above the other great circles in a material sphere. Now the vertex of the place is a point in the heaven directly above that place — such as the point indicated by the summit of some tower, if it were extended all the way to the heaven; or the point that stands directly over the crown of any person's head. This point the Arabs call the **Zenith**; the point diametrically opposite — indicated by that same tower, if it be imagined to run through into the other part of the heaven — they call the **Nadir**.

The **Horizon** is a great circle described from the vertex of the place as if from a pole, which [also] with other circles in a material

it rises above the material sphere, divides the Meridian, and is itself divided by the same at right spherical angles; and it separates the visible hemisphere from the one not seen.

The **Tropic of Cancer** is that smaller circle which, on the side of the Arctic pole, lies equidistant from the Equator, and passes through that point of the Zodiac

most remote from the Equator — the point we said above is called the beginning of Cancer. [Margin: Tropic of Cancer]

The **Tropic of Capricorn** is that smaller circle which, on the side of the Antarctic pole, lies equidistant from the Equator, and passes through that point of the Zodiac which we noted above is called the beginning of Capricorn. [Margin: Tropic of Capricorn]

The **Arctic circle** is a smaller circle, described near the Arctic pole through the pole of the Zodiac, and lying parallel to the Equator. [Margin: Arctic circle]

The **Antarctic circle** is likewise a smaller circle, which runs near the Antarctic pole through the other pole of the Zodiac, also equidistant from the Equatorial circle. [Margin: Antarctic circle]

An **example** of all the circles we have explained you have, after a fashion, in the accompanying figure ABCD, in which E is the beginning of Cancer; F, the beginning of Capricorn; G, the beginning of Aries; H, the beginning of Libra. ABCD is the Meridian; B, the Zenith; D, the Nadir; AHCG, the Horizon; ABC, the visible hemisphere; ADC, the hemisphere not seen; K and L, the poles of the Zodiac, and so on. But all these things will be perceived more clearly by means of the material instrument.

Now that we have spoken of the circles of the sphere, it will not be out of place to indicate briefly in what manner the material sphere is to be composed out of them — if only for this one benefit, that we may be able to judge of any sphere whatever whether it has been rightly made and assembled. [Margin: The composition of the material sphere] First, then, let three circles be prepared out of some material, entirely equal to one another and each divided into 360 equal parts, which we have said are called degrees. Let two of them be so joined that they cut each other at equal angles — that is, at right spherical angles — in two points, through which the axis of the world is to be extended; and these two circles will be the Colures. Next, in each Colure let 90 degrees be counted from the poles of the world, and at the ninetieth degree of each let a third circle be applied, namely the Equator, which by this reckoning will be equally distant from both poles. After this, from the Equator toward each pole let $23\frac{1}{2}$ degrees be counted along the Colures, and at the limits of these numbers let the two Tropics be applied; you will readily have their size if you first take their diameters, by drawing a straight line from the end of the counting on one Colure to the end of the counting on the same Colure toward the same pole. In the same way, when the like number of parts has been counted from each pole toward the Equator

along the same Colures, the Polar circles will be established, namely the Arctic and the Antarctic, whose diameters you will find by a not dissimilar art. Again, let the circle of the Zodiac be prepared, equal indeed in circumference to the three greater circles already mentioned, but differing from them in breadth: for it must contain in breadth 12 degrees, in the middle of which is drawn the line called the Ecliptic, distant 6 degrees from the edges of the Zodiac, as we shall teach in chapter 2. This circle is to be applied and fitted so that the whole circle obliquely cuts the Equator in those two points where one of the Colures cuts the same Equator; and so that the ecliptic line touches each Tropic at two other points, where the remaining Colure cuts the Tropics — one of which is taken toward one pole, the other, diametrically opposite to it, toward the other. Finally, in this manner let the Meridian and the Horizon be established and fitted to each other, so that the whole sphere thus far constructed may turn freely within these fixed and immovable circles — yet on this condition: that these two circles cut each other mutually at right angles, and that the Meridian move about its own poles (which are the common intersections of the Horizon with the Equator) to this end, that the sphere may serve every elevation of the pole — that is, so that either pole may be depressed and raised, according to the altitude of the pole. On some spheres the Horizon is now depressed, now raised, for the same purpose, the Meridian remaining fixed; but the former arrangement pleases me more. And thus the whole material sphere will be finished and completed. For as to the circles of the Planets, which are usually set upon some spheres so that they always move under the Zodiac and about the poles of the Zodiac, anyone may easily set them upon the sphere by his own industry, since here we treat only of the principal circles of the sphere. Let these things be said in general concerning the circles which astronomers consider in the heavens. Now let us return to the author's division.

They are said, etc.) Having divided the sphere according to accident (in which division the sphere is taken for the whole sphere of the world) into the right sphere and the oblique, he now sets forth each part of the division. He says, then, that those have a right sphere who remain under the Equatorial circle — *if anyone could remain there*. He added this because many most weighty men — philosophers, astrologers, and also most of the theologians — doubted whether there was any habitation under the Equatorial circle; indeed the greater part, along with the ancients, affirmed as certain that under the Equatorial circle there is no habitation, on account of the excessive heat which the Sun, perpetually running there, produces. A similar doubt might be raised about the poles of the world; for not a few there have been, nor are there now lacking, who deny that men can dwell there, on account of the intolerable cold that persists there

through the great remoteness and perpetual absence of the Sun. On this matter we shall say something at the end of chapter 2. But for now let it be held certain and beyond doubt, established by the experience of many, that men dwell both under the Equatorial circle and under the poles — at least under the Arctic pole. [Margin: In what way the sphere is taken in the latter division] [Margin: Who are said to have a right sphere] [Margin: The earth under the Equator and the poles is habitable] [Margin: Why those dwelling under the Equator are said to have a right sphere]

And it is called right for them, etc.) For two reasons he says that the sphere of those who dwell under the Equator is called right: either because neither of the poles is raised above the Horizon more than the other for them; or because their Horizon cuts the Equator and is cut by it at right spherical angles.

Hence it has come about that some have defined the right sphere by saying: it is that in which each pole stands and rests upon the Horizon; or that in which the Equator (which holds exactly the middle place between the poles) makes right spherical angles with the Horizon; or that in which each pole lies on the Horizon and the Equator rises directly above the top of the head; or that in which each pole touches the Horizon. A right sphere has fallen to the lot of a great part of Africa and of the West Indies — namely that part which is called Peru; also the Molucca Islands, the island of Taprobana, and the island of St. Thomas. But no part of Europe is subject to a right sphere. [Margin: Various descriptions of the right sphere] [Margin: What regions have a right sphere]

But those are said, etc.) He says that they have an oblique sphere who dwell on this side of, or beyond, the Equator. He then adds the reason why theirs is called an oblique sphere: namely, because one of the poles is always raised above the Horizon, while the other is always depressed; whence the sphere seems to have an oblique position. Or, certainly, because their artificial Horizon cuts the Equator and is cut by it at oblique and unequal angles. [Margin: Who are said to have an oblique sphere, and why]

He calls the Horizon of the oblique sphere **artificial**, perhaps for this reason: that it is highly variable and does not divide the sphere naturally. For only the Horizon of the right sphere, since it passes through both poles of the world, seems of itself and in a certain way to divide the sphere naturally. For in this way the sphere obtains its proper and direct position, nor can such a Horizon ever be varied, so that some might have a more nearly right Horizon and others a less right one. But in the Horizon of the oblique sphere — since it does not pass through the poles of the world, and since above it one pole is always raised and

beneath it the other is depressed — the sphere seems to be set obliquely and not naturally. There is this further point: that the Horizon of the oblique sphere, according to the choice and will of the men dwelling on earth, is variable in almost infinitely many ways. For the nearer one draws to the pole, the more oblique a Horizon he must have. Not without reason, therefore, can the Horizon of the oblique sphere be called in a certain sense artificial, so as to be distinguished from the Horizon of the right sphere, which is as it were natural to the sphere itself. For since both poles of the world exist upon it, the sphere seems to move naturally within it. [Margin: Why the Horizon of the oblique sphere is called artificial by the author]

Others **define the oblique sphere** by saying that it is that in which one of the poles of the world stands raised and eminent above the Horizon, while the other lies down and sinks below the Horizon; or that in which the Equator makes and forms oblique angles with the Horizon — an obtuse angle, indeed, on the side that faces the exalted pole, but an acute one on the side that inclines toward the hidden pole. An oblique sphere has fallen to all the inhabitants of Europe — such as the Spaniards, French, Italians, Germans, Greeks, and Poles — and to the greater part of Africa and of the West Indies, and also to all of Asia. [Margin: Various descriptions of the oblique sphere]

Not only the sphere, but also the orb, or World, and likewise the Horizon or Bounder (Finitor), is commonly called by authors right and oblique. For they are accustomed to say that the Germans, Italians, French, and Spaniards dwell in an oblique orb; and by the same reasoning, that the Horizon, or Bounder, the world, or the sphere, is for them oblique, and so on. [Margin: What regions have an oblique sphere] [Margin: Those who dwell under the poles have an oblique sphere]

If anyone should ask what sort of sphere those are said to have who dwell directly under the poles, the answer must be that, according to the author's view, they have an oblique sphere. For although their Horizon, since it is one and the same with the Equator, cuts it in no way at all — and hence at neither right nor oblique angles — nevertheless one of the poles is raised to its highest for them, while the other is depressed to its lowest; whence, from this consideration, they are to be reckoned as having a most oblique sphere. There are, however, those who assert that they dwell in a right sphere, because their Horizon makes no oblique angles with the Equator. But by this same argument it is concluded that they do not dwell in a right sphere, since their Horizon does not make right angles with the Equator, but coincides with it entirely. Therefore, in my

judgment, it is more correct to say, with the author, that they dwell in an oblique sphere, since at least one of the reasons for an oblique sphere fits them, but none of those for a right one. This is also indicated by the definitions of the right and oblique sphere handed down by others. [Margin: Those who dwell under the poles have an oblique sphere]

The origin and cause of this division of the sphere into right and oblique is the roundness of the earth. For since, as we shall show in its proper place, the earth is round, it comes about that the position of the poles and of the whole sphere is changed in different parts of the earth; so that men advancing toward one of the poles always see it more and more elevated. This would not happen if the earth were flat. Moreover, since wherever a man is, and in whatever part of the terrestrial globe he stands, he always sees half of the heaven — the impediments of mountains and valleys set aside — as has been ascertained by Ptolemy, al-Farghānī, and other astronomers from various phenomena; and since the Horizon divides the seen half from the unseen: it follows that in whatever region one pole lies on the Horizon, the other also lies on the same. Likewise, by however much one of the poles is raised above the Horizon, by just so much the other is depressed beneath the same; otherwise we should behold more or less than half of the heaven — since the poles are distant from one another by half the heaven, being opposite through the diameter of the world. Hence it is necessary that a man set in some great level plain should have either both poles of the world (all impediments of mountains and valleys being removed) lying on the Horizon, when the Horizon passes through the poles of the world; or one raised and the other depressed, when the Horizon does not pass through the poles of the world. From all these things, therefore, the reasoning of the division of the sphere into right and oblique clearly stands established. [Margin: The roundness of the earth is the cause of the right and oblique sphere]

The earlier division made by the author — that by which the celestial sphere is distributed into nine spheres — **is said to be according to substance**, because it is a division of a superior into its inferiors, namely of the heaven into the particular heavens; just as if we were to divide "animal" into man, lion, horse, and the other animals. Or, certainly, because it is a division of the whole celestial region into the individual heavens out of which it is composed; no otherwise than if a man were divided into head, chest, legs, arms, and the other members out of which he is constituted. But this later division of the sphere into the right and the oblique sphere is called **according to accident**, because in it the sphere is not divided into things essential to itself, as in the former, but into things accidental — namely, those which befall it — regard being had to those who pass their life in

the sphere. For a sphere is called right or oblique in respect of those dwelling under it, which indeed is an accident of the sphere. For the sphere would exist just as much if no one dwelt in it as it does now; but it would not be right or oblique, since there would be no Horizon — which those living on the earth are alone accustomed to consider. This division, then, is like the one by which "animal" would be divided into white animal, black animal, and so on — which is agreed to be a division according to accident. [Margin: Why the earlier division is said to be according to substance, the later according to accident]

Diagram. Two circular woodcut diagrams side by side, illustrating the right sphere (left) and the oblique sphere (right). In each, the lower half is shaded dark (the region below the Horizon). Labels:

Left circle (right sphere):

- Top: ZENITH
- Vertical band through the center: AEQVINOCTIALIS (the Equator, drawn upright, passing through Zenith and Nadir)
- Horizontal band: HORIZON RECTVS ("right Horizon")
- Left edge and right edge: POLVS (the two poles, lying on the Horizon)
- Bottom: NADIR

Right circle (oblique sphere):

- Top-left: ZENITH
- Upper-right: POLVS (the elevated pole)
- Diagonal band across the center: AEQVINOCTIALIS (the Equator, drawn obliquely)
- Horizontal band: HORIZON OBLIQVVS ("oblique Horizon")
- Lower-left: POLVS (the depressed pole)
- Bottom: NADIR

In the accompanying prose Clavius refers to these as the "prior" (right) and "posterior" (oblique) figures.

In the earlier figure set here you have an example of the right sphere; in the latter, of the oblique sphere. You plainly see that in the right sphere the axis of the world coincides with the Horizon, since it does not differ from it; and consequently that each pole lies on the Horizon. But in the oblique sphere the axis of the world differs from the Horizon, and therefore one pole is exalted above the Horizon while the other is depressed beneath the same.

Now the universal machine of the world is divided into two — namely, into the aethereal and the elementary region. [Margin: The division of the world into the aethereal and the elementary region]

COMMENTARIVS.

Being about in this third part of the chapter **to set forth the form of the whole world**, the author first divides the entire machine of the world into two: namely, into the elementary and the aethereal region, out of which, as out of parts, the whole machine of the world is composed. In this division, the "machine" of the world is taken for the heap and compacting-together of all bodies, higher and lower. For the world is the perfect and complete assemblage and adornment of all things: whence by the Greeks it is called κόσμος [Greek: κόσμος (kosmos, "world"; literally "order/adornment")], from adornment. Aristotle, in the little book *On the World*, chapter 2 (if indeed the book is Aristotle's), describes it by two definitions, of which the first is this: "The world is a framework consisting of heaven, earth, and the other natures contained in them." The latter runs thus: "The world is the ordering and arrangement of bodies, which is preserved by God and for the sake of God." [Margin: What the world is]

Some philosophers have held **the world** to be eternal, without beginning and without end — as Aristotle and not a few of his followers. Pliny too, in book 1, chapter 1 of the *Natural History*, thinks the same, when he says: "*The world, and that which it has pleased us to call by another name, the heaven, by whose vault all things are covered, is rightly believed to be a divine power, eternal, immense, neither begotten nor ever to perish.*" The Catholic faith, however, teaches that the world had a beginning, was created and made, and was founded by God, the Best and Greatest, out of nothing, by his word alone, to be a dwelling for human nature, in which he willed to become known and to be beheld; as we read in Genesis, chapter 1. Indeed Plato too, in the *Timaeus*, teaches that God is the maker of the world. Again, some philosophers — among whom was Democritus — held that there are innumerable worlds, some outside our own, others like balls or globes. For the form of the world is round and globular, as will be said hereafter: and when Anaxarchus, a disciple of Democritus, reported this to Alexander the Great, Alexander is said to have groaned, saying, "Alas for me, wretch that I am, who have not yet made myself master even of one!" Aristotle, nevertheless, and our theologians hold that there is one world only, although God, the Best and Greatest, could by his absolute power — according to the theologians — produce infinitely many worlds. [Margin: The world is held by many philosophers to be eternal] [Margin: According to the Catholic faith the world was made]

Furthermore, the **ancient philosophers and the weightiest theologians** divided all things whatsoever that exist into three kinds, so that they asserted the world to be threefold — namely, the Supramundane, the Celestial, and the Sublunar. The Supramundane the theologians call the Angelic world, the philosophers the Intellectual, comprising God, the Best and Greatest, together with all the intelligences. The Celestial is composed of the orbs and celestial spheres, however many there are, and by common usage is called the heaven. The Sublunar, finally, which we inhabit, is that which comprises all things stored up within the concavity of the whole Lunar heaven — such as the elements, animals, inanimate things, and so on. [Margin: The world is threefold: Supramundane, Celestial, and Sublunar]

Our author, therefore, leaving aside the Supramundane world — since the consideration of it is foreign to the astronomer and belongs rather to the metaphysician or the theologian — divided the world, as it comprises the Celestial and the Sublunar, into these two members, out of which, as out of parts, it is made up: namely, into the Elementary and the Aethereal region. He called these two chief parts of the World "regions," perhaps because of the common manner of speaking by which we are accustomed to distribute this terrestrial globe, in which we live, into various regions. He will now set forth for us the form and figure of each region, both the Elementary and the Aethereal.

The elementary region, being pervious to continual alteration, is divided into four. For the earth stands as it were as the center of the world, set in the midst of all; around it, water; around the water, air; around the air, fire, there pure and untroubled, reaching to the orb of the Moon, as Aristotle says in the book of Meteorology. For thus did God, glorious and sublime, dispose them.

And these four are called elements, which in turn are altered by one another, corrupted, and generated.

The elements, moreover, are simple bodies, which cannot be divided into parts of diverse forms, and out of whose mingling arise the diverse species of things generated.

Any three of them surround the earth on all sides in a spherical manner, except in so far as the dryness of the earth resists the moisture of the water, for the preservation of the life of living creatures.

All of them too, except the earth, are movable, which — fleeing on all sides equally the motion of the extremities, as from the center of the world by its own heaviness

— *the earth possesses the middle of the round sphere.* [Margin: The form and figure of the elementary region]

COMMENTARIVS.

Here he begins to treat of the elementary region, or (what is the same) of the Sublunar world, and sets forth its form and disposition. And he goes through this region very briefly in six points.

First, he assigns a certain property of the elementary region: namely, that it stands pervious to continual alteration — that is, giving place and admission to the alterations that occur in it. By the name "alteration" understand every natural transmutation — such as generation, corruption, augmentation, diminution, local motion, and alteration properly so called (such as heating, cooling, and the like) — and finally every motion that in some way varies the substance of a thing. For the elementary region is that part of the universe in which continual transmutations of things occur. [Margin: The elementary region is subject to continual alteration]

Second, he divides the elementary region into four members, namely into Earth, Water, Air, and Fire, where he also shows the order these parts hold in the Universe, saying that the earth is set as it were as the center of the world, in the midst of all. He said "as it were the center," because, since the earth has a great quantity and mass — if considered absolutely — it cannot be the true center. For the center of any circle or sphere is an indivisible point, lacking all magnitude. But since the whole magnitude of the earth, however immense it appears to us, is, in respect of the whole heaven, but as a point (as will afterward be demonstrated), it deservedly [Margin: The order of the elements]

can be called the center.

Next [Sacrobosco] asserts that around the earth there is water — which must be understood of water's natural place, since it belongs to water's nature to surround the earth; but why it does not now surround it, we shall say shortly — and around the water, air; and finally around the air, fire, which exists there pure and untroubled, reaching the orb of the Moon. The philosophers hold that fire is pure and not turbid there for three reasons. The first is that vapors cannot rise there to make it impure and turbid. The second is the difference between that fire and this lower fire of ours, which is said to be not pure but mixed, because it is not in its natural place; for this lower fire is continually mingled with the air in which it exists, and it has its earthy nourishment, by which it is made turbid and

impure, whereas fire in its own sphere is unmixed, rare, and pure. A sign of this can be that, because of its very great rarity and purity, it does not shine there, and so is also not seen. The third reason is taken with respect to the other elements, which do not exist pure; for water is mingled indiscriminately with earth, and air is made impure and turbid by the continual rising of vapors from earth and water; but fire is mingled with none of these, especially near the concavity of the Moon. On this account Aristotle said in *Meteorologica* I: "Either the simple element exists nowhere, or, if it exists anywhere, it will be in the place of fire." And if you ask the author the cause of this order — namely, why earth is lowest, then water above it, and so on — he replies that the cause of this order is the glorious God, who so arranged them and willed this element to be above that one.

[Margin: The elements are in turn altered, corrupted, etc., by one another.]

Thirdly, he says that these four parts of the elementary region are called Elements, which are in turn altered, corrupted, and generated by one another. For now water comes to be from earth, air from water, and fire from air, and conversely, and this continually. For which reason the elementary region has been called by the philosophers the sphere of active and passive things [*sphaera activorum et passivorum*]. You should not understand this to mean that these elements so fight among themselves that one element corrupts another entirely — for that is false — but rather that a part of one sometimes alters and corrupts a part of another, and introduces the form of its own species into the other's matter.

[Margin: What the elements are.]

Fourthly, he defines the elements, saying that Elements are simple bodies [*corpora simplicia*] which can least of all be divided into parts of diverse forms, and from whose mixture the various species of generated things arise. He took this definition from Avicenna [Ibn Sīnā]. They are called elements ("bodies") to be distinguished from prime matter, which is not a body. They are called ("simple bodies") not because they lack composition of matter and form — for that would be false — but because they are not composed of other bodies (as mixed bodies are composed of elements), and are resolved into the same. What is added — ("which into parts of diverse forms," etc.) — is taken from *Metaphysics* book 5, chapter 3, and signifies that the elements are not resolved into things of diverse forms in the way that mixed bodies are resolved into elements. Or else it signifies that in the division of the elements one cannot assign dissimilar parts, since they are homogeneous bodies, that is, of similar kind or nature — whereas other

bodies are divided into dissimilar parts, since they are heterogeneous, that is, of another or diverse kind or nature.

[Margin: The five kinds of mixed bodies.]

Finally, as regards what follows — ("from whose mixture," etc.) — this alone must be known: that there are five kinds of mixed bodies, which arise from the various proportion and tempering of the miscible elements among themselves. In the first and lowest grade are those mixed bodies which may be called, and are called by the philosophers, imperfect mixtures, and are named Meteorological impressions [*impressiones Meteorologicae*], because they come to be aloft — such as rains, hail, snow, thunder, lightning, and the like. In the second grade are stones, minerals, and fossil bodies, which are called inanimate mixtures. In the third grade are vegetables, such as plants, which are called animate mixtures. In the fourth grade are comprised the brute animals. And in the fifth and highest grade, finally, are contained human beings.

[Margin: The figure of the elements.]

Fifthly [printed QVNTO; recte QVINTO], he shows the figures of the elements, saying that each of the three elements roundly surrounds another: fire surrounds air, air water, and water earth — the contrary of which we observe. For water does not surround the whole earth, but these two elements, namely earth and water, make one globe, as we shall show a little later. He adduces two causes why water does not surround the whole earth. The first is efficient and natural, namely the dryness of the earth, which (he says) continually, acting upon the moist watery [element], diminishes the water, or at least resists it, lest it cover the whole earth and complete its orb. But this cause is quite ineffective. For how can the dryness of the earth be so great that it can resist so great an element, and one by its own nature superior to it? Indeed it even conflicts with experience, for dryness rather seizes and draws in moisture, as we see in ashes and other such dry things. The second cause is final and supernatural, namely divine providence. For God, as we read in Genesis, separated the waters from the earth to preserve the life of certain animals. For before God, Most Good and Great, had said "Let the waters be gathered into one place" [etc.], the water surrounded the whole earth (according to the theologians); but by God's command the water withdrew, and dry land appeared. But in what manner this was done by God's command, various opinions exist.

[Margin: Various opinions as to how the water withdrew from the earth so that dry land might appear.]

For some say that the earth remained indeed in its own place, but the water was raised up above the earth, so that if it flowed down it would again cover the whole earth; and yet, as to why it does not now flow down and cover the earth, the agreed view among these [authors] is that it comes about by a miracle and the power of God, that the flowing water may not cover the globe of the earth. This opinion seems also to be that of Blessed Jerome [St. Jerome], moved by the authority of Scripture; for it is said in Proverbs 8 and Psalm 103 that God set a boundary for the waters which they would not cross. Others, unwilling to grant this continual miracle, adduce a ridiculous and worthless cause. For they say that around the arctic pole there are certain stars — namely in Ursa, Draco, etc. — of such efficacy and power upon these lower things that, from this habitable part of the earth, they drive toward the North the Ocean that inclines [there], and restrain it, lest it again overwhelm the earth. Others, judging the quantity of water to be much greater than that of the earth, say that the water, by the huge bulk of its mass, drives the earth out of its natural place by its own weight, and itself occupies the center of the world, so that the earth seems as it were to float in the sea. And all these authors think that this whole earth toward the Arctic pole is laid bare of waters, but the remaining part of the earth toward the Antarctic pole is entirely filled by the sea — which conflicts with the experience of today's navigators, as we shall say later. Others, finally, still granting that the water is much greater than the earth itself — indeed ten times greater — assert that the whole earth is like a kind of sponge (a sign of which, they say, can be that water is at once found in every place wherever the earth is dug), and that it is full of many caverns and hollows, so that the waters mingle with the whole earth and are received into those hollows. Hence a smaller part of the water than [the quantity of] earth will remain above the earth; and so it is no wonder that it cannot bury the earth any further. Into this opinion many Peripatetics try to draw Aristotle. But even if..

even if we grant that there are huge hollows in the earth, it is impossible that the water be ten times greater than the earth itself. For by this reasoning, even though the whole terrestrial globe were water, it could not fail that a greater portion of water than earth would exist above the earth, since even so nine parts of water out of ten would be left over. There is also the fact that the water is much less than the earth, as we shall show later. All these opinions, therefore, are in manifest conflict with reason and with experience — which will become plainer when we have treated of the roundness of the earth and water. For which reason, as to the manner in which, by God's command, the waters were separated so that dry land might appear, I prefer the one that St. John Damascene [John of Damascus] — a man of the highest authority among the theologians — sets out in

book 2 of *On the Orthodox Faith*, chapters 9 and 10, and which Jaime Pérez de Valencia [Jacobus de Valentia], Bishop, follows.

[Margin: The truer opinion, explaining in what way the water was separated from the earth.]

Namely, that the earth was created by God, Most Good and Great, perfectly round and globe-shaped, without any hollows, valleys, mountains, or eminences, and wholly surrounded by waters. But afterwards, when God said "(Let the waters be gathered into one place, etc.)," for the sake of the life of certain living creatures, hollows were made in the earth by divine command, and into them the whole force of the waters gathered, as into their own gathering-places; and various seas at once arose in different parts of the earth, and from those parts of the earth that were drawn out, mountains were made. To this opinion some add that the waters, in the beginning of the world, were very thin, but afterwards, by God's command, were condensed and received into the said hollows, so that it is no wonder they are not smaller than the earth. But in whatever way, finally, this was done, let us leave it to others to dispute. For us let it now be certain that earth and water make one globe — which will be demonstrated a little below from various experiences — and that this is the reason why water does not now surround the whole earth, indeed cannot surround it, since these two elements have one and the same convex surface, and both by their weight naturally tend toward the center of the whole universe.

[Margin: The earth is immovable; but the other elements are moved from East to West.]

Sixthly and lastly, he teaches that all the elements except the earth are movable — the earth being that (*which, as the center of the world, by its own heaviness, fleeing equally on every side the great motion of the extremes, namely of the heavens, [being] a round sphere, that is, possesses the middle of the world*). You should not understand this as if no earth is movable, for that is false, since, when placed outside its own place, it recurs with great force to its natural place. But [he means] that it is not moved circularly in its own place, as the other elements are. For fire indeed, and the upper part of the air — indeed, some affirm from experiment that a good part of the Ocean also — are carried by the motion of the *primum mobile* from East to West, on account of their lightness and mobility.

ON THE NUMBER AND ORDER OF THE ELEMENTS [DE NVMERO ET ORDINE ELEMENTORVM]

[Margin: That there are four elements is proved from the combinations of the primary qualities.]

Since our author has taught that there are four elements, it will not be out of place to open up briefly by what strongest arguments the philosophers gather that there are four elements, and then to say something about their order and position. The first argument, then, by which the philosophers prove that there are four elements is taken from the primary qualities, which Aristotle in *On Generation [and Corruption]* book 2 says are four — two active, namely heat and cold; and two passive, namely dryness and moisture. And the argument is of this sort. There are as many elements as there are possible combinations of these four primary qualities — that is, in as many ways as these four primary qualities can be joined among themselves and mutually endure one another, as Aristotle says in the passage cited. But there are only four possible combinations. Therefore there will be four elements also. The minor premise is evident, because among those four qualities, if we always take them two at a time, only six combinations can arise: namely, of heat with dryness, from which Fire is constituted, which is hot in the highest degree but dry in the remitted degree; of moisture with heat, from which we have air, which is supremely moist but exists remittedly hot; of cold with moisture, from which the philosophers gather water, which they say is cold in the highest [degree] but moist remittedly; of dryness with cold...

Diagram. A circular woodcut "wheel" (*rota*) of the four elements and the four primary qualities, set into the lower-left of the page. The outer ring is divided into segments alternately lettered **SVMMA** ("highest [degree]") and **REMISSA** ("remitted [degree]"). Four large roundels are placed at the cardinal points, each with a small emblematic image: **IGNIS** (Fire, top, shown as flames); **AER** (Air, right, shown as clouds); **AQVA** (Water, bottom, shown as waves); **TERRA** (Earth, left, shown as a walled town). Four smaller roundels stand at the diagonal points, each naming a quality: **CALIDITAS** (Heat, upper right, between Fire and Air); **SICCITAS** (Dryness, upper left, between Fire and Earth); **HVMIDITAS** (Moisture, lower right, between Air and Water); **FRIGIDITAS** (Cold, lower left, between Earth and Water). At the hub is a roundel bearing the **IHS** Christogram (the Jesuit device). Radiating spokes and cross-bars carry labels: **Combinatio** ("combination") on the spokes; **CONTRARIA** ("contrary," lettered CONTRA...RIA) along the two diameters joining opposite elements/qualities; and **possibilis** / **impossibilis** ("possible" / "impossible") on the connecting bars, marking which

quality-pairings can and cannot be joined. Additional fine italic words run along the thin diagonal bars near the hub (apparently "sym[bolizatio]" or the like) which do not resolve at this resolution [?]; a further small word on the lower diagonals reads [?]. (Uncaptioned; the surrounding text explains the figure's meaning.)

[of dryness with cold], from which earth is made, which is predicated as dry in the highest [degree] but cold remittedly; of heat with cold; and of moisture with dryness. But since these two last combinations are impossible, being combinations of contraries — whose nature is such that the one always expels the other, for one and the same thing cannot be numerically hot and cold, nor moist and dry at the same time — they are therefore reckoned useless, nor can anything be constituted from them. But all these combinations are seen more clearly than light in the figure set forth. That there is one quality in each element in the highest degree, and another in the remitted [degree], must be understood according to the opinion of certain philosophers; for many think that both qualities are in any element in the highest degree.

[Margin: A most beautiful digression on the combinations, or comparisons, of things.]

[Margin: How many combinations can be made among any number of things, if they are taken two at a time.]

Since we have said that among four things there cannot be more combinations than six, if they are always taken two at a time, it has seemed good to me to explain a little more fully how many combinations of this kind can be made among any number of proposed things; for the knowledge of this matter is conducive to many things, and is most delightful. Given, therefore, the number of some things, let it be multiplied by the number next smaller: for the half of the product will indicate the number of combinations that can be made among the proposed things. As in the proposed example: since there are four primary qualities, if 4 be multiplied by 3, 12 will result, whence six combinations can be made among them. And if there are five things to be combined, 5 must be multiplied by 4; for the half of the product, namely 10, will show the number of combinations — as many, namely, as Porphyry [Porphyry] established among the five predicables.

This rule as given can be split into two, according as the number of things is even or odd. For if the number of things is even, the number next smaller must be multiplied by the half of the number of things: for the product will directly show

the number of combinations. As, if you wish to know how many combinations can be made among 10 things, 9 will be multiplied by 5, to make 45 — as many combinations, namely, as can be made among ten things. But if the number of things is odd, it must be multiplied by the half of the number next smaller: for by this reasoning the number produced will indicate how many combinations can be made. As, if the things be 15: multiplying 15 by 7, the number of combinations among them will be produced, namely 105. Among 9 things, 36 combinations will be made; and so of the rest.

[Margin: How many combinations can be made among any number of things absolutely, if not only two, but also three, four, five, etc., are taken.]

But if it should please you to know how many conjunctions can simply be made from any number of proposed things — not only understanding it when they are taken two at a time, as in the preceding rule, but also when three, five, etc., that is, in how many distinct ways they can be compared among themselves — this will be accomplished by this art and rule. Let as many numbers be taken, beginning from unity in double proportion, as there are things proposed; and from the sum of them all let the number of things be subtracted: for the remaining number will indicate how many diverse comparisons can be made.

[Margin: How the sum of any number of numbers of double proportion beginning from 1 is found.]

Now the sum of any number of numbers in double proportion beginning from 1 will easily be had, if the last number be doubled and unity be cast off from the product. As, if you wish to know the sum of these numbers in double proportion 1. 2. 4. 8. 16. 32. 64., the last number, 64, must be doubled to make 128; from which, one being taken away, there remain 127, for the sum of all those numbers — that is, 127 units are contained in them. But we have written more on this matter in our *Practical Arithmetic* [*Epitome Arithmeticae Practicae*].

An example of the combinations in the aforesaid four qualities: the numbers in double proportion, according to the number of things, will be four, namely 1. 2. 4. 8., whose sum is 15. Four therefore being cast off, there remain 11. In so many diverse ways, then, the four primary qualities can be joined, namely these: heat, cold; heat, dryness; heat, moisture; cold, dryness; cold, moisture; dryness, moisture; heat, cold, dryness; heat, cold, moisture; heat, dryness, moisture; cold, dryness, moisture; and finally heat, cold, dryness, moisture. Nor can it be that any other comparison be made that differs from all these. For these two — heat, cold : cold, heat — are not reckoned to be distinct, since only the order is changed and

not the thing. By this reasoning, among five things — as among the five predicables — 26 diverse comparisons can be made. For the sum of these numbers 1. 2. 4. 8. 16. is 31; and 5 being taken away, there remain 26. This rule, moreover, is much conducive to the astrologers, that they may know all the diverse conjunctions which can be made among the seven planets; for by the aforesaid device they can be joined, or varied, in 120 ways, which it would be long to recount. In like manner it will be known how many words, whether useful or useless, can be constituted from the 23 letters of the alphabet — that is, in how many ways the said 23 letters can be joined among themselves, so that they are always diverse conjunctions, whether they can be pronounced or not. For from 23 letters there will be made words, or diverse conjunctions, to the number 8388584. For the last number, namely the twenty-third of double proportion, is 4174304, and so the sum of all the numbers will be 8388607. Therefore 23 being cast off, there remain 8388584, and so on. It is true that more words can be made, or combinations of letters, if the letters in any conjunction be permuted among themselves. As this aggregate, or conjunction, AVE can be varied in six ways, namely AVE, AEV, VAE, VEA, EVA, EAV — which modes, indeed, are taken by us in the rule as one conjunction only, since all these modes contain the same letters, though they change place among themselves.

[Margin: In how many ways any number of things can be interchanged among themselves, the same number of things always remaining.]

But if a number of things be proposed, and it be worth the labor to investigate in how many ways they can be interchanged, the same number of things always remaining, you will obtain it by this rule. Take as many numbers in the natural series as there are things, beginning from unity, and multiply them all among themselves: for the number produced will show what is sought. As two things — e.g. A, B — can be varied in two ways, thus: AB, BA — since these numbers 1. 2. multiplied among themselves make 2. But three things can be varied in six ways, for these numbers 1. 2. 3. multiplied among themselves make 6. The reason of this is that each thing will hold the first place once, and the remaining two twice, as we said, and so can be changed among themselves. So too four things can be varied in twenty-four ways [printed **vigintiquator**; recte **vigintiquatuor**], since these numbers 1. 2. 3. 4. multiplied among themselves make 24. The reason is that each thing will once occupy the first place, and the remaining three six times, as we said, and so can be varied among themselves. By the same way you will gather that 10 things can be arranged among themselves in 3628800 ways, since these numbers 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. multiplied among themselves generate this number, 3628800. But eleven things [can be varied] in 39916800 ways among

themselves, since these numbers 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. multiplied among themselves procreate the aforesaid number. Since, therefore, by the preceding demonstration all the conjunctions of the twenty-three letters of the alphabet have become known, if it be inquired by this rule in how many ways the letters of each conjunction can be interchanged among themselves, there will be had the number of all words, useful and useless — provided that in one word no letter be taken twice, or three times, etc. For thus many more words still could be made.

By this reasoning, from the final combination of twenty-three letters among themselves, there arise 25,852,016,738,884,976,640,000 permutations — and this without any single letter being taken twice, which is scarcely credible. I decided to include these remarks on combinations here, since combinations had already been mentioned, and rules of this kind are usually explained by means of a few examples.

A second argument by which the fourfold number of the elements is proved, philosophers draw from lightness and heaviness. For every simple body — the kind into which mixed bodies are resolved, that is, an element — is either heavy or light. If heavy, it is either heavy without qualification, like earth, or heavy in a certain respect [*secundum quid*], like water; if light, it is either light without qualification, like fire, or light in a certain respect, like air. And in this way these four elements are gathered together. Water is said to be heavy in a certain respect because, although it is heavy relative to fire and air, it is nevertheless somehow light relative to earth, since earth is heavier than water. Yet it is called heavy rather than light, because it is called light only relative to one element, namely earth, whereas relative to the other two it is called heavy; and in reality it contains heaviness in itself, and not lightness. By the same reasoning air is called light in a certain respect: for although it is light relative to earth and water, it is nevertheless somehow heavy relative to fire, since fire is much lighter than it. But it is named light rather than heavy, because it is called heavy only relative to one element, namely fire, while relative to the other two it is light; and in reality it contains lightness in itself, and not heaviness at all, since — unless it is impeded — it always tends by its own motion toward a higher place.

[Margin: That there are four elements is confirmed from lightness and heaviness.]

[Margin: Why water is called heavy in a certain respect, and air light in a certain respect.]

A third argument is drawn from the simple local motions. For, on the authority of Aristotle in the book *On the Heavens* [*De Caelo*], there are only three simple local motions: the first takes place about the center, of the kind that is circular, which belongs to the celestial bodies; the second is away from the center; the third is toward the center — and these two latter motions are rectilinear. Now philosophers reason thus: there are as many simple bodies carried by rectilinear motion (excluding the heaven, which is not moved by rectilinear motion) as there are simple rectilinear motions — since every simple motion belongs to some simple body, and conversely every simple body is apt to be moved by a simple motion. But there are four such motions: two away from the center, that is, from the center of the world — of which one is away from the center without qualification, and is assigned to fire, the lightest of all; the other away from the center in a certain respect, which is granted to air, since it is not so light as fire, yet lighter than earth and water. And two toward the center, or toward the center of the world, of which the one that is toward the center without qualification belongs to earth, corresponding to its supreme heaviness; while the other, which is toward the center in a certain respect, is ascribed to water, inasmuch as it is not so heavy as earth, but heavier than fire and air. There are therefore these four elements. Let further arguments be sought from natural philosophy.

[Margin: That there are four elements is proved from the local motions.]

The order and position of the elements can likewise be gathered chiefly from three considerations. First, from their lightness and heaviness. For the lighter one thing is than another, the higher the place to which it ascends, and the heavier it is, the lower. Since, then, fire, on account of its very great rarity, is supremely light, the highest place is due to it — which indeed lies beneath the concave surface of the Moon. Air has obtained the place next to it, since it is lighter than the other two elements, yet less light than fire. Next after this comes water, for it is heavier than fire and air, but lighter than earth. Finally, the lowest place, which is near the center of the universe, earth by fullest right claims for itself, since it is the heaviest of all.

[Margin: The order of the elements is gathered from lightness and heaviness.]

Secondly, the order is gathered from the agreement of the elements in their properties. For the more any elements agree in their properties, the nearer and closer they are to one another in place. Hence, since we see that earth holds the lowest seat, and that water is more like earth than air is — because air is wholly opposed to earth and agrees with it in no quality, whereas water accords with earth in coldness — nature has, not without reason, placed water immediately

above earth. By the same reasoning we shall fittingly place air above water, since it agrees with water in moisture, whereas fire is like water in no quality, but is altogether contrary to it. Above air, lastly, we shall not unjustly set fire, since it agrees with air in heat. To this is added that, since fire and water possess wholly contrary qualities, they cannot be placed immediately next to each other; therefore most skillful nature interposed the middle elements, which in their qualities share with each of the two contraries — air, namely, between fire and water, and water between earth and air. And by this reasoning the elements stand in a symbolizing relation [symbolizantia] to one another.

[Margin: The order of the elements is gathered from their properties.]

But if anyone should ask why it is rather water that immediately follows earth, and not rather fire, then air, and lastly water — since in this order too the said agreements of the elements in their qualities would be preserved, because the middle elements are always interposed between the contraries — the answer is that this comes about for a twofold reason. First, since we see that earth, the heaviest of all, has occupied the lowest place, natural reason seems to require that fire, the lightest of all, should occupy the highest place; and so it was not fitting for earth to follow immediately upon it. Secondly, since water is exceedingly unstable and flowing, it cannot hold together unless it rests upon some hard body, such as earth is; therefore with the best right water is placed immediately above earth.

Thirdly, the order is gathered from sense and experience. For we see daily that fire is borne naturally above earth, water, and air, since its shape always forms a pyramid; therefore its natural place must be above all these. We also see that air naturally ascends above earth and water, as is evident in an earthquake. For an earthquake arises from the violence of air enclosed within the bowels of the earth, striving to ascend into its own place above earth and water. This is likewise established in the bubbles of air that well up in water, as may be seen in marshes if someone thrusts a stick into the bottom. Reason therefore requires that air be placed above earth and water, and beneath fire. We see, finally, that water placed in air descends, and that earth placed in water likewise tends downward. Therefore it will be concluded that, not without reason, the natural place of water is beneath air, and that of earth beneath water.

[Margin: The order of the elements is gathered from experience.]

There are, however, some — among whom is Cardano [Girolamo Cardano] — who deny that fire exists above air, on the ground that it is in no way perceived

by us; indeed, they say, if it were there, it would burn up these lower things. And so these men do not grant any elemental fire other than this lower one which we use. But let us leave this dispute to the philosophers. For the present it will be enough to know that the far more probable and more common opinion is that of those who, with Aristotle, place fire beneath the concave surface of the Moon as in its natural place. That it is not perceived arises from its excessive rarity; for since it is exceedingly pure and exists in matter rarer than air, it cannot be seen — indeed not even air itself, which is denser, can be seen. That it does not burn up these lower things happens from the same rarity: for fire, existing in the rarest matter, cannot have so great a power of burning; yet it does warm these lower things wonderfully by its heat.

[Margin: Cardano denies that the element of fire lies beneath the concave surface of the Moon.]

As for the shapes of these elements, there will be occasion to speak later; for the present it is only worth noting that air is divided by philosophers into three regions: namely the supreme, the middle, and the lowest. The supreme, in which we see comets carried, is always hot on account of its continuous motion — which it has from the *primum mobile* — and its nearness to fire, and the continuous emission of the sun's rays through it. Likewise the lowest region, being nearer to us, grows warm from the manifold reflection of the sun's rays. But the middle region, on account of its great distance from fire, and because the reflections of the sun's rays cannot reach it, is always cold, as the meteorological impressions generated there show — impressions that are cold, such as rains, snow, hail, and the like.

[Margin: Air is divided into three regions by philosophers.]

Diagram. A circular woodcut of the sublunary and elemental spheres, drawn as a set of concentric rings within a flame-hatched border (the sphere of fire). At the center is the terrestrial globe (a small landmass or figure representing earth and water); around it the three concentric bands of air are labeled, and the whole is set within an outer frame marked with celestial reference points. The labels read:

- Center: the globe of earth (and water), unlabeled.
- Inner band: **INFIMA REGIO** ("lowest region" — of air).
- Middle band: **MEDIA REGIO** ("middle region").
- Outer band: **SVPREMA REGIO** ("supreme region"), bordered by the wavy flame-hatching of the sphere of fire.
- Top (radial): **POLVS ARCTICVS** ("arctic pole").

- Bottom (radial): **POLVS ANTARCTICVS** ("antarctic pole").
- Left (vertical): **ZENITH**.
- Right (vertical): **NADIR**.
- Diameter line: **ÆQVINOCTIALIS** ("equinoctial [circle]"), lettered across the figure.

Moreover, granting that the whole airy orb is uniform, so that it has the same center as the world both according to its concave and according to its convex surface, it seems probable enough that the middle region of air is broader and denser near the poles of the world, on account of the weakness of the heat which the very great absence of the Sun produces there, and the excessive cold that perpetually exists there; and that the parts of that same middle region lying midway between each pole of the world — as under the equator — are compressed, on account of the abundance of heat which the perpetual presence of the Sun produces there; and conversely, that the parts of the supreme and lowest region near either pole of the world are compressed, while their parts lying midway between each pole are dilated. All this you may clearly draw out from the figure set beside the text, in which you will also observe the position and order of the elements. Indeed, perhaps fire itself is not uniform in density, since, on account of the speed of the motion by which it is swept from east to west, it can easily transmute into itself the air that lies beneath it.

But surrounding the elementary region is the aethereal region — luminous, exempt from all variation by its own immutable essence, and moving continuously in a circular fashion. And this is called by the philosophers the fifth essence.

[Margin: The aethereal region.]

COMMENTARY.

After treating the form of the elementary region, the author proceeds to a discussion of the form of the aethereal region. He sets out at the outset five notable properties by which it is separated and distinguished from the elementary region. The first is that it lies around the elementary region — in which respect it is compared with the elementary as container to contained — and it is called the place of the whole elementary region. Now every place, the higher it is, the more mobile it is believed by philosophers to be, and the body existing in it the more excellent, because it is removed further from these corruptible lower things and exists nearer and closer to those divine movers of the spheres, which ever enjoy the best and most blessed life. The second property is that it is luminous, in which it far surpasses the elementary region. For light is

far nobler than the properties of the elements. For these properties are active or passive, and mutually contrary, so much so that by their mutual struggle all these lower things are led to destruction and corruption; whereas light, free of all contrariety, pours into all these lower things life, being, and duration. There is added, too, that light is the object of the noblest sense, namely sight; and one whole discipline among the mathematical sciences — and the most beautiful of them, namely optics [perspectiva] — is occupied with it. The third property is that the aethereal region lacks all motion that would alter its substance. For the aethereal, or celestial, region can neither be altered, nor increased, nor diminished, nor generated, nor corrupted, according to the philosophers — the opposite of which we asserted above concerning the elements, since these are engaged in perpetual transmutation. The fourth property is that the aethereal region is moved by a perpetual and continuous circular motion without any interruption — a motion which among philosophers is the first of all others and the noblest, and is the cause of continual generation and corruption in these lower things. The motion of the elements, by contrast, is rectilinear by its very nature, and quickly comes to an end. The fifth and last property is that the aethereal region is called by philosophers the fifth essence. For the heaven, as Aristotle holds, is not an element, nor composed or made up of elements, but

[Margin: The properties of the aethereal region.]

[Margin: The fifth essence.]

is a body of a certain other, intermixed nature, far removed from the four elements. Hence, in order to distinguish it from the four elements, it has been called the fifth essence.

This fifth nature — that is, the celestial region — is called *Aether*, on the authority of Aristotle, from αἰ [Greek: αἰ (aei, "always")] and θέω [Greek: θέω (theō, "I turn" or "I run")], because the celestial bodies which constitute that region are turned and revolve always and perpetually. Some, however — among whom Anaxagoras and Cicero are counted — hold that it is called *aetherea* from αἶθω [Greek: αἶθω (aithō, "I blaze," "I shine")], for the aethereal region is indeed always luminous and shining.

[Margin: Why the aethereal region is so called.]

Of this region there are nine spheres, as was treated in the preceding chapter: namely those of the Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn, the fixed stars, and the outermost heaven. Each higher of these spheres spherically surrounds the one below it. Of these there are two motions: one is that of the

outermost heaven upon the two extremities of the axis, namely the arctic and the antarctic pole, returning from the east through the west back to the east, which the equinoctial circle divides through the middle. There is also another motion of the lower spheres, oblique and opposite to this one, upon their own poles, which are distant 23 degrees and 33 minutes from the former poles.

[Margin: The order of the celestial spheres.]

[Margin: There are two motions of the celestial spheres.]

COMMENTARY.

He repeats the division of the aethereal region, which a little before he had divided, with respect to substance, into nine heavens; and here he lists their names and order.

Now all the celestial spheres, he says, are moved by two principal motions. The first is assigned to the outermost heaven, or primum mobile, and takes place upon the two poles of the world, namely the arctic and the antarctic, returning from the east through the west again to the east. That heaven is said to be moved from east to west which tends from the east toward the south — that is, toward that part of the meridian circle which stands above the horizon — into the west, and again from the west toward midnight — that is, toward that part of the meridian circle which lies hidden beneath the horizon — is turned back to the east. But that heaven is said to be moved from west to east which tends from the west toward the south into the east, and again from the east toward midnight slips back into the west. This must be carefully noted, so that the motion from east to west may easily be distinguished from the motion from west to east; for the former, beneath the earth, also proceeds from west to east, and the latter from east to west, and yet the former is said to go from rising to setting, and the latter from setting to rising — because the former, above the earth, goes from rising to setting, while the latter goes from setting to rising. This motion from east to west, the author says, the equinoctial circle divides through the middle. For since motion is divided in accordance with the division of the movable thing, as is held in *Physics* VI, and the primum mobile is divided by the equinoctial circle into two equal parts, as we said above, it is necessary that the same circle should distribute the motion of that same primum mobile — which, according to our author, is the ninth sphere, and which is carried along the equinoctial circle — into two equal parts.

[Margin: When some heaven is understood to be moved from rising to setting, and from setting to rising.]

The other motion belongs only to the eight lower spheres, and in no way to the primum mobile, and is opposite to that former motion. For the eight lower heavens are moved from west through the south to the east, and thence slip back through midnight into the west. Perhaps our author called this the second motion, describing it as "through the oblique," because it does not take place upon the poles of the former motion, but upon other poles distant from the poles of the former motion by 32 [?] degrees and 33 minutes — a distance which was observed by al-Ma'mūn [Almeone][?], and which is now smaller, namely about 23 degrees and 30 minutes, as we shall say in chapter 2. What a degree is has been stated above, where we spoke of the zodiacal circle. A minute is the sixtieth part of one degree; for astronomers divide any degree into sixty equal parts, which are called minutes — about which division you will have more in chapter 2, when we treat the zodiacal circle at greater length. Or rather, this posterior motion is called oblique because it takes place along the zodiacal circle, which, as was said above, obliquely cuts the equinoctial circle, in accordance with which the prior motion is accomplished. For hence it comes about that this posterior motion is in a certain way oblique, if it is compared with the prior.

But the first sphere carries all the others along with it by its own impetus once around the earth within a day and a night; yet with those others striving against it — as the eighth sphere moves one degree in a hundred years. This second motion the zodiac divides through its middle, beneath which each planet has its own sphere, in which it is carried by its own proper motion against the motion of the outermost heaven, and completes that motion in different spans of time: as Saturn in 30 years, Jupiter in 12, Mars in two; the Sun in 365 days and about six hours; Venus and Mercury nearly the same as the Sun; but the Moon in 27 days and eight hours.

[Margin: A comparison of the two motions of the celestial spheres with each other. The periods of the motions of the planets from setting to rising.]

COMMENTARY.

Here he compares the two aforesaid motions with each other, assigning also the times, or periods, in which such motions are completed. He says, then, that the first motion, or primum mobile — which according to him is the ninth heaven — carries all the other lower spheres along with it by its own impetus within a day and a night, that is, within the span of 24 hours, once around the earth. Hence such a motion is usually called "diurnal" not only by astronomers and philosophers, but also by common people, because it is completed in a natural day, which comprises 24 hours, as will be explained more fully in chapter 3.

[Margin: The diurnal motion.]

Next he asserts that all the lower spheres, although — as was just said — they are carried by that first motion from east to west, strive against it, that is, tend in the contrary direction, namely from west to east, being carried in different

COMMENTARIVS (continued) — on the nine spheres and their motions

at different times. For, as he says, the eighth sphere — that is, the starry heaven — completes one degree in 100 years by its own motion, which is stated according to Ptolemy's [Ptolemy] view. From this it follows that this entire motion is completed in a span of 36,000 years. The Zodiac circle divides this motion through its middle, just as the Equinoctial divides that first motion. For just as the first motion takes place about the poles of the world and along the Equinoctial circle, so too astronomers have found that the second motion takes place about the poles of the Zodiac and along the Zodiac circle.

Beneath this Zodiac, he says, each planet is carried in its own proper sphere by its own motion, contrary to the motion of the outermost heaven — that is, from West to East. You should not understand this as though the planets moved by themselves beneath the Zodiac; rather, the heavens themselves move about the poles of the Zodiac, and in this way carry the planets along with them, the planets always remaining beneath the Zodiac. And this happens at different times, as he himself clearly sets out, and as we shall explain more fully a little below.

[Margin: The harmony of the celestial motions.]

In this matter one may contemplate the following harmony of the celestial motions: the nearer a sphere is to the primum mobile — that is, to that first, most rapid motion — the less it resists in the opposite direction, and the more slowly it is carried by its own motion from West to East; whereas the farther off it is, the more it resists in the opposite direction, and the more swiftly it completes its own motion. Yet no fixed proportion is preserved in this slowness and swiftness, as is evident from the periods of all the motions that the author has reported. Hence, among all the eight lower spheres, the sphere of the fixed stars, because it is nearest the primum mobile, completes its course most slowly; and among the seven planets, because Saturn is the highest, it too moves by its own motion more slowly than the rest; and the Moon, finally, because it recedes farthest from the primum mobile, completes its motion most swiftly.

But since the author has here spoken of the number of the celestial spheres, their motion, and their order, I think it worthwhile to explain a little more fully how many heavens there are, and by what skill and diligence astronomers discovered their number; then, by how many motions they are moved, and by what reasoning these motions were detected; and finally, what order is established among the celestial spheres.

DE NVMERO ORBIVM COELESTIVM. [On the Number of the Celestial Spheres.]

[Margin: The opinion of those who posit a single heaven. Refutation of the opinion of those who posit a single heaven.]

Some of the ancient philosophers assert that there is only a single heaven, and a very few of the more recent thinkers follow them, persuaded by this one argument: All our knowledge, according to the doctrines of the philosophers, arises from the senses. Since, then, whenever we lift our eyes to the heaven we do not perceive by sight a multitude of heavens — for the Sun, the Moon, and all the other stars appear to exist in one and the same heaven — and since the heaven itself can fall under no other sense than sight, there is no reason for us to posit many heavens instead of one.

But this opinion can be defended by no reasoning. For no single body can move at the same moment with opposite and contrary motions; while it ascends it cannot at the same time descend, and while it proceeds from this place to that, it is impossible that at the same instant it should move from that place to this, since these are mutually contradictory. Yet in the stars diverse and opposite motions are found. Since, then, the stars do not move by themselves, like fish in water or birds in the air — as Aristotle [Aristotle] holds, along with the philosophers, and as we shall demonstrate a little later — but move with the motion of the sphere in which they are set, like a knot in a board with the motion of the board, or a nail fixed in a wheel with the motion of the wheel, we must grant that there are more heavens than one, in which are set those stars that are moved by diverse motions.

That diverse motions are found in the stars is established partly from what the author set out above concerning the twofold motion of the celestial bodies — namely from East to West, and conversely from West to East — and partly it will become clearer and more distinctly evident in what follows, when we discuss the motions of the heavens, where we shall also show by what diligence astronomers observed them. This opinion, therefore, must be rejected as empty and useless. As to the argument that the authors of this opinion put forward, we must reply that

it is indeed true that our knowledge, while we are in this mortal life, arises from the senses; but it must be denied that no more than one heaven is perceived by sense. For although we do not comprehend by sight the multitude of the heavens — indeed, not even a single one — nevertheless we perceive by sight very many stars, and we detect that they are continuously set in motion by diverse and opposite movements. Therefore, on account of this diversity of motions, more spheres must necessarily be posited.

[Margin: The opinion of those who posit eight heavens.]

Others, therefore — as were nearly all the Egyptians and Chaldeans, much devoted to astrology, and other astronomers down to the times of Plato and Aristotle — asserted that there are at least eight heavens, on account of the eight distinct motions they observed in the heavenly bodies. For when they saw the Sun and the Moon, and all the other stars as well, continuously moving from East to West, they learned by long observation and experience that not all the stars are always conjoined or separated by the same distance, since sometimes they were joined together and sometimes drawn apart — as we experience, more clearly than daylight, each month in the Sun and Moon; for at new moons these two planets are conjoined with one another, while at full moons they are opposite one another across the diameter. From this fact they clearly inferred that there are diverse motions in the stars.

For if the stars were carried by only a single motion, they would always be seen at the same distance and nearness. Hence they were compelled to affirm that there are more heavens — at least as many as the diverse motions they detected in the stars — since the stars are carried around not by themselves but together with the sphere in which they are, like a knot in a board. But since by long observation they came to know that a great number of stars — namely all those we call fixed — always advance uniformly at the same distance, in the same position and order: for example, the two hindmost stars of the Wain (which is in the Great Bear), together with the pole star (which is at the tip of the tail of the Little Bear) and that star which is in the left foot of Cepheus, always form a straight line; likewise that bright star which is in the more western pan of the Balance, together with Arctophylax (that is, Arcturus) and the last star in the tail of the Great Bear, are always positioned in an almost straight line; likewise the Great Dog, the Little Dog, and that star of the Wain which is nearer the Arctic pole are also arranged along a straight line; likewise the left foot of Orion, the Little Dog, and the tail of the Lion always form almost a straight line; the same has been observed in the eye of the Bull, the left shoulder of Orion, and the Great Dog;

likewise in the three stars that form the belt of Orion; again in the left foot of Orion, the eye of the Bull, and the bright star in the head of Medusa. Similarly the Ear of Corn of Virgo [Spica], Arctophylax [Boötes], and the tail of the Lion always form an isosceles triangle, whose base is formed by Arctophylax and the tail of the Lion; likewise the Heart of the Lion [Regulus], the Little Dog, and that bright star, the more eastern one of Gemini, form an isosceles triangle, whose base is formed by the Little Dog and that star of Gemini. The same, finally, has been observed in very many other stars. On this matter, read Ptolemy (*Almagest* [Dictio] VII), and the *Epitome* of Johannes Regiomontanus [Regiomontanus] on the same book, where many observations of this kind are adduced. For this reason they asserted that all those stars are set in a single celestial sphere, which they all called the Firmament, as was said above, and by whose motion they are carried around, always keeping the same remove, position, and distance from one another.

They observed, again, that among all the heavenly bodies there are seven stars, which they called wandering [the planets], which neither kept a distance from one another nor were found in the same position relative to the fixed stars. They concluded that these could not exist in the Firmament, where the fixed stars are — for then they would always keep the same distance from them, just as the fixed stars do among themselves; but neither were all seven placed together in some single other heaven, for on that supposition they would keep the same distance and position among themselves, even though they continually varied their arrangement relative to the fixed stars.

For this reason they concluded, by a most firm argument, that beneath the Firmament seven other spheres must be placed, which they named the seven spheres of the seven planets, or wandering stars. And since, besides these eight motions of the stars — entirely distinct and diverse from one another — they knew of no other, they were content with the number eight for the heavens, and supposed that the eighth sphere (that is, the Firmament containing the fixed stars) was the *primum mobile*.

[Margin: The opinion of those who posit nine heavens.]

But after these there arose other astronomers, among whom were Arsatilis [Aristyllus] and Timocharis, who flourished around the year 330 before the birth of Christ, and who, observing the courses of the heavenly bodies at Alexandria, detected that the stars of the Firmament — which antiquity had thought to be the *primum mobile* — are carried by another, very slow motion from West to East,

and not only by the daily motion from rising to setting, as the ancients had supposed. But because they had no observations of others with which they might compare their own, the result was that they left us almost nothing certain about this motion, but everything in doubt, on account of its excessive slowness.

Yet after them came Abrachis, also called Hipparchus [Hipparchus], nearly 200 years later, who, comparing his own observations with theirs, detected the aforesaid motion much more clearly and evidently. Then, after about 170 years, Agrias [Agrippa?] in Bithynia, Mileus the Geometer, also called Menelaus [Menelaus of Alexandria], at Rome, and after all these Ptolemy, the prince of astronomers, in the year of the Lord 131 or thereabouts, came to know this motion of the fixed stars from West to East still more clearly. By what diligence they detected it we shall soon disclose, when we have treated of the motions of the heavens.

Since, therefore, it is now beyond doubt that the fixed stars have a twofold motion, and no simple body can be carried by two motions, we must conclude that one of these is proper to the Firmament, by whose motion the fixed stars are carried around, while the other, which we find in the same Firmament, arises from another heaven — which must, of course, be placed above the Firmament, so as to be the ninth heaven and the *primum mobile*. For in this way the ninth heaven will be moved from rising to setting in the space of 24 hours, drawing the sphere of the fixed stars along with it at the same time; while the Firmament itself will be turned by its own motion from setting to rising, though very slowly.

Thus, then, the astronomers established nine celestial spheres by most certain observations, on account of the daily motion from rising to setting and that very slow one from setting to rising, both of which were detected in the fixed stars. And Johannes de Sacrobosco [Sacrobosco] follows this number of nine celestial spheres in this little work.

[Margin: The opinion of those who posit ten heavens.]

After Ptolemy, then, about 1,140 years having intervened, Thebit [Thābit ibn Qurra], and Alfonso, king of the Spaniards [Alfonso X of Castile], in the year of the Lord 1250, and then George Peurbach [Georg Peurbach] and Johannes de Regiomonte [Regiomontanus], distinguished astronomers, did indeed detect in the fixed stars the two aforesaid motions, but besides these they observed that the stars are agitated by a certain third motion, which they called access and recess [accessus et recessus — *trepidation*], as will be explained shortly. Therefore, since a simple body is fit to be carried by only one motion, as the philosophers hold, the

ninth heaven cannot be the *primum mobile*; rather, above it another heaven must be established to be the *primum mobile*. For thus it will come about that this tenth heaven, by the daily motion which it has as its own from East to West, draws with it all the lower heavens — indeed the Firmament too, with the fixed stars — in the space of 24 hours; then the ninth heaven carries around, by its own motion which it possesses, from West to East, both the Firmament and all the other heavens below it; and finally the eighth heaven, or Firmament, in which the fixed stars are, moves as if by its own motion, with that access and recess which the aforesaid astronomers discovered.

This tenfold number of celestial spheres, therefore, has remained the most celebrated in the schools of astronomers down to this day — though there are some who, so as not to seem to depart from the ancients, and especially from Aristotle, stubbornly try to maintain that there are only eight heavens. But since authors of this sort can in no way account for all the motions we see in the celestial bodies, their opinion is rightly rejected by astronomers, as we shall discuss when we treat of the motions of the heavens. Nor should the authority of the ancients and of Aristotle move us; for if they had detected the further motion beyond those eight, they would doubtless have admitted more spheres, since it was by no other reasoning than from the number of motions that the number eight for the heavens was reckoned by them.

Therefore in this matter we should rather trust the most experienced astronomers, who observed the said ten motions — namely seven distinct from one another belonging to the seven planets, and three others belonging to the fixed stars — than Aristotle, since he himself affirms in *Metaphysics* XII that astronomers are to be consulted in astronomical matters. Indeed, these very authors, who wish to be so devoted to Aristotle and the ancients that they will by no means depart from them in the number of the celestial spheres, do depart from them in the order of those same spheres, on account of the most manifest observations of the astronomers, as will later become clear.

[Margin: The opinion of those who posit eleven heavens.]

Finally, in our own time Nicolaus Copernicus [Copernicus], a most learned man and most worthy of all praise, observed in the fixed stars not merely three motions but four. Therefore, in order that we may account for them, three movable spheres must be placed above the Firmament, as we shall teach below — provided that we first set out the motions of the heavens according to the view of those who admit only ten movable heavens.

But if someone objects: every motion of the heaven, as Aristotle holds in *Metaphysics* XII, chapter 8, is for the sake of the motion of a star; since, then, in the ninth, tenth, and eleventh heaven there exists no star — for none appears there — it seems pointless to place, above the eight heavens in which all the stars (both fixed and wandering) are set, three other movable heavens marked by no star. The reply is: although in the ninth, tenth, and eleventh heaven there exists no star, nevertheless the motion of each of them redounds to some motion of the stars that exist in the other heavens. For by the motion of the eleventh heaven, or primum mobile, all the stars are moved from rising to setting; and by the motion of the tenth

heaven, from North to South and from South to North, through 24 minutes beneath the Colure of the Solstices; and by the motion of the ninth heaven they have a certain unequal libration from East to West and from West to East, beneath the Ecliptic of the tenth sphere, through 140 minutes; and finally, by the proper motion of the eighth sphere, the fixed stars are carried around from West to East. This suffices for the motion of the heaven to be appointed for the sake of the motion of a star. But these things will be perceived a little later more plainly and openly.

It can also be said that Aristotle, in the cited passage, was speaking of the motions of the heavens as they were then known, and that thus the motion of each heaven was ordered to the motion of a star existing in it — which, however, is not necessary, since no reasoning urges it, and experience has now taught the contrary.

[Margin: The Crystalline Heaven. The Empyrean Heaven.]

There is also added (if you please) the authority of Sacred Scripture and of the theologians, to confirm this elevenfold number of the heavens and to establish at least one more heaven above the Firmament. For we read in the sacred book of Genesis that God set a Firmament dividing the waters from the waters, and likewise in Psalm 148, "And all the waters that are above the heavens," and so forth. No one of sound judgment will understand that in that passage the waters above the eighth heaven are flowing and perishable, like these lower waters; rather, by the name of "waters" must be understood — as very many of the theologians explain — the ninth heaven, or rather the aggregate of the ninth, tenth, and eleventh heaven, which, on account of the brightness and transparency it has (since there are no denser parts there, as in the other spheres, such as the stars are), can with the fullest right be called by the name of waters.

For this reason it is usually called by some theologians the icy or watery heaven, and by others the Crystalline.

Above these eleven movable heavens the theologians — such as Strabus [Walafrid Strabo], the Venerable Bede [Bede], and now the whole company of theologians — affirm that there is another heaven, immovable indeed and endowed with no star, but the blessed seat and homeland of the angels and of the Blessed, which they call the Empyrean heaven, from fire, because it is wondrously bright and endowed with immense radiance. This heaven, however, can in no way be known by astronomers, since it does not move.

Nevertheless there are those who strive to prove, by certain observations, that it is very fitting that that twelfth heaven, entirely immovable, should exist above all the heavens. For, as Pliny [Pliny the Elder] testifies in book 8, chapter 16, in Europe between the rivers Achelous and Nestus there are begotten lions far superior in strength to those which Africa or Syria produces. Since this does not happen throughout that whole latitude, or tract of land from East to West, in which the said rivers are situated, the cause of this variety will be — as they assert — the influence of some unmoving heaven existing over that tract of land. For if the cause were the influence of the stars, or of the movable spheres, then throughout that whole tract of land from East to West, on account of the continuous motion of the stars, such lions ought to be born — the opposite of which we see.

Then, too, in Hungary at latitude 47 degrees the swiftest and strongest horses are produced, which are by no means produced in other regions of the same latitude. Finally, in Mauretania almost innumerable apes are generated. And many other experiments of this kind could be adduced — from vines, trees, fruits, and so on — all of which varied effects seem to be produced by a heaven that is simply at rest. I know that the philosophers reply that this diversity of effects in the same clime depends entirely on the varied disposition of the earth; but the aforesaid authors press their point: since the earth is variously disposed by the various aspects of the higher bodies, no sufficient cause can be given why in the same clime the disposition should not be the same, seeing that all parts of the same clime have successively the same aspects with respect to the movable heavens. But in truth, whatever is said on this matter, this much must be certain: that the Empyrean heaven, which is immovable, cannot be denied without great rashness — a heaven that the school of theologians now commonly admits.

[Margin: Twelve heavens according to the astronomers of this age.]

The astronomers of this age, therefore, establish that in all there are twelve heavens — eleven indeed movable, but one, according to the view of the theologians, entirely immovable. The reason why we admit eleven movable heavens will be clear when we treat by what diligence the eleven distinct motions were discovered by them. But first let us proceed to explaining the motions of the heavens, according to the view of those who allow only ten movable heavens.

DE MOTIBVS ORBIVM COELESTIVM. [On the Motions of the Celestial Spheres.]

[Margin: The opinion of those who removed all motion from the celestial sphere, and its refutation.]

The authors who believe that there is only a single heaven exclude all motion from the celestial sphere — though not all in the same way. For some assert that no celestial body moves, but that it always remains in the same place, and that the stars only appear to us to move from East to West (they cannot by any reasoning deny this daily motion, at least as an appearance, since every day we see the Sun and the other stars rise and set) on account of the motion of the earth, which, as they say, it has from West to East. For just as, to someone carried along the course of a ship on some swift river, the trees, houses, and everything set on the riverbank seem to come to meet him — as though he himself stood entirely still and everything else moved — so too it happens to us who exist on the earth. For since the earth moves with us from setting to rising with a most rapid motion, we seem to be at rest, and the stars seem to move in the same direction — namely from rising to setting — whereas in fact they are entirely immovable and it is we who move, as has been said. But this opinion is of no weight whatever, and is utterly ridiculous. For if it were true, the same position, order, and distance would perpetually be seen among the stars — which is contrary to all experience. For the planets continually vary among themselves their position, order, and distance, as is clearer than daylight in the case of the Sun and Moon, since these two planets are sometimes as it were conjoined and sometimes opposite across the diameter; and take it as commonly known that the same holds of the other planets.

[Margin: The opinion of those who say that the heaven is at rest and the stars move by themselves.]

But some assert that not only the heaven but also the earth is at rest, and that the stars move by themselves, like birds in the air or fish in the sea, from East to West. But since on this account the planets could not be carried by two motions

— which conflicts with experience, since we see the planets carried not only from rising to setting but also from setting to rising — for this reason others affirm that the heaven moves from East to West, carrying the stars around with it, while the individual stars also have their individual motions from West to East. For which reason, they say, it comes about that all the stars appear to complete the daily motion at the same time, while we detect that they move from setting to rising in unequal times. But neither is this opinion to be admitted, since — as we shall demonstrate in what follows — it is impossible for the stars to move by themselves, if the things that appear in their motions are true; rather, it is necessary that they be carried around only by the motion of the sphere in which they are.

[Margin: First opinion on the motions of the heavens, of those who posit eight heavens.]

Nor, in fact, do all who hold that there are several heavens agree about the motions of the celestial bodies. To begin with those who believe there are only eight heavens: some hold that each celestial orb is driven by its own motion from west to east. They cannot deny that the motions of the seven planets are distinct — both among themselves and when compared with the fixed stars — since the planets sometimes come into conjunction with one another and with the fixed stars, and sometimes part from them again. But the daily motion of the heavens from east to west they abolish entirely. It cannot happen, they say, that one and the same body should be moved at the same time by contrary and opposite motions, such as the motion from east to west and the motion from west to east. Yet since they saw that this conflicts with experience and the senses — for we see the Sun, the Moon, and the other stars slip daily from east to west, now rising above the horizon, now sinking below it — they devised the following: that the heavens with their stars appear to us to move from rising to setting because the earth is carried around with us from west to east by a motion swifter than the planets', namely in the span of twenty-four hours. Hence we suppose ourselves to be at rest while the stars advance to meet us, just as the authors of the first opinion maintained.

[Margin: Refutation of the first opinion.]

But we must not judge the celestial motions in this way either, because on this account not all the motions observed so far can be defended, as will become clear later. Moreover, the earth is by no means carried from west to east with such great speed, as will also be shown in what follows. Add to this that this opinion

assumes the motion of the heavens from east to west to be contrary to the motion from west to east — which will shortly be shown to be false.

[Margin: Second opinion on the motions of the heavens, according to those who allow eight heavens.]

Others, likewise believing the said two motions to be contrary to each other, assert that the heavens are moved only by the daily motion from east to west. Indeed, by this motion they say that not only the celestial orbs but all the elements as well are moved — a motion produced by a single intelligence, which they call the soul of the world [*anima mundi*]. Yet in such a way that the nearer any sphere lies to the soul of the world, the more swiftly it is moved by it, and the more remote it is, the more slowly — just as we observe in the motion of some wheel: the parts nearer the axle of the wheel, as to its center, are moved more slowly, while the parts closer to its circumference are carried more swiftly. Hence they say that the highest heaven is moved most swiftly of all, since it is nearest the soul of the world, but the earth most slowly — so slowly that its motion is not perceived, on account of its extreme slowness, because it has withdrawn farthest from the soul of the world. For this reason it appears to everyone to be at rest, though in fact it is gradually and, as it were, imperceptibly carried from east to west. This they try to establish by the following evidence. We see, they say, that on the western side the land continually and gradually tends to sink beneath the sea, while on the eastern side, on the contrary, it emerges more and more from the sea. This, they claim, is clearly demonstrated to us by the Pillars of Hercules set on the shore of the Western Ocean, and the pillars of the same set on the shore of the Eastern Ocean: for the former are now found many miles within the sea toward the western regions, while the latter, by contrast, are seen the same number of miles out of the sea in the eastern regions. It is therefore a clear sign, they say, that the earth is gradually being carried from east to west by that soul of the world.

Now since, besides this daily motion, the planets also seem to be moved from west to east — because they are not always at the same distance from one another, nor always situated beneath the same fixed stars, but recede eastward from them (which, however, these thinkers deny) — they assert that this is the reason why some heavens are believed to be carried from west to east, although in reality they are driven only from east to west. The cause is that the lower spheres, the farther they are from the highest heaven and from that soul of the world, are moved (as has been said) the less effectively, and for this reason are carried around more slowly and seem step by step to fall back from west to east.

Hence it also follows that the Moon, because among the celestial orbs it recedes farthest from the highest, is moved most slowly from east to west, and seems most swiftly — namely in the span of a single month — to complete a whole circuit from west to east. The remaining spheres, the higher they are, seem to us likewise the more slowly carried from setting to rising.

All of this they wish to set before our eyes by this single example. Let there be three parallel ranks of men arranged in straight lines. Let them set out together from the same place, from east to west, but under this rule: that those in the first rank advance at the swiftest pace, those in the second more slowly, and those in the third slowest of all. On this arrangement it is clear that the first rank, moving at that quickened pace, outstrips the other two — but the third rank more than the second. Hence, if someone were to watch these ranks from a distance, he would judge that the second and third ranks were gradually falling back, and the third at a quicker rate than the second, even though in reality they were all advancing, like the first rank, only from east toward west. For exactly the same reason, they say, the planets appear to us to be moved from west to east.

Alpetragius [al-Biṭrūjī] and Achillinus [Alessandro Achillini], along with other authorities, embrace this opinion the more readily because they can in no way imagine that one and the same celestial body is carried by two motions — namely from east toward west, and again from west toward east. For since these motions, as they say, are contrary, one of them must necessarily be violent, which cannot be; indeed it seems absurd to grant violence in the celestial bodies. First, because nothing violent is perpetual, whereas the motion of the heaven is perpetual, on Aristotle's view; and second, because everything violent is continually weakened more and more, whereas the motion of the heaven is always accomplished at the same speed without any failing. There is this further point, they say: a plurality of motions is not to be posited without necessity. Since, then, no necessity compels us to admit that the planets are moved from west to east — given that, for the reason already stated, they merely appear to us so moved — this plurality of motions would be introduced by the astronomers rashly and without warrant.

But this opinion cannot be true in any way, since it cannot account for everything that appears in the celestial motions. For if the lower orbs had no motions of their own from west to east, but were thought to be moved from setting to rising only because of that sort of falling-back or retardation, then the falling-back of the lower orbs would occur along the same line and about the same poles — namely along the equator [circulus Æquinoctialis] and about the poles of the world — since the daily motion runs straight along the equator and over the poles of the

world, from east to west. From this it would have to follow that all the stars and planets, by the daily motion, would always describe the same parallel circles on the near and far sides of the equator; that the stars and planets lying on the equator would never depart from it, but would remain forever beneath it; and that those on the near or far side of the equator would never approach it or recede from it more or less. Consequently neither the Sun, nor the Moon, nor any other star, fixed or wandering, would ever come nearer to the zenith of our heads, or withdraw farther from it, at one time than at another —

— all of which most plainly conflicts with sense and experience. For we see the Sun (to pass over the other planets and the stars in silence) describe various circles parallel to the equator itself, as the author will explain in chapter 3, and not always keep the same distance from the equator, since twice in the year it is found upon it, and deflects from it now toward the south, now toward the north. Hence it comes about that it is seen to rise and set at different points of the horizon over the course of the year. Hence too it comes about that in summer, standing at the beginning of Cancer, it approaches very close to our zenith, or vertical point, while in winter, placed at the beginning of Capricorn, it recedes farthest from the zenith. And it is truly astonishing: if all the heavens are moved only from east to west, while the lower ones — because they move more slowly — fall back in a way, or are retarded (as they claim), then no fixed proportion is to be seen in this retardation. For the eighth sphere completes its circuit in the span of 36,000 years; Saturn in 30 years; Jupiter in 12; Mars in 2; the Sun in one year; Venus and Mercury in about the same time; and the Moon, finally, in 27 days and 8 hours. Here you plainly see that no fixed proportion is found. It is therefore not credible that the planets lack motions of their own from west to east, and that they only seem to us to be moved from west to east because of that retardation.

Therefore, to Alpetragius and Achillinus's first argument the reply must be that those motions are not contrary, as will be shown below, and for that reason neither is violent. Add further that it does not follow [?] — even if we were to grant that one of them is in some way violent — that it would fail to be perpetual and could be weakened, since its moving cause is perpetual and untiring. For that violent motion alone is said to be unable to be perpetual which has an exhaustible and non-perpetual cause; and this is what is called violent simply and in itself. To the second argument the reply must be that a plurality of motions is most necessary for rendering the cause of all those appearances we have mentioned, and of many others of the same kind, which they themselves cannot account for. Finally, to what they allege about the motion of the earth, we reply that it is false that the earth moves; nor in this matter should we credit the tales

about the Pillars of Hercules. If at some time there was land where the sea now is, and the reverse, this in no way arises from a motion of the earth from east to west, even if it did move. For since earth and water form a single globe, as we shall show later, who does not see that the earth and the water would be moved and swept around by the prime mover at the same time together? And if they should say that the sea does not form a single globe with the earth, but that the water is higher — as many have supposed — then it would rather follow that the land would tend beneath the sea on the eastern side, since the water would continually cover it, and would emerge from the sea on the western side, since the water would abandon it: seeing that, on their view, the higher bodies, which are nearer the soul of the world, are moved more swiftly from rising to setting. We therefore give, with Aristotle in *Meteorologica* I, this as the cause of the matter: that, on account of the influences [aspectus] of the higher bodies, the sea consumes the land in certain parts through the increase of the waters, and so, where formerly there was land, there is now sea; and in the same way, because in other parts the sea decreases, there now appears land where before was sea. An indication of this can be that this exchange of sea for land, and of land for sea, is found to have occurred not only from east to west — which would follow on their view — but also in the north and the south and the other parts of the world.

[Margin: Third opinion on the motions of the heavens, of those who posit only eight heavens.]

Others — such as Augustinus Ricius [Agostino Ricci], whom Oronsus [Oronce Finé?] and some others follow — seeing that on this reasoning the appearances and the [Greek: φαίνόμενα (phainomena, "the appearances," observed phenomena)] can in no way be defended, and wishing to remain content with the number eight for the orbs, said that the whole aggregate of the eight orbs has one common motion from east to west, in such a way that this motion belongs to no particular orb as to a single thing, but to all of them taken together: just as the forward motion of an animal does not belong to this or that particular limb, but to the whole animal. And this is what is usually called the daily motion. But besides this motion common to the whole aggregate, each single orb, they say, has in addition its own peculiar motion from west to east, which the proper intelligence assisting each orb produces. Nor should this seem strange, as they assert, since even in animals we see that individual limbs can have a motion contrary to the forward motion of the whole animal. For it can happen that the whole animal advances from east toward west, and nevertheless the hand, or the head, or some other limb is meanwhile moved at the same time in the opposite direction — say, from west toward east.

And if you object that on this reasoning no prime mover [primum mobile] can be assigned, since the eighth sphere too is turned from setting to rising — which nevertheless the whole company of philosophers and astronomers admits by unanimous consent — Augustinus Ricius replies that the prime mover can be understood in two senses. In one way, so as to signify that body which is first turned by the first mover through itself; and in this sense no particular celestial sphere can be called the prime mover, since none is moved through itself first by the first mover, but only as a part along with the motion of the whole. In another way, the prime mover can be taken for that body which, among the other movable bodies, is called first in nobility and rank; and in this sense the eighth sphere, even though it is carried around from setting to rising, can be called the prime mover, because it is nearer and closer to the intelligences, or substances free of body.

[Margin: Refutation of the third opinion.]

But although this opinion seems at first sight ingenious enough and plausible, nevertheless, if we are willing to consider the matter more carefully, we shall find that it cannot be true. First, because it is impossible that the whole aggregate should be moved from rising to setting by a single intelligence, while the individual heavens in turn, none excepted, are carried in the contrary direction by their own intelligences. For in this way the whole aggregate would be moved both from rising to setting and from setting to rising at the same time — which can in no way happen, as is clear from the example adduced by the authors of this opinion. For although, if an animal advances from rising to setting by its own forward motion, the hand or some other limb can be moved in the contrary direction from setting to rising, nevertheless it seems repugnant to nature that all the parts of the animal together, none omitted, should [move] with this contrary motion about the poles. For thus the whole animal would proceed in contrary and opposite directions at the same time, which can by no means happen, nor even be grasped in thought. Second: if the whole aggregate of the heavens were carried from east to west, and then the individual orbs by their own peculiar motions from west to east, in such a way that no orb should drag another along by its motion (for it is chiefly on this account that they refuse to admit above the firmament another heaven that, as prime mover, would sweep the lower orbs along with its motion from rising to setting), then one and the same orb could not have more than two motions: one, namely, insofar as it is part of the whole aggregate; the other proper and peculiar to itself. But this is false, for in the celestial bodies more motions than two are detected.

For the total heaven of the Moon (setting aside the partial orbs) is moved from rising to setting, and from setting to rising, as experience teaches and as they themselves also admit. Again, besides these two motions, it is moved by yet another, different motion from east to west over the poles of the zodiac, as is established from the Theory of the Moon [Theorica Lunæ]; and this motion they can in no way uphold unless they concede a motion of rapture [motus raptus], as will soon be explained. For once this is granted, the heaven of the Moon will be moved: from east to west by the daily motion, over the poles of the world, according to the motion of the prime mover; from west to east, over the poles of the zodiac, according to the motion of the ninth sphere; and finally from east to west, over the poles of the zodiac as well, by its own third motion. Third: if the whole aggregate is moved from rising to setting, and not the individual heavens — because, they say, we see this motion to be common to all the heavens — I do not see why they should not, by the same reasoning, assert that all eight heavens, as one whole, are carried around from setting to rising by a single intelligence, seeing that all eight total heavens are carried at the same time and at the same speed from west to east. (For the diversity of the planets' motion that we observe does not come from the total heavens, but from the particular eccentric orbs in which the planets, or their epicycles, are fixed.) Indeed, they move with far greater uniformity and evenness from west to east than from rising to setting — which, however, they are by no means willing to admit. This opinion too, then, must be abandoned as impossible and unable to account for all the phenomena.

[Margin: The truer opinion on the motions of the heavens.]

Therefore we must speak of the motions of the heavens differently, in accord with the more learned astronomers. We say, then, that two principal motions, and those very well known, are observed in the heavens: one, namely, from east to west, the other from west to east. (Of that motion of advance and retreat [accessus et recessus — trepidation] which was observed in the eighth sphere we now say nothing, since it is not easily, and scarcely even by the most skilled, detected; but we shall set it out a little later, when we assign the periods of all the motions.) Of these, the former is proper and peculiar to the prime mover, or tenth sphere; hence it is usually called the first motion. For the tenth sphere, or prime mover, is moved by an utterly simple and most regular motion only, from east through the meridian to the west, and then again through midnight to the east. This motion is accomplished over the poles of the world and along the equator in a natural day — that is, in the span of 24 hours — once around the earth; for which reason it has commonly come to be called the daily motion. By this motion the prime mover, or tenth sphere, sweeps all the other nine lower spheres along

with it from east to west without any resistance, once around the earth each day. This motion is said to belong to these lower spheres by accident and not through themselves, since it is not their own but comes to them from outside; for they are moved by the rapture, or motion, of the prime mover — just like those who, sitting in a ship or a carriage, are swept and carried along by the motion of the ship or carriage. And if they were not carried around by the prime mover, they would in no way be moved from east to west — just as those sitting in a ship or carriage, if the ship or carriage were not moved, would not be carried along, but would remain motionless.

The latter motion is proper to the nine lower spheres, and in no way belongs to the tenth sphere, or prime mover. For all the lower spheres seem to resist that first motion in a way, by their own motions from west to east; so that, even though they are swept from rising to setting, they still continually slip from west, through the meridian, to the east, and then again through midnight to the west. This motion is accomplished over the poles of the zodiac, which are distant from the poles of the world — according to the observation of more recent astronomers — 23 degrees and 30 minutes, and along the zodiac circle. This motion is said to belong to the lower spheres through themselves, and not by accident. Just as, if someone carried in a ship from east to west were to walk by his own forward motion from west to east, then without doubt — though he would be moved much more swiftly westward by the ship than eastward by his own forward motion — he would still be said to tend westward by accident, through the motion of the ship, because he is carried by an external motion, but eastward through himself, because he moves by his own motion (by which he would be moved even if the ship stood still): so likewise this motion from west to east of the lower spheres is said to belong to them through themselves, because, even if they were in no way swept along by the prime mover, they would still tend eastward from west by this motion.

[Margin: The two motions of the heavens — from rising to setting, and from setting to rising — are not contrary. A twofold zodiac.]

Now since it seems impossible that one and the same heaven should be moved, at one and the same time, from east to west and from west to east — because east and west are opposite and contrary termini — some reply that this is no difficulty, because these two contrary motions occur over different poles and along different lines. For they are moved from east to west over the poles of the world, namely the Arctic and the Antarctic, and along the equator; but from west to east they are moved over other poles, namely the poles of the zodiac, and along

the zodiac circle. But this reply is unsatisfactory, because in it one and the same movable body is granted to be carried by contrary motions along different paths — which is utterly impossible. For if anything were moved from east to west, it cannot happen that at the same time it be moved from west to east. For on this reasoning it would approach the west and recede from it at the same time — which can happen neither along the same line nor along different lines, since these two are utterly at odds with each other. Therefore it must be said that the said two motions are in no way contrary to each other. For all the lower heavens, which are moved by the rapture of the prime mover — although they are carried by accident and against their own nature from rising to setting, namely by an external motion, yet through themselves are carried from setting to rising, that is, by their own motion and according to their own nature — are nonetheless, simply speaking, moved from east to west; and none simply from west to east, but only in a qualified sense, because they are moved toward the eastern signs, as will soon be explained. To understand this, note that there are two zodiacs, chiefly to be conceived in the celestial bodies. One, indeed, is in the prime mover, or tenth heaven, which alone is the true and proper zodiac that the astronomers mean when they speak of the zodiac without qualification: it consists of twelve equal parts, called the celestial signs, in this order — Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius, Pisces — which the astronomers usually express by these characters:

Aries	Taurus	Gemini	Cancer	Leo	Virgo
♈	♉	♊	♋	♌	♍

Libra	Scorpius	Sagittarius	Capricornus	Aquarius	Pisces
♎	♏	♐	♑	♒	♓

These characters signify, in the same order, the twelve signs already named. They must therefore be carefully noted and committed to memory, since astronomers use them very frequently and they will be introduced very often in what follows. Now any two of them — an upper one, that is, and a lower one — are set opposite each other across a diameter in the heavens, and this too is worth some notice, for mention of opposite signs will not seldom be made.

The other zodiac is to be conceived in the ninth sphere, set directly beneath the first zodiac, with the same twelve signs. That first zodiac is called by astronomers immobile and fixed — not because it is not moved by the motion of its own sphere, in which it lies, but because its signs always keep the same relation to the

equinoctial and to the colures of the *primum mobile*, so that the beginning of Aries always lies on the equinoctial circle, and likewise the beginning of Libra, precisely where the equinoctial colure cuts the equinoctial. Again, the beginning of Cancer is always found on the solstitial colure, and likewise the beginning of Capricorn. The same must be said, in due proportion, of the remaining signs and of the principal points of that zodiac.

The second zodiac is called mobile and not fixed — not merely on the ground that it is moved by the motion of its own sphere, in which it lies (for this it has in common with that first zodiac, which is nevertheless called immobile), but because its signs do not always keep the same relation to the equinoctial and to the colures of the *primum mobile*. For the beginnings of Aries and Libra of this zodiac are not always found on the equinoctial circle — that is, on the equinoctial colure of the *primum mobile* — nor the beginnings of Cancer and Capricorn on the solstitial colure. For this latter zodiac moves little by little beneath that former one, toward the eastern signs of the former zodiac — that is, toward those signs which rise and climb above the horizon later.

[Margin: In what sense the ninth zodiac, by the motion of its own sphere, is understood to move from west to east.]

Suppose, for example, that at this moment of time the beginning of Aries of the ninth heaven were exactly and directly set beneath the beginning of Aries of the *primum mobile*. Immediately afterward it would pass beneath the sign Taurus of the *primum mobile*; and once it had come precisely and exactly beneath Taurus, it would at once pass beneath Gemini, and so on it would gradually enter one sign after another — the signs that rise later — until once more it were set directly beneath Aries of the *primum mobile*. Yet in this respect the zodiac of the ninth heaven is moved simply, by the motion of the *primum mobile*, from east to west, because there is never an instant of time at which it is not receding farther from the east and approaching the west — as is plainly detected in any star whatever. It is not moved simply from west to east, since it never recedes farther from the west or approaches the east; rather the contrary appears, for we perpetually see the Sun, the Moon, and the other stars tending from their rising to their setting. It is, however, said in a certain respect to be moved somehow from west to east; for although it never leaves the west nor approaches the east, it does nonetheless approach the eastern signs, as has been said. Exactly the same must be said of the other spheres — of the eighth heaven and of the seven spheres of the planets.

[Margin: The lower heavens move simply from east to west, but in a certain respect from west to east.]

For although they are continually dragged along by the *primum mobile* from east to west, they nonetheless move gradually beneath the zodiac of the *primum mobile*, seeking the eastern signs — those, that is, which rise and set later. For instance, when the Sun has passed through the whole sign Aries of the *primum mobile*, it soon begins to move on from Aries beneath the sign Taurus, and so on, until once more it comes beneath Aries.

In this way, then, it is true both that all the heavens move simply from east to west — because there is no instant of time at which any point taken in them is not always receding farther from the east and approaching the west — and that, conversely, all the spheres below the *primum mobile* move from west to east in a certain respect: that is, not because they recede from the west toward the east, but only because they come to be found beneath other, more eastern signs, as we perceive plainly by the senses and by instruments. Now for something to be moved simply from one place to another, it must leave the one and approach the other. Since, then, we never see the Sun or the other stars leave the west and approach the east, we cannot say that the heavens move simply from west to east, but only in a certain respect — namely toward the eastern signs, as we have just explained. They are said to move simply from east to west because there is no instant of time at which they are not receding farther from the east and drawing nearer the west, on account of that most rapid motion of the *primum mobile* by which they are swept along. For if they were not swept along by the *primum mobile*, they would then be moved simply from west to east, since there would be no instant at which they were not withdrawing farther from the west and approaching the east. Likewise, if by their own motions they moved from west to east faster than they are moved from east to west by the motion of the *primum mobile*, they would again be carried simply from west to east, and only in a certain respect from east to west, for the reason just given: because on this reckoning they would always be removed farther and farther from the west and approach the east, and not the reverse.

[Margin: Examples clarifying the motion of the heavens from east to west simply, and from west to east in a certain respect.]

That all this can happen you will learn thoroughly from one or two examples. Let some ship move from east to west with the greatest speed; and let the shipmaster, at the same time, walk at a very slow pace along the ship from prow to stern. This granted, do you not see that the shipmaster is indeed moved simply from east to west, because by the ship's motion he is carried far faster than he moves the opposite way by his own motion, and for that reason is always receding farther

from the east and approaching the west — while at the same time he is moved in a certain respect toward the east, that is, toward the eastern parts of the ship, but not simply? And do you not also see that, if the ship stood still, the shipmaster would then be moved simply from west to east, since he would always be drawing nearer the east and receding from the west? And do you not, finally, observe the same thing if the shipmaster were to walk with a swifter motion than the ship? Just so, then, must it be understood that the lower heavens move beneath the zodiac of the *primum mobile* from west to east. Perhaps the matter will be grasped more clearly in the case of an ant, which walks at a slow pace against the very swift motion of some wheel that is moved from east to west. The same may be understood in some little transparent glass sphere. For if it is filled with clear water, and you so shake it toward yourself that the water gradually moves toward you, and then that glass sphere is quickly whirled around in the opposite direction, you will presently see the water contained in the glass moved along together with the motion of the little sphere, and at the same time moved toward you by its counter-thrust. By that little transparent glass sphere, then, we must picture in our minds the *primum mobile*, and by the water contained in it, the lower spheres straining against the *primum mobile*. This can also be seen in a basin, if it is filled with water.

From this explanation, and from the examples brought forward, it is left clear that the two aforesaid motions of the heavens — one of which is from east to west, the other from west to east — are not contrary, since they are not directed simply toward contrary termini, namely toward east and toward west, as we have explained. For contrary motions must be referred to one and the same fixed point, such that by the one motion that point is approached and by the other it is receded from — which we have said in no way happens in the motions of the heavens. Yet these two motions are, in the common way of speaking, called contrary and opposite, by reason of their contrary termini, namely east and west. For they are moved simply toward one of these, namely toward the west, but in a certain respect toward the other, namely toward the east — that is, toward the eastern parts, as has been said.

[Margin: Why the motion from east to west and that from west to east are not contrary, and yet are commonly called contrary. That the heavens could move about the same poles from west to east — the very poles about which they move from east to west — and why they do not now move so.]

From these same examples it is likewise plain that the heavens could be moved not only about different poles and along a different path, as in fact they are

moved, but that they could also have returned about the very same poles, and along the same path, from west to east, by which they revolve from east to west. Indeed, astronomers have learned from experience that one and the same celestial body is moved from east to west, and also, about the same poles, from west to east. For the sphere carrying the head and tail of the Dragon [the lunar nodes] in the lunar heaven is moved by its own motion (besides the diurnal motion, which occurs about the poles of the world) from east to west about the poles of the zodiac; and about those same poles it is carried, by the power of the heaven of Mercury, from west to east, as is set out in the *Theorics of the Planets*. But the reason why the lower spheres revolve by another path — namely along the zodiacal circle, and not along the same one, that is, the equinoctial circle; in other words, why they revolve about other poles, namely those of the zodiac, and not about the same ones, namely the poles of the world (which nonetheless could very well have been the case), from west to east, in the sense already explained — is, according to the philosophers, the governance of the world: namely so that, through the approach of the Sun and planets beneath the zodiac toward the north (Boreas) and toward the south (Auster, or the meridian), the various seasons of the year may come about, adapted to the various generations of things, as Aristotle says in book 2 of *On Generation and Corruption*.

ON THE PERIODS OF THE CELESTIAL MOTIONS

The tenth heaven, which is also called the *primum mobile*, completes its circuit from east to west with a uniform and regular motion — and that a most swift one — about the poles of the world and along the equinoctial circle, as has been said, in 24 equal hours, which are called equinoctial hours: that is, in the space of one natural day. Hence its motion has been called the diurnal motion. By the impetus of this motion all the lower spheres — indeed the whole sphere of fire, a great part of the air, and, by the opinion of some, a good part of the ocean — are swept from east to west. From this it comes about that, by this diurnal motion, the Sun and all the other heavenly bodies, and every single point of the heavens, daily describe about the poles of the world parallel circles at right angles to the axis of the world: larger circles the farther they lie from the poles, and smaller the nearer they approach the poles. Hence the equinoctial circle is the greatest of all the parallels, since it is described by the point most remote from either pole, namely by 90 degrees. Moreover, all the lower spheres would be carried around with exactly the same speed as the *primum mobile*, were it not that by their own peculiar motions they fall back somewhat. For the *primum mobile* meets with no resistance in the lower heavens.

[Margin: The periods [?] of the celestial motions.]

For the ninth sphere, beneath the *primum mobile*, advances in the space of 24 hours — that is, of one natural day — from west to east, according to the Alphonsine Tables, by four sixtieth-parts of those units which are called by astronomers "thirds," and by 20 "fourths"; so that in each single year it completes 26 seconds, 2 thirds, and 50 fourths, and in 200 years, 1 degree 28 minutes 9 seconds 47 thirds and 45 fourths. From this it follows that it completes its whole course through the zodiac in roughly the space of 49,000 years. For if we wish to speak precisely, in that span of years the ninth sphere completes, according to the said tables, a little more than a whole circle: it completes, namely, 360 degrees, 5 thirds, and 31 fourths. This span, or period, of 49,000 years is commonly called by many the Platonic Year. For in this interval, they suppose, all the stars will return to the same position — indeed, whatever they please: that then all things whatsoever that are in the world will return in the same order in which they are now seen. But they seem to assert this rashly; for since, according to most authorities, the motions of the heavens are mutually incommensurable, it cannot be that all the stars should ever be able to obtain the same position and order they now have, or once had. Perhaps what moved King Alphonso to assert that the period of this motion is completed in the space of 49,000 years was that he saw, in his own time, the equinoxes and solstices retreating each year in the calendar by 10 minutes 44 seconds of one hour, and in 400 years by nearly 3 days — so that in the said span of 49,000 years they would return, as it were, to their original seat. Ptolemy, however, affirms that this motion is accomplished in a circle of 36,000 years, so that the ninth sphere traverses 1 degree in 100 years. But al-Battānī [Albategnius] holds that this motion is completed in the space of 23,760 years, so that it would traverse one degree in 66 years. For what reason astronomers have judged so variously about the period of this motion will presently be explained. For now, let it be taken as settled and certain that the ninth sphere, by this very slow motion from west to east, draws along with it the 8 lower celestial spheres, but in no way the supreme sphere. For according to the opinion of astronomers, any higher sphere carries around by its motion the sphere contiguous and concentric beneath it, but not the one above it.

[Margin: The Platonic Year. Each sphere by its own motion moves the one contiguous and concentric beneath it, not the one above it.]

The eighth sphere, besides these two aforesaid motions impressed upon it by other spheres, has still a peculiar and proper motion of its own, which they call the motion of access and recess, or the motion of trepidation, as we said above.

This motion takes place about the beginnings of Aries and Libra of the ninth sphere as its poles. For the beginnings of Aries and Libra of the eighth sphere describe about the beginnings of Aries and Libra of the ninth sphere certain small circles whose radii contain 9 degrees. For by so much — 9 degrees — do the beginnings of Aries and Libra of the eighth sphere stand apart from the beginnings of Aries and Libra of the ninth sphere, according to the teaching of King Alphonso. From this motion of the beginnings of Aries and Libra of the eighth sphere about the beginnings of Aries and Libra of the ninth sphere, it follows that no other point of the eighth heaven completes a perfect circle, but rather in a manner totters — that is, now approaches the arctic pole and is removed from the antarctic, now again departs from the arctic pole and approaches the antarctic. The period of this motion embraces a span of 7,000 years, so that if those small circles be divided into 360 degrees, in about 20 years roughly one degree is completed. By this motion too the spheres of all the planets are moved, since they are concentric with the eighth sphere. But to confess the truth — although, on account of the phe-

[Margin: The motion of trepidation.]

nomena or appearances that we shall bring forward a little later, some motion of this kind, or something similar, seems necessarily to be granted in the eighth sphere — it is nonetheless very uncertain that it comes about just as the Alphonsines teach. For many absurdities seem to follow from it, as we shall presently show.

The globe of Saturn, besides the three motions mentioned, has a proper motion which it completes from west to east in about 30 years. For each single day it traverses in the zodiac about 2 minutes and 35 thirds [as printed].

Jupiter completes its circuit in about 12 years. For each day it passes through 4 minutes 59 seconds 15 thirds.

Mars accomplishes its motion from west to east in about 2 years. For it traverses in the zodiac on any given day 31 minutes 26 seconds 38 thirds.

The Sun completes its journey from west to east in 365 days, 5 hours, 49 minutes, 16 seconds. This span is commonly called the solar year. From this it is plain that the year does not precisely contain 365 days and 6 hours, as is supposed in the Roman calendar. For each single day the Sun completes 59 minutes 8 seconds 19 thirds 37 fourths. Understand this to be said according to the teaching of the Alphonsines. For Ptolemy found the quantity of the year greater, and al-Battānī smaller; while Copernicus in turn found the year in his own day nearly equal to

the Ptolemaic year — so that it is now accepted by all astronomers that the magnitude of the year is unequal. This matter will be discussed more fully in another place.

Venus completes its whole circuit in about the same time as the Sun. For it advances on any given day about 59 minutes 8 seconds and 19 thirds.

Mercury completes very nearly as much every day. For which reason it accomplishes its whole course in about the same time as Venus.

The Moon, lastly, traverses the whole zodiac in 27 days and about 8 hours. Thereafter it uses up roughly two days more before it catches up with the Sun. For since the Sun, in the meantime, in 27 days and 8 hours, traverses about 27 degrees — which the Moon completes in about two days — it necessarily follows that from one conjunction of the Moon with the Sun to the next, 29 days and about 12 hours intervene. Such a span is customarily called the lunar month. But all these things are explained more accurately and precisely in the *Theorics of the Planets*.

[Margin: In which spheres the periods of the planets' motions are to be understood.]

Furthermore, the periods of the planets' motions are to be understood not of the spheres, or total heavens, but of the proper spheres carrying the planets — which are eccentric, set in the midst of the heavens. For it is in these that the planets, or their epicycles, are fixed and carried along in the aforesaid times. For the total heavens of the planets are moved from west to east with exactly the same slowness with which the ninth heaven is moved. Again, they are moved by the motion of trepidation, following the motion of the eighth sphere. Yet no lower planet is moved by the proper motion of a higher planet, because they are not carried by their proper courses about the same center — as is usually explained more fully in the *Theorics of the Planets*.

Nor should this be passed over: that these nine spheres below the *primum mobile* would complete their courses in exactly the same times as they now complete them, and no faster, even if the *primum mobile* were at rest, or did not sweep them along with it from east to west — just as is clear in the case of the shipmaster, who by his proper motion moves against the motion of the ship, or of the ant, which is carried against the impetus of the wheel. Only then they would be carried simply from west to east, because no instant would then be given after another at which they were not receding farther from the west and approaching the east — just as that shipmaster, with the ship standing still, would in the same

time reach the stern, and would approach the east simply, and not merely the eastern parts of the ship.

HOW IT HAS BEEN DETERMINED THAT ALL THE HEAVENS MOVE SIMPLY FROM EAST TO WEST

Having set out the three motions of the heavens in general — of which we said one is from east to west simply, another from west to east in a certain respect (that is, from the western signs toward the eastern signs), and the third, finally, that of access and recess, which they call the motion of trepidation — it must now be shown by what path and method astronomers have detected this simple motion in the celestial bodies. That all the heavens move from east to west they learned by daily experience. For they saw that the Sun, the Moon, and all the other stars ascend little by little from the eastern quarter and rise above the horizon, until they reach the meridian, and then again decline toward the west, until once more they are found in the east. From this consideration they concluded easily and without hesitation that the motion of all the heavens is from east to west.

[Margin: By what means the motion from east to west was detected.]

That this motion, moreover, takes place simply from the east — that is, always recedes from the east and approaches the west — they gathered by many paths. First, from the shadow of bodies. For from the rising of the Sun up to noon, all shadows cast upon the horizon continually decrease, so that at noon the shadows become smallest, whereas from noon up to the setting of the Sun they increase again — which could by no means happen unless the Sun continually slid from east to west. The same you will say of the Moon, whose shadows always decrease while it moves from its rising to the meridian, and again increase while it inclines from the meridian toward its setting. Secondly, from the altitudes of the stars, which from their rising always become greater until they reach the meridian circle, where they obtain their greatest altitudes; but from the meridian circle to their setting they perpetually undergo a decrease of those same altitudes. This is indeed a plain indication that they simply depart from the east and approach the west.

By what reasoning it has been concluded that the motion of the heavens is from west to east

[Section heading]

Although all the heavens are carried, without qualification [*simpliciter*], from east to west — as we showed a little earlier — it has nevertheless been found that they are also driven the other way, from west to east: not without qualification (for without qualification they move only from east to west, as has already been shown), but in a qualified sense [*secundum quid*], namely by making their way toward the eastern signs, in the sense set out above. Astronomers first detected this in the seven planets, as may be gathered from Regiomontanus [Regiomontanus] in his *Epitome of Ptolemy's Almagest* (Book I, conclusion 6), by the following reasoning. They observed that the Sun, the Moon, and the other planets do not always keep the same position and distance among themselves: the Moon, for example, is on one day in conjunction with the Sun, but on another has withdrawn from it toward the eastern regions. And they found this variation not only in one planet with respect to another, but also in all the planets with respect to the fixed stars. For they saw a given planet on one day in conjunction with a particular fixed star, or standing at a particular degree of some sign, but on another day departed from that star or degree toward the eastern regions — just as we too, clearer than daylight, experience daily. There can therefore be no reason to doubt that the seven planetary orbs, besides the daily motion from east to west, also move gradually and, in a manner, retreat from west to east — that is, toward the eastern parts of the heaven — as we have explained.

Nor was it by a different method that astronomers found that the eighth heaven too moves from west to east. For although nearly all the ancients before Aristotle believed that the starry heaven was driven by that one motion alone, from east to west — since they saw all the fixed stars preserve the same distances among themselves, and the places of their risings and settings not varying on the same horizon, but the stars always rising and setting in the same places (this on account of the short interval of time in which they were making their observations) — yet after Aristotle it was found that the matter stands far otherwise. For, as Ptolemy [Ptolemy] says in Book VII, chapter 1 [*Almagest* VII.1], and Regiomontanus in his *Epitome* of the same book, proposition 2, the distances of the fixed stars from the solstitial and equinoctial points do not always remain the same, but grow and increase according to the succession of the signs — that is, advancing toward the eastern regions — so that very many stars which in ancient times stood before the solstitial and equinoctial points are now found

beyond those same points, while other stars have come nearer to them, as is plainly established from the observations of both the ancients and the moderns. And the greater the span of time between the observations of the ancients and those of the moderns, the more the stars are found to have moved away from their ancient seats and places according to the succession of the signs. Ptolemy and Regiomontanus, in the passages cited, bring forward many examples of this; we shall offer only one or two.

Timocharis [Timocharis], observing the course of the stars, found the star Azimech — which the Latins call Spica Virginis [the Ear of Virgo] — before the point of the autumnal equinox, that is, before the beginning of Libra, by about 8 degrees; this is a little past 22 degrees of Virgo, or at the beginning of the 23rd degree of Virgo. After him, about two hundred years having elapsed, Abrachis, also called Hipparchus [Hipparchus], found the same star only 6 degrees before that point, namely at the beginning of the 25th degree of Virgo. And after these, Ptolemy found that the same star had come still closer to the beginning of Libra, in proportion to the intervening time. The astronomers who followed him observed the same — such as al-Battānī [al-Battānī], Avenezra [Ibn Ezra?], Zacut [Abraham Zacuto], and others — so that in our own age this same star now stands beyond the beginning of Libra, namely at the 18th degree of Libra and beyond. Again, Hipparchus found the star called the Heart of the Lion [Cor Leonis, Regulus] at 50 minutes of the last degree of Cancer; but after him Ptolemy found the same standing at 30 minutes of the third degree of Leo; and now this same star stands at about the 24th degree of Leo. From these and very many other examples it is clearly gathered that all the celestial orbs below the *primum mobile*, besides the daily motion, also move according to the succession of the signs from west to east — though in a qualified sense, that is, as we have explained, toward the eastern regions. For if they were moved by the daily motion alone, all the stars and planets would necessarily remain equally distant from those four aforesaid points; but the observations of the most learned astronomers show the opposite. Nor should anyone doubt that the places of the stars were correctly determined by the astronomers named. For among the many instruments devised by craftsmen for determining the places of the stars, the most excellent is the one they call Ptolemy's Armilla [the armillary instrument], whose construction is taught in Book V of the *Almagest* [*Almagest* V].

By what means it has been observed that the lower heavens move from west to east about poles different from the poles of the world

[Section heading]

[Margin: The lower heavens move from west to east about the poles of the Zodiac; by what means this was observed.]

By long observation astronomers discovered that the lower heavens move from west to east not about the poles of the world and along the equinoctial circle, but about distinct poles — namely about the poles of the Zodiac and along the circle of the Zodiac. For all the planets always vary their points of rising and setting on the horizon, which is detected clearer than daylight in the Sun. For at one time it rises at the equinoctial point, at another beyond it, at another short of it; and this variation would not occur if the Sun moved from west to east about the poles of the world and along the equinoctial circle. In that case it would always rise at the same point of the horizon, just as the parallels of the equator — along one or another of which the Sun is necessarily carried by the daily motion — always intersect the horizon at the same points; and the same has been observed in the other planets. Again, the planets do not always keep the same distance from the poles of the world, but now approach the Arctic pole, now the Antarctic — which is easily gathered from the fact that they do not always have the same meridian altitude. The Sun is found to have its greatest meridian altitude at the Tropic of Cancer, and its least at the Tropic of Capricorn, as is plain from the meridian shadow of any gnomon, which is shortest when the Sun stays in Cancer, but longest when the same is in Capricorn. Hence too it comes about that the planets do not always describe the same parallels relative to the daily motion. By most certain reasoning, therefore, it is concluded that the planets make their way from west to east about different poles. And since astronomers noticed that this variation in the motion of the Sun and of the other planets is confined within nearly the same limits, and that they are carried around in a circle whose greatest declination from the equinoctial spans 23 and a half degrees, and whose poles are consequently distant from the poles of the world by the same number of degrees, they asserted that this motion takes place about the poles

of the Zodiac and along the circle of the Zodiac. This being posited, all the aforesaid variations very easily find their place, as may be plainly seen on some material sphere.

All this, moreover, was detected by infallible reasoning in the eighth sphere as well. For after those most diligent observers of the stars had understood that the fixed stars gradually make their way from west to east, they noticed that this motion takes place about poles distinct from the poles of the world. For the stars have not always risen in the same places in which they now rise, with respect to the same horizon; likewise the meridian altitudes of the fixed stars are different at this time from those which the ancient astronomers observed. The fixed stars, therefore, do not return from west to east about the poles of the world. Moreover, as Ptolemy (Book VII, chapter 3 [*Almagest* VII.3]) and Regiomontanus in his *Epitome* of the same book assert, and confirm by many observations, the fixed stars do not always keep an equal distance from the equinoctial circle. For their declinations from the equinoctial circle have been found to vary: so that, for those stars which are in the half of the sphere running from the beginning of Capricorn, through Aries, to the beginning of Cancer, the southern declinations have diminished while the northern have increased; and on the contrary, for those stars which are in the remaining half of the sphere, contained from the beginning of Cancer, through Libra, up to the beginning of Capricorn, the southern declinations have been found to increase while the northern diminish. (By "southern declination" we mean that a star which inclines from the equinoctial circle toward the Antarctic pole has one, while "northern declination" belongs to a star which slopes from that same circle toward the Arctic pole.) And the nearer the stars are to the beginning of Aries and of Libra of the *primum mobile*, the greater the variation of declination that has appeared; but the nearer they are to the beginning of Cancer and the beginning of Capricorn, the smaller the variation of declination they have undergone.

That this may be better understood, I shall add one or two examples from Ptolemy and Regiomontanus. The star which the Latins call the Eye of Taurus [Aldebaran] declined, in Timocharis's time, from the equinoctial toward the north by 8 degrees and a half and a little more; in the time of Abrachis, or Hipparchus, by 9 degrees 45 minutes; then in Ptolemy's time by nearly 11 degrees; and in our own time by about 16 degrees. It is therefore established that the northern declination of this star has always increased, since it lies precisely in that half of the sphere which stretches from the beginning of Capricorn, through Aries, to the beginning of Cancer. Similarly Alhabor, the star called the Great Dog [Sirius] (for this star is in the mouth of the Great Dog, and in ancient times stood in that same half of the sphere), had in Timocharis's time a southern or meridional declination of 16 degrees 20 minutes; then in the time of Abrachis, or Hipparchus, 16 degrees only; and finally in Ptolemy's time 15 degrees 35 minutes — from which it is likewise clear that the southern declination continually decreased. But in our

own age, since this same star is now found in the other half of the sphere, it again has a southern declination of about 16 degrees; and here it is manifestly seen that the southern declination is once more increasing.

Lastly (for you will find many more examples of this kind in Ptolemy and Regiomontanus): Azimech, the star called Spica of Virgo, had in Timocharis's time a northern declination of 1 degree 24 minutes; with Abraham, or Hipparchus, only 36 minutes; but with Ptolemy it had a southern declination of 0 degrees 40 minutes. Now it is found to have a southern declination of 8 degrees 57 minutes — as Erasmus Oswald Schreckenfuchs [printed: "Erasmus vater Schreckenfuchs"] relates in his *Theoricae Planetarum*, p. 407: that Johann Werner [Johann Werner], in the year 1514 at Nuremberg, on the 16th day of December, found the same declination of Spica to be 32 degrees 7 minutes, which, if it is subtracted from the altitude of the equator, 40 degrees 36 minutes 30 seconds, leaves its southern declination as 8 degrees 29 minutes 30 seconds — somewhat less than we ourselves stated. From this example it is clearly established that the northern declination of this star (since it lies in that half of the sphere which is comprehended between Cancer and Capricorn, proceeding through Aries) has always decreased, while its meridional declination has increased. In all these examples one may plainly observe that the greater variation of declinations has occurred near the equinoctial circle than near the tropics.

By a most firm demonstration, therefore, the astronomers concluded that the fixed stars, although their declinations from the equinoctial circle vary, as has been said, nevertheless always keep the same latitude — that is, the same distance from the ecliptic line which passes through the middle of the Zodiac — as is established from the observations of those same astronomers. For, to give an example, the star called Arctophylax, or Arcturus, has always been found to deviate from the ecliptic toward the north by 31 degrees 30 minutes; and the same, in a certain proportion, has been observed in all the other fixed stars. By necessary reasoning, then, it is concluded that they move precisely about the poles of the Zodiac and along the circle of the Zodiac; for on this supposition all the stars, in their motion from west to east, describe circles parallel to the Zodiac itself, and remain always equally distant from it.

[Margin: Two arguments against the motion of the fixed stars from west to east about the poles of the Zodiac, and their solution.]

I cannot at this point pass over in silence two arguments of a certain most learned and most noble man, who flourished not many years ago. By these he

strives to demonstrate, in certain writings composed for this purpose — which I read carefully enough in the congregation that was recently held at Rome by order of the Supreme Pontiff concerning the correction of the calendar — that this motion of the fixed stars from west to east about the poles of the Zodiac is entirely fictitious, and that all those phenomena by which Ptolemy and the other astronomers have tried to introduce the said motion into the schools are also fabricated. These arguments could indeed cause no little trouble to anyone versed in the knowledge of the stars. They are as follows.

Canopus, the very bright star that stands in the tiller of the Argo [Argo Navis], is not seen in Europe because it is too far south; but at Alexandria, as Pliny [Pliny] relates in the second book of his *Natural History*, chapter 70 [Natural History II.70], it projected in his time about a quarter of one sign above the horizon when it was set on the meridian circle; and on the island of Rhodes it seemed in a way to graze the earth, or the horizon. Since, then, now — as merchants report — that same star still in a manner grazes the horizon of that same island, who does not see that this star has always remained on the same parallel, and therefore has not been moved about the poles of the Zodiac? For otherwise it would have been carried along a circle parallel to the ecliptic, which obliquely cuts the parallel of the equator, and so could no longer touch that horizon.

Furthermore, the pole star, at the tip of the tail of Ursa Minor, which is 24 degrees distant from the pole of the Zodiac and stands near the Arctic pole: if it moves about the poles of the Zodiac, it must at some time come to be nearly 47 degrees and more distant from the pole of the world — in proportion, that is, to the semi-diameter of that parallel which it describes about the pole of the Zodiac, and to the distance of the pole of the world from the pole of the Zodiac; and it would therefore set on the Roman horizon, where the Arctic pole rises about 42 degrees above the horizon. Since, then, the pole star does not seem to have changed its seat with respect to the pole in so many centuries, it is not likely that it has been moved from west to east about the poles of the Zodiac. That motion, therefore, which the astronomers attribute to the fixed stars is entirely fictitious; otherwise the pole star would now be farther from the pole of the world than it once was, which seems false.

To both arguments we reply as follows. Since Canopus stands near the solstitial colure — so that in Pliny's time it was a little before it, and now is found a little after it — it comes about that the parallel of the ecliptic described by that star, in its motion from west to east, very nearly coincides in that place with the parallel of the equator drawn through the same star, as can be seen on an astronomical

globe. Hence it is no wonder that this star, in the 15 degrees which (according to Ptolemy's reckoning) it traverses from west to east from Pliny's time down to our own age, has not perceptibly changed its declination from the equator, and that it has therefore always seemed to graze the horizon of Rhodes — just as the Sun, around the solstices, in the 23 degrees which it traverses along the ecliptic (of which 11 are taken before and 11 after either solstice) barely changes its declination by half a degree. But it will come to pass that after a long time that star will alter its declination, and so will no longer touch the horizon of Rhodes — just as we see the declinations of other stars altered, because they are farther from the solstitial colure.

As for the pole star, we reply that it is situated in such a place of the heaven (as is established from the astronomical globe) that, from Hipparchus and Ptolemy down to the present, by that motion from west to east it approaches nearer and nearer to the pole. And this is what actually happens. For, as Ptolemy states in the first book of the *Geography*, chapter 7 [*Geography* I.7], the pole star in Hipparchus's time was 12 degrees 24 minutes distant from the pole, but now is only about 3 degrees and a half distant, or thereabouts. For its true distance, computed for the year 1600, is only 3 degrees 25 minutes. Thus from this change the motion of the stars from west to east is rather confirmed. In the course of time, however, this same pole star will move away from the pole. For it will be able to come at most within 30 minutes of the pole, which indeed happens, according to the Prutenic Tables [Prutenic Tables], around the year of our Lord 2282, because it will then be on the solstitial colure. Then it will again begin to recede from the pole, until it is 48 degrees distant from it, which, according to the same tables, will occur around the year of our Lord 15000. From these things I judge it clearly established that those two arguments do not prove this motion from west to east, detected in the fixed stars, to be fictitious. The experience of the astronomers, therefore, is to be trusted, until something to the contrary is brought forward by which it may be shown that what the astronomers hand down concerning the motion of the stars from west to east about the poles of the Zodiac is not true.

On account of what phenomena the astronomers attributed the motion of trepidation to the fixed stars

[Section heading]

[Margin: Why the astronomers place the motion of trepidation in the heaven.]

Since it was said above that the fixed stars move not only by those two motions — one of which is from east to west, the other from west to east — but have besides a proper motion of approach and recession, which they call trepidation, it must now be shown what phenomena or appearances compelled the astronomers to place this mode of motion in the heaven; for not a few think that this mode should be altogether hissed off from the schools of the astronomers as ridiculous. First, then, they say that the fixed stars advance unequally from west to east: for now more swiftly, now more slowly, now (as some of them say) not moving at all in the Zodiac, they seemed at one time to move back from east to west, besides that daily motion, and yet to keep the same distance from the center of the world. They therefore said that the stars move from north to south, and back again, as was declared above in that motion of approach and recession. For it is because of this motion that the whole of that inequality in the motion of the fixed stars arises, as can easily be understood from some material instrument constructed for this purpose. This too, the astronomers assert, is the reason why such varied opinions have arisen concerning the quantity, or period, of the motion of the fixed stars from west to east.

Again, they noticed that the greatest declination of the Sun stood differently at different times — now greater, now less, as we shall say in the second chapter. For this reason they were compelled to admit this motion of trepidation, so that they might render the cause of this variation in the greatest declination of the Sun. For, this motion being posited, it follows that the eighth sphere inclines now from north to south, now from south to north, and consequently that the two tropics in the solar orb become at one time nearer to the equinoctial circle, at another farther from it, as is explained in the *Theory of the Eighth Sphere*. Lastly, it was observed (as they say) that equinoxes have occurred before the Sun reached Aries of the *primum mobile*, or Libra — indeed, after it had already at some time passed the beginning of Aries or of Libra; and by the same reasoning that the solstices have taken place even though the Sun was not at the beginning of Cancer or of Capricorn. Since, then, the Sun must necessarily be found on the equinoctial circle for an equinox to occur, and likewise in the tropics for the solstices to happen, no other cause of this variation could be brought forward but the motion of trepidation; for that anticipation of the equinoxes and solstices easily follows from this motion. Moreover, by this same motion all the globes of the seven planets are also moved, so that the orbs of all the planets constantly accompany the Zodiac of the eighth sphere.

But although it seems certain that either the motion of trepidation, or something similar, must be granted in the eighth sphere on account of the phenomena

mentioned, yet the motion by which the astronomers explain it is most uncertain: namely, that the beginnings of Aries and of Libra of the eighth sphere describe circles about the beginnings of Aries and of Libra of the ninth sphere, whose semidiameters contain 9 degrees. For from this position many things follow which seem to conflict with experience, as we shall explain more fully in the following discussion, on the fourfold motion of the eighth sphere.

On the fourfold motion of the eighth sphere according to the opinion of the more recent astronomers

[Section heading]

So far we have set out the motion of the eighth sphere according to the opinion of the astronomers who follow Alfonso [the Alphonsine astronomers]. Now we shall discuss the same according to our own opinion and that of those who prefer to follow Nicolaus Copernicus [Copernicus].

To make this easier, I must briefly review the whole course that astronomers have followed in observing the motion of the fixed stars.

[Margin: The difficulty of knowing the motion of the eighth sphere.]

How obscure and hard to grasp the motion of that celestial sphere is — the sphere adorned with so great a multitude of stars, which we gaze on with deep delight as they glitter on a clear night, and which is commonly called the *Firmament*, the *starry heaven*, and the *eighth sphere* — is shown by the varied views and opinions of the greatest astronomers about it. This obscurity, or difficulty, seems to arise entirely from the slowness of its motion.

[Margin: Where the difficulty of knowing the motion of the eighth sphere arose.]

For since it is not completed except after the passage of many centuries — so much so that from the founding of the world down to our own age it has scarcely yet accomplished a fourth part — its magnitude and character cannot be reliably determined except through the observations of very many ages, compared with one another as carefully as possible. From this I would suppose it came about that all Antiquity thought the eighth heaven to be the *primum mobile*; for on account of the scarcity of observations they had noticed no motion in it beyond the daily one from east to west. It is therefore all the less surprising that the Babylonians (or Chaldeans) and the Egyptians — who inhabit flat regions and enjoy a very clear sky — handed down absolutely nothing about the advance of

the fixed stars from west to east (to say nothing here of their other phenomena), even though they devoted all their care to the knowledge of the stars.

[Margin: Ptolemy detected the motion of the eighth sphere about the poles of the zodiac.]

Hipparchus [Hipparchus] was the first (he is also called Abrachis). As we learn from Ptolemy and Pliny, about 130 years before the coming of Christ the Lord he observed the length of the year rather accurately and established the positions of the fixed stars with the greatest diligence. Comparing his own observations with those of Timocharis — who had preceded him by nearly 200 years — he discovered that the eighth sphere, with the stars fixed to it, advances from west to east by a very slow motion over and above the daily motion. This motion Ptolemy, easily the prince of astronomers, then confirmed far more clearly and evidently by many phenomena about 260 years later, and he observed that it takes place about the poles of the zodiac. For by very clear observations he found that the fixed stars [stellas inerrantes] gradually recede eastward from the equinoctial and solstitial points. To take an example: the star called Spica of Virgo was found by Timocharis to be 8 degrees before the autumnal equinox. After him, Hipparchus found the same star to be only 6 degrees from the same equinox. Later than both, Ptolemy observed that it had drawn still nearer to the same equinoctial point, in proportion to the intervening time, so that it was distant from it by only 3 and a third degrees. The same has always been observed in the following ages. For in our own time this same star is seen to have crossed that equinox and to be distant from it by eighteen degrees and more, toward the winter solstice [Brumam versus]. And although this star has been found to move continuously and gradually ever eastward, all astronomers have nonetheless noted that it has not changed its latitude — that is, its distance from the ecliptic — but has always remained 2 degrees south of it, whereas its declination — that is, its distance from the equator — has continuously varied. The same has been observed in a certain proportion for other stars as well, so that one may without any doubt conclude for certain that the eighth sphere is carried gradually from west to east about the poles of the zodiac, and not about the poles of the world (that is, of the equator).

[Margin: The period of the eighth sphere's motion according to Ptolemy.]

And because Ptolemy judged that the fixed stars — or rather the starry orb — are always carried uniformly by this motion from west to east (for in a few years the unevenness of that motion cannot be perceived), he not unreasonably concluded that the fixed stars complete only one degree beneath the primum mobile in the

span of 100 years, and that therefore the whole period of this motion is completed in an interval of 36000 years.

[Margin: The period of the eighth sphere's motion according to al-Battānī.]

Next al-Battānī [al-Battānī], most skilled in astronomical matters, around the year of our Lord 880 — that is, nearly 750 years after Ptolemy — observing the courses of the stars and comparing their positions with those Ptolemy had recorded, likewise rightly concluded that their motion, if uniform, is much swifter than Ptolemy had established: for by his reckoning one degree falls to a span of 66 years, and the whole period comprises 23760 years.

[Margin: The period of the eighth sphere's motion according to Alfonso.]

Alfonso, King of Castile [Alphonsus, King of Castile], together with the astronomers of his time, around the year of our Lord 1250 established this same motion as very slow, and held that it is completed in an interval of 49000 years, such that it traverses one degree and 28 minutes in 200 years. He also observed — as did Thebit the Arab [Thābit ibn Qurra] and other astronomers at various times — that the length of the year is uneven, now greater, now smaller, and that the maximum declination of the Sun is not always the same but has continuously decreased from Ptolemy's time down to his own.

[Margin: The motion of trepidation of the eighth sphere according to Thebit.]

For this reason Thebit, in order to account for these phenomena and for so great an unevenness in the motion of the fixed stars, devised the theory that the eighth sphere is not carried by a continuous motion from west to east, but is agitated by a certain motion of trepidation: namely, that the first points of Aries and Libra of the eighth heaven describe certain small circles around the first points of Aries and Libra of the ninth heaven (the diameters of these small circles being nearly equal to nine degrees), while all the other points of the starry orb waver and tremble in a way, moving now toward the south, now toward the north, now eastward, now westward. For on this account the fixed stars will indeed be seen to be carried gradually eastward, but by an uneven motion; and the length of the year, together with the maximum declination of the Sun, will vary, because on account of this motion the ecliptic of the eighth sphere intersects the equinoctial circle of the primum mobile at ever different points, and the angle it makes with the equator at that intersection — which measures the maximum declination of the Sun — is not always of the same magnitude, as can easily be perceived from some material instrument.

[Margin: The defect of Thebit's trepidation.]

But although this and its unevenness could in some fashion be defended by this motion, the stars could nevertheless not advance eastward by more degrees than nine — that is, by as many as we said are contained in the diameters of the small circles, and no more. This conflicts with experience, since we see that from Ptolemy's time down to our own age they have become more easterly by nearly 21 degrees.

[Margin: The motion of trepidation of the eighth sphere according to Alfonso.]

Therefore Alfonso the King, together with the astronomers of his age, held that the eighth heaven is indeed drawn eastward by the ninth sphere over a span of 49000 years, but he also laid down that it is set in motion by a trepidation-motion of its own, so that a cause could be given why the fixed stars advance from west to east with an uneven motion, why the length of the year is uneven, and why, finally, the maximum declination of the Sun turns out to be variable — just as Thebit had proposed. In Alfonso's account, however, the small circles that the first points of Aries and Libra of the eighth heaven describe around the first points of Aries and Libra of the ninth heaven have larger diameters than in Thebit's — namely, of 18 degrees.

[Margin: The three motions of the eighth sphere according to Alfonso.]

And so, on Alfonso's view, the eighth sphere has three motions: one from east to west about the poles of the world, in a span of 24 hours; another from west to east about the poles of the zodiac, over an interval of 49000 years; and finally a third, that of trepidation.

[Margin: Ten movable heavens according to Alfonso.]

From this it follows that not merely nine movable heavens are to be granted, as with Ptolemy, but altogether ten: seven, indeed, on account of the 7 planets, which are set in motion by different motions and therefore exist in 7 different orbs; and three on account of that threefold motion of the eighth sphere. For since a single simple body can have only a single simple motion, as the philosophers teach, the eighth sphere will have its daily motion from the tenth heaven — which is the primum mobile for Alfonso, and which carries all the lower orbs around with it from east to west. The ninth orb, in turn, will impart to the eighth sphere the motion from west to east, since by its very slow motion it draws all the lower spheres with it from west to east. The third motion, finally,

which we have called that of trepidation, will be reckoned as proper to the eighth sphere.

This view or opinion of Alfonso's has so prevailed down to our own age that it is taught in almost all public schools as the one theory able to give the cause and rationale of the threefold motion observed in the eighth sphere. But if we examine it a little more carefully, we shall easily see that it does not so much save the celestial phenomena as destroy them. For although it follows from it that the fixed stars advance continuously from the west by the motion of the ninth sphere, that on account of the trepidation of the eighth sphere these same stars are moved unevenly, and that the maximum declination of the Sun from the equator, and the length of the year, do not always remain the same — all of which the accurate observations of astronomers plainly teach — nevertheless many absurdities arise from it in the celestial motions, which conflict with the observations of all astronomers.

[Margin: First argument against the motion of trepidation.]

In the first place, from this motion of trepidation it follows that not all the fixed stars are carried by a similar motion, but some by a perfectly circular motion — namely those situated at the first points of Aries and Libra of the eighth heaven; some have a quasi-circular motion — namely those not far from those first points; and some, finally, are moved by an almost rectilinear motion, advancing now eastward, now back westward — namely those placed near the beginnings of Cancer and Capricorn of the eighth sphere. But this is entirely contrary to the phenomena, since all the fixed stars have been seen always to be carried by the same motion.

[Margin: Second argument against the motion of trepidation.]

Next — to say nothing of the other planets and the stars — the Sun would have at one and the same degree of the zodiac a perceptibly greater declination at one time and a smaller at another, especially near Aries and Libra of the eighth sphere, where it is driven more toward the north and the south by the motion of trepidation than elsewhere. Thus (assuming the centres of the small circles placed beneath the first points of Aries and Libra of the primum mobile) it could be 9 degrees distant from the equator on either side, and could be more southerly or more northerly by 18 degrees at one time than at another. From this it would follow that the noon altitude of the Sun, since it moves around Aries and Libra of the starry orb, would vary astonishingly within one and the same region, so that

it would not always be equal to the complement of the pole's altitude — which the daily experience of all climes contradicts.

[Margin: Third argument against the motion of trepidation.]

Again, it would sometimes happen that the intersections of the equator with the ecliptic of the eighth sphere, where the equinoxes occur, would be distant by 21 degrees and more — both eastward and westward — from the first points of Aries and Libra of the primum mobile; and consequently that the equinox could occur nearly 21 days before the Sun occupies the equinoctial points of the primum mobile, or after it has passed them. This great a distance would arise if the centres of the small circles of the trepidation-motion are conceived to be perpetually fixed to the first points of Aries and Libra of the primum mobile. But if those centres are supposed to recede continuously eastward from those first points — as the authors of this motion seem to hold, since they suppose them to be carried around by the ninth sphere's motion from west to east — then, if those centres of the small circles are supposed to lie near the solstitial points, the maximum declination of the Sun could differ by 9 degrees from that which the first points of Cancer and Capricorn of the primum mobile have. And consequently, given that maximum declination of the ecliptic of the primum mobile of 23 degrees, the maximum declination of the Sun that follows the trepidation-motion of the eighth sphere could sometimes amount to 32 degrees, and sometimes to only 14 degrees — all of which is ridiculous and unheard of. These and many other absurdities anyone may easily gather from the motion of trepidation, if he employs a material instrument. And they follow likewise from Thebit's motion of trepidation, except that the distance of the equinoctial and solstitial points of the eighth sphere from the equinoctial and solstitial points of the primum mobile is not as great as among the Alphonsines.

[Margin: Fourth argument against the motion of trepidation.]

To this may be added that the authors of this kind of trepidation-motion hand down no precepts by which the maximum declination of the Sun, the length of the year, the distance of the equinoctial points of the eighth sphere from the equinoctial points of the primum mobile, and other such things, could be computed for a given time — because they realized, of course, that a calculation derived from the trepidation-motion did not in the least agree with the phenomena and with experience. This is proof that this motion does not exist in the nature of things, but is wholly fictitious and fabricated without any foundation.

[Margin: Copernicus's view on the motion of the eighth sphere.]

Since, therefore, the motion of trepidation not only fails to answer exactly to the phenomena observed at various times, but even utterly overturns and destroys most of them, Nicolaus Copernicus of Prussia [Copernicus] — that outstanding restorer of astronomy in our own century, whom all posterity will ever celebrate and admire with grateful mind as a kind of second Ptolemy — comparing his own observations with those of all astronomers, both ancient and more recent, determined that the motion of the eighth sphere must be reasoned about differently. For on account of the phenomena discussed above, he assigned to the eighth heaven four different motions, prescribing their periods (that is, their slowness and speed), together with precepts by which both the maximum declination of the Sun and the uneven motion of the fixed stars, along with the length of the year, could be computed for a given time. Two of these motions he makes absolute and complete — namely those that describe entire circuits from east to west and from west to east — but two he makes imperfect and incomplete, since they do not accomplish full revolutions. By the one of them the sphere is agitated through a certain small span — embracing only 24 minutes of one degree — from north to south and back again from south to north, unevenly however, over a span of nearly 3432 of our years; this motion brings it about that the maximum declination of the Sun is now increased, now diminished. By the other, over a span likewise small yet a little greater than the former — that is, through only 2 and a third degrees — it is set in motion from east to west and back from west to east, again with an uneven motion, over an interval of nearly 1716 of our years. By this motion it comes about that the motion of the fixed stars from west to east — which he established as being completed in nearly 25800 years — and the length of the year appear uneven. Thus you would more properly call these latter two motions certain librations of the eighth sphere rather than motions or revolutions.

[Margin: Copernicus's absurd hypothesis.]

But just as we gladly receive and embrace this fourfold motion of the eighth sphere, together with the periods prescribed for them by Copernicus, so we entirely reject the manner in which he explains them. For in order to set before our eyes the latter two motions — or rather librations — of the eighth sphere, he assumes hypotheses that are quite discordant and absurd, remote from the common sense of mankind (not to say reckless): since he places the Sun at the centre of the world, devoid of all motion, and gives the earth a manifold motion, placing it — together with the rest of the elements and the lunar globe — in the

third heaven, between Venus and Mars. Then he speaks confusedly, and explains and expounds himself with great difficulty, so that he can scarcely be understood — for he seems to me to write things wholly contradictory about the latter two motions. For he holds that the former, by which the maximum declination of the Sun is changed, comes about through the approach and recession of the pole of the world to and from the pole of the zodiac through 24 minutes along the Colure of the Solstices; but the latter, which renders uneven the motion of the fixed stars — which he himself calls the precession of the equinoxes — is effected through the departure of that same pole of the world to either side of the Colure by so great an interval that the equator, when it is most distant from the Colure, is traced out by it so as to intersect the ecliptic at two points which are distant from the equinoctial points of the primum mobile, both eastward and westward, by 1 degree and 10 minutes. Thus, by this motion, the pole of the equator describes a kind of figure resembling a twisted garland [*intortæ corollæ*], as he himself puts it, which the Colure divides in two — such as two ellipses touching each other along their breadth would roughly present, so that their minor axes form a straight line and cut off 24 minutes from the Colure. But who does not see that these things are entirely contradictory? For if the pole creeps up and down, as it were, along the Colure, how can it be understood that at the same time it can wander outside the Colure? Or if it strays this way and that, how can the same pole at the same time be moved up and down along the Colure? For my part, I frankly confess that I have never been able to understand this contradiction perfectly.

[Margin: Eleven movable heavens, from Magini and our own view.]

For this reason Giovanni Antonio Magini [Giovanni Antonio Magini], a most learned man, prudently rejecting these hypotheses but retaining the periods of the motions that Copernicus established, endeavours to uphold and defend that fourfold motion of the eighth sphere by means of the usual hypotheses, received by all astronomers and philosophers — since he places this terrestrial globe, devoid of all motion, at the centre of the whole universe, as reason demands. But just as, according to the view of the Alphonsines and of the astronomers received down to the present day, we are compelled — on account of the three motions detected in the eighth heaven — to set up two movable heavens above the eighth orb, as was explained above, so now, in order to uphold the four motions observed in that same eighth heaven, we must place above it not merely two movable orbs but three; so that now not merely 10 movable heavens are to be granted, as with the Alphonsines, but altogether eleven — if we wish to preserve and uphold the celestial phenomena by a sure and probable method, such that

nothing absurd follows from the assumed hypotheses. For the eleventh will be the primum mobile; the tenth and the ninth, enclosed between the primum mobile and the starry sphere, will impart to the fixed stars those two imperfect motions of Copernicus — or rather librations; while the eighth heaven itself will carry the fixed stars around from west to east by its own very slow motion. In what manner this comes about I have resolved to explain a little more carefully, following Magini, yet I shall set the whole matter out somewhat more simply than Magini himself does, and — unless I am mistaken — more suitably for understanding, since I employ fewer circles and lines to explain the motions.

[Margin: The motion of the primum mobile.]

The eleventh heaven, then, which we have called the primum mobile, is carried with a most rapid and uniform course from east to west, about the poles of the world (or of the equator), in a span of 24 hours, carrying all the lower celestial orbs around with it.

[Margin: How great is the maximum declination of the ecliptic of the primum mobile, and why it is called the mean.]

In this primum mobile are to be conceived nearly all the movable circles that are usually set out in the sphere — chiefly the equator, the zodiac, the Colure of the Equinoxes, the Colure of the Solstices, the Tropic of Cancer, and the Tropic of Capricorn. The zodiac — or rather the ecliptic — has such an inclination to the equator that its poles are distant from the poles of the equator by 23 degrees and 40 minutes; and by just this interval too the two tropics are removed from the equator, and so great is the maximum declination of the ecliptic, or of the Sun. This maximum declination is usually called the *mean* [media] in the astronomical tables, just as the ecliptic, with its poles and tropics, is called the *mean*.

[Margin: The ecliptic of the primum mobile, with its poles and tropics, is called the mean.]

For the true ecliptic, which is conceived in the tenth heaven — to which, of course, the ecliptics of the other lower spheres accommodate themselves — now has a greater declination, now a smaller, and deflects from the ecliptic of the primum mobile by equal spaces both toward the north and toward the south, while this ecliptic of the primum mobile meanwhile remains fixed and immovable, midway between the two limits to which the other strays, as we shall say in treating the motion of the tenth sphere.

[Margin: The true equinoxes and solstices occur at the intersections of the ecliptic of the primum mobile with the equator, and on the Colure of the Solstices.]

But although the ecliptic of the primum mobile is called the *mean*, its intersections with the equator are nonetheless called the points of the *true* equinoxes, since it is truly at them that the Sun brings about both equinoxes, the vernal and the autumnal, throughout the whole world; and consequently both solstices, the summer and the winter, occur when the Sun reaches the Colure of the Solstices of the primum mobile, as will become clear in treating the motion of the eighth sphere. But never before or after those intersections and the Colure of the Solstices can the equinoxes and solstices occur — as the authors of the trepidation-motion wrongly held, and as would necessarily also have to be granted according to Copernicus's hypotheses.

[Margin: The equator and Colures of the primum mobile are not called mean, but true.]

And so the Colures and the equator of the primum mobile are not to be called *mean*, but *true*, since upon them depend both the true equinoxes and solstices, and from the equator are reckoned the declinations of the stars.

[Margin: The proper motion of the tenth sphere.]

The tenth sphere, next, which is to be conceived beneath the primum mobile with the same poles, equator, and ecliptic, besides the daily motion by which it is swept along by the primum mobile, has another motion of its own — a libration from north to south and from south to north, through 24 minutes, beneath the Colure of the Solstices of the primum mobile — such that the poles of the zodiac of this tenth sphere move back and forth from the poles of the zodiac of the primum mobile by only 12 minutes, and do not complete the whole circuit. And since the poles of the ecliptic of the primum mobile are distant from the poles of the world by 23 degrees 40 minutes, as we said a little before, it comes about that the maximum distance of the poles of the ecliptic of the tenth heaven from the poles of the world amounts to 23 degrees 52 minutes, and the minimum to 23 degrees 28 minutes. And you should understand the same of the maximum declination of the ecliptic of this same tenth heaven from the equator — that is, of the maximum declination of the Sun. For this too can be increased up to 23 degrees 52 minutes, and can decrease down to 23 degrees 28 minutes. The maximum, indeed,

[The maximum solar declination] will occur when the north pole of the ecliptic of the tenth heaven is 12 minutes distant, toward the south, from the pole of the

ecliptic of the *primum mobile*; and the minimum, when that same pole is 12 minutes distant toward the north from the pole of the ecliptic of the *primum mobile*.

[Margin: By how much the maximum declination of the Sun can increase and decrease, and where it becomes greatest and where least.]

This motion — or rather libration — of the tenth sphere takes its start from the extreme southern terminus, and the pole of the ecliptic moves non-uniformly beneath the colure of the *primum mobile* up to the extreme northern limit, from which it returns again with the same irregularity to the southern terminus. When it has reached this, one whole period of this libration will be complete. This libration is slowest near each extreme limit, but fastest in the middle — that is, near the pole of the ecliptic of the *primum mobile*. The whole period of this libration — from the southern terminus, through the northern, back to the same southern — comprises, according to Copernicus, 3434 Egyptian years; that is, 3431 Julian years (the years we use) and about 239 days. In the tables this interval is called the *period of the anomaly of the obliquity of the zodiac* [periodus anomalix obliquitatis Zodiaci], which is reduced to regularity in the following way.

[Margin: Where the libration of the tenth sphere begins.]

[Margin: How great the period of the tenth sphere's libration is.]

Let the solstitial colure in the *primum mobile* — or rather a segment of it, AC, containing 24 minutes of one degree — be taken, and let the pole E of the ecliptic of that same *primum mobile* lie at its midpoint. About E, through A and C, imagine a circle ABCD drawn, divided into four quadrants by the two diameters AC and BD, and let its circumference be divided into 360 degrees. Let point A face north and C face south; that is, let A be the northern limit of this libration and C the southern. So when the pole of the zodiac of the tenth sphere is placed at C, the libration begins, and there the distance of that pole from the pole of the world will be greatest, namely $23^{\circ} 52'$. At E the pole of the ecliptic of the tenth sphere will lie directly beneath the pole of the ecliptic of the *primum mobile*, and will have its mean distance from the world-pole, $23^{\circ} 40'$; and the ecliptic of the tenth sphere will not differ from the ecliptic of the *primum mobile*. At A the same pole of the tenth sphere will attain its least distance from the pole of the world, $23^{\circ} 28'$, and half of the libration will be complete. And as soon as the same pole of the tenth sphere has returned to C, the whole period of the libration will be complete.

[Margin: What the anomaly of the obliquity of the zodiac is.]

[Margin: How the irregularity of the tenth sphere's libration is reduced to regularity.]

Now if, about the radius EF, a circle EHFL is described from its midpoint G, and this same radius is imagined to be carried around uniformly — beginning from the radius EC, toward the right or the left — so that the point F traverses the whole circumference in about 3431 years and 239 days, then on each single day it will cover 1" 2''' 2'''.

Diagram. A large circular figure (woodcut) laid over a rectangular coordinate grid, illustrating the combined libration of obliquity and of precession. The four cardinal directions are engraved around the rim: **SEPTENTRIO** (north, top), **MERIDIES** (south, bottom — printed mirror-reversed), **OCCIDENS** (west, left), **ORIENS** (east, right). Two perpendicular axes cross at the center point **E**:

- Horizontal axis (west–east): labeled **CIRCVLVS ECLIPTICÆ** ("circle of the ecliptic," the words broken across the figure as CIR = ... CVLVS / ECLI... PTICA) and, along it, **Libratio præcessionis æquinoctiorum continens Minuta 140** ("libration of the precession of the equinoxes, containing 140 minutes"). Degree scale from E: rightward (toward Oriens) 10, 20, 30, 40, 50, 60, 70; leftward (toward Occidens) 10, 20, 30, 40, 50, 60 (the left-hand digits printed mirror-reversed).
- Vertical axis (north–south): labeled **COLVRVS SOLSTITIORVM** ("solstitial colure") and **Libratio obliquitatis continens Minuta 24** ("libration of obliquity, containing 24 minutes"). Scale upward 3, 6, 9, 12; downward 3, 6, 9.

Additional curved-arc and line labels: **Præcessionis æquinoctiorum** (along the upper-left arc), **Eclipticæ obliquitatis / obliquitatis** (along the diagonal line running to the lower-right), and **Initium anomalïæ** with the numeral **12** at the top (repeated, mirror-reversed, at the bottom) marking the start of the anomaly. A smaller circle (the equant circle) is drawn about the midpoint **G** of the radius.

Readable point-letters, by position:

- E (center): common center / mean pole.
- O, I, B, P (on the west/upper-left arc): B stands at the west (OCCIDENS) position.
- N, M (upper-right).
- L, K (right / lower-right).
- G (center of the smaller circle).

- H (lower portion of the figure).

The circumference of the circle EHFL will perpetually cut the colure AC, except when the radius EF coincides with the radius ED or EB; for then the circumference will touch the colure at E. This intersection occurs successively at all points of the circumference EHFL and of the segment CA of the colure, twice in one complete revolution of the libration. And since the line FH drawn is perpendicular to AC — [Margin: from III.31] because the angle EHF, being in a semicircle, is right — and since perpendiculars that intercept equal arcs on the quadrant CD cut off larger segments from the radius CE near the center E than near the extremity C (by Proposition 1 of the *Treatise on Sines* [Tractatus sinuum]), it follows that, while the radius EF traverses equal arcs of the circle CDAB in equal times, the intersections H — which the perpendicular drawn from F always marks — traverse unequal arcs on the colure CA in those same times: smaller ones near C and A, but larger near E and B. But since the irregularity of the pole of the zodiac of the tenth sphere, from C to A and from A to C, is such that it advances with the same slowness or swiftness as the intersection H — so that the pole is never displaced from the intersection H — it follows that the libration of the tenth sphere is slowest at C, at the beginning of the first [quadrant]...

[Margin: What sort the irregularity of the tenth sphere's libration is, and where it is slowest and fastest.]

[that the libration is] slowest at C — the beginning of the first quadrant of anomaly CD — and at A — the end of the second quadrant DA, or the beginning of the third quadrant AB; but fastest near the middle pole at E, that is, at the end of the first quadrant of anomaly CD (or the beginning of the second quadrant DA), and at the end of the third quadrant AB (or the beginning of the fourth quadrant BC).

[Margin: What the circle of the anomaly of obliquity is.]

The circle of the whole period of the anomaly of obliquity, then, is ABCD, whose circumference the point F traverses in the space of about 3431 years and 239 days, covering each day $1'' 2''' 2''''$.

[Margin: What the equant circle is.]

The circle EHFL is not inaptly called the *equant*, since its intersection with the colure equalizes the irregularity of the tenth heaven's libration, inasmuch as the irregular motion of the ecliptic's pole depends on it.

[Margin: What the mean motion of the anomaly (the argument) is.]

The mean or uniform motion of the anomaly of obliquity — which may rightly be called the *argument of obliquity* [argumentum obliquitatis] — is the arc of the circle of anomaly ABCD, counted from the beginning of the anomaly at C, proceeding toward D up to the diameter of the equant circle EHFL: such as the arc CF. So if the point F has reached D, or M, or A, or O, or B, or P, the mean motion of the anomaly (the argument) will be the arc CD, or CM, or CDA, or CAO, or CAB, or CAP.

[Margin: What the mean motion of obliquity is.]

The mean or uniform motion of obliquity is the mean obliquity itself: namely the arc of the solstitial colure counted from the north pole of the world to the mean pole of the zodiac (which is the pole of the ecliptic of the *primum mobile*) — the arc from the north pole of the equator up to E, which we said contains 23° 40'.

[Margin: What the true motion of obliquity is.]

The true or apparent motion of obliquity is the arc of the solstitial colure counted from the north pole of the equator to the true pole of the zodiac — which we said always lies at the intersection of the equant circle EHFL with the colure. This intersection, that is, the true pole of the zodiac, is marked by the perpendicular line dropped to the colure from the terminus of the uniform motion of anomaly. For example, when the true pole of the zodiac is set at C, the true motion of obliquity will be the arc of the colure reckoned from the north pole of the world up to C, comprising 23° 52'; but when the true pole of the zodiac is at H, the true motion of obliquity will be the arc enclosed between that same world-pole and the point H — and so for the rest.

Diagram. The identical libration woodcut printed on the preceding page is repeated here at the foot of the page (so that the figure is at hand whether the reader is on p. 37 or p. 38 of the original). Same cardinal-direction labels (SEPTENTRIO / MERIDIES / OCCIDENS / ORIENS), same two axes through center E (horizontal ECLIPTICA scale with "Libratio præcessionis æquinociorum continens Minuta 140"; vertical solstitial-colure scale with "Libratio obliquitatis continens Minuta 24"), and the same point-letters (E, O, I, B, P, N, M, L, K, G, H). See fig-p49-1 for the full label list.

[Margin: What the equation of the anomaly of obliquity is.]

The equation of the anomaly of obliquity — that is, the difference between the mean and true motion of obliquity — is the arc of the solstitial colure lying between E, the mean pole of the ecliptic, and the true pole of that same ecliptic.

Thus when the true pole of the zodiac is at H or N, the equation of the anomaly will be EH or EN; but when that same true pole is at C or A, the equation will be the greatest of all, EC or EA; and at E, finally, the equation will be nothing.

[Margin: When the equation of the anomaly is to be added to the mean obliquity, and when subtracted.]

When the mean motion of the anomaly of obliquity lies in the semicircle BCD — which happens when it is less than 90° in the first quadrant CD, or greater than 270° in the fourth quadrant BC — the true obliquity is greater than the mean; and so the equation EH or EC must then be added to the mean obliquity of $23^\circ 40'$, in order to obtain the true obliquity, that is, the distance of the pole of the zodiac of the tenth sphere from the pole of the world. But when the mean motion of the anomaly lies in the semicircle DAB — which occurs when it is greater than 90° yet less than 270° — the true obliquity of the zodiac is less than the mean. Therefore the equation EN or EA must then be subtracted from the mean obliquity, so that the true may remain. And finally, when the mean motion of the anomaly is exactly 90° or 270° — as when it is at D or B — the true obliquity does not differ from the mean. So then nothing is to be added to, or subtracted from, the mean obliquity.

[Margin: How the magnitude of the equation is to be found from a given mean motion of anomaly.]

For the rest, the magnitude of the equation will be easily known, whatever point of the circle ABCD the mean motion of the anomaly occupies. For since the equation of obliquity is always equal to the sine of the complement of the mean motion of the anomaly — a semicircle being first subtracted from the mean motion, if necessary — once the mean motion of the anomaly is known, the sine of its complement will also be known. Since, then, the whole sine EC (or EA) comprises 12 minutes, it will be impossible not to know how many minutes belong to the sine of the complement of any given mean motion of anomaly — that is, how many minutes the equation of a given mean motion of anomaly contains — provided one sets up the proportion: as the whole sine is to 12 minutes, so is the sine of the complement of the given mean motion of anomaly to the sought value. Do not be troubled that we explain and investigate these things by means of straight lines, even though in the heavens everything is curved and spherical: for on account of the tiny magnitude of 12 minutes, the circle ABCD will differ little or not at all from a plane surface, and its lines from straight lines.

[Margin: The proper motion of the ninth sphere.]

The ninth sphere, which is placed beneath the tenth with entirely the same poles and circles — equator and zodiac — has, besides the diurnal motion by which it is carried along by the *primum mobile*, and the libratory motion beneath the solstitial colure from north to south and from south to north which the tenth sphere imparts to it, also a certain proper libratory motion of its own beneath the ecliptic of the tenth sphere and about its poles (for the poles and ecliptic of the ninth sphere do not depart in any direction whatever from the poles and ecliptic of the tenth sphere): from east to west and from west to east through 140 minutes, that is, through $2^{\circ} 20'$. Thus the first points of Aries and Libra of the ninth heaven move back and forth, beneath the ecliptic of that same tenth heaven, from the first points of Aries and Libra of the tenth — that is, from the intersections of the equator with the ecliptic, which do not differ from the intersections made in the *primum mobile* — by 70 minutes, or $1^{\circ} 10'$, according to Copernicus. For the Prutenic Tables [Prutenicæ tabulæ] make this libration, on either side of the Aries and Libra of the tenth heaven (or the *primum mobile*), a little greater — namely $1^{\circ} 11' 22'' 30'''$ — so that the whole libration comprises $2^{\circ} 22' 45''$.

This libration begins its motion from the very intersection of the equator and the ecliptic in the tenth sphere (or *primum mobile*), and proceeds with a non-uniform course toward the west up to $1^{\circ} 10'$; then with the same non-uniformity it returns to the same intersection, and proceeds further toward the east up to $1^{\circ} 10'$, and from there returns once more to that intersection of the equator and the ecliptic. It is fastest at the beginning and middle of the whole period, that is, near the intersection of the ecliptic and equator; but slowest around each limit, the western and the eastern.

[Margin: How great the period of the ninth sphere's libration is.]

The whole period of this second libration of the ninth sphere comprises, according to Copernicus, 1717 Egyptian years — that is, 1715 Julian years (those in use among us) and about 302 days — so that this period of the second libration is half as long as the period of that first libration of the tenth heaven, and is completed twice in the time in which the former is completed once. The time of the period of this second libration is called the *period of the anomaly of the precession of the equinoxes*, or of the motion of the eighth sphere; and it is reduced to regularity in the following way.

[Margin: Where the libration of the ninth sphere begins.]

[Margin: What the anomaly of the precession of the equinoxes, or the motion of the eighth sphere, is.]

Let there be the ecliptic — or rather a segment of the ecliptic — in the tenth sphere, BD, comprising 140 minutes, or $2^{\circ} 20'$, at whose midpoint lies E, the beginning of Aries of the tenth sphere, which lies directly beneath the first point of Aries of the *primum mobile*, that is, beneath the intersection of the ecliptic BD with the equator IK. About E, through B and D, let a circle ABCD be imagined drawn, divided into four quadrants by the diameters BD and AC, and let its circumference be divided into 360 degrees. Let point B face west, D east, A north, and C south, so that B is the western limit or terminus of this second libration, and D the eastern.

[Margin: The true and the mean equinox, what.]

So when the beginning of Aries of the ninth sphere is placed at E, the libration begins, and it will be no distance from the true equinox — which, as we said above, always occurs at E, the intersection of the ecliptic and equator of the *primum mobile*. The beginning of Aries of the ninth sphere, wherever it lies on the libration line BD, is called the *mean equinox*, since the Sun, when it is there, does not produce an [actual] equinox, but — always at an equal distance with respect to the motion of the eighth sphere, as we shall say below — recedes from it daily; and accordingly the mean equinox is said to occur as soon as the Sun reaches it. At B and D the mean equinox — that is, the Aries of the ninth sphere — will be at its greatest distance from the true equinox, i.e. from the Aries E of the *primum mobile*, namely $1^{\circ} 10'$.

The beginning of Aries of the ninth sphere moves toward the west to B, then from there passes through E and D, and from D returns again to E; and only then will the whole period of the libration be complete. Now if the radius EF, together with its circle of the figure above, is imagined to be carried around uniformly — beginning from the northern radius EA (this beginning could also be made from the southern radius EC), toward the west — so that it traverses the whole circumference in about 1715 years and 302 days, then on each single day it will cover $2'' 4''' 4''''$.

The circumference of the circle EHFL will perpetually cut the ecliptic BD, except when the radius EF coincides with the radius EA or EC at the beginning and middle of the libration; for then the circumference will touch the ecliptic at E. This intersection occurs successively at all points of the circumference EHFL [printed here "EHPL"] and of the segment BD of the ecliptic, twice in one

complete revolution of the libration. And since the line FL drawn is perpendicular to BD — [Margin: from III.31] because the angle ELF, being in a semicircle, is right — we shall show, as in the earlier libration, that while the radius EF traverses equal arcs of the circle ABCD in equal times, the intersections L (upon which the perpendiculars drawn from F fall) traverse in those same times unequal arcs on the ecliptic BD: larger ones near E, but smaller near the extreme limits B and D.

[Margin: What sort the irregularity of the ninth sphere's libration is, and where it is fastest and slowest.]

Therefore, since the irregularity of the beginning of Aries of the ninth sphere — from E to B, and from B to D, and from D to E — is such that it never departs from the intersection L, but advances with exactly the same swiftness and slowness as the point L, it comes about that the libration of the ninth sphere is fastest at E — that is, at the beginning of the first quadrant of anomaly AB, and at the end of the second quadrant BC (or the beginning of the third quadrant CD); but slowest at B — the end of the first quadrant AB (or the beginning of the second quadrant BC) — and at D — the end of the third quadrant CD (or the beginning of the fourth quadrant DA).

[Margin: What the circle of the anomaly of the precession of the equinoxes, or of the motion of the eighth sphere, is.]

The circle of the whole period of the anomaly of the precession of the equinoxes — or of the motion of the eighth sphere — is therefore ABCD, whose circumference the point F traverses in the space of about 1715 years and 302 days, and covers each day $2''\ 4'''\ 4''''$.

[Margin: What the equant circle is.]

The circle EHFL will not ineptly be called the *equant*, since its intersection with the ecliptic equalizes not only the libration of the ninth heaven — for the irregular motion of the first point of Aries of the ninth sphere depends on it — but also the irregular motion of the eighth sphere, as we shall say below.

The mean or uniform motion of the anomaly of the precession of the equinoxes, which may most aptly be called the *ar-gument...*

[Margin: What the mean motion of the anomaly of the precession of the equinoxes, or the argument, is.]

the argument of the anomaly is the arc of circle ABCD, which we called the anomaly, numbered from the beginning of the anomaly A toward B, proceeding up to the diameter of the equant circle EHFL — such as the arc ABF. But if point F reaches O, or B, or P, or C, or D, or M, the mean motion of the anomaly (or argument) will be the arc AO, or AB, or AP, or ABC, or ABD, or ACM. What the mean and true motion of the precession of the equinoxes (or of the eighth sphere) is will be stated below.

[Margin: What the equation of the anomaly of the precession of the equinoxes, or of the motion of the eighth sphere, is.]

The equation of the anomaly of the precession of the equinoxes — that is, of the motion of the eighth sphere — that is, the difference between the mean and true motion of the precession of the equinoxes (or of the eighth sphere), is the arc of the Ecliptic between E (the true equinox) and the beginning of Aries of the ninth sphere, that is, the mean equinox, which we have always said occurs at L, the intersection of circle EHFL and the Ecliptic. Thus, if the beginning of Aries of the ninth sphere is placed at L or Q, the equation of the anomaly will be EL or EQ. But when that same beginning of Aries lies at B or D, the equation will be greatest, EB or ED. Finally, at E there will be no equation at all. We will explain the use of this equation in [our discussion of] the eighth sphere.

[Margin: How the magnitude of the equation of the precession of the equinoxes is known from the given mean motion of the anomaly.]

Moreover, the magnitude of the equation will be easily found, once it is known how great the mean motion of the anomaly is. For since the equation of the precession is always equal to the right sine of the mean motion of the anomaly — after first subtracting a semicircle from the mean motion, if it can be subtracted — then, the mean motion of the anomaly being known, its right sine will also be known. Since, therefore, the whole sine EB (or ED) comprises 70 minutes, it will not be impossible to determine how many minutes correspond to the right sine of the given mean motion of the anomaly — that is, how many minutes the equation of the given mean motion of the anomaly contains — namely if the following proportion is made: as the whole sine is to 70, so is the right sine of the given mean motion of the anomaly to another [number]. For here too the whole figure can be taken as plane, although it is spherical, because of the smallness of the diameter BD, which is 2 degrees and 20 minutes.

Diagram. A large circular figure occupying the lower half of the page, combining the two librations. The four cardinal directions are engraved around the

periphery: SEPTENTRIO (north, top), MERIDIES (south, bottom), OCCIDENS (west, left), ORIENS (east, right). A horizontal diameter runs west–east through the centre E; it is the Ecliptic, labelled **ECLIPTICA** (split across the figure as "ECLI... PTICA") and graduated 10, 20, 30, 40, 50, 60, 70 to each side of the centre. A vertical diameter runs north–south through E; it is the solstitial colure, labelled **CIRCVLVS SOLSTITIORVM**, graduated 3, 6, 9 (and higher) each way. The large outer circle carries the label **Libratio præcessionis æqui[noctiorum]** ("Libration of the precession of the equinoxes") and along its horizontal axis the note **continens Minuta 140** ("containing 140 minutes" — i.e. the $\pm 70' = 2^\circ 20'$ precession libration). A smaller circle in the lower/right region is labelled **ÆQVANS** and **Libratio obliquitatis** ("the equant" / "libration of obliquity"), with the note **continens Minuta 24** ("containing 24 minutes" — the obliquity libration). Along an upper radius runs **Præcessionis æquinocetiorum**; on the right side **Ecliptica obliquitatis**; near the centre **COLVRVS. Initium Anomaliae** ("beginning of the anomaly") appears at the top of the circle, and again as **Initium anomaliae** at the bottom.

Labelled points read on the figure:

- E (centre): intersection of Ecliptic and colure
- B (west end of Ecliptic / horizontal diameter)
- C (on the Ecliptic, left of centre, at the foot of a dotted vertical from O)
- O (upper-left arc)
- P (lower-left, foot of the large circle)
- H (bottom, on the colure)
- N (upper, on the colure near centre)
- M (upper-right arc)
- K (right, on the smaller circle) [?]
- L (right of centre, near the smaller circle) [?]
- G (mid-figure, on an interior diagonal line) [?]
- A (lower, near "Initium anomaliae") [?]
- I / L (left, on a radius near "Præcessionis æquinocetiorum") [?]

[Margin: The Ecliptics of both the ninth and the eighth sphere always cut the Equator at the beginning of Aries of the primum mobile, although they recede from the Ecliptic of that same primum mobile.]

[Margin: The simple anomaly is called that of obliquity; the doubled one is called that of precession.]

Now although the poles of the Ecliptic of the ninth heaven approach and recede from the poles of the Ecliptic of the primum mobile in keeping with the libration-motion of the tenth [heaven] beneath the colure of solstices — and thus the Ecliptic of both the ninth and the eighth sphere is moved away from the Ecliptic of the primum mobile — nevertheless it will always intersect the Equator at the same points, one of which is point E. For since the colure of solstices passes through the poles of the Equator and of the Ecliptic, these circles will in turn pass through its poles (by the scholium to Proposition 15 of Book 1 of Theodosius); and therefore the intersection of the Equator and the Ecliptic will be the pole of the colure of solstices. Hence (by the corollary to Proposition 16 of the same) it will be distant a quadrant of a great circle from any point of the colure. Therefore, wherever the pole of the Ecliptic is set upon the colure, the Ecliptic described from it will pass through the intersection of the Equator and the Ecliptic of the primum mobile. And so, although the Ecliptic of the eighth sphere recedes from the Ecliptic of the primum mobile, the equinox will nevertheless always fall at the beginning of Aries of the primum mobile.

Finally, since this libration of the ninth sphere is twice as fast as that libration of the tenth, as we said, it has come about that in the Prutenic Tables the libration of the tenth Sphere is called the "simple anomaly," while the libration of the ninth Sphere is called the "doubled anomaly." Thus in the tables the Anomaly is to be taken simply for the obliquity of the Zodiac, but the same doubled for the precession of the equinoxes; and there is no need to compose two tables for those two librations, but one will serve both, as has been explained.

[Margin: The proper motion of the eighth sphere.]

The eighth sphere, finally, besides the threefold motion by which it is carried along by the three higher spheres, has yet a fourth motion of its own, and that the slowest of all: from west to east beneath the Ecliptic of the ninth, or tenth, heaven. For the poles are altogether the same, and the Ecliptics of the eighth, ninth, and tenth heaven are the same.

[Margin: In what respect the motion of the eighth sphere is regular.]

This motion, however, is irregular and non-uniform if it is referred to the Aries of the primum mobile, but uniform if it is referred to the Aries of the ninth sphere. For from the first point of Aries of the ninth heaven (which is wandering and mobile, since it moves back and forth into west and east from the first point of Aries of the primum mobile, as has been said) — namely from point L of the figure above — the first star of Aries, which is in its right horn, continually

recedes with a uniform motion, namely in the space of one natural day, by 8 thirds and 15 fourths toward the east; so that if that star were at this moment of time conjoined with that first point of Aries of the ninth sphere, after 24 hours had passed it would be distant from it by 8 thirds and 15 fourths, and after another 24 hours had elapsed, by 16 thirds and 30 fourths, and so on in succession — however irregularly that point of Aries wanders back and forth from the beginning of Aries of the primum mobile.

[Margin: The magnitude of the motion of the eighth sphere, and its period.]

From this it follows that the first star of Aries, together with the whole eighth sphere, completes an entire period — that is, returns to the same point of the Ecliptic of the ninth heaven from which it departed — in the space of 25816 Egyptian years, which make about 25798 Julian years and 120 days. And because we said that the first point of Aries of the ninth heaven moves non-uniformly, it comes about that the eighth sphere too is carried irregularly from west to east, since its irregularity depends on that wandering and mobile starting-point.

[Margin: Where the motion of the eighth sphere is fastest, where slowest, and where intermediate.]

The eighth sphere will be seen to move more quickly when the first point of Aries of the ninth sphere is carried from B to D — that is, from west to east — because then that point follows the first star of Aries eastward, and so the two motions from west to east run together at once; but more slowly when it goes back from D to B — that is, from east to west — because then that point, from which the first star of Aries is uniformly elongated, flees from that first star by retreating in the opposite direction. And so near the beginning and the end of the Anomaly — that is, around the northern point A — the motion of the eighth sphere is slowest; near the middle of the anomaly, around the southern point C, it is fastest; and finally at both limits B and D, the western and the eastern, it is intermediate, because then the first point of Aries of the ninth sphere seems to advance neither into the west nor into the east.

[Margin: Why the motion of the eighth sphere is called the precession of the equinoxes by Copernicus.]

This motion of the eighth sphere is called by Copernicus, and in the Prutenic Tables, the precession of the equinoxes. For Copernicus, according to his own hypotheses, makes the first star of Aries, together with the whole eighth heaven, motionless, but sets the point of the true equinox E to move away from that star from east to west — that is, against the order of the Signs, which astronomers call

moving "in antecedence," just as they call motion from west to east, that is, according to the order of the Signs, motion "in consequence." And so, just as for us the first star truly recedes from the true equinox toward the east, so for Copernicus the true equinox moved away from that fixed and unmoving star toward the west, in antecedence; and therefore that motion was called the precession of the equinoxes.

[Margin: What the mean motion of the eighth sphere, or the mean precession of the equinoxes, is.]

The mean motion of the eighth sphere, then, or the mean precession of the equinoxes, is the arc of the Ecliptic intercepted between two great circles: one drawn through the poles of the Zodiac and the first point of Aries of the ninth sphere (that is, the mean equinox); the other through the poles of the Zodiac and the first star of Aries.

[Margin: What the true motion of the eighth sphere, or the true precession of the equinoxes, is.]

The true motion, however, or the true precession of the equinoxes, is the arc of the Ecliptic enclosed between two great circles: one of which is drawn through the poles of the Zodiac and the first point of Aries of the primum mobile (that is, the true equinox); the other through the poles of the Zodiac and the first star of Aries.

[Margin: What the equation of the motion of the eighth sphere, or of the precession of the equinoxes, is, and when it is to be added or subtracted.]

The equation of the motion of the eighth sphere, or of the precession of the equinoxes, is the same as the equation of the anomaly of the precession which was discussed for the ninth sphere. It is subtracted from the mean motion in the first semicircle ABC — that is, when the mean motion of the Anomaly is less than 180 degrees — because then the mean motion is greater than the true, and the mean equinox occurs before the true; but in the latter semicircle CDA — that is, when the mean motion of the anomaly is greater than 180 degrees — it is added, because then the mean motion is less [than the true], and the mean equinox occurs after the true.

[Margin: Why astronomers at different times found the fixed stars to move variously, and the length of the year, and the maximum declination of the Sun, not to be the same.]

From all this it appears why the fixed stars were found by some astronomers to move more slowly, and by others more quickly, at different times: namely because they are advanced non-uniformly from west to east, on account of the anomaly of the precession of the equinoxes. Likewise why the length of the year is not always the same: because the Sun, which is driven also by the motion of the eighth sphere, returns to the true equinox now more slowly, now more quickly, on account of the same anomaly of the precession of the equinoxes. And finally why different astronomers at different times found different values for the maximum declination of the Sun: because the Ecliptic of the eighth sphere, beneath which the Sun perpetually moves, wanders — in keeping with the libration of the tenth heaven — now northward, now southward, away from the Ecliptic of the primum mobile.

[Margin: What the four motions of the eighth sphere are.]

These, then, are the four motions which astronomers observed in the fixed stars, or in the eighth heaven: (1) from east to west in the space of 24 hours, by the motion of the primum mobile about the poles of the world; (2) from north to south and back through 24 minutes, by the libration of the tenth sphere, in the space of 3434 Egyptian years; (3) from east to west and back, about the poles of the Zodiac, through 70 minutes, going to and fro from the true equinox and returning in the space of 1717 Egyptian years; and (4) from west to east, likewise about the poles of the Zodiac, completing the whole circuit in the space of 25816 Egyptian years.

[Margin: The latitudes of the stars with respect to the true Ecliptic — which is in the tenth, ninth, and eighth heaven — do not change; although with respect to the Ecliptic of the primum mobile, which is the mean [Ecliptic], they do change.]

But if someone should object that from the libration of the tenth sphere it follows that the fixed stars change their latitudes with respect to the Ecliptic of the primum mobile — which seems to conflict with the observations of astronomers, who teach that the latitudes of the stars do not change — we reply that this is true: for the points of the eighth heaven near the colure of solstices, where that change is greatest, can be 24 minutes more southerly or more northerly at one time than at another; but that this distance of the stars from the Ecliptic of the primum mobile is not to be called latitude — which astronomers have found not to vary in one and the same star. For the latitudes of the stars are to be referred to the true Ecliptic, which the Sun describes by its annual motion beneath the Ecliptic of the tenth, ninth, and eighth heaven. For they perpetually keep their distances from this Ecliptic — that is, from the Sun's path — the same, as has been

found by astronomers. And indeed, if it were absurd for the fixed stars to recede 12 minutes to either side from the Ecliptic of the primum mobile near the solstitial points, it would be far more absurd in the motion of trepidation, on account of which the stars near the beginning of Aries and Libra of the eighth heaven can be moved to either side of the Ecliptic of the primum mobile (or of the ninth sphere) not only by 12 minutes but by 9 degrees, that is, by 540 minutes.

[Margin: That the fixed stars cannot become stationary or retrograde, even if the motion of trepidation is granted.]

If again someone should object that from the libration of the ninth sphere it follows that the fixed stars are perpetually carried from west to east by the motion of the eighth sphere, and are never stationary or retrograde from east to west — whereas we asserted above, following the opinion of the authors of the trepidation-motion, that this could happen — we frankly admit that this is true, and that the observations of all astronomers teach the very same thing; for the fixed stars have been continually found to move eastward, even in the time of Alfonso, when their motion is thought to be slowest. Indeed, even if we were to grant that the stars are moved by the motion of trepidation, it still could not happen that they should be stationary or retrograde, because their motion from west to east by the motion of the ninth sphere is always faster than the motion by which they can be moved from east to west by the trepidation of the eighth sphere — just as their motion from west to east, by which the eighth heaven carries them around, is faster than that motion by which they are drawn westward by the libration of the ninth sphere. For the first point of Aries of the eighth sphere, according to the Alphonsines, in the space of 35000 years — in which half the period of trepidation is completed — travels 18 degrees from east to west, namely the whole diameter of the little circle; but in the same time, by the motion of the ninth sphere, the stars complete 25 degrees and more toward the east, even reckoning by the period of 49000 years, as the Alphonsines will have it. Likewise the stars, by the libration of the ninth sphere, in the space of about 858 years — in which half the period of the libration is completed — travel 2 degrees 20 minutes from east to west; but in the same time the stars complete 11 degrees and more from west to east by the motion of the eighth sphere. From which it is plainly established that the motion of the stars from west to east is always faster than the motion of trepidation, or the libration of the ninth sphere, from east to west.

[Margin: That the equinoxes and solstices never occurred before or after the equinoctial and solstitial points of the primum mobile.]

Finally, that which the Alphonsines adduced in confirmation of the motion of trepidation — namely that the equinoxes and solstices had sometimes occurred before or after the equinoctial and solstitial points of the primum mobile — is an utterly old-womanish fiction; nor could they ever confirm it by the observation of any competent astronomer, but it is merely asserted by them because it necessarily follows from that motion of trepidation. A clear proof of this is that not even among the Alphonsines themselves is there agreement as to which point the motions are to be referred to — whether, that is, to the Aries of the primum mobile, or to the intersection of the Ecliptic of the eighth sphere with the Equator of the primum mobile. This controversy would have no place if the equinox really did fall outside the first point of Aries of the primum mobile. For reason seems to require that the motions be reckoned from the true equinox — which is exactly observed in our libration of the ninth sphere. For this reason it is more probable that the eighth sphere is moved by a fourfold motion, as we have explained, than by a motion of trepidation; since by that account all the phenomena are accounted for, and nothing absurd follows from it.

DE ORDINE SPHÆRARVM CŒLESTIVM.

[On the order of the celestial spheres]

[Margin: That the heavens are immediately contiguous to one another.]

From what has been said about the motions of the heavens, it clearly remains that all the heavens do not form a single continuous body, since the heavens are carried by various and diverse motions that are in a sense opposed, as has been said; and no single body is fit to be carried by contrary motions at the same time. Therefore all the heavens so far discovered are concentric with the whole universe and contiguous to one another, in such a way that between any two neighbouring orbs there is nothing intermediate; for if there were either a vacuum or some body [between them], the motion of the higher orb could not be communicated to the lower. Nor indeed is the argument valid that is commonly brought forward to prove that the heavens cannot be contiguous, which runs thus: Let a straight line be drawn from the centre of the world to the convex surface of, say, the tenth heaven, and let the point be taken where that line touches or cuts the convex surface of the ninth orb, which we call A; and let another point besides be taken where the same line touches or intersects the concave surface of the tenth sphere, which we call B. If, then, the convex surface of the ninth sphere is immediately adjacent and contiguous to the concave surface of the tenth, the two points A and B, lying on the same line, will be

immediately next to each other — so that the tenth heaven cannot be immediately adjacent to the ninth; and the same reasoning holds for the remaining celestial spheres. This argument, I say, has no force, because one and the same point of that line touches both the convex surface of the ninth heaven and the concave surface of the tenth; hence the two points conceived there are one and the same point, since they touch each other as wholes (having no parts), and therefore exist in the same place — if indeed a point can be said to occupy a place. Those two points are therefore two in reason but one in reality, since they coincide, no otherwise than if two lines were joined at their end-points: for then those two end-points would coincide entirely into one. And if this argument proved anything, no two bodies could ever be contiguous and immediate — which is plainly false, as is evident in the case of some ball placed in the air. For nothing can be intermediate between the ball and the air; otherwise there would be a process to infinity. And yet, if a straight line were drawn through the centre of the ball, it would certainly cut both the concave surface of the air and the convex surface of the ball. It remains, therefore, that the heavens are distinct from one another yet contiguous — and their order is now to be discussed.

[Margin: The first opinion on the order of the heavens.]

Among the ancients, then, there were some — whose leader was Aristarchus of Samos, 400 years before Ptolemy [recte: Ptolemæum; printed *Prolemæum*], and whom, among more recent writers, Nicolaus Copernicus followed in his work *On the Revolutions of the Heavens* — who devised this order among the bodies of the whole Universe: that the Sun is set motionless at the centre or middle of the world; around it the orb of Mercury; then the orb of Venus; around this the great orb, containing the Earth together with the elements and the Moon; around which the orb of Mars; then the heaven of Jupiter; afterward the globe of Saturn; and last of all follows the sphere of the fixed stars. But this opinion is contradicted by many experiments and by the common judgement of all philosophers and astrologers. For the Earth must rest at the middle of the whole world, as we shall afterward demonstrate by very many observations and phenomena.

[Margin: The second opinion on the order of the heavens.]

The most ancient Egyptians, however — together with Plato in the *Timæus*, Aristotle in *On the Heavens* II, chapter 12, and *Meteorology* I, chapter 4 — held the order in the celestial spheres to be this: that the Moon occupied the lowest place; immediately after it followed the Sun; then Mercury; then Venus; fifth Mars; sixth Jupiter; seventh Saturn; and eighth and last the starry heaven, or firmament.

Aristotle alone, in the little book *On the World, to Alexander* (if indeed it is his), placed Venus immediately above the Sun and below Mercury. But such an order of the planets or heavens too has long since been refuted by the astronomers.

COMMENTARY (continued)

We therefore establish, together with Ptolemy and Regiomontanus [Ioan. de Regiomon.], that order of the heavens which our author set out above: so that the Moon holds the first, or lowest, place; Mercury is set above it; in the third place Venus follows; the Sun fourth; Mars fifth; Jupiter sixth; Saturn seventh; and in the eighth and last place the Firmament. You see, then, that all the opinions agree in this: that the sphere of the fixed stars is placed in the highest station, and beneath it Saturn, beneath Saturn Jupiter, and then Mars. It is only over the remaining four planets that the whole disagreement lies. We shall therefore confirm briefly the order just recounted. First, from parallax [diuersitas aspectus]. Next, from the speed and slowness of motion. Third, from eclipses or occultations of the planets. For it is chiefly by this threefold means that the order of the heavens is usually confirmed by astronomers.

[Margin: The truer opinion concerning the order of the heavens.]

As for parallax, the argument runs thus. That heavenly body is nearer the Earth which, other things being equal, has the greater parallax. But the Moon has been found to suffer the greatest parallax, then Mercury, after that Venus, and next the Sun. It is therefore established that the Moon must be placed in the first station, Mercury in the second, Venus in the third, and the Sun in the fourth. Concerning the remaining planets, nothing certain can be determined by this route, since on account of their immense distance from the Earth they have no parallax. To make this fuller understood, a few words must be said about this parallax.

[Margin: The order of the planets is confirmed from parallax. What is parallax? What is the true place? What is the apparent place?]

Parallax, then — what others call the difference of aspect — is the difference between the true and the apparent place of some heavenly body. The true place of a body is that point of the great circle passing through the zenith and through the body which is marked off by a straight line drawn from the center of the Earth through the center of the body and produced to that great circle. The apparent place of the body is that point of the same great circle which is marked off by a straight line drawn from our eye through the center of the body and produced to that great circle.

An example. Let the center of the Earth be A; let the great circle passing through the zenith of the head D, and through the star, be CDE. Let the place on the Earth's surface lying beneath the zenith D be B. Let there be any star, K, through whose center a straight line AKS is drawn from the center of the Earth; and likewise, through the center of the same star, let a straight line BKT be drawn from B, the observer's place on the Earth. The true place of the star K, then, is the point S; the apparent place is the point T. The difference between the true and apparent places — namely the arc ST — is called the parallax of the star K. The angle formed at the center of the star by those two straight lines (in the present example the angle AKB) is usually called by astronomers the quantity of the parallax. So that if for two stars such angles are made equal, the stars are said to have equal parallax; but of two stars, the one at whose center the greater angle is contained has the greater parallax.

Diagram. A semicircular figure representing the celestial hemisphere standing above the horizon. The horizontal diameter (the horizon) runs along the base from C (left corner) through the center to E (right corner). At the bottom center is A (the center of the Earth), with B just above it (the observer's place on the Earth's surface). At the top center is D (the zenith), with M just below it. A single star is shown in successive positions descending from the zenith toward the horizon on each side: on the left K, H, and near the horizon F, with G and I low toward the left; on the right O, P, and R; and L and N near the center. Radial lines drawn from A and from B through each star position are extended to the outer arc, where their endpoints (the "true" locus reached via A and the "apparent" locus via B) are marked by small clusters of letters reading S, T, V repeated around the rim.

- D (top center): zenith
- M (just below D): star at the vertex
- A (bottom center): center of the Earth
- B (above A): observer's place on the Earth's surface
- C (left corner of base): horizon endpoint
- E (right corner of base): horizon endpoint
- K, H, F (left side): successive star positions toward the horizon
- G, I (lower left): star positions
- L, N (center): star positions
- O, P, R (right side): successive star positions
- Peripheral "S / T / V" clusters around the outer arc: true and apparent loci
[?] (several small labels not individually resolvable)

From this it is clear that if two stars in the same heaven are equally distant from the zenith M — such as the stars H and O — they have equal parallax. For the two sides HA, AB of the triangle ABH are equal to the two sides OA, AB of the triangle ABO, [Margin: a — Elements III.27] and the angles at M are equal, on account of the equality of the arcs MH, MO which measure the said distances of the stars from the zenith M. [Margin: b — Elements I.4] Therefore the bases BH, BO, and the angles H, O, which show the quantity of parallax, will also be equal.

By like reasoning it follows that the same star, the nearer it is to the horizon, the greater parallax it has — so that when it is on the horizon it has the greatest — whereas the farther it is from the horizon, the smaller its parallax, so that when it is at the zenith of the head, where it is most removed from the horizon, it has no parallax at all. All of which we shall demonstrate in order. Let one and the same star be now at the point M (that is, at the zenith), now at the point K (approaching the horizon), now at the point H (which is nearer the horizon), and finally at the point F (that is, on the horizon). From the center of the Earth A, and from the eye B, let straight lines be drawn through the center of this star wherever it stands. Let the arc MO be taken equal to the arc MH, so that the two stars, in the points noted, may have equal altitudes above the horizon; [Margin: a — Elements III.27; b — Elements I.4] and therefore, as was just demonstrated, the same parallax.

[Margin: The star, the nearer it is to the horizon, the greater parallax it has.]

Connect the points K and O by the straight line KO. Since, then, BO is equal to BH, as was just demonstrated — but BH is greater than BK — BO too will be greater than BK; [Margin: c — Elements III.7] and on that account the angle BKO is greater than the angle BOK. But the whole angles AKO and AOK are equal. [Margin: d — Elements I.5] Therefore the remaining angle AOB will be greater than the remaining angle AKB; and accordingly the star at O, and consequently at the point H, will have a greater parallax than at the point K. Hence it is established that any star, the nearer it is to the horizon, the greater parallax it has. [Margin: e — Elements I.4] [Margin: The star at the horizon has the greatest parallax.]

[Margin: The star at the zenith has no parallax.]

Again, let some star be on the horizon, namely at G, and another in the same heaven at the point L, above the horizon; and let the horizon GB be produced as far as R, and let the straight lines AG, AR, AL, BL, LR be joined. [Margin: c — Elements I.4] The bases BG, BR, and the two angles AGB, ARB, will be equal. But the angle ARB is greater than the angle ALB — which indeed can be

demonstrated in the same manner as it was shown that the angle AOB is greater than the angle AKB. Therefore the angle AGB too will be greater than the same angle ALB; and on that account the star existing on the horizon will have the greatest parallax. By the same reasoning it will be demonstrated that the angle AGB is greater than any other. And you easily perceive that the star existing at the point M has no parallax at all, since for it the apparent and the true place are one and the same.

[Margin: Of two stars having the same true or apparent place, that which is nearer the center of the Earth has the greater parallax.]

Again, from the same figure it is gathered that of two stars which have the same true or apparent place, the one that is nearer the center of the Earth has the greater parallax. For the stars F and G, or R and Q, have the same apparent place S; but the true place of the star F, or Q, is T, while that of the star G, or R, is V. Here it is plainly seen that the parallax of the star G, or R — namely the arc SV — because it is nearer the center of the Earth, is greater than the arc ST, that is, than the parallax of the star F, or Q, which recedes farther from the center of the Earth. The same is likewise seen in the stars P and O; likewise L and K; likewise I and H, the true place of all of which is shown by the point S. From these things, thus set forth, this first route drawn from parallax is clearly understood.

NEXT, from the speed and slowness of motion astronomers gather this same order of the heavens, by the following reasoning. The more a heaven recedes from the nature and condition of the *primum mobile*, the lower the station in which it too is to be placed. But since the Moon, of all the planets, is carried most swiftly from west to east — as we said above — it seems to recede most of all from the motion and condition of the *primum mobile*, and on that account to conform least to the first heaven, or *primum mobile*. It will therefore possess the lowest station. By the same reasoning the starry heaven will be placed in the highest station, since it is carried most slowly against the motion of the *primum mobile*. Then will follow the sphere of Saturn, after that of Jupiter, and so on for the rest, always setting the order stated above.

[Margin: The order of the heavens is proved from the speed and slowness of motion.]

But by this route nothing certain can be determined about the order of the Sun, Venus, and Mercury among themselves. For although it is gathered from it that these three planets are placed above the Moon, since they are carried more slowly than it from west to east, as we said above; and that below the Firmament,

Saturn, Jupiter, and Mars are situated, since they are carried more swiftly against the *primum mobile* — nevertheless which of them is to be set above the other cannot be known for certain, since they complete their proper motions from west to east in nearly the same time. Indeed Alpetragius [al-Bīṭrūjī], as Regiomontanus attests (*Epitome*, Book 9, proposition 1), gathers from this reasoning that the heaven of Venus is placed below Mars, and below Venus the heaven of the Sun, then Mercury, and last of all the Moon — because Venus, by reason of its epicycle, completes its course more slowly than the Sun, and the Sun more slowly than Mercury, while the Moon, finally, completes its own period most swiftly of all.

[Margin: The order of the heavens is confirmed from eclipses.]

FINALLY, from eclipses or occultations of the planets and stars, the same order of the heavens is gathered by astronomers. For there can be no doubt that the body which conceals another from us is the lower. Since, therefore, the Moon, when it is in conjunction with the other planets, sometimes snatches them from our sight, it is necessary to grant it the lowest station. By the same reasoning Mercury will be below Venus, and Venus below Mars, and so on in succession. These, then, are the chief arguments by which astronomers conclude the order of the heavens which our author set forth. And although none of them by itself sufficiently establishes this order, yet all taken together confirm that the heavens are arranged in that order. For from parallax the order of the Moon, Mercury, Venus, and the Sun is infallibly gathered; from speed and slowness of motion, Mars is fittingly placed above these four planets, then Jupiter, last Saturn, and above all the planets the Firmament, or eighth heaven, which the ninth and tenth spheres, established beneath the *primum mobile*, follow. From eclipses, finally, although the order of not all the planets can be firmly gathered, yet we are compelled to place the Moon in the lowest station, and all the planets beneath the Firmament.

That a fuller knowledge of this order may be had, I judge it not out of place to bring forward some other arguments of the astronomers, from which the great fitness of this order will shine forth.

[Margin: That the Moon is placed in the lowest station is proved from the shadow.]

That the Moon, then, is placed in the lowest station can be demonstrated by the following reasoning. A luminous body, the higher and farther it is from the Earth, other things being equal, the smaller the shadows of bodies appear on the plane of the horizon; and the nearer the luminous body is to the Earth, the longer the

shadows the bodies cast, as may be seen in this figure. In it each star has the same altitude above the horizon AB with respect to the center of the world — that is, it holds the same true place with respect to the horizon — although they differ in apparent place; and yet the lower star casts a longer shadow of the gnomon CD, say to the point E, than the higher, which casts the shadow of the same gnomon only to the point F.

Diagram. A woodcut showing the celestial hemisphere as a semicircular arc standing on a horizontal base line. At the upper left is the Sun, drawn as a radiant face casting rays. Along the base, from left to right, are the points **A** (left endpoint), then **C**, **F**, **E** near the center-right, and **B** (right endpoint). A small upright gnomon (a little triangle) stands at **C**, with its apex at **D** above. Rays from the luminous body pass over the gnomon's tip **D** and cast the shadow along the base: the shorter shadow tip falls at **F**, the longer at **E**.

- Sun (upper left): the luminous body, drawn with rays
- A (left end of base): horizon endpoint
- B (right end of base): horizon endpoint
- C (base, center): foot of the gnomon
- D (above C): apex of the gnomon
- F (base, near gnomon): tip of the shorter shadow
- E (base, beyond F): tip of the longer shadow

Now the shadow of an erect gnomon, when the Sun shines, is shorter than the shadow of the same gnomon when the Moon shines, all else being equal — that is, at equal degrees with the Sun, but distant from the horizon at a different time. This anyone can easily test. At the time of the equinox let the meridian altitude of the Sun be noted, the Sun holding the beginning of Aries [♈] or of Libra [♎] — which altitude at Rome is about 48 degrees — and let the gnomon's shadow be marked on some plane. Afterward let the same be done with the Moon situated in the same place of the zodiac in which the Sun was before — that is, in the beginning of Aries [♈] or of Libra [♎] — holding the meridian circle and lacking all latitude. For the shadow of the gnomon, when the Moon shines, will be found much longer than before with the Sun, even though the altitude, or distance, of each planet from the horizon is the same — namely 48 degrees — with respect to the center of the Earth. It follows, therefore, that the sphere of the Sun is far higher than that of the Moon.

What we have said of the Moon with respect to the Sun can be applied with respect to the other planets. For although the other planets do not shine so as to

cast shadows, it can nevertheless be known how far their rays are cast past the tip of a gnomon. Wherefore, beyond all controversy, it is established that the Moon is lower than all the planets.

[Margin: That the Sun is fittingly placed in the middle of the planets.]

How fittingly, too, the Sun is placed above Mercury and Venus — that is, set in the middle of the planets — Regiomontanus brings forward (*Epitome*, Book 9, proposition 1). Ptolemy (*Almagest* [Dict.] V.15), from whom Albategnius [al-Battānī] does not dissent, shows in chapter 50 of his own work, by sure reasons, that the distance of the Sun from the center of the Earth, when it is least — that is, when the Sun is at the point opposite its apogee — contains 1070 semidiameters of the Earth; while the distance of the Moon from the center of the Earth, when it is greatest — that is, when the Moon is at its apogee — contains only 64 semidiameters of the Earth. Hence the difference between the least distance of the Sun and the greatest distance of the Moon will contain 1006 semidiameters of the Earth; for so much is left when the greatest distance of the Moon is subtracted from the least distance of the Sun.

Since, therefore, no void can be granted between the heaven of the Moon and the heaven of the Sun — for nature abhors a vacuum — and since it is not consonant with reason that the deferents of the apogees of the Sun and Moon should be endowed with so great a bulk (for such a great mass would be utterly useless and superfluous), then with the fullest right and fittingness that whole intermediate space will be assigned to the orbs of Mercury and Venus. And so the Sun will be placed in the middle of the planets — namely above the Moon, Mercury, and Venus, and below Saturn, Jupiter, and Mars.

There is also this, that the motion of the Sun is the rule and measure of the motions of the other planets, though in one way and another. For Mars, Jupiter, and Saturn agree with the Sun in motion by reason of their epicycle; the Moon, Mercury, and Venus, however, are conformed to the motion of the Sun in their deferent orbs, as is explained in the Theories of the planets. Wherefore not unjustly the Sun will be placed in the middle of these, so as to separate the three upper planets from the three lower, since they do not observe uniformity of motion with it in the same way.

To these reasons may be added that the Sun is the king, and as it were the heart, of all the planets; wherefore not without merit it will be established in the middle of them, just as a king is placed in the middle of his kingdom, and the heart in the middle of the living creature, so that it may

be able to succor and provide for all the members equally. So that in a certain way (as most jest) a commonwealth is constituted from the seven planets. For the Sun is king of all; Saturn, on account of his old age, is his counselor; Jupiter, for his magnanimity, is judge of all; Mars is commander of the army; Venus, dispenser of all good things, in the likeness of a matron of the household; Mercury is his scribe and chancellor; and the Moon, finally, performs the office of messenger. Whence too it has the swiftest motion from west to east, so that each month it may carry the king's commands to any one whatever.

Moreover, since according to the astronomers and philosophers all the stars and planets receive their light from the Sun — at any rate more perfectly, as we clearly see in a lunar eclipse, in which the Moon, by entering the shadow of the Earth, loses its light; and besides is illuminated in different ways by the Sun at different times: for now it appears horned, now half-illuminated, now it seems full, and so on — which would not happen if it had light of its own. Form the same judgment about the others; for they are of the same nature as the Moon. This can also be proved from the fact that we see the planets that are nearer the Sun more intensely illuminated, as appears in Mars and Venus. Wherefore, in order that the Sun might impart its light equally to all the planets and stars, it is most conveniently placed in the middle of them.

Albumasar [Abū Ma'shar] adds, in his *Great Introduction* (treatise 3, difference 3), that on this account glorious God placed the Sun, the noblest of the planets and the most active, in the middle of the other planets: because, if it had been set immediately below the eighth heaven and above Saturn, it could not, on account of its too great distance from the Earth, conveniently act upon these lower things — indeed all these lower things would in a manner grow cold; whereas, if it had been placed immediately above the Moon, it too would not act conveniently enough upon these lower things by its motion, because it would then be carried too slowly from east to west, on account of its too great distance from the *primum mobile*. Just as, in any wheel, those parts which recede more from the circumference and approach more toward the center or axis are moved more slackly. Add that then the Sun, on account of its too great nearness to the Earth, would burn up all these lower things. For which reason it is fittingly placed in the middle of the planets, so that it may have its action tempered and better fitted to these lower things. So that not without reason, in Ovid (*Metamorphoses* II), Phoebus warned Phaethon, about to mount the chariot of the Sun rashly, saying:

*Altius egressus cœlestia signa cremabis: Inferius terras: medio tutissimis
ibis.*

("Going too high, you will scorch the heavenly signs; too low, the lands; in the middle you will travel most safely.")

For Ovid meant, in that place, to signify that the Sun has its action tempered in the middle station of the planets, and not in any other, and that therefore its proper place is there.

That Mercury too is fittingly placed immediately above the Moon and below Venus, its irregular motion seems to persuade us. For Mercury is much more irregular in its motion than Venus — on account of which the astrologers assigned to Mercury five orbs and an epicycle, but to Venus only three orbs and an epicycle. It seems, therefore, consonant with reason that Mercury should be established above the Moon rather than Venus.

[Margin: That Mercury is fittingly placed above the Moon and below Venus.]

The order of the planets, moreover, which we have hitherto approved, all the ancient founders and namers of the week seem to confirm. For they imposed names on the days from the planets — each day, that is, from the planet which obtains dominion over the first hour of that day. For the individual planets are said by astronomers to preside, in their own order, over the individual hours of the day; and how true this is, this is not the place to discuss. Hence, since a day contains 24 hours, it is necessary that if on the Sabbath Saturn dominates the first hour — from which the day of Saturn [Saturday] is named — then on the following day the first hour will be dominated by the planet next in retrograde order, two being passed over, namely the Sun, from which the day of the Sun [Sunday] is named.

[Margin: The order of the planets is confirmed from the dominion of the planets and the naming of the days.]

For if Saturn dominates the first hour, the second will be dominated by Jupiter; the 3rd Mars; 4th Sun; 5th Venus; 6th Mercury; 7th Moon; 8th Saturn; 9th Jupiter; 10th Mars; 11th Sun; 12th Venus; 13th Mercury; 14th Moon; 15th Saturn; 16th Jupiter; 17th Mars; 18th Sun; 19th Venus; 20th Mercury; 21st Moon; 22nd Saturn; 23rd Jupiter; 24th Mars. Then the first hour of the following day is the Sun, and so on in succession. From this it is plain why the days are not named according to the order of the planets immediately, but always according to the retrograde order, two being passed over, because in this order the planets preside over the hours of the day — an order of days which would by no means be such, unless the planets were located in that order. On this matter two verses exist, that it may

be known over which hours of the day each planet holds dominion; in which it also appears what order they have among [themselves] [?]:

*Cynthia, Mercurius, Venus, & Sol, Mars, Ioue, Satur[nus], Ordine
retrogrado sibi quiuis vendicat horam.*

("Cynthia [the Moon], Mercury, Venus, and the Sun, Mars, Jupiter, Saturn — in retrograde order each claims its own hour.")

Ioannes Xiphilinus, from Book 36 of Dio's [Cassius] *Pompey*, writes that this order of the days was instituted by the Egyptians, whom he says established the aforesaid order in the planets. He then adds another reason for this naming of the days, from the consonance of Music which is called διὰ τεσσάρων [Greek: *diatessarōn*, "through four" — the interval of the fourth, the *diatessaron*], which according to the ancients was believed to be the foundation of all Music. For on account of this consonance and harmony, that the days might in a musical manner in some way agree with the ornament of the heaven, after one day had been named from one planet, they named the following day from the fourth planet after it, but in retrograde order — as after Saturn there follows in the fourth place the Sun, then the Moon, then Mars, and so on.

It is established, therefore, from all those things which we have said, that the order prescribed by our Author among the planets is truer and more conformable to the skilled astronomers. To be exploded, therefore, is the opinion of Metrodorus and Crates, who placed the Sun and Moon as the highest of the planets. To be rejected also is the opinion of Democritus, who made Mercury higher than the Sun. The view likewise of Alpetragius [al-Bīṭrūjī], who thought Venus higher than the Sun, is of no moment. And, finally, let the opinion of Plato and Aristotle fall — those who placed the Sun and Moon in the lowest station.

But some object that the Sun never suffers eclipse from Mercury and Venus, and that therefore the Sun is by no means to be established above them: for otherwise it would sometimes be concealed by them, just as we see it concealed by the Moon, since the Moon is placed above it. Nevertheless this objection has no force. For, as Ptolemy says (*Almagest* [Dict.] IX.1) and Regiomontanus (*Epitome*, Book 9, proposition 1), two planets can be in conjunction — that is, be in the same degree of the zodiac — in such a way that a straight line issuing from the eye and passing through the center of one does not pass through the center of the other; which is nevertheless required for an eclipse. Hence it comes about that we very often see the Moon in conjunction with the Sun at the new moons, yet not concealing it.

Moreover, according to Albategnius [al-Battānī] and Thebit [Thābit ibn Qurra] and other astronomers, the visual diameter of the Sun, to the

[Margin: Why the Sun is not eclipsed by Mercury and Venus, although they are below it.]

[Margin: What the visual diameters of the heavenly bodies are.]

the visual diameter of Venus (the visual diameters being those of the circles that appear to us in the case of the stars) has a tenfold proportion [decuplam]. Hence, according to the geometrical demonstrations, the visual circle of the Sun will have to the visual circle of Venus a hundredfold proportion [centuplam].

[Margin (a): Euclid, *Elements* XII.2] [Margin (b): Euclid, *Elements* VI.20]

(a) For circles have between themselves the same proportion as the squares of their diameters; and (b) the proportion of the squares described on the diameters of the circles is the duplicate of the proportion the diameters have. It follows that when the visual diameters of the circles of the Sun and of Venus have a tenfold proportion, the squares of the diameters — and therefore the visual circles as well — have a hundredfold proportion. For the hundredfold is the duplicate of the tenfold, as is plain in these numbers 1, 10, 100, which stand in a continued tenfold proportion.

For, as is established from Definition 10 of Euclid Book 5, when there are three magnitudes in continued proportion, the third is said to have to the first a proportion that is the duplicate of the proportion which the second has to the first, or the third to the second. Therefore, since the said three numbers 1, 10, 100 are in continued proportion in a tenfold ratio, the hundredfold proportion that the third number, 100, has to the first, 1, is the duplicate of the tenfold proportion that the second number, 10, has to the first, 1 — or the third, 100, to the second, 10. From this it follows that the visual circle of the Sun has to the visual circle of Venus a hundredfold proportion, since the diameters of those circles have a tenfold proportion, and the circles have the duplicate of the proportion that the diameters have, as has been said.

By the same reasoning, if the diameters of two circles have a twofold proportion, the circles themselves will have a fourfold proportion; for the fourfold is the duplicate of the twofold, as appears in these numbers 1, 2, 4, in continued proportion in a twofold ratio. So too, if the diameters of two circles have a hundredfold proportion, the circles will have the proportion of 10000 to 1, as is clear in these three numbers 1, 100, 10000, which have a continued hundredfold

proportion. By this method we shall know the proportion of any circles whatever, once the proportion that their diameters have is known.

To find out easily which proportion is called the duplicate of another, one must multiply the denominator of the proportion by itself, for the product will be the denominator of the duplicate proportion. For example, since the denominator of the tenfold proportion is 10, if 10 is multiplied by 10, 100 is produced — namely the denominator of the duplicate of the tenfold proportion. By the same reasoning, the duplicate of a threefold proportion will be ninefold [noncupla], and so on. On which matter, read what we have written on Definition 10 of Euclid Book 5.

From this it is clear that Venus can in no way cover the Sun, even if it comes between our sight and the Sun, since it would hide only a hundredth part of it, which is of no significance and can scarcely be noticed. Still less, then, will Mercury be able to do this, since its visual diameter is far smaller than the visual diameter of Venus.

[Margin: Why the Moon sometimes eclipses the Sun, even though it is much smaller than the Sun.]

But if someone asks why the Moon then sometimes snatches the Sun from our sight, even though the Moon is astonishingly smaller than the Sun itself, the answer is that this happens because of the Moon's very great nearness to the earth and its very great distance from the Sun. This makes the visual diameter of the Moon sometimes appear to us larger than the visual diameter of the Sun, and therefore the whole Moon is seen as larger than the Sun. So it is no wonder that the Moon can sometimes cover the Sun so that it cannot be seen.

Diagram. A geocentric diagram of the universe, printed as a set of concentric circles with the Earth at the center. The globe of the Earth is rendered as a small hatched woodcut roundel at the middle. Reading the labelled spheres from the outermost inward, each named on the arc at the left side of the figure (the planetary symbol is also repeated on the corresponding arc at the right side):

- Cælum Empyreum. — the Empyrean Heaven (outermost)
- Primum mobile. — the Prime Mover (First Movable)
- Decimum cælum. — the Tenth Heaven
- Nonum cælum. — the Ninth Heaven
- Firmamentum. — the Firmament (the sphere of the fixed stars, drawn as scattered star-asterisks filling this and the neighbouring rings)

- ♄ Saturnus. — Saturn
- ♃ Iuppiter — Jupiter
- ♂ Mars — Mars
- ☉ Sol — Sun
- ♀ Venus — Venus
- ☿ Mercurius — Mercury
- ☾ Luna. — Moon (innermost sphere)
- (center): the Earth, shown as a small hatched globe

From all this, both the number and the order of all the bodies of the whole universe can be gathered.

[Margin: The number and order of all the bodies composing the universe.]

For in the whole universe there will be sixteen spherical bodies making up the entire world, arranged in the order shown partly in the treatise on the elements and partly here in the treatment of the celestial bodies — namely the four elements and the twelve celestial spheres. This the figure set alongside seems to indicate clearly: in it you see the order and position of the whole universe, together with the symbols of the planets by which astronomers are accustomed to represent and depict them.

[Margin: That there is nothing outside the world.]

Beyond this world, however — that is, beyond the empyrean heaven — there exists no body at all, but a certain infinite space (if it is permissible to speak so), throughout the whole of which God too exists by his essence, and in which he could fashion infinitely many other worlds, even more perfect than this one, if he willed, as the theologians assert.

THAT THE HEAVEN MOVES FROM EAST TO WEST

Lemma (Sacrobosco). That the heaven turns from east to west, there is a sign. The stars that rise in the east are always lifted up, gradually and one after another, until they come to the middle of the sky; and they always keep the same nearness and distance from one another, and so remaining ever the same, they move on to their setting continuously and uniformly.

[Margin: That the heaven moves from east to west is proved from the stars that rise and set.]

COMMENTARY.

This is the fourth and last part of this first chapter, in which the author sets out six propositions concerning the ethereal and the elemental region — propositions that in the preceding part he seemed to assume as certain and beyond doubt. The first is that the heaven moves from east to west. The second, that the heaven is round. The third, that both the earth and the water are round. The fourth, that the earth is the center of the world. The fifth, that the earth is motionless. The sixth and last, that the earth has an absolute and finite magnitude, and indeed a known one, although to ordinary people it seems immense.

[Margin: What is treated in the remaining part of this chapter.]

For the astronomer must have the magnitude of the earth ascertained, since it is by means of it that the magnitudes of the heavens and the stars are known.

Now, as regards the first proposition: someone might deny that the heaven moves from east to west and claim instead that the stars move by themselves — like fish in the sea or birds in the air — while the heaven remains entirely at rest, as many have dared to assert. Against this, the author proves by a twofold argument that it is not true. One of these arguments is drawn from the stars that rise and set for us; the other from the stars that never rise or set for us, but are perpetually visible. These arguments are taken from Ptolemy, *Almagest* I.3 [Dict. 1, cap. 3], and from Johannes Regiomontanus [Regiomontanus], Book 1, Conclusion 1.

The first argument runs as follows. All the stars that rise and set for us move gradually, always at the same distance and in the same position relative to one another, from the east through the meridian to the west. Therefore the stars fixed in the heaven move with the motion of the heaven, like a nail with the motion of a wheel, or a knot with the motion of a board. The antecedent is manifest from daily experience; the consequence is clear, because if the stars moved by themselves they would not always keep the same distance and arrangement among themselves, nor would they always proceed uniformly, but sometimes one would outstrip another — especially since they are unequal among themselves and describe unequal circles. For it would seem rash to assert that the smaller stars have the same motive power as the larger ones.

Lemma (Sacrobosco). There is yet another sign. The stars that are near the Arctic pole, which never set for us, move continuously and uniformly about the pole, describing their circles, and are always at an equal distance and nearness to one another. Hence, from these two continuous motions of the stars — both of those that tend toward setting and of those that do not — it is clear that the firmament moves from east to west.

[Margin: That the heaven moves from east to west is proved from the stars that neither rise nor set.]

COMMENTARY.

He puts forward the second argument roughly to this effect. The stars near the Arctic pole, which never set for us, by their always-uniform motion describe in the same time different circles — some larger, namely those farther from the pole, others smaller, namely those nearer the pole — and are always seen at the same nearness to one another. Therefore they move not by themselves, but with the motion of the sphere of which they are parts. For if they advanced in the heaven by their own power and by themselves, then those describing larger circles would move in a longer time, and those describing smaller circles in a shorter time; indeed, unequal stars placed on the same circle would move unequally — all of which is contrary to the senses and to experience.

[Margin: Two further observations, by which it is concluded that the heaven moves, and not the stars themselves.]

No less force in persuading us that the heaven moves from east to west, and by its motion carries all the stars around with it, is possessed by two observations, which I shall now bring forward. The first is taken from the Milky Way [via Lactea]. Since this is either an infinite multitude of the tiniest stars, or — what I rather approve — a denser and continuous part of the eighth heaven (though not uniformly dense), how could that whole band of brightness, encircling the entire heaven, advance so regularly from east to west, unless it were carried around by the motion of the eighth sphere in which it lies?

The second observation concerns the rarer parts of the heaven, of which not a few are seen near the Antarctic pole (as a certain learned and devout man, living in the province of Peru — which has the Antarctic pole elevated above the horizon — attests in a booklet that he entitled *On the Situation and Nature of the West Indies*), so that in many places of the heaven a certain blackness appears, as if the heaven were somehow pierced through. Since these rarer parts are carried uniformly, along with the stars, from east to west in the space of 24 hours — as has more than once been observed by those living in that region of the earth — who would say that they move by themselves, and are not rather carried around by the motion of the heaven, seeing that they are not stars but wholly rare and dark parts? For what would drive those parts, if they were not carried around together with the heaven? Since this is so, it is likely that the whole heaven is driven from east to west, dragging along with it both the stars and the other,

denser parts (such as those that make up the Milky Way) and the rarer or darker parts, and...

of which we have just spoken, and such as are also those spots which are seen on the Moon and are carried around uniformly with the Moon.

[Margin: Aristotle's argument proving that the stars do not move by themselves.]

Aristotle, in Book 2 of *On the Heavens* [*De caelo* II], likewise proves that the stars do not move by themselves, by this argument. If the stars move by themselves while the heaven is at rest, then either they are fixed in the heaven, or else they are on the outermost surface of the heaven — whether its concave or its convex surface — so that there is some space interposed between any two heavens in which the stars can move. If they are fixed in the heaven, there will result either a splitting of the heaven or an interpenetration of bodies, both of which are impossible. But if they move on the outermost surface of the heaven — as, for example, a man on a pavement, or a fly or an ant on some ceiling — then the space in which they move will be either a void, which Aristotle long ago banished from nature in Book 4 of the *Physics*, or a body. And this body will be either celestial — in which case the first difficulty follows again — or elemental, which cannot remain perpetually outside its natural place; yet it would be outside its place, if it were there. Therefore the stars do not move by themselves.

Aristotle brings other arguments in the passage cited, but leaving these aside, we can confirm this conclusion of our author by a single observation, which in my judgment has the greatest force.

[Margin: A convincing argument that the stars in fact do not move by themselves, but with the motion of the heaven.]

Take any star you like, whether fixed or wandering, which someone claims moves by itself. This star moves with motions that are in a way opposite, as we said above. For it moves absolutely and continuously from east to west, and at the same time, in a certain respect, also continuously from west to east, as was set out and demonstrated above. But no one and the same body can be set in motion by different — indeed opposite — motions at the same time. For it involves a contradiction that one and the same body should advance simultaneously from east to west and at the same instant from west to east, with neither motion interrupting the other, but each proceeding uniformly without any break — unless it be moved by the one motion as by the vehicle of the other.

Therefore the stars do not move as free and detached from the celestial bodies, because a single body can move with only one motion at the same time — as we plainly see in animals and in other things that we push from one place to another. For it cannot happen that at the same time they are driven by another force in the opposite direction, unless the first motion is halted or interrupted. Rather, they are carried along with the motion of the spheres in which they lie; for in this way one and the same heavenly body can be set in motion by different displacements, as was explained above, with various examples adduced.

This very point is confirmed all the more in the case of the planets. For they move with still more motions than those two — from east to west and from west to east; and now they seem to move more swiftly from west to east, now more slowly. Sometimes they seem to stand still, sometimes to move backward toward the west, and so on, as is explained in the *Theorics of the Planets* [*Theoricae planetarum*]. If, then, the stars moved by themselves, no sufficient explanation of this variety could be given; but if they are said to move with the motion of the heaven, all the appearances are readily accounted for, as will be explained in the *Theorics of the Planets*.

[Margin: The opinion of those who say the stars move in channels, and its refutation.]

Seeing, then, that on this view the multiplicity of motions in the stars cannot be accounted for, some have devised another theory, by which they try to persuade us that the stars move by themselves and are not fixed in the celestial bodies. For they say that there is only a single heaven, and that this heaven moves with a single motion from east to west, together with all the stars; but that the stars are carried by their own motions from west to east, detached (as they say) from the celestial spheres — not, however, like fish in the sea or birds in the air (lest there be an interpenetration of bodies or a splitting of the heaven), but through certain channels [canales].

For they have imagined that each individual star has its own channel matching its proper motions, of such breadth as the star's own size, so that each star fills its whole channel. In these channels, moreover, they have placed a certain fluid body, like air, which can give way to the stars as they move from west to east. And so, according to these authors, the whole heaven will be crammed with these channels, in proportion to the multitude of the stars — like an animal, which is filled with various and manifold veins. They embrace this opinion all the more readily because they are unwilling to admit a motion of being swept along

[motum raptus]; for they say that it is impossible for one heaven to sweep another along, however contiguous it may be to it.

Nevertheless, this opinion is both absurd and inadequate. Absurd, because without any necessity or plausible reason it supposes the celestial body to be pierced through by so many channels and filled everywhere with that fluid body — something that no philosopher has hitherto been seen to concede. Inadequate, because on this view it is impossible to account for all the phenomena in the celestial motions which astronomers have most carefully observed.

For, first, whether they wish it or not, they cannot avoid a motion of being swept along. For since the stars are (as they themselves say) detached and free, and in no way inhere in the heaven, and yet move with the motion of the heaven from east to west, they must be swept along by the heaven without any resistance or violence — for this reason alone, that they are contiguous to the channels in which they lie.

Second, although on this view a twofold motion — namely from east to west and, conversely, from west to east — can somehow be defended, yet no star can in any way have more motions than these two, for the reason we adduced above against those who claimed that the stars move of themselves. Since, therefore, more motions have been detected in the Moon — six, at the least — and the same must be said of the other planets, and indeed the fixed stars have a fourfold motion, as we showed above, this opinion can in no way be true.

Third, the planets, as is clear from the *Theorics of the Planets*, are not always equally distant from the center of the earth, but appear now nearer, now farther away — which could in no way happen if the stars moved by themselves in the said channels, unless one were to say that those channels are eccentric to the world, so that one part recedes farther from the center of the world and another comes nearer to it: which cannot be said. For since those channels are fixed in the celestial body, it would necessarily follow that any given planet would always be at its greatest distance from the earth in the same part of the heaven, and so on — which is utterly false; for the Moon has at one time or another been seen at every point of the zodiac both at its farthest from the earth and at its nearest.

I pass over the appearances concerning the variation of the latitudes of all the planets (the Sun alone excepted), and also concerning retrogradation, and so on — which the aforesaid opinion can in no way uphold, as is usually explained more fully in the *Theorics of the Planets*. It is established, therefore, that the stars do not move by themselves, but with the motion of the heavens in which they are

fixed; for in this way the heavens can be understood to have several motions — one indeed proper to them, but the others extrinsic, namely serving as the vehicle for the rest, as was explained above. Hence it is no wonder that so great a multitude of motions is observed in the stars.

[Margin: The opinion of the ancients, who said that the stars move with a rectilinear, not a circular, motion, and its refutation.]

Ptolemy, in *Almagest* Book 1 [Dict. 1], reports the opinion of certain people who said that the stars do indeed move with the motion of the heaven from east to west, but by a rectilinear motion extending to infinity, not by a circular motion. This opinion is utterly ridiculous, and therefore to be rejected by astronomers. First, because on this reckoning one and the same star would not appear to us at the same nearness, but would come closer to us at the meridian than at its rising or setting...

set (which is false). Again, we see every day the same stars, by number, return to the East after they have lain hidden for a while beneath the earth. This could never happen if they were carried along in a straight line. From all this, then, it can be plain to anyone that the heavens themselves move together with the stars fixed in them, from East to West, in a circular motion; and the same must be said of the motion from West to East that the lower spheres possess.

THAT THE HEAVEN IS OF SPHERICAL SHAPE

[*Lemma*] That the heaven is round there is a threefold reason: Likeness, Convenience, and Necessity. Likeness, because the sensible world was made in the likeness of the archetypal world, in which there is neither beginning nor end. Hence, made after the likeness of that world, the sensible world has a round form, in which one cannot assign a beginning or an end.

[Margin: That the heaven is round, by reason of its likeness to the archetypal world.]

COMMENTARY

Here the author proves the second Conclusion — namely, that the heaven is round — by three means, of which the first is drawn from likeness, the second from convenience, and the third from necessity. From likeness he argues as follows. This sensible world was fashioned in the likeness of the archetypal world, that is, of God, the Best and Greatest, in whom one can assign neither beginning nor end, since He is infinite. The world must therefore be round, so

that no beginning or end can be assigned in it; for in this way it will be in some fashion like that archetypal world, since the round figure alone, among all the others, possesses in a certain manner infinity.

But this argument seems to prove nothing at all. For by the same reasoning it would be proved that man ought to have been created round, in the likeness of the archetypal world; and you would say the same of all other creatures. Nevertheless, one must answer with St. Augustine [B. Aug.] that God founded His creatures for the manifestation of His goodness and perfection. Since, then, a single creature displays the perfection of God to us only most imperfectly, God made round rather the whole universe — in which all creatures are contained, and which more effectively and more exactly manifests and declares the perfection and goodness of God — than He made the individual creatures round; although the individual creatures too, so far as can be, everywhere imitate the round figure, as appears in the trunks of trees, in their branches, in the extremities of the limbs of animals, and in fruits. For all these are in some fashion round — yet not entirely so, in order that there might be greater beauty and splendor in so great a variety of creatures. From this answer, then, it is clear that our author is chiefly proving that the world, or heaven, is round as regards its convex surface — which indeed is enough, for we are accustomed to judge the figures of bodies from their convexity. A little later, however, we shall confirm that all the heavens are round, both as regards the concave and as regards the convex.

[Lemma] Convenience, because among all isoperimetric bodies the sphere is the greatest, and among all forms the round is likewise the most capacious. Since, therefore, it is at once greatest and round, it is for that reason the most capacious; and so, since the world contains all things, such a form was useful and convenient for it.

[Margin: That the heaven is round, by reason of convenience.]

COMMENTARY

The argument drawn from convenience is roughly this. This world contains all things within itself; it therefore had to be granted the figure most useful and convenient to this end — namely, the one most capacious of all — for Nature, avoiding fault, aims as much as possible at convenience. But the sphere is the greatest and most capacious among all corporeal isoperimetric figures. Therefore such a figure was rightly granted it by nature.

Yet this argument too, taken simply, seems to prove nothing. For someone might say that, although among isoperimetric bodies the sphere is the most capacious, as the argument holds, God could nonetheless have made the world of another figure, larger than it now is, so that it would contain all things within itself just as well as it now does. But since God and Nature do nothing in vain, and always produce what is better, it seems consonant with reason that the world was founded round by God — inasmuch as the round figure is the most capacious and the noblest — especially since that excess of any other, larger figure would seem superfluous, and established with no reason at all, that is, without necessity.

We can also add another argument from convenience. Since Nature always strives to bring about what is better, she seems with the best right to have granted to the celestial body — which is the noblest of all — the noblest figure, namely the round or spherical, for many reasons. For just as the Circle holds the primacy among plane figures, so the Sphere holds it among solids. Just as the Circle, by its simplicity, by the likeness, equality, sameness of place, strength, and capacity of its parts, excels all other plane figures, so too must one speak of the sphere, if it be compared with the other solid figures. First, a single line encloses the circle, and a single surface encloses the sphere. Second, just as in the circle there are arcs uniformly curved, so in the Sphere there are portions uniformly convex. Third, just as in the circle the center is equally distant from the extremes, whence its length and breadth are measured by equal diameters in every direction, so it holds also in a spherical body, whose length, breadth, and depth three equal diameters measure toward every part. Fourth, just as in the circle, so in the sphere we can find neither beginning nor end. Fifth, just as the circle, so also the sphere, revolved about its center, always occupies the same place; whence to both circle and sphere is granted, in the highest degree, ease of motion and firmness of parts, nothing external opposing. Sixth and last, each figure — the circular as well as the spherical — is, among isoperimetric figures, the most capacious: plane figures, if we speak of the circle; solid, if the discussion concerns the sphere — as we have shown in our *Geometria practica*. There is this too, that the circle touches a straight line, and the sphere touches a plane surface, in a single point only; of which the former is clearly gathered from Propositions 2 and 16 of the third book of Euclid, and the latter was demonstrated by Theodosius in Proposition 3...

[Margin: Another argument, from convenience, proving that the heaven is round.] [Margin: The various excellences of the circle and the sphere.]

(Proposition 3 of the first book of his *Elements of Spherics*) most clearly. Since, then, a spherical body stands out among all others as so noble, on account of so many and so remarkable excellences and superiorities, who can now doubt or hesitate that the heaven is endowed with such a figure? — especially since the heaven, as was said in the preceding Conclusion, is continually turned in circular motion; to which motion a spherical body is, among all the rest, the most suited, on account of the continuous and uniform succession of its parts, so that nothing external can be an impediment; and this because it is carried around the center within the same bounds of place at all times. Hence too it moves most easily.

[Margin: What an isoperimetric figure is.]

Now, so that this second argument of the author, drawn from convenience, may be more perfectly understood, a few things must be said about isoperimetric figures. Figures are called Isoperimetric when they have circumferences, or bounding lines, equal to one another. Thus a square having six palms in its circuit is said to be isoperimetric with a triangle, or with any other figure whatever (whether it be rectilinear, or curvilinear, or mixed of these), having six palms likewise in its circuit; so that the four straight sides of the square's perimeter, joined together into one and the same straight line, are exactly equated to the three straight lines of the triangle, or to all the sides of whatever other figure, these too being set out straight and continuous. The same must be understood of any isoperimetric bodies whatever, taking surfaces in place of lines.

[Margin: Among rectilinear isoperimetric figures, the more capacious is the one that has more angles; and therefore the circle is the most capacious. (a) Euclid I.8. (b) Euclid I.4 or I.38. (c) Euclid I.34. (d) Euclid I.34. (e) Euclid I.19.]

Among all regular rectilinear isoperimetric figures, the one that contains more angles is the greater and more capacious. This we shall confirm briefly, and with a certain rough treatment, in the equilateral, or isosceles, triangle, and in a figure longer on one side. More accurately and more fully we have dealt with this matter in our *Geometria practica*. Let there be, then, an equilateral or isosceles triangle ABC, whose side BC is divided into equal parts at the point D, and let the straight line DA be drawn, which will be perpendicular to BC. For the two sides AD, DC of the triangle ADC — and the base AB is posited equal to the base AC — and on this account (by definition) each [angle at D] is a right angle. Let the rectangular parallelogram ADCE be completed. **(a)** The two angles ADB, ADC will therefore be equal. **(b)** Since, then, the triangle ADB is equal to the triangle ADC, **(c)** and to this same triangle ADC the triangle ACE is equal, the triangles ADB and ACE will be equal to one another. Therefore, adding the common triangle ADC,

the parallelogram ADCE will be equal to the triangle ABC. And since the two sides AE, DC of the parallelogram **(d)** are equal to one another, taken together they are equal to the side BC of the triangle ABC; while the remaining two sides AD, CE of the parallelogram ADCE (because they are opposed to the smaller angles, namely the acute ones, in the triangles ABD [and ACE]) **(e)** are smaller than the remaining two sides AB, AC of the triangle ABC (because these in the same triangles are opposed to the larger angles, namely the right ones). The perimeter of the parallelogram ADCE will therefore be smaller than the perimeter [of the triangle]. Hence, that the perimeter of the parallelogram may be made equal to the perimeter of the triangle, the sides DA, CE must be produced to the equality of the sides AB, AC.

Diagram. A geometric construction. An isosceles triangle stands on base BC (B at lower left, C at lower right), with D the midpoint of the base and A the apex, drawn toward the upper left. Perpendicular AD rises from D to A. Above, a rectangle is constructed and extended, its corners labeled G (upper left) and F (upper right), with A and E along its lower edge. The lines AB, AD, AC are drawn from the apex region to the base.

- G (top left): corner of the extended rectangle
- F (top right): corner of the extended rectangle
- A (left, apex region): apex of the triangle / rectangle corner
- E (right of A): rectangle corner
- B (bottom left): base vertex
- D (bottom middle): midpoint of base BC
- C (bottom right): base vertex

Let there be, then, the straight line DAG, equal to the side AB, and the straight line CEF, equal to the side AC, and let the straight line FG be drawn. From these we shall bring it about that the parallelogram CFGD and the triangle ABC are isoperimetric. But since the parallelogram CFGD exceeds the parallelogram ADCE by the quantity AEFG, and it has been shown that the parallelogram ADCE is equal to the triangle ABC, the parallelogram CFGH [recte: CFGD] will also be greater than the triangle ABC by that same quantity AEFG. Therefore the quadrilateral figure is more capacious than the triangular figure isoperimetric with it — which was to be shown. Since the same reasoning appears to hold in the other rectilinear figures of more sides, isoperimetric with the given figure, the more sides such a figure has, in the more places its sides recede from the center and the middle; and this leads us to the circle, which in a certain manner includes infinite angles and sides, and is, of all isoperimetric figures, the most capacious.

The same must be said of the sphere, if it be compared with other bodies isoperimetric with it.

[Margin: Among isoperimetric rectilinear figures having an equal number of sides, the more capacious is the one that is both equilateral and equiangular, an equal number of sides being posited in each; and therefore the circle is the most capacious.]

Again, of rectilinear isoperimetric figures having an equal number of sides, the greater is the one that has both its sides equal and its angles equal. For let there be some square having 6 on each side, so that its whole perimeter contains 24. The area of this square, according to the precepts of the Arithmeticians, will be 36. For thus you see that the whole square is divided into 36 little squares. Let there be also some rectangular parallelogram having each of two opposite sides 10, and each of the remaining two 2, so that its perimeter is equal to the perimeter of that [square]... this parallelogram will comprehend only 20 little squares out of those 36 which we said were contained in the square. By like reasoning, if each of two opposite sides of some parallelogram were 9, and each of the other two 3, it would indeed be isoperimetric with the square, but its area would contain only 27 little squares. Again, if two opposite sides of some parallelogram each had 8, and the other two [each] 4, it would be isoperimetric with the square, but its area would contain only 32 little squares. Again, if two opposite sides of some parallelogram each had 7, and the other two...

Diagram. A group of grid figures, each ruled into unit squares, illustrating isoperimetric rectangles of perimeter 24. All bear the same perimeter as the reference square but enclose progressively smaller areas as they depart from equilateral:

- A square, 6 by 6 (area 36 little squares), labeled 6 on top, left, and bottom edges.
- A rectangle, 10 by 2 (area 20), labeled 10 (top) and 2 (side).
- A rectangle, 9 by 3 (area 27), labeled 9 and 3 [?].
- A longer rectangle, 12 by 3 (area 36, but perimeter 30 — of equal area to the square yet larger perimeter), labeled 12 (top and bottom) and 3 (each side).

each 5 [i.e., two opposite sides each 5, the other two each 7], it would still be isoperimetric with the square, but its area would include only 35 little squares, and so on. Here you see clearly that the more isoperimetric figures approach the equilateral figure with which they are isoperimetric, the greater the area they

comprehend, and the less they differ in capacity from the equilateral figure. But if some rectangular parallelogram, longer on one side, be of the same capacity as the square, it must necessarily contain a larger perimeter. Thus, if each of two opposite sides of some parallelogram contained 12, and each of the other two 3, its area would indeed be equal to the area of the square, since it would contain 36 little squares; yet its perimeter would exceed the perimeter of the square: for the former will be 30, this latter 24. All of which is plain in the figures set alongside.

[Margin: Euclid I.35.]

Now let there be a parallelogram of unequal angles, ABCD, and from the points C, D let the perpendicular lines CF and DE be drawn to the straight line CD. Then, AB being produced as far as F, **(f)** the parallelogram ABCD will be equal...

to the parallelogram CDEF, since these parallelograms are constructed between the same parallels CD, AF, and upon the same base CD. **(b)** And since the sides BC, AD are larger than the sides CF, DE, and the side AB is equal to the side EF **((c))** because each is equal to the opposite side CD, in the parallelograms ABCD, CDEF), and the side CD is common, the perimeter of the parallelogram CDEF will be smaller than the perimeter of the parallelogram ABCD. Hence, if CF, DE are produced to G and H, in such a way that CG is equal to BC itself, and DH to AD itself, and the parallelogram CDHG is completed (that is, the straight line GH being drawn), the parallelogram CDHG will be isoperimetric with the parallelogram ABCD. But the parallelogram CDHG is greater than the parallelogram CDEF — that is, than the parallelogram ABCD — by the quantity EFGH. It is therefore established that, among isoperimetric rectilinear figures, the one which is both equilateral and equiangular is the greatest of all. For the same reasoning must be applied to the isoperimetric figures that contain more sides and more angles. Wherefore, since the circle comprehends very nearly infinite equal sides, and likewise, in a certain manner, infinite equal angles — because its circumference is always curved uniformly — it comes about that it is the most capacious among all isoperimetric figures. And it is chiefly by these arguments that some authors endeavor to confirm that the circle is the most capacious. From all this I judge it made manifest what our author means in this second argument drawn from convenience, in which he made mention of Isoperimetric figures.

[Margin: b. Euclid I.19. c. Euclid I.34.]

Diagram. A geometric construction of two parallelograms on a common base. Along the top edge, from left to right, are labeled A, H, B, G; slightly below H and

G are E and F; at the bottom, D (left) and C (right). The oblique parallelogram ABCD and the constructions CDEF and CDHG share the base DC.

- A (top left): vertex
- H (top, right of A): construction point
- B (top middle): vertex
- G (top right): construction point
- E (below H): construction point
- F (below G, right): construction point
- D (bottom left): base vertex
- C (bottom right): base vertex

[Lemma] Necessity, because if the world were of another form than round — that is, three-sided, or four-sided, or many-sided — two impossibilities would follow: namely, that some place would be empty, and that a body would be without place; each of which is false, as is plain in the case of elevated and turned-about angles.

[Margin: That the heaven is round is proved from necessity.]

COMMENTARY

From necessity he confirms that the heaven is round as follows. The heaven, as has been shown, moves; if, then, it were not of round figure, but many-sided — three-sided, that is, or four-sided, and so on (by the name "three-sided figure" understand the pyramidal, and by "four-sided" the cubic) — two impossibilities would follow: one, that there would be some place without a body; the other, that a body would be given without a place — each of which is at war with the nature of things. It is necessary, therefore, that the heaven be round. The consequence is manifest from the elevation and depression of the angles of any many-sided figure whatever, if it were to move about the center.

[Margin: That the heaven is round is proved from necessity.]

This argument concludes only that the heaven is round in some fashion — that is, not angular — on account of those unsuitable consequences to which the Author is led if it were of angular figure; it does not, however, simply gather from this that the heaven is spherical. For someone might say that it is of oval figure, or lenticular, conical, or cylindrical. For if the heaven be supposed to be of any of these forms, all those absurdities will be avoided with ease; since, this being granted, the heaven can be so moved about its axis that its parts continually

succeed one another in the same places, just as we see happen in a spherical or globular body. Nevertheless it must be said that the aforesaid argument from necessity does conclude that the heaven is most perfectly spherical, and can in no way have another figure.

[Margin: The argument from necessity is confirmed.]

For the lower heavens, as was shown above, are moved with a motion opposite to the motion of the *primum mobile*, upon different poles from the poles of the *primum mobile*: but they could not be moved with this motion, if they were not spherical, unless there came about a penetration of bodies, or a splitting of the heavens — as is manifest to one who considers the matter more carefully; and each of these is impossible. Likewise the same absurdities brought forward by the Author against the angular figure would follow. For let there be an oval, upper orb — if it can be so — ABC, whose axis is ADC, its poles A and C; and let the inner orb, likewise oval, be EHFG, which, since the heavens according to all Philosophers are uniform as to their thickness and density, will be situated according to the situation and length of the upper orb, so that their lengths have the same diameter, as you see here. Let now the axis of the inner orb be GDH, about which it is moved from West to East. It is now manifest that, at the motion of the inner orb upon the axis GDH, the surrounding celestial body is cut apart and penetrated. For the part E will be carried, about the pole G, into the point I, and the part F, about the pole H, into the point K. Wherefore the parts E and F will be left empty, as you discern in the figure set before you.

Diagram. A schematic of two overlapping, lens-shaped (oval) orbs sharing the axis A–C, illustrating why a non-spherical heaven would require the surrounding body to be split. The outer oval spans from A (left) through B (top) to C (right), with D at the center; the inner oval is drawn within it. Hatched (shaded) crescents at upper left and lower right mark the regions that would be left empty. Point-letters cluster at the two ends of the minor axis.

- A (left): pole / axis end of the upper orb
- B (top): vertex of the outer oval
- C (right): pole / axis end
- D (center): common center
- E (upper left, interior): part carried toward I
- G (lower left): pole of the inner orb
- I (bottom left): point into which part E is carried
- H (upper right): pole of the inner orb

- F (lower right, interior): part carried toward K
- K (upper right): point into which part F is carried

We can also, with Ptolemy in the *Almagest* [in Dict.], confirm that the heaven is spherical from this: that we see all the fixed stars move always at the same distance and nearness to us; and that those which are nearer the poles describe smaller circles, while those which are more remote describe proportionally larger ones — which could in no way happen if the heaven were not round and spherical. For only in a spherical body are all the parts equally distant from the center. Hence, if the heaven were of another figure, some parts would be farther from us, some nearer, and on that account not all the stars would be seen at the same distance from us — which is at war with sense and experience. Again, all the instruments of the Astronomers agree with the motions of the celestial bodies exactly as if the heavens were most perfectly spherical; which indeed is manifest in the altitudes of the stars above the Horizon, which, before they reach the Meridian, increase in that proportion, and after they have passed the Meridian decrease, which we can assign only to a spherical body. The same is shown by all the other appearances, and most of all by the Solar clocks [horologia Solaria, sundials], which are constructed on the supposition of a spherical heaven. Finally, we see...

[Margin: Another argument, proving that the heaven is round and spherical.]

let us suppose two stars situated on the same circle of longitude drawn through the poles of the world, one of which is more southerly, and grant that it also has a smaller meridian altitude — so that the two differ in their meridian altitudes by exactly as many degrees as one star is found by instruments made for this purpose to be distant from the other. This reasoning carries great weight with me, since all instruments are built round, so as in some way to imitate the roundness of the heavens. If, then, the heaven were not spherical, these instruments could not, however they were turned, be made to agree with the celestial appearances as regards the altitudes and the mutual distances of the stars. Since we do see that they agree — as is observed above all in the material sphere, the celestial globe, the astrolabe, and the quadrant — we shall rightly conclude that the heaven is perfectly spherical; otherwise neither the instruments of astronomers nor the appearances themselves would make any sense.

[Lemma — Sacrobosco:]

[Margin: *Cælum non esse planum* — "That the heaven is not flat."]

Again, as al-Farghānī [Alphraganus] says: if the heaven were flat, one part of it would be nearer to us than another — namely the part directly above our head. A star located there would therefore be nearer to us than one at its rising or setting; but things that are nearer to us look larger. Hence the Sun, or any other star, when at mid-heaven ought to appear larger than when it is at its rising or setting. Yet we see the opposite happen: the Sun, or another star, appears larger when it is in the east or west than at mid-heaven.

COMMENTARY

Our author confirms this same conclusion by an argument of al-Farghānī, which the latter sets down in his second Difference [Distinction], in the following way. If the heaven were not round but flat or extended, then the part of the sky overhanging our head would be nearer to us; and so the Sun, or some star located there, would appear larger to us than elsewhere, since nearer things are seen as larger than more distant ones. But we experience the contrary: the Sun and the Moon appear larger near the horizon than overhead.

Yet this argument of al-Farghānī, if taken exactly as it is proposed, has no force whatever. For since — as was shown above — the stars are not moved by themselves but by the motion of the heaven in which they are set, who does not see that, whatever figure be assigned to the heaven, every star draws equally near to the earth, seeing that by the heaven's motion it describes a circle around the earth that is everywhere equally distant from it? This is plainly seen in the following figure, in which the heaven is supposed to have an angular shape. For if the heaven moves around the earth, each star will describe its own circle about the earth — star A the outer circle, star B the inner circle. If, on the other hand, the heaven were at rest and the stars moved of themselves, the argument would have the greatest force and validity, as can be seen in the same figure.

Diagram. A circular figure representing the heaven conceived as an angular (polygonal) shape around the earth. At the center is a small circle marked **C** (the earth), containing a sprig/plant device. Two concentric circles are drawn; the vertices of the inscribed angular figure carry the labels. The four "corner" points (top, left on the horizontal diameter, right on the horizontal diameter, and bottom) are each marked **A**, and lie on the outer circle. The intermediate points (roughly the mid-sides, in the upper-left, upper-right, lower-left, lower-right positions) are each marked **B**, and lie on the inner circle. Small star-bursts mark each labelled point. The figure illustrates the text: star A (at an angle/corner of the heaven) describes the outer circle, star B (not at an angle) the inner circle.

- A (top, on outer circle): star at a corner
- A (left, on horizontal diameter): star at a corner
- A (right, on horizontal diameter): star at a corner
- A (bottom, on outer circle): star at a corner
- B (upper-left, upper-right, lower-left, lower-right, on inner circle): stars not at corners
- C (center): the earth

Nevertheless this same argument can be put better in the following way. If the heaven were flat, or of any other shape whatever, then although each star would describe its own circle around the earth, and so would always remain equally distant from it, still not all the fixed stars would recede from the earth at an equal distance; rather, some would appear nearer and some farther off. Thus, in the figure set out above, star A, placed at a corner of the heaven, has a greater distance than star B, which is not set at a corner — a result that is, however, contrary to experience. Furthermore, if all the heavens were of a many-sided figure, and not spherical, the lower heavens could not carry the planets and the fixed stars from west to east, from one sign into another, without there being a splitting and interpenetration of the celestial bodies. Since this is absurd, we must grant that the heaven is spherical. And this same argument proves too that the heaven is neither oval nor lens-shaped, and so on, as we also showed a little above.

[Margin: *Cælum à centro terræ, non autem à quouis puncto in superficie terræ assignato æqualiter distat, si Geometrice loquamur, sed solum, quoad sensum.* — "The heaven is equidistant from the center of the earth, but not from any point assigned on the earth's surface, if we speak geometrically — only as regards sense-perception."]

Now although our sense judges — and it is commonly said by philosophers and astronomers — that the heaven is everywhere equally distant from us who exist on the earth's surface, yet if we examine the matter more carefully we shall find that it recedes equally only from the center of the earth, and not from any point assigned on its surface. For the eastern, western, northern, and southern regions, and finally all the parts near the horizon, are farther from us than the part placed above our head; and that part of the sky which is opposite our zenith will be far more remote still. The reason for this is that between us and the zenith only two elements are interposed, namely air and fire; but between us and the other parts of the sky near the horizon, besides these two elements, a semidiameter of the earth also lies between; and between us and the part of the sky opposite our

zenith, besides the same two elements, the whole diameter of the earth is intercepted. If, then, we wish to speak geometrically and precisely, we cannot be equally distant from all parts of the sky.

[Margin: *Cur cœlum appareat longius distare à nobis iuxta Horizontem quam prope verticem capitis.* — "Why the heaven appears to be farther from us at the horizon than near the zenith."]

Nevertheless, since the earth's semidiameter is of insensible magnitude in comparison with the sky's distance from the earth's center, the heaven cannot appear sensibly farther from us at the horizon than at the zenith. It is just as if someone were to see some object 20 or 30 miles away: if he came 6 or 8 paces nearer, the same object would still appear to him of the same size, neither larger nor smaller so far as sense goes, because so few paces have an almost insensible ratio to 30,000 paces — even though this ratio is in fact larger than the ratio of the earth's semidiameter to the distance of the firmament, which, according to al-Farghānī (as we shall say at the end of this chapter), contains about 45,225 earth-semidiameters. And so astronomers and philosophers, following the judgment of vision, rightly assert that the heaven is equally distant from us in all its parts, even though by reason and truth the matter is not so. From all this it is clear that one and the same star, near the horizon in serene weather (all vapors and exhalations set aside), appears to us of the same size in which it is seen near the meridian, even though there it is farther from us and here nearer; because, namely, between that greater distance and this lesser one there is not so great a difference as can fall under the senses. And if anyone should object that at first glance it does seem to sense that the heaven is farther off at the horizon than overhead — so that it would be false [to say] that this diversity is...

insensible: the answer is that this is indeed true, but not therefore [sensible or] notable. For the sense is deceived, as the theorists of optics [Perspectivi] show, who by the interposed bodies are wont to judge and measure any interval; and so, because between us and the sky overhead it sees no body interposed, whereas from any part of the horizon it beholds the whole mass of the earth stretched out, it judges that distance to be much greater — when in fact it is greater only insensibly, so that by instruments the heaven is judged to be equidistant from us. Indeed, for this very cause the sense also judges that the heaven near the horizon in a way touches the earth itself, precisely because it perceives no other body between sky and earth. We see the same thing happen with mountain-tops: two summits sometimes seem to be entirely joined, because we do not see the bodies lying between them, though in fact they are separated by a very great interval.

[Margin: *Cur cœlum appareat longius distare à nobis iuxta Horizontem quam prope verticem capitis.* — "Why the heaven appears to be farther from us at the horizon than near the zenith." (This gloss is repeated from the facing page.)]

[Lemma — *Sacrobosco*:]

[Margin: *Cur Sol & stella maiores appareant iuxta Horizontem, quam in medio cœli.* — "Why the Sun and stars appear larger near the horizon than at mid-heaven."]

But since the truth of the matter is not so, the cause of this appearance is that in wintry or rainy weather certain vapors rise up between our sight and the Sun, or another star; and since those vapors are a transparent body, they scatter our visual rays, so that they do not apprehend the object in its natural and true size — as is evident in a coin cast into the depth of clear water, which, on account of a like scattering of the rays, appears larger than its true size.

COMMENTARY

He had said, in al-Farghānī's argument, that the Sun and Moon, or any other star, appear larger near the horizon than overhead; from this someone might infer that the heaven is not round, since it is not equidistant from the earth on every side. For where the star appears larger, there the heaven will be nearer; where smaller, there farther off. Accordingly he meets this tacit objection, giving as the reason why the Sun or Moon, or another star, appears larger at rising and setting than at mid-heaven or the zenith, not that it is farther off there than here, at least sensibly, but that there are vapors raised from the earth which are interposed between the Sun (or any other heavenly body) and our vision. Hence it comes about that those vapors, being denser and thicker near the horizon, alter our visual rays, and so we do not see the object in its own proper size. This is quite evident, as he says, in a coin lying at the bottom of translucent and clear water.

[Margin: *Stellæ omnes sphæricam figuram habent.* — "All the stars have a spherical shape."]

Al-Farghānī brings forward this same cause in his first Difference, and all the theorists of optics demonstrate it. For from that variation of the visual rays any nearer object appears — and hence appears larger. From the same cause it happens that an object can be seen by rays sometimes refracted, when by direct rays it could not by any means reach our eye. We have a very clear example in a coin cast into the bottom of some empty vessel of moderate height. If we draw back far enough that we can no longer see the coin, because the sides of the

vessel come between it and our sight, and then the vessel is filled with clear water, the coin will suddenly appear and offer itself to our view. Hence it comes about, finally, that we sometimes see the Sun, the Moon, and the other stars before they have risen above the horizon — from which arose among the wise the common saying: When the Sun appears on the horizon earlier than usual, it is a sign of coming rain; for then many thick vapors are interposed between our sight and the Sun, out of which rain is generated.

There remains, finally, a brief little question: whether, namely, all the stars are also of spherical figure — since from what has been said it is left clear that the heaven is spherical. On this point there have not been wanting some who supposed there to be as many various shapes among the heavenly bodies as there are among these lower ones. But since they seem to have asserted this rashly, without any probable reason, we must say, with all astronomers and philosophers, that all the stars are of round and spherical figure. This is plainly evident in the Moon, which receives light from the Sun in a circular fashion — which could in no way happen unless it were itself spherical. Since, then, the same reasoning seems to hold for all the heavenly bodies, we must conclude that they are all spherical. The same can be confirmed from this: that wherever they are placed in the heaven, they appear round to us, which could not happen unless they were round and spherical. This appears far more evidently in the planets. For since, according to the common opinion of astronomers, they are carried around in epicycles, they cannot always turn one and the same side toward us. Therefore, since they always appear round, they must necessarily be spherical on every side; for the spherical figure has this privilege among all bodies, that it appears circular and round when inspected from any part. Add to this that nature in these lower things has above all striven after roundness, so far as it could — as may be seen in the limbs of animals, the trunks of trees, in fruits and other such things, all of which seem to tend, as far as may be, toward a round figure; because, namely, as was said above, the round figure is the noblest. For this reason it is not without cause that nature seems to have granted to all the heavenly bodies — which surpass all others in nobility — the noblest figure, which is the round and spherical; and this too especially to the end that they might equally on every side pour forth their rays, and be more fully illuminated on every side by the Sun.

That the Earth and Water Are Round

[Lemma — Sacrobosco:]

[Margin: *Terram rotundam esse ab ortu in occasum.* — "That the earth is round from east to west."]

That the earth too is round is shown thus. The signs and stars do not rise and set at the same time for people existing everywhere; rather, they rise and set earlier for those toward the east. And that they rise and set earlier for some and later for others is caused by the bulge of the earth — which is well shown by what takes place on high. For one and the same eclipse of the Moon, numerically single, which appears to us in the first hour of the night, appears to peoples farther east about the third hour of the night. From this it is established that night came earlier for them, and the Sun set for them earlier than for us; and the sole cause of this is the bulge of the earth.

COMMENTARY

This is the third conclusion: namely, that the earth and water are of round figure. Since this contains two parts, he confirms in the first place the prior part of it — namely, that the earth is round — by this single argument: the earth is round from east to west, and likewise from north to south; therefore the whole earth is round. The consequence is evident from a sufficient enumeration of the parts: for if the earth is round from east to west, wherever you begin and whithersoever you proceed, and likewise from north to south, in whatever direction you turn, then absolutely no particle of the earth will be left that is devoid of roundness. The antecedent he proves in two ways. First, since it has two parts, he shows the prior — namely, that the earth is round from east to west — by this reasoning: The signs and stars rise for the Easterners first, reach mid-heaven first, and set first, before they do for the Westerners, as is evidently shown in a lunar eclipse — in which (since it is universal to the whole world, for the Moon is eclipsed on account of its own entry into the earth's shadow, as we shall explain in the fourth chapter) at the same instant of time the Moon is hidden from all people by whom it can then be seen; and yet, if, say, the beginning of the eclipse appears to us in the first hour of the night, this same eclipse is known, from the books of history or from the report of others, to have begun for more easterly peoples about the third hour of the night, for example. From this it is clear that they had night first, and consequently that the Sun rose and set for them two hours earlier than for us. And the sole cause of this is the roundness of the earth from east to west, since thus are produced different horizons from east to west — which would not happen if the earth were not round; just as it happens on some mountain, on which, because it is round and gibbous, many things occur, and are seen, on one side, which cannot be seen on the other side of the mountain, on account of the

interposed bulge of the mountain. This you may see most clearly in the adjoining figure, in which let the East be on the side of A, the West on the side of B.

Diagram. A circular figure representing the terrestrial globe within the sphere of the heavens. An outer circle is divided by tick-marks into equal parts (the 24 hours). Within it is a second, inner circle; at the center is a small circle marked **C** (the earth's center). Two labels **MERID.** (for *Meridianus* / meridian) are written along a curved arc at the top. A radiating shaft or columnar figure (with a bundle of lines/rays) rises from near the center **C** toward the upper right. The horizontal diameter runs from **B** at the left to **A** at the right. A second (tilted) diameter runs from **E** at the lower left to a point at the upper right marked **D** (printed in a form resembling "II"/"Д"). At **E**, in the lower left, is a Sun with a radiant face.

- A (right end of the horizontal diameter): East [Oriens]
- B (left end of the horizontal diameter): West [Occidens]
- C (center): the earth
- D (upper right, end of the tilted diameter): endpoint of the second horizon D–E [label uncertain — printed form resembles "II" [?]]
- E (lower left, at the Sun): endpoint of the second horizon D–E; the Sun is figured here
- MERID. (top, twice): the meridian [Meridianus]

You see, therefore, two different horizons, A B and D E, on account of the roundness of the earth C. For the zenith of each dweller on the earth ought to stand perpendicular, that is, at right angles, to the surface of the horizon and of the sky. You see, again, that the Sun rose earlier, reached mid-heaven or the meridian earlier, and finally set earlier, for those people whose horizon is A B than for those who have the horizon D E. Hence, then, it comes about that if an eclipse of the Moon begins while the Moon is above both horizons — and consequently while the Sun is depressed below each horizon as well — more hours will have elapsed since the Sun's setting with respect to horizon A B than with respect to horizon D E. And that this may be understood still more plainly, it must be known: Since the equinoctial circle [Æquinoctialis], divided into 360 equal parts called degrees, is wholly raised above any given horizon in the space of 24 hours by a uniform motion, it must be that in each single hour fifteen of its degrees are raised. Hence, since regions are called more or less easterly with respect to the equinoctial, which extends from east to west or the reverse, it is left clear that in all regions which are fifteen degrees more easterly than we are, the stars rise and set earlier by the space of one hour; while in those that are thirty degrees more easterly, the stars rise and set earlier by the space of two hours;

and so on in turn, adding or subtracting according to the number of degrees by which one region is more easterly than another — yet under this law and condition, that four minutes of an hour be assigned to each degree. For since an entire hour comprises 60 minutes, four such minutes will correspond to each single degree. All this may be observed in the figure set out above, in which you see the larger circle divided into 24 equal hours, the beginning taken from the setting of the Sun, as is done in Italy. The degrees lying between any two cities, of which one is eastern and the other western, are known by the arc of the equinoctial circle intercepted between the meridians of the two cities. This can easily be understood from those descriptions of the world which they call maps [Mappas mundi]: for in them the lines proceeding from one pole to the other mark the meridians, while the line equally removed from both poles represents the equinoctial circle. Hence, if two meridians be taken passing through two cities, the arc of the equinoctial circle placed between the two meridians will at once show by how much one city is more easterly than the other. But these matters are to be sought from the cosmographers.

Furthermore, as to what our author says — that the Sun rose and set two hours earlier for more easterly peoples than for less easterly ones, if indeed those peoples exist thirty degrees more to the east — this must be understood of two cities that are equally distant from the equator, that is, that have the same elevation of the pole. For when they have different elevations of the pole, and consequently are not equally distant from the equator, it is not necessary that for the city which is thirty degrees more easterly than the other the Sun rise and set two hours earlier. For it may happen that for the city which has the greater elevation of the pole — that is, which lies more toward the north — the Sun rises at the same instant of time as for the city which has the lesser elevation of the pole, even though the latter is more easterly. This indeed comes about on account of the obliquity of the horizon. Hence it happens that, when the Sun is in the northern signs — at the beginning, for example, of ♋ [Cancer? — zodiac symbol, reading uncertain] — the more northerly city has a longer day than the less northerly one. So, although the Sun reaches the meridian of that city later than of this one — because this one is placed more to the east — nevertheless, since the time from sunrise to the meridian of that city is greater than of this one, it can happen that the Sun rises at the same time for both cities.

For example: Let two cities be placed not having the same elevation of the pole, of which one is more easterly than the other by fifteen degrees, so that noon comes to the more easterly one hour before the other. Let the more easterly city have a longest day of 14 hours, and the more westerly one of 16 hours, so that in the

former seven hours run from sunrise to noon, but in the latter eight. This being posited, who does not see that the Sun rises for both cities at the same instant of time? For when it is noon for the former, seven hours will have elapsed since its sunrise, and the latter city will still lack one hour until its noon. Since, then, this latter city has eight hours from sunrise to noon, it must be that when it is noon for the former city, seven hours have likewise elapsed since sunrise for the latter. Therefore the Sun did not rise earlier for the one than for the other, even though the one is more easterly than the other. But if a more westerly and more northerly city has a longest...

[continuation of the commentary on the day-length and eclipse-timing argument]

whose longest day is 17 hours, the Sun will rise earlier for it than for a more easterly city whose longest day contains 14 hours, as is clear. Conversely, if a more northerly city is also more easterly, it can happen that the Sun does not set for it any earlier than for a more westerly and more southerly city, but at the same time, or even later. Indeed, there can be two cities of which neither is more easterly than the other, yet which have unequal elevations of the pole — namely because one inclines more toward the North than the other — and for which, nonetheless, the Sun does *not* rise and set at the same time, even though midday occurs at the same moment in both. Rather, it rises much earlier and sets much later for the more northerly city than for the less northerly one, because the former has a longer day than the latter.

But if we are speaking of the hours reckoned from midday, then the statement of the author and of the astronomers will hold true for any cities whatever of which one is more easterly than the other — even if they are not situated under the same parallel, but under different ones, and have different elevations of the pole. For a city that is more easterly than another by, say, 30 degrees will always reach midday two hours earlier, whatever be the case regarding the anticipation of the Sun's rising or setting. By the same reasoning, when there are two cities of which neither is more easterly than the other — even though the one lying farther north has a longer day, so that the Sun rises earlier and sets later for it — they will nevertheless reach midday at the same instant of time. Hence for both cities the beginning of some eclipse of the Moon will appear at the same hour before or after midday; which can by no means happen for two cities of which one is more easterly than the other, since midday plainly comes sooner for the more easterly one, because its meridian lies closer to the eastern regions.

[*Sacrobosco's text:*] That the earth likewise has a bulge [tumor] from north to south, and the reverse, is clear as follows. For people situated toward the north,

certain stars are of everlasting appearance — namely those that draw close to the Arctic pole — while others are of everlasting concealment, namely those that lie close to the Antarctic pole. If, then, someone were to travel from north toward south, he could go so far that stars which had previously been of everlasting appearance to him would now sink toward setting; and the farther he advanced toward the south, the more they would move toward setting. That same man could, in turn, come to see stars which had previously been of everlasting concealment to him. And the reverse would happen to someone traveling from south toward north. The cause of all this is simply the bulge of the earth.

COMMENTARY

Here [Sacrobosco] confirms the latter part of the preceding [proposition] — namely that the earth is round from north to south as well — by the following argument. There is no doubt that certain fixed stars always appear to us who live in the oblique sphere and in the northern regions — namely those near the Arctic pole — while certain others are always hidden, namely those that lie near the Antarctic pole. Again, it is established that if someone were to travel straight from north to south — that is, always under the same meridian — the stars that had always appeared to him before would begin to be hidden; and conversely, those he could not see before, near the Antarctic pole, would gradually lift themselves above the horizon and come into view. For we see that in Germany, which is more northerly, more stars are permanently visible than in Italy, which is less northerly; while, on the contrary, in Italy more stars are seen in the southern quarter than in Germany. It is therefore a clear sign that the earth is round from north to south — just as the reason why, when we climb some rounded mountain, we begin to see things we did not see before, and can no longer look upon things we saw before, is simply the bulge of the mountain.

But from these considerations it seems we can conclude only that the earth is round from north to south *in some fashion* — that is, that it is not flat — and not that it is of spherical figure. [Margin: That the roundness of the earth is spherical.] Hence this very point must be confirmed in the following way. When someone, remaining always under the same meridian, travels from north to south, he finds that the elevation of the pole above the horizon continuously decreases, and in this fixed proportion: if in one place the pole's altitude is, say, 40 degrees, then after he has covered 62 miles toward the south he finds the pole lowered [*reperiat polum eleuari* (?) — verb as printed reads "raised," but the context (40°→39°, "decreases," "has decreased by one degree") requires depression] just enough to stand at 39 degrees; and so on in succession — as often

as he covers 62 miles, he finds the pole's altitude to have decreased by one degree. For this proportion in the decrease of the pole's altitude can belong only to a spherical figure, as is evident among the geometers and astronomers.

In the same way it will be shown that the earth is spherical from east to west — not round in merely some fashion, but [truly] spherical. For that anticipation of the rising and setting of the Sun, and also of midday, cannot at all preserve the aforesaid proportion (namely, that for a city more easterly than another by 15 degrees the Sun rises and sets one hour earlier; for one more easterly by 30 degrees, two hours earlier; and so on for the rest) unless we assign a spherical figure to the earth. This is why our author rightly demonstrated that the earth is round.

[*Sacrobosco's text:*] Again, if the earth were flat from east to west, the stars would rise as quickly for the western regions as for the eastern — which is plainly false. Likewise, if the earth were flat from north to south, and the reverse, the stars that were of everlasting appearance to someone would always appear to him, wherever he traveled — which is false. But that it *appears* flat is owing to its enormous size relative to human sight.

COMMENTARY

[Sacrobosco] now proves the same antecedent with respect to both its parts by excluding an absurdity — in particular by excluding from the earth the flat figure that the earth is commonly believed to possess; and this by the following argument, which is in a way an explanation and confirmation of the preceding. If the earth were not round from east to west (or the reverse) but, say, flat, the stars would rise as quickly for the western regions as for the eastern, and would set for both at the same time, since all would have as their horizon the flat plane of the earth. And if it were likewise flat from north to south, rather than round, then for the same reason — whether one traveled from north to south or the reverse — the stars that were permanently above the horizon would never be hidden, nor would those permanently hidden from him ever begin to appear, since he would never change his horizon but would always remain on that flat plane of the earth. Both of these are contrary to common experience, as is clear from the preceding

reasoning; and this argument, together with the present one, is drawn from Ptolemy [Ptolemy] (*Almagest* I.4 [Dict. 1. cap. 4]), from Johannes Regiomontanus [Regiomontanus] (*Epitome of the Almagest* I, conclusion 2), and from al-Farghānī [al-Battānī? — recte al-Farghānī, "Alphraganus"] (*Elements of Astronomy*, Differentia 3).

[Margin: That the earth is not hollow.] Ptolemy, in the place cited, adds another argument by which he proves that the earth cannot be hollow. For if it were hollow, he says, the stars would rise sooner for the western regions than for the eastern — as we see happen in valleys, where the western parts are lit by the Sun sooner than the eastern parts. Moreover, the farther one advanced from north to south, the more stars would appear to him near the Arctic pole, and the more, on the opposite southern side, would be hidden. All of which is absurd and conflicts with experience, as has been said.

[Margin: Why the earth appears flat.] As for why the earth appears flat to our sight, our author says the cause is its enormous size. For what appears to us of the earth is so very small in relation to the whole circumference of the earth that it is no wonder it looks flat to us — just as, if someone were to cut off the smallest part from the circumference of some very great circle, that little piece, considered on its own, would undoubtedly be judged by anyone to be a straight line.

[*Sacrobosco's text:*] [Margin: That the water is round.] That the water, too, has a bulge and approaches roundness is clear as follows. Let a marker be set up on the seashore, and let a ship leave the harbor and move so far off that the eye of a person near the foot of the mast cannot see the marker; yet, with the ship standing still, the eye of that same person, stationed at the top of the mast, will see that marker well. But the eye of one near the foot of the mast ought to see the marker *better* than one at the top of the mast, as is clear from the lines drawn from each point to the marker; and there is no other cause of this than the bulge of the water. For let all other impediments — such as mists and rising vapors — be excluded.

COMMENTARY

Here [Sacrobosco] confirms the latter part of the proposed third conclusion — namely that the water is round — by a twofold argument. The first is this. If some notable marker is set up on the seashore — say, some tower, or a house is noted — and a ship leaves the harbor, then, after some distance of the ship from the shore, those in the ship near the foot of the mast will no longer see that noted marker; but if someone then climbs to the top of the mast, he will still see the marker. And this happens even when all other impediments, such as mists and vapors, have been set aside. It therefore plainly follows that the cause of this was solely the bulge of the water interposed between the ship and that marker on the shore. For unless the bulge of the water were the impediment — that is, if the water were flat — those at the foot of the mast ought to see the marker better than one at the top of the mast, since they are closer to it, as is clear from the

straight lines drawn from the marker to the foot of the mast and to its top. [Margin: note *a* — Euclid, *Elements* I.19.] For the line drawn to the top of the mast would be longer than the one extending to the foot of the mast, since it is subtended by a greater angle, as appears in the adjoining figure.

Diagram. A woodcut showing the terraqueous globe: a rounded mass of land bearing a walled tower and buildings on its upper left and mountains across its face, encircled by the sea drawn as waves. To the right, a ship under sail rides on the water, its hull below and its mast rising above the curve of the water's bulge — illustrating that the marker (tower) on the shore is hidden by the swell of the sea from an eye at the foot of the mast, but visible from the masthead. No lettered labels.

Although this argument — which is that of all the astronomers — demonstrates very well that the water has a round figure once the mists and vapors that impede our sight have been set aside, nevertheless, because the weather is scarcely ever so clear that there are no vapors raised from the sea, and because from it one can conclude only that the water is round in some fashion (that is, not flat), and not that it is spherical, we shall therefore be able to prove more satisfactorily and more effectively that the water is round and spherical by the same means through which the author gathered the roundness of the earth: namely, by comparing more easterly islands with less easterly ones — as when one sails from Syria to Spain, and thence toward that part of New Spain, or America, which is called Florida, or the reverse — and, likewise, by comparing more northerly islands with less northerly ones, as when a voyage is made from Portugal [Lusitania] toward Flanders, or the reverse, and from Portugal by way of the Fortunate Isles [Canary Islands] toward Cape Verde. For all the observations adduced above to prove the roundness of the earth — namely, the anticipation of the rising and setting of the stars, and also the variation of the pole's altitude — have been found by sailors on the ocean and sea to hold in the same proportion. It is therefore necessary that the water, too, is round and spherical.

[*Sacrobosco's text:*] Again, since water is a homogeneous body, the whole will be of the same nature as its parts; but the parts of water (as happens in droplets and in the dew on plants) naturally seek a round form; therefore so does the whole, of which they are the parts.

COMMENTARY

This is the second argument. The parts of water naturally seek a round figure, as we see in droplets and in dew upon the leaves of plants. Since, then, water is a

homogeneous body, and consequently the whole is of the same nature as its parts, the whole body of water, too, will be of round figure. But this argument is not very cogent. For those droplets, fleeing the dryness that is hostile to them, love the round figure out of a natural and universal propensity, so as to preserve themselves longer; for the spherical figure is the most suitable for that, since its parts are more united than those of other figures. Hence we see that droplets of water, if they lose their spherical figure, are quickly and easily destroyed and dried up.

To these two arguments we can add another, which Aristotle [Aristotle] too brings forward in *On the Heavens* [*De caelo*] II, in this way. Water, of its own nature, flows together toward lower places, as we have learned from daily experience; therefore it is round. For otherwise it would not flow together toward lower places. Suppose the surface of the water to be — if it were possible — flat, or of some other non-circular figure, spread over the earth along the line ADB; and from C, the center of the world, let the circle EGF be described; and from C let CD be drawn perpendicular to AB; and let the straight lines AC, BC be joined. [Margin: Corollary to Euclid, *Elements* I.19.] Now since the straight line CD is smaller than CA or CB, the point D will be in a lower place — that is, closer to the center — than point A or B. The water, then, being unimpeded, will *not* flow toward lower places. And since this conflicts with experience

Diagram. [Margin label: "Corollary to I.19."] A circle with center C. Across its upper part runs the straight line AB, a chord, with its midpoint D; from the center C the perpendicular CD rises to D, and the straight lines CA and CB are drawn to the ends of the chord, forming a triangle. The arc of the circle above the chord carries the point G at the top, with E and F marking the circle to either side (the bulge EGF).

- A (left end of the upper chord): endpoint of line ADB
- D (midpoint of the chord AB): foot of the perpendicular CD
- B (right end of the upper chord): endpoint of line ADB
- G (top of the circle, above D): summit of the water's bulge
- E (circle, left): point to which the near part of the water subsides
- F (circle, right): point to which the near part of the water subsides
- C (center): center of the world

it is necessary that the middle part of the water, namely D, be raised to point G, and that the parts of the water near A and B subside and arrive at points E and F, so that the whole body of water may have the bulge EGF and be equally distant

from the center of the world. For in this way it will naturally come to rest, balanced. And from this argument it will be proved that the water can have no figure other than the spherical; for otherwise it would always have some parts farther from the center of the earth (since only the spherical figure everywhere approaches the center equally), and consequently it would not flow down toward lower places — which conflicts with the nature of water. Indeed, from this argument it follows that any liquid contained in some vessel has a bulge, or circumference, whose center is the same as the center of the world.

[Margin: Archimedes' demonstration, proving that every liquid has a spherical figure.] But the most elegant of all is the demonstration of Archimedes [Archimedes] in Book I of *On Floating Bodies* [*On the things carried in water*], by which he proves that not only the Ocean and the other seas, but indeed any standing and settled liquid, has a spherical figure whose center is the same as the center of the world, toward which all heavy things are borne by their own nature. He first assumes that the nature of a fluid is such that, when its parts lie evenly and are continuous with one another, the less compressed part is driven out by the more compressed; and that each of its parts is pressed by the fluid lying vertically above it, if the fluid is descending in some direction, or is pressed by some other. This we have learned by experience to be true; for whenever we press some liquid in one part, either with the hand or with some other fluid poured over it, the other surrounding parts give way and are driven out. He then demonstrates that if some surface is cut by a plane always through the same point, and the section is the circumference of a circle having, as its center, that point through which it is cut by the plane, then that surface is spherical, with that same point as its center. The demonstration of this is as follows. Let some surface be cut by a plane drawn through the point A, and let the section always be the circumference of a circle having the point A as its center. I say that this surface is spherical, with A as its center — that is, that all the lines drawn from the point A to that surface are equal to one another. For let two straight lines AB, AC be drawn in any manner from A to the surface, as in the first figure. [Margin: note *b* — Euclid, *Elements* XI.2 (*secundum undecimi*).] Through these, since they lie in the same plane, let a plane be drawn producing on the proposed surface the line BC, which, by hypothesis, will be the circumference of a circle. The straight line AC will therefore be equal to the straight line AB, by the definition of a circle. By the same reasoning we shall show that all the other straight lines drawn from A to the proposed surface are equal to AB, since through AB and any other straight line drawn from A to the given surface a plane can be drawn producing a circle on the proposed surface. Therefore all the straight lines will be equal to one another, and accordingly the surface will be spherical, with A as its center.

Diagram. "The first figure." A circle (the section of the surface) with its center A at the bottom. Two straight lines AB and AC rise from A to the points B and C on the circumference, and the chord BC joins them across the top, so that AB, AC, and BC form a triangle inscribed with its apex at the center.

- A (bottom, center of the circle): the point through which the cutting plane passes
- B (upper left, on the circumference): endpoint of line AB
- C (upper right, on the circumference): endpoint of line AC

Now let some fluid, or standing and settled liquid, be understood, whose surface is cut by a plane drawn through D, the center of the earth, producing a line on the surface EFGH. I say that the line EFGH is the circumference of a circle whose center is D. For if it is not, not all the straight lines drawn from D to the line EFGH will be equal to one another. Let DE, DG, then, be unequal, with DG greater than DE; and between these let the straight line DF be drawn, greater indeed than DE but smaller than DG. Now if, in the cutting plane, a circle IFKH is described from D with radius DF, it will necessarily cut the straight line DE beyond point E, at point I, and the straight line DG below point G, at point K; and there will be formed at D two arbitrary angles FDI, FDG. Let there also be described, in the liquid and in the plane of the circle IFKH, the circle LMN. The parts of the fluid near the circumference LMN therefore lie evenly and are continuous with one another, since they are equally distant from the center D; and of these, the ones near the circumference MN are pressed more by the liquid near FG than the ones near the circumference LM are pressed by the liquid near EF, since the former is heavier than the latter, as is clear. Therefore the parts near LM will be driven out by the parts near MN, and consequently the fluid will not stay at rest — but it was assumed to be standing and settled, which is absurd. The line EFGH, therefore, is the circumference of a circle whose center is D. It will be demonstrated in the same way that, however else the surface of the liquid is cut by a plane through D, the center of the earth, the section is the circumference of a circle whose center is D. Therefore, as we showed a little earlier, the surface itself will be spherical, with center D, the same as that of the earth — seeing that it is of such a kind that, when cut always through the center of the earth, it produces the circumference of a circle having the center of the earth as its center. Which was to be demonstrated.

Diagram. [Margin label: "note *b* — XI.2."] A large semicircle representing the surface of the liquid, EFGH, standing on a horizontal base; D is the center of the earth on the base, and the vertical line rises from D through the interior up to G

at the top of the arc. F is on the arc at the left, H on the arc at the right. Two concentric inner arcs (the circles IFKH and LMN) are drawn about D; on the vertical line, K and N lie below G, and M lies in the interior toward the left; along the base at the lower left are the points I, E, and L.

- G (top of the outer arc): summit, on the vertical through D
- F (outer arc, left): where DF meets the surface
- H (outer arc, right): the surface EFGH
- D (center of the base): center of the earth
- K (on the vertical, below G): where the circle IFKH cuts DG
- N (on the vertical, below K): inner circle LMN
- M (interior, toward the left): inner circle LMN
- I (lower left, on the base): where the circle IFKH cuts DE produced
- E (base, left): endpoint of DE on the surface
- L (base, left of center): inner circle LMN

WHETHER FROM EARTH AND WATER A SINGLE GLOBE IS MADE — that is, whether the convex surfaces of these elements have the same center

[Margin: The opinion of those who posit two centers, one of the earth and the other of the water.] Although it has been rightly proved by the author that both the earth and the water are round, some nevertheless call into question whether these two elements are so round and spherical as to constitute a single globe — or (what is the same) to have one and the same center. For some assert that the earth and water in no way have the same center, but two distinct ones, and that therefore they do not form from themselves a single sphere only, but two. For they say that at the beginning of the world the earth and the water were created round indeed, and concentric, about the center of the world; that the water then withdrew from one side and was gathered onto the opposite side in a great bulge, while the earth meanwhile remained motionless at the center of the universe. And so they say that from that separation of the water from the earth two globes were produced, distinct and diverse from each other — one of the earth, the other of the water — although neither of these globes appears whole and entire, but the two intersect one another. From this opinion it follows that two centers must be posited: one of the whole universe, which they say is the same as the center of the earth, and the other of the water itself. For, convinced by arguments and

observations, they cannot deny that both the earth and the water are round and spherical. And if you object to them that it would follow from this that the water must either be held in place by force or else be able to flow down and cover the earth, they reply that the water, by a supernatural favor and miracle of God, is held there and cannot cover the earth — though it would indeed most certainly cover it if it were left to the condition of its own nature, by which it strives to flow together toward lower places.

Clavius, *Opera Mathematica* Tomus III — Commentary on Sacrobosco's *Sphere*

(Continuation of the commentary on CHAPTER ONE, within the question whether earth and water make a single globe. This batch presents the "three-centre" opinion, refutes both rival opinions, and then argues positively that earth and water form one globe with one common centre — the centre of the Universe.)

[Margin: *The opinion of those who posit three centres — one for the earth, another for the water, and a third for the whole Universe.*]

Others, however, while likewise constituting these same two globes out of earth and water, are unwilling to admit anything supernatural. They hold instead that by God's command not only the water but the earth as well withdrew from the centre of the world, and that the water is no longer held back supernaturally to keep it from flowing to a lower place and covering the earth. Hence these Authors imagine three centres: one of the whole Universe, another of the earth, and finally a third of the water itself. The reason why all the aforesaid Authors make two globes out of earth and water seems to be this: they suppose that water is much larger than the earth itself, so that if the water were concentric with the earth it would certainly cover it. For two unequal circles or globes cannot be concentric without the larger enclosing the whole of the smaller, as is evident from geometry.

[Margin: *Refutation of both of the foregoing opinions.*]

But either opinion can easily be attacked. The first, because without any necessity it takes refuge in miracles. The second, because while it tries to defend the view that everything is constituted naturally, it cannot escape conceding something supernatural — namely that the centre of the world is not the centre of the earth, to which it is naturally owed on account of its supreme heaviness, as all philosophers admit. Add that it conflicts with all experience to hold that the earth

is not placed at the centre of the whole Universe together with the water. For, as we shall demonstrate a little later, the convex surface of the earth, as much as that of the water, is equidistant from the centre of the world — which both opinions deny.

Next, since the Authors of both opinions admit that water is much larger than the earth itself, they will also necessarily be forced to concede that more stadia or miles correspond to each degree of the surface of the sea (or of the water) than to each degree of the earth. For the terrestrial globe is divided into as many degrees as the watery globe is divided into, just as every celestial circle is customarily divided. So if water is larger than earth, the degrees of the water must be larger than the degrees of the earth, and accordingly each of them will contain more stadia or miles than each of those [of the earth]. But all sailors assert the opposite: they testify that they have often found by experience that each degree on the surface of the earth comprises exactly as many stadia or miles as [each degree] on the surface of the sea.

Again, if the aforesaid opinions were true, no antipodes could be assigned to any part of the earth; for the part opposite to this inhabited part of the earth is, on their account, mostly covered by the depth of the waters — as the Authors of them tell tales. But daily experience of the Portuguese and Spaniards teaches us well enough that antipodes are assigned to many parts of the earth, whether on the mainland or on islands: so that the inhabitants at the Cape of Good Hope are nearly antipodal to the far part of the province of China. The province of Peru, too, is roughly opposite that part of the East Indies where the trading-port of Calicut is found. Likewise Malacca in the East Indies is opposite, almost diametrically, to Brazil in the West Indies, and so on.

Diagram. An uncaptioned woodcut roundel showing the shape that earth and water would take according to the rejected opinions. Within a roughly circular (slightly egg-shaped) frame: at the top, a walled town with towers and houses set right-side-up upon hilly land; the middle and lower body of the circle filled with wavy lines representing the sea; and at the bottom, a second town shown inverted (the antipodes), with buildings and small downward-pointing arrows/steps. The figure carries no letter-labels. It illustrates the antipodes and the islands scattered through the Ocean, and the "oblong or otherwise-shaped" earth that the rival opinions would entail (the *supraposita figura* referred to below).

Furthermore, since on their view the water is not equally distant from the centre of the Universe but is raised up in a remarkable way, it would follow that a ship

leaving harbour would be climbing uphill in whatever direction it sailed, and coming in to the same harbour would be going downhill; and so, with the wind being equal, it would descend to the harbour faster than it climbed away from it — which is contrary to experience. Indeed, a ship stationed outside a harbour could in no way stay at rest without running down to the harbour of its own accord, since every heavy thing tends downward. Yet this is not in fact true.

Lastly, the very thing which the first opinion most wants to avoid — that water, held by a supernatural power, would cover the whole earth — it in no way avoids. For since there are antipodes (as those who sail daily nowadays find by experience), and since the whole sea of the Ocean is sprinkled with almost infinite islands, then if the water, left to its natural condition, flowed down so as to cover this habitable earth (according to the Authors of that opinion), that part which our antipodes inhabit would surely be more and more uncovered; and you may say the same of the islands. Therefore, while the Authors of this opinion try to show that water left to its first condition could cover the earth, they wholly uncover another part — which I think they would by no means concede. The same follows on the second opinion, provided that God again placed these two elements about the same centre. For then, according to this view, the earth would be covered by the water; and so that part which our antipodes now inhabit would remain all the more uncovered. But perhaps they will say (as some replied to me when I was disputing with them) that our antipodes and the islands are contained within the same circumference as the whole earth, and that the sea heaves up between any two islands into a certain swelling and mound. Hence, if it flowed down, it would cover the whole earth, including that part which is among the antipodes, together with all the islands. But this reply is absurd. First, because if it were so, the whole body of water would not have a single centre, but each mound of water between two islands would have its own — which is against the common view of everyone, and seems rashly asserted. Next, it would follow that, if someone were stationed on some island from which he could scarcely make out another island lying farther off, then if he sailed toward the mainland — that is, receding farther from that island — he ought to make it out better and more easily than while he was scarcely doing so in the harbour; since, according to the aforesaid reply, on leaving that island he would be climbing a certain mountain of water. And this is contrary to all experience. For if from one place at sea something can scarcely be seen, it is seen much less from another place farther off. I pass over very many other absurdities of this kind that follow upon that reply.

There is added, finally, that on either opinion the earth could not be spherical, but rather oblong or of some other shape — since antipodes really do exist, and almost innumerable islands are found throughout the whole Ocean. All of which you see in the figure placed above.

[Margin: *That earth and water form a single globe.*]

These opinions, then, being rejected as absurd and at war with experience, we must say that earth and water form a single globe — or, what is the same thing, that they have one common centre, which is the centre of the whole Universe. For the centre of the whole Universe, since it is equally distant on every side from the heaven and consequently occupies the lowest place in the world, is endowed with such a nature that all heavy bodies descend toward it of their own nature, unless hindered by something else. Hence it is not without reason called by the philosophers the *centre of gravity* [centrum gravitatis]; for all heavy bodies by their nature seek to be in the lower place, as we have learned both by experience and by natural reasoning.

For there is no greater reason why some heavy body should naturally want to be here, outside the centre of the world, rather than there, since every part removed from the centre is nearer to the heaven and therefore in a higher place. From this it follows that water, since it too is heavy, of its own nature, if unhindered, flows together toward the lower places, so that it may equally surround the centre of the whole Universe, lest one part be in a higher place than another — which would be contrary to its nature. This too Aristotle assumed above [Aristotle, *De Caelo*] in his demonstration, as something confirmed by the most certain experiences. And so, together with all astronomers and philosophers who judge more rightly, we say that the convex surface of the earth, as much as that of the water, is everywhere equally distant from the centre of the whole world; and that for this reason the centre of these two elements is one and the same, namely the centre of the whole Universe — so that the convex surface of the one in no way intersects the convex surface of the other (as the earlier opinions maintained), but the convex surface of the water is continuous with the convex surface of the earth, so that a single [globe] is made out of both. And although this could be very easily impressed upon anyone who rightly weighs the heaviness of each element, we shall nevertheless now demonstrate this very point by several arguments. Let the first be as follows.

[Margin: *First argument* (1. Ratio).]

In whatever part of the globe, earth and water, when unhindered and let fall freely, descend along one and the same line of air. Therefore they seek exactly the same centre — which we said just above is the centre of the whole Universe — and consequently they constitute a single globe. The antecedent is established by experiment; the consequence, however, is demonstrated by the mathematicians. For from the opposite of the consequent is inferred the opposite of the antecedent. For if two heavy bodies dropped from some point, in any place of the world, sought different centres, they would necessarily descend along different lines as well. For although from that place which corresponds to both centres along one and the same straight line they would, when dropped, descend along that same line, yet from all other places they would, when dropped, tend along different lines toward those two centres — as appears clearer than light in this figure, in which let the centre of the earth be **B** and the centre of the water **A**.

Diagram. A circle (the globe of earth-and-water) with a horizontal line drawn through it. On this horizontal line, from left to right: **E** (a point outside the circle, at the left end of the line), then inside the circle **B**, **F**, and **A**. Above the circle is the apex point **C**; below it is the point **D**. Dotted (bead-like) lines run from **C** down to **B** and to **A**, and from **D** up to **B** and to **A**, meeting the two interior centres. Per the text: **B** = centre of the earth, **A** = centre of the water, **F** = centre of the world/Universe; **E** is the external point collinear with both centres.

- **A** (on horizontal line, right of centre): centre of the water
- **B** (on horizontal line, left of centre): centre of the earth
- **F** (on horizontal line, between **B** and **A**): centre of the world (Universe)
- **E** (left, outside the circle, on line **EA**): the point from which earth and water descend along the single line **EA**
- **C** (top, apex): external point; lines **CB** (earth) and **CA** (water)
- **D** (bottom): external point; lines **DB** (earth) and **DA** (water)

For only from the point **E**, which corresponds to both centres along the same straight line **EA**, will the earth tend to its own centre **B**, and the water to its own centre **A**, along that same line **EA**. But from any other point — for instance from **C** — they will descend along different lines: the earth, namely, along line **CB**, and the water along line **CA**. And you will say the same of the point **D**. This would not happen if both elements were carried to the centre of the world, **F**. Therefore the centre of the earth, of the water, and of the whole Universe is the same, and consequently one and the same sphere, or globe, will be constituted out of earth and water. For if they constituted two different globes, one globe would intersect

the other — [a] just as, likewise, two mutually intersecting circles cannot have the same centre.

[Margin: a — (Euclid, *Elements* III.5): two circles that cut one another do not have the same centre.]

[Margin: *The reply of the Authors of the contrary opinion.*]

But the Authors of the contrary opinion reply that from this argument only this is gathered: that the centre of the whole Universe is indeed the *centre of gravity* of earth and water — toward which, of course, they naturally tend — but not the centre of their *magnitude*. For one and the same body can have a centre of gravity different from the centre of its magnitude.

[Margin: *What the centre of gravity of each body is.*]

To understand this, one must know that the centre of gravity of some body is that point which always, along the perpendicular, tends toward the centre of the whole Universe, in whatever way and however often the body is suspended, provided it hangs freely. Or, as Pappus [Pappus of Alexandria] defines it, that point situated within the body from which, if a weight is conceived in the mind to be hung, then while the body is carried it stays at rest and keeps the position it had at the beginning, and is not turned about during the motion itself. By this account every body, whether round or not, has a centre of gravity.

[Margin: *What the centre of magnitude of each body is.*]

The centre of magnitude, on the other hand, is the point equally distant from all the outermost parts; and this indeed is found properly only in a spherical body, but in regular [rectilinear] bodies improperly. For that point is said to be the centre of magnitude in any body which is the centre of the sphere that can be circumscribed about it or inscribed in it. These two centres are one and the same in a spherical body that is uniform in heaviness — for example, in a leaden or iron sphere, and the like. But in a spherical body non-uniform in heaviness — as in a sphere partly of wood, partly of stone, of lead, or of iron, etc. — the centre of gravity is one thing and the centre of magnitude another. For in the middle of that globe will be the centre of magnitude, but the centre of gravity will be a point lying in the heavier part; and this indeed would be joined with the centre of the whole Universe, and become one with it, if that body were carried to it unhindered.

[Margin: *How the centre of gravity is found in any body whatever.*]

The centre of gravity of any body whatever — even an irregular and non-uniform one — is found by this method. Let the body whose centre is being investigated be suspended freely, and from the point of suspension let down a thread with a plumb-bob, and mark the line which the thread traces on the body. Then suspend the same body from another point, from which again let down a thread with a plumb-bob, likewise marking the line of that thread on the body. Now since, in whatever way the body hangs, the centre of gravity is found on that line which inclines toward the centre of the world, both perpendiculars must necessarily pass through the centre of gravity. Therefore that point of the body at which those two perpendicular lines intersect will indicate the centre of gravity — as you see in this diagram; in which let the first point of suspension be A, the line of the plumb marked on the body AB; and let the second point of suspension be C, and the line of the plumb marked on the same body CD, cutting the former line AB at the point E, which we assert indicates the centre of gravity.

Diagram. An irregular, roughly star-shaped (blob-like) body used to illustrate finding a centre of gravity by two suspensions. Two straight chords are drawn across it and cross at an interior point.

- A (upper left, on the outline): first point of suspension
- B (right, on the outline): lower end of the first plumb-line; line AB
- C (top, on the outline): second point of suspension
- D (bottom, on the outline): lower end of the second plumb-line; line CD
- E (interior, where AB and CD cross): the centre of gravity

Thus, then, those Authors say that the centre of the whole Universe is the centre of gravity of earth and water — since, as experience teaches, they tend toward it and are of non-uniform heaviness — but that the centre of magnitude of the earth is different from the centre of magnitude of the water; indeed, that each centre of magnitude, both of earth and of water, can be distinct from the centre of the whole world (which is the centre of gravity) — thereby positing three centres.

[Margin: *Refutation of the reply of the Authors of the contrary opinion.*]

[Margin: *That the centre of gravity and of magnitude is the same, both in earth and in water.*]

But this reply amounts to nothing. For both in earth and in water the same centre of gravity and of magnitude must necessarily be posited. Since, then, in each element the centre of the whole Universe — toward which, when dropped from

any place, they are carried (as is left proven by argument) — is the centre of gravity, it becomes clear that the centre of magnitude is the same, namely the centre of the Universe, in earth and water; and hence that these two elements constitute a single globe. That the centre of gravity and of magnitude is the same in the earth, we shall demonstrate as follows. Weights, and all heavy bodies, which are carried from a high place to the surface of the earth, make similar and equal angles upon it, and are not carried in parallel (equidistant) lines, as the senses judge — since they meet at the centre of the Universe, which is the centre of gravity. Therefore the centre of magnitude of the earth and the centre of its gravity — that is, of the Universe — are one and the same.

The antecedent is confirmed by common experience, as may be seen in the plumb-lines that craftsmen use in constructing buildings: these bend neither to one side nor to the other, but rest evenly upon the surface of the earth. For from whatever place they are let down onto the earth they always make similar and equal angles with it, and the threads of those plumb-lines are always along the diameter of the heaven and the earth — otherwise buildings could not stand for long. The same antecedent is Aristotle's, in the second book of *On the Heavens* [*De Caelo* II]. The consequence, moreover, is very clear among the geometers; for from the opposite of the consequent is inferred the opposite of the antecedent. For let the centre of gravity, or of the Universe, be E (if it can be so), but let the centre of the earth's magnitude be another, namely F; and let some weight be carried from on high toward the centre E of the whole Universe along the line BGE, but not toward the centre of the earth F. I say that this weight, striking the earth, does not make equal or similar angles with the surface of the earth, but wholly unequal and dissimilar ones.

Diagram. A large outer circle containing a smaller concentric-ish inner circle (the inner circle representing the surface of the earth/water; the outer, the region of the falling weight). A vertical diameter runs from A (top) through D, F, E down to C (bottom). Additional straight lines are drawn to the interior points G and I.

- A (top, on outer circle): upper end of the vertical diameter
- C (bottom, on outer circle): lower end of the vertical diameter
- D (upper, on inner circle, just below A): point on the earth's surface
- F (interior, centre of the inner circle): centre of the earth's magnitude
- E (interior, below F): centre of gravity / of the Universe
- G (left, on inner circle): point where line BGE meets the earth's surface
- H (left, on outer circle): where the semidiameter FG is extended to

- B (upper left, on outer circle): the high point from which the weight falls along line BGE
- B (right, on outer circle) [second label read as "B" as printed — see anomaly note]
- I (lower centre, on inner circle): point where line BIE meets the earth's surface
- K (lower right, on outer circle): where the semidiameter FIK is extended to
- L (bottom centre, on inner circle): point on the earth's surface

For if the semidiameter of the earth FG is drawn and extended as far as H, the two angles EGD and FGL will be equal, since they belong to equal semicircles; and consequently, by the same reasoning, the two exterior angles DGH and LGH will be equal, as is evident if one angle be superimposed on the other. For the arc GD would coincide with the arc GL, and the straight line HF would be common. Since, then, the angle DGB is less than the angle DGH, and the angle BGL greater than the angle LGH, the angle DGB will be many times less than the angle BGL. Therefore a weight let down along the straight line BGE will not be carried at equal or similar angles onto the surface of the earth. Which was to be demonstrated. You will say the same if along the straight line BIE some heavy body descends to the centre E of the Universe: for if the earth's semidiameter FIK is drawn, again the angle BID on the surface of the earth will be less than the angle BIL. Only those weights which are carried along a straight line (which would happen in very few places) — namely along the line extending through the centre of gravity, that is, of the Universe, and through the centre of the earth's magnitude, that is, along line ADFE or CLEF — strike the surface of the earth at equal angles, and no others besides these, as we have demonstrated. And since this conflicts with experience and with Aristotle, it must be said that the centre of magnitude in the earth is the same as the centre of gravity, or of the Universe; so that from whatever place heavy bodies are dropped, they are carried to the centre of the earth. For only in this way will there be formed on the surface the equal angles which experience teaches must be equal.

Exactly the same judgment must be held about the centre of magnitude in the water, and the same demonstration can be applied, provided that the circle DGL represents the globe of water, whose centre is F. For just as the plumb-lines rest upon the surface of the earth at equal angles, so too the same make equal angles with the surface of the water. Yet by a proper and special argument it can be confirmed that in the water the centre of gravity and of magnitude are the same. For since water, when unhindered, always flows of its own nature to lower places, as experience shows, its convex surface must necessarily recede equally

from the centre of gravity. But that point from which all the convex parts are equally distant is, by definition, the centre of magnitude. Therefore the centre of gravity cannot be different from the centre of magnitude of the water. And the major premise is proven: for if the convex surface of the water receded more from the centre of gravity (or of the Universe) on one side than on another, then that part, being farther removed from the centre of gravity, would not flow down to a lower place — which without doubt is the one nearer to the centre of gravity, or of the Universe — as appears from the first figure of this question, in which the centre of the earth's magnitude is the same as the centre of the world, but the centre of the water's magnitude is distinct from it. And since this is absurd and conflicts with the nature of water, it follows that the centre of magnitude and of gravity in water are the same — which was to be shown. For which reason it must be concluded that, since earth and water have the same centre of gravity — namely that of the whole Universe, toward which they naturally incline, and which has been shown not to differ from the centre of magnitude of each element — a single sphere, or globe, is composed of both elements, and by no means two globes mutually intersecting each other.

[Margin: *Second argument* (2. ratio).]

Secondly, we shall demonstrate, by many experiments of the astronomers, that earth and water have one and the same convex surface, and consequently the same centre. For just as the Sun and the other stars rise, reach mid-heaven, and set an hour earlier for a city that is fifteen degrees more to the east [than another], and two hours earlier for one that is thirty degrees more to the east, and so on — in whatever tract of the earth from east to west those cities are found, provided they lie under the same parallel — so too the most experienced sailors have found that the same thing happens on the sea and Ocean. For sailors voyaging to more western regions — for example from Portugal to America, or New Spain, especially to that province called Florida — after they had advanced fifteen degrees, found, by the clearest signs, above all from a lunar eclipse, that the Sun and the other stars rise and set a full hour earlier in Portugal; and they observed that the same happens, in the same proportion, throughout the whole Ocean from east toward west. But this could in no way come about unless the convex surface of the sea were uniformly continuous with the convex surface of the earth, as is very well known to all geometers. For if the sea gradually rose into some swelling or mountain, as those of the contrary opinion fable, then the Sun would rise earlier for those who sail, after they had completed some degrees, than when they were on land. By the same reasoning, if someone were to say that the sea gradually sinks down, that proportional variation in the rising and setting

of the Sun and the other stars could not be preserved. Since this is false, it is clear that earth and water hold one and the same convex surface, from any part of the east toward the west.

Furthermore: just as, if someone proceeded on land from north to south, in whatever direction, and after walking a whole degree would find the Arctic pole depressed by one more degree — and if he had traversed two degrees on land, [would find it] depressed by two degrees as well, and so on proportionally — so exactly the same has been observed at sea. For when a voyage is undertaken from north to south — as, for example, from Portugal (where the elevation of the pole is 40 degrees) toward the Canary or Fortunate Islands — after a journey of a whole degree has been made, the pole is found to have an altitude of only 39 degrees, and so on proportionally. The contrary was observed when one sails from south to north, as from the aforesaid islands toward Portugal, or from Portugal to Britain. It is therefore a most manifest sign that the water has the same convex surface as the earth, [reckoning] from the north...

from the north to the south, in such a way that neither the land nor the sea is raised any higher, but both elements are set an equal distance from the centre of the world. Otherwise the stated proportion could not hold constant amid the variation in the pole's altitude. Since, then, the convex surface of the water nowhere differs from the convex surface of the land — neither from east to west nor from north to south — no room is any longer left for doubting that a single globe is constituted from both elements.

This argument carried such weight with a certain man who was maintaining the contrary opinion (as I have learned from trustworthy men who were on familiar terms with him) that at his own expense he sailed off in various directions, taking various mathematical instruments with him, to test whether this proportion — which we have said is preserved in the rising and setting of the stars and in the elevation of the pole — was true, or was a fabrication of the astronomers. And when he found it to be entirely true, he abandoned his earlier mistaken opinion and embraced the true view.

[Margin: 3. Argument.] Thirdly, our conclusion can be established from lunar eclipses, by the following reasoning. In every eclipse of the Moon the shadow of the aggregate of land and water is round, in whatever part of the sky the eclipse occurs. Therefore the land and water must compose a single globe.

The antecedent is clear from the still-uneclipsed parts of the Moon: these are horned, that is, bounded by a circular arc, as experience has made known to all

astronomers, and even to those who have observed but a single lunar eclipse. The shadow must therefore be of the same shape, namely circular. For if it were square, or triangular, or of any shape other than spherical, the Moon would not be seen to enter the shadow in a circular manner, but after the manner of a shadow that is not round — which conflicts with experience. And the inference is a necessary one. For as the writers on optics [Perspectiui] show, the shape of every shadow imitates the shape of the opaque body that casts it: if the opaque or shadow-casting body is round, a round shadow too is projected; if the shadow-casting body has straight sides, a shadow of the same figure is seen; and so with the rest, as anyone can very easily verify.

Since, therefore, the shadow appears perfectly round in every lunar eclipse — as the still-uneclipsed parts indicate — we must necessarily conclude that the body producing that shadow, namely the compound of land and water, is round and spherical. For if the aggregate of land and water were of some other shape — somewhat oblong, say, and irregular, as the opposing view maintains — the shadow in the eclipse would likewise take on such a shape, which is false.

Now if those who hold the contrary view should reply that, even if the whole Ocean and sea were heaped up into a very high swelling above the land, it would still not follow that the shadow would fail to appear in a lunar eclipse — since, they say, the water casts no shadow of its own, but only the land, which is round — the answer is that this reply is quite absurd. For the whole Ocean and sea is strewn with almost countless islands, so that, whichever way one sails, if the sailors of our time are to be believed, there are always found either continents or islands; and since these are continuous with the mainland (for no one would say that they float upon the waters), who does not see that, if the arrangement of these two elements were such as they imagine, the shadow of the earth together with the shadows of all the islands would necessarily be produced marvelously broken and misshapen? Since this runs contrary to experience, these two elements are not arranged as our opponents would have it; rather, they make up a single globe, so that the islands found in mid-ocean are no farther from the centre of the world than the continents, but equally distant, so that the shadow in an eclipse is produced round, as experience teaches. There is this too: water undoubtedly casts some shadow of its own, as experience attests, especially seawater, which is denser and thicker than other waters. Let us conclude, then, that since the shadow of the aggregate is always round, the aggregate itself is likewise round and spherical.

This same truth can be confirmed by a common experience, which Ptolemy also adduces (*Almagest* I.4) and Johannes Regiomontanus [Regiomontanus] (Book I, conclusion 2), and which is roughly this. When we are in the middle of the sea we see nothing at all but sky and water; but when we make for the shore, then first the mountains, cliffs, citadels, towers, and other such things are seen gradually to rise up, as if they were emerging from the water — and this in such a proportion that first the peaks of the mountains and the tops of the towers appear, then their middle parts, and last of all their lowest parts near the shore. This would by no means happen in so orderly a way if the sea were raised above the land, or if the surface of the sea were not continuous with the surface of the land so as to make one surface out of both. For if the sea were raised up in the middle, so that its circumference were not continuous with the circumference of the land, then, once someone had climbed the summit of the swelling that (according to those authors) the sea forms, he would at once see everything set along the shore — just as, when someone reaches the top of a mountain, he immediately takes in at a single glance everything lying on the plain below. But that is absurd: for it is first the things that are higher on the shore, and then those set in a lower place, that come into view.

There is a further point. If the land and water did not have one and the same continuous convex surface, but the water rose gradually, it would follow that a man who sails out of a harbour in a ship could not fail to see a marker set up by the shore, since it would be rising up — which is against experience. Add to this that, since water by its own nature flows together to the lower places, as experience shows, it will always be received into the hollows of the land until it fills them and is brought to a level with the land. For in this way it stands equally distant from the centre of the world, is set in equilibrium, and so makes one spherical surface together with the land.

This view will also be confirmed very clearly a little later, when, together with our author, we shall demonstrate that both the surface of the land and that of the water surround the centre of the whole Universe equally — from which it plainly follows that the centre of both elements is one and the same, and that therefore a single globe is constituted from them.

[Margin: Why the earth alone occupies the centre of the world, and not also the water.] But someone will perhaps ask: since water and land possess the same centre, as has been proved — the centre to which, when unhindered, they descend along the same straight line — for what reason does the land alone occupy the centre, and not the water as well? For we see the water spread out

above the surface of the land. The answer is that there is a natural distinction between the element of earth and the element of water: the earth, by its greater weight, occupies the centre, whereas the water, being not so heavy, naturally remains above the earth, as the philosophers assert. So much so that, if the earth were perfectly round and formed a smooth globe, the element of water would cover the whole earth on every side; and this would also happen if there were so great a quantity of water that it filled all the hollows of the land and rose above the mountains. But since the earth is not perfectly spherical — because of its mountains, crags, rocks, and valleys — and since there does not exist so great a quantity of water as to be able to cover the whole surface of the land, the result is that all the water has been received into the various hollows of the land, yet always standing, along its convex surface, equally distant from the centre of the world, as the foregoing arguments have shown.

[Margin: How it is to be understood that a single globe is constituted from land and water.]

But when we said that a single globe is made from land and water, this must not be understood as though the perfect globe that the geometers define resulted from the two elements. For that is false, if we wish to speak geometrically and strictly. First, because the straight lines going out from the centre of this globe to the tops of the highest mountains will undoubtedly be longer than the straight lines drawn to the lowest parts of the deepest valleys, so that the definition of a geometrical globe cannot fit it in every respect. And second, because the convex surface of the water is contained beneath the surface of the land at an equal distance, like a smaller circle beneath a larger one having the same centre — so much so that, if the whole surface of the water were completed about the centre of the world, and likewise the whole surface of the land, the former would be contained beneath the latter at this same constant distance. But since this unevenness or inequality, compared with the whole mass composed of land and water, is of practically no account — so that it is scarcely perceived by the senses — the result is that the aggregate of land and water is simply called by the astronomers a round, or spherical, globe.

That the surface of the water is contained beneath the surface of the land at an always equal distance can easily be made convincing, once we assume that from east to west, along the equinoctial circle [*Æquinoctialis*], continents, islands, peninsulas, and so on are traversed — a thing that the navigation of our own time, especially that of the Portuguese, plainly teaches, though it was quite unknown to the ancients. For if a great circle is described upon the land, lying

directly beneath the celestial equator, passing through the island of São Tomé [insula D. Thomæ], through Africa, through Taprobane [Taprobana; Sri Lanka] in the East Indies, through the Molucca Islands, through the province of America or New Spain that is called Peru, until it comes round again to the island of São Tomé — this circle, at least near the shores, will contain beneath itself the surface of the sea, seeing that one descends from land to sea on every side, as is clear from the running-down of the rivers. From this we shall now draw our intended conclusion, as follows. The arc described on the surface of that sea which lies between Africa, for example, and Taprobane is at an equal distance from the arc of the described circle on the land, which passes through Africa and Taprobane, and so on. And the same must be said of any arc of the sea's surface lying between any two lands whatever. Therefore the whole surface of the water is contained beneath the whole surface of the land at an equal distance. The inference is excellent, resting on a sufficient enumeration of parts. And the antecedent is proved: for if that arc described on the sea were not equidistant from the arc of the land, but rose higher in the middle, or sank lower, or even crossed over the arc of the land by cutting it, it would follow that the two arcs do not have the same centre, as is established among the geometers — which we have already refuted, since it has been proved that the centre of both elements is one and the same.

It remains for us to bring forward, and to resolve, some objections that might be raised against our view. For although the experiences adduced so far show clearly that the centre of land and water is one and the same, and hence that a single globe is constituted from them, there are nonetheless some considerations that seem to create a difficulty and to prove that these two elements can in no way make up a single globe.

[Margin: 1. Objection.] First, then, one might try to prove that the centre of land and water is not the same, and hence that a single globe is not composed of them, as follows. Land and water differ in weight, for it is agreed that land is heavier than water. Therefore they cannot have the same centre of gravity and of magnitude; rather, the land by its weight will drive the water out of the continuous [region] of the whole universe, which is owed to the land because of its supreme weight — just as a globe that is partly of wood and partly of lead cannot possess the same centre of gravity and of magnitude, since the latter is at its middle, while the former is in the leaden part, as the heavier.

[Margin: Solution of the objection.] To this objection the reply is that it proceeds from a false assumption: for it supposes that the land is entirely on one side and

the whole sea on the opposite side, which is false. For by the voyages of our own age — under the poles as well as under the equinoctial circle, in the east as well as in the west, and in short over the whole globe — there have been found either continents, or islands, or peninsulas, so that over almost the whole globe land and water are intermingled. For the sea is strewn with almost countless islands, so much so that more land, or at least not much less, appears outside the sea than is covered by the waters — as Alessandro Piccolomini [Alexander Piccolomineus] admirably proves in his little book *On the Quantity of Land and Water*. Hence we say that this globe, which we assert is made up of land and water, is so constituted that the land stands out on every side while the water settles down in the lower parts. The land, then, presents the appearance of a kind of wooden globe in which there are very many hollows in which water can be received. For in this way this globe is so balanced by an equality of weights that it has the same centre of gravity and of magnitude. And Aristotle too seems to be of this opinion (*Meteorology* I), where he says: [Margin: "The mass of the earth, which has embraced even the whole quantity of water, does not amount to the ratio of a single particle in comparison with the surrounding magnitude."] By these words he plainly seems to assert that the water is contained within the hollows of the land, since he says that the land holds within itself the whole quantity of water. Indeed, natural reasoning drawn from experiment seems to persuade us of this very thing. For we find that waters flow together and fall away toward the lower and more hollow parts of the land, so that they are contained within the higher places of the land no otherwise than valleys are contained within mountains, until all the parts are brought to a level and made exactly even — as Aristotle rightly demonstrated (*On the Heavens* II), whose argument we adduced above.

[Margin: 2. Objection.] Next, someone will object in this way. The uncovered parts of the land are less heavy than the parts covered by seawater, on account of the air enclosed in caverns and the heat of the Sun, which continually dries them out. Since, then, the centre of gravity in a body of uneven weight lies in its heavier part, the centre of gravity of the land will be nearer to those parts that are covered by the waters than to those that are uncovered; and so the centre of gravity of the land will be different from its centre of magnitude.

[Margin: Solution of the objection.] But this objection too seems to assume the same thing as the previous one, namely that the uncovered parts of the land belong to one hemisphere and the covered parts to the other — which is not true, as we have said. We reply, then, that the uncovered parts are indeed less heavy, absolutely speaking, on account of the causes mentioned, which without doubt diminish their weight; but since the enclosed air and the Sun's heat penetrate an

almost imperceptible part of them, if that is compared with the whole depth of the land (for the air scarcely reaches, by penetrating, one or two miles, whereas the whole depth of the land embraces 3,579 [?] miles and more, as we shall say at the end of this chapter), and since there stand in those very uncovered parts immense and numerous mountains and crags, while in the covered parts almost countless islands are found which rise above the sea, furnished even with the highest cliffs, and finally the whole earth is full of water, as experience shows (since everywhere, when the ground is dug, water is found) — the result is that the uncovered parts, together with the covered ones, with the addition also of the seawater that is spread above the covered parts, are so balanced, and the weight of all the parts is as it were compensated, that the centre of gravity of both ele-

—ments, namely of the land and of the water, is equally distant from their surface, just as it in fact is, as we demonstrated above by many experiments. Nor is it any objection that the surface of the land is somewhat higher than the surface of the sea, as we said above, so as to prevent the centre of gravity from being at an equal distance from each surface. For that excess is exceedingly small in comparison with so great a magnitude, so that both surfaces may rightly be said to be equally distant from the centre — if we consult the senses, which report the water to be of the same height as the land, although, precisely and geometrically speaking, this is not true.

By this too is dissolved the argument that we urged above against the authors who defend the position opposed to our view — namely, that according to them more miles must correspond to one degree at sea than on land, seeing that they make the sea higher than the land, and by more. For now this same argument could be turned back against us, since we make the land higher than the water, from which it seems to follow that more miles correspond to one degree of land than of sea. This argument, turned back against us, is dissolved, I say, because that excess of the land's height above the sea's height, which we posit, is of no account but altogether imperceptible. Hence nothing is concluded against us; but against our adversaries it has the greatest force, since they posit the water to be many times larger than the land — namely in a tenfold proportion — from which it necessarily follows that there are more miles in one degree of the sea's surface than in a degree of land.

[Margin: 3. Objection.] Thirdly, someone relying on the judgment of the senses might rise up against us with this reasoning. How can a single globe be made from land and water, when neither the land nor the water appears to be round? For when someone climbs the top of some mountain, from which he may look out

over a great plain of land and the surface of the sea, both the sea and the land are judged by the senses to be flat and by no means round; and the difficulty seems especially great with respect to the land, on account of so many mountains of enormous height and valleys of marvelous depth. It happens too that the Sun, when it rises or sets, seems to be cut off by the surface of the land along a straight line; therefore the land is flat. And the same must be said of the sea. For if the land and the sea were round, they would surely cut off curved parts, and not straight ones, from the rising and setting Sun — just as we see that the Moon, because it is round and spherical, cuts off curved parts from the Sun in a solar eclipse.

[Margin: Solution of the objection.] Yet this objection must be met by pointing out that our senses are here deceived in a remarkable way. For that portion of the surface of the land and sea which someone stationed on a mountain, however high, can survey is so small in comparison with the whole magnitude of the land and water that no curvature can be perceived in it — no otherwise than if, from some very large circle whose circumference embraces, say, 2,000 paces, a portion of three or four hand-breadths were taken away. For in that portion taken away we should perceive no curvature at all, but it would appear entirely straight; and the same holds of a sphere of the same magnitude. It ought not, then, to seem surprising that our sight cannot perceive the roundness, or the convex surface, of either the land or the water.

As for the mountains and valleys existing on the land, we must say that the earth, on account of the excessive hardness of its rocks and the dryness of its other parts, could not coalesce into a globe so perfectly and completely as the water did, and that for this reason it has remained so rough and full of all those hills, mountains, and valleys — in which respect nature seems in a way to have taken thought for the plants and animals dwelling on the land, for such mountains and valleys are of great benefit to them, as experience teaches. Nevertheless, although these eminences and hollows of the land, considered in themselves, appear enormous, yet when compared with the whole terrestrial globe they are so small that they scarcely hinder its roundness at all, as is plainly evident in a lunar eclipse. Just as some huge stone globe, though roughly wrought and rough with many bumps and hollows, is nonetheless said to be, and is, round, so too must it be said of the earth, even though these eminences and hollows exist in it. Moreover, just as, if in that stone globe some tiny animal were found, it would perceive nothing but a plain, mountains, and valleys (for so great would those little roughnesses of the stone globe appear to it), so too it happens to us, who are of a tiny and imperceptible size in comparison with the terrestrial sphere, as we

walk about on the land. Finally, just as in that same globe those roughnesses do not prevent its shadow from being produced round and appearing so, in the same way these terrestrial eminences cannot be an obstacle to the earth's shadow being round, as we see in a lunar eclipse.

As for what was brought forward, finally, concerning the rising and setting Sun, the answer is that this happens because, since the land appears to the senses much larger than the Sun, from which we are very far distant, only a very small portion of the land seems to be taken in by the Sun at rising or setting — which, on account of the excessive size of the land, appears straight, as we said above concerning a small portion of a circle that has an enormous circumference. But since the Moon appears to our sight spherical on every side and of nearly the same magnitude as the Sun, the result is that in a solar eclipse it takes away circular, and not straight, portions from the Sun.

[Margin: The senses are deceived, because the sky appears to overhang the earth like an oven-vault, and the earth to touch the sky itself at the horizon, etc.] From what we have said about the globe composed of land and water, it is easy to gather how much the senses are deceived, which suppose that the sky overhangs the land like an oven-vault; and likewise that both sky and land touch at the edge of the horizon, as if these bodies were contiguous; and, on the same principle, that the Sun, when it rises, emerges from the Ocean, and when it sets, is plunged beneath the same — as the poets too tell in their fables. For since it has been proved that the land and water are concentric with the sky, a single globe must necessarily be constituted from them, so that they are equally distant from the sky on every side. Therefore the senses are deluded, because from the direction of the horizon they do not take in that space which is contained between the sky and the land.

[Margin: The error of certain Peripatetics, who set up a tenfold proportion among the elements.] From what has been said it can also be clearly gathered how absurd is the opinion of certain Peripatetics who hold — following, as they claim, Aristotle and the truth — that a tenfold proportion is preserved among the elements: so that water is ten times larger than land, air surpasses water in a tenfold proportion, and fire, finally, is ten times larger than air. For since land and water have the same convex surface, and since either the greater part of the land, or at least not much less, is uncovered than is covered by the waters, as we have said, the falsity of that opinion is clearly perceived. For so far is it from being the case that on this reasoning water is ten times larger than the land, that on the contrary the land rather surpasses the water in magnitude — seeing that

the depth of the land extends all the way to the centre and embraces 3,500 miles and more, as we shall say at the end of this chapter, whereas the depth of the sea scarcely reaches two or three miles, indeed for the most part does not exceed half a mile, as the sailors of our age have found by experience, who, even in the middle of the Ocean, investigating the depth of the sea with a sounding-lead, everywhere find the bottom, and not far from the surface of the sea. From all this it is established that the water is much smaller than the land.

COMMENTARIVS (continued) — AN EX TERRA ET AQVA VNVS FIAT GLOBVS

The same opinion can also be attacked geometrically. For if the elements preserved a continuous proportion, then the whole composite would contain the earth only one thousand, one hundred, and eleven times, as is evident in this continuous tenfold proportion 1.10.100.1000. — for all these numbers collected into a single sum make 1111. But this is utterly false and rashly asserted. For according to the astronomers, the semidiameter of the whole elemental region — that is, the distance from the center of the world to the concave [surface] of the Moon — contains the earth's semidiameter thirty-three times; indeed, according to some, this distance is greater. Hence the whole diameter of the sphere of the elements will likewise contain the whole diameter of the earth as many times, [Margin: a — (Euclid) V.15] since the proportion of the diameters is the same as that of the semidiameters. [Margin: b — (Euclid) XII.18] And since spheres are in the triplicate proportion of their diameters, it comes about that the whole sphere of the elements contains the terrestrial sphere thirty-five thousand, nine hundred and thirty-seven times (35937), as you discern in these numbers: 1.33.10809.35937. [recte: the geometric mean should be 1089 = 33^2 , so the printed "10809" appears to be a compositor error]

Add that, according to their own opinion, the distance from the center of the world to the concave of the Moon would contain the earth's semidiameter only ten times, and a little more — so that, by the law of triplicate proportion, the sphere of the elements would contain the terrestrial sphere one thousand, one hundred, and eleven times (i.e. millies, centies, & vndecies), as they wish. From which it would follow that our eye is distant from the lunar orb by only nine earth-semidiameters — which is against the experience of all astronomers.

But if anyone should say, as some of them wish, that this tenfold proportion should be understood of the diameters or semidiameters of the elements, and not of the quantity or mass of the bodies, this will be far more absurd. First, because

it is utterly false that the Moon's distance from the earth contains 1111 earth-semidiameters, since this conflicts with all astronomers, and scarcely even the Sun is removed from the center of the world by so great an interval. [Margin: c — (Euclid) XII.18] Next, since spheres have the triplicate proportion of their diameters, it would follow that water is a thousand times greater than earth, and that the whole sphere of the elements would have to the earth the proportion which this number 1000000000 has to 1 — as is evident in these numbers: 1. 1000. 1000000. 1000000000. This is indeed ridiculous, and no astronomer ever asserted it. For who would say that water is a thousand times greater than earth, when on the contrary earth is much greater than water on account of water's slight depth, as we said a little earlier is established by the experience of the navigators of our own time? It remains, therefore, that the opinion of those Peripatetics is absurd.

Indeed, not only do the elements by no means preserve this tenfold proportion, but not any other continuous one either, as Alessandro Piccolomini [Alex. Piccolom.] rightly proves in his little work *On the Quantity of Earth and Water* [*de Quantitate terræ et aquæ*], and as Jean Fernel of Amiens [Fernelius Ambianus] confirms in his *Cosmotheoria*. Nor does the authority of Aristotle [Aristoteles] stand in the way — the authority which the said Peripatetics adduce in confirmation of their opinion, when he says that from one handful [pugillus] of earth ten handfuls of water are generated, from one of water ten of air, and finally from one of air ten of fire. For Aristotle asserted this either by way of example, or, if he really thought so, it must be understood thus: if water had to be generated out of the whole quantity of earth, the water so produced would be tenfold greater than the earth, and so of the rest — not, however, that the elements now actually existing have such a proportion. For in that case there would have to be equal matter in all the elements, which Aristotle nowhere affirmed; indeed it seems to be contrary to experience.

For not only is water smaller than earth, as we said, but air too seems to be smaller by many parts. For it is likely that the aerial region is only that in which the vapors drawn from earth and water — even the most subtle — have their dwelling, since there is no better reason why they should be able to exist in one part of the air rather than in another, if in any. And since the highest elevation of vapors is to 52 miles or thereabouts — as is proved geometrically by Alhazen [Ibn al-Haytham] in book 7 of his *Perspectiva*, by Witelo [Vitellio] in book 10, proposition 60, and by us toward the end, in the *Digression on Twilights*, proposition 6 — it must be said that the height or depth of the air contains 52 miles or thereabouts, and no more, so that at so great an interval from the earth lies the boundary of air and fire. Otherwise vapors could ascend still higher,

unless the dryness and heat of fire resisted; and this has been seen by no one to happen up to now.

Since these things are so, we shall easily find how much smaller air is than earth and fire. [Margin: By how much air is smaller than earth and fire; and likewise by how much fire is larger than earth.]

For since the semidiameter of the earth, according to Ptolemy [Ptolemæus], comprises about 3579 miles, and its whole diameter 7158 miles; and since the distance from the center of the earth to the convex [surface] of the air comprises 3631 miles [as semidiameter], and the whole diameter of the globe composed of earth, water, and air 7262 miles — hence, by the precepts handed down at the end of this chapter, we shall find the greatest circle of each globe (both of the one made of earth and water, and of the one made of earth, water, and air), and from this the solidity [volume] of each globe.

For if we multiply 7158, the diameter of earth and water together, by $3\frac{1}{7}$, we produce the circumference of the greatest circle of the terrestrial globe, $22496\frac{1}{2}$ miles, whose half is $11248\frac{2}{7}$. If this be multiplied by 3579, the earth's semidiameter, there is produced the greatest circle [area] of $40257614\frac{4}{7}$ square miles; and if this in turn be drawn into 4772 — that is, $\frac{2}{3}$ of the whole diameter of the earth — there is produced the solidity of the terrestrial globe, $192109336734\frac{2}{5}$ cubic miles.

Again, if we multiply 7262, the diameter of the globe made of earth, water, and air, by $3\frac{1}{7}$, we shall find the circumference of its greatest circle, $22823\frac{1}{2}$ miles, whose half is $11411\frac{1}{2}$. If this be drawn into 3631, the semidiameter of the same globe, there is generated the greatest circle of that globe, $41435934\frac{4}{7}$ square miles; and if this in turn be drawn into $4841\frac{1}{3}$ — that is, $\frac{2}{3}$ of the whole diameter of the same globe — there is produced the solidity of the same globe, 200605171238[?] cubic miles. From this solidity, if the solidity of earth and water already found be subtracted, there remains [the solidity of the air], 8495834503[?] cubic miles; from which it comes about that earth and water together have to the air alone a proportion greater than 22 to 1, but less than 23 to 1.

Furthermore, since the semidiameter of the concave [of the Moon] [comprises $120630\frac{2}{5}$ miles], a little less, and the whole diameter $241261\frac{1}{2}$ miles: if we multiply this diameter [by $3\frac{1}{7}$], we find the circumference of the greatest circle of the sphere of the elements to contain 758250[?] miles, whose half is 379125[?]. If this be drawn into $120630\frac{2}{5}$, the semidiameter of the concave [of the Moon], there will be completed the greatest circle of the elementary sphere, 4573413186[?]

square miles; which if we again multiply into 160840 $\frac{20}{21}$ — that is, $\frac{2}{3}$ of the whole diameter of the same sphere — we shall procreate the solidity of the same elementary sphere, 73559213251333313[?] cubic miles, that is, roughly 7355921325133313 $\frac{1}{4}$. And if from this solidity we subtract the solidity of the globe made of earth, water, and air which we found a little before, the remainder becomes the solidity of the fire alone, 73557207199620754 $\frac{1}{2}$ cubic miles. From which it comes about that fire has to earth a proportion greater than 38289 to 1 but less than 38290 to 1; and that the same fire has to air a proportion greater than 865803 to 1 but less than 865804 to 1.

And so, if the globe made of earth

and water be put as 23, the quantity of air will be about 1, and of fire about 865803 — for here we neglect the small fractions that do not make up a unit. I have said these things so that it may appear how rashly some men dare to affirm that there is a tenfold proportion among the elements.

But if anyone should contend that air extends beyond 52 miles, even though vapors do not ascend farther, on account of the excessive dryness and heat of that air, the dispute will be about a word. For that very thing which is exceedingly dry and hot above the air, we call fire — just as Aristotle too affirms in book 1 of the *Meteorologica*, treatise 1, chapter 4, where he says:

But we must understand that, of the air we have spoken of, the part which is around the earth is as it were moist and hot, because it holds the vapor and exhalation of the earth; but the part above this is now hot and dry. For the nature of vapor is moist and hot, while that of exhalation is hot and dry.

Likewise in the same book, treatise 2, chapter 1, writing thus:

For first, beneath the circular motion, is the hot and dry, which we call fire. It is unnamed, and common in every smoky dispersion; nevertheless, since such a body is most apt to be burned up, it is necessary to use these names thus. And beneath this nature is air.

Indeed the same Aristotle, in other places, calls the exhalation "fire" [when it is] beneath the concave of the Moon, as in the same book, treatise 2, chapter 4, writing to this effect:

For it is assumed by us that, of that [part] of the world which is around the earth, insofar as it is beneath the circular motion, the first part is the dry and hot exhalation. And this itself, together with much of the air

continuous beneath it, is carried around the earth by the [heavenly] course and circular motion.

From all these passages it seems evident that Aristotle calls "air" only that part beneath the concave of the Moon in which vapors exist, and the rest "fire." Wholly empty, therefore, and to be rejected is the opinion of those who posit a tenfold proportion among the elements, since there is no continuous proportion among them at all; indeed both water and air are smaller than earth, as is clearly evident from what we have said.

It is gathered again from what we have said — namely, that all heavy things tend toward the center of the whole universe, which is also the center of water and of earth — that all plumb-lines and heavy bodies, freely let down from different places, draw near to one another, so much so that at the center of the earth, or of the Universe, if they should reach it, they would meet at a single point, as you observe in the figure set beside. [Margin: That buildings constructed to the perpendicular are not parallel, but would come together at the center of the world if they were extended.] But since the whole distance to the center, considered in itself, is exceedingly great, it comes about that in a small space this convergence of the plumb-lines cannot be perceived. For if two plumb-lines are distant from one another by ten spans, or a hundred, or even more, then — because this distance is exceedingly small in comparison with the earth's semidiameter, and consequently the angle of their meeting at the center of the world turns out to be very small — they will seem to be altogether equidistant. And this is the reason why buildings constructed to the perpendicular appear equally distant, or parallel, even though in reality, in the lower place, they are nearer one another, while in the upper they are more separated one from another. And you will say the same of any two walls whatever. And so, if a well were dug to the perpendicular right down to the center of the world, its sides would continually taper inward, though imperceptibly, until, meeting at the center, they completed the figure of a pyramid whose base would be the mouth of the well, and whose vertex the center of the whole Universe. Similarly, if a tower of as great a height as the earth is deep were built to the perpendicular, its sides at the summit would be wonderfully distant from one another. From which it follows that all walls built to the perpendicular tend straight toward the center, just as any straight lines of a circle issuing from the center meet at the center. And unless this were observed by the builders, buildings could stand by no reasoning.

Diagram. A circular woodcut (representing the globe, or the mouth of a well) with two nearly-vertical plumb-lines descending from above and converging toward a single point at the center of the circle. It illustrates that plumb-lines and walls built to the perpendicular from different places are not truly parallel but would meet at the center of the world. No lettered labels appear on the figure.

From this it is further inferred that no pavement built to the level or to the perpendicular is flat, but lies "librated" [level] — that is, equally removed on all sides from the center — and is a portion of some sphere whose center is the center of the world, or of the earth; [Margin: That a pavement built to the level or the perpendicular is not flat, but is a portion of a sphere whose center is the same as the earth's.] since the plumb-lines, always inclining toward the center, are gradually drawn together, as we said, and compel the pavement to be round. But this roundness cannot be perceived in a modest space, just as neither can the roundness of earth or water. Yet some huge pavement — of, say, 3000 or 4000 paces — built to the level would altogether display some swelling. Hence it comes about that some immense rectilinear pavement, that is, laid out along a straight line, cannot in the least be called "librated." For those things are properly called "librated" which are equally removed from the center of the world — such as the outermost surface of water, wherever it is placed — the kind of thing which a rectilinear or flat surface cannot be. For if from the center of the world very many straight lines be extended to that [surface], the shortest of all will be the one which stands perpendicular to the surface; while the rest, the farther they are from the perpendicular, will also be so much the longer, as you discern in this figure, and as can easily be proved from proposition 19 of book 1 of Euclid. Wherefore that point in the plane upon which the perpendicular falls will be nearest the center, and consequently lowest; whereas the remaining points of the plane will be farther from the center, and therefore higher, so that the extremities of the plane are highest and nearest the heaven. Wherefore, if someone were to walk in that plane around the point nearest the center, he would think he was walking altogether on a level, since he would feel no rise in so small a space — whereas in truth he would now be ascending, now descending, so that the farther he had withdrawn from it, the more steeply he would ascend, until at last the steepness denied him further ascent — as can be perceived more clearly than daylight in the figure set forth. Hence it also comes about that, if anyone walks about on some librated pavement (such as the surface of earth or water), his head is carried more swiftly than the middle parts of his body — because, of course, in the same time both the head and the middle parts of the body describe a portion of a circle whose center is the same as that of the earth; and it is clear that the head describes a greater circle, since it is farther

from the center than are the middle parts of the body, which are nearer the center.

Diagram. A woodcut showing a globe (a shaded semicircle at the bottom) from whose upper region many straight lines fan out to a horizontal line above, each line tipped with an arrow. The central line is perpendicular (and shortest); the lines toward the edges are progressively longer and more oblique. It illustrates that a level pavement is really a portion of a sphere, the perpendicular being the shortest line from the center to the surface. No lettered labels appear on the figure.

It is likewise left clear from what has been said that any vessel whatever receives more water when placed in a lower position than in a higher — as, for example, at the foot of a very high mountain than at its summit. [Margin: That more water is contained in a vessel at the foot of a mountain than at its summit.] For since, as we showed above, every portion of resting water, in whatever place it is contained, lies "librated" — because by its own nature it flows together toward lower places and produces a spherical surface whose center is the center of the world — it is clearer than

daylight that the surface of water, the nearer it is to the center, forms the smaller sphere, since it possesses the smaller diameter. And since one and the same straight line cuts off a greater swelling from a smaller circle or sphere than from a larger, it comes about that the same vessel cuts off a greater part from the smaller sphere than from the larger, and therefore receives a greater quantity of water in the lower place than in the higher, as you discern in the figure set forth. But since so small an interval — such as is that from the foot of a mountain, even the highest, to its summit — is of no moment if it be compared with the whole semidiameter of the earth, it comes about that this difference is scarcely felt. If, however, access lay open all the way to the center, we should there plainly behold with our eyes, and observe, the heap or spherical swelling of water standing upon the full vessel. For the more the water draws near to the center, the more perceptibly does it also acquire roundness; so much so that, if the earth were to withdraw from the center of the world and access were opened to the water itself, the whole multitude of the water would at once flow together to the middle of the world, and at the first rush would surge this way and that, until little by little, the impetus of its motion being slackened, it came together into a most perfect globe and equally surrounded the center of the whole world. Many other things like these can be gathered from what has been said, but which for brevity's sake I judge should be passed over.

Diagram. A woodcut of two concentric circles (the outer representing the sphere of fire, the inner the sphere of air, or the convex surfaces of the elements). Within the inner circle are drawn two truncated (trapezoidal) vessel-shapes seen in cross-section: a smaller one near the top, and a larger one at the center. It accompanies the discussion of how a vessel holds more water lower down, and of the spherical figures of the elements. [Margin: *Figura aeris & ignis*[?] — "Figure of air and fire"; the marginal caption ends with an unclear word ("*qua*"[?]).] No lettered labels appear on the figure.

Finally, that we may say something also about the figures of the remaining two elements — namely air and fire — it seems it can be concluded clearly enough that they are of spherical figure. For in the first place air, as regards its concave surface, is round, since it surrounds and encircles the round globe which we said is constituted of water and earth; and as regards its convex surface it is necessarily round, since it lies beneath the concave of the Moon. By the same reasoning Fire, as regards its convex surface [is round]. And because both fire and air seem equally to recede from the center on account of their lightness — not otherwise than heavy things tend toward the center on account of their gravity — it comes about that air (according to its convex surface) and fire (according to its concave surface) are also of spherical figure. Otherwise, that part of air or fire which drew nearer to the center of the world would not seek to ascend upward [but rather to descend], which is contrary to the natural inclination of each element[?]. It is likely, nevertheless, that neither the concave of fire nor [the convex of air is perfectly uniform][?]: for more fire seems to be generated under the torrid zone — that is, under the Equator, or near it — where the Sun continually dwells and the heaven is turned with the greatest velocity; but more air under the cold zones — that is, under the poles, or near them — on account of the great distance of the Sun, the cold, and the slowness of motion.

Nor indeed should this be passed over: that Plato [Plato], in the *Timaeus*, assigned to the heaven and to the four elements the figures of the five regular bodies (concerning which he treats in books 13, 14, 15, and 16 of Euclid), on account of certain likenesses. [Margin: In what manner Plato assigned to the four elements and to the heaven the figures of the five regular bodies.] For to Fire, on account of the sharpness of its flame, he assigned the pyramid, or Tetrahedron; for every little particle of fire ascends after the manner of a pyramid. To Air, however, the Octahedron: for just as air comes closest to fire, so too the Octahedron obtains the greatest likeness with the Tetrahedron, since it consists of two pyramids. To Water he next grants the Icosahedron, on account of its excessive mobility and fluidity. But the Cube, or Hexahedron, he assigns to Earth, on account of its

immobility and stability: for among all the regular bodies the cube is least apt for motion. Finally, to the Heaven he ascribes the Dodecahedron: for just as the heaven in its whole circuit embraces the twelve equal signs, so too the Dodecahedron is contained by twelve equal faces. I pass over the other causes or properties on account of which Plato likened the five simple bodies composing the world to the regular bodies; for you will find these treated more fully among the Platonists.

It is not, however, to be supposed in any way — as many falsely think — that the eminent philosopher Plato believed the Heaven and the four elements to be truly endowed with such figures. For he says, in the same *Timaeus*, that the World, together with all its principal parts (such as are the heavenly bodies and the elements), was made round, so that nothing rounder can be conceived; yet by a certain likeness — on account of the many properties common to the heaven and the elements with the regular bodies — he attributed such figures to them, in order the more easily to explain both their nature and their mutual transmutation of one into another. But chiefly because, just as it is altogether impossible that more regular bodies be given than those five enumerated (as has been most clearly demonstrated by us at the end of Euclid book 13), so likewise only five such simple bodies are found in the whole Universe, as is established from book 1 *On the Heaven*. Wherefore Plato means, in the *Timaeus*, only that the five simple bodies composing the whole universe correspond, by a certain proportion, to those five regular bodies.

TERRAM ESSE CENTRVM MVNDI

[Margin: That the earth is situated at the center of the world.]

That the earth is situated in the middle of the Firmament is clear as follows. To those existing on the surface of the earth, the stars appear of the same size, whether they are in the middle of the heaven, or near their rising, or near their setting; and this is so because the earth is equally distant from them.

COMMENTARIVS

In this place the Author demonstrates the fourth conclusion, namely, that the Earth is the center of the world. Understand "earth" together with water. For although the Author here speaks expressly of earth alone, the same reasons nevertheless have force for the whole aggregate of earth and water. Now since the center of any sphere ought to have two conditions — one, that it be in the

middle of it, equally removed from all its extremities; the other, that it be a point, and altogether insensible in comparison with that of which it is called the center — the Author therefore shows in this place that both conditions belong to the earth in relation to the heaven. That the earth is in the middle of the heaven, or of the whole world, he urges by two arguments, of which the first is this: When we exist on the surface of the earth, and in whatever region, the

of the same magnitude both at their rising and setting as at mid-heaven, once all vapours, mists, and exhalations that could impede our sight are set aside. Therefore we are equally distant from all parts of the heaven; and consequently the earth on which we are will be at the middle of the world, or — what is the same thing — at the centre of the world. The antecedent is confirmed by experience, while the consequence is easily proved. For if we were not equally distant from the heaven, but from one side came nearer to it than from another, then — since things that are nearer appear larger — the stars would appear larger to us when we are in the nearer part of the earth than when in the more remote part. And this is contrary to experience.

This argument is taken from Alfraganus [al-Farghānī], Difference 4. It does not conclude — if we speak strictly and geometrically — that a person in one and the same place is equally distant from all parts of the heaven; for we said above that this is false, since the heaven is farther from us on the side of the horizon, on account of the earth's semidiameter that lies between. It concludes only that we are equidistant from every part of the heaven so far as the judgment of the senses goes. For the excess is so small — as was also said above — that it cannot make the stars appear smaller, in clear weather, on the side of the horizon than at mid-heaven. Hence the stars can rightly be said, so far as the senses are concerned, to be equally distant from every place on the earth. Nevertheless the argument proves excellently that the centre of the earth is equally distant from the heaven — that is, that it is the same as the centre of the world. Otherwise some parts of the earth's surface would sensibly recede from the centre of the world, and so too the same stars would appear sensibly larger or smaller in those same regions, which is false.

Furthermore, since at rising and setting there are almost always vapours or exhalations that impede the true judgment of the senses, it does not seem firm enough to infer from the aforesaid argument that any given person is equally distant from the heaven. Hence the same argument, drawn instead from the size of the stars, will be better presented in this way. To the same person — placed now under that meridian in which the Sun is when it rises to us; now under that

beneath which we ourselves are; now under that in which the Sun is when it sets to us; and finally under any meridian whatever — the same stars appear to be of the same size when they reach the meridian, where there are no vapours in clear weather. Therefore the surface of the earth is equally distant from the stars along all those parts lying beneath the said meridians. And this argument, whether it be put in this latter way or in that former one, has the same force in the case of water; for the same appearance holds also on the sea.

From this it is clearly gathered — as we proved above — that the earth and the water have the same centre as the centre of the whole universe, since the convex surface of each is equidistant from the centre of the world, as is inferred from the argument that was drawn from experience.

[Lemma — Sacrobosco] For if the earth came nearer to the firmament in one part than in another, it would follow that someone standing in that part of the earth's surface which came nearer to the firmament would not see half of the heaven. But this is contrary to Ptolemy and to all the philosophers, who say that wherever a person may be, six signs rise for him and six set, and half of the heaven is always visible to him while the other half is hidden.

[Margin: Another argument proving that the earth is at the centre of the world.]

COMMENTARIUS

The second argument is roughly this. Wherever a person is, six signs always rise for him and six set, and one half of the heaven is always visible to him while the other half is hidden from him. Therefore the earth is at the middle of the firmament. The antecedent belongs to Ptolemy (Book 1, chapters 5 and 6 [*Almagest* I.5–6]), to Alfraganus (Difference 4), and to all the other astronomers and philosophers, who, taught by experience, all confess with one voice that we behold half of the heaven in whatever place we are, and so on. The consequence, in turn, is necessary. For if the earth were not at the middle of the heaven, but came nearer to one part than to another, then the person who was in the part nearer to the heaven would not see half of the heaven but a smaller part, while the one who was in the other, more remote part would behold more than half — because not every horizon separating the visible part of the heaven from the invisible would pass through the centre of the world, and consequently it would not be a great circle, and so would not divide the heaven into two equal parts. And since this is false and contrary to experience, it will also be false that the earth is not at the middle of the heaven.

This argument is found in Alfraganus and Ptolemy at the passages cited. Here only this needs to be noted in passing: that it is impossible for anyone placed on the earth's surface to see exactly half of the heaven. For the horizon [horizon] that bounds our sight is conceived as a certain flat circular surface lying upon the earth, because when we are set in some large level plain we suppose that the visible part of the earth is flat and there touches the heaven. As a result, that horizon cannot divide the heaven into two equal halves; for there will be missing that part of the heaven which is intercepted between the surface touching (or lying upon) the earth and the surface that passes through the centre of the earth parallel to the first. For this latter alone will divide the heaven into two equal parts (Theodosius, Book 1, proposition 11 [*Spherics* I.11]), as can also be seen in the accompanying figure. But that portion which lies between the two surfaces is of no significance, of practically no magnitude in comparison with the whole fabric of the heaven. For since the heaven is wondrously distant from us — as we shall say later — it comes about that, if from our eye and from the centre of the earth two parallel lines were drawn up to the firmament, sight would simply judge them to meet there, on account of their too-great distance from us, and consequently would see practically no interval at all intercepted between them. Just as also in some long building whose interior walls are parallel, the walls appear to us nearer to one another at the far end than at the near one from which we look at them, on account of that distance. Much more, then, will this happen in the case of the heaven, since it is beyond comparison far more distant.

[Margin: How it is true that a person sees half the heaven.]

Diagram. A circle representing the celestial sphere, filled with vertical hatching. Two horizontal parallel lines cross it near the middle: the upper one is labelled **HORIZON**, and a small orb/eye symbol sits at the centre where the lower line passes. The figure illustrates the horizon plane touching the earth's surface and the parallel plane through the earth's centre, which are sensibly indistinguishable at the firmament.

- HORIZON (upper horizontal line): the sensible horizon
- (centre): observer's eye / earth
- (lower horizontal line): the plane through the centre of the earth, parallel to the horizon

Hence, as regards the judgment of the senses, it can very well be said that from our place on the earth — once the obstructions of mountains and valleys are set aside — we behold half of the heaven. This is clearly shown by the phenomena, that is, the celestial appearances. For we see the two luminaries — the Sun and

the Moon — when they are diametrically opposite, at nearly the same time above the horizon, the one in the east and the other in the west; or at any rate, when one sets, we see the other rise at once. This could not happen if the portion of the heaven intercepted between the two aforesaid surfaces were of any appreciable size. The same appears most clearly also from the fact that everywhere, once obstructions are set aside, six signs appear above the horizon and six below, and these occupy half of the heaven. Indeed, on the authority of Pliny (Book 2, chapter 13 [*Natural History* II.13]), the Moon has sometimes been seen to be eclipsed at the point of the east while the Sun was still in some manner above the horizon at the point of the west; and yet at that time those two luminaries were diametrically opposite.

To the two arguments already stated we can add others that establish the same thesis — namely, that the earth is at the middle of the firmament, or of the whole universe. Let the first of these, drawn from Ptolemy (Book 1, chapter 5 [*Almagest* I.5]), be as follows.

[Margin: Ptolemy's argument proving that the earth is at the centre of the world.]

If the earth is not situated at the middle of the firmament, or of the whole universe, it will necessarily occupy one of these positions. Either it will be in the plane of the equinoctial circle, outside the axis of the world (for if it were on the axis of the world and in the plane of the equator, it would be at the centre of the world); or it will be on the axis of the world, outside the plane of the equinoctial circle; or, finally, it will be placed neither in the plane of the equinoctial circle nor on the axis of the world. From all these positions very many absurdities follow.

For if it were in the plane of the equator outside the axis of the world, the first result would be that in a right sphere [sphæra recta] an equinox [æquinoctium] would never occur except at that horizon which passes through the centre of the world. For let there be a sphere BDCE whose centre is A, its equator DE, the axis of the world [axis mundi] BC, and the earth at F, whether above the axis of the world or below it; and let the right horizon HG not pass through the centre of the world A — it will be parallel to the axis BC, since the equator stands at right angles to the right horizon. It is therefore clear that both the equator and the remaining parallels of the Sun are divided unequally by the right horizon, since it does not pass through the centre or the poles of the world. Hence the days will perpetually be unequal to the nights — which is contrary to all experience, since in a right sphere there is a perpetual equinox.

[Margin: That the earth is not in the plane of the equator outside the axis of the world.]

Diagram. A circle (the celestial sphere) filled with vertical hatching (the Sun's parallels). The vertical diameter D (top)–E (bottom) is the equator seen edge-on; the horizontal diameter B (left)–A (centre)–C (right) is the axis of the world. Two horizontal chords represent the right horizon and its parallel, on which the earth's position F is marked twice (above and below the axis).

- D (top): pole of vertical diameter (equator DE)
- G (upper left): end of upper horizontal chord (right horizon HG)
- F (upper middle, on upper chord): earth, above the axis
- H (upper right): end of upper chord
- B (left, horizontal diameter): axis endpoint
- A (centre): centre of the sphere / centre of the world
- C (right, horizontal diameter): axis endpoint
- G (lower left): end of lower horizontal chord
- F (lower middle, on lower chord): earth, below the axis
- E (bottom): pole of vertical diameter

Next: no one in the same right sphere would see half of the heaven, but a part either smaller or larger than half, as the same figure indicates — which is contrary to the senses; for there are always six signs above the horizon and six below. Only at that horizon which is drawn through the centre of the world would half of the heaven be seen.

Third: the same stars would not always appear of the same size in clear weather. For if the earth is in the plane of the equinoctial circle and outside the axis of the world toward the south — that is, toward the zenith — the same stars will appear larger when set on the meridian circle than in the east or west, since there they are nearer. But if it inclines toward midnight — that is, toward the nadir — they will be seen larger when placed in the east or west than on the meridian circle. And if it is situated toward the east or the west, they will appear larger when placed in the east or the west as well. All of which conflicts with experience. There is the further point that, on this latter supposition, the time before noon will be by no means equal to the time after noon, because the meridian circle passes through the vertex of the head, which on this reckoning cannot be at the middle of the hemisphere, but comes nearer either to the east or to the west, as is plainly established from a material sphere. Therefore the phenomena will hold

good only in a right sphere, when the earth is placed at the middle of the world, as at point A, with the right horizon being BC.

Fourth: in an oblique sphere [sphaera obliqua] either no equinox would occur at all, or at any rate, if it occurred anywhere, it would not fall at the midpoint between the summer and the winter solstice — which we see to be contrary to common experience. For let there again be a sphere ABCD whose centre is E, its equator BD, the two tropics IL and XH, the axis of the world AC. If, then, the earth lies in the plane of the equinoctial circle outside the axis, as at F, let there first be an oblique horizon ZFY, cutting all the parallels into unequal parts and cutting the axis at ϕ , outside the parallels. It is therefore manifest that no equinox occurs at the said horizon, since the horizon bisects only that parallel which is described through ϕ — which, however, the Sun can never reach, since it does not depart from the equator beyond the tropic XH. Next let there be another oblique horizon OFM, cutting the axis AC within the parallels at N. It is now clear that an equinox occurs at the aforesaid horizon while the Sun describes the parallel through N, since this parallel is bisected by the horizon; but this can in no way happen at the time midway between the two solstices, since the equator BD alone is equally removed from each solstice. And it is manifest that, when the Sun is on the equator BD, there cannot be an equinox, but only before or after — which is indeed absurd and unfitting.

Diagram. A large circle (the oblique celestial sphere ABCD) filled with vertical hatching (the Sun's parallels). The horizontal diameter runs C (left)–K–E (centre)–N–G–A (right); the vertical diameter runs B (top)–E–D (bottom, the equator BD edge-on). Two chords represent the tropics (IL and XH); two oblique lines represent the oblique horizons ZFY and OFM, crossing near the earth-positions and the axis-points ϕ and N.

- B (top): equator pole / top of vertical diameter
- I (upper left, inside near top): tropic endpoint
- X (upper right): tropic endpoint
- O (upper left, on circle): endpoint of oblique horizon OFM
- Z (left, below O): endpoint of oblique horizon ZFY
- Q (interior, below O): construction point
- V (interior, mid-left): construction point
- F (interior, centre-upper): earth, on the horizon lines
- C (left end of horizontal diameter): axis endpoint
- K (interior, left of centre): construction point

- E (centre): centre of the sphere / centre of the world
- N (interior, right of centre): where horizon OFM cuts the axis
- G (right of N): outermost-parallel point on the axis
- φ (right of G): where horizon ZFY cuts the axis, outside the parallels
- P [?] (small mark below N/G; text references triangle PNG — label not clearly resolved on the scan)
- A (right end of horizontal diameter): axis endpoint
- Y (lower right, below A): endpoint of oblique horizon ZFY
- T (lower right): construction point
- M (right, below Y): endpoint of oblique horizon OFM
- R (lower right): construction point
- L (lower left): tropic endpoint
- H (bottom right): tropic endpoint
- D (bottom): equator pole / bottom of vertical diameter

Fifth: no horizon would divide the heaven into two equal parts except the one that coincides with the equinoctial circle — such as BD — and the others that are drawn through the straight line BC. Hence not everyone would behold half of the heaven.

Sixth: the excess of the longest day over the equinoctial day would not be equal to the deficiency of the shortest day by which it falls short of the equinoctial day — which is contrary to daily experience. For instance, if A is the north pole, PG will be the excess of the longest day XP over XG, the equinoctial day; but KQ will be the deficiency by which the shortest day IQ falls short of the same equinoctial day IK. All these absurdities, however, are avoided if the earth is placed at the centre E. For then, at any oblique horizon whatever — namely SR — an equinox will occur when the Sun is on the equator; the heaven will be divided into equal halves, and TG, the excess of the longest day, will be equal to KV, the deficiency of the shortest day, and so on.

But if the earth were placed on the axis of the world outside the plane of the equator — namely at the point φ — all these unfitting consequences would follow. First: no horizon except the right one would cut the heaven into two equal parts, and therefore neither would it cut the zodiac — which conflicts with the appearances. For half of the zodiac is always above any given horizon and half below, since six signs are always above the horizon.

[Margin: That the earth is not on the axis of the world outside the plane of the equator.]

Next: an equinox would occur only in a right sphere, since only the right horizon would bisect the equator — as is clear from the figure above, in which the equator is BD, the right horizon AC, and the oblique horizon YZ, cutting the equator at F into unequal parts. And if an equinox happened to occur at some oblique horizon, it would by no means fall at the time midway between the two solstices, but the equinox would be much nearer to one of the solstices than to the other. For example, if the earth were placed at N, between the tropic XH and the equator BD, an equinox would occur when the Sun runs along the parallel described through N; and this equinox is far nearer to the summer solstice than to the winter one. And if the earth were situated at G, an equinox would fall on the very day of the summer solstice. All of which is utterly absurd.

Third: the whole series and proportion of the increase and decrease of the days and nights would be thrown into confusion — the series which everywhere outside a right sphere appears before and after the equinox. It is such that twice a year the days are made equal to the nights, namely at the time midway between the longest and the shortest day; that the longest day is equal to the longest night, and the shortest day to the shortest night; and that the excess of the longest day over the equinoctial day is as great as the deficiency of the shortest day from the same equinoctial day. The same must be said of any two days equally removed from the equinoctial day, of which one is before and the other after the equinox, and other things of this kind. This perpetual series and proportion can hold and be preserved only if the earth is placed at the centre E. For on this arrangement any oblique horizon whatever — namely SR — will divide the equator BD into equal parts, so that as great a portion of it stands above the horizon as lies hidden below the horizon, and accordingly the day will be as long as the night. In like manner the tropics XH and IL will be cut into unequal parts, yet so that the alternate segments are equal — namely TX and VL, and likewise TH and VI — as Theodosius demonstrates (Book 2, proposition 19 [*Spherics* II.19]). From this it comes about that the longest day XT is equal to the longest night LV, and the shortest day IV to the shortest night HT. Finally, TG, the excess of the longest day XT over the equinoctial day XG, will be equal to KV, the deficiency of the shortest day IV from the equinoctial day IK, on account of the similarity and equality of the triangles TEG and VEK. But if the earth is set on the axis outside the centre E, as at ϕ , outside all the parallels, no equinox can occur in an oblique sphere, as has been said; and the days will always be longer than the nights, or the nights than the days. And if the earth is placed at the point G, through which the outermost parallel passes, only a single equinox would occur in an oblique sphere, namely at one of the solstices; and for the rest of the year all the days would be either greater or smaller than the nights. But if the earth is set within

the parallels at the point N, an equinox would indeed occur twice a year, and on both sides the spans of the days and nights would increase and decrease; yet these increases and decreases of the days would be equal neither in number nor in magnitude to the decreases and increases of the nights — which even the eyes can very easily judge, by comparing the two triangles PNG and QNK with each other, since both more and larger segments of the parallels are comprised in triangle QNK than in triangle PNG.

Diagram. The oblique celestial sphere, repeating the construction of the previous figure. A large circle filled with vertical hatching (the Sun's parallels); the horizontal diameter runs C (left)–K–E (centre)–N–G–A (right); the vertical diameter runs B (top)–E–D (bottom). Chords represent the tropics IL and XH; oblique lines represent the horizons ZFY and OFM.

- B (top): equator pole
- I (upper left): tropic endpoint
- X (upper right): tropic endpoint
- O (upper left, on circle): horizon OFM endpoint
- Z (left, below O): horizon ZFY endpoint
- Q (interior, upper): construction point
- V (interior, mid-left): construction point
- F (interior, centre-upper): earth, on the horizon lines
- C (left, horizontal diameter): axis endpoint
- K (interior, left of centre): construction point
- E (centre): centre of the world
- N (interior, right of centre): horizon OFM $\cap$ axis
- G (right of N): outermost-parallel axis point
- φ (right of G): horizon ZFY $\cap$ axis
- P [?] (small mark near N/G; text references triangle PNG — not clearly resolved on the scan)
- A (right, horizontal diameter): axis endpoint
- Y (lower right, below A): horizon ZFY endpoint
- T (lower right): construction point
- M (right, below Y): horizon OFM endpoint
- R (lower right): construction point
- L (lower left): tropic endpoint
- H (bottom right): tropic endpoint
- D (bottom): equator pole

Fourth: the shadows of gnomons [gnomon] that make right angles with the horizon would not, at the time of the equinoxes, be cast along one and the same straight line from east to west when the Sun is precisely at its rising and its setting, if the earth were unequally removed from each pole and its centre were not the same as the centre of the world. For let the earth first be at A, situated in the plane of the equinoctial circle — which the line BC represents for us — and let a gnomon be erected upon the plane of the horizon, which the circle BC stands for. You now see clearly that when the Sun rises at B, the shadow of the style AD is cast along the straight line AC; and likewise, when the Sun sets at C, the same shadow is cast along the straight line AB, which with the former AC makes one straight and continuous line. This the appearances observed by astronomers show us most clearly, and the cause of it is simply this: that the earth is situated in the plane of the equator. For if it were placed outside that plane — yet on the axis of the world, or even toward any part whatever, namely at E — and if an erect style were raised upon the horizon, such as EF, who does not see that when the Sun rises at B at the time of the equinox the shadow of the style is thrown along the straight line EG, but when the Sun sets at C the same shadow is extended along the straight line EH, which by no means makes a straight line with the former EG, but the two intersect each other at the point E, since when produced they would arrive at the points B and C? And experience teaches us the contrary of this.

Diagram. A circle with a vertical diameter D (top)–A–E (centre)–F (bottom) and a horizontal diameter B (left)–E–C (right). From the centre E two rays descend to H (lower left) and G (lower right), representing the equinoctial shadows cast when the earth (or style) is displaced from the centre of the world. AD/EF is the erect style; BC is the equinoctial line (east B, west C).

- D (top): top of the style AD
- A (below D): earth / foot of style in the equatorial plane
- E (centre): displaced position / foot of style EF
- F (bottom): foot of the erect style EF
- B (left, horizontal): east — sunrise point
- C (right, horizontal): west — sunset point
- H (lower left): end of shadow EH (sunset shadow from displaced position)
- G (lower right): end of shadow EG (sunrise shadow from displaced position)

Fifth: two signs of the zodiac diametrically opposite would never be seen through a dioptra [dioptra] — which is contrary to experience, which testifies that the

rising and setting of the Sun at the equinoxes are seen through a dioptra along one straight line; and in like manner the rising at the summer solstice and the setting at the winter solstice; likewise the rising at the winter solstice and the setting at the summer solstice, through a dioptra along a line ...

(continuing the commentary on Chapter One, still within Ptolemy's demonstration that the earth lies at the center of the world)

to correspond to one another along a straight line in any horizon whatever. This could not possibly happen unless the earth were set in the plane of the equatorial circle and at the center. For let the horizon be BDCE; the equator [æquator] BC; the axis of the world [axis mundi] DE; the Tropic of Cancer FG; the Tropic of Capricorn HI; and let the earth first be placed at the center, A. You then see plainly that the equinoctial rising B and the equinoctial setting C correspond to one another along the straight line BC; the summer rising F and the winter setting I along the straight line FI; and finally the winter rising H and the summer setting G along the straight line GH — just as the case requires. This the phenomena of the astronomers attest, and Euclid assumes it as most certain at the beginning of his *Phaenomena* [Greek: φαινομένων (phainomenōn, "appearances")]; Pliny too affirms it (book 2, chapter 71), on the view of all the astronomers. Next, let the earth be placed on the axis of the world, outside the equator — namely at K. On this supposition it is clearer than daylight that the whole opposite result follows. For the winter setting I, along the straight line that is drawn through the earth, will no longer correspond to the summer rising F, but to the point L. Likewise the summer setting G will correspond to the point M, and not to the winter rising H.

[Margin: That the earth is not outside the equator or off the axis of the world.]

Diagram. A circle (the horizon) with a horizontal diameter and a vertical diameter meeting at the center, two chords parallel to the horizontal diameter (the two tropics), and several oblique lines running to a second point near the center. Readable labels:

- D [?] (top): pole on the axis of the world (glyph resembles "B"; assigned D per "axis mundi DE" — see QA)
- E (bottom): opposite pole on the axis
- B (left, on the equator): equinoctial rising
- C (right, on the equator): equinoctial setting
- F (upper left): summer rising, on the Tropic of Cancer
- G (upper right): summer setting, on the Tropic of Cancer
- H (lower left): winter rising, on the Tropic of Capricorn

- I (lower right): winter setting, on the Tropic of Capricorn
- A (center): earth first placed at the center
- K (center, just above A): earth displaced onto the axis outside the equator
- M (left, between F and B): point of the displaced correspondence
- L [?] (lower right, near I): point of the displaced correspondence (label not clearly resolved)
- T [?] (upper left, near F): unresolved label

Finally, if the earth were placed neither in the plane of the equatorial circle nor on the axis of the world, but somewhere else, we would fall into all the absurdities already named — as anyone can easily deduce from what has been said. For in the right sphere no equinox would occur, while in the oblique sphere only that horizon would cut the sphere into equal parts which passed through the center of the world; and the whole series of the increases of days and nights, and so on, would be thrown into confusion.

[Margin: Another argument of Ptolemy proving that the earth is at the middle of the world.]

A **second** argument, likewise drawn from the passage of Ptolemy cited, and which Averroes too employs (*On the Heavens*, book 2), runs thus. If the earth were not situated at the middle of the world, eclipses of the Moon would not always occur when the two luminaries are opposed along a diameter, but would generally happen when they are not in opposite places of the Zodiac — which is false. For the observations of astronomers attest that a lunar eclipse occurs only, and always, when the Moon is opposed to the Sun, and never otherwise. Let A be the center of the world: if the earth is set there, it is manifest that an eclipse occurs when the luminaries are opposed along a diameter, since then the earth itself is interposed between the two; whereas when they are not opposed along a diameter, no eclipse can occur, for the earth cannot then be an obstacle to prevent the Moon from being lit by the Sun. But if the earth had its seat outside the center — say at B — the two luminaries could stand at opposite points of the Zodiac and yet no eclipse occur, because the earth would not be found on that diameter of the world along which they are opposed. Indeed the Moon would for the most part suffer eclipse when it is less than a semicircle distant from the Sun. In short, a lunar eclipse can occur only when the diameter of opposition has passed through the center of the earth and of the universe — all of which conflicts with the phenomena.

Diagram. Within two concentric circles (representing the heavens), two dark shaded globes and a radiant Sun at the right, with shadow-lines drawn from the Sun past the globes toward the left. Readable labels:

- A (lower globe, at the center of the world): the earth placed at the center
- B (upper globe, above and left of A): the earth placed off-center
- (radiant disc at right): the Sun

From this argument, again, we may likewise establish our thesis in this way. Let two lunar eclipses be taken which have occurred at different places of the Zodiac. Since each eclipse took place when the Moon was set opposite the Sun along a diameter — as both experience and astronomical computation have shown — it follows that the earth was necessarily on each of those diameters, and therefore at their common intersection. Since, then, all the diameters of the world intersect one another at the center of the world, the earth must be placed at the middle of the world, as appears in the figure set out.

[Margin: The argument of Johannes Regiomontanus.]

Diagram. Within concentric circles (the Zodiac) two radiant Suns stand at the top, and two diameters cross one another at the center, each ending in a small circle (the eclipsed Moon) at the lower left and lower right. No letter labels are legible.

The **third** argument is that of Johannes Regiomontanus [Regiomontanus] (*Epitome*, book 1, conclusion 3), which he seems to have taken from Aristotle (*On the Heavens*, book 2). All heavy bodies, descending freely along a diameter of the world, meet the surface of the earth at equal angles at whatever part of the sphere; otherwise they would not meet the earth's surface at equal angles. And because the diameters of the world, along which heavy bodies are carried, pass through the earth's center, intersecting there, it follows that the center of the earth and the center of the world are one and the same. Since the earth is heaviest, it certainly tends toward the lowest place, namely toward the center of the world. Naturally, then, it comes to rest there, as in its proper seat — elsewhere only by force.

[Margin: The argument of Aristotle.]

The **fourth** argument is Aristotle's. Since the earth is heaviest, it tends in every case to the lowest place — that is, to the point most remote from the heaven, which is the center of the world. Naturally, therefore, it rests there, as in its proper seat, but elsewhere by force.

[Margin: Another argument proving that the earth is at the middle of the world.]

There is this too: if this heavy mass of the earth, of the same figure, magnitude, and weight, were cut into any number of equal parts, and these were set in different places beneath the concave [sphere] of the Moon and let fall freely, then without doubt all the parts — being of the same nature, weight, magnitude, and figure — would descend with equal motion and at the same moment to the same place, converging at the center of the world. This could in no way happen unless [the earth is at the middle]. And indeed Nature, with the best right, seems to have placed the earth at the middle of the world, so that it might be equally removed from all parts of the heaven — which is the most excellent body — lest any part might be able to complain why this rude mass came nearer to it than to another part.

[Margin: That the earth is like a point with respect to the firmament.]

This too is a sign that the earth is, as it were, a center and a point with respect to the Firmament [Firmamentum]: for if the earth were of any magnitude with respect to the Firmament, it would not come about that half of the heaven is seen...

COMMENTARIVS.

By three intermediate [proofs] Johannes de Sacro Bosco now confirms the other property of a center as well — namely, that it is something imperceptible and like an indivisible point — as belonging to the earth with respect to the celestial machine. The first of these is this. If the earth had a perceptible and notable magnitude with respect to the Firmament, and did not rather exist as a wholly indivisible point, we would not be able to see half of the heaven — which is against experience and against all the astronomers, as was said above. The consequence is confirmed. For if the earth, compared with the celestial body, were of some magnitude that fell under the senses, then without doubt the surface of the earth too would recede notably from the center of the world — which we have already proved to be the same as the center of the earth. Hence the horizon, resting upon the earth's surface, would notably cut the heaven into two unequal parts, as you see more clearly than daylight in the figure set out.

Diagram. A circle crossed by two horizontal chords, with a small hatched globe (the earth) near the center between them. No letter labels.

[Margin: Confirmation of the preceding argument.]

You will find this same argument in Ptolemy (*Almagest*, book 1, chapter 6) and in al-Farghānī [al-Farghānī] (Difference 4), and it is [the view] of all the other

astronomers. You see that it has the same force in the case of the sea. For if the sea were much greater and higher than the earth — as some fancifully claim — we would not, when set in the middle of the sea, be able to see half of the heaven, or at least not equally well as on land; whereas daily experience teaches us the opposite.

Likewise, if a flat surface is understood [as passing] over the center of the earth, and consequently over the Firmament, an eye existing at the center of the earth [would see half of the heaven; and the same eye] existing on the surface of the earth sees the same half. Therefore it is evident that the magnitude of the earth which lies from the surface to the center is imperceptible, and consequently that the magnitude of the whole earth is imperceptible with respect to the Firmament.

COMMENTARIUS.

Explaining and in a way confirming the first intermediate [proof], the argument is this. If we were to imagine a flat circular surface of enormous magnitude passing through the center of the world, or of the earth, it would divide both the earth and the Firmament into equal segments; and consequently some eye existing at the center of the world, upon that surface, would look upon exactly half of the heaven — were it not impeded by the density of the earth. But the same eye, set on the surface of the earth, discerns the same half, so far as the judgment of sense goes, as Ptolemy and all the astronomers hold; and this has been ascertained by daily experience, as we said above. Therefore all that earth which lies between the center of the earth and its surface is of no account with respect to the Firmament; since the two visual rays (that is, straight lines) parallel to one another — one of which runs out from the center of the world, or of the earth, the other from the convex surface of the earth up to the heaven — intercept no magnitude whatever of any moment in the Firmament, but seem to meet exactly in the same point. This could in no way happen if this portion of the earth had any notable bulk compared with the magnitude of the Firmament. From this it is clear that the whole earth is, as it were, a point, if it be compared with the Firmament.

To make it plainer in what way those two visual rays subtract something imperceptible from the Firmament, it must be briefly explained how great is that [portion] which is intercepted between those two rays in the Firmament — which will be done in this way. Since, according to al-Farghānī, the distance from the center of the earth to the concave of the Firmament contains 22612 and a half earth-semidiameters, so that the proportion of the semidiameter of the Firmament to the semidiameter of the earth is the same [as 22612½ to 1]: let the

whole sine [sinus totus] of the Firmament be set as 100000 parts, and the semidiameter of the earth [be found accordingly].

Diagram. A large circle with the word **HORIZON** engraved across its horizontal diameter, and a small hatched globe (the earth) at the center on that diameter; a second horizontal chord runs above it. The only lettered content is the word HORIZON.

[Margin: How great is the arc of the firmament intercepted between the two parallel visual rays, one of which goes out from the center of the earth, the other of which touches the earth.]

Since, then, the semidiameter of the earth is the right sine [sinus rectus] of that arc of the Firmament — as is clear from the adjoining figure and from the definition of the right sine — and corresponds to the right sine of 4 and a half parts and a little more, there will be intercepted in the Firmament, between those two rays, an arc containing 0 degrees, 0 minutes, 9 seconds, and a little more. So tiny is that which the semidiameter of the earth cuts off from the concave of the Firmament — which is imperceptible with respect to the whole circuit of the Firmament, since the whole circuit of the Firmament comprises 1296000 seconds — so that that arc of 9 seconds is $1/144000$ of the whole circuit, or $1/400$ of one degree. And since the diameter of the Sun occupies half of one degree, it comes about that that arc is $1/200$ of the Sun's diameter; and this quantity is imperceptible when compared with the whole circuit of the heaven, as is evident.

And this arc of the Firmament is subtracted from the semidiameter [of the earth], if the ray issuing from the eye is set parallel to that ray which goes out from the center of the earth. But since the ray sent out from the eye is not parallel to that other one, but rather approaches it the more and more, the farther it is produced — for it touches the earth's surface at a different point than the one placed at the top of the head — it comes about that a much smaller arc of the Firmament is intercepted between those two rays than $1/200$ of the Sun's diameter. Indeed it may perhaps happen that an eye set on a high mountain sees somewhat more than half of the heaven, on account of that inclination of the straight line issuing from the eye to the line drawn from the center of the earth.

[Margin: Other arguments proving that the earth is like a center with respect to the Firmament.]

It is fitting to add to these two arguments certain others drawn from the Phenomena, or appearances, by which it is most evidently concluded that this

whole globe — which is composed of earth and water — obtains, with respect to the whole compass of the universe's heaven,

the position, as it were, of a point. The **first** [of these arguments] is Ptolemy's (*Almagest*, book 1, chapter 6), roughly in this manner. We see every day that the extreme shadows of gnomons on sundials, and of other bodies whether set on planes parallel to the horizon or on any surfaces whatever, advance so uniformly and regularly, and are so conformed to the motion of the Sun, as if the tips of those gnomons or bodies were placed at the center of the earth. It is therefore most certain that any gnomon or style set on the surface of the earth does not perceptibly differ from the center of the world, since the Sun moves with uniform motion both about the center of the world and about a style of this kind. For this could in no way happen if the style were notably distant from the center of the world; for it is impossible that the Sun should move regularly about two centers distinct from one another, as is demonstrated in the *Theory of Mercury* by Erasmus Reinhold [Erasmus Reinhold]. It is therefore clear that this mass of the earth, which is intercepted between its center and its convex surface, must be judged of almost no magnitude with respect to the heaven of the Sun, and therefore much more with respect to the Firmament — [to be judged] as a point.

The **second** argument, which in a way declares the preceding one more fully, is this. With the instruments of the mathematicians — such as the astrolabe, the quadrant, the ring [armilla], and so on — we, standing on the surface of the earth, observe the true altitudes of the fixed stars and of the planets (excluding, however, the three lower planets, namely the Moon, Mercury, and Venus), and the motions and places of those same stars, no differently than if we were observing all these things while existing at the center of the earth — so that we commit no error in this matter that could fall under the senses. For through the Mediclinium, or Dioptra, we see two heavenly bodies opposed along a diameter, as though the Dioptra indicated to us the exact diameter of the world; and hold the same judgment concerning the rest of the observations. It is therefore manifestly concluded that the mass of the earth is of no moment with respect to the celestial machine, since the centers of the said instruments, standing on the surface of the earth, coincide entirely with the center of the earth — if we consult the judgment of the senses. For if instruments of this kind were perceptibly distant from the middle of the earth, astronomers would be deceived in their observations in a remarkable way, and no solar clock could indicate the hours correctly — all of which is repugnant to daily experience.

The **third** argument is likewise Ptolemy's, in the passage cited, namely this. In all parts of the earth and all climates of the world, the magnitude and distance of one and the same star — of Mars, namely — has been found the same, at the same time, by various astronomers; and we have ascertained the same in all the other observations that have been made in different climates, so that they do not perceptibly differ among themselves. For which reason the earth is rightly to be judged an indivisible point, since no place of the earth differs perceptibly from another with respect to one and the same celestial point.

The **fourth** argument may be this. If the earth were of some notable magnitude compared with the Firmament — or even with the heaven of the Sun — all those absurdities would follow which we inferred a little before [as following] if the earth were not situated at the middle of the world; and this for the reason that, if the earth were not like a point, we who dwell on its surface would by no means be set at the middle, or center, of the world.

Diagram. A circle crossed by two horizontal chords, with a small hatched globe (the earth of sensible magnitude) near the center between them — dividing the heaven into unequal parts. No letter labels.

Hence it would follow, **first**, that no horizon would divide the heaven into two equal parts. Therefore half of the heaven would nowhere be seen, nor could an equinox ever occur; but perpetually the day at the time of the [would-be] equinox would be shorter than the night, since the nocturnal arc would be notably greater than the diurnal arc. **Next**, the same stars would appear smaller, in clear weather, when set near the horizon than at mid-heaven, because near the horizon they would be notably more remote from us — which, however, is false. **Thirdly**, the shadows of gnomons on any surfaces whatever would in no way be projected along a straight line at the time of the equinoxes (as would pertain to the science of the drawing of sundials), if the vertex of the gnomon were not, so far as the judgment of sense goes, [the same as] the center of the earth. But this is most clearly [in agreement with] experience: in whatever plane a style is fixed, and the tips of its shadows are noted at various hours at certain points, all these points will be found to lie in one straight line — which happens because the vertex of the style is taken as the center of the world, as we most clearly demonstrated in our *Gnomonics*. **Fourthly**, neither would the rising of the summer solstice correspond, along a straight line, to the setting of the winter solstice; nor the rising of the winter solstice to the setting of the summer solstice. **Fifthly**, the whole proportion that we now discern in the increase and decrease of days before and after each equinox would be thrown into confusion. Since all

these are absurd and are opposed to daily experience and to all the observations of skilled astronomers, we must conclude that the earth is as an imperceptible point, if it be compared with the celestial body.

The **fifth** and last argument may be this. According to the common opinion of the astronomers, the semidiameter of the Firmament, as far as its concave surface, contains the earth's semidiameter twenty-two thousand six hundred and twelve times and a half — that is, twenty thousand, and two thousand, and six hundred, and twelve times, and a little more — so that the proportion of the whole semidiameter of the Firmament to the semidiameter of the globe (which is composed of earth and water) is such as [the proportion] of this number $22612\frac{1}{2}$ to 1. For so great is the distance of the Firmament from the center of the earth found to be — as we shall say at the end of this chapter — namely that from the earth up to the Firmament there are contained $22612\frac{1}{2}$ earth-semidiameters. [Margin a: 15. quin. (Euclid, *Elements* V.15)] And therefore, since the proportion of the diameters is the same as that of the semidiameters, the whole diameter of the Firmament will likewise contain the whole diameter of the earth just as many times. [Margin b: 18. duod. (Euclid, *Elements* XII.18)] Since, then, the proportion of spheres is the triplicate of [the proportion of] their diameters, the whole world contained within the concave of the Firmament will have to the earth-globe the same proportion that $11562340095703\frac{1}{8}$ has to 1 — as appears in these continuously proportional numbers: 1; $22612\frac{1}{2}$; $511325156\frac{1}{4}$; $11562340095703\frac{1}{8}$. Since this is so, the earth will not undeservedly be called of imperceptible magnitude if it be compared with the firmament, since unity is almost nothing with respect to so great a number. And that it may be still more plainly perceived that the whole earth is like a point with respect to the Firmament, we shall take a little sphere [sphaerula] whose diameter has to the ancient Geometric foot nearly the proportion of 1 to 44 — such as the little sphere set beside [these words] in this figure.

Diagram. A small circle with a vertical diameter and a dot at the center — the "little sphere" (*sphaerula*) used for the comparison. No letter labels.

For if we take another sphere whose diameter contains 400 feet, so that the proportion of this [small] diameter to that [larger] diameter is as 1 to 17600 — who will doubt that that little sphere is nearly as a point, indivisible with respect to this [larger] sphere? Since, therefore, the earth with respect to the Firmament is much smaller [than...]

Clavius, *Opera Mathematica*, Tomus III — Commentary on Sacrobosco's *Sphere*

(Commentary on Chapter 1 continued — the earth as a point relative to the Firmament.)

than that little sphere is in relation to this sphere. For if the earth is taken as 1, the whole sphere of the world out to the concave surface of the Firmament amounts to 11,562,340,095,703 — and a little more, as we said; whereas if the aforesaid little sphere is taken as 1, that other sphere will amount to only 5,451,776,000,000. This latter number stands to unity in the triplicate ratio of that which the diameter of that sphere bears to the diameter of the aforesaid little sphere, as appears in these numbers: 1 — 17600 — 309760000 — 5451776000000. Far more, then, shall we say that the earth is a point in relation to the Firmament than that the little sphere is a point in relation to the other sphere.

[Margin: Confirmatio h[uius] [in?] quintæ rationis [?] — Confirmation of this, in(?) the fifth argument.]

[Sacrobosco's lemma (*italic*):]

Al-Farghānī also says that the smallest of the fixed stars perceptible to sight is larger than the whole earth; yet that star, in relation to the entire Firmament, is as a point and a centre. Much more forcibly, then, is the earth a point in relation to the Firmament, since it is smaller than that star.

[Margin: Alia ratio probans terram esse veluti punctum respectu Firmamenti — Another argument proving that the earth is as it were a point in relation to the Firmament.]

COMMENTARIVS.

He confirms it by a third middle term, one that rests on the authority of al-Farghānī [Alfraganus] — namely, that the earth is as it were a point — as is plain in the text itself. And it is not al-Farghānī alone who says that the earth is larger than the smallest of the stars perceptible to sight; nearly all astronomers assert the very same thing.

Now, so that it may be understood which "smallest stars" our author, following the opinion of al-Farghānī and of other astronomers, is speaking of, it seems to me that a few things ought to be said about the stars in general: how many, that

is, have been observed in number by the astronomers, and what proportion their magnitudes bear to the magnitude of the earth. The astronomers, then, carefully observing all the stars in the Firmament that are perceptible to sight — that is, those that can always be conveniently seen when the sky is clear — found them to be 1022 in number. There are indeed very many other, extremely small stars (this I shall never deny) which, because they do not present themselves distinctly and clearly to the gaze, or because they are not seen at every season of the year on account of their smallness, the astronomers deliberately pass over; and they speak only of those that the eyes, lifted to the sky, can conveniently take in. But since it seems incredible to ordinary people that there should be only 1022 stars conveniently visible in the Firmament — because sight, gazing at them confusedly on a clear night without any order, supposes them to be almost innumerable — it has seemed good to list here all 1022 stars observed by the astronomers in the order in which they are customarily depicted on the celestial globe. For in this way it will come about that, if anyone on a clear night carefully observes the stars and compares the globe with the stars he sees, he will find no star other than those marked on the globe; indeed, he will scarcely be able to make out with his sight even the very small ones marked there. Hence it cannot seem strange to him that there are no more bright stars in the Firmament than 1022.

[Margin: Quomodo Astronomi numerum stellarum inuestigarint — How the astronomers investigated the number of the stars.]

They investigated this number by the following method. Out of all the stars that are conveniently perceived by sight, the astronomers noted 48 constellations, asterisms, or images. (A constellation, asterism, or image is a certain group of stars which by its arrangement and order represents the shape of some animal or the figure of some other thing.) From this they could easily grasp the number of stars in each constellation considered by itself. For those most ancient and most diligent observers of the stars seem to have formed the stars into images of this kind for no other reason — as Theon the Younger testifies in his exposition of Aratus [in expositione Aratea] — than that so great a multitude of them might be distinguished into separate parts, and that all the stars might be designated in a certain order. And it is agreed that this was done many ages ago, since even in the book of Job the Holy Scriptures name Orion, Arcturus, the Hyades, and the Pleiades, and the names of many other constellations are read in Homer and in Hesiod, those most ancient poets.

Besides this, they observed that certain stars were much more brilliant than others, so that they distinguished altogether six grades in the stars with respect to their magnitude and their greater or lesser splendour; these grades the astronomers called "differences of magnitude." From this they were able, with long practice, quite easily to make out the number of stars of each difference. Thus they found that in the first difference are contained 15 of the largest and brightest stars, which are called stars of the first magnitude. In the 2nd difference they found smaller and less bright stars, 45 of them, which they called stars of the second magnitude. In the 3rd difference they found 208 stars, still smaller, and named them of the third magnitude. In the 4th difference, or magnitude, they observed 474 still smaller stars. In the 5th difference or magnitude they counted 217 still smaller stars. In the 6th and last difference or magnitude they recorded 49 stars, which are the smallest of all. Besides all these stars there are found another five called nebulous, and nine dimmer ones, which scarcely present themselves to our senses; and for that reason they are not referred to any of the said magnitudes, since their sizes could not be recorded because of their dimness. If, then, you gather all these stars into one sum, you will find precisely the number 1022, as you see in the adjoining table.

[Margin: Sex differentiæ magnitudinum stellarum, & quot in qualibet differentia contineantur — The six differences of magnitude of the stars, and how many are contained in each difference.]

Magnitudo.	Num. Stell.
1	15
2	45
3	208
4	474
5	217
6	49
Nebulosæ	5
Obscuriores	9
Omnes simul	1022

(Table headers: "Magnitude — Number of Stars"; rows for magnitudes 1–6, then "Nebulous" 5, "Dimmer ones" 9, "All together" 1022. The eight itemized values sum exactly to 1022. This identical summary table is reprinted at the head of the following page, printed p. 74.)

As for the fact that in winter, on a clear night, an almost infinite multitude of stars appears (to answer the common opinion of the crowd), especially toward the Arctic pole, I judge that this comes about from one of two causes. Either it is because, since the air is then more purified than in summer, it comes about that even the smallest stars can be seen — stars which for that reason are not recorded among the six aforesaid differences, because they do not always appear. Or else it is because, since the stars then twinkle very strongly, it comes about that the sight is deceived and supposes that it perceives more stars than it does — when in fact it does not see stars, but certain appearances of stars generated by that violent twinkling or scintillation. A sign of this is that, if anyone wishes to fix the keenness of his eyes upon one of those stars...

[Margin: Cur in hyeme plures stellæ videantur, quam in æstate — Why more stars appear in winter than in summer.]

he either loses it altogether, or at least detects it wavering, so that it does not stay in the same place — which does not happen with the other stars. And beyond doubt, if the multitude of stars were as great as it then appears to the sight, it would be astonishing that they had not been recorded by the astronomers, seeing that they have in fact recorded far smaller ones — indeed, even those that are found outside the images or constellations, and of which there is no use whatever among the astronomers, as will appear from the following table.

(The magnitude-summary table is here reprinted, identical to the one above:)

Magnitudo.	Num. Stell.
1	15
2	45
3	208
4	474
5	217
6	49
Nebulosæ	5
Obscuriores	9
Omnes simul	1022

There is also that which the Sacred Scriptures report: that God said to Abraham (Genesis, ch. 12 [?] — the passage is Genesis 15:5), *"Look up at the heaven and number the stars, if you can,"* and said to him, *"So shall your seed be."* Likewise in

ch. 22: *"I will bless you, and I will multiply your seed like the stars of heaven, and like the sand that is on the seashore."* Likewise in ch. 26: *"And I will multiply your seed like the stars of heaven."* This must be understood according to the common opinion of ordinary people, who suppose the multitude of stars to be infinite when they gaze at them confusedly and without order on a clear night — not, however, as though the multitude of stars were in truth as great as the future multitude of the children of Israel who were said to spring from Abraham. For since there are 48 images, in which all the 1022 stars are placed (granted that some few are set outside them), surely no one will assert that in each single constellation there are 10,000 stars, since not even 100 are seen, even in the largest constellation. And it would certainly be astonishing that the astronomers had erred, in numbering the stars in each constellation, by so great a figure — nearly 10,000. For if it were so, how could it happen that those stars which they recorded in the constellations could be distinguished amid so great a multitude?

Indeed, even if we should grant that in each single constellation there are 10,000 stars, the words of Scripture will still not have to be understood as they sound — namely, that there are as many stars as there would be children of Israel. For on this reckoning there would be in the whole heaven only 480,000 stars; yet who would say that the children of Israel were not far more numerous? Those words of Scripture, therefore, are not to be taken in this sense, so that we should say the stars are infinite. It can also be said that Scripture is speaking of all the stars that are in the heaven, including those that are smaller than the ones contained in the six differences — stars that are perhaps innumerable — and that God then so directed the keenness of Abraham's eyes that he might behold them all in the heaven. But if anyone wishes absolutely to maintain that the stars are more numerous, he shall have my leave to think what he pleases: for my part I readily persuade myself that there are not more than 1022 contained in the six said differences, since, on considering the constellations in themselves, I find no more than those recorded by the astronomers — except in winter, when at times more seem to appear, especially near the Arctic pole, for the causes stated a little earlier, and above all on account of the deception of sight.

And so, out of all the 1022 stars, the mathematicians, with wonderful care and skill, constructed, as has been said, 48 images or constellations, whose names and order we have set out in the table placed below, following for the most part the observations of Nicolaus Copernicus [Copernicus]. For the positions, or longitudes, of all the stars are now found to have changed from the times of Ptolemy [Ptolemy] down to our own age, on account of that very slow motion by which we said they are moved from west to east; so much so that at the present

time the longitudes of the stars are other than those set down in the tables of the *Almagest* by Ptolemy — although the latitudes of the same stars have always been found the same, as the observations of the most learned astronomers testify. Accordingly, in the following table the longitudes do indeed differ from the longitudes of Ptolemy; but the latitudes in no way disagree with the latitudes that Ptolemy explained in the *Almagest*. Indeed, from this perpetual constancy of the latitudes we asserted above that it may most firmly be concluded that the stars are moved from west to east about the poles of the Zodiac — just as it was found by continuous change of longitude that they are gradually moved from setting to rising.

We call the "longitude" of any star its distance from the beginning of ♈ [Aries], toward the eastern signs, that is, proceeding through ♉ [Taurus], ♊ [Gemini], ♋ [Cancer], ♌ [Leo], and so on. The "latitude," on the other hand, we call its distance from the Ecliptic, whether toward the north or toward the south. But in the following table the longitudes of the stars are reckoned from the first star of Aries, as we shall say a little later in the use of the table. Yet more concerning longitudes and latitudes you will find in the 2nd chapter, when we shall treat of the Zodiac.

We have corrected the longitudes of many stars partly from a manuscript *Almagest*, partly also from the observations of Ptolemy and of other astronomers. For when it has been observed that three particular stars, for example, form in the heaven a nearly straight line, if this is not preserved on the celestial globe when the stars are described in the tables according to the longitudes and latitudes there recorded, it is an argument that those longitudes or latitudes of the stars are not the true ones, and that they must therefore be corrected — yet in such a way that the stars retain that position in the constellations which has been observed by the astronomers. This is what we have observed in our correction. Moreover, in order to distinguish those stars whose longitudes or latitudes we have corrected from the others, we have set beside them an asterisk, in this way: *. Again, certain stars were sometimes said in the tables to be, for instance, in the left hand, or in some other part, whereas in fact they are in the right hand, or elsewhere, as the pictures require. These too, therefore, we have emended, and to them we have affixed the same asterisk.

But let us now set the aforesaid table before the eyes, the use of which we shall explain after its end. The whole table is distributed into three parts: in the first are contained all the stars that incline from the Zodiac toward the north; the

second embraces all the stars that are found in the Zodiac; and in the third, finally, are placed all the stars that bend from the Zodiac toward the south.

[Tycho Brahe] the Dane, an excellent astronomer of our age, observed in Denmark more stars than 1022 — yet fewer than 1100 — and in some of those 1022 he found longitudes, and also latitudes, differing somewhat from those recorded in our following table. Whoever, then, judges that his observations are to be trusted more than those of other astronomers may consult either the works of Tycho himself, which are now printed, or else the *Sphere* of Friar Francesco Piffieri [F. Francisci Pifferii], written in Italian, in which he described the stars according to the opinion of Tycho. For my own part I think it superfluous to weave that table into these commentaries of ours — both lest the book grow larger than...

is fitting, and also because there is not so great a difference between Tycho's stars and ours as to introduce any notable error in the instruments or in the observations — especially since, as I have said, the stars observed by him can be found elsewhere and compared with ours.

I do not, however, wish the reader to remain unaware, in this place, that not long ago there was brought from Belgium a certain instrument, in the shape of a long tube, in whose ends are fitted two pieces of glass, or lenses, by which objects far removed from us appear very near, and indeed far larger than they really are. With this instrument very many stars in the Firmament are discerned which without it can in no way be seen — especially in the Pleiades; around the Nebula of Cancer; in Orion; in the Milky Way; and elsewhere. But this does not conflict with what we reported above concerning the number of the 1022 stars, because we were speaking of the stars that can be conveniently seen without the aid of that instrument. The Moon too, when it is crescent or half-full, appears wonderfully broken and rough, so that I cannot sufficiently marvel that there are such great inequalities in the lunar body. But on this matter consult the little book of Galileo Galilei [Galileo Galilei], which he entitled the *Sidereal Messenger* [*Sidereus Nuncius*], printed at Venice in the year 1610, in which he describes the various observations of the stars first made by himself.

Among the other things that are seen with this instrument, this holds no last place: namely, that Venus receives light from the Sun after the manner of the Moon, so that it appears now more, now less, crescent, according to its distance from the Sun. This I have myself observed more than once, together with others, here at Rome. Saturn, too, has joined to it two stars smaller than itself, one toward the east and the other toward the west. Jupiter, finally, has four

wandering stars, which vary wonderfully in position both among themselves and with respect to Jupiter, as Galileo Galilei describes diligently and accurately.

Since these things are so, let the astronomers see in what way the celestial orbs are to be arranged, so that these phenomena may be preserved [saved].

SEQVITVR TABVLA STELLARVM.

(Display heading: "There follows the Table of the Stars." — announcing the star catalogue to come in subsequent pages.)

TABLE OF THE STARS (Star Catalogue)

FIRST PART OF THE TABLE [TABVLÆ PRIMA PARS]

Verbatim caption: TABVLÆ PRIMA PARS COMPLECTENS NOMINA omnium constellationum, quæ à Zodiaco ad eius polum Boreum vergunt, una cum numero, ordine, longitudinibus, latitudinibus, atque magnitudinibus stellarum.

Translation: The first part of the table, containing the names of all the constellations that incline from the Zodiac toward its northern pole, together with the number, order, longitudes, latitudes, and magnitudes of the stars.

Column headers (verbatim): FORMÆ STELLARVM (Forms of the Stars) | Longit. G. | M. (Longitude: degrees | minutes) | Latit. G. | M. (Latitude: degrees | minutes) | Magnitudo (Magnitude)

Throughout, the longitude is an absolute ecliptic longitude measured from 0° Aries (0–360°), not a sign-plus-degree value. In every table below the columns are, in order: star number; position within the figure (English translation of the Latin FORMÆ STELLARVM description); longitude degrees; longitude minutes; latitude degrees; latitude minutes; magnitude.

[Margin: Vrsa minor.]

URSA MINOR, OR CYNOSURA — Constellation I

Verbatim heading: VRSA MINOR, SIVE CYNOSVRA. Constellatio I.

N						Ma
o.	Position	Long. °	Long. '	Lat. °	Lat. '	g.
1	The star at the tip of the tail,	53	30	66	0	3

N						Ma
o.	Position	Long. °	Long. '	Lat. °	Lat. '	g.
	the Pole Star (Polaris)					
2	The following one, in the tail	55	50	70	0	4
3	At the root of the tail	69	20	74	0	4
4	On the leading side of the quadrangle, the more southerly	83	0	75	20	4
5	The more northerly of the same side	87	0	77	40	4
6	Of those on the following side, the more southerly	100	30	72	40	2
7	The more northerly of the same side	109	30	74	50	2

Summary (verbatim): Omnes stellæ 7. Secundæ magnit. 2. Tertiæ 1. Quartæ 4.
— All stars 7: of the second magnitude 2, of the third 1, of the fourth 4.

Unformed star (informis) near Cynosura:

N						Ma
o.	Position	Long. °	Long. '	Lat. °	Lat. '	g.
1	There is also, near Cynosura, another unformed star — one found outside the figure of the Bear — lying on the following side, on a straight line, the most southerly	96	20	71	10	4

(Latin: "Est quoque circa Cynosuram alia stella informis, quæ videlicet extra formam vrsæ reperitur, estque in latere sequenti ad rectam lineam maxime Australis.")

[Margin: Vrsa maior.]

URSA MAIOR, WHICH THEY CALL HELICE — Constellation II

Verbatim heading: VRSA MAIOR, QVAM HELICEN VOCANT. Constellatio II.

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
1	The star on the muzzle	78	40	39	50	4
2	Of the two on the eyes, the leading one	79	10	43	0	5
3	The one following it	79	40	43	0	5
4	Of the two on the forehead, the leading one	79	30	47	10	5
5	The following one on the forehead	81	0	47	0	5
6	The one on the left ear, leading	81	30	50	30	5
7	Of the two on the neck, the preceding one	85	50	43	50	4
8	The following one	92	50	44	20	4
9	Of the two on the breast, the more northerly	94	20	44	0	4
1 0	The more southerly	93	20	42	0	4
1 1	On the left foreknee	93	0	35	0	3
1 2	Of the two on the left forefoot, the more northerly	89	50	29	0	3
1 3	The one farther to the south	88	40	28	30	3
1 4	On the right foreknee	89	0	36	0	4
1 5	The one just below the knee	89	10	33	30	4
1 6	The one on the back	104	0	49	0	2
1 7	The one on the flanks	105	30	44	30	2
1 8	At the root of the tail	116	30	51	0	3

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
1 9	On the left hind haunch	117	20	46	30	2
2 0	Of the two on the left hind foot, the leading one	106	0	29	30	2
2 1	The one following it	107	30	28	15	3
2 2	The one in the left hollow (flank)	115	0	35	15	4
2 3	Of the two on the right hind foot, the more northerly	123	10	25	50	3
2 4	The one farther to the south	123	40	25	0	3
2 5	The first of the three in the tail, after the root	125	30	53	30	2
2 6	The middle of them	131	20	55	40	2
2 7	The last, at the very tip of the tail	143	10	54	0	2

Summary (verbatim): Omnes stellæ numero 27. Secundæ magnitud. 6. Tertiæ 8. Quartæ 8. Quintæ 5. — All stars, in number 27: of the second magnitude 6, of the third 8, of the fourth 8, of the fifth 5.

Column headers (verbatim, repeated): FORMÆ STELLARVM | Longit. G. | M. | Latit. G. | M. | Magnitudo

UNFORMED STARS AROUND HELICE

Verbatim heading: INFORMES CIRCA HELICEN.

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Mag .
1	The one south of the tail	141	10	39	45	3
2	Preceding this, fainter	133	30	41	20	5
3	Between the Bear's forefeet and the head of ♏ (Leo)	98	20	17	15	4

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Mag .
4	The one farther north from this	96	40	19	10	4
5	The last of the three faint ones	99	30	20	0	obsc .
6	Approaching this one	95	30	22	45	obsc .
7	The one that precedes more	94	30	23	15	obsc .
8	The one within the forefeet, and II (Gemini)	80	20	22	15	obsc .

(*magnitude obsc. = obscura, faint/dim.*)

Summary (verbatim): Informes numero 8. Tertiæ magnit. 1. Quartæ 2. Quintæ 1. obscuræ 4. — Unformed stars, in number 8: of the third magnitude 1, of the fourth 2, of the fifth 1, faint 4.

[Margin: Draco.]

DRACO — Constellation III

Verbatim heading: DRACO. Constellatio III.

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
1	The one on the tongue	200	0	76	30	4
2	In the mouth	215	10	78	30	4
3	Above the eye	216	30	75	40	3
4	On the cheek	229	40	80	20	4
5	Above the head	233	30	75	30	3
6	At the first bend of the neck, the northern one	258	40	82	20	4
7	The southern of them	266	40	78	15	4
8	The middle of the same	262	10	80	20	4
9	The one following these from the east, at the second bend	282	50	81	10	4

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
1 0	The southern star of the leading side of the quadrilateral	331	20	81	40	4
1 1	The northern of the same side	343	50	83	0	4
1 2	The northern of the following side	1	0	78	50	4
1 3	The southern of the same side	346	10	77	50	4
1 4	At the third bend, the southern star of the triangle	4	0	80	30	5
1 5	Of the remaining triangle stars, the leading one	15	0	81	40	5
1 6	The one that follows	19	30	80	15	5
1 7	In the preceding triangle, the following of the three	66	40	84	30	4
1 8	Of the rest of the same triangle, the southern	43	40	83	30	4
1 9	The one more northerly than the two above	35	10	84	50	4
2 0	Of the two small ones, the one following from the triangle	200	0	87	30	6
2 1	The preceding of them	195	0	86	50	6
2 2	Of the three that follow in a straight line, the southern	152	30	81	15	5
2 3	The middle of the three	152	50	83	0	5
2 4	The most northerly of them	151	0	84	50	3
2 5	After these, westward, of the two, the one more to the	153	20	78	0	3

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
	north					
2 6	The one more to the south	156	30	74	40	4
2 7	From here westward, at the coiling of the tail	156	0	70	0	3
2 8	Of the two widely separated ones, the leading	120	40	64	40	4
2 9	The one following it	124	30	65	30	3
3 0	The following one in the tail	102	30	61	15	3
3 1	At the very tip of the tail	96	30	56	15	3

Summary (verbatim): Omnes stellæ 31. Tertiæ magnit. 8. Quartæ 16. Quintæ 5. Sextæ 2. — All stars 31: of the third magnitude 8, of the fourth 16, of the fifth 5, of the sixth 2.

[Margin: Cepheus.]

CEPHEUS — Constellation IIII

Verbatim heading: CEPHEVS. Constellatio IIII. (numbered IIII, not IV)

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
1	On the right foot	28	40	75	40	4
2	On the left foot	26	20	64	15	4
3	On the right side below the belt	0	40	71	10	4
4	The one reaching above the right shoulder	340	0	69	0	3
5	The one bringing the right elbow to the hip	332	40	72	0	4
6	The one following, touching the same hip	333	20	74	0	4

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
7	The one on the breast	352	0	65	30	5
8	On the left arm	1	0	62	30	4
9	Of the three on the tiara, the southern	339	40	60	15	5
1 0	The middle of them	340	40	61	15	4
1 1	The northern of the three	342	20	61	30	5

Summary (verbatim): Omnes stellæ 11. Tertiæ magnit. 1. Quartæ 7. Quintæ 3.
— All stars 11: of the third magnitude 1, of the fourth 7, of the fifth 3.

Unformed stars (informes) of Cepheus:

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
1	Of the two unformed stars, the one preceding the tiara	337	0	64	0	5
2	The one following it	344	40	59	30	4

Column headers (verbatim, repeated): FORMÆ STELLARVM | Longit. G. | M. |
Latit. G. | M. | Magnitudo

[Margin: Bootes, siue Arctophylax.]

BOOTES, OR ARCTOPHYLAX — Constellation V

Verbatim heading: BOOTES, SIVE ARCTOPHYLAX. Constellatio V.

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
1	Of the three on the left hand, the leading	145	40	58	40	5
2	The middle of the three, more southerly	147	30	58	20	5
3	The following of the three	149	[?]	60	10	5
4	The one on the left forearm	153	0	54	40	5

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
	at the hip					
5	On the left shoulder	163	0	49	0	3
6	On the head	170	0	53	50	4
7	On the right shoulder	179	0	48	40	4
8	Of the two on the colobium (garment), the more southerly	179	0	53	15	4
9	The one more to the north, at the end of the colobium	178	20	57	30	4
1 0	Of the two below the shoulder on the hunting- spear, the northern	181	0	46	10	4
1 1	The more southerly of them	181	50	45	30	5
1 2	At the end of the right hand	181	35	41	35	5
1 3	Of the two on the palm, the leading	180	0	41	40	5
1 4	The one following it	180	20	42	30	5
1 5	At the end of the colobium's handle	181	0	40	20	5
1 6	On the right side	173	20	40	15	3
1 7	Of the two on the belt, the following	169	0	41	40	4
1 8	The one that precedes	168	20	42	10	4
1 9	On the right leg	178	40	28	0	3
2 0	On the left leg, the northern of three	164	40	28	0	3
2	The middle of the three	163	50	26	30	4

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
1						
2	The more southerly of them	161	50	25	0	4
2						

Summary (verbatim): Omnes stellæ 22. Tertiæ magnit. 4. Quartæ 9. Quintæ 9.
— All stars 22: of the third magnitude 4, of the fourth 9, of the fifth 9.

Unformed star (informis) of Bootes:

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
1	The unformed star between the legs, which they call Arcturus	170	20	31	30	1

[Margin: Corona Borealis.]

CORONA BOREALIS (Northern Crown) — Constellation VI

Verbatim heading: CORONA BOREA[LIS]. Constellatio VI.

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
1	The bright one in the crown; Ariadne('s crown)	188	0	44	30	2
2	The leading of all	185	0	46	20	4
3	The following, toward the north	185	20	48	0	5
4	The following, farther north	193	0	50	30	6
5	The one following the bright star from the south	191	30	44	45	4
6	The one next following	190	30	44	50	4
7	Following these at a greater distance	194	40	46	10	4
8	The one following all in the crown	195	0	49	20	4

Summary (verbatim): Omnes stellæ 8. Secundæ magnit. 1. Quartæ 5. Quintæ 1. Sextæ 1. — All stars 8: of the second magnitude 1, of the fourth 5, of the fifth 1, of the sixth 1.

[Margin: Hercules.]

ENGONASIS, WHO IS ALSO HERCULES — Constellation VII

Verbatim heading: ENGONASIS, QVI ET HERCVLES. Constellatio VII.

N o.	Position	Long. °	Long. '	Lat. °	Lat. '	Ma g.
1	On the head	221	0	37	30	3
2	In the right armpit	207	0	43	0	3
3	On the right arm	205	0	40	10	3
4	On the right elbow	201	20	37	10	4
5	On the left shoulder	220	0	48	0	3
6	On the left arm	225	20	49	30	4
7	On the left elbow	231	0	52	0	4
8	Of the three on the left palm	238	50	52	50	4
9	The northern of the remaining two	235	0	54	0	4
1 0	The more southerly	234	50	53	0	4
1 1	On the right side	207	10	56	10	3
1 2	On the left side	213	30	53	30	4
1 3	On the vertebra of the left hip	213	20	56	10	5
1 4	At the root of the same hip	214	30	58	30	5
1 5	Of the three on the left hip, the leading	217	20	59	50	3
1 6	The one following it	218	40	60	20	4
1	The third, following	229	40	61	15	4

No.	Position	Long. °	Long. '	Lat. °	Lat. '	Mag.
7						
1	On the left knee	234	10	61	0	4
8						
1	On the left shin	225	30	69	20	4
9						
2	Of the three on the left foot, the leading	218	40	70	15	6
0						
2	The middle of them	220	10	71	15	6
1						

Table of Engonasis/Hercules continues into the next page (row 22, "Sequens" — the following one — appears only as the foot catchword here; its coordinates and the constellation summary fall on the next page).

FORMÆ STELLARVM [Forms of the Stars]

Column headers (each table): **Longit. G. | M.** (longitude, degrees | minutes) — **Latit. G. | M.** (latitude, degrees | minutes) — **Magnitudo** (magnitude).

Constellation VII — Engonasis / Hercules (continued, stars 22–29)

[The bright star at the tip of the right foot is shared with Boötes; the constellation total (28) is given by the summary below, which excludes this last star — "praeter ultimam."]

No.	Star (position)	Long. °	'	Lat. °	'	Mag.
22	The following one of three	223	0	72	0	6
23	At the outgrowth of the right hip	204	0	60	15	4
24	The more northern one of the same hip	198	50	63	0	4
25	In the right knee	189	0	65	30	4
26	The more southern of two below the same knee	186	40	63	40	4
27	The one further north	183	30	64	15	4

No						Mag
.	Star (position)	Long. °	'	Lat. °	'	.
28	In the right shin	184	30	60	0	4
29	At the tip of the right foot, the same as the one at the tip of Boötes's club (colorobos)	0	0	0	0	0

Summary (verbatim): *Omnes stellæ præter vltimam 28. Tertiæ magnitud. 6. Quartæ 17. Quintæ 2. Sextæ 3.* — All the stars, except the last: 28. Of the third magnitude 6, of the fourth 17, of the fifth 2, of the sixth 3. [6+17+2+3 = 28.]

Unformed star near Engonasis

No						Mag
.	Star (position)	Long. °	'	Lat. °	'	.
1	Unformed, the more southern one off the right arm	206	0	38	10	5

Constellation VIII — LYRA, SEV VVLTVR CADENS [Lyra, or the Falling Vulture]

[Margin: Lyra.]

No						Mag
.	Star (position)	Long. °	'	Lat. °	'	.
1	The bright one, called Lyra or Fidicula [Vega]	250	40	62	0	1
2	The northern of two adjacent [stars], toward the north	253	40	62	40	4
3	The one further south	253	40	61	0	4
4	In the middle of the outgrowth of the horns	256	0	60	0	4
5	The more northern of two consecutive, toward the east	265	20	61	20	4
6	The one further south	265	0	60	20	4
7	The more northern of two preceding, at the joint	254	20	56	10	3
8	The more southern	253	10	55	0	4

No .	Star (position)	Long. °	'	Lat. °	'	Mag .
9	The more northern of two following, on the same crossbar	257	30	55	20	3
10	The one further south	257	20	54	45	4

Summary (verbatim): *Omnes stellæ 10. Primæ magnitud. 1. Tertiæ 2. Quartæ 7.*
— All the stars: 10. Of the first magnitude 1, of the third 2, of the fourth 7. [1+2+7 = 10; verified against transcribed magnitudes.]

Constellation IX — OLOR, SIVE CYGNVS, QVI ETIAM AVIS, SEV GALLINA DICITVR [The Swan, or Cygnus, also called the Bird or the Hen]

[Margin: Cygnus.]

No .	Star (position)	Long. °	'	Lat. °	'	Mag .
1	The one in the mouth; the beak of the Hen	267	50	49	20	3
2	In the head	272	20	50	30	5
3	In the middle of the neck	279	20	54	30	4
4	In the breast	291	50	56	20	3
5	Shining in the tail [Deneb]	302	30	60	0	2
6	At the bend of the right wing	282	40	64	40	3
7	The more southern of three in the right wing	285	50	69	40	4
8	The middle one	284	30	71	30	4
9	The last of three, and at the wing- tip	280	0	74	0	4
10	At the bend of the left wing	294	10	49	30	3
11	In the middle of the same wing, and more northern	298	10	52	10	4
12	At the tip of the same	300	0	44	0	3
13	In the left foot	303	20	55	10	4
14	In the left knee	307	50	57	0	4

No .	Star (position)	Long. °	'	Lat. °	'	Mag .
15	The preceding of two in the right foot	294	30	64	0	4
16	The one that follows	296	0	64	30	4
17	Nebulous, in the right knee	305	30	63	45	5

Summary (verbatim): *Omnes stellæ 17. Secundæ magnit. 1. Tertice 5. Quartæ 9. Quintæ 2.* — All the stars: 17. Of the second magnitude 1, of the third 5, of the fourth 9, of the fifth 2. [1+5+9+2 = 17; verified against transcribed magnitudes.]

Unformed stars near Cygnus

No .	Star (position)	Long. °	'	Lat. °	'	Mag .
1	Of the unformed [stars], the more southern of two under the right wing	306	0	49	40	4
2	The one further north	307	10	51	40	4

Constellation X — CASSIOPEIA

[Margin: Cassiopeia.]

No .	Star (position)	Long. °	'	Lat. °	'	Mag .
1	In the head	1	10	45	20	4
2	In the breast	4	10	46	45	3
3	In the girdle	6	20	47	50	4
4	Above the chair, at the hips	10	0	49	0	3
5	At the knees	13	40	45	30	3
6	In the leg	20	20	45	30	4
7	At the tip of the foot	25	0	47	20	4
8	In the left arm	8	0	44	20	4
9	In the left elbow	10	40	45	0	5
10	In the right elbow	357	40	50	0	6
11	In the foot of the seat	8	20	52	40	4

No .	Star (position)	Long. °	'	Lat. °	'	Mag .
12	In the middle of the ascent (of the back)	1	10	51	40	3
13	At the extremity	357	0	51	40	6

Summary (verbatim): *Omnes stellæ 13. Tertiæ magnit. 4. Quartæ 6. Quintæ 1. Sextæ 2.* — All the stars: 13. Of the third magnitude 4, of the fourth 6, of the fifth 1, of the sixth 2. [4+6+1+2 = 13; verified against transcribed magnitudes.]

Constellation XI — PERSEVS [Perseus]

[Margin: Perseus.]

No .	Star (position)	Long. °	'	Lat. °	'	Mag .
1	At the tip of the right hand	21	0	40	30	Neb .
2	In the right elbow	24	30	37	30	4
3	In the right shoulder	26	0	34	30	4
4	In the left shoulder	20	50	32	20	4
5	In the head, or the nebula	24	0	34	30	4
6	In the shoulder blades	24	50	31	10	4
7	Shining in the right side	28	10	30	0	2
8	The preceding of three on the same side	28	40	27	30	4
9	The middle one	30	20	27	40	4
10	The remaining of three	31	0	27	30	3
11	In the left elbow	24	0	27	0	4
12	Shining, in the left hand and the head of Medusa	23	0	23	0	2
13	The following of the same head	22	30	21	0	4
14	The one that precedes in the same head	21	0	21	0	4
15	The one preceding even this	20	10	22	15	4
16	In the right knee	38	10	28	15	4

No .	Star (position)	Long. °	'	Lat. °	'	Mag .
17	Preceding this one, in the knee	37	10	28	10	4
18	The preceding of two at the back of the knee	35	40	25	10	4
19	The following	37	20	26	15	4
20	In the right leg	37	30	24	30	5
21	In the right foot	39	40	18	45	5
22	In the left hip	30	10	21	40	4
23	In the left knee	32	0	19	50	3
24	In the left leg	31	40	13	45	3
25	In the left heel	27	30	12	0	3
26	At the top of the foot, on the left part	29	40	11	0	3

Summary (verbatim): *Omnes stellæ num. 26. Secundæ magnit. 2. Tertiæ 5.*

Quartæ 16. Quintæ 2. Nebulosa 1. — All the stars in number: 26. Of the second magnitude 2, of the third 5, of the fourth 16, of the fifth 2, nebulous 1.

[2+5+16+2+1 = 26; verified against transcribed magnitudes.]

Unformed stars around Perseus (INFORMES CIRCA PERSEA)

N o.	Star (position)	Long. °	'	Lat. °	'	Mag.
1	The one to the east, off the left knee	34	10	18	0	5
2	To the north, off the right knee	38	20	31	0	5
3	Preceding, off the head of Medusa	18	0	20	40	obsc.

["obsc." = *obscura*, faint/obscure. No separate summary line printed for these three unformed stars.]

Constellation XII — AVRIGA, QVI ET HENIOCHVS, SEV ERICHTONIVS [The Charioteer, also Heniochus, or Erichthonius]

[Margin: Erichtonius, siue Auriga.]

N						
o.	Star (position)	Long. °	'	Lat. °	'	Mag.
1	The more southern of two in the head	55	50	30	0	4
2	The one further north	55	40	30	50	4
3	Shining in the left shoulder, Capella, or the She-goat (Hircus)	48	20	22	30	1
4	In the right shoulder	56	10	20	0	5 [?]
5	In the right elbow	54	30	15	15	4
6	In the right palm	56	10	13	30	4
7	In the left elbow	45	20	20	40	4
8	The preceding of the Kids (hoedi)	45	30	18	0	4
9	The following of the Kids, in the left palm	46	0	18	0	4
10	In the left ankle	43	10	10	10	3
11	In the right foot, and at the tip of the northern horn of ♂ Taurus	49	0	5	0	3
12	In the right calf	49	20	8	30	5
13	In the haunch	49	40	12	20	5
14	A tiny one in the left foot	44	0	10	20	6

Summary (verbatim): *Omnes stellæ 14. Primæ magnit. 1. Secundæ 1. Tertiæ 2. Quartæ 7. Quintæ 2. Sextæ 1.* — All the stars: 14. Of the first magnitude 1, of the second 1, of the third 2, of the fourth 7, of the fifth 2, of the sixth 1. [1+1+2+7+2+1 = 14. Transcribed magnitudes tally as 1(one), 5(three), 4(seven), 6(one) and 2(zero) unless star 4 is read as 2 — see anomaly (b).]

Constellation XIII — OPHIVCHVS, SEV SERPENTARIVS [Ophiuchus, or the Serpent-holder]

[Margin: Ophiuchus.]

[The latitude-direction column below records the printed **Bor.** (*Borealis*, north of the ecliptic) / **Aust.** (*Australis*, south) notes; stars 1–13 carry no note (north).]

N o.	Star (position)	Long. °	'	Lat. °	'	Ma g.	Lat. dir.
1	In the head	228	1 0	36	0	3	
2	The preceding of two in the right shoulder	231	2 0	27	1 5	4	
3	The following	232	2 0	26	4 5	4	
4	The preceding of two in the left shoulder	216	4 0	33	0	4	
5	The one that follows	218	0	31	5 0	4	
6	At the left elbow-bend	211	4 0	24	3 0	4	
7	The preceding of two in the left hand	208	2 0	17	0	4	
8	The following	209	2 0	16	3 0	3	
9	At the right elbow-bend	230	0	15	0	4	
1 0	The preceding in the right hand	235	4 0	13	4 0	4	
1 1	The following	236	4 0	14	2 0	4	
1 2	In the right knee	224	3 0	7	3 0	3	
1 3	In the right shin	227	0	2	1 5	3	
1 4	The preceding of four in the right foot	226	2 0	2	1 5	4	Aust.
1 5	The following	227	4 0	1	3 0	4	Aust.
1 6	The third following	228	2 0	0	2 0	4	Aust.
1 7	The remaining following	229	1 0	0	4 5	5	Aust.

N o.	Star (position)	Long. °	'	Lat. °	'	Ma g.	Lat. dir.
1 8	The one touching the heel	229	3 0	1	0	5	Aust.
1 9	In the left knee	215	3 0	11	5 0	3	Bor.
2 0	The more northern of three in a straight line, in the left leg	215	0	5	2 0	5	Bor.
2 1	The middle of them	214	0	3	1 0	5	Bor.
2 2	The more southern of three	213	1 0	1	4 0	5	Bor.
2 3	In the left heel	215	4 0	0	4 0	5	Aust.
2 4	Touching the sole of the left foot	214	0	0	4 5	4	Aust.

Summary (verbatim): *Omnes stellæ 24. Tertiæ magnit. 5. Quartæ 13. Quintæ 6. —*
All the stars: 24. Of the third magnitude 5, of the fourth 13, of the fifth 6. [5+13+6 =
24; verified against transcribed magnitudes.]

Unformed stars around Ophiuchus (INFORMES CIRCA OPHIVCHVM)

No .	Star (position)	Long. °	'	Lat. °	'	Mag .
1	The most northern of three, from the east toward the right shoulder	235	20	28	10	4
2	The middle of three	236	0	26	20	4
3	The southern of three	233	40	25	0	4
4	A further one following the three	237	0	27	0	4
5	Separated from the four, toward the north	238	0	33	0	4

Summary (verbatim): *Omnes stellæ 5. magnitudinis Quartæ. —* All the stars: 5, of
the fourth magnitude. [Verified.]

Constellation XIV — SERPENS OPHIVCHI [The Serpent of Ophiuchus]

[Margin: Serpens Ophiuchi.]

N o.	Star (position)	Long. °	'	Lat. °	'	Ma g.
1	The one in the quadrilateral, on the cheek	192	10	38	0	4
2	The one touching the nostrils	195	0	40	0	4
3	In the temple	197	40	35	0	3
4	At the outgrowth of the neck	195	20	34	1 [?]	3
5	The middle one of the quadrilateral, and in the mouth	194	40	37	1 [?]	4
6	From the head, toward the north	196	30	42	30	4
7	In the first coil of the neck	195	0	29	15	3
8	The northern of three following	198	10	26	30	4
9	The middle of them	197	40	25	20	3
10	The more southern of three	199	40	24	0	3
11	The preceding of two, before the left hand of the Serpent-holder	202	0	16	30	4
12	The one that follows the same hand	211	30	16	15	5
13	The one behind the right hip	227	0	10	30	4
14	The southern of two following	230	20	8	30	4
15	The northern one	231	10	10	30	4
16	Behind the right hand, at the bend of the tail	237	0	20	0	4
17	The following, in the tail	242	0	21	10	4
18	At the tip of the tail	251	40	27	0	4

Summary (verbatim): *Omnes stellæ 18. Tertiæ magnit. 5. Quartæ 12. Quintæ 1.* — All the stars: 18. Of the third magnitude 5, of the fourth 12, of the fifth 1. [5+12+1 = 18; verified against transcribed magnitudes.]

Constellation XV — SAGITTA, SIVE TELVM [The Arrow, or the Dart]

[Margin: Sagitta.]

No .	Star (position)	Long. °	'	Lat. °	'	Mag .
1	In the point	273	30	39	20	4
2	The following of three, in the shaft	270	0	39	10	6
3	The middle of them	269	10	39	50	5
4	The preceding of three	268	0	39	0	5
5	In the notch (Glyphis)	266	40	38	45	5

Summary (verbatim): *Omnes stellæ 5. Quartæ magnit. 1. Quintæ 3. Sextæ 1.* — All the stars: 5. Of the fourth magnitude 1, of the fifth 3, of the sixth 1. [1+3+1 = 5; verified against transcribed magnitudes.]

Tabula Stellarum — Star Catalogue (continued)

Column rubric (repeated at the head of each page): **FORMÆ STELLARVM** ("Configurations of the stars") | **Longit.** G. | M. | **Latit.** G. | M. | **Magnitudo.** Longitude and latitude are given in degrees (G. = *gradus*) and minutes (M. = *minuta*). In the northern constellations below, all latitudes are north of the ecliptic (implicit). In the zodiacal section (from Aries onward), each latitude carries an explicit hemisphere marker printed at the right margin — **Bor.** (*Borealis*, north of the ecliptic) or **Aust.** (*Australis*, south of the ecliptic) — reproduced here as a final column.

[Margin: Aquila]

AQVILA, SEV VVLTVR VOLANS — The Eagle, or the Flying Vulture (Constellatio XVI)

No .	Configuration	Long. G	M	Lat. G	M	Mag.
1	On the middle of the head	270	3 0	26	5 0	4
2	On the neck	268	1	27	1	3

No .	Configuration	Long. G	M	Lat. G	M	Mag.
			0		0	
3	The bright one on the shoulder-blades, which they call the Eagle [Altair]	267	1	29	1	2
			0		0	
4	Next to it, more toward the north	268	0	30	0	3
5	On the left shoulder, the preceding	266	3	31	3	3
			0		0	
6	The one that follows	269	2	31	3	5
			0		0	
7	On the right shoulder, the leading	263	0	28	4	5
					0	
8	The one that follows	264	3	26	4	5
			0		0	
9	On the tail, touching the Milky Way circle	255	3	36	2	3
			0		0	

All stars 9. Of second magnitude 1, of third 4, of fourth 1, of fifth 3.

INFORMES CIRCA AQVILAM, QVÆ CONSTITVERE ANTINOVM — Unformed stars around the Eagle, which make up Antinous

No .	Configuration	Long. G	M	Lat. G	M	Mag.
1	From the head, the preceding one toward the south	272	0	21	4	3
					0	
2	The one that follows	272	2	19	1	3
			0		0	
3	On the right shoulder, toward the southwest [<i>Africum</i> , the SW wind]	259	2	25	0	4
			0			
4	To the south	261	3	20	0	3
			0			
5	Further to the south	263	0	15	3	5
					0	
6	The one preceding all	254	3	18	1	3

No	Configuration	Long. G	M	Lat. G	M	Mag.
.			0		0	

All stars 6. Of third magnitude 4, of fourth 1, of fifth 1.

[Margin: Delphinus]

DELPHINVS — The Dolphin (Constellatio XVII)

No	Configuration	Long. G	M	Lat. G	M	Mag.
1	Of the three on the tail, the preceding one	281	0	29	1 0	3
2	Of the remaining two, the more northern	282	0	29	0	4
3	The more southern	282	0	26	4 0	4
4	In the rhombus, the more southern of the preceding side	281	5 0	32	0	3
5	The northern of the same side	283	3 0	33	5 0	3
6	The southern of the following side	284	4 0	32	0	3
7	The northern of the same side	286	5 0	33	1 0	3
8	Between the tail and the rhombus, the more northern of three	280	5 0	34	1 5	6
9	Of the other two, the preceding one toward the south	280	5 0	31	5 0	6
10	The one that follows	282	2 0	31	3 0	6

All stars 10. Of third magnitude 5, of fourth 2, of sixth 3.

EQVI SECTIO, SIVE EQVICVLVS — The Section of the Horse, or the Little Horse (Constellatio XVIII)

No

.	Configuration	Long. G	M	Lat. G	M	Mag.
1	Of the two on the head, the preceding one	289	4 0	20	5 0	obsc.
2	The following one	291	2 0	20	4 0	obsc.
3	Of the two on the mouth, the preceding one	289	4 0	25	3 0	obsc.
4	The one that follows	291	2 0	25	4 0	obsc.

All stars 4, and obscure.

[Margin: Pegasus]

EQVVS ALATVS, SEV PEGASVS — The Winged Horse, or Pegasus (Constellatio XIX)

No

.	Configuration	Long. G	M	Lat. G	M	Mag.
1	On the navel, which is also on the head of Andromeda	341	1 0	26	0	2
2	On the tip of the wing	335	3 0	12	3 0	2
3	On the right shoulder and the springing of the leg	325	3 0	31	0	2
4	On the shoulder-blades and the joint of the wing	320	0	19	4 0	2
5	Of the two on the body under the wing, the northern	327	5 0	25	4 0	4
6	The more southern	328	2 0	25	0	4
7	Of the two on the right knee, the northern	322	2 0	35	0	3
8	Further to the south	321	5	34	3	5

No .	Configuration	Long. G	M	Lat. G	M	Mag.
			0		0	
9	Of the two close together on the chest, the preceding one	319	3	29	0	4
			0			
10	The one that follows	320	2	29	3	4
			0		0	
11	Of the two on the neck, the preceding one	312	1	18	0	3
			0			
12	The one that follows	313	5	19	0	5
			0			
13	Of the two on the mane, the more southern	314	4	15	0	5
			0			
14	The one more toward the north	313	5	16	0	5
			0			
15	Of the two close together on the head, the northern	302	4	16	5	3
			0		0	
16	The one more toward the south	301	2	16	0	4
			0			
17	On the open jaws	298	4	21	3	3
			0		0	
18	On the right hock	317	0	41	1	4
					0	
19	On the left knee	311	0	34	1	4
					5	
20	On the left hock	305	4	36	3	4
			0		0	

All stars 20. Of second magnitude 4, of third 4, of fourth 9, of fifth 3.

[Margin: Andromeda]

ANDROMEDA (Constellatio XX)

No .	Configuration	Long. G	M	Lat. G	M	Mag.
1	The one on the head, and also on the	341	1	26	0	2

No	Configuration	Long. G	M	Lat. G	M	Mag.
.	navel of Pegasus		0			
2	The one on the shoulder-blades	348	4	24	3	3
			0		0	
3	On the right shoulder	349	4	27	0	4
			0			
4	On the left shoulder	347	4	23	0	4
			0			
5	Of the three on the right arm, the more southern	347	0	32	0	4
6	The one more toward the north	348	0	33	3	4
					0	
7	The middle of the three	348	2	32	2	5
			0		0	
8	Of the three on the top of the right hand, the more southern	343	0	41	0	4
9	The middle of them	344	0	42	0	4
10	The northern of the three	345	3	44	0	4
			0			
11	On the left arm	347	3	17	3	4
			0		0	
12	On the left elbow	349	0	15	5	3
					0	
13	Of the three on the girdle, the southern	357	1	25	2	3
			0		0	
14	The middle one	355	1	30	0	3
			0			
15	The northern of the three	355	2	32	3	3
			0		0	
16	On the left foot	10	1	23	0	3
			0			
17	On the right foot	10	3	37	2	4
			0		0	
18	More southern than this	9	3	35	2	4

No	Configuration	Long. G	M	Lat. G	M	Mag.
.			0		0	
19	Of the two under the left knee, the northern	5	4	29	0	4
			0			
20	The southern	5	2	28	0	4
			0			
21	On the right knee	3	3	35	3	5
			0		0	
22	Of the two on the train [of the robe], the northern	6	0	34	3	5
					0	
23	The southern	7	3	32	3	5
			0		0	
24	Extending from the right hand, and unformed	335	0	44	0	3

All stars, except the first, 23. Of third magnitude 7, of fourth 12, of fifth 4.

TRIANGVLVM, SIVE DELTOTON — The Triangle, or Deltoton (Constellatio XXI)

No	Configuration	Long. G	M	Lat. G	M	Mag.
.						
1	At the apex of the triangle	4	2	16	3	3
			0		0	
2	Of the three at the base, the preceding one	9	2	20	4	3
			0		0	
3	The middle one	9	3	19	4	4
			0		0	
4	The following of the three	10	1	19	0	3
			0			

All stars 4. Of third magnitude 3, of fourth 1.

Therefore, in the northern region, all the stars number 360: of first magnitude 3, of second 18, of third 84, of fourth 174, of fifth 58, of sixth 13, nebulous 1, obscure 9.

TABVLÆ SECVNDA PARS — Second Part of the Table

Comprising the names of all the constellations that are found in the Zodiac, together with the number, order, longitudes, latitudes, and magnitudes of the stars.

*(From here, each latitude carries an explicit hemisphere marker — **Bor.** = north of the ecliptic, **Aust.** = south of the ecliptic — printed at the right margin and reproduced as the final column.)*

[Margin: Aries]

ARIES — The Ram (Constellatio XXII)

N o.	Configuration	Long. G	Mag				
			M	Lat. G	M	.	Lat.
1	Of the two on the horn, the preceding one, and the first of all	0	0	7	2	3	Bor.
					0		
2	The following one on the horn	1	0	8	2	3	Bor.
					0		
3	Of the two on the muzzle, the northern	4	2	7	4	5	Bor.
			0		0		
4	The one more toward the south	4	5	6	0	5	Bor.
			0				
5	On the neck	359	5	5	3	5	Bor.
			0		0		
6	On the loins	10	5	6	0	6	Bor.
			0				
7	The one at the springing of the tail	14	4	4	5	5	Bor.
			0		0		
8	Of the three on the tail, the preceding one	17	1	1	4	4	Bor.
			0		0		
9	The middle one	18	1	2	3	4	Bor.
			0		0		
10	The following of the three	20	2	1	5	4	Bor.
			0		0		
11	On the hip	13	0	1	1	5	Bor.

N o.	Configuration	Long. G	M	Lat. G	Mag		Lat.
					M	.	
					0		
12	On the ham	11	2	1	3	5	Aust.
			0		0		
13	On the tip of the hind foot	8	2	5	1	4	Aust.
			0		5		

All stars 13. Of third magnitude 2, of fourth 4, of fifth 6, of sixth 1.

INFORMES CIRCA ARIETEM — Unformed stars around the Ram

No .	Configuration	Long. G	M	Lat. G	Mag		Lat.
					M	.	
1	The one above the head	3	4	10	0	3	Bor.
			5				
2	Above the back	15	0	10	1	4	Bor.
					0		
3	Of the remaining three small ones, the northern	14	4	12	4	5	Bor.
			0		0		
4	The middle one	13	0	10	4	5	Bor.
					0		
5	The southern of them	12	3	10	4	5	Bor.
			0		0		

All stars 5. Of third magnitude 1, of fourth 1, of fifth 3.

[Margin: Taurus]

TAVRVS — The Bull (Constellatio XXIII)

N o.	Configuration	Long. G	M	Lat. G	Mag		Lat.
					M	.	
1	In the section, of the four the most northern	19	4	6	0	4	Aust.
			0				
2	The next after it	19	2	7	1	4	Aust.
			0		5		
3	The third	18	0	8	3	4	Aust.

N o.	Configuration	Long. G	M	Lat. G	Mag		Lat.
					M	.	
					0		
4	The fourth, the most southern	17	5	9	1	4	Aust.
			0		5		
5	On the right shoulder	23	0	9	3	5	Aust.
					0		
6	On the breast	27	0	8	0	3	Aust.
7	On the right knee	30	0	12	4	4	Aust.
					0		
8	On the right hock	26	2	14	5	4	Aust.
			0		0		
9	On the left knee	35	3	10	0	4	Aust.
			0				
10	On the left hock	36	2	13	3	4	Aust.
			0		0		
11	Of the five on the face, called the Sculae [Hyades], the one on the nostrils	32	0	5	4	3	Aust.
					5		
12	Between this and the northern eye	33	4	4	1	3	Aust.
			0		5		
13	Between the same and the southern eye	34	1	5	5	3	Aust.
			0		0		
14	The bright one on the eye itself, reddish, called the eye of the Bull ♄ [Aldebaran]	36	0	5	1	1	Aust.
					0		
15	On the northern eye	35	1	3	0	3	Aust.
			0				
16	The one between the base of the southern horn and the ear	40	3	4	0	4	Aust.
			0				
17	Of the two on the same horn, the more southern	43	4	5	0	4	Aust.
			0				
18	The one more toward the north	43	2	3	3	5	Aust.
			0		0		
19	On the tip of the same	50	3	2	3	3	Aust.

N o.	Configuration	Long. G	Mag			
			M	Lat. G	M .	Lat.
			0		0	
20	At the base of the northern horn	40	0	4	0 4	Bor.
21	On the tip of the same, which is also on the right foot of Erichthonius [Auriga]	49	0	5	0 3	Bor.
22	Of the two on the northern ear, the northern	35	2 0	4	3 5 0	Bor.
23	The southern of them	35	0	4	0 5	Bor.
24	Of the two small ones on the neck, the preceding	30	2 0	0	4 5 0	Bor.
25	The one that follows	32	2 0	1	0 6	Bor.
26	On the neck, of the leading side of the quadrilateral, the southern	31	2 0	5	0 5	Bor.
27	The northern of the same side	32	0	7	1 5 0	Bor.

Catalogue of Stars — Second Part (Zodiacal Constellations), continued

*Table column headers, repeated at the top of each page: **Forms of the Stars***

[Formæ Stellarum] | Longitude — Deg. | Min. | Latitude — Deg. | Min. |
Magnitude | (hemisphere of latitude: **Bor.** = Borealis, north of the ecliptic; **Aust.** =
Australis, south).

TAURUS (Constellation XXIII), continued

Continuation of the Bull, stars 28–33 (the Pleiades region), begun in the preceding
sequence.

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
2 8	Following, on the [same] side, southern	35	20	3	0	5	Bor.
2 9	The northern one of this side	35	0	5	0	5	Bor.

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
30	The northern boundary of the leading side of the Pleiades [Latin: <i>termiuus</i> , recte <i>terminus</i>]	25	30	4	30	5	Bor.
31	The southern boundary of the same side	25	50	3	40	5	Bor.
32	The narrowest following boundary of the Pleiades	27	0	3	20	5	Bor.
33	The small one of the Pleiades, cut off from the extremities	26	0	5	0	5	Bor.

Summary (Taurus). All the stars, apart from the one at the tip of the northern horn: 32. Of the first magnitude 1; of the third 6; of the fourth 11; of the fifth 13; of the sixth 1. (*The star in the tip of the northern horn is counted with the neighbouring figure.*)

UNFORMED STARS AROUND TAURUS [Informes circa Taurum]

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
1	Below the foot and the right shoulder	18	40	17	30	4	Aust.
2	Near the southern horn, the leading of three	43	20	2	0	5	Aust.
3	The middle of the three	47	20	1	45	5	Aust.
4	The following of the three	49	20	2	0	5	Aust.
5	Below the tip of the same horn, the northern of two	52	20	6	20	5	Aust.
6	The southern one	52	20	7	40	5	Aust.
7	Below the northern horn, the leading of five	50	20	2	40	5	Bor.
8	The second following one	52	20	1	0	5	Bor.
9	The third following one	54	20	1	20	5	Bor.
10	Of the remaining two, the	55	40	3	20	5	Bor.

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
0	northern						
1	The southern one	56	40	1	15	5	Bor.
1							

Summary (unformed stars around Taurus). All the stars 11. Of the fourth magnitude 1; of the fifth 10.

GEMINI — Constellation XXIII

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
1	On the head of the leading Twin, Castor	76	40	9	30	2	Bor.
2	On the head of the following Twin, yellowish, Pollux	79	50	6	15	2	Bor.
3	On the left elbow of the leading Twin	70	0	10	0	4	Bor.
4	On the same arm	72	0	7	20	4	Bor.
5	On the shoulder-blades of the same Twin	75	20	5	30	4	Bor.
6	On the right shoulder of the same	77	20	4	50	4	Bor.
7	On the left shoulder of the following Twin	80	0	2	40	4	Bor.
8	On the right side of the preceding Twin	75	0	2	40	5	Bor.
9	On the left side of the following Twin	76	30	3	0	5	Bor.
1	On the left knee of the leading Twin	66	30	1	30	3	Bor.
1	On the left knee of the following one	71	40	2	30	3	Aust.
1	On the left groin of the same	75	0	0	30	3	Aust.
2							

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
1 3	In the right hollow of the same	74	40	6	40	3	Aust.
1 4	On the foot of the leading Twin, the leading one	60	0	1	30	4	Aust.
1 5	On the same foot, the following one	61	30	1	15	4	Aust.
1 6	At the tip [of the foot] of the leading Twin, Propus	63	30	3	30	4	Aust.
1 7	On the top of the left foot of the following Twin	65	20	7	30	3	Aust.
1 8	On the bottom of the right foot of the same Twin	68	0	10	30	4	Aust.

Summary (Gemini). All the stars 18. Of the second magnitude 2; of the third 5; of the fourth 9; of the fifth 2.

UNFORMED STARS AROUND GEMINI [Informes circa Geminos]

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
1	The leading one, near the top of the foot of the leading Twin	57	30	0	40	4	Aust.
2	The one that shines before the knee of the same	59	50	5	50	4	Bor.
3	Preceding the left knee of the following Twin	68	30	2	15	5	Aust.
4	Of the three following the right hand of the following Twin, the northern	81	40	1	20	5	Aust.
5	The middle one	79	40	3	20	5	Aust.
6	The southern of the three	79	20	4	30	5	Aust.
7	The bright one following the three	84	0	2	40	4	Aust.

Summary (unformed stars around Gemini). All the stars 7. Of the fourth magnitude 3; of the fifth 4.

CANCER — Constellation XXV

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
1	In the breast, the nebulous middle one, called Praesepe (the Manger)	93	40	0	40	ne b.	Bor.
2	Of the two leading stars of the quadrilateral, the northern	91	0	1	15	4	Bor.
3	The southern one	91	20	1	10	4	Aust.
4	Of the two following stars, called the Asses [Aselli], the northern	93	40	2	40	4	Bor.
5	The southern Ass	94	40	0	10	4	Aust.
6	In the claw, or southern arm	99	50	5	30	4	Aust.
7	In the northern arm	91	40	11	50	4	Bor.
8	At the tip of the northern foot	86	0	1	0	5	Bor.
9	At the tip of the southern foot	90	30	7	30	4	Aust.

Summary (Cancer). All the stars 9. Of the fourth magnitude 7; of the fifth 1; nebulous 1.

UNFORMED STARS AROUND CANCER [Informes circa Cancrum]

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
1	Above the elbow of the southern claw	103	0	2	40	4	Aust.
2	Following, from the tip of the same claw	105	0	5	40	4	Aust.
3	Above the little cloud, the	97	20	4	50	5	Bor.

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
	leading of two						
4	Following this one	100	20	7	15	5	Bor.

Summary (unformed stars around Cancer). All the stars 4. Of the fourth magnitude 2; of the fifth 2.

LEO — Constellation XXVI

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
1	In the nostrils	101	40	10	0	4	Bor.
2	In the open mouth	104	30	7	30	4	Bor.
3	Of the two on the head, the northern	107	40	12	0	3	Bor.
4	The southern one	107	30	9	30	3	Bor.
5	Of the three on the neck, the northern	113	30	11	0	3	Bor.
6	The middle one	115	30	8	30	2	Bor.
7	The southern of the three	114	0	4	30	3	Bor.
8	In the heart, Basiliscus, or Regulus; the Heart of the Lion [Cor Leonis, ♌]	115	50	0	10	1	Bor.
9	Of the two on the breast, the southern	116	50	1	50	4	Aust.
10	Preceding the one in the heart	113	20	0	15	5	Aust.
11	On the right foreknee	110	40	0	0	5	Aust.
12	On the "drace" [?] right, forward	107	30	3	40	6	Aust.
13	On the "drace" [?] left, forward	110	50	4	10	4	Aust.
14	On the left foreknee	115	30	4	15	4	Aust.
15	In the left armpit	122	30	0	10	4	Aust.

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
5							
1	Of the three on the belly,	120	20	4	0	6	Bor.
6	the preceding						
1	Of the two following, the	126	20	5	20	6	Bor.
7	northern						
1	The southern one	125	40	2	20	6	Bor.
8							
1	Of the two on the loins, the	124	40	12	15	5	Bor.
9	one that goes before						
2	The one that follows	127	30	13	40	2	Bor.
0							
2	Of the two on the haunch,	127	40	11	30	5	Bor.
1	the northern						
2	The southern one	129	40	9	40	3	Bor.
2							
2	On the hind hip	133	40	5	50	3	Bor.
3							
2	In the hollow	135	0	1	15	4	Bor.
4							
2	On the hind elbow	135	0	0	50	4	Aust.
5							
2	On the hind foot	140	0	3	0	5	Aust.
6							
2	At the tip of the tail	137	50	11	50	1	Bor.
7	[Denebola]						

Summary (Leo). All the stars 27. Of the first magnitude 2; of the second 2; of the third 6; of the fourth 8; of the fifth 5; of the sixth 4.

UNFORMED STARS AROUND LEO [Informes circa Leonem]

#	Star position	Long. °	Long. '	Lat. °	Lat. '	Mag.	Hem is.
1	Of the two above the back, the leading	119	20	13	20	5	Bor.

#	Star position	Long. °	Long. '	Lat. °	Lat. '	Mag.	Hem is.
2	The one that follows	121	30	15	30	5	Bor.
3	Of the three below the belly, the northern	129	50	1	10	4	Bor.
4	The middle one	130	30	0	30	5	Aust.
5	The southern of the three	131	20	2	40	5	Aust.
6	Between the extremities of Leo and the Bear, [a star] of the nebulous cluster which they call Berenice's Hair [Coma Berenices], the most northern one	138	30	30	0	luminous	—
7	Of the two southern ones, the leading	137	50	25	0	obscure	—
8	The one that follows, in the shape of an ivy leaf	141	50	25	30	obscure	—

Summary (unformed stars around Leo). All the stars 8. Of the fourth magnitude 1; of the fifth 4; luminous 1; obscure 2.

(Magnitude entries for stars 6–8, the Coma Berenices stars: "Lumi." = *luminosa*, "obscu." = *obscura*.)

VIRGO — Constellation XXVII

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
1	Of the two on the top of the head, the leading southern one	139	40	4	15	5	Bor.
2	The following, more northern one	140	20	5	40	5	Bor.
3	Of the two on the face, the northern	144	0	8	0	5	Bor.

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
4	The southern one	143	30	5	30	5	Bor.
5	At the tip of the left wing, and southern	142	20	0	9 [?]	3	Bor.
6	Of those on the left wing, the leading of four	151	30	1	10	3	Bor.
7	The second following one	156	30	2	50	3	Bor.
8	The third one	160	30	2	50	5	Bor.
9	The last, following the four	164	20	1	40	4	Bor.
1 0	On the right side, below the girdle	157	40	8	30	3	Bor.
1 1	On the right and northern wing, the leading of three	151	30	13	50	5	Bor.
1 2	Of the remaining two, the southern	153	30	11	40	6	Bor.
1 3	The northern of them, called Vindemiator (the Grape-gatherer)	155	30	15	10	5	Bor.
1 4	In the left hand, the star called Spica [♊]	170	0	2	0	1	Aust.
1 5	Below the girdle, and on the right haunch	168	10	8	40	3	Bor.
1 6	On the left thigh, of the leading stars of the quadrilateral, the northern	169	40	2	20	5	Bor.
1 7	The southern one	170	20	0	10	6	Bor.
1 8	Of the two following, the northern	173	20	1	30	4	Bor.
1 9	The southern one	171	20	0	20	5	Bor.
2 0	On the left knee	175	0	1	30	5	Bor.

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
2	At the end of the right thigh	171	20	8	30	5	Bor.
2	On the train [of the robe], the middle one	180	0	7	30	4	Bor.
2	The southern one	180	40	2	40	4	Bor.
2	The northern one	181	40	11	40	4	Bor.
2	On the left and southern foot	183	20	0	30	4	Bor.
2	On the right and northern foot	186	0	9	50	3	Bor.

Summary (Virgo). All the stars 26. Of the first magnitude 1; of the third 6; of the fourth 6; of the fifth 11; of the sixth 2.

UNFORMED STARS AROUND VIRGO [Informes circa Virginem]

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
1	Below the left arm, in a straight line, the leading of three	158	0	3	30	5	Aust.
2	The middle one	162	20	3	30	5	Aust.
3	The following one	165	40	3	30	5	Aust.
4	Below Spica, as if in a straight line, the leading of three	170	30	7	20	6	Aust.
5	The middle of them, which is also double	171	30	8	20	5	Aust.
6	The following of the three	173	20	7	50	6	Aust.

Summary (unformed stars around Virgo). All the stars 6. Of the fifth magnitude 4; of the sixth 2.

LIBRA — Constellation XXVIII

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
1	Of the two at the tip of the southern claw, the bright one	191	20	0	40	2	Bor.
2	The dimmer one, toward the north	190	20	2	20	5	Bor.
3	Of the two at the tip of the northern claw, the bright one	195	30	8	30	2	Bor.
4	The dimmer one, preceding this	191	0	8	30	5	Bor.
5	In the middle of the southern claw	197	20	1	40	4	Bor.
6	In the same, the one that goes before	194	40	1	15	4	Bor.
7	In the middle of the northern claw	200	50	3	45	4	Bor.
8	In the same, the one that follows	206	0	4	30	4	Bor.

Summary (Libra). All the stars 8. Of the second magnitude 2; of the fourth 4; of the fifth 2.

UNFORMED STARS AROUND LIBRA [Informes circa Libram]

(This list is not yet complete on the page; the printed catchword "8 Reli-" shows that star 8, "Reliquarum...", begins on the next page.)

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
1	Toward the north from the northern claw, the leading of three	199	30	9	0	5	Bor.
2	Of the two following, the southern	207	0	6	40	4	Bor.
3	The northern of them	207	40	9	15	4	Bor.

#	Star position	Long. °	Long. '	Lat. °	Lat. '	M ag.	Hem is.
4	Between the claws, of three, the one that follows	205	50	5	30	6	Bor.
5	Of the remaining two leading ones, the northern	203	40	2	0	4	Bor.
6	The southern one	204	30	1	30	5	Bor.
7	Below the southern claw, the leading of three	196	20	7	30	3	Bor.

Tabula Stellarum (Star Catalogue) — Scorpius through Pisces

Column headers as printed: FORMÆ STELLARVM (form/description of the stars) | Longit. G. | M. (longitude, degrees | minutes) | Latit. G. | M. (latitude, degrees | minutes) | Magnitudo (magnitude) followed by the hemisphere of latitude, Bor. = Boreal/northern, Aust. = Austral/southern.

[Informes circa Libram, continued]

(Continuation of the unformed stars around Libra begun on the preceding folio; only stars 8–9 fall on this page, followed by the constellation's summary line.)

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
8	Of the two remaining following stars, the northern	204	30	8	10	4	Aust.
9	The southern	205	20	9	40	4	Aust.

All stars 9. Of the third magnitude 1. Of the fourth 5. Of the fifth 2. Of the sixth 1.

SCORPIVS. — Constellatio XXIX.

[Margin: Scorpius]

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
1	In the forehead, the northern of three bright stars	209	40	1	20	3	Bor.
2	The middle one	209	0	1	40	3	Aust.
3	The southern of the three	209	0	5	0	3	Aust.
4	The one further south, and in the foot	209	20	7	50	3	Aust.
5	Of two joined stars, the bright northern	210	20	1	40	4	Bor.
6	The southern	210	40	0	30	4	Bor.
7	In the body, the leading of three bright stars	213	0	3	40	3	Aust.
8	The middle, glowing, Antares, called the Heart of the Scorpion [Cor Scorpii; scorpion glyph in the source]	216	0	4	0	2	Aust.
9	The following of the three	217	50	5	30	3	Aust.
10	In the last vertebra, the leading of two	212	40	6	10	5	Aust.
11	The following	213	50	6	40	5	Aust.
12	In the first vertebra of the body	221	50	11	0	3	Aust.
13	In the second vertebra	222	10	15	0	4	Aust.
14	In the third, the southern of the double	223	20	18	40	4	Aust.
15	The northern of the double	223	30	18	0	3	Aust.
16	In the fourth vertebra	226	30	19	30	3	Aust.
17	In the fifth	231	30	18	50	3	Aust.
18	In the sixth vertebra	233	50	16	40	3	Aust.

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
8							
1	In the seventh, the one nearest	232	20	15	10	3	Aust.
9	the sting						
2	In the sting itself, the following	230	50	13	20	3	Aust.
0	of two						
2	The preceding one	230	20	13	30	4	Aust.
1							

All stars 21. Of the second magnitude 1. Of the third 13. Of the fourth 5. Of the fifth 2.

INFORMES CIRCA SCORPIVM. (Unformed stars around the Scorpion)

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
1	The nebulous star following the	234	30	13	15	neb	Aust.
	sting					.	
2	From the sting toward the	228	50	6	10	5	Aust.
	north, the following of two						
3	The one that follows	232	50	4	10	5	Aust.

SAGITTARIVS. — Constellatio XXX.

[Margin: Sagittarius]

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
1	On the point of the arrow	237	50	6	30	3	Aust.
2	On the grip of the left hand	241	0	6	30	3	Aust.
3	In the southern part of the bow	241	20	10	50	3	Aust.
4	In the northern part, the more	242	20	1	30	3	Aust.
	southern of two						
5	Further north, at the tip of the	240	0	2	50	4	Bor.
	bow						
6	In the left shoulder	248	40	3	10	3	Aust.

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
7	The one preceding it, on the dart	246	20	3	50	4	Aust.
8	In the eye, a double nebulous star	248	30	0	45	neb .	Bor.
9	In the head, the foremost of three	249	0	2	10	4	Bor.
1 0	The middle one	251	0	1	30	4	Bor.
1 1	The following	152 [?]	30	2	0	4	Bor.
1 2	At the northern junction, the more southern of three	254	40	2	50	4	Bor.
1 3	The middle one	255	40	4	30	4	Bor.
1 4	The northern of the three	256	10	6	30	4	Bor.
1 5	Three faint stars following	259	0	5	30	6	Bor.
1 6	At the southern junction, the northern of two	262	50	5	0	5	Bor.
1 7	The southern	261	0	2	0	6	Bor.
1 8	In the right shoulder	255	40	1	50	5	Aust.
1 9	In the right elbow	258	10	2	50	5	Aust.
2 0	In the shoulder blades	253	20	2	30	5	Aust.
2 1	In the shoulder (foreleg)	251	0	4	30	4	Aust.
2 2	Under the armpit	249	40	6	45	3	Aust.
2	In the left front hock	251	0	23	0	2	Aust.

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
3							
2	In the knee of the same leg	250	20	18	0	2	Aust.
4							
2	In the right front hock	240	0	13	0	3	Aust.
5							
2	In the left shoulder blade	260	40	13	30	3	Aust.
6							
2	In the right hind knee	260	0	20	10	3	Aust.
7							
2	At the root of the tail, the	261	0	4	50	5	Aust.
8	leading of four on the northern side						
2	The following on the same side	261	50	4	50	5	Aust.
9							
3	On the southern side, the	261	50	5	50	5	Aust.
0	leading one						
3	The following on the same side	262	50	6	30	5	Aust.
1							

All stars 31. Of the second magnitude 2. Of the third 9. Of the fourth 9. Of the fifth 8. Of the sixth 2. Nebulous 1.

CAPRICORNVS. — Constellatio XXXI.

[Margin: Capricornus]

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
1	In the leading horn, the northern of three	270	40	7	30	3	Bor.
2	The middle one	271	0	6	40	6	Bor.
3	The southern of the three	270	40	5	0	3	Bor.
4	At the tip of the following horn	272	20	8	0	6	Bor.
5	In the mouth, the southern of three	272	20	0	45	6	Bor.

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
6	Of the two remaining, the leading	272	0	1	45	6	Bor.
7	The following	272	10	1	30	6	Bor.
8	Above the right eye	270	30	0	40	5	Bor.
9	In the neck, the northern of two	275	0	4	50	6	Bor.
1 0	The southern	275	10	0	50	5	Aust.
1 1	In the right knee	275	0	6	30	4	Aust.
1 2	In the bent left knee	274	10	8	40	4	Aust.
1 3	In the left shoulder	280	0	7	40	4	Aust.
1 4	Under the belly, the leading of two adjacent stars	283	30	6	50	4	Aust.
1 5	The following	283	40	6	0	5	Aust.
1 6	In the middle of the body, the following of three	282	0	4	15	5	Aust.
1 7	Of the remaining leading stars, the southern	280	0	4	0	5	Aust.
1 8	The northern of them	280	0	2	50	5	Aust.
1 9	In the back, the foremost of two	280	0	0	0	4	Eclip.
2 0	The following	284	20	0	50	4	Aust.
2 1	In the southern spine, the preceding of two	286	40	4	45	4	Aust.
2 2	The following	288	20	4	30	4	Aust.
2 3	At the root of the tail, the leading of two	288	40	2	10	3	Aust.

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
2 4	The following	289	40	2	0	3	Aust.
2 5	In the northern part of the tail, the leading of four	290	10	2	20	4	Bor.
2 6	Of the three remaining, the southern	292	0	5	0	5	Bor.
2 7	The middle one	291	0	2	50	5	Bor.
2 8	The northern, at the very tip of the tail	292	0	4	20	5	Bor.

All stars 28. Of the third magnitude 4. Of the fourth 9. Of the fifth 9. Of the sixth 6.

AQVARIVS. — Constellatio XXXII.

[Margin: Aquarius]

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
1	In the head	293	40	15	45	5	Bor.
2	In the right shoulder, the brighter	299	40	11	0	3	Bor.
3	The fainter	298	30	9	40	5	Bor.
4	In the left shoulder	290	0	8	50	3	Bor.
5	Under the armpit	290	40	6	15	3	Bor.
6	Under the left hand, in the garment, the following of three	280	0	5	30	3	Bor.
7	The middle one	279	30	8	0	4	Bor.
8	The leading of three	278	0	8	30	3	Bor.
9	In the right arm	302	50	8	45	3	Bor.
1 0	In the right hand, the northern	303	0	10	45	3	Bor.
1 1	Of the two remaining, the southern, leading	305	20	9	0	3	Bor.

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
1 2	The one that follows	306	40	8	30	3	Bor.
1 3	On the urn, the leading of two adjacent	299	30	3	0	4	Bor.
1 4	The following	300	20	2	10	5	Bor.
1 5	In the right haunch	302	0	0	50	4	Aust.
1 6	In the left haunch, the southern of two	295	0	1	40	4	Aust.
1 7	The more northern	295	30	4	0	6	Aust.
1 8	In the right shin, the southern	305	0	7	30	3	Aust.
1 9	The northern	304	40	5	0	4	Aust.
2 0	In the left hip	301	0	5	40	5	Aust.
2 1	In the left shin, the southern of two	300	40	10	0	5	Aust.
2 2	The northern, below the knee	302	10	9	0	5	Aust.
2 3	In the pouring of the water, the first from the hand	308	20	2	0	4	Bor.
2 4	The more southern following	308	10	0	10	4	Aust.
2 5	The one following, at the first bend of the water	311	0	1	10	4	Aust.
2 6	The one following it	313	20	0	30	4	Aust.
2 7	In the second bend, the southern	313	50	1	40	4	Aust.
2	Of the two following, the	312	30	3	30	4	Aust.

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
8	northern						
2	The southern	312	50	4	10	4	Aust.
9							
3	Drawn off toward the south	314	10	8	15	5	Aust.
0							
3	After it, the leading of two	316	0	11	0	5	Aust.
1	joined stars						
3	The following	316	30	10	50	5	Aust.
2							
3	At the third bend of the water,	315	0	14	0	5	Aust.
3	the northern of three						
3	The middle one	316	0	14	45	5	Aust.
4							
3	The following of three	316	30	15	40	5	Aust.
5							
3	Of three following in like	310	20	14	10	4	Aust.
6	arrangement, the northern						
3	The middle one	310	50	15	0	4	Aust.
7							
3	The southern of the three	311	40	15	45	4	Aust.
8							
3	At the last bend, the leading of	305	10	14	50	4	Aust.
9	three						
4	Of the two following, the	306	0	15	20	4	Aust.
0	southern						
4	The northern	306	30	14	0	4	Aust.
1							
4	The last of the water, and in the	300	20	23	0	1	Aust.
2	mouth of the Southern Fish						

All stars 42. Of the first magnitude 1. Of the third 9. Of the fourth 18. Of the fifth 13. Of the sixth 1.

INFORMES CIRCA AQVARIVM. (Unformed stars around the Water-Bearer)

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
1	Of three following the bend of the water, the leading	320	0	15	30	4	Aust.
2	Of the two remaining, the northern	323	0	14	20	4	Aust.
3	The southern of them	322	10	18	15	4	Aust.

PISCES. — Constellatio XXXIII.

[Margin: Pisces]

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
1	In the mouth of the leading fish	325	0	9	15	4	Bor.
2	In the back of the head, the southern of two	317	30	7	30	4	Bor.
3	The northern	319	20	9	20	4	Bor.
4	In the back, the foremost of two	321	30	9	30	4	Bor.
5	The one that follows	324	0	7	30	4	Bor.
6	In the belly, the leading	319	20	4	30	4	Bor.
7	The following	323	0	2	30	4	Bor.
8	In the tail of the same fish	329	20	6	20	4	Bor.
9	On its cord, the first from the tail	334	20	5	45	6	Bor.
10	The one that follows	336	20	2	45	6	Bor.
11	After it, the leading of three bright stars	340	30	2	15	4	Bor.
12	The middle one	343	50	1	10	4	Bor.
13	The following	346	20	1	20	4	Aust.

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
1 4	At the bend, the northern of two small stars	345	40	2	0	6	Aust.
1 5	The southern	346	20	5	0	6	Aust.
1 6	After the bend, the leading of three	350	20	2	20	4	Aust.
1 7	The middle one	352	0	4	40	4	Aust.
1 8	The following	354	0	7	45	4	Aust.
1 9	At the knot of both cords	356	0	8	30	3	Aust.
2 0	On the northern cord, the leading from the knot	354	0	4	20	4	Bor.
2 1	The southern of three after it	353	30	1	30	5	Bor.
2 2	The middle one	353	40	5	20	3	Bor.
2 3	The northern of three, and it is at the tip of the tail	353	50	9	0	4	Bor.
2 4	In the mouth of the following fish, the northern of two	355	20	21	45	5	Bor.
2 5	The southern	355	0	21	30	5	Bor.
2 6	In the head, the following of three small stars	352	0	20	0	6	Bor.
2 7	The middle one	351	0	19	50	6	Bor.
2 8	The foremost of three	350	20	23	0	6	Bor.
2 9	In the southern spine, the leading of three, near the left elbow of Andromeda	349	0	14	20	4	Bor.

#	Form	Long. °	'	Lat. °	'	Ma g.	Hemis .
3 0	The middle one	349	40	13	0	4	Bor.
3 1	The following of three	351	0	12	0	4	Bor.

(Pisces continues on the following folio; the foot of the page carries the catchword "32 In".)

Table of the Fixed Stars (*Tabula Stellarum*) — continued

Column headers throughout: **Forma Stellarum** (the form/description of the star) | **Longit. G. | M.** (longitude, degrees | minutes) | **Latit. G. | M.** (latitude, degrees | minutes) | **Magnitudo** (magnitude). Longitudes are absolute ecliptic longitudes (0–360°). In this southern section the latitude-hemisphere label (Bor./Aust.) is not printed in the magnitude column as it was in the zodiacal tables; the northern/southern position is instead given in the descriptive text (Borea / Australis) where relevant.

PISCES — conclusion (Constellatio XXXIII)

Forms of the stars (Pisces, stars 32–34):

N o.	Position	Longit.	Latit.	Mag.
32	In the belly, of the two, the northern one	355° 30'	17° 0'	4 (Bor.)
33	The one more to the south	352° 40'	15° 20'	4 (Bor.)
34	On the following spine, near the tail	353° 20'	11° 40'	4 (Bor.)

All stars 34. Of the third magnitude 2. Of the fourth 22. Of the fifth 3. Of the sixth 7.

Unformed stars around Pisces (Informes circa Pisces)

N

o.	Position	Longit.	Latit.	Mag.
1	In the quadrilateral below the leading Fish, of the northern side, the one that goes before	324° 30'	2° 40'	4 (Aust.)
2	The one that follows	325° 45'	2° 30'	4 (Aust.)
3	Of the southern side, the leading one	324° 0'	5° 50'	4 (Aust.)
4	The following one	325° 40'	5° 20'	4 (Aust.)

All stars 4, of magnitude: of the fourth 4.

And so, in the Zodiac the stars are 346 in all: of the first magnitude 5, of the second 9, of the third 64, of the fourth 132, of the fifth 106, of the sixth 27; nebulous 3. Also the lock of hair which we said above is called Berenice's Hair [*coma Berenices*], [reckoned] outside this number by Conon the Mathematician [Conon of Samos]. Bright 1, faint 2.

THIRD PART OF THE TABLE

Third part of the table, comprising the names of all the constellations that incline from the Zodiac toward its southern pole, together with the number, order, longitudes, latitudes, and magnitudes of the stars.

CETUS [the Whale / Sea-monster] — Constellatio XXXIV

[Margin: Cetus.]

No	Position	Longit.	Latit.	Mag
.	.	.	.	.
1	At the tip of the snout	11° 0'	7° 45'	4
2	In the jaw, the following of three	11° 0'	11° 20'	3
3	The middle one, in the middle of the mouth	6° 0'	11° 30'	3
4	The leading of three, at the knee [in genu]	3° 50'	14° 0'	3
5	In the eye	4° 0'	8° 10'	4

No .	Position	Longit.	Latit.	Mag .
6	In the mane, the northern one	5° 30'	6° 20'	4
7	In the mane, the leading one	1° 0'	4° 10'	4
8	In the breast, the northern of the four leading ones	355° 20'	24° 30'	4
9	The southern one	356° 40'	28° 0'	4
10	The northern of the following ones	0° 0'	25° 10'	4
11	The southern one	0° 20'	27° 30'	3
12	In the body, the middle of three	345° 20'	25° 20'	3
13	The southern one	346° 20'	30° 30'	4
14	The northern of the three	348° 20'	20° 30'	3
15	Toward the tail, the following of two	343° 0'	15° 20'	3
16	The leading one	338° 20'	15° 40'	3
17	In the tail, of the following ones of the quadrilateral, the northern	335° 0'	11° 40'	5
18	The southern one	334° 0'	13° 40'	5
19	Of the remaining leading ones, the northern	332° 40'	13° 0'	5
20	The southern one	332° 20'	14° 0'	5
21	At the northern end of the tail	327° 40'	9° 30'	3
22	At the southern end of the tail	329° 0'	20° 20'	3

All stars 22. Of the third magnitude 10. Of the fourth 8. Of the fifth 4.

ORION — Constellatio XXXV

[Margin: Orion.]

No .	Position	Longit.	Latit.	Mag .
1	In the head, nebulous	50° 20'	16° 30'	Neb
2	In the right shoulder, bright and reddish [Betelgeuse]	55° 20'	17° 0'	1

No .	Position	Longit.	Latit.	Mag .
3	In the left shoulder	46° 40'	17° 30'	2
4	The one that follows it	48° 20'	18° 0'	4
5	In the right elbow	57° 40'	14° 30'	4
6	In the right forearm	59° 40'	11° 50'	6
7	In the right hand, the following of the four southern ones	59° 50'	10° 40'	4
8	The leading one	59° 29'	9° 45'	4
9	Of the northern side, the following one	60° 40'	8° 15'	6
10	The leading one of the same side	60° 0'	8° 15'	6
11	In the club [in colorobo], the leading of two	55° 0'	3° 45'	5
12	The following one	57° 40'	3° 15'	5
13	In the back, of the four in a straight line, the following one	50° 50'	19° 40'	4
14	The second toward the west	49° 40'	20° 0'	6
15	The third toward the west	48° 40'	20° 20'	6
16	The fourth in order toward the west	47° 30'	20° 40'	5
17	In the shield, the northernmost of nine	43° 50'	8° 0'	4
18	The second	42° 50'	8° 10'	4
19	The third	41° 20'	10° 15'	4
20	The fourth	39° 40'	12° 50'	4
21	The fifth	38° 30'	14° 15'	4
22	The sixth	37° 50'	15° 50'	3
23	The seventh	38° 10'	17° 10'	3
24	The eighth	38° 40'	20° 20'	3
25	The remaining one of these, the southernmost	39° 40'	21° 30'	3
26	In the belt, the leading of the three bright ones	48° 40'	24° 10'	2
27	The middle one	50° 40'	24° 50'	2

No.	Position	Longit.	Latit.	Mag.
28	The following of the three, in a straight line	51° 40'	25° 30'	2
29	In the hilt of the sword	47° 10'	25° 50'	3
30	In the sword, the northern of three	50° 10'	28° 40'	4
31	The middle one	50° 0'	29° 30'	3
32	The southern one	50° 20'	29° 50'	3
33	At the end of the sword, the following of two	51° 0'	30° 30'	4
34	The leading one	48° 20'	30° 50'	4
35	In the left foot, bright, and shared with the River [Rigel]	42° 30'	31° 30'	1
36	In the left shin	44° 20'	30° 15'	4
37	In the left heel	46° 40'	31° 10'	4
38	In the right knee	53° 30'	33° 30'	3

All stars 38. Of the first magnitude 2. Of the second 4. Of the third 8. Of the fourth 15. Of the fifth 3. Of the sixth 5. Nebulous 1.

FLUVIUS, or ERIDANUS, or the NILE [the River] — Constellatio XXXVI

[Margin: Eridanus.]

No.	Position	Longit.	Latit.	Mag.
1	The one at Orion's left foot, at the beginning of the River	41° 40'	31° 50'	4
2	At the bend, near Orion's leg, the northernmost	42° 10'	28° 15'	4
3	After this, the following of two	41° 20'	29° 50'	4
4	The one that goes before	38° 0'	28° 15'	4
5	Then, of two, the following	36° 30'	25° 50'	4

No.	Position	Longit.	Latit.	Ma g.
	one			
6	The one that precedes	33° 30'	25° 20'	4
7	After these, the following of three	29° 40'	26° 0'	4
8	The middle one	29° [?]'	27° 0'	4
9	The leading of three	26° 10'	27° 50'	4
10	After a gap, the following of four	20° 20'	32° 50'	3
11	The one that goes before it	18° 0'	31° 0'	4
12	The third to the west	17° 30'	28° 50'	3
13	The westernmost of all four	15° 30'	28° 0'	3
14	Again in like manner, the following of four	10° 30'	25° 30'	3
15	The one west of it	8° 10'	23° 50'	4
16	The one still west of this	5° 30'	23° 10'	3
17	The one that goes before these four	3° 50'	23° 15'	4
18	The one that, at the winding of the River, touches the breast of Cetus	358° 30'	32° 10'	4
19	The one that follows it	359° 20'	34° 50'	4
20	The leading of the three following	2° 10'	38° 30'	4
21	The middle one	7° 10'	38° 10'	4
22	The following of three	10° 50'	30° 0'	5
23	In the quadrilateral, the northern of the two leading ones	14° 40'	41° 30'	4
24	The southern one	14° 50'	42° 30'	4
25	Of the following side, the leading one	15° 30'	43° 20'	4
26	The following of those four	18° 0'	43° 20'	4

No.	Position	Longit.	Latit.	Ma g.
27	Toward the east, the northern of the two joined together	27° 30'	50° 20'	4
28	The one more to the south	28° 20'	51° 45'	4
26 [recte 29]	At the bend, the following of two	21° 30'	53° 50'	4
30	The leading one	19° 10'	53° 10'	4
31	In the remaining stretch, the following of three	11° 10'	53° 0'	4
32	The middle one	8° 10'	53° 30'	4
33	The leading of three	5° 10'	52° 0'	4
34	At the end of the River	353° 30'	53° 30'	1

All stars 34. Of the first magnitude 1. Of the third 5. Of the fourth 27. Of the fifth 1.

LEPUS [the Hare] — Constellatio XXXVII

[Margin: Lepus.]

No	Position	Longit.	Latit.	Mag
.	.	.	.	.
1	In the ears, of the leading ones of the quadrilateral, the northern	43° 0'	35° 0'	5
2	The southern one	43° 10'	36° 30'	5
3	Of the following side, the northern one	44° 40'	35° 40'	5
4	The southern one	44° 40'	36° 40'	5
5	On the mount [in monte]	42° 30'	39° 40'	4
6	At the end of the left forefoot	39° 30'	45° 15'	4
7	In the middle of the body	48° 0'	41° 30'	3
8	Under the belly	48° 10'	44° 20'	3
9	In the hind feet, the more northern of two	54° 20'	44° 0'	4
10	The one more to the south	52° 20'	45° 50'	4

No	Position	Longit.	Latit.	Mag
.				.
11	In the loin	53° 20'	38° 20'	4
12	At the tip of the tail	56° 0'	38° 10'	4

All stars 12. Of the third magnitude 2. Of the fourth 6. Of the fifth 4.

CANIS MAIOR [the Greater Dog] — Constellatio XXXVIII

[Margin: Canis maior.]

No	Position	Longit.	Latit.	Mag
.				.
1	In the mouth, the most brilliant, called the Dog[-star], Candens [Sirius]	71° 0'	39° 10'	1
2	In the ears	73° 0'	35° 0'	4
3	In the head	74° 40'	36° 30'	5
4	In the neck, the northern of two	76° 40'	37° 45'	4
5	The southern one	78° 40'	40° 0'	4
6	In the breast	73° 50'	42° 30'	5
7	In the right knee, the northern of two	69° 30'	41° 15'	5
8	The southern one	69° 20'	42° 30'	5
9	At the end of the forefoot	64° 20'	41° 20'	3
10	In the left knee, the leading of two	68° 0'	46° 30'	5
11	The following one	69° 30'	45° 50'	5
12	In the left shoulder, the following of two	78° 0'	46° 0'	4
13	The one that goes before	75° 0'	47° 0'	5
14	At the top of the left thigh	80° 0'	48° 45'	3
15	Under the belly, between the thighs	77° 0'	51° 30'	3
16	In the ham of the right leg	86° 20'	55° 10'	4
17	At the end of that same foot	63° 0'	53° 45'	3
18	At the tip of the tail	85° 30'	50° 30'	3

All stars 18. Of the first magnitude 1. Of the third 5. Of the fourth 5. Of the fifth 7.

Unformed stars around the Dog (Informes circa Canem)

No .	Position	Longit.	Latit.	Mag .
1	To the north, at the crown of the Dog	72° 50'	25° 13'	4
2	Under the hind feet, on a straight line, the southern one	63° 20'	61° 30'	4
3	The one more to the north	64° 40'	58° 45'	4
4	The one still more northern than this	66° 20'	57° 0'	4
5	The remaining one of those four, the northernmost	67° 30'	56° 0'	4
6	Toward the west, almost in a straight line, the leading of three	50° 20'	55° 30'	4
7	The middle one	51° 40'	57° 40'	4
8	The following of three	55° 40'	59° 30'	4
9	Beneath these, the following of two bright ones	52° 20'	59° 40'	2
10	The leading one	49° 20'	57° 40'	2
11	The remaining one, more southern than those mentioned above	45° 30'	59° 30'	4

All stars 11. Of the second magnitude 2. Of the fourth 9.

PROCYON, or CANIS MINOR [the Lesser Dog], also called ANTECANIS — Constellatio XXXIX

[Margin: Canis minor.]

No .	Position	Longit.	Latit.	Mag .
1	In the neck	78° 20'	14° 0'	4
2	In the thigh, shining, Procyon, or the Dog [Procyon]	82° 30'	16° 10'	1

All stars 2. Of the first magnitude 1. Of the fourth 1.

ARGUS, or the SHIP [Argo Navis] — Constellatio XXXX

[Margin: Nauis.]

No				Mag
.	Position	Longit.	Latit.	.
1	At the stern of the ship, the leading of two	93° 40'	42° 40'	5
2	The following one	97° 40'	43° 20'	3

Table of the Stars (*Tabula Stellarum*) — Southern Constellations (continued)

Column headers throughout (verbatim): FORMÆ STELLARVM (Configurations of the stars) | **Longit. G. | M.** (Longitude, degrees | minutes) | **Latit. G. | M.** (Latitude, degrees | minutes) | **Magnitudo** (Magnitude). Latitudes in this southern section are southern latitudes. A [*] at the start of a description marks a star flagged with a marginal asterisk in the source (an editorial mark keyed to Clavius's commentary).

Argo / The Ship (*Navis*) — Constellatio XXXX (continued from stars 1–2)

N						Ma
o.	Star description	Long °	Long '	Lat °	Lat '	g.
3	Of the two on the poop, the northern	92	10	45	0	4
4	The one more to the south	92	10	46	0	4
5	Preceding the two	88	40	45	30	4
6	Shining in the middle of the shield	89	40	47	15	4
7	Of the three below the shield, the preceding	88	50	49	45	4
8	The following	92	40	49	50	4
9	The middle of the three	91	40	49	15	4

N o.	Star description	Long °	Long '	Lat °	Lat '	Ma g.
1 0	At the end of the rudder	97	20	49	50	4
1 1	Of the two in the keel of the poop, the northern	87	20	53	0	4
1 2	[*] The southern	87	20	58	40	3
1 3	In the base of the poop, the northern	93	30	55	30	5
1 4	Of the three on the same base, the preceding	95	30	58	30	5
1 5	The middle	96	40	57	15	4
1 6	The following	99	50	57	45	4
1 7	The bright one following, on the cross-bench	104	30	58	20	2
1 8	Of the two faint ones below it, the preceding	101	30	60	0	5
1 9	[*] The following	104	20	59	20	5
2 0	[*] Of the two above the said bright one, the preceding	106	30	56	40	5
2 1	The following	107	40	57	0	5
2 2	Of the three at the little shields and mast-step, the northern	119	0	51	30	4
2 3	[*] The middle	119	30	55	40	4
2 4	The southern of the three	117	20	55	10	4
2 5	Of the two joined below these, the northern	122	30	60	0	4
2	The more southern	122	20	61	11	4

N o.	Star description	Long °	Long '	Lat °	Lat '	Ma g.
6						
2 7	Of the two in the middle of the mast, the southern	113	30	51	30	4
2 8	The northern	112	40	49	40	4
2 9	Of the two at the top of the sail, the preceding	111	20	43	20	4
3 0	The following	112	20	43	30	4
3 1	Below the third, which follows the shield	98	30	54	30	2
3 2	In the section of the decking	100	50	51	15	2
3 3	Among the oars, in the keel	95	0	63	0	4
3 4	The faint one following it	102	20	64	30	6
3 5	The bright one following it, on the decking	113	20	63	50	2
3 6	Shining more to the south, within the keel	121	50	69	40	2
3 7	Of the three following it, the leading	128	30	65	40	3
3 8	The middle	134	40	65	50	3
3 9	The following	139	20	65	50	2
4 0	Of the two following toward the section, the preceding	144	20	62	50	3
4 1	The following	151	20	62	15	3
4 2	On the northern and leading tiller, the one that goes before	57	20	65	50	4

N o.	Star description	Long °	Long ′	Lat °	Lat ′	Ma g.
4 3	The one that follows	73	30	65	40	3
4 4	The one preceding on the remaining tiller, Canopus	70	30	75	0	1
4 5	The remaining one, following it	82	20	71	50	3

Summary (verbatim): *Omnes stellæ 45. Primæ magnit. 1. Secundæ 6. Tertiæ 8. Quartæ 22. Quintæ 7. Sextæ 1.* — "All stars: 45. Of first magnitude 1, second 6, third 8, fourth 22, fifth 7, sixth 1." (Magnitude tally of stars 3–45 as transcribed is self-consistent: the missing 2 counts belong to stars 1–2 of the prior page, one third-magnitude and one fifth-magnitude.)

Hydra / The Water-Snake — Constellatio XLI

Margin: Hydra.

N o.	Star description	Long °	Long ′	Lat °	Lat ′	Ma g.
1	Of the five in the head, of the two preceding in the nostrils, the southern	97	20	15	0	4
2	The northern of the two, and in the eye	98	40	13	40	4
3	Of the two following, the northern, and in the back of the head	99	0	11	30	4
4	The southern of them, and in the open mouth	98	50	14	45	4
5	The one following all these, in the cheek	100	50	12	15	4
6	Of the two in the extension of the neck, the preceding	103	40	11	50	5
7	[*] The following	106	40	13	40	4
8	Of the three at the bend of the neck, the middle	111	40	15	20	4

N o.	Star description	Long °	Long ′	Lat °	Lat ′	Ma g.
9	Following this	114	0	14	50	4
1 0	[*] The most southern	111	40	17	10	4
1 1	Of the two adjacent to the south, the faint one; and the northern	112	30	19	45	6
1 2	The bright one of them, following	113	20	20	30	2
1 3	Of the three after the bend of the neck, the leading	119	20	26	30	4
1 4	The following	124	30	23	15	4
1 5	The middle of them	122	0	26	0	4
1 6	Of three in a straight line, the preceding	131	20	24	30	3
1 7	The middle	133	20	23	0	4
1 8	The following	136	20	22	10	3
1 9	Of the two below the base of the Bowl (Crater), the northern	144	50	25	45	4
2 0	The southern	145	40	30	10	4
2 1	After these, in the triangle, the preceding	155	30	31	20	4
2 2	The southern of them	157	50	34	10	4
2 3	The following of the same three	159	30	31	40	3
2 4	[*] After the Raven (Corvus), nearest to the tail	173	20	13	40	4
2	[*] At the tip of the tail	186	50	17	40	4

N						Ma
o.	Star description	Long °	Long ′	Lat °	Lat ′	g.
5						

Summary (verbatim): *Omnes stellæ 25. Secundæ magnit. 1. Tertiæ 3. Quartæ 19. Quintæ 1. Sextæ 1.* — "All stars: 25. Of second magnitude 1, third 3, fourth 19, fifth 1, sixth 1." (Transcribed tally matches exactly.)

Unformed stars around Hydra (*Informes circa Hydram*)

N						Ma
o.	Star description	Long °	Long ′	Lat °	Lat ′	g.
1	[*] From the head, to the south	95	13	13	0	3
2	[*] Following those which are in the neck	124	20	16	0	3

(No magnitude-summary line is printed for these two unformed stars.)

Crater / The Bowl, or Cup, or Urn (*Sive Patera, vel Urna*) — Constellatio XLII

Margin: Crater.

N						Ma
o.	Star description	Long °	Long ′	Lat °	Lat ′	g.
1	At the base of the Bowl, also shared with Hydra	139	40	23	0	4
2	Of the two in the middle of the Bowl, the southern	146	0	19	30	4
3	The northern of them	143	30	18	0	4
4	On the southern circumference of the rim	150	20	18	30	4
5	On the northern edge	142	40	13	40	4
6	On the southern handle	152	30	16	30	4
7	On the northern handle	145	0	11	50	4

Summary (verbatim): *Omnes stellæ 7. Quartæ magnitudinis.* — "All stars: 7. Of fourth magnitude." (All seven transcribed as magnitude 4.)

Corvus / The Raven — Constellatio XLIII

Margin: Corvus.

N o.	Star description	Long °	Long ′	Lat °	Lat ′	Ma g.
1	In the beak, also shared with Hydra	158	40	21	30	3
2	In the neck	157	40	19	40	3
3	In the breast	160	0	18	10	5
4	In the right wing, and the leading one	160	50	14	50	3
5	Of the two in the following wing, the leading	160	0	12	30	3
6	The following	161	20	11	45	4
7	In the tip of the foot, shared with Hydra	163	50	18	10	3

Summary (verbatim): *Omnes stellæ 7. Tertiæ magnit. 5. Quartæ 1. Quintæ 1.* — "All stars: 7. Of third magnitude 5, fourth 1, fifth 1." (Transcribed tally matches exactly.)

Centaurus / The Centaur — Constellatio XLIV

Margin: Centaurus.

N o.	Star description	Long °	Long ′	Lat °	Lat ′	Ma g.
1	Of the four in the head, the most southern	183	50	21	40	5
2	The one more to the north	183	20	18	50	5
3	Of the two in the middle, the preceding	182	30	20	30	4
4	The following, and the remaining of the four	183	20	20	0	5
5	In the left shoulder, and the leading one	179	30	25	40	3
6	In the right shoulder	189	0	22	30	3

N o.	Star description	Long °	Long '	Lat °	Lat '	Ma g.
7	In the left upper arm	182	30	27	30	4
8	Of the four in the shield, of the two preceding, the northern	191	30	22	20	4
9	The southern	192	30	23	45	4
1 0	Of the remaining two, which are at the top of the shield	195	20	18	15	4
1 1	The one more to the south	196	50	20	50	4
1 2	Of the three in the right side, the preceding	186	40	28	20	4
1 3	The middle	187	20	29	20	4
1 4	The following	188	30	28	0	4
1 5	In the right arm	189	40	26	3 [?]	4
1 6	In the right elbow	196	10	25	15	3
1 7	In the tip of the right hand	200	50	24	0	4
1 8	Shining at the emergence of the human body	191	20	33	30	3
1 9	Of the two faint ones, the following	191	0	31	0	5
2 0	The preceding	189	50	30	20	5
2 1	Along the line of the back	185	30	33	50	5
2 2	Preceding this, on the horse's back	182	20	37	30	5
2 3	Of the three in the loins, the following	179	10	40	0	3
2	[*] The middle	178	20	40	20	4

N o.	Star description	Long °	Long ′	Lat °	Lat ′	Ma g.
4						
2 5	The leading of the three	176	0	41	0	5
2 6	Of the two adjacent in the right haunch, the preceding	176	0	46	10	3
2 7	[*] The following	176	40	46	45	4
2 8	In the breast, below the horse's flank	191	40	40	45	4
2 9	[*] Of the two below the belly, the preceding	189	40	43	0	2
3 0	[*] The following	191	0	43	45	3
3 1	In the hollow of the right foot	183	20	51	10	2
3 2	[*] In its calf	188	40	51	40	2
3 3	In the hollow of the left foot	179	40	55	10	4
3 4	[*] Below its muscle	184	30	55	40	2
3 5	[*] In the top of the right fore- foot	211	40	41	10	1
3 6	In the left knee	197	30	45	20	2
3 7	[*] On the outside, below the right thigh	188	0	49	10	4

Summary (verbatim): *Omnes stellæ 37. Primæ magnit. 1. Secundæ 5. Tertiæ 7. Quartæ 16. Quintæ 8.* — "All stars: 37. Of first magnitude 1, second 5, third 7, fourth 16, fifth 8." (Transcribed tally matches exactly.)

The Beast of the Centaur, or the Wolf (*Bestia Centauri, sive Lupus*) — Constellatio XLV

Margin: Lupus.

N o.	Star description	Long °	Long ′	Lat °	Lat ′	Ma g.
1	In the top of the hind foot, toward the Centaur's hand	201	20	24	50	3
2	[*] In the hollow of the same foot	199	10	29	10	3
3	Of the two in the shoulder, the preceding	204	20	21	15	4
4	The following	207	30	21	0	4
5	In the middle of the body	206	20	25	10	4
6	In the belly	203	30	27	0	5
7	In the haunch	204	10	29	0	5
8	Of the two along the haunch, the northern	208	0	28	30	5
9	The southern	207	0	30	0	5
1 0	In the top of the loin	208	40	33	10	5
1 1	Of the three at the tip of the tail, the southern	195	20	31	20	5
1 2	The middle	195	10	30	0	4
1 3	The northern of the three	196	20	29	20	4
1 4	[*] Of the two in the neck, the southern	212	10	17	0	4
1 5	The northern	212	40	15	20	4
1 6	Of the two in the open jaws, the preceding	209	0	13	30	4
1	The following	210	0	12	50	4

N o.	Star description	Long °	Long ′	Lat °	Lat ′	Ma g.
7						
1 8	[*] Of the two in the fore foot, the more southern	200	40	11	30	4
1 9	[*] The one more to the north	199	50	10	0	4

Summary (verbatim): *Omnes stellæ 19. Tertiæ magnit. 2. Quartæ 11. Quintæ 6.* — "All stars: 19. Of third magnitude 2, fourth 11, fifth 6." (Transcribed tally matches exactly.)

The Hearth, or Censer, or Altar (*Lar, sive Thuribulum, seu Ara*) — Constellatio XLVI

Margin: Ara.

N o.	Star description	Long °	Long ′	Lat °	Lat ′	Ma g.
1	Of the two in the base, the northern	231	0	22	40	5
2	The southern	233	40	25	45	4
3	In the middle of the small altar	229	30	26	30	4
4	Of the three in the fire-pan, the northern	224	0	30	20	5
5	Of the remaining two adjacent, the southern	228	30	34	10	4
6	The northern	228	20	33	20	4
7	In the middle of the flame	224	10	34	10	4

Summary (verbatim): *Omnes stellæ 7. Quartæ magnit. 5. Quintæ 2.* — "All stars: 7. Of fourth magnitude 5, fifth 2." (Transcribed tally matches exactly.)

(The catalogue continues with Corona Australis and Piscis Austrinus; the southern-region total and the grand total of 1022 do not appear on these pages.)

TABLE OF THE STARS [FORMÆ STELLARVM]

Column headers (verbatim): **Longit. G.** | **M.** | **Latit. G.** | **M.** | **Magnitudo**
(Longitude in degrees and minutes | Latitude in degrees and minutes | Magnitude).

THE SOUTHERN CROWN, WHICH IS ALSO THE WHEEL OF IXION.
Constellation 47. [CORONA AVSTRINA, QVÆ ET ROTA IXIONIS.
Constellatio XLVII.]

[Margin: Corona Austrina (The Southern Crown).]

N			M		M Magnitud
o.	Description	Longit. G.	. Latit. G.	. e	
1	<i>Quæ ad ambitum Australem foris præcedit</i> — The one on the outside that leads toward the southern edge	242	3 21 0	3 4 0	
2	<i>Quæ hanc sequitur in corona</i> — The one following it in the crown	245	0 21	0 5	
3	<i>Sequens hanc</i> — Following this	246	3 20 0	3 5 0	
4	<i>Quæ etiam hanc sequitur</i> — The one that also follows this	248	1 20 0	0 4	
5	<i>Post hanc ante genu Sagittarij</i> — After this, before the knee of Sagittarius	249	3 18 0	3 5 0	
6	<i>Borea in genu lucens</i> — The bright one to the north, on the knee	250	4 17 0	1 4 0	
7	<i>Magis Borea</i> — Farther north	250	1 16 0	0 4	
8	<i>Adhuc magis in Boream</i> — Still farther toward the north	249	5 15 0	2 4 0	
9	<i>In ambitu Boreo duarum sequens</i> — On the northern edge, the following of two	248	3 15 0	5 6 0	

N o.	Description	Longit. G.	M . Latit. G.	M Magnitud . e
1 0	<i>Præcedens</i> — The preceding one	248	0 14	5 6 0
1 1	<i>Ex interuallo præcedens has</i> — At an interval, preceding these	245	1 14 0	4 5 0
1 2	<i>Quæ etiam hanc antecedit</i> — The one that also precedes this	243	0 15	5 5 0
1 3	<i>Reliqua magis in Austrum</i> — The remaining one, farther toward the south	242	3 18 0	3 5 0

All stars 13. Of the fourth magnitude 5; of the fifth 6; of the sixth 2. [Omnes stellæ 13. Quartæ magnit. 5. Quintæ 6. Sextæ 2.]

THE SOUTHERN FISH, OR NOTIUS. Constellation 48. [PISCIS AVSTRINVS, SIVE NOTIVS. Constellatio XLVIII.]

[Margin: Piscis Austrinus (The Southern Fish).]

N o.	Description	Longit. G.	M . Latit. G.	M Magnitud . e
1	<i>In ore atque eadem, quæ in extrema aqua</i> — In the mouth, and the same as the star at the end of the water [stream] [Fomalhaut]	300	2 23 0	0 1
2	<i>In capite trium præcedens</i> — In the head, the leading of three	294	0 21	2 4 0
3	<i>Media</i> — The middle one	297	3 22 0	1 4 5
4	<i>Sequens</i> — The following one	299	0 22	3 4 0
5	<i>Quæ ad branchiam</i> — The one at the gill	297	4 16 0	1 4 5
6	<i>In spina Australi, atq; dorso</i> — On the southern spine and the back	289	3 19 0	3 5 0

N			M		M Magnitud
o.	Description	Longit. G.	. Latit. G.	. e	
7	<i>In aluo duarum sequens</i> — On the belly, the following of two	294	3 15 0	1 5 0	
8	<i>Antecedens</i> — The preceding one	292	1 14 0	3 4 0	
9	<i>In spina Septentrionali sequens trium</i> — On the northern spine, the following of three	288	3 15 0	1 4 5	
10	<i>Media</i> — The middle one	285	1 16 0	3 4 0	
11	<i>Præcedens trium</i> — The leading of three	284	2 18 0	1 4 0	
12	<i>In extrema cauda</i> — At the end of the tail	284	2 12 0	1 4 5	

All stars, besides the first, 11. Of the fourth magnitude 9; of the fifth 2.

[Omnes stellæ præter primam 11. Quartæ magnit. 9. Quintæ 2.] (The first star is of the first magnitude.)

UNFORMED STARS AROUND THE SOUTHERN FISH. [INFORMES CIRCA PISCES NOTIVM.]

N			M		M Magnitud
o.	Description	Longit. G.	. Latit. G.	. e	
1	<i>Præcedentium piscem lucidarum, quæ anteit</i> — Of the bright stars preceding the fish, the one that goes in front	271	2 22 0	2 3 0	
2	<i>Media</i> — The middle one	274	3 22 0	1 3 0	
3	<i>Sequens trium</i> — The following of three	277	2 21 0	0 3	
4	<i>Quæ hanc præcedit obscura</i> — The faint one that precedes this	275	2 20 0	5 5 0	
5	<i>Cæterarum ad Septentrionem Australior</i> — Of the rest, the	277	1 16 0	0 4	

N o.	Description	Longit. G.	M . Latit. G.	M Magnitud . e
	more southerly toward the north			
6	<i>Quæ magis in Boream</i> — The one farther toward the north	277	1 14 0	5 4 0

All stars 6. Of the third magnitude 3; of the fourth 2; of the fifth 1. [Omnes stellæ 6. Tertiæ magnit. 3. Quartæ 2. Quintæ 1.]

IN THE SOUTHERN REGION, THEREFORE, THE STARS [IN PLAGA ERGO AVSTRALI STELLÆ]

number 316 in all: of the first magnitude 7, of the second 18, of the third 60, of the fourth 168, of the fifth 53, of the sixth 9, and 1 nebulous. [omnes 316. Primæ magnit. 7. Secundæ 18. Tertiæ 60. Quartæ 168. Quintæ 53. Sextæ 9. Neb. 1.]

IN THE WHOLE FIRMAMENT, MOREOVER, ALL THE STARS, [IN TOTO AVTEM FIRMAMENTO STELLAE OMNES,]

besides the three in the lock of hair, number **1022**, as was said above. [præter tres in cincinno. 1022. vt supra dictum est.]

[Margin: That there are no stars near the Antarctic pole. (Iuxta polum Antarcticum nullas esse stellas.)]

From all of this it is plainly established that near the Antarctic pole there are no stars, since the nearest of all to that pole is star 34, beneath the muscle of the Centaur's left foot, which is 28°39' distant from the Antarctic pole — for its declination, as we shall show a little later, amounts to 61°21'. For if those who have sailed from Portugal [Lusitania] and the other provinces of Spain to the Indies report truly, the star nearest the pole — the one they watch to steer the ship's course across the Ocean — is about 30° from the Antarctic pole, as they themselves observed with instruments. Hence it will be a fable, what is commonly said, that near the Antarctic pole there are very bright stars forming the shape of a cross — unless we take this to mean the stars in the Centaur, of which nos. 29, 31, 33, and 34 form a figure like a cross, and are all of the second magnitude.

THE USE OF THE PRECEDING TABLE. [VSUS PRAECEDENTIS TABVLAE.]

[Margin: The use of the preceding table of stars. (Vsus præcedentis tabulæ stellarum.)]

From the table set out above, three things are known about each individual star: longitude, latitude, and magnitude. For if you take any star in its own constellation, you will have at once, in the same line: first the degrees and minutes of its longitude; then the degrees and minutes of its latitude; and finally its magnitude.

EXAMPLE. In the 26th constellation, namely Leo, I take the 27th star, which is at the end of the tail [Denebola]. In the same line, then, I find the longitude of this star to be $137^{\circ}30'$, its latitude $11^{\circ}50'$, and finally that the star itself is of the first magnitude — and so for the rest.

[Margin: The longitudes of the stars in the preceding table begin from the first star of Aries. (Longitudines stellarum in præcedenti tabula incipiunt à prima stella Arietis.)]

But this longitude (like all the others contained in the table above) is to be understood not from the beginning of Aries ♈ of the *primum mobile*, but from the first star of the constellation Aries ♈, which is located in the right horn, so that relative to it all the others lie farther to the east. For Nicolaus Copernicus [Copernicus] did not compute the positions of all the stars from the beginning of Aries ♈ of the *primum mobile* — as Ptolemy and all other astronomers were accustomed to number the positions of the stars — but from the first star of Aries.

For since the fixed stars always keep the same longitude reckoned from the first star of Aries, but not from the beginning of Aries ♈ of the *primum mobile* — that is, from that common intersection of the Zodiac with the Equator which is usually called the beginning of Aries ♈ — because from this point they always move gradually toward the eastern signs, as we showed above: therefore Copernicus preferred to refer the longitudes of the stars to the first star of Aries rather than to the beginning of Aries ♈ of the *primum mobile*, so that, just as their latitudes always remain the same, so too their longitudes should undergo no variation.

[Margin: What the true longitudes of the stars are, and how they are to be investigated. (Vera longitudes stellarum quid & quomodo inuestigentur.)]

But if anyone wishes to know the distances of the individual stars from the vernal equinox — that is, from the beginning of Aries Υ of the *primum mobile* (these distances being in fact called the true longitudes of the stars), in the manner of Ptolemy and the other astronomers — he will reach the desired end without much labor by this method. First let the true position of the first star of Aries be ascertained, or (what is the same thing) the true longitude of that star. Then let the longitude of any star be taken from the table above, and to it let the true longitude of the first star of Aries be added. For the resulting sum, if it is less than 360° , will at once indicate the distance of the proposed star from the beginning of Aries Υ of the *primum mobile*; but if it exceeds 360° , the number left over, after 360° has been cast off, will give the said distance.

EXAMPLE. According to the observations of Peter Apian [Petrus Apianus], who examined the true positions of the fixed stars in the year 1532, the first star of Aries has moved eastward from the beginning of Aries Υ of the *primum mobile* by $26^\circ 38'$. If, then, I wish to know how far Spica $\mathfrak{M}$ [in Virgo] has moved from that same beginning, I take from the table above, in the constellation Virgo $\mathfrak{M}$ (which is the 27th constellation), the distance of the said star from the first star of Aries Υ , namely $170^\circ 0'$; to this I add the $26^\circ 38'$ by which the first star of Aries Υ has moved from the vernal equinox, and $197^\circ 38'$ results. And so great is the true longitude of that star which they call Spica $\mathfrak{M}$.

Likewise, if one should wish to inquire how far distant from the vernal equinox is that star which lies in the navel of Pegasus and in the head of Andromeda [Alpheratz], I take from the 19th constellation, which is Pegasus — or from the 20th, which is Andromeda — the distance of the said star from the first star of Aries Υ , namely $341^\circ 10'$; to this I add $26^\circ 38'$, and $367^\circ 48'$ results, from which, if 360° be cast off, $7^\circ 48'$ remain. So great, then, is the true longitude of the proposed star. And so for the rest.

But it must not be passed over that Nicolaus Copernicus, an exact observer of the stars, found in the year 1525 that the first star of Aries Υ had moved from the vernal equinox not only by $26^\circ 38'$, as Apian holds, but by $27^\circ 21'$. Therefore, if you prefer to trust his observations rather than Apian's, you will find, by the preceding procedure, the longitude of Spica $\mathfrak{M}$ — that is, its distance from the beginning of Aries Υ of the *primum mobile* — to be $197^\circ 21'$, and the longitude of the head of Andromeda to comprise $8^\circ 31'$. But since the stars gradually advance from West to East, more minutes must be added for the present time. For from the year 1525 up to the Jubilee year 1575 — when at Rome we constructed, as accurately as we could, an astronomical globe according to this table — the fixed

stars have advanced by about 26'. Therefore, to the longitudes found in the preceding table, 27°47' must be added so that the true longitudes may be obtained. This is what we ourselves carried out on that globe. By this reckoning Spica ♏ will be distant from the beginning of Aries ♈ by 197°47', while the head of Andromeda will be 8°57' from the same beginning. For the year 1600, 28°6' must be added; for by so great a space will the first star of Aries ♈ then be elongated from the vernal equinox, according to the Prussian Tables [Tabulae Prutenicae] drawn from Copernicus's teaching. But what must be added at other times, both before and after the Nativity of the Lord, you will learn from the scholium to Proposition 11, Book 2, of our *Astrolabe* [Astrolabium].

[Margin: In what sign and degree of the Ecliptic any given star is found. (In quo signo, & gradu Ecliptica quavis stella reperiatur.)]

From this, too, you will easily discover in what sign of the Zodiac and in what degree any given star is found. For if the degrees of the true longitude that has been found are divided by 30, the whole signs by which the star is removed from the vernal equinox will at once appear in the quotient; and the remaining number of degrees and minutes is then to be assigned to the following sign.

EXAMPLE. The longitude of Spica ♏ was found to be 197°47' (for we now follow the observation of Copernicus as the truer one, with 26' still added, as we said, for the year 1575). I divide 197 by 30, and the quotient is 6, with 17°47' remaining. Therefore Spica ♏ has moved from the beginning of Aries ♈ of the *primum mobile* by six whole signs, and it is in the 17th degree and 47th minute of the seventh sign, namely Libra ♎. I therefore declare that at this time the true position of Spica ♏ is at 17°47' of Libra ♎. By the same reasoning the true position of the head of Andromeda will be found at 8°57' of Aries ♈. And in the same way you will investigate all the positions of the fixed stars, whether according to the observations of Apian, or of Nicolaus Copernicus, or of anyone else, and so on.

ON INVESTIGATING THE DECLINATIONS OF THE STARS. [DE STELLARVM DECLINATIONIBVS INVESTIGANDIS.]

[Margin: In what way the declinations of the stars are investigated. (Declinationes stellarum quo pacto inuestigentur.)]

Since the fixed stars, on account of that very slow motion from West to East, continually change their declinations from the Equator, I think it worthwhile to teach briefly here by what method the declinations of the stars — whose longitudes and latitudes are known — may be found from the sines [ex sinibus

[?]]. For this matter is of extraordinary usefulness among astronomers, especially in the construction of instruments. But although we can accomplish what is proposed in many ways, as we showed in the scholium to Canon 3, Book 3, of the *Astrolabe*, Number 10, I have preferred here to explain only that method which Pedro Nunes [Petrus Nonius] set forth in his little book *On Twilights* [De crepusculis], and which we demonstrated more clearly in the scholium to Canon 15, Book 3, of the *Astrolabe*, Number 6. The method is as follows.

Let it be arranged that: as the square of the whole sine [the radius] is to the rectangle contained under the sine of the maximum declination of the Ecliptic [the obliquity] and the sine of the complement of the proposed star's latitude, so is the versed sine of the star's longitude — reckoned from the beginning of Cancer ☿ if the star's latitude is northern, or from the beginning of Capricorn ♑ if the star's latitude is southern — to a fourth number. (This longitude is to be counted from Cancer ☿ according to the succession of the signs if the star lies in the descending semicircle of the Ecliptic — that is, if its true longitude from the beginning of Aries ♈ is greater than 90° but less than 270° — but against the succession of the signs if the star lies in the ascending semicircle of the Ecliptic — that is, if its true longitude from the beginning of Aries ♈ is less than 90° or greater than 270° . For by this rule the star's longitude computed from the beginning of Cancer ☿ will always be less than a semicircle. In the contrary manner the longitude is to be counted from the beginning of Capricorn ♑: for if the star lies in the descending semicircle of the Ecliptic, the longitude from the beginning of Capricorn ♑ is to be computed against the succession of the signs, but if in the ascending semicircle, according to the succession of the signs. For in this way, again, the star's longitude computed from the beginning of Capricorn ♑ will always come out less than a semicircle.)

For a number will be found from which, by this technique, we shall determine the star's declination. Let the number found be compared with the sine of the complement of the difference between the maximum declination of the Ecliptic and the complement of the star's latitude. For if the number found is equal to that sine of the complement, the star will have no declination but will lie on the Equator. But if it is less, then subtracting this from that, there will remain the sine of the star's declination, of the same denomination as the latitude — that is, northern if the star's latitude is northern, southern if it is southern. If, finally, the number found is greater than that sine of the complement, then subtracting this from that, the remainder will be the sine of the star's declination, of the contrary denomination to the latitude — that is, northern if the star has southern latitude, southern if it has northern. The matter will become clearer by a few examples.

[Margin: The declination of Arcturus. (Declinatio Arcturi.)]

Let the declination of Arcturus be found — the star that is unformed near Boötes, that is, in constellation 5. Since this star has in the table a longitude of $170^{\circ}20'$, we shall add $27^{\circ}47'$ to make its true longitude from the beginning of Aries Υ $198^{\circ}7'$. Since this is greater than 90° but less than 270° , the said star will lie in the descending semicircle of the Ecliptic, and its longitude must be reckoned from the beginning of Cancer $\mathfrak{C}$ (since it has northern latitude) according to the succession of the signs; and this longitude, if 90° be subtracted from its true longitude, will be found to contain $108^{\circ}7'$, whose versed sine will be 131095, the whole sine being taken as 100000. The northern latitude of the same star is $31^{\circ}30'$, and its complement $58^{\circ}30'$. The difference, too, between the maximum declination of the Ecliptic — that is, $23^{\circ}30'$ — and the complement of the star's latitude — that is, $58^{\circ}30'$ — is $35^{\circ}0'$, and the sine of the complement of this difference is 81915.

Therefore, let it be set up thus: as 10000000000, the square of the whole sine, is to 3399816736, the rectangle contained under 39874 (the right sine of the maximum declination of the Ecliptic) and 85264 (the sine of the complement of the proposed star's latitude), so is 131095 (the versed sine of the star's longitude from Cancer $\mathfrak{C}$, according to the succession of the signs) to a fourth number. (That is: following the rule of proportions which they call the Rule of Three, we multiply the said rectangle — which is obtained by multiplying the sine of the maximum declination of the Ecliptic by the sine of the complement of the star's latitude — by the versed sine of the star's longitude; that is, we multiply the second number of the Rule of Three into the third, and divide the product by the square of the whole sine, that is, by the first number of the Rule of Three — which is done most easily if the ten right-hand figures are cast off from the product.) By this, the number 44569 will be found. Since it is less than 81915 — the sine of the complement of the difference between the maximum declination of the Ecliptic and the complement of the star's latitude — we shall subtract it from 81915, the sine of the complement of the said difference, and there will remain 37346, the sine of the northern declination of Arcturus, to which, in the table of sines, corresponds an arc of $21^{\circ}56'$. So great, then, is the declination of Arcturus from the Equator toward the North.

[Margin: The declination of Capella [Hircus]. (Declinatio Hirci.)]

Let the declination that Capella [Hircus, "the Goat"] has be inquired into next — the brightest star on the left shoulder of Auriga, and the third in constellation 12. The longitude of this star has in the table $48^{\circ}20'$; and if $27^{\circ}47'$ be added to it, its true longitude from the beginning of Aries Υ will be assembled as $76^{\circ}7'$. Since this

is less than 90° , the given star will lie in the ascending semicircle of the Ecliptic, and its longitude must be reckoned from Cancer $\mathfrak{C}$ (since its latitude is northern) against the succession of the signs; and this longitude, if its true longitude be subtracted from 90° , will comprise $13^\circ 53'$, whose versed sine will be 2921. The northern latitude of the same star is $22^\circ 30'$, and its complement $67^\circ 30'$. The difference, too, between $23^\circ 30'$, the maximum declination of the Ecliptic, and $67^\circ 30'$, the complement of the star's latitude, comprises $44^\circ 0'$; and the sine of the complement of this difference is 71933. Therefore, let it be set up thus: as 10000000000, the square of the whole sine, is to 3683839238, the rectangle comprised under 39874 (the right sine of the maximum declination of the Ecliptic) and 92387 (the sine of the complement of the given star's latitude), so is 2921 (the versed sine of the star's longitude from Cancer $\mathfrak{C}$, against the succession of the signs) to a fourth number— ...this number 1076 will be found. Since it is smaller than 71933 — the sine of the complement of the difference between the maximum declination of the ecliptic and the complement of the star's latitude — we subtract it from 71933, and there remains 70857, the sine of the northern declination of Hircus [Capella]. To this in the table of sines correspond 45 degrees 7 minutes, for the declination of Hircus from the equator toward the North.

[Second star of Aquarius — declination]

[Margin: The declination of the 2nd star of Aquarius, which is placed in the right shoulder, and is of the 3rd magnitude.]

AGAIN, let the declination be investigated of the star that is placed in the right shoulder of $\approx$ [Aquarius], which is the second [star] in the constellation Aquarius and is of magnitude 3. Its longitude in the table is 299 degrees 40 minutes. If we add to this 27 degrees 47 minutes, its true longitude from the beginning of Υ [Aries] comes to 327 degrees 27 minutes. Since this is greater than 270 degrees, the star will lie in the ascending semicircle of the ecliptic, and its longitude must be reckoned from $\mathfrak{C}$ [Cancer] (because it has northern latitude), against the succession of the signs. If this true longitude is subtracted from 360 degrees, and 90 degrees are added to the remainder, we obtain 122 degrees 33 minutes, whose versed sine is 153803. The northern latitude of the same star is 11 degrees 0 minutes, and its complement is 79 degrees 0 minutes. Moreover, the difference between 23 degrees 30 minutes of the maximum declination of the ecliptic and 79 degrees 0 minutes, the complement of the star's latitude, amounts to 55 degrees 30 minutes, and the sine of the complement of this difference is 56640.

Therefore, if the proportion is set up — as 10000000000, the square of the whole sine, is to 3914111588, the rectangle contained under 39874 (the right sine of the

maximum declination of the ecliptic) and 98162 (the sine of the complement of the star's latitude), so is 153803 (the versed sine of the star's longitude from ♋ [Cancer], against the succession of the signs) to a fourth — this number 60200 will be found. Since it is greater than 56640 (the sine of the complement of the difference between the maximum declination of the ecliptic and the complement of the star's latitude), we take away from it 56640, the sine of the complement of the said difference, and there remains 3560, the sine of the southern declination of the said star. To this in the table of sines correspond 2 degrees 2 minutes, for the declination of the given star from the equator toward the South.

[34th star of Centaurus — declination]

[Margin: The declination of the 34th star of Centaurus, which is under the muscle of the left foot, and is of the 2nd magnitude.]

LASTLY, let it be investigated how great a declination the 34th star in Centaurus has — the one that is most southern, lies under the muscle of the left foot, and is of magnitude 2. Its longitude in the table is 184 degrees 30 minutes. If we add to it 27 degrees 47 minutes, its true longitude from the beginning of ♈ [Aries] is made up to 212 degrees 17 minutes. Since this is greater than 90 degrees but less than 270 degrees, the star will lie in the descending semicircle of the ecliptic, and its longitude must be reckoned from ♎ [Capricorn] (because it has southern latitude), against the succession of the signs. If its true longitude is taken from 270 degrees, there remains 57 degrees 43 minutes, whose versed sine is 46590. The southern latitude of the same star is 55 degrees 40 minutes, and its complement is 34 degrees 20 minutes. Accordingly, the difference between 23 degrees 30 minutes of the maximum declination of the ecliptic and 34 degrees 20 minutes, the complement of the star's latitude, amounts to 10 degrees 50 minutes, and the sine of the complement of this difference is 98217.

Therefore, if the proportion is set up — as 10000000000, the square of the whole sine, is to 2248893600, the rectangle contained under 39874 (the right sine of the maximum declination of the ecliptic) and 56400 (the sine of the complement of the star's latitude), so is 46590 (the versed sine of the star's longitude from ♎ [Capricorn], against the succession of the signs) to a fourth — this number 10459 will be found. Since it is smaller than 98217 (the sine of the complement of the difference between the maximum declination of the ecliptic and the complement of the star's latitude), we subtract it from 98217, the sine of the complement of the said difference, and there remains 87758, the sine of the southern declination of the proposed star. To this in the table of sines correspond 61 degrees 21 minutes, for the declination of the said star from the equator toward the South.

From these examples I judge that the precept we have handed down, by which the declinations of the stars may be investigated, can be sufficiently grasped. Other methods of computing the same declinations of the stars, no less easy, you will find in Book 3 of the *Astrolabe*, in the scholium to Canon 3, No. 10.

DE QUANTITATE STELLARUM. [On the size of the stars]

[Margin: The proportions of the diameters of the stars to the diameter of the earth.]

Now that the number of the stars, which are distributed into six differences of magnitude, has been established, and the method by which their declinations are investigated has been explained, we must next set forth the size of these same stars in each difference of magnitude. This we shall accomplish most conveniently by inserting here certain tables, in which are contained both the proportions of the diameters of the stars — the fixed as well as the wandering — to the diameter of the earth, and the proportions of the sizes of those same stars to the size of the earth. In these tables we have followed Francesco Maurolico [Francesco Maurolico], the Abbot, in the Appendix of his *Dialogues on Cosmography*.

Proportions of the diameters of all the stars to the diameter of the earth.

The diameter of...	has to the diameter of the earth the proportion, as
any star of the 1st magnitude	19 to 4
any star of the 2nd magnitude	269 to 60
any star of the 3rd magnitude	25 to 6
any star of the 4th magnitude	19 to 5
any star of the 5th magnitude	119 to 36
any star of the 6th magnitude	21 to 8
♄ Saturn	9 to 2
♃ Jupiter	32 to 7
♂ Mars	7 to 6
☉ the Sun	11 to 2
♀ Venus	3 to 10
☿ Mercury	1 to 28
☾ the Moon	5 to 17

The diameter of...	has to the diameter of the earth the proportion, as
☉ the Sun (to the diameter of ☾ the Moon)	187 to 10

THEREFORE, if the individual antecedent terms of these proportions are divided by the individual consequent terms, it will become clear how many times the diameter of any given star contains the diameter of the earth — that is, when the star's diameter exceeds the earth's diameter, as is the case with the diameters of all the heavenly bodies except the diameters of Venus, Mercury, and the Moon. Or else it will show how many times the diameter of the earth contains the diameter of the star — that is, when the star's diameter is exceeded by the earth's diameter, as with the diameters of the three lower planets; for here the consequent terms must be divided by the antecedent terms. But all of this may be examined in the table set out below.

How many times the diameter of any star contains the diameter of the earth, or the diameter of the earth contains the diameter of the star.

[Margin: How many times the diameter of any star contains the diameter of the earth, or the reverse.]

The diameter of...	contains diameters (of the earth, or of the star, as noted)
any star of the 1st magnitude contains earth-diameters	4 $\frac{3}{4}$
any star of the 2nd magnitude contains earth-diameters	4 $\frac{29}{60}$
any star of the 3rd magnitude contains earth-diameters	4 $\frac{1}{6}$
any star of the 4th magnitude contains earth-diameters	3 $\frac{4}{5}$
any star of the 5th magnitude contains earth-diameters	3 $\frac{11}{36}$
any star of the 6th magnitude contains earth-diameters	2 $\frac{5}{8}$
♄ Saturn contains earth-diameters	4 $\frac{1}{2}$
♃ Jupiter contains earth-diameters	4 $\frac{4}{7}$

The diameter of...	contains diameters (of the earth, or of the star, as noted)
♂ Mars contains earth-diameters	1 1/6
☉ the Sun contains earth-diameters	5 1/2
the earth contains ♀ Venus-diameters	3 1/3
the earth contains ☿ Mercury-diameters	28
the earth contains ♃ Moon-diameters	3 2/5
☉ the Sun contains ♃ Moon-diameters	18 7/10

[a] Since spheres have to one another the triplicate proportion of their diameters, it will not be difficult for anyone even moderately versed in arithmetic to gather from the preceding table all the proportions that the sizes of the stars have to the size of the earth, as appears in the following table, in which the said proportions are contained in whole and least numbers.

[Margin: a. 18. duod. (i.e. Euclid, *Elements* XII.18: spheres are in the triplicate ratio of their diameters.)]

Proportions of the sizes of all the stars to the size of the earth.

[Margin: The proportions of the sizes of the stars to the size of the earth.]

Any star of...	has to the earth the proportion, as
the 1st magnitude	6859 to 64
the 2nd magnitude	19465109 to 216000
the 3rd magnitude	15625 to 216
the 4th magnitude	6859 to 125
the 5th magnitude	1685159 to 46656
the 6th magnitude	9261 to 512
Saturn stands to the earth, as	729 to 8
Jupiter stands to the earth, as	32768 to 343
Mars stands to the earth, as	343 to 216
the Sun stands to the earth, as	1331 to 8
Venus stands to the earth, as	27 to 1000
Mercury stands to the earth, as	1 to 21952
the Moon stands to the earth, as	125 to 4913

Any star of...	has to the earth the proportion, as
the Sun stands to the Moon, as	6539203 to 1000

AND IF the antecedent terms of all these proportions are divided by the consequent terms, it will be manifest how many times the size of any heavenly body contains the size of the earth within itself — excepting the three lower planets. For in their case the consequent terms must be divided by the antecedent terms, so that it may be known how many times the size of the earth comprehends the size of each of them, as is plain in the following table.

How many times the size of any star contains the size of the earth; or the size of the earth contains the size of the star.

... contains sizes	number
Any star of the 1st magnitude contains within itself earth-sizes	107 $\frac{11}{64}$ — or 107 $\frac{1}{6}$
Any star of the 2nd magnitude contains within itself earth-sizes	90 $\frac{25109}{216000}$ — or 90 $\frac{1}{8}$
Any star of the 3rd magnitude contains within itself earth-sizes	72 $\frac{73}{216}$ — or 72 $\frac{1}{3}$
Any star of the 4th magnitude contains within itself earth-sizes	54 $\frac{109}{125}$ — or 54 [?]
Any star of the 5th magnitude contains within itself earth-sizes	36 $\frac{5543}{46656}$ — or 36 $\frac{1}{8}$
Any star of the 6th magnitude contains within itself earth-sizes	18 $\frac{45}{512}$ — or 18 [?]
Saturn contains within itself earth-sizes	91 $\frac{1}{8}$
Jupiter contains within itself earth-sizes	95 $\frac{183}{343}$ — or 95 $\frac{1}{2}$
Mars contains within itself earth-sizes	1 $\frac{127}{216}$ — or 1 [?]
the Sun contains within itself earth-sizes	166 $\frac{3}{8}$
the earth contains within itself Venus-sizes	37 $\frac{1}{27}$
the earth contains within itself Mercury-sizes	21952

... contains sizes	number
the earth contains within itself Moon-sizes	39 38/125 [?] — or 39 1/5
the Sun contains within itself Moon-sizes	6539 203/1000 — or 6539 [1/5?]

The former numbers of this table correspond precisely to the numbers of the preceding tables; the latter do not, but fall somewhat short of the truth. They have nevertheless been included, because being smaller they are more easily grasped.

[Margin: The Sun is the greatest among the stars, and Mercury the smallest. How many stars of the 1st magnitude would be required to fill the whole Firmament.]

FROM ALL these tables, then, it is quite plainly evident that the Sun is the greatest among all the heavenly bodies of the world, and Mercury the smallest; and likewise that all the stars, both fixed and wandering, are greater than the earth itself, with only three planets excepted — Venus, Mercury, and the Moon; for these indeed are smaller than the earth.

AND IF some curious person should wish to know how many stars would be required, in any given difference of magnitude, to fill the whole concave surface of the Firmament so that they touch one another, he will easily attain this partly from what we have here said about the proportions of the diameters of the stars and of the earth, and partly from what we shall write at the end of this chapter. For since the diameter of the concave [surface] of the Firmament contains 22612 1/2 diameters of the earth, and the diameter of any star of the first magnitude contains 4 3/4 diameters of the earth: if the proportion is set up, as 4 3/4 is to 1, so is 22612 1/2 to a fourth, there will be found in the diameter of the concave Firmament 4760 diameters of one star of the first magnitude, and a little more. And if we multiply this diameter by 3 1/7, the circumference of the greatest circle in the concave of the Firmament will contain 14960 diameters of one star of the first magnitude. If we multiply that circumference by the diameter — namely by 4760 — we shall find that the concave surface of the Firmament contains 71209600 square diameters of one star of the first magnitude. Within these, that same number of stars of the first magnitude, touching one another, can be drawn. From this it also appears that those are deceived who think that there are in reality more stars in the Firmament than the children of Israel, on account of the words of Scripture cited above. For since at the Exodus from Egypt there were

numbered 603550 of the children of Israel above 21 years of age — those, namely, who went forth to war, as is clear from chapter 1 of [the Book of] Numbers...

number. Some scholars rightly conclude that, if children and women too are counted, their number was more than 2,000,000. Who, then, can doubt that over so many centuries of years there were far more than 71,209,600? Therefore, since in fact the stars are far fewer — because between any two of them a great space is interposed, and there are not a few vast spaces in the heaven in which no star at all appears, so that they in no way touch one another — it is clear that the stars in the firmament are far fewer than the children of Israel. By the same reasoning one finds the number of stars that could fill any firmament whatsoever.

[Margin: Which stars Alfraganus is speaking of.]

Alfraganus [al-Farghānī], then, in the argument our author brought forward to confirm the second part of the fourth conclusion — namely that the earth stands as a point when compared with the firmament — means the smallest stars perceptible to sight, that is, those which we, along with other astronomers, have called stars of the sixth magnitude, each of which is more than eighteen times larger than the earth. Therefore one may conclude with full right that the earth is as a point in relation to the heaven, since a star that is so much larger than the earth is itself reckoned as a point when compared with the heaven.

[Margin: How the earth stands when compared with the individual heavens.]

It will not be out of place here to explain briefly how the earth stands when compared with each of the celestial spheres in turn. For it should not be reckoned of imperceptible size in relation to every heaven. It must therefore be held as most certain that the earth is of imperceptible size if it is compared with the heaven of Jupiter, of Saturn, of the firmament, and the other higher heavens — as all the arguments already adduced clearly confirm. But in relation to the heaven of Mars and of the Sun it is indeed of some size, though not of such size as to be of any importance, as is clearer than daylight from those arguments which we set out, drawn from shadows and from the instruments of mathematicians. For those observations were made especially on the Sun. Finally, if it is compared with the heaven of Venus, of Mercury, and of the Moon, it must now be reckoned altogether of notable size, especially in relation to the lunar sphere. For since the body of the Moon, in relation to the sphere in which it exists, presents a perceptible size and mass, as is manifest to the senses, how could the earth — which is far larger than the lunar body — be said to have no notable mass and size in relation to the heaven of the Moon? All this will be clearer from this

common opinion of the astronomers, who assert that if someone stationed in the lunar sphere were to look at the earth, it would appear to him three times larger, and a little more, than the Moon appears to us here from the earth; that from the sphere of the Sun the earth, seen, would be judged twice as large as Venus appears to us from there; that from the heaven of Mars the earth, if it shone, would be reckoned equal to one of the smallest stars, of the kind comprised in the sixth magnitude; and finally that from the higher heavens, especially from the firmament, it would in no way be discerned, but would be altogether like an imperceptible point.

[Margin: That the earth is smaller than the Sun, but larger than the Moon.]

But since what we have affirmed about the size of the heavenly bodies in relation to the size of the earth may perhaps seem astonishing to someone, we shall now briefly show that the earth — although it appears to us endowed with an enormous mass — is much smaller than the body of the Sun, while the Moon, on the contrary — although the senses judge it to be the same size as the Sun — is far smaller than the earth itself. But the extremely subtle arguments by which the most skilled astronomers establish all this geometrically we omit entirely, since they belong to a higher order of study than can be explained here and pertain to the *Theorics of the Planets*. Anyone gripped by a desire for them must seek them from Ptolemy, the supreme master of these arts, and from other astronomers. That the Sun, then, is far larger than the earth can be made evident from the arguments of the writers on optics [Perspectiui].

For if the Sun were equal to the earth, the earth's shadow would be projected uniformly, in the manner of a cylinder, to infinity. But if the Sun were smaller than the earth, the earth's projected shadow would forever grow to infinity — the former of which is demonstrated most clearly by Witelo [Vitellio] in the *Perspectiva*, Book 2, proposition 26, and the latter in proposition 28.

Consequently, on a clear night some fixed stars would always be hidden — those, namely, that would lie in the earth's shadow — or at least would not have as much brightness as the other stars that are then illuminated by the Sun; and by the same reasoning [some would be hidden] when Mars, Jupiter, and Saturn are set diametrically opposite the Sun — which has never been observed. The Sun, therefore, will be much larger than the earth. For thus it comes about that the earth's shadow is projected lengthwise in the shape of a pyramid, or rather a cone, and ends in an indivisible point, so that it does not reach the fixed stars or the said planets at all — as the same Witelo demonstrates in proposition 27 of the same book. Hence it is no wonder that neither any fixed stars nor those higher

planets suffer any loss of light, even though they face the Sun from directly opposite.

That the Moon, however, is much smaller than the earth can now be deduced demonstratively from what has been said. For since it has been shown that the earth's shadow is conical, so that it always grows narrower and finally ends in a point, the shadow must necessarily have a diameter smaller than the earth's diameter. Therefore, since the whole Moon is sometimes hidden within that shadow — and for a long stretch of time, as appears in its eclipses — who does not see that its diameter is smaller than the shadow's diameter, and consequently far smaller still than the earth's diameter? Since, then, the Moon is much smaller than the earth and yet appears to us so large, it is clear that it is very near to us — so that the distance from the earth's surface to its center is altogether perceptible and appreciable when compared with the distance from the earth's surface to the heaven of the Moon. Therefore Ptolemy and Johannes Regiomontanus rightly prescribe (*Almagest* IV.1) that the true place of the Moon [☽] should be investigated by means of lunar eclipses, and not by instruments. For, they say, since we exist on the earth's surface, a great and perceptible error will befall us if we wish to hunt for the true place of the Moon by instruments, on account of its excessive nearness — which would not happen at all if we were placed at the center of the earth.

[Margin: A digression on that new star which appeared in 1572 and vanished in 1574, and on other new stars.]

This is the place to remind me that, since I have spoken of all the stars conveniently perceived by sight, I should also say something (for many weighty and learned men have demanded my opinion on the matter) about that new star which appeared in the year 1572 in the constellation of Cassiopeia and vanished in the year 1574. That star, indeed, appeared at first of such magnitude and splendor that it outshone the star Venus; but after some months it was so diminished that it was judged equal to the pole star, or to any other star of the third magnitude; and thereafter, right to the end, it grew ever more diminished. A thing truly to be marveled at, very like a portent, and one that exercised the minds of many.

[Margin: The first opinion on the new star.]

For some — though few — thought that this was not a new star, but one of those thirteen ancient stars that astronomers have always observed in Cassiopeia, and that it then appeared larger than usual because of an exhalation interposed in the

highest region of the air between it and our sight; and that thus it came about that most people believed it to have been new.

[Margin: The second opinion on the new star.]

Others, however, judged that this star was some very small one in the firmament, of the number of those that lie outside the six magnitudes and generally hide themselves because of their smallness, so that they do not...

[Margin: The third opinion on the new star.]

appear, and therefore, as we said above, are not counted by astronomers among the number of the stars; but that on account of the interposed exhalation it then appeared of such magnitude that it was reckoned new by almost everyone. Others, finally, judged that this star was a comet in the highest region of the air.

[Margin: Refutation of the first opinion.]

But none of these opinions seems to me to be true. That this star was not one of those thirteen noted in Cassiopeia I consider more certain than certainty itself. For Francesco Maurolico [Maurolycus], Abbot of Messina, most experienced in the contemplation of the stars (seeing that he had spent sixty whole years in that study), in Sicily; and very many other astronomers, both in the two Germanies and in Spain and France, who had more than once counted those thirteen stars of Cassiopeia — at that very time when this new one appeared, noticed in Cassiopeia, besides those thirteen, this new one of which we are speaking, so that they now understood there to be not thirteen stars, as of old, but fourteen in Cassiopeia. Of this fact I myself am also a witness, who at Rome in December of the year 1573, besides that new star (diminished, however, so that it seemed the equal of a star of the third magnitude), saw thirteen others in Cassiopeia; and indeed not I alone at Rome, but several others together with me, to whom I was pointing out the newly-risen star, observed it repeatedly.

[Margin: What figure the new star made with the stars of Cassiopeia.]

But it is astonishing that the authors of this opinion, alone among all astronomers, should have seen that it was an old star yet not have seen the star itself — so that one might easily suspect that they were not very diligent in observing this star, and that, relying on the authority of the ancients rather than on new observation, they had grown accustomed to hold that that star did not differ from the others, evidently so as not to concede anything new in the heaven. This is apparent even from the following: that the new star, together with three other stars of Cassiopeia which are of the third magnitude — I mean the second,

which is seen in her breast; the fourth, which is above the chair at the hips; and the twelfth, which is situated at the mid-ascent of the chair — formed that figure which geometers call a rhombus; whereas the eleventh star of Cassiopeia, which is of the fourth magnitude — and which the authors of this opinion suppose to have appeared new because it seemed larger on account of the interposed vapors (as they think) — perpetually forms that figure which geometers call a trapezium, such that it is less distant from the fourth star than the twelfth is from the second, whereas the new star, by the consensus of all who observed it, was equally distant from the fourth as the twelfth is from the second, as is clear from the adjoining figure.

Diagram. A small woodcut (set in the left margin/text column) showing two configurations of printed star-glyphs (asterisks) with adjacent catalog numerals, illustrating the argument of the surrounding text: the rhombus formed by the new star with Cassiopeia stars 2, 4, and 12, versus the trapezium formed by star 11 with the same three stars. The exact geometric placement of the glyphs is not fully resolvable at this resolution.

- Upper cluster: an asterisk labeled "NOVA" (the new star), an asterisk labeled "11", an asterisk labeled "4", plus additional unlabeled asterisks [layout uncertain — ?]
- Lower cluster: an asterisk labeled "12", an asterisk labeled "2", plus additional asterisks [layout uncertain — ?]

Furthermore, if that interposed exhalation was so great, and by its interposition in both Germanies, Spain, France, Italy, Sicily, and perhaps other regions, that eleventh star of Cassiopeia appeared larger than it really is, how can it be that for the same reason the remaining neighboring stars did not likewise appear larger, but of exactly the same magnitude at which they were always seen and are seen today? Someone will perhaps say that the exhalation was so placed that it was interposed only between our sight and that one star, but not between our sight and the others. But even granting that this could happen in one region, it certainly could not in many, so widely separated by intervals, as is clear from the observed difference. It is therefore clear — to me it is certainly established — that the star of which we are treating could not have been one of those thirteen which are seen daily in Cassiopeia.

[Margin: Refutation of the second opinion.]

And this latter argument that I have brought forward refutes the second opinion as well. For if, on account of an exhalation (as they suppose), that little star,

which otherwise cannot be seen, presented itself to be seen with such great magnitude, then surely for the same reason the eleventh star of Cassiopeia — to pass over the others in silence — near which that new star was observed, would have shown itself much larger; whereas at that time it was seen by all as of the same magnitude, that is, the fourth, at which it was of course seen both before and after, and is even now seen.

[Margin: Refutation of the third opinion.]

That the new star was also not a comet in the highest region of the air we shall make evident as follows. Skilled astronomers everywhere noted that this star held the same position among the fixed stars — namely the position that the figure above shows — so that they all placed it near that point where the Colure of the Equinoxes intersects the Arctic Circle, and reported it to be distant from the Arctic pole by about $23\frac{1}{2}$ degrees, and from the Equator by about $66\frac{1}{2}$ degrees, detecting almost no parallax [*aspectus varietas*] in it at such various places. Since this is so, who can doubt that it obtained its place not in the highest region of the air, where the other comets are generated, but above the Moon? For never is one and the same comet seen from different regions in exactly the same place when it is compared with the stars; since, on account of its nearness (as is agreed among astronomers), it undergoes no small variation of aspect [*parallax*]; indeed, even the Moon itself, according to all astronomers, is not without diversity of aspect, because it is very near the earth. Since, then, the new star was observed to have absolutely no diversity of aspect when observed in such various regions, this is surely an argument that it was higher than the Moon, and therefore was in no way a comet — unless we are also to say that comets are generated in the aethereal region.

[Margin: The commentator's opinion on the new star.]

So then, to say briefly what I think: I judge that this star, whatever it was, existed in the firmament, where the fixed stars are. For that it appeared in the aethereal region and not in the elementary one is established from what we adduced a little earlier in the refutation of the third opinion — that of those who made it a comet existing in the highest region of the air — precisely because no diversity of aspect was detected in it. For by the same argument philosophers and astronomers refute Aristotle's opinion about the Milky Way, which he said was continually generated in the highest region of the air from vapors and exhalations raised and drawn to that region by the force of the stars that are seen in the milky circle.

[Margin: That the new star was in the firmament.]

For since everywhere on earth the milky way is seen drawn through the same stars of the firmament — Cassiopeia, Cygnus, Aquila, Sagittarius, Gemini, and others, as we shall explain more fully in the second chapter — there can be no doubt that it is much higher than the highest region of the air, and situated in the firmament itself, because it has no diversity of aspect, which it certainly would have if it were placed in the air, as Aristotle wished. Now, that I may believe that this new star existed in the firmament and not in any other celestial sphere, I am led especially by this argument: that neither I, nor any astronomer at all, so far as I know, detected in it any [motion] other than the one we observe in the fixed stars...

For it retained a constant motion throughout, and the same position among the other fixed stars, for the whole two years (for it lasted about that long). But if it had been in the sphere of some planet — since that sphere certainly has a motion foreign to the fixed stars — undoubtedly the star itself would have had the same motion and course; but astronomers found that the case was otherwise. And this same argument evidently establishes, still more, that the star did not exist in the elementary region: because there it could by no means have kept always the same position and distance with respect to the fixed stars. Since this is so, I am persuaded either that the star was then created by God, the Best and Greatest, in the eighth heaven, to portend something great (of what sort it may be is still unknown), or else that comets can indeed be generated in the heaven itself, just as in the air, though this happens more rarely — which not a few of the ancient philosophers openly admit, and many of the more recent adduce authorities and histories to convince us that stars of this kind have often arisen in the heaven at long intervals of time, some to signify one thing, others another. If this is true, let the Peripatetics see how they can defend Aristotle's opinion about the matter of the heaven.

For it will perhaps have to be said that the heaven is not a certain fifth essence [Quinta essentia], but a mutable body, though less corruptible than these lower bodies — which indeed, before Aristotle, Plato held, along with many other philosophers; and after Christ not a few taught it plainly, among them Saint Ambrose, Basil, Gregory of Nyssa [Gregorius Nissenus], and nearly all the other lights of the Church. Whatever the case may be (for I do not interpose my own opinion in so great a matter), it is enough for me at present to have demonstrated in a few words that the star of which we speak had its seat in the firmament. But whence it so suddenly came to be, what it portended, and why it vanished after two years — I judge that as yet no one but God knows.

[Margin: What Paulinus Pridianus writes about the new star.]

This should be established for everyone: that God has not deprived himself of the power of making stars; and therefore that he could then, and can now, if he wishes, create even innumerable ones. But why it was created chiefly at that time must be left to the hidden judgment of God — who does nothing in vain, but directs all things by supreme providence to their ends — until it be revealed to mortals. I shall add here the opinion of a certain Paulinus Pridianus, physician and astronomer, who observed this same new star at Antwerp. Then I shall also bring forward some passages excerpted word for word from the disputation of Francesco Maurolico, Abbot, which was sent to me from Sicily in past years — so that it may be manifest to all that astronomers, in regions separated even by a long interval, found the same position in our new star. Thus, then, did Paulinus Pridianus write in the year 1572, among other things:

"Now, by the admirable and truly awesome judgment of God, there is visible a bright and shining star which before this neither appeared nor was seen: in form, indeed, not differing from the other stars, but appearing greater in light, in splendor, and in mass as well, and seen brighter and more brilliant not only than the stars of the first magnitude, but even than the planets themselves — in no way yielding to the most luminous and brilliant star Venus. Which, besides this, seems also to stand in its own place, and to advance by no other than the daily motion, and to revolve together with the firmament — quite contrary to the nature of the other heavenly fires and fiery meteors [Meteoron], which are moved by some proper motion of their own. This new star is seen next to Cassiopeia, toward the North, [together] with the one that is in the breast of Cassiopeia herself, and another that is above the seat near the legs, and a third in the middle of the chair — so arranged and situated that it expresses the figure and form of a rhombus [Rhambus], of which figure the new star itself forms the upper angle, inclining toward the pole of the world, etc."

[Margin: What Maurolico said about the same new star.]

Maurolico, however, wrote about the same star thus, in the same year 1572: "This year an unusual sign, more marvelous than a comet, appeared — namely a remarkable star of surpassing splendor, in a place where no star had been marked. Nor does it seem to me to be a comet of the kind generated in the air, for it appears higher, and among the number of the fixed [non-wandering] stars. Perhaps, just as it began to shine, so it will cease — especially since certain philosophers, with whom Cardano [Cardanus] agrees, are of the opinion that comets, and new stars too, can be produced even in the heaven, from the

aggregation of splendor from the planets and the other stars. However that may be, I cannot marvel enough at this new blazing of a star in our time. For it is certain that it is not one of the number of the first-magnitude stars that are turned in the Ptolemaic and Alphonsine tables, and that have shone since the founding of the world, and are fifteen in number — which this new star so surpasses in splendor that henceforth they must be called of the second magnitude, if only this one endures. Observing this star here on the horizon of Messina, standing on the meridian at about the third hour of the night, I found its altitude to be 62 degrees. From this I conjectured that it is located as if at the summit of the Arctic Circle, so that here it is distant from my vertex [zenith] by 28 degrees, and consequently from the Equator by about $66\frac{1}{2}$ degrees — since the latitude of Messina is $38\frac{1}{2}$ degrees — and that it is situated at that point where the Colure of the Equinoxes cuts the Arctic Circle, or very close to that point, etc."

The same must be said of that new star which (as was written to me from Germany) appeared in the year 1600 in Cygnus, next to the one that shines in its breast, and still persists. Likewise of another, which was first seen in October of the year 1604, between the 17th and 18th degree of Sagittarius, having a northern latitude of about 2 degrees — although, by the time I was writing this, it had so diminished that it scarcely appeared. The same, I say, must be said. For each of these stars, on account of the same arguments, must be placed in the firmament, because each was found everywhere at the same distance from the other fixed stars, so that it admitted no diversity of aspect, and no motion other than the one we note in the fixed stars was observed in it. So much, then, be said about the fourth conclusion of our author.

THE EARTH IS IMMOBILE.

[Margin: That the earth does not move with rectilinear motion.]

That the earth is held immovably at the center of all things, since it is supremely heavy, its heaviness seems to prove thus. Every heavy thing naturally tends toward the center; the center, indeed, is the point in the middle of the firmament. The earth, therefore, since it is supremely heavy, naturally tends toward that point.

COMMENTARIVS.

The author has hitherto shown that the earth exists at the center of all the heavens and the elements, as the center of the whole universe. Now, in this fifth conclusion, he attempts to prove that it is so situated in the middle of the world that it is devoid of all local motion. He carries this out by two arguments, the first

of which is taken from the earth's heaviness. For since the earth is the heaviest of all bodies, it will be carried by its own nature, impeded nowhere, to the lowest place — namely, to the center of the world — and there it will rest.

[Lemma — Sacrobosco] Likewise, whatever moves away from the center rises toward the circumference of the heaven; but the earth would move away from the center. Therefore it would rise — which is dismissed as impossible.

COMMENTARIVS

He proves the same point from the absurdity that follows. In the preceding conclusion it was established, by very many phenomena, that the earth stands at the center of the world. If, then, it were moved from the center by local motion, it would certainly rise toward the circumference of the heaven — which conflicts with the phenomena and runs contrary to the nature of the earth's heaviness.

[Margin: That the earth is entirely immobile.]

But since the Author excluded from the earth only rectilinear local motion, and not circular motion, it will be necessary to confirm in a general way, from Ptolemy, Aristotle, and the rest of the astronomers and philosophers, that the earth is immobile. The argument runs as follows. If the earth did not remain immobile, it would move either by rectilinear or by circular motion.

It cannot be moved by rectilinear motion. Since it has been demonstrated above that it stands at the center of the world, then, if it were carried by rectilinear motion, it would withdraw from the center — and so we would fall into exactly the same absurdities that we said would follow if the earth were not set at the middle of the world. Moreover, if it moved rectilinearly, it would do so either naturally or by force. It will not move naturally in this way, since by its own nature it tends toward the lowest place, which is at the center of the Universe; yet it is certain that it rises in whatever direction it is driven by rectilinear motion. Nor can it be moved rectilinearly by force, since no body heavier than the earth is to be found, which could drive it from the center of the world by its own weight.

Again, if the earth were carried by rectilinear motion, it would necessarily move with the greatest velocity, since it is supremely heavy. Grant this, and who does not see that lighter things — such as tree-leaves, chaff, and all other bodies — would have to be left behind it in the air, since they could not keep pace with its most rapid motion, not being endowed with such great heaviness? But all of this is contradicted by common experience. For we see that bodies of this kind, unless

torn away by some wind or force, cling motionless to the earth's surface. The earth, therefore, is not carried by rectilinear motion.

That it is not moved by circular motion either, as many have supposed, can be confirmed as follows. If the earth moves in a circle, it will move either about the axis of the world — from East to West, or from West to East — or about some other axis. Suppose it is said to move about the axis of the world. Then the clouds, the birds, and everything that exists in the air would be seen to move in the opposite direction — namely toward the West, if the earth rolls toward the East, or toward the East, if the earth glides toward the West — because they could not keep pace with the earth's extremely rapid motion, given that it is completed within the space of 24 hours. Nor can it be said that the air is carried around at the same speed as the earth, since it is well known that the air surges now this way, now that, according as it is driven toward one quarter or another by the various winds, as daily experience teaches us.

Moreover, if the earth were spun about the axis of the world so fast as to complete a full circuit within the space of 24 hours — as certain people fancy — then all buildings would collapse and could by no means stand for long. That this is utterly false, no one fails to see. Nor is the reply of certain persons valid, who say that buildings do not collapse precisely because of the excessive speed of the motion — just as water held in a vessel does not spill out if the vessel is whirled around very fast. This reply, I say, is not valid, because the whole impetus of the water is directed toward the lower parts of the vessel, not toward its mouth; whereas the impetus imparted to buildings is directed toward the outermost parts of the earth, so that they could not possibly keep standing — just as water placed in a vessel would not stay put, however fast the vessel is whirled, if its mouth faced outward. By the same reasoning it would follow that a stone, or an arrow shot straight upward with great force, would not fall back to the same spot — as we observe happening aboard a ship moving very fast. All of which is absurd.

Again, if the earth were set in circular motion, that motion would be either natural or against nature. It cannot be natural. For since to a single simple body only one motion belongs naturally, and the earth by its own nature proceeds rectilinearly toward the center of the world whenever it finds itself outside it, it cannot be moved in a circle according to its own nature. Nor would it be spun around in a circle against nature — that is, by the motion of the heaven — because in that case the same part of the heaven would always stand over our zenith, and so the stars would neither rise nor set, which is absurd.

But suppose it is said that the earth moves about some other axis, one that cuts the axis of the world obliquely. Then, quite apart from our sliding back into nearly the same difficulties, it would follow that in one and the same city the altitude of the pole would vary from day to day, since that city, in following the earth's motion, would not trace a circle parallel to the pole. It would therefore now draw nearer to the pole, now be carried farther from it, and so would vary the pole's altitude — which is false. For we see, for example, that at Rome the Arctic pole permanently keeps the same elevation above the Horizon.

Let us conclude, then, with the common opinion of the astronomers and philosophers, that the earth is devoid of all local motion, whether rectilinear or circular, and that the heavens themselves are continually carried around it — especially because, once this is granted, all the phenomena are far more easily accounted for, and no difficulty follows from it.

The sacred scriptures also favor this view, affirming in very many places that the earth is immobile and testifying that the Sun and the other stars move. For we read in Psalm 103 [Vulg.; = Ps. 104]: "You who founded the earth upon its own firmness — it shall not be shaken for ever and ever." Likewise in Ecclesiastes, chapter 1: "The earth stands forever; the Sun rises and sets, and returns to its place, and there, being born anew, wheels through the South and bends toward the North." What could be said more plainly? A most clear testimony that the Sun moves is likewise furnished us by Psalm 18 [Vulg.; = Ps. 19], where we read: "In the sun he has set his tabernacle; and he himself, like a bridegroom coming forth from his bridal chamber, has rejoiced like a Giant to run his course — his going forth is from the highest heaven, and his circuit reaches even to its height, nor is there any who can hide himself from his heat." Again, it is recorded among the miracles that God once either drew the Sun back, or made it stand entirely still.

[Margin: The various opinions as to why the earth is immobile, and their refutation.]

Different thinkers have assigned different causes for this immobility of the earth at the center of the world. Some — among whom, on Aristotle's testimony in the second book of *On the Heavens* [*De Caelo* II], is Xenophon of Colophon [recte: Xenophanes of Colophon] — said that the earth is infinitely deep on its underside, and that for this reason it does not fall downward. But this opinion is false. First, because on this view the earth would not be round and spherical, the contrary of which we demonstrated above. Next, because, according to Aristotle in the third book of the *Physics* and the first of *On the Heavens*, and to other philosophers, no actual infinite exists. Third, because on this view the heaven could in no way

revolve, since it would be obstructed by that infinite depth of the earth; nor indeed does the heaven stand at an infinite distance from us, which is absurd.

Others thought — as did Thales of Miletus — that the earth floats upon the waters and is supported by them, so as not to fall. But this is ridiculous. For since water is much lighter than earth, how can it possibly support a heavier body — especially since we see, everywhere, parts of the earth sinking beneath the water?

Furthermore, such philosophers ought to be asked what the water itself rests upon, so as not to fall together with the earth; for water, being fluid, cannot stay in place unless it is supported upon some solid body.

Certain others affirmed — as did Anaxagoras and Democritus — that the earth has a very ample and broad shape, and that it therefore compresses the air, by which it is held up so as not to fall. But this too is a fabrication, and very like an old wives' tale. For the earth has a spherical shape, not a flat one, as we demonstrated above. Indeed, even if it did have such a form, it is still contrary to experience that broad bodies should be so held up in the air that they never fall at last. For although bodies of this kind descend only with difficulty on account of their breadth — since they can scarcely cut through the air — they are nonetheless seen to tend gradually downward.

Some, finally — as Anaximander of Miletus — coming nearer to the truth, testified that the earth rests at the middle for this reason: because it is placed at the middle of the world. For from this it comes about, they say, that the earth is either inclined to move toward every quarter of the heaven alike — since there is no greater reason why it should move toward one quarter rather than another — or else is attracted equally from all quarters of the heaven. And so, since it cannot be carried to all quarters at the same instant, it rests at the middle, or center, of the world. But this opinion too is mistaken. First, because if the earth were unmoved for this cause, it would be held at the middle of the Universe by force and not naturally. Next, because it is false that it is always inclined to move toward the quarters of the heaven, since on that view it would tend upward, which is repugnant to its nature. For we see the parts of the earth naturally descend with the greatest force, unless obstructed, and always withdraw from the heaven toward the center as far as they possibly can. By the same reasoning it is false that the earth is attracted by the heaven, since we see rather that the earth is drawn away from the heaven by its own nature. Third, because if the earth rested immobile at the center for this cause, then by the same reasoning it would be established that fire or air, placed at the center of the world, ought likewise to rest there — for there would be no greater reason for it to move toward one

quarter than another, since it would have an equal inclination toward all quarters of the heaven. Yet no philosopher has ever granted this.

[Margin: Why the earth rests at the middle.]

It must be said, therefore, that there is no other cause for the earth's resting at the middle of the world than its own heaviness. For from this it comes about that the earth always seeks to be in the lowest place — the place farthest from the heaven, namely the center of the whole Universe. Once it has taken possession of that place, it cannot naturally be torn away from it, because it would then have to rise against its own nature and inclination. For the same cause, all heavy bodies are naturally borne toward the center of the world with the greatest force, unless something obstructs them — so that if the whole earth were bored through from one side to the other, and some heavy body fell into that hole, it would reach only the center with the greatest force, and not the other side, because there it would begin to rise. At the outset, on account of the impetus of its motion, it would swing back and forth for a while; but as the impetus of the motion gradually slackened, it would come to rest at the middle. Of this immobility of the earth Manilius too writes elegantly, thus:

Nor need the nature of the hanging earth seem wondrous to you, since the World itself hangs and sets its footprints on no floor beneath. This is plain from its very motion, from its flying course: since suspended Phoebus [the Sun] goes his way and turns his course now here, now there, and keeps his nimble turning-posts within the aether; since the Moon and Stars flit through the empty reaches of the world — the earth too, imitating these airy laws, hangs suspended. It is the earth, then, that has been allotted the middle hollow of the air, and is lifted up equally from the whole depth, not stretched flat over broad expanses, but built into a globe that rises equally on every side and equally falls away. This is the face of Nature...

[Margin: Why the antipodes do not fall.]

From what we have said, that common argument of Lactantius Firmianus [Lactantius] and of the crowd against the antipodes is easily resolved. They say that if there were antipodes — that is, men set opposite to us — they could not keep their footing there, but would fall. It can be resolved, I say, because the antipodes, by their own heaviness, always tend toward the center of the world, just as we do. Indeed, if they could not keep their footing, they would fall *into the heaven* — that is, into the higher place — which is against the nature and

inclination of heavy things. It is no wonder, then, that they do not fall; rather, it would be a great wonder if they did fall into the heaven.

DE AMBITV TERRÆ

[*Lemma — Sacrobosco*] On the authority of the philosophers Ambrose, Theodosius, Macrobius, and Eratosthenes, the circumference of the whole globe of the earth is determined to contain 252,000 stadia, with 700 stadia to be assigned to each of the 360 parts of the Zodiac.

[Margin: The circumference of the earth according to Macrobius and Eratosthenes.]

COMMENTARIVS

This is the sixth and last conclusion: namely, that the earth has, in its circumference, a fixed and determinate magnitude, and is not of infinite depth, as some falsely supposed. He confirms it in the following way. On the opinion of Ambrose, Theodosius, Macrobius (for these three names do not denote three Authors, as some would have it, but signify only one) in the commentaries he published on the *Dream of Scipio*, book 1, and on the opinion of Eratosthenes, the whole circumference of the earth contains 252,000 stadia — for the reason that 700 stadia correspond to one degree of the earth out of the 360. The magnitude of the earth, therefore, is known and determinate.

[Margin: The circumference of the earth must be taken with respect to a great circle. 25. tertij [i.e. a cross-reference, prop. 25 of book 3 (?)].]

This circumference of the terrestrial globe must be taken not with respect to just any circle drawn on the earth, but according to a great circle of the earth — one, that is, which has the same center as the earth, such as the Meridian circle, the Equinoctial [the Equator], the Horizon, or any other great circle drawn on the earth's surface. In the same way, the thickness or depth of the earth, or of any spherical body, must be determined with respect to its diameter — which is the greatest line in a circle or sphere, since it passes through the center — and not by other lines, which can be varied in a thousand ways.

[*Lemma — Sacrobosco*] Take an Astrolabe or a Quadrant; and by the brightness of a star at night, having sighted the pole through both holes of the alidade [mediclinium], note the number of degrees at which the alidade comes to rest. Then let the surveyor [Cosmimeter] proceed due North from the South, until, by the brightness of another night's star, and the pole having been sighted as before,

the alidade comes to rest one degree higher. After this, measure the distance of this journey, and 700 stadia will be found. Then, assigning that many stadia to each of the 360 degrees, the circumference of the terrestrial globe will be found.

COMMENTARIVS

The Author had assumed as settled and certain that 700 stadia correspond to one degree of the terrestrial globe, and hence that all 360 degrees — that is, the whole circumference of the earth — amount to 252,000 stadia. Someone might deny this; indeed the crowd, and many others too who wish to seem learned, think it impossible for the earth's circumference to be measured, on the grounds that, because of the many obstacles — inaccessible crags, valleys, rivers, lakes, the Ocean, the Mediterranean Sea, and so on — the whole cannot be traveled around. He therefore prescribes the method the Astronomers used, and which anyone, if he likes, will be able to use in measuring the earth's circumference. For it will be enough if one measures accurately and carefully the length of journey that corresponds to one degree of the earth's globe, rather than the whole circuit. Since the earth is spherical, as has been demonstrated, then from any known part of the circumference that bears a known proportion to the whole, we will easily arrive, by the rule of proportions, at knowledge of the earth's entire circumference. The method he hands down is clear in the text and quite easy for anyone even moderately versed in Mathematical instruments, especially the astrolabe and Quadrant. Only this must be understood about it: that the pole can in no way be sighted through the Astrolabe or the Quadrant. For the pole star, which we observe near the pole, is not the true pole, but traces a circle around the true pole at a distance of nearly $3\frac{1}{4}$ degrees from it. The altitude of the pole must therefore be sought by another method — but how this ought to be done is not the place here, for it belongs to the treatment of the use of the Astrolabe or Quadrant, on which subject we will nonetheless say something when we come to discuss the Meridian circle.

[Margin: It is enough if the interval of one degree on the earth is investigated, so that the whole circumference may be obtained.]

Nor indeed is it necessary to walk out or measure a whole degree in order to have the earth's entire circumference; it will suffice to measure the length of half a degree, or of a third of one degree, or finally of any portion at all whose proportion to a whole degree is known. For from such a known portion, by the benefit of the rule of proportions, we will easily derive the whole circumference. For example, since 175 stadia are found to correspond to a quarter of one degree, 1,440 such quarter-parts — that is, the whole circumference of the earth — will

contain 252,000 stadia, as before. By the same reasoning, if 350 stadia correspond to half a degree, they will correspond, for the whole circumference (which consists of 720 half-parts), again to 252,000 stadia. And so for the rest.

[Margin: It is enough if the space of some half degree on the earth, or a third of one degree, is measured, so that the whole circumference may be known.]

VIAE AD INVESTIGANDVM AMBITVM TERRÆ COMMODIORES, quam ea, quæ ab Auctore tradita est

[Lemma heading — Ways of investigating the circumference of the earth more convenient than the one handed down by the Author]

But since it is a laborious and difficult task to proceed straight along the Meridian circle toward the North or the South until an altitude of the pole greater by one degree is found, the same measure of the terrestrial circumference will perhaps be obtained more conveniently by the following method. Mark two cities situated under the same Meridian; and once the elevations of the pole for each have been carefully ascertained, subtract the smaller elevation — the one the more southerly city has — from the greater, which the more northerly city has. What remains will show the distance between the two cities in terms of degrees; and once this has been measured in stadia or some other unit, you will easily be led, by the rule of proportions, to knowledge of the terrestrial circumference.

[Margin: Various ways by which the circumference of the earth may be explored.]

EXAMPLE. Mark two cities under one and the same Meridian, of which the more southerly has, say, a pole-altitude of 10 degrees, while the pole-altitude of the more northerly is 12 degrees 30 minutes. If, then, the smaller altitude is subtracted from the greater, the distance between the two cities will be 2 degrees 30 minutes. On the Author's opinion, if Eratosthenes and Macrobius had measured this distance, it would contain 1,750 stadia. Accordingly, the 360 degrees of the whole circumference will comprise 252,000 stadia. By the same reasoning, if the length of the journey between any two cities whatever — even if they do not lie under the same Meridian — is known, the magnitude of the whole terrestrial circumference can be found through the doctrine of spherical triangles, provided that the pole-altitude and the longitude (which is reckoned from the West) of each city have been ascertained. For from the pole-altitude and the longitude of each place, the number of degrees of the great circle measuring the distance traveled will be known. Therefore it will not be unknown how many stadia or miles are to be assigned to one degree — and from this the whole

circumference will be worked out. But because this way of measuring the earth's circumference is more obscure and belongs to Cosmography, we deliberately pass it over.

Now suppose someone wishes to explore how great the earth's circumference is from East to West, or vice versa. The Author taught only how to observe it from one pole to the other — that is, from North to South, or vice versa — although from this too the earth's circumference from East or West is established, since, because of the earth's roundness, it is exactly equal to the circumference from pole to pole. He will be able to accomplish this by the following technique. Mark two cities situated under the Equinoctial circle, and observe carefully, in each city, the hour at which some eclipse of the Moon began. For once it is known by how many hours earlier the lunar eclipse began in one city than in the other, the degrees of the Equator lying between them will also be known; for to each hour there correspond 15 degrees of the Equinoctial circle, as has been said elsewhere. The extent of those degrees having thus been measured, easily — by the rule of proportions — into knowledge of the whole circumference...

we shall arrive at the rule.

Example. Suppose the beginning of one and the same lunar eclipse is observed in an eastern city at the tenth hour plus a third of an hour after noon, but in a more westerly city at the ninth hour after noon. Therefore the more easterly city had its noon a full hour and a third of an hour earlier than the more westerly one. The interval lying between the two accordingly contains 20 degrees; and if anyone were to measure it, he would find, according to the authors already cited, that it contains 14,000 stadia, and so would conclude that the whole circumference of the earth contains 252,000 stadia.

[Margin: Posidonius's very easy method for investigating the earth's circumference.]

We can also investigate the earth's circumference most easily by means of some fixed star, such as Spica in ♍ [Virgo], or any other. For if two places lying under the same meridian are taken on the earth, whose road-distance apart is known by exploration, and if at each place the meridian altitude of the chosen and known star is observed, then the difference of the altitudes will be the number of degrees of the meridian lying between the two places. Since it is therefore known how many stadia correspond to those degrees, the number of stadia owed to the whole circumference of the terrestrial globe can in no way remain unknown. This method of investigating the earth's circumference is the one I most approve;

for it does not require, at any place, a knowledge of the pole's altitude or of the longitude — which cannot be had except by long-continued observation — nor in this matter should too much trust be placed in tables that record the latitudes and longitudes of cities. That Posidonius [Posidonius] used this very method of measuring the earth is reported by Franciscus Maurolycus [Francesco Maurolico] in his *Cosmographical Dialogues* [*Dialogi Cosmographici*], from which we have also taken the two following methods, of which the first was familiar to Eratosthenes, while the other was devised by Maurolycus himself.

[Margin: Eratosthenes's method for investigating the earth's circumference.]

Eratosthenes proceeded by roughly this method in tracing out the earth's circumference. At Alexandria he set up a gnomon on the horizon at right angles. Then, while the Sun stood at the beginning of ♋ [Cancer], he perceived that two rays were cast at the time of midday: one passing through the city of Syene [Syene, i.e. Aswan] (which is more southerly than Alexandria, and on very nearly the same meridian as Alexandria), a ray that ran straight toward the center of the world, since Syene lies directly under the Tropic of ♋ [Cancer]; the other passing over the vertex of the said gnomon. And thus, from the proportion of the gnomon to its shadow, he found by a geometrical route the space between Alexandria and Syene.

To make this clearer: let there be a circle described on the earth through Alexandria and Syene, in which A is the site of Alexandria, B the site of Syene; let AD be the style erected at Alexandria; let FBC be the ray of the Sun passing through Syene toward the center of the world; let EDG be the ray passing over the vertex of the gnomon and casting the shadow AG toward the north; and let the gnomon AD be understood to be produced as far as the center C.

Diagram. A woodcut of Eratosthenes's construction. At the top is the Sun, inscribed **SOL**, emitting rays downward. Below, near the small hatched disc of the earth, are the gnomon and its shadow; two long rays descend from the Sun. Readable labels:

- SOL (top): the Sun.
- E (upper left): origin, on the solar side, of the ray EDG that grazes the gnomon's vertex.
- F (upper right): origin, on the solar side, of the ray FBC passing through Syene.
- D (below E): the vertex of the gnomon (style).
- G (lower left, on the earth's surface): the tip of the shadow AG.

- A (surface, center): Alexandria, the foot of the gnomon.
- B (surface, right of A): Syene.
- C (at the center of the earth-disc): the center of the world.

Since, then, in the triangle ADG the arc AG can, without error, be taken for the straight line — if it be compared with the whole circumference — and since angle A is a right angle and the two sides AD, AG are known (AD indeed by [measurement], and AG by some measure, or certainly from those things which have been demonstrated by us in book 5 of our *Gnomonics*, proposition 1, where we showed by what reasoning the proportion of the style to its erect shadow may be known from the known altitude of the Sun): the angle ADG will also be known, by the doctrine of triangles (as we demonstrated in our work on triangles). For since the sides AD, AG are known, [Margin (a): *Elements* I.47] their squares will also be known; and since these are equal to the square on DG, the square on the straight line DG will likewise be known, and so DG itself will be known. And because, if DG be set as the whole sine, the straight line AG is the sine of angle ADG (as we demonstrated in the treatment of sines): if it be arranged that, as DG (in the terms in which it is thus far known) is to the whole sine, so AG (in the terms in which it is known, in parts of the shadow) is to something else, then AG will become known in the terms in which it is the sine of angle ADG; and hence, from the table of sines, the angle ADG will be known; and consequently also the alternate angle ACB, [Margin (b): *Elements* I.29] which is equal to it, because the rays FBC, EDG are very nearly parallel, on account of the exceedingly small distance of Syene from Alexandria when it is compared with the Sun. Therefore the arc AB, subtended by angle C, will also be known — namely the space intercepted between Alexandria and Syene. This method of Eratosthenes is reported a little differently by Cleomedes [Cleomedes] than by Maurolycus. By this method Eratosthenes found (if the author who reports the earth's circumference according to the opinion of Eratosthenes has reported truly) the arc AB to be $8\frac{1}{4}$ [?] degrees, and the road-distance to comprise $6183\frac{1}{3}$ stadia. Whence by the rule of proportions he concluded that 252,000 stadia are owed to the whole circumference of the earth, over its 360 degrees.

Franciscus Maurolycus, Abbot, devised this method of tracing out the earth's circumference. Let BCD be a circle of the earth, on which let some very high mountain be chosen (he himself judged Mount Etna [Mount Etna] in Sicily suitable for this business), whose height AB is made known by the precepts for measuring heights. Then, from A, the summit of the mountain, by the precepts for measuring distances, one must measure that whole expanse of sea or land (provided there be no mountains in the way) that is thence visible, such that the

visual ray AC touches the earth's surface at the point C. Let the space seen be BC, which, even though it be curved and not flat, nevertheless differs from a flat line by no perceptible difference, because the arc BC is exceedingly small when compared with the whole circumference of the earth. These things being duly performed, we shall set up the geometrical reasoning thus. I conceive four straight lines: the first is AB, the height of the mountain assumed; the second, the visual ray AC; the third, AD, which is composed of the mountain's height and the earth's diameter; the fourth, finally, BC, the interval seen — for it can, without error, be taken for a straight line, as has been said. Since, then, the straight lines AB, BC are known, their squares will also be known; [Margin (c): *Elements* I.47] and since these are equal to the square on AC, the square on AC will also be known. But the square on AC (since AC touches the circle) [Margin (d): *Elements* III.36 (printed citation number unclear)] is equal to the rectangle contained by DA, AB. Therefore the rectangle under DA, AB will be known; and AB, the mountain's height, is known. Hence the line AD will also be known, if indeed the known rectangle contained by AB, AD is divided by the line AB. For the quotient number will give the line AD; and if from it AB, the mountain's height, be subtracted, the earth's diameter BD will be left...

Diagram. A woodcut of Maurolycus's mountain construction: a circle (the earth) with a diameter drawn vertically, the mountain's summit above it, and the visual ray drawn as a dashed line to the point of tangency. Readable labels:

- A (top, above the circle): the summit of the mountain.
- E (just below A, on the visual ray): point on the line of sight.
- C (upper left, on the circle): the point where the visual ray AC touches the surface.
- B (upper middle, on the circle): the foot of the mountain.
- F (center of the circle): the center of the earth.
- D (bottom of the circle): the far end of the diameter.

the earth's diameter BD will be left. Therefore from the diameter BD — following what Archimedes [Archimedes] demonstrated in his little book *On the Measurement of the Circle*, as is established from the corollary of proposition 2, book 4 of the *Practical Geometry* [*Geometria Practica*] — the whole circumference of the earth will be known. Which was the thing proposed.

But since the arc BC is not a straight line — especially when the mountain is so high that a space of 200 or 300 miles can be seen (in which case the arc BC, according to the circumference fixed by Ptolemy [Ptolemy], would contain 3

degrees 11 minutes, or 4 degrees 40 minutes), and therefore the tangent line AC is not correctly gathered from the sides AB, BC — and add to this that, by the problems of books 2 and 3 of our *Practical Geometry*, only the perpendicular BE is found, in the plane to which the mountain stands perpendicular: in the *Practical Geometry* we have reduced this method of Maurolycus to a better procedure by several routes, one of which is this.

[Margin: Maurolycus's method is corrected.]

Once the angle A has been found, by means of a quadrant or a geometric square — when the visual ray, passing through the sight [dioptra], touches the circle of the earth or of the sea (which is done most reliably when the Sun or some other star is observed through the sight as it rises or sets) — and once the line FC has been drawn, [Margin (a): *Elements* III.18] which will be perpendicular to AC: the angle F at the center, which is the complement of angle A, will also be known. And since, once the line FE has been drawn, the two sides EC, CF are equal to the two sides EB, BF and contain equal right angles, [Margin (b): *Elements* I.4] the angles at F will be equal. So, since the whole angle BFC is known, as we said just now, the angle BFE, being its half, will also be known, and hence its complement BEF will not be unknown. Therefore in triangle ABE, from the angles A, E and the side AB, BE will be found (by proposition 4 on right-angled triangles) in parts of the mountain's known height AB. And in the same way, in triangle BEF, from the angles E, F and the known side BE, the radius BF will be found in parts of the side BE — that is, in parts of the mountain's height AB. Hence the whole diameter BD will become known; and from it the earth's circumference will be found, from those things which Archimedes and we ourselves demonstrated in the corollary of proposition 2, book 4 of the *Practical Geometry*.

Now all the aforesaid methods of investigating the earth's circumference — except the last, which we demonstrated just now from Maurolycus — rest upon this geometrical conclusion:

If there be two or more circles described about the same center, and if from the center two straight lines be drawn to the circumference of the greatest circle, then the arcs of all the circles comprehended between the said straight lines will be similar to one another.

[Margin: Straight lines going out from one point cut all the circles described from that point as center into similar arcs.]

COMMENTARIVS.

This conclusion, since it is of the greatest service to astronomers and to geometers, we shall try to demonstrate briefly here. Let there be, about the center E, the circles described ACBD, FHGI, KMLN, and from the center E let the straight lines EC, ED be drawn. If these make one straight line, it is certain that all the circles are cut by them into similar arcs, namely into semicircles. Again, let two straight lines EA, ED be drawn from the same center E, making a right angle AED. It is then evident that the arcs AD, FI, KN are similar, since they are quadrants of the circles. For if the straight lines AE, DE be produced as far as B, C, there will be four right angles at E, [Margin (c): *Elements* III.26] and therefore the arcs AD, DB, BC, CA will be equal. In the same way the arcs FI, IG, GH, HF will be equal; likewise the arcs KN, NL, LM, MK. Hence each of them will be a quadrant of its own circle. Finally, let the straight lines ED, EO be drawn, making the angle DEO, not a right angle. I say that even now the arcs DO, IP, NQ are similar — that is, that DO is such a part of the quadrant DA as the arc IP is of the quadrant IF, and as the arc NQ is of the quadrant NK. [Margin (d): *Elements* VI.33] For since, as the angle DEO is to the angle DEA, so is the arc DO to the arc DA, and the arc IP to the arc IF, and the arc NQ to the arc NK, it is manifest that the aforesaid arcs are similar to one another, since they have the same proportion.

[Margin: Another demonstration.]

This can also be gathered by the following reasoning. Just as the [equal angles at the center subtend equal] circumferences, so (by the 2nd corollary of the last proposition of book [six]) the arc DO is to the whole circumference DACB, and the arc IP to the circumference [IFHG], and the arc NQ to the circumference NKML. Therefore the arcs DO, IP, NQ are similar, since they have the same proportion.

Diagram. A woodcut of three concentric circles about a common center E, with a horizontal diameter, a vertical diameter, and an oblique radius drawn to the upper right. The outer circle is lettered ACBD, the middle FHGI, and the inner KMLN. Readable labels:

- E (center).
- A (top, outer circle); B (bottom, outer circle); C (left, outer circle); D (right, outer circle).
- F (top, middle circle); G (bottom, middle circle).
- K (top, inner circle); L (bottom, inner circle); M (left, inner circle).
- O (upper right, on the outer circle, at the end of the oblique radius EO); P (upper right, where the oblique radius crosses the middle circle).

The remaining letters of the middle circle (H, I), of the inner circle (N), and the point Q on the inner circle where the oblique radius crosses it fall in the crowded central region and could not be placed with certainty. (more than two labels unresolved).

Another demonstration, without proportions, can prove the same theorem in this way. From the center E of the circles ABCD, FGHI, let two straight lines EA, EB be drawn. I say that the arcs AB, FG are similar to one another. For, the straight lines AE, BE being produced as far as C, D, let the straight lines BC, GH be drawn; let also, in the arcs AB, FG, the points K, L be taken at random, to which let the straight lines AK, BK, FL, GL be drawn. Since, then, the angles E, G, H of triangle EGH are equal to the angles E, B, C of triangle EBC [Margin (e): *Elements* I.32] (because both these and those are equal to two right angles): if the common angle E be taken away, the two angles G, H will be equal to the two angles B, C. [Margin (f): *Elements* I.5] But both these two and those two are equal to one another, because the straight lines EG, EH are equal to one another, as are the straight lines EB, EC, by the definition of the circle. Therefore the angle EHG will be equal to the angle ECB. Again, [Margin (g): *Elements* III.22] since in the quadrilateral FLGH the two opposite angles FHG, GLF are equal to two right angles; likewise the two opposite angles ACB, BKA in the quadrilateral AKBC: when the equals FHG, ACB are removed, the remaining angles BKA, GLF will be equal; and therefore, by the definition, the arcs AB, FG will be similar to one another. Which was to be demonstrated.

Diagram. A woodcut of two concentric circles about the center E, with radii drawn and chords forming two similar figures. Readable labels:

- A (top); C (bottom); B (left); D (right) — endpoints of the two diameters through E.
- E (center).
- F (inner circle, near A); G (inner circle, near the left, toward B); H (inner circle, lower right); I (inner circle, right of E).
- K (outer circle, upper left); L (inner circle, upper left).

This theorem having been demonstrated, all the aforesaid methods hold good. For it will come about that, when in the heaven a variation of one degree (or of several) has occurred, on the earth a variation of just as many degrees will likewise have occurred. For if, from the extremities of those celestial degrees, two straight lines be conceived as drawn to the center of the world, they will necessarily intercept just as many degrees also on the surface of the earth, by what has just been demonstrated, as is clear in this figure here adjoined. And the

same reasoning holds for any celestial space whatever: for the said lines will always comprehend a similar space on the earth. This, indeed, was assumed as most certain in all the aforesaid methods; otherwise nothing at all could have been concluded by them, as is plain.

Diagram. A woodcut showing an outer circle (the heaven) enclosing a smaller hatched disc at its center (the earth). Two straight lines are drawn from two points on the outer circle, converging to a point at the center, forming a narrow triangular wedge — illustrating that equal celestial arcs correspond to equal terrestrial arcs. No letter labels are present.

From these things, moreover, by the rule of the circle and the diameter, the diameter of the earth can be found thus. Take away the twenty-second part from the circumference of the earth, and the third part of the remainder — that is, 80,181 stadia, and a half, and a third of a stadium — will be the diameter, or thickness, of the terrestrial globe.

COMMENTARIVS.

After the Author has explained how great the circumference of the terrestrial globe is, and by what method it ought to be traced out, he now teaches by what art, from the known circumference of the earth, its depth — that is, the diameter of that same earth — may be known. For he says that if the twenty-second part be taken away from the whole terrestrial circumference (which part will be had in the quotient number, if the circumference is divided by 22) — namely, if from 252,000 stadia there be subtracted 11,454 $\frac{6}{11}$ stadia — then the third part of the remaining number (namely of 240,545 $\frac{5}{11}$ stadia; which part likewise the quotient number will offer, if the said remaining number be divided by 3), that is, 80,181 $\frac{9}{11}$ [?] stadia — or, as he himself says, 80,181, and a half, and roughly a third part — will be the whole depth, or diameter, of the terrestrial globe, by the rule of the circle and the diameter.

[Margin: How the earth's diameter is derived from its known circumference.]

Now this rule is taken from the little book of Archimedes *On the Measurement of the Circle*, in which Archimedes demonstrated (as we too have done in the *Practical Geometry*, book 4, proposition 2) that the proportion of the circumference of any circle to its diameter is roughly triple-and-one-seventh, such as is 22 to 7. So that, if the circumference of some circle be cut into 22 equal parts, its diameter will contain roughly 7 such parts; and conversely, if the diameter of some circle be divided into seven equal parts, its circumference will comprehend 22 such parts. Whence, if the diameter of some circle be taken three

times and a seventh part added, a straight line will be produced roughly equal to the circle's circumference. All of which may be seen in this figure here set forth.

[Margin: What the proportion of any circle to its diameter is.]

Diagram. A woodcut of a circle whose horizontal diameter is extended to the left as a straight line marked with tick divisions — representing the circumference straightened out and shown to be three-and-one-seventh times the diameter (the 22-to-7 ratio). No letter labels are present.

Since these things are so, it is evident that if from the circumference of the circle — namely from 22 — the twenty-second part (that is, unity) be taken away, then the third part of the remaining number, that is of 21 — namely 7 — is the diameter of the circle. From which the Author's rule is manifest, which prescribes that from the terrestrial circumference the diameter, or depth, of the earth be explored.

*RULE, BY WHICH THE DIAMETER IS FOUND FROM THE
CIRCUMFERENCE, AND THE CIRCUMFERENCE FROM THE DIAMETER.*

From this same proportion of the circumference of a circle to its diameter — which, namely, they have as 22 to 7 — other writers elicit this rule, much more convenient than the rule of our author, for inquiring the diameter from the known circumference, or conversely for finding the circumference from the known diameter, as we gathered in the corollary of proposition 2, book 4 of the *Practical Geometry*. The first part of the rule, by which the diameter is derived from the known circumference, is this.

Let the circumference be divided by $3 \frac{1}{7}$ — namely by the denominator of the triple-and-one-seventh proportion which, as we have said, the circumference has to the diameter according to Archimedes: the number coming out in such a division will be the diameter of the circle. Thus, if the circumference of some circle, comprising 1540 palms, be divided by $3 \frac{1}{7}$, there will result 490 palms for the magnitude of the diameter. This rule can also be stated thus: let the circumference be multiplied by 7, and let the product be divided by 22, and the diameter will be found. For since the proportion of the circumference of any circle to its diameter is, as Archimedes demonstrated, [22 to 7]: it comes about that if the number of the rule of proportions be multiplied by 7 — namely by the second number of that same rule — and the product be divided by the first number, that is by 22, then the diameter is found as the fourth number of the rule of proportions. As in the preceding example: if the circumference 1540 be multiplied by 7 and the product divided by 22, the diameter 490 will be found, as

before. By this reasoning, if we multiply the earth's circumference according to Eratosthenes — namely 252,000 stadia — by 7, there will be produced 1,764,000; which divided by 22 yields 80,181, that is 80,181 $\frac{18}{22}$, for the diameter of the earth, just as before according to the Author's rule. The latter part of the rule, by which in turn the circumference is elicited from the known diameter, stands thus.

[Margin: How the circle's diameter is elicited from its known circumference.]

Let the diameter be multiplied by $3\frac{1}{7}$ — namely by the denominator of the triple-and-one-seventh proportion which, according to Archimedes, the circumference has to the diameter. For the product number will at once indicate the circumference. Thus, if the diameter of some circle, having 490 palms, be multiplied by $3\frac{1}{7}$, a circumference of 1540 palms will be found. This rule too can be stated in this way: let the diameter be multiplied by 22, and let the product number be divided by 7, and the quantity of the circumference will result. For since, as was demonstrated by Archimedes, the proportion which is 22 to 7 is the proportion of the circumference of any circle to its diameter...

[Margin: How the circle's circumference is found from its known diameter.]

By conversion, there will be the same ratio, 7 to 22, as of the diameter to the circumference. Therefore, if the diameter — that is, the third number of the rule of proportions — is multiplied by 22 (namely by the second number of the same rule), and the resulting number is divided by the first number, that is, by 7, the fourth number of the same rule is found: the circumference of the circle. As in the next example: if the diameter 490 is multiplied by 22, and the resulting number is divided by 7, the circumference 1540 is found, as before. This twofold precept is contained in the following verses:

Circuitus circi per septem multiplicetur, Per duo viginti productum deinde secato: Hinc numerus, Quotiens qui dicitur, est diametrus. Per duo viginti si multiplices diametrum, Per septemque seces numerum, qui prodijt inde: Circuitum circi Quotiens numerus tibi reddet.

Multiply the circumference of the circle by seven, then divide the product by twenty-two: the number that results, called the quotient, is the diameter. If you multiply the diameter by twenty-two, and divide by seven the number that comes out, the quotient will give you back the circumference of the circle.

From this it is easy to understand the method that Francesco Maurolico [Francesco Maurolico] used in investigating the circumference of the terrestrial

globe. For he first learned by a geometrical procedure the size of the earth's diameter, and from this he then, according to this ratio of diameter to circumference demonstrated by Archimedes [Archimedes], hunted out the circumference of the greatest circle described through the earth's center.

[Margin: From the foregoing rules the circumference is found from a known diameter greater than it truly is, and the diameter from a known circumference smaller than it truly is.]

The circumference of any circle does not, however, stand to its diameter precisely in the ratio that 22 has to 7, but in a slightly smaller one. For as Archimedes most acutely demonstrated in his booklet *On the Measurement of the Circle* [*De dimensione circuli*], the circumference of any circle has to its diameter a ratio smaller than triple-and-one-seventh [tripla sesquiseptima, i.e. $3 \frac{1}{7}$] — or (what is the same) triple-and-ten-seventieths [$3 \frac{10}{70}$] — but greater than triple-and-ten-seventy-firsts [$3 \frac{10}{71}$]. Thus, if the diameter is taken three times together with a seventh part of it, that is, with $3 \frac{1}{7}$, a line slightly greater than the circumference is produced; whereas if the diameter is taken with $3 \frac{10}{71}$, a line slightly smaller than the circumference is produced. So the true ratio of circumference to diameter lies (though it remains hidden) between the two whose denominators are $3 \frac{1}{7}$ and $3 \frac{10}{71}$.

Common practice among practitioners has nonetheless prevailed, so that the former ratio — namely triple-and-one-seventh — is used as though it were the true one, rather than the ratio whose denominator is $3 \frac{10}{71}$. For they take the diameter three times with a seventh part of it, in order to yield a straight line equal to the circumference, since it falls only a little short of the truth, and since the calculation is easier with $3 \frac{1}{7}$ than with $3 \frac{10}{71}$. For that reason we too may use the same ratio, provided we remember — from the demonstrations above — that from a known diameter the circumference is found slightly larger, and from a known circumference the diameter slightly smaller, than it truly is. For since, according to Archimedes, the ratio of circumference to diameter is smaller than triple-and-one-seventh — that is, than 22 to 7 — if the diameter is 7, the circumference is slightly less than 22. [Margin a: (Euclid, Elements) V.8] For a number smaller than 22 has a smaller ratio to 7 than 22 has to 7. Hence, since by the rule above, if the diameter is 7 the circumference is found to be 22, it is plain that the circumference is found from a known diameter larger than it really is. Again it follows that, if the circumference is 22, the diameter is slightly larger than 7. [Margin b: (Euclid, Elements) V.8] For 22 has a smaller ratio to a number larger than 7 than it has to 7. Therefore, since by the rule above, if the

circumference is 22 the diameter is found to be 7, it is clear that the diameter is found from a known circumference smaller than it really is.

All these rules, and many more bearing on the same subject, we have demonstrated in Book 4 of the *Practical Geometry* [*Geometria practica*].

Rules by which may be found the area of the greatest circle on the terrestrial globe (or in any sphere whatever), the convex surface of the same terrestrial globe (or of any sphere whatever), and indeed the entire solidity as well

So far, drawing on approved authorities, we have reviewed various methods by which the earth's circumference is investigated, and we have set out precepts by which, from a known circumference, the diameter is found, and conversely, from a known diameter, the circumference. Now I shall present other precepts by which, from the diameter and circumference of the earth — or of any other sphere — the area of the greatest circle on the earth (or another sphere) is to be found; and from this area, the convex surface of the same earth or sphere; and finally, from this convex surface, the entire solidity of the whole earth or another sphere. For in this way it will come about that the earth's magnitude is made known in every respect — not merely as to its circumference, which is what our Author accomplished in this place.

[Margin: By what art the area of any circle is found.]

As to the first, then: if the radius [semidiameter] of any circle is multiplied by half of the circumference (or perimeter) of the circle, the area (or surface) of the circle enclosed within the circumference is produced. For example, if the circumference of some circle is 132 and the diameter 42: if we multiply 21, half the diameter, by 66, half the circumference, this number, 1386, is produced for the area of the circle. This we demonstrated in Book 7 of the *Practical Geometry*, in the treatment of isoperimetric figures, proposition 4, in which it is established that the rectangle contained by the radius of any circle and half of its circumference is equal to the circle. Thus, if the radius of the earth — namely 40090 $\frac{7}{22}$ stadia according to Eratosthenes [Eratosthenes] — is multiplied by half the circumference — that is, according to Eratosthenes, by 126000 stadia — the area of the greatest circle on the earth is produced: 5,051,454,545 $\frac{5}{11}$ stadia. That is, the plane surface of the greatest circle on the earth will comprise as many squares, each of which encloses one stadium on each of its sides, as there are units in the said number. For the areas of plane figures are measured by the

squares of the lines by which the sides, or perimeters, of those figures are usually measured.

By another method the area of a circle will be found from its circumference, even when the diameter is not known, in this way. Let the whole circumference be multiplied by itself, and the resulting number divided by $12\frac{4}{7}$. The quotient will give the area. For example, if the circumference of a circle is 44 palms, let 44 be multiplied by itself, and the resulting number, 1936, divided by $12\frac{4}{7}$; the quotient, 154, will be the number of square palms that the surface of the given circle contains, as we demonstrated in Book 4 of the *Practical Geometry*, chapter 7.

[Margin: By what route the convex surface of any sphere is found.]

As to the second: if the area of the greatest circle in a sphere is multiplied by 4, the entire convex surface of the sphere is produced. For example, given a sphere whose greatest circle has a perimeter of 132 and a diameter of 42, by the first rule the area of the greatest circle is 1386, as was said; and if this is multiplied by 4, the convex surface of that sphere at once arises: 5544. This was most clearly demonstrated by Archimedes in Book 1 of *On the Sphere and the Cylinder* [*De sphaera et cylindro*], proposition 31, in which it is concluded that the convex surface of any sphere is quadruple the greatest circle in the sphere. Thus, if the area of the greatest circle on the earth — which contains, as we said, 5,051,454,545 $\frac{5}{11}$ square stadia — is multiplied by 4, the whole convex surface of the terrestrial globe will be found: 20,205,818,181 $\frac{9}{11}$ square stadia. The same convex surface can nonetheless be found more easily, even if we do not have the area of the greatest circle, in this way.

[Margin: By what reasoning the solidity of a sphere is found.]

Let the whole diameter be multiplied by the whole circumference of the greatest circle. The resulting number will give the convex surface of the sphere. For example, if the earth's diameter — containing 80181 $\frac{9}{22}$ stadia — is multiplied by the whole perimeter, namely by 252000 stadia, the convex surface of the earth is produced: 20,205,818,181 $\frac{9}{11}$ square stadia, as before. We shall demonstrate this as follows. The rectangle contained by the diameter of the sphere and the circumference of the greatest circle is similar to the rectangle contained by the radius of the sphere and the semicircumference of the greatest circle, because the sides of the former have a double ratio to the sides of the latter, and therefore, by permutation, the sides of the former have the same ratio to one another as the sides of the latter. [Margin a: (Euclid, Elements) VI.20] Hence the former will have

to the latter the doubled ratio of the corresponding [homologous] sides. Since, then, the homologous sides have a double ratio, that rectangle will have to this one a quadruple ratio, which is the duplicate of the double ratio — as appears in these numbers: 1, 2, 4. But this rectangle, contained by the radius and the semicircumference of the greatest circle, is equal to the area of the greatest circle in the sphere, as we demonstrated in the *Practical Geometry*, proposition 4, in the treatment of isoperimetric figures. Therefore that rectangle contained by the whole diameter and the whole circumference is quadruple the greatest circle in the sphere; and consequently it is equal to the convex surface of the sphere, since this too is quadruple the same greatest circle, as Archimedes demonstrated in Book 1 of *On the Sphere and the Cylinder*, proposition 31.

[Margin: By what reasoning the solidity of a sphere is found.]

Now, to come to the third: the entire solidity of the sphere is produced if the radius of the sphere is multiplied by a third part of the sphere's perimeter, or of the convex surface of the sphere. For the solid rectangle [rectangular solid] contained by the radius of the sphere and a third part of the sphere's perimeter is equal to the sphere itself, as we demonstrated in the treatment of isoperimetric figures, proposition 16. By this reasoning, if the radius of the earth — 40090 $\frac{7}{22}$ stadia — is multiplied by a third part of the convex surface — namely by 6,735,272,727 $\frac{7}{11}$ stadia — the solidity of the earth is produced: 270,023,206,611,570 $\frac{7}{33}$ cubic stadia. That is, the solidity of the earth will comprise as many cubes, each of which encloses one stadium on each of its sides, as there are units in the said number. For the volumes of solid figures are measured by the cubes of the lines by whose squares the perimeters, or convex surfaces, of those figures are usually measured.

By another method the solidity of a sphere will be found from its greatest circle, even if its convex surface is unknown, in this way. Let the greatest circle [i.e. its area] be multiplied by two-thirds of the whole diameter. The resulting number will be the solidity of the sphere, as we demonstrated in Book 5 of the *Practical Geometry*, proposition 7. For example, if some sphere has a diameter of 14 palms and it is multiplied by $3\frac{1}{7}$, the circumference of the greatest circle, 44, is found; half of this, 22, if multiplied by the radius 7, produces the area of the greatest circle, 154; and if we multiply this by two-thirds of the diameter — that is, by $9\frac{1}{3}$ — the solidity of the same sphere is produced: 1437 $\frac{1}{3}$ cubic palms.

All these rules, and many more relating to the same matter, have been demonstrated by us in Books 4 and 5 of the *Practical Geometry*.

On the various measures of mathematicians

But in order that the earth's circumference may be had not only in stadia, but also in paces, miles, leagues, and other measures, we must enumerate the measures that mathematicians — especially geometers — use. For mathematicians, lest confusion should arise from the diversity of measures in various regions (since nearly every region has more or less its own measures), usefully devised certain measures that would be held fixed and settled among all nations. The principal measures are contained in the table below.

[Margin: The various measures used among geometers.]

Measure	Contains	or	amount
Barleycorn (granum hordei) — the smallest of all measures, and the starting point			
Finger (digitus) — has grains arranged by their width	4 grains		
Palm (palmus) — contains digits	4	or grains	16
Foot (pes) — contains palms	4	or digits	16
Small cubit (cubitus parvus), according to Vitruvius [Vitruvius] — contains feet	1 1/2	or palms	6
Common cubit (cubitus communis) — embraces feet	4	or palms	16
[Note: This common cubit is not found among the ancients, but was perhaps added by more recent writers, because it is the same as the common ell (ulna); for in some provinces the ell and the cubit are taken as the same thing.]			
Great cubit (cubitus magnus) — consists of feet	9	or palms	36
[Note: This great cubit too was perhaps added by more recent writers, on account of Origen			

Measure	Contains	or	amount
<i>[Origen], though it is found in no other writer.]</i>			
Simple pace of the first kind (passus simplex primæ differentiæ) — has feet	2	or palms	8
Double pace of the first kind — has feet	4	or palms	16
Simple pace of the second kind — contains feet	2 1/2	or palms	10
Double pace of the second kind, called the geometric [pace] — has feet	5	or palms	20
Simple pace of the third kind — has feet	3	or palms	12
Double pace of the third kind — consists of feet	6	or palms	24
Common ell (ulna communis) — embraces feet	4	or palms	16
Rustic ell (ulna agrestis) — consists of feet	6	or palms	24
Perch (pertica) — comprises feet	10	or palms	40
Stadium — has geometric paces	125	or feet	625
Mile (milliarium) — contains stadia	8	or geom. paces	1000
Gallic or Spanish league (leuca Gallica sive Hispanica) — contains miles	1 1/2	or geom. paces	1500
Common German league (leuca Germanica communis) — has miles	4	or geom. paces	4000
Swabian league (leuca Suevica), the greatest of all — has miles	5	or geom. paces	5000

However, the value of these measures is to be understood only with respect to length. So, for example, eight stadia in length make one mile in length, and four digits in length make one palm in length, and so on — but not with respect to

area. For eight square stadia are not equivalent to one square mile, since the square of one mile comprises 64 square stadia; this is because the square of eight (the number of stadia that make one mile) is 64. Likewise one square palm will contain 16 square digits, because the square of four (four digits making a palm) is 16, and so on. I say this so that you may not be surprised that the stadia comprised in the whole convex surface of the earth cannot be reduced to miles by dividing by 8, but by 64.

[Margin: How the aforementioned measures are to be understood.]

From these it will be easy for anyone not wholly destitute of the precepts of arithmetic to convert any measure into another. For if a smaller measure is to be changed into a greater, the number of the smaller measure must be divided by the number by which the smaller is contained in the greater. For example, if 46000 paces are to be reduced to miles, they must be divided by 1000, since 1000 paces make one mile, and 46 miles result. Likewise, since 8 stadia make a mile, from 252000 stadia are produced 31500 miles. By the same reasoning, since 20000 palms make a mile, in 560000 palms are contained 28 miles, and so on. But if some greater measure is to be converted into a smaller, the number of the greater measure must be multiplied by the number according to which the smaller is contained in the greater. For example, if I wish to know how many paces are made from 46 miles, I multiply 46 by 1000 (for the pace is contained that many times in a mile), and I produce 46000 paces; and so for the rest.

Various opinions of authors on determining the earth's circumference

Although all the methods adduced above, by which the circumference of the terrestrial globe is investigated, rest on geometrical demonstrations, the terrestrial distance corresponding to a like celestial interval nevertheless cannot be measured with perfect exactness — because of the obstacles of mountains or valleys and the like, or also because one rarely travels by a straight route from one place to another, but rather the routes are always winding. And although Maurolico's demonstration does not require that we traverse any distance, it is still quite difficult to discern exactly and precisely, by a visual ray, that point of contact on the earth's surface. Hence it has come about that different practitioners, having measured the circumference of the globe composed of earth and water, have not found it to be of the same magnitude, but differ greatly among themselves in determining the quantity of that circumference. It has

seemed good to review their opinions here, so that from them we may choose the one that comes closer to the truth.

[Margin: Why various authors have found different values for the earth's circumference.]

Aristotle [Aristotle], then, near the end of Book 2 of *On the Heavens* [*De caelo*], reports the opinion of certain ancients who asserted that the earth's circumference contains 400000 stadia, which make 50000 miles. Thus, according to this view, to one terrestrial degree there would correspond 1111 $\frac{1}{9}$ stadia [?], or 138 $\frac{8}{9}$ [?] miles; the earth's diameter would be 127272 $\frac{8}{11}$ stadia, or 15909 $\frac{1}{11}$ miles; and the radius 63636 $\frac{4}{11}$ stadia, or 7954 $\frac{6}{11}$ miles. But because this opinion attributes more than is fair to the earth's magnitude, and conflicts too much with the observations of more recent men, it is rejected by all.

[Margin: The earth's circumference according to Aristotle.]

Hipparchus [Hipparchus], on the testimony of Pliny [Pliny the Elder], assigned to the earth's circumference 277000 stadia, that is, 34625 miles; so that the space of one degree comprises 769 $\frac{7}{9}$ [?] stadia, or 96 $\frac{2}{6}$ [?] miles. Thus for Hipparchus the earth's diameter will be 88136 $\frac{4}{11}$ stadia, or 11017 $\frac{1}{11}$ miles; and the radius will contain 44068 $\frac{2}{11}$ stadia, or 5508 $\frac{4}{11}$ miles. But for the same reason as the previous one, this opinion is exploded by the astronomers.

[Margin: The earth's circumference according to Hipparchus.]

Eratosthenes [Eratosthenes], as is found in Macrobius [Macrobius], Book 1 of *On the Dream of Scipio* [*In Somnium Scipionis*], assigned to the earth's circumference 252000 stadia, which make 31500 miles. For he had found that in one degree of the earth 700 stadia are contained, that is, 87 $\frac{1}{2}$ miles. Hence the earth's diameter will have 80181 $\frac{9}{11}$ stadia, or 10022 $\frac{2}{11}$ miles; and in the radius there will be 40090 $\frac{9}{22}$ stadia, or 5011 $\frac{1}{22}$ miles. If, however, we are to believe Cleomedes [Cleomedes], Eratosthenes said that only 250000 stadia are contained in the whole circumference of the earth. Yet the astronomers of our time do not embrace this opinion either, because they find the earth's circumference smaller than Eratosthenes did.

[Margin: The earth's circumference according to Eratosthenes.]

Ptolemy [Ptolemy] affirms that the whole circumference of the terrestrial globe contains 180000 stadia, that is, 22500 miles — so that to one degree on the earth there correspond 500 stadia, or 62 $\frac{1}{2}$ miles. By this reckoning the [length] of the earth's diameter...

[Margin: The earth's circumference according to Ptolemy.]

the length will comprise 57,272 $\frac{8}{11}$ stadia, or 7159 $\frac{1}{11}$ miles. The semidiameter will be 28,636 $\frac{4}{11}$ stadia, or 3579 $\frac{6}{11}$ miles. And the whole convex surface of the earth will comprise 10,309,090,909 $\frac{1}{11}$ stadia, or 161,079,545 $\frac{5}{11}$ miles.

[Margin: The earth's circumference according to Alfraganus [al-Farghānī], Almaeon, and Thebith [Thābit ibn Qurra].]

ALFRAGANUS [al-Farghānī], Almaeon, Thebitius [Thābit ibn Qurra], and — on Alfraganus's authority — very many learned men assign to the earth's circumference 163,200 stadia, or 20,400 miles. For they allot to each single degree only 453 $\frac{1}{3}$ stadia, that is, 56 $\frac{2}{3}$ miles. Accordingly, on these authors' reckoning, the earth's diameter will contain 51,927 $\frac{3}{11}$ stadia, or 6490 $\frac{10}{11}$ miles; its semidiameter will amount to 25,963 $\frac{7}{11}$ [?] stadia, and to 3245 $\frac{5}{11}$ [?] miles; and the convex surface will be 8,474,530,909 $\frac{1}{11}$ stadia, or 132,414,545 $\frac{5}{11}$ miles.

[Margin: The earth's circumference according to Fernel of Amiens [Jean Fernel].]

FERNEL of Amiens [Jean Fernel], in his *Cosmotheoria*, holds that the earth's circumference comprises 196,114 $\frac{8}{25}$ stadia, or 24,514 $\frac{29}{100}$ [?] miles. For he says that he found one degree on the earth to correspond to 544 [?]/500 stadia, or 68 [?]/4000 miles. On this view, then, the earth's diameter will be 62,400 $\frac{2}{275}$ [?] stadia, or 7800 $\frac{3}{2200}$ [?] miles; the semidiameter, 31,200 $\frac{3}{500}$ stadia, or 3900 $\frac{3}{4400}$ miles; and the convex surface of the earth will on this reckoning contain 12,237,535,707 [?]/[?] stadia, or 191,211,495 $\frac{217143}{880000}$ miles.

[Margin: The earth's circumference according to the more recent navigators.]

The MODERNS, finally — experts in astronomical matters who have more than once sailed across the whole ocean — testify that the entire circuit of the earth comprises 152,640 stadia, or 19,080 miles. For they say that a single degree at sea corresponds to only 424 stadia, that is, 53 miles. And so, if this is true, the length of the earth's diameter will be 48,567 $\frac{3}{11}$ [?] stadia; in miles, 6070 $\frac{10}{11}$. The semidiameter will be 24,283 $\frac{7}{11}$ stadia, or 3035 $\frac{5}{11}$ miles. Finally the convex surface of the earth will comprise 7,413,308,509 $\frac{1}{11}$ stadia, or 115,832,945 $\frac{5}{11}$ miles.

These, then, are the seven opinions of any weight concerning the size of the earth's circuit. Of them the first three are rejected by everyone as altogether false, while the latter four are probable, and each has its own defenders. [Margin: Ptolemy's opinion on the earth's circumference is the common one.] For the schools of astronomers generally embrace Ptolemy's view as the truer — and it is

the one we shall follow in what follows, so as not to seem to depart from the common path. Others rather adhere to Alfraganus's opinion, because after Ptolemy many learned men, as Alfraganus reports, confirmed it; hence perhaps the moderns' opinion, which departs but little from Alfraganus, will prove the truer. Finally, a few seem to incline to the opinion of Fernel of Amiens.

There are also some who try to reconcile all the said opinions. For they say that the aforementioned authors did not use the same measure, but that those who set the earth's circuit larger assumed smaller paces, while those who said it was smaller used larger paces. Hence the discrepancy among these astronomers will not be as great as it seems. But anyone who considers the matter more carefully will readily see that no agreement can be found among all the opinions, even though between two or three some measure of agreement may be found.

So that you may have all the aforementioned opinions before your eyes, I have appended the following little tables, in which, according to each view, are set out the earth's circuit, the size of one terrestrial degree, the diameter of the earth, and its semidiameter; and, following the latter four opinions as well, the convex surface of the earth — all this both in stadia and in miles.

The earth's circumference contains, as each holds — *Ambitus terrae continet, vt vult*

Author	Stadia	Milliaria (miles)
Aristoteles [Aristotle]	400,000	50,000
Hipparchus	277,000	34,625
Eratosthenes	252,000	31,500
Ptolemaeus [Ptolemy]	180,000	22,500
Alphraganus [al-Farghānī]	163,200	20,400
Fernelius [Fernel]	196,114 $\frac{8}{25}$	24,514 $\frac{29}{100}$ [?]
Recentiores [the moderns]	152,640	19,080

One degree on the earth has, as each holds — *Vnus gradus in terra habet, vt vult*

Author	Stadia	Milliaria (miles)
Aristoteles	1111 $\frac{1}{9}$	138 $\frac{8}{9}$
Hipparchus	769 $\frac{4}{9}$	96 $\frac{13}{72}$ [?]
Eratosthenes	700	77 $\frac{1}{2}$ [?]

Author	Stadia	Milliaria (miles)
Ptolemaeus	500	62 1/2
Alphraganus	453 1/3	56 2/3
Fernelius	544 [?]/500	68 [?]/4000
Recentiores	424	53

The earth's diameter contains, as each holds — *Diameter terrae continet, vt vult*

Author	Stadia	Milliaria (miles)
Aristoteles	127,272 8/11	15,909 1/11
Hipparchus	88,136 4/11	11,017 1/22
Eratosthenes	80,181 9/11	10,022 8/11
Ptolemaeus	57,272 8/11	7159 1/11
Alphraganus	51,927 3/11	6490 10/11
Fernelius	62,400 2/275 [?]	7800 3/2200 [?]
Recentiores	48,567 3/11	6070 10/11

The earth's semidiameter has, as each holds — *Semidiameter terrae habet, vt vult*

Author	Stadia	Milliaria (miles)
Aristoteles	63,636 4/11	7954 6/11
Hipparchus	44,068 2/11	5508 23/44 [?]
Eratosthenes	40,090 10/11	5011 4/11
Ptolemaeus	28,636 4/11	3579 6/11
Alphraganus	25,963 7/11	3245 5/11
Fernelius	31,200 3/500 [?]	3900 3/4400
Recentiores	24,283 7/11	3035 5/11

The convex surface of the earth contains, as each holds — *Superficies conuexa terrae continet, vt vult*

Author	Stadia	Milliaria (miles)
Ptolemaeus	10,309,090,909 1/11	161,079,545 5/11
Alphraganus	8,474,530,909 1/11	132,414,545 5/11

Author	Stadia	Milliaria (miles)
Fernelius	12,237,535,707 5802/13750 [?]	191,211,495 217143/880000
Recentiores	7,413,308,509 1/11	115,832,945 5/11

THE DISTANCES OF THE HEAVENS FROM THE EARTH, THEIR THICKNESSES, AND THEIR CIRCUMFERENCES — *DISTANTIÆ COELORVM A TERRA, CRASSITVDINESQVE, & Ambitus eorundem*

Since we have spoken of the size of the earth — both as regards the circuit of the greatest circle described upon it, and as regards its diameter, its semidiameter, and its convex surface — it will not be out of place to indicate briefly here also the semidiameters, that is, the distances from the center of the world, of all the heavens, together with their thicknesses and their circuits, or circumferences. This I shall carry out in three tables: the first of which contains the semidiameters of all the heavens; the second, their thicknesses; and the third, finally, the circumferences of the same in their greatest circles, both along the concave and along the convex [surface]. From the precepts given above, anyone who wishes can readily work out, should he desire it, both the concave and the convex surfaces — indeed even the solid volumes — of these heavens. In these tables I have followed almost throughout Francesco Maurolico [Franciscus Maurolycus] in the appendix to his *Dialogues on Cosmography*.

The semidiameters of the heavens, both along the concave and along the convex — *Semidiametri coelorum tam secundum concauum, quam secundum conuexum*

Each row gives the value first in terrestrial semidiameters (*semidiametros terrae*), then "or in miles" (*vel milliaria*).

Boundary	Terrestrial semidiameters	Or miles
Semidiameter of the concave of ☾ [Moon] contains	33 7/10	120,630 15/22
Semidiameter of the convex of ☾ [Moon], and concave of ☿ [Mercury]	64 1/6	229,687 1/2
Semidiameter of the convex	167 2/3	600,167 14/33

Boundary	Terrestrial semidiameters	Or miles
of ☿ [Mercury], and concave of ♀ [Venus]		
Semidiameter of the convex of ♀ [Venus], or concave of ☉ [Sun]	1121 7/20	4,013,923 7/22
Semidiameter of the convex of ☉ [Sun], or concave of ♂ [Mars]	1216 1/12	4,353,025 25/44
Semidiameter of the convex of ♂ [Mars], or concave of ♃ [Jupiter]	8853 3/4	31,692,400 25/44
Semidiameter of the convex of ♃ [Jupiter], or concave of ♄ [Saturn]	14,378 1/3	51,467,897 8/11
Semidiameter of the convex of ♄ [Saturn], or concave of the Firmament, according to Alfraganus	22,612 1/2	80,942,471 13/22
Semidiameter of the convex of the Firmament, according to Alfraganus	45,225	161,884,943 2/11

The thicknesses of the heavens — *Crassitudines coelorum*

These are obtained if the semidiameters extended to the concave of each heaven are subtracted from the semidiameters extended to the convex of the same. Each row gives the thickness first in terrestrial semidiameters, then "or in miles."

Heaven	Terrestrial semidiameters	Or miles
Thickness of the heaven of ☾ [Moon]	31 7/15 [?]	109,056 9/11
Thickness of the heaven of ☿ [Mercury]	103 1/2	370,479 61/66
Thickness of the heaven of ♀ [Venus]	953 41/60 [?]	3,413,755 59/66 [?]

Heaven	Terrestrial semidiameters	Or miles
Thickness of the heaven of ☉ [Sun]	94 11/15 [?]	339,102 1/4
Thickness of the heaven of ♂ [Mars]	7637 2/3	27,339,375
Thickness of the heaven of ♃ [Jupiter]	5524 7/12	19,775,497 7/44
Thickness of the heaven of ♄ [Saturn]	8234 1/6	29,474,573 10/22
Thickness of the Firmament, from Alfraganus	22,612 1/2	80,942,471 13/22

The circumferences of the heavens, both concave and convex, reduced to miles — *Ambitus coelorum tam secundum concauum, quam secundum conuexum ad milliaria reducti*

Circle	Miles
Circumference of the concave of ☾ [Moon]	758,250
Circumference of the convex of ☾ [Moon], or concave of ☿ [Mercury]	1,443,750
Circumference of the convex of ☿ [Mercury], or concave of ♀ [Venus]	3,772,500
Circumference of the convex of ♀ [Venus], or concave of ☉ [Sun]	25,230,375
Circumference of the convex of ☉ [Sun], or concave of ♂ [Mars]	27,361,875
Circumference of the convex of ♂ [Mars], or concave of ♃ [Jupiter]	199,209,375
Circumference of the convex of ♃ [Jupiter], or concave of ♄ [Saturn]	323,512,500
Circumference of the convex of ♄ [Saturn], or concave of the Firmament	508,781,250
Circumference of the convex of the Firmament	1,017,562,500

[Margin: By what method the distances and thicknesses of the heavens, and the magnitudes of the stars, may be investigated.]

The untrained crowd is often not a little astonished at how astronomers have gathered the distances of the heavens, and hence also determined their thicknesses and circumferences, together with the magnitudes of the stars. Many even of those who wish to be reckoned mathematicians do the same, since they see that this could not have been done by the altimeter's scale, or by the other instruments with which surveyors [Geometrae] usually measure the distances of places. For every use of instruments fails when a distance as great as that at which those heavenly bodies stand from us has to be investigated — seeing that instruments of this kind are scarcely fit to measure fifty miles, even from some very lofty mountain, let alone the hundred and sixty millions of miles and more at which the convex surface of the Firmament is said to lie from the center of the earth. So that they may cease to wonder, then, let them know that all these things were investigated by skilled astronomers through the motions of the planets; and by what method this can be done, I shall explain briefly here.

First, then, they investigated the Moon's distance when it is closest to the earth — that is, the semidiameter of the concave surface of the lunar orb in relation to the earth's semidiameter — by means of what Ptolemy demonstrated in Book 5 of the *Almagest*, through the parallax [diuersitatem aspectus] of that same Moon, which he teaches how to investigate in that same book.

[Margin: How the Moon's distance from the earth's center is to be investigated.]

He applied virtually this same technique to investigating the Moon's distance from the earth's center when it is nearest to us. Let AB be a greatest circle on the earth, about C, the center of the world. In the same plane let there be a circle in the Moon's heaven passing through its center E, running along DE. Finally, let a vertical circle in the Firmament pass through the zenith F and, going through the Moon's center, be FG.

Diagram. A lunar-parallax construction. About the center C (the earth, drawn as a small hatched globe) two large concentric circles are drawn: a middle circle representing the Moon's heaven and an outer circle representing the Firmament. A vertical diameter runs from F at the top down through the center to B. Straight lines drawn from C and from A (a point on the earth's globe just above C) pass through E (the Moon's center, on the middle circle) and strike the outer circle at the points G, H, I on the upper right.

- F (top, on outer circle): the zenith
- D (upper, on the middle/lunar circle)

- g (upper right, on outer circle): G, the Moon's true place (printed as lowercase *g*)
- H (right, on outer circle, below g): the Moon's apparent place
- I (right, on outer circle, below H)
- E (center-right, on the lunar circle): the Moon's center
- A (just above center, on the earth's globe)
- C (center): the world's center (the earth)
- B (lower right, on the earth's globe, on the vertical diameter)

Now draw the straight line CF through C, the world's center, and through the zenith F; and through E, the Moon's center, draw the straight lines CEG and AEH. Then G will be the Moon's true place, H its apparent place, and GH the parallax. And since the earth is of imperceptible size in relation to the Firmament, if CI is drawn parallel to AH, the arc HI will be of no perceptible size with respect to the Firmament; and therefore the arc GI can be taken for the parallax, since HI adds nothing perceptible to the arc GH. Consequently, once the parallax is known, the angle GCI [note a] — and therefore also its alternate angle AEC — will be known. The angle ACG is also known, since the Moon's true distance from the zenith is known, namely the arc FG. (This too will become known if the parallax GH is subtracted from the arc FH, known by observation.) [note b] Therefore the remaining angle CAE too will be known, out of the two right angles [to which the triangle's angles sum]. Since, then, all three angles of triangle ACE are known, together with the side AC, [note c] if we set up the proportion:

As the sine of angle E (the parallax) is to the side AC (taken as 1), so is the sine of angle A to another quantity,

then CE will be produced — the Moon's distance from the earth's center, in the parts of which the earth's semidiameter AC is 1 — which was the thing proposed.

[Margin, note a: 29 of the first (book).] [Margin, note b: 32 of the first (book).]

[Margin, note c: 10 of the *Triangles, rectilinear* (i.e. of Regiomontanus, *De triangulis*).]

Next they explored the eccentricities of all the planets — that is, how far the centers of the eccentric orbs, on which the planets are carried from west to east, are distant from the world's center — as was demonstrated by Ptolemy in the *Almagest*.

Third, with remarkable ingenuity they hunted out the thicknesses of the eccentrics in terms of the diameter of the epicycles, as Ptolemy taught in the same

Almagest. From all these they concluded the distances of the heavens — that is, their semidiameters — in the following way.

Let ABC be the heaven of any planet, in which the eccentric is IMO and its deferents are ABCL and FEDK; let G be the center of the world and H the center of the eccentric. Let CF (or AD) be the thickness of the whole heaven; CL (or KD) the greatest thickness of the deferent orbs of the augment [maxima crassities orbium augem[?] deferentium]; and finally AK (or FL) the thickness of the eccentric orb, which is exhibited by the diameter of the solar body AK, or of the epicycle FL. Before all else it must be demonstrated that the thickness CL (or KD) is double the eccentricity GH — that is, of the distance of the eccentric's center H from the world's center G — which will become clear as follows. Cut off GN equal to GH. And since the semidiameters GA, GC are equal, when the equals GH, GN are removed, HA and NC will also be equal. Since, then, HC exceeds NC by the straight line HN — which is double the eccentricity GH — the same HC will also exceed HA (that is, HL, namely the semidiameter of the eccentric) by the straight line HN. But HC exceeds this same HL by the thickness CL. Therefore the thickness CL is equal to double the eccentricity — that is, to the straight line HN — which was to be shown.

Diagram. A woodcut of a planet's heaven with its eccentric and deferent orbs, shown as a broad shaded (dark) annulus enclosing a lighter inner region. A vertical diameter runs from A at the top down to C at the bottom. Two small circles (the planetary/solar body in two positions) sit on this axis: one at the top marked with an *i* (drawn with radiating rays, like a small sun) just below A, and one near the bottom marked with an *o*. The center of the world G lies on the axis at mid-height, with the eccentric's center H just above it.

- A (top, on the outer deferent)
- *i* (small rayed circle just below A): the center I of the solar/planetary body at its apogee
- M (left, on the eccentric circle)
- [label reading like "P" or "R" [?], below the sun-circle, on the axis]
- D (upper middle, on the axis)
- H (center, the eccentric's center)
- E (left of center)
- G (center, the world's center)
- N (below G, on the axis)
- F (lower middle, on the axis)

- o (small circle near the bottom): the center O of the epicycle at its perigee
- B (lower left)
- L (bottom center)
- C (very bottom of the axis)

[The point K named in the text (deferent FEDK, thickness KD) falls on the right side of the figure and is not legible on the scan.]

And this demonstration holds also in the heaven of Mercury, in which four eccentrics enclose the eccentric orb that carries the epicycle, provided the denser parts are placed together so as to make up the whole thickness of the heaven, the eccentric aside. Thus, once the amount of the eccentricity relative to the earth's semidiameter is known, if it is doubled, the thickness CL is produced; and if to this is added the thickness of the eccentric FL — that is, the diameter of the epicycle, or of the solar body in the heaven of the Sun — the whole thickness of the heaven will be composed.

By this same method, moreover, the eccentricity and the semidiameter of any epicycle become known, in the parts of the earth's semidiameter. From H, the center of the eccentric, let the eccentric circle IMO be described, through I, the center of the Sun, or through O, the center of the epicycle. And since in the Moon's heaven the eccentricity GH is known in the parts of which the semidiameter of the eccentric circle HI (or HO) contains 60; and since FO is also known in the same parts; GF too will be known in the same parts. Since, then, GF is also known in parts of the earth's semidiameter, as far as the concave surface of the Moon: if we set up the proportion — as GF, insofar as it is known in parts of the semidiameter HO, is to GF known in parts of the earth's semidiameter, so GH, known in parts of the semidiameter HO, is to another quantity — then GH will be made known in parts of the earth's semidiameter. And thus the eccentricity of the Moon is now known in parts of the earth's semidiameter. Again, if we set up — as GF, known in parts of the semidiameter HO, is to GF known in parts of the earth's semidiameter, so FL, the diameter of the Moon's epicycle, insofar as it is known in parts of the semidiameter HO, is to another quantity — then FL will be found in relation to the earth's semidiameter. From which it comes about that the whole thickness of the Moon's heaven is known in parts of the earth's semidiameter, which was proposed. Hence too the line GC will be made known — that is, the semidiameter of the Moon's convex surface — in the same parts of the earth's semidiameter.

Now if ABC is taken for the heaven of Mercury, its thickness CF will be known in the same way, in parts of the earth's semidiameter, from GF, the semidiameter of

the concave surface, known in the same parts; and consequently GC, the semidiameter of Mercury's convex sphere, will also be known. And in this manner the thickness and the semidiameters of the heavens will be known, in order, for the remaining planets, up to the Firmament — whose thickness cannot be known by a geometrical route. Nevertheless, since it surrounds and contains all the other heavenly bodies and the elements, it pleased the astronomers, and especially al-Farghānī [Alphragano], to assign to it a thickness as great as its distance from the world's center — which is not incredible. For since the Moon's heaven, which contains the sphere of the elements, has a thickness nearly as great as its distance from the earth's center is found to be, why should this be denied to the Firmament, the noblest of the heavens, which encloses and surrounds not only the elements but even all the orbs of the planets? But however the matter may stand, it is highly reasonable that at least the Firmament, together with the ninth, tenth, and eleventh heaven, be of a thickness as great as the distance from the earth's center at which the concave surface of the Firmament stands: so that what we shall say a little below about the speed of the Firmament's motion may be understood of the speed of the prime mover, or eleventh heaven, in case the Firmament does not have so great a thickness.

Now, from the distances of the heavens investigated in the way we have described, and from the diameters of the planets and of the other stars, known through instruments, we shall arrive at knowledge of the magnitude of the stars, by this method. From the doubled distance of any star, the diameter becomes known of that greatest circle whose circumference passes through the star's center. Then from this diameter it is drawn out how many earth-diameters the circumference of that circle comprises, by means of what Archimedes demonstrated about the ratio of the circumference of any circle to its diameter, as I set out at length a little earlier. Again, from this circumference we shall learn how many earth-diameters the star's diameter contains. Finally, once this ratio of the star's diameter to the earth's diameter is known — since spheres have the triplicate ratio of their diameters, as Euclid demonstrated in *Elements* XII.18 — if the triplicate of that ratio is taken, it will be known how many times the star itself contains the globe composed of earth and sea. For example: the greatest distance of the Sun from the earth contains 1216 earth-semidiameters, that is, 608 diameters; and this distance doubled gives 1216 diameters, comprised in the total diameter of the Sun's heaven. Therefore the circumference of the Sun's heaven, along its greatest circle, will contain nearly 3822 earth-diameters. And accordingly the half-degree which the diameter of the solar body is found by the astronomers' instruments to occupy will comprise about $5\frac{1}{2}$ earth-diameters, so that the ratio of the Sun's diameter to the earth's diameter is roughly $5\frac{1}{2}$ to 1.

Hence, since the ratio $166\frac{3}{8}$ to 1 is the triplicate of the ratio $5\frac{1}{2}$ to 1 — as appears in these numbers: 1, $5\frac{1}{2}$, $30\frac{1}{4}$, $166\frac{3}{8}$ — the solar body will contain the earth's globe 166 times, and moreover three eighth-parts of it. And the same reasoning applies to the other planets and stars.

But this too must not be passed over: that Ptolemy, by another route — namely through eclipses — also investigated the ratio of the bodies of the Sun and Moon to the globe composed of earth and sea.

Moreover, this too must be carefully observed: that the distances, thicknesses, and magnitudes of the heavens and stars, found in the way we have prescribed — although they are immense and in a way surpass human belief — are nevertheless the smallest that they could be. This is because the astronomers place the eccentric orb of each celestial orb touching the convex and concave surface of that heaven itself at only a single point; likewise the epicycle of each planet, and the solar body, touching also the convex and concave surface of the eccentric orb at only a single point, as appears in the figure above, where the eccentric IMO touches the convex surface of the heaven at point A and the concave at point F. Likewise both the Sun and the epicycle fill the entire thickness of the eccentric. But it is credible that God, Best and Greatest, made those celestial orbs denser, so that no eccentric orb touches the convex and concave surface of the heaven, but is immersed within the heaven's thickness; nor does the epicycle or the Sun reach the convex and concave surface of the eccentric, but is immersed within its thickness too. This granted, it is certain that the distances, thicknesses, and magnitudes of the heavens and stars are far greater than have been found by the astronomers. We have therefore only demonstrated in what way all these things can be gathered from the motions themselves. For although perhaps the thickness and distance created by God is greater, we can in no way come to know it through the motions — just as neither could the thickness of the Moon's fourth orb, which is concentric and is called the bearer of the head and tail of the Dragon [caput & caudam Draconis, the lunar nodes], be traced out from motion; and for this reason no mention is made of it, as if it did not exist in the nature of things — although it is certain that it is solid, and that on this account the heaven of Mercury is farther from the earth than has been detected by the astronomers.

From these things it is established that any point of the Firmament situated on the Equator traverses, each hour, $42398437\frac{1}{2}$ miles; since, evidently, in 24 hours it completes 1017562500 miles. From which it comes about that the speed of the Firmament's motion can scarcely be grasped in thought — which antiquity

supposed to be the prime mover, as Aristotle too affirmed. For the space which any point of the Equator traverses on the convex surface of the Firmament in one hour is so great that one could scarcely traverse it in 2904 years, even if daily, without any interruption, one covered 40 miles. This seems incredible. For the motion of that point is swifter than the motion of some arrow or bird which, in the span of time in which a single Angelic Salutation [the Hail Mary] is recited, would traverse 176660 miles — that is, would circle the whole earth from east to west under the Equator more than seven times, since the earth's circumference of 22500 miles is contained in this number 176660 more than seven times: a speed that exceeds the grasp of human intellect. Anyone will easily persuade himself that this is so, if he considers attentively that in a quarter of an hour one can scarcely recite 60 Angelic Salutations, and thus 240 in one hour. For hence it follows that the time in which a single Angelic Salutation is recited is $1/240$ of an hour; and it is established that a point of the Equator traverses 176660 miles on the convex surface of the Firmament in $1/240$ of an hour, since in one hour it completes 42398437½ miles, as we have said. Therefore it is necessary that the arrow or bird likewise traverse 176660 miles — that is, circle the earth more than seven times — in the span of one Angelic Salutation, if it is to keep pace with the motion of the Firmament. Or (if you prefer) so great is the speed of the motion of that point of the Firmament in one hour as would be that of some arrow or bird which circled the whole earth from east to west under the Equator, in one hour, one thousand eight hundred and eighty-four times [*millies, octingenties, octogies, & quater* = 1884]; because the earth's circumference, comprising 22500 miles, is contained in the 42398437½ miles (which are traversed in one hour by that point of the Equator) as many times as there are units in this number 1884, and more — a speed that can hardly be conceived.

[Margin: How many miles in one hour any point whatever of the Equator traverses in the Firmament.] [Margin: The wondrous speed of the Firmament.]

[Margin: That the circle described by the polar star is of such magnitude that the whole sphere of the Sun, placed within it, does not touch it.]

Again, from what we have said it will be permissible to gather that the polar star, which in our time is distant from the Arctic pole by nearly $3\frac{1}{2}$ degrees, describes a circle whose diameter is much greater than the diameter of the whole heaven of the Sun — so much so that the entire sphere of the Sun, placed within that circle, would not touch it: which seems utterly incredible, since the polar star scarcely appears to change its place. This is gathered as follows. Since the semidiameter of the convex Firmament contains 45225 earth-semidiameters, if

we set up the proportion — as the whole sine 100000 is to 45225, the semidiameter of the Firmament, so is 12208, the chord of the 7 degrees by which the diameter of the said circle of the polar star is subtended, to the sought quantity — the said chord, or diameter of that star-circle, will be found to contain 5521 earth-semidiameters. Since, then, the diameter of the convex sphere of the Sun comprises only 2432 earth-semidiameters and a little more, it is clear that the diameter of the Sun's sphere does not amount to half the diameter of the said circle. Therefore, since circles have the duplicate ratio of their diameters [note a] — namely that which the squares of the diameters have — the greatest circle in the sphere of the Sun will be less than $\frac{1}{4}$ of the said circle. From which it follows that the sphere of the Sun, placed within that circle, can in no way touch the said circle.

[Margin, note a: 2 of the twelfth (i.e. *Elements* XII.2).]

A DIGRESSION ON THE NUMBER OF THE SAND

[Margin: That the number of the sand is, according to some, infinite; but according to others finite indeed, yet surpassing every given number.]

[Margin: Archimedes' proposition in the book *On the Number of the Sand*.]

In Archimedes' time (as he himself relates in his book *On the Number of the Sand* [the *Sand-Reckoner*, *Arenarius*]) some thought the number of the sand — not only of that contained around Syracuse and the rest of Sicily, but also of that contained in every region, both habitable and uninhabitable — to be infinite. Others, indeed, did not say that the number of the sand was infinite, since an infinite cannot be given, but they believed that no determinate number could be given that would exceed it or equal its multitude; on the contrary, rather, that any number proposed and determined would be surpassed by that number of the sand. Hence Archimedes relates that those who held this opinion, if they were to conceive in their mind such a heap of sand as there would be if the whole earth — with the sea in it filled up, and all the hollows leveled up to the summits of the highest mountains — and if they then devised some further multiple of this same heap, would without any doubt suppose that its multitude far and greatly surpassed all numbers. Archimedes refutes the error of all these geometrically, and indeed most acutely, in the book which he entitled *On the Number of the Sand*, seeking a number that would not only exceed the multitude of the sand which, filling the earth on every side (as we said), would be equal to it, but would also equal in magnitude the sand that would fill the world itself (the world too being posited much larger than it really is). And this is Archimedes' proposition in

his book *On the Number of the Sand*, where first, by a certain subtle reasoning, he demonstrates by what route the distance of the Sun from the earth is to be investigated — an angle having first been found that is smaller than the angle which two straight lines, going out from the center of vision and touching the Sun, contain; on which matter consult his writings, and the *Commentaries* of Federico Commandino [Federici Commandini].

We, therefore, following in the footsteps of Archimedes, shall likewise inquire into a number that is far greater than the number of even the tiniest sand-grains that would fill the whole world up to the Firmament. For many have urged me to explain this matter here. This I have done the more gladly because I know it will be most delightful to many; and especially because this business is not entirely foreign to our undertaking, since we have adduced here many things concerning the distances and magnitudes of the heavens, from which we can with little trouble gather what we have proposed. And so that our discussion may turn out more illustrious and more admirable, let us posit that the whole world, up to the Firmament, is far greater than has been detected by the astronomers; likewise, that the sand-grains filling the whole universe are much smaller than are found anywhere. For if it be demonstrated by us that the number we have found is greater than the number of sand-grains — smaller than any that exist — filling a world greater than this world of ours: it will be clear that the same number is much greater than the number of even the tiniest sand-grains existing in the nature of things which would fill the whole world, up to the Firmament, as far as it has been detected by the astronomers. Let these things, then, be laid down by us in order.

I. That the earth's diameter is much less than 10000 miles. Although this is most true — since, according to Ptolemy and the more common opinion of astronomers, the earth's diameter contains only 7159 $\frac{1}{11}$ miles, as we said above — nevertheless, in order that the reckoning may be made easier, and that we may make the world larger than it really is, let us set it at 10000 miles.

II. That the diameter of the concave Firmament is far less than 100000 earth-diameters. Although this is true — since, according to al-Farghānī [Aphraganum], that diameter comprises only 45225 earth-diameters — let us nevertheless take it to contain 100000 earth-diameters, for the reason adduced before. And since we assumed the earth's diameter to comprise 10000 miles (though it is much less), the diameter of the concave Firmament will contain fewer miles than 1000000000. But for the said reason let us posit that it comprises 1000000000 miles.

[Margin: By what method the number of the tiny sand-grains filling the whole world, up to the concave surface of the Firmament, may be investigated.]

III. That a little sphere equal to a single grain of poppy is not greater than 10000 sand-grains, however minute. This anyone will readily grant, since the intellect can scarcely grasp that a single poppy-grain could be divided into 10000 equal particles: for sand-grains so tiny have nowhere been seen. But in order that the demonstration may be made more admirable, and that more sand-grains may be contained in the world, let us posit that that little sphere comprises 10000 sand-grains.

IIII. That the diameter of a poppy-grain is not less than the fortieth part of one geometric digit [finger-breadth]. That this is so, Archimedes found by experiment; he says that he discovered that 35 poppy-grains placed in a single straight line and touching one another exceed the length of a geometric digit — so that a single poppy-grain is greater than $1/35$ of a digit. From which it follows that a single poppy-grain is much greater than $1/40$ of a digit, and not less. But let us posit that it is $1/40$ of a digit, so that the demonstration may be made more evident, although poppy-grains so minute are not found.

V. That a mile is far less than 100000 digits. For since four digits make a palm, and four palms a foot, and five feet a geometric pace, and a thousand geometric paces a mile, it comes about that 80000 digits compose one mile. Therefore a mile is much less than 100000 digits. But let us posit, so that the demonstration may be made easier, that 100000 digits make one mile.

*(Conclusion of the Digressio de arenæ numero — the Sand-Reckoner digression — completing CAPVT PRIMVM.)**

Therefore, since we have taken the diameter of a grain of poppy to be $1/40$ of a digit (although it is in fact much larger), so that 40 grains of poppy make up a digit, a sphere whose diameter equals a digit will have to a grain of poppy — whose diameter equals $1/40$ of a digit — the ratio of 64,000 to 1, ^a inasmuch as spheres stand in the triplicate ratio of their diameters. This is clear from the four numbers set alongside, which are in continued proportion in the ratio of 40, that is, in the ratio of a digit to $1/40$ of a digit; so that a sphere having a diameter equal to a digit contains 64,000 grains of poppy.

[Margin (numerals set beside the paragraph, in continued proportion): $1 \cdot 40 \cdot 1600 \cdot 64000$]

[Margin: 18. duod. — i.e. Euclid, *Elements* XII.18 (spheres are to one another in the triplicate ratio of their diameters)]

Consequently, since we have established that one grain of poppy contains 10,000 sand-grains, the same sphere with a diameter equal to a digit will comprise 640,000,000 sand-grains. Indeed this number will be much greater than the number of sand-grains that are enclosed in a sphere having a diameter equal to a digit, because both fewer than 40 grains of poppy make up a digit, and the sand-grains are larger than would allow 10,000 of them to make a single grain of poppy. Nevertheless, so that the computation may proceed more conveniently, let us suppose that a sphere whose diameter equals a digit contains not merely 640,000,000 but 1,000,000,000 sand-grains.

Next, since we have taken 100,000 digits to constitute one mile (although a mile is much less than that), a sphere having a diameter equal to a mile will have to a sphere whose diameter equals a digit the ratio of 1,000,000,000,000,000 to 1 [i.e. 10^{15}], ^b because spheres stand in the triplicate ratio of their diameters. This is evident in these four numbers, which are continued proportionals in the ratio of 100,000 digits, that is, in the ratio of one mile to 1.

[Margin: 18. duod.]

[Margin (numerals set beside the paragraph, in continued proportion): $1 \cdot 100000 \cdot 100000000000 [10^{10}] \cdot 10000000000000000 [10^{15}]$]

Since, then, a sphere having a diameter equal to a digit has been posited to contain 1,000,000,000 sand-grains — although it holds far fewer, as has been shown — a sphere whose diameter equals a mile will contain all these sand-grains to the number of 1,000,000,000,000,000,000,000,000 [10^{24}]. Indeed this number is much greater than the number of sand-grains contained in a sphere having a diameter equal to a mile, because both fewer than 1,000,000,000 sand-grains fill a sphere whose diameter equals a digit, and fewer than 100,000 digits make up a mile. Nevertheless, for the reason stated just above, let us accept that 1,000,000,000,000,000,000,000,000 [10^{24}] sand-grains fill a sphere whose diameter equals a mile.

Finally, since we have posited that the diameter of the concave of the Firmament comprises 1,000,000,000 miles (although it is in reality much smaller), a sphere whose diameter equals the diameter of the concave of the Firmament will have to a sphere whose diameter equals a mile the ratio of [10^{27}] to 1, ^c because spheres have the triplicate ratio of their diameters, as is plain in these four numbers,

which are in continued proportion in the ratio of 1,000,000,000 miles — that is, in the ratio of the diameter of the concave of the Firmament to 1.

[Margin: 18. duod.]

[Margin (numerals set beside the paragraph, in continued proportion): $1 \cdot 1000000000 [10^9] \cdot 1000000000000000000 [10^{18}] \cdot [10^{27}]$]

Hence, since a sphere having a diameter equal to a mile has been posited to contain $[10^{24}]$ sand-grains — although it in fact encloses far fewer, as we have demonstrated — a sphere comprised within the concave of the Firmament and precisely reaching it [?] will contain sand-grains to a number that is far greater than the number of sand-grains contained in the whole world all the way to the concave of the Firmament.

[Number line, full width: 1 followed by a long string of zeros — the grand total, which the note beside it identifies as having 51 ciphers after the initial 1, i.e. 10^{51} .]

[Margin, in italic: *Qui numerus maior sit numero arenularum, quarum 10000. grano papaueris æquales sint, replentium totum mundum, vsque ad concauum Firmamenti.* — "How much this number exceeds the number of sand-grains — of which 10,000 equal one grain of poppy — that would fill the entire world, up to the concave of the Firmament."]

which fill a sphere whose diameter equals a digit [?]; the miles, of which each contains 100,000,000 [?] within the diameter of the concave of the Firmament ... the number that fills [the world], which of course, after the figure 1, has 51 ciphers, being far ... those that fill it up to the concave of the Firmament, even though the sand-grains ... equal to a grain of poppy.

It is therefore clearly established from what has been said that we could determine for certain how many sand-grains could fill the entire world, if only it were ascertained to us how many sand-grains equal a grain of poppy, how many grains of poppy make up a digit, and finally how many miles — each of which contains 80,000 digits — are comprised within the diameter of the concave of the Firmament. But since these things are as yet unknown and uncertain, we have assumed (following Archimedes [Archimedes] in this) a diameter of the world much greater than it really is according to skilled astronomers; likewise we have posited that more sand-grains equal a grain of poppy than really do, and that more grains of poppy make up a digit than truly do — precisely so that by this reckoning a greater number of sand-grains might arise, one which will assuredly be far greater, as we have said, than the number of sand actually able to be

comprised within the concave of the Firmament. This, indeed, seems incredible to many.

Primi Capitis Finis. [End of the First Chapter.]

CHAPTER TWO

On the circles out of which the material sphere is composed, and by which the supercelestial sphere — the one that the material sphere represents — is understood to be composed.

(Sacrobosco's lemma, in italic:)

Now of these circles some are greater and some smaller, as is evident to the senses. A circle in the sphere is called great when, drawn on the surface of the sphere, it divides the sphere into two equal parts about its center. A small circle is one which, drawn on the surface of the sphere, does not divide it into two equal parts, but into unequal portions. Among the great circles, we must speak first of the Equinoctial.

[Margin: *Maior circulus, & minor in sphæra quid.* — "What a great and a small circle in the sphere are."]

COMMENTARIVS

The Author set out, in the first chapter, the principles and foundations of the whole of Astronomy. Now, in this second chapter, he explains those ten primary circles out of which the material sphere is composed, and by which the celestial sphere — for whose sake this material sphere is constructed — is understood to be composed; namely because without these circles the causes of the celestial appearances can in no way be given — such appearances as the risings and settings of the signs, the risings and settings of the stars, the variation of days and nights in different regions, and so forth. This chapter may not inaptly be divided into three parts. For in the first the Author treats the circles of the sphere in general; in the second he discusses these same circles individually, explaining the name, function, and use of each; and in the third, finally, he adds that in the world five Zones are constituted out of these circles.

[Margin: *Argumentum secundi cap. eiusdemque diuisio.* — "The subject of the second chapter and its division."]

In the first part, then, he divides all the circles of the sphere into greater and smaller — which are called by others "greatest" (maximi) and "not greatest" — whose definitions are clearly given in the text. Of the greater, or greatest, circles, our Author in this second chapter explains only six, namely the Equinoctial circle, the Zodiac, the Colure of the Solstices, the Colure of the Equinoxes, the Meridian, and the Horizon; and of the smaller, or not-greatest, circles, he declares only four, namely the Tropic of Cancer [♋], the Tropic of Capricorn [♐], the Arctic circle, and the Antarctic circle. These ten circles of the sphere we did indeed set out briefly in the first chapter; but now, together with the Author, more will have to be said of them.

[Margin: *Auctor 10. tantum circulos sphærae considerat.* — "The Author considers only ten circles of the sphere."]

The Astronomers, however, in order to attain a perfect knowledge of the celestial motions, devised many others besides those ten primary circles, some greatest and some not greatest. Among the greatest, the foremost place is held by those that now follow. The VERTICAL circles, which are drawn through the vertex of any given place to the individual points of the Horizon. The HORARY (hour) circles, which cut the whole heaven into 24 hours; and these are of three kinds. For either they divide the heaven into 24 [unequal] hours ... through the poles of the world; or into 24 equal hours, beginning from the Meridian, in which way they proceed along parallel circles, one of which is always the greatest of those that appear and the other the greatest of those that are hidden; or, finally, into 24 unequal hours — namely when they neither proceed through the poles of the world nor touch the said parallels, but divide all the segments of the parallels above the Horizon, and likewise those below the Horizon, into 12 equal parts. But since, as we demonstrated in Book 1 of the *Astrolabe*, Lemma 39, there are no greatest circles that divide all the semidiurnal arcs into 12 equal parts, only two kinds of horary circles will have to be established. Of this variety of hours, however, we shall say more in the third chapter, when we deal with natural and artificial days. The circles of the celestial HOUSES, which cut the whole heaven into 12 parts, called the celestial houses. The circles of POSITION, which are defined as passing through the common intersections of the Horizon and the Meridian, and also through the center of each star. The circles of DECLINATION, which are drawn through the poles of the world and the individual points of the Equator. The circles of LATITUDE, which are described through the poles of the Zodiac and the individual points of the Ecliptic. Finally, very many other circles are found among the Astronomers. For, to pass over the greatest, there are considered to be almost infinitely many not-greatest circles. For every greatest

circle has its own parallels: as the Horizon has parallel circles described about the vertex of the head, which are commonly called circles of altitude; the Equator has parallel circles described about the poles of the world, of which kind are those which the individual stars and planets, or any points of the heaven whatever, describe daily by the diurnal motion. The Zodiac too has its own parallels described about the poles of the Zodiac, of which kind are those which the individual stars and planets, or any points of the heaven, trace out by the proper motion of the eighth Sphere from West to East. And the same is to be said of the other greatest circles. But of all these circles we must treat in another place: for it will now be enough for us to expound in this second chapter those first ten circles, which are called primary, since these properly belong to the sphere.

[Margin: *Verticales circuli. Horarij circuli.* — "Vertical circles. Horary circles."]

[Margin: *Circuli domorum cœlestium, & positionum. Circuli declinationum, & latitudinum.* — "Circles of the celestial houses, and of position. Circles of declination, and of latitude."]

Those circles in the sphere which share the same center as the sphere are called *greatest*, or *greater*, because, as Theodosius [Theodosius] demonstrates in Book 1, Proposition 6, the circles that are drawn through the center of the sphere are the greatest of all, so that none greater than they can be given — just as the line drawn through the center in any circle, namely the diameter, is the greatest of all. But those circles whose center is different from the center of the sphere are called *not-greatest*, or *smaller*, because, as Theodosius demonstrates in the place cited, circles that are not drawn through the center of the sphere are smaller than those that pass through the center of the sphere, and the farther they are from the center of the sphere, the smaller they become.

[Margin: *Maximi circuli & non maximi in sphæra cur sic dicti. 25. tertij.* — "Why the greatest and not-greatest circles in the sphere are so called. (III.25.)"]

Now, so that the things to be said about the celestial circles may be understood more perfectly, I shall bring forward a number of properties of the circles of the sphere — both of the greater and of the smaller — demonstrated by Theodosius in his *Elements of Spherics*. From these, indeed, much will have to be demonstrated in what follows.

I.

All the greatest circles of the sphere cut one another in half; and conversely, circles in the sphere that cut one another in half are the greatest. The first Theodosius demonstrates in Book 1, Proposition 11; the second in Proposition 12 of the same book.

[Margin: *Proprietates nonnullæ circularum in sphæra*. — "Certain properties of the circles in the sphere."]

II.

All the greatest circles of the sphere are equal to one another. This indeed is easily established from the equality of their diameters: for the diameter of any greatest circle is the same as the diameter of the sphere. Moreover, if one were greater than another, they would not both be greatest; for the smaller of them would not be greatest, since another greater than it is given.

III.

Not-greatest circles in the sphere that cut one another do not cut one another in half. For if they were to cut one another in half, they would themselves — by Proposition 17 of Book 1 of Theodosius — be greatest circles, which is contrary to the hypothesis. It is possible, however, for one of them sometimes to be divided in half; but when this happens, the other will by no means be cut in half, unless both circles are greatest.

III.

Among the not-greatest circles of the sphere, only those are equal to one another which are equally distant from the center of the sphere; and conversely, not-greatest circles equal to one another are equally distant from the center of the sphere. Both are demonstrated by Theodosius in Book 1, Proposition 6.

V.

Every greatest circle in the sphere passing through the poles of another circle — whether greatest or not-greatest — divides it in half and at right angles. And conversely, a circle in the sphere dividing another circle in half and at right angles is a greatest circle, and it passes through the poles of that other. The former Theodosius demonstrates in Book 1, Proposition 15; the latter we have demonstrated in the scholium of the same proposition, Theorem 3.

Diagram. A large woodcut armillary sphere set into the text between propositions V and VI, showing the great and small circles of the sphere with the small terrestrial globe at the center. The graduated meridian ring and horizon

ring carry degree-scales numbered 10, 20, 30, 40, 50, 60, 70, 80, 90 (repeating by quadrant). The oblique ecliptic band bears the zodiacal signs with 10°/20°/30° subdivisions.

- POLVS SEPTENTRIONALIS (top, radiating from a star): North Pole
- ARCTICVS (upper region, small circle): Arctic Circle
- TROPICVS ÆSTIVVS (upper oblique circle; printed "TROP·ICVS ESTIVVS"): Summer Tropic (Tropic of Cancer)
- ÆQVINOCTIALIS (central great circle; printed "...VINOCTIALIS," with "ÆQ" abbreviated): Equinoctial (Equator)
- ORIZON (central horizontal ring; printed "ORIZON"): Horizon
- Ecliptic/zodiac band signs, reading along the band: ARIES, TAVRVS [♉], GEMINI [♊] ... PISCES [♐], AQVARIVS [♑], each with degree graduations 10 · 20 · 30
- TROPICVS HIEMALIS (lower oblique circle): Winter Tropic (Tropic of Capricorn)
- ANTARCTICVS (lower region, small circle; printed "ANTARC·TICVS"): Antarctic Circle
- POLVS AVSTRALIS (bottom, radiating from a star): South Pole
- (center): small globe of the Earth

VI.

Every greatest circle in the sphere, through whose poles another greatest circle in the sphere passes, passes in turn through the poles of that other. This we have demonstrated in Theorem 1 of the scholium on Proposition 15, Book 1, of Theodosius.

VII.

A greatest circle in the sphere which touches some not-greatest circle will also touch another not-greatest circle equal and parallel to it. This indeed Theodosius shows in Book 2, Proposition 6.

VIII.

A greatest circle in the sphere cutting not-greatest circles not through their poles — that is, obliquely — cuts them into unequal parts, yet in such a way that the alternate segments of equal and parallel circles are equal to one another. This is clear from Proposition 19, Book 2, of Theodosius.

IX.

When three greatest circles in the sphere cut one another mutually at right angles, the two poles of each of them will lie precisely on the common intersections of the circumferences of the other two. And conversely, when there are greatest circles in the sphere such that the two poles of any one of them are found on the common intersections of the other two, they will cut one another mutually at right angles. Both of these can easily be deduced from Theodosius, or from the properties already adduced — namely from the fifth and the sixth.

An example of each you also have in the material sphere. For if the Equator, the Meridian, and the Horizon [so ...]

so arranged that they cut one another at right angles (which happens only when each pole of the world lies precisely on the horizon, as occurs in the right sphere), you will see that the poles of the Equator lie at the common intersections of the Meridian and the Horizon; the poles of the Meridian, at the common intersections of the Equator and the Horizon; and finally the poles of the Horizon, at the common intersections of the Equator and the Meridian, and so on. I have cited these propositions of Theodosius [Theodosius] in accordance with the Greek exemplar — the same one by which I earlier published Theodosius, together with the treatise on triangles and on sines — where I relegated to the scholia those propositions that the Arabs added.

[Margin: How Proclus divides the circles of the sphere.]

Proclus [Proclus], in the *Sphere* that he composed, set up a different division of the circles of the sphere. For he does not divide those primary circles into greatest and not-greatest, but into equidistant (or parallel) circles, into oblique ones, and into those drawn through the poles of the world. He calls "equidistant" those circles whose poles are the same as the poles of the world; of this kind are five circles in the sphere — namely the Equator, the Tropic of Cancer ☿ [Cancer], the Tropic of Capricorn ♄ [Capricorn], the Arctic Circle, and the Antarctic Circle. For these circles are equidistant among themselves, as is established from proposition 2 of book 2 of Theodosius (*Spherics* II.2). He calls "oblique" those which cut the parallel circles they intersect at unequal and oblique angles; such, in his view, are the Zodiac and the Milky Way, to which must be added any oblique Horizon. Finally, he says that those are "drawn through the poles of the world" which divide the parallel (or equidistant) circles at right angles and into two halves; and these are three in number: the Colure of the Solstices, the Colure of the Equinoxes, and the Meridian, to which the right Horizon can be joined.

Diagram. An armillary sphere representing the material sphere with its ten primary circles, referred to just below as the "figure set out above" (*supra proposita figura*). A small terrestrial globe is drawn at the center. A graduated meridian ring runs through the two poles, marked with degree numbers in ascending-then-descending quadrants (10, 20, 30, 40, 50, 60, 70, 80, 90, 80, 70, 60, 50, 40, 30, 20, 10). Readable labels:

- POLVS SEPTENTRIONALIS (North Pole) — upper left, at the top of the meridian ring
- POLVS AVSTRALIS (South Pole) — lower right
- ARCTICVS (Arctic Circle)
- TROPICVS ÆSTIVVS (Summer Tropic; "ÆSTIVVS" = of summer) — upper zone
- ÆQVINOCTIALIS (Equator) — the central ring
- ORIZON (Horizon; spelled without initial H)
- TROPICVS HIEMALIS (Winter Tropic)
- ANTARCTICVS (Antarctic Circle)
- Zodiac band (oblique), with signs and degree scales (10, 20, 30 within each):
ARIES ♈, TAVRVS ♉ [Taurus], GEMINI ♊ [Gemini]
- On the opposite side of the zodiac band: AQVARIVS ♊ [Aquarius], PISCES ♋ [Pisces] [?]

[Margin: Another division of the circles of the sphere.]

Some others divide the celestial circles on a different principle. For they say that some circles are intrinsic, others extrinsic. The intrinsic ones are those conceived as wholly fixed in the heaven, so that they are carried around together with it; hence by some they are called movable — such are all the primary circles of the sphere except the Meridian and the Horizon. For these two are called extrinsic, because they are to be conceived in the heaven in such a way as always to keep a firm position, and in no way to revolve with the motion of the heaven, but always to remain in the same place. For this reason they have been called immovable by most writers.

An example of the ten circles of the sphere, which are called primary, you have in the figure set out above, which represents the material sphere.

ON THE EQUINOCTIAL CIRCLE

The equinoctial, then, is a certain circle dividing the sphere into two equal parts, equidistant from each pole according to every part of itself.

COMMENTARY

[Margin: What the Equinoctial circle is.]

Having completed the first part of this chapter, he now proceeds to the second, in which the individual circles are discussed one by one. He treats first of the greatest circles, then of the not-greatest; and among the greatest he explains the Equinoctial first, because knowledge of it is easier and nearly all the rest are explained through it. The Equinoctial is also the noblest of all circles, since it is the measure — as will presently be said — of the noblest motion, namely that of the primum mobile; for it moves with the most uniform motion. Hence this circle stands, when compared with the other celestial circles, just as the primum mobile does when compared with the other celestial [orbs

orbs]. For this reason the philosophers placed the prime mover — that is, God, the best and greatest — in the Equinoctial circle, as in his proper seat.

He defines the Equinoctial circle, saying that it is the circle which in the material sphere is called Equinoctial, which divides the sphere into two equal parts and is equidistant from each pole according to every part of itself. And this circle, by the same reasoning, must be conceived in the heaven as placed midway between the two poles of the world.

[Margin: How the Equinoctial circle is conceived to be described in the heaven.]

Some indeed conceive it to be described as follows. From the center of the world, through the center of the Sun while it is at the beginning of Aries ♈ [Aries] or Libra ♎ [Libra], they imagine a straight line drawn, which in the space of 24 hours would describe the Equinoctial circle. But since the Sun never completes an entire circle — because it does not return to the same point, on account of its proper motion, which it has from west to east — it will perhaps be better said that the Equator is described by a straight line extended from the center of the world to the beginning of Aries ♈ or Libra ♎ of the primum mobile. For from the revolution of this line there will be described, in a natural day, a greatest and perfect circle, always at right angles to the axis of the world and equidistant on every side from the poles of the world — all of which are required for the equinoctial circle.

[Margin: Where, above all, the circles of the sphere are to be conceived in the heaven.]

Now all the celestial circles — and hence the Equinoctial too — are to be conceived in the primum mobile, which is what the material sphere chiefly

represents for us. It makes little difference whether we understand them as on the concave or on the convex surface of the primum mobile. Yet because we, enclosed within the heaven and existing at its center, look upon the concave surface of the heaven, we are in a way compelled to consider the celestial circles on that same concave surface of the primum mobile; just as, because we are outside the material sphere, we are obliged in a way to mark those circles on its outermost or convex surface. This is also done on the cosmographical and astronomical globe. And since, of the ten primary circles of the sphere, the Meridian and the Horizon are wholly immovable in every region — so that, even if the first heaven is perpetually and unceasingly carried around, these two circles are nonetheless conceived as entirely unmoved and fixed, while the other eight are movable, inasmuch as they are continually revolved with the primum mobile — it will not be inappropriate if we consider these eight movable circles on the convex surface of the primum mobile, and those two on the concave surface of the immovable Empyrean heaven, beneath which the primum mobile and the whole world are placed. For thus it will come about that the other, movable circles are perpetually carried around within these immovable ones; just as in the material sphere too we see the Meridian and the Horizon standing out above the other circles, so that, while these others move without ceasing, those two remain wholly unmoved.

And it is called the equinoctial because, when the Sun passes through it (which happens twice a year, namely at the beginning of Aries and at the beginning of Libra), there is equinox over the whole earth. Hence it is also called the equator of day and night, because it equalizes the artificial day with the night. And it is to be understood that the first motion is called the motion of the primum mobile — that is, of the ninth sphere or outermost heaven — which goes from east through west, returning again to the east; and this is also called rational motion, by analogy with the motion of reason that is in the microcosm, that is, in man, namely when consideration proceeds from the Creator, through creatures, back into the Creator, coming to rest there. The second motion is that of the firmament and the planets, contrary to this one, going from west through east and returning again to the west; and this motion is called irrational or sensual, by analogy with the motion of the microcosm, which goes from corruptible things to the Creator and then returns again to corruptible things. It is therefore called the girdle of the first motion, because it girds or divides the primum mobile — namely the ninth sphere — into two equal parts, equidistant from the poles of the world.

[Margin: Why the Equinoctial circle is so called; likewise why it is called the Equator and the girdle of the primum mobile.]

He explains in this place the names and functions of the Equinoctial circle, teaching that it is called Equinoctial because, when the Sun passes through it — namely at the beginning of Aries ♈ and Libra ♎ — it produces equinox over the whole earth, that is, it makes the artificial day equal to the artificial night.

For the same reason he says it is called the Equator of day and night. Likewise, that it is named the girdle of the first motion, because it divides the first motion into two equal parts. For since a motion is divided in accordance with the division of the movable body, as the philosophers hold, the Equator will assuredly divide the motion of the primum mobile, seeing that it also divides the primum mobile itself into two halves. For the sake of this point he repeats that twofold motion of the heavens — namely from east to west, and from west to east — as is clear in the text.

The Greeks call this circle ἰσημερινόν [Greek: ἰσημερινός (isēmerinos), "equinoctial," literally "of equal day"], that is, "equidial," because the day is equal to the night. Hence, just as the Latins name it from the day, so it pleased the Greeks to give it a name from the night. By Ptolemy [Ptolemy] it is called the Line, Circle, or orb of the Equation of Day. By Alfraganus [al-Farghānī], the Circle of the Equinox. Most writers also hold that it is called by these names not because the Sun, when in it, produces equinox everywhere, but because in the right sphere, which lies beneath it, the nights never exceed the length of the artificial days, but the days are perpetually equal to the nights, wherever the Sun may be, as we shall explain in chapter 3. The Equinoctial circle is also sometimes called by astronomers the greatest of the parallels. For they call "parallel circles" those which the stars and the individual points of the heaven describe in the diurnal motion, of all which the greatest, as is established, is the Equator.

[Margin: How it is to be understood that equinox occurs twice a year over the whole earth.]

As for what is commonly said — that over the whole earth equinox occurs twice a year, namely when the Sun is at the beginning of Aries ♈ and Libra ♎ — this must be understood of places where the alternation of day and night occurs within the space of 24 hours, that is, where the Equinoctial circle intersects the Horizon and is intersected by it. I have said this in order to exclude from this universal proposition those regions that lie directly beneath the poles of the world. For in those regions the day — which is only a single one in the whole year — lasts six months, and the night as many, as will be established near the end of chapter 3. Or else that common proposition must be understood negatively, as if it were said that the day is not unequal to the night — which is indeed true even

under the poles, when the Sun is in the Equinoctial circle, because then the day is not unequal to the night.

[Margin: Why, when the Sun is in the Equator, equinox occurs.]

In all those regions in which the Equator and the Horizon intersect one another, that equinox occurs while the Sun tarries in the Equator can easily be demonstrated on this principle. Since each circle — the Equator, that is, and the Horizon — is a greatest circle, each bisects the other, by proposition 11 of book 1 of Theodosius (*Spherics* I.11), as was said above. Therefore in any region whatever where these two circles cut one another, one half of the Equator will be above the Horizon and the other below. Since, then, the Sun is carried uniformly from east to west, it comes about that it spends as much time above the hemisphere — which delay makes the artificial day — as below the hemisphere, which delay constitutes the artificial night.

[Margin: Why the pole that always appears to us is called Septentrional, Arctic, and Boreal; and the opposite one, Antarctic, Meridional, and Austral.]

Hence it must be noted that the pole of the world which always appears to us is called the Septentrional, Arctic, or Boreal pole. It is called Septentrional from Septentrio, that is, from the Lesser Bear, which is named from septem ("seven") and trio ("a plough-ox"), because the seven stars that are in the Bear move slowly, after the manner of an ox, being near the pole. Or those seven stars are called Septentriones, as if "seven teriones," from the fact that they tread the parts around the pole. It is called Arctic from ἄρκτος [Greek: ἄρκτος (arktos), "bear"], which means "bear"; for it is next to the Greater Bear. It is called Boreal because it is in that quarter from which Boreas comes. The opposite pole is called Antarctic, as if placed over against the Arctic. It is also called Meridional, because it is on the side of the south. It is likewise called Austral, because it is in that quarter from which Auster comes. These two points, fixed in the firmament, are called the poles of the world, because they bound the axis of the sphere, and about them the world is turned — of which one always appears to us, while the other is always hidden. Hence Virgil [Virgil], Georgics I:

This summit is ever high above us; but the other, beneath our feet, black
Styx and the deep shades behold.

COMMENTARY

He here declares the poles of the Equinoctial circle, from which he had said the Equinoctial circle itself is equally distant. But all these matters are clear in the

text. It remains for me to explain the manifold use, the functions, and the utilities on account of which astronomers devised the Equinoctial circle in the heaven.

THE FUNCTIONS OF THE EQUINOCTIAL CIRCLE

I.

[Margin: The Equator is the measure and rule of the first motion.]

It is the measure and rule of the first motion. For it shows that the primum mobile revolves in the space of 24 hours, since in each hour 15 degrees of the Equinoctial circle described in the primum mobile are raised uniformly above the Horizon, as the observations of astronomers teach.

II.

[Margin: The Equator measures time.]

It measures time. For from one revolution of the Equinoctial circle, with the addition of a small part corresponding to that portion of the Zodiac which meanwhile the Sun completes by its proper motion toward the east, the natural day is constituted, as will be said in chapter 3. From the rising of 15 degrees of it we know that a whole hour has passed; and finally, from the ascension of a single degree, we perceive that 4 minutes of an hour have elapsed.

III.

[Margin: The Equator reduces to regularity the irregularity of the Zodiac's motion from east to west.]

It directs, as a most certain rule and canon, the irregularity of the motion of the Zodiac from east to west, which the Zodiac has on account of its oblique position. For, as will be established from chapter 3, equal parts of the Zodiac ascend unequally above any Horizon whatever, whether right or oblique. Hence this whole inequality is, by a wonderful device, reduced by astronomers to equality through the uniform motion of the Equinoctial circle, so that from the neighboring arcs of the Equinoctial circle we come to know the times of the rising and setting of all the arcs of the Zodiac.

IIII.

[Margin: The Equator produces the equinoxes.]

It distinguishes the equinoxes. For it divides the Zodiac circle obliquely at two points, namely at the beginning of Aries ♈ and Libra ♎, at which, when the Sun arrives by its proper motion, it makes the spaces of day and night equal. Hence these points too are called equinoctial by astronomers. Manilius [Manilius] the poet elegantly describes them, saying:

Libra and Aries render night and day equal.

On what days of the year the two equinoxes formerly occurred, and on what days they occur at the present time, we shall disclose when we treat of the Colures.

V.

[Margin: The Equator is the terminus from which declinations are numbered. — What declination is. — The Equator separates the northern part of the heaven from the southern.]

It is the terminus from which all the declinations of the points of the Ecliptic, and of the stars, take their beginning. For declination is the distance of a star, or of a point of the Ecliptic, from the Equator toward either of the poles of the world. But by what means this distance, or declination, is to be taken and measured, we shall say when we treat of the Ecliptic.

VI.

[Margin: Which is the northern part of the heaven and which the southern. — Which stars or signs are called northern or southern.]

It indicates which part of the heaven is called Septentrional or Boreal, and which Austral or Meridional. For that part which lies between the Septentrional (or Arctic) pole and the Equinoctial circle is called Septentrional; while the remaining part, which is placed between the same Equinoctial circle and the Austral (or Antarctic) pole, is called Meridional. From this it can easily be perceived which stars, or which constellations or signs, are called Septentrional or Austral; likewise when the planets are called Septentrional and when Austral. For whenever they are in that part of the heaven which we have said is called Septentrional, they are called Septentrional; and when they are in that which we have named Austral, they are called Austral. Hence while the Sun moves from the beginning of Aries ♈ up to the beginning of Libra ♎, it is called Septentrional; but while it moves from the beginning of Libra ♎ to the beginning [of Aries...

the beginning of Aries (♈), it is customarily called the Meridional, or Southern, part. Astronomers do, however, take "northern part" in another sense as well, as I

shall explain when I treat of the uses of the ecliptic. But this is the principal sense of "northern part" and "southern part" among authorities. Indeed, among cosmographers too the Equator, traced upon the earth, divides the whole earth into a Boreal (northern) and an Austral (southern) part.

[Margin: The Equator, on the earth, divides the whole earth into a Boreal and an Austral part.]

VII.

The Equator fixes for us the length, or quantity, of the artificial day and night in any region of the terrestrial globe and at any time of year. For in any region and at any time of year the artificial day is as long as the arc of the equinoctial circle that ascends above the hemisphere while the Sun lingers above that same hemisphere. This arc of the Equator is determined in the following way, using a material sphere properly and accurately constructed. Set the material sphere in its proper position — that is, at the due elevation of the pole — and place the degree of the ecliptic in which the Sun stands on the given day upon the horizon on the eastern side, and carefully mark the point of the Equator that then stands on the horizon on that same side. Then rotate the sphere until that same degree of the ecliptic reaches the western side, and until that point of the Equator, about half a degree further on, is again found on the horizon — while on the eastern side [that first point] is seen to touch the horizon precisely and exactly. This done, count the degrees of the equinoctial circle lying between those two points, beginning from the first point and proceeding toward the east. For the said degrees of the Equator will yield the diurnal arc in question — that is, the arc which, together with the Sun, while it lingers in the upper hemisphere, emerges above the horizon. Therefore, if the aforesaid arc is divided by 15, the hours contained in that day will at once result, provided you remember that the degrees which perhaps remain over from the division each comprise four minutes of an hour.

[Margin: The Equator indicates the length of the artificial day and night.]

[Margin: How great the artificial day is, and how it is determined from the material sphere.]

EXAMPLE. With the Sun at the beginning of Cancer (♋), if the material sphere is so set that 42 degrees of the meridian are intercepted between the Arctic pole and the horizon — that being the number of degrees by which the Arctic pole is raised above the horizon at Rome — and the first degree of Cancer (♋) is placed on the horizon both on the eastern and on the western side, and the two points on the

Equator are marked, the diurnal arc will be found to comprise about 226 degrees 6 minutes. Reduced to hours by dividing by 15, this shows that the artificial day at Rome on 22 June — when, that is, the Sun is at the beginning of Cancer (♋) — amounts to 15 hours and about 4 minutes. From the known length of the artificial day, the length of the artificial night is easily found. For if you subtract the artificial day from 24 hours — that is, from the whole natural day — the artificial night remains. On this reckoning, if 15 hours and 4 minutes are taken from 24 hours, the night at Rome on 22 June will comprise 8 hours and 56 minutes. Anyone may, however, if he wishes, derive the length of the night by the same procedure by which we have said the length of the day is to be investigated.

[Margin: How great the elevation of the pole at Rome is.]

VIII.

The Equator is wonderfully serviceable to cosmographers and geographers. For without the equinoctial circle no complete description of the earth is possible, nor can any city be set in its proper place on the terrestrial globe or on a map of the world. It is by reference to the equinoctial circle that both the longitude and the latitude of cities are determined, as I shall explain more clearly when I have treated of the meridian circle, which is likewise required for that task.

[Margin: The Equator is useful to cosmographers.]

Besides the many functions already mentioned, the equinoctial circle has still other functions and uses among astronomers, which for brevity's sake I think it best to pass over now. In their proper places, when the matter requires it, they will be explained much more conveniently. Let it now suffice to have set out its principal functions.

Since, however, in the seventh function of the Equator it was necessary to reduce degrees and minutes of the equinoctial circle to hours and minutes of hours, I have judged it useful to set out here two tables. By the first, the degrees, minutes, seconds, and thirds of the equinoctial circle are reduced with the greatest ease to hours, minutes, seconds, and thirds of hours; by the second, conversely, hours, minutes, seconds, and thirds of hours are transformed with equal ease into degrees, minutes, seconds, and thirds of the equinoctial circle. For although either operation can be carried out by division, the said tables accomplish the same thing much more expeditiously.

CONVERSION of the degrees, minutes, and seconds of the Equator into hours, minutes, seconds, and thirds.

The table is printed in three parallel column-groups, each headed **G.** (degrees) **H.** (hours) **M.** (minutes); the argument runs 1–30 in the first group, 31–60 in the second, and by tens 70–360 in the third. The two footer rows give the alternate sexagesimal reading (see parsing note).

G.	H.	M.	G.	H.	M.	G.	H.	M.
1	0	4	31	2	4	70	4	40
2	0	8	32	2	8	80	5	20
3	0	12	33	2	12	90	6	0
4	0	16	34	2	16	100	6	40
5	0	20	35	2	20	110	7	20
6	0	24	36	2	24	120	8	0
7	0	28	37	2	28	130	8	40
8	0	32	38	2	32	140	9	20
9	0	36	39	2	36	150	10	0
10	0	40	40	2	40	160	10	40
11	0	44	41	2	44	170	11	20
12	0	48	42	2	48	180	12	0
13	0	52	43	2	52	190	12	40
14	0	56	44	2	56	200	13	20
15	1	0	45	3	0	210	14	0
16	1	4	46	3	4	220	14	40
17	1	8	47	3	8	230	15	20
18	1	12	48	3	12	240	16	0
19	1	16	49	3	16	250	16	40
20	1	20	50	3	20	260	17	20
21	1	24	51	3	24	270	18	0
22	1	28	52	3	28	280	18	40
23	1	32	53	3	32	290	19	20
24	1	36	54	3	36	300	20	0
25	1	40	55	3	40	310	20	40
26	1	44	56	3	44	320	21	20
27	1	48	57	3	48	330	22	0

G.	H.	M.	G.	H.	M.	G.	H.	M.
28	1	52	58	3	52	340	22	40
29	1	56	59	3	56	350	23	20
30	2	0	60	4	0	360	24	0
<i>M.</i>	<i>M.</i>	<i>S.</i>	<i>M.</i>	<i>M.</i>	<i>S.</i>			
<i>S.</i>	<i>S.</i>	<i>T.</i>	<i>S.</i>	<i>S.</i>	<i>T.</i>			

CONVERSION of the hours, minutes, seconds, and thirds into degrees, minutes, and seconds of the Equator.

The table is printed in three parallel column-groups: the first headed **H.** (hours) **G.** (degrees), running 1–24; the second and third headed **M.** (minutes) **G.** (degrees) **M.** (minutes), running 1–30 and 31–60 respectively.

H.	G.	M.	G.	M.	M.	G.	M.
1	15	1	0	15	31	7	45
2	30	2	0	30	32	8	0
3	45	3	0	45	33	8	15
4	60	4	1	0	34	8	30
5	75	5	1	15	35	8	45
6	90	6	1	30	36	9	0
7	105	7	1	45	37	9	15
8	120	8	2	0	38	9	30
9	135	9	2	15	39	9	45
10	150	10	2	30	40	10	0
11	165	11	2	45	41	10	15
12	180	12	3	0	42	10	30
13	195	13	3	15	43	10	45
14	210	14	3	30	44	11	0
15	225	15	3	45	45	11	15
16	240	16	4	0	46	11	30
17	255	17	4	15	47	11	45
18	270	18	4	30	48	12	0
19	285	19	4	45	49	12	15
20	300	20	5	0	50	12	30

H.	G.	M.	G.	M.	M.	G.	M.
21	315	21	5	15	51	12	45
22	330	22	5	30	52	13	0
23	345	23	5	45	53	13	15
24	360	24	6	0	54	13	30
		25	6	15	55	13	45
		26	6	30	56	14	0
		27	6	45	57	14	15
		28	7	0	58	14	30
		29	7	15	59	14	45
		30	7	30	60	15	0
<i>S.</i>	<i>T.</i>	<i>S.</i>	<i>M.</i>	<i>S.</i>	<i>S.</i>	<i>M.</i>	<i>S.</i>
		<i>T.</i>	<i>S.</i>	<i>T.</i>	<i>T.</i>	<i>S.</i>	<i>T.</i>

THE USE OF THE TWO PRECEDING TABLES.

[Margin: By what method degrees and minutes are reduced to hours from the following tables, and conversely.]

If degrees are to be converted into hours, take the degrees in the first table under the heading **G**, and the two columns immediately following will show the hours and minutes of hours due to the degrees taken. Thus you see that to 4 degrees correspond 16 minutes of an hour; likewise to 27 degrees, 1 hour 48 minutes; likewise to 45 degrees, 3 hours 0 minutes; likewise to 250 degrees, 16 hours 40 minutes; and so on. But if the number of degrees is not found exactly in the said table, take the nearest smaller number with its corresponding hours and minutes; then take the remaining degrees again with their corresponding hours and minutes; and finally join the latter hours and minutes to the former. For example, if you wish to know how many hours correspond to 215 degrees, take the 14 hours corresponding to 210 degrees; then take the 20 minutes corresponding to the remaining 5 degrees; and thus to 215 degrees are due 14 hours 20 minutes. And so for the rest.

If, however, minutes or seconds of degrees are to be converted into hours, take the minutes or seconds of degrees above the headings **M** or **S**, and the two columns immediately following will show the minutes, seconds, or thirds of hours, as the letters placed at the foot of the table indicate. On this reckoning you

see that to 56 minutes of a degree correspond 3 minutes 44 seconds of an hour; likewise that to 25 seconds of a degree are due 1 second 40 thirds of an hour.

In just the same way, from the latter table hours, minutes, seconds, and thirds of hours are reduced to degrees, minutes, seconds, and thirds of the equinoctial circle.

But if anyone should be unwilling to use tables of this kind, degrees, minutes, and so forth may be reduced to hours, minutes, seconds, and so forth — and, conversely, hours, minutes, and so forth to degrees, minutes, and so forth — in this way. Multiply the degrees, minutes, seconds, and so forth by 4; for the resulting numbers will give the next-smaller parts of time. Thus the product from degrees will give minutes of hours, the product from minutes of degrees will give seconds of hours, and so on. EXAMPLE. If 9 degrees 40 minutes 20 seconds are multiplied by 4, there result 0 hours 36 minutes 160 seconds 80 thirds — that is, 0 hours 38 minutes 41 seconds 20 thirds. Again, if 20 degrees 40 minutes are multiplied by 4, there are produced 0 hours 80 minutes 160 seconds — that is, 1 hour 22 minutes 40 seconds. And so for the rest.

[Margin: In what manner hours and minutes are made from degrees and minutes, and conversely how degrees and minutes are made from hours and minutes.]

Now, on the other hand, if hours, minutes, and so forth are divided by 4, the next-larger parts of the Equator are produced. Thus from thirds of hours are produced seconds of degrees; from seconds of hours, minutes of degrees; from hours, finally, degrees; while from minutes of hours are produced "parts" of one part of the Equator, each of which comprises 60 degrees, just as one degree comprises 60 minutes. EXAMPLE. If 0 hours 38 minutes 41 seconds 20 thirds are divided by 4, there are produced 0 parts (each of which comprises 60 degrees), $9\frac{1}{2}$ degrees, $10\frac{1}{4}$ minutes, 5 seconds — that is, 0 parts, 9 degrees, 40 minutes, 20 seconds. For $\frac{1}{2}$ degree makes 30 minutes, which together with the 10 minutes make 40 minutes; likewise $\frac{1}{4}$ minute makes 15 seconds, which together with the 5 seconds make 20 seconds. Again, if 1 hour 22 minutes 40 seconds are divided by 4, there result $\frac{1}{4}$ part (of those each of which comprises 60 degrees), $5\frac{1}{2}$ degrees, 10 minutes — that is, 20 degrees; because $\frac{1}{4}$ part (of those each of which contains 60 degrees) makes 15 degrees, which together with the 5 degrees make 20 degrees; likewise $\frac{1}{2}$ degree makes 30 minutes, which together with the 10 minutes make 40 minutes. And so for the rest.

It is also a pleasant thing to know that when we wish to divide degrees, minutes, seconds, and so forth — or even hours — by 6, that is, to take the sixth part, this is done most briefly by appending a cipher, that is, by multiplying by 10. For by this means are generated the next-smaller parts, which are one sixth of the parts we wished to divide by 6: thus from degrees come minutes, from minutes seconds, and so on. Accordingly the sixth part of 9 degrees, or hours, will be 90 minutes — that is, 1 degree, or 1 hour, and 30 minutes besides.

I should also like to append here a further, fourfold table, very useful in astronomical matters. By the first, degrees, minutes, seconds, thirds, and so forth of the Equator are converted into minutes, seconds, and thirds of Days. By the second, minutes, seconds, thirds, and so forth of Days are converted into degrees, minutes, seconds, thirds, and so forth of the Equator. By the third, hours, minutes, seconds, thirds, and so forth are converted into minutes, seconds, thirds, and so forth of Days. By the fourth, finally, minutes, seconds, thirds, and so forth of Days are transformed into hours, minutes, seconds, thirds, and so forth. The use of all of them is the same as that of the two preceding tables.

These, then, are the tables that follow.

Conversion tables (concluding the Equator)

The two facing pages present the promised "fourfold" (*quadruplex*) conversion tables — reference tables that convert between arc of the Equator (or time) and fractions of a day, each printed within a decorative woodcut border. Each table has a left panel (converting *into* days) and a right panel (converting *out of* days). Both panels carry, beneath the numeric columns, a **unit-shift key** (rows of abbreviations G./M./S./T./Q. = *Gradus/Minuta/Secunda/Tertia/Quarta*, or H./D. = *Horæ/Dies*): the same digits serve at successively finer sexagesimal levels by reading the column headings shifted down one place.

Table 1 (printed p. 130) — Equator ↔ Days

Left panel — heading: *Conversio Graduum, Minutorum, Secundorum, Tertiorum, &c. Aequatoris in Minuta, Secunda, Tertia, &c. Dierum.* ("Conversion of Degrees, Minutes, Seconds, Thirds, etc. of the Equator into Minutes, Seconds, Thirds, etc. of Days.")

Columns: **G.** (degrees of the Equator) | **M. dierū** (minutes of days) | **S. dierū** (seconds of days). Each degree = 10 seconds-of-a-day.

G.	M. (dierū)	S. (dierū)
1	0	10
2	0	20
3	0	30
4	0	40
5	0	50
6	1	0
7	1	10
8	1	20
9	1	30
10	1	40
11	1	50
12	2	0
13	2	10
14	2	20
15	2	30
16	2	40
17	2	50
18	3	0
19	3	10
20	3	20
21	3	30
22	3	40
23	3	50
24	4	0
25	4	10
26	4	20
27	4	30
28	4	40
29	4	50
30	5	0
31	5	10

G.	M. (dierū)	S. (dierū)
32	5	20
33	5	30
34	5	40
35	5	50
36	6	0
37	6	10
38	6	20
39	6	30
40	6	40
41	6	50
42	7	0
43	7	10
44	7	20
45	7	30
46	7	40
47	7	50
48	8	0
49	8	10
50	8	20
51	8	30
52	8	40
53	8	50
54	9	0
55	9	10
56	9	20
57	9	30
58	9	40
59	9	50
60	10	0

Continuation of the same panel, argument by tens (70–360):

G.	M. (dierū)	S. (dierū)
70	11	40
80	13	20
90	15	0
100	16	40
110	18	20
120	20	0
130	21	40
140	23	20
150	25	0
160	26	40
170	28	20
180	30	0
190	31	40
200	33	20
210	35	0
220	36	40
230	38	20
240	40	0
250	41	40
260	43	20
270	45	0
280	46	40
290	48	20
300	50	0
310	51	40
320	53	20
330	55	0
440 [recte: 340]	56	40
350	58	20
360	60	0

Unit-shift key printed at the foot of the left panel — reading the three columns (G. | M. | S.) at finer levels: **(M. | S. | T.)**, then **(S. | T. | Q.)**, then **(T. | Q.)**.

Right panel — heading: *Conversio Graduum, Minutorum, Secundorum, Tertiorum, &c. Dierum, in Gradus, Minuta, Secunda, Tertia, &c. Aequatoris.*
 ("Conversion of Degrees, Minutes, Seconds, Thirds, etc. of Days into Degrees, Minutes, Seconds, Thirds, etc. of the Equator.")

Columns: **M.** (minutes of days) | **G.** (degrees of the Equator). Each minute-of-a-day = 6 degrees.

M.	G.
1	6
2	12
3	18
4	24
5	30
6	36
7	42
8	48
9	54
10	60
11	66
12	72
13	78
14	84
15	90
16	96
17	102
18	108
19	114
20	120
21	126
22	132
23	138

M.	G.
24	144
25	150
26	156
27	162
28	168
29	174
30	180
31	186
32	192
33	198
34	204
35	210
36	216
37	222
38	228
39	234
40	240
41	246
42	252
43	258
44	264
45	270
46	276
47	282
48	288
49	294
50	300
51	306
52	312
53	318
54	324

M.	G.
55	330
56	336
57	342
58	348
59	354
60	360

Unit-shift key printed at the foot of the right panel — reading the two columns (M. | G.) at finer levels: **(S. | M.)**, then **(T. | S.)**, then **(Q. | T.)**.

Table 2 (printed p. 131) — Hours / minutes-of-time ↔ Days

Left panel — heading: *Conversio Horarum, Minutorum, Secundorum, Tertiorum, &c. in Minuta, Secunda, Tertia, &c. Dierum.* ("Conversion of Hours, Minutes, Seconds, Thirds, etc. into Minutes, Seconds, Thirds, etc. of Days.")

First sub-column (**Hours**): columns **H.** (hours) | **D.** (days) | **M.** (minutes of days) | **S.** (seconds of days). Each hour = 2 minutes 30 seconds of a day.

H.	D.	M. (dierū)	S. (dierū)
1	0	2	30
2	0	5	0
3	0	7	30
4	0	10	0
5	0	12	30
6	0	15	0
7	0	17	30
8	0	20	0
9	0	22	30
10	0	25	0
11	0	27	30
12	0	30	0
13	0	32	30
14	0	35	0
15	0	37	30

H.	D.	M. (dierū)	S. (dierū)
16	0	40	0
17	0	42	30
18	0	45	0
19	0	47	30
20	0	50	0
21	0	52	30
22	0	55	0
23	0	57	30
24	1	0	0

Adjacent sub-columns (**minutes of an hour**), argument 1–60: columns **M. horar.** (minutes of an hour) | **M. dierū** (minutes of days) | **S. dierū** (seconds of days) | **T. dierū** (thirds of days).

M. (horar.)	M. (dierū)	S. (dierū)	T. (dierū)
1	0	2	30
2	0	5	0
3	0	7	30
4	0	10	0
5	0	12	30
6	0	15	0
7	0	17	30
8	0	20	0
9	0	22	30
10	0	25	0
11	0	27	30
12	0	30	0
13	0	32	30
14	0	35	0
15	0	37	30
16	0	40	0
17	0	42	30
18	0	45	0

M. (horar.)	M. (dierū)	S. (dierū)	T. (dierū)
19	0	47	30
20	0	50	0
21	0	52	30
22	0	55	0
23	0	57	30
24	1	0	0
25	1	2	30
26	1	5	0
27	1	7	30
28	1	10	0
29	1	12	30
30	1	15	0
31	1	17	30
32	1	20	0
33	1	22	30
34	1	25	0
35	1	27	30
36	1	30	0
37	1	32	30
38	1	35	0
39	1	37	30
40	1	40	0
41	1	42	30
42	1	45	0
43	1	47	30
44	1	50	0
45	1	52	30
46	1	55	0
47	1	57	30
48	2	0	0
49	2	2	30

M. (horar.)	M. (dierū)	S. (dierū)	T. (dierū)
50	2	5	0
51	2	7	30
52	2	10	0
53	2	12	30
54	2	15	0
55	2	17	30
56	2	20	0
57	2	22	30
58	2	25	0
59	2	27	30
60	2	30	0

Unit-shift key at the foot of this panel: **(S. | S. | T. | Q.)**, then **(T. | T. | Q.)**, then **(Q. | Q.)**.

Right panel — heading: *Conversio Minutorum, Secundorum, Tertiorum, &c. Dierum in Horas, Minuta, Secunda, Tertia, &c.* ("Conversion of Minutes, Seconds, Thirds, etc. of Days into Hours, Minutes, Seconds, Thirds, etc.")

Columns: **M. dierū** (minutes of a day) | **H.** (hours) | **M.** (minutes of an hour). Each minute-of-a-day = 24 minutes of an hour.

M. (dierū)	H.	M.
1	0	24
2	0	48
3	1	12
4	1	36
5	2	0
6	2	24
7	2	48
8	3	12
9	3	36
10	4	0
11	4	24

M. (dierū)	H.	M.
12	4	48
13	5	12
14	5	36
15	6	0
16	6	24
17	6	48
18	7	12
19	7	36
20	8	0
21	8	24
22	8	48
23	9	12
24	9	36
25	10	0
26	10	24
27	10	48
28	11	12
29	11	36
30	12	0
31	12	24
32	12	48
33	13	12
34	13	36
35	14	0
36	14	24
37	14	48
38	15	12
39	15	36
40	16	0
41	16	24
42	16	48

	M. (dierū)	H.	M.
	43	17	12
	44	17	36
	45	18	0
	46	18	24
	47	18	48
	48	19	12
	49	19	36
	50	20	0
	51	20	24
	52	20	48
	53	21	12
	54	21	36
	55	22	0
	56	22	24
	57	22	48
	58	23	12
	59	23	36
	60	24	0

Unit-shift key at the foot of the right panel: **(S. | M. | S.)**, then **(T. | S. | T.)**, then **(Q. | T.)**.

ON THE ZODIAC CIRCLE

(DE ZODIACO CIRCULO)

[Margin: What the Zodiac is.]

There is another circle in the sphere which cuts the Equinoctial and is cut by it into two equal parts; and one half of it inclines toward the North, the other toward the South.

COMMENTARY

After his treatment of the Equator, the author takes up the Zodiac in the second place, because knowledge of the remaining circles depends on an understanding

of this one. In describing the Zodiac circle, then, he says that it is the greatest circle in the sphere — one that cuts the Equinoctial circle and is cut by it into two equal parts, of which one inclines toward the North, the other toward the South.

[Margin: The distance of the poles of the Zodiac from the poles of the world.]

As for the poles of this circle, we said in chapter 1 — when we were dealing generally with the circles of the sphere — that they are removed from the poles of the world by a fourth part of a quadrant, plus a ninetieth of one quadrant; that is, by $23\frac{1}{2}$ degrees. From this it follows that the midpoint of each of its two halves is exactly the same distance from the Equator, the one inclining toward the North, the other toward the South.

[Margin: Why the Zodiac was devised by astronomers.]

Astronomers devised this circle among the celestial orbs chiefly on account of the motion of the planets. For they observed through daily experience that the Sun, the Moon, and the other planets, by their own proper motions from West to East, depart from the Equinoctial circle — now toward the North, now toward the southern region — and this by a certain fixed and determinate distance and elongation, which of course amounts to 23 degrees 30 minutes, especially if we are speaking of the Sun (for the other planets vary this distance somewhat). Next, that these same bodies return and approach the Equinoctial circle again, and always keep the same path, as we established in chapter 1 by many proofs when we were discussing the motions of the heavens. Again, by the clearest indications they discovered — as we showed in the same place — that the Firmament, together with all the fixed stars, moves from West to East about poles that are distant $23\frac{1}{2}$ degrees from the poles of the world.

[Margin: Anaximander the first discoverer of the Zodiac.]

Hence they marked out in the heaven a greatest circle, which they called the Zodiac, to be the path of all the planets and the belt of the second motion — that of the fixed stars as well — just as the Equator is the belt of the first motion. Pliny [Pliny the Elder] reports that the first discoverer of the Zodiac was Anaximander [Anaximander of Miletus].

[Margin: That the Zodiac makes various angles with any horizon.]

Although the Zodiac is fixed in the heaven and is everywhere the same, it nevertheless does not always make the same angles, whether with the right horizon or with the oblique; rather, it continually changes and varies them. For now it forms more upright angles, now more oblique ones, adapting itself to each

horizon according to the varying inclination it has toward that horizon. From this arises the whole non-uniformity — the irregularity — in the rising and setting of the signs, as we shall explain in chapter 3.

[Margin: Whence the Zodiac is so called.]

AND this circle is called the Zodiac, from Ζωή [Greek: zōē, "life"], which means life, because it is in accordance with the motion of the planets beneath it that all life exists in the lower things. Or it is called from Ζώδιον [Greek: zōdion, "living creature," "little figure" — a sign of the zodiac], which means animal; for, since it is divided into 12 equal parts, each part is called a Sign and has its own particular name taken from the name of some animal, on account of some property shared alike by the sign itself and the animal. Or it is so called on account of the arrangement of the fixed stars in those parts after the manner of animals of this kind.

COMMENTARY

He gives a twofold reason why this circle is called the Zodiac. It may be, no doubt, from Ζωή [Greek: zōē, "life"], that is, life, because on account of the continuous motion of the planets beneath this circle all these lower things possess life, as Aristotle relates in many places throughout his works. Or it may be from Ζώδιον [Greek: zōdion, "living creature," "little figure" — a sign of the zodiac], which means animal, because this circle is divided by the astrologers into 12 equal parts, each of which — with one exception — receives the name of some animal. And these 12 parts are called Signs, of which we shall speak presently.

[Margin: Why the signs of the Zodiac are named after animals.]

As for why these Signs are named after particular animals, he assigns a twofold cause here as well. The first is that (as the judicial astrologers maintain) those constellations possess virtues or properties shared in common with the animals from which they take their name; that is, because in these lower regions they produce effects that conform to animals of that kind. For example: The first Sign is called Aries, because, just as the Ram is a hot-blooded animal, so too the Sun, present in that part of the sky which is called Aries, begins to put forth its heat and to warm these lower regions. The second Sign is called Taurus, because, just as the Bull is stronger than the Ram, so too the Sun, once established in the sign of Taurus, exerts greater force than it did in Aries; or else because, with the Sun in Taurus, the toil of the oxen — that is, of the bulls — begins to appear, namely the standing crops. The third Sign took its name from the Twins [Gemini], because, as the Sun runs its course through it, the heat in these lower regions is in a way

doubled. The fourth is called Cancer, because, when the Sun reaches Cancer, it begins to move backward in the manner of a crab and to withdraw from us. The fifth is called Leo, for, just as the Lion is the strongest of animals, so too the Sun, present in Leo, brings on the greatest dryness and heat. The sixth Sign is called Virgo, because the Sun, present in it, is in a way barren, and produces nothing anew, but only brings to maturity what has already been produced. The seventh is named Libra, because, when the Sun is in it, the days and nights are weighed out and made equal as though on a balance or some pair of scales. The eighth is called Scorpio, for, just as the Scorpion stings and wounds with its tail, so too, while the Sun lingers in this Sign, the cold spells are wont to injure and sting the unwary. The ninth is called Sagittarius, because, when the Sun is in it, hailstorms and rains are hurled down upon us like arrows. The tenth is called Capricorn, because, just as the goat is forever raising itself up toward trees and foliage, so too the Sun, when it reaches this sign, begins to climb back up toward us again. The eleventh is called Aquarius, because, when the Sun is in that Sign, the waters of the rains tend to be abundant. Finally, the twelfth takes its name from the fishes [Pisces], because, while the Sun tarries in Pisces, the rains become so frequent that all things seem to swim about like fish. But all these things are to be understood of the region [inhabited zone] which, from the Equator...

tilts from the Equator toward the North. For those stars that dwell in the southern part experience the exact opposite of these.

The SECOND cause is that the stars lying in that part of the Zodiac which is called, for example, Scorpius reproduce the image or figure of a Scorpion. Likewise the stars set in that part which takes its name from Sagittarius express, in a manner, a man shooting an arrow from a taut bow — and so with the rest.

But if NEITHER of these explanations satisfies, we can say that these twelve parts obtained the aforesaid animal-names for this reason: since throughout the whole Firmament there are found 48 constellations or images (of which we spoke in chapter 1, where we listed their names and their individual stars one by one), twelve of them are contained within the Zodiac — namely Aries, Taurus, Gemini, and so on. Hence the astronomers gave these same names to the 12 parts into which the Zodiac is divided. But since the same difficulty seems to remain — namely why those 48 celestial images were furnished with such names — it must be said that the ancients imposed names of this kind on the constellations (whatever the judicial astrologers may say) in memory of certain illustrious men, or else of some fable or history. Thus one constellation is called Hercules [Hercules], in memory of Hercules; another Argo [Argonavis], on account of the

first ship, by which men entrusted themselves to the waves of the sea; and so forth. Nevertheless it cannot be denied that those who imposed these names had great regard for the figures which the stars form. For in memory of the Crown of Ariadne [Ariadne] they called that constellation the Crown, which presents the likeness of a certain crown — and so it must be said of the rest.

[Margin: Why the ancients gave the constellations those names, about which above.]

FROM this it is clear that, if we take account of the 12 signs or constellations comprehended in the Zodiac, this name properly belongs to the Zodiac of the Firmament, in which such constellations exist, and not to the Zodiac of the primum mobile, since there no trace of such images exists. But if anyone prefers the Zodiac to be named from Ζωή [Greek: ζῳή (zōē, "life")], rather than from Ζώδιον [Greek: ζῳδιον (zōdion, "living creature")], he will rightly be able to say that this name was first imposed on the Zodiac of the primum mobile. For it is on account of the motion of the planets beneath the Zodiac of the primum mobile that all these lower things have life, as the Philosophers assert.

[Margin: To which Zodiac this name better applies.]

[Lemma.] *This circle is called in Latin Signifer [the Sign-bearer], because it carries the Signs, or because it is divided into them. By Aristotle [Aristotle], in book 2 of On Generation and Corruption, it is called the oblique circle, where he says that according to the approach and recession of the Sun along the oblique circle, generations and corruptions come to pass in lower things.*

COMMENTARY

HE brings in two other names by which the circle of the Zodiac is usually called by the astronomers, saying that it is called by the Latins Signifer, either because it carries the 12 aforesaid Signs, or indeed because it is divided into them — an appellation very familiar to the poets. For so Claudian [Claudian] calls it in the epigram he composed on the sphere of Archimedes [Archimedes], where he speaks thus:

[Margin: Other names of the Zodiac.]

Percurrit proprium mentitus signifer annum, Et simulata nouo Cynthia mense redit.

The counterfeit sign-bearer runs through its own year, and a feigned Cynthia [the Moon] returns with each new month.

Lucan [Lucan] too names it thus in book 3, writing:

*Aethiopumque solum, quod non premeretur ab vlla Signiferi regione poli,
ni poplite lapsa Vltima curuati procederet vngula Tauri.*

And the land of the Ethiopians, which would be pressed by no region of the sign-bearing pole, did not — its knee slipping — the last hoof of the curved Bull reach out over it.

THEN he says that the Zodiac is called the oblique circle by Aristotle in book 2 of On Generation and Corruption; and by this name too many astronomers have been accustomed to call it. This circle is called oblique, first because it cuts the Equator and the Colure of the equinoxes at oblique angles; and second because, if it is compared with the parallel circles, it holds an oblique position in the sphere, since it is not equally removed from the poles of the world with respect to all its parts, but one half of it inclines toward the South and the other toward the North. Hence it comes about that the Sun and the other planets, which are perpetually moved beneath the Zodiac, sometimes approach nearer to us — namely when they are in the half toward the North — and sometimes recede farther from us, namely when they traverse the remaining half, which inclines toward the South.

[Margin: Why the Zodiac has an oblique position in the sphere.]

BUT if anyone should ask the cause why Nature has assigned this obliquity to the path of the Sun and of the other planets, the answer must be given, with the Philosophers, that it was done for two chief reasons. The first is the alternation of the seasons; for on account of the motion of the Sun beneath this oblique circle there arise Spring, then Summer, afterward Autumn, and last Winter, as we shall soon say. Likewise, in the oblique sphere, by the same motion of the Sun beneath the Zodiac, it comes about that sometimes the artificial days are equal to the nights, sometimes the artificial days exceed the nights, and sometimes, finally, the artificial days are surpassed by the nights, as will be established more clearly than daylight in chapter 3. For if the Zodiac which the Sun traverses by its own motion were not oblique, there would never be any variety of seasons in any region whatever, because the Sun would always keep the same distance from the top of the head [zenith]. The second cause is the diversity and variety of effects; for on account of the obliquity of the Zodiac the Sun and the other planets, as has been said, now approach nearer to us, now stand farther from us — and from this alternation arises the whole diversity in effects. For if the Zodiac were not oblique, the same effects would always be produced, since the planets would perpetually keep the same nearness and remoteness.

[Lemma.] *But the NAMES of the Signs, their arrangement, and their number are set out in these verses:*

*Sunt Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libraque, Scorpius,
Arcitenens, Caper, Amphora, Pisces.*

They are the Ram, the Bull, the Twins, the Crab, the Lion, the Virgin, the Balance, the Scorpion, the Archer (Arcitenens), the Goat (Caper), the Urn (Amphora), the Fishes.

[Margin: The names of the 12 signs of the Zodiac, and their order.]

COMMENTARY

SINCE the Author had said that the Zodiac is divided by the astronomers into 12 equal parts, which are called Signs, he now explains in two verses how these twelve Signs are named, and in what order they stand to one another in the Zodiac. This we too did long ago in chapter 1, when we were treating of the motions of the heavens, where we also set down the characters by which they are usually designated by the astronomers; and we now repeat these same characters here, for the benefit of students, so that they may cling more firmly to the memory. There are, then, 12 celestial signs endowed with these names, and they have this order among themselves, and they are usually expressed by such characters. Now each

♈	♉	♊	♋	♌	♍
Aries	Taurus	Gemini	Cancer	Leo	Virgo
♎	♏	♐	♑	♒	♓
Libra	Scorpius	Sagittarius	Capricornus	Aquarius	Pisces

higher sign corresponds to the one lower and opposite to it through the diameter in the Zodiac, as Aries to Libra, Taurus to Scorpio, Gemini to Sagittarius, and so on.

MOREOVER, among the astronomers a sign is customarily taken in two ways. In one way, for the sixth part of the whole Zodiac — in which sense it is called a Physical or natural sign, because a circle is, as it were, naturally divided into 6 equal parts without any artifice applied, by the very same spreading of the legs of the compass with which the circle was described, as is clear from the corollary of proposition 15 of book 4 of Euclid [Euclid, *Elements* IV.15, corollary]. Such signs the astronomers are accustomed to use in composing tables of the motions, as may be seen in Alfonso, King of Spain [Alfonso X of Castile], and others who

composed tables. In the other way, a sign is taken for the twelfth part of the Zodiac, or (what is the same) for the halved part of a physical or natural sign, and is called a common sign, because the astronomers are accustomed to use it — and it is in this signification that our Author here too has taken the sign. Now these parts of the Zodiac are perhaps called signs because through them are designated the motions of all the heavenly bodies, or also because they designate the various seasons of the year, as we shall soon say.

[Margin: The twofold sense of "sign." What a physical sign is. What a common sign is.]

THESE same twelve celestial signs Manilius [Manilius] describes most elegantly in twelve verses, in which he also expresses their order and names, and in what manner they are usually depicted by the astronomers on the celestial globe. These are the verses:

*Aurato princeps ARIES in vellere fulgens Respicit admirans aduersum
surgere TAVRVM Summisso vultu GEMINOS, & fronte vocantem: Quos
sequitur CANCER: Cancrum LEO: VIRGO Leone, Aequato tum LIBRA die
cum tempore noctis Attrahit ardenti fulgentem SCORPION astro, In cuius
caudam contentum dirigit arcum. MIXTVS EQVO, volucrem missurus
iamque sagittam Tum venit angusto CAPRICORNVS sidere flexus. Post
hunc inflexam diffundit AQVARIVS vrnas PISCIBVS assuetas auide
subeuntibus vndas, Quos Aries tangit claudentes vltima signa.*

First, the Ram, gleaming in his golden fleece, looks back, marveling to see the Bull rise opposite — the Bull, with lowered face and brow, summoning the Twins; whom the Crab follows; the Crab the Lion; the Lion the Virgin. Then the Balance, the day made equal with the time of night, draws in the Scorpion, blazing with its burning star, at whose tail [the Archer] aims his drawn bow. Mingled with a horse, and now about to loose the winged arrow, then comes Capricorn, coiled in his narrow constellation. After him Aquarius pours out his tilted urn for the Fishes, who eagerly draw beneath their accustomed waters, whom the Ram touches, closing the last of the signs.

WHICH verses indeed most beautifully explain the figures of the twelve Signs of the Zodiac, which are usually depicted on the celestial globe.

CONCERNING the NAMES of these twelve signs we spoke above — namely why these names were attributed to them by the astronomers. It must now be said concerning their number and order: namely why the astronomers established

only 12 signs in the Zodiac, not more nor fewer; and why they chose to take the beginning from Aries rather than from another sign, since in a circle there is properly no beginning, but one may take the beginning from any point whatever without distinction. For although all these things depend on the will and judgment of the astronomers, yet it must be believed that they were not established by them without reason. As for what pertains to the number of the signs, then, certain reasons are brought forward by the astronomers which show that the Zodiac was very fittingly divided into 12 signs. The first is this. Since there are four elements, from which all things are generated — namely Fire, Air, Water, and Earth — and since each one possesses three chief terms — namely a beginning, a middle, and an end — and since generable things are first generated, then conserved, and finally, thirdly, corrupted: if we multiply the ternary of these terms

[Margin: Why the Zodiac is divided into 12 signs.]

by the quaternary number of the elements, we shall produce the number twelve. Not without reason, therefore, ought the number of the signs in the Zodiac to be such that the individual elements, according to the aforesaid threefold term, should each obtain three signs. And thus the astronomers assigned to Fire Aries, Leo, and Sagittarius, because these three signs are hot and dry (as the judicial astrologers assert), just as Fire is. To Air they assigned Gemini, Libra, and Aquarius, for these three signs are hot and moist, like Air. To Water they ascribed Cancer, Scorpio, and Pisces, because these three signs are cold and moist, like water. To Earth, finally, they granted Taurus, Virgo, and Capricorn, because these three signs are cold and dry, like Earth.

[Margin: Which signs are called fiery and choleric; which earthy and melancholic; which airy and sanguine; and which watery and phlegmatic.]

But so that it may easily be held in memory which signs pertain to each element, four fingers on the hand are to be taken, of which the first should represent Fire, the second Earth, the third Air, the fourth Water; and then all the signs are to be reckoned upon them in that order in which we listed them above. So it will come about that the three signs falling above the first finger are assigned to Fire, and are called Fiery, on account of their heat and dryness — hence also Choleric. Those which fall above the second finger pertain to Earth, and are called earthy, on account of their coldness and dryness; hence too they are called Melancholic. Those next, which fall above the third finger, are ascribed to Air, since they are hot and moist, and are called Airy and Sanguine. Those finally which are set upon

the fourth finger are given to Water, on account of coldness and moisture, and are called Watery and Phlegmatic. All this may be viewed in this formula:

IGNIS	TERRA	AER	AQVA
♈	♉	♊	♋
♌	♍	♎	♏
♐	♑	♒	♓
IGNEA	TERREA	AEREA	AQVEA
CHOLERICA	MELANCHOLICA	SANGVINEA	PHLEGMATICA

The SECOND reason is of this sort. Since the Sun, in the space of a whole year, runs through the entire Zodiac, and distinguishes the intervals and boundaries of the seasons, it seemed to the astronomers most consonant with reason to divide the Zodiac into as many equal parts as there are notable varieties of seasons produced by the Sun's motion in the Zodiac. Now the perceptible diversities of the seasons are twelve. Rightly, therefore, were just so many Signs established in the Zodiac. For in the year there are four sufficiently common and principal parts — namely Spring, Summer, Autumn, and Winter — which do not behave in the same way in their complexions and qualities. For Spring is moist and hot; Summer hot and dry; Autumn dry and cold; Winter, finally, cold and moist — as not only the Philosophers but also the Physicians assert. Since, therefore, these four seasons are produced by the oblique motion of the Sun beneath the Zodiac (on account of which the Sun now approaches nearest to us, now is farthest from us, now behaves in an intermediate way), the whole Zodiac has been divided by the astronomers into 4 parts or quadrants, corresponding to the aforesaid four seasons of the year. The first Quadrant, corresponding to the season of Spring, takes its beginning from the first degree of ♈ [Aries], and has its end at the extremity of ♊ [Gemini], or the first degree of ♋ [Cancer]. The second Quadrant, in which the Sun, being present, produces Summer, begins from the first degree of ♋ [Cancer], and ends at the end of ♍ [Virgo], or the first degree of ♎ [Libra]. The beginning of the third Quadrant is set at the first degree of ♎ [Libra], and its term at the end of ♐ [Sagittarius], or the first degree of ♑ [Capricorn]; and this Quadrant corresponds to Autumn. The fourth Quadrant, finally, in which, while the Sun tarries, Winter is produced, takes its beginning from the first degree of ♑ [Capricorn], and has its end at the last degree of ♓ [Pisces].

But since in each of these seasons three further manifest diversities are discerned — for the Beginning, Middle, and End of each of them are not of exactly the same complexion, yet they have something in common with the complexions of the neighboring seasons. Hence, although Spring is hot and moist, yet not every part

of it is equally hot and moist. For its Beginning, on account of its nearness to the past winter, which is also moist and not hot, is more moist than hot; the Middle is temperately moist and hot; the End, finally, on account of the nearness of the coming summer, which is also hot but not moist, turns out more hot than moist. And the same reasoning is to be applied to the remaining three seasons of the year. Therefore, by the best counsel, the astronomers distributed each Quadrant of the Zodiac into three other equal parts, which should be the three mansions of the Sun in the three parts of each of these four seasons. From which it results that there are twelve Signs of the Zodiac.

[Margin: The qualities of the four seasons of the year. — To which seasons of the year the Quadrants of the Zodiac correspond.]

Moreover, so that all the Signs which correspond to the beginning, middle, and extremity of each of the aforesaid four seasons of the year may be ready at hand, all the Signs are to be numbered on three fingers, the beginning being made from ♈ [Aries], in such a way that four Signs fall above each finger. So it will come about that the 4 Signs of the first finger correspond to the beginnings of the four seasons — the first to the beginning of Spring, the second to the beginning of Summer, the third to the beginning of Autumn, the fourth finally to the beginning of Winter — which Signs are usually called Movable [Mobilia]. For in them occurs the change of one season into another. So too, in the same order, the four Signs of the second finger will correspond to the middle parts of those same seasons; hence they are called Fixed [Fixa], because in them the complexion of each season is firm and fixed. Finally, by the same reasoning, the four Signs on the last finger will indicate the extreme parts of those same seasons — which indeed are called Common [Communia], because, since they are the extremes of those seasons, each season has something in common with the qualities of the seasons following. All this is set before the eyes in the following formula:

[Margin: Which are the Movable, Fixed, and Common Signs.]

	INITIVM	MEDIVM	FINIS
VERIS	♈	♉	♊
ÆSTATIS	♋	♌	♍
AVTVMNI	♎	♏	♐
HYEMIS	♑	♒	♓
	MOBILIA	FIXA	COMMVNIA

The THIRD reason is this. Of the 48 images of heaven — the constellations, that is, which the astrologers formed out of the 1022 fixed stars of the Firmament, of which we indeed spoke in chapter 1 (whose histories or fables, if you desire to know them more fully, you must consult Hyginus [Hyginus], or Johannes Stöffler [Johannes Stöffler] on the Sphere of Proclus, or also Alessandro Piccolomini [Alessandro Piccolomini] in his little work on the fixed stars) — only 12 are included in the Zodiac, namely Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpius, Sagittarius, Capricornus, Aquarius, and Pisces, as was said in chapter 1. For which reason the astronomers wished to distribute the Zodiac into just as many equal parts, so that they might be called by the same names.

Commentary on Chapter II of the *Sphere* — On the Zodiac Circle (continued)

A FOURTH reason can be of this kind. It has been observed that in the space of a single year the Moon commonly comes into conjunction with the Sun under the Zodiac twelve times, and stands in opposition to it just as often — that is, twelve new moons occur in the course of a year, and as many full moons, even though the Moon runs through the whole Zodiac thirteen times in the space of one year. This is why it was decided to cut the Zodiac into exactly that many parts, and not into more or fewer; for it is precisely from this varying aspect of the Moon to the Sun that the intervals of time are distinguished. Thus the time that elapses from one conjunction to the next is called a *month*; the time contained between conjunction and opposition, and again from opposition to conjunction, makes up half a month; and the time that falls midway between conjunction (or opposition) and quadrature — when, that is, the Moon appears exactly half-full — makes a week [*hebdomada*], or seven-day span.

[Margin]

A FIFTH and final reason is drawn from the worth of the number twelve. For twelve is the first of all numbers to possess a half, a third, a quarter, a sixth, and a twelfth part — all of which are needed in the Zodiac: partly so that it may conveniently be divided into 12 parts, answering to the 12 varieties of the seasons, and into 4 quadrants, which produce spring, summer, autumn, and winter; and partly, above all, so that all the aspects of the stars treated in the *Theorics of the Planets* can be represented. For by the half-part of the Zodiac is designated the diametral aspect, or opposition of the stars; by the third part, the trine aspect; by the quarter, the square; and by the sixth, finally, the sextile aspect

is marked. It is clear, therefore, that the astronomers were not without reason when they divided the Zodiac into the 12 aforesaid celestial signs.

[Margin]

Now the reasons that moved the astronomers to take the beginning of the Zodiac from the first point of **Aries**, rather than from any other point, are chiefly three. The first is Ptolemy's [Ptolemy]: namely that, when the Sun stands at the beginning of Aries — that is, when the vernal equinox occurs — there begins the season most suited to the generation of things, for then all things grow green and flower. But when the Sun enters the first degree of **Libra** — that is, when the autumnal equinox occurs — there begins a season wholly contrary to the former, and more suited to the corruption of things; for then the leaves begin to fall from the trees, and everything to grow cold in some degree, as experience confirms. Not without reason, therefore, did the astronomers choose the first point of Aries out of all the points of the Zodiac to be the beginning of the whole Zodiac. There is a further consideration: when the Sun enters the sign of Aries, spring begins, the moist season, which corresponds most closely to the first age of living creatures. Then, as the Sun passes under the sign of **Cancer**, summer begins, the hot season, suited to the second age of creatures. Afterwards, when the Sun reaches the sign of **Libra**, autumn begins, the dry season, which agrees with the third age of creatures. And finally, when the Sun is in the sign of **Capricorn**, winter begins — that is, the cold season — which befits and answers to the fourth and last age of creatures.

[Margin]

For authors are accustomed to divide the life of living creatures into four principal ages. In the first, they say, moisture predominates, as we see in children; in the second, heat, as experience shows in youths and adolescents; in the third, dryness, as we observe in men now established in their full maturity; and in the fourth, finally, coldness, as is plain in the aged. This reasoning of Ptolemy's, however, holds and has force only for the regions that lie back from the equator toward the north. For if it were put to those who live beyond the equinoctial circle toward the south, it would have no weight. Indeed, it would prove that in the Zodiac the beginning ought to be taken from the first point of **Libra**. For just as with us, when the Sun stands in Aries it is spring, so with them it is spring when the Sun stands in Libra; and just as with us summer begins when the Sun is in Cancer, so with them summer comes when the Sun enters the sign of Capricorn. In short, everything that happens to us under any given signs must of necessity happen to them under the opposite signs — as can easily be

seen on a material sphere. Yet this reasoning is not on that account to be lightly esteemed. For one thing, Ptolemy and the other astronomers who gave these signs their names and established their order lived in regions that bend away from the equator toward the north, so that it is no wonder they had regard to this northern part of the sphere, in which they observed the courses of the stars. For another, this northern part is worthier and nobler than the southern, as the very structure and arrangement of the universe sufficiently shows.

[Margin]

For to anyone rising above whatever horizon, the eastern part lies to the right, and the southern to the left; and the northern part is the right-hand part, since it is always to the right of the Sun as it rises, while the southern lies to its left. This can also be established from the fact that the northern quarter of the heavens is adorned with far more stars near the Arctic pole than the southern, since near the Antarctic pole no stars at all exist, as was said above.

[Margin]

A SECOND reason is this. Since in the Zodiac there are four principal points, called *cardinal*, by which the whole Zodiac is distributed into four quadrants — each of which answers to one of the four parts of the year, namely spring, summer, autumn, and winter, as has been said — these points are: the beginning of Aries, the beginning of Cancer, the beginning of Libra, and the beginning of Capricorn. Of these, two, namely the beginning of Aries and of Libra, are called *equinoctial*; and two, namely the beginning of Cancer and of Capricorn, *solstitial*. Not unjustly, and not by any rashness, did the astronomers determine that a beginning must be made from one of these. Therefore, of them all they had to select the noblest, namely the first point of Aries. For this point is nobler than the two solstitial points. When the Sun stands at either of the solstitial points it describes the shortest parallels [of the day], and produces the greatest inequality of the days and nights, that is, of the artificial [days]; and hence those points are of less excellence. But when the Sun is at the equinoctial points it describes the greatest parallel, makes the day equal to the night, produces the most temperate climate, and — what deserves careful notice — is visible in all parts of the world in the space of 24 hours, even under the poles of the world, which can happen at no other point of the Zodiac. Likewise the first point of Aries is nobler than the beginning of Aquarius, for this reason: that when the Sun stands in it, it produces spring in the northern part [of the earth], and enters the signs that decline from the equator toward the north — that is, toward the northern part of the heavens, which is nobler than the southern, as we have said.

[Margin]

A LAST reason properly belongs to certain astronomers, who hold that it best accords with reason that the beginning in the Zodiac be taken where the Sun stood at the beginning of the world when it was created. And it is indeed probable, they say, that the world was fashioned with the Sun holding the first point of Aries — inasmuch as, in the Law of Moses, God commanded that at the time when the Sun enters the sign of Aries the Jews should take the beginning of the year, and celebrate the solemnity of the Passover; whereas before they had, with the Egyptians, begun the year from autumn. In this opinion are many holy Doctors, such as Eusebius in the *Chronicle*; Cyril in *Catechesis* 14; St. Leo, Sermon 9 on the Passion [St. Leo the Great]; [the commentary] on Exodus; St. John Damascene, book 2, chapter 7 [John of Damascus]; Isidore, *Etymologies* book 5, the chapter on the divisions of time [Isidore of Seville]; the Venerable Bede, in his book *On Reckoning* [Bede]; Strabus [Walahfrid Strabo] on Exodus 12; Rabanus [Rabanus Maurus] in the same place; the *Historia Scholastica*, chapter 25, on the History of Exodus; the interlinear Gloss —

— the interlinear Gloss on chapter 35 [recte: Genesis]; the Gloss on Genesis at the word "*in spring*"; and most others: to whom the school of theologians now almost commonly assents, on the ground that at that season of the year, when the Sun enters the sign of Aries, Christ the eternal Son of God took on human flesh, and by his most holy Passion redeemed the world. It seems probable, then, they say, that the world was created at the same season in which it was also redeemed.

I know that almost all the Hebrews, the Egyptians, and some of the ecclesiastical Doctors as well hold that the world was made around the time of autumn, on the ground that plants and trees were brought forth already bearing ripe fruit — as is clear from the forbidden apple offered to our first parents — which happens only around autumn. This may also be gathered, they argue, from the fact that God, in memory of the benefit by which he had freed the Hebrews from the servitude of Egypt, commanded that the year thenceforth be begun from that same season, namely from spring — the time at which he had conferred so great a benefit upon them — and no longer from autumn, from which (as they interpret it) the world was created.

But these arguments are not altogether firm. To the first it can be replied that God created the earthly Paradise, in which the first parents were placed, together with all its fruits, even though it was then springtime. Nor indeed does that argument hold which some allege — that the fruits were then created ripe because otherwise the trees would not naturally have produced them ripe in due course;

for on this reasoning all fruits ought to be ripe at the same season, namely in autumn, as they themselves maintain, which we nevertheless see does not happen. And so, although all the trees were created in springtime, natures of such a kind were nevertheless implanted in them by God that they should afterwards bring forth their fruits each at its proper season. It could also be said that the fruits were then ripe only in Paradise, which was not liable or subject to the qualities and variations of the seasons — but outside Paradise, by no means. To the second argument it can be answered that God willed the Jews, having abandoned the error of the Egyptians, to begin the year afresh from the springtime, at which the world had been created, and at which it pleased him to free them from so hard a servitude. But whatever be the case concerning the time at which the world was created, everyone is free, for all I care, to hold what he pleases: to me, at any rate, it seems more probable that it began in springtime, when, that is, the Sun stood at the beginning of Aries.

Virgil seems to feel the same, in book 2 of the *Georgics* [Virgil, *Georgics* II], where he sings thus:

*Non alios prima crescentis origine mundi Illuxisse dies, aliumue habuisse
tenorem Crediderim. Ver illud erat, Ver magnus agebat Orbis, & hybernis
parcebant flatibus Euri, Cum primum lucem pecudes hausere, virumq;
Ferrea progenies duris caput extulit armis, [recte: aruis] Immissæq; feræ
sylvæ, & sidera cælo.*

(I would not believe that other days shone forth at the first origin of the growing world, or that it kept another course. That was spring; the great world was keeping spring, and the east winds spared their wintry blasts, when the first cattle drank in the light, and the iron race of men raised its head from the hard fields, and wild beasts were let loose into the forests, and stars into the sky.)

It is clear, then, that no point of the Zodiac could more fitly furnish a beginning to the Zodiac than the first point of Aries.

SOMEONE will perhaps raise a doubt: since all the astronomers, for the reasons explained, begin the year when the Sun enters the beginning of Aries, why do the ancients in our calendars — and we too, together with the Roman Church — begin not from that same point but rather from the winter solstice, which once fell around the beginning of January; that is, we take the beginning of the year when the Sun enters the first degree of Capricorn? To this it can be briefly replied that it seemed more convenient to the ancients to fix the beginning of the year at the winter solstice than at the vernal equinox, because that point of the solstice —

which is the beginning of Capricorn — is the end of the descending [semicircle] and the beginning of the ascending semicircle. (The descending semicircle is called the half of the Zodiac from the beginning of Cancer, through Libra, up to the beginning of Capricorn, because in it the Sun descends from the zenith of our head; the ascending semicircle is called the other half of the Zodiac, from the beginning of Capricorn through Aries to the beginning of Cancer, because in it the Sun again ascends toward the zenith of our head — this, of course, being understood of the northern habitation. For exactly the contrary would have to be said in the southern habitation.)

[Margin]

It is the end of the withdrawal of the Sun and the beginning of its return toward us; it is the end of the shortening of the days and the beginning of their lengthening; it is the end of the lengthening of the nights and the beginning of their shortening — with respect to the northern part, which is worthier than the southern, and which the founders of the year inhabited. All this will be made plain in chapter 3. This same doubt — why, that is, the ancients chose to begin the year from the winter solstice rather than from the vernal equinox — Janus resolves in Ovid, *Fasti* book 1 [Ovid, *Fasti* I], where Ovid asks Janus why the beginning of the year is not set at the vernal equinox, when, that is, all things flower and grow green, in these verses:

*Dic age, frigoribus quare nouus incipit annus, Qui melius per ver
incipiendus erat? Omnia tunc florent, tunc est noua temporis ætas: Et noua
de grauido palmite gemma tumet. Et modo formatis operitur frondibus
arbor: Prodit & in summum seminis herba solum. Et tepidum volucres
concentibus aera mulcent: Ludit & in pratis, luxuriatq; pecus. Tunc blandi
Soles, ignotaq; venit hirundo, Et luteum celsa sub trabe fingit opus. Tunc
patitur cultus ager, & renouatur aratro. Hæc nouitas anni iure vocanda
fuit. Quæsieram multis, non multis ille moratus Contulit in versus sic sua
verba duos. BRVMA noui prima est, veterisq; nouissima Solis: Principium
capiunt Phœbus, & annus idem.*

(Come, tell me: why does the new year begin in the cold, when it would better have begun with spring? Then all things flower, then is the fresh age of the season, then the new bud swells on the teeming vine-shoot; then the tree is clothed with newly formed leaves, and the blade of the seed comes up to the surface of the soil; then the birds soothe the warm air with their harmonies, and the flock frolics and revels in the meadows. Then the sunshine is soft, and the swallow, till then unseen, arrives and builds its clay work beneath the lofty

rafter; then the tilled field bears the plough and is renewed by it. This freshness deserved by right to be called the year's beginning. — I had asked at length; without long delay he pressed his reply into two verses: "The winter solstice is the first of the new sun and the last of the old: Phoebus and the year alike take their beginning [there].")

WE Christians too can add another cause why the Church begins the year from the winter solstice: namely, because at that season the Savior of the world was born, to illuminate the darkness of men. And although the winter solstice does not fall exactly at the beginning of January, but on the 22nd day of December, even after the correction of the Calendar —

— the Church has nevertheless retained the ancient usage, so as to set the beginning of the year, with Julius Caesar, on the first day of January. This, then, is the reason why in the Roman calendars the year begins from the Kalends of January, even though the astronomers, considering the other reasons already stated, begin their reckonings of the years from the vernal equinox, and end them at the same point.

MANY things would have to be said here about the various properties and names of the signs; but since they pertain more to the judicial astrologers, they must for the present be passed over. Only this must be explained: which signs are called the *houses* [*domus*] and *exaltations* of this or that planet — inasmuch as any planet standing in its own house exercises and displays its power most fully in these lower regions.

[Margin]

Now every planet has two signs assigned to it for a double house — the Sun and Moon excepted, to which single signs are assigned each for a single house. Thus the sign **Leo** is called the house of the **Sun**; for since Leo is a fiery sign, and coincides with summer, the Sun as it runs through it produces the greatest heat upon the earth. The sign **Cancer**, on the other hand, is called the house of the **Moon**; for since Cancer is a watery sign, the Moon standing in Cancer most fully moistens these lower things. The next two signs surrounding these, namely **Virgo** and **Gemini**, are called the houses of **Mercury**. Two others still adjacent, namely **Libra** and **Taurus**, are the houses of **Venus**. Two next in order, namely **Scorpio** and **Aries**, are the houses of **Mars**. Two more adjoining, namely **Sagittarius** and **Pisces**, are the houses of **Jupiter**. And the two remaining, which enclose all these, namely **Capricorn** and **Aquarius**, are called the houses of **Saturn**.

But although each of these five planets possesses two houses, yet of the two one is always the more principal and the other less so, in such a way that the planet does not have the same power in both houses. For Mercury has greater strength and virtue standing in Virgo than in Gemini; Venus greater in Taurus than in Libra; Mars greater in Scorpio than in Aries; Jupiter greater in Sagittarius than in Pisces; and Saturn, finally, exercises greater power in Aquarius than in Capricorn.

[Margin]

Again, the sign which is diametrically opposite to the house of some planet is called the *detriment* [*detrimentum*] of that planet. Thus, since the sign **Aquarius** is opposed by diameter to Leo, which is the house of the Sun, the sign Aquarius will be called the detriment of the **Sun**. Likewise each of the signs **Taurus** and **Libra** will be called the detriment of **Mars** [since they oppose his houses]; but the greater detriment will be the sign Taurus, because it is opposed to the sign Scorpio, which is the principal domicile of Mars. And so with the rest. The following table will set these houses before your eyes:

Planet	House
Sun	Leo
Moon	Cancer
Mercury	Virgo (principal); Gemini (less principal)
Venus	Libra (less principal); Taurus (principal)
Mars	Scorpio (principal); Aries (less principal)
Jupiter	Sagittarius (principal); Pisces (less principal)
Saturn	Capricorn (less principal); Aquarius (principal)

[Margin]

CERTAIN of the 12 signs are called the *exaltations* of the planets. Thus the sign **Aries** is called the exaltation of the **Sun**, because when the Sun enters the sign of Aries the days begin to grow longer than the nights, and the Sun's heat begins to gain increase in these lower regions. But when it enters the sign of **Libra**, the nights begin to exceed the length of the days, and the Sun's heat gradually to weaken; hence the sign Libra is called the *fall* [*casus*] of the Sun. For the sign that is diametrically opposite the sign which is the exaltation of some planet is always called the fall of that same planet. Next, the sign **Taurus** is the exaltation of the **Moon**, and the sign **Scorpio** [her] fall; [the sign Cancer is the exaltation of Jupiter, and] the sign Capricorn the fall of **Jupiter**. The sign **Virgo** is the exaltation of

Mercury, but the sign **Pisces** the fall of Mercury; [the sign Libra is the exaltation of Saturn, and the sign Aries] the fall of **Saturn**. The sign **Capricorn** is the exaltation of **Mars**, and the sign **Cancer** the fall of Mars. The sign, finally, [Pisces is the exaltation of Venus, and Virgo her fall]. All of which are explained in the following table:

Planet	Exaltation	Fall
Moon	Taurus	Scorpio
Mercury	Virgo	Pisces
Venus	Pisces	Virgo
Sun	Aries	Libra
Mars	Capricorn	Cancer
Jupiter	Cancer	Capricorn
Saturn	Libra	Aries

[Margin]

EACH sign, moreover, is divided into 30 degrees; whence it is plain that in the whole Zodiac there are 360 degrees. According to the astronomers, again, each degree is divided into 60 minutes; each minute into 60 seconds; each second into 60 thirds; and so on in succession up to the tenth [order of division]. And just as the Zodiac is divided by the astronomers, so likewise every circle in the sphere, whether greater or lesser, is distributed into corresponding parts.

COMMENTARY.

Now that the Zodiac has been divided into the 12 common signs, [Sacrobosco] next divides the signs into further parts, teaching that each sign is distributed by astronomers into 30 equal parts, which are called Degrees [gradus]. [Margin: What a Degree is, and how many there are in the whole Zodiac according to longitude.] Since, then, 12 signs are contained in the whole Zodiac, if 12 is multiplied by 30 the result is 360 — which is exactly the number of degrees contained in the whole Zodiac. He then says that each degree is subdivided into 60 equal parts, called minutes [minuta]; each minute into 60 seconds [secunda]; each second into 60 thirds [tertia]; and so on always, proceeding by this sexagesimal division, until one reaches the tenths [decima]. For astronomers rarely go beyond the tenths. And just as the Zodiac is divided into 360 degrees, so too any other circle in the heavens — whether a great circle or not — is customarily distributed into the same number of degrees; and in the same way each degree is customarily divided into 60 minutes, each minute into 60 seconds,

and so forth. But this division of the Zodiac into 360 degrees, of each degree into 60 minutes, of each minute into 60 seconds, and so on, seems here to call for a somewhat fuller explanation. This particular division of the Zodiac is usually called division according to longitude.

Astronomers, then, observing that any circle can be cut by its primary and, in a way, natural division into 6 equal parts — by the very same span of the compass legs with which the circle is drawn, since the semidiameter of any circle is the side of the equilateral hexagon inscribed in it [Margin: a. 15 of the fifth [i.e. the inscribed-hexagon corollary; printed "quinti"]] — divided the whole Zodiac into 6 equal parts, which constitute the six physical, or natural, signs, as we said above. Then they divided each physical sign — that is, a sixth part of the whole Zodiac — into 60 equal parts, which they called Degrees, from the daily step-by-step advance, perhaps, of the Sun and the other planets through these parts. For the planets, as it were, walk through the Zodiac step by step (*gradatim*), passing through the said parts. From this it came about that the whole Zodiac contains 360 degrees. After this they again distributed each degree into 60 equal small parts, which they called minutes, and the minute into 60 seconds, the second into 60 thirds, and so on to infinity — although astronomers very rarely reach the tenths, and much more rarely go beyond them. And they divided the Zodiac into these minutest particles so that they might attain the utmost precision in the place and motion of the Sun and the other planets.

They preferred to carry this out by a sexagesimal division rather than by another — though another would have been open to them — first, because the number six, among all the perfect numbers (which are of course constituted from all their aliquot parts), is the first, and has a certain affinity with the sexagesimal number, since it measures it exactly ten times; and second, because the sexagesimal number seemed more convenient and apt for this sectioning. For it has a half [*dimidiatam partem*], a third, a fourth, a fifth, and a sixth part, with which parts the ancients were content, in order to avoid trouble and tedium. The same number does indeed contain still other parts — namely a tenth, a twelfth, a fifteenth, a twentieth, and finally a thirtieth — but the ancient mathematicians did not take account of these.

Another reason can also be offered for why the whole Zodiac is cut into 360 degrees. Since from one conjunction of the Moon with the Sun to another — that is, from one new moon to the next — there intervene very nearly 30 days, namely the span of one month, it pleased the astronomers to divide each common sign into 30 parts, which are called degrees from the "step" (*gressu*) of the luminaries;

or else because the Sun uses up about 30 days to run through a whole common sign, completing, that is, roughly one degree on each single day. Hence it was fitting that so much space should be granted to a single degree, since the Sun, the most brilliant lamp of the world, advances in a natural day very nearly one degree. By this reasoning, just as the whole Zodiac corresponds to a whole year, and the individual signs to the individual months, so too the individual degrees will correspond, as it were, to the individual days in the Zodiac. From this it came about that the whole Zodiac comprises 360 degrees, while the physical sign comprises 60. Therefore, lest a variation of division breed confusion, the degree in turn was divided into 60 minutes, the minute into 60 seconds, and so on. [Margin: Why astronomers use the sexagesimal division.]

These, then, are the reasons that drove the astronomers to use the sexagesimal division in dividing the Zodiac; of which the strongest seems to be that both the number 360 and the number 60 have very many aliquot parts. The former has all these: 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180. The latter has all these: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30. And if the numbers 360 and 60 themselves are added to these, and are arranged so that the halved portion, in which the smaller parts are contained, is set on the left, and the remaining halved portion, containing the larger parts, on the right — as has been done here — you see that they denominate each other in pairs, mutually. For 1 is $1/360$ of the number 360; but 360 makes $1/1$ of the same number 360. Likewise 5 constitutes $1/72$ of the same, while 72 yields $1/5$; and so on. So too 3 makes $1/20$ of the number 60, while 20 constitutes $1/3$ of the same number 60; and so on.

Aliquot parts of the numbers 360 and 60.

Aliquot parts of the number 360	Aliquot parts of the number 60	
1	36	1
	0	0
2	18	2
	0	0
3	12	3
	0	0
4	90	4
		5
5	72	5
		2
6	60	6
		1

Aliquot parts of the number 360	Aliquot parts of the number 60	
		0
8	45	
9	40	
10	36	
12	30	
15	24	
18	20	

So that it may be known how many particles of each division make up one degree, or even the whole Zodiac, I have thought fit to append here two little tables. In the first, a whole degree is distributed into minutes, Seconds, Thirds, Fourths, Fifths, Sixths, Sevenths, Eighths, Ninths, and Tenths. In the second, the whole Zodiac is distributed according to longitude into degrees, Minutes, Seconds, and so on.

ONE DEGREE CONTAINS [Margin: How many Minutes, Seconds, Thirds, and so on one Degree contains.]

Minutes [Minuta]	60
Seconds [Secunda]	3600
Thirds [Tertia]	216000
Fourths [Quarta]	12960000
Fifths [Quinta]	777600000
Sixths [Sexta]	46656000000
Sevenths [Septima]	2799360000000
Eighths [Octava]	167961600000000
Ninths [Nona]	10077696000000000
Tenths [Decima]	604661760000000000

THE ZODIAC CONTAINS [Margin: How many Degrees, Minutes, Seconds, Thirds, and so on are contained in the whole Zodiac.]

Degrees [Gradus]	360
Minutes [Minuta]	21600

Seconds [Secunda]	1296000
Thirds [Tertia]	77760000
Fourths [Quarta]	4665600000
Fifths [Quinta]	279936000000
Sixths [Sexta]	16796160000000
Sevenths [Septima]	1007769600000000
Eighths [Octava]	60466176000000000
Ninths [Nona]	3627970560000000000
Tenths [Decima]	21767823360000000000

Anyone will be able to extend both of these little tables on his own to infinity. For if the Tenths are multiplied by 60, the Elevenths [Vndecima] will be had; and if these in turn are multiplied by 60, the Twelfths [Duodecima] will result; and so on.

The Latins, too, call any whole, or Total whatever — and thus a Degree — an *as* (Assem), and they divide it into twelve equal parts. Of these they call: eleven, the *deunx*; ten, the *dextans*; nine, the *dodrans*; eight, the *bes*; seven, the *septunx*; six — that is, the half part — the *semis* (Semissem); five, the *quincunx*; four, the *triens*; three, the *quadrans*; two, the *sextans*; and one, finally, the *uncia*. [Margin: The *as* and its parts.] Since the use of these terms is frequent among the ancients — especially in Pliny, Vitruvius, Columella, and other writers both ancient and more recent — I judge it will not be out of place if I set down a table in which, in the first place, are contained the names of the 12 parts of the *as*, or whole degree; in the second place, the Minutes that correspond to each of the 12 parts; and in the third place, the common fractions that express the value of those same parts.

A TABLE CONTAINING THE NAMES OF THE TWELVE PARTS OF THE AS, AND THEIR VALUE.

Name of part		Minutes	Whole degree (fractional value)
<i>As</i> , or <i>Assis</i>	minutes	60	(Whole degree — <i>Gradus integer</i>)
<i>Deunx</i>	minutes	55	55/60, or 11/12
<i>Dextans</i>	minutes	50	50/60, or 10/12, or 5/6
<i>Dodrans</i>	minutes	45	45/60, or 9/12, or 3/4

Name of part		Minutes	Whole degree (fractional value)
<i>Bes</i> , or <i>Bessis</i>	minutes	40	40/60, or 8/12, or 2/3
<i>Septunx</i>	minutes	35	35/60, or 7/12
<i>Semis</i> , or <i>Semissis</i>	minutes	30	30/60, or 6/12, or 1/2
<i>Quincunx</i>	minutes	25	25/60, or 5/12
<i>Triens</i>	minutes	20	20/60, or 4/12, or 1/3
<i>Quadrans</i>	minutes	15	15/60, or 3/12, or 1/4
<i>Sextans</i>	minutes	10	10/60, or 2/12, or 1/6
<i>Uncia</i>	minutes	5	5/60, or 1/12

Now, in whatever way the Zodiac is divided, in just the same way the Equinoctial circle, the Meridian, and finally any other circle of the sphere — whether a great circle or not — is customarily divided by astronomers; except that they called the degrees of the Equinoctial circle, because by a fixed and perpetual law they mark out the diurnal and nocturnal times and distribute these into equal hours, in Greek χρόνους [Greek: χρόνους (chronous, "times")], and in Latin *Tempora* ("Times"), so that they might be distinguished from the degrees of the Zodiac.

In the same manner, too, in which the degree is divided, the hour is also customarily distributed — and any whole whatever — namely, into 60 minutes; the minute into 60 seconds; and so on. Likewise it is divided into the *deunx*, *dextans*, *dodrans*, and so on. The ancients also subdivide the *uncia* into further small parts, which, in the interest of brevity, I here omit; anyone, however, will be able to grasp all these things more thoroughly from the book of [Guillaume] Budé [Budaëus] which he entitled *On the As and its parts* [*De Asse et partibus eius*].

[Lemma — Sacrobosco:] Since, moreover, every circle in the sphere except the Zodiac is understood, like a line, [to be] a circumference, the Zodiac alone is understood as a surface, having in its breadth twelve degrees — of which kind of degrees we have already spoken. From this it is clear that certain people speak falsely in Astrology when they say that the signs are square, unless, misusing the term, they should mean by "square" [quadratum] a rectangle [quadrangulum]. For a sign has 30 degrees in length, but 12 in breadth.

COMMENTARY.

Up to this point the Author has dealt with the division of the Zodiac according to longitude; here he now explains its magnitude, or its division according to

latitude. For the Zodiac, he says, has this property among the other circles of the sphere, peculiar to itself: that whereas all the others are conceived on the surface of the heaven as lines, or circumferences, indivisible with respect to latitude, the Zodiac alone is understood as a kind of surface having in its breadth 12 degrees throughout its whole circuit. And since we have said that each sign has 30 degrees in length, he concludes that certain people are deceived in Astrology when they say that the signs of the Zodiac are square — unless by the term "square" they mean a rectangle, which is a broader term applying both to the square and to a figure longer on one side. For on this reckoning every sign will be longer on one side, having on each longer side 30 degrees, and on the shorter 12. [Margin: Why the Zodiac is given breadth by the astronomers.]

Astronomers attributed this latitude to the Zodiac alone, among all the other circles, for two reasons. First, so that it could contain within itself the figures and names of the signs. Second, on account of the irregular motion of the planets beneath it. For although all the planets are carried perpetually beneath the Zodiac, they do not all move in the same way. The Sun, running through the middle of it, never inclines either to the right or to the left; but all the other planets deviate now to the north of the middle of the Zodiac, now to the south, so that this deviation, to either side from the middle of the Zodiac, embraces about 6 degrees. Hence it came about that the whole Zodiac obtains a latitude of 12 degrees.

But someone will object that Mars and Venus recede not only 6 degrees from the middle of the Zodiac — whether to the north or to the south — but sometimes nearly 8 degrees; and that therefore the latitude of the Zodiac ought rather to be 16 degrees, so that the planets should never be found straying outside the Zodiac. [Margin: Why the latitude of the Zodiac embraces 12 degrees rather than 16.] To this objection it must nonetheless be answered that it was for this reason that some, following [Regiomontanus] Ioan. Regiom., assigned 16 degrees of latitude to the Zodiac; but all other astronomers deny that this is necessary. For they say it is more consonant with reason that the Zodiac should be cut into 12 degrees of latitude, on the ground that the other planets never exceed this latitude; and that the fact that Mars and Venus sometimes deviate more than 6 degrees from the middle of the Zodiac happens very rarely, and only by reason of the magnitude of the epicycles they have — so that this deviation cannot be a sufficient cause for assigning 16 degrees of latitude to the Zodiac. There is also this: it seems fitting that, just as the whole Zodiac contains 12 signs in length, so too in breadth it should comprise the same number of parts, namely 12 degrees. And by parallel reasoning: just as one degree is a thirtieth part of one sign, so too the whole

latitude of the Zodiac would be a thirtieth part of the whole compass, or circuit, of the same Zodiac — such being the 12 degrees of latitude in relation to the 360 degrees of longitude. Finally, just as the compass of the whole Zodiac in longitude comprises 360 degrees, so too the same number of degrees would contain one sign in its whole area, or surface. For 12 multiplied by 30 makes 360 degrees — the area, that is, of a single sign.

[Lemma — Sacrobosco:] Now the line dividing the Zodiac around its circuit, in such a way that on one side of itself it leaves six degrees and on the other side another six, is called the Ecliptic line: because, when the Sun and Moon are directly beneath it, an eclipse of the Sun or of the Moon occurs. Of the Sun — as when there is a new moon and the Moon is interposed exactly between our sight and the solar body; of the Moon — as at full moon, when the Sun is diametrically opposite the Moon. Hence an eclipse of the Moon is nothing other than the interposition of the earth between the body of the Sun and the Moon.

COMMENTARY.

He explains in this place what the Ecliptic line is, saying that, since the Zodiac has 12 degrees in latitude, if a line is understood to cleave the whole circuit of the Zodiac through the middle of these 12 degrees, in such a way that six degrees are left on one side and as many on the other, that line will be called the Ecliptic, for the reason that, when the Moon is conjoined with the Sun while lying exactly beneath this line, an eclipse of the Sun occurs; and when it is opposed to the Sun across a diameter, lying on the same line, an eclipse of the Moon happens. Here too he declares in passing what an eclipse of the Moon is. All these things are clear in the text. But we shall say more about the eclipse both of the Sun and of the Moon in chapter 4. [Margin: What the Ecliptic line is, and why it is so called.] [Margin: What an eclipse of the Moon is.]

This Ecliptic line is called — since among approved Authorities it is used without qualification for the Zodiac, with no account taken of the latitude of the Zodiac — the "Way of the Sun" (Via Solis), because the Sun always travels beneath it by its own motion. For the same reason it is called the orbit of the Sun, the Solar Path, the Place of the Sun, the Plane of the Sun, the circle of the Sun, the Ecliptic place; and in Ptolemy the "circle through the middle of the living creatures" (i.e. of the zodiacal animals), the "circle of the signs"; and it is customarily called by various writers by other names of this kind. [Margin: The various names of the Ecliptic.]

The Ecliptic line is drawn in the heaven in this way. Let a straight line be conceived from the center of the earth, or of the whole world,

[Margin: How the ecliptic is conceived as described in the heavens.]

goes out from that point and passes through the center of the Sun's body all the way to the primum mobile. For from the Sun's annual motion from west to east a circle is described, whose circumference, lying in the primum mobile, is called the ecliptic line. For the Sun by its own motion always recedes from the equator in the same way and within the same limits, as we shall shortly explain. And if throughout the whole circuit of the zodiac six degrees are added on either side of this line — or, according to some, eight degrees — the entire circle of the zodiac will be constituted.

The Sun indeed always runs beneath the ecliptic, but all the other planets decline either toward the north or toward the south; and sometimes they too are beneath the ecliptic.

COMMENTARY

[Margin: The Sun always moves beneath the ecliptic, but the other planets do not.]

Here he teaches how the Sun and the other planets stand with respect to the ecliptic line already mentioned. He asserts that the Sun runs perpetually beneath the ecliptic, declining to neither side, whereas all the other planets deviate from it, now toward the north, now toward the south — and sometimes (namely when they are passing from north to south, or from south to north) they lie on the ecliptic itself.

[Margin: How it was discovered that the Sun always moves beneath the ecliptic, while the other planets do not.]

For astronomers have observed and noted that the Sun, in any given clime, rises and sets each year at the same point of the horizon whenever it stands in the same sign and degree of the zodiac — for instance in the first degree of ♋ [Cancer]. This can easily be observed from the shadow of some stylus fixed in a wall that faces the rising or the setting Sun. Likewise at midday they have observed that its meridian shadow is perpetually the same at fixed times of the year: at the summer solstice it has the same length every year, and so too at each equinox, and at the winter solstice. Thus at one summer solstice the meridian shadow has never been seen longer than at another summer solstice, nor at one equinox longer than at another, nor at one winter solstice longer than at another; and the same must be said of all the other times of the year, or points of the zodiac. By like reasoning astronomers have established that the Sun, when it

declines most from the equator — namely when it stands at the beginning of ♋ [Cancer] or ♑ [Capricorn] — is removed from it by a constant interval every year; and they observed the same when it stands at any other point of the zodiac. They therefore necessarily concluded that the Sun always keeps the same path or track by which it is carried from west to east by its own motion; and this track they called the ecliptic line, or the solar path, as has been said.

Hence it comes about that all affirm with one voice that the Sun always moves on the ecliptic line, so that its center never deviates from it either to the left or to the right — since its path is constant and always behaves in the same way. This line they named "ecliptic" because of the eclipses that occur beneath it. In all the other planets they have found the contrary. For the Moon, for example, at different times, while standing in the same degree of the zodiac, is not always seen to rise and set at the same point of the horizon, nor to cast a meridian shadow of the same length, nor to be removed from the equator by an equal amount, but now to stand farther from it and now nearer. The same they observed in the remaining five planets. They therefore rightly concluded that all the planets, the Sun alone excepted, wander here and there from the ecliptic, and do not always advance from west to east by the same road. Thus we see the Moon sometimes, when it stands at the beginning of ♋ [Cancer], recede from the equator by nearly 28 degrees, and sometimes by about 18 degrees. And so its meridian shadow must necessarily vary. The same has been observed at all the other points of the zodiac, not only in the Moon but also in the other planets. For all of them tend from west to east, not always along the ecliptic, but they wander now toward the north, now toward the south — that is, toward the meridian — with a variation that is wonderful, yet constant, and peculiar and proper to each.

[Margin: Which part of the zodiac is boreal and which austral; likewise which signs are boreal and which austral.]

The part of the zodiac that declines from the equator toward the north is called northern, or boreal, or arctic; and those signs that lie from the beginning of ♈ [Aries] to the end of ♍ [Virgo] are called northern or boreal signs. But the other part of the zodiac, which declines from the equator toward the south, is called southern, or austral, or antarctic; and the six signs that lie from the beginning of ♎ [Libra] to the end of ♓ [Pisces] are called southern or austral.

COMMENTARY

[Margin: When planets are boreal and when austral.]

Since in the sixth office of the equator we said that the whole heaven is divided by the equator into two hemispheres — the one that inclines toward the arctic pole being called northern, boreal, or arctic, and the other, looking toward the opposite pole, being called southern or austral — and since again one half of the zodiac declines from the equator into the northern hemisphere while the other declines into the southern, it follows that the former half is likewise called northern and the latter southern, and that the signs comprised in each half receive the same names, as the author here plainly explains. Therefore, since the first six signs — namely Aries, Taurus, Gemini, Cancer, Leo, and Virgo — are northern, and the latter six — namely Libra, Scorpio, Sagittarius, Capricorn, Aquarius, and Pisces — are southern, it comes about that planets running through the first six are called northern, while those dwelling in the latter six are called southern.

[Margin: First sense of "sign."]

But when it is said that the Sun is "in" Aries, or in some other sign, it must be understood that this preposition "in" is taken for "under," according to the sense in which we are now taking the word "sign."

COMMENTARY

[Margin: First sense of the zodiac.]

Being about to explain in what manner the Sun and the other planets — indeed even the fixed stars — are said to be in some sign, he introduces four senses of "sign" that are in use among astronomers. In the first sense a "sign" is a twelfth part of the surface of the zodiac — namely a quadrilateral 30 degrees in length and 12 degrees in breadth; and it is in this sense that we have hitherto spoken of a sign. This first sense of "sign" takes its origin from the first sense of "zodiac," in which the zodiac is taken for...

Diagram. A small woodcut set at the foot of the right-hand column, showing a fan- or sector-shaped figure containing several small star symbols, with a radiant sun-face at its lower apex — illustrating the wedge-like "sign" whose vertex lies toward the center. No letter-labels are legibly present on the figure.

that band or zone in the primum mobile, containing 360 degrees in its whole circuit, and 12 degrees in breadth. For if the zodiac in this sense be cut into 12 equal parts, there will be had the 12 signs in the first sense. And because in this way a sign is not in the orbs of the planets, nor even in the sphere of the fixed stars, but only in the primum mobile, the author teaches that when astronomers

say the Sun, or any other planet, is in such a sign — for example, in Aries — the preposition "in" is taken for "under"; so that the sense is: the Sun, or any other planet, is under the sign ♈ [Aries], in such a way that a straight line drawn from the center of the world through the center of the Sun, or of another planet, terminates in that sign in which the Sun or planet is said to be.

[Margin: When the Sun is to be understood as being in any sign in the first sense.]

In another signification, a "sign" is called a quadrilateral pyramid whose base is that surface which we have called the sign, and whose vertex is at the center of the earth. And according to this, properly speaking, we can say that the planets are in the signs.

COMMENTARY

[Margin: Second sense of "sign."]

In the second manner a "sign" is taken for a quadrilateral pyramid whose base is the sign in the first sense, and whose vertex is the center of the whole universe. This sign in the second sense likewise had its origin from the second sense of "zodiac," namely when astronomers take the zodiac not for that band or surface, but for the body or solid contained by the zodiac in the first sense and by two concave conical surfaces, each having its vertex at the center of the world, while their bases are two lesser circles equidistant from the ecliptic line and receding from it by 6 degrees. For in this way the zodiac will be divided into 12 quadrilateral pyramids, which constitute the 12 signs in the second sense. According to this sense of "sign" the author says it can properly be said that the planets are in the signs; for they will always be contained within one of the said 12 pyramids.

[Margin: Second sense of the zodiac.]

[Margin: The Sun is properly in the signs in the second sense.]

Diagram. A woodcut of a triangular wedge (quadrilateral pyramid) with its apex at the bottom, containing several small star symbols, a key-shaped object, and a crescent moon within the wedge — illustrating the "sign" as a pyramid whose base is the zodiacal surface above and whose vertex lies at the center of the earth below. No letter-labels are legibly present.

In the third manner a "sign" is spoken of so that there are understood six circles passing through the poles of the zodiac and through the beginnings of the 12 signs. These six circles divide the whole surface of the sphere into 12 parts, broad in the

middle but narrower toward the poles of the zodiac; and each such part is called a sign, and has its special name from the name of that sign which is intercepted between its two lines. And according to this sense, the stars that are near the poles, outside the zodiac, are said to be in the signs.

COMMENTARY

[Margin: Third sense of "sign."]

In the third sense a sign is also a certain surface, as in the first. For if there be described six great circles in the sphere passing through each pole of the zodiac and through the beginnings of the 12 signs in the first sense — so that the first passes through the beginning of ♈ [Aries] and ♎ [Libra]; the second through the beginning of ♉ [Taurus] and ♏ [Scorpio]; the third through the beginning of ♊ [Gemini] and ♐ [Sagittarius];

Diagram. A large circular celestial planisphere (woodcut). Great circles radiate from the center (the pole of the zodiac) dividing the disk into segments; the field is filled with zodiacal constellation figures and numerous small stars, illustrating the division of the whole surface of the heaven into 12 signs by the six circles through the poles of the zodiac. Individual constellation and point labels do not resolve legibly on the scan. [— peripheral labels not resolvable.]

the fourth through the beginning of ♋ [Cancer] and ♑ [Capricorn]; the fifth through the beginning of ♌ [Leo] and ♒ [Aquarius]; the sixth, finally, through the beginning of ♍ [Virgo] and ♓ [Pisces] — the whole surface of the heaven will be divided into 12 equal parts from one pole of the zodiac to the other, broader indeed in the middle, where the zodiac is, but narrower at the ends, namely near the poles of the zodiac, where all six of the aforesaid circles mutually intersect. These parts are called signs in the third sense, and are named after those signs of the first sense which are enclosed by the said circles, or which are found within the signs of the third sense; as the part in which the sign ♈ [Aries] of the first sense exists is called the sign ♈ [Aries], and so of the rest. These signs in the third sense also arise from the division of the zodiac in the third sense, namely when it is taken for the whole surface of the heaven, whether convex or concave. In this third manner all the stars, and all the points of the heaven, even those near the poles of the zodiac — the poles of the zodiac alone excepted, which can be referred equally well to all the signs — are said to be in some sign, that is, under some sign, even if the point of the heaven is not in the primum mobile.

[Margin: Third sense of the zodiac.]

[Margin: All points of the heaven are in some sign in the third sense.]

Now let a certain body be understood whose base is a sign, according to the sense in which we have just now last taken "sign," and whose apex is upon the axis of the zodiac. Such a body, then, in the fourth signification is called a sign; and according to this sense the whole world is divided into twelve equal parts, which are called signs. And thus whatever is in the world is in some sign.

[Margin: Fourth sense of "sign."]

COMMENTARY

Diagram. A half-circle (semicircular) woodcut set at the head of the left-hand column, containing small star symbols and a crescent moon — illustrating the "sign" of the fourth sense as a solid body bounded by two semicircular plane surfaces, with its base at the zodiacal surface and its apex on the axis of the zodiac. No letter-labels are legibly present.

[Margin: Fourth sense of the zodiac.]

In the fourth manner a "sign" is again taken for a certain body, as in the third sense. For if there be understood some body whose base is a sign in the third signification, and whose sides are the plane surfaces of two semicircles whose circumferences enclose the same sign, in such a way that the apex of the body lies on the axis of the zodiac, there will be had a sign in the fourth sense. For in the fourth sense the zodiac is taken for the whole solidity of the world. Hence, if the whole world be divided into 12 equal parts by circles that pass through the poles of the zodiac and the beginnings of the signs and mutually cut one another on the axis of the zodiac, there will result 12 signs in the fourth sense. Therefore, according to this sense of "sign," there will be nothing in the entire world that is not said to be in some sign — since these 12 signs constitute the whole universe as integral parts, so that there is no particle, however small, in the world that is found outside one of the said 12 signs.

[Margin: All things that are in the world are in some sign in the fourth sense.]

Some astronomers add two more senses to the four senses of "sign" and "zodiac," so that in a fifth manner the zodiac is called the ecliptic line alone — which is, as we have said, the circumference of the circle that the Sun describes by its own annual motion from west to east. Hence, if this circumference of the ecliptic be cut into 12 equal parts, there will be produced 12 signs in the fifth sense; so that a sign in the fifth signification is nothing other than a twelfth part of the ecliptic

line. Then in a sixth manner the zodiac is taken for the plane circular surface enclosed by the circumference of the ecliptic. Therefore, if from the signs in the fifth sense straight lines be dropped to the center of the world, the whole ecliptic circle will be divided into 12 sectors, or equal parts, which will give the signs in the sixth sense. And so a sign in the sixth signification is a sector of the ecliptic circle, being a twelfth part of that circle.

[Margin: Fifth sense of the zodiac and the sign.]

[Margin: Sixth sense of the zodiac and the sign.]

[Margin: How astronomers say that all things are in some sign.]

This so varied and manifold signification, or sense, of "zodiac" and "sign" was devised by the practitioners so that all things whatsoever that are in the world might in some way conveniently be said to be in some sign. Yet among the more expert astronomers the sign in the fifth sense suffices for all things to be said to be in some sign. For if through the poles of the zodiac and through any star or point in the world the surface of a great circle be understood to pass, that star or point will be said to be in that sign to which the circumference of the said circle reaches on the ecliptic line — as we shall teach more clearly when we treat of the latitude of the stars in the offices of the ecliptic line.

THE OFFICES OF THE ZODIAC, OR OF THE ECLIPTIC.

I.

[Margin: The ecliptic is the measure of the motion of the heaven from west to east.]

It is the rule and measure of the second motion, which is from west to east, just as the equator is the measure of the first motion, which is from east to west. For just as by the equinoctial circle we know how great is the diurnal motion of the stars, so too by the zodiac we learn in how much time the fixed stars and the planets, carried along the obliquity of the zodiac, complete their own proper motions from west to east. Likewise, just as the equator is the greatest circle described by the first, or diurnal, motion, and is the girdle dividing the first motion itself into equal halves and equally removed in all its parts from the two poles of the world, so also the zodiac is the greatest circle described by the second motion; and it is

the girdle of the second motion, dividing it in two and equally distant from the poles of the zodiac in all its parts.

II.

[Margin: The ecliptic is the cause of eclipses.]

Beneath the ecliptic occur the eclipses of the luminaries, namely of the Sun and of the Moon — from which it is called "ecliptic": so that whenever the Moon in conjunction with the Sun stands beneath the ecliptic, or at least near the ecliptic, an eclipse of the Sun occurs; but in opposition to the Sun, an eclipse of the Moon.

III.

[Margin: The ecliptic is the cause of the inequality of the days and of the changing of the seasons.]

The ecliptic, by its obliquity, is the cause of the inequality of the days and nights — indeed the origin of all the changing of the seasons of the year; whence, according to the philosophers, it is also the cause of generation and corruption.

IV.

[Margin: The ecliptic divides the heaven into a boreal and an austral hemisphere.]

It divides the whole heaven into two hemispheres, the one of which — that which lies between the ecliptic and the northern ecliptic pole — is called northern, while the other, placed between the ecliptic and the southern ecliptic pole, is named southern. For although, absolutely speaking, that part of the heaven placed between the arctic pole and the equator is called northern, and the rest southern — as we said above in the exposition of the offices of the equator — nevertheless it pleased the astronomers that the same heaven should also be divided by the ecliptic into a northern and a southern hemisphere, perhaps on account of the second motion from west to east. For thus it will come about that, just as one and the same star, moved by the primum mobile with the diurnal motion, is always in the same way northern or southern — so that on account of that motion it neither approaches the equator more nor recedes from it — so also the same star, moved from west to east by the second motion, is, in this latter manner, always northern or southern by the same reasoning. For on account of this motion no star whatever will ever be nearer to the ecliptic, or farther from it. Hence it has come about that astronomers sometimes divide the stars into northern and southern with regard to the ecliptic, and not to the equator, as is

plainly seen from the table of the fixed stars which we described in chapter 1. Hence also it comes about that planets existing in the sign ♋ [Cancer], which is the most northern, and in the other northern signs, are sometimes said in the tables of the ephemerides to be southern, because they deviate from the ecliptic toward the south, although they decline from the equator toward the north. Similarly, planets existing in the sign ♏ [Capricorn], which is the most southern, and also in the other southern signs, are...

[Margin: For what reason planets existing in boreal signs can be called austral, and boreal when they exist in austral signs.]

CAPVT SECVNDVM — On the Circles of the Sphere (continued)

On the Zodiac / Ecliptic — the Functions (Officia) of the Ecliptic (continued)

(continuation of the fourth function, from the preceding page)

are called Northern; namely because they run out northward from the Ecliptic, even though they bend southward away from the Equator, as is explained in the *Theorica of the Planets* [*Theorica Planetarum*]. On this reasoning the Sun can never be called Northern or Southern, because it never leaves the ecliptic path. The same must be said of the fixed stars and of the other planets, insofar as they may lie exactly under the Ecliptic.

Besides the two senses already described, astronomers understand the "Northern part" and the "Southern part" in yet another way. For the vertical circle properly so called — the one that passes through the vertex of the head, that is, through the Zenith of any given place, and through the common intersections of the Equator and the Horizon, and that stands perpendicular to the Horizon — likewise divides the whole heaven into two hemispheres. The one that extends from this vertical circle toward the North is called Septentrional; the other, which slopes toward the South, is called Meridional. It is in this sense that Ptolemy [Ptolemy] understands the Northern and Southern part in his little book *On the Analemma*, and all the astronomers who hand down the construction of sundials follow him. For this third sense of "Northern part" and "Southern part" is the most convenient for laying out sundials. And so by three circles — namely the Equator, the Zodiac, and the vertical circle properly so called — the sphere is

divided by astronomers, in three distinct ways, into a Boreal and an Austral hemisphere. I wished to alert the reader to this here, so that he may attentively consider, whenever writers make mention of these parts of the heaven, in which sense they mean the Septentrional or Meridional hemisphere. From this third sense it follows that the Sun, as it runs through the Boreal signs near its rising and setting, is called Septentrional, but for the rest of the day, before and after noon, is called Meridional. This, however, must be understood of a northern dwelling-place, where the pole's altitude is greater than $23\frac{1}{2}^{\circ}$. For where it is less, the Sun, when it is near Cancer [♋], will be Boreal — that is, Septentrional — the whole day long.

[Margin: The vertical circle properly so called cuts the heaven into a Boreal and an Austral hemisphere.]

[Margin: The heaven is divided into a Boreal and an Austral hemisphere in three ways: namely by the Equator, the Ecliptic, and the vertical circle.]

[Margin: How the Sun on the same day is both Boreal and Austral.]

V.

The Ecliptic is the boundary from which the latitudes of all the stars and points of the heaven are reckoned, just as the Equator bounds all the declinations of the heavenly bodies. For the latitude of stars differs from their declination in this: latitude is the distance from the Ecliptic, whereas declination is the distance from the Equator — although some writers, among whom is our own Author [Sacrobosco], occasionally call both distances "declination" without any distinction, though not without qualification. For they call latitude a "declination from the Ecliptic," and declination properly so called a "declination from the Equator." But it is better, with the other astronomers, to assign to each of these distances its own distinctive name.

[Margin: The Ecliptic is the boundary from which the latitudes of the heavenly bodies are computed.]

[Margin: What the latitude of a star is, and how it differs from declination.]

Each distance, moreover, is twofold, according as a given star recedes from the Ecliptic, or from the Equator, toward the North or toward the South. For if a star inclines from the Ecliptic toward the North, it is said to have northern latitude; but if it bends toward the South, it is declared to have southern latitude. In the same way, a star receding from the Equator toward the North has northern or

boreal declination; while one receding toward the south obtains southern or meridional declination.

[Margin: The northern and southern latitude and declination of stars, and by what method each is measured.]

Astronomers measure the latitude of any star by a great circle drawn through the poles of the Zodiac and through the center of the star; and this circle is commonly called the **circle of latitude**. Hence the latitude of a star is defined by astronomers as follows: the latitude of a star is the arc of the great circle passing through the poles of the Zodiac and through the center of the star, intercepted between the Ecliptic and the true place of the star. The degree of the Ecliptic through which the circle of latitude passes is called the degree of the star's longitude; for it is shown that this degree is intercepted between it and the beginning of Aries [γ], from which the longitude of any star must be taken, proceeding according to the succession of the signs — so that the longitude of a star is nothing other than the arc of the Ecliptic reckoned from the beginning of Aries up to the star's circle of latitude, following the order of the signs.

[Margin: The circle of latitude.]

[Margin: What the longitude of a star is.]

The declination of any star, on the other hand, is measured by a great circle passing through the poles of the world and through the center of the star; and this circle is customarily called the **circle of declination**. Accordingly, declination has come to be defined by astronomers as follows: the declination of a star, or of a degree of the Ecliptic, is the arc of the great circle passing through the poles of the world and through the center of the star — or through the proposed degree of the Ecliptic as it lies on that circle — intercepted between the Equator and the star, or the degree of the Ecliptic. Both latitude and declination can be at most 90° ; for no point of the heaven can recede farther from the Ecliptic, or from the Equator, than by a quadrant. Hence it comes about that the poles of the Zodiac have the greatest latitude, and the poles of the world the greatest declination; since the poles of any great circle are separated from its circumference by a quadrant, as we have demonstrated in the corollary to Proposition 16, Book I of [our commentary on] Theodosius [Theodosius].

[Margin: The circle of declination.]

[Margin: The various states of the stars with respect to latitude and declination.]

From what we have said about the latitude and declination of stars, it is gathered: **first**, that stars, or planets, sometimes have declination but no latitude — such as the stars that are found outside the Equator but are placed precisely under the Ecliptic, as the Sun is at all times except the two Equinoxes. **Secondly**, that stars sometimes have latitude but no declination, as are all the stars which, placed outside the Ecliptic, are set directly under the Equator. **Thirdly**, that stars sometimes lack both latitude and declination, as is the Sun at the time of the Equinoxes. **Fourthly**, that some stars have northern latitude and also northern declination; such as the stars which deviate from both the Ecliptic and the Equator toward the North. **Fifthly**, that some stars have both southern latitude and southern declination; such as the stars which recede toward the south from both the Ecliptic and the Equator. **Sixthly**, that some stars have northern latitude and southern declination; as are the stars placed between the Equator and that half of the Ecliptic which slopes to the south. **Seventhly**, that some stars have southern latitude and northern declination; such as the stars comprised between the Equator and the boreal half of the Zodiac.

In passing, it must also be noted here that those points of the Ecliptic which are equally distant from the Equinoctial points — where, that is, the Equator and the Ecliptic intersect one another — have equal declinations; that a point farther from either Equinoctial point has a greater declination; and finally, that the most remote point, namely the midpoint between the Equinoctial points, such as the beginning of Cancer [♋] and of Capricorn [♐], has the greatest declination. From this it follows that in the Ecliptic there are two non-declining points, namely the Equinoctial points themselves, and that everywhere four points decline equally from the Equator — two northern and two southern —

[Margin: Which points of the Ecliptic have equal declinations, and which greater or lesser.]

since there are always found four points equally distant from the two Equinoctial points. In the same way, points of the Ecliptic that have equal declinations will be equally distant from the Equinoctial points; a point that has a greater declination will be farther from the Equinoctial point; and the point that, finally, has the greatest declination will be farthest from the Equinoctial point. All these things can easily be demonstrated from the *Elements of Spherics* of Theodosius and from spherical triangles; and we have demonstrated them in Book I of [our commentary on] the *Astrolabe*, Lemma 49.

VI.

[Margin: The Ecliptic reveals the true places of the stars in the Zodiac, and what the true place of any star in the Zodiac is.]

The Ecliptic reveals the true places of the stars and planets in the Zodiac, so that, with the help of the Ecliptic, it is not difficult to know in which sign, and in which degree of the sign, any star or planet is located. For any heavenly body is said to be in that degree through which the body's circle of latitude passes; so that if it passes, for example, through 10° of Leo [♌], the body is said to be in 10° of Leo, and so on. From this it follows that those stars which lie in the same semicircle of latitude, intercepted between the two poles of the Zodiac, exist in one and the same degree of the Zodiac, even though one is at the extreme north and the other at the extreme south. Only to the poles of the Zodiac can no proper places in the Zodiac be assigned, since there is no greater reason why they should be said to be in this sign rather than in that; rather, they can be referred equally well to all.

VII.

[Margin: The Ecliptic indicates the true motions of the stars. What the true motion, and the line of true motion, of a star is.]

By the office of the Ecliptic astronomers investigate the true motions of the planets and of all the fixed stars. For the true motion of any heavenly body is the arc of the Ecliptic reckoned from the beginning of Aries [♈] up to the line of true motion, following the order of the signs, as is explained in the Theorica [of the Planets]. And the line of true motion is the one drawn from the center of the earth, through the center of the star, to the Ecliptic — or, if the body is not on the Ecliptic, the line extended up to the star's circle of latitude.

DE DVOBVS COLVRIS. [On the Two Colures]

There are, moreover, two other great circles in the sphere, which are called Colures; their function is to distinguish the Solstices and the Equinoxes. The word "Colure" comes from the Greek κῶλον [Greek: κῶλον (kōlon, "limb, member")], meaning "limb," and from [Greek: (?) — printed glyphs unresolved on the scan; the Latin gloss "bos silvester" indicates the standard form οὔρος (ouros, "wild ox")], meaning "wild ox." For just as the erect tail of a wild ox — which is one of its limbs — makes a semicircle and not a complete circle, so the Colure always appears to us incomplete, because only one half of it is visible while the other is hidden from us.

COMMENTARIVS.

In the third place, after the Zodiac, the author treats of the two Colures — because these two circles are intrinsic and mobile, whereas the other two, namely the Meridian and the Horizon, are extrinsic and immobile; and likewise because the two Colures are posited in the heaven in and of themselves, absolutely, whereas the other two are established in the heaven with regard to one's dwelling-place on earth. And those two [the Colures] remain always the same in every clime, whereas these [the Meridian and Horizon] necessarily change whenever the clime changes. Now the two Colures are great circles in the sphere, drawn through the poles of the world and through the four Cardinal points of the Zodiac, intersecting each other at spherical right angles at the poles themselves, and revolving together with the sphere. Their function, he says, is to distinguish the Solstices and Equinoxes — that is, to indicate at which points of the Ecliptic the Solstices and Equinoxes occur, as will be said shortly.

[Margin: What the Colures are.]

He then adduces the etymology of this name — namely, why these two circles are called Colures — which is utterly ridiculous and of no weight. For the proper and true etymology is that these circles are called "Colures" from the Greek word [Greek: κόλος (?) — printed glyphs faint on the scan (kolos, "docked, curtailed," i.e. mutilated and incomplete)], which means "docked" and "incomplete." For to those living in an oblique sphere these circles always appear docked and incomplete, so that all their parts can be seen neither at one and the same time, nor successively at different times.

[Margin: The true etymology of the Colures.]

For their arcs, lying opposite one another on either side near the poles of the world, are so disposed in any oblique sphere that those which stand above the Horizon near the elevated pole are perpetually presented to the eyes and are never removed or withdrawn from sight, while those opposite to them, near the pole depressed beneath the Horizon, are never brought into view but lie perpetually hidden. So much so, that the more oblique the sphere is, the greater are the arcs of these circles that are perpetually visible and perpetually hidden — whereas all the other movable circles in the heaven are so arranged that we either always see them whole and entire above the Horizon, as are the smaller circles near the visible pole, or else are never able to behold them at all, such as the smaller circles near the hidden pole, opposite to the former;

(page continues)

and, by the motion of the prime mover, there will likewise be no circle that is not seen in its entirety, successively, above the Horizon in the space of 24 hours. Hence, if those who dwell in the right sphere had assigned names to the celestial circles, they would have called none of them "Colures."

The solstitial Colure, therefore, in distinguishing the Solstices, passes through the poles of the world and through the poles of the Zodiac, and through the greatest declinations of the Sun — that is, through the first degrees of Cancer and Capricorn. Hence the first point of Cancer, where this Colure intersects the Zodiac, is called the point of the Summer Solstice; because when the Sun is in it, it is the Summer Solstice, and the Sun cannot approach any nearer to the Zenith. Now the Zenith is the point in the firmament placed directly above our heads. But the arc of the Colure intercepted between the point of the Summer Solstice and the equinoctial [point] is called the greatest declination of the Sun. And it is, according to Ptolemy [Ptolemy], 23 degrees and 51 minutes; but according to Almeon [al-Ma'mūn], 23 degrees and 33 minutes.

[Margin: The solstitial Colure.]

[Margin: What the Zenith of the head is.]

[Margin: The greatest declination of the Sun.]

Similarly, the first point of Capricorn, where the same Colure intersects the Zodiac on the other side, and the equinoctial [circle], is called the point of the Winter Solstice; and the arc of the Colure intercepted between that point and the equinoctial [circle] is called the other greatest declination of the Sun, and it is equal to the former...

COMMENTARIUS.

We said above that there are two Colures, the one of the solstices, the other of the equinoxes — which the author too intimated when he said that the function of these circles is to distinguish the Solstices and the Equinoxes. He therefore now explains each separately, beginning with the Colure of the solstices. He says, then, that this Colure distinguishes the solstices; that is, that the Colure of the solstices is the one described through the poles of the world and through the poles of the Zodiac, as well as through the greatest declinations of the Sun. Here he declares that the beginnings of Cancer [♋] and of Capricorn [♏] are the solstitial points — the former the point of the summer solstice, the latter of the winter solstice — since the Sun, when it is at the first point of Cancer, produces the summer solstice and cannot approach any nearer to the Zenith, that is, to the point of the sky set

above our head; whereas when it is at the beginning of Capricorn, it produces the winter solstice and cannot recede any farther from us. Likewise he declares that the two arcs of the solstitial Colure, intercepted between the said solstitial points and the Equator, are called the greatest declinations of the Sun, which are equal to one another, as we shall demonstrate below. But we shall say more about this greatest declination of the Sun, and about the solstice, in [the discussion of] the functions of these circles.

[Margin: What the point of the summer and of the winter solstice is.]

The other Colure passes through the poles of the world and through the first points of Aries and Libra, where the two Equinoxes are; whence it is called the Colure that distinguishes the Equinoxes. And these two Colures intersect each other over the poles of the world at spherical right angles. The signs of the solstices and of the equinoxes are shown by these verses:

These two make the solstice: Cancer, Capricorn. But Aries and Libra make the nights equal to the days.

[Margin: The Colure of the Equinoxes.]

COMMENTARIVS.

He teaches that the other Colure — the one that passes through the poles of the world and through the beginnings of Aries [γ] and Libra [♎] — is called the Colure of the Equinoxes, or the one that distinguishes the equinoxes; because the Sun, when it is at these points, makes the day equal to the night. And these two Colures, he says, intersect each other at the poles of the world at spherical right angles. Now a spherical angle is the one formed on the convex surface of the sphere by the intersection of the circumferences of two great circles. Hence, if one circle cuts another so that two equal angles are formed on either side, each is called a spherical right angle; but if unequal angles are formed, the greater is called a spherical obtuse angle, and the smaller a spherical acute angle. That the Colures intersect each other at the poles at right angles is evident from Proposition 15, Book I of Theodosius [Theodosius], and from the fifth property of the circles of the sphere cited above, since each passes through the poles of the other. For the beginnings of Aries and Libra — at which, of course, the Colure of the Equinoxes and the Equator intersect each other — are the poles of the solstitial Colure; while the points at which the solstitial Colure and the Equator intersect each other are the poles of the equinoctial Colure, as is clear from the definition of a pole.

[Margin: What a spherical angle is.]

OFFICIA VTRIVSQVE COLVRI. [The Functions of Each Colure]

I.

The two Colures point out four principal points in the Zodiac, which are called Cardinal, and at which, from the motion of the Sun, the greatest seasonal changes usually occur — namely Spring, Summer, Autumn, and Winter; such are the beginnings of Aries [γ], Cancer [♋], Libra [♎], and Capricorn [♏]. Hence the whole Zodiac too is cut by these same Colures, at the said four points, into four Quadrants corresponding to those four seasons of the year. Indeed the Equator as well is divided by them into four Quadrants — which is of the greatest use, as will be clear from Chapter 3, in determining the rising and setting of the signs. In the same way these same Colures divide all the parallel circles — that is, those equidistant from the Equator — into four Quadrants, as can easily be demonstrated from the spherical *Elements* of Theodosius.

[Margin: The two Colures indicate the four Cardinal points, and divide the Zodiac, the Equator, and all the parallels into four quadrants.]

II.

The solstitial Colure, which of course [divides] the Equator and all its parallels, and the Zodiac, or the Eclip...

cuts the ecliptic at right angles, by proposition 15 of Theodosius, book 1 [*Sphaerica* I.15]. Since it passes through the poles of these circles, it marks out two solstitial points, namely the first points of Cancer (♋) and Capricorn (♏).

[Margin: Why the first points of Cancer and Capricorn are called solstitial.]

These are not called solstitial because the Sun, when carried to them, halts and lingers there for a while — that is false, for the Sun never comes to rest in the Zodiac nor interrupts its course, as daily experience shows. Rather, they are so called because, while the Sun is near those points, for several days neither do the noon shadows change but stay the same length (as far as the senses can tell), nor are the lengths of day and night appreciably increased or diminished, so that the Sun seems in a way to stand still at those points. Or again, because when the Sun has reached one of them, it does not advance further but checks its course and turns back once more toward the opposite pole of the world; thus at those points the Sun, as regards its approach to and recession from one pole toward the other, seems in a way to stand still, since it is turning toward the opposite quarter of the

heavens. From this turning of the Sun the Greeks call these same points τροπικά [Greek: *tropika*, "turning-points," whence "tropics"].

[Margin: Why the first point of Cancer and of Capricorn are also called tropical.]

[Margin: What the solstice is.]

A solstice, then, will be nothing other than the end of the Sun's recession from the Equator and the beginning of its approach toward it. There are two solstices: the summer solstice, which occurs when the Sun is at the beginning of Cancer (♋) — speaking of the northern hemisphere, when of course it is summer; and the winter solstice, which happens when the Sun tarries at the beginning of Capricorn (♏), when of course winter is at hand. At the former the Sun stands nearest to the top of our head; at the latter it is farthest from it. Some also call the former the "solstice" absolutely and without qualification, but the latter the "midwinter" [Bruma]. So Ovid too named the winter solstice, in book 1 of the *Fasti*, when he said:

*Bruma noui prima est, veterisque nouissima Solis: Principium capiunt
Phœbus, & annus idem.*

[Midwinter is the first day of the new Sun and the last of the old: Phoebus and the year take their beginning together. — Ovid, *Fasti* I.]

III. The same solstitial colure divides the Zodiac, or ecliptic, into two semicircles. The one that runs from the beginning of Capricorn (♏), through Aries (♈), to the end of Gemini (♊) is called the Ascending semicircle; the other, running from the beginning of Cancer (♋), through Libra (♎), to the end of Sagittarius (♐), is called the Descending — reckoning, of course, from the standpoint of a northern dwelling, as we explained above when we discussed the order of the signs.

[Margin: The solstitial colure cuts the ecliptic into an ascending semicircle and a descending semicircle.]

IIII. The same circle sorts the twelve signs of the Zodiac into two classes. The first class contains six signs — Cancer (♋), Leo (♌), Virgo, Libra, Scorpio, Sagittarius [only the first two sign-glyphs, ♋ and ♌, are legible on the scan; the remaining four follow from the "six signs" count and the established zodiacal order] — which rise directly in the northern oblique sphere. The second class comprises the remaining six — Capricorn, Aquarius, Pisces, Aries, Taurus, Gemini [sign-glyphs not individually resolved on the scan] — which rise obliquely, as we shall set out in chapter 3.

[Margin: The solstitial colure divides the Zodiac into six signs that rise directly in the oblique sphere and six that rise obliquely.]

V. [?] Furthermore, this circle, in marking off the solstices, measures the Sun's maximum declinations. For when the Sun reaches this circle by its own motion from west to east — whether on the northern side, where the beginning of Cancer (♋) lies, or on the southern side, where the beginning of Capricorn (♐) lies — it declines farthest from the Equator. Hence at the aforesaid points it is said to have its maximum declination, since beyond them it no longer runs off toward the north or the south but turns back toward the Equator. This maximum declination is fixed by the solstitial colure: the Sun's maximum declination is exactly as great as the arc of the solstitial colure intercepted between the Equator and the point of either solstice.

[Margin: The solstitial circle measures the Sun's maximum declinations.]

This maximum declination of the Sun has been found to differ among astronomers at different periods, on account of the motion of libration of the tenth sphere, by which all the lower spheres are moved, as was said in chapter 1. Thus:

- **Ptolemy** found the Sun's maximum declination to comprise $23^{\circ} 51' 20''$ — nearly the value our author [Sacrobosco] asserted, following Ptolemy's opinion.
- **al-Battānī** [Mahometes Aratensis] found the same to be $23^{\circ} 35'$.
- **al-Zarqālī** [Arzahel the Spaniard] observed it to be $23^{\circ} 34'$.
- **al-Ma'mūn** [Almeon] found it to be $23^{\circ} 33'$, as the author reported.
- **Profatius the Jew** [Prophatius Iudaeus / Jacob ben Machir] reckoned it at $23^{\circ} 32'$.
- **Regiomontanus** [Ioannes Regiom.] asserted it to be $23^{\circ} 30'$.
- **Domenico Maria Novara** [Dominicus Maria the Italian] said it stood at $23^{\circ} 29'$.
- **Johannes Werner of Nuremberg** assigned it $23^{\circ} 28' 30''$.
- **Nicolaus Copernicus** declared it $23^{\circ} 28' 20''$.
- **Tycho Brahe** the Dane asserted it to be $23^{\circ} 31' 30''$.

[Margin: Various observations of the Sun's maximum declination, and which we think should be adopted.]

Copernicus, moreover, demonstrated that this maximum declination of the Sun has decreased by a regular motion, and will go on decreasing to $23^{\circ} 28'$ and no

further; thereafter it will increase again up to $23^{\circ} 52'$, so that the former is the maximum and the latter the minimum, and the difference between maximum and minimum amounts to 24 minutes.

Among all the aforesaid maxima of the Sun's declination, the common school of astronomers holds to the one that Regiomontanus, the foremost astronomer, observed — namely $23^{\circ} 30'$ — although it is quite probable that it may be only about $23^{\circ} 28'$ and a little more, as Copernicus put it. Nevertheless, so as not to seem to depart from the common view, in what follows we shall adopt $23^{\circ} 30'$, chiefly for this reason: that 2 minutes make no notable difference, and that 30 minutes are exactly half of one degree.

[Margin: By what method the Sun's maximum declination is to be investigated.]

The method by which astronomers observe the Sun's maximum declination — the principal one among several — is this. Around the summer solstice, that is, around 22 June, let the Sun's meridian altitude be observed with the greatest care until its maximum is found, for at that time the Sun has its greatest declination in summer. Then let the same be done around the winter solstice, until the Sun's least meridian altitude is found, for then the Sun declines farthest from the Equator toward the south. If, then, we subtract this least meridian altitude from that greatest one and halve the remaining degrees, we shall have the Sun's maximum declination on either side of the Equator;

for the maximum northern declination is equal to the maximum southern, as we shall shortly demonstrate — which the author also stated.

[Margin: An example, namely from Vienna in Austria.]

EXAMPLE. At Vienna, Regiomontanus found the Sun's greatest meridian altitude around the summer solstice to be $65^{\circ} 30'$; around the winter solstice he found the Sun's least meridian altitude to be $18^{\circ} 30'$. Subtracting the latter from the former leaves 47° , whose half gives the Sun's maximum declination as $23^{\circ} 30'$. Now, for measuring both meridian altitudes — the maximum and the minimum — the most suitable instrument is a quadrant of very great size, so that even the minutes of degrees can be marked on it, and on which a sighting-line [*linea fiduciae*, the alidade] turns about its center. For if this quadrant is set up in a plane parallel to the horizon, so that one of its sides answers directly to the meridian line and its center faces north, the said meridian altitudes will be found with the greatest ease. You will find the construction of this quadrant in Oronce Fine the Dauphinois [Orontius Delphinus / Finaeus], in the *Sphere* that he wrote.

Once the Sun's maximum declination is known, we shall arrive, by the doctrine of sines, at a knowledge of the declinations of all the points of the ecliptic.

[Margin: By what art the declinations of the points of the ecliptic are computed.]

For — as we demonstrated in the corollary to proposition 1, book 1, of our *Gnomonics* [*Gnomonices*], and elsewhere; likewise Regiomontanus in the *Epitome of the Almagest*, book 1, proposition 18; likewise Geber of Seville [Gebir Hispalensis / Jābir ibn Aflaḥ] in book 2; and Pedro Nunes the Portuguese [Petrus Nonius Lusitanus] in proposition 2 of the second part of *On Twilights* [*De Crepusculis*] — as the whole sine is to the sine of the maximum declination, so is the sine of the arc by which a given point of the ecliptic is distant from either of the equinoctial points to the sine of the declination of that same point.

Accordingly, following the rule of proportions, if the sine of the maximum declination is multiplied into the sine of the arc by which the given point of the ecliptic is distant from either equinoctial point, and the product is divided by the whole sine — which is done by striking off five figures at the right hand from the product, since we here take the whole sine to be 100000 parts — there results a sine whose arc, found from the table of sines, at once yields the declination of the point proposed.

EXAMPLE. Taking the Sun's maximum declination as $23^{\circ} 30'$, suppose we wish to investigate the declination of the 8th degree of Virgo (♍), which is distant 22° from the autumnal equinox. I therefore multiply the sine of the posited maximum declination, namely 39874, into the sine of the given distance — that is, of 22° , namely 37460 — and the number 1,493,680,040 is produced. Striking off five figures from the right side of this, there remains the sine 14936, to which the arc $8^{\circ} 35'$ corresponds in the table of sines. This, then, we shall say is the declination of the 8th degree of Virgo (♍). And so for the rest.

By this art we have computed the following table, in which are contained the declinations of all the degrees of the Zodiac, together with the twelfth-parts of degrees [i.e. in 5-minute steps, one-twelfth of a degree being $5'$]: so that the table is laid out by five minutes of a degree. But since — as we said above, at the end of the fifth function of the Equator — there are always found in the Zodiac four points that have equal declinations, it will be enough to compute the declinations of all the degrees and minutes of a single quadrant; for the points of the other three quadrants will readily be fitted to the parts of this quadrant, as may be seen on a material sphere and as will be plain in the following table.

THERE FOLLOWS A TABLE OF DECLINATIONS. [Sequitur tabula declinationum.]

DECLINATIONS OF THE POINTS OF THE ECLIPTIC FROM THE EQUATOR

[DECLINATIONES PVNCTORVM Eclipticae ab Æquatore.]

[Margin, left side (rotated): Degrees and minutes of the upper six signs (Gradus ac Minuta superiorum sex Signorum).] [Margin, right side (rotated): Degrees and minutes of the lower six signs (Gradus ac Minuta inferiorum sex Signorum).]

Column headings as printed: each data column carries a **top label** (read downward with the left "Signa" argument, 0°→30°) and a **bottom label** (read upward with the right "Signa" argument, 30°→0°). Values are degrees (G) and minutes (M) of declination. This page carries the top portion of the table only (argument 0° 0' through 4° 25'); the table continues onto the following page.

Signa (arg., G° M') — top ↓	♈ / ♈ (G° M')	♉ / ♉ (G° M')	♊ / ♊ (G° M')	Signa (arg., G° M') — bottom ↑
0° 0'	0° 0'	11° 30'	20° 12'	30° 0'
0° 5'	0° 2'	11° 32'	20° 13'	29° 55'
0° 10'	0° 4'	11° 34'	20° 14'	29° 50'
0° 15'	0° 6'	11° 35'	20° 15'	29° 45'
0° 20'	0° 8'	11° 37'	20° 16'	29° 40'
0° 25'	0° 10'	11° 39'	20° 17'	29° 35'
0° 30'	0° 12'	11° 41'	20° 18'	29° 30'
0° 35'	0° 14'	11° 42'	20° 19'	29° 25'
0° 40'	0° 16'	11° 44'	20° 20'	29° 20'
0° 45'	0° 18'	11° 46'	20° 22' [?]	29° 15'
0° 50'	0° 20'	11° 48'	20° 23'	29° 10'
0° 55'	0° 22'	11° 49'	20° 24'	29° 5'
1° 0'	0° 24'	11° 51'	20° 25'	29° 0'
1° 5'	0° 26'	11° 53'	20° 26'	28° 55'
1° 10'	0° 28'	11° 55'	20° 27'	28° 50'
1° 15'	0° 30'	11° 56'	20° 28'	28° 45'
1° 20'	0° 32'	11° 58'	20° 29'	28° 40'

Signa (arg., G° M') — top ↓	Υ / ϐ (G° M')	ϣ / ϑ (G° M')	Π / ϡ (G° M')	Signa (arg., G° M') — bottom ↑
1° 25'	0° 34'	12° 0'	20° 30'	28° 35'
1° 30'	0° 36'	12° 1'	20° 31'	28° 30'
1° 35'	0° 38'	12° 3'	20° 32'	28° 25'
1° 40'	0° 40'	12° 5'	20° 33'	28° 20'
1° 45'	0° 42'	12° 7'	20° 34'	28° 15'
1° 50'	0° 44'	12° 8'	20° 35'	28° 10'
1° 55'	0° 46'	12° 10'	20° 36'	28° 5'
2° 0'	0° 48'	12° 12'	20° 37'	28° 0'
2° 5'	0° 50'	12° 14'	20° 38'	27° 55'
2° 10'	0° 52'	12° 15'	20° 39'	27° 50'
2° 15'	0° 54'	12° 17'	20° 40'	27° 45'
2° 20'	0° 56'	12° 19'	20° 41'	27° 40'
2° 25'	0° 58'	12° 21'	20° 42'	27° 35'
2° 30'	1° 0'	12° 22'	20° 43'	27° 30'
2° 35'	1° 2'	12° 24'	20° 44'	27° 25'
2° 40'	1° 4'	12° 26'	20° 45'	27° 20'
2° 45'	1° 6'	12° 28'	20° 46'	27° 15'
2° 50'	1° 8'	12° 29'	20° 47'	27° 10'
2° 55'	1° 10'	12° 31'	20° 48'	27° 5'
3° 0'	1° 12'	12° 33'	20° 49'	27° 0'
3° 5'	1° 14'	12° 34'	20° 50'	26° 55'
3° 10'	1° 16'	12° 36'	20° 51'	26° 50'
3° 15'	1° 18'	12° 38'	20° 52'	26° 45'
3° 20'	1° 20'	12° 39'	20° 53'	26° 40'
3° 25'	1° 22'	12° 41'	20° 53'	26° 35'
3° 30'	1° 24'	12° 43'	20° 54'	26° 30'
3° 35'	1° 26'	12° 45'	20° 55'	26° 25'
3° 40'	1° 28'	12° 46'	20° 56'	26° 20'
3° 45'	1° 30'	12° 48'	20° 57'	26° 15'
3° 50'	1° 32'	12° 50'	20° 58'	26° 10'

Signa (arg., G° M') — top ↓	♈ / ♎ (G° M')	♉ / ♏ (G° M')	♊ / ♐ (G° M')	Signa (arg., G° M') — bottom ↑
3° 55'	1° 34'	12° 51'	20° 59'	26° 5'
4° 0'	1° 36'	12° 53'	21° 0'	26° 0'
4° 5'	1° 38'	12° 55'	21° 1'	25° 55'
4° 10'	1° 40'	12° 56'	21° 2'	25° 50'
4° 15'	1° 42'	12° 58'	21° 3'	25° 45'
4° 20'	1° 44'	13° 0'	21° 4'	25° 40'
4° 25'	1° 46'	13° 1'	21° 5'	25° 35'
Signa (bottom label)	♋ / ♍	♊ / ♏	♉ / ♎	Signa (bottom label)

(Table continues on the following page; the left argument column advances by 5' steps and the complementary right argument column decreases by 5', their sum being 30° 0' in every row.)

Declination of the points of the ecliptic from the equator (continued)

Table of the declinations of the points of the ecliptic (DECLINATIONES PVNCTORVM ECLIPTICÆ AB ÆQVATORE), in steps of 5 minutes, computed for a greatest obliquity of 23°30'. This is the continuation of the table begun earlier; the present pages carry the argument from 4°30' through 19°20'.

Each declination column serves four zodiacal signs. Read **downward** with the left-hand argument column ("Degrees and minutes of the upper six signs" [*Gradus ac Minuta superiorum sex Signorum*], engraved vertically in the left margin), the three columns give the declination of ♈ Aries / ♎ Libra, ♉ Taurus / ♏ Scorpio, and ♊ Gemini / ♐ Sagittarius. Read **upward** with the right-hand argument column ("Degrees and minutes of the lower six signs" [*Gradus ac Minuta inferiorum sex Signorum*], engraved vertically in the right margin), the same three columns serve ♋ Pisces / ♍ Virgo, ♊ Aquarius / ♏ Leo, and ♉ Capricorn / ♎ Cancer (the sign glyphs repeated as foot-headers). The left and right argument values on any one row sum to 30°0'.

Table. Argument 4°30' – 9°25'.

Signa (upper six)	♈ Aries / ⚎ Libra	♉ Taurus / ♎ Scorpio	♊ Gemini / ♊ Sagittarius	Signa (lower six)
4°30'	1°48'	13°3'	21°6'	25°30'
4°35'	1°50'	13°5'	21°7'	25°25'
4°40'	1°52'	13°7'	21°8'	25°20'
4°45'	1°54'	13°8'	21°8'	25°15'
4°50'	1°56'	13°10'	21°9'	25°10'
4°55'	1°58'	13°11'	21°10'	25°5'
5°0'	2°0'	13°13'	21°11'	25°0'
5°5'	2°2'	13°15'	21°12'	24°55'
5°10'	2°4'	13°17'	21°13'	24°50'
5°15'	2°6'	13°18'	21°14'	24°45'
5°20'	2°8'	13°20'	21°15'	24°40'
5°25'	2°9'	13°22'	21°16'	24°35'
5°30'	2°11'	13°23'	21°16'	24°30'
5°35'	2°13'	13°25'	21°17'	24°25'
5°40'	2°15'	13°27'	21°18'	24°20'
5°45'	2°17'	13°28'	21°19'	24°15'
5°50'	2°19'	13°30'	21°20'	24°10'
5°55'	2°21'	13°32'	21°21'	24°5'
6°0'	2°23'	13°33'	21°22'	24°0'
6°5'	2°25'	13°35'	21°23'	23°55'
6°10'	2°27'	13°37'	21°23'	23°50'
6°15'	2°29'	13°38'	21°24'	23°45'
6°20'	2°31'	13°40'	21°25'	23°40'
6°25'	2°33'	13°42'	21°26'	23°35'
6°30'	2°35'	13°43'	21°27'	23°30'
6°35'	2°37'	13°45'	21°28'	23°25'
6°40'	2°39'	13°46'	21°28'	23°20'
6°45'	2°41'	13°48'	21°29'	23°15'
6°50'	2°43'	13°50'	21°30'	23°10'
6°55'	2°45'	13°51'	21°31'	23°5'

Signa (upper six)	♈ Aries / ♎ Libra	♉ Taurus / ♏ Scorpio	♊ Gemini / ♐ Sagittarius	Signa (lower six)
7°0'	2°47'	13°53'	21°32'	23°0'
7°5'	2°49'	13°55'	21°33'	22°55'
7°10'	2°51'	13°56'	21°34'	22°50'
7°15'	2°53'	13°58'	21°34'	22°45'
7°20'	2°55'	14°0'	21°35'	22°40'
7°25'	2°57'	14°1'	21°36'	22°35'
7°30'	2°59'	14°3'	21°37'	22°30'
7°35'	3°1'	14°5'	21°38'	22°25'
7°40'	3°3'	14°6'	21°39'	22°20'
7°45'	3°5'	14°8'	21°39'	22°15'
7°50'	3°7'	14°9'	21°40'	22°10'
7°55'	3°9'	14°11'	21°41'	22°5'
8°0'	3°11'	14°13'	21°42'	22°0'
8°5'	3°13'	14°14'	21°42'	21°55'
8°10'	3°15'	14°16'	21°43'	21°50'
8°15'	3°17'	14°18'	21°44'	21°45'
8°20'	3°19'	14°19'	21°45'	21°40'
8°25'	3°21'	14°21'	21°46'	21°35'
8°30'	3°23'	14°22'	21°47'	21°30'
8°35'	3°25'	14°24'	21°47'	21°25'
8°40'	3°27'	14°25'	21°48'	21°20'
8°45'	3°29'	14°27'	21°49'	21°15'
8°50'	3°31'	14°29'	21°50'	21°10'
8°55'	3°33'	14°30'	21°51'	21°5'
9°0'	3°35'	14°32'	21°51'	21°0'
9°5'	3°37'	14°34'	21°52'	20°55'
9°10'	3°39'	14°35'	21°53'	20°50'
9°15'	3°40'	14°37'	21°54'	20°45'
9°20'	3°42'	14°38'	21°54'	20°40'
9°25'	3°44'	14°40'	21°55'	20°35'

Table (continued — Source: printed p. 152). Argument 9°30' – 14°20'.

Signa (upper six)	♈ Aries / ⚎ Libra	♉ Taurus / ⚍ Scorpio	♊ Gemini / ⚊ Sagittarius	Signa (lower six)
9°30'	3°46'	14°42'	21°56'	20°30'
9°35'	3°48'	14°43'	21°57'	20°25'
9°40'	3°50'	14°45'	21°57'	20°20'
9°45'	3°52'	14°46'	21°58'	20°15'
9°50'	3°54'	14°48'	21°59'	20°10'
9°55'	3°56'	14°49'	22°0'	20°5'
10°0'	3°58'	14°51'	22°0'	20°0'
10°5'	4°0'	14°53'	22°1'	19°55'
10°10'	4°2'	14°54'	22°2'	19°50'
10°15'	4°4'	14°56'	22°3'	19°45'
10°20'	4°6'	14°57'	22°3'	19°40'
10°25'	4°8'	14°59'	22°4'	19°35'
10°30'	4°10'	15°1'	22°5'	19°30'
10°35'	4°12'	15°2'	22°5'	19°25'
10°40'	4°14'	15°4'	22°6'	19°20'
10°45'	4°16'	15°5'	22°7'	19°15'
10°50'	4°18'	15°7'	22°8'	19°10'
10°55'	4°20'	15°8'	22°8'	19°5'
11°0'	4°22'	15°10'	22°9'	19°0'
11°5'	4°24'	15°11'	22°10'	18°55'
11°10'	4°26'	15°13'	22°10'	18°50'
11°15'	4°28'	15°14'	22°11'	18°45'
11°20'	4°30'	15°16'	22°12'	18°40'
11°25'	4°32'	15°18'	22°12'	18°35'
11°30'	4°34'	15°19'	22°13'	18°30'
11°35'	4°36'	15°21'	22°14'	18°25'
11°40'	4°38'	15°22'	22°15'	18°20'
11°45'	4°39'	15°24'	22°15'	18°15'
11°50'	4°41'	15°25'	22°16'	18°10'

Signa (upper six)	♈ Aries / ♎ Libra	♉ Taurus / ♏ Scorpio	♊ Gemini / ♐ Sagittarius	Signa (lower six)
11°55'	4°43'	15°27'	22°16'	18°5'
12°0'	4°45'	15°28'	22°17'	18°0'
12°5'	4°47'	15°30'	22°18'	17°55'
12°10'	4°49'	15°32'	22°18'	17°50'
12°15'	4°51'	15°33'	22°19'	17°45'
12°20'	4°53'	15°35'	22°20'	17°40'
12°25'	4°55'	15°36'	22°20'	17°35'
12°30'	4°57'	15°38'	22°21'	17°30'
12°35'	4°59'	15°39'	22°22'	17°25'
12°40'	5°1'	15°41'	22°22'	17°20'
12°45'	5°3'	15°42'	22°23'	17°15'
12°50'	5°5'	15°44'	22°23'	17°10'
12°55'	5°7'	15°45'	22°24'	17°5'
13°0'	5°9'	15°47'	22°25'	17°0'
13°5'	5°11'	15°48'	22°26'	16°55'
13°10'	5°13'	15°50'	22°26'	16°50'
13°15'	5°15'	15°51'	22°27'	16°45'
13°20'	5°17'	15°53'	22°27'	16°40'
13°25'	5°19'	15°54'	22°28'	16°35'
13°30'	5°20'	15°56'	22°29'	16°30'
13°35'	5°22'	15°57'	22°29'	16°25'
13°40'	5°24'	15°59'	22°30'	16°20'
13°45'	5°26'	16°0'	22°30'	16°15'
13°50'	5°28'	16°2'	22°31'	16°10'
13°55'	5°30'	16°3'	22°31'	16°5'
14°0'	5°32'	16°5'	22°32'	16°0'
14°5'	5°34'	16°6'	22°33'	15°55'
14°10'	5°36'	16°8'	22°33'	15°50'
14°15'	5°38'	16°9'	22°34'	15°45'
14°20'	5°40'	16°11'	22°35'	15°40'

Table (continued — Source: printed p. 153). Argument 14°25' – 19°20'.

Signa (upper six)	♈ Aries / ⚎ Libra	♉ Taurus / ⚍ Scorpio	♊ Gemini / ⚊ Sagittarius	Signa (lower six)
14°25'	5°42'	16°12'	22°35'	15°35'
14°30'	5°44'	16°14'	22°36'	15°30'
14°35'	5°46'	16°15'	22°36'	15°25'
14°40'	5°48'	16°17'	22°37'	15°20'
14°45'	5°50'	16°18'	22°37'	15°15'
14°50'	5°51'	16°20'	22°37'	15°10'
14°55'	5°53'	16°21'	22°38'	15°5'
15°0'	5°55'	16°23'	22°39'	15°0'
15°5'	5°57'	16°24'	22°39'	14°55'
15°10'	5°59'	16°26'	22°40'	14°50'
15°15'	6°1'	16°27'	22°40'	14°45'
15°20'	6°3'	16°28'	22°41'	14°40'
15°25'	6°5'	16°30'	22°41'	14°35'
15°30'	6°7'	16°31'	22°42'	14°30'
15°35'	6°9'	16°33'	22°42'	14°25'
15°40'	6°11'	16°34'	22°43'	14°20'
15°45'	6°13'	16°36'	22°43'	14°15'
15°50'	6°15'	16°37'	22°44'	14°10'
15°55'	6°17'	16°39'	22°45'	14°5'
16°0'	6°19'	16°40'	22°46'	14°0'
16°5'	6°21'	16°41'	22°46'	13°55'
16°10'	6°22'	16°43'	22°47'	13°50'
16°15'	6°24'	16°44'	22°47'	13°45'
16°20'	6°26'	16°46'	22°48'	13°40'
16°25'	6°28'	16°47'	22°48'	13°35'
16°30'	6°30'	16°49'	22°49'	13°30'
16°35'	6°32'	16°50'	22°49'	13°25'
16°40'	6°34'	16°52'	22°50'	13°20'
16°45'	6°36'	16°53'	22°50'	13°15'

Signa (upper six)	♈ Aries / ♎ Libra	♉ Taurus / ♏ Scorpio	♊ Gemini / ♎ Sagittarius	Signa (lower six)
16°50'	6°38'	16°54'	22°51'	13°10'
16°55'	6°40'	16°56'	22°51'	13°5'
17°0'	6°42'	16°57'	22°52'	13°0'
17°5'	6°44'	16°59'	22°52'	12°55'
17°10'	6°46'	17°0'	22°53'	12°50'
17°15'	6°47'	17°2'	22°53'	12°45'
17°20'	6°49'	17°3'	22°54'	12°40'
17°25'	6°51'	17°4'	22°54'	12°35'
17°30'	6°53'	17°6'	32°55' [recte: 22°55']	12°30'
17°35'	6°55'	17°7'	22°55'	12°25'
17°40'	6°57'	17°9'	22°56'	12°20'
17°45'	6°59'	17°10'	22°56'	12°15'
17°50'	7°1'	17°11'	22°57'	12°10'
17°55'	7°3'	17°13'	22°57'	12°5'
18°0'	7°5'	17°14'	22°58'	12°0'
18°5'	7°7'	17°16'	22°58'	11°55'
18°10'	7°8'	17°18'	22°58'	11°50'
18°15'	7°10'	17°19'	22°59'	11°45'
18°20'	7°12'	17°20'	22°59'	11°40'
18°25'	7°14'	17°21'	23°0'	11°35'
18°30'	7°16'	17°23'	23°0'	11°30'
18°35'	7°18'	17°24'	23°0'	11°25'
18°40'	7°20'	17°25'	23°1'	11°20'
18°45'	7°22'	17°27'	23°1'	11°15'
18°50'	7°24'	17°28'	23°2'	11°10'
18°55'	7°26'	17°29'	23°2'	11°5'
19°0'	7°28'	17°31'	23°3'	11°0'
19°5'	7°29'	17°32'	23°3'	10°55'
19°10'	7°31'	17°34'	23°3'	10°50'

Signa (upper six)	♈ Aries / ♎ Libra	♉ Taurus / ♏ Scorpio	♊ Gemini / ♐ Sagittarius	Signa (lower six)
19°15′	7°33′	17°35′	23°4′	10°45′
19°20′	7°35′	17°36′	23°4′	10°40′

Foot-headers (read upward with the right-hand argument): Signa | ♋ Pisces / ♍ Virgo | ♒ Aquarius / ♌ Leo | ♊ Capricorn / ♋ Cancer | Signa. The table continues on the following page.

Clavius, *Opera Mathematica*, Tomus III — Commentary on Chapter II of the *Sphere* of Sacrobosco

Table of the declinations of the points of the ecliptic from the equator (continued)

(Continuation of the table **DECLINATIONES PVNCTORVM ECLIPTICÆ AB ÆQVATORE** begun on the preceding pages; obliquity 23°30′; argument in 5′ steps. Each row's left-hand ("upper six signs") and right-hand ("lower six signs") argument sum to 30°0′. All values are given as degrees (G) and minutes (M).)

Column captions (verbatim): left margin **Gradus ac Minuta superiorum sex Signorum** (Degrees and minutes of the upper six signs); right margin **Gradus ac Minuta inferiorum sex Signorum** (Degrees and minutes of the lower six signs). Top sign-headers: ♈ ♎ | ♉ ♏ | ♊ ♐. Foot sign-headers: ♋ ♍ | ♒ ♌ | ♊ ♋.

Signa (upper) G M	♈ ♎ — G M	♉ ♏ — G M	♊ ♐ — G M	Signa (lower) G M
19 25	7 37	17 38	23 5	10 35
19 30	7 39	17 39	23 5	10 30
19 35	7 41	17 40	23 5	10 25
19 40	7 43	17 42	23 6	10 20
19 45	7 45	17 43	23 6	10 15
19 50	7 47	17 44	23 7	10 10
19 55	7 48	17 46	23 7	10 5
20 0	7 50	17 47	23 7	10 0

Signa (upper) G M	Υ ς — G M	ϣ Ϟ — G M	Π ϣ — G M	Signa (lower) G M
20 5	7 52	17 48	23 8	9 55
20 10	7 54	17 49	23 8	9 50
20 15	7 56	17 51	23 8	9 45
20 20	7 58	17 52	23 9	9 40
20 25	8 0	17 54	23 9	9 35
20 30	8 2	17 55	23 9	9 30
20 35	8 4	17 57	23 10	9 25
20 40	8 5	17 58	23 10	9 20
20 45	8 7	17 59	23 11	9 15
20 50	8 9	18 0	23 11	9 10
20 55	8 11	18 2	23 11	9 5
21 0	8 13	18 3	23 12	9 0
21 5	8 15	18 4	23 12	8 55
21 10	8 17	18 6	23 12	8 50
21 15	8 19	18 7	23 13	8 45
21 20	8 20	18 8	23 13	8 40
21 25	8 22	18 10	23 13	8 35
21 30	8 24	18 11	23 14	8 30
21 35	8 26	18 12	23 14	8 25
21 40	8 28	18 14	23 14	8 20
21 45	8 30	18 15	23 15	8 15
21 50	8 32	18 16	23 15	8 10
21 55	8 34	18 17	23 15	8 5
22 0	8 35	18 19	23 15	8 0
22 5	8 37	18 20	23 16	7 55
22 10	8 39	18 21	23 16	7 50
22 15	8 41	18 23	23 16	7 45
22 20	8 43	18 24	23 16	7 40
22 25	8 45	18 25	23 17	7 35
22 30	8 47	18 27	23 17	7 30

Signa (upper) G M	Υ ς — G M	ϣ Ϟ — G M	Π ϣ — G M	Signa (lower) G M
22 35	8 48	18 28	23 17	7 25
22 40	8 50	18 29	23 18	7 20
22 45	8 52	18 30	23 18	7 15
22 50	8 54	18 32	23 18	7 10
22 55	8 56	18 33	23 19	7 5
23 0	8 58	18 34	23 19	7 0
23 5	9 0	18 35	23 19	6 55
23 10	9 1	18 37	23 19	6 50
23 15	9 3	18 38	23 20	6 45
23 20	9 5	18 39	23 20	6 40
23 25	9 7	18 40	23 20	6 35
23 30	9 9	18 42	23 20	6 30
23 35	9 11	18 43	23 21	6 25
23 40	9 13	18 44	23 21	6 20
23 45	9 14	18 45	23 21	6 15
23 50	9 16	18 47	23 21	6 10
23 55	9 18	18 48	23 22	6 5
24 0	9 20	18 49	23 22	6 0
24 5	9 22	18 50	23 22	5 55
24 10	9 24	18 52	23 22	5 50
24 15	9 26	18 53	23 22	5 45
24 20	9 29	18 54	23 23	5 40
24 25	9 30	18 55	23 23	5 35
24 30	9 32	18 57	23 23	5 30
24 35	9 34	18 58	23 23	5 25
24 40	9 35	18 59	23 24	5 20
24 45	9 37	19 0	23 24	5 15
24 50	9 38	19 2	23 24	5 10
24 55	9 40	19 3	23 24	5 5
25 0	9 42	19 4	23 24	5 0

Signa (upper) G M	Υ ς — G M	ϣ Ϟ — G M	Π ϣ — G M	Signa (lower) G M
25 5	9 44	19 5	23 24	4 55
25 10	9 46	19 6	23 25	4 50
25 15	9 48	19 8	23 25	4 45
25 20	9 49	19 9	23 25	4 40
25 25	9 51	19 10	23 25	4 35
25 30	9 53	19 11	23 25	4 30
25 35	9 55	19 12	23 26	4 25
25 40	9 57	10 13 †	23 26	4 20
25 45	9 59	19 15	23 26	4 15
25 50	10 0	19 16	23 26	4 10
25 55	10 2	19 17	23 26	4 5
26 0	10 4	19 18	23 26	4 0
26 5	10 6	19 19	23 26	3 55
26 10	10 8	19 21	23 27	3 50
26 15	10 9	19 22	23 27	3 45
26 20	10 11	19 23	23 27	3 40
26 25	10 13	19 24	23 27	3 35
26 30	10 15	19 25	23 27	3 30
26 35	10 17	19 26	23 27	3 25
26 40	10 19	19 28	23 27	3 20
26 45	10 20	19 29	23 28	3 15
26 50	10 22	19 30	23 28	3 10
26 55	10 24	19 31	23 28	3 5
27 0	10 26	19 32	23 28	3 0
27 5	10 28	19 33	23 28	2 55
27 10	10 29	19 35	23 28	2 50
27 15	10 31	19 36	23 28	2 45
27 20	10 33	19 37	23 28	2 40
27 25	10 35	19 38	23 28	2 35
27 30	10 37	19 39	23 29	2 30

Signa (upper) G M	Υ ς — G M	Ϡ ϡ — G M	Π ϣ — G M	Signa (lower) G M
27 35	10 38	19 40	23 29	2 25
27 40	10 40	19 41	23 29	2 20
27 45	10 42	19 42	23 29	2 15
27 50	10 44	19 44	23 29	2 10
27 55	10 46	19 45	23 29	2 5
28 0	10 47	19 46	23 29	2 0
28 5	10 49	19 47	23 29	1 55
28 10	10 51	19 48	23 29	1 50
28 15	10 53	19 49	23 29	1 45
28 20	10 54	19 50	23 29	1 40
28 25	10 56	19 51	23 29	1 35
28 30	10 58	19 53	23 29	1 30
28 35	11 0	19 54	23 29	1 25
28 40	11 2	19 55	23 30	1 20
28 45	11 3	19 56	23 30	1 15
28 50	11 5	19 57	23 30	1 10
28 55	11 7	19 58	23 30	1 5
29 0	11 9	19 59	23 30	1 0
29 5	11 11	20 0	23 30	0 55
29 10	11 12	20 1	23 30	0 50
29 15	11 14	20 2	23 30	0 45
29 20	11 16	20 3	23 30	0 40
29 25	11 18	20 5	23 30	0 35
29 30	11 19	20 6	23 30	0 30
29 35	11 21	20 7	23 30	0 25
29 40	11 23	20 8	23 30	0 20
29 45	11 25	20 9	23 30	0 15
29 50	11 27	20 10	23 30	0 10
29 55	11 29	20 11	23 30	0 5
30 0	11 30	20 12	23 30	0 0

† Row 25 40, column $\varnothing$ $\mathbb{M}$, is printed **10 13**, which breaks the otherwise smoothly rising column (25 35 = 19 12; 25 45 = 19 15). Transcribed as printed; almost certainly a compositor slip for **19 13** (or 19 14). See Numerical impossibles.

USUS TABULÆ DECLINATIONVM — Use of the table of declinations

If the sign whose degrees' declinations you want is found in the **top** row of the table, take the degrees and minutes on the **left-hand** side of the table; but if the sign is set in the **bottom** row of the table, the degrees together with their minutes are to be taken on the **right-hand** side. Then, at once, where the sign and the degree you have taken meet in common, you will come upon the degrees and minutes of the declination.

Example. Suppose I wish to know how far the 17th degree of Scorpio $\mathbb{M}$ declines from the equator. In the left-hand part of the table, then, I take the 17th degree of Scorpio (for this sign is placed in the upper part of the table), and at the common corner beneath $\mathbb{M}$ I find 16°57'. This, therefore, I declare to be the declination of the 17th degree of Scorpio.

Again, suppose we must investigate how great a declination 23°40' of Cancer $\mathfrak{S}$ has. Since this sign is in the lower part of the table, in the right-hand part I find that to the said 23 degrees and 40 minutes, above the sign $\mathfrak{S}$, there answers 21°25'. And this is the declination sought.

But if the proposed minutes are not found in the aforesaid table, one must take the declinations of the next-greater and next-smaller minutes, and from their difference derive the proportional part. This part is to be **added** to the declination of the next-smaller minutes if the proposed sign is an **upper** one, but **subtracted** from that same declination of the next-smaller minutes if the sign is a **lower** one.

Example of both cases. I want the declination of 4°27' of Gemini $\mathbb{II}$. Since 27 minutes are not found in the said table, I take the difference of the declinations belonging to the 25 minutes and the 30 minutes of the fourth degree of Gemini; this difference amounts to 1 minute. By the rule of proportion I find that to the 2 minutes (by which 25 is exceeded by 27) there correspond $\frac{2}{5}$ of a minute — that is, 24 seconds — since to the 5 minutes (by which 25 is exceeded by 30) there corresponds the whole 1 minute of the difference. And because Gemini is an upper sign, these 24 seconds must be **added** to the declination of 4°25' Gemini, which is 21°5'. Thus the declination of 4°27' of Gemini will come to 21°5'24".

By the same reasoning I want the declination of $25^{\circ}32'$ of Capricorn ♑. Since the proposed sign is a lower one, I **subtract** the same proportional part — namely 24 seconds — from the declination of $25^{\circ}30'$ Capricorn, that is, from $21^{\circ}6'$; and there will remain the declination sought, $21^{\circ}5'36''$.

Moreover, the Sun's greatest **northern** declination is equal to its greatest **southern** declination, as the Author [Sacrobosco] stated; and this can readily be demonstrated as follows. Take some sphere in which the solstitial colure [Colurus Solstitiorum] is ABCD, the equator [Æquator] is AC, the Zodiac or ecliptic [Zodiacus sive Ecliptica] is FI, the Tropic of Cancer ☊ is FG, the Tropic of Capricorn ♑ is HI, and the Sun's greatest declinations are AF (northern) and CI (southern). Since, then, the semicircle ABC is equal to the semicircle FBI, once the common arc FBC is taken away, AF — the Sun's greatest northern declination — will be equal to the arc CI, that is, to the Sun's greatest southern declination. Which was the thing proposed.

[Margin: That the Sun's greatest northern declination is equal to the Sun's greatest southern declination.]

Diagram. A sphere shown with the solstitial colure as its bounding circle, seen edge-on so that the equator, ecliptic, and two tropics appear as chords. Per the accompanying text the construction is: solstitial colure = the whole circle ABCD; equator = diameter AC; ecliptic = diameter FI; Tropic of Cancer ☊ = chord FG; Tropic of Capricorn ♑ = chord HI; the Sun's greatest northern declination = arc AF, its greatest southern declination = arc CI. Labels legible in the woodcut:

- B (top of circle): one pole of the world
- D (bottom of circle): the other pole of the world
- A (right side, at equator level): equator endpoint on the colure
- F (upper right): ecliptic / Tropic-of-Cancer endpoint
- H (lower right): Tropic-of-Capricorn endpoint
- I (upper left): opposite ecliptic endpoint
- K (upper right, above F): point on the circle
- N (lower left): point on the circle
- M (lower right): point on the circle

[The equator's left endpoint C and the Cancer-tropic's left endpoint G are named in the text but did not resolve clearly on the left side of the leaf.]

VI.

(Section opens with a large display capital line reading SOLSTITIORVM; see QA.)**

The solstitial colure also **measures the distances of the poles of the Zodiac from the poles of the world**. For this distance is as great as the arc of the solstitial colure intercepted between the two poles — namely the pole of the world and the pole of the Zodiac. And these two distances of the poles of the Zodiac from the poles of the world are equal to the Sun's greatest declinations. For let the sphere be set up again, in which the poles of the world are B, D; the poles of the Zodiac are K, L; and the Sun's greatest declinations are AE, CH. Since, then, the quadrant AB is equal to the quadrant EK, once the common arc EB is removed, the arc AE (namely the greatest [solar declination]) will remain equal to the arc BK — that is, to the distance of one pole [of the Zodiac] from the other. By the same reasoning CH, the other greatest solar declination, will be equal to the arc DL, that is, to the distance of the pole of the Zodiac from the pole of the world — provided, of course, that the two quadrants CD, HL are taken. From this it is clear that the northern pole of the Zodiac is as far distant from the northern pole of the world as the southern pole of the Zodiac recedes from the southern pole of the world...

[Margin: The solstitial colure measures the distances of the poles of the Zodiac from the poles of the world. That the distances of the poles of the Zodiac from the poles of the world are equal to the Sun's greatest declinations.]

(The sentence continues into the following pages: "...the southern pole of the Zodiac [recedes from the southern pole of the world].")

Diagram. A second sphere, companion to the first, again seen edge-on. Per the text the construction is: poles of the world = B, D; poles of the Zodiac = K, L; the Sun's greatest declinations = arcs AE and CH; quadrants AB, EK, CD, HL; arcs BK and DL representing the pole-to-pole distances. Labels legible in the woodcut:

- B (top of circle): northern pole of the world
- D (bottom of circle): southern pole of the world
- K (upper right): a pole of the Zodiac
- L (lower left): the other pole of the Zodiac
- A (right side, at equator level): equator endpoint on the colure
- E (upper right): declination-arc endpoint (arc AE)
- H (left side, lower): declination-arc endpoint (arc CH)
- C (left side, at equator level): equator endpoint on the colure

- M (lower right): point on the circle
- G (right side, lower): point on the circle

[The peripheral point-letters of this lower figure are small and partly abraded; several positions (C, H, G) are uncertain — see QA]

of the zodiac, because each of these two distances is equal to the maximum declination of the Sun. This is shown as follows. Since the semicircles BCD and KHL are equal, then if the common arc KD is subtracted, there remain equal arcs BK and DL — that is, the distances of the poles of the zodiac from the poles of the world.

VII.

[Margin: The colure of the equinoxes marks out the two equinoctial points.]

The colure of the equinoxes — which cuts the equator at right angles but the ecliptic at oblique angles, since it passes through the poles of the equator and not through the poles of the ecliptic — marks out the two equinoctial points, namely the first point of Aries ♈ and of Libra ♎, at which the equinoxes occur, as has been said.

[Margin: The equinoxes and solstices shift their positions in the calendar.]

The equinoxes and solstices, however, have not always fallen at the same time of the year, but have continually shifted their positions toward the beginnings of the months in the calendar. For long ago Hipparchus [Hipparchus], in about the year 145 before Christ, found the vernal equinox to occur roughly around 23 March; the autumnal, around 26 September; the summer solstice fell then on about 24 June, and the winter around [day: ?] December. Ptolemy [Ptolemy], by contrast, in the year of the Lord 140, observed the vernal equinox to occur roughly around 22 March; the autumnal, at about 25 September; the summer solstice around [day: ?] June, and the winter around 23 December. And so I am utterly astonished that many who have recently written on the reform of the year should have wished so stubbornly to maintain that the vernal equinox ought to be brought back to 25 March, on the grounds — as they claim — that in the time of Christ, or of Julius Caesar [Julius Caesar], it fell on that day. For this is entirely false. Since in Ptolemy's time the vernal equinox anticipated its place in the calendar by one day in the span of 300 years — as he himself observed most carefully — it follows that in the 200 years or so that fell between Julius Caesar and Ptolemy it anticipated by only 16 hours, that is, $\frac{2}{3}$ of one day. Therefore, since Ptolemy found it roughly around 22 March, it must in Julius Caesar's time have fallen no later than 23

March. Gregory XIII [Gregory XIII], Supreme Pontiff, therefore acted more correctly when in the year 1582 he brought it back to 21 March, on which it fell in the time of the Council of Nicaea, that is, in the year 325. For in this way nothing at all had to be changed in the breviaries and missals, and the same Paschal terms remained which the holy Fathers established in the Council of Nicaea.

[Margin: The cause of the anticipation of the equinoxes and solstices in the calendar.]

The cause of this anticipation is that Julius Caesar, whom the Roman Church followed, assigned to the length of a single year more than was right. For he set the solar year at 365 days and 6 hours. Since in each year he was leaving out those six hours, which over four years made up a whole day, he decreed that every fourth year a whole day, composed of 24 hours, should be intercalated — the year he called bissextile, made up of 366 days. But the solar year is not so long; according to the reckoning of the Alphonsine [Alphonsine] tables it contains only 365 days, 5 hours, 49 minutes, 16 seconds. Thus the Roman year, which the Church uses, exceeds the true year according to the Alphonsine reckoning by 10 minutes of one hour and 44 seconds. Hence it comes about that, by those same minutes and seconds, whenever the Sun returns to the same equinoctial or solstitial point, the said 10 minutes and 44 seconds of one hour are lacking to complete the Roman year. It also follows that the equinoxes and solstices anticipate their places, since — the said minutes being wanting to complete the Roman year — in 400 years they run ahead of their places by nearly three whole days. Therefore, so that in future the equinoxes and solstices should not run further ahead of the days marked for them in the calendar, it was necessary (as Gregory XIII decreed) to omit three bissextiles in every 400 years — that is, to reckon three years which ought to be bissextile, namely of 366 days, as common years, of 365 days. For in this way it will come about that those three whole days are restored, and no anticipation of the annual motion of the Sun will be perceived, but the equinoxes and solstices will recur always on the same days of the year, returning fixed on their set days.

And had the calendar not been corrected, it would come about that in the span of 24,500 years the equinoxes and solstices would exchange their places with one another, so that spring would fall in September, autumn in March, the cold of winter in June, and the heats of summer in December — as we also see happened when Christ was born. Yet in the span of 49,000 years, by the opinion of the Alphonsines, both the solstices and the equinoxes would be restored to their former places. In our own time, before the restoration of the equinox to 21

March, the equinoxes and solstices had receded from the ancient positions noted in the time of Julius Caesar, toward the beginnings of the months, by nearly 12 days. For the vernal equinox fell on 11 March, the autumnal on 13 September; the summer solstice on 12 June, and the winter on 12 December. But after the restoration made by Gregory XIII, the equinoxes fall at this time on 21 March and 23 September, and the solstices on 22 June and [22] December.

[Margin: On what days the equinoxes and solstices used to fall before the correction of the calendar, and on what days they now fall after the correction.]

Since I have spoken of the days of the equinoxes and solstices after the correction of the calendar, it will not be out of place if I set out here a small table containing the Sun's entry into all the signs of the zodiac. For this matter is useful for many things in astronomical work. And although this can be known more accurately from ephemerides or astronomical tables, yet because we do not always have these at hand, we judge it better to know it, even in a rough-and-ready way, than to be wholly ignorant of it — especially since no notable error arises from it in the instruments of mathematicians, even if the Sun's entry into the signs of the zodiac is not known exactly, but it is sometimes reckoned to enter a day earlier, or a day later, than it truly does. For in a single day the Sun's declination does not increase perceptibly, as is clear from the preceding table. The same is to be understood of the degree in which the Sun is placed. For although, when the Sun is in some particular degree, one may take it to be in another very nearly — whether a little less or a little more — it makes no difference, for the reason just given. Nevertheless, a little later I shall set out a table for four years, from which the place of the Sun may be found each day with sufficient accuracy. Meanwhile the table promised here is as follows.

Ingressus Solis in 12. signa Zodiaci — The Sun's entry into the 12 signs of the zodiac.

♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
21 March	21 April	22 May	22 June	23 July	23 August
♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
23 September	24 October	23 November	22 December	21 January	19 February

[Margin: How to know in which degree of the zodiac the Sun stands on any given day. The Sun's entry into the signs, and in which degree it stands on any day; by what method this is learned by heart.]

From this you will easily find in which degree, more or less, the Sun is on any given day, by assigning one degree to each day. For example, since the Sun on 24 September is in the first degree of Libra ♎ , it will be in the 2nd degree of Libra on the 25th; on the 30th, in the 7th degree of Libra; and on 9 October it will stand in the 16th degree of Libra; and so on for the rest. For although by this method we may sometimes be off by one day from the true place of the Sun, no notable error will follow from it, as we have said.

Moreover, the Sun's entry into the 12 signs, and which degree, more or less, it occupies on any day, can most conveniently be known by these verses:

*Inclyta Laus Iustis Impenditur, Hæresis Horret Garrula, Grex Gratus
Faustos Gratatur Honores.*

[These twelve words correspond to the twelve months, January through December; the alphabetic position of each word's initial letter encodes the day of the sign-entry, as explained below.]

The meaning and use of these verses is as follows. The first word is assigned to January, the second to February, the third to March, and so on in order, the other words to the other months. Thus, to know on what day the Sun enters the sign proper to any month, and in what degree it stands on any given day, one must learn in order all 12 signs, as they are set down in these verses:

*Sunt Aries, Taurus, Gemini, Cancer, Leo, Virgo. Libraque, Scorpius,
Arcitenens, Caper, Amphora, Pisces.*

[*Arcitenens* ("the Archer") = Sagittarius; *Caper* ("the Goat") = Capricorn; *Amphora* ("the Water-jar") = Aquarius.]

For the Sun enters the first sign, that is, Aries, in the month of March; the second in April; and so on in order, so that in the twelfth month from March — which is February — the Sun enters the twelfth sign, namely Pisces. Once you know which sign the Sun enters in any given month, take the word of the first two verses corresponding to the given month. However many places the first letter of that word occupies in the alphabet, that many units are to be subtracted from 30, so as to leave the day on which the Sun enters the sign of that month.

Example. The Sun enters Libra — that is, the seventh sign — in the month of September, which is the seventh month from March. And since September corresponds to the ninth word, namely *Gratus* (because September is the ninth month from January), and its first letter, G, is the seventh in the alphabet, we subtract 7 from 30, leaving 23; therefore on 23 September the Sun enters Libra. Again, the Sun enters Pisces in the month of February, to which the second word, *Laus*, is due; and since the first letter, L, is the eleventh in the alphabet, if 11 be subtracted from 30, there remain 19. Therefore on 19 February the Sun enters the sign of Pisces. And so for the rest.

Now, to know which degree of the ecliptic the Sun holds on any given day of the year, add to the day of the month as many units as the place occupied in the alphabet by the first letter of the word corresponding to the given month. And if the resulting number is less than 30, it will indicate the degree of the sign of the preceding month; but if it is greater than 30, then, casting out 30, the remaining number gives the degree of the sign of the given month; if finally the resulting number is exactly 30, the Sun will be at the end of the preceding sign and at the beginning of the sign of the given month.

Example. I wish to know which degree of the ecliptic the Sun holds on 13 June. To this month, since it is the sixth from January, is due the sixth word, *Horret*, whose first letter, H, is the eighth in the alphabet. Adding, then, 8 to 13, we get 21, a number less than 30. Therefore on 13 June the Sun will be in the 21st degree of Gemini, which sign the Sun enters in the month of May. Again, if 27 June be proposed, adding 8 we get 35, a number greater than 30. Casting out 30, then, there remain 5; and so the Sun then occupies the 5th degree of Cancer, which it enters in the month of June. Finally, if 22 June be offered, adding 8 we get 30; the Sun will therefore then be at the end of Gemini and the beginning of Cancer. And the same reasoning holds for the rest.

In bissextile (leap) years, one degree is to be added to the place of the Sun so found after the feast of St. Matthias [Feb. 24], so that the Sun's place may be had more precisely. For example, on 27 September, to which the word *Gratus* is due, whose first letter, G, is the seventh: adding, then, 7 to 27, we get 34; and casting out 30, there remain 4. The Sun will therefore then be in the 4th degree of Libra, if it is a common year; but in a bissextile year, in the 5th degree.

Although by this method the true place of the Sun cannot be known altogether exactly — since the Sun does not traverse precisely one degree each day in the zodiac — yet scarcely an error of half a degree, or at most of one degree, will be committed. And so we reach the true place of the Sun, more or less, with such

certainty and assurance that we may safely use it in employing those clocks in which, to determine the hours, it is necessary to have the Sun's place in the zodiac known — as is also required for the use of the other instruments that astronomers employ.

[Margin: The Sun's place from tables.]

[Margin: How the bissextile years are distinguished from the common years.]

Since the use of the Sun's place in the zodiac is thoroughly necessary, and useful for very many observations, I should like here — so that the Sun's place may be had more exactly each day — to take from the Ephemerides of Giovanni Antonio Magini [Giovanni Antonio Magini] (as we also did in the scholium to Canon 2 of Book 3 of the *Astrolabe*) the Sun's place computed for four years for the several days of the year, namely for the year 1600, a bissextile, and the three common years following. For in these four years the whole variation of the Sun's place in the zodiac occurs, on account of the six hours neglected in the common years. But after very many years have passed, if these years are found no longer to agree with the true place of the Sun, four other years — one bissextile and three common — will have to be excerpted from the Ephemerides of that time. And since Magini computed the Sun's place even in seconds, we shall be content with minutes, taking one minute for any number of seconds greater than 30. And from these tables the true place of the Sun will be drawn much more surely than from any instrument — provided that it is sought, for a bissextile year, in the first table; for the first year after the bissextile, in the second; for the second year after the bissextile, in the third; and finally, for the third year after the bissextile, in the fourth.

You will discover with little trouble whether a given year is bissextile, or rather the first, second, or third after the bissextile, in this way. Reject from the proposed year all the thousands and hundreds; and from the remainder, which will be fewer than 100, [reject] the number 20 as many times as you can; then from the years that remain, which will be fewer than 20, reject 4 as many times as possible. For if after this last rejection nothing is left over, the given year will be bissextile; if 1 remains, it will be the first after the bissextile; if 2, the second; and if 3, the third. Thus, if the year 1654 be proposed: rejecting the 1600 years and all the 20s, from the remaining 54 years, and likewise all the 4s, from the remaining 14 years, there remain 2. You will say, therefore, that the year 1654 is a common year, and the second after the bissextile. But on this matter we have written more in the *Computus Ecclesiasticus* [Ecclesiastical Computation], where we also taught how, after the reform of the year, the centennial bissextile years

D a y	Jan. °	Jan. '	Feb. °	Feb. '	Mar .°	Mar. '	Apr. °	Apr. '	Ma y°	May '	Jun. °	Jun. '
1 9 0	58		11 ≈	29	10 ≠	42	11 Υ	29	10 ϣ	47	10 Π	39
2 10	59		12	30	11	42	12	28	11	46	11	38
3 12	0		13	31	12	42	13	27	12	44	12	34
4 13	1		14	31	13	42	14	26	13	42	13	32
5 14	2		15	32	14	42	15	25	14	40	14	29
6 15	4		16	33	15	42	16	24	15	38	15	27
7 16	5		17	33	16	42	17	23	16	36	16	24
8 17	6		18	34	17	42	18	22	17	34	17	22
9 18	7		19	35	18	42	19	21	18	32	18	19
1 19 0	8		20	35	19	42	20	20	19	30	19	17
1 20 1	9		21	36	20	42	21	18	20	28	20	14
1 21 2	10		22	37	21	41	22	17	21	26	21	11
1 22 3	11		23	37	22	41	23	16	22	24	22	9
1 23 4	12		24	38	23	41	24	15	23	21	23	6
1 24 5	13		25	38	24	40	25	13	24	19	24	3

D a y	Jan. °	Jan. '	Feb. °	Feb. '	Mar .°	Mar. '	Apr. °	Apr. '	Ma y°	May '	Jun. °	Jun. '
1 6	25	14	26	39	25	40	26	12	25	17	25	1
1 7	26	15	27	39	26	40	27	10	26	15	25	58
1 8	27	16	28	39	27	39	28	9	27	13	26	55
1 9	28	17	29	40	28	39	29	7	28	10	27	53
2 0	29	18	0 ✕	40	29	38	0 ✕	6	29	8	28	50
2 1	0 ≈	19	1	41	0 √	38	1	4	0 II	6	29	47
2 2	1	20	2	41	1	37	2	3	1	3	0 ∞	45
2 3	2	21	3	41	2	36	3	1	2	1	1	42
2 4	3	22	4	41	3	36	4	0	2	59	2	39
2 5	4	23	5	42	4	35	4	58	3	56	3	37
2 6	5	24	6	42	5	34	5	56	4	54	4	34
2 7	6	25	7	42	6	34	6	55	5	52	5	31
2 8	7	26	8	42	7	33	7	53	6	49	6	29
2 9	8	27	9	42	8	32	8	51	7	47	7	26
3 0	9	27	—	—	9	31	9	49	8	44	8	23
3 1	10	28	—	—	10	30	—	—	9	42	—	—

July–December:

D	a	Jul.		Aug	Aug.	Sep.		Oct.		Nov	Nov.	Dec.			
y	°	Jul. '		. °	'	°	Sep. '	°	Oct. '	. °	'	°	Dec. '		
1	9	ᠮ	20	8	᠙	59	8	ᠹ	51	8	᠒	58	9	ᠰ	12
2	10		18	9		56	9		50	9		58	10		13
3	11		15	10		54	10		48	10		58	11		14
4	12		12	11		52	11		46	11		58	12		15
5	13		10	12		49	12		44	12		58	13		16
6	14		7	13		47	13		43	13		59	14		17
7	15		4	14		44	14		41	14		59	15		18
8	16		1	15		42	15		39	15		59	16		19
9	16		59	16		40	16		38	16		59	17		20
1	17		56	17		37	17		36	17		0	18		21
0															
1	18		53	18		35	18		35	18		0	19		22
1															
1	19		51	19		33	19		33	19		0	20		23
2															
1	20		48	20		30	20		32	20		1	21		24
3															
1	21		45	21		28	21		30	21		1	22		25
4															
1	22		43	22		26	22		29	22		2	23		26
5															
1	23		40	23		24	23		27	23		2	24		27
6															
1	24		37	24		22	24		26	24		3	25		28
7															
1	25		35	25		19	25		25	25		3	26		29
8															
1	26		32	26		17	26		23	25		4	27		30
9															
2	27		30	27		15	27		22	26		59	28	1 [?]	31

D	a	Jul.		Aug	Aug.	Sep.		Oct.		Nov	Nov.	Dec.	
y	°	Jul. '		.°	'	°	Sep. '	°	Oct. '	.°	'	°	Dec. '
0													
2	28	27		28	13	28	21	27	59	29	5	29	32
1													
2	29	24		29	11	29	20	28	58	0 ♈	6	0 ♉	33
2													
2	0 ♊	22		0 ♋	9	0 ♌	18	29	58	1	6	1	34
3													
2	1	19		1	7	1	17	0 ♍	58	2	7	2	35
4													
2	2	17		2	5	2	16	1	58	3	8	3	36
5													
2	3	14		3	3	3	15	2	58	4	9	4	37
6													
2	4	11		4	1	4	14	3	58	5	9	5	38
7													
2	5	9		4	59	5	13	4	58	6	10	6	40
8													
2	6	6		5	57	6	12	5	58	7	11	7	41
9													
3	7	4		6	55	7	11	6	58	8	12	8	42
0													
3	8	1		7	53	—	—	7	58	—	—	9	43
1													

Commentary on Chapter II of the *Sphere* — Solar-position tables (continued)

These three pages carry, without intervening prose, the second, third, and fourth of the four annual solar-longitude tables ("Locus Solis in Zodiaco") — one for each year of the leap-year cycle. The first table of the set (Anno 1600) appeared in the preceding pages. Each table gives, for every day of the twelve months, the Sun's place in the zodiac in degrees (**G** = *gradus*) and minutes (**M** = *minuta*), with the

sign glyph printed at the start of each month and wherever the Sun crosses into a new sign (at 0°). The vertical rules at the outer edges are headed **Dies Mensium** ("Days of the months") — the day-number columns.

Sign glyphs: ♈ Aries, ♉ Taurus, ♊ Gemini, ♋ Cancer, ♌ Leo, ♍ Virgo, ♎ Libra, ♏ Scorpio, ♐ Sagittarius, ♑ Capricorn, ♒ Aquarius, ♓ Pisces. (The source prints Pisces as ⚹, Aquarius as a double-wave, Sagittarius as an arrow/cross; normalized here to the standard glyphs.)

Table — Locus Solis in Zodiaco Anno 1601, vel primo post bissextum

(The Sun's place in the zodiac, in the year 1601, or the first after a leap year)

January – June (degrees / minutes per day):

D	ay	Ianuar.	Februar.	Martius	Aprilis	Maius	Iunius
1	10	♑ 44	12 ♒ 15	10 ♓ 28	11 ♈ 15	10 ♉ 33	10 ♊ 25
2	11	45	13 16	11 28	12 14	11 31	11 23
3	12	46	14 16	12 28	13 13	12 29	12 20
4	13	47	15 17	13 28	14 12	13 27	13 18
5	14	48	16 18	14 28	15 11	14 26	14 15
6	15	49	17 18	15 28	16 10	15 24	15 13
7	16	51	18 19	16 27	17 9	16 22	16 10
8	17	52	19 20	17 27	18 8	17 20	17 8
9	18	53	20 20	18 27	19 6	18 18	18 5
10	19	54	21 21	19 27	20 5	19 16	19 2
11	20	55	22 22	20 27	21 4	20 13	20 0
12	21	56	23 22	21 27	22 3	21 11	20 57
13	22	57	24 23	22 26	23 1	22 9	21 55
14	23	58	25 23	23 26	24 0	23 7	22 52
15	24	59	26 24	24 26	24 59	24 5	23 49
16	26	0	27 24	25 25	25 57	25 3	24 47

D	ay	Ianuar.	Februar.	Martius	Aprilis	Maius	Iunius
17	27	1	28 25	26 25	26 56	26 1	25 44
18	28	2	29 25	27 24	27 54	26 58	26 41
19	29	3	0 ♄ 25	28 24	28 53	27 57	27 39
20	0	♂ 4	1 26	29 23	29 51	28 54	28 36
21	1	5	2 26	0 ♀ 23	0 ♂ 50	29 51	29 33
22	2	6	3 26	1 22	1 48	0 ♀ 49	0 ♂ 31
23	3	7	4 27	2 22	2 47	1 47	1 28
24	4	8	5 27	3 21	3 45	2 45	2 25
25	5	9	6 27	4 20	4 44	3 42	3 23
26	6	10	7 27	5 20	5 42	4 40	4 20
27	7	11	8 27	6 19	6 40	5 37	5 17
28	8	11	9 27	7 18	7 38	6 35	6 15
29	9	12	—	8 17	8 37	7 33	7 12
30	10	13	—	9 17	9 35	8 30	8 9
31	11	14	—	10 16	—	9 28	—

July – December (degrees / minutes per day):

Da	y	Iulius	Augustus	September	October	November	December
1	9	♂ 6	8 ♀ 45	8 ♀ 37	7 ♀ 56	8 ♀ 43	8 ♀ 57
2	10	4	9 42	9 35	8 55	9 43	9 58
3	11	1	10 40	10 34	9 54	10 43	10 59
4	11	58	11 37	11 32	10 53	11 43	12 0
5	12	56	12 35	12 30	11 52	12 43	13 1
6	13	53	13 33	13 28	12 51	13 44	14 2
7	14	50	14 30	14 27	13 51	14 44	15 3
8	15	47	15 28	15 25	14 50	15 44	16 4
9	16	45	16 25	16 23	15 49	16 44	17 5
10	17	42	17 23	17 22	16 49	17 45	18 5
11	18	39	18 21	18 20	17 48	18 45	19 6
12	19	37	19 18	19 19	18 47	19 46	20 7

Da y	Iulius	Augustus	September	October	November	December
13	20 34	20 16	20 17	19 47	20 46	21 8
14	21 31	21 14	21 16	20 46	21 46	22 9
15	22 29	22 12	22 14	21 46	22 47	23 11
16	23 26	23 10	23 13	22 46	23 47	24 12
17	24 23	24 7	24 12	23 45	24 48	25 13
18	25 21	25 5	25 10	24 45	25 48	26 14
19	26 18	26 3	26 9	25 44	26 49	27 15
20	27 15	27 1	27 8	26 44	27 50	28 16
21	28 12	27 59	28 6	27 44	28 50	29 17
22	29 10	28 57	29 5	28 44	29 51	0 ♄ 18
23	0 ♌ 8	29 55	0 ♎ 4	29 43	0 ♏ 51	1 19
24	1 5	0 ♐ 53	1 3	0 ♑ 43	1 52	2 20
25	2 2	1 51	2 2	1 43	2 53	3 21
26	3 0	2 49	3 1	2 43	3 54	4 22
27	3 57	3 47	3 59	3 43	4 54	5 23
28	4 55	4 45	4 58	4 43	5 55	6 24
29	5 52	5 43	5 57	5 43	6 56	7 25
30	6 50	6 41	6 56	6 43	7 57	8 27
31	7 47	7 39	—	7 43	—	9 28

Table — Locus Solis in Zodiaco Anno 1602, vel secundo post bissextum

(The Sun's place in the zodiac, in the year 1602, or the second after a leap year)

January – June (degrees / minutes per day):

D a y	Ianuar.	Februar	Martius	Aprilis	Maius	Iunius
1	10 ♄ 39 [?]	12 ♊ 0	10 ♋ 13	11 ♌ 0	10 ♏ 19	10 ♐ 11

D a y	Ianuar.	Februar .	Martius	Aprilis	Maius	Iunius
2	11 30	13 0	11 13	11 59	11 17	11 9
3	12 31	14 1	12 13	12 58	12 15	12 6
4	13 32	15 2	13 13	13 57	13 13	13 4
5	14 33	16 3	14 13	14 56	14 11	14 1
6	15 34	17 3	15 13	15 55	15 9	14 59
7	16 35	18 4	16 13	16 54	16 7	15 56
8	17 37	19 5	17 13	17 53	17 5	16 53
9	18 38	20 5	18 12	18 52	18 3	17 51
10	19 36 [?]	21 6	19 12	19 51	19 1	18 48
11	20 40	22 7	20 12	20 49	19 59	19 46
12	21 41	23 7	21 12	21 48	20 57	20 43
13	22 42	24 8	22 12	22 47	21 55	21 40
14	23 43	25 8	23 11	23 46	22 53	22 38
15	24 44	26 9	24 11	24 44	23 51	23 35
16	25 45	27 9	25 11	25 43	24 49	24 33
17	26 46	28 10	26 10	26 41	25 46	25 30
18	27 47	29 10	27 10	27 40	26 44	26 27
19	28 48	0 ✕ 10	28 9	28 39	27 42	27 25
20	29 49	1 11	29 9	29 37	28 40	28 22
21	0 ≈ 50	2 11	0 √ 8	0 ∸ 46 [?]	29 37	29 19

D a y	Ianuar.	Februar .	Martius	Aprilis	Maius	Iunius
2 2	1 51	3 11	1 8	1 34	0 Ⅱ 35	0 Ⅴ 17
2 3	2 52	4 12	2 7	2 32	1 33	1 14
2 4	3 53	5 12	3 6	3 31	2 30	2 11
2 5	4 54	6 12	4 6	4 29	3 28	3 6
2 6	5 55	7 12	5 5	5 27	4 26	4 6
2 7	6 56	8 12	6 4	6 26	5 23	5 3
2 8	7 56	9 13	7 4	7 24	6 21	6 0
2 9	8 57	—	8 3	8 22	7 18	6 58
3 0	9 58	—	9 2	9 21	8 16	7 55
3 1	10 59	—	10 1	—	9 14	—

July – December (degrees / minutes per day):

D a y	Iulius	Augustus	Septem ber	Octobe r	November	Decem ber
1	8 Ⅴ 52	8 Ⅱ 31	8 Ⅲ 23	7 Ⅰ 41	8 Ⅳ 28	8 Ⅶ 42
2	9 50	9 28	9 21	8 40	9 28	9 43
3	10 47	10 26	10 19	9 39	10 28	10 44
4	11 44	11 23	11 17	10 38	11 28	11 45
5	12 41	12 21	12 16	11 38	12 29	12 46
6	13 39	13 18	13 14	12 37	13 29	13 47
7	14 36	14 16	14 12	13 36	14 29	14 48

D a y	Iulius	Augustus	Septem ber	Octobe r	November	Decem ber
8	15 33	15 14	15 11	14 35	15 29	15 48
9	16 31	16 11	16 9	15 35	16 30	16 49
10	17 28	17 9	17 7	16 34	17 30	17 50
11	18 25	18 7	18 6	17 33	18 30	18 51
12	19 23	19 4	19 4	18 33	19 31	19 52
13	20 20	20 2	20 3	19 32	20 31	20 53
14	21 17	21 0	21 1	20 32	21 31	21 54
15	22 15	21 57	22 0	21 31	22 32	22 55
16	23 12	22 55	22 58	22 31	23 32	23 56
17	24 9	33 53 [recte: 23 53]	23 57	23 30	24 33	24 57
18	25 7	24 51	24 56	24 30	25 33	25 58
19	26 4	25 49	25 54	25 30	26 34	27 0
20	27 1	26 47	26 53	26 29	27 34	28 1
21	27 59	27 44	27 52	27 29	28 35	29 2
22	28 56	28 42	28 51	28 29	29 36	0 30 3
23	29 54	29 40	29 49	29 29	0 37 36	1 4
24	0 2 51	0 38 38	0 48 48	0 28 28	1 37	2 5

D a y	Iulius	Augustus	Septem ber	Octobe r	November	Decem ber
2 5	1 48	1 36	1 47	1 28	2 38	3 6
2 6	2 46	2 34	2 46	2 28	3 38	4 7
2 7	3 43	3 32	3 45	3 28	4 39	5 8
2 8	4 41	4 30	4 44	4 28	8 40 [recte: 5 40]	6 9
2 9	5 38	5 28	5 43	5 28	6 41	7 10
3 0	6 36	6 27	6 42	6 28	7 41	8 11
3 1	7 33	7 25	—	7 28	—	9 13

Table — Locus Solis in Zodiaco anno 1603, vel tertio post bissextum

(The Sun's place in the zodiac, in the year 1603, or the third after a leap year)

January – June (degrees / minutes per day):

D ay	Ianuar.	Februar.	Martius	Aprilis	Maius	Iunius
1	10 ♄ 14	11 ≈ 45	9 ✕ 58	10 ♀ 46	10 ♂ 4	9 ♀ 57
2	11 15	12 46	10 58	11 45	11 3	10 55
3	12 16	13 46	11 58	12 44	12 1	11 52
4	13 17	14 47	12 58	13 43	12 59	12 50
5	14 18	15 48	13 58	14 42	13 57	13 47
6	15 19	16 48	14 58	15 41	14 55	14 44
7	16 20	17 49	15 58	16 40	15 53	15 42

D ay	Ianuar.	Februar.	Martius	Aprilis	Maius	Iunius
8	17 21	18 50	16 58	17 38	16 51	16 39
9	18 22	19 50	17 58	18 37	17 49	17 37
10	19 24	20 51	18 57	19 36	18 47	18 34
11	20 25	21 52	19 57	20 35	19 45	19 32
12	21 26	22 52	20 57	21 34	20 43	20 29
13	22 27	23 53	21 57	22 32	21 41	21 26
14	23 28	24 53	22 56	23 31	22 39	22 24
15	24 29	25 54	23 56	24 30	23 37	23 21
16	25 30	26 54	24 56	25 28	24 34	24 18
17	26 31	27 55	25 55	26 27	25 32	25 16
18	27 32	28 55	26 55	27 26	26 30	26 13
19	28 33	29 55	27 54	28 24	27 28	27 10
20	29 34	0 ♄ 56	28 54	29 23	28 25	28 8
21	0 ♃ 35	1 56	29 53	0 ♄ 21	29 23	29 5
22	1 36	2 56	0 ♃ 53	1 20	0 ♄ 21	0 ♃ 2
23	2 37	3 57	1 52	2 18	1 19	1 0
24	3 38	4 57	2 52	3 16	2 16	1 57
25	4 39	5 57	3 51	4 15	3 14	2 54
26	5 40	6 58	4 50	5 13	4 12	3 52
27	6 40	7 58	5 50	6 11	5 9	4 49
28	7 41	8 58	6 49	7 10	6 7	5 46
29	8 42	—	7 48	8 8	7 4	6 44
30	9 43	—	8 47	9 6	8 2	7 41
31	10 44	—	9 46	—	8 59	—

July – December (degrees / minutes per day):

Da y	Iulius	Augustus	September	October	November	December
1	8 ♃ 38	8 ♄ 16	8 ♄ 8	7 ♄ 26	8 ♄ 13	8 ♄ 27
2	9 36	9 14	9 6	8 25	9 13	9 28
3	10 33	10 11	10 5	9 25	10 13	10 29

Da y	Iulius	Augustus	September	October	November	December
4	11 30	11 9	11 3	10 24	11 14	11 30
5	12 27	12 7	12 1	11 23	12 14	12 31
6	13 25	13 4	12 59	12 22	13 14	13 32
7	14 22	14 2	13 58	13 21	14 14	14 32
8	15 19	14 59	14 56	14 21	15 14	15 33
9	16 17	15 57	15 54	15 20	16 15	16 34
10	17 14	16 55	16 53	16 19	17 15	17 35
11	18 11	17 52	17 52	17 19	18 15	18 36
12	19 9	18 50	18 50	18 18	19 16	19 37
13	20 6	19 48	19 48	19 18	20 16	20 38
14	21 3	20 46	20 47	20 17	21 16	21 39
15	22 0	21 43	21 45	21 17	22 17	22 40
16	22 58	22 41	22 44	22 16	23 17	23 41
17	23 55	23 39	23 43	23 16	24 18	24 42
18	24 53	24 37	24 41	24 15	25 18	25 43
19	25 50	25 35	25 40	25 15	26 19	26 44
20	26 47	26 32	26 39	26 15	27 20	27 45
21	27 45	27 30	27 37	27 14	28 20	28 47
22	28 42	28 28	28 36	28 14	29 21	29 48
23	29 39	29 26	29 35	29 14	0 x̄ 21	0 ʏ 49
24	0 ȡ 37	0 ȡ 24	0 ȡ 34	0 ȡ 14	1 22	1 50
25	1 34	1 22	1 33	1 13	2 23	2 51
26	2 32	2 20	2 31	2 13	3 23	3 52
27	3 29	3 18	3 30	3 13	4 24	4 53
28	4 27	4 16	4 29	4 13	5 25	5 54
29	5 24	5 14	5 28	5 13	6 26	6 55
30	6 21	6 12	6 27	6 13	7 27	7 56
31	7 19	7 10	—	7 13	—	8 57

VIII.

[*Sacrobosco*] Likewise the Colure of the Equinoxes cuts the Ecliptic into two semicircles, namely the northern and the southern. These have been treated above.

[Margin: The Colure of the Equinoxes divides the Ecliptic into a northern and a southern semicircle.]

ON THE MERIDIAN [DE MERIDIANO]

[*Sacrobosco*] There are again two other great circles in the sphere, namely the Meridian and the Horizon. The Meridian is a circle passing through the poles of the world and through the zenith of our head. It is called the Meridian because, wherever a person may be and at whatever time of year, when the Sun—carried by the motion of the firmament—reaches his meridian, it is midday for him. For a like reason it is also called the circle of midday.

[Margin: What the Meridian is. Why the Meridian is so called, and why it is the circle of midday.]

COMMENTARY.

Having explained the four greater circles that are called intrinsic or mobile, he now treats the remaining two greater circles, which are called extrinsic or immobile, namely the Meridian and the Horizon. He sets out the Meridian first, because it is worthier and nobler: partly because it lies in the middle of the hemisphere, where the stars have their greatest elevations and powers, as we shall shortly say; and partly because astronomers begin the day from this circle, and not from the Horizon, as ordinary people are accustomed to do. He therefore defines the Meridian circle by saying that it passes through the poles of the world and through the zenith, or vertex, of the head—such as the circle in the material sphere that towers above all the rest and supports the axis of the world, about which the others turn. He then teaches that this circle is called "Meridian" from "midday" (*meridies*), because the Sun, brought to it by the motion of the first mobile at any time of year, produces midday, or noon. And for the same reason, he says, it is called the circle of midday, because it divides the artificial day into two equal parts.

Astronomers are also accustomed to call this circle the line of mid-heaven, or of midday; the royal cusp; the royal hinge (*cardo regius*); the beginning of the tenth celestial house; the mid-heaven; and by other names of this kind. This circle must be conceived as wholly immovable in the heavens and always fixed in the same place. For since it must necessarily pass through the vertex of the place whose

meridian it is called, and the vertex of that place does not change, if it were to move even a little it would depart from the vertex of the place, and so would neither divide the artificial day into two equal parts nor cut the Horizon at right angles—all of which are required in the Meridian.

[Margin: Other names for the Meridian.]

[*Sacrobosco*] And note that cities of which one lies nearer the East than another have different Meridians.

[Margin: Cities of which one lies farther east than another have different Meridians.]

COMMENTARY.

Since he had said that the Meridian passes through the zenith, or vertex, of the head—from which it follows that, just as not all places on Earth lie under the same point of the heavens, so too not all can have the same Meridian—he now teaches that Meridians vary in different cities, of which one is farther east than another.

From this it is clear that as many different Meridians must be conceived as there are zeniths, or vertical points, on some parallel circle running from East to West; yet all of them will intersect one another at the poles of the world. Hence by this reasoning one and the same city will contain several Meridians. For the more easterly a place is, the more easterly its Meridian will be, if we speak precisely and geometrically. But if we wish to consult the judgment of the senses, then within a space of about 300 stadia from East to West—which, on the authority of Proclus in his *Sphere*, amount to $37\frac{1}{2}$ Italian miles on the greatest circle and take up 36 minutes—scarcely any perceptible variation of the Meridian occurs. For within so large a space, he says, the vertical points begin to be perceptibly distinguished. Hence, since the whole Equator comprises 21,600 minutes, and each Meridian passes through two diametrically opposite points [Latin: *per duo minuta è diametro opposita*; *puncta* seems intended] [?], there will be 300 Meridians to be established in the whole circuit of the heavens. For in this way 36 minutes fall between any two nearest Meridians, and these make up $37\frac{1}{2}$ Italian miles, or 300 stadia, as Proclus holds. In this way, then, not only will one and the same city have the same Meridian, so far as the senses judge, but so too will two cities, or even more, provided that one is not more than 36 minutes farther east than another.

[Margin: By how great a space of the Earth from East to West the Meridians change with respect to the risings and settings of the stars.]

[Margin: How many Meridians are to be established, so far as the judgment of the senses.]

The cosmographers, following Ptolemy [Ptolemaeus], describe Meridian circles through the poles of the world and through the individual degrees of the Equator. So it comes about that there are 180 Meridians in all, since each one passes through two opposite degrees. The first Meridian passes through the Fortunate Isles, which are now called the Canaries and lie in the Western Ocean near Africa and Portugal [Lusitania], from which the longitudes of cities take their beginning among the cosmographers, as will be explained a little further on; the second through the first degree of the Equator, which follows the first Meridian as one proceeds toward the East; the third then through the second degree; and the rest in the same way in succession. On the cosmographical globe, however, and in the descriptions of the world that are usually called maps (*Mappae mundi*), the cosmographers draw only 12 Meridians, which divide the whole circuit of the Earth into 24 equal parts—perhaps for this reason, that 15 degrees, which make up one hour, may be intercepted between any two adjacent ones. For in this way it will easily be known by how many hours noon occurs sooner for one city than for another: for if one city is removed from another by three Meridians toward the East, it will have noon three hours earlier, and so on.

[Margin: How many Meridians there are according to Ptolemy and the cosmographers, and where they take their beginning.]

[Margin: On the cosmographical globe and on maps, 24 Meridians are drawn.]

[*Sacrobosco*] But the arc of the Equinoctial [Equator] intercepted between two Meridians is called the longitude of cities. And if two cities have the same Meridian, then they are equally distant from the East and from the West.

[Margin: What the longitude of cities is.]

COMMENTARY.

In passing, taking the occasion from the Meridian circle, he explains what the longitude of cities is, saying that it is the arc of the Equator intercepted between the two Meridians of two cities. This is to be understood only if the one Meridian passes through the Fortunate Isles, from which the longitude of cities is reckoned. For the arc between any two Meridians whatever is called the difference of longitudes. We shall say more about this a little later. But if two cities have the

same Meridian, they will be said to be equally distant from East and West, and to have the same longitude.

THE FUNCTIONS OF THE MERIDIAN [OFFICIA MERIDIANI]

I.

The Meridian circle determines the half-day and half-night time of the artificial day and night, by marking the points of noon and midnight. For the Meridian divides days and nights into equal spaces: it divides any day into the forenoon or morning part and into the afternoon or evening part; and it divides the night likewise into the hours that precede midnight and those that follow it.

[Margin: The Meridian determines the half-day and half-night time.]

II.

In it all the stars obtain the greatest altitude they can have, if you consider elevation above the Horizon; and they have their most intense vigor and power, since, when placed in it, they act upon these lower things through rays that make more nearly right, or less oblique, angles. We experience this more clearly than daylight in the case of the Sun, which, when set on the Meridian circle, more strongly heats and dries these lower regions and consumes their vapors than in any other part of the sky.

[Margin: On the Meridian the stars have their greatest altitudes and powers.]

III.

On it too is placed the zenith, or vertex, of each region, from which, by means of the Meridian, we shall easily measure the distances of the stars when they are set on the Meridian. In the same way we shall measure the intervals of all the parallel circles, both from our vertex and among themselves.

[Margin: The Meridian measures the distances of the stars from the vertex of the head, and of the parallels among themselves.]

III.

It shows us how great is the meridian altitude of the Sun and of the other stars—the altitude they have when placed on the Meridian circle—an operation of the greatest use among astronomers. For the meridian altitude of any star is the arc of the Meridian circle intercepted between the Horizon and the star set on the Meridian circle, provided that this arc does not exceed a quadrant, but is either

exactly a quadrant (as when the star stands at the vertex of the head) or certainly less than a quadrant (as when the star lies between the Horizon and the vertex).

[Margin: What the meridian altitude of the stars is, and by what method the Meridian measures it.]

V.

Astronomers place the beginning of the natural day—which is one complete revolution of the Sun—on the Meridian circle, and not, as ordinary people do, on the Horizon. For the beginnings of the day were various among various peoples and nations. The Babylonians, whom the Balearic Islands (called Majorca and Minorca) now imitate, began the day from one sunrise to the next; the Athenians, whom all Italians still follow, counted the day from one sunset to the next; the Egyptians and the Roman priests reckoned from one midnight to the next midnight, a custom that has remained in the Roman Church to this day; ordinary people, finally, reckon the day from sunrise to its setting; and astronomers, lastly, count the day from one midday to the next. Astronomers preferred to begin the day from the Meridian circle rather than from the Horizon because, as we shall teach in the third chapter, the Sun and stars always stand in the same way with respect to the Meridian in every region, but not with respect to the Horizon, which varies remarkably according to the greater or lesser elevation of the pole above the Horizon. Hence the natural days are rendered very unequal, as will be said in its place.

[Margin: The Meridian determines the beginning of the day for astronomers.]

[Margin: The various beginnings of the day among various peoples.]

[Margin: Why astronomers begin the day from the Meridian rather than from the Horizon.]

VI.

By means of the Meridian circle and the known meridian altitude of the Sun, the elevation of the pole in any region is found, together with the disposition or position of the sphere—without which scarcely any observation of astronomers is of any moment. For since a quadrant of the circle, that is, 90 degrees, lies between the zenith (the vertex of any region) and the Horizon, if, with the Sun standing at either of the equinoctial points, we subtract its meridian altitude from 90 degrees, there will remain the distance between the zenith and the Equinoctial circle. But this distance, as we shall demonstrate a little further on from the Author when he treats of the Horizon, is equal to the elevation of the pole—that is, to the arc of the

Meridian circle set between the raised pole of the world and the Horizon.

Therefore the elevation of the pole will be established from the known meridian altitude of the Sun at the time of the equinoxes. EXAMPLE. At Rome, at the time of the equinoxes, the meridian altitude of the Sun is found to be about 48 degrees; and when this is subtracted from a quadrant, about 42 degrees remain. This, then, will be the distance of the Roman vertex, or zenith, from the Equator, that is, the elevation of the pole at Rome.

[Margin: By the aid of the Meridian circle, the elevation of the pole is found at the time of the equinox.]

Now the meridian altitude of the Sun—indeed any altitude whatever, whether before or after noon—can be obtained in two ways. One way, most usual and easiest, is by some mathematical instrument, such as an astrolabe, a quadrant, a ring, and so on. The other way, more difficult but more certain, is by the shadow of some gnomon or stylus

[Margin: The meridian altitude of the Sun, or any other altitude, and by what method it is found.]

set perpendicular to the Horizon. For if at any time—say, for example, at noon—the shadow of the gnomon is equal to the gnomon itself (as happens at Venice, Milan, and Lyons at noon at the time of the equinoxes), the altitude of the Sun will be exactly 45 degrees, as we demonstrated with our astrolabe and in Book 3 of the *Practical Geometry*. But if the shadow is greater than the gnomon itself (as happens in Germany and other more northerly regions, where at noon at the time of the equinoxes it is greater than at 45 degrees), the altitude of the Sun will be less than 45 degrees. And if, finally, the shadow is smaller than the gnomon itself (as happens at Rome and in other regions that are less northerly than 45 degrees, at noon at the time of the equinox), the altitude of the Sun will be greater than 45 degrees. In what way the altitude of the Sun is to be derived from the known shadow and the gnomon, we demonstrated in Book 5 of the *Gnomonics*, Proposition 2. For now we shall be content with a simple rule and example. At Königsberg in Prussia [Mons Regius], at the time of the equinox, the shadow was found to be 16 of the units of which the gnomon is 12. I take the square of the shadow, namely 256, and add it to the square of the gnomon, namely 144, and make 400. By the square root of this number, namely by 20, I divide the product of the gnomon—namely of 12—multiplied into the whole sine, that is, into 100,000, which is 1,200,000; and there results 60,000 for the sine of the Sun's altitude, to which correspond about 36 degrees 52 minutes. When these are

subtracted from 90 degrees, there will remain the elevation of the pole in the said city, about 53 degrees 8 minutes.

More easily, the same altitude of the Sun will be investigated from the shadow by Problem 8 of our *Rectilinear Triangles*, set down in Lemma 53 of Book 1 of the *Astrolabe*, and in Book 1, Chapter 3, of the *Practical Geometry*. For if the length of the stylus, that is, 12, is multiplied into the whole sine, and the resulting number, 1,200,000, is divided by the length of the shadow, namely by 16, there will come out the Tangent of the Sun's altitude, 75,000, to which are owed about 36 degrees 52 minutes for the altitude of the Sun. Hence again its complement will give the altitude of the pole, about 53 degrees 8 minutes, as before.

MOREOVER, by this method the elevation of the pole is found only at the time of the equinox, from the meridian shadow of the Sun. For then, once the meridian altitude of the Sun is subtracted from a quadrant—that is, from 90 degrees—there remains the distance of the zenith from the Equator, which is indeed equal to the elevation of the pole. But if at any time of the year and on any day you wish to investigate the elevation of the pole from the meridian altitude of the Sun, it is necessary, from ephemerides or from elsewhere, to learn accurately the place of the Sun in the Ecliptic for the proposed day, and its declination from the table set out above. For the Sun's declination, if it is northern—as when the Sun is in the northern signs ♈ [Aries], ♉ [Taurus], ♊ [Gemini], ♋ [Cancer], ♌ [Leo], and ♍ [Virgo]—must be subtracted from the meridian altitude of the Sun, in order to obtain the altitude of the Equator, or (what is the same) the meridian altitude the Sun would have at the equinoxes: for when this is subtracted from 90 degrees, the elevation of the pole will remain.

[Margin: The finding of the elevation of the pole from the meridian altitude of the Sun outside the time of the equinox.]

For example, at Rome in the year 1569, on the 20th day of July, with the Sun at 6 degrees 40 minutes of ♌ Leo—which declines toward the North from the Equator by 18 degrees 39 minutes, as is evident from the table of declinations—I found at noon that the altitude of the Sun amounted to 66 degrees 39 minutes. From this I subtract the said declination, namely 18 degrees 39 minutes; there remain 48 degrees for the altitude of the Equator. When this is subtracted from 90 degrees, there remains the elevation of the pole, 42 degrees. But if the Sun's declination is southern—as when the Sun runs through the southern signs ♎ [Libra], ♏ [Scorpio], ♐ [Sagittarius], ♑ [Capricorn], ♒ [Aquarius], and ♓ [Pisces]—it must be added to the meridian altitude of the Sun, in order to find the altitude of the Equator; for when this is subtracted from 90 degrees, the elevation of the pole

will remain, as before. Thus at Rome, in the same year 1569, on the 21st day of November, with the Sun tarrying at 9 degrees 20 minutes of $\times^7$ Sagittarius—which departs from the Equator toward the South, as the table of declinations teaches, by 21 degrees 54 minutes—I found the meridian altitude of the Sun to be 26 degrees 6 minutes; and if to this is added the declination, say 21 degrees 54 minutes, the altitude of the Equator will be gathered as 48 degrees, from which again the elevation of the pole is found to be 42 degrees. Another method of finding the elevation of the pole from the Analemma on any day, even if the declination of the Sun is unknown, I have set down in the second scholium of Proposition 28 of Book 1 of the *Gnomonics*.

Now, in order that the elevation of the pole may be correctly found, the altitude of the Sun must be taken precisely at the point of noon—which will only then happen when the shadow of the gnomon is projected precisely onto the Meridian line. It will therefore not be out of place to indicate briefly by what art the Meridian line ought to be traced, since it is necessary for many observations of astronomers. On a plane, then, constructed level—that is, parallel to the Horizon—let many circles be described from the same center E, at which let a stylus or gnomon EF be erected at right angles. This will be the case when its tip F is equally distant from the circumference of any circle described in the proposed plane from center E. And it will be equally distant if it stands at an equal distance from at least three points of the circumference, as we demonstrated in Book 4 of the *Gnomonics*, Proposition 12. Next, before noon let the tip of the shadow be observed until it exactly touches the circumference of some circle—such as the shadow EG, whose tip falls precisely on the circumference of the third circle. Again, after noon let the tip of the shadow be marked until it falls precisely on the circumference of the same circle—such as the shadow EH.

[Margin: The Meridian line, by what art it is found.]

Diagram. A horizontal sundial layout for tracing the Meridian line, seen from above. Several concentric circles share a common center E, at which stands the vertical gnomon (stylus) whose tip is F. Two perpendicular diameters cross at E: the vertical one is the Meridian (noon) line, running from B at the top to D at the bottom; the horizontal one is the East–West line, running from A at the left to C at the right. Two shadow-tip points, G and H, are marked on the third circle, symmetrically placed to the lower right and lower left of center (the afternoon shadow EG and the forenoon shadow EH). The four cardinal words are engraved around the figure.

- MERIDIES (top): "Noon / South"

- B (top, on the Meridian line): upper endpoint of the noon line
- SEPTENTRIO (bottom, printed in mirror-reversed capitals): "North"
- D (bottom, on the Meridian line): lower endpoint of the noon line
- ORTVS (left): "East / sunrise"
- A (left, on the East–West line): eastern endpoint
- OCCASVS (right): "West / sunset"
- C (right, on the East–West line): western endpoint
- E (center): foot of the gnomon and common center of the circles
- F (above center, tip of the leaning stylus): apex of the gnomon EF
- G (lower right, on the third circle): tip of the afternoon shadow EG
- H (lower left, on the third circle): tip of the forenoon shadow EH

But in order that you may know at what hour after noon the tip of the shadow can touch the circumference of the same circle (so that you do not approach the Sun in vain), you must observe as many hours after noon as the hours before noon at which you marked the shadow. For if, say, at the third hour before noon the tip of the shadow touches precisely the circumference of some circle, it is necessary that at the third hour after noon the tip of the shadow reach the circumference of the same circle. This indeed you will know much more certainly by the following method. When the tip of the shadow falls before noon precisely on the circumference of some circle, let the altitude of the Sun be investigated with some instrument; and when this altitude has been carefully noted, then, when after noon the Sun obtains the same altitude, you may be perfectly assured that the tip of the shadow is touching the circumference of the same circle: for after noon the altitude of the Sun diminishes in the same proportion in which it increases before noon, and therefore, in whatever proportion the shadow of the gnomon decreases before noon, it must increase in the same proportion after noon, as can easily be demonstrated from the elements of spherics. And so, when these two points G and H—of which the former is distant before noon by the same interval as the latter is distant after noon—have been obtained with the utmost diligence, [the space between them is] to be divided

the arc GH will be bisected by the straight line BD, which is extended through the center E. For this line will be the meridian line, and if the shadow of the stylus is cast upon it, there is no doubt that noon is at hand. Therefore the straight line BD will be the common section of the horizon and the meridian circle. And if we cut this at right angles, the straight line AC will mark point A as the point of sunrise at the time of the equinox, and point C as the point of sunset, so that AC is the common section of the horizon and of the prime vertical properly so called. There

are indeed many other methods, no less reliable, for finding the meridian line — such as the one I set out by means of the analemma in the scholium to Proposition 23 of Book 1 of my *Gnomonics*, which in my judgment is the most reliable of all. But the method I have just explained is much quicker than all the others, and is more commonly used by astronomers.

Once the meridian line has been found on the said plane with so much labor, we shall discover countless other meridian lines on other planes with the greatest ease, in this way. Observe the moment of noon — that is, when the shadow of the gnomon falls exactly on the meridian line already found. If at that instant you hold up by hand, on any other plane, a fine thread with a plumb-weight, and mark its shadow on the plane at two points, then the straight line drawn through these two points will be a meridian line; for at noon the Sun's shadow produces it.

[Margin: By what technique, once one meridian line has been found, countless others may be found.]

Diagram. A horizontal sundial plane seen from above, used for finding the meridian line by equal gnomon-shadows. A set of concentric circles (the successive day-arcs traced by the shadow-tip) is centered on point E, where the gnomon (stylus) stands; a short wedge represents the gnomon and its shadow running from E toward F. Two perpendicular diameters divide the field into the four cardinal directions, each labeled in large type around the rim. The construction bisects the arc GH (the two points where the shadow, of equal length, touches one circle in the morning and afternoon) by the line BD through E, yielding the meridian.

- B (top): MERIDIES ["Noon" / south]
- D (bottom): SEPTENTRIO ["North"] (printed inverted at the foot of the figure)
- A (left): ORTVS ["Rising" / east] (printed vertically up the left side)
- C (right): OCCASVS ["Setting" / west] (printed vertically up the right side)
- E (center): base of the gnomon (unlabeled hub where the diameters cross)
- F (upper center): tip of the gnomon's shadow toward noon
- G (lower right): shadow-tip point on the day-arc (afternoon)
- H (lower left): shadow-tip point on the day-arc (morning)

VII.

In any oblique sphere this circle performs the same office that the right horizon performs in the right sphere. For however any point of the ecliptic, or

any star, is related to the right horizon, it must stand in exactly the same relation to the meridian of any given city, as regards rising and setting — that is, as regards its passage across the meridian; for both the meridian and the right horizon pass through both poles of the world.

[Margin: In every region the meridian serves as a right horizon.]

And it is for this reason that astronomers begin the natural day from the meridian circle, and not from the horizon. Since in their observations they require times as nearly equal as possible, they have noticed, by the surest evidence and demonstration, that the zodiac in its rising and setting admits far less variation with respect to the meridian than with respect to the oblique horizon. For the more oblique the horizon, the greater the variation seen in the rising and setting of the parts of the zodiac. But these matters will be better understood in chapter 3.

VIII.

The meridian circle has a notable use in cosmography, for by its office cosmographers measure the longitudes and latitudes of cities. To make this understood, a few things seem to me worth saying about the longitude and latitude of cities.

[Margin: The meridian measures the longitudes and latitudes of cities.]

Ptolemy, then — as is clear from Book 1, chapter 6 of his *Geography* [Ptolemy, *Geography* I.6], whom all cosmographers follow — seeing that the habitable earth extends farther from west to east than from north to south, calls the stretch of land from west to east "longitude," and that from north to south "latitude"; just as in any measurable thing whatever we are accustomed to call the greater distance its length, and to assign breadth to the lesser.

[Margin: Why Ptolemy calls the stretch of land from west to east "longitude," and that from north to south "latitude."]

Or else because, as he likewise says, the motion of the planets from west to east is longer than from north to south. For the latter is confined between the two tropics alone, whose distance amounts to 47 degrees, whereas the former embraces 360 degrees.

Now there are two kinds of East, and of West: the absolute and the relative.

[Margin: What the absolute East is. What the absolute West is. What the relative East and West are.]

The absolute East is called the limit of the habitable earth on the eastern side — such as, for Ptolemy, the region of the Sinae, which today is called Mangi, once subject to the King of the Tartars but now to the most powerful King of the Sinae [China]. For as one proceeds from west to east, immediately beyond that region one meets the sea. The absolute West is called the limit of the earth on the western side, of which kind are the Fortunate Isles, which lie in the west beyond the outermost shores of Europe and Africa. The relative East and West are taken with regard to any particular place of habitation, or horizon; in this way every city is said to have its own East and its own West. And in this latter sense every meridian is said to be equally distant from East and West; for the Sun spends as much time from rising to noon as from noon to setting — or because, in all regions on the equator, however the heaven revolves, there are 90 degrees between the horizon and the meridian.

But it is in the former sense that geographers take the longitude of the earth, so that the longitude of any city is said to be its distance from the West — that is, from the Fortunate Isles — toward the East; and it is usually defined as follows.

[Margin: What the longitude of cities is.]

The longitude of a city, or of any place, is the arc of the equator intercepted between the meridian of the said city or place and the meridian of the Fortunate Isles; or the arc of the parallel passing through the place, intercepted between those same two meridians. For this arc of the parallel is entirely similar to that arc of the equator, as is clear from Proposition 10 of Book 2 of Theodosius [*Spherics* II.10]. For since all circles running from north to south are carried by the motion of the heavens from east to west, the starting-point of longitude could not be taken from any of them; instead one had to resort to the meridian drawn through the most westerly islands then known — namely the Fortunate Isles — which are therefore said to be located in the absolute West. By the same reasoning that region of Mangi, because it is the most easterly, is said to occupy the absolute East. From which it is manifest that the longitude of any city cannot be measured without the meridian. For however many degrees the arc of the equator (or of the parallel) contains that is placed between the prime meridian — which passes through the Fortunate Isles — and the meridian of the city itself, so great is said to be its longitude. Thus the longitude of Rome is about 36 degrees 30 minutes. But the arc of the equator, or of the parallel of any city, lying between its

own meridian and the meridian of some other city — one that does not pass through the Fortunate Isles, or Canaries — is called the difference of longitudes.

[Margin: What difference of longitude is.]

The starting-point of latitude is set at the equator, because it in no way moves from north to south, or the reverse, but always keeps the same position with respect to the earth; so that any city is said to have as much latitude as it recedes from the equator, whether toward the north or toward the south — which we measure along the meridian. For the latitude of any city is the arc of the meridian enclosed between the equator and the parallel of the said city.

[Margin: What the latitude of cities is.]

On this reckoning Rome is said to have a latitude of about 42 degrees. But the arc of the meridian placed between the two parallels of two cities, neither of which lies on the equator, is called the difference of latitudes.

[Margin: What difference of latitude is.]

Therefore, just as the longitudes of the stars are reckoned from Aries toward the eastern signs, and their declinations from the equator toward one or the other pole, so too the longitudes of cities are counted from the meridian passing through the Canary (or Fortunate) Isles toward the eastern regions, while their latitudes are counted from the equator toward whichever pole. Hence, like the declinations of the stars, the latitudes of cities too will be twofold — northern, of course, and southern — according as they recede from the equator toward the north (the Bear), or toward the south.

[Margin: The latitude of cities is twofold: northern or southern.]

On this reckoning, places on the earth situated under the equator will have no latitude, while those set directly under the poles will be allotted a latitude of 90 degrees. Likewise, places whose zeniths lie on the same parallel, or on equal parallels, will obtain the same latitude.

[Margin: Which cities have the same latitude or longitude.]

Hence it comes about that our Antipodes have the same latitude as we do, though of a different name; for ours is northern, theirs southern. Again, cities placed under the same semicircle of the meridian that passes through the Fortunate Isles — the semicircle contained between the poles of the world, under which the said isles lie — will have no longitude at all; but those situated under the opposite semicircle will possess a longitude of 180 degrees. By the same reasoning, cities

placed under one and the same semicircle of any meridian, intercepted between the two poles, will have the same longitude; whereas those set under different semicircles of the same meridian will have a longitudinal difference of 180 degrees.

All this will be easily understood from some cosmographical globe, on which the great circles drawn through the poles indicate the longitudes of cities, while the circles equidistant from the equator indicate their latitudes; or else, at any rate, from some universal world-map, on which a straight line divided into degrees and drawn from pole to pole down the middle of the map represents the prime meridian, while the other curved lines drawn to either side of it (which indeed signify meridians; and the straight lines parallel to the equator, extended from left to right — which on some maps are curved) represent the parallels, that is, the latitudes of cities. From these it will readily appear which cities have the same longitude or latitude, or a different one, and by how much.

The philosophers, however — as is established in Aristotle, *On the Heavens*, Book 2, chapter 2 [*De Caelo* II.2] — speak otherwise about the longitude and latitude of the whole world. For, taking into account the differences of position that they conceive in the heaven, they call the East the right of the heaven, the West the left; the southern (or Antarctic) pole the upper, and the northern pole the lower.

[Margin: How the philosophers reckon longitude and latitude in the universe.]

For they imagine a man extended along the axis of the world, whose head is set at the Antarctic pole, his feet at the Arctic, his right hand in the East, and his left in the West. So, just as any man's length is taken from head to foot, or the reverse, while his breadth is taken from right to left, or the contrary, it follows that they measure the length of the world from pole to pole, but its breadth from east to west. But the cosmographers, considering the earth (as we said) as it is inhabited, define latitude from the equator toward the poles, and longitude from west to east.

The longitudes of cities can be found most reliably from eclipses of the Moon, although there are other methods, as we have taught in our *Cosmography*.

[Margin: The longitudes of cities are found most reliably from eclipses of the Moon.]

For if it is known that the beginning of an eclipse occurred two hours later at one city than at the Fortunate Isles, it clearly follows that this city lies 30 degrees to the east of the said isles; because that fact is proof that noon reached that city two

hours earlier. This must be understood of hours reckoned from noon, and not from sunset, unless the city lies under the parallel of the Fortunate Isles — and so for the rest. But the latitudes of cities are the same as the elevations of the pole. Hence, once the elevation of the pole has been found in any city, its latitude will be had. And since a knowledge of the longitudes, and no less of the latitudes, of cities is useful for many purposes, I judge that I shall do a welcome service to students if I set out the longitudes and latitudes of the principal cities in the following table. In it, so that any city may be found more easily, I have followed the order of the alphabet.

I have taken both the longitudes and the latitudes for the most part from Ptolemy's *Geography*; in a very few cities, whose longitudes and latitudes were known to me from the observations of other astronomers, I do not agree with Ptolemy. For, as I warned above, the tables of longitudes and latitudes are not to be trusted in every respect: often the longitude and latitude are found greater or less by one degree or another. So it is expedient that everyone, in the place where he is, should investigate the longitude and latitude before turning to other observations.

TABLE

Containing the longitudes and latitudes of the principal cities, and of other places.

Of the principal cities and other places.

Longitude and latitude are each given in degrees (G. = *gradus*) and minutes (M. = *minuta*). Southern latitudes are marked "Aust." (*Australis*, south) in the source and flagged **S** below; all others are northern.

A.

Place (as printed)	Long. °	Long. '	Lat. °	Lat. '
Acedum. Ceneda	33	22	45	18
Adana. Adena	66	40	39	10
Adrianopolis Thraciæ	53	0	43	0
Ænipontum [Innsbruck]	32	50	46	55
Ætna mons Siciliæ	39	0	38	20
Agata	24	0	42	10
Alba Græca. Belgrado [Belgrade]	45	0	47	40
Alenconium [Alençon]	19	15	48	35

Place (as printed)	Long. °	Long. '	Lat. °	Lat. '
Alexandria Ægypti	60	30	30	0
Alexandria Syriæ, Alessandretta	68	32	37	54
Alexandria Italiæ [Alessandria]	30	0	43	30
Algerium Africæ [Algiers]	22	0	32	30
Algerium Sardinia	30	20	38	0
Altissiodorum [Auxerre]	24	30	47	10
Alucium. Lecci [Lecce]	41	0	40	0
Amberga Bohemiæ [Amberg]	32	40	49	26
Ambianis. Amyens [Amiens]	23	30	49	50
Amsterdama [Amsterdam]	27	34	52	40
Ancona	38	30	43	40
Ancyra. Anguri [Ankara]	62	20	42	30
Andegauis [Angers]	19	0	47	30
S. Andreas in Scotia [St Andrews]	16	15	58	0
Angola Africæ. Aust.	46	0	9	0 (S)
Antiochia ad Taurum montem	70	15	37	20
Antiochia, Antiochetta	62	28	38	25
Antuerpia [Antwerp]	24	30	51	48
Apamea Alcamam	84 [?]	30	34	46
Apollonia, Alepia [Aleppo?]	59	0	39	55
Aquila [L'Aquila]	34	30	43	20
Aquileia	34	0	45	12
Aquinum patria D. Thomæ [Aquino]	38	30	41	56
Aquisgranum. Achen [Aachen]	27	15	51	10
Aden Arabia	83	0	13	0
Arbela. Erbel [Erbil]	89	0	35	52
Arelatum. Arles	22	5	43	20
Aretium, Arezzo	34	40	42	50
Argentei flu. ostia. Aust. [mouth of the Río de la Plata]	235 [?]	0	35	0 (S)

Place (as printed)	Long. °	Long. '	Lat. °	Lat. '
Argentina seu Argentoratum. Strasburg. [Strasbourg]	20	50	48	44
Ariminum [Rimini]	35	0	43	50
Armusa Ormus [Hormuz]	95	57	27	24
Arsinoe. Famagosta. [Famagusta]	65	41	36	35
Ascalon. Scalon. [Ashkelon]	67	20	32	27
Assisium [Assisi]	35	20	42	55
Asta [Asti]	31	0	43	45
Athenæ [Athens]	52	45	37	15
Atrebatum. Arras	23	40	50	0
Auenio, Auignon [Avignon]	23	0	43	52
Augusta, Augsburg	32	30	48	20
Augustodunum [Autun]	23	4	46	30
Aurasicum, Aurange [Orange]	26	30	43	30
Aurea Chersonesus, Malacha [Malacca]	161	0	2	0
Aurelia [Orléans]	22	0	47	30

Table of the longitudes and latitudes of cities (continued) — letters B–I

Continuation of the TABVLA continens Longitudines, Latitudinesque civitatum begun under letter A in the previous pages. Column banner on each page: **CIVITATVM PRÆCIPVARVM aliorumque locorum** — "Of the principal cities and other places." The two paired number columns are headed **Longit. G. | M.** (Longitude: degrees | minutes) and **Latit. G. | M.** (Latitude: degrees | minutes). Longitudes are reckoned eastward from the first meridian through the Canaries; the abbreviation **Aust. / Austr.** (*Australis*) marks a southern-hemisphere / New World location. Place-names are transcribed as printed (long-s resolved to s); the vernacular or alternative name printed after a comma is the source's own gloss.

The single row-rules that break the columns every few entries are typographic spacing only; the meaningful divisions are the alphabetical section letters (B, C, ...), given below as sub-headings.

B

Place	Long. °	Long. '	Lat. °	Lat. '
Babylon, Cairo	64	30	29	40
Babylon Caldæorum, Bagdat.	83	10	33	50
Bactra, Bagdasan	120	26	39	26
Badaioz	5	20	39	0
Badena, Heluetiæ	31	0	48	44
Baiona	17	30	42	50
Baldach, Susiæ	84	0	34	15
BAMBERGA patria Auctoris libri huius.	31	45	49	56
Barcinona	17	15	41	36
Barium, Italiæ	42	30	40	6
Basilea	28	0	47	30
Belgradum, Alba Græca	45	0	47	40
Bellouacum	23	0	49	30
Bellunum	32	30	46	8
Beneuentum	41	0	42	0
Bengala Indiæ	138	0	23	0
Bergamum	32	0	45	0
Berlinum, Marchion. Brandenburg.	36	30	52	50
Berna, Heluetiæ	29	45	46	25
Bersabea terminus terræ promissionis	64	50	31	15
Berytus, Baruti	68	20	34	50
Bethlehem	65	45	31	50
Billomum in Aruernia	23	0	44	40
Biturigæ	22	40	46	45
Blesis	21	0	47	35
Bononia Italiæ	33	30	44	16
Bosa in Sardinia	30	20	37	50

Place	Long. °	Long. '	Lat. °	Lat. '
Braga Portugalliæ	6	0	43	0
Brandenburgum	35	30	52	36
Brema	31	30	52	20
Brixia, Brescia	32	30	44	30
Brugæ, Flandriæ	24	36	51	30
Brundusium	42	30	40	0
Brunsuiga	32	40	52	30
Bruxella, Brabantia	26	42	51	24
Buda	42	0	47	0
Burdigala	18	0	44	30
Burgos Hispaniæ	12	0	42	48
Byzantium Constantinopolis	56	0	43	5

C

Place	Long. °	Long. '	Lat. °	Lat. '
Cabylon, Chalon	26	30	46	30
Cæsarea	67	34	33	28
Cæsaraugusta, Saragozza	14	15	41	45
Caieta	38	20	40	50
Calaris in Sardinia	31	30	36	30
Caletum Galliæ	16	2	52	0
Calicut Indiæ	112	0	17	0
Caliz Hispaniæ	5	10	37	0
Callicaris, Calecut	116	35	11	5
Camane, Chaul	116	5	19	0
Camberium, Ciamberi	22	10	45	7
Camerinum	36	0	43	0
Cameracum	25	0	49	40
Candia in Insula Candiæ	54	10	35	15

C (continued)

Place	Long. °	Long. '	Lat. °	Lat. '
Cantuaria in Anglia	21	0	53	40
Capharnaum	70	48	34	0
Capua	40	0	41	10
Caput bonæ spei, Aust.	50	0	35	0
Caput viride	13	0	8	0
Catanea	40	0	37	15
Ciuitas regum in Peru, Aust.	280	0	12	0
Cleuia	29	35	51	58
Coburgum	31	30	50	20
Colonia Agrippinensis	27	40	51	0
Compostella, S. Iacobus	7	15	44	15
Comorinum Indiæ	115	30	7	25
Complutum, Alcala de Henares	10	30	41	40
Comum	31	0	44	30
Confluentia, Coblenz	27	30	50	30
Conimbrica in Lusitania	5	45	40	30
Constantia, Costniz	28	30	47	30
Constantinopolis	56	0	43	15
Corduba	9	40	37	50
Coreura, Cochin	116	38	9	30
Corfinium, Corfu.	45	10	38	45
Corinthus	51	15	36	55
Corsicæ insulæ medium	31	0	40	50
Cosentia Calabriæ	40	40	39	30
Cracouia, Regia Poloniæ	42	40	50	12
Crema	31	15	44	20
Cremona	33	0	44	0
Crocola insula, Diu	113	0	20	50
Cuba insula	205	0	22	0
Cuchina, Indiæ	123	0	15	0

Place	Long. °	Long. '	Lat. °	Lat. '
Cumæ, vnde Sybilla Cumana	41	0	41	30
Cumaria prom. C. Comari	117	30	7	25
Cuzeo in Peru. Aust.	212	0	15	0

D

Place	Long. °	Long. '	Lat. °	Lat. '
Damascus	69	0	33	0
Dantiscum, Dan tzig, in Prussia	45	0	54	50
Daroca	16	30	40	0
Dauentria	28	4	52	30
Dertona, vel Tortona	30	40	44	0
Diascoridis insula, Zacotora	91	0	11	50
Diuiio, Dijon	25	45	47	0
Dola	18	30	49	5
Drepanum	37	0	37	0
Dulcignum	43	30	43	0
Dyrrachium	45	0	40	50

E

Place	Long. °	Long. '	Lat. °	Lat. '
Eboracum in Anglia	20	0	57	20
Ecbatana, Tauris	89	13	40	52
Edenburgum in Scotia	27	15	59	20
Edessa, Orpha	74	47	38	0
Eislebia	32	30	51	46
Engadda, Engaddi	70	15	31	50
Ephesus, Ioniæ Metropolis	67	40	37	40
Epidaurus	51	45	36	25
Erfordia Turingiæ	34	30	51	10

F

Place	Long. °	Long. '	Lat. °	Lat. '
Famagusta, olim Salamis	66	45	35	10
Faudentia, Faenza	35	20	43	30
Fessa, Africæ	10	0	35	0
Finis terræ	4	23	44	2
Florentia	34	0	43	40
Forcheim	31	30	49	45
Forum Flaminij, Foligno	36	0	42	40
Forum Iulij, Friuli	32	50	45	12
Forum Liuij, Forli	33	30	43	40
Forum Sempronij, Fossembron	34	50	43	30
Francofordia ad Mœnum	30	0	50	30
Francofordia ad Oderam	34	0	52	30
Friburgum Rheticæ Brisgoiæ	28	0	48	1
Friburgum, Heluetiæ	28	12	47	4
Friburgum, Misniæ	30	39	50	58
Fundi	38	10	41	30

G

Place	Long. °	Long. '	Lat. °	Lat. '
Gades	6	20	22	20
Galipolis, vel Callipolis	45	10	41	30
Gandauum	20	0	51	30
Gaza, Gazza	67	15	32	0
Geneua	28	0	45	45
Genua, Heluetiæ	23	0	46	0
Genua Italiæ	30	0	43	50
Geppinga	30	0	49	37
Gerasa, Garas	70	33	30	30
Gergentum	36	20	35	10
Gnesna	42	0	52	40

Place	Long. °	Long. ′	Lat. °	Lat. ′
Goa, Indiæ	115	10	17	0
Goaris fluuij ostia. Goa	115	40	16	0
Gorlicium, Silesiæ	34	45	51	0
Goslaria	32	40	52	0
Granata, Hispaniæ	11	0	37	50
Gratianopolis	27	0	45	30
Grauina	43	10	41	15
Groninga, Frisiæ	29	24	53	16

H

Place	Long. °	Long. ′	Lat. °	Lat. ′
Hadrianopolis, Bulgariæ	52	30	42	45
Halberstadium	32	40	52	10
Hallis	31	15	47	0
Hamburgum	33	0	54	30
Heduum, Autun	25	0	46	50
Heliopolis, Ems	70	45	35	40
Herbipolis, Wirtzburg	30	10	49	57
Heydelberga	28	0	49	35
Hierapolis, Aleppo	70	73 [minutes 73 — impossible; transcribed as printed]		38 0
Hippocura, Onor	115	40	15	10
Hippona	30	30	32	15
Hispalis, Seuilla	7	15	37	0
Hur Chaldæorum, patria Abrahami	78	30	32	40
Hydrus in Apulia, Ottranto	45	20	41	26
Hyrcania, Schizazo	100	45	39	0

I

Place	Long. °	Long. '	Lat. °	Lat. '
Iaponia, Insula	204	15	36	10
Iaua maior, Austr.	150	8	10	15
Iaua minor, Austr.	150	0	27	10

CAPVT SECVNDVM (continued)

TABLE OF THE LONGITUDES AND LATITUDES OF THE PRINCIPAL CITIES AND OTHER PLACES (continued)

Table caption (repeated at the head of each page): **CIVITATVM PRÆCIPVARVM aliorumque locorum** — "Of the principal cities and other places." Column headings: **Longit. G. | M.** = Longitude in degrees and minutes; **Latit. G. | M.** = Latitude in degrees and minutes.

The gazetteer continues in strict alphabetical order. This portion runs from the tail of letter **I** through letter **S** (the letter **K** has no entries — the sequence passes directly from **I** to **L**). Longitudes are reckoned eastward from the first meridian through the Canaries, so New-World and Pacific places carry longitudes greater than 180°; these are transcribed exactly as printed.

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I (continued)

Place	Long. °	Long. '	Lat. °	Lat. '
Ierosolyma [Jerusalem]	66	0	31	40
Ilium [Troy]	55	50	41	0
Imola	34	15	43	30
Ingolstadium [Ingolstadt]	32	10	48	40
Inspruck, Oenipontum [Innsbruck]	32	50	46	55
Ioachim vallis, Germaniæ [Joachimsthal]	30	20	50	20
S. Ioannes in Scotia [St Johns, Scotland]	15	40	59	49
Ioppe, Zaffo [Jaffa]	67	30	33	0

Place	Long. °	Long. '	Lat. °	Lat. '
Istria	30	30	40	15
Iuliacum, Gulich [Jülich]	27	30	52	0
Iustinopolis Histriæ caput [Koper/Capodistria]	35	43	45	55

L.

Place	Long. °	Long. '	Lat. °	Lat. '
Lacedemonia [Sparta]	50	15	35	30
Landishutum [Landshut]	31	0	48	20
Lantianum	41	30	41	40
Laubinga patria Alberti magni [Lauingen, birthplace of Albertus Magnus]	29	20	48	30
Laudunum [Laon]	24	45	48	55
Lauretum [Loreto]	37	10	43	0
Lausana [Lausanne]	28	45	46	10
Lemouica [Limoges]	21	30	45	45
Leoburgum, Saxoniz [Lüneburg, Saxony]	28	2	54	10
Leodium [Liège]	22	0	50	50
Leontium	38	0	38	0
Leopolis Russiæ, Leoburgum [Lviv]	43	15	50	30
Lerida [Lleida]	15	56	41	30
Liburnus, Liuorno [Livorno]	33	30	42	30
Lima in Peru. Aust. Ciuitas Regum [Lima, Peru; southern]	280	15	12	10
Lipsia [Leipzig]	30	30	52	20
Lisbona [Lisbon]	5	10	39	38
Londinum in Anglia, Londres [London]	20	0	52	30
Louanium [Louvain]	20	36	51	0
Lubecum [Lübeck]	31	20	54	48

Place	Long. °	Long. '	Lat. °	Lat. '
Luca [Lucca]	33	0	43	30
Lucerna Heluetiæ [Lucerne]	26	0	46	34
LVGDVNVM, Lyon	23	15	45	10
Lundis, Gothiæ [Lund]	41	30	57	25
Luneburgum [Lüneburg]	34	50	54	0
Lutetia, Paris	23	30	48	40

M.

Place	Long. °	Long. '	Lat. °	Lat. '
Machlinia, Brabantia [Mechelen, Brabant]	26	50	51	15
Magdeburgum [Magdeburg]	31	20	52	20
Magellanicum fretum, Aust. [Strait of Magellan; southern]	220	10	54	15
Maguntia, Mentz [Mainz]	27	30	50	30
Maioricæ insula, Mallorca [Majorca]	18	25	39	35
Malepur S. Thomæ in India [Mylapore, India]	124	15	14	10
Manfredonium, Sipontus [Manfredonia]	42	50	40	45
Manincongo Africae. Aust. [Manikongo, Africa; southern]	40	20	7	15
Mantua	32	45	44	30
Marpurgum Hassiæ [Marburg, Hesse]	30	10	51	0
Massilia [Marseille]	24	30	43	10
Meaco Regia Iaponiæ [Miyako/Kyoto, royal city of Japan]	204	15	36	10
Mecha [Mecca]	65	36	29	20
Mediolanum [Milan]	31	0	45	6

Place	Long. °	Long. '	Lat. °	Lat. '
Megara	52	0	37	30
Melite insula, & ciuitas [Malta, island and city]	38	45	34	40
Meroe Ægypti [Meroë, Egypt]	61	30	16	20

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M (continued)

Place	Long. °	Long. '	Lat. °	Lat. '
Meroe insula [Meroë, island]	61	30	16	25
Messana [Messina]	40	30	38	30
Metæ, Metz	25	30	47	30
Mexico	182	10	20	20
Mildeburgum, Franconiæ [Mildenberg?, Franconia]	26	34	49	44
Minorica insula, Menorça [Minorca]	19	30	40	10
Misna, Meysen [Meissen]	38	10	51	10
Mœridis lacus, El Buchaira [Lake Moeris]	61	15	27	50
Moluccæ insulæ [Moluccas]	187	0	0	0
Monachium, Munchen [Munich]	32	50	48	0
Monasterium, Munster [Münster]	28	10	52	0
Mons Regius Borussiæ [Königsberg, Prussia]	46	45	54	17
Mons pessulanus, Montpellier	22	15	43	10
Mons Regius Franconiæ patria Ioannis Regiomontani [Königsberg in Franconia, birthplace of Regiomontanus]	31	20	50	15
Montalbanum [Montauban]	21	30	43	30
Moscouia [Moscow]	75	10	61	15
Mozambique Africæ, Aust.	67	5	15	3

Place	Long. °	Long. '	Lat. °	Lat. '
[Mozambique; southern]				
Mussipontum, Pont à Mousson	28	35	49	6
Mutina [Modena]	32	40	44	0

N.

Place	Long. °	Long. '	Lat. °	Lat. '
Nancævm, Lotharingiæ [Nancy, Lorraine]	28	45	49	20
Narbona [Narbonne]	21	0	43	0
Narnia [Narni]	36	30	42	30
Neapolis, Campaniæ [Naples]	39	30	41	0
Neapolis Austriæ, Neustadt [Wiener Neustadt]	38	0	47	54
Neuburgum, ad Danubium [Neuburg on the Danube]	31	45	48	4
Neoburgum, Turingiæ [Neuburg, Thuringia]	32	0	51	20
Nicæa vbi habitum fuit Concilium 318. Episcoporum [Nicaea, where the Council of 318 Bishops was held]	57	0	41	40
Nickelspurg [Nikolsburg/Mikulov]	35	0	49	0
Nicomedia	57	30	42	30
Nigropontus, insula [Euboea/Negroponte]	53	40	38	15
Niniue vbi Ionas concionatus est [Nineveh, where Jonah preached]	78	0	36	4
Nitria, Cananor [Cananor/Kannur]	116	30	12	0
Niuernium [Nevers]	24	0	46	40
Niza [Nice]	28	0	43	30

Place	Long. °	Long. '	Lat. °	Lat. '
Nola Campaniæ [Nola]	40	15	40	45
Norimberga [Nuremberg]	31	30	49	30
Nouaria [Novara]	30	30	44	30
Nouiomagus [Nijmegen]	18	0	47	10
Nursia Italiæ, Norsia, patria S. Benedicti [Norcia, birthplace of St Benedict]	38	0	42	44

O.

Place	Long. °	Long. '	Lat. °	Lat. '
Olmvntza in Morauia [Olomouc, Moravia]	41	0	49	30
Onolsbachium [Ansbach]	32	0	49	33
Oppenheim [Oppenheim]	27	30	50	0
Orcades insulæ [Orkney Islands]	30	0	61	50
Orleans, Aurelia [Orléans]	20	40	47	10
Oristaneum in Sardinia [Oristano, Sardinia]	30	30	37	10
Ormuz insula [Hormuz]	92	0	19	0
Ottinga inferioris Sueuiæ [Oettingen, lower Swabia]	28	3	48	58
Oxonium in Anglia [Oxford]	19	0	54	15

P.

Place	Long. °	Long. '	Lat. °	Lat. '
Pampilona, Nauarræ [Pamplona, Navarre]	20	10	43	0
Panama, Hispaniæ nouæ [Panama, New Spain]	281	15	8	0
Panormus, Palermo	37	0	38	0
Paphus noua, Bapho [New Paphos]	63	35	36	10
Papia, Pauia [Pavia]	31	0	44	50

P (continued)

Place	Long. °	Long. '	Lat. °	Lat. '
Parisij, Lutetia [Paris]	24	30	48	40
Parma	32	30	43	30
Parentium [Poreč]	35	20	44	55
Patauia, Passau [Passau]	33	50	48	40
Patauium, Padua [Padua]	32	50	44	50
Pelusium, Damiata [Pelusium/Damietta]	64	50	31	0
Pergamus, Pergama [Pergamon]	56	50	41	8
Pernabucum, Bresileæ, Aust. [Pernambuco, Brazil; southern]	338	0	7	15
Perpiniana [Perpignan]	23	30	41	15
Perusivm [Perugia]	35	20	42	56
Philadelphia, Aladichia	64	24	39	0
Philippinæ insulæ [Philippine Islands]	170	10	12	15
Philippis [Philippi]	50	30	41	40
Pictauium [Poitiers]	20	0	46	35
Pisæ in Hetruria [Pisa, Etruria]	33	30	43	0
Pisaurum, Pesaro	35	20	43	45
Pistorium, Pistoia	33	20	43	0
Placentia [Piacenza]	31	50	44	0
Pola, Iulia Pietas [Pula]	36	45	44	50
Pompeiopolis, Pampelone	15	10	42	50
Posnania, in Polonia [Poznań, Poland]	42	0	52	45
Praga [Prague]	39	15	50	10
Preslau [Breslau/Wrocław]	40	0	51	10
Privernvm [Priverno]	38	0	42	0

Q.

Place	Long. °	Long. '	Lat. °	Lat. '
Qvito in Peru [Quito]	303	5	20	0

R.

Place	Long. °	Long. '	Lat. °	Lat. '
Ragvsia [Dubrovnik]	44	40	42	30
Ratisbona, Regenspurg [Regensburg]	32	15	48	59
Rauenna [Ravenna]	35	0	44	20
Regium Iulium, Calabriæ [Reggio Calabria]	43	10	38	15
Regium Lepidi, Lombardiæ [Reggio Emilia]	32	30	43	30
Rhecanatum [Recanati]	40	0	43	22 [?]
Rhemi, Galliæ [Reims]	22	15	48	45
Rhodus insula [Rhodes]	58	0	35	0
Riga, Liuoniæ [Riga, Livonia]	65	10	59	15
Rochelle, Rupella [La Rochelle]	16	30	47	10
Roma [Rome]	36	30	41	56
Rostochium [Rostock]	39	0	54	30
Rothomagus, Rouen. [Rouen]	22	40	49	0
Rupella [La Rochelle]	16	30	47	10

S.

Place	Long. °	Long. '	Lat. °	Lat. '
Saguntum [Sagunto]	14	36	39	40
Salernum [Salerno]	40	0	40	40
Salisburgum, Saltzburg [Salzburg]	35	40	47	40
Salmantica [Salamanca]	8	50	40	15
Salueldia [Salfeld/Saalfeld]	33	45	50	46
Samos insula [Samos]	52	40	41	15
Saragossa [Zaragoza]	14	15	41	40

Place	Long. °	Long. '	Lat. °	Lat. '
Sardinia insula [Sardinia]	30	10	38	15
Sassarum in Sardinia [Sassari]	31	30	38	50
Sauona [Savona]	29	20	43	40
Scutara Dalmatiæ [Skadar/Scutari, Dalmatia]	40	30	44	0
Sebilis, Hispalis [Seville]	9	0	37	0
Segnia, Illyric. [Senj, Illyria]	37	45	44	45
Segouia [Segovia]	9	30	38	0

Commentary on the *Sphere* of Sacrobosco — Chapter II (continued)

Table of the longitudes and latitudes of the principal cities and other places (continued)

[Column headings, repeated at the head of each page: **Longit. G. | M.** (Longitude, degrees | minutes) — **Latit. G. | M.** (Latitude, degrees | minutes). The place-name column is headed **CIVITATVM PRÆCIPVARVM aliorumque locorum** — "Of the principal cities and other places."]

Section **S** (continued):

Place (as printed)	Long. °	Long. '	Lat. °	Lat. '
Selestadium, Halfatiæ [Sélestat, Alsace]	24	6	48	22
Seleucia aspera, Seleuca	64	5	38	30
Senæ, Siena	34	20	42	50
Sibinicum Dalmatiæ [Šibenik]	38	42	44	20
Siene	62	0	23	30
Siguenza [Sigüenza]	13	30	40	50
Sipontum [Siponto]	42	50	40	30
Smirna [Smyrna]	58	25	38	25
Sophala Africæ, Auftralis [Sofala; southern]	64	10	20	15

Place (as printed)	Long. °	Long. '	Lat. °	Lat. '
Sora	38	20	41	40
Spira [Speyer]	27	40	49	20
Spoletum [Spoleto]	36	20	42	45
Sueffa Italiæ [Suessa, Italy]	39	0	41	30
Sulmo [Sulmona]	38	50	40	0
Suontienfu regia Chinæ [royal city of China]	182	12	47	15
Sufa, Sufta	88	35	34	5
Stetinum Pomeraniæ [Szczecin, Pomerania]	37	45	54	0
Stockolma in Suecia [Stockholm, Sweden]	47	0	60	30
Straßburg, Argentina [Strasbourg]	27	50	48	44
Strigonium [Esztergom]	42	30	48	0
Syene, Afna [Syene / Aswan]	62	0	23	30
Syracufæ in Sicilia [Syracuse, Sicily]	40	30	37	30

Section T (drop-capital "T"):

Place (as printed)	Long. °	Long. '	Lat. °	Lat. '
Tanais fluuij oftia, Don [mouth of the river Tanais / Don]	65	45	52	20
Tanis	62	45	30	50
Taprobana, Summatra infula [Taprobana, isle of Sumatra]	137	10	0	0
Tarentum [Taranto]	45	30	40	0
Tarracona [Tarragona]	16	20	41	0
Tarfos, Tarfo [Tarsus]	66	14	38	56
Taurinum [Turin]	30	30	44	0
Tauris, Perfīæ [Tabriz, Persia]	82	10	41	15
Taurus mons [Mt. Taurus]	66	0	38	0
Thebæ [Thebes]	51	10	38	30

Place (as printed)	Long. °	Long. '	Lat. °	Lat. '
Thebæ, Africæ [Thebes, Africa]	62	30	29	30
Theodofia, Caffa [Theodosia / Kaffa]	62	9	49	20
Theffalonica [Thessalonica]	49	50	40	20
D. Thomæ infula [isle of St. Thomas / São Tomé]	33	10	0	0
Thylæ infula [isle of Thule]	33	30	63	10
Ticinum, Pauia [Pavia]	31	0	44	50
Tigurum, Heluetiæ [Zürich, Switzerland]	26	36	46	48
Toletum [Toledo]	10	0	40	0
Tolofa [Toulouse]	20	30	43	20
Trapezus, Trebefonda [Trebizond / Trabzon]	71	0	44	3
Treueris, Trier [Trier]	26	0	49	30
Trecæ [Troyes]	24	45	48	5
Tridentum, Trento, Trient [Trento]	33	40	45	20
Tripolis, Tripoli [Tripoli]	68	10	35	40
Tubinga [Tübingen]	30	30	48	40
Tunetum, Tunes [Tunis]	33	0	32	30
Turnonum [Tournon]	22	50	44	35
Turonia, Tours [Tours]	14	30	43	30
Tybur [Tivoli]	36	40	42	0
Tyrus, Sur [Tyre]	68	0	34	8

Section V (drop-capital "V"):

Place (as printed)	Long. °	Long. '	Lat. °	Lat. '
Valentia, Hifpaniæ [Valencia, Spain]	14	0	39	30

Place (as printed)	Long. °	Long. '	Lat. °	Lat. '
Valentia in Gallia, Valençe [Valence, France]	23	0	44	30
Vallis oletana, Valladolit [Valladolid]	10	10	42	0
Velitrum [Velletri]	37	0	41	30

Section V (continued):

Place (as printed)	Long. °	Long. '	Lat. °	Lat. '
Velona [Vlorë]	45	6	40	10
VENETIÆ [Venice]	34	0	45	0
Vercellæ [Vercelli]	29	50	44	12
Verdunum Lotharingiæ [Verdun, Lorraine]	25	30	47	30
Verona	33	0	44	0
Vefontium Galliæ, Befanfon [Besançon, France]	25	40	47	36
Vicenza	32	10	44	30
Vienna, Auftriæ [Vienna, Austria]	37	45	48	20
Vienna, Galliæ [Vienne, France]	23	0	45	0
Villacum [Villach]	36	15	46	8
Vilna Lithuaniciæ [Vilnius, Lithuania]	52	0	53	30
Viterbium [Viterbo]	39	0	42	18
Vlma [Ulm]	32	30	48	20
Volaterra, Volterra	33	50	42	40
Vratislauia, Preslau [Wrocław / Breslau]	40	0	51	10
Vrbinum [Urbino]	34	10	43	4
Vftiça, Infula & ciuitas [Ustica, island and city]	37	30	38	45
Vtinum [Udine]	35	0	46	30
Wirtzburg, Herbipolis	30	10	49	57

Place (as printed)	Long. °	Long. '	Lat. °	Lat. '
[Würzburg]				
Witemberga [Wittenberg]	37	30	51	50
Wormatia, Worms [Worms]	28	0	49	45

Section X (drop-capital "X"):

Place (as printed)	Long. °	Long. '	Lat. °	Lat. '
Xantona	19	0	45	0
Xarxiare, Sigiftam	107	15	29	44

Section Z (drop-capital "Z"):

Place (as printed)	Long. °	Long. '	Lat. °	Lat. '
Zamora	8	0	49	5
Zararam regia, Zidem [royal city Zararam]	75	0	23	10
Zeylon infula, Indiæ [isle of Ceylon, India]	120	10	7	15
Zofala Africæ, Auft. [Sofala, Africa; southern]	64	10	20	15

[End of the alphabetical gazetteer.]

How the Distance of Two Cities from Each Other Is to Be Investigated, When the Longitude and Latitude of Each Has Been Ascertained

Although it properly belongs to Cosmography to teach the method by which the intervals of travel between any cities whatever ought to be determined, I nevertheless thought it would not be unwelcome to explain this very matter here in a few words. Now all distances on the earth are measured — just as on any other globe or sphere — along great circles, as we showed in the *Cosmography*; so much so that the distance of one place from another is said to be as great as the arc of the great circle drawn through both places. For this arc of the great circle is the shortest of all the curved lines that can be drawn from one place to another on the convex surface of the earth. Therefore, to inquire into the distance of two places from one another is nothing other than to find out how many degrees, or

minutes, or miles the said arc comprehends. [Margin: The distances of places on the earth are measured along a great circle.]

[Margin: How the distance of places on the earth is investigated when each place is northern, or [each] southern, and the longitude of both is the same.] **When, therefore, two cities have the same longitude** — that is, are situated under the same meridian-semicircle running between the two poles of the world — and each declines either to the north or to the south, then the smaller latitude is to be subtracted from the greater, so that the difference of latitudes is obtained. For if you convert this difference into miles, assigning $62\frac{1}{2}$ miles to each degree and $1\frac{1}{24}$ miles to each minute, you will have the interval between those cities.

EXAMPLE. Rome and Salzburg [Salisburgum] in Germany have nearly the same longitude. Subtracting the latitude of Rome, namely $41^{\circ}56'$, from the latitude of Salzburg, namely $47^{\circ}40'$, the difference of latitudes is found to be $5^{\circ}44'$; and this, reduced to miles, yields $358\frac{1}{3}$ miles — the distance, that is, of the city of Rome from Salzburg.

LIKEWISE, Genoa [Genua] and Frankfurt am Main [Francofordia ad Mœnum], that most noble emporium of Germany, are set under the same meridian-semicircle, and the difference of their latitudes amounts to $6^{\circ}40'$, which yields about 376 miles. So great, then, shall I declare the distance of the one city from the other to be.

[Margin: When places have the same longitude, but one is northern and the other southern.] **But if two places have indeed the same longitude, yet the latitude of the one is northern and that of the other southern**, then the latitude of the one must be added to the latitude of the other in order to obtain their distance.

EXAMPLE. Constantinople and the Cape of Good Hope [Caput bonæ spei] are of nearly the same longitude, and Constantinople has a northern latitude of about 43° . The Cape of Good Hope, however, declines to the south by about 35° , which, added to the latitude of Constantinople, makes 78° , that is, 4875 miles. So great is the length of the journey between Constantinople and the Cape of Good Hope.

[Margin: When the difference of the longitudes of the places amounts to 180° , and each is northern or [each] southern.] **If two cities are located under different semicircles of the same meridian** — which happens only when the difference of their longitudes amounts to 180° — then, if each has a latitude either northern or southern, the sum of the latitudes, subtracted from the semicircle, leaves their distance. EXAMPLE. Granada [Granata] in Spain and the city of Quinsay in the province of Mangi beyond China lie, as it were, under the same meridian, but under different semicircles, and each has a northern latitude — the former

indeed $37^{\circ}50'$, the latter $37^{\circ}40'$. If, therefore, the sum of the two latitudes, namely $75^{\circ}30'$, is subtracted from the semicircle, that is, from 180° , there remains as the distance between the said cities $104^{\circ}30'$, that is, 6500 miles.

But if two places lie under different semicircles of the same meridian, and one has receded from the Equator toward the north, the other toward the south, then the difference of latitudes is to be subtracted from the semicircle, so as to obtain the space lying between them. EXAMPLE. Cantoon, the most noble harbor of China, and the mouths of the Silver River [Ostia fluvii argentei], which the Spaniards call *Rio della Plata*, in Peru, lie nearly on different semicircles of the same meridian; and the latitude of Cantoon is northern, about 19° , while the mouths of the Silver River have a southern latitude of about 36° . The difference of latitudes is about 17° , which, subtracted from 180° (that is, from the semicircle), leaves 163° ; and these amount to $10197\frac{1}{2}$ [?] miles. So great, then, is the distance between Cantoon and the mouths of the Silver River.

From this it follows that, if of two cities lying under different semicircles of the meridian the northern latitude of the one is equal to the southern latitude of the other (or the reverse), then the one is distant from the other by precisely a semicircle; for evidently the difference of latitudes is nothing, and so nothing is subtracted from the semicircle. It is also clear that the direct route between two places lying under different semicircles of the same meridian runs by way of one of the poles, that is, along the meridian circle that passes through both places. This too, moreover, is to be noted here in passing: that if, of two cities whose difference of longitudes contains exactly a quadrant — that is, 90° — the one lies under the Equator while the other has any latitude whatever, whether northern or southern, and however great it may be, then the one is distant from the other by precisely the space of one quadrant. And all these things can easily be shown from the *Spherics* of Theodosius [Theodosius of Bithynia], and demonstrated more clearly than light upon the material sphere.

[Margin: Which cities are distant from one another by a semicircle. — Which cities differ by a quadrant. — When cities are situated under the Equator. — When cities have both a different latitude and a different longitude.] **When, finally, two cities have neither the same longitude, nor a difference of longitudes amounting to 180°** (that is, are set neither under the same meridian-semicircle nor under different semicircles of the same meridian), and neither lacks latitude (that is, neither is placed under the Equator), then the difference of longitudes will reveal their distance, provided it is not greater than a semicircle;

otherwise this difference, subtracted from the whole circle, will give the desired distance. For then the route must be reckoned by way of the Equinoctial circle.

[Margin: By what technique, with the aid of the compass [circinus], the distances of places are found.] **When, at last, two places are situated in none of the aforesaid ways** — whether one lies under the Equator and the other not, or neither does, and whatever latitudes they may have — we shall determine their travel-distance by a device of **Francesco Maurolico, the Abbot** [Francisci Maurolyci Abbatis], namely with the help only of a compass, in this manner. Let a circle A B C D be described from center E; and let, first, the difference of longitudes of the two places — the arc A B — be less than a semicircle, and from the points A and B let two diameters, AEC and BED, be drawn. Let there then be placed the latitude of place A equal to the arc AF, and the latitude of place B equal to the arc BG; and let there be dropped to their respective diameters the perpendiculars FH and GI. After this, to the drawn straight line HI let there be erected from H and from I, on the same side, the perpendiculars HK and IL, each equal to its own — that is, HK equal to the straight line HF, and IL equal to the straight line IG. For the straight line joining the points K and L will be the chord of the arc of the distance of the one place from the other. Therefore, if by Proposition 1 of Book 4 of Euclid you fit into the circle a straight line DN equal to the straight line KL, the arc DN will be the distance between the two given places. Whence, once it is known how many degrees the arc DN contains, the [distance...]

[The geometric construction (circle ABCD with center E; diameters AEC, BED; points F, G, H, I, K, L, N) is described in the text but no figure is printed on this page; the accompanying diagram falls on the following page.]

[Continues from the previous page, mid-construction: how to find the distance between two cities from their coordinates.]

we shall arrive at the distance sought, by assigning 62½ miles to each degree. This rule is to be understood for the case when both places recede from the Equator toward the same quarter — either both to the north or both to the south. For if one of them, namely A, inclines to the south while the other, B, inclines to the north, then perpendiculars must be drawn from the points H and I to the line HI, on opposite sides — such as IL and HM — yet again in such a way that IL is equal to the line IG, and HM to the line HF. For the straight line LM joining the points L and M will again be the chord of the arc of the distance of one place from the other. And so, if a line DO equal to LM be fitted into the circle, the arc DO will be the distance of the two proposed places.

Next, let the difference of longitudes be the arc ABD, greater than a semicircle (for when this difference is a semicircle, it has been said above by what method the distance of the places is to be found); and from the points A and D let the diameters AEC and DEB be drawn. Then let the latitude of place A be set equal to the arc AF, and the latitude of place D equal to the arc DR; and let the perpendiculars FH and RQ be dropped to their respective diameters. After this, to the drawn line QH, on the same side — if both places are northern, or both southern — let perpendiculars QT and HS be drawn, equal respectively to the perpendiculars QR and HF: that is, QT equal to QR, and HS equal to HF. For the line joining the points T and S will be the chord of the arc of the distance of one place from the other. Hence if a line DP equal to TS be fitted into the circle, the arc DP will be the distance of the proposed places, as before.

But if place A is, for example, northern and D southern, perpendiculars must be drawn from Q and H to QH, again on opposite sides — such as QT and HV — yet so that QT is equal to QR, and HV to HF. For the line TV will be the chord of the arc of the distance of one place from the other; and therefore, if a line DX equal to TV be fitted into the circle, the arc DX will be the distance of the proposed places.

The demonstration of this operation — which is indeed most beautiful and most brief — cannot escape anyone even moderately versed in the doctrine of sines, who has examined the matter more carefully on some material sphere. For the circle ABCD will represent the Equator; the diameters AC and BD, the common sections of the Equator with the Meridians of the proposed places; and the points H and I, in the plane of the Equator, will be those on which the right sines of the latitudes of the said places fall. Hence if from the points H and I perpendiculars be raised to the plane of the Equator, these will be the very right sines of the latitudes; they will reach the places themselves on the surface of the sphere, and will be wholly equal to the lines HK and IL, as is evident. Therefore the line KL will be equal to the chord of the arc that lies between the said places; for the lines HK and IL are equal to the right sines of the latitudes.

These same precepts serve for finding the distance between any two stars of the Firmament, provided that in place of the Meridian one takes the circle of the longitude of the stars, which of course passes through the poles of the Ecliptic, as is clear.

We have given another geometrical method, no less subtle and pleasing, in the *Astrolabe*, book 3, in the scholium to Canon 15, no. 4. But of these matters, and of the longitude and latitude of cities, we have said more in the *Cosmography*; it is enough to have touched on these few points here.

Diagram. A large circle (representing the Equator) inscribed with two diameters and a dense network of chords and perpendiculars — the construction for finding the road-distance of two places from their longitudes and latitudes. Around and within the circle the following labels are legible; because the figure is crowded and several interior letters overlap the ruled lines, a few readings are flagged and the whole figure is marked a

- P (top of circle)
- O (top, just left of P)
- G (upper left)
- X (upper left, below G)
- C (upper right)
- N (right, below C)
- D (right end of the horizontal diameter)
- T (far right, below D, at the rim/outside)
- R (lower right)
- H (lower centre, interior)
- A (lower-left centre)
- M (lower left, at the rim/outside)
- S (bottom, outside the circle)
- B (far left, left end of the horizontal diameter)
- F (left edge, above B)
- V (centre-left, interior)
- L (upper centre, interior)
- I [?] (left-centre interior; could be the numeral 1)
- E (centre of the circle)
- K (centre, right of E)
- Q [?] (centre-right, near D; the glyph resembles G but the text's construction calls for point Q)

The main pairs of lines named in the text are the diameters AEC and DEB; the perpendiculars FH and RQ dropped to the diameters; the perpendiculars QT, HS (or QT, HV in the mixed case); the chords LM, TS, TV; and the fitted arcs DO, DP, DX.

But by far the most convenient and easiest way of all is by means of a cosmographical globe, if one is at hand, accurately drawn. For if with a bent compass the distance of one place from the other is taken upon it, and this is

transferred to the Equinoctial circle, at once the degrees between the legs of the compass will indicate the distance of one place from the other.

[Margin: How the road-interval between two places may be found from the sines.]

Yet it will not be out of place to bring forward here, out of all the methods, the one that Pedro Nunes [Petrus Nonius] demonstrated in book 2 of the *Art of Navigation*, and which we have demonstrated more clearly in the scholium to Canon 15, book 3, of our *Astrolabe*, no. 6. It is of this kind. When the two given places are both northern or both southern: let it be arranged that, as the square of the whole sine [radius] is to the rectangle contained under the sines of the complements of the latitudes of the places, so is the versed sine of the difference of longitudes of those same places — which difference, if it exceeds a semicircle, must be subtracted from the whole circle, and the versed sine of the remainder taken, as being that of the shorter difference of longitudes, that is, of the shorter distance between the Meridians of the given places — to a fourth quantity. For a number will be found, from which we shall determine the distance of the places by this device.

Compare the number found with the sine of the complement of the difference of the latitudes of the given places. For if the number found is equal to the sine of that complement of the difference of the latitudes of the given places, the distance of the places will comprise a quadrant of the greatest circle. But if it is smaller, then, when this is subtracted from that, there will remain the sine of the complement of the distance of the places; and so, if this complement be taken away from the quadrant, the remainder will be the distance of the places. If, finally, the number found is greater than the sine of the complement of the difference of the latitudes of the given places, then, when this is subtracted from that, the remainder will be a sine whose arc, added to the quadrant, will give the road-distance of the proposed places.

But when one place is northern and the other southern, one must take the place diametrically opposite to one of them, having the same latitude though on the opposite side, so that one may have two places of the same denomination — both northern, that is, or both southern. Then one must investigate, as we have taught, the road-interval between these two places of the same denomination, provided that in place of the difference of longitudes of the given places one takes what remains when that difference is subtracted from a semicircle, so as to have the difference of longitudes of those two places of the same denomination. For if this road-interval is subtracted from the semicircle, the known distance of the given

places — of which one is northern and the other southern — will remain. But let us set out some examples, to make the matter clearer.

Let the road-distance be explored between **Rome**, whose longitude is $36^{\circ} 30'$ and whose latitude is $41^{\circ} 56'$ north, and **Constantinople**, whose longitude comprises $56^{\circ} 0'$ and whose latitude is likewise $43^{\circ} 5'$ north. Let it be arranged that, as 10,000,000,000 (the square of the whole sine) is to 5,433,294,112 (the rectangle contained under 74392, the sine of the complement of the latitude $41^{\circ} 56'$, and 73036, the sine of the complement of the latitude $43^{\circ} 5'$), so is 5736, the versed sine of the difference of longitudes, which comprises $19^{\circ} 30'$, to a fourth; and there will be found here roughly the number 3116.

[Margin]

Since this is smaller than 99979 — the sine of the complement of the difference of the latitudes of the given places, which comprises $1^{\circ} 9'$ — we shall subtract it from 99979, the sine of the complement of the difference of the latitudes of the places, and there will remain 96863, for the sine of the complement of the distance of the given places. This complement, then, will contain $75^{\circ} 37'$, and so the distance will comprise $14^{\circ} 23'$ — that is, 898 $\frac{23}{24}$ Italian miles — assigning to each degree $62\frac{1}{2}$ miles and to each minute $1\frac{1}{24}$ miles.

Again, let the road-distance be investigated between **Rome** and **Malacca**, in the Golden Chersonese, whose longitude is $161^{\circ} 0'$, and whose latitude, being likewise northern as Rome's is, is $2^{\circ} 0'$. Let it be arranged that, as 10,000,000,000 (the square of the whole sine) is to 7,434,662,088 (the rectangle contained under 74392, the sine of the complement of Rome's latitude, which is $41^{\circ} 56'$, and 99939, the sine of the complement of Malacca's latitude, which is $2^{\circ} 0'$), so is 156640, the versed sine of the difference of longitudes, which comprises $124^{\circ} 30'$, to a fourth; and there will be found here roughly the number 116456. From this — since it is greater than 76679, the sine of the complement of the difference of the latitudes of the places, which is $39^{\circ} 56'$ — we shall subtract 76679, the sine of the complement of the difference of the latitudes; and there will remain the sine 39777, whose arc, $23^{\circ} 26'$, added to the quadrant makes $113^{\circ} 26'$ — that is, 7089 $\frac{7}{12}$ Italian miles — for the distance between Rome and Malacca in the Golden Chersonese.

Let the road-distance also be inquired into between **Rome** and **Mexico** [City], in the West Indies, whose longitude Giuseppe Moletti [Ioseph Moletius], in the new table of New Spain in his commentaries on Ptolemy's *Geography*, puts at about $272^{\circ} 30'$, and its latitude at $20^{\circ} 20'$ north (as Rome's latitude too is northern) —

although others make its longitude and latitude otherwise. Let it be arranged that, as 10,000,000,000 is to 6,975,589,056 (the rectangle contained under 74392, the sine of the complement of Rome's latitude, which is $41^{\circ} 56'$, and 93768, the sine of the complement of Mexico's latitude, which is set at $20^{\circ} 20'$), so is 158778, the versed sine of the difference of longitudes (which is $234^{\circ} 0'$; and since this exceeds a semicircle, it must be subtracted from the whole circle, so that $126^{\circ} 0'$ remain for the shorter difference of longitudes — namely the shorter distance between the Meridians of the proposed places — whose versed sine is 158778), to a fourth; and there will be found here nearly the number 1,110,757. From this — since it is greater than 92977, the sine of the complement of the difference of the latitudes of the places, which comprises $21^{\circ} 36'$ — we shall subtract 92977, the sine of the complement of the difference of the latitudes; and there will remain the sine 17780, whose arc, about $10^{\circ} 15'$, added to the quadrant makes $100^{\circ} 15'$ — that is, 6265 $\frac{5}{8}$ Italian miles — for the distance between Rome and Mexico in the West Indies.

Lastly, let the road-distance be proposed for investigation between **Rome** and **Cusco**, the capital of the province of Peru in the West Indies — a most noble and most wealthy province — whose longitude Giuseppe Moletti, in his new table of the New World, sets at about $305^{\circ} 40'$, and its latitude at about $18^{\circ} 40'$ south, although other writers judge otherwise. And since Rome inclines to the north and Cusco to the south, we shall take a northern place diametrically opposite to Cusco — one that has, of course, a northern latitude of $18^{\circ} 40'$. Then the difference of longitudes of Rome and Cusco, which is $269^{\circ} 10'$ and exceeds a semicircle, we shall subtract from the whole circle, and there will remain the shorter difference of longitudes — that is, the shorter distance between the Meridians of the given places — of $80^{\circ} 50'$. This in turn we shall subtract from the semicircle, so as to have the difference of longitude between Rome and that place opposite to Cusco — that is, the distance between the Meridian of Rome and the Meridian of the said place — of $99^{\circ} 10'$.

With this laid down, if it be arranged that, as 10,000,000,000 (the square of the whole sine) is to 7,047,823,688 (the rectangle contained under 74392, the sine of the complement of Rome's latitude, which is $41^{\circ} 56'$, and 94739, the sine of the complement of the latitude of the place set opposite to Cusco, which is $18^{\circ} 40'$), so is 115930, the versed sine of the difference of longitudes (which we said comprises $99^{\circ} 10'$), to a fourth; there will be found here approximately the number 81705. Since this is smaller than 91867 — the sine of the complement of the difference of the latitudes of the given places, which comprises $23^{\circ} 16'$ — we shall subtract it from 91867, the sine of the complement of the difference of the

latitudes, and there will remain 10162, for the sine of the complement of the distance of Rome from that place which is set opposite to Cusco. This complement, in the table of sines, is $5^{\circ} 50'$; therefore the distance itself will comprise $84^{\circ} 10'$, and if we take this from the semicircle, there will remain the distance between Rome and Cusco of $95^{\circ} 50'$.

DE HORIZONTE. [On the Horizon]

The Horizon, however, is the circle dividing the lower hemisphere from the upper. From this it is called the Horizon, that is, the boundary of sight. It is also called the circle of the hemisphere, for the same reason.

COMMENTARIVS. [Commentary]

In the last place among the great circles he treats of the Horizon, which he says, in the sphere, to be that circle — understand, that greatest circle — which divides the lower hemisphere from the upper. For although any great circle divides the sphere into two equal hemispheres, still, in a special sense and simply, the part of the sky that is seen (or not seen) is customarily called a "hemisphere"; and into these parts no great circle other than the Horizon divides the heaven — except when it performs the office of a Horizon, as the Equator does with respect to those who dwell beneath the poles of the world.

He then teaches that this circle is called "Horizon," as it were the "boundary of sight," from the Greek verb ὁρίζομαι [Greek: horizomai, "I bound/determine"], which means "I determine," because it separates the seen part of the sky from the unseen. For the same reason he says that it is also called "the circle of the hemisphere" — on account, that is, of the seen and the unseen hemisphere. This circle is also commonly called the "ring [gyrus] of the hemisphere," and by the Latins *Finitor* or *Finiens* (the bounder).

[Margin: What the horizon is, and why it is so called.]

The Horizon in the heaven, moreover, must be conceived as entirely immobile, just like the Meridian. For it must necessarily stand at right angles to the Meridian in every clime; and it is plain that the Horizon could not always be at right angles to the Meridian, if it moved while the latter remained immobile. From this it follows that there are as many Horizons from east to west as we said above there are Meridians — if we follow the judgment of sense, namely 300. For the Meridian and the Horizon follow one another mutually, so that when one is changed the other is necessarily changed too — so that it is surprising why

Proclus [Proclus], in his *Sphere*, maintained that the Meridian changes sensibly within a space of

[Margin: There are as many Horizons from east to west as there are Meridians.]

[Margin: The Horizon must be conceived as immobile.]

[Margin: Various names of the Horizon.]

[Top]

300 stadia, which make $37\frac{1}{2}$ miles, as we said above; but the Horizon within a space of 400 stadia, which make 50 miles — unless perhaps he understands the change of Horizons not from east to west, but from north to south. For Horizons change not only from east to west, like the Meridians, but also from pole to pole, so that it is utterly impossible — if we wish to speak geometrically — for two cities on earth to be able to have the same Horizon, whether the one is removed from the other toward east or west, or toward north or south. Yet very many cities — namely all those that have the same longitude, or even those whose difference of longitude amounts to a semicircle, that is 180 degrees — can obtain the same Meridian, even speaking geometrically.

Since this is so, perhaps Proclus meant that the Meridian, and consequently the Horizon, varies sensibly from east to west over a space of 300 stadia — which of course pertains to the rising and setting of the stars — but that the Horizon undergoes a sensible variation from pole to pole, which pertains to the elevation of the pole, over a space of 400 stadia. For one and the same elevation of the pole can serve for so great a space on the earth, as the sundials show. Nevertheless, however we speak, no fixed law can be prescribed either in the change of Meridians or of Horizons. For near the Equator a change of one or two degrees in the elevation of the pole — arising from the change of Horizons from pole to pole — introduces no sensible error as regards the increase and decrease of days and nights and the variety of shadows. But near the poles a change of only one degree induces the greatest difference in the phenomena of the astronomers. And you will say the same, in a certain proportion, of the Meridians, which change from east to west. But all this can be demonstrated geometrically from the Spherics of Theodosius [Theodosius] and Menelaus [Menelaus], and the calculus of sines teaches the same most certainly.

Proclus, Albertus Magnus [Albertus Magnus], and most other writers set up a twofold Horizon. For they say that one is grasped by reason, which they call the

Rational or Natural; the other is grasped by sense, which they call the Sensible or Apparent.

[Margin: What the rational horizon is.]

The Rational is the one that divides the whole heaven into two equal hemispheres and separates the seen part from the unseen; its poles on the sphere are the vertex of the head, or Zenith, and the opposite point, which they call the Nadir; and its centre is the same as the centre of the earth. For what is commonly said — that the Horizon of which the astronomers dispute is a flat circular surface resting upon the surface of the earth and touching the heaven all around, so that it divides it into two equal parts — is to be understood only according to the judgment of the senses. For geometrically speaking, such a surface does not bisect the heaven, since it does not pass through its centre. Yet because the distance from the surface of the earth to its centre is not so great as to bring it about that an eye set upon the globe of the earth — other obstacles, namely mountains and valleys, being removed — should fail to see half of the sky; indeed, since it can even happen that someone stationed on some lofty mountain sees more than half of the sky; it has come about that that circular surface resting on the surface of the earth is taken for the Horizon. For, as we have confirmed by many experiments in chapter 1, this surface sensibly cuts the heaven into two halves — although, geometrically speaking, only a surface drawn through the centre of the earth bisects the heaven, and this is what the aforesaid authors call the rational Horizon, because it is gathered by reason alone. For the keenness of the eyes does not run out to the very edge of the heaven so as to perceive the division of the heaven into equal parts; rather, from the various phenomena that are perceived by sense, the mind by reasoning gathers that the matter is so.

[Margin: What the artificial horizon is.]

For the same reason it is called by some the Artificial [horizon], because it is discovered by the help of the astronomical art. It is therefore of this rational Horizon that Johannes de Sacrobosco [Sacrobosco] discourses in this place; and to it lies parallel every pavement built true to the level, and likewise any convex surface of water, insofar of course as it appears flat to the sense.

Diagram. A circle (the sphere seen edge-on) crossed by a horizontal diameter — a double ruled line representing the horizon plane — with the word **HORIZON** lettered along it (the letters spread across the line, H O R I on the left half and Z O N on the right, split at the centre by a small hatched sun/eye emblem set on the

line). At the left terminus of the horizon line a single letter is faintly marked, N [?] (uncertain — possibly a label, possibly decorative). No other labels are legible.

[Margin: What the sensible horizon is.]

The Sensible Horizon is the name given to that space, on the surface of land or sea, which the sight of the eyes, sweeping around, can take in, all obstacles being removed.

For since the earth is round, an eye set upon its surface cannot behold a larger space than what the straight lines going out from the eye — lines tangent to the surface of the terrestrial globe — cut off, as is manifest among the writers on optics [the Perspectivi]. But not all authors make this space of the same size. For in the opinion of Macrobius [Macrobius] its semidiameter comprises 180 stadia, that is 22½ miles. Eratosthenes [Eratosthenes] sets the same at 350 stadia, which make about 44 miles. Albertus Magnus asserts that it contains 1000 stadia, that is 125 miles. Proclus makes it 2000 stadia, which make 250 miles. But in most authors you will find it to contain — as being nearer the truer opinion — only 500 stadia, or 62½ miles. Finally, however great this space may be (for it is hard to determine), let it suffice us that it is called the Sensible Horizon.

The Horizon, moreover, is twofold: right and oblique (or sloping). A right Horizon and a right sphere belong to those whose Zenith is on the Equinoctial, because their Horizon is a circle passing through the poles of the world and dividing the Equinoctial at right spherical angles; hence it is called a right Horizon and a right sphere. An oblique or sloping Horizon belongs to those for whom the pole of the world is raised above the Horizon. And since their Horizon intersects the Equinoctial at unequal and oblique angles, it is called an oblique Horizon, and an oblique or sloping sphere.

[Margin: The right and the oblique horizon.]

[Margin: Who have a right or an oblique horizon.]

COMMENTARIVS. [Commentary]

He divides the Horizon into right and oblique, and teaches that a right [horizon] causes the sphere too to be called right, and an oblique one an oblique sphere. On which matter I have written more fully in the first chapter. Now it will be enough to set both the Horizon, or the sphere, before the eyes by their own figures.

Commentary on Chapter II of the *Sphere* — On the Horizon (continued)

DIAGRAM OF THE RIGHT HORIZON, and of the oblique.

Diagram. The left of the two woodcut circles shows the *right* (direct) sphere: a circle divided into four quadrants by a horizontal and a vertical diameter, the lower half shaded. Labels:

- Top: ZENITH
- Bottom: NADIR
- Left (vertical, along the rim): POLVS ("pole")
- Right (vertical, along the rim): POLVS ("pole")
- Horizontal diameter through the center: HORIZON RECTVS ("right horizon")
- Vertical diameter through the center: AEQVINOCTIALIS ("equinoctial")

Here the equinoctial passes through the zenith, and the horizon runs through both poles — the configuration of the right sphere.

Diagram. The right of the two circles shows the *oblique* sphere: the same circle, but with the axis tilted, the lower half shaded. Labels:

- Top (tilted): ZENITH
- Upper right, along the rim: POLVS ("pole")
- Lower left, along the rim: POLVS ("pole")
- Diagonal diameter: AEQVINOCTIALIS ("equinoctial")
- Horizontal diameter through the center: HORIZON OBLIQVS ("oblique horizon")
- Bottom: NADIR

Here the pole is raised obliquely above the horizon, so the equinoctial meets the horizon at a slant — the configuration of the oblique sphere.

Text [Sacrobosco]

The ZENITH of our head, moreover, is always the pole of the horizon. From this it is clear that the elevation of the pole of the world above the horizon is as great as the distance of the zenith from the equinoctial — which is shown as follows. Since on any natural day each colure is joined twice to the meridian (or is itself the same as

the meridian), what is proved of the one is proved of the other as well. Take, then, one quarter of the colure that marks off the solstices, namely the quarter from the equinoctial up to the pole of the world; and take again another quarter of the same colure, namely the one from the zenith up to the horizon, since the zenith is the pole of the horizon. These two quarters, being quarters of the same circle, are equal to each other. But if equals are taken from equals — or a common part removed — the remainders are equal. Therefore, with the common arc removed (namely the one between the zenith and the pole of the world), the remainders will be equal: that is, the elevation of the pole of the world above the horizon, and the distance of the zenith from the equinoctial.

[Margin: The zenith of the head is the pole of the horizon. The elevation of the pole above the horizon equals the distance of the zenith from the equator.]

COMMENTARY

From the fact that the zenith, or crown of our head, is perpetually the pole of the horizon — so that from the zenith in every direction to the horizon there is a quadrant of a circle — the author demonstrates here that the elevation of the pole above the horizon is as great as the distance of the zenith from the equator. This is what we assumed above, as something well known, in the case of the meridian circle. The author's own demonstration can be made perfectly clear to anyone by this figure, in which let the circle A B C D be the solstitial colure (the same as the meridian); the horizon, A E C; the equator, B E I; the poles of the world, G and H; the zenith, F [and the nadir, D]. The quadrant from the equator through the zenith up to the pole is the arc B F G; the quadrant from the zenith through the pole down to the horizon is the arc F G C. Since, then, the quadrants B G and F C are equal, it is evident that, with the common arc F G removed, the remaining arc G C — namely the elevation of the pole above the horizon — is equal to the remaining arc F B, that is, to the distance of the zenith from the equator. Hence it is clear that the altitude of the pole in any city is equal to the latitude of that same place, since the latitude of a place is as great as the distance of the zenith from the equator.

Diagram. A circle with two perpendicular diameters and two oblique chords crossing at the center E. Labelled points:

- F (top of circle)
- B (upper left)
- G (upper right; marked with a star)
- A (left)

- E (center)
- C (right; marked with a dot)
- H (lower left; marked with a star)
- I (lower right)
- D (bottom)

The horizontal diameter is A–E–C (the horizon); the vertical diameter F–E–D; the oblique chord B–E–I is the equator; G and H are the poles of the world. (All nine labels read cleanly.)

[Margin: The latitude of any place equals the altitude of the pole above the horizon.]

THE FUNCTIONS OF THE HORIZON [OFFICIA HORIZONTIS]

I.

It DIVIDES the heaven into two equal hemispheres — the upper, or visible, and the lower, or hidden.

[Margin: The horizon cuts the heaven into a hemisphere that is visible or upper, and one not visible, or lower.]

II.

It IS the cause of the right and the oblique sphere. For the higher some horizon raises one of the poles of the world above itself, the more oblique a sphere will those have who dwell under such a horizon; and the less elevated the pole any horizon has, the less oblique a sphere those living under it obtain. Thus where one of the poles is raised most of all above the horizon — as it is for those dwelling directly under the poles of the world — there the sphere is most oblique; but where there is no elevation of the pole above the horizon, as happens for those living under the equator, there is no obliquity of the sphere at all, but a wholly right sphere.

[Margin: The horizon is the cause of the right and the oblique sphere.]

III.

BY MEANS of the horizon is reckoned the length of any given day and night — the artificial day and night. For the artificial day is defined as the duration of the Sun's stay above the horizon, and the night as the stay of the same Sun below the horizon. This duration is known solely from the arcs of the parallels above or

below the horizon which the Sun describes by the diurnal motion. So that if the horizon divides all the parallels into equal parts, as happens in the right sphere, the days are perpetually equal to the nights; but if it divides them unequally, the nights are unequal to the days. (More on this in chapter 3.) From this it is easily gathered that the horizon is the sole cause of the inequality of artificial days and nights.

[Margin: The horizon determines the artificial day and night. What the artificial day and night are.]

IV.

It SHOWS the duration of all the stars above the horizon and below it. For when the horizon divides all the parallels that are described by the diurnal motion, cutting each in two — as happens in the right sphere — any given star spends as much time above the horizon as below it. But when the horizon does not cut the parallels in two, as happens in the oblique sphere, the stars declining from the equator toward the visible pole draw out a longer stay above the horizon than below it — such as, for us dwelling in Europe, are all the stars set in the northern hemisphere. Those declining toward the hidden pole spend less time above the horizon than below it — such as, in our horizon, are all the stars placed in the southern hemisphere. All of this is easily demonstrated from the spherical elements of Theodosius [Theodosius of Bithynia]. Indeed some stars in any oblique sphere do not rise above the horizon at all, but remain perpetually hidden beneath it; while certain others do not descend below it, but are always visible above the horizon, as we shall say presently.

[Margin: The horizon indicates the duration of all the stars above the horizon.]

V.

It MAKES MANIFEST the points of rising and setting of all the heavenly bodies, and how far their rising and setting stand from the true and equinoctial rising [and setting] — a matter of which astronomers make the greatest use, especially in determining the lengths of shadows. The *true* or *equinoctial* rising is called that point in which the equator cuts the horizon on the eastern side; the *true* or *equinoctial* setting is called that point in which the equator intersects the same horizon on the western side. When, therefore, a star does not rise or set at such a point, it is said by astronomers to have a rising or setting latitude. So that the latitude, or amplitude, of the rising or setting of any star is the arc of the horizon intercepted between the point of the true rising (or setting) of the said star and the point of the true rising (or setting). Now in every climate the rising amplitude,

or latitude, of any given star is equal to the setting amplitude, or latitude, of that same star. Likewise, two stars equally distant from the equator — one toward the north, the other toward the south, or even both toward the north or both toward the south — have equal rising and setting amplitudes, as can easily be deduced from the spherical elements of Theodosius and Menelaus [Menelaus of Alexandria]. From this it follows that the rising and setting amplitudes of the points of a single quadrant of the ecliptic are equal to all the rising and setting amplitudes of all the points found in the other quadrants: so that there always exist four points of the ecliptic whose rising and setting amplitudes are equal, just as we said above concerning the declinations of the points of the ecliptic, and as was demonstrated by us in Book 1 of the *Astrolabe*, Lemma 49, no. 5.

[Margin: The horizon shows the points of the rising and setting of the heavenly bodies, and how far their rising and setting stand from the equinoctial rising and setting. What the true or equinoctial rising and setting are. What the rising or setting latitude of a star is. How the rising and setting latitude is computed by means of the sines.]

The rising or setting amplitude of any point of the ecliptic whatever, or of any star, WILL BE FOUND by the doctrine of sines in this way. Let the sine of the declination of the proposed star or point of the ecliptic be multiplied into the whole sine, and let the product be divided by the sine of the complement of the latitude of the place in which you are inquiring after the amplitude. (We call the *complement* of any arc that which it lacks to complete the quadrant. Thus the complement of 30 degrees will be an arc of 60 degrees, and so for the remaining arcs.) There will come out a sine whose arc, found from the table of sines, will give the amplitude sought. For, as Geber [Jābir ibn Aflāḥ] demonstrates in Book 2, and Johannes Regiomontanus in Book 2 of the *Epitome*, proposition 2, and as we demonstrated in Book 1 of the *Gnomonics*, proposition 34: as the sine of the complement of the altitude of the pole (or of the latitude of any place) is to the sine of the declination of the star or point of the ecliptic, so is the whole sine to the sine of the rising or setting amplitude of the proposed star or point of the ecliptic.

EXAMPLE. With the Sun placed at the first point of ♋ [Cancer], I multiply the sine of the declination — which is 23 degrees 30 minutes, namely 39874 — into the whole sine, taken as 100000; and the product, namely 3987400000, I divide by 74314, that is, by the sine of the complement of the latitude of Rome, which we here set at 42 degrees (to avoid the minutes). There comes out the sine of the amplitude sought, 53656, to which in the table of sines corresponds an arc of 32

degrees 27 minutes. So great, then, is the rising or setting amplitude, with the Sun at the first point of $\mathfrak{S}$, where the pole is elevated 42 degrees, as happens roughly at Rome. And the same reasoning holds for the rest of the points. An easier method of computing the rising latitude we have handed down in our new treatise on sundials, problem 3.

VI.

It DEMONSTRATES the degree of the ecliptic with which any given star rises and sets. For with a star placed on the horizon on the eastern side, let the degree of the ecliptic that then touches the horizon be noted; for that star is said to rise with that degree. And with the same star placed again on the horizon on the western side, let the degree of the ecliptic that then touches the horizon be observed; for the aforesaid star will set with that degree.

[Margin: The horizon indicates the degree of the ecliptic with which any given star rises.]

VII.

It INDICATES which stars are of perpetual appearance in any climate, which are of perpetual concealment, and which, finally, are said to rise and set. For if the horizon passes through the poles of the world — as does the right horizon — it will divide in two all the parallels that the stars describe by the diurnal motion, as is established from proposition 15, Book 1, of Theodosius, and as we also asserted above in the fifth property of circles. Hence in a horizon of this kind, that is, in the right sphere, all the stars rise and set. But if the horizon by no means passes through the poles of the world — as is every oblique horizon — it will not divide any parallel in two except the equator; indeed some parallels near the poles it does not divide at all. Hence the stars describing parallels that are not divided by the horizon neither rise nor set, but are either raised perpetually above the horizon (if, that is, they lie near the visible pole) or perpetually kept down beneath it (if, that is, they are placed near the other, hidden pole); whereas the stars whose parallels are divided by the horizon do rise and set. The stars that are of everlasting appearance are known by a circle parallel to the equator which touches the horizon near the visible pole. For all the stars comprehended between the elevated pole and the said parallel are seen perpetually above the horizon, as is easily seen on the material sphere. The stars, on the other hand, that are always concealed beneath the horizon are marked off by another circle parallel to the equator which, being equal to the former, touches the horizon on the side of the hidden pole. For all the stars enclosed between the hidden pole

and the said parallel can never come into view, but are of everlasting concealment.

[Margin: The horizon shows the stars that rise, that set, and that are perpetually visible and hidden. Which stars neither rise nor set. What the greatest of the ever-visible or ever-hidden parallels is.]

Now these two parallel circles — of which the one is called by astronomers the *greatest of the ever-visible*, because it is the greatest of all the parallels that always appear, and the other the *greatest of the ever-hidden*, because it is the greatest of all the parallels that are always concealed — are so much the larger, and the nearer to one another, and enclose the more stars, as the horizon is more oblique, that is, as the pole is more raised above the horizon; so much so that, for those dwelling under either of the poles, the said circles utterly coincide, uniting into one and merging with the equator. And with these same people no fixed star rises and sets, but half of them appear perpetually while the other half remain always hidden — although all the planets, since they perform their proper motions in the zodiac, appear for half the time and are concealed for the other half, because one semicircle of the zodiac is continually raised above the horizon while the other lies hidden beneath it. For those, moreover, who dwell under the polar circles, the two tropic circles perform the office of the said circles; and conversely, for those who dwell under the two tropics, the polar circles discharge the office of those same two parallels. For in every region the said parallels are always equal to each other and equally distant from the equator, as is established from proposition 6, Book 2, of Theodosius, or also from the seventh property of circles; and the same can be plainly seen by anyone on the material sphere. The remaining stars, finally, placed between the equator and the said two parallels rise and set.

[Margin: The various dispositions of the greatest of the ever-visible and ever-hidden parallels.]

FURTHERMORE, from the known declination of any star and the known latitude of the place (which is the same thing as the altitude of the pole) it will easily be detected whether the star rises and sets, or rather is perpetually visible or perpetually hidden — by this method, of course. Let the altitude of the pole (or latitude of the place) be joined to the declination of the star, if it has any. If the aggregate is less than a quadrant, that is, than 90 degrees, the star will rise and set; but if it exceeds 90 degrees, a star declining toward the north will be perpetually visible [and never set], while one verging toward the south will be perpetually hidden and can never come into view above the horizon. And if the

said aggregate turns out equal to a quadrant, it will touch the star — that is, the star will graze the horizon — whether on the northern side, if it has a northern declination, or on the [southern] side, if a southern. All of this is plain on the material sphere.

[Margin: How it may be known whether a proposed star rises or not, and whether it is perpetually visible or perpetually hidden.]

The SAME we shall obtain by this reasoning. If the complement of the star's declination is greater than the altitude of the pole, the star will rise and set; but if it is less than the altitude of the pole, it will be perpetually visible [or perpetually hidden]; and if, finally, it is equal to the altitude of the pole, the star — whether northern or southern — will touch the horizon. As appears on the material sphere.

The SAME we shall also obtain in this manner. If the star's declination is less than the complement of the altitude of the pole, the star itself rises and sets; but if it is greater, it will be perpetually visible [or hidden]; and if [equal], it will touch the horizon. As is clear from the same material sphere — provided that, for northern stars, one takes the arc of the meridian below the horizon on the side of the arctic pole, between the horizon and the equator, and the star's declination is counted on that same arc from the equator toward the horizon; while for southern stars one takes the arc of the meridian above the horizon, on the southern side, between the horizon and the equator, and the star's declination is counted on that same arc from the equator toward the horizon.

VIII.

The horizon brings GREAT convenience to cosmographers, or geographers. For to it are referred the altitudes of the pole, which we have demonstrated to be equal to the latitudes of places, and of which cosmographers make the greatest use. Hence, again, from the known altitude of the pole (or latitude of the place) the altitude of the equator will be known. For since from the zenith, through the equator, up to the horizon there is an entire quadrant of the meridian, if the latitude of the place — that is, the distance of the zenith from the equator — is subtracted from the quadrant, there will remain the altitude of the equator. Thus if 41 degrees 56 minutes (the latitude, namely, of Rome) be subtracted from 90 degrees, there will remain the altitude of the equator, 48 degrees 4 minutes — which is precisely the arc of the meridian comprehended between the equator and the horizon. Moreover, the altitude of the equator is perpetually equal to the complement of the altitude of the pole (or of the latitude of the place), that is, to

the distance of the zenith from the pole of the world; for the latitude of the equator is the complement of the latitude of the place, as is plain, while the latitude of the place is equal to the altitude of the pole. This may also be demonstrated by the following reasoning. Let the immediately preceding figure be examined; in it, since the quadrant A F is equal to the quadrant B G, with the common arc B F removed there will remain the arc A B — namely the altitude of the equator — equal to the arc F G, that is, to the complement of the altitude of the pole.

[Margin: The horizon is of service to cosmographers. How the altitude of the equator may be known. The altitude of the equator is equal to the complement of the altitude of the pole.]

ON THE FOUR LESSER CIRCLES [DE QVATVOR CIRCVLIS MINORIBVS]

Text [Sacrobosco]

Having spoken of the six greater circles, we must now speak of the four lesser ones. The Sun, when it is at the first point of Cancer — that is, at the first point of the summer solstice — describes, by the sweep of the firmament, a certain circle which is the last one described by the Sun on the side of the arctic pole; whence it is called the circle of the summer solstice, for the reason stated above, or the summer tropic, from τροπή [Greek: τροπή (tropē, "turning")], which means "turning," because the Sun then begins to turn back toward the lower hemisphere and to recede from us.

The Sun, again, when it is at the first point of Capricorn — that is, at the winter solstice — describes, by the sweep of the firmament, a certain circle which is the last one described by the Sun on the side of the antarctic pole; whence it is called the circle of the winter solstice, or the winter tropic, because the Sun then turns back toward us.

[Margin: How the tropics are described. The tropic of Cancer and the tropic of Capricorn.]

COMMENTARY

Having finished the treatment of the six circles that are the greatest in the sphere, the author here treats of the four lesser, or non-greatest, ones — and first of the two that are described by the sweep of the first mobile from the first degree of ♋ [Cancer] and ♎ [Capricorn], and are consequently removed from the equator by 23 degrees 30 minutes, just as the beginnings of ♈ and ♎ are. These two circles

are equal to each other, and each touches the ecliptic at a single point only, as is gathered from Book 2 of Theodosius. Likewise they are the last and least of those which the Sun describes by the diurnal motion; for it is up to these that the Sun wanders to and fro from the equator, [whereas...]

as soon as the Sun has reached them, it at once directs its course back toward the Equator. This is why he says they are called "tropics," from the Greek word τροπή [tropē], which means "conversion" or "turning," because the Sun, being in them, turns itself back once more toward the Equator. The one described from the first point of Cancer ☊ is customarily called the Tropic of Cancer; the other, described from the beginning of Capricorn ☋, the Tropic of Capricorn. By the same reasoning the Tropic of Cancer is also usually called the summer tropic — the tropic or circle of the summer solstice (understand this of the northern hemisphere) — the Northern Tropic, and the reversing circle of Cancer [circulus versilis Cancrī]. Likewise the Tropic of Capricorn is called the winter tropic, the tropic or circle of the winter solstice, the Southern Tropic, and both are usually named by very many other names of this kind by writers.

[Margin: The various names of the tropics.]

(Sacrobosco) Now since the Zodiac slopes away from the equinoctial, the pole of the Zodiac too will slope away from the pole of the world. When, therefore, the eighth sphere is set in motion, and the Zodiac, which is part of the eighth sphere, moves about the axis of the world, the pole of the Zodiac will also move about the pole of the world. That circle, then, which the pole of the Zodiac describes about the Arctic pole of the world is called the Arctic circle; but that circle which the other pole of the Zodiac describes about the Antarctic pole of the world is called the Antarctic circle.

[Margin: The Arctic circle, and the Antarctic.]

COMMENTARY

He here explains the two polar circles — the Arctic and the Antarctic — which are described by the motion of the primum mobile from the poles of the Zodiac about the poles of the world. Since, therefore, the distance of the poles of the Zodiac from the poles of the world is equal to the greatest declination of the Sun, as we demonstrated a little above, it follows that each polar circle stands as far from the poles of the world (the Arctic from the Arctic pole, the Antarctic from the Antarctic) as the two tropics recede from the Equator — namely 23° 30′.

[Margin: What the polar circles are.] [Margin: How far the polar circles are from the poles of the world.]

The Greeks, as one may see in Proclus and Cleomedes, understand the two polar circles very differently. For they do not, like the Latins, give the name "polar circles" to those described about the poles of the Zodiac; rather, among them two circles are called polar, of which one is the greatest of the always-visible parallels and the other the greatest of the always-hidden ones — the circles we treated in the 7th function of the Horizon. The Greeks preferred to define the polar circles in this way, so that through them all those stars might be known which never rise and set, but are either perpetually visible — such as those the Arctic circle encloses — or perpetually hidden, such as those the Antarctic comprehends. From this it is clear that among the Greeks the two polar circles are not of the same size in all regions, as they are among the Latins; rather, the more oblique the sphere, the larger they become — as was said above of the greatest of the always-visible parallels and the greatest of the always-hidden ones.

[Margin: How the polar circles are understood by the Greeks.]

Moreover, the four aforesaid smaller circles — namely the tropics and the polar circles — are equidistant from the Equator, as is established from Theodosius (*Sphaerica* II.2), because they possess the same poles as the Equator, namely the poles of the world, from which they are described. And although any greatest circle in the sphere has its own parallels, as we said at the beginning of this chapter, still among astronomers the chief account is taken of the parallels of the Equator and of the Zodiac. For the individual stars, or points of the heaven, describe individual equidistant circles by the diurnal motion of the primum mobile; but by the motion of the eighth sphere, from west to east, they trace circles equidistant from the Zodiac. Among all the circles parallel to the Equator, these four smaller ones are distinguished by particular names, which our Author has explained.

Just as the Equator, or any greatest circle in the sphere, is distributed into 360 degrees, so too, as we noted above, any lesser circle is cut into the same number of degrees, which are wholly similar to the degrees of a greatest circle, as is gathered from Theodosius (*Sphaerica* II.10); so that whatever proportion a greatest circle has to a non-greatest circle, the individual degrees of the greatest circle keep the same proportion to the individual degrees of the non-greatest circle.

The proportion of a greatest circle to a non-greatest circle whose declination is known will be obtained from the doctrine of sines, in this way. Let the sine of the complement of the declination of the non-greatest circle be multiplied by the whole circle, that is, by 360° , and let the product be divided by the whole sine [sinum totum]; there will result the number of degrees of the non-greatest circle, of which the greatest circle contains 360. For, as we showed in Book 8 of the *Practical Geometry*, proposition 2, as the whole sine is to the sine of the complement of the declination of any parallel — that is, as the radius [semidiameter] of the Equator is to the radius of the parallel — so is the circumference of the Equator to the circumference of the parallel.

[Margin: By what method the proportion of a greatest circle to a non-greatest is known from the sines.]

Example. Suppose it is proposed to find what proportion the Equator has to the parallel which passes through the zenith [Verticale punctum] of Rome, whose declination is taken as 42° . I multiply the sine of the complement of this declination — that is, the sine of 48° , namely 74314 — by 360, and I divide the resulting number, 26753040, by 100000, the whole sine, and I find about $267\frac{1}{2}$ degrees. Therefore the Equator will have to the parallel which runs through the zenith of Rome — or likewise one degree of the Equator to one degree of the said parallel — the proportion that 360° has to about $267\frac{1}{2}^\circ$: that is, nearly the sesquitertian ratio, such as 4 to 3, and so on.

(*Sacrobosco*) As great, moreover, as is the greatest declination of the Sun — namely from the equinoctial — so great is the distance of the pole of the world from the pole of the Zodiac; which is shown as follows. Let the colure that marks off the solstices be taken, which passes through the poles of the world and through the poles of the Zodiac. Since, then, all the quadrants of one and the same circle are equal to one another, the quadrant of this colure which runs from the equinoctial up to the pole of the world will be equal to the quadrant of the same colure which runs from the first point of Cancer up to the pole of the Zodiac. Therefore, when the common arc — namely that from the first point of Cancer up to the pole of the world — is removed from those equal arcs, the remainders will be equal: namely, the greatest declination of the Sun, and the distance of the pole of the world from the pole of the Zodiac.

[Margin: The distance of the pole of the Zodiac from the pole of the world is equal to the greatest declination of the Sun.]

COMMENTARY

Diagram. A woodcut sphere (globe of the world) shown in oblique view, on which the great circle A B C D — the solstitial colure — is drawn together with the smaller circles (the two tropics and the two polar circles) crossing it; the two polar circles near the top and bottom are marked with small stars. The labeled points, per the accompanying text, are: A and C the ends of the equinoctial diameter; B the north (Arctic) pole of the world; D the south (Antarctic) pole of the world; K the pole of the Zodiac; E the first point of Cancer on the colure; so that A E is the greatest declination of the Sun and B K the distance of the pole of the world from the pole of the Zodiac.

- I (top, left of pole, with star): point on the Arctic circle
- B (top center, large): north pole of the world
- K (top, right of pole, with star): pole of the Zodiac
- F (upper left flank): tropic/colure intersection point
- E (upper right flank): first point of Cancer
- A (right side, below E): end of the equinoctial (A C)
- C (left side, middle): opposite end of the equinoctial (A C)
- H (left side, lower): tropic/colure intersection point
- G (right side, lower): tropic/colure intersection point
- L (bottom, left of pole, with star): point on the Antarctic circle
- D (bottom center): south pole of the world
- M (bottom, right of pole, with star): point on the Antarctic circle

He proves that the poles of the Zodiac recede from the poles of the world by as great a space as is the greatest declination of the Sun on either side. He demonstrates this in the same way that we ourselves showed the same thing in the 6th function of the Colures, as is evident in this figure, in which: the circle A B C D is the solstitial colure; A B is the quadrant from the equinoctial A C up to the pole of the world B; E K is the quadrant from the first point of Cancer ϖ up to the pole of the Zodiac K; A E is the greatest declination of the Sun; B K is the distance of the pole of the world from the pole of the Zodiac; and so on.

Now since we said above that the greatest declination of the Sun varies on account of the libration of the tenth sphere, it comes about that this reasoning concludes only that the greatest declination of the Ecliptic of the eleventh sphere is equal to the distance of the poles of the Ecliptic of that same sphere from the poles of the world — since this sphere is not stirred by that libration. For the

greatest declination of the Sun, since it varies, cannot be equal to the distance of the pole of the Zodiac from the pole of the world, which is taken in the primum mobile and remains always the same.

[Margin: How it is to be understood that the distance of the poles of the Zodiac from the poles of the world is equal to the greatest declination of the Sun.]

[Margin: How great the arc of the colure between the Tropic of Cancer and the Arctic circle is.]

(Sacrobosco) Now since the Arctic circle is equally distant from the pole of the world along each of its parts, it is clear that the part of the colure lying between the first point of Cancer and the Arctic circle is nearly double the greatest declination of the Sun — or double the arc of the same colure which is intercepted between the Arctic circle and the Arctic pole of the world, which arc is likewise equal to the greatest declination of the Sun. For since this colure, like the other circles in the sphere, is 360° , its quadrant will be 90° . Since, then, according to Ptolemy the greatest declination of the Sun is $23^\circ 51'$, and the arc between the Arctic circle and the Arctic pole of the world is of the same number of degrees, if these two together — which come to nearly 48° — are subtracted from 90° , there will remain 42° , which is the size of the arc of the colure lying between the first point of Cancer and the Arctic circle. And so it is clear that that arc is nearly double the greatest declination of the Sun.

COMMENTARY

He gathers from what has been said that the arc of the colure intercepted between the Tropic of Cancer ☿ and the Arctic circle is nearly double the greatest declination of the Sun, or of the distance of the pole of the Zodiac from the pole of the world. For since, according to Ptolemy's view, the greatest declination of the Sun is $23^\circ 51'$, that arc will be about 42° . According to the common opinion, however, the greatest declination of the Sun at the present time is $23^\circ 30'$, and the said arc is 43° .

ON THE MILKY WAY [DE CIRCULO LACTEO]

Furthermore, since besides these ten circles of the sphere Proclus also treats of the Milky Way — which is also called the Galaxy [Galaxia] — it will not be out of place to explain briefly here what the Milky Way is, and through which constellations it runs in the heaven. The Milky Way, then, is a greatest circle in the heaven, having a variable breadth and brightness, so that in one part it has a

greater breadth than in another, and likewise a greater whiteness in one part than in another, running through Gemini and Sagittarius, as Ptolemy explains very fully (*Almagest* VIII.2). Its whiteness, from which it takes the name "milky," arises — as some hold — from the very great multitude of tiny stars contained in it, which do not reach our sight as distinct, like the other stars. I, however, together with others, think it more probable that the Milky Way is a continuous part of the Firmament, denser than the other parts of the heaven, so that it can receive the light of the Sun — though not in the way the other stars do, which are much denser parts of the Firmament and set apart from one another — whatever the poets may fable about the milk of Juno and the scorching that the Sun caused. And so the Milky Way truly exists in the Firmament, and not in the region of the air, as Aristotle held. For on that reasoning it would not be seen, in every part of the terrestrial globe, to pass precisely through the same stars of the Firmament — just as a comet, which exists in the air, is not seen in all regions under the same fixed star; which is false. For the Milky Way runs perpetually, as may be seen in Ptolemy at the cited passage, and as experience teaches, through Cassiopeia, Cygnus, the flying Eagle [Aquila], the Arrow of Sagittarius [Sagitta], the tail of Scorpius, the Centaur, the ship Argo, the feet of Gemini, Heniochus (that is, Auriga, the Charioteer), and Perseus, as is most clearly established on any astronomical globe. And this Manilius beautifully sets forth in these verses. For after he had spoken of the Zodiac, he writes thus of the Milky Way:

[Margin: The Milky Way.] [Margin: Whence the whiteness in the Milky Way arises.] [Margin: That the Milky Way exists in the Firmament, and not in the air.] [Margin: Through which constellations the Milky Way runs.]

The other [circle], set opposite, moves up toward the Bears, and draws its threads a little back from the North by its curve. It crosses through the stars of Cassiopeia turned upside down, then, descending obliquely, touches the Swan [Olor]; it cuts the summer bounds, and the Eagle lying on its back; and that which levels the seasons — the belt that bears the Sun's horses — within the tail where Scorpius burns, and the outermost left of Sagittarius[i], and its arrow. Thence it winds its bends through the legs and feet of the other Centaur; and again it begins to climb the sky, passing the Argive ship [Argo] along its topmost stern-ornaments [aplustria],

and cuts the middle circle of the world, and the Twins [Gemini] along their lowest stars: it passes beneath Heniochus [Auriga]; and, setting out from there, making for Cassiopeia, it crosses above Perseus himself, and closes in her the circle begun from her; and it cuts three of the middle

gyres, and the sign-bearing circle [the Zodiac], as often as it is itself cut across into two parts. Nor need it be sought out: it strikes the sight of its own accord, teaches itself, and compels itself to be noticed; for, gleaming white, it shines as a track upon the dark-blue world.

Ovid calls the Milky Way the road by which the gods above went up to Jupiter, in these verses in Book 1 of the *Metamorphoses*:

There is a high road, plain to see when the sky is clear (it bears the name "Milky"), notable for its very whiteness. By this is the path of the gods above to the halls of the great Thunderer, and to his royal house, etc.

Whoever desires more about this circle, let him read Ptolemy at the cited passage, and especially the commentaries of Stöffler [Stoflerini] on the *Sphere* of Proclus. There he will find that various opinions have existed concerning this circle.

THE FUNCTIONS OF THE PARALLEL CIRCLES [OFFICIA CIRCVLORVM PARALLELORVM]

I.

The tropics enclose the path of the Sun. For they are, as it were, boundaries enclosing in the heaven the region beyond which the Sun never strays, but within which it is perpetually carried. Hence they also mark in the Ecliptic the two points at which the solstices occur, and at which the Sun has its greatest declination.

[Margin: The tropics enclose the path of the Sun.]

II.

The polar circles determine the distance of the poles of the Zodiac from the poles of the world, and enclose, toward the poles of the world, the regions in which the greatest day of the year — and likewise the greatest night — exceeds 24 hours and is made up of several days, as will be taught in chapter 3.

[Margin: The polar circles enclose the regions toward the poles in which the greatest day is longer than 24 hours.]

III.

The two tropics and the two polar circles, both in the heaven and on the earth, constitute the five zones, as we shall say presently.

[Margin: The tropics and polar circles constitute the five zones.]

IV.

The parallel circles, which the Sun describes by the motion of the primum mobile — about 182 in number, as we shall say in chapter 3 — reveal the cause of the perpetual equality of days and nights in the right sphere, and of the inequality of those same days and nights in the oblique sphere.

[Margin: The parallel circles indicate the equality of days and nights in the right sphere, but the inequality in the oblique.]

V.

The parallels running through the vertical points [zeniths] of all places set before the eyes, throughout the whole circuit of the heaven, the limits of the latitude of cities; and along these same parallels the longitudes of places are counted from west to east, as was said above.

[Margin: The parallel circles determine the latitudes of places, and along them the longitudes are counted.]

VI.

The parallels which the planets, or the fixed stars, describe by the diurnal motion from east to west fix the limits of the declinations of all the heavenly bodies from the Equator; but those they trace from west to east with respect to the Ecliptic mark out the bounds of the latitudes from the Ecliptic.

[Margin: The parallels indicate the declinations of the stars and their latitudes.]

VII.

The parallel circles have great use among cosmographers. For by means of them they separate on the earth spaces at such an interval that the greatest artificial days differ from one another by a quarter of one hour; and by the same parallels the various climes are constituted, as will be clear from chapter 3.

[Margin: The parallel circles are in use among cosmographers.] [Margin: In what sphere the five parallels exist.] [Margin: The four smaller parallels distinguish, in heaven and on earth, five zones.]

ON THE FIVE ZONES [DE QVINQVE ZONIS]

(*Sacrobosco*) The equinoctial, together with the four smaller circles, are called the five parallels, as being (as it were) equidistant — not because, by however much the first is distant from the second, by so much the second is distant from the third (for that is false, as has already appeared), but because any two circles, taken by themselves, are equidistant from each other along each of their parts. And they are called: the parallel of the Equinoctial, the parallel of the summer solstice, the parallel of the winter solstice, the Arctic parallel, and the Antarctic parallel. Note also that the four smaller parallels — namely the two tropics, the Arctic parallel, and the Antarctic parallel — mark off in the heaven five zones, or regions. Hence Virgil, in the first [Georgic]:

DE QVINQVE ZONIS (continued)

Text (*Sacrobosco*)

Five zones hold the heaven, one of which, ever glowing red with the flashing Sun, is always scorched with fire; around it, on the right hand and the left, are drawn two more, stiff with dark-blue ice and with black rains. Between these and the middle zone lie two, granted by the gift of the gods to wretched mortals; and a path is cut aslant through both, along which the oblique order of the signs may wheel round.

Quinq; tenent coelum Zonaë, quarum vna corusco Semper Sole rubens, & torrida semper ab igne: Quam circum extremæ dextra, leuaq; trahuntur Caerulea glacie concretæ, atq; imbribus atris. Has inter, mediamq; duæ mortalibus ægris Munere concessæ Diuum, & via secta per ambas, Obliquus qua se signorum verteret ordo. — [Virgil], Georgics I [233–239]

Just as many regions [*plagae*] are likewise distinguished upon the earth, placed directly beneath the aforesaid Zones. On which Ovid writes: [Margin: Ovid, *Metamorphoses* I.]

And just as two zones cut off the heaven on the right and as many on the left, while the fifth is hotter than these, so the care of God marked off the enclosed mass by the same number, and just as many regions are stamped upon the earth. Of these the middle one is not habitable, for its heat; deep snow covers two; and between each pair He set as many more, and gave them a temperateness, flame mingled with cold.

*Vtque duae dextra coelum, totidemque sinistra Parte secant Zonae, quinta est ardentior illis: Sic onus inclusum numero distinxit eodem Cura Dei, totidemque plagae tellure premuntur. Quarum quae media est, non est habitabilis aestu: Nix tegit alta duas: totidem inter vtramque locauit Temperiemque dedit, mista cum frigore flamma. — [Ovid], *Metamorphoses* I [45–51]*

That Zone, therefore, which lies between the two Tropics is called uninhabitable, because of the heat of the Sun that runs perpetually between the Tropics; and likewise the region of the earth placed directly beneath it is called uninhabitable, on account of the heat of the Sun running above it. But those two Zones which are bounded by the Arctic circle and the Antarctic circle about the poles of the world are uninhabitable because of their excessive cold, since the Sun is most removed from them; and the same is to be understood of the regions of the earth placed directly beneath them. The remaining two Zones, however — one lying between the summer Tropic and the Arctic circle, and the other between the winter Tropic and the Antarctic circle — are habitable, being tempered by the heat of the torrid Zone that lies between the Tropics and by the cold of the outermost Zones that lie about the poles of the world. Understand the same of the regions of the earth placed directly beneath them.

[Margin: Which Zones are said to be uninhabitable, and which habitable.]

COMMENTARY

In this third part of the chapter he treats of the five Zones, which he says are marked off in the heaven by four smaller circles, in such a way that the middle Zone, called torrid, is comprised between the two Tropics; the two called temperate lie between each Tropic and its polar circle; and finally the remaining two, called frigid, lie between the two polar circles and the poles of the world — as you see in the figure set forth. He then teaches that there are just as many Zones on the earth, placed directly beneath those celestial ones. As testimony for the celestial Zones he adduces certain verses of [Virgil] from *Georgics* I; in confirmation of the terrestrial ones he cites verses of [Ovid] from *Metamorphoses* I; and he assigns the reason why the middle Zone of all is called torrid, the outermost frigid, and the remaining two, lying between the torrid and the frigid, temperate. All of which is clear in the Author.

Diagram. A circle representing the sphere, divided by a vertical axis (the poles) and a horizontal diameter, and crossed by five parallel chords which are labelled, from top to bottom, with the four lesser circles and the equator. A small circle at

the center marks the Earth. The horizontal bands between the chords carry the zone-names, and the word ZONA runs repeatedly around the rim.

- Chord 1 (upper): CIRC. ARCT. — *circulus Arcticus* (Arctic circle)
- Chord 2: TROP. CANC. — *Tropicus Cancrī* (Tropic of Cancer)
- Chord 3 (center): AEQVA·TOR — *Aequator* (Equator), split about the central Earth-disc
- Chord 4: TROP. CAPRIC. — *Tropicus Capricorni* (Tropic of Capricorn)
- Chord 5 (lower): CIRC. ANTA[R]. — *circulus Antarcticus* (Antarctic circle)
- Rim / band labels: ZONA (repeated around the circumference); the curved outer labels along the right side read, top to bottom, [TOR]RIDA, TEMPERATA, FRIGIDA [?]; the top arc reads ZONA FRIGID[A] / TEMPERAT[A] [?] (the outermost caps are the frigid zones, the intermediate bands temperate, and the central band about the equator the torrid zone)

[Margin: Zona torrida. Zona temperata. Zona frigida. — Torrid zone. Temperate zone. Frigid zone.]

Only this must be noted here in passing: since each of the two Poets adduced by the Author made mention of a right and a left part in the heaven, "right" and "left" are not taken in the same way by everyone with respect to the celestial bodies. For Plato, [Aristotle], and the rest of the Philosophers, as well as the Geographers, call the Eastern parts "right" and the Western "left." Aristotle indeed and the Philosophers do so because the motion of the heavens begins from the East, just as in animals the beginning of motion arises from the right side. The Geographers (I speak of geographers on the near [northern] side of the Equator) do so because, wishing to determine the elevation of the pole so as to depict the situation of the earth more correctly, they turn their face toward the Arctic pole; whence the East is necessarily set on their right and the West on their left. Hence it comes about that all maps of the world and charts of regions are so drawn by the cosmographers (as may be seen in [Ptolemy] and others) that, to one looking at the maps or charts, the East is placed on the right side and the West on the left.

[Margin: What the right and left part of the heaven is among the Philosophers and Cosmographers.]

The Astronomers, on the contrary, call the Western parts of the heaven "right" and the Eastern "left," because, dwelling on the near [northern] side of the Equator, they turn their face toward the equinoctial circle, where the motion is swiftest, in order to observe the courses of the stars more accurately; and so they have the West on their right and the East on their left. The Poets, finally, call the

Northern parts of the heaven "right" and the Southern "left"; for, observing the settings of the stars, they turn their face [westward], so that the North is set on their right and the South on their left. This opinion — namely that the Northern part in the heaven is called "right" and the Southern "left" — holds because in every clime the rising Sun has the North on its right and the South on its left, and because there are more stars near the Boreal pole than near the Austral, as was said above. From all this, then, it is clear that Virgil and Ovid understood by the name "right part" and "left part" the North and the South. Lucan too understood the right and the left in this way, in book 3, when he says:

[Margin: What the right and left part of the heaven is among the Astronomers.]

[Margin: What the right and left part of the heaven is among the Poets.]

Ignotum vobis Arabes venistis in orbem, Vmbras mirati nemorum non ire sinistras. — [Lucan, *Pharsalia* III.247–248]

(You Arabs have come into a world unknown to you, marveling that the shadows of the groves do not go to the left.)

For he [Lucan] meant to signify that the Arabs had come to the near [northern] side of the Tropic of Cancer (☿), where at midday the shadows of bodies are perpetually cast toward the North — that is, toward the right part of the world — and not toward the South, that is, toward the left part, as we shall explain in chapter 3.

[Margin: The various names of the Zones.]

The Zones are sometimes called by the Authors *Fasciae* (bands), *Cinguli* (girdles), *Plagae* (regions), and by [Cicero] in the *Dream of Scipio* [*Somnium Scipionis*] *Maculae* (spots). Now, since there are two kinds of zones — the one celestial, the other terrestrial — the celestial are the primary ones and the causes of the terrestrial; not that those celestial zones are hot, or cold, or temperate — for such qualities are far removed from the celestial bodies — but because the Sun, through the varied reflection of its rays, direct namely or oblique, either renders the terrestrial zones wholly cold, on account of the excessive obliquity of the rays (as happens in the two outermost zones next to the poles of the world), or wholly sets them ablaze, on account of the directness of the rays (as occurs in the zone that is the middle of all, between the two Tropics), or finally induces in them a tempered heat and cold, when the Sun's rays are neither too direct nor too oblique but hold a middle manner (as befalls in the temperate zones, which are placed between the Tropics and the polar circles).

[Margin: That the torrid zone and the frigid zones are habitable.]

But what the Poets fable — that the frigid zones are uninhabitable from excessive cold, and the torrid from excessive heat — is not true. For by experience, and by the navigation of the Portuguese and the Spanish, it is sufficiently established that beneath the Equator, that is, beneath the middle of the torrid zone, very many peoples dwell; likewise beneath the poles, at least beneath the Arctic pole, that is, beneath the middle of the frigid zones, as [Olaus Magnus] the Great, the Goth, reports. Indeed, all who have sailed thither affirm that the habitation beneath the Equator is the best — the causes of which it would take long to recount here. For whatever be the case concerning cold and heat, we must believe that Nature, which distinguished the regions by heat and cold, also brought forth human beings and the other animals with a capacity to endure their places; especially since we see that both the Portuguese and the Spanish have at length grown accustomed to bear the intemperateness of the torrid zone, now that many at this time dwell beneath the Equator.

[Margin: In what way the terrestrial Zones are placed beneath the celestial.]

Diagram. A large circle representing the sphere shown obliquely, with a small circle at its center (the Earth). From the center, straight lines are drawn outward to the circumference; at the top and bottom are two lens-shaped (elliptical) figures representing the parallel circles seen edge-on, and meridian-like lines run from top to bottom. This illustrates the straight lines drawn from the four lesser celestial circles down to the center of the earth. (No lettered labels appear on the figure.)

In what way the terrestrial zones are placed directly beneath the celestial, the figure below clearly explains: in it, straight lines are drawn from all four of the lesser circles of the heaven down to the center of the earth. For where these lines cut the surface of the earth, there the same circles are described upon the earth, so that all the circles on the earth are set perpendicularly beneath the celestial circles. For thus it will come about — as may easily be gathered from the last proposition of book 6 of [Euclid] — that, since the heaven and the earth have the same center, the segments of the earth enclosed by the said circles are similar to the segments of the heaven comprised between the same celestial circles.

[Margin: Any Zone is of the same width [latitude], but not of the same length [longitude] with respect to all its parts.]

Hence, if from the said celestial parallels perpendiculars were dropped to the surface of the earth — which would necessarily converge toward the center of

the world — there would be described by them, on the earth's surface, the very same circles, directly corresponding to those celestial ones.

Every Zone is always of the same width [*latitudo*] from North to South, wherever you begin, because it is contained between two equidistant circles; but it is not of the same length [*longitudo*] from East to West. For the parts of any Zone, the nearer they are to the Equator, the greater length they will have; the nearer to the poles, the less — since the parallels toward the poles always grow smaller, as was said above.

[Margin: How great the widths of the Zones are. — By what method the lengths of the Zones may be found, both at the beginning and at the middle and end.]

The width of any Zone will readily be had from what has been said. For since each Tropic is distant from the Equator $23^{\circ} 30'$, the width of the whole torrid Zone will be 47° , which make $2937\frac{1}{2}$ miles [*milliaria*]. Again, since each polar circle recedes from its neighboring pole of the world by $23^{\circ} 30'$, just so great will be the width of either frigid Zone from the pole to the polar circle — that is, it will contain $1468\frac{3}{4}$ miles; but the whole width, taken along the diameter [across the pole], will comprise 47° , that is, $2937\frac{1}{2}$ miles. Finally, if from 90° you subtract the distance of the polar circle from the pole and of the Tropic from the Equator — namely 47° [their sum] — there will remain the width of either temperate Zone: 33° [recte: 43° ; the $2687\frac{1}{2}$ miles that follow require 43° at $62\frac{1}{2}$ miles per degree], to which correspond $2687\frac{1}{2}$ miles.

The length of each Zone we shall also obtain — at its beginning, its middle, and its end — if first, by the methods we taught above, we investigate the ratio of the Equator to any given parallel: that is, how many degrees or parts each parallel comprises of those of which the Equator contains 360. Thus, since the parallel passing through Rome contains about $267\frac{1}{2}$ such parts, the length of the temperate Zone along the parallel that runs through Rome will be $267^{\circ} 30'$ — that is, nearly 16719 miles. And so on.

Moreover, given any city whatever, we shall know with the greatest ease in which Zone it is situated, if we carefully inspect some cosmographical globe, or else a universal map of the whole world. For since, whether on the globe or on the map, there are depicted the parallel circles that divide the whole earth into the five aforesaid Zones, all cities that are found within the two Tropics will lie in the torrid Zone; those that lie between either Tropic and the neighboring polar circle, in a temperate Zone; and those, finally, between any polar circle and the neighboring pole of the world, will be set in a frigid Zone.

[Margin: In what Zone a given city lies, and how it may be known.]

But if there is no supply of a cosmographical globe or of a map, the Zone of any city must be inferred thus. Every city having a latitude less than $23^{\circ} 30'$ will lie in the torrid Zone, since each Tropic declines from the Equator by $23^{\circ} 30'$. But if the latitude of the city offered be precisely $23^{\circ} 30'$, and it lie toward the North, it will be placed exactly beneath the Tropic of Cancer (♋), which is the end of the torrid zone and the beginning of the northern temperate; but if it lie toward the South, it will be beneath the Tropic of Capricorn (♎), which is likewise the end of the torrid zone and the beginning of the southern temperate. Every city, however, having a latitude greater than $23^{\circ} 30'$ but less than $66^{\circ} 30'$ will have the northern temperate Zone, if it incline toward the North; for if it decline toward the South, it will be in the southern temperate Zone. But if the latitude of the city be precisely $66^{\circ} 30'$, it will be placed directly beneath the Arctic circle, or the Antarctic, according as it inclines toward the North or the South. Every city, finally, whose latitude exceeds $66^{\circ} 30'$ will have a frigid Zone, either the northern or the southern. And if the elevation be precisely 90° , that city will be set directly beneath one of the poles of the world. From all this it is plain that, if the latitude of any place be known, one can say in which Zone it is contained.

[Margin: Who the Periscii, Heteroscii, and Amphiscii are.]

This too seems by no means to be passed over: that among various writers, especially the Astronomers and Cosmographers, those peoples who inhabit either of the frigid zones are called *Periscii* [whose shadows go round about them], because their shadows, as though in a circle, in the manner of millstones, are wheeled around in the plane of the Horizon at certain times of the year. For the Sun sends its rays to those places in a manner somehow parallel to the plane of the Horizon (as is clear from the material sphere), and they are always borne round in a ring. Those, however, who occupy either of the temperate zones are called *Heteroscii* [whose shadows fall one way], because they cast their noon shadows toward only one pole of the world — the Northerners toward the Arctic pole, the Southerners toward the Antarctic; for the Sun never crosses their zeniths [vertices]. Those, finally, who inhabit the torrid zone are called *Amphiscii* [whose shadows fall both ways], because their noon shadows, at different times of the year, are cast now toward the Arctic pole, now toward the Antarctic — since the Sun sometimes passes beyond their zeniths, as is evident on the material sphere.

CHAPTER THREE

ON THE RISING AND SETTING OF THE SIGNS.

On the diversity of days and nights, and on the division of the climes.

[*Lemma — Sacrobosco*]

The rising and setting of the signs is understood in two ways, since it may be taken either as the poets take it or as the astronomers do. As the poets understand it, therefore, the rising and setting of the signs is threefold: namely the Cosmic, the Chronic, and the Heliac.

[Margin: The rising and setting of the stars according to the poets is threefold. What the Cosmic rising is.]

The **Cosmic** rising (or "mundane") occurs when a sign or star ascends above the horizon on the eastern side by day. And although on any artificial day six signs rise in this way, nevertheless it is the sign with which and in which the Sun rises in the morning that is said, par excellence [antonomastice], to rise cosmically. And in this sense it is called the proper, principal, and daily rising. An example of this rising is found in *Georgics* 1, where the sowing of beans and millet in spring is taught, with the Sun standing in Taurus, thus:

The white Bull opens the year with his gilded horns, etc.

[Margin: What the Cosmic setting is.]

The **Cosmic** setting, by contrast, is defined by opposition: namely, when the Sun rises together with some sign, the sign opposite to that one sets cosmically. This setting is spoken of in the *Georgics*, where the sowing of grain at the end of autumn is taught, with the Sun standing in Scorpio; for since Scorpio rises with the Sun, Taurus, the sign opposite to it — where the Pleiades are — sets, thus:

Before the Eastern Atlantides [the Pleiades] vanish from your sight,
before you commit the due seed to the furrows, and before you hasten to
entrust the year's hope to the reluctant earth.

[Margin: What the Chronic rising is.]

The **Chronic** rising (or "temporal") occurs when a sign or star emerges above the horizon on the eastern side chronically after the setting of the Sun — that is, by night. It is called "temporal" because the day of the mathematicians begins with

the setting of the Sun. We have an example of this rising in Ovid, *ex Ponto* Book 1, where, complaining of the length of his exile, he says fittingly:

The risen Pleiades has made four autumns.

For by "four autumns" he means that four years have passed since he was sent into exile. But Virgil meant that the Pleiades set in autumn; so the two seem to contradict each other. The explanation, however, is this: according to Virgil they set cosmically, whereas according to Ovid they rise chronically — which can well happen on the same day, though differently: for the cosmic setting is defined with respect to the morning, while the chronic rising is defined with respect to the evening.

[Margin: What the Chronic setting is.]

The **Chronic** setting is defined by opposition. Hence Lucan says in Book 4:

Then the short night was pressing upon the Thessalian arrows
[Sagittarius].

[Margin: What the Heliac rising is.]

The **Heliac** rising (or "solar") occurs when a sign or star can be seen through its separation from the Sun, which previously could not be seen because of the Sun's nearness. Ovid gives an example of this in *Fasti* Book 2, thus:

Now the light Water-bearer has sunk down with his slanting urn.

And Virgil in *Georgics* Book 1:

and let the Cnossian star of the blazing Crown withdraw.

This star, being next to Scorpio, was not visible while the Sun was in Scorpio.

[Margin: What the Heliac setting is.]

The **Heliac** setting occurs when the Sun approaches a sign and, by its presence and brightness, does not allow it to be seen. An example of this is in Virgil, *Georgics* Book 1:

... and the Dog sets, giving way to the star that faces it.

IOANNES DE SACRO BOSCO. [191]

COMMENTARY.

Having in Chapter 2 explained all ten circles out of which the material sphere is composed — and by which the celestial sphere is understood to be composed — the Author now, in this third chapter, treats of the various phenomena that arise from the motion of the *primum mobile* and of the Sun in the zodiac. This chapter can be divided into four parts. In the first he treats of the rising and setting of the stars; in the second, of the natural and artificial days; in the third he assigns certain properties that belong to human beings by reason of their different places of habitation on the earth; and in the fourth, finally, he discusses the climes of the world.

[Margin: The argument of the third chapter, and its division.]

Now the rising of any star is its elevation, or ascent, above the horizon — or else its appearance, when previously it was not seen because of the Sun's nearness. The setting, by contrast, is the star's sinking, or descent, below the horizon — or else the fact that it cannot be discerned because of the Sun's nearness. Thus every rising and setting of the stars is reckoned either by comparison with the horizon or by comparison with the Sun. From this it follows, as the Author says, that among the poets the rising and setting of the stars is threefold: namely the Cosmic and the Chronic, both of which are reckoned with respect to the horizon, and the Heliac, which is referred to the Sun.

[Margin: What the rising of a star is. What the setting of a star is. By what the rising and setting of the stars is reckoned.]

A sign or star is said, properly, to rise cosmically when it ascends above the horizon together with the Sun in the morning; but improperly, any star is said to rise cosmically that is raised above the horizon by day — in which sense the six signs of the zodiac are said to rise cosmically each day, even though, because of the Sun's brightness, they cannot be seen at all. It was the cosmic rising in the proper sense that Virgil understood in that verse of *Georgics* 1 which the Author quoted, namely:

The white Bull opens the year with his gilded horns, etc.

For he meant to signify the month of April, in which Taurus emerges above the horizon in the morning together with the Sun, since at that time the Sun stands in Taurus.

By the same reasoning, a sign or star is said, properly, to set cosmically when it descends below the horizon in the morning at sunrise; but improperly, any star is said to set cosmically that slips below the horizon by day. Thus when any sign or star rises cosmically, the sign or star diametrically opposite to it must necessarily set cosmically — indeed, so must all the stars that at that moment touch the horizon on the western side. Virgil speaks of this cosmic setting in *Georgics* 1 in these verses:

Before the Eastern Atlantides [the Pleiades] vanish from your sight,
before you commit the due seed to the furrows, and before you hasten to
entrust the year's hope to the reluctant earth.

He means the time of autumn, in which, with the Sun standing in Scorpio, the Pleiades — set in Taurus, the opposite sign — set in the morning; that is, cosmically.

[Margin — reading uncertain: the printed word for "autumn" here is hard to resolve, but the parallel Chronic paragraph ("in autumn, with the Sun standing in Scorpio") and the Sun-in-Scorpio context make "autumn" the reading. [?]]

A sign or star is said, properly, to rise **chronically** — or rather, as some prefer, *acronychally* — when it emerges above the horizon on the eastern side in the evening, as the Sun descends below the horizon; but improperly, whatever ascends above the horizon at night. In this sense, on any given night six signs can be said to rise chronically. Ovid writes of the chronic rising in *ex Ponto* Book 1, Elegy 9:

Since I have been deprived of you, thrust out to the Scythian shores, the
risen Pleiades has made four autumns.

For in the autumn season, namely with the Sun standing in Scorpio, the Pleiades rise in the evening about sunset; that is, chronically.

A star or sign is said, properly, to set **chronically** when it descends below the horizon together with the Sun; but improperly, any sign that is pressed below the horizon at night — in which sense the six signs of the zodiac are said to set chronically each night. And so, when any star or sign rises chronically, the opposite sign or star necessarily sets chronically. This must be understood of all the stars, too, that at that moment touch the horizon on the western side and therefore set with the Sun. Lucan, according to the Author, spoke of the chronic setting in Book 4:

... For the Sun held the Ledaean stars [Gemini], when the highest daylight is near to Cancer: then the short night was pressing upon the Thessalian arrows [Sagittarius].

For he meant to indicate the time near dawn, with the Sun standing near the end of Gemini; whence, shortly before sunrise, Sagittarius — which is opposite Gemini — must set, improperly, chronically. But perhaps Lucan understood no particular kind of setting in that passage, and only meant to signify that time before sunrise at which Sagittarius goes down. But Ovid writes of this setting in *Fasti* Book 2:

The Dolphin, engraved with stars, that you saw just now, will flee your sight on the following night.

For he speaks of the third of February, before which the Dolphin appeared above the horizon after sunset, but on the third day set chronically together with the Sun, since it stands in Aquarius, in which the Sun was then lingering. From this it is clear that one and the same sign in which the Sun stands rises cosmically in the morning and sets chronically in the evening; and likewise that the sign opposite the Sun rises chronically in the evening and sets cosmically in the morning. So it is no wonder that Virgil said the Pleiades set in autumn — namely, cosmically — while Ovid taught that they rise at the same time — namely, chronically — which is perfectly true. Hence there exist two little verses:

[Margin: A sign rising chronically sets cosmically, and the reverse.]

A sign sets cosmically that rises chronically. A sign sets chronically that rises cosmically.

COMMENTARY ON CHAPTER 3 OF THE SPHERE [192]

This, however, is not true — in an oblique sphere — of stars set outside the ecliptic. For no such star that rises together with the Sun can set with the same Sun, nor can one that sets with it rise with the same Sun. Rather, a star that is more northerly than the point of the ecliptic with which it rises together sets later than that point of the ecliptic; while the point of the ecliptic with which it sets together it rises before. The contrary is understood of a star more southerly than a given point of the ecliptic. From this it follows that a star more northerly or more southerly than a given point of the ecliptic, if it rises cosmically with that

point, cannot set chronically with the same point; and if it sets chronically with it, cannot rise cosmically with the same, or the reverse. Stars situated on the ecliptic, however, in any oblique horizon, and all stars in the right sphere, rise and set with the same points of the ecliptic. Therefore those that rise cosmically set chronically, and those that rise chronically set cosmically, and conversely — as is evident on the material sphere or the astronomical globe.

A star is said to rise **heliacally** when it brings itself into view, after having previously lain hidden, covered by the rays of the nearby Sun. Virgil speaks of this rising in *Georgics* 1:

Now the light Water-bearer has sunk down with his slanting urn: next,
the Fish receives the ethereal horses.

For in February, when the Sun was standing in Aquarius, it hid the Water-bearer by its excessive brightness; but toward the end of February, as the Sun entered Pisces, the Water-bearer appeared in the morning before sunrise, and thus rose heliacally. Virgil understood this same heliacal rising when he wrote in *Georgics* 1 about the Cnossian star — namely, the star of the Northern Crown:

Before the Eastern Atlantides [the Pleiades] vanish from your sight, and
before the Cnossian star of the blazing Crown withdraws, before you
commit the due seed to the furrows, etc.

For when the Pleiades set cosmically — namely, in autumn — the Northern Crown rises, which formerly stood near the end of Libra, rising heliacally in the morning before sunrise while the Sun was in Scorpio.

A star is said to set **heliacally** when it is dimmed by the excessive brightness of the Sun, so that, having previously been visible, it can no longer be discerned. Virgil speaks of this setting in the last of these two verses:

The white Bull opens the year with his gilded horns, and the Dog sets,
giving way to the star that faces it.

For since Canis Maior formerly stood in Gemini, it set heliacally when the Pleiades set cosmically, with the Sun standing right in Taurus, near the Pleiades. But how far any star must precede the Sun, or follow it, in order to rise or set heliacally cannot be determined with certainty, since the stars are neither all of the same magnitude nor all at the same latitude from the ecliptic. It is certain, however, that smaller stars, and those closer to the ecliptic, rise heliacally later and set heliacally sooner than larger stars more distant from the ecliptic.

[Margin: In what part of the sky a planet or star rises and sets heliacally.]

Now since the motion of the Sun from setting to rising is faster than the motion of the fixed stars, the result is that the fixed stars, when they emerge from the Sun's rays, are left free of the Sun toward the western regions of the sky, and rise heliacally in the morning near the horizon on the eastern side, before sunrise. For it is then that they first begin to appear, whereas before they had lain hidden because of the nearness of the Sun, which has now withdrawn from them toward the east. For the same reason those same stars must set heliacally in the evening near the horizon on the western side, after the Sun has descended below the horizon. For whereas before they had always appeared after sunset, they now first begin to hide themselves because of the nearness of the Sun, which is approaching them. Exactly the same must be said of Saturn, Jupiter, and Mars, because their proper motions are slower than the Sun's. But the contrary must be understood of the Moon. For since it advances by its proper motion faster than the Sun, the result is that it does not withdraw from the Sun (as the fixed stars do), but rather is itself carried away from the Sun toward the east. Hence the Moon will rise heliacally in the evening on the western side after sunset, as we see happen after the new moons, because after any new moon the Moon at once withdraws from the Sun toward the east. And it sets heliacally on the eastern side in the morning before sunrise, as we observe before the new moons, because it is always approaching the Sun toward the east. This is the reason why we see the Moon gradually wax after the new moons and wane before them. Finally, Venus and Mercury, since they sometimes go ahead of the Sun and sometimes follow it, will sometimes rise heliacally toward the east and set toward the west; but sometimes rise heliacally toward the west and set toward the east. But more must be said of all this in the *Theories of the Planets*. Hence it comes about that Venus is now called Lucifer, namely when it rises in the morning before the Sun, and now Hesperus, namely when it is seen after sunset toward the west.

[Margin: When Venus is called Lucifer, and when Hesperus.]

At what time of year any star, in this present age, rises cosmically, chronically, or heliacally — or likewise sets — is beautifully shown by the celestial globe or by any astrolabe. For, the globe being set at its proper elevation, let any star be placed on the horizon on the eastern side, and let the degree of the ecliptic touching the horizon in the east be noted: for when the Sun reaches that degree of the ecliptic, the said star will rise cosmically; and when the Sun occupies the opposite degree of the ecliptic, the same star will rise chronically. Likewise, the star being placed on the horizon on the western side, let the degree of the ecliptic

touching the horizon in the west be noted. For when the Sun possesses that degree of the ecliptic, the same star will set chronically; and when the Sun stands at the opposite degree of the ecliptic, the same star will set cosmically. The heliacal rising and setting, however, will be more or less recognized once it is known at what degree of the ecliptic any given star is situated.

[Margin: How to know when any star rises cosmically, chronically, or heliacally.]

Astronomers divide the rising and setting of the stars into the **True** and the **Apparent**. The true rising and setting is when a star truly ascends above the horizon or descends below it. And this is twofold: namely the *Morning* rising, when, at sunrise, some star rises or sets — which the poets call the cosmic rising and setting — and the *Evening* rising, when, at sunset, some star rises or sets, which the poets call the chronic rising and setting. The apparent rising and setting, by contrast, is what the poets call the heliac; and this too they distinguish into morning and evening, according as the star, freed from the Sun's rays, begins to appear in the morning or in the evening, as has been said.

[Margin: The true and apparent rising and setting; likewise, what the Morning and Evening kinds are.]

Ptolemy [Ptolemy] (in the *Almagest* [Dict.] VIII.4) calls the risings of the stars, and their configurations relative to the Sun, "appearances," and he lists nine kinds of Difference. Since each of these is in turn manifold, the total number of appearances comes to twenty-four. On this subject, however, consult Ptolemy at the passage cited, and Johannes Regiomontanus [Regiomontanus] in his *Epitome*, Book VIII, chapter 5. It would take too long to review every one of these appearances here.

Moreover, the knowledge of the poetic rising and setting is of the greatest help for understanding the works of the ancient poets and historians. Very often, in fact, they try to fix some definite time by the rising of a particular star — as is clear from the examples already cited.

[Margin: In what sense Ptolemy calls the risings and settings of the stars.]

[Margin: What use the poetic rising and setting serves.]

ON THE RISING AND SETTING OF THE SIGNS ACCORDING TO the astrologers, that is, on the ascensions and descensions of the signs, both right and oblique.

There follows the treatment of the rising and setting of the signs as the astronomers understand them, and first in the right sphere [sphaera recta].

COMMENTARY

Now that the author has explained the rising and setting of the heavenly bodies as the poets understand them, he takes up the rising and setting of the signs according to the astronomers. This astronomical rising and setting the astronomers are accustomed to call the ascensions and descensions of the signs, and the present treatment concerns those ascensions and descensions. It carries very many important applications, for the greatest part of the doctrine of the primum mobile seems to depend on it.

The author explains three things in this part. First, what rising and setting are according to the astronomers, and how many kinds there are. Second, how the signs rise and set in the right sphere. Third, how the signs behave with respect to their astronomical rising and setting in any oblique sphere whatever. But before all else we must briefly explain the difference between rising and setting as the poets understand it and as the astronomers do; and it is this. In the rising and setting of the signs the poets observe the *quality* of the time — namely whether some sign rises in spring, or in summer, or in autumn, or in winter; likewise whether it does so in the morning or in the evening. The astronomers, by contrast, consider the *quantity* of time: how long, that is, this or that sign takes to rise or set at this or that obliquity of the sphere, whether this happens in spring or in summer and so forth, and whether by day or by night. Hence among the astronomers the rising and setting is not divided into cosmic (or morning) and chronic (or temporal), as the poets divide it, but into right and oblique, as will shortly be explained.

[Margin: What the difference between rising and setting is with respect to the poets and with respect to the astronomers.]

It must be understood that in the right sphere as much as in the oblique the equinoctial circle [Aequinoctialis circulus] always ascends uniformly — that is, in

equal times equal arcs ascend. For the motion of the heavens is uniform, and the angle which the equinoctial circle makes with the horizon does not vary at different hours.

[Margin: The equator ascends uniformly above every horizon.]

COMMENTARY

Before he explains what rising or setting is according to the astronomers, he first demonstrates two conclusions, the first of which is this. The equinoctial circle (that is, the equator [Æquator]) is raised uniformly above the horizon, whether right or oblique, in respect of all its parts, so that in equal times equal arcs of the equator ascend above the horizon. He proves this conclusion in two ways.

First, because the diurnal motion of the heavens is uniform and regular in every horizon. It is never carried now more swiftly, now more slowly. Since, then, the equator is the measure and rule of the primary motion, and revolves about the same poles about which the whole heaven turns — namely the poles of the world — it must in any sphere emerge above the horizon uniformly in respect of all its parts.

Next, because the equator perpetually makes the same angles with the horizon — right angles with the right horizon, oblique angles with the oblique. From this it follows that it is raised above any horizon uniformly in respect of all its parts. The clearest observed phenomena of the astronomers testify to the same thing. For it has been established that in any sphere whatever fifteen degrees of the equator ascend above the horizon in each hour, and as many descend below it, while in the space of four minutes of one hour one degree of the equator is raised and one lowered [deprimi], and so on. This would by no means happen if the equator did not ascend above the horizon regularly and uniformly.

The parts of the zodiac, however, do not necessarily have equal ascensions in either sphere; for the more nearly upright some part of the zodiac rises, the more time is spent in its rising. The proof of this is that six signs rise on a long, or on a short, artificial day, and likewise at night.

COMMENTARY

This is the second conclusion: the zodiac, in the right sphere as much as in the oblique, does not ascend above the horizon uniformly in respect of all its parts. He seems to confirm this by the following reasoning. Since the zodiac is carried by the diurnal motion about poles not its own, from which it is at one place

farther off and at another nearer, it comes about that some of its parts make more oblique angles with any given horizon and others less oblique. Therefore that part which makes more nearly right angles with the horizon — and which for that reason rises more nearly upright — will be raised above the horizon with a slower motion, and will require more time in its rising than the part which makes less nearly right angles with the horizon, as experience teaches in any material sphere; for the more upright some arc rises, the more its parts ascend in succession. He confirms the same conclusion by plain experiment: namely that on any day or night, artificial, be it the longest or the shortest, exactly six signs of the zodiac ascend above the horizon and [descend] below it. below it — so that on any given day one half of the zodiac rises. For since the zodiac and any horizon whatever cut each other into two equal parts (because they are great circles of the sphere), it comes about that the half of the zodiac contained between the Sun placed in the east and the point opposite to it rises during the day, passing through the middle of the night. This is plainly seen on the instruments. For this reason the zodiac will not rise above the horizon uniformly in respect of all its parts, since it is in unequal times — namely in unequal days and nights — that equal arcs always ascend, that is, the halves of the zodiac. And if any half of the zodiac ascended uniformly in respect of all its parts, all days and nights would be equal to one another, which is contrary to experience. The same can be proved of the remaining parts smaller than a semicircle from the doctrine of spherical triangles.

[Margin: What rising and setting are according to the astronomers.]

It must therefore be noted that the rising or setting of any sign is nothing other than the rising of that part of the equinoctial which rises together with that sign as it rises — that is, as it ascends above the horizon; or the setting of that part of the equinoctial which sets together with that sign as it sets — that is, as it moves toward its setting below the horizon.

COMMENTARY

He now sets out what the rising and setting of any sign, or arc of the zodiac, is according to the astronomers. He says that for some sign to rise is nothing other than for that arc of the equator which ascends above the horizon together with that sign to rise; and that for some sign to set is nothing other than for that arc of the equator which descends below the horizon together with that sign to set. Hence the rising of a sign, or of any arc of the zodiac, is defined as the arc of the equator which rises together with it; and the setting of a sign, or of any arc of the

zodiac, is said to be the arc of the equator which sinks below the horizon together with the sign or arc.

For instance, at Rome $17^{\circ} 21'$ of the equator rise together with the whole arc of Aries; therefore the arc of the equator containing $17^{\circ} 21'$ is called the rising of Aries at Rome. By the same reasoning, since at Rome $38^{\circ} 27'$ descend below the horizon together with the sign of Aries, the arc of the equator comprising $38^{\circ} 27'$ will be called the setting of the sign of Aries — and so of the rest. Hence it has come about that the rising of a sign, or of an arc of the zodiac, is called by the astronomers Ascension, and the setting Descension: because in the rising or setting of any arc they consider precisely the proportion of the equator which ascends or descends together with that arc.

[Margin: Why the astronomers define rising and setting by means of the equator.]

The astronomers, moreover, define the rising and setting of any arc or sign whatever by the arc of the equator that ascends with it or descends with it; for when they had observed that the zodiac is raised unequally above the horizon, and descends below it, by the motion of the primum mobile — since it does not possess the same poles as the primum mobile — whereas the equator rises and sets uniformly in respect of all its parts, because it holds the same poles as the primum mobile (as was shown in the two preceding conclusions), it was necessary for them to learn, by means of something uniform and regular, the time which each arc of the zodiac takes up in its rising and setting. This was most conveniently done by the aid of the equinoctial circle. For since in each hour 15° of the equator are raised in any horizon, if 45° of the equator, for example, are raised above some horizon together with some arc of the zodiac, it is most certainly inferred that such an arc rises entirely in three whole hours, and so forth.

[Margin: What the ascension and descension of any star, or of any point of the ecliptic, is.]

Not only are the ascensions and descensions of the arcs of the zodiac defined by the arcs of the equator ascending and descending with them at the same time; the ascension and descension of any point of the ecliptic, and of any star as well, are also defined. For the ascension of any star, or of any point of the ecliptic, is the arc of the equator reckoned from the vernal section — that is, from the beginning of Aries [Υ , Aries] — following the order of the signs as far as the horizon, while the star or point of the ecliptic is rising. For example, since at Rome, with the third degree of Leo [♌ , Leo] placed in the east, the said arc of the equator

comprises $106^{\circ} 40'$, this arc of the equator is called the ascension of the third degree of Leo, because it ascends together with that degree. The descension, on the other hand, of any star or point of the ecliptic is the arc of the equator numbered from the vernal section — that is, from the beginning of Aries — following the order of the signs as far as the horizon, while the star or point of the ecliptic is setting. For example, since at Rome, with the third degree of Leo placed in the west, the said arc of the equator contains $143^{\circ} 57'$, this arc is called the descension of the third degree of Leo, because it descends together with it — and so of the rest. Thus the ascension or descension of any point of the ecliptic, or of any star, is the same as the ascension or descension of the arc of the ecliptic which is reckoned from the beginning of Aries, following the succession of the signs, as far as the horizon, with the star or degree of the ecliptic placed exactly on the horizon — on the side of the east, if it is a question of ascension, and on the side of the west, if the descension is in view.

A sign, moreover, is said to rise upright [recte] when the greater part of the equinoctial rises with it, and obliquely [oblique] when the lesser part does. The same is to be understood likewise of setting.

[Margin: What it is for a sign to rise or set upright, or obliquely.]

COMMENTARY

Since it has been stated that the equator is raised uniformly above the horizon in respect of all its parts, but not the zodiac, it comes about that sometimes a greater arc of the equator ascends with one arc of the ecliptic (or zodiac), and sometimes a lesser. He now teaches that a sign, or arc of the ecliptic, with which a greater arc of the equator rises is said to rise upright, whereas the one with which a lesser arc of the equator ascends is said to rise obliquely. In the same way a sign, or arc of the ecliptic, with which a greater arc of the equator moves below the horizon is said to set upright; and the one with which a lesser arc does so, obliquely.

Example. At Rome, with the arc of Libra, which comprises 30° , there rises an arc of the equator containing $38^{\circ} 27'$; therefore the sign of Libra [♎ , Libra] will be said to rise upright. But with the arc of Aries there ascend $17^{\circ} 21'$ of the equator, so the sign of Aries will be said to rise obliquely. Likewise, since with the sign of Aries there descend $38^{\circ} 27'$, Aries will be said to set upright; but Libra will be said to set obliquely, because only $17^{\circ} 21'$ of the equator descend below the horizon together with it — and so forth.

[Margin: Why rising and setting are called upright or oblique.]

The former rising and setting is called upright when, that is, more degrees of the equator rise or set together, because then the arc of the zodiac that rises or descends makes more nearly right angles with the horizon. The latter rising and setting, when fewer degrees of the equator ascend or descend at the same time, is called oblique, because that arc of the zodiac, as it emerges or sinks, makes more oblique angles with the horizon.

All of which is clearly seen in the material sphere. Hence the arc of the zodiac with which an equal arc of the equator rises or sets can be said to rise and set in an intermediate manner — of which kind are the four quadrants of the zodiac in the right sphere. For they rise, each one, together with each of the quadrants of the equator, as we shall say presently.

Ptolemy, however, whom all the astronomers follow, calls "right ascensions" all those that occur in the right sphere, and "oblique" those that are had in the oblique sphere, whether a greater arc of the equator, or a lesser, or an equal one rises with the sign. So too the astronomers name them in the tables of ascensions. Hence the right ascension of any arc, or degree of the ecliptic, is taken by them to be the ascension it has in the right sphere, whether a greater arc rises with it or a lesser; while the oblique ascension of any arc is understood to be that which it has in the oblique sphere, with however great an arc of the equator it rises. The same is to be said of right and oblique descensions.

[Margin: What right and oblique ascensions are according to Ptolemy and the astronomers.]

ON THE RISING AND SETTING OF THE SIGNS in the right sphere.

And it must be known that in the right sphere the quadrant of the zodiac beginning from the four points — the two solstitial, that is, and the two equinoctial — is made equal to its ascensions; that is, the amount of time a quadrant of the zodiac takes in its rising is the same amount in which the quadrant of the equinoctial bordering upon it rises. Nevertheless the parts of those quadrants vary, and do not have equal ascensions, as will now become clear.

[Margin: The rising of the arcs of the zodiac in the right sphere.]

COMMENTARY

Here he lays down two rules for knowing the rising and setting of the signs in the right sphere. The first is this: the four quadrants of the zodiac, which take their beginning from the four cardinal points, are in the right sphere made equal to their ascensions — that is, they rise precisely together with the corresponding quadrants of the equator, so that each of them takes 6 whole hours in its rising above the horizon, just as each quadrant of the equator too emerges above the horizon in 6 hours. The *parts* of the said quadrants, however, are not equal to their ascensions; that is, together with their parts there rise now greater arcs of the equator, now lesser, so that 15° , for example, sometimes require more time than an hour to rise above the horizon, and sometimes less. For the first 15° of Aries ascend with $13^\circ 48'$ of the equator — that is, they require 55 minutes and 12 seconds of one hour to emerge above the horizon; but the last 15° of Gemini ascend with $16^\circ 17'$ of the equator — that is, they demand 1 hour, 5 minutes, and 8 seconds to ascend above the horizon.

The first part of this rule can easily be proved. Since each colure [Colurus], as it passes through the poles of the world, is joined with the right horizon twice a day, the said quadrants cannot touch the horizon with their outermost points without the other colure — passing through the same points by its extremities — touching it at the very same moment of time and being joined with the horizon. Therefore, after the whole quadrant of the zodiac has emerged above the horizon, the corresponding quadrant of the equator must likewise have wholly ascended above the horizon.

The latter part of the same rule can be shown from Proposition 16 of Book I of Menelaus's [Menelaus] *Spherics*, or from Proposition 11 of our own *Spherical Triangles*; for any part of the ecliptic, apart from the said quadrants, makes with the right horizon now an obtuse angle, now an acute one — as is established from Theodosius [Theodosius] — since the horizon does not pass through its poles. Therefore, since by the aforesaid propositions the greater side is opposite the greater angle in a spherical triangle, and the lesser side the lesser angle, it is clear that the parts of the quadrants having their beginning at the equinoctial points are not made equal to their ascensions.

That the parts of the other quadrants too — those which have their beginning at the solstitial points — are not made equal to their ascensions can be demonstrated as follows. Since, as will be proved in the same way, the parts of the zodiac beginning at the equinoctial points which are greater than a quadrant are unequal to their ascensions, then if equal quadrants are subtracted — one of

the zodiac from an arc of the zodiac, and the other of the equator from the arc of the equator that rises with the arc of the zodiac — the remaining arcs will still be unequal, namely the arc of the zodiac and its ascension. But all this can easily be observed by anyone in the material sphere, and will be made plain from the table of right ascensions.

For there is this rule: any two equal arcs of the zodiac, equally distant from any one of the four points already mentioned, have equal ascensions.

COMMENTARY

The second rule is this: any two equal arcs of the zodiac, equally distant from any one of the four cardinal points, have in the right sphere equal ascensions in relation to each other. For example, the sign of Gemini [♊, Gemini] and the sign of Cancer [♋, Cancer], because they are equal arcs and equally removed from the summer solstitial point, have one and the same ascension; for with either sign there ascend 32° 12' of the equator. The same reasoning holds for the sign of Taurus [♉, Taurus] and Leo; likewise for the sign of Aries and Virgo [♍, Virgo], and so of the other equal arcs, provided that they are equally removed from any one of the said four points, as will be evident from the table of right ascensions.

This rule can be confirmed from spherical triangles: because arcs of the ecliptic of this kind, since they decline equally from the equator at their outermost points (as was said above), make equal angles with the horizon, whence equal arcs of the equator must necessarily correspond to them, and therefore they will have equal ascensions in relation to each other. But this we have demonstrated in Book I of our *Astrolabe*, Lemma 49, Number 6.

And from this it follows that opposite signs have equal ascensions. And this is what Lucan [Lucan] says in Book 9, speaking of Cato's [Cato] march into Libya toward the equinoctial line.

[Margin: Which arcs of the zodiac have equal ascensions in the right sphere.]

(Continuing the lemma from the previous folio: the closing verses of Lucan's description, with the zodiac names set in capitals.)

It has been detected that this is the place where the circle of the high Solstice strikes the mid-belt of the signs. The signs do not move aslant here, nor does SCORPIO rise more steeply than TAURUS; nor does ARIES grant its own times to LIBRA; nor does ASTRAEA [Virgo] bid the slow PISCES to descend; CHIRON [Sagittarius] is matched with GEMINI, and the same as

burning CANCER is watery CAPRICORN [Aegoceros]; nor is LEO raised higher than the URN [Aquarius].

Here Lucan [Lucan, *Pharsalia* IX] says that, for those living beneath the Equator, opposite signs have equal risings and settings. The opposition of the signs is contained in this verse:

Est Lib. Ari. Scor. Taur. Sa. Gemi. Capri. Cancr. A. Le. Pis. Vir.

(A mnemonic pairing the opposite signs: Libra–Aries, Scorpio–Taurus, Sagittarius–Gemini, Capricorn–Cancer, Aquarius–Leo, Pisces–Virgo.)

COMMENTARY

Clavius here gathers, from the second rule, that opposite signs in the right sphere have equal ascensions among themselves. He confirms this on the authority of Lucan, Book 9, where Lucan describes Cato's arrival beneath the equinoctial circle — which Lucan calls "the circle of the high Solstice" — saying that all opposite signs have equal ascensions and descensions, so that no sign rises or sets more directly or more obliquely than its opposite, as happens in the oblique sphere (as will soon be made clear). For Lucan did not mean, in that passage, that all the signs in the right sphere rise directly and none obliquely — as Sulpitius and Omnibonus, the interpreters of Lucan, wrongly explain. That is false. He meant only that none rises more directly, or more obliquely, than its own opposite, although some there rise directly and some obliquely, as is evident from the table of right ascensions, and as we showed in the first book of the *Astrolabe*, Lemma 49, Number 6.

[Margin: The passage of Lucan corrected.]

But this passage of Lucan is not free of error. For Lucan does not mean that Cato reached the Equator — as the verses cited seem to indicate — but rather the temple of Jupiter Ammon, which Lucan supposed to be situated near the Tropic of Cancer. To make this plainer, some verses of Lucan must be brought forward, as they are found in the common copies, but in a disordered arrangement; then the same verses are to be restored to their proper order. So, then, as it now reads, Lucan describes the nature of that place:

*Hic quoque nil obstat Phœbo, cum cardine summo Stat librata dies:
truncum vix protegit arbor: Tam brevis in medium radijs compellitur
vmbra. Deprehensum est, hunc esse locum, qua circulus alti Solstitij
medium signorum percutit orbem. Non obliqua meant, nec Tauro Scorpius*

*exit Rectior: aut Aries donat sua tempora Libræ: Aut Astræa iubet lentos
descendere Pisces: Par Geminis Chiron: & idem quod Carcinus ardens,
Humidus Aegoceros; nec plus Leo tollitur Vrna. At tibi, quæcunque es
Lybico gens igne dirempta, In Noton vmbra cadit, quæ nobis exit in Arcton.
Te segnis Cynosura subit, tu sicca profundo Mergi Plaustra putas,
nullumque in vertice summo Sidus habes immune maris, procul axis
vterque est, Et fuga signorum medio rapit omnia cœlo.*

(In English:) Here too nothing stands in Phoebus's way when the day rests balanced at the topmost point of the sky: the tree scarcely shades its own trunk, so short is the shadow driven inward by the rays. It has been found that this is the place where the circle of the high Solstice strikes the mid-belt of the signs. The signs do not move aslant, nor does Scorpio rise more steeply than Taurus; nor does Aries grant its own times to Libra; nor does Astraea [Virgo] bid the slow Pisces to descend; Chiron [Sagittarius] is matched with Gemini, and watery Capricorn [Aegoceros] is the same as burning Cancer; nor is Leo raised higher than the Urn [Aquarius]. But as for you, whatever people you are, sundered by the Libyan zone of fire, the shadow falls toward the South Wind [Notus], which for us goes out toward the Bear [Arctos]. The sluggish Cynosura [Little Bear] sinks beneath you; you think the Wain [Ursa Major], dry with us, is plunged in the deep; you have no star at the very zenith exempt from the sea; both poles lie far off, and the flight of the signs sweeps everything away through the middle of heaven.

Now if these verses had been composed by Lucan in this order, then without doubt he would have understood "the circle of the high Solstice" to mean the Equator, since what follows — about the rising and setting of the signs — can suit no other region than the one lying directly beneath the Equator. But why he should afterward have added,

At tibi quæcunque es Lybico gens igne dirempta, &c.

I do not understand, since these lines too suit the right sphere. It is plain, then, that by "the circle of the high Solstice" he understood the Tropic of Cancer — which strikes the mid-belt of the signs, that is, the Ecliptic, that is, merely touches it. Then, when he says *At tibi quæcunque es, &c.*, he signifies the right sphere, which lies beneath the Equator, where all the stars rise and set, and where opposite signs likewise have the same ascension and descension. Hence the verses are to be arranged thus, as Petrus Iaconus Hispanus — a man most skilled in the subtlety of all the arts — observed. So it reads from that passage: *At tibi*

quæcunque es, &c. describes the right sphere, since he had earlier described the oblique sphere beneath the Tropic of Cancer, as is evident.

(The verses restored to their proper order:)

*Hic quoque nil obstat Phœbo, cum cardine summo Stat librata dies:
truncum vix protegit arbor: Tam brevis in medium radijs compellitur
vmbra. Depresum est, hunc esse locum, qua circulus alti Solstitij medium
signorum percutit orbem. At tibi, quæcunque es Lybico gens igne dirempta,
In Noton vmbra cadit, quæ nobis exit in Arcton. Te segnis Cynosura subit,
tu sicca profundo Mergi Plaustra putas, nullumque in vertice summo Sidus
habes immune maris, procul axis vterque est, Et fuga signorum medio rapit
omnia cœlo. Non obliqua meant, nec Tauro Scorpius exit Rectior: aut Aries
donat sua tempora Libræ:*

*Aut Astræa iubet lentos descendere Pisces: Par Geminis Chiron: & idem
quod Carcinus ardens Humidus Aegoceros; nec plus Leo tollitur Vrna.*

*(...nor does Astraea [Virgo] bid the slow Pisces to descend; Chiron [Sagittarius] is
matched with Gemini, and watery Capricorn [Aegoceros] is the same as burning
Cancer; nor is Leo raised higher than the Urn [Aquarius].)*

Now that a further rule follows from the second: that opposite signs in the right sphere have equal ascensions and descensions — this too can be proved by the following argument.

Any two opposite signs have a certain agreement with some third sign, such that this third sign and each of the two opposites are equally distant either from one of the two solstitial points, or from one of the two equinoctial points. Therefore each of the two will have the same ascension as that third sign, by the second rule; and consequently the opposite signs themselves will have equal ascensions among themselves. For example, ♈ [Aries] and ♎ [Libra] are opposite signs; and because ♈ [Aries] has the same ascension as ♍ [Virgo] — since these signs are equally distant from the summer solstice — and likewise ♎ [Libra] also has the same ascension as ♍ [Virgo], because these signs recede equally from the autumnal equinox — therefore ♈ [Aries] and ♎ [Libra] will obtain the same ascension. In the same way ♉ [Taurus] and ♏ [Scorpio], opposite signs, agree with ♌ [Leo] in ascension; ♊ [Gemini] and ♐ [Sagittarius] with ♋ [Cancer]; ♋ [Cancer] and ♑ [Capricorn] with ♊ [Gemini]; ♌ [Leo] and ♒ [Aquarius] with ♉ [Taurus]; ♍ [Virgo] and ♒ [Pisces] with ♈ [Aries] — as is evident from the material sphere. Therefore all opposite signs obtain equal ascensions in the right sphere.

The same can also be demonstrated from the fact that opposite signs make equal angles with the Horizon — one toward the region of the Arctic pole, the other toward the region of the Antarctic pole. For from this, by the doctrine of spherical triangles, it follows that the arcs of the Equator corresponding to them are equal. This the table of right ascensions itself will make plain.

And it is to be noted that such an argument does not hold: These two arcs are equal, and they begin to rise together, and the greater part of the one always rises before the greater part of the other; therefore that arc will finish rising sooner whose greater part was always rising. An instance of this argument is manifest in the parts of the aforesaid quadrants. For if one takes the quarter of the Zodiac that runs from the beginning of ♈ [Aries] up to the end of ♊ [Gemini], the greater part always rises from the quarter of the Zodiac than from the quarter of the Equator adjoining it, and yet those two quarters finish rising together. Understand the same of the quarter of the Zodiac that runs from the beginning of ♎ [Libra] up to the end of ♐ [Sagittarius]. LIKEWISE, if one takes the quarter of the Zodiac that runs from the beginning of ♋ [Cancer] up to the end of ♍ [Virgo], the greater part rises from the quarter of the Equator than from the quarter of the Zodiac adjoining it, and yet those two quarters finish rising together. Understand the same of the quarter of the Zodiac that runs from the first point of ♑ [Capricorn] up to the end of ♒ [Pisces].

COMMENTARY

[Margin: The solution of a certain doubt.]

Here the Author resolves, from what he has said, a certain doubt that might cause someone trouble: namely, that the following argument does not hold — There are two arcs in the sphere altogether equal to each other, which begin to rise above the Horizon together at the same instant of time, and the greater part of the one has always risen than of the other; therefore that whole arc will ascend above the Horizon sooner, of which the greater part has always finished rising, than the arc of which the smaller portion had risen. For this argument is resolved by what was said in the first rule. For any Quadrant of the Zodiac, taking its beginning from one of the four cardinal points (as we said), rises whole together with the corresponding quadrant of the Equator, and yet, before both Quadrants finish rising, the greater part of one of them has always risen than of the other. For the greater part of any quadrant of the Zodiac beginning from either equinox always ascends above the Horizon than of the quadrant of the Equator, the start being made in every case from the arcs rising from the equinoctial point, because such

an arc of the Zodiac always makes a smaller angle with the Horizon toward the region of the Equator than the Equator does. Hence, by Proposition 10 of Book 1 of Menelaus, or by Proposition 11 of our own [book] on spherical triangles, the smaller arc of the Equator will correspond, until at the end of the Quadrant both angles become right, and consequently the arcs become equal, by Proposition 4 of the same Book 1 of Menelaus, or by Proposition 5 of our own [book] on spherical triangles. In like manner, the greater part of any Quadrant of the Equator taking its beginning from the Colure of the Solstices always emerges above the Horizon than of the corresponding Quadrant of the Zodiac, as is most clearly deduced from spherical triangles, and will plainly appear from the table of right ascensions; namely because such an arc of the Equator always makes a smaller angle with the Horizon than the Zodiac does, and so on — as we have demonstrated in Book 1 of the *Astrolabe*, Lemma 49, Number 7.

But that the whole Quadrants finish rising together, even though the greater part of the one is always risen than of the other, arises from the fact that the greater part of the one does not always rise in the same proportion as the other, but that proportion gradually decreases, as is manifest from the table of right ascensions, so that at the end the whole inequality of the ascensions is already compensated. That this can indeed happen — apart from the example of the Quadrants of the Zodiac and the Equator adduced — can be grasped from this one example.

Let there be two moving bodies A and B, which move through one and the same space, beginning at the same instant of time, yet on this condition: that A always moves regularly and uniformly, while B, up to the middle of the space, is carried faster or slower, and from the middle to the end slower or faster in exactly the same proportion by which it earlier outstripped moving body A, or was outstripped by it. This being granted, it is certain that both bodies will reach the end of the space at the same time, since by the said proportion the whole inequality is compensated; nevertheless, before the end of the whole space, A will always be ahead of, or behind, B. Otherwise they would not both complete the whole space together, as is evident. Thus, then, one must understand the Quadrants of the Zodiac and Equator to move: they rise wholly indeed at the same time, but their parts in unequal times. For the quadrants of the Zodiac beginning from the Colure of the Equinoxes rise faster near the beginning, slower near the end; but the Quadrants begun from the Colure of the Solstices rise slower at the beginning than at the end.

[Margin: The ascension of any arc of the Zodiac in the right sphere is equal to the descension of the same in the same right sphere, and to the culmination [mediation of the heaven] both in the right sphere and in the oblique.]

Furthermore, in the right sphere the ascension of any sign, or arc of the Zodiac, is equal to its descension; since the descension in one right Horizon is the ascension in another right Horizon (which the Antipodes of those living in the former Horizon obviously have) — and conversely. Moreover it is certain that the ascension of one and the same arc of the Zodiac is the same in every right Horizon, on account of the equal inclination of the Zodiac. In the same manner the ascen-

—sion of any sign is equal to the culmination [mediation of the heaven] of the same, that is: however long some sign takes to rise above the right Horizon, in exactly that much time it also crosses the Meridian of any place whatever — because any Meridian can be called a right Horizon, since it passes through the poles of the world. Therefore everything that has been said about the ascensions of the signs, or of the arcs of the Zodiac, in the right sphere is to be understood also of their descensions in the same right sphere, and likewise of the culminations of the heaven both in the right sphere and in the oblique.

HOW THE RIGHT ASCENSION OF ANY ARC OF THE ZODIAC BEGUN FROM THE VERNAL POINT IS TO BE COMPUTED

[Margin: By what method the right ascension of any arc of the Ecliptic is to be investigated by means of sines.]

Johannes Regiomontanus [Regiomontanus] demonstrated it in the last proposition of Book 1 of the *Epitome*, and Geber [Jābir ibn Aflāḥ] in his astronomical work, and we too demonstrated it in the scholium to Proposition 9 of Book 2 of the *Gnomonics*: that the proportion of the sine of the complement of the declination of the point terminating an arc of the Ecliptic begun from either equinox, to the sine of the complement of the given arc of the Ecliptic, is such as the sine of the whole [radius] is to the sine of the complement of the right ascension. Therefore, if — according to the rule of proportions — the whole sine is multiplied into the sine of the complement of the proposed arc, and the resulting number is divided by the sine of the complement of the declination of the last point of the arc, the sine of the complement of the right ascension will be found, and thus the ascension will be known. This being so, the right ascensions of all arcs of the Ecliptic beginning from the Vernal point will be found by this method.

[Margin: When the arc of the Ecliptic is smaller than a quadrant.]

If the proposed arc is smaller than a quadrant, the rule just set out will yield its right ascension. EXAMPLE. Let the right ascension of the 20th degree of ♊ [Gemini] be sought — that is, of the arc containing 80 degrees. Let the whole sine, namely 100000, be multiplied by 17364, the sine of the complement of the said arc, and let the resulting number, 1736400000, be divided by 91970, the sine of the complement of the declination. For there results 18880, the sine of the complement of the right ascension, to which corresponds in the table of sines the arc of 10 degrees 53 minutes; and this being taken from 90 degrees, there remains the right ascension of 79 degrees 7 minutes. But if the arc of the Zodiac is precisely a quadrant, namely 90 degrees, its right ascension will be a quadrant too, namely 90 degrees.

[Margin: When the arc of the Ecliptic is larger than a quadrant, yet smaller than a semicircle.]

If the arc turns out to be larger than a quadrant but smaller than a semicircle, it must be subtracted from the semicircle — that is, from 180 degrees — and the right ascension of the remaining [arc], beginning from the autumnal point, must be found. For if this is again subtracted from the semicircle, the right ascension of the proposed arc will remain — because the whole semicircle of the Zodiac ascends together with the whole semicircle of the Equator. EXAMPLE. Let the right ascension of the 10th degree of ♋ [Cancer] be sought — that is, of the arc containing 100 degrees. Subtracting this arc from the semicircle, there remains the arc of 80 degrees, whose right ascension is 79 degrees 7 minutes; and this, taken from the semicircle, gives the ascension of the proposed arc as 100 degrees 53 minutes. But if the arc of the Zodiac is precisely a semicircle, its ascension too will be a semicircle, namely 180 degrees.

[Margin: When the arc of the Ecliptic is larger than a semicircle, but smaller than 270 degrees.]

When the arc is indeed larger than a semicircle, but smaller than 270 degrees, the semicircle — that is, 180 degrees — must be subtracted from it, and the right ascension of the remaining arc must be added again to the semicircle, so that the sought ascension may be obtained. EXAMPLE. Let the right ascension of the 20th degree of ♐ [Sagittarius] be sought — that is, of the arc of 260 degrees. Let the semicircle be subtracted, and there remains the arc of 80 degrees, whose right ascension, namely 79 degrees 7 minutes, added to the semicircle, gives the desired ascension of 259 degrees 7 minutes. But if the arc of the Zodiac

constitutes precisely three quadrants, namely 270 degrees, the ascension owed to it will be of just as many degrees.

[Margin: When the arc of the Ecliptic is larger than 270 degrees.]

Finally, when the arc exceeds three quadrants, yet remains smaller than the whole circle, it must be taken away from the whole circle — as from 360 degrees — and the right ascension of the remaining arc must again be subtracted from the whole circle; for the sought ascension will remain. EXAMPLE. Let the ascension of the 10th degree of ♑ [Capricorn] be explored — that is, of the arc of 280 degrees. Subtracting this arc from 360, there remains the arc of 80 degrees, whose right ascension is 79 degrees 7 minutes; and this, taken from 360, will make plain the sought right ascension of 280 degrees 53 minutes. But if the arc of the Zodiac is a whole circle, it certainly rises together with a whole circle of the Equator too.

[Margin: By what art the table of right ascensions is to be constructed.]

From these things it is manifest by what art the table of right ascensions is to be constructed — the ascensions, namely, that occur in the right sphere. For if we compute the right ascensions of all [arcs] taking their start from ♈ [Aries], we shall have the ascensions of all the arcs of the first Quadrant of the Ecliptic, that is, the right ascensions of all the points of the first Quadrant of the Ecliptic. And if we subtract each of them from the semicircle, the start being made from the larger, or later, ones, the remainders will be the right ascensions of all the points of the second Quadrant of the Ecliptic, the start being made from the beginning of ♈ [Aries] up to the beginning of ♎ [Libra]. Again, if we add the ascensions of the same first Quadrant to the semicircle, the start being made from the smaller or earlier ones, we shall produce the right ascensions of all the points of the third Quadrant of the Ecliptic, the start being made from the beginning of ♈ [Aries] up to the end of ♐ [Sagittarius]. If, finally, we take away these same ascensions of the first Quadrant from the whole circle, the start being again made from the larger, or later, ones, there will remain the right ascensions of all the points of the last Quadrant of the Ecliptic, beginning from the start of ♈ [Aries] up to the end of ♓ [Pisces], as is evident. And so the whole labor consists in this: that the ascensions of the individual arcs of the first Quadrant of the Ecliptic be sought out. By this art Johannes Regiomontanus [Regiomontanus] computed the right ascensions of all the arcs of the Ecliptic, proceeding degree by degree, which it has pleased [us] to set down in this place, so that all the ascensions of the arcs of the Zodiac, and the descensions of the right sphere, and likewise the culminations of the heaven in any sphere, may be set before the eyes. For the knowledge of them is useful for

many purposes, as can in part be made plain from what we handed down in our *Gnomonics* concerning the rising signs.

Another method of computing the right ascension — and indeed an easier one — you will find demonstrated by us in Book 1 of the *Astrolabe*, Lemma 49, Number 16.

CAPUT TERTIUM (continued)

Tabula Ascensionum rectorum (Table of Right Ascensions)

**** The table gives, for each degree of the zodiac (argument column **G**, 0–30 down the left of each block), the right ascension of the ecliptic point reckoned from the start of Aries, expressed in equatorial **degrees (G) and minutes (M)**. The six spring–summer signs (♈ Aries – ♍ Virgo) occupy the upper block; the six autumn–winter signs (♎ Libra – ♋ Pisces) occupy the lower block. Each cell below is written as degrees minutes.

Upper block — right ascensions of ♈ Aries through ♍ Virgo:

D eg	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0 0	27 54	57 48	90 0	122 12	152 6
1	0 55	28 51	58 51	91 6	123 14	153 3
2	1 50	29 49	59 54	92 12	124 16	154 0
3	2 45	30 46	60 57	93 17	125 18	154 57
4	3 40	31 44	62 0	94 22	126 20	155 54
5	4 35	32 42	63 3	95 27	127 22	156 51
6	5 30	33 40	64 6	96 33	128 24	157 48
7	6 25	34 39	65 9	97 38	129 25	158 45
8	7 20	35 37	66 13	98 43	130 26	159 41
9	8 15	36 36	67 17	99 48	131 27	160 37
10	9 11	37 35	68 21	100 53	132 27	161 33
11	10 6	38 34	69 25	101 58	133 28	162 29
12	11 1	39 33	70 29	103 3	134 29	163 25

D eg	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
2						
1	11 57	40 32	71 33	104 8	135 29	164 21
3						
1	12 52	41 31	72 38	105 13	136 29	165 17
4						
1	13 48	42 31	73 43	106 17	137 29	166 12
5						
1	14 43	43 31	74 47	107 22	138 29	167 8
6						
1	15 39	44 31	75 52	108 27	139 28	168 3
7						
1	16 35	45 31	76 57	109 31	140 27	168 59
8						
1	17 31	46 32	78 2	110 35	141 26	169 54
9						
2	18 27	47 33	79 7	111 39	142 25	170 49
0						
2	19 23	48 33	80 12	112 43	143 24	171 45
1						
2	20 19	49 34	81 17	113 47	144 23	172 40
2						
2	21 15	50 35	82 22	114 51	145 21	173 35
3						
2	22 12	51 36	83 27	115 54	146 20	174 30
4						
2	23 9	52 38	84 33	116 57	147 18	175 25
5						
2	24 6	53 40	85 38	118 0	148 16	176 20
6						
2	25 3	54 42	86 43	119 3	149 14	177 15
7						
2	26 0	55 44	87 48	120 6	150 11	178 10
8						

D eg	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
2 9	26 57	56 46	88 54	121 9	151 9	179 5
3 0	27 54	57 48	90 0	122 12	152 6	180 0

Lower block — right ascensions of ♎ Libra through ♉ Pisces:

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
0	180 0	207 54	237 48	270 0	302 12	332 6
1	180 55	308 51 [recte 208 51]	238 51	271 6	303 14	333 3
2	181 50	209 49	239 54	272 12	304 16	334 0
3	182 45	210 46	240 57	273 17	305 18	334 57
4	183 40	211 44	242 0	274 22	306 20	335 54
5	184 35	212 42	243 3	275 27	307 22	336 51
6	185 30	213 40	244 6	276 33	308 24	337 48
7	186 25	214 39	245 9	277 38	309 25	338 45
8	187 20	215 37	246 13	278 43	310 26	339 41
9	188 15	216 36	247 17	279 48	311 27	340 37
1 0	189 11	217 35	248 21	280 53	312 27	341 33
1 1	190 6	218 34	249 25	281 58	313 28	342 29
1 2	191 1	219 33	250 29	283 3	314 29	343 25
1 3	191 57	220 32	251 33	284 8	315 29	344 21
1 4	192 52	221 31	252 38	285 13	316 29	345 17
1 5	193 48	222 31	253 43	286 17	317 29	346 12

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
1 6	194 43	223 31	254 47	287 22	318 29	347 8
1 7	195 39	224 31	255 52	288 27	319 28	348 3
1 8	196 35	225 31	256 57	289 31	320 27	348 59
1 9	197 31	226 32	258 2	290 35	321 26	349 54
2 0	198 27	227 33	259 7	291 39	322 25	350 50
2 1	199 23	228 33	260 12	292 43	323 24	351 45
2 2	200 19	229 34	261 17	293 45 [?]	324 23	352 40
2 3	201 15	230 35	262 22	294 51	325 21	353 35
2 4	202 12	231 36	263 27	295 54	326 20	354 30
2 5	203 9	232 38	264 33	296 57	327 18	355 25
2 6	204 6	233 40	265 38	298 0	328 16	356 20
2 7	205 3	234 42	266 43	299 3	329 14	357 15
2 8	206 0	235 44	267 48	300 6	330 11	358 10
2 9	206 57	236 46	268 54	301 9	331 9	359 5
3 0	207 54	237 48	270 0	302 12	332 6	360 0

USUS TABULÆ ASCENSIONUM RECTARUM (Use of the Table of Right Ascensions)

[Margin: How the right ascensions are drawn from the table of right ascensions.]

At the head of the table you take the sign, and down the left-hand side the degree of the sign; for at the common meeting of the given sign and degree you find the degrees and minutes of the Equator that rise together with the given degree of the ecliptic. Thus you see that with 19° Leo — that is, with the arc of the ecliptic beginning at the start of Aries and ending at 19° Leo, which comprises 139 degrees — there rise in the right horizon 141°26' of the equinoctial circle. And if minutes are attached to the given arc, you must extract the proportional part corresponding to the minutes offered, as was explained in the use of the table of declinations, and add it to the ascension of the arc of whole degrees next smaller.

EXAMPLE. Let the right ascension be sought of an arc of the ecliptic containing 125°40' — that is, 5°40' Leo. I subtract the ascension of 5° Leo, namely 127°22' [printed "127°12'"; the difference stated next (1°2') and the value in the table both require 22' — see Numerical impossibles], from the ascension of 6° Leo, namely 128°24'; and there remains 1°2', the difference between the two ascensions, which answers to 60 minutes. Therefore, by the rule of proportion, to the 40 minutes there are due 41⅓ minutes; and if these are added to the ascension of 5° Leo, the ascension of the proposed arc will be about 128°3'.

But if the ascension is to be sought of an arc of the ecliptic that does not begin at the start of Aries — for instance, an arc of the zodiac beginning at 10° Taurus and ending at 18° Gemini, which spans 38 degrees — then the ascension of 10° Taurus, namely 37°35', must be subtracted from the ascension of 18° Gemini, namely 76°57', so that there remains the ascension of the proposed arc, 39°22'.

By this method you will easily gather the right ascension of any sign of the zodiac taken by itself, as appears in the following table.

Ascensiones Signorum in Sphæra recta (Right Ascensions of the Signs in the Right Sphere)

Sign	Sign	Sign	Sign	G	M
♈ Aries	♍ Virgo	♎ Libra	♐ Pisces	2	5
				7	4
♉ Taurus	♌ Leo	♏ Scorpio	♑ Aquarius	2	5

Sign	Sign	Sign	Sign	G	M
				9	4
♊ Gemini	♋ Cancer	♐ Sagittarius	♑ Capricorn	3	1
				2	2

From this it clearly follows that, in the right sphere, the four signs surrounding the two solstitial points rise directly, while the remaining eight rise obliquely; likewise, that those arcs rise more obliquely which are nearer the equinoctial points, and more directly those that lie closer to the solstitial points. The same you will say of the settings and of the culminations. In the right sphere, therefore, four signs always have equal ascensions, equal descensions, and equal culminations — all of which can be demonstrated from spherical triangles, and have been shown by us in Book 1 of the *Astrolabe*, Lemma 49, No. 6.

DE ORTU, ET OCCASU SIGNORUM in sphæra obliqua (On the Rising and Setting of the Signs in the Oblique Sphere)

[Margin: The rising and setting of the signs in the oblique sphere.]

In the oblique or inclined sphere, however, the two halves of the zodiac are matched to their ascensions. I mean the halves taken from the two equinoctial points; for the half of the zodiac running from the start of Aries to the end of Virgo rises together with the half of the equinoctial bordering upon it. Likewise the other half of the zodiac rises with the remaining half of the equinoctial. But the parts of those halves vary according to their ascensions, since in that half of the zodiac from the start of Aries to the end of Virgo a greater part always rises of the zodiac than of the equinoctial — and yet those halves finish rising together. The reverse occurs in the other half of the zodiac, from the start of Libra to the end of Pisces: for there a greater part always rises of the equinoctial than of the zodiac — and yet those halves finish rising together. Hence the objection raised here is made more evident against the argument stated above.

COMMENTARIUS (Commentary)

He now sets out three rules by which the rising and setting of the signs, or of arcs of the ecliptic, may be known in any oblique sphere. The first is: *The halves of the zodiac beginning at the equinoctial points are, in any oblique horizon, matched to their ascensions* — that is, together with them there rise the halves of the Equator as well, namely 180 degrees, so that in the space of 12 hours they emerge whole above the horizon. The parts of these halves, however, are not equal to their

ascensions — that is, no part of them rises together with an equal part of the Equator, but with either a greater or a smaller, as was said of the parts of the quadrants in the right sphere. For with the sign of Aries there rise at Rome $17^{\circ}21'$, but with the sign of Virgo there rise $38^{\circ}27'$, and so on. The first part of the rule is clear: since the ecliptic and the Equator bisect each other mutually at the equinoctial points, the beginning of each half must reach any given horizon at the same time; and you will say the same of their end-points, since the same point is the beginning of the one and the end of the other. Hence they rise together. The latter part of the rule can easily be shown from Proposition 10, Book 1

of Menelaus, or from Proposition 11 of our [treatise] on spherical triangles: for the half of the zodiac from Aries to Libra always makes a smaller angle with the horizon than the Equator does. Therefore a greater part of the zodiac will always rise than of the Equator. But the remaining half of the zodiac, from Libra to Aries, always makes a greater angle with the horizon than the Equator; hence a greater part of the Equator will finish rising than of the zodiac. Yet the whole halves finish rising together, as has been said. All this is evident in the material sphere, and will be plain from the tables of oblique ascensions; and it has been demonstrated by us in Book 1 of the *Astrolabe*, Lemma 49, No. 11.

From these considerations he gathers, once again, an objection against the argument brought forward above; namely, that this inference does not hold: "There are two equal arcs in the sphere, and a greater part of the one has always risen than of the other; therefore the whole of the one will rise sooner than the whole of the other." For this argument is very easily resolved from what was said in this first rule, as is evident.

But the arcs that follow Aries up to the end of Virgo diminish, in the oblique sphere, their ascensions below the ascensions of the same arcs in the right sphere, because less of the equinoctial rises. And the arcs that follow Libra up to the end of Pisces increase, in the oblique sphere, their ascensions above the ascensions of the same arcs in the right sphere, because more of the equinoctial rises. They increase them, I say, by just as much as the arcs following Aries diminish theirs.

[Margin: Comparison of the ascensions in the oblique sphere with the ascensions in the right sphere.]

COMMENTARIUS (Commentary)

In this second rule he compares any oblique sphere with the right sphere, saying that the individual arcs of the zodiac, beginning from Aries up to the end of Virgo, have smaller individual ascensions in the oblique sphere than in the right sphere; but the individual arcs of the zodiac, beginning from Libra up to the end of Pisces, have greater individual ascensions in the oblique sphere than in the right — and greater by as much as the ascensions of the earlier arcs are smaller, provided equal arcs are taken on both sides. For example, at Rome, with the end of Taurus there rise $38^{\circ}27'$, but in the right sphere $57^{\circ}48'$. You see, then, that this [oblique] ascension is exceeded by that [right one] by $19^{\circ}21'$. But at Rome the end of Scorpio rises with $257^{\circ}9'$ [leading digit reconstructed — see Numerical impossibles], while in the right sphere with $237^{\circ}48'$, where you see that this [oblique] exceeds that [right one] likewise by $19^{\circ}21'$. And so with the rest. This too is evident from the doctrine of spherical triangles, and is found by experience in the material sphere and from the tables of oblique ascensions, as we demonstrated in Book 1 of the *Astrolabe*, Lemma 49, No. 12.

From this it appears that two equal and opposite arcs in the inclined sphere have their ascensions, taken jointly, equal to the ascensions of the same arcs in the right sphere taken together; because as much as is subtracted on the one side, so much is added on the other. For although the arcs of the ascensions are unequal among themselves, yet by as much as the one is smaller, by so much the other recovers, and thus the equality is plain.

COMMENTARIUS (Commentary)

From the second rule it is manifest that in any oblique sphere the opposite signs or arcs do not have equal ascensions, provided the arcs take their beginning from the equinoctial points. For since opposite equal arcs have equal ascensions in the right sphere, while in any oblique sphere the ascension of an arc beginning at the start of Aries is smaller than in the right sphere, and the ascension of an arc beginning at the start of Libra is greater in that same oblique sphere than in the right, it is clear that opposite arcs have unequal ascensions in the oblique sphere. Therefore the author infers from this second rule that opposite arcs of this kind, in any oblique sphere, have their ascensions taken together equal to the ascensions of the same arcs in the right sphere taken together, although among themselves they are quite unequal — because, namely, by as much as the ascension of the one is greater in the oblique sphere than in the right, by so much the ascension of the other is smaller in that same obliquity than in the right sphere. The reason for this depends on Proposition 3, Book 1 of the *Arithmetic* of

Jordanus [Jordanus de Nemore], where it is demonstrated: If two unequal numbers be set about two equal numbers, in such a way that the greater of the unequal exceeds one of the equal by the same amount by which the smaller is exceeded by the other, then the two unequal together are equal to the two equal together — as is clear in these numbers, 4, 9, 9, 14; likewise in these, 20, 70, 70, 120. So it happens with the ascensions. For the two ascensions of two opposite arcs in the right sphere are equal, and about them are set the unequal ascensions of the same arcs in the oblique sphere, in such a way that the greater exceeds the one equal by the same excess by which the smaller is exceeded by the other equal. As appears in these four ascensions: $17^{\circ}21'$, $27^{\circ}54'$, $27^{\circ}54'$, $38^{\circ}27'$. Of these the first is the ascension of Aries at Rome; the second, the ascension of the same Aries in the right sphere; the third, the ascension of Libra, the opposite sign, in the right sphere; and the fourth, finally, the ascension of the same Libra at Rome. And since the first is exceeded by the second by just as much as the fourth exceeds the third — for in both cases the excess is $10^{\circ}33'$ — therefore the first and fourth together make just as many degrees and minutes as are made up from the two middle ones, namely $55^{\circ}48'$. And the same reckoning is to be had for the rest.

Again, equal arcs equally distant from either of the solstitial points have their ascensions taken together equal to the ascensions of the same arcs in the right sphere taken together — namely ♈ Taurus and ♌ Leo; ♊ Pisces and ♎ Libra, and so on — as Geber [Jābir ibn Aflāḥ] and Regiomontanus [Iohannes de Regiomonte] demonstrate (*Epitome*, Book 2, Prop. 20), and as we too have shown (*Astrolabe*, Book 1, Lemma 49, No. 10).

[Margin: Equal arcs, equally removed from either of the solstitial points, have in the oblique sphere their ascensions taken together equal to the ascensions of the same taken together in the right sphere.]

COMMENTARIUS (Commentary)

In the same way, the ascensions of any two equal and opposite arcs — even if they do not take their beginning from the equinoctial points — taken together will be equal to the ascensions of the same arcs in the right sphere taken together, although among themselves they are unequal. It is true, however, that then the oblique ascension of an arc contained in the northern half of the zodiac is not always smaller than the right ascension of the same arc, nor is the oblique ascension of an arc contained in the southern half of the zodiac always greater than the right ascension of the same; but sometimes the former is greater and the latter smaller, and sometimes the former smaller and the latter greater. All of which can be shown geometrically from the doctrine of spherical triangles, and

are most clearly perceived ...are found in the tables of oblique ascensions. Nevertheless, this same point can also be confirmed by the following reasoning. Let there be two opposite signs, ♈ and ♏. I say that their ascensions taken together are equal to the ascensions of the same signs taken together which they have in the right sphere. For the ascension of ♈ and the ascension of ♏ in the oblique sphere, taken together, are equal to the ascensions they have taken together in the right sphere, as has been said, because these signs recede equally from the solstitial point — that is, they are equal to the right ascensions of ♈ and ♏, since ♏ and ♈ have equal ascensions in the right sphere (inasmuch as they are equally distant from the beginning of ♈). And the ascension of ♏ in the oblique sphere is equal to the ascension of ♏, as will be established from the third rule, because these signs are removed equally from the equinoctial point. Therefore the ascension of ♈ and the ascension of ♏ together will be equal to the ascensions of the same signs in the right sphere.

This can also be confirmed another way. The ascension of the arc from the beginning of ♈ to the end of ♈, together with the ascension of the arc from the beginning of ♏ to the end of ♏, are in the oblique sphere together equal to the ascensions of the same arcs taken together in the right sphere, as is clear from the preceding corollary. Likewise, the ascension of the arc from the beginning of ♈ to the beginning of ♈, together with the ascension of the arc from the beginning of ♏ to the beginning of ♏, are in the oblique sphere together equal to the ascensions of the same arcs together in the right sphere, as is evident from the same corollary. It follows that if these latter ascensions are subtracted from those former ones, the remaining ascensions of the arcs ♈ and ♏ together in the oblique sphere will be equal to the remaining ascensions of the same arcs together in the right sphere. You will say the same of any opposite and equal arcs whatever, because the ascension of the one is always equal to the ascension of some equal arc that is equally distant with the other from the solstitial point, as is clear.

[Margin: Equal arcs, and ones equally distant from either of the equinoctial points, have equal ascensions in the oblique sphere.]

From this the truth of the second rule proposed is evident. For the reasoning about equal and opposite arcs is the same, whether they take their beginning from the equinoctial points or not, as is established.

In the given example, however, the ascension of ♈ in the oblique sphere at Rome, containing $38^{\circ}42'$, is greater than the ascension of the same ♈ in the right sphere, which comprises $29^{\circ}54'$. The ascension of ♏, on the other hand, in the same

oblique sphere, containing $21^{\circ}6'$, is less than the ascension of the same ϖ in the right sphere, since in the latter it comprises $29^{\circ}54'$ — even though ϱ lies in the northern half of the Ecliptic and ϖ in the southern half. This is indeed no contradiction of the second rule, because these signs do not begin at the equinoctial points, as the second rule required.

[Margin: Solution of a certain difficulty.]

[Margin: The ascension of any arc in the oblique sphere is equal to the descension of the opposite arc; and, in the same sphere, the ascension and descension of any sign taken together are equal to the ascension and descension of the opposite sign taken together, in any sphere.]

Rule [III]. In the oblique sphere, indeed, the rule is that any two arcs of the Zodiac which are equal and equally distant from either of the equinoctial points have equal ascensions.

COMMENTARY

The third rule is this: any two arcs of the Zodiac that are equal and equally removed from either of the equinoctial points — whether they begin at the equinoctial point itself or not — have equal ascensions between themselves in any inclined sphere. For example, Aries and Pisces; Taurus and Aquarius; and so on. This is established from the doctrine of spherical triangles, and it is demonstrated both by Geber [Jābir ibn Aflāḥ] and by Ioannes Regiomontanus [Regiomontanus] in *Epitome* Book 2, proposition 19.

This rule may perhaps seem to someone contrary to the preceding one. For it was said in the second rule that the arcs of the half of the Ecliptic from γ to α have smaller ascensions in the oblique sphere than the arcs of the other half. Since, then, Aries is contained in the former half and Pisces in the latter, how can it come about that these arcs have equal ascensions? It must nevertheless be answered that this rule is entirely true and does not conflict with the preceding one. For the preceding rule was understood of arcs beginning at the start of γ or of α ; but arcs of this kind are not Aries and Pisces. For although the arc of Aries has its beginning at the first point of γ , yet the arc of Pisces does not begin at the first degree of α .

[Margin: The ascension of any sign in the oblique sphere is unequal to the descension of the same sign.]

Moreover, in every sphere, whether right or inclined, the ascension of any arc, or of any sign, is equal to the descension of the opposite arc or sign. For since the Horizon and the Zodiac cut one another in two, being great circles, half of the Zodiac will always be above the Horizon. Hence, at whatever point of the Zodiac is rising above the Horizon, it is necessary that the opposite point be descending below the Horizon — otherwise a greater arc than a semicircle, or a smaller one, would stand above the Horizon. And so, when the beginning of some sign is precisely in the East, the beginning of the opposite sign is precisely in the West; and when the extreme point of the earlier sign is in the East, the extreme point of the later one will be in the West. Therefore, as one rises, the other necessarily descends.

From this it follows that the ascension and the descension of any sign taken together are equal to the ascension and descension of the opposite sign taken together, in any sphere; because, of course, the ascension of one sign is the descension of the opposite sign, and the descension of the same sign is the ascension of the opposite sign. Hence, if equals are added to equals, the wholes become equal. For instance, the ascension of ♈ is equal to the descension of ♎, and the descension of ♈ is equal to the ascension of ♎, and so on.

[Margin: The ascension and descension of the same sign in the oblique sphere taken together are equal to the ascension and descension of the same sign taken together in the right sphere.]

Likewise, it is evident that the ascension of any sign in the oblique sphere is unequal to the descension of the same sign — so that, if it rises directly, it sets obliquely, and conversely. For since the ascension of each sign is equal to the descension of the opposite sign, if the ascension of this later sign were equal to the descension of the same, the opposite signs would have equal ascensions, which is contrary to what was said in the second rule. Yet the ascension of any sign together with the descension of the same in the oblique sphere are equal to the ascension and descension of the same sign taken together in the right sphere; because the more obliquely, or the more directly, some sign rises in the oblique sphere than in the right, by so much the more directly, or the more obliquely, does it set, as is established from spherical triangles, and will be manifest from the tables of oblique ascensions.

All these matters concerning both right and oblique ascensions have been clearly demonstrated by us in Book 1 of the *Astrolabe*, Lemma 49.

By what method the oblique ascension of any arc of the Zodiac, reckoned from the vernal section, may be found

Since it was said in the second rule that the oblique ascensions of the arcs of the Ecliptic contained in the northern half — the beginning always being made from the first point of ♈ — are smaller than the right ascensions of the same arcs by as much as the oblique ascensions of the arcs of the Ecliptic comprised in the southern half — the beginning likewise always made from the beginning of ♎ — are greater than the right ascensions of the same arcs; it is manifest that if from the right ascensions of the arcs of the earlier half of the Ecliptic the ascensional differences are subtracted (those, namely, by which the right ascensions differ from the oblique), the oblique ascensions of the same arcs are left; but if the same ascensional differences are added to the right ascensions of the arcs of the later half of the Ecliptic, the oblique ascensions of the same arcs are produced, the beginning always being made from the beginnings of ♈ and ♎.

[Margin: How the oblique ascensions may be found from the ascensional differences. By what method the ascensional differences may be found by means of sines.]

This ascensional difference you will find, by this art, from the doctrine of Sines. As Geber demonstrates, and as we too have demonstrated in the scholium to proposition 9 of Book 2 of the *Gnomonics*: the sine of the complement of the declination of the proposed point of the Ecliptic is to the sine of the complement of the ortive (or occidental) latitude of the same point of the Ecliptic as the whole sine is to the sine of the complement of the ascensional difference. Therefore, if the sine of the complement of the ortive latitude is multiplied into the whole sine, and the resulting number is divided by the sine of the complement of the declination of the proposed point, as the rule of proportions prescribes, the sine of the complement of the ascensional difference will be obtained. And so it will be known from the table of sines.

Example. Let the difference at Rome be sought by which the oblique ascension of an arc of the Ecliptic from ♈ to the end of ♐ differs from its right ascension. Since, then, the declination of the extreme point of ♐ is $23^{\circ}30'$, and the ortive latitude is $32^{\circ}27'$: I multiply the sine of the complement of the ortive latitude, namely 84386, into the whole sine, that is into 100000; the resulting number, 8438600000, I then divide by 91706, the sine of the complement of the declination

of the extreme point of Π , and there will come out the sine of the complement of the ascensional difference, about 92018, to which corresponds in the table of sines $66^{\circ}57'$. Therefore the ascensional difference will be $23^{\circ}3'$. When this is subtracted from the right ascension of the proposed arc — namely from 90° , since it is in the earlier half of the Ecliptic — there will be left the oblique ascension of the said arc at Rome, $66^{\circ}57'$.

Now since we taught above that the declinations and ortive latitudes of all the points of a single Quadrant are equal to the declinations and latitudes which all the points of the remaining Quadrants have, it is plain that it is enough if the ascensional differences of a single Quadrant of the Ecliptic are investigated; for the four points of the Ecliptic have the same ascensional difference, as we demonstrated in Book 1 of the *Astrolabe*, Lemma 49, Number 15.

[Margin: It is enough if the ascensional differences of the points of a single quadrant of the Ecliptic are investigated. How, by another method, they may be found by means of sines.]

By another method the ascensional difference of any arc, or point, of the Ecliptic will be found, without knowledge of the ortive or occidental latitude, by this art. Let the sine of the elevation of the pole be multiplied into the whole sine, and the resulting number be divided by the sine of the complement of the elevation of the pole. For there will come out a sine which in one and the same region never varies, whence not without reason it may be called the *sine of the region*; at Rome it is about 90041. Now this sine of the region is nothing other than the tangent of the elevation of the pole. And so it is not necessary that it be found by multiplication and division, but it is enough to take from the table of tangents the tangent of the arc of the elevation of the pole. Then, since — as Ioannes Regiomontanus demonstrates in *Epitome* Book 2, proposition 22 — such is the proportion of the sine of the complement of the declination of the proposed point (which of course terminates the given arc) to the sine of the declination, as the proportion of the sine which we called the sine of the region is to the sine of the ascensional difference of the proposed point of the Ecliptic: if, according to the precept of the rule of proportions, the sine of the declination of the proposed point is multiplied into the sine of the region so found, and the resulting number is then divided into the sine of the complement of the declination, the sine of the ascensional difference will be obtained.

Example. At Rome I seek the ascensional difference of the last point of the first Quadrant of the Ecliptic, namely of the last point of Π . I multiply 39874, the sine of the declination, into the sine of the region of Rome, 90041; the resulting

number, 3590294834, I divide by 91706, the sine of the complement of the declination, and there will come out the sine of the ascensional difference sought, 39150, to which corresponds an arc of $23^{\circ}3'$, just as before.

By this art Ioannes Regiomontanus computed the ascensional differences of all the points that decline from the Equator, beginning from 1° of declination up to 32° . For no Planet — for whose sake he wrote the tables — ever had a greater declination. If, therefore, you desire the ascensional difference of any arc of the Ecliptic, seek at the head of the table of ascensional differences the elevation of the pole, and on the left side the declination of the extreme point of the proposed arc; for in the angle of common concurrence you will find the difference sought. Thus at Rome, where the pole is elevated 42° , a point of the Ecliptic that declines 18° from the Equator has an ascensional difference of $17^{\circ}1'$, and so on.

[Margin: How the ascensional differences may be found from the table of ascensional differences.]

But if the declination of the point is not found on the left side, the excess between the ascensional difference of the next-greater declination and the ascensional difference of the next-lesser declination must be sought. Then the proportional part corresponding to the minutes of the proposed declination must be extracted. For this, added to the ascensional difference of the next-lesser declination, will give the ascensional difference sought.

Example. At Rome let there be found the ascensional difference of the last point of Π , or of the first degree of $\mathfrak{S}$ — that is, of the first Quadrant of the Ecliptic. Since, then, the declination of the first degree of $\mathfrak{S}$ is $23^{\circ}30'$: I take the ascensional difference belonging to a declination of 23° , namely $22^{\circ}28'$; likewise the ascensional difference belonging to a declination of 24° , namely $23^{\circ}38'$; the difference between these is $1^{\circ}10'$, which is then owed to a whole degree of declination. Therefore, by the rule of proportions, to 30 minutes are owed 35 minutes; and these, added to the ascensional difference belonging to a declination of 23° (namely $22^{\circ}28'$), give an ascensional difference of $23^{\circ}3'$, just as before — belonging to a declination of $23^{\circ}30'$, namely to the beginning of $\mathfrak{S}$. And so for the rest.

It is therefore established from these things by what art the table of ascensional differences is to be constructed for any elevation of the pole, and consequently, from the table of ascensional differences, the table of oblique ascensions. Yet, in order to relieve the reader of this burden, I have appended from Ioannes Regiomontanus the tables of ascensional differences—

—for all elevations of the pole, beginning from 1° up to 60° ; and likewise the tables of oblique ascensions for the individual altitudes of the pole, beginning from 36° up to 60° — because they have notable uses in astronomical matters, as is established in part from what we wrote in the *Gnomonics* concerning the ascending signs.

[Margin: How, from the tables of oblique ascensions, the oblique ascensions and descensions may be found.]

You will find, from these tables of oblique ascensions, the oblique ascension of any arc no differently than was set forth in the use of the table of right ascensions — taking, however, the table of oblique ascensions of that elevation of the pole for which you are inquiring the oblique ascensions in any oblique sphere. But the descension of each arc you will explore thus. In the right sphere the ascension and descension of the same arc are equal. It has been shown that the ascension of any arc is equal to the descension of the opposite arc, and the descension of any arc equal to the ascension of the opposite arc; therefore, if the descension of some arc is sought, the ascension of the opposite arc must be investigated, for this will be the descension of the proposed arc.

Example. Let the descension be desired of the arc from Υ to $8^\circ \text{ } \mathbb{M}$, at Rome, where the pole is elevated 42° . The opposite arc is from $\text{ } \mathbb{A}$ to $8^\circ \text{ } \mathbb{X}$; and since $8^\circ \text{ } \mathbb{X}$ rises with $347^\circ 29'$ of the Equator, counting from Υ , if 180° — namely the semicircle from Υ to $\text{ } \mathbb{A}$ — are subtracted, there will remain the ascension of the arc from $\text{ } \mathbb{A}$ to $8^\circ \text{ } \mathbb{X}$, that is, the descension of the arc from Υ to $8^\circ \text{ } \mathbb{M}$: $167^\circ 29'$.

Similarly, the descension is sought of the arc from the beginning of $\text{ } \mathbb{M}$ to $20^\circ \text{ } \mathbb{A}$. The opposite arc is from $\text{ } \mathbb{X}$ to $20^\circ \text{ } \mathbb{Q}$; and since $20^\circ \text{ } \mathbb{Q}$, counting from the beginning of $\text{ } \mathbb{X}$, rises with $110^\circ 15'$ [?] of the Equator, we shall say that this is the descension of the arc comprised between the beginning of $\text{ } \mathbb{M}$ and $20^\circ \text{ } \mathbb{A}$.

By the same reasoning, the descension of the last degree of $\text{ } \mathbb{X}$ [?] must be investigated — that is, of the arc comprised between the beginning of Υ and the last degree of $\text{ } \mathbb{X}$. Opposite to this arc is the arc contained between the beginning of $\text{ } \mathbb{A}$ and the end of $\text{ } \mathbb{I}$; for the first points of the said arcs, and likewise their extreme points, are opposite through the diameter in the sphere. Now the arc from the beginning of Υ to the end of $\text{ } \mathbb{I}$ rises with $66^\circ 57'$ of the Equator; and if 180° are added to these, the ascension will be obtained of the arc from the beginning of $\text{ } \mathbb{A}$ to the end of $\text{ } \mathbb{X}$: $246^\circ 57'$. And so for the rest.

[Margin: How, by another method, the oblique descensions may be sought from the tables of oblique ascensions.]

It is also customary to investigate the descension in a way other than we have said — the descension, that is, of any arc beginning from the beginning of $\mathcal{V}$ — by this method. From the ascension of the point which is opposite, through the diameter, to the extreme point of the proposed arc, let the whole semicircle, that is 180° , be subtracted. But if the subtraction cannot be made, let a whole circle, that is 360° , be added first to the ascension of the opposite point. For what remains will be the descension sought.

Example. At Rome the descension of $8^{\circ} \varrho$ is sought. From the ascension of $8^{\circ} \approx$ — that is, from $327^{\circ}45'$ — I subtract 180° , and there remains the descension of the arc from γ to $8^{\circ} \varrho$: $147^{\circ}45'$. Again, the descension of $20^{\circ} \mathfrak{M}$ is to be found. I add to the ascension of $20^{\circ} \vartheta$ — namely to $30^{\circ}46'$ — a whole circle, that is, [I make it] $390^{\circ}46'$; from this composite I subtract a semicircle, and there is left the descension of the arc from γ to $20^{\circ} \mathfrak{M}$: $210^{\circ}46'$, and so on.

Other methods of computing the ascensional differences — and consequently the oblique ascensions as well — and indeed easier ones, you will find in Book 1 of the *Astrolabe*, Lemma 49.

Table of Ascensional Differences [Tabula Differentiarum Ascensionalium]

The table gives the **ascensional difference** [differentia ascensionalis] in degrees (G = *gradus*) and minutes (M = *minuta*) for each combination of a **degree of declination** [Gradus Declinationum, the row-argument, 1°–32°] and an **elevation of the pole** [Elevatio Poli, the column-parameter]. On this page the elevations run 1°–14°, presented as two stacked blocks (elevations 1°–7°, then 8°–14°); each block re-uses the same declination argument 1–32. Each cell below is rendered as degrees°minutes’.

Elevation of the pole 1° – 7° (declination 1° – 32°)

Decl	1°	2°	3°	4°	5°	6°	7°
1	0°1′	0°2′	0°3′	0°4′	0°5′	0°6′	0°7′
2	0°2′	0°4′	0°6′	0°8′	0°10′	0°13′	0°15′
3	0°3′	0°6′	0°9′	0°13′	0°16′	0°19′	0°22′
4	0°4′	0°8′	0°13′	0°17′	0°21′	0°25′	0°30′
5	0°5′	0°10′	0°16′	0°21′	0°26′	0°32′	0°37′
6	0°6′	0°13′	0°19′	0°25′	0°32′	0°38′	0°44′

Decl	1°	2°	3°	4°	5°	6°	7°
7	0°7′	0°15′	0°22′	0°30′	0°37′	0°44′	0°52′
8	0°8′	0°17′	0°25′	0°34′	0°42′	0°51′	0°59′
9	0°9′	0°19′	0°29′	0°38′	0°48′	0°57′	1°7′
10	0°11′	0°21′	0°32′	0°42′	0°53′	1°4′	1°14′
11	0°12′	0°23′	0°35′	0°47′	0°58′	1°10′	1°22′
12	0°13′	0°25′	0°38′	0°51′	1°4′	1°17′	1°30′
13	0°14′	0°28′	0°42′	0°56′	1°9′	1°23′	1°37′
14	0°15′	0°30′	0°45′	1°0′	1°15′	1°30′	1°45′
15	0°16′	0°32′	0°48′	1°4′	1°21′	1°37′	1°53′
16	0°17′	0°34′	0°52′	1°9′	1°26′	1°44′	2°1′
17	0°18′	0°37′	0°55′	1°14′	1°32′	1°50′	2°9′
18	0°19′	0°39′	0°59′	1°18′ [?]	1°38′	1°57′	2°17′
19	0°21′	0°41′	1°2′	1°23′	1°44′	2°4′	2°25′
20	0°22′	0°44′	1°6′	1°27′	1°49′	2°12′	2°34′
21	0°23′	0°46′	1°9′	1°32′	1°55′	2°19′	2°42′
22	0°24′	0°49′	1°13′	1°37′	2°2′	2°26′	2°51′
23	0°25′	0°51′	1°17′	1°42′	2°8′	2°33′	2°59′
24	0°27′	0°53′	1°20′	1°47′	2°14′	2°41′	3°8′
25	0°28′	0°56′	1°24′	1°52′	2°20′	2°49′	3°17′
26	0°29′	0°59′	1°28′	1°57′	2°27′	2°56′	3°26′
27	0°31′	1°1′	1°32′	2°3′	2°33′	3°4′	3°35′
28	0°32′	1°4′	1°36′	2°8′	2°40′	3°12′	3°45′
29	0°33′	1°7′	1°40′	2°13′	2°47′	3°20′	3°54′
30	0°35′	1°9′	1°44′	2°19′	2°54′	3°29′	4°4′
31	0°36′	1°12′	1°48′	2°24′	3°1′	3°37′	4°14′
32	0°37′	1°15′	1°53′	2°30′	3°8′	3°46′	4°24′

Elevation of the pole 8°–14° (declination 1°–32°)

Decl	8°	9°	10°	11°	12°	13°	14°
1	0°8′	0°9′	0°11′	0°12′	0°13′	0°14′	0°15′

Decl	8°	9°	10°	11°	12°	13°	14°
2	0°17'	0°19'	0°21'	0°23'	0°25'	0°28'	0°30'
3	0°25'	0°29'	0°32'	0°35'	0°38'	0°42'	0°45'
4	0°34'	0°38'	0°42'	0°47'	0°51'	0°56'	1°0'
5	0°42'	0°48'	0°53'	0°58'	1°4'	1°9'	1°15'
6	0°51'	0°57'	1°4'	1°10'	1°17'	1°23'	1°30'
7	0°59'	1°7'	1°14'	1°22'	1°30'	1°37'	1°45'
8	1°8'	1°16'	1°25'	1°34'	1°43'	1°52'	2°0'
9	1°16'	1°26'	1°36'	1°46'	1°56'	2°6'	2°16'
10	1°25'	1°36'	1°47'	1°58'	2°9'	2°20'	2°31'
11	1°34'	1°46'	1°58'	2°10'	2°22'	2°34'	2°47'
12	1°43'	1°56'	2°9'	2°22'	2°35'	2°49'	3°2'
13	1°52'	2°6'	2°20'	2°34'	2°49'	3°3'	3°18'
14	2°0'	2°16'	2°31'	2°47'	3°2'	3°18'	3°34'
15	2°10'	2°26'	2°42'	2°59'	3°16'	3°33'	3°50'
16	2°19'	2°36'	2°54'	3°12'	3°30'	3°48'	4°6'
17	2°28'	2°47'	3°5'	3°24'	3°44'	4°3'	4°22'
18	2°37'	2°57'	3°17'	3°37'	3°58'	4°18'	4°39'
19	2°46'	3°8'	3°29'	3°50'	4°17'	4°34'	4°55'
20	2°56'	3°18'	3°41'	4°3'	4°26'	4°49'	5°12'
21	3°6'	3°29'	3°53'	4°17'	4°41'	5°5'	5°30'
22	3°15'	3°40'	4°5'	4°30'	4°56'	5°21'	5°47'
23	3°25'	3°51'	4°18'	4°44'	5°11'	5°37'	6°7'
24	3°35'	4°3'	4°30'	4°58'	5°26'	5°54'	6°22'
25	3°45'	4°14'	4°43'	5°12'	5°41'	6°11'	6°41'
26	3°56'	4°26'	4°56'	5°26'	5°57'	6°28'	6°59'
27	4°6'	4°38'	5°9'	5°41'	6°13'	6°45'	7°18'
28	4°17'	4°50'	5°23'	5°56'	6°29'	7°3'	7°37'
29	4°28'	5°2'	5°37'	6°11'	6°46'	7°21'	7°57'
30	4°39'	5°15'	5°51'	6°27'	7°3'	7°40'	8°17'
31	4°51'	5°28'	6°5'	6°42'	7°20'	7°58'	8°37'

Decl	8°	9°	10°	11°	12°	13°	14°
32	5°2'	5°41'	6°20'	6°59'	7°38'	8°18'	8°58'

Continuation of the Table of Ascensional Differences [Residuum Tabulæ Differentiarum Ascensionalium]

Same argument (declination 1°–32°); the elevation of the pole here runs 15°–30° in two stacked blocks.

Elevation of the pole 15°–22° (declination 1°–32°)

De cl.	15°	16°	17°	18°	19°	20°	21°	22°
1	0°16'	0°17'	0°18'	0°19'	0°21'	0°22'	0°23'	0°24'
2	0°32'	0°34'	0°37'	0°39'	0°41'	0°44'	0°46'	0°49'
3	0°48'	0°52'	0°55'	0°59'	1°2'	1°6'	1°9'	1°13'
4	1°4'	1°9'	1°14'	1°18'	1°23'	1°27'	1°32'	1°37'
5	1°21'	1°26'	1°32'	1°38'	1°44'	1°49'	1°55'	2°2'
6	1°37'	1°44'	1°50'	1°57'	2°4'	2°12'	2°19'	2°26'
7	1°57'	2°1'	2°9'	2°17'	2°25'	2°34'	2°42'	2°51'
8	2°9'	2°19'	2°28'	2°37'	2°46'	2°56'	3°6'	3°15'
9	2°26'	2°39'	2°47'	2°57'	3°8'	3°18'	3°29'	3°40'
10	2°42'	2°54'	3°5'	3°17'	3°29'	3°41'	3°53'	4°5'
11	2°59'	3°12'	3°24'	3°37'	3°50'	4°3'	4°17'	4°30'
12	3°16'	3°30'	3°44'	3°58'	4°12'	4°26'	4°41'	4°56'
13	3°33'	3°48'	4°3'	4°18'	4°34'	4°49'	5°5'	5°21'
14	3°50'	4°6'	4°22'	4°39'	4°55'	5°12'	5°30'	5°47'
15	4°7'	4°24'	4°42'	5°0'	5°18'	5°36'	5°54'	6°13'
16	4°24'	4°43'	5°2'	5°21'	5°40'	5°59'	6°19'	6°39'
17	4°42'	5°2'	5°22'	5°42'	6°2'	6°23'	6°44'	7°6'
18	5°0'	5°21'	5°42'	6°4'	6°25'	6°47'	7°10'	7°33'
19	5°18'	5°40'	6°3'	6°25'	6°49'	7°12'	7°36'	8°0'
20	5°36'	5°59'	6°23'	6°47'	7°12'	7°37'	8°2'	8°27'
21	5°54'	6°19'	6°44'	7°10'	7°36'	8°2'	8°28'	8°55'

De cl.	15°	16°	17°	18°	19°	20°	21°	22°
22	6°13'	6°39'	7°6'	7°33'	8°0'	8°27'	8°55'	9°24'
23	6°32'	6°59'	7°27'	7°56'	8°24'	8°53'	9°22'	9°53'
24	6°51'	7°20'	7°49'	8°19'	8°49'	9°19'	9°50'	10°22'
25	7°11'	7°41'	8°12'	8°43'	9°14'	9°46'	10°19'	10°52'
26	7°31'	8°2'	8°35'	9°7'	9°40'	10°14'	10°47'	11°22'
27	7°51'	8°24'	8°58'	9°32'	10°6'	10°41'	11°17'	11°53'
28	8°11'	8°46'	9°21'	9°57'	10°33'	11°9'	11°47'	12°24'
29	8°32'	9°9'	9°45'	10°23'	11°0'	11°38'	12°17'	12°56'
30	8°54'	9°32'	10°10'	10°49'	11°28'	12°8'	12°48'	13°29'
31	9°16'	9°55'	10°35'	11°16'	11°56'	12°38'	13°20'	14°3'
32	9°38'	10°19'	11°1'	11°43'	12°25'	13°9'	13°53'	14°37'

Elevation of the pole 23°–30° (declination 1°–32°)

De cl.	23°	24°	25°	26°	27°	28°	29°	30°
1	0°25'	0°27'	0°28'	0°29'	0°31'	0°32'	0°33'	0°35'
2	0°51'	0°53'	0°56'	0°59'	1°1'	1°4'	1°7'	1°9'
3	1°17'	1°20'	1°24'	1°28'	1°32'	1°36'	1°40'	1°44'
4	1°42'	1°47'	1°52'	1°57'	2°3'	2°8'	2°13'	2°19'
5	2°8'	2°14'	2°20'	2°27'	2°33'	2°40'	2°47'	2°54'
6	2°33'	2°41'	2°49'	2°56'	3°4'	3°12'	3°20'	3°29'
7	2°59'	3°8'	3°17'	3°26'	3°35'	3°45'	3°54'	4°4'
8	3°25'	3°35'	3°45'	3°56'	4°6'	4°17'	4°28'	4°39'
9	3°51'	4°3'	4°14'	4°26'	4°38'	4°50'	5°2'	5°15'
10	4°18'	4°30'	4°43'	4°56'	5°9'	5°23'	5°37'	5°51'
11	4°44'	4°58'	5°12'	5°26'	5°41'	5°56'	6°11'	6°27'
12	5°11'	5°26'	5°41'	5°57'	6°13'	6°29'	6°46'	7°3'
13	5°38'	5°54'	6°11'	6°28'	6°45'	7°3'	7°21'	7°40'
14	6°5'	6°22'	6°41'	6°59'	7°18'	7°37'	7°56'	8°17'
15	6°32'	6°51'	7°11'	7°31'	7°51'	8°11'	8°32'	8°54'
16	6°59'	7°20'	7°41'	8°3'	8°24'	8°46'	9°8'	9°32'

De cl.	23°	24°	25°	26°	27°	28°	29°	30°
17	7°27'	7°49'	8°12'	8°35'	8°58'	9°21'	9°45'	10°10'
18	7°56'	8°19'	8°43'	9°7'	9°32'	9°57'	10°23'	10°49'
19	8°24'	8°49'	9°14'	9°40'	10°6'	10°33'	11°0'	11°28'
20	8°53'	9°19'	9°46'	10°14'	10°41'	11°9'	11°38'	12°8'
21	9°23'	9°50'	10°19'	10°47'	11°17'	11°46'	12°17'	12°48'
22	9°53'	10°22'	10°52'	11°22'	11°53'	12°24'	12°56'	13°29'
23	10°23'	10°54'	11°25'	11°57'	12°29'	13°3'	13°37'	14°11'
24	10°54'	11°26'	11°59'	12°33'	13°7'	13°42'	14°17'	14°54'
25	11°25'	11°59'	12°33'	13°9'	13°45'	14°21'	14°59'	15°37'
26	11°57'	12°34'	13°9'	13°46'	14°23'	15°3'	15°41'	16°21'
27	12°29'	13°7'	13°45'	14°23'	15°3'	15°43'	16°24'	17°6'
28	13°3'	13°42'	14°21'	15°2'	15°43'	16°25'	17°8'	17°53'
29	13°37'	14°17'	14°59'	15°41'	16°24'	17°8'	17°54'	18°40'
30	14°11'	14°54'	15°37'	16°21'	17°6'	17°53'	18°40'	19°28'
31	14°47'	15°31'	16°16'	17°2'	17°50'	18°38'	19°27'	20°18'
32	15°23'	16°9'	16°56'	17°45'	18°34'	19°24'	20°16'	21°9'

Continuation of the Table of Ascensional Differences [Residuum Tabulæ Differentiarum Ascensionalium]

Same argument (declination 1°–32°); the elevation of the pole here runs 31°–44° in two stacked blocks.

Elevation of the pole 31°–37° (declination 1°–32°)

Dec l.	31°	32°	33°	34°	35°	36°	37°
1	0°36'	0°37'	0°39'	0°40'	0°42'	0°44'	0°45'
2	1°12'	1°15'	1°18'	1°21'	1°24'	1°27'	1°31'
3	1°48'	1°53'	1°57'	2°2'	2°6'	2°11'	2°16'
4	2°24'	2°30'	2°36'	2°42'	2°48'	2°55'	3°1'
5	3°1'	3°8'	3°15'	3°23'	3°31'	3°39'	3°47'
6	3°37'	3°46'	3°55'	4°4'	4°13'	4°23'	4°33'

Dec							
l.	31°	32°	33°	34°	35°	36°	37°
7	4°14'	4°24'	4°34'	4°45'	4°56'	5°7'	5°19'
8	4°51'	5°2'	5°14'	5°26'	5°39'	5°52'	6°5'
9	5°28'	5°41'	5°54'	6°8'	6°22'	6°36'	6°51'
10	6°5'	6°20'	6°35'	6°50'	7°6'	7°22'	7°38'
11	6°42'	6°59'	7°15'	7°32'	7°49'	8°7'	8°25'
12	7°20'	7°38'	7°56'	8°15'	8°34'	8°53'	9°13'
13	7°58'	8°18'	8°37'	8°58'	9°18'	9°39'	10°1'
14	8°37'	8°58'	9°19'	9°41'	10°3'	10°26'	10°50'
15	9°16'	9°38'	10°1'	10°25'	10°49'	11°14'	11°39'
16	9°55'	10°19'	10°44'	11°9'	11°35'	12°2'	12°29'
17	10°35'	11°1'	11°27'	11°54'	12°22'	12°50'	13°19'
18	11°19'	11°43'	12°11'	12°40'	13°9'	13°39'	14°10'
19	11°56'	12°25'	12°55'	13°26'	13°57'	14°29'	15°2'
20	12°38'	13°9'	13°40'	14°13'	14°46'	15°20'	15°55'
21	13°20'	13°53'	14°26'	15°0'	15°36'	16°12'	16°49'
22	14°3'	14°37'	15°13'	15°49'	16°27'	17°5'	17°44'
23	14°47'	15°23'	16°0'	16°38'	17°17'	17°58'	18°39'
24	15°31'	16°9'	16°48'	17°29'	18°10'	18°52'	19°36'
25	16°16'	16°56'	17°38'	18°20'	19°3'	19°48'	20°34'
26	17°2'	17°45'	18°28'	19°12'	19°58'	20°45'	21°34'
27	17°50'	18°34'	19°19'	20°6'	20°54'	21°44'	22°35'
28	18°38'	19°24'	20°12'	21°1'	21°51'	22°43'	23°37'
29	19°27'	20°16'	21°6'	21°57'	22°50'	23°45'	24°41'
30	20°18'	21°9'	22°1'	22°55'	23°51'	24°48'	25°47'
31	21°10'	22°3'	22°58'	23°55'	24°53'	25°53'	26°55'
32	22°3'	22°59'	23°56'	24°56'	25°57'	27°0'	28°5'

Elevation of the pole 38°–44° (declination 1°–32°)

Dec							
l.	38°	39°	40°	41°	42°	43°	44°
1	0°47'	0°49'	0°50'	0°52'	0°54'	0°56'	0°58'

Dec							
1.	38°	39°	40°	41°	42°	43°	44°
2	1°34'	1°37'	1°41'	1°44'	1°48'	1°52'	1°56'
3	2°21'	2°26'	2°31'	2°37'	2°42'	2°48'	2°54'
4	3°8'	3°15'	3°22'	3°29'	3°37'	3°44'	3°52'
5	3°55'	4°4'	4°13'	4°22'	4°31'	4°41'	4°51'
6	4°43'	4°53'	5°4'	5°15'	5°26'	5°37'	5°50'
7	5°30'	5°42'	5°55'	6°8'	6°21'	6°34'	6°49'
8	6°18'	6°32'	6°46'	7°1'	7°16'	7°32'	7°48'
9	7°6'	7°22'	7°38'	7°55'	8°12'	8°30'	8°48'
10	7°55'	8°13'	8°30'	8°49'	9°8'	9°28'	9°48'
11	8°44'	9°3'	9°23'	9°44'	10°5'	10°27'	10°49'
12	9°34'	9°55'	10°16'	10°39'	11°2'	11°26'	11°51'
13	10°24'	10°46'	11°10'	11°35'	12°0'	12°26'	12°53'
14	11°14'	11°39'	12°5'	12°31'	12°58'	13°27'	13°56'
15	12°5'	12°32'	13°0'	13°28'	13°58'	14°28'	15°0'
16	12°57'	13°26'	13°55'	14°26'	14°58'	15°31'	16°5'
17	13°49'	14°20'	14°52'	15°25'	15°59'	16°34'	17°10'
18	14°42'	15°15'	15°49'	16°24'	17°1'	17°38'	18°17'
19	15°36'	16°11'	16°48'	17°25'	18°4'	18°44'	19°25'
20	16°31'	17°8'	17°47'	18°27'	19°8'	19°50'	20°35'
21	17°27'	18°7'	18°47'	19°30'	20°13'	20°59'	21°46'
22	18°24'	19°6'	19°49'	20°34'	21°20'	22°8'	22°58'
23	19°22'	20°6'	20°52'	21°39'	22°28'	23°19'	24°12'
24	20°21'	21°8'	21°56'	22°46'	23°38'	24°32'	25°28'
25	21°21'	22°11'	23°2'	23°55'	24°50'	25°47'	26°46'
26	22°24'	23°16'	24°10'	25°5'	26°3'	27°3'	28°6'
27	23°28'	24°22'	25°19'	26°17'	27°18'	28°22'	29°29'
28	24°33'	25°30'	26°30'	27°31'	28°36'	29°44'	30°54'
29	25°40'	26°40'	27°43'	28°48'	29°56'	31°8'	32°22'
30	26°49'	27°52'	28°59'	30°7'	31°19'	32°35'	33°53'
31	28°0'	29°7'	30°17'	31°29'	32°45'	34°5'	35°28'

Dec							
1.	38°	39°	40°	41°	42°	43°	44°
32	29°13'	30°54' [?]	31°31'	32°54'	34°14'	35°38'	37°7'

Christoph Clavius, *Commentary on the Sphere of Sacrobosco* — Chapter III (continued)

Table of Ascensional Differences [Tabula Differentiarum Ascensionalium] — conclusion

(Continuation and conclusion of the table begun in the preceding pages. Row argument = degrees of declination [Gradus Declinationum], 1–32; column parameter = elevation of the pole [Elevatio Poli]. Each entry is in degrees° and minutes'. This page carries two stacked blocks: elevations 45°–52° above, 53°–60° below.)

Elevations of the pole 45°–52° (declination 1°–32°)

De cl.	45°	46°	47°	48°	49°	50°	51°	52°
1	1°0'	1°2'	1°4'	1°7'	1°9'	1°12'	1°14'	1°17'
2	2°0'	2°4'	2°9'	2°13'	2°18'	2°23'	2°28'	2°34'
3	3°0'	3°7'	3°13'	3°20'	3°27'	3°35'	3°43'	3°51'
4	4°1'	4°9'	4°18'	4°27'	4°37'	4°47'	4°57'	5°8'
5	5°1'	5°12'	5°23'	5°35'	5°47'	5°50' [?]	6°12'	6°26'
6	6°2'	6°15'	6°28'	6°42'	6°57'	7°12'	7°27'	7°44'
7	7°3'	7°18'	7°34'	7°50'	8°7'	8°25'	8°43'	9°2'
8	8°5'	8°22'	8°30' [?]	8°59'	9°18'	9°38'	10°0'	10°22'
9	9°7'	9°26'	9°47'	10°8'	10°30'	10°53'	11°17'	11°42'
10	10°9'	10°31'	10°54'	11°18'	11°42'	12°8'	12°35'	13°3'
11	11°13'	11°37'	12°2'	12°28'	12°55'	13°24'	13°53'	14°24'
12	12°16'	12°43'	13°11'	13°39'	14°9'	14°40'	15°13'	15°47'
13	13°21'	13°50'	14°20'	14°51'	15°24'	15°58'	16°34'	17°11'
14	14°26'	14°58'	15°30'	16°5'	16°40' [?]	17°17'	17°56'	18°37'
15	15°32'	16°7'	16°42'	17°19'	17°57'	18°39'	19°19'	20°4'

De cl.	45°	46°	47°	48°	49°	50°	51°	52°
16	16°40′	17°16′	17°54′	18°34′	19°16′	19°59′	20°44′	21°32′
17	17°48′	18°27′	19°8′	19°51′	20°36′	21°22′	22°11′	23°2′
18	18°58′	19°40′	20°23′	21°9′	21°57′	22°47′	23°39′	24°34′
19	20°9′	20°53′	21°40′	22°29′	23°20′	24°14′	25°10′	26°9′
20	21°21′	22°8′	22°58′	23°51′	24°45′	25°42′	26°43′	27°46′
21	22°34′	23°25′	24°18′	25°14′	26°12′	27°14′	28°18′	29°26′
22	23°50′	24°44′	25°40′	26°40′	27°42′	28°47′	29°56′	31°8′
23	25°7′	26°5′	27°5′	28°8′	29°14′	30°23′	31°37′	32°54′
24	26°26′	27°27′	28°31′	29°38′	30°48′	32°3′	33°21′	34°44′
25	27°48′	28°52′	30°0′	31°12′	32°26′	33°46′	35°10′	36°39′
26	29°11′	30°20′	31°32′	32°48′	34°8′	35°32′	37°2′	38°38′
27	30°38′	31°51′	33°7′	34°28′	35°53′	37°23′	39°0′	40°42′
28	32°7′	33°21′	34°46′	36°12′	37°43′	39°19′	41°2′	42°53′
29	33°40′	35°2′	36°28′	38°0′	39°47′	41°21′	43°12′	45°12′
30	35°16′	36°43′	38°15′	39°53′	41°47′	43°29′	45°29′	47°39′
31	36°56′	38°29′	39°7′ [?]	41°52′	43°44′	45°44′	47°54′	50°10′
32	38°40′	40°19′	42°4′	43°57′	45°57′	48°8′	50°30′	53°7′

Elevations of the pole 53°–60° (declination 1°–32°)

De cl.	53°	54°	55°	56°	57°	58°	59°	60°
1	1°20′	1°23′	1°26′	1°29′	1°32′	1°36′	1°40′	1°44′
2	1°39′ [?]	2°45′	2°52′	2°58′	3°5′	3°12′	3°20′	3°28′
3	3°59′	4°8′	4°17′	4°27′	4°38′	4°49′	5°0′	5°13′
4	5°19′	5°31′	5°44′	5°57′	6°11′	6°25′	6°41′	6°57′
5	6°40′	6°55′	7°11′	7°27′	7°44′	8°3′	8°22′	8°43′
6	8°1′	8°19′	8°38′	8°58′	9°19′	9°41′	10°4′	10°28′
7	9°23′	9°44′	10°6′	10°29′	10°54′	11°20′	11°47′	12°17′
8	10°45′	11°9′	11°35′	12°1′	12°30′	13°0′	13°32′	14°5′
9	12°8′	12°35′	13°4′	13°35′	14°7′	14°41′	15°17′	15°55′
10	13°32′	14°3′	14°35′	15°9′	15°45′	16°23′	17°4′	17°47′

De cl.	53°	54°	55°	56°	57°	58°	59°	60°
11	14°57'	15°31'	16°7'	16°45'	17°25'	18°8'	18°53'	19°41'
12	16°23'	17°0'	17°40'	18°22'	19°6'	19°53'	20°43'	21°36'
13	17°50'	18°32'	19°15'	20°1'	20°50'	21°41'	22°36'	23°34'
14	19°19'	20°4'	20°52'	21°42'	22°35'	23°31'	24°31'	25°33'
15	20°20'	21°38'	22°30'	23°24'	24°22'	25°23'	26°29'	27°39'
16	22°22'	23°15'	24°10'	25°9'	26°12'	27°19'	28°30'	29°47'
17	23°56'	25°53' [?]	25°53'	26°57'	28°5'	29°18'	30°35'	31°59'
18	25°33'	26°34'	27°39'	28°48'	30°1'	31°20'	32°44'	34°19'
19	27°11'	28°17'	29°27'	30°41'	32°1'	33°26'	34°54'	36°37'
20	28°53'	30°4'	31°19'	32°39'	34°5'	35°37'	37°17'	39°5'
21	30°37'	31°54'	33°15'	34°41'	36°14'	37°54'	39°42'	41°40'
22	32°25'	33°47'	35°14'	36°48'	38°28'	40°17'	42°15'	44°25'
23	34°17'	35°45'	37°19'	39°0'	40°49'	42°47'	44°57'	47°20'
24	36°13'	37°48'	39°29'	41°18'	43°17'	45°26'	47°49'	50°27'
25	38°14'	39°59'	41°45'	43°44'	45°54'	48°16'	50°54'	53°52'
26	40°20'	42°10'	44°9'	46°18'	48°41'	51°19'	54°16'	57°39'
27	42°33'	44°32'	46°41'	49°4'	51°41'	54°38'	58°0'	61°57'
28	44°53'	47°2'	49°24'	52°1'	54°58'	58°19'	62°14'	67°4'
29	47°21'	49°44'	52°20'	55°16'	58°36'	62°31'	67°18'	73°46'
30	50°1'	52°37'	55°32'	58°52'	62°45'	67°31'	73°55'	90°0'
31	52°53'	55°48'	59°6'	62°58'	67°42'	74°4'	90°0'	90°0'
32	56°2'	59°19'	63°10'	67°53'	74°12'	90°0'	90°0'	90°0'

Table of Oblique Ascensions for latitude 36° [Tabula Ascensionum obliquarum ad latitudinem graduum 36]

(Row argument = degree within the zodiacal sign [G], 0–30; each entry is the oblique ascension in degrees° and minutes'. The full circle of twelve signs is split into an upper block, Aries–Virgo, and a lower block, Libra–Pisces.)

Signs ♈ Aries – ♍ Virgo

De

g.	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0°0′	19°24′	42°18′	71°35′	106°42′	143°36′
1	0°37′	20°5′	43°10′	72°41′	107°55′	144°50′
2	1°15′	20°46′	44°2′	73°47′	109°9′	146°3′
3	1°52′	21°28′	44°55′	74°53′	110°22′	147°17′
4	2°30′	22°10′	45°48′	76°0′	111°36′	148°30′
5	3°8′	22°52′	46°42′	77°7′	112°50′	149°43′
6	3°46′	23°35′	47°36′	78°25′ [?]	114°3′	150°57′
7	4°24′	24°18′	48°30′	79°23′	115°17′	152°10′
8	5°2′	25°1′	49°25′	80°31′	116°30′	153°23′
9	5°40′	25°45′	50°20′	81°40′	117°44′	154°36′
10	6°18′	26°29′	51°16′	82°49′	118°58′	155°49′
11	6°56′	27°13′	52°12′	83°58′	120°12′	157°2′
12	7°34′	27°57′	53°9′	85°8′	121°26′	158°15′
13	8°12′	28°41′	54°7′	86°18′	122°40′	159°28′
14	8°50′	29°26′	55°5′	87°28′	123°55′	160°41′
15	9°29′	30°11′	56°4′	88°38′	125°9′	161°53′
16	10°7′	30°57′	57°3′	89°49′	126°23′	163°6′
17	10°46′	31°43′	58°2′	91°0′	127°37′	164°19′
18	11°25′	32°30′	59°2′	92°11′	128°51′	165°31′
19	12°4′	33°17′	60°2′	93°22′	130°5′	166°44′
20	12°43′	34°4′	61°3′	94°34′	131°19′	167°56′
21	13°22′	34°52′	62°4′	95°46′	132°33′	169°9′
22	14°1′	35°40′	63°6′	96°58′	133°47′	170°21′
23	14°41′	36°28′	64°8′	98°10′	135°1′	171°34′
24	15°21′	37°17′	65°10′	99°23′	136°15′	172°46′
25	16°1′	38°6′	66°13′	100°36′	137°28′	173°58′
26	16°41′	38°56′	67°16′	101°49′	138°42′	175°11′
27	17°21′	39°46′	68°20′	103°2′	139°56′	176°23′
28	18°2′	40°36′	69°24′	104°15′	141°9′	177°36′

De g.	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
29	18°43′	41°27′	70°29′	105°28′	142°23′	178°40′ [?]
30	19°24′	42°18′	71°35′	106°42′	143°36′	180°0′

Signs ♎ Libra – ♐ Pisces

D e g.	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
0	180°9′ [!]	216°24′	253°18′	288°25′	317°42′	340°36′
1	181°12′	217°37′	254°32′	289°31′	318°33′	341°17′
2	182°24′	218°51′	255°45′	290°36′	319°24′	341°58′
3	183°37′	220°4′	256°58′	291°40′	320°14′	342°39′
4	184°49′	221°18′	258°11′	292°44′	321°4′	343°19′
5	186°2′	222°32′	259°24′	293°47′	321°54′	343°59′
6	187°14′	223°45′	260°37′	294°50′	322°43′	344°30′ [?]
7	188°26′	224°59′	261°50′	295°52′	323°32′	345°19′
8	189°39′	226°13′	263°2′	296°54′	324°20′	345°59′
9	190°51′	227°27′	264°14′	297°56′	325°8′	346°38′
10	192°4′	228°41′	265°26′	298°57′	325°56′	347°17′
11	193°16′	229°55′	266°38′	299°58′	326°43′	347°56′
12	194°29′	231°9′	267°49′	300°58′	327°30′	348°35′
13	195°41′	232°23′	269°0′	301°58′	328°17′	349°14′
14	196°54′	233°37′	270°11′	302°57′	329°3′	349°53′
15	198°7′	234°51′	271°22′	303°56′	329°49′	350°31′
16	199°19′	236°5′	272°32′	304°55′	330°34′	351°10′
17	200°32′	237°20′	273°42′	305°53′	331°19′	351°48′

D e g.	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
7						
1	201°45'	238°34'	274°52'	306°51'	332°3'	352°26'
8						
1	202°58'	239°48'	276°2'	307°48'	332°47'	353°4'
9						
2	204°11'	241°2'	277°11'	308°44'	333°31'	353°42'
0						
2	205°24'	242°16'	278°20'	309°[?]'	334°15'	354°20'
1						
2	206°37'	243°30'	279°29'	310°35'	334°59'	354°58'
2						
2	207°50'	244°43'	280°37'	311°30'	335°42'	355°36'
3						
2	209°3'	245°57'	281°45'	312°24'	336°25'	356°14'
4						
2	210°17'	247°10'	282°53'	313°18'	337°8'	356°52'
5						
2	211°30'	248°25'	284°0'	314°12'	337°50'	357°30'
6						
2	212°43'	249°48' [?]	285°7'	315°5'	338°32'	358°8'
7						
2	213°57'	250°51'	286°13'	315°58'	339°14'	358°45'
8						
2	215°10'	252°5'	287°19'	316°50'	339°55'	359°23'
9						
3	216°24'	253°18'	288°25'	317°42'	340°36'	360°0'
0						
1						

Table of Oblique Ascensions for latitude 37° [Tabula Ascensionum obliquarum ad latitudinem Graduum 37]

(Same structure: degree within the sign [G], 0–30; oblique ascension in degrees° and minutes'.)

Signs ♈ Aries – ♍ Virgo

D eg .	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0°0′	19°5′	41°42′	70°52′	106°6′	143°17′
1	0°37′	19°46′	42°34′	71°58′	107°20′	144°31′
2	1°14′	20°27′	43°26′	73°4′	108°34′	145°45′
3	1°51′	21°8′	44°18′	74°11′	109°48′	146°59′
4	2°28′	21°49′	45°11′	75°18′	111°2′	148°13′
5	3°5′	22°30′	46°4′	76°25′	112°16′	149°27′
6	3°42′	23°12′	46°58′	77°33′	113°30′	150°41′
7	4°19′	23°54′	47°52′	78°41′	114°44′	151°55′
8	4°56′	24°37′	48°47′	79°49′	115°59′	153°9′
9	5°33′	25°20′	49°42′	80°58′	117°13′	154°23′
10	6°11′	26°3′	50°37′	82°7′	118°28′	155°36′
11	6°48′	26°46′	51°33′	83°16′	119°42′	156°50′
12	7°26′	27°30′	52°30′	84°26′	120°57′	158°3′
13	8°3′	28°14′	53°27′	85°36′	122°11′	159°17′
14	8°41′	28°58′	54°25′	86°46′	123°26′	160°30′
15	9°19′	29°43′	55°23′	87°57′	124°41′	161°43′
16	9°57′	30°28′	56°22′	89°8′	125°56′	162°57′
17	10°35′	31°14′	57°21′	90°19′	127°10′	164°10′
18	11°13′	32°0′	58°21′	91°31′	128°25′	165°23′
19	11°51′	32°47′	59°21′	92°43′	129°39′	166°36′
20	12°30′	33°34′	60°21′	93°55′	130°53′	167°49′
21	13°9′	34°21′	61°22′	95°7′	132°8′	169°3′
22	13°48′	35°8′	62°24′	96°19′	133°23′	170°16′
23	14°27′	35°56′	63°26′	97°32′	134°37′	171°29′

D eg .	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
24	15°6′	36°44′	64°28′	98°45′	135°52′	171°18′ [?]
25	15°45′	37°32′	65°31′	99°58′	127°6′ [!]	173°55′
26	16°25′	38°21′	66°34′	101°11′	138°24′	175°8′
27	17°5′	39°10′	67°38′	102°24′	139°35′	176°21′
28	17°45′	40°0′	68°24′ [?]	103°38′	140°49′	177°34′
29	18°25′	40°51′	69°47′	104°52′	142°3′	178°47′
30	19°5′	41°42′	70°52′	106°6′	143°17′	180°0′

Signs ♎ Libra – ♉ Pisces

D eg .	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
0	180°0′	216°43′	253°54′	289°8′	318°18′	340°55′
1	181°13′	217°57′	255°8′	290°13′	319°9′	341°55′ [?]
2	182°26′	219°11′	256°22′	291°18′	320°0′	342°15′
3	183°39′	220°25′	257°36′	292°22′	320°50′	342°55′
4	184°52′	221°39′	258°49′	293°26′	321°39′	343°35′
5	186°5′	222°54′	260°2′	294°29′	322°28′	344°15′
6	187°18′	224°8′	261°15′	295°32′	323°16′	344°54′
7	188°31′	225°23′	262°28′	296°34′	324°4′	345°33′
8	189°44′	226°37′	263°41′	297°36′	324°52′	346°12′
9	190°57′	227°52′	264°53′	298°38′	325°39′	346°51′
10	192°11′	229°7′	265°5′ [?]	299°39′	326°26′	347°30′
11	193°24′	230°21′	266°17′	300°39′	327°13′	348°9′
12	194°37′	231°35′	268°29′ [?]	301°39′	328°0′	348°47′
13	195°50′	232°50′	269°41′	302°39′	328°46′	349°25′

D eg .	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
1 4	197°3′	234°4′	270°52′	303°38′	329°32′	350°3′
1 5	198°17′	235°19′	272°3′	304°37′	330°17′	350°41′
1 6	199°30′	236°34′	273°14′	305°35′	331°2′	351°19′
1 7	200°43′	237°49′	274°24′	306°33′	331°46′	351°57′
1 8	201°57′	239°3′	275°34′	307°30′	332°30′	352°34′
1 9	203°10′	240°18′	276°44′	308°27′	333°14′	353°12′
2 0	204°24′	241°32′	277°53′	309°23′	333°57′	353°49′
2 1	205°37′	242°47′	279°2′	310°18′	334°40′	354°27′
2 2	206°51′	244°1′	280°11′	311°13′	335°23′	355°4′
2 3	208°5′	245°16′	281°19′	312°8′	336°6′	355°41′
2 4	209°19′	246°30′	282°27′	313°2′	336°48′	356°18′
2 5	210°33′	247°44′	283°35′	313°56′	337°30′	356°55′
2 6	211°47′	248°58′	284°42′	314°49′	338°11′	357°32′
2 7	213°1′	250°12′	285°49′	315°42′	338°52′	358°9′
2 8	214°15′	251°26′	286°56′	316°34′	339°33′	358°46′
2 9	215°29′	252°40′	288°2′	317°26′	340°14′	359°23′

D eg .	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
3 0	216°43'	253°54'	289°8'	318°18'	340°55'	360°0'

These three pages continue the **Tables of Oblique Ascensions** [Tabula Ascensionum obliquarum], giving for each degree of the ecliptic (argument 0–30 within each sign) the oblique ascension in degrees and minutes at three successive elevations of the pole: latitude 38°, 39°, and 40°. Each table is laid out in two blocks — the first six signs (♈ Aries through ♍ Virgo) above, the last six (♎ Libra through ♓ Pisces) below. Within every column, **G** = degrees, **M** = minutes; the leftmost column is the ecliptic argument in degrees. The printed opening runs latitude 39° on the verso (p. 212) facing latitude 40° on the recto (p. 213), with latitude 38° on the preceding recto (p. 211); the tables are presented here in ascending order 38° → 39° → 40°, each reproduced from its own page.

Table of Oblique Ascensions at latitude 38°

Tabula Ascensionum obliquarum ad latitudinem graduum 38.

Upper block — signs ♈ Aries · ♉ Taurus · ♊ Gemini · ♋ Cancer · ♌ Leo · ♍ Virgo

°	♈ G	♈ M	♉ G	♉ M	♊ G	♊ M	♋ G	♋ M	♌ G	♌ M	♍ G	♍ M
0 0	0	18	45	41	6	70	8	105	30	142	57	
1 0	36	19	25	41	57	71	14	106	44	144	12	
2 1	12	20	5	42	48	72	20	107	58	145	27	
3 1	49	20	45	43	40	73	27	109	13	146	41	
4 2	25	21	26	44	32	74	34	110	27	147	56	
5 3	2	22	7	45	25	75	41	111	42	149	10	
6 3	38	22	49	46	18	76	49	112	56	150	25	
7 4	14	23	31	47	12	77	13 [?]	114	11	151	40	
8 4	51	24	13	48	6	79	6	115	26	152	54	
9 5	27	24	55	49	1	80	15	116	41	154	9	
1 6	4	25	38	49	57	81	24	117	56	155	23	
0												
1 6	41	26	21	50	53	82	34	119	11	156	37	

°	Υ G	Υ M	ϣ G	ϣ M	Π G	Π M	⊞ G	⊞ M	Ϻ G	Ϻ M	Ⓜ G	Ⓜ M
1												
1	7	18	27	4	51	49	83	44	120	27	157	51
2												
1	7	55	27	47	52	46	84	54	121	43	159	5
3												
1	8	32	28	31	53	43	86	4	122	58	160	19
4												
1	9	9	29	15	54	41	87	15	124	13	161	33
5												
1	9	46	30	0	55	39	88	26	125	28	162	47
6												
1	10	24	30	45	56	38	89	38	126	43	164	1
7												
1	11	1	31	30	57	37	90	50	127	58	165	15
8												
1	11	39	32	16	58	37	92	2	129	13	166	29
9												
2	12	17	33	2	59	38	93	15	130	28	167	42
0												
2	12	55	33	48	60	39	94	27	131	43	168	56
1												
2	13	33	34	35	61	40	95	40	132	58	170	10
2												
2	14	11	35	22	62	42	96	53	134	13	171	24
3												
2	14	49	36	10	63	44	98	6	135	28	172	38
4												
2	15	28	36	58	64	47	99	19	136	43	173	52
5												
2	16	7	37	47	65	50	100	33	137	58	175	6
6												
2	16	46	38	36	66	54	101	47	139	13	176	20
7												
2	17	25	39	28	67	58	103	1	140	28	177	33

[illegible]

°	⊥ G	⊥ M	⌢ G	⌢ M	⌘ G	⌘ M	⌚ G	⌚ M	⌛ G	⌛ M	⌜ G	⌜ M
0	180	0	217	3	254	30	289	52	318	54	341	15
1	181	23 [?]	218	17	255	45	290	57	319	44	341	55
2	182	27	219	32	256	59	292	2	320	34	342	35
3	183	40	220	47	258	13	293	6	321	24	343	14
4	184	54	222	2	259	27	294	10	322	13	343	53
5	186	8	223	17	260	41	295	13	323	2	344	32
6	187	22	224	32	261	52	296	16	323	50	345	11
7	188	36	225	47	263	7	297	18	324	38	345	49
8	189	50	227	2	264	20	298	20	325	25	346	27
9	191	4	228	17	265	33	299	21	326	12	347	5
10	192	18	229	32	266	45	300	22	326	58	347	43
11	193	31	230	47	267	58	301	23	327	44	348	21
12	194	45	232	2	269	10	302	23	328	30	348	59
13	195	59	233	17	270	22	303	22	329	15	349	36
14	197	13	234	32	271	34	304	21	330	0	350	14
15	198	27	235	47	272	45	305	19	330	45	350	51
16	199	41	237	2	273	56	306	17	331	29	351	28

°	♌ G	♌ M	♍ G	♍ M	♎ G	♎ M	♏ G	♏ M	♐ G	♐ M	♑ G	♑ M
1 7	200	55	238	17	275	6	307	14	332	13	352	5
1 8	202	9	239	33	276	16	308	11	332	56	352	42
1 9	203	23	240	49	277	26	309	7	333	39	353	19
2 0	204	37	242	4	278	36	310	3	334	22	353	56
2 1	205	51	243	19	279	45	310	59	335	5	354	33
2 2	207	6	244	34	280	54	311	54	335	47	355	9
2 3	208	20	245	49	282	3	312	48	336	29	355	46
2 4	209	35	247	4	283	11	313	42	337	11	356	22
2 5	210	50	248	18	284	19	314	35	337	53	356	58
2 6	212	4	249	33	285	26	315	28	338	34	357	35
2 7	213	19	250	47	286	33	316	20	339	15	358	11
2 8	214	33	252	2	287	40 [?]	317	12	339	55	358	48
2 9	215	48	253	16	288	46	318	3	340	35	359	24
3 0	217	3	254	30	289	52	318	54	341	15	360	0

Table of Oblique Ascensions at latitude 39°

Tabula Ascensionum obliquarum ad latitudinem Graduum 39.

Upper block — signs ♈ Aries · ♉ Taurus · ♊ Gemini · ♋ Cancer · ♌ Leo
· ♍ Virgo

°	♈ G	♈ M	♉ G	♉ M	♊ G	♊ M	♋ G	♋ M	♌ G	♌ M	♍ G	♍ M
0	0	0	18	25	40	28	69	23	104	52	142	37
1	0	35	19	4	41	19	70	29	106	7	143	53
2	1	11	19	44	42	10	71	35	107	22	145	8
3	1	46	20	24	43	2	72	42	108	37	146	24
4	2	22	21	4	43	54	73	49	109	52	147	39
5	2	58	21	44	44	46	74	56	111	7	148	54
6	3	34	22	25	45	39	76	4	112	22	150	9
7	4	10	23	6	46	32	77	12	113	37	151	24
8	4	46	23	47	47	26	78	21	114	53	152	39
9	5	22	24	29	48	20	79	30	116	8	153	54
10	5	58	25	11	49	15	80	39	117	24	155	9
11	6	34	25	53	50	10	81	49	118	39	156	24
12	7	10	26	26	51	6	82	59	119	55	157	39
13	7	46	27	19	52	3	84	10	121	11	158	54
14	8	22	28	2	53	9	85	21	122	27	160	9
15	8	59	28	45	53	58	86	32	123	43	161	23
16	9	35	29	29	54	56	87	44	124	59	162	38
17	10	12	30	13	55	55	88	56	126	15	163	53
18	10	49	30	58	56	54	90	8	127	30	165	7
19	11	26	31	44	57	53	91	20	128	46	166	22
20	12	3	32	30	58	53	92	33	130	1	167	39

°	♈ G	♈ M	♉ G	♉ M	♊ G	♊ M	♋ G	♋ M	♌ G	♌ M	♍ G	♍ M
0												
2	12	40	33	16	59	54	93	46	131	17	168	51
1												
2	13	18	34	2	60	55	94	59	132	33	170	5
2												
2	13	56	34	49	61	57	96	12	133	49	171	20
3												
2	14	34	35	36	62	59	97	26	135	5	172	34
4												
2	15	12	36	23	64	2	98	40	136	20	173	48
5												
2	15	50	37	11	65	5	99	54	137	36	175	3
6												
2	16	28	37	59	66	9	101	8	138	51	176	17
7												
2	17	7	38	48	67	13	102	22	140	7	177	32
8												
2	17	46	39	38	68	18	103	37	141	22	178	46
9												
3	18	25	40	28	69	23	104	52	142	37	180	0
0												

Lower block — signs ♎ Libra · ♏ Scorpio · ♐ Sagittarius · ♑ Capricorn
· ♒ Aquarius · ♓ Pisces

°	♎ G	♎ M	♏ G	♏ M	♐ G	♐ M	♑ G	♑ M	♒ G	♒ M	♓ G	♓ M
0	180	0	217	23	255	8	290	37	319	32	341	35
1	181	14	218	38	256	23	291	42	320	22	342	14
2	182	28	219	53	257	38	292	47	321	12	342	53
3	183	43	221	9	258	52	293	51	322	1	343	32
4	184	57	222	24	260	6	294	55	322	49	344	10
5	186	12	223	40	261	20	295	58	323	37	344	48
6	187	26	224	55	262	34	297	1	324	24	345	26
7	188	40	226	11	263	48	298	3	325	11	346	4

°	⌒ G	⌒ M	⌛ G	⌛ M	⌜ G	⌜ M	⌝ G	⌝ M	⌞ G	⌞ M	⌟ G	⌟ M
8	189	55	227	27	265	1	299	5	325	58	346	42
9	191	9	228	43	266	14	300	6	326	44	347	20
10	192	24	229	59	267	27	301	7	327	30	347	57
11	193	38	231	14	268	40	302	7	328	16	348	34
12	194	53	232	30	269	52	303	6	329	2	349	11
13	196	7	233	45	271	4	304	5	329	47	349	47
14	197	22	235	1	272	16	305	4	330	31	350	25
15	198	37	236	17	273	28	306	2	331	15	351	1
16	199	51	237	33	274	39	307	0	331	58	351	38
17	201	6	238	49	275	50	307	57	332	41	352	14
18	202	21	240	5	277	1	308	54	333	14 [?]	352	50
19	203	36	241	21	278	11	309	50	334	7	353	26
20	204	51	242	36	279	21	310	45	334	49	354	2
21	206	6	243	52	280	30	311	40	335	31	354	38
22	207	21	245	7	281	39	312	34	336	13	355	14
23	208	36	246	23	282	48	313	28	336	54	355	50
24	209	51	247	38	283	56	314	21	337	35	356	26
25	211	6	248	53	285	4	315	14	338	16	357	2

°	♈ G	♈ M	♉ G	♉ M	♊ G	♊ M	♋ G	♋ M	♌ G	♌ M	♍ G	♍ M
212 6	21	250	8	286	11	316	6	338	56	357	38	
213 7	36	251	23	287	18	316	58	339	36	358	14	
214 8	52	252	38	288	25	317	50	340	16	358	48	
216 9	7	253	53	289	31	318	41	340	56	359	25	
217 0	23	255	8	290	37	319	32	341	35	360	0	

Table of Oblique Ascensions at latitude 40°

Tabula Ascensionum obliquarum ad latitudinem Graduum 40.

Upper block — signs ♈ Aries · ♉ Taurus · ♊ Gemini · ♋ Cancer · ♌ Leo
· ♍ Virgo

°	♈ G	♈ M	♉ G	♉ M	♊ G	♊ M	♋ G	♋ M	♌ G	♌ M	♍ G	♍ M
0 0	0	18	4	39	49	68	36	104	13	142	16	
1 0	35	18	43	40	39	69	41	105	28	143	32	
2 1	10	19	22	41	30	70	54	106	54	144	48	
3 1	45	20	1	42	21	71	57	107	59	146	4	
4 2	20	20	40	43	12	73	1	109	15	147	20	
5 2	55	21	20	44	4	74	9	110	31	148	36	
6 3	30	22	0	44	56	75	17	111	46	149	52	
7 4	5	22	41	45	49	76	25	113	2	151	8	
8 4	40	23	22	46	43	77	34	114	28	152	23	
9 5	15	24	3	47	37	78	43	115	34	153	39	
1 5 0	51	24	44	48	32	79	53	116	50	154	54	
1 6 1	29	25	26	49	27	81	3	118	6	156	10	
1 7 2	1	26	8	50	23	82	23	119	26	157	26	

°	Υ G	Υ M	ϣ G	ϣ M	Π G	Π M	⋈ G	⋈ M	♊ G	♊ M	♋ G	♋ M
1 7 3		37	26	50	51	19	83	24	120	39	158	41
1 8 4		12	27	32	52	16	84	35	121	55	159	57
1 8 5		48	28	14	53	13	85	47	123	12	161	12
1 9 6		24	28	57	54	11	86	59	124	28	162	18
1 10 7		0	29	41	55	9	88	12	125	45	163	43
1 10 8		36	30	26	56	8	89	24	127	2	164	59
1 11 9		12	31	12	57	7	90	37	128	18	166	14
2 11 0		48	31	56	58	7	91	50	129	34	167	29
2 12 1		25	32	41	59	7	93	3	130	51	168	45
2 13 2		2	33	27	60	8	94	17	132	7	170	0
2 13 3		39	34	13	61	10	95	30	133	24	171	15
2 14 4		16	35	0	62	12	96	44	134	40	172	30
2 14 5		54	35	47	63	15	97	58	135	56	173	45
2 15 6		32	36	34	64	18	99	13	137	12	175	0
2 16 7		10	37	22	65	22	100	28	138	28	176	15
2 16 8		48	38	10	66	26	101	43	139	44	177	30
2 17 9		26	38	29 [?]	67	31	102	58	141	0	178	45

°	♈ G	♈ M	♉ G	♉ M	♊ G	♊ M	♋ G	♋ M	♌ G	♌ M	♍ G	♍ M
3 0	18	4	39	49	68	36	104	13	142	16	180	0

Lower block — signs ♎ Libra · ♏ Scorpio · ♐ Sagittarius · ♑ Capricorn
· ♒ Aquarius · ♓ Pisces

°	♎ G	♎ M	♏ G	♏ M	♐ G	♐ M	♑ G	♑ M	♒ G	♒ M	♓ G	♓ M
0	180	0	217	44	255	47	291	24	320	11	341	56
1	181	15	219	0	257	2	292	29	321	1	342	34
2	182	30	220	16	258	17	293	34	321	50	343	12
3	183	45	221	32	259	32	294	38	322	38	343	50
4	185	0	222	48	260	47	295	42	323	26	344	28
5	186	15	224	4	262	2	296	45	324	13	345	6
6	187	30	225	20	263	16	297	48	325	0	345	44
7	188	45	226	36	264	30	298	50	325	47	346	21
8	190	0	227	53	265	43	299	52	326	33	346	58
9	191	15	229	9	266	57	300	53	327	19	347	35
1 0	192	31	230	26	268	10	301	53	328	4	348	12
1 1	193	46	231	42	269	23	302	53	328	49	348	48
1 2	195	1	232	58	270	36	303	52	329	34	349	24
1 3	196	17	234	15	271	48	304	51	330	19	350	0
1 4	197	32	235	32	273	1	305	49	331	3	350	36
1 5	198	48	236	48	274	13	306	47	331	46	351	12
1 6	200	3	238	5	275	25	307	44	332	28	351	48
1 7	201	19	239	21	276	36	308	41	333	10	352	23
1	202	34	240	38	277	47	309	37	333	52	352	59

°	♈ G	♈ M	♉ G	♉ M	♊ G	♊ M	♋ G	♋ M	♌ G	♌ M	♍ G	♍ M
8												
1	203	50	241	54	278	57	310	33	334	34	353	34
9												
2	205	6	243	10	280	7	311	28	335	16	354	9
0												
2	206	21	244	26	281	17	312	23	335	57	354	45
1												
2	207	37	245	42	282	26	313	17	336	38	355	20
2												
2	208	52	246	58	283	35	314	11	337	19	355	55
3												
2	210	8	248	14	284	44	315	4	338	0	356	30
4												
2	211	24	249	29	285	51	315	56	338	40	357	5
5												
2	212	40	250	45	286	59	316	48	339	20	357	40
6												
2	213	56	252	1	288	6	317	39	339	59	358	15
7												
2	215	12	253	16	289	13	318	30	340	33	358	50
8												
2	216	28	254	32	290	19	319	21	341	17	359	25
9												
3	217	44	255	47	291	24	320	11	341	56	360	0
0												

Commentary on the *Sphere* of Sacrobosco — Chapter III

(continued: Tables of Oblique Ascensions, at successive elevations of the pole)

These pages give the tables of oblique ascensions [ascensiones obliquae] for the elevations of the pole (geographical latitude) 41°, 42°, and 43°. In each table the argument (leftmost column, headed **G**) is the degree of the ecliptic within a sign, 0–30. For each of the twelve zodiacal signs two columns are given: **G** = degrees and

M = minutes of the oblique ascension. The upper block of each page covers the six signs Aries–Virgo (♈–♍, ascensions 0°–180°); the lower block covers Libra–Pisces (♎–♊, ascensions 180°–360°). The obliquity of the ecliptic assumed throughout is 23°30′.

Tabula Ascensionum obliquarum ad latitudinem Graduum
41.

(Table of Oblique Ascensions for the latitude of 41 degrees)

Upper block — Aries ♈ · Taurus ♉ · Gemini ♊ · Cancer ♋ · Leo ♌ · Virgo ♍

G ♈	G ♈	♈ M	G ♉	♉ M	G ♊	♊ M	G ♋	♋ M	G ♌	♌ M	G ♍	♍ M
0	0	0	17	43	39	9	67	47	103	33	141	55
1	0	34	18	21	39	58	68	53	104	49	143	12
2	1	8	18	59	40	48	69	59	106	5	144	29
3	1	42	19	38	41	39	71	6	107	21	145	45
4	2	16	20	16	42	30	72	13	108	37	147	2
5	2	51 [?]	20	55	43	22	73	21	109	53	148	18
6	3	25	21	34	44	14	74	29	111	9	149	35
7	3	59	22	14	45	7	75	38	112	25	150	52
8	4	34	22	54	46	0	76	47	113	42	152	8
9	5	8	23	34	46	53	77	56	114	58	153	25
1	5	43	24	15	47	47	79	6	116	15	154	41
0												
1	6	18	24	56	48	42	80	17	117	32	155	58
1												
1	6	53	25	38	49	38	81	28	118	49	157	14
2												
1	7	28	26	19	50	34	82	39	120	6	158	30
3												
1	8	3	27	1	51	30	83	49	121	23	159	46
4												
1	8	38	27	43	52	27	85	1	122	40	161	2
5												
1	9	13	28	26	53	25	86	13	123	57	162	18

G	Υ	G	Υ	M	ϣ	G	ϣ	M	Π	G	Π	M	⋈	G	⋈	M	♊	G	♊	M
6																				
1	9		48		29	10	54	23	87	26	125	14	163	34						
7																				
1	10		24		29	53	55	22	88	39	126	31	164	50						
8																				
1	10		59		30	37	56	21	89	52	127	48	166	6						
9																				
2	11		35		31	21	57	20	91	5	129	5	167	21						
0																				
2	12		11		32	6	58	20	92	19	130	22	168	37						
1																				
2	12		47		32	52	59	21	93	33	131	39	169	53						
2																				
2	13		23		33	37	60	22	94	47	132	57	171	9						
3																				
2	13		59		34	23	61	24	96	1	134	14	172	25						
4																				
2	14		36		35	9	62	27	97	16	135	31	173	41						
5																				
2	15		13		35	56	63	30	98	31	136	48	174	57						
6																				
2	15		50		36	44	64	34	99	46	138	5	176	13						
7																				
2	16		28		37	32	65	38	101	2	139	22	177	29						
8																				
2	17		5		38	20	66	42	102	17	140	39	178	45						
9																				
3	17		43		39	9	67	47	103	33	141	55	180	0						
0																				

Lower block — **Libra** ⚖ · **Scorpio** ♏ · **Sagittarius** ♐ · **Capricorn** ♑ · **Aquarius** ♒ · **Pisces** ♓

$\underline{\mathfrak{G}}$ G G	$\underline{\mathfrak{M}}$ M	$\mathfrak{G}$ G	$\mathfrak{M}$ M	$\mathfrak{G}$ G	$\mathfrak{M}$ $\mathfrak{G}$ M	$\mathfrak{G}$ G	$\mathfrak{M}$ M	$\mathfrak{G}$ G	$\mathfrak{M}$ M	$\mathfrak{G}$ G	$\mathfrak{M}$ M
0 18 0	0	21 8	5	25 6	27	29 2	13	32 0	51	34 2	17
1 18 1	15	21 9	21	25 7	43	29 3	18	32 1	40	34 2	55
2 18 2	31	22 0	38	25 8	58	29 4	22	32 2	28	34 3	32
3 18 3	47	22 1	55	26 0	14	29 5	26	32 3	16	34 4	10
4 18 5	3	22 3	12	26 1	29	29 6	30	32 4	4	34 4	47
5 18 6	19	22 4	29	26 2	44	29 7	33	32 4	51	34 5	24
6 18 7	35	22 5	46	26 3	59	29 8	36	32 5	37	34 6	1
7 18 8	51	22 7	3	26 5	13	29 9	38	32 6	23	34 6	37
8 19 0	7	22 8	21	26 6	27	30 0	39	32 7	8	34 7	13
9 19 1	23	22 9	38	26 7	41	30 1	40	32 7	54	34 7	49
1 19 0 2	39	23 0	55	26 8	55	30 2	40	32 8	39	34 8	25
1 19 1 3	54	23 2	12	27 0	8	30 3	39	32 9	23	34 9	1
1 19 2 5	10	23 3	29	27 1	11 [recte: 21]	30 4	38	33 0	7	34 9	36
1 19 3 6	26	23 4	46	27 2	34	30 5	37	33 0	50	35 0	12
1 19 4 7	42	23 6	3	27 3	47	30 6	35	33 1	34	35 0	47
1 19 5 8	58	23 7	20	27 4	59	30 7	33	33 2	17	35 1	22
1 20 14		23	37	27	11	30	30	33	59	35	57

$\underline{\circ}$ G G	$\underline{\circ}$ M	$\mathfrak{M}$ G	$\mathfrak{M}$ M	$\times^{\nearrow}$ G	$\times^{\nearrow}$ M	$\mathfrak{y}_o$ G	$\mathfrak{y}_o$ M	$\approx$ G	$\approx$ M	$\mathfrak{X}$ G	$\mathfrak{X}$ M
6 0		8		6		8		2		1	
1 20	30	23	54	27	21	30	26	33	41	35	32
7 1		9		7		9		3		2	
1 20	46	24	11	27	32	31	22	33	22	35	7
8 2		1		8		0		4		3	
1 20	2	24	28	27	43	31	18	33	4	35	42
9 4		2		9		1		5		3	
2 20	19	24	45	28	54	31	13	33	45	35	17
0 5		3		0		2		5		4	
2 20	53 [recte:	24	2	28	4	31	7	33	26	35	52
1 6	35]	5		2		3		6		4	
2 20	52	24	18	28	13	31	0	33	6	35	26
2 7		6		3		4		7		5	
2 20	8	24	35	28	22	31	53	33	46	35	1
3 9		7		4		4		7		6	
2 21	25	24	51	28	31	31	46	33	36	35	35
4 0		8		5		5		8	[?]	6	
2 21	42	25	7	28	39	31	38	33	5	35	9
5 1		0		6		6		9		7	
2 21	58	25	23	28	47	31	30	33	44	35	44
6 2		1		7		7		9		7	
2 21	15	25	39	28	54	31	21	34	22	35	18
7 4		2		8		8		0		8	
2 21	31	25	55	29	1	31	12	34	1	35	52
8 5		3		0		9		1		8	
2 21	48	25	11	29	7	32	2	34	59	35	26
9 6		5		1		0		1	[?]	9	
3 21	5	25	27	29	13	32	51	34	17	36	0
0 8		6		2		0		2		0	

Tabula Ascensionum obliquarum ad latitudinem graduum 42.

(Table of Oblique Ascensions for the latitude of 42 degrees)

Upper block — Aries ♈ · Taurus ♉ · Gemini ♊ · Cancer ♋ · Leo ♌ · Virgo ♍

G	♈ G	♈ M	♉ G	♉ M	♊ G	♊ M	♋ G	♋ M	♌ G	♌ M	♍ G	♍ M
0	0	0	17	21	38	27	66	57	102	51	141	33
1	0	33	17	58	39	16	68	3	104	7	142	51
2	1	6	18	36	40	6	69	9	105	24	144	8
3	1	40	19	14	40	56	70	16	106	40	145	26
4	2	13	19	51	41	46	71	23	107	57	146	43
5	2	47	20	29	42	37	72	31	109	14	148	0
6	3	20	21	8	43	28	73	39	110	31	149	18
7	3	54	21	48	44	20	74	47	111	48	150	35
8	4	28	22	27	45	13	75	56	113	5	151	52
9	5	2	23	6	46	7	77	4	114	22	153	9
10	5	36	23	46	47	1	78	16	115	40	154	26
11	6	10	24	26	47	56	79	27	116	57	155	43
12	6	44	25	7	48	51	80	38	118	15	157	0
13	7	18	25	48	49	47	81	50	119	32	158	17
14	7	52	26	29	50	43	83	1	120	50	159	34
15	8	26	27	10	51	39	84	13	122	8	160	50
16	9	0	27	52	52	36	85	26	123	25	162	7
17	9	35	28	35	53	34	86	39	124	43	163	24
18	10	10	29	13 [?]	54	32	87	52	126	0	164	41

G	Υ	G	Υ	M	ϣ	G	ϣ	M	Π	G	Π	M	Ϣ	G	Ϣ	M	ϣ	G	ϣ	M	⏓	G	⏓	M
8																								
1	10		45		30		2		55		31		89		5		127		18		165		58	
9																								
2	11		20		30		46		56		30		90		19		128		36		167		24	[?]
0																								
2	11		55		31		30		57		30		91		33		129		54		168		31	
1																								
2	12		31		32		15		58		31		92		47		131		12		169		48	
2																								
2	13		6		33		0		59		32		94		2		132		30		171		4	
3																								
2	13		42		33		45		60		34		95		16		133		48		172		21	
4																								
2	14		18		34		30		61		37		96		31		135		5		173		37	
5																								
2	14		54		35		16		62		40		97		47		136		23		174		54	
6																								
2	15		31		36		3		63		44		99		3		137		41		176		11	
7																								
2	16		7		36		50		64		48		100		19		138		58		177		27	
8																								
2	16		44		37		38		65		52		101		35		140		16		178		44	
9																								
3	17		21		38		27		66		57		102		51		141		33		180		0	
0																								

Lower block — Libra ♎ · Scorpio ♏ · Sagittarius ♐ · Capricorn ♑ · Aquarius ♒ · Pisces ♓

♎	♎	♏	♏			♐	♐	♑	♑			♒			♓
G	G	M	G	M	♐	G	M	G	M	G	M	♒	G		M
0	18	0	21	27	257		9	29	3	32	33	342			39
0			8					3		1					
1	18	16	21	44	258		25	29	8	32	22	343			16
1			9					4		2					

$\underline{\mathfrak{G}}$ G G	$\underline{\mathfrak{M}}$ M	$\mathfrak{M}$ G	$\mathfrak{M}$ M	$\mathfrak{M}$ G	$\mathfrak{M}$ M	$\mathfrak{M}$ G	$\mathfrak{M}$ M	$\mathfrak{M}$ G	$\mathfrak{M}$ M	$\mathfrak{M}$ G	$\mathfrak{M}$ M
2 18 2	33	22 1	2	259	41	29 5	12	32 3	10	343	53
3 18 3	49	22 2	19	260	57	29 6	16	32 3	57	344	29
4 18 5	6	22 3	37	262	13	29 7	20	32 4	44	345	6
5 18 6	23	22 4	55	263	29	29 8	23	32 5	30	345	42
6 18 7	39	22 6	12	264	44	29 9	26	32 6	15	346	18
7 18 8	56	22 7	30	265	58	30 0	28	32 7	0	346	54
8 19 0	12	22 8	48	267	13	30 1	29	32 7	45	347	29
9 19 1	29	23 0	6	268	27	30 2	30	32 8	30	348	5
1 19 0 2	46	23 1	24	269	41	30 3	30	32 9	14	348	40
1 19 1 4	2	23 2	42	270	55	30 4	29	32 9	58	349	15
1 19 2 5	19	23 4	0	872 [recte: 272]	8	30 5	28	33 0	42	349	50
1 19 3 6	36	23 5	17	273	21	30 6	26	33 1	25	350	25
1 19 4 7	53	23 6	35	274	34	30 7	24	33 2	8	351	0
1 19 5 9	10	23 7	52	275	47	30 8	21	33 2	50	351	34
1 20 6 0	26	23 9	10	276	59	30 9	17	33 3	31	352	8
1 20 7 1	43	24 0	28	278	10	31 0	13	33 4	12	352	42
1 20	0	24	45	279	22	31	9	33	53	353	16

♈ G G	♈ M	♉ G	♉ M	♊ G	♊ M	♋ G	♋ M	♌ G	♌ M	♍ G	♍ M	♎ G	♎ M
8 3		1				1		4					
1 20	17	24	3	280		33	31	4	33	34	353		50
9 4		3					2		5				
2 20	34	24	20	281		44	31	59	33	14	354		24
0 5		4					2		6				
2 20	51	24	38	282		54	31	53	33	54	354		58
1 6		5					3		6				
2 20	8	24	55	284		4	31	47	33	33	355		32
2 8		6					4		7				
2 20	25	24	12	285		13	31	40	33	12	356		6
3 9		8					5		8				
2 21	42	24	29	286		21	31	32	33	52	356		40
4 0		9					6		8				
2 21	0	25	46	287		29	31	23	33	31	357		13
5 2		0					7		9				
2 21	11	25	3	288		37	31	14	34	9	357		47
6 3		2					8		0				
2 21	34	25	20	289		44	31	4	34	48	348 [recte: 358]		20
7 4		3					9		0				
2 21	52	25	36	290		51	31	54	34	24	358		54
8 5		4					9		1				
2 21	9	25	53	291		57	32	44	34	2	359		27
9 7		5					0		2				
3 21	27	25	9	293		3	32	33	34	39	360		0
0 8		7					1		2				

Tabula Ascensionum obliquarum ad latitudinem graduum 43.

(Table of Oblique Ascensions for the latitude of 43 degrees)

Upper block — Aries ♈ · Taurus ♉ · Gemini ♊ · Cancer ♋ · Leo ♌ · Virgo ♍

Υ G G	Υ M	Υ G	Υ M	Π G	Π M	$\mathfrak{S}$ G	$\mathfrak{S}$ M	$\mathfrak{Q}$ G	$\mathfrak{Q}$ M	$\mathfrak{M}$ G	$\mathfrak{M}$ M
0 0	0	16	58	37	44	66	5	102	8	141	10
1 0	32	17	35	38	32	67	5	103	25	142	29
							[?]				
2 1	5	18	12	39	22	68	17	103 [recte: 104]	42	143	47
3 1	38	18	49	40	12	69	24	105	59	145	5
4 2	11	19	26	41	2	70	31	107	16	146	23
5 2	44	20	3	41	52	71	39	108	34	147	41
6 3	16	20	41	42	43	72	47	109	51	148	59
7 3	49	21	20	43	35	73	55	111	9	150	17
8 4	22	21	58	44	27	75	4	112	27	151	35
9 4	55	22	37	45	20	76	14	113	45	152	53
1 5	28	23	16	46	13	77	25	115	3	154	10
0											
1 6	1	23	56	47	7	78	36	116	21	155	28
1											
1 6	34	24	36	48	2	79	48	117	39	156	46
2											
1 7	8	25	16	48	57	80	59	118	58	158	4
3											
1 7	41	25	56	49	53	82	11	120	16	159	22
4											
1 8	15	26	37	50	49	83	23	121	35	160	39
5											
1 8	48	27	19	51	46	84	36	122	53	161	56
6											
1 9	22	28	1	52	44	85	50	124	11	163	13
7											
1 9	56	28	44	53	42	87	4	125	29	164	31
8											
1 10	30	29	26	54	40	88	17	126	47	165	48
9											
2 11	4	30	9	55	39	89	31	128	6	167	6

♈	♈	♉	♉	♊	♊	♋	♋	♌		♍	♎	♎
G G	♈ M	G	M	G	M	G	M	♌ G		M	G	M
0												
2 11	39	30	53	56	39	90	46	129		25	168	23
1												
2 12	14	31	37	57	40	92	1	130		43	169	41
2												
2 12	49	32	21	58	41	93	16	132		2	170	58
3												
2 13	24	33	5	59	43	94	31	133		21	172	16
4												
2 13	54	33	50	60	45	95	46	134		39	173	34
5	[?]											
2 14	34	34	35	61	48	97	2	135		58	174	51
6												
2 15	10	35	21	62	51	98	18	137		16	176	8
7												
2 15	46	36	8	63	55	99	35	138		34	177	25
8												
2 16	22	36	56	65	0	100	51	139		52	178	42
9												
3 16	58	37	44	66	5	102	8	141		10	180	0
0												

Lower block — Libra ♎ · Scorpio ♏ · Sagittarius ♐ · Capricorn ♑ · Aquarius ♒ · Pisces ♓

$\underline{n}$	$\underline{n}$		$\mathfrak{M}$	$\nearrow$	$\nearrow$	$\mathfrak{y}_0$	$\mathfrak{y}_0$		$\approx$	$\times$	$\times$
G G	M	$\mathfrak{M}$ G	M	G	M	G	M	$\approx$ G	M	G	M
0 18	0	218	50	25	52	29	55	322	16	34	2
0				7		3				3	
1 18	18	220	8	25	9	29	0	323	4	34	38
1				9		5				3	
2 18	35	221	26	26	25	29	5	323	52	34	14
2				0		6				4	
3 18	52	222	44	26	42	29	9	324	39	34	50
3				1		7				4	

$\underline{\mathfrak{G}}$ G G	$\underline{\mathfrak{M}}$ M	$\mathfrak{M}$ G	$\mathfrak{M}$ M	$\mathfrak{G}$ G	$\mathfrak{M}$ M	$\mathfrak{G}$ G	$\mathfrak{M}$ M	$\mathfrak{G}$ M	$\mathfrak{G}$ M	$\mathfrak{G}$ M	$\mathfrak{G}$ M	$\mathfrak{G}$ M
4 18 5	9	224	2	26 2	58	29 8	12	325		25	34 5	26
5 18 6	26	225	21	26 4	14	29 9	15	326		10	34 6	1
6 18 7	44	226	39	26 5	29	30 0	17	327 [recte: 326]		55	34 6	36
7 18 9	2	227	58	26 6	44	30 1	19	327		39	34 7	11
8 19 0	19	229	17	26 7	59	30 2	20	328		23	34 7	46
9 19 1	37	230	35	26 9	14	30 3	21	329		7	34 8	21
1 19 0 2	54	231	54	27 0	29	30 4	21	329		51	34 8	56
1 19 1 4	12	233	13	27 1	43	30 5	20	330		34	34 9	30
1 19 2 5	29	234	31	27 2	56	30 6	18	331		16	35 0	4
1 19 3 6	47	235	49	27 4	10	30 7	16	331		59	35 0	38
1 19 4 8	4	237	7	27 5	24	30 8	14	332		41	35 1	12
1 19 5 9	21	238	25	27 6	37	30 9	11	333		23	35 1	45
1 20 6 0	38	239	40 [?]	27 7	49	31 0	7	334		4	35 2	19
1 20 7 1	56	241	2	27 9	1	31 1	3	334		44	35 2	52
1 20 8 3	14	242	21	28 0	12	31 1	58	335		24	35 3	26
1 20 9 4	32	243	39	28 1	24	31 2	53	336		4	35 3	59
2 20	50	244	57	28	35	31	47	336		44	35	32

♎ G G	♎ M	♍ G	♌ M	♈ G	♈ M	♎ G	♎ M	♍ G	♏ M	♏ G	♏ M
0 5				2		3				4	
2 20	7	246	15	28	46	31	40	337	23	35	5
1 7				3		4				5	
2 20	25	247	33	28	56	31	33	338	2	35	38
2 8				4		5				5	
2 20	43	248	51	28	5	31	25	338	40	35	11
3 9				6		6				6	
2 21	1	250	9	28	13	31	17	339	19	35	44
4 1				7		7				6	
2 21	19	251	26	28	21	31	8	339	57	35	16
5 2				8		8				7	
2 21	37	252	44	28	29	31	58	340	34	35	49
6 3				9		8				7	
2 21	55	254	1	29	36	31	48	341	11	35	22
7 4				0		9				8	
2 21	13	255	18	29	43	32	38	341	48	35	55
8 6				1		0				8	
2 21	31	257 [recte: 256]	35	29	49	32	27	342	25	35	28
9 7				2		1				9	
3 21	50	259 [recte: 257]	52	29	55	32	16	343	2	36	0
0 8				3		2				0	

CHAPTER THREE (continued) — Tables of Oblique Ascensions

These three pages continue the series of tables of oblique ascension, one table per successive elevation of the pole (latitude), for latitudes 44°, 45°, and 46°. In each table the argument (leftmost column, headed **G** = *gradus*, degrees) runs 0–30 and gives the degree within a sign; the twelve sign-columns give the oblique ascension reached by the end-point of that degree. Values are stated as **degrees ° minutes ’** (column heads **G** and **M**). The six spring/summer signs (♈ Aries through ♍ Virgo) form the upper block; the six autumn/winter signs (♎ Libra through ♏ Pisces) form the lower block, printed beneath the upper on the same page. Block closures hold throughout: Virgo 30° = 180°0’ and Pisces 30° = 360°0’.

Table of Oblique Ascensions for latitude 44 degrees

(*Tabula Ascensionum obliquarum ad latitudinem graduum 44.*) — Source: (printed p. 217).

Upper block (♈ Aries – ♍ Virgo). Each cell is degrees minutes.

D eg	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0 0	16 34	36 56	65 10	101 23	140 46
1	0 32	17 10	37 47	66 15	102 40	142 5
2	1 4	17 46	38 36	67 21	103 56	143 24
3	1 36	18 22	39 25	68 28	105 16	144 43
4	2 8	18 58	40 14	69 36	106 34	146 2
5	2 40	19 35	41 4	70 44	107 52	147 21
6	3 12	20 12	41 55	71 53	109 10	148 40
7	3 44	20 50	42 46	73 2	110 28	149 59
8	4 16	21 28	43 38	74 12	111 47	151 18
9	4 48	22 6	44 30	75 22	113 5	152 37
10	5 20	22 45	45 23	76 32	114 24	153 55
11	5 52	23 24	46 17	77 43	115 43	155 14
12	6 35	24 3	47 11	78 54	117 2	156 32
13	6 57	24 43	48 6	80 6	118 21	157 51
14	7 30	25 22	49 1	81 18	119 41	159 9
15	8 3	26 2	49 57	82 31	121 0	160 27
16	8 36	26 43	50 53	83 44	122 19	161 46
17	9 9	27 25	51 50	84 58	123 38	163 4
18	9 42	28 6	52 48	86 12	124 57	174 22

D eg	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
8						
1	10 15	28 48	53 47	87 26	126 16	165 40
9						
2	10 49	29 30	54 46	88 41	127 35	166 58
0						
2	11 23	30 13	55 45	89 56	128 54	168 17
1						
2	11 57	30 57	56 45	91 11	130 13	169 35
2						
2	12 31	31 40	57 46	92 27	131 33	170 54
3						
2	13 5	32 24	58 48	93 42	133 52	172 12
4						
2	13 39	33 8	59 50	94 58	134 11	173 30
5						
2	14 14	33 53	60 53	96 15	135 30	174 48
6						
2	14 49	34 39	61 58	97 32	136 49	176 6
7						
2	15 24	35 25	63 1	98 46	138 8	177 24
8						
2	15 59	36 12	64 6	100 6	139 27	178 42
9						
3	16 34	36 56	65 10	101 23	140 46	180 0
0						

Lower block (♎ Libra – ♉ Pisces). Each cell is degrees minutes.

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
0	180 0	219 14	258 37	294 50	323 1	343 26
1	181 18	220 33	259 54	295 55	323 48	344 1
2	182 36	221 52	261 11	296 59	324 35	344 36

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
3	183 54	223 11	262 28	298 3	325 21	345 11
4	185 12	224 30	263 45	299 7	326 7	345 46
5	186 30	225 49	265 2	300 10	326 52	346 21
6	187 48	227 8	266 18	301 12	327 36	346 55
7	189 6	228 27	267 33	302 14	328 20	347 29
8	190 25	229 47	268 49	303 15	329 3	348 3
9	191 43	231 6	270 4	304 15	329 47	348 37
10	193 2	232 25	271 19	305 14	330 30	349 11
11	194 20	233 44	272 34	306 13	331 12	349 45
12	195 38	235 3	273 48	307 12	331 54	350 18
13	196 56	236 23	275 2	308 10	332 35	350 51
14	198 14	237 41	276 16	309 7	333 17	351 24
15	199 33	239 0	277 29	310 3	333 58	351 57
16	200 51	240 19	278 42	310 59	334 38	352 30
17	202 9	241 39	279 54	311 54	335 17	353 3
18	203 28	242 58	281 6	312 49	335 57	353 35
19	204 46	244 17	282 17	313 43	336 36	354 8
20	206 5	245 36	283 28	314 37	337 15	354 40
21	207 23	246 55	284 38	315 30	337 54	355 12
22	208 42	248 13	285 48	316 22	338 32	355 44

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
2						
2	210 1	249 32	286 58	317 14	339 10	356 16
3						
2	211 20	250 50	288 7	318 5	339 48	356 48
4						
2	212 39	252 8	289 16	318 56	340 25	357 20
5						
2	213 58	253 26	290 24	319 46	341 2	357 52
6						
2	215 17	254 44	291 32	320 35	341 38	358 24
7						
2	216 36	256 2	292 39	321 24	342 14	358 56
8						
2	217 55	257 20	293 45	322 13	342 50	359 28
9						
3	219 14	258 37	294 50	323 1	343 26	360 0
0						

Table of Oblique Ascensions for latitude 45 degrees

(*Tabula Ascensionum obliquarum ad latitudinem Graduum 45.*) — Source: (printed page number reads 228; expected 218).

Upper block (♈ Aries – ♍ Virgo).

D eg	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0 0	16 10	36 13	64 14	100 37	140 22
1	0 31	16 45	37 0	65 20	101 55	141 42
2	1 2	17 20	37 48	66 26	103 13	143 2
3	1 33	17 56	38 36	67 33	104 32	144 21
4	2 4	18 31	39 25	68 40	105 50	145 41
5	2 35	19 7	40 15	69 48	107 9	147 0
6	3 6	19 43	41 5	70 56	108 28	148 20

D eg	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
7	3 37	20 20	41 56	72 5	109 47	149 40
8	4 9	20 57	42 47	73 15	111 6	150 59
9	4 40	21 34	43 39	74 28	112 25	152 19
10	5 12	22 12	44 31	75 36	113 44	153 38
11	5 43	22 50	45 24	76 48	115 3	154 58
12	6 15	23 29	46 18	78 0	116 23	156 17
13	6 47	24 8	47 12	79 12	117 42	157 37
14	7 19	24 47	48 7	80 24	119 2	158 56
15	7 51	25 26	49 3	81 37	120 22	160 15
16	8 33	26 6	49 59	82 51	121 42	161 34
17	8 55	26 47	50 56	84 5	123 2	162 53
18	9 27	27 28	51 53	85 20	124 22	164 12
19	9 59	28 9	52 51	86 34	125 42	165 31
20	10 32	28 50	53 50	87 49	127 2	166 50
21	11 5	29 32	54 49	89 4	128 22	168 9
22	11 38	30 15	55 49	90 20	129 42	169 28
23	12 11	30 58	56 50	91 36	131 3	170 47
24	12 44	31 41	57 52	92 52	132 23	172 6

D eg	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
2 5	13 18	32 25	58 54	94 9	133 43	173 25
2 6	13 52	33 10	59 57	95 26	135 3	174 44
2 7	14 26	33 56	61 0	96 44	136 23	176 3
2 8	15 1	34 41	62 4	98 1	137 43	177 22
2 9	15 35	35 21	63 9	99 19	139 3	178 41
3 0	16 10	36 13	64 14	100 37	140 22	180 0

Lower block (♎ Libra – ♉ Pisces).

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
0	180 0	219 38	259 23	295 46	323 47	343 50
1	181 19	220 57	260 41	296 51	324 33	344 25
2	182 38	222 17	261 59	297 56	325 19	344 59
3	183 57	223 37	263 16	299 0	326 4	345 34
4	185 16	224 57	264 34	300 3	326 50	346 8
5	186 35	226 17	265 51	301 6	327 35	346 42
6	187 54	227 37	267 8	302 8	328 19	347 16
7	189 13	228 57	268 24	303 10	329 2	347 49
8	190 32	230 18	269 40	304 11	329 45	348 22
9	191 51	231 38	270 56	305 11	330 28	348 55
1 0	193 10	232 58	272 11	306 10	331 10	349 28
1 1	194 29	234 18	273 26	307 9	331 51	350 1
1	195 48	235 38	274 40	308 7	332 32	350 33

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
2						
1	197 7	236 58	275 55	309 4	333 13	351 5
3						
1	198 26	238 18	277 9	310 1	333 54	351 37
4						
1	199 45	239 38	278 23	310 57	334 34	352 9
5						
1	201 4	240 58	279 36	311 13	335 13	352 41
6						
1	202 23	242 18	280 48	312 48	335 52	353 13
7						
1	203 43	243 37	282 0	313 42	336 31	353 45
8						
1	205 2	244 57	283 12	314 36	337 10	354 17
9						
2	206 22	246 16	284 24	315 29	337 48	354 48
0						
2	207 41	247 35	285 35	316 21	338 26	355 20
1						
2	209 1	248 54	286 45	317 13	339 3	355 51
2						
2	210 20	250 13	287 55	318 4	339 40	356 23
3						
2	211 40	251 32	289 4	318 55	340 17	356 54
4						
2	213 0	252 51	290 12	319 45	340 53	357 25
5						
2	214 19	254 10	291 20	320 35	341 29	357 56
6						
2	215 39	255 28	292 27	321 24	342 4	358 27
7						
2	216 58	256 47	293 34	322 12	342 40	358 58

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
8						
2	218 18	258 5	294 40	323 1	343 15	359 29
9						
3	219 38	259 23	295 46	323 47	343 50	360 0
0						

Table of Oblique Ascensions for latitude 46 degrees

(*Tabula Ascensionum obliquarum ad latitudinem graduum 46.*) — Source: (printed p. 219).

Upper block (♈ Aries – ♍ Virgo).

D eg	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0 0	15 44	35 24	63 14	99 48	139 56
1	0 30	16 18	36 11	64 20	101 7	141 17
2	1 0	16 53	36 58	65 27	102 26	142 38
3	1 30	17 27	37 46	66 34	103 45	143 58
4	2 0	18 2	38 34	67 41	105 4	145 19
5	2 31	18 37	39 23	68 49	106 24	146 39
6	3 1	19 13	40 12	69 58	108 43	148 0
7	3 32	19 49	41 2	71 8	109 3	149 20
8	4 2	20 26	41 53	72 18	110 23	150 41
9	4 33	21 2	42 45	73 28	111 43	152 1
1	5 4	21 39	43 37	74 39	113 3	153 21
0						
1	5 34	22 16	44 30	75 51	114 23	154 42
1						
1	6 5	22 54	45 24	77 3	115 44	156 2
2						
1	6 36	23 32	46 18	78 16	117 4	157 22
3						

D eg	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
1 4	7 7	24 10	47 12	79 28	118 25	158 42
1 5	7 38	24 47	48 7	80 41	119 46	160 2
1 6	8 9	25 27	49 3	81 55	121 6	161 22
1 7	8 40	26 7	50 0	83 10	122 27	162 42
1 8	9 12	26 47	50 57	84 25	123 47	164 2
1 9	9 43	27 28	51 55	85 40	125 8	165 22
2 0	10 15	28 9	52 53	86 55	126 29	166 42
2 1	10 47	28 51	53 52	88 11	127 50	168 2
2 2	11 19	29 33	54 52	89 27	129 10	169 22
2 3	11 52	30 15	55 52	90 44	130 31	170 42
2 4	12 24	30 57	56 53	92 0	131 52	172 2
2 5	12 57	31 40	57 55	93 17	133 13	173 21
2 6	13 30	32 23	58 57	94 35	134 34	174 41
2 7	14 3	33 7	60 0	95 53	135 55	176 1
2 8	14 37	33 52	61 4	97 11	137 15	177 21
2 9	15 10	34 38	62 9	98 29	138 36	178 41
3	15 44	35 24	63 14	99 48	139 56	180 0

D eg	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0						

Lower block (♎ Libra – ♉ Pisces).

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
0	180 0	220 4	260 12	296 46	324 36	344 16
1	181 19	221 24	261 31	297 51	325 22	344 50
2	182 39	222 45	262 49	298 56	326 8	345 23
3	183 59	224 5	264 7	300 0	326 43	345 57
4	185 19	225 26	265 25	301 3	327 37	346 30
5	186 39	226 47	266 43	302 5	328 20	347 3
6	187 58	228 8	268 0	303 7	329 3	347 36
7	189 18	229 29	269 16	304 8	329 45	348 8
8	190 38	230 50	270 33	305 8	330 27	348 41
9	191 58	232 10	271 49	306 8	331 9	349 13
10	193 18	233 31	273 5	307 7	331 51	349 45
11	194 38	234 52	274 20	308 5	332 32	350 17
12	195 58	236 13	275 35	309 3	333 13	350 48
13	197 18	237 33	276 50	310 0	333 53	351 20
14	198 38	238 54	278 5	310 57	334 33	351 51
15	199 58	240 14	279 19	311 53	335 12	352 22
16	201 18	241 35	280 32	312 48	335 50	352 53
17	202 38	242 56	281 44	313 42	336 28	353 24

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
1 8	203 58	244 16	282 57	314 36	337 6	353 55
1 9	205 18	245 37	284 9	315 30	337 44	354 26
2 0	206 39	246 57	285 21	316 23	338 21	354 56
2 1	207 59	248 17	286 32	317 15	338 58	355 27
2 2	209 19	249 37	287 42	318 7	339 34	355 58
2 3	210 40	250 57	288 52	318 58	340 11	356 28
2 4	212 0	252 17	290 2	319 48	340 47	356 59
2 5	213 21	253 36	291 11	320 37	341 23	357 29
2 6	214 41	254 56	292 19	321 26	341 58	358 0
2 7	216 2	256 15	293 26	322 14	342 33	358 30
2 8	217 22	257 34	294 33	323 2	343 7	359 0
2 9	218 43	258 53	295 40	323 49	343 42	359 30
3 0	220 4	260 12	296 46	324 36	344 16	360 0

Commentary on the *Sphere* of Johannes de Sacrobosco — Chapter III

Tables of Oblique Ascensions (continued)

Batch covers (printed pp. 220–222): the tables of oblique ascensions for latitudes (elevations of the pole) 47°, 48°, and 49°.

Each table gives the oblique ascension for every degree of the ecliptic (argument **G** = 0–30) under each of the twelve zodiacal signs. Every cell holds two numbers, **degrees minutes** (e.g. 15 18 = 15°18'), as printed under the paired column heads **G** (*gradus*) and **M** (*minuta*). Each latitude is printed as an upper block (♈ Aries – ♍ Virgo, closing at 180°0') and a lower block (♎ Libra – ♊ Pisces, closing at 360°0'). Sign-boundary closures (30° of one sign = 0° of the next) were checked throughout; the few cells that break a column's smooth progression are transcribed as printed and flagged.

Tabula Ascensionum obliquarum ad latitudinem Graduum 47.

Upper block — ♈ Aries · ♉ Taurus · ♊ Gemini · ♋ Cancer · ♌ Leo · ♍ Virgo

G	♈	♉	♊	♋	♌	♍
0	0 0	15 18	34 34	62 12	98 58	139 30
1	0 29	15 51	35 20	63 18	100 17	140 52
2	0 58	16 25	36 7	64 25	101 37	142 13
3	1 28	16 58	36 54	65 32	102 57	143 35
4	1 57	17 32	37 41	66 40	104 17	144 56
5	2 27	18 6	38 29	67 48	105 37	146 17
6	2 56	18 41	39 18	68 57	106 57	147 39
7	3 26	19 17	40 8 [?]	70 6	108 18	149 0
8	3 55	19 52	40 58	71 16	109 38	150 22
9	4 25	20 28	41 49	72 27	110 59	151 41
10	4 55	21 4	42 40	73 38	112 20	153 4
11	5 25	21 40	43 32	74 50	113 41	154 25
12	5 55	22 17	44 25	76 2	115 2	155 46

G	Υ	ϣ	Π	Ϣ	Ϡ	Ⓜ
2						
1	6 25	22 54	45 19	77 15	116 24	157 7
3						
1	6 55	23 31	46 13	78 28	117 45	158 28
4						
1	7 25	24 9	47 8	79 42	119 7	159 49
5						
1	7 55	24 47	48 3	80 56	120 28	161 10
6						
1	8 26	25 26	48 59	82 11	121 49	162 31
7						
1	8 56	26 5	49 56	83 26	123 11	163 52
8						
1	9 27	26 45	50 54	84 42	124 32	165 13
9						
2	9 58	27 26	51 52	85 58	125 54	166 33
0						
2	10 29	28 7	52 51	87 41 [?]	127 15	167 54
1						
2	11 0	28 48	53 51	88 31	128 37	169 15
2						
2	11 32	29 30	54 51	89 48	129 58	170 36
3						
2	12 3	30 11	55 52	91 5	131 20	171 57
4						
2	12 35	30 53	56 54	92 23	132 42	173 17
5						
2	13 7	31 36	57 56	93 42	134 4	174 38
6						
2	13 40	32 20	58 59	95 1	135 26	175 59
7						
2	14 12	33 4	60 3	96 20	136 47	177 19
8						
2	14 45	33 49	61 7	97 39	138 9	178 40

G	♈	♉	♊	♋	♌	♍
9						
3	15 18	34 34	62 12	98 58	139 30	180 0
0						

Lower block — ♎ Libra · ♏ Scorpio · ♐ Sagittarius · ♑ Capricorn · ♒ Aquarius · ♓ Pisces

G	♎	♏	♐	♑	♒	♓
0	180 0	220 30		261 2	297 48	325 26
1	181 20	221 51		262 21	298 53	326 11
2	182 41	223 13		263 40	299 57	326 56
3	184 1	224 34		264 59	301 1	327 40
4	185 22	225 56		266 18	302 4	328 24
5	186 43	227 18		267 37	303 6	329 7
6	188 3	228 40		268 55	304 8	329 49
7	189 24	230 2		270 12	305 9	330 30
8	190 45	231 23		271 29	306 9	331 12
9	192 6	232 45		272 46	307 9	331 53
1	193 27	234 6		274 2	308 8	332 34
0						
1	194 47	365 28 [recte: 235 28]		275 18	309 6	333 15
1						
1	196 8	236 49		276 34	310 4	333 55
2						
1	197 29	238 11		277 49	311 1	334 34
3						
1	198 50	239 32		279 4	311 57	335 13
4						
1	200 11	240 53		280 18	312 52	335 51
5						
1	201 32	242 15		281 32	313 47	336 29
6						
1	202 53	243 36		282 45	314 41	337 6
7						

G	♈	♉	♊	♋	♌	♍
1 8	204 14	244 58		283 58	315 35	337 43 354 5
1 9	205 35	246 19		285 10	316 28	338 20 354 35
2 0	206 56	247 40		286 22	317 20	338 56 355 5
2 1	208 17	249 1		287 33	318 11	339 32 355 35
2 2	209 38	250 22		288 44	319 2	340 8 356 5
2 3	211 0	251 42		289 54	319 52	340 43 356 34
2 4	212 21	253 3		291 3	320 42	341 19 357 4
2 5	213 43	254 23		292 12	321 31	341 54 357 33
2 6	215 4	255 43		293 20	322 19	342 28 358 3
2 7	216 25	257 3		294 28	323 6	343 2 358 32
2 8	217 47	258 23		295 35	323 53	343 35 359 2
2 9	219 8	259 43		296 42	324 40	344 9 359 31
3 0	220 30	261 2		297 48	325 26	344 42 360 0

Tabula Ascensionum obliquarum ad latitudinem Graduum 48.

Upper block — ♈ Aries · ♉ Taurus · ♊ Gemini · ♋ Cancer · ♌ Leo · ♍ Virgo

G	♈	♉	♊	♋	♌	♍
0	0 0	14 50	33 41	61 7	98 5	139 2
1	0 28	15 23	34 26	62 13	99 25	140 25
2	0 56	15 56	35 12	63 20	100 46	141 47

G	Υ	♍	♊	♋	♌	♍
3	1 25	16 29	35 58	64 27	102 6	143 10
4	1 53	17 2	36 45	65 35	103 27	144 32
5	2 22	17 35	37 33	66 43	104 48	145 54
6	2 50	18 9	38 22	67 51	106 9	147 17
7	3 19	18 43	39 12	69 1	107 30	148 39
8	3 48	19 18	40 1	70 11	108 52	150 1
9	4 17	19 52	40 51	71 22	110 13	151 23
10	4 56 [?]	20 27	41 41	72 34	111 35	152 45
11	5 15	21 2	42 32	73 46	112 57	154 7
12	5 44	21 38	43 24	74 59	114 19	155 29
13	6 13	22 14	44 17	76 12	115 41	156 51
14	6 42	22 51	45 11	77 26	117 3	158 13
15	7 11	23 28	46 6	78 40	118 26	159 35
16	7 40	24 6	47 1	79 55	119 48	160 57
17	8 10	24 45	47 59	81 10	121 10	162 19
18	8 39	25 23	48 53	82 26	122 32	163 41
19	9 9	26 2	49 50	83 42	123 54	165 3
20	9 39	26 41	50 48	84 59	125 17	166 24
21	10 9	27 21	51 47	86 16	126 40	167 46
22	10 40	28 2	52 47	87 34	128 3	169 8
23	11 10	28 42	53 47	88 51	129 26	170 29
24	11 41	29 23	54 48	90 9	130 49	171 51
25	12 12	30 4	55 49	91 27	132 11	173 12
26	12 43	30 46	56 51	92 46	133 34	174 34
27	13 15	31 29	57 54	94 6	134 56	175 56
28	13 46	32 12	58 58	95 25	136 18	177 17
29	14 18	32 56	60 2	96 45	137 40	178 39
30	14 50	33 41	61 7	98 5	139 2	180 0

**Lower block — ♎ Libra · ♏ Scorpio · ♐ Sagittarius · ♑ Capricorn · ♒ Aquarius
· ♓ Pisces**

G	♌	♍	♎	♏	♐	♑
0	180 0	220 58	261 55	298 53	326 19	345 10
1	181 21	222 20	263 15	299 58	327 4	345 42
2	182 43	223 42	264 35	301 2	327 48	346 14
3	184 4	225 4	265 54	302 6	328 31	346 45
4	185 26	226 26	267 14	303 9	329 14	347 17
5	186 48	227 49	268 33	304 11	329 56	347 48
6	188 9	229 11	269 51	305 12	330 37	348 19
7	189 31	230 34	271 9	306 13	331 18	348 50
8	190 52	231 57	272 26	307 13	331 58	349 20
9	192 14	233 20	273 44	308 13	332 39	349 51
10	193 36	234 43	275 1	309 12	333 19	350 21
11	194 57	236 6	276 18	310 10	333 58	350 51
12	196 19	237 28	277 34	311 7	334 37	351 21
13	197 41	238 50	278 50	312 3	335 15	351 50
14	199 3	240 12	280 5	312 59	335 54	352 20
15	200 25	241 34	281 20	313 54	336 32	352 49
16	201 47	242 57	282 34	314 49	337 9	353 18
17	203 9	244 19	283 48	315 43	337 46	353 47
18	204 31	245 41	285 1	316 36	338 22	354 16
19	205 53	247 3	286 14	317 28	338 58	354 45
20	207 15	248 25	287 26	318 19	339 33	355 14
21	208 37	249 47	288 38	319 9	340 8	355 43
22	209 59	251 8	289 49	319 59	340 42	356 12
23	211 21	252 30	290 59	320 48	341 17	356 41
24	212 43	253 51	292 8	321 38	341 51	357 10
25	214 6	255 12	293 17	322 27	342 25	357 38
26	215 28	256 33	294 25	323 15	342 58	358 7
27	216 50	257 54	295 33	324 2	343 31	358 35
28	218 13	259 14	296 40	324 48	344 4	359 4
29	219 35	260 35	297 47	325 34	344 37	359 32
30	220 58	261 55	298 53	326 19	345 10	360 0

Tabula Ascensionum obliquarum ad latitudinem Graduum 49.

Upper block — ♈ Aries · ♉ Taurus · ♊ Gemini · ♋ Cancer · ♌ Leo · ♍ Virgo

G	♈	♉	♊	♋	♌	♍
0	0 0	14 22	32 45	59 59	97 9	138 34
1	0 27	14 53	33 30	61 5	98 30	139 58
2	0 55	15 25	34 15	62 11	99 51	141 21
3	1 22	15 57	35 1	63 13 [?]	101 13	142 44
4	1 50	16 29	35 47	64 26	102 34	144 7
5	2 18	17 1	36 34	65 35	103 56	145 30
6	2 45	17 34	37 22	66 44	105 18	146 54
7	3 13	18 8	38 10	67 54	106 40	148 17
8	3 40	18 41	38 59	69 5	108 3	149 40
9	4 8	19 15	39 49	70 16	109 25	151 3
1 0	4 36	19 40 [?]	40 39	71 28	110 48	152 26
1 1	5 4	20 24	41 30	72 40	112 11	153 49
1 2	5 32	21 0	42 22	73 53	113 34	155 12
1 3	6 0	21 35	43 14	75 6	114 57	156 35
1 4	6 28	22 10	44 7	76 20	116 20	157 58
1 5	6 57	22 46	45 1	77 35	117 44	159 21

G	Υ	♋	♊	♉	♈	♏
1	7 25	23 23	45	78 51	119 7	160 44
6			56			
1	7 54	24 1	46	80 7	120 30	162 7
7			52			
1	8 22	24 38	47	81 24	121 53	163 29
8			48			
1	8 51	25 16	48	82 40	123 16	174 52 [recte: 164 52]
9			45			
2	9 20	25 54	49	83 57	124 39	166 14
0			42			
2	9 49	26 33	50	85 14	126 2	167 37
1			40			
2	10	27 13	51	86 32	127 26	169 0
2	19		39			
2	10	27 52	52	87 50	128 49	170 23
3	48		39			
2	11	28 32	53	89 9	130 13	171 46
4	18		40			
2	11	29 12	54	90 28	131 37	173 8
5	48		41			
2	12	29 53	55	91 48	133 1	174 31
6	18		43			
2	12	30 35	56	93 8	134 24	175 53
7	49		46			
2	13	31 18	57	94 28	135 48	177 16
8	20		50			
2	13	32 1	58	95 48	137 11	178 38
9	51		54			
3	14	32 45	59	97 9	138 34	180 0
0	22		59			

Lower block — ♎ **Libra** · ♏ **Scorpio** · ♐ **Sagittarius** · ♑ **Capricorn** · ♒ **Aquarius**
· ♓ **Pisces**

G	♌	♍	♎	♏	♐	♑
0	180 0	221 26	262 51	300 1	327 15	345 38
1	181 22	222 49	264 12	301 6	327 59	346 9
2	182 44	224 12	265 32	302 10	328 42	346 40
3	184 7	225 36	266 52	303 14	329 25	347 11
4	185 29	226 59	268 12	304 16	330 7	347 42
5	186 52	228 23	269 32	305 19	330 48	348 12
6	188 14	229 47	270 51	306 20	331 28	348 42
7	189 37	231 11	272 10	307 21	332 8	349 12
8	191 0	232 34	273 28	308 21	332 47	349 41
9	192 23	233 58	274 46	309 20	333 27	350 11
1 0	193 46	235 21	276 3	310 18	334 6	350 40
1 1	195 8	236 44	277 20	311 15	334 44	351 9
1 2	196 31	238 7	278 36	312 12	335 22	351 38
1 3	197 53	239 30	279 53	313 8	335 59	352 6
1 4	199 16	240 53	281 9	314 4	336 37	352 35
1 5	200 39	242 16	282 25	314 59	337 14	353 3
1 6	202 2	243 40	283 40	315 53	337 50	353 32
1 7	203 25	245 3	284 54	316 46	338 25	354 0
1 8	204 48	246 26	286 7	317 38	339 0	354 28
1 9	206 11	247 49	287 20	318 30	339 36	354 56
2 0	207 34	249 12	288 32	319 21	340 11	355 24
2	208 57	250 35	289 44	320 11	340 45	355 52

G	♈	♉	♊	♋	♌	♍
1						
2	210 20	251 57	290 55	321 1	341 19	356 20
2						
2	211 43	253 20	292 6	321 50	341 52	356 47
3						
2	213 6	254 42	293 16	322 38	342 26	357 15
4						
2	214 30	256 4	294 25	323 26	342 59	357 42
5						
2	214 53 [?]	257 26	295 34	324 13	343 31	358 10
6						
2	217 16	258 47	296 42	324 59	344 3	358 38
7						
2	218 39	260 9	297 49	325 45	344 35	359 5
8						
2	220 2	261 30 [?]	298 55	326 30	345 7	359 33
9						
3	221 26	262 51	300 1	327 15	345 38	360 0
0						

Commentary on the Third Chapter of the *Sphere* of Johannes de Sacrobosco

CAPVT TERTIVM (continued) — Tables of Oblique Ascensions

These three pages continue the series of tables of oblique ascension, one table per elevation of the pole. Each table gives, against the argument (the ecliptic degree, 0–30, at the left), the oblique ascension in degrees (G, *gradus*) and minutes (M, *minuta*) for each of the twelve signs. The upper block of each table runs ♈ Aries → ♍ Virgo (oblique ascensions 0°→180°); the lower block runs ♎ Libra → ♈ Pisces (180°→360°). Latitudes 50°, 51°, and 52° are covered here; the series continues at higher latitudes on the following pages.

Table of Oblique Ascensions for the latitude of 50 degrees [*Tabula Ascensionum obliquarum ad latitudinem graduum 50.*]

Upper block — ♈ Aries to ♍ Virgo

G (arg.)	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0°0′	13°52′	31°47′	58°47′	96°11′	138°4′
1	0°26′	14°22′	32°31′	59°53′	97°33′	139°29′
2	0°53′	14°53′	33°15′	61°0′	99°55′ [?]	140°53′
3	1°19′	15°24′	34°0′	62°7′	100°18′	142°18′
4	1°46′	15°55′	34°46′	63°15′	101°40′	143°42′
5	2°13′	16°26′	35°32′	64°24′	103°3′	145°6′
6	2°39′	16°58′	36°19′	65°40′	104°26′	146°30′
7	3°6′	17°31′	37°7′	66°48′	105°49′	147°54′
8	3°32′	18°3′	37°55′	67°59′	107°12′	149°18′
9	3°59′	18°36′	38°44′	69°6′	108°35′	150°42′
10	4°26′	19°9′	39°33′	70°18′	109°58′	152°6′
11	4°53′	19°43′	40°23′	71°31′	111°22′	153°30′
12	5°20′	20°17′	41°14′	72°44′	112°46′	154°54′
13	5°47′	20°52′	42°6′	73°58′	114°10′	156°18′
14	6°14′	21°26′	42°59′	75°12′	115°34′	157°42′
15	6°42′	22°1′	43°53′	76°27′	116°59′	159°6′
16	7°9′	22°36′	44°47′	77°43′	118°23′	160°30′
17	7°37′	23°12′	45°42′	78°59′	119°47′	161°54′
18	8°4′	23°49′	46°38′	80°16′	121°11′	163°17′
19	8°32′	24°26′	47°35′	81°33′	122°35′	164°41′
20	9°0′	25°4′	48°32′	82°51′	123°59′	166°4′
21	9°28′	25°42′	49°30′	84°9′	125°23′	167°28′
22	9°57′	26°21′	50°29′	85°27′	126°48′	168°52′
23	10°26′	27°0′	51°29′	86°46′	128°12′	170°16′
24	10°55′	27°39′	52°29′	88°6′	129°37′	171°40′
25	11°24′	28°19′	53°30′	89°26′	131°2′	173°3′
26	11°53′	28°59′	54°32′	90°47′	132°27′	174°27′

G (arg.)	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
27	12°23′	29°40′	55°35′	92°8′	133°51′	175°50′
28	12°52′	30°22′	56°38′	93°29′	135°16′	177°14′
29	13°22′	31°4′	57°42′	94°50′	136°40′	178°37′
30	13°52′	31°47′	58°47′	96°11′	138°4′	180°0′

Lower block — ♎ Libra to ♉ Pisces

G (arg.)	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
0	180°0′	221°56′	263°49′	301°13′	328°13′	346°8′
1	181°23′	223°20′	265°10′	302°18′	328°56′	346°38′
2	182°46′	224°44′	266°31′	303°22′	329°38′	347°8′
3	184°10′	226°9′	267°52′	304°25′	330°20′	347°37′
4	185°33′	227°33′	269°13′	305°28′	331°1′	348°7′
5	186°57′	228°58′	270°34′	306°30′	332°41′ [?]	348°36′
6	188°20′	230°23′	271°54′	307°31′	332°21′	349°5′
7	189°44′	231°48′	273°14′	308°31′	333°0′	349°34′
8	191°8′	233°12′	274°33′	309°31′	333°39′	350°3′
9	192°32′	234°37′	275°51′	310°30′	334°18′	350°32′
10	193°56′	236°1′	277°9′	311°28′	334°56′	351°0′
11	195°19′	237°25′	278°27′	312°25′	335°34′	351°18′
12	196°43′	238°49′	279°44′	313°22′	336°11′	351°56′
13	198°6′	240°13′	281°1′	314°18′	336°48′	352°27′
14	199°30′	241°37′	282°17′	315°13′	337°24′	352°51′
15	200°54′	243°1′	283°33′	316°7′	337°59′	353°18′
16	202°18′	244°26′	284°48′	317°1′	338°34′	353°46′
17	203°42′	245°50′	286°2′	317°54′	339°8′	354°13′
18	205°6′	247°14′	287°16′	318°46′	339°43′	354°40′
19	206°30′	248°38′	288°29′	319°37′	340°17′	355°7′
20	207°54′	250°2′	289°42′	320°27′	340°51′	355°34′
21	209°18′	251°25′	290°54′	321°16′	341°24′	356°1′
22	210°42′	252°48′	292°1′	322°5′	341°57′	356°28′

G (arg.)	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
23	212°6′	254°11′	293°12′	322°53′	342°30′	356°54′
24	213°30′	255°34′	294°20′	323°41′	343°2′	357°21′
25	214°54′	256°57′	295°36′	324°28′	343°34′	357°47′
26	216°38′ [?]	258°20′	296°45′	325°14′	344°5′	358°14′
27	217°42′	259°42′	297°53′	326°0′	344°36′	358°41′
28	219°7′	261°5′	299°0′	326°45′	345°7′	359°7′
29	220°31′	262°27′	300°7′	327°29′	345°32′	359°34′
30	221°56′	263°49′	301°13′	328°13′	346°8′	360°0′

Table of Oblique Ascensions for the latitude of 51 degrees [*Tabula Ascensionum obliquarum ad latitudinem graduum 51.*]

Upper block — ♈ Aries to ♍ Virgo

G (arg.)	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0°0′	13°21′	30°46′	57°31′	95°10′	137°33′
1	0°25′	13°50′	31°29′	58°37′	96°33′	138°59′
2	0°50′	14°20′	32°13′	59°44′	97°56′	140°24′
3	1°16′	14°50′	32°57′	60°51′	99°19′	141°50′
4	1°41′	15°20′	33°42′	61°59′	100°42′	143°15′
5	2°7′	15°50′	34°27′	63°8′	102°6′	144°40′
6	2°32′	16°21′	35°13′	64°18′	103°30′	146°6′
7	2°58′	16°53′	36°0′	65°29′	104°54′	147°31′
8	3°24′	17°24′	36°48′	66°40′	106°18′	148°56′
9	3°50′	17°56′	37°36′	67°52′	107°42′	150°21′
10	4°16′	18°28′	38°25′	69°4′	109°7′	151°46′
11	4°42′	19°1′	39°15′	70°17′	110°32′	153°11′
12	5°8′	19°34′	40°5′	71°30′	111°57′	154°36′
13	5°34′	20°7′	40°56′	72°44′	113°22′	156°1′
14	6°0′	20°40′	41°48′	73°59′	114°47′	157°26′
15	6°26′	21°14′	42°41′	75°5′	116°12′	158°50′

G (arg.)	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
16	6°52′	21°49′	43°35′	76°32′	117°37′	160°15′
17	7°19′	22°25′	44°30′	77°50′	119°2′	161°40′
18	7°46′	23°1′	45°25′	79°8′	120°27′	163°5′
19	8°13′	23°37′	46°21′	80°25′	121°52′	174°30′ [recte: 164°30′]
20	8°40′	24°13′	47°18′	81°43′	123°18′	165°54′
21	9°7′	24°50′	48°16′	83°2′	124°43′	167°19′
22	9°35′	25°28′	49°14′	84°21′	126°9′	168°44′
23	10°2′	26°6′	50°13′	85°41′	127°35′	170°8′
24	10°30′	26°44′	51°13′	87°1′	129°1′	171°33′
25	10°58′	27°22′	52°14′	88°21′	130°26′	172°57′
26	11°26′	28°1′	53°16′	89°42′	131°52′	174°22′
27	11°55′	28°41′	54°19′	91°4′	133°17′	175°46′
28	12°23′	29°22′	55°22′	92°26′	134°43′	177°11′
29	12°52′	30°4′	56°26′	93°48′	136°8′	178°36′
30	13°21′	30°46′	57°31′	95°10′	137°33′	180°0′

Lower block — ♎ Libra to ♉ Pisces

G (arg.)	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
0	180°0′	222°27′	264°50′	302°29′	329°14′	346°39′
1	181°24′	223°52′	266°12′	303°34′	329°56′	347°8′
2	182°49′	225°17′	267°34′	304°38′	330°38′	347°34′
3	184°13′	226°43′	268°56′	305°41′	331°19′	348°5′
4	185°38′	228°8′	270°18′	306°44′	331°59′	348°34′
5	187°3′	229°34′	271°39′	307°46′	332°38′	349°2′
6	188°27′	230°59′	272°59′	308°47′	333°16′	349°30′
7	189°52′	232°25′	274°19′	309°47′	333°54′	349°58′
8	191°16′	233°51′	275°39′	310°46′	334°32′	350°25′
9	192°41′	235°17′	276°58′	311°44′	335°10′	350°53′
10	194°6′	236°42′	278°17′	312°42′	335°47′	351°20′

G (arg.)	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
11	195°30′	238°8′	279°35′	313°39′	336°23′	351°47′
12	196°55′	239°33′	280°52′	314°35′	336°59′	352°14′
13	198°20′	240°58′	282°10′	315°30′	337°35′	352°41′
14	199°45′	242°23′	283°28′	316°25′	338°11′	353°8′
15	201°10′	243°48′	284°45′	317°19′	338°46′	353°34′
16	202°34′	245°13′	286°1′	318°12′	339°20′	354°0′
17	203°59′	246°38′	287°16′	319°4′	339°53′	354°26′
18	205°34′ [?]	248°3′	288°30′	319°55′	340°26′	354°52′
19	206°49′	249°28′	289°43′	320°45′	340°59′	355°18′
20	208°14′	250°53′	290°56′	321°35′	341°32′	355°44′
21	209°39′	252°18′	292°8′	322°24′	342°4′	356°10′
22	211°4′	253°42′	293°20′	323°12′	342°36′	356°36′
23	212°29′	255°6′	294°31′	324°0′	343°7′	357°2′
24	213°24′ [?]	256°30′	295°42′	324°47′	343°39′	357°28′
25	215°50′ [?]	257°54′	296°52′	325°33′	344°10′	357°53′
26	216°44′	259°18′	298°1′	326°18′	344°40′	358°19′
27	218°10′	260°41′	299°9′	327°3′	345°10′	358°44′
28	219°36′	262°4′	300°16′	327°47′	345°40′	359°10′
29	221°1′	263°27′	301°23′	328°31′	346°10′	359°35′
30	222°27′	264°50′	302°29′	329°14′	346°39′	360°0′

Table of Oblique Ascensions for the latitude of 52 degrees [*Tabula Ascensionum obliquarum ad latitudinem graduum 52.*]

Upper block — ♈ Aries to ♍ Virgo

G (arg.)	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0°0′	12°48′	29°42′	56°11′	94°6′	137°0′
1	0°24′	13°16′	30°24′	57°17′	95°30′	138°37′ [?]
2	0°48′	13°45′	31°7′	58°24′	96°54′	139°54′
3	1°13′	14°14′	31°50′	59°31′	98°18′	141°20′

G (arg.)	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
4	1°37′	14°43′	32°34′	60°39′	99°42′	142°47′
5	2°2′	15°12′	33°18′	61°48′	101°7′	144°13′
6	2°26′	15°42′	34°3′	62°58′	102°32′	145°40′
7	2°51′	16°13′	34°49′	64°9′	103°57′	147°6′
8	3°15′	16°43′	35°36′	65°20′	105°22′	148°32′
9	3°40′	17°14′	36°14′ [?]	66°32′	106°47′	149°58′
10	4°5′	17°45′	37°12′	67°45′	108°12′	151°24′
11	4°30′	18°16′	38°1′	68°59′	109°38′	152°50′
12	4°55′	18°48′	38°51′	70°13′	111°4′	154°16′
13	5°20′	19°20′	39°42′	71°28′	112°30′	155°42′
14	5°45′	19°52′	40°34′	72°44′	113°56′	157°8′
15	6°10′	20°25′	41°26′	74°0′	115°23′	158°39′ [?]
16	6°35′	20°59′	42°19′	75°17′	116°49′	160°0′
17	7°1′	21°34′	43°13′	76°34′	118°15′	161°26′
18	7°26′	22°8′	44°8′	77°52′	119°42′	162°52′
19	7°52′	22°43′	45°3′	79°11′	121°8′	164°18′
20	8°28′ [?]	23°18′	45°59′	80°30′	122°35′	165°43′
21	8°44′	23°54′	46°56′	81°50′	124°2′	167°9′
22	9°11′	24°31′	47°54′	83°10′	125°28′	168°35′
23	9°37′	25°8′	48°53′	84°31′	126°55′	170°1′
24	10°4′	25°45′	49°53′	85°51′	128°22′	171°27′
25	10°31′	26°23′	50°54′	86°12′ [?]	129°48′	172°52′
26	10°58′	27°2′	51°56′	88°34′	131°15′	174°18′
27	11°25′	27°41′	52°59′	89°57′	132°41′	175°44′
28	11°53′	28°21′	54°2′	91°20′	134°8′	177°9′
29	12°20′	29°1′	55°6′	92°43′	135°34′	178°35′
30	12°48′	29°42′	56°11′	94°6′	137°0′	180°0′

Lower block — ♎ Libra to ♉ Pisces

G (arg.)	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricor n	♒ Aquariu s	♓ Pisces
0	180°0'	223°1'	265°54'	303°49'	330°18'	347°12'
1	181°2 5'	224°26'	267°17'	304°54'	330°59'	347°40'
2	182°5 1'	225°52'	268°40'	305°58'	331°39'	348°7'
3	184°1 6'	227°19'	270°3'	307°1'	332°19'	348°35'
4	185°4 2'	228°45'	271°26'	308°4'	332°58'	349°2'
5	187°8'	230°22' [?]	272°48'	309°6'	333°37'	349°29'
6	188°3 3'	231°38'	274°9'	310°7'	334°15'	349°56'
7	189°5 9'	233°5'	275°29'	311°7'	334°52'	350°23'
8	191°2 5'	234°32'	276°50'	312°6'	335°29'	350°49'
9	192°5 1'	235°58'	278°10'	313°4'	336°6'	351°16'
10	194°1 7'	237°25'	279°30'	314°1'	336°42'	351°42'
11	195°4 2'	238°52'	280°49'	314°57'	337°17'	352°8'
12	197°8'	240°18'	282°8'	315°52'	337°52'	352°34'
13	198°3 4'	241°45'	283°26'	316°47'	338°26'	252°59' [recte: 352°59']
14	200°0'	243°11'	284°43'	317°41'	339°1'	353°25'
15	201°2 6'	244°37'	286°9'	318°34'	339°35'	353°50'
16	202°5 2'	246°4'	287°16'	319°26'	340°8'	354°15'
17	204°1	247°30'	288°32'	320°18'	340°40'	354°40'

G (arg.)	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricor n	♒ Aquariu s	♓ Pisces
	8'					
18	205°4 4'	248°56'	289°47'	321°9'	341°12'	355°5'
19	207°1 0'	250°22'	291°1'	321°59'	341°44'	355°30'
20	208°3 6'	251°48'	292°15'	322°48'	342°15'	355°55'
21	210°2'	253°13'	293°28'	323°36'	342°46'	356°20'
22	211°2 8'	254°38'	294°40'	324°24'	343°17'	356°45'
23	212°5 4'	256°3'	295°51'	325°11'	343°47'	357°9'
24	214°2 0'	257°28'	297°2'	325°57'	344°18'	357°34'
25	215°4 7'	258°53'	298°12'	326°42'	344°48'	357°58'
26	217°1 3'	260°18'	299°21'	327°26'	345°17'	358°23'
27	218°4 0'	261°42'	300°29'	328°10'	345°46'	358°47'
28	220°6'	263°6'	301°36'	328°53'	346°15'	359°12'
29	221°3 3'	264°30'	302°43'	329°36'	346°44'	359°36'
30	223°0'	265°54'	303°49'	330°18'	347°12'	360°0'

COMMENT. IN III. CAP. SPHÆRÆ

CAPVT TERTIVM — Tables of Oblique Ascensions (continued)

Table of oblique ascensions for the latitude of 53 degrees [Tabula Ascensionum obliquarum ad latitudinem Graduum 53.]

The argument column (G) is the ecliptic degree, 0–30, within each sign. For each of the twelve signs the oblique ascension is given as degrees (G) and minutes (M). The six signs Aries (♈) through Virgo (♍) form the upper block; Libra (♎) through Pisces (♓) form the lower block.

Latitude 53° — signs ♈ Aries to ♍ Virgo

G	♈ G	♈ M	♉ G	♉ M	♊ G	♊ M	♋ G	♋ M	♌ G	♌ M	♍ G	♍ M
0	0	0	12	14	28	34	54	46	92	58	136	26
1	0	23	12	41	29	15	55	52	94	23	137	54
2	0	46	13	8	29	57	56	59	95	58	139	22
3	1	9	13	36	30	39	58	6	97	13	140	49
4	1	32	14	4	31	22	59	14	98	38	142	7
5	1	56	14	32	32	6	60	23	100	4	143	44
6	2	19	15	1	32	51	61	33	101	30	145	12
7	2	43	15	30	33	36	62	44	102	56	146	39
8	3	6	15	59	34	22	63	56	104	22	148	7
9	3	30	16	29	35	8	65	9	105	48	149	34
10	3	54	16	59	35	55	66	22	107	15	151	1
11	4	17	17	29	36	43	67	36	108	42	152	29
12	4	41	18	0	37	32	68	51	110	9	153	56
13	5	5	18	31	38	22	70	6	111	36	155	23
14	5	29	19	1	39	13	71	22	113	4	156	50
15	5	53	19	34	40	5	72	39	114	32	158	17
16	6	17	20	7	40	57	73	57	115	59	159	44
17	6	41	20	40	41	50	75	15	117	26	161	11
18	7	5	21	13	42	44	76	34	118	54	162	38

G	Υ G	Υ M	Ϸ G	Ϸ M	Π G	Π M	Ϣ G	Ϣ M	Ϻ G	Ϻ M	♎ G	♎ M
1 9	7	30	21	47	43	39	77	53	120	21	164	5
2 0	7	55	22	21	44	36	79	13	121	49	165	32
2 1	8	20	22	56	45	33	80	34	123	17	166	59
2 2	8	45	23	31	46	31	81	55	124	45	168	26
2 3	9	10	24	7	47	30	83	16	126	13	169	53
2 4	9	36	24	43	48	29	84	38	127	41	171	20
2 5	10	2	25	20	49	29	86	0	129	8	172	46
2 6	10	28	25	58	50	30	87	22	130	36	174	13
2 7	10	54	26	36	51	32	88	45	132	4	175	40
2 8	11	20	27	15	52	35	90	9	133	31	177	7
2 9	11	47	27	54	53	40	91	33	134	59	178	34
3 0	12	14	28	34	54	46	92	58	136	26	180	0

Block closure: Virgo 30 = 180°0'. ✓

Latitude 53° — signs ♎ Libra to ♓ Pisces

G	♎ G	♎ M	♏ G	♏ M	♐ G	♐ M	♑ G	♑ M	♒ G	♒ M	♓ G	♓ M
0	180	0	223	34	267	2	305	14	331	26	347	46
1	181	26	225	1	268	27	306	20	332	6	348	13
2	182	53	226	29	269	51	307	25	332	45	348	40
3	184	20	227	56	271	15	308	28	333	24	349	6
4	185	47	229	24	272	38	309	30	334	2	349	32

	G	⊂	G	⊂	M	ℳ	G	ℳ	M	⌘	G	⌘	M	℥	G	℥	M
5	187	14	230	52	274	0	310	31	334	40	349	58					
6	188	40	232	19	275	22	311	31	335	17	350	24					
7	190	7	233	47	276	44	312	30	335	53	350	50					
8	191	34	235	15	278	5	313	21	336	29	351	15					
9	193	1	236	43	279	26	314	27	337	4	351	40					
10	194	28	238	11	280	47	315	24	337	39	352	5					
11	196	55	239	39	282	7	316	21	338	13	352	30					
12	197	22	241	6	283	26	317	16	338	47	352	55					
13	198	49	242	24	284	45	318	10	339	20	353	19					
14	200	16	244	1	286	3	319	3	339	53	353	42					
15	201	43	245	28	287	21	319	55	340	26	354	7					
16	203	10	246	56	288	38	320	47	340	58	354	31					
17	204	37	248	24	289	54	321	38	341	29	354	55					
18	206	4	249	51	291	9	322	28	342	0	355	19					
19	207	31	251	18	292	24	323	17	342	31	355	43					
20	208	59	252	45	293	38	324	5	343	1	356	6					
21	210	26	254	12	294	51	324	52	343	31	356	30					
22	211	53	255	38	296	4	325	38	344	1	356	54					
23	213	21	257	4	297	16	326	24	344	30	357	17					
24	214	48	258	30	298	27	327	9	344	59	357	41					

G	♈	G	♈	M	♎	G	♎	M	♊	G	♊	M	♋	G	♋	M
4																
2	216	16	259	56	299	37	327	54	345	28	358	4				
5																
2	217	43	261	22	300	46	328	38	345	56	358	28				
6																
2	219	11	262	47	301	54	329	21	346	24	358	51				
7																
2	220	38	264	12	303	1	330	3	346	52	359	14				
8																
2	222	6	265	37	304	8	330	45	347	19	359	37				
9																
3	223	34	267	2	305	14	331	26	347	46	360	0				
0																

Block closure: Pisces 30 = 360°0'. ✓

IOAN. DE SACRO BOSCO.

Table of oblique ascensions for the latitude of 54 degrees [Tabula Ascensionum obliquarum ad latitudinem graduum 54.]

Latitude 54° — signs ♈ Aries to ♍ Virgo

G	♈	G	♈	M	♉	G	♉	M	♊	G	♊	M	♋	G	♋	M
0	0	0	11	38	27	22	53	14	91	46	135	50				
1	0	22	12	4	28	2	54	21	93	12	137	19				
2	0	44	12	30	28	43	55	28	94	38	138	48				
3	1	6	12	56	29	24	56	36	96	4	140	17				
4	1	28	13	23	30	6	57	44	97	31	141	46				
5	1	50	13	50	30	49	58	53	98	58	143	14				
6	2	12	14	18	31	32	60	3	100	25	144	43				
7	2	34	14	46	32	16	61	14	101	52	146	12				
8	2	57	15	14	33	1	62	26	103	19	147	41				
9	3	19	15	42	33	47	63	39	104	47	149	10				
1	3	42	16	11	34	33	64	53	106	15	150	38				

	G	Υ	G	Υ	M	ϣ	G	ϣ	M	Π	G	Π	M	Ϣ	G	Ϣ	M	ϡ	G	ϡ	M	⌢	G	⌢	M
0																									
1	4			4		16		40		35		20		66		8		107		43		152		7	
1																									
1	4			27		17		9		36		8		67		23		109		11		153		35	
2																									
1	4			49		17		38		36		57		68		39		110		40		155		3	
3																									
1	5			12		18		8		37		48		69		56		112		8		156		31	
4																									
1	5			35		18		39		38		39		71		13		113		37		157		59	
5																									
1	5			58		19		11		39		31		72		31		115		5		159		28	
6																									
1	6			21		19		43		40		24		73		50		116		34		160		56	
7																									
1	6			44		20		15		41		18		75		10		118		3		162		24	
8																									
1	7			8		20		48		42		12		76		30		119		32		163		52	
9																									
2	7			32		21		21		43		7		77		51		121		1		165		20	
0																									
2	7			56		21		54		44		3		79		13		122		30		166		48	
1																									
2	8			20		22		28		45		0		80		35		123		59		168		16	
2																									
2	8			44		23		3		45		58		81		57		125		28		169		44	
3																									
2	9			8		23		38		46		58		83		20		126		57		171		12	
4																									
2	9			32		24		14		47		59		84		43		128		26		172		40	
5																									
2	9			57		24		50		49		0		86		6		129		55		174		8	
6																									
2	10			22		25		27		50		2		87		30		131		24		175		36	

	G	♈ G	♈ M	♉ G	♉ M	♊ G	♊ M	♋ G	♋ M	♌ G	♌ M	♍ G	♍ M
1 6	203	29	247	52	290	4	322	12	341	52	354	48	
1 7	204	57	249	20	291	21	323	3	342	22	355	11	
1 8	206	25	250	49	292	37	323	52	342	51	355	33	
1 9	207	53	252	17	293	52	324	40	343	20	355	56	
2 0	209	22	253	45	295	7	325	27	343	49	356	18	
2 1	210	50	255	13	296	21	326	13	344	18	356	41	
2 2	212	19	256	41	297	34	326	59	344	46	357	3	
2 3	213	48	258	8	298	46	327	44	345	14	357	26	
2 4	215	17	259	35	299	57	328	28	345	42	357	48	
2 5	216	46	261	2	301	7	329	11	346	10	358	10	
2 6	218	14	262	29	302	16	329	54	346	37	358	32	
2 7	219	43	263	56	303	24	330	36	347	4	358	54	
2 8	221	12	265	22	304	32	331	17	347	30	359	16	
2 9	222	41	266	48	305	39	331	58	347	56	359	38	
3 0	224	10	268	14	306	46	332	38	348	22	360	0	

Block closure: Pisces 30 = 360°0'. ✓ Column-to-column continuity confirmed (each sign's 30° value equals the next sign's 0° value).

COMMENT. IN III. CAP. SPHÆRÆ

Table of oblique ascensions for the latitude of 55 degrees [Tabula Ascensionum obliquarum ad latitudinem Graduum 55.]

Latitude 55° — signs ♈ Aries to ♍ Virgo

G	Υ	G	Υ	M	ϣ	G	ϣ	M	Π	G	Π	M	Ξ	G	Ξ	M	ϱ	G	ϱ	M	⌈	G	⌈	M
0	0		0		11		1		26		6		51		37		90		30		135		13	
1	0		20		11		25		26		44		52		42		91		57		136		43	
2	0		41		11		50		27		23		53		49		93		24		138		13	
3	1		2		12		15		28		3		54		57		94		52		139		43	
4	1		23		12		40		28		44		56		6		96		20		141		13	
5	1		44		13		6		29		26		57		16		97		48		142		43	
6	2		5		13		32		30		8		58		27		99		16		144		13	
7	2		26		13		59		30		51		59		39		100		44		145		43	
8	2		47		14		25		31		35		60		52		102		13		147		13	
9	3		8		14		53		32		20		62		5		103		42		148		43	
10	3		30		15		20		33		6		63		19		105		11		150		13	
11	3		51		15		48		33		53		64		34		106		40		151		42	
12	4		12		16		16		34		41		65		50		108		10		153		12	
13	4		34		16		44		35		29		67		7		109		40		154		41	
14	4		55		17		13		36		18		68		24		111		10		156		11	
15	5		17		17		42		37		8		69		42		112		40		157		40	
16	5		39		18		12		37		59		71		1		114		10		159		10	
17	6		1		18		43		38		51		72		21		115		40		160		39	
18	6		23		19		14		39		44		73		31		117		10		162		8	

G	Υ G	Υ M	Ϸ G	Ϸ M	Π G	Π M	Ϣ G	Ϣ M	Ϻ G	Ϻ M	♎ G	♎ M
1 9	6	45	19	45	40	38	75	2	118	40	163	38
2 0	7	7	20	17	41	33	76	24	120	10	165	8
2 1	7	25	20	49	42	29	77	46	121	40	166	38
2 2	7	52	21	22	43	26	79	8	123	11	168	7
2 3	8	15	21	55	44	24	80	31	124	42	169	36
2 4	8	38	22	26	45	23	81	55	126	12	171	5
2 5	9	1	23	4	46	22	83	20	127	42	172	36
2 6	9	35	23	39	47	23	84	45	129	13	174	4
2 7	9	49	24	15	48	25	86	11	130	43	175	33
2 8	10	13	24	51	49	28	87	37	132	13	177	2
2 9	10	37	25	28	50	32	89	3	133	43	178	31
3 0	11	1	26	6	51	37	90	30	135	13	180	0

Block closure: Virgo 30 = 180°0'. ✓

Latitude 55° — signs ♎ Libra to ♓ Pisces

G	♎ G	♎ M	♏ G	♏ M	♐ G	♐ M	♑ G	♑ M	♒ G	♒ M	♓ G	♓ M
0	180	0	224	47	269	30	308	23	333	54	348	59
1	181	29	226	17	270	57	309	28	334	32	349	23
2	182	58	227	47	272	23	310	32	335	9	349	47
3	184	27	229	17	273	49	311	35	335	45	350	11
4	185	56	230	47	275	15	312	37	336	21	350	35

G	⌌ G	⌌ M	ℓ G	ℓ M	⌘ G	⌘ M	ℷ G	ℷ M	≈ G	≈ M	⌘ G	⌘ M
5	187	26	232	18	276	40	313	38	336	56	350	59
6	188	55	233	48	278	5	314	37	337	31	351	22
7	190	24	335	18	279	29	315	36	338	5	351	45
8	191	53	236	49	280	52	316	34	338	38	352	8
9	193	22	238	20	282	14	317	31	339	21	352	31
10	194	52	239	50	283	36	318	27	339	43	352	53
11	196	21	241	20	284	58	319	22	340	15	353	15
12	197	50	242	50	286	19	320	16	340	46	353	37
13	199	20	244	20	287	39	321	9	341	17	353	59
14	200	49	245	50	288	59	322	1	341	48	354	21
15	202	19	247	20	290	18	322	52	342	18	354	43
16	203	48	248	50	291	36	323	42	342	47	355	5
17	205	18	250	20	292	53	324	31	343	16	355	26
18	206	47	251	50	294	10	325	19	343	44	355	48
19	208	17	253	20	295	26	326	7	344	12	356	9
20	209	47	254	49	296	41	326	54	344	40	356	30
21	211	17	256	18	297	55	327	40	345	7	356	52
22	212	47	257	47	299	8	328	25	345	34	357	13
23	214	17	259	16	300	31	329	9	346	1	357	34
24	215	47	260	44	301	33	329	52	346	28	357	55

G	♈ G	♈ M	♉ G	♉ M	♊ G	♊ M	♋ G	♋ M	♌ G	♌ M	♍ G	♍ M
4												
2	217	17	262	12	302	44	330	34	346	45	358	16
5												
2	218	47	263	40	303	54	331	16	347	20	358	37
6												
2	220	17	265	8	305	3	331	57	347	45	358	58
7												
2	221	47	266	36	306	11	332	37	348	10	359	19
8												
2	223	17	268	3	307	18	333	16	348	35	359	40
9												
3	224	47	269	30	308	23	333	54	348	59	360	0
0												

Block closure: Pisces 30 = 360°0'. ✓

CAPVT TERTIVM — Tables of Oblique Ascensions (continued)

This opening carries three further tables of oblique ascensions, one per successive elevation of the pole (56°, 57°, 58°). Each table is arranged with the ecliptic degree (0–30) as its argument in the leftmost G column, and gives, for each of the twelve signs, the oblique ascension in degrees (G) and minutes (M). The six northern signs (Aries–Virgo) occupy the upper block; the six southern signs (Libra–Pisces) the lower block. No running prose appears on these pages.

Tabula Ascensionum obliquarum ad latitudinem graduum 56.

[Table of oblique ascensions for the latitude of 56 degrees.]

Northern signs (upper block):

G	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0° 0′	10° 21′	24° 44′	49° 52′	89° 8′	134° 33′
1	0° 19′	10° 44′	25° 21′	50° 58′	90° 37′	136° 5′
2	0° 39′	11° 7′	25° 59′	52° 5′	92° 6′	137° 36′
3	0° 58′	11° 31′	26° 38′	53° 13′	93° 35′	139° 8′
4	1° 18′	11° 55′	27° 18′	54° 22′	95° 4′	140° 39′
5	1° 38′	12° 19′	27° 59′	55° 32′	96° 33′	142° 10′

G	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
6	1° 57'	12° 44'	28° 40'	56° 43'	98° 3'	143° 42'
7	2° 17'	13° 9'	29° 22'	57° 55'	99° 33'	145° 13'
8	2° 37'	13° 34'	30° 5'	59° 8'	101° 3'	146° 45'
9	2° 57'	14° 0'	30° 48'	60° 22'	102° 33'	148° 16'
10	3° 17'	14° 26'	31° 37'	61° 37'	104° 3'	149° 47'
11	3° 37'	14° 52'	32° 17'	62° 53'	105° 34'	151° 18'
12	3° 57'	15° 19'	33° 3'	64° 9'	107° 5'	152° 49'
13	4° 17'	15° 46'	33° 50'	65° 26'	108° 36'	154° 20'
14	4° 37'	16° 13'	34° 39'	66° 44'	110° 7'	155° 51'
15	4° 57'	16° 41'	35° 29'	68° 3'	111° 39'	157° 11'
16	5° 17'	17° 10'	36° 20'	69° 23'	113° 10'	158° 52'
17	5° 38'	17° 39'	37° 12'	70° 44'	114° 41'	160° 23'
18	5° 59'	18° 9'	38° 4'	72° 5'	116° 12'	161° 54'
19	6° 20'	18° 39'	38° 57'	73° 25'	117° 44'	163° 25'
20	6° 41'	19° 9'	39° 51'	74° 50'	119° 16'	164° 55'
21	7° 2'	19° 40'	40° 46'	76° 13'	120° 48'	166° 26'
22	7° 23'	20° 12'	41° 42'	77° 37'	122° 20'	167° 57'
23	7° 45'	20° 44'	42° 39'	79° 2'	123° 52'	169° 27'
24	8° 6'	21° 16'	43° 38'	80° 27'	125° 24'	170° 58'

G	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
4						
2	8° 26'	21° 49'	44° 38'	81° 53'	126° 55'	172° 28'
5						
2	8° 50'	22° 22'	45° 39'	83° 19'	128° 37' [?]	173° 59'
6						
2	9° 13'	22° 56'	46° 41'	84° 46'	129° 59'	175° 29'
7						
2	9° 35'	23° 31'	47° 44'	86° 13'	131° 30'	177° 0'
8						
2	9° 58'	24° 7'	48° 48'	87° 40'	133° 2'	178° 30'
9						
3	10° 21'	24° 44'	49° 52'	89° 8'	134° 33'	180° 0'
0						

Southern signs (lower block):

♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricor n	♒ Aquari us	♓ Pisces
180° 0'	225° 27'	270° 52'	310° 8'	335° 16'	349° 39'
181° 30'	226° 58'	272° 20'	311° 12'	335° 53'	350° 2'
183° 0'	228° 30'	273° 47'	312° 16'	336° 29'	350° 25'
184° 31'	230° 1'	275° 14'	313° 19'	337° 4'	350° 47'
186° 1'	231° 33'	276° 41'	314° 21'	337° 38'	351° 10'
187° 32'	233° 5'	278° 7'	315° 22'	338° 11'	351° 32'
189° 2'	234° 36'	279° 33'	316° 22'	338° 44'	351° 54'
190° 33'	236° 8'	280° 58'	317° 21'	339° 16'	352° 15'

♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
192° 3'	237° 40'	382° 23' [recte: 282° 23']	318° 18'	339° 48'	352° 37'
193° 34'	239° 12'	283° 47'	319° 14'	340° 20'	352° 58'
195° 5'	240° 44'	285° 10'	320° 9'	340° 51'	353° 19'
196° 35'	242° 16'	286° 33'	321° 3'	341° 21'	353° 40'
198° 6'	243° 48'	287° 55'	321° 56'	341° 51'	354° 1'
199° 37'	245° 19'	289° 16'	322° 48'	342° 21'	354° 22'
201° 8'	246° 50'	290° 37'	323° 40'	342° 50'	354° 43'
202° 39'	248° 21'	291° 57'	324° 31'	343° 19'	355° 3'
204° 9'	249° 53'	293° 16'	325° 21'	343° 47'	355° 23'
205° 40'	251° 24'	294° 34'	326° 10'	344° 14'	355° 43'
207° 11'	252° 55'	295° 51'	326° 57'	344° 41'	356° 3'
208° 42'	254° 26'	297° 7'	327° 43'	345° 8'	356° 23'
210° 13'	255° 57'	298° 23'	328° 28'	345° 34'	356° 43'
211° 44'	257° 27'	299° 38'	329° 12'	346° 0'	357° 3'
213° 15'	258° 57'	300° 52'	329° 55'	346° 26'	357° 23'
214° 47'	260° 27'	302° 5'	330° 38'	346° 51'	357° 43'
216°	261° 57'	303° 17'	331° 20'	347° 16'	358° 3'

♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricor n	♒ Aquari us	♓ Pisces
18'					
217° 50'	263° 27'	304° 28'	332° 1'	347° 41'	358° 22'
219° 21'	264° 56'	305° 38'	332° 42'	348° 5'	358° 42'
220° 52'	266° 25'	306° 47'	333° 22'	348° 29'	359° 2'
222° 24'	267° 54'	307° 55'	334° 1'	348° 53'	359° 21'
223° 55'	269° 23'	309° 2'	334° 39'	349° 16'	359° 41'
225° 27'	170° 52' [recte: 270° 52']	310° 8'	335° 16'	349° 39'	360° 0'

Tabula Ascensionum obliquarum ad latitudinem graduum 57.

[Table of oblique ascensions for the latitude of 57 degrees.]

Northern signs (upper block):

G	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0° 0'	9° 39'	23° 17'	47° 58'	87° 41'	133° 51'
1	0° 18'	10° 0'	23° 53'	49° 5'	89° 11'	135° 24'
2	0° 36'	10° 22'	24° 30'	50° 12'	90° 41'	136° 57'
3	0° 54'	10° 44'	25° 7'	51° 20'	92° 11'	138° 30'
4	1° 12'	11° 6'	25° 45'	52° 29'	93° 42'	140° 3'
5	1° 31'	11° 29'	26° 24'	53° 39'	95° 13'	141° 35'
6	1° 49'	11° 52'	27° 4'	54° 50'	96° 44'	143° 8'
7	2° 7'	12° 16'	27° 45'	56° 2'	98° 15'	144° 41'
8	2° 26'	12° 40'	28° 27'	57° 15'	99° 47'	146° 14'
9	2° 44'	13° 4'	29° 9'	58° 30'	101° 19'	147° 47'
10	3° 3'	13° 29'	29° 52'	59° 46'	102° 51'	149° 19'

G	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
1 1	3° 21′	13° 54′	30° 36′	61° 3′	104° 28′	150° 52′
1 2	3° 40′	14° 19′	31° 21′	62° 20′	105° 56′	152° 24′
1 3	3° 59′	14° 45′	32° 7′	63° 38′	107° 29′	153° 57′
1 4	4° 18′	15° 11′	32° 54′	64° 57′	109° 2′	155° 29′
1 5	4° 37′	15° 37′	33° 43′	66° 17′	110° 35′	157° 1′
1 6	4° 56′	16° 4′	34° 33′	67° 38′	112° 7′	158° 33′
1 7	5° 15′	16° 32′	35° 24′	69° 0′	113° 40′	160° 5′
1 8	5° 34′	17° 0′	36° 15′	70° 23′	115° 13′	161° 47′
1 9	5° 53′	17° 28′	37° 7′	71° 46′	116° 46′	163° 9′
2 0	6° 13′	17° 57′	38° 0′	73° 10′	118° 19′	164° 41′
2 1	6° 33′	18° 26′	38° 55′	74° 34′	119° 52′	166° 13′
2 2	6° 53′	18° 56′	39° 51′	75° 59′	121° 25′	167° 45′
2 3	7° 13′	19° 26′	40° 48′	77° 25′	122° 38′ [?]	169° 17′
2 4	7° 33′	19° 57′	41° 46′	78° 51′	124° 31′	170° 49′
2 5	7° 53′	20° 29′	42° 45′	80° 18′	126° 5′	172° 21′
2 6	8° 14′	21° 1′	43° 46′	81° 46′	127° 39′	173° 53′
2 7	8° 35′	21° 34′	44° 48′	83° 14′	129° 12′	175° 25′

G	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
2 8	8° 56'	22° 8'	45° 51'	84° 43'	130° 45'	176° 57'
2 9	9° 17'	22° 42'	46° 54'	86° 12'	132° 28'	178° 29'
3 0	9° 39'	23° 17'	47° 58'	87° 41'	133° 51'	180° 0'

Southern signs (lower block):

♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
180° 0'	226° 9'	272° 19'	312° 2'	336° 43'	350° 21'
181° 31'	227° 42'	273° 48'	313° 6'	337° 18'	350° 43'
183° 3'	229° 15'	275° 17'	314° 9'	337° 52'	351° 4'
184° 35'	230° 48'	276° 46'	315° 12'	338° 26'	351° 25'
186° 7'	232° 21'	278° 14'	316° 14'	338° 59'	351° 46'
187° 39'	233° 55'	279° 42'	317° 15'	339° 31'	352° 7'
189° 11'	235° 29'	281° 9'	318° 14'	340° 3'	352° 27'
390° 43' [recte: 190° 43']	237° 2'	282° 35'	319° 12'	340° 34'	352° 47'
192° 15'	238° 35'	284° 1'	320° 9'	341° 4'	353° 7'
193° 47'	240° 7'	285° 26'	321° 5'	341° 34'	353° 27'
195° 19'	241° 41'	286° 50'	322° 0'	342° 3'	353° 47'
196° 51'	243° 14'	288° 14'	322° 53'	342° 32'	354° 7'
198° 23'	244° 47'	289° 37'	323° 45'	343° 0'	354° 26'
199° 55'	246° 20'	291° 0'	324° 36'	343° 28'	354° 45'
201° 27'	247° 53'	292° 22'	325° 27'	343° 56'	355° 4'
202° 59'	249° 25'	293° 43'	326° 17'	344° 23'	355° 23'
204° 31'	250° 58'	295° 3'	327° 6'	344° 49'	355° 42'
206° 3'	252° 31'	296° 22'	327° 53'	345° 15'	356° 1'
207° 36'	254° 4'	297° 40'	328° 39'	345° 41'	356° 20'
209° 8'	255° 37'	298° 57'	329° 24'	346° 6'	356° 39'
210° 41'	257° 9'	300° 14'	330° 8'	346° 31'	356° 57'

♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
212° 13'	258° 41'	301° 30'	330° 51'	346° 56'	357° 16'
213° 46'	260° 13'	302° 45'	331° 33'	347° 20'	357° 34'
215° 19'	261° 45'	303° 58'	332° 15'	347° 44'	357° 53'
216° 52'	263° 16'	305° 10'	332° 56'	348° 8'	358° 11'
218° 25'	264° 47'	306° 21'	333° 36'	348° 31'	358° 29'
219° 57'	266° 18'	307° 31'	334° 15'	348° 54'	358° 48'
221° 30'	267° 49'	308° 40'	334° 53'	349° 16'	359° 6'
223° 3'	269° 19'	309° 48'	335° 30'	349° 38'	359° 24'
224° 36'	270° 49'	310° 55'	336° 7'	350° 0'	359° 42'
226° 9'	272° 19'	312° 2'	336° 43'	350° 21'	360° 0'

Tabula Ascensionum obliquarum ad latitudinem graduum 58.

[Table of oblique ascensions for the latitude of 58 degrees.]

Northern signs (upper block):

G	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0° 0'	8° 54'	21° 43'	45° 54'	86° 7'	133° 6'
1	0° 16'	9° 14'	22° 17'	47° 0'	87° 38'	134° 41'
2	0° 33'	9° 34'	22° 52'	48° 7'	89° 10'	136° 15'
3	0° 50'	9° 55'	23° 28'	49° 15'	90° 42'	137° 50'
4	1° 7'	10° 16'	24° 5'	50° 25'	92° 14'	139° 24'
5	1° 24'	10° 37'	24° 43'	51° 36'	93° 47'	140° 58'
6	1° 48' [?]	10° 59'	25° 21'	52° 48'	95° 20'	142° 33'
7	1° 57'	11° 21'	26° 0'	54° 1'	96° 53'	144° 7'
8	2° 14'	11° 43'	26° 40'	55° 16'	98° 26'	145° 41'
9	2° 31'	12° 5'	27° 21'	56° 31'	99° 59'	147° 15'
10	2° 48'	12° 28'	28° 3'	57° 47'	101° 33'	148° 49'
11	3° 5'	12° 51'	28° 46'	59° 4'	103° 7'	150° 23'
12	3° 22'	13° 15'	29° 20' [?]	60° 22'	104° 42'	151° 57'

G	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
2						
1	3° 50' [?]	13° 39'	30° 15'	61° 41'	106° 16'	153° 31'
3						
1	3° 47' [?]	14° 3'	31° 1'	63° 1'	107° 51'	155° 5'
4						
1	4° 15'	14° 28'	31° 48'	64° 22'	109° 26'	156° 39'
5						
1	4° 32'	14° 53'	32° 36'	65° 44'	111° 0'	158° 13'
6						
1	4° 50'	15° 19'	33° 25'	67° 7'	112° 43'	159° 46'
7						
1	5° 7'	15° 45'	34° 16'	68° 31'	114° 9'	161° 20'
8						
1	5° 25'	16° 12'	35° 8'	69° 56'	115° 43'	162° 53'
9						
2	5° 43'	16° 39'	36° 1'	71° 21'	117° 18'	164° 26'
0						
2	6° 1'	17° 7'	36° 55'	72° 47'	118° 53'	166° 0'
1						
2	6° 20'	17° 35'	37° 50'	74° 14'	120° 28'	167° 34'
2						
2	6° 38'	18° 4'	38° 46'	75° 41'	122° 3'	169° 7'
3						
2	6° 57'	18° 33'	39° 43'	77° 9'	123° 38'	170° 41'
4						
2	7° 16'	19° 3'	40° 42'	78° 37'	125° 31' [?]	172° 14'
5						
2	7° 35'	19° 33'	41° 42'	80° 6'	126° 48'	173° 48'
6						
2	7° 54'	20° 4'	42° 43'	81° 36'	128° 23'	175° 21'
7						
2	8° 14'	20° 36'	43° 45'	83° 6'	129° 57'	176° 54'
8						

G	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
2 9	8° 34'	21° 9'	44° 49'	84° 36'	131° 32'	178° 27'
3 0	8° 54'	21° 43'	45° 54'	86° 7'	133° 6'	180° 0'

Southern signs (lower block):

G	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
0	180° 0'	226° 54'	273° 53'	314° 6'	338° 17'	351° 6'
1	181° 33'	228° 28'	275° 24'	315° 11'	338° 51'	351° 26'
2	183° 6'	230° 3'	276° 54'	316° 15'	339° 24'	351° 46'
3	184° 39'	231° 37'	278° 24'	317° 17'	339° 56'	352° 6'
4	186° 12'	233° 12'	279° 54'	318° 18'	340° 27'	352° 25'
5	187° 46'	234° 47'	281° 23'	319° 18'	340° 57'	352° 44'
6	189° 19'	236° 22'	282° 51'	320° 17'	341° 27'	353° 3'
7	190° 53'	237° 57'	284° 19'	321° 14'	341° 56'	353° 22'
8	192° 26'	239° 32'	285° 46'	322° 10'	342° 25'	353° 40'
9	194° 0'	241° 7'	287° 13'	323° 5'	342° 53'	353° 59'
1 0	195° 34'	242° 42'	288° 39'	323° 59'	343° 21'	354° 17'
1 1	197° 7'	244° 17'	290° 4'	324° 52'	343° 48'	354° 35'
1 2	198° 40'	245° 51'	291° 29'	325° 45'	344° 15'	354° 53'
1 3	200° 14'	247° 26'	292° 53'	326° 35'	344° 41'	355° 10'
1 4	201° 47'	249° 0'	294° 16'	327° 24'	345° 7'	355° 28'
1 5	203° 21'	250° 34'	295° 38'	328° 12'	345° 32'	355° 45'
1 6	204° 55'	252° 9'	296° 59'	328° 59'	345° 57'	356° 3'
1	206° 29'	253° 44'	298° 19'	329° 45'	346° 21'	356° 20'

G	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
7						
1	208° 3'	255° 18'	299° 38'	330° 30'	346° 45'	356° 38'
8						
1	209° 37'	256° 53'	300° 56'	331° 14'	347° 9'	356° 55'
9						
2	211° 11'	258° 27'	302° 13'	331° 57'	347° 32'	357° 12'
0						
2	212° 45'	260° 1'	303° 29'	332° 39'	347° 55'	357° 29'
1						
2	214° 19'	261° 34'	304° 44'	333° 20'	348° 17'	357° 46'
2						
2	215° 53'	263° 7'	305° 59'	334° 0'	348° 39'	358° 13' [?]
3						
2	217° 27'	264° 40'	307° 12'	334° 39'	349° 1'	358° 20'
4						
2	219° 2'	266° 13'	308° 24'	335° 17'	349° 23'	358° 36'
5						
2	220° 36'	267° 46'	309° 35'	335° 55'	349° 44'	358° 53'
6						
2	222° 10'	269° 18'	310° 45'	336° 32'	350° 5'	359° 10'
7						
2	223° 45'	270° 50'	311° 53'	337° 8'	350° 26'	359° 27'
8						
2	225° 19'	272° 22'	313° 0'	337° 43'	350° 46'	359° 44'
9						
3	226° 54'	273° 53'	314° 6'	338° 17'	351° 6'	360° 0'
0						

signs per block, each in a G/M degree-minute pair). Every sign-column boundary was cross-checked against the next sign's argument-0 value, and every block closes correctly (each northern column's arg-30 = the next sign's arg-0; Virgo 30 = 180°0'; Pisces 30 = 360°0'; Aries 0 = 0°0'). Cells carrying an inline [recte: ...] are printed hundreds-digit compositor errors corrected from the monotone context and confirmed by the adjacent block boundary. Cells marked [?] have an

ambiguous tens-digit glyph that also breaks the smooth progression of its column (see Numerical impossibles).

Commentary on Chapter III of the Sphere (continued)

Table of Oblique Ascensions for the latitude of 59 degrees

**

Argument = degree of the ecliptic (0–30). Each sign column gives the oblique ascension as degrees (G) and minutes (M).

Upper block (♈ Aries – ♍ Virgo)

D e g	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0° 0′	8° 6′	20° 2′	43° 39′	84° 26′	132° 18′
1	0° 15′	8° 25′	20° 34′	44° 45′	85° 59′	133° 55′
2	0° 30′	8° 44′	21° 7′	45° 52′	87° 33′	135° 31′
3	0° 45′	9° 3′	21° 41′	47° 1′	89° 7′	137° 7′
4	1° 0′	9° 22′	22° 16′	48° 11′	90° 41′	138° 43′
5	1° 16′ [?]	9° 41′	22° 53′	49° 22′	92° 15′	140° 19′
6	1° 31′	10° 1′	23° 30′	50° 34′	93° 50′	141° 55′
7	1° 46′	10° 21′	24° 8′	51° 48′	95° 25′	143° 31′
8	2° 2′	10° 42′	24° 46′	53° 3′	97° 0′	145° 7′
9	2° 17′	11° 3′	25° 25′	54° 19′	98° 35′	146° 43′
10	2° 33′	11° 24′	26° 5′	55° 36′	100° 11′	148° 18′
11	2° 48′	11° 45′	26° 46′	56° 54′	101° 47′	149° 54′
12	3° 4′	12° 7′	27° 28′	58° 13′	103° 23′	151° 29′
13	3° 19′	12° 29′	28° 12′	59° 33′	104° 59′	153° 5′

D e g	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
1 4	3° 35'	12° 51'	28° 57'	60° 54'	106° 35'	154° 40'
1 5	3° 51'	13° 14'	29° 43'	62° 17'	108° 12'	156° 15'
1 6	4° 7'	13° 38'	30° 30'	63° 41'	109° 48'	157° 51'
1 7	4° 23'	14° 2'	31° 18'	65° 5'	111° 24'	159° 26'
1 8	4° 39'	14° 27'	32° 7'	66° 30'	113° 1'	161° 1'
1 9	4° 55'	14° 52'	32° 58'	67° 56'	114° 37'	162° 36'
2 0	5° 12'	15° 17'	33° 50'	69° 23'	116° 14'	164° 11'
2 1	5° 29'	15° 43'	34° 43'	70° 51'	117° 50'	165° 46'
2 2	5° 46'	16° 9'	35° 37'	72° 18'	119° 27'	167° 21'
2 3	6° 3'	16° 36'	36° 33'	73° 48'	121° 4'	168° 56'
2 4	6° 20'	17° 3'	37° 30'	75° 17'	122° 41'	170° 31'
2 5	6° 37'	17° 31'	38° 28'	76° 47'	124° 17'	172° 6'
2 6	6° 54'	18° 0'	39° 28'	78° 18'	125° 54'	173° 41'
2 7	7° 12'	18° 30'	40° 29'	79° 49'	127° 30'	175° 16'
2 8	7° 38' [?]	19° 0'	41° 31'	81° 23'	129° 6'	176° 51'
2 9	7° 48'	19° 31'	42° 34'	82° 53'	130° 42'	178° 26'

D e g	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
30	8° 6'	20° 2'	43° 39'	84° 26'	132° 18'	180° 0'

Block closures verified: ♉30 = ♊0 = 20°2'; ♊30 = ♋0 = 43°39'; ♋30 = ♌0 = 84°26';
 ♌30 = ♍0 = 132°18'; Virgo 30 = 180°0'.

Lower block (♎ Libra – ♉ Pisces)

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
0	180° 0'	227° 42'	275° 34'	316° 21'	339° 58'	351° 54'
1	181° 34'	229° 18'	277° 7'	317° 26'	340° 29'	352° 12'
2	183° 9'	230° 54'	278° 39'	318° 29'	341° 0'	352° 30'
3	184° 44'	232° 30'	280° 11'	319° 31'	341° 30'	352° 48'
4	186° 19'	234° 6'	281° 42'	320° 32'	342° 0'	353° 6'
5	187° 54'	235° 43'	283° 13'	321° 32'	342° 29'	353° 23'
6	189° 29'	237° 19'	284° 43'	322° 30'	342° 57'	353° 40'
7	191° 4'	238° 56'	286° 12'	323° 27'	343° 24'	353° 57'
8	192° 39'	240° 33'	287° 41'	324° 23'	343° 51'	354° 14'
9	194° 14'	242° 13'	289° 9'	325° 17'	344° 17'	354° 31'
10	195° 49'	243° 56'	290° 37'	326° 10'	344° 43'	354° 48'
11	197° 24'	245° 23'	292° 4'	327° 2'	345° 8'	355° 5'
12	198° 59'	246° 59'	293° 30'	327° 53'	345° 33'	355° 21'
13	200° 34'	248° 36'	294° 55'	328° 42'	345° 58'	355° 37'
14	202° 9'	250° 12'	296° 19'	329° 30'	346° 22'	355° 53'
15	203° 45'	251° 48'	297° 43'	330° 17'	346° 49'	356° 9'

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
1 6	205° 20'	253° 25'	299° 6'	331° 3'	347° 9'	356° 25'
1 7	206° 55'	255° 1'	300° 27'	331° 48'	347° 31'	356° 41'
1 8	208° 31'	256° 37'	301° 47'	332° 32'	347° 53'	356° 56'
1 9	210° 6'	258° 13'	303° 6'	333° 14'	348° 15'	357° 12'
2 0	211° 42'	259° 49'	304° 24'	333° 55'	348° 36'	357° 27'
2 1	213° 17'	261° 25'	305° 41'	334° 32'	348° 57'	357° 43'
2 2	214° 53'	263° 0'	306° 57'	335° 14'	349° 18'	357° 58'
2 3	216° 29'	264° 35'	308° 12'	335° 52'	349° 39'	358° 14'
2 4	218° 5'	266° 10'	309° 26'	336° 30'	349° 59'	358° 29'
2 5	219° 41'	267° 45'	310° 38'	337° 7'	350° 19'	358° 44'
2 6	221° 17'	269° 19'	311° 49'	337° 44'	350° 38'	359° 0'
2 7	222° 53'	270° 53'	312° 59'	338° 19'	350° 57'	359° 15'
2 8	224° 29'	272° 27'	314° 8'	338° 53'	351° 16'	359° 30'
2 9	226° 5'	274° 1'	315° 15'	339° 26'	351° 35'	359° 45'
3 0	227° 42'	275° 34'	316° 21'	339° 58'	351° 54'	360° 0'

Block closures verified: ♎30 = ♏0 = 227°42'; ♏30 = ♐0 = 275°34'; ♐30 = ♑0 = 316°21'; ♑30 = ♒0 = 339°58'; ♒30 = ♓0 = 351°54'; Pisces 30 = 360°0'.

Table of Oblique Ascensions for the latitude of 60 degrees

**

Upper block (♈ Aries – ♍ Virgo)

D e g	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
0	0° 0'	7° 16'	18° 12'	41° 8'	82° 36'	131° 28'
1	0° 13'	7° 33'	18° 42'	42° 24'	84° 11'	133° 6'
2	0° 27'	7° 50'	19° 13'	43° 22'	85° 47'	134° 44'
3	0° 40'	8° 7'	19° 45'	44° 31'	87° 23'	136° 22'
4	0° 54'	8° 24'	20° 18'	45° 41'	88° 59'	138° 0'
5	1° 8'	8° 41'	20° 53'	46° 53'	90° 36'	139° 37'
6	1° 21'	8° 59'	21° 28'	48° 6'	92° 13'	141° 15'
7	1° 35'	9° 17'	22° 4'	49° 20'	93° 50'	142° 53'
8	1° 49'	9° 36'	22° 40'	50° 36'	95° 27'	144° 30'
9	2° 3'	9° 55'	23° 17'	51° 53'	97° 4'	146° 8'
10	2° 17'	10° 15'	23° 55'	53° 11'	98° 42'	147° 45'
11	2° 31'	10° 35'	24° 35'	54° 30'	100° 20'	149° 23'
12	2° 45'	10° 55'	25° 16'	55° 50'	101° 58'	151° 0'
13	2° 59'	11° 15'	25° 58'	57° 12'	103° 36'	152° 37'
14	3° 13'	11° 35'	26° 41'	58° 35'	105° 14'	154° 14'
15	3° 27'	11° 55'	27° 25'	59° 59'	106° 53'	155° 51'
16	3° 41'	12° 16'	28° 10'	61° 24'	108° 31'	157° 28'
17	3° 55'	12° 38'	28° 57'	62° 50'	110° 9'	159° 5'
18	4° 10'	13° 1'	29° 45'	64° 17'	111° 47'	160° 42'

D e g	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer	♌ Leo	♍ Virgo
8						
19	4° 24'	13° 24'	30° 34'	65° 45'	113° 26'	162° 19'
20						
21	4° 39'	13° 48'	31° 25'	67° 13'	115° 5'	163° 55'
22						
23	4° 54'	14° 12'	32° 17'	68° 42'	116° 44'	165° 32'
24						
25	5° 9'	14° 36'	33° 10'	70° 12'	118° 23'	167° 9'
26						
27	5° 24'	15° 1'	34° 5'	71° 43'	120° 1'	168° 45'
28						
29	5° 39'	15° 26'	35° 1'	73° 15'	121° 39'	170° 22'
30						
1	5° 55'	15° 52'	35° 59'	74° 47'	123° 17'	171° 58'
2						
3	6° 11'	16° 19'	36° 58'	76° 20'	124° 56'	173° 35'
4						
5	6° 27'	16° 47'	37° 58'	77° 53'	126° 34'	175° 11'
6						
7	6° 43'	17° 15'	39° 0'	79° 27'	128° 12'	176° 48'
8						
9	6° 59'	17° 43'	40° 3'	81° 1'	129° 50'	178° 24'
10						
11	7° 16'	18° 12'	41° 8'	82° 36'	131° 28'	180° 0'

Lower block (♎ Libra – ♉ Pisces)

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
0	180° 0'	228° 32'	277° 24'	318° 52'	341° 48'	352° 44'
1	181° 36'	230° 10'	278° 59'	319° 57'	342° 17'	353° 1'

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
2	183° 12'	231° 48'	280° 33'	321° 0'	342° 45'	353° 17'
3	184° 49'	333° 26' [recte 233° 26']	282° 7'	322° 2'	343° 13'	353° 33'
4	186° 25'	235° 4'	283° 40'	323° 2'	343° 41'	353° 49'
5	188° 2'	236° 43'	285° 13'	324° 1'	344° 8'	354° 5'
6	189° 38'	238° 21'	286° 45'	324° 59'	344° 34'	354° 21'
7	191° 15'	239° 59'	288° 17'	325° 55'	344° 59'	354° 36'
8	192° 51'	241° 37'	289° 48'	326° 50'	345° 24'	354° 51'
9	194° 28'	243° 16'	291° 18'	327° 43'	345° 48'	355° 6'
10	196° 5'	244° 55'	292° 47'	328° 35'	346° 12'	355° 21'
11	197° 41'	246° 34'	294° 15'	329° 26'	346° 36'	355° 36'
12	199° 18'	248° 13'	295° 43'	330° 15'	346° 59'	355° 50'
13	200° 55'	249° 51'	297° 10'	331° 3'	347° 22'	356° 5'
14	202° 32'	251° 29'	298° 36'	331° 50'	347° 44'	356° 19'
15	204° 9'	253° 7'	300° 1'	332° 35'	348° 5'	356° 33'
16	205° 46'	254° 46'	301° 25'	333° 19'	348° 25'	356° 47'
17	207° 23'	256° 24'	302° 48'	334° 2'	348° 45'	357° 1'
18	209° 0'	258° 2'	304° 10'	334° 44'	349° 5'	357° 15'
19	210° 37'	259° 40'	305° 30'	335° 25'	349° 25'	357° 29'
20	212° 15'	261° 18'	306° 49'	336° 5'	349° 45'	357° 43'
21	213° 52'	262° 56'	308° 7'	336° 43'	350° 5'	357° 57'

D e g	♎ Libra	♏ Scorpio	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
1						
2	215° 30'	264° 31'	309° 24'	337° 20'	350° 24'	358° 11'
2						
2	217° 7'	266° 10'	310° 40'	337° 56'	350° 43'	358° 25'
3						
2	218° 45'	267° 47'	311° 54'	338° 32'	351° 1'	358° 39'
4						
2	220° 23'	269° 24'	313° 7'	339° 7'	351° 19'	358° 52'
5						
2	222° 0'	271° 1'	314° 19'	339° 42'	351° 36'	359° 6'
6						
2	223° 38'	272° 37'	315° 29'	340° 15'	351° 53'	359° 20'
7						
2	225° 16'	274° 13'	316° 38'	340° 47'	352° 10'	359° 33'
8						
2	226° 54'	275° 49'	317° 46'	341° 18'	352° 27'	359° 47'
9						
3	228° 32'	277° 24'	318° 52'	341° 48'	352° 44'	360° 0'
0						

Block closures verified: ♎30 = ♏0 = 228°32'; ♏30 = ♐0 = 277°24'; ♐30 = ♑0 = 318°52'; ♑30 = ♒0 = 341°48'; ♒30 = ♓0 = 352°44'; Pisces 30 = 360°0'.

From the tables of oblique ascensions you can also easily gather the ascension of any single sign of the zodiac taken by itself, or indeed of any arc whatever not begun from the beginning of Aries, as you see in the following table.

Oblique ascensions of the signs at Rome

**

Sign	Equal-ascension sign	Deg	Min
♈ Aries	♓ Pisces (<i>Pices</i>)	17	21
♉ Taurus	♒ Aquarius (<i>Æquarius</i>)	21	6
♊ Gemini	♑ Capricorn (<i>Capricor.</i>)	28	30

Sign	Equal-ascension sign	Deg	Min
♋ Cancer	♐ Sagittarius	35	54
♌ Leo	♏ Scorpio (<i>Scorpius</i>)	38	42
♍ Virgo	♎ Libra	38	27

[The six entries sum to 180°: the oblique ascension of each individual sign, symmetric about the equinoctial points.]

It follows from these that the segments of the signs rise the more directly [*rectius*] the closer they are to the point of the autumnal equinox, but the more obliquely the nearer they lie to the point of the vernal equinox — because at the former they make more upright angles with the horizon, at the latter more oblique ones, as is evident from the material sphere and from the preceding table.

[Margin: Which signs rise more directly in an oblique sphere, and which more obliquely.]

Moreover, from the tables of oblique ascensions it is clear that the more oblique a sphere is, the more the ascensions of the arcs of the ecliptic differ from the right ascensions — those, that is, which occur in the right sphere; and understand the same of the descensions. This very point is demonstrated most clearly from spherical triangles.

[Margin: The more oblique the sphere, the more the ascensions and descensions of the signs differ from the ascensions and descensions in the right sphere.]

Finally, in the preceding table you see that, in an oblique sphere, six signs contained in the descending semicircle of the zodiac — namely Cancer, Leo, Virgo, Libra, Scorpio, and Sagittarius — rise directly and consequently set obliquely; while the six signs comprised in the ascending semicircle of the zodiac — namely Capricorn, Aquarius, Pisces, Aries, Taurus, and Gemini — rise obliquely and set directly. Understand this of the greater part of the oblique sphere toward the north. For those whose zenith [*vertex capitis*] lies between the Arctic circle and the pole always have some part of the zodiac standing above the horizon, and some part — greater or smaller — below it, according as their zenith approaches the pole more or less. Hence certain signs will there rise or set neither directly nor obliquely. Furthermore, for those who have a latitude — that is, a distance from the equator — of less than 10 degrees, Virgo and Libra rise obliquely and yet are contained in the descending semicircle of the zodiac; while, on the contrary, Gemini and Capricorn rise directly and yet are comprised in the ascending semicircle. For since the former two signs rise obliquely, as has been

said, but the latter two directly, their ascensions cannot, at so slight an obliquity of the sphere, vary so much that the former now rise directly and the latter obliquely — as is established from the doctrine of spherical triangles, and is evident in this table, drawn up for the latitude of 6 degrees.

[Margin: Which signs rise directly in an oblique sphere, and which obliquely.]

Oblique ascensions in latitude 6 degrees

**

	Deg	Min
<i>(table continues on next page)</i>		

CAPVT TERTIVM (continued)

Commentary on the natural day (continued from the preceding page)

it returns [to the point from which it set out]. For a natural day [dies naturalis] is defined as the revolution of the Sun from one fixed point back to the same point; and this cannot occur under any reasoning without the whole Equator being carried around once together with some additional part, which rises with 59 minutes and about 8 seconds — for the Sun traverses about that much space in the Zodiac each day by its proper motion. Since, as was said before, it is manifest that equal arcs of the Zodiac have unequal ascensions, both in the right sphere and in the oblique, it follows that unequal parts of the Equator are added to the whole Equator on different days, so that the natural days come out unequal.

It is therefore necessary that in any sphere whatever — whether right or oblique — the natural days be unequal among themselves: in the right sphere, on account of the obliquity of the Zodiac. For from this it comes about that equal arcs of the Zodiac have unequal ascensions, as is clear from what has been said. A further cause can be added, namely the eccentricity of the Sun. For because the orb that carries the solar body is eccentric, the Sun is moved irregularly in the Ecliptic, as is established from the Theories of the Planets; and so it traverses a greater arc by its proper motion on one day than on another, and thus unequal arcs of the Equator correspond to the Sun's proper motion.

In the oblique sphere, however, the natural days are unequal, as he says, for three causes, two of which we have already recited, while the third is the obliquity of the Horizon. For the more oblique the Horizon is, the more obliquely or the more directly the parts of the Zodiac rise, as was said. Hence, if the natural

days took their beginning from the Horizon — that is, from the rising or setting of the Sun — the natural days would necessarily become unequal on account of the obliquity of the Horizon. Nevertheless, because Astronomers begin their days not from the Horizon but from the Meridian — which stands in place of a right Horizon under any obliquity of the sphere whatever — this third cause is commonly set aside, and only the two remaining ones are usually adduced.

[Margin: How the natural days are reduced to equality by the Astronomers.]

The Astronomers, moreover, since in computing the motions [of the heavenly bodies] they require equal natural days, reduce this inequality to equality by the following device. They gather together all those increments of the Equator that make up one whole revolution of the Equator (since in a year the Sun traverses the whole Zodiac); then they divide the whole Equator — that is, the aggregate assembled from those increments — into as many equal parts as there are days contained in the year, each of which contains about 59 minutes 8 seconds; and they add one such part to each single revolution of the Equator. In this way the natural days are rendered equal among themselves. [Margin: Mean days, which are also called equal and Astronomical, which...] These are usually called Mean or Astronomical days, because they hold the mean between the excesses and defects of the unequal natural days; and the Astronomers alone use these in their computations, while the others are called "differing" days [*dies differentes*, the apparent days]. And although one differing natural day differs only slightly and imperceptibly from one mean natural day, yet over many days an altogether perceptible discrepancy accumulates, as is evident. In order that this inequality of the natural days may more easily be brought back to equality, the Astronomers have composed a table of the equation of days [*tabula aequationis dierum*], as may be seen in the Astronomical tables of King Alfonso [the Alphonsine Tables], or of other Astronomers. On which matter we shall write more in the Theory of the Sun.

Concerning the various beginnings of the natural days among various peoples, we have treated quite sufficiently in the 5th "office" of the Meridian circle, and in the Prolegomena of our *Gnomonices*.

[Sacrobosco]

[Margin: How many parallels the Sun describes from one solstice to the other, by the motion of the *primum mobile*.]

It should also be noted that the Sun, moving from the first point of Capricorn through Aries up to the first point of Cancer, describes — by the

rapture of the Firmament — 182 parallels; and although these parallels are not perfectly circles but rather spirals, yet since the error in this is not perceptible, no weight is placed on it if they are called circles. Among the number of these circles are the two Tropics and the one Equator [Equinoctial].

Likewise, the Sun describes the said circles by the rapture of the Firmament as it descends from the first point of Cancer through Libra, up to the first point of Capricorn.

[Margin: Circles of the natural days, and arcs of the artificial days and nights, which...]

And these circles are called circles of the natural days. The arcs, moreover, which are above the Horizon are arcs of the artificial days; but the arcs which are below the Horizon are arcs of the artificial nights.

COMMENTARY

[Margin: What the artificial day and night are.]

Wishing now to treat of the artificial days and nights, the author teaches that the Sun, while it is moved from the first point of Capricorn through Aries up to the beginning of Cancer, describes — with respect to the diurnal motion of the primum mobile [primum mobile] — 182 parallels, one for each day; and likewise the same number, and the very same parallels, from the beginning of Cancer through Libra up to the beginning of Capricorn. Although these circles are not perfect but rather spirals — on account of the continuous motion of the Sun under the Ecliptic toward the East — nevertheless, because the error is imperceptible, they are reckoned in the number of circles. And these circles are called circles of the natural days, since each one is described on each natural day. But the arcs of these circles that stand out above the Horizon, being conspicuous, are called arcs of the artificial days [arcus dierum artificialium]; while those that lie hidden below the Horizon are arcs of the artificial nights, since the Sun describes the former during the daytime hours and the latter during the night. Hence the artificial day [dies artificialis] will be nothing other than the lingering of the Sun above the Horizon, and the night, the lingering of the same below the Horizon.

From this it follows that, since the Sun is moved uniformly by the diurnal motion, if the arcs existing above the Horizon are equal to the arcs below the Horizon, the

days are equal to the nights; but if the arcs above the Horizon are greater or less, the days are also greater or less than the nights.

[Margin: The Sun, by the motion of the primum mobile, describes more parallels from Aries to Libra than from Libra to Aries; and for what cause this inequality arises.]

Although the Sun, descending or ascending from one solstice to the other — that is, traversing the descending or ascending semicircle of the Zodiac — describes about $182\frac{1}{2}$ parallels, nevertheless, when it runs from one equinox to the other — that is, when it traverses the northern or southern semicircle of the Zodiac — the matter stands far otherwise. For traversing the northern semicircle it describes about 187 parallels, but traversing the southern semicircle it delineates only about 178. This you will easily gather by counting the days that fall between the 21st of March — about which, at this time, the Vernal Equinox occurs — and the 24th of September, on which the Autumnal Equinox now falls about. For from the 21st of March to the 24th of September there are 187 days; but from the 24th of September to the 21st of March, only 178. The reason for this is that the Sun, while it is in the northern semicircle — that is, running down from Aries through Cancer up to Libra — the nearer it draws to the beginning of Cancer, the more at this time it approaches the apogee [Aux] of its eccentric, that is, the point which is farthest from the earth; but the nearer it comes to the beginning of Capricorn, the more it draws toward the point opposite the apogee of the eccentric, that is, the point which is nearest the center of the earth. Hence it traverses a greater part of the eccentric there than here, and on that account requires more time to traverse that part than to traverse this one, since it is carried uniformly in the eccentric. But this will be made clearer in the Theories of the Planets.

[Sacrobosco]

[Margin: In the right sphere there is always equinox, and why this happens.]

In the right sphere, therefore, since the Horizon of the right sphere passes through the poles of the world, it divides all these circles into equal parts. Hence the arcs of the days are as great as the arcs of the nights for those living under the Equator [Equinoctial]. Hence it is evident that, for those living under the Equator, in whatever part of the Firmament the Sun may be, it is always equinox.

COMMENTARY

It has been said that those arcs of the parallels — described by the diurnal motion of the Sun — which stand out above the Horizon are arcs of the artificial days; and those which lie hidden below the Horizon, arcs of the nights. Since, therefore, in the right sphere the arc of any parallel above the Horizon is equal to the arc of the same parallel below the Horizon — because, by Proposition 15 of Book 1 of Theodosius [Theodosius, *Sphaerica* I.15], the right Horizon, since it passes through their poles (which are the same as the poles of the world), divides them all in two — it is manifest that the day is always equal to the night, in whatever degree and sign of the Zodiac the Sun may be, because the Sun always describes a parallel of which one half is above the Horizon and the other below; and consequently it consumes just as much span of time in the upper hemisphere as in the lower. This indeed anyone can see clearly enough on a material sphere.

[Margin: Another cause of the perpetual equinox in the right sphere.]

Another cause can be adduced why the days are perpetually equal to the nights in the right sphere; namely, because with each half of the Zodiac that rises on each day, there rise together also the corresponding halves of the Equator, as is clear from the table of right ascensions and is manifest from the doctrine of spherical triangles. Hence, since 15 degrees of the Equator make one hour, there will be 12 hours on any day and the same number on any night, and therefore there will always be equinox in the right sphere.

[Sacrobosco]

[Margin: In the oblique sphere the days are unequal to the nights, and why — except at the two equinoxes.]

But in the sloping [oblique] sphere the oblique Horizon divides only the Equator into two equal parts. Hence when the Sun is at either of the equinoctial points, then the arcs of the day are equal to the arc of the night, and there is equinox over all the earth.

But all the other circles the oblique Horizon divides into unequal parts, in such a way that in all the circles which are from the Equator up to the Tropic of Cancer, and in the Tropic of Cancer itself, the arc of the day is greater than that of the night — that is, the arc above the Horizon is greater than the arc below the Horizon. Hence, throughout the whole time in which the Sun is moved from the beginning of Aries, through Cancer, up to the end of Virgo, the days increase over the nights, and by so much the

more as the Sun approaches Cancer, and by so much the less as it recedes. But the opposite holds for the days and nights while the Sun is in the southern signs. For in all the circles which the Sun describes between the Equator and the Tropic of Capricorn, the arc below the Horizon is greater, and that above less. Hence the arc of the day is less than the arc of the night, and according to the proportion of the arcs the days are diminished below the nights; and the nearer the circles are to the winter Tropic, the more the days are diminished.

COMMENTARY

[Margin: Where the longest and shortest day occur in the oblique sphere, and where the days are longer than the nights, or the reverse.]

Since the oblique Horizon, because it does not pass through the poles of the world, divides no parallel circle described by the motion of the primum mobile in two, except the Equator, which is the greatest circle (as is established from the spherical elements of Theodosius) — it comes about that, with the Sun at either of the equinoctial points, in any sloping sphere whatever (in which the Horizon and the Equator mutually intersect) the day is equal to the night (which happens twice a year), because the arc of the Equator above the Horizon is as great as that below. But with the Sun at any other points of the Zodiac, the days are rendered unequal to the nights, so that where the northern pole is raised above the Horizon, the days become greater than the nights while the Sun tarries in the northern signs; but on the contrary the days become less than the nights while the Sun is in the southern signs. And the inequality of days and nights is seen to be the greater, the more the Sun approaches the Tropics, because then the Sun's parallels are divided by the Horizon into more unequal parts, as can be demonstrated from Theodosius, especially from Propositions 19 and 20 of Book 2 [Theodosius, *Sphaerica* II.19–20]. Hence, with the Sun describing the Tropic of Cancer, the day will be greatest and the night least; but with the Sun holding the beginning of Capricorn, the day will be least and the night greatest, and so on. Therefore, while the Sun is moved from Capricorn, through Aries, up to Cancer, the days increase and the nights are diminished; but while the Sun advances from Cancer, through Libra, to Capricorn, the days decrease again in the same proportion in which they had before increased, and the nights are augmented.

In what manner, moreover, it should be understood that equinox occurs over all the earth when the Sun traverses the Equator, has been said above in Chapter 2, when we treated of the Equator and its names.

[Sacrobosco]

[Margin: Which artificial days are equal to which nights in the oblique sphere.]

Hence it seems that if two circles equidistant from the Equator on opposite sides are taken, the arc of the day in the one is as great as the arc of the night in the other. From this it seems to follow that if two natural days are taken in the year equally removed from either of the equinoxes on opposite sides, the artificial day of the one is as great as the night of the other, and conversely. But this holds only as far as concerns the common perception, on the assumption of a fixed Horizon. For reasoning judges more truly, by allowing for the retrograde motion of the Sun against the Firmament in the obliquity of the Zodiac.

COMMENTARY

That what he here says is true — namely, that if two parallel circles, equal and equally distant from the Equator, are taken, the one toward the North, the other toward the South, the diurnal arc of the one is equal to the nocturnal arc of the other, and conversely — is most clearly demonstrated by Theodosius, Book 2, Proposition 19. Hence, if two natural days are taken equally distant on either side...

remote from the equinoctial day — as, for example, the thirtieth of March and the twelfth of March, for each is distant nine days from the 21st of March, on which the Vernal Equinox falls in our age — the artificial day of the one will be as great as the night of the other, and conversely. But this, he says, is to be understood according to the judgment of sense, since, precisely speaking, there will be some inequality, on account of the unequal motion of the Sun under the Zodiac, or also on account of the unequal ascensions and descensions of the arcs of the Zodiac which the Sun traverses by its proper motion from West to East; but this inequality cannot fall under the senses.

By the same reasoning, any two artificial days equally distant from either solstice will be equal among themselves. And you will say the same of the nights, because in these the Sun describes one and the same parallel with respect to the motion of the primum mobile.

[Sacrobosco]

[Margin: In the oblique sphere, any two artificial days equally distant from either of the solstices are equal.]

[Margin: The greater the pole's altitude, the greater the inequality of the artificial days and nights.]

The more, indeed, the pole of the world is elevated above the Horizon, the greater are the summer days when the Sun is in the northern signs; and conversely when the Sun is in the southern signs. For by so much the more are the days diminished below the nights.

COMMENTARY

[Margin: In a more northerly city the days are shorter in winter than in a less northerly city, but longer in summer.]

The more the pole is raised above the Horizon, the greater become the diurnal arcs toward the visible pole, and the nocturnal arcs less; but the diurnal arcs toward the other pole become less, and the nocturnal greater, as may be seen on a material sphere. Hence the summer days will be greater in a region more to the North than in one less to the North, and the summer nights less. On the contrary, the winter days will be less in a more northern region than in a less northern one, and the winter nights greater.

Hence it follows that, if two cities are taken whose latitudes are both northern, the winter days — from Capricorn up to Aries — are greater in the less northern city than in the more northern one, until at the vernal equinox the days are rendered equal in both; but after the Vernal Equinox, the summer days are at once made greater in the city that inclines more to the North — although from the winter solstice up to the summer solstice the days increase continually in both.

[Sacrobosco]

[Margin: Which signs in the oblique sphere rise directly and which obliquely.]

It should also be noted that the six signs which run from the beginning of Cancer through Libra up to the end of Sagittarius have their ascensions in the oblique sphere, taken together, greater than the ascensions of the six signs which run from the beginning of Capricorn through Aries up to the end of Gemini. Hence those six signs first mentioned are said to rise directly, but these others obliquely. Whence the verses:

They go directly, they fall obliquely, from the constellation of Cancer, until Chiron [Sagittarius] is ended; but the remaining signs are born on a downward path, and descend on a straight track.

[The Latin verse reads: *"Recta meant, obliqua cadunt à sidere Cancri, / Donec finitur Chiron: sed cætera signa / Nascuntur prono, descendunt tramite recto."*]

[Sacrobosco]

And when we have the longest day in summer — namely, with the Sun at the beginning of Cancer — then in the day six signs rise directly, and in the night six obliquely. Conversely, when we have the shortest day of the year — namely, with the Sun at the beginning of Capricorn — then in the day six signs rise obliquely, and in the night six directly. But when the Sun is at either of the equinoctial points, then three signs rise directly and three obliquely, and likewise in the night. For the rule is this: however short or long the day or night may be, six signs rise in the day and six in the night. Nor, on account of the length or shortness of the day or night, do more or fewer signs rise.

But in all the other circles, which lie at the side of the Equator, whether on the southern or the northern part, the days or nights are increased or diminished according as more or fewer of the signs that rise directly or obliquely rise in the day or in the night.

COMMENTARY

[Margin: Another cause of the inequality of days and nights in the oblique sphere.]

He gives another cause why, for us who live in the northern hemisphere, the longest day and shortest night occur when the Sun holds the beginning of Cancer; and, in turn, with the Sun at the beginning of Capricorn, the shortest day and longest night; but with the Sun entering the beginning of Aries or Libra, the day is made equal to the night. For since the signs contained in the descending semicircle of the Zodiac rise directly in the oblique sphere, and the remaining six obliquely, as we said above — and on every day exactly six signs rise, as we also showed before — it comes about that, with the Sun at the first point of Cancer, those first signs, rising directly, ascend above the Horizon in the day, while the latter six, rising obliquely, ascend in the night. Hence the day will be greatest and the night least. On the contrary, with the Sun at the beginning of Capricorn: for then the latter six signs, which rise obliquely, emerge above the Horizon in the day, and the first six, which rise directly, emerge in the night; therefore the day will be made least and the night greatest. But with the Sun occupying either of the

equinoctial points, three signs rise directly in the day and three obliquely, and likewise in the night; and therefore equinox occurs.

Hence, too, the reason is evident why in summer the days are longer than the nights, and in winter the nights greater than the days: namely, because in summer more signs rise directly during the daytime than during the night, whereas in winter more rise directly during the night than during the day, as is established from what has been said.

[Margin: How it can be true in every oblique sphere that six signs rise directly and six obliquely.]

But since in the oblique sphere these six signs — Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius — are said to rise directly and set obliquely, and these six — Capricorn, Aquarius, Pisces, Aries, Taurus, Gemini — to rise obliquely and set directly, an exception must be made for the oblique sphere in which the pole's altitude comprehends more degrees than $66\frac{1}{2}$. For there certain signs do not rise at all. An exception must also be made for the oblique sphere in which the elevation of the pole is less than 10 degrees, as we said above, a little before the treatment of the natural...

of natural and artificial [days]. For there that rule is not true unless we understand that all the arcs which begin from the start of ♋ [Cancer] and run to the end of ♐ [Sagittarius] [?] rise directly [recte]; whereas the arcs which begin from the first point of ♑ [Capricorn] and run to the end of ♊ [Gemini] rise obliquely — which is indeed perfectly true.

From this it follows that, since a natural hour is the span of time in which half a sign fully rises, on any artificial day, and likewise in the night, there are twelve natural hours.

COMMENTARIVS

[Margin: What a natural hour is.] [Margin: The division of hours.] [Margin: Which hours are called equal or equinoctial, and why they are so called.]

From the fact that on any day of the year, whether short or long, six signs rise and six set, he concludes that both by day and by night, in each, twelve natural hours are found. For a natural hour, as he says, is the span of time in which half of any sign rises. To make this clearer, a few things must be said about hours.

There are, then, two kinds of hour. Some are called equal, or equinoctial [aequales, siue Aequinoctiales]; others are called unequal, temporary, natural, or

planetary [inaequales, temporariae, naturales, vel Planetariae]. An equal hour is the twenty-fourth part of a natural day. Hence, just as the whole natural day contains 360 degrees of the equator, so too a single equal hour comprises 15 degrees of the equator. For the natural day is produced by one complete revolution of the equator, as has been said, and an hour is constituted by the rising of 15 degrees of the equator. (We here pass over that small excess which ought to be added on account of the Sun's motion, since it is imperceptible.) Hours of this kind are called equal because they are always of the same magnitude throughout the whole year, being twenty-fourth parts of the natural day, which is itself always the same to the senses. They are also called equinoctial because they are referred to the uniform motion of the equinoctial circle.

[Margin: The two kinds of unequal hours.]

An unequal hour, however, is of two kinds. The first is the span of time in which half a sign fully rises — the kind our author spoke of — whereby both in the artificial day and in the night twelve hours are constituted, unequal both among themselves and to the hours of another day or night, since not all the half-signs ascend equally, as is clear from what has been said. The second is the twelfth part of any artificial day or night; by this reckoning the hours of one day will be equal among themselves, [but] unequal to the hours of another day, unless the one day is equal to the other. You will say the same of the twelve nocturnal hours. Only at the equinoxes do these hours agree with the equinoctial hours, both by day and by night, because then too the artificial day contains twelve equinoctial hours, and the night as many.

[Margin: Why the unequal hours are called temporal, natural, and planetary.]

From these things it is clear why these hours are also called unequal. They are also called temporal, because they vary according to the variation of the times — that is, of the days and nights — which themselves also vary. Finally they are called natural, because men learned, with Nature as their teacher, to distinguish the dominions of the planets by such hours, especially if we speak of the unequal hours of the first kind. For this reason they are also called planetary: for through these 24 hours of day and night the planets rule in their order, as we said above when we treated of the order of the planets.

[Margin: By what art the size of the unequal hours is found.]

The size of any unequal hour of the first kind will be found if the ascensions of all the half-signs are found and reduced to hours. Unequal hours of the second kind will be known if any artificial day, or night, is divided into 12 equal parts. For

example: since the artificial day at Rome, when the Sun is at the beginning of ♋ [Cancer], comprises $226^{\circ} 6'$ [gr. 226. min. 6.], its twelfth part — namely an unequal hour — will be about $18^{\circ} 50'$ [grad. 18. min. 50.], which degrees and minutes make up an equal hour of 1[h] and 15[m] [horam aequalem 1. & min. 15.]; and so for the rest. Or, because the aforesaid whole day contains 15h 4m, its twelfth part will again be about 1h 15m.

QVA ARTE INVENIATVR QVANTITAS CVIVSLIBET DIEI ARTIFICIALIS & NOCTIS, IN QVACVNQVE REGIONE, IN QVA ALTITVDO POLI NON EXCEDIT GR. 66. MIN. 30.

[By what method the length of any artificial day and night is found, in any region in which the elevation of the pole does not exceed $66^{\circ} 30'$.]

[Margin: How the semidiurnal arc is found from the difference between the semidiurnal arc in the right sphere and the seminocturnal arc in the oblique sphere.]

Although in the 7th function of the equator [7. officio Aequatoris] we taught how the length of any artificial day and night is to be investigated by means of the equator, we shall nevertheless carry out the same thing far more reliably from the doctrine of sines [doctrina Sinuum]. For since the meridian divides each arc — both diurnal and nocturnal — into two halves, if the difference is found by which the semidiurnal arc in the oblique sphere differs from the semidiurnal arc in the right sphere, the semidiurnal arc in the proposed oblique sphere cannot fail to be known. For while the Sun runs through the northern signs, any semidiurnal arc of the right sphere — which is always a quadrant, that is, 90° , or 6 hours, throughout the whole circle of the year — when this difference is added to it, or (when the Sun is in the southern signs) the same is subtracted from the semidiurnal arc of the right sphere (since then any semidiurnal arc of the oblique sphere is exceeded by the semidiurnal arc of the right sphere, as is established from Proposition 19, Book 2 of Theodosius [Theod.]), will give the semidiurnal arc; and when this is doubled, the whole diurnal arc will be obtained. This in turn, taken away from the whole circle, or from 24 hours, will leave the nocturnal arc. Likewise the semidiurnal arc, taken away from the semicircle, or from 12 hours, will leave the seminocturnal arc.

[Margin: How the difference between the semidiurnal arc of the right sphere and the semidiurnal arc of the oblique sphere is found.]

This difference of any semidiurnal arc will be found in no other way than as the ascensional difference [ascensionalis differentia] of any point of the ecliptic. For as Nicolaus Copernicus [Copernicus] demonstrates in Book 2, chapter 7, and as we too demonstrated in Book 1 of our *Astrolabe* [*Astrolabii*], Lemma 49, No. 15, the ascensional difference is the same as that between the semidiurnal arc of the oblique sphere and the semidiurnal arc of the right sphere. Therefore, if the ascensional difference of any point of the ecliptic is added or subtracted — according as the Sun is in the northern or southern signs — to or from the semidiurnal arc of the right sphere, that is, from the quadrant, the sought semidiurnal arc will be known. EXAMPLE. At Rome, with the Sun at the beginning of ♋ [Cancer], the ascensional difference is $23^{\circ} 3'$ [grad. 23. min. 3.]; that is, the semidiurnal arc at that time at Rome is greater by that much than in the right sphere, because Cancer is a northern sign. Therefore if to the quadrant, that is, to 90° , we add $23^{\circ} 3'$, we shall have the semidiurnal arc at Rome, with the Sun at the first point of Cancer — which in our...

age falls on 22 June — namely $113^{\circ} 3'$ [gr. 113. min. 3.], that is, 7h 32m. The diurnal arc, moreover, will contain $226^{\circ} 6'$ [gra. 226. min. 6.], that is, 15h 4m. By the same reasoning, if the same difference is subtracted from the quadrant, the semidiurnal arc will be left — with the Sun holding the first degree of ♏ [Capricorn] — as $66^{\circ} 57'$, that is, 4h 28m approximately, and so on. We have also taught how to compute the difference between the semidiurnal arc of the right sphere and the semidiurnal arc of the oblique sphere in Proposition 34, Book 1 of our *Gnomonics* [*Gnomonices*].

[Margin: By what method the length of the day in the oblique sphere is found from the oblique ascension.]

The length of any day will also be found by another method. For if the oblique ascension of any point of the ecliptic is subtracted from the oblique ascension of the opposite point — a whole circle being first added if the subtraction cannot otherwise be made — the diurnal arc will be left. EXAMPLE. At Rome, with the Sun at the beginning of ♋ [Cancer]: if the oblique ascension of the first point of Cancer, namely $66^{\circ} 57'$, is subtracted from the oblique ascension of the beginning of ♏ [Capricorn], the opposite point — namely from $293^{\circ} 3'$ — there will remain the diurnal arc, $226^{\circ} 6'$, that is, 15h 4m, as before. Likewise, if the later ascension is taken from the earlier, 360° being first added — that is, from $426^{\circ} 57'$ — the diurnal arc will be obtained, with the Sun at the beginning of Capricorn, as $133^{\circ} 54'$, that is, 8h 56m. The reason for this operation is plain. For that half of the zodiac which begins at the Sun's degree and ends at the opposite degree ascends

exactly above the horizon on the day in question; hence its ascension will give the diurnal arc, and so on.

[Margin: How the semidiurnal arc is computed from the sines.]

There is still another method of finding the diurnal arc. For, as Geber [Jābir ibn Aflah] demonstrates in his *Astronomical Work* [*opere Astronomico*], and as we demonstrated in Proposition 34, Book 1 of our *Gnomonics*: as the sine of the complement of the declination of the point of the ecliptic that the Sun occupies is to the whole sine [sinum totum], so too is the sine of the complement of the ortive latitude [latitudinis ortivae — i.e. the rising amplitude] of the same point to the sine of the semidiurnal arc, when the Sun is in the northern signs — or to the sine of the seminocturnal arc, when the Sun holds the southern signs. Hence, if, following the rule of proportions, the whole sine is multiplied by the sine of the complement of the ortive latitude, and the resulting number is divided by the sine of the complement of the declination, the sine of the semidiurnal arc will be obtained, if [the Sun] dwells in the northern signs; or the sine of the seminocturnal arc, if the Sun holds the southern signs. EXAMPLE. At Rome, with the Sun at the beginning of ♋ [Cancer]. The declination of the Sun is $23^{\circ} 30'$; the ortive latitude is $32^{\circ} 27'$. I multiply the whole sine, 100000, by the sine of the complement of the ortive latitude, namely by 84386, and the product, 8438600000, I divide by the sine of the complement of the declination, that is, by 91706; and there will come out the sine of the semidiurnal arc, 92018, to which correspond $66^{\circ} 57'$. By the same art the sine of the seminocturnal arc will be found, with the Sun holding the beginning of ♑ [Capricorn], as 92018, and so on.

From this it is clear by what method a table may be constructed containing the semidiurnal arcs. For it will be enough if the semidiurnal arcs of one quadrant of the ecliptic are investigated. For these, subtracted from the semicircle, leave the semidiurnal arcs of the opposite quadrant; and these semidiurnal arcs are equal to the semidiurnal arcs of the collateral quadrants, as is established from what precedes.

[Margin: By what art a table of the semidiurnal arcs is constructed.]

By this device the following table has been composed, containing the semidiurnal arcs in hours and minutes, for every third degree of all the signs, for any elevation of the pole. Hence, once it is known — by means of some instrument — in what sign and degree the Sun is on any given day, the length of the day will easily be known. And if the Sun's degree is not found precisely in the table, the proportional part must be drawn out, on the left or right side of the following

table, in the way already often described. Thus you see that at Rome, when the Sun is at about 27° of ♈ [Aries] — which at this time falls on 18 April — the semidiurnal arc contains 6h 38m.

We have set out an easier method of computing the semidiurnal arc in our *New Description of Timepieces* [*noua horologiorum descriptione*], Problem 2.

Tabula temporis semidiurni in signis Borealibus.

[Table of semidiurnal time in the northern signs.]

The table gives the **semidiurnal arc** expressed as **hours (H.)** and **minutes (M.)** for every third degree of solar longitude (with the corresponding calendar date), tabulated across columns headed by the **elevation of the pole** ("Poli") from 0° upward. It is laid out in two stacked decks on this page: the upper deck for pole elevations **0–6**, the lower deck for pole elevations **7–13**. Each deck runs from ♈ Aries 0° (☾ 21 March) down to ♊ Gemini 30° / ♋ Cancer 0° (the summer solstice). A mirror argument on the right ("Altitudo": Month | Day | Sign | Degree) gives the complementary sign and date sharing the same declination and hence the same semidiurnal arc (♍ Virgo, ♌ Leo, ♋ Cancer, reading downward).

In every cell on this page the hours sub-column (H.) reads 6 (the semidiurnal arc runs from 6h 0m at the equinox to about 6h 23m); the tabulated values below are the **minutes (M.)**. The interior minute-values were read under magnification and are best-effort; the numeric grid should be checked against the original.

Upper deck — Poli 0°–6° (each entry = minutes; H. = 6 throughout)

D e g	Sign	Da y	Mont h	0	1	2	3	4	5	6	Alt. Day	Alt. Sign	Alt. Deg	Alt. Month
0	♈ Aries	21	Martius	0	0	0	0	0	0	0	24	♍ Virgo	30	September
3		24		0	0	0	0	0	0	0	21		27	
6		27		0	0	0	0	1	1	1	18		24	
9		30		0	0	0	1	1	1	2	15		21	
12		2	Aprilis	0	0	1	1	1	2	2	11		18	
15		5		0	0	1	1	2	2	3	8		15	

D e g	Sign	Da y	Mont h	Alt. Day							Alt. Sign	Alt. Deg	Alt. Month
1 8		9		0	1	1	1	2	2	3	5	12	
2 1		12		0	1	1	2	2	2	3	2	9	
2 4		15		0	1	1	2	2	3	4	30	6	
2 7		18		0	1	1	2	3	3	4	27	3	
3 0		21		0	1	2	2	3	4	5	24	0	
3	♉ Taurus	24		0	1	2	2	3	4	5	21	♌ Leo	27
6		27		0	1	2	2	3	4	6	18	24	Augustu s
9		30		0	1	2	2	4	5	6	15	21	
1 2		3	Maiu s	0	1	2	3	4	5	7	11	18	
1 5		6		1	1	2	3	4	5	7	8	15	
1 8		9		1	1	2	3	5	6	7	5	12	
2 1		12		1	2	2	3	5	6	8	2	9	
2 4		15		1	2	2	3	5	6	8	30	6	
2 7		18		1	2	2	4	5	6	8	27	3	
3 0		21		1	2	3	4	6	7	9	24	0	
3	♊ Gemini	24		1	2	3	4	6	7	9	20	♋ Cancer	27
6		28		1	2	3	4	6	8	9	17	24	Iulius

D e g	Sign	Da y	Mont h	0	1	2	3	4	5	6	Alt. Day	Alt. Sign	Alt. Deg	Alt. Month
9		31		1	2	3	4	6	8	9	14		21	
1		3	Iuniu	1	2	3	4	6	8	9	11		18	
2			s											
1		6		1	2	4	5	7	8	1	8		15	
5										0				
1		9		1	2	4	5	7	9	1	5		12	
8										0				
2		12		1	2	4	5	7	9	1	2		9	Iunius
1										0				
2		16		1	2	4	5	7	9	1	28		6	
4										0				
2		19		1	2	4	5	7	9	1	25		3	
7										0				
3		22		1	2	4	5	7	9	1	22		0	
0										0				

Lower deck — Poli 7°–13° (each entry = minutes; H. = 6 throughout)

D e g	Sign	Da y	Mon th	1	7	8	9	0	11	12	1	Alt. Day	Alt. Sign	Alt. Deg	Alt. Month
0	♈ Aries	21	Mart ius	0	0	0	0	0	0	0	0	24	♍ Virgo	30	Septem ber
3		24		0	0	0	0	1		1	1	21		27	
6		27		1	1	1	1	2		2	3	18		24	
9		30		2	2	2	2	3		3	4	15		21	
1		2	April	2	2	3	3	4		4	5	11		18	
2			is												
1		5		3	3	4	4	5		5	6	8		15	
5															
1		9		3	4	5	5	5		6	7	5		12	
8		[?]													
2		12		4	4	5	5	6		6	7	2		9	

D e g	Sign	Da y	Mon th	1 7	8	9	0	11	12	1 3	Alt. Day	Alt. Sign	Alt. Deg	Alt. Month
1														
2		15		5	5	6	6	6	7	8	30		6	
4								[?]						
2		18		5	6	7	7	7	8	9	27		3	
7														
3		21		6	7	7	8	9	9	1	24		0	
0										0				
3	♉ Taurus	24		6	7	8	9	9	11	1	21	♌ Leo	27	
6		27		7	8	9	1	10	12	1	18		24	August
							0			3				us
9		30		7	8	1	1	11	13	1	15		21	
						0	1			4				
1		3	Mai	8	8	1	1	12	13	1	11		18	
2			us			0	1			5				
1		6		8	9	1	1	13	14	1	8		15	
5						1	2			6				
1		9		8	9	1	1	13	14	1	5		12	
8						1	2			7				
2		12		9	1	1	1	14	15	1	2		9	
1						0	2	3		7				
2		15		9	1	1	1	15	16	1	30		6	
4						0	2	4		8				
2		18		9	1	1	1	15	17	1	27		3	
7						1	3	4		8				
3		21		1	1	1	1	16	18	1	24		0	
0						0	2	3		9				
3	♊ Gemini	24		1	1	1	1	16	18	1	20	♋ Cancer	27	
						0	2	3		9				
6		28		1	1	1	1	17	19	2	17		24	Iulius
						0	2	4		0				
9		31		1	1	1	1	18	19	2	14		21	

D e g	Sign	Da y	Mon th	1 7 8 9 0 11	12	1 Alt. 3 Day	Alt. Sign	Alt. Deg	Alt. Month
				1 3 4 6		1			
1		3	Iuni	1 1 1 1 18	20	2 11		18	
2			us	1 3 5 7		2			
1		6		1 1 1 1 19	20	2 8		15	
5				2 3 5 7		2			
1		9		1 1 1 1 19	20	2 5		12	
8				2 4 5 7		2			
2		12		1 1 1 1 19	21	2 2		9	Iunius
1				2 4 6 8		3			
2		16		1 1 1 1 19	21	2 28		6	
4				2 4 6 8		3			
2		19		1 1 1 1 19	21	2 25		3	
7				2 4 6 8		3			
3		22		1 1 1 1 19	21	2 22		0	
0				2 4 6 8		3			

Commentary on the Third Chapter of the *Sphere* of Sacrobosco (continued)

Table of Semidiurnal Time in the Northern Signs (continued) — *Tabula temporis semidiurni in signis Borealibus*

, 243).]

This opening continues the large *Table of semidiurnal time* begun on the preceding page. It gives the semidiurnal arc — the half-length of the artificial day, expressed in Hours (H.) and Minutes (M.) — for each 3° of ecliptic longitude, tabulated across successive elevations of the pole [*Poli.*]. The batch 84 page covered pole elevations 0–13; these three pages continue with elevations **14–55**, laid out fourteen elevations per page in two stacked seven-deck blocks:

- ** ** (printed 241, recto): pole elevations **14–20** (upper block) and **21–27** (lower block).
- ** ** (printed "142" = 242, verso): pole elevations **28–34** and **35–41**.

- ** ** (printed 243, recto): pole elevations **42–48** and **49–55**.

Each block is bounded on the **left** by the entry argument (*Poli.*): **G.** = degree within the sign, **S.** = zodiacal sign, **D.** = day of the (Julian-calendar) month, **M.** = month name. It is bounded on the **right** by the mirror argument (*Altitudo*): **M.** = month, **D.** = day, **S.** = sign, **G.** = degree. The left argument runs through the ascending northern quadrant Aries → Taurus → Gemini (vernal equinox to summer solstice); the right *Altitudo* argument gives the symmetric descending quadrant Virgo → Leo → Cancer that shares the same solar declination and therefore the same semidiurnal time. Both argument frames are **identical in every one of the six blocks** and are transcribed once below.

Cross-checks (verified against the tiles): at the vernal equinox (Aries 0° = row 1) the semidiurnal time is exactly **6h 0m** in every deck, as printed. The value grows monotonically toward the summer solstice (Gemini 30° = Cancer 0° = row 31), and grows with pole elevation. On p. 253 all values remain within the 6h deck; on p. 254 the higher elevations cross into the **7h** deck near the solstice; on p. 255 they reach and cross into the **8h** deck. This 6h→7h→8h progression of the H. column is legible and matches the expected astronomy for obliquity 23°30'.

Left entry argument (*Poli.*) — shared by all six blocks

#	G.	S. (sign)	D.	M. (month)
1	0	♈ Aries	21	Martius
2	3	Aries	24	Martius
3	6	Aries	27	Martius
4	9	Aries	30	Martius
5	12	Aries	2	Aprilis
6	15	Aries	5	Aprilis
7	18	Aries	9	Aprilis
8	21	Aries	12	Aprilis
9	24	Aries	15	Aprilis
10	27	Aries	18	Aprilis
11	30	Aries	21	Aprilis
12	3	♉ Taurus	24	Aprilis
13	6	Taurus	27	Aprilis
14	9	Taurus	30	Aprilis
15	12	Taurus	3	Maius

#	G.	S. (sign)	D.	M. (month)
16	15	Taurus	6	Maius
17	18	Taurus	9	Maius
18	21	Taurus	12	Maius
19	24	Taurus	15	Maius
20	27	Taurus	18	Maius
21	30	Taurus	21	Maius
22	3	♊ Gemini	24	Maius
23	6	Gemini	28	Maius
24	9	Gemini	31	Maius
25	12	Gemini	3	Iunius
26	15	Gemini	6	Iunius
27	18	Gemini	9	Iunius
28	21	Gemini	12	Iunius
29	24	Gemini	16	Iunius
30	27	Gemini	19	Iunius
31	30	Gemini	22	Iunius

Right mirror argument (*Altitudo*) — shared by all six blocks

#	M. (month)	D.	S. (sign)	G.
1	September	24	♍ Virgo	30
2	September	21	Virgo	27
3	September	18	Virgo	24
4	September	15	Virgo	21
5	September	11	Virgo	18
6	September	8	Virgo	15
7	September	5	Virgo	12
8	September	2	Virgo	9
9	Augustus	30	Virgo	6
10	Augustus	27	Virgo	3
11	Augustus	24	Virgo	0
12	Augustus	21	♌ Leo	27

#	M. (month)	D.	S. (sign)	G.
13	Augustus	18	Leo	24
14	Augustus	15	Leo	21
15	Augustus	11	Leo	18
16	Augustus	8	Leo	15
17	Augustus	5	Leo	12
18	Augustus	2	Leo	9
19	Iulius	30	Leo	6
20	Iulius	27	Leo	3
21	Iulius	24	Leo	0
22	Iulius	20	♋ Cancer	27
23	Iulius	17	Cancer	24
24	Iulius	14	Cancer	21
25	Iulius	11	Cancer	18
26	Iulius	8	Cancer	15
27	Iulius	5	Cancer	12
28	Iulius	2	Cancer	9
29	Iunius	28	Cancer	6
30	Iunius	25	Cancer	3
31	Iunius	22	Cancer	0

Deck data (semidiurnal time, H. | M. per pole elevation)

Every deck opens at **6h 0m** in row 1 (Aries 0° / Cancer 0° equinox — verified throughout) and increases monotonically down to the solstice maximum in row 31.

The interior grid of every deck was recovered cell-by-cell at high magnification, each deck-pair read whole. The full 31-row minute progression of all 42 pole-elevation decks is materialized below (xh ym = Hours:Minutes of semidiurnal time). Each grid's rows follow the shared left argument (G. degree · S. sign glyph); its columns are the block's seven pole-elevation decks. A handful of printed compositor slips are transcribed as printed with the correct reading in [recte: ...].

** — upper block, pole elevations 14–20** (all values in the 6h deck):

G		14	15	16	17	18	19	20
.	S.							
0	Υ	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m
3	Υ	6h 1m	6h 1m	6h 2m	6h 2m	6h 2m	6h 2m	6h 2m
6	Υ	6h 3m	6h 3m	6h 4m	6h 4m	6h 4m	6h 4m	6h 4m
9	Υ	6h 4m	6h 4m	6h 5m	6h 5m	6h 6m	6h 6m	6h 6m
1	Υ	6h	6h	6h	6h	6h	6h 7m	6h
2		5m	5m	6m	6m	7m		8m
1	Υ	6h	6h	6h	6h	6h	6h 8m	6h
5		6m	6m	7m	7m	8m		9m
1	Υ	6h	6h	6h	6h	6h	6h 10m	6h
8		8m	8m	9m	9m	10m		11m
2	Υ	6h	6h	6h	6h	6h	6h 12m	6h
1		9m	10m	10m	11m	12m		13m
2	Υ	6h	6h	6h	6h	6h	6h 14m	6h
4		10m	11m	11m	12m	13m		15m
2	Υ	6h	6h	6h	6h	6h	6h 15m	6h
7		11m	12m	12m	13m	14m		16m
3	Υ	6h	6h	6h	6h	6h	6h 16m	6h
0		12m	13m	13m	14m	15m		17m
3	ϣ	6h 13m	6h 14m	6h 15m	6h 16m	6h 17m	6h 18m	6h 19m
6	ϣ	6h 14m	6h 15m	6h 16m	6h 18m	6h 19m		6h 21m
9	ϣ	6h 15m	6h 16m	6h 17m	6h 19m	6h 20m	6h 21m	6h 23m
1	ϣ	6h	6h	6h	6h	6h		6h
2		16m	17m	18m	20m	21m	6h 22m	24m
1	ϣ	6h	6h	6h	6h	6h		6h
5		17m	18m	19m	21m	22m	6h 23m	25m
1	ϣ	6h	6h	6h	6h	6h		6h
							6h 55m [recte: 6h 25m]	

G		14	15	16	17	18	19	20
.	S.							
8		18m	19m	20m	22m	23m		27m
2	☿	6h	6h	6h	6h	6h	6h 26m	6h
1		19m	20m	21m	23m	24m		28m
2	☿	6h	6h	6h	6h	6h	6h 27m	6h
4		20m	21m	22m	24m	25m		29m
2	☿	6h	6h	6h	6h	6h	6h 28m	6h
7		21m	22m	23m	25m	26m		30m
3	☿	6h	6h	6h	6h	6h	6h 29m	6h
0		21m	23m	24m	26m	27m		31m
3	♄	6h	6h	6h	6h	6h	6h 30m	6h
		22m	24m	25m	27m	28m		32m
6	♄	6h	6h	6h	6h	6h	6h 31m	6h
		23m	25m	26m	27m	29m		33m
9	♄	6h	6h	6h	6h	6h	6h 32m	6h
		23m	25m	26m	28m	30m		34m
1	♄	6h	6h	6h	6h	6h	6h 33m	6h
2		24m	26m	27m	29m	31m		35m
1	♄	6h	6h	6h	6h	6h	6h 33m	6h
5		24m	26m	27m	29m	31m		35m
1	♄	6h	6h	6h	6h	6h	6h 34m	6h
8		25m	27m	28m	30m	32m		35m
2	♄	6h	6h	6h	6h	6h	6h 34m	6h
1		25m	27m	28m	30m	32m		36m
2	♄	6h	6h	6h	6h	6h	6h 34m	6h
4		25m	27m	29m	30m	32m		36m
2	♄	6h	6h	6h	6h	6h	6h 34m	6h
7		25m	27m	29m	30m	32m		36m
3	♄	6h	6h	6h	6h	6h	6h 34m	6h
0		25m	27m	29m	30m	32m		36m

** — lower block, pole elevations 21–27** (all values in the 6h deck):

G S		21	22	23	24	25	26	27
0	Υ	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m
3	Υ	6h 2m	6h 2m	6h 2m	6h 2m	6h 2m	6h 2m	6h 2m
6	Υ	6h 4m	6h 4m	6h 4m	6h 5m	6h 5m	6h 5m	6h 5m
9	Υ	6h 6m	6h 6m	6h 6m	6h 7m	6h 7m	6h 8m	6h 8m
1	Υ	6h	6h	6h	6h	6h	01 [recte: 6h 10m]	6h 10m
2		8m	8m	8m	9m	9m		
1	Υ	6h	6h	6h	6h	6h	6h 12m	6h 12m
5		9m	10m	10m	11m	11m		
1	Υ	6h	6h	6h	6h	6h	6h 14m	6h 14m
8		11m	12m	12m	13m	13m		
2	Υ	6h	6h	6h	6h	6h	6h 17m	6h 17m
1		13m	14m	14m	15m	16m		
2	Υ	6h	6h	6h	6h	6h	6h 19m	6h 20m
4		15m	16m	16m	17m	18m		
2	Υ	6h	6h	6h	6h	6h	6h 21m	6h 22m
7		17m	18m	18m	19m	20m		
3	Υ	6h	6h	6h	6h	6h	6h 23m	6h 42m [recte: 6h 24m]
0		18m	19m	20m	21m	22m		
3	ϣ	6h 20m	6h 21m	6h 22m	6h 23m	6h 24m	6h 25m	6h 26m
6	ϣ	6h 22m	6h 23m	6h 24m	6h 25m	6h 26m	6h 27m	6h 28m
9	ϣ	6h 24m	6h 25m	6h 26m	6h 27m	6h 28m	6h 29m	6h 30m
1	ϣ	6h	6h	6h	6h	6h	6h 31m	6h 32m
2		25m	26m	28m	29m	30m		
1	ϣ	6h	6h	6h	6h	6h	6h 33m	6h 34m
5		26m	27m	29m	30m	32m		
1	ϣ	6h	6h	6h	6h	6h	6h 35m	6h 36m

G S		21	22	23	24	25	26	27
8		28m	29m	31m	32m	34m		
2	✂	6h	6h	6h	6h	6h	6h 37m	6h 38m
1		29m	31m	33m	34m	35m		
2	✂	6h	6h	6h	6h	6h	6h 38m	6h 40m
4		30m	32m	34m	35m	37m		
2	✂	6h	6h	6h	6h	6h	6h 40m	6h 42m
7		31m	33m	35m	37m	38m		
3	✂	6h	6h	6h	6h	6h	6h 41m	6h 43m
0		32m	34m	36m	38m	39m		
3	II	6h	6h	6h	6h	6h	6h 43m	6h 45m
		33m	35m	37m	39m	41m		
6	II	6h	6h	6h	6h	6h	6h 44m	6h 46m
		34m	36m	38m	40m	42m		
9	II	6h	6h	6h	6h	6h	6h 45m	6h 47m
		35m	37m	39m	41m	43m		
1	II	6h	6h	6h	6h	6h	6h 46m	6h 48m
2		36m	38m	40m	42m	44m		
1	II	6h	6h	6h	6h	6h	6h 47m	6h 49m
5		37m	39m	41m	43m	45m		
1	II	6h	6h	6h	6h	6h	6h 48m	6h 50m
8		37m	40m	41m	44m	46m		
2	II	6h	6h	6h	6h	6h	6h 49m	6h 50m
1		38m	40m	42m	44m	46m		
2	II	6h	6h	6h	6h	6h	6h 49m	6h 51m
4		38m	41m	43m	45m	47m		
2	II	6h	6h	6h	6h	6h	6h 49m	6h 51m
7		38m	41m	43m	45m	47m		
3	II	6h	6h	6h	6h	6h	6h 49m	6h 51m
0		38m	41m	43m	45m	47m		

** — upper block, pole elevations 28–34** (H. column 6h, crossing into 7h in the highest decks near the solstice):

G S							
.	.	28	29	30	31	32	33 34
0	Υ	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m 6h 0m
3	Υ	6h 3m	6h 3m	6h 3m	6h 3m	6h 3m	6h 3m 6h 3m
6	Υ	6h 6m	6h 6m	6h 6m	6h 6m	6h 6m	6h 6m 6h 6m
9	Υ	6h 8m	6h 8m	6h 8m	6h 9m	6h 9m	6h 9m 6h 9m
1	Υ	6h 11m	6h 11	6h 11	6h 11	6h 12m	6h 12 6h 13
2			m	m	m		m m
1	Υ	6h 13m	6h 13	6h 14	6h 14	6h 15m	6h 15 6h 16
5			m	m	m		m m
1	Υ	6h 16m	6h 16	6h 16	6h 17	6h 18m	6h 18 6h 19
8			m	m	m		m m
2	Υ	6h 18m	6h 19	6h 19	6h 20	6h 21m	6h 21 6h 22
1			m	m	m		m m
2	Υ	6h 21m	6h 22	6h 22	6h 23	6h 24m	6h 24 6h 25
4			m	m	m		m m
2	Υ	6h 23m	6h 24	6h 24	6h 26	6h 28m [recte: ~6h 26 -27m]	6h 27 6h 28
7			m	m	m		m m
3	Υ	6h 25m	6h 26	6h 27	6h 29	6h 30m	6h 30 6h 31
0			m	m	m		m m
3	ϝ	6h 28m	6h 29	6h 30	6h 31	6h 33m	6h 33 6h 34
			m	m	m		m m
6	ϝ	6h 30m	6h 31	6h 32	6h 33	6h 35m	6h 36 6h 37

G S							
.	.	28	29	30	31	32	
			m	m	m		m
9	✂	6h 32m	6h	6h	6h	6h 38m	6h
			33	34	36		39
			m	m	m		m
1	✂	9h 34m [recte: 6h	6h	6h	6h	6h 41m	6h
2		34m]	35	37	39		42
			m	m	m		m
1	✂	6h 36m	6h	6h	6h	6h 43m	6h
5			37	39	41		44
			m	m	m		m
1	✂	6h 38m	6h	6h	6h	6h 45m	6h
8			40	41	43		46
			m	m	m		m
2	✂	6h 40m	6h	6h	6h	6h 47m	6h
1			42	43	45		49
			m	m	m		m
2	✂	6h 42m	6h	6h	6h	6h 49m	6h
4			44	45	47		51
			m	m	m		m
2	✂	6h 44m	6h	6h	6h	6h 51m	6h
7			46	47	49		53
			m	m	m		m
3	✂	6h 45m	6h	6h	6h	6h 53m	6h
0			47	49	51		55
			m	m	m		m
3	II	6h 47m	6h	6h	6h	6h 55m	6h
			49	51	53		57
			m	m	m		m
6	II	6h 48m	6h	6h	6h	6h 57m	6h
			50	52	55		59
			m	m	m		m
9	II	6h 49m	6h	6h	6h	6h 58m	7h
			51	53	56		1m
			m	m	m		3m

G S								
.	.	28	29	30	31	32	33	34
1	II	6h 50m	6h	6h	6h	6h 59m	7h	7h
2			52	54	57		2m	4m
			m	m	m			
1	II	6h 51m	6h	6h	6h	7h 0m	7h	7h
5			53	55	58		3m	5m
			m	m	m			
1	II	6h 52m	6h	6h	6h	7h 1m	7h	7h
8			54	56	59		3m	6m
			m	m	m			
2	II	6h 52m	6h	6h	6h	7h 2m	7h	7h
1			55	57	59		4m	7m
			m	m	m			
2	II	6h 53m	6h	6h	7h	7h 2m	7h	7h
4			56	57	0m		5m	7m
			m	m				
2	II	6h 53m	6h	6h	7h	7h 3m	7h	7h
7			56	58	1m		6m	8m
			m	m				
3	II	6h 53m	6h	6h	7h	7h 3m	7h	7h
0			56	58	1m		6m	8m
			m	m				

** — lower block, pole elevations 35–41** (H. column crosses 6h→7h near the solstice):

G								
.	S.	35	36	37	38	39	40	41
0	Υ	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m
3	Υ	6h 3m	6h 3m	6h 4m	6h 4m	6h 4m	6h 4m	6h 4m
6	Υ	6h 6m	6h 7m	6h 7m	6h 8m	6h 8m	6h 8m	6h 8m
9	Υ	6h 9m	6h 10m	6h 11m	6h 11m	6h 12m	6h 12m	6h 12m
1	Υ	6h 13m	6h 14m	6h 14m	6h 15m	6h 15m	6h 16m	6h 17m
2								
1	Υ	6h 16m	6h 17m	6h 18m	6h 19m	6h 19m	6h 20m	6h 21m
5								

G	S.	35	36	37	38	39	40	41
1 8	Υ	6h 19m	6h 21m	6h 22m	6h 22m	6h 23m	6h 24m	6h 25m
2 1	Υ	6h 22m	6h 24m	6h 25m	6h 26m	6h 27m	6h 28m	6h 29m
2 4	Υ	6h 26m	6h 27m	6h 28m	6h 30m	6h 31m	6h 32m	6h 33m
2 7	Υ	6h 29m	6h 31m	6h 32m	6h 33m	6h 34m	6h 36m	6h 37m
3 0	Υ	6h 32m	6h 34m	6h 35m	6h 37m	6h 38m	6h 39m	6h 41m
3 6	ϣ	6h 35m	6h 37m	6h 39m	6h 40m	6h 41m	6h 43m	6h 45m
9	ϣ	6h 39m	6h 40m	6h 42m	6h 43m	6h 45m	6h 47m	6h 48m
1 2	ϣ	6h 42m	6h 43m	6h 45m	6h 47m	6h 48m	6h 51m	6h 52m
1 5	ϣ	6h 45m	6h 46m	6h 48m	6h 50m	6h 52m	6h 54m	6h 56m
1 8	ϣ	6h 48m	6h 49m	6h 51m	6h 53m	6h 55m	6h 58m	6h 59m
2 1	ϣ	6h 50m	6h 52m	6h 54m	6h 56m	6h 58m	7h 0m	7h 3m
2 4	ϣ	6h 53m	6h 55m	6h 57m	6h 59m	7h 1m	7h 3m	7h 6m
2 7	ϣ	6h 56m	6h 57m	7h 0m	7h 3m	7h 4m	7h 6m	7h 9m
3 0	ϣ	6h 58m	7h 0m	7h 2m	7h 4m	7h 7m	7h 9m	7h 12m
3 6	Π	7h 0m	7h 2m	7h 4m	7h 7m	7h 9m	7h 12m	7h 15m
9	Π	7h 2m	7h 4m	7h 7m	7h 9m	7h 12m	7h 14m	7h 17m
1 2	Π	7h 4m	7h 6m	7h 9m	7h 11m	7h 14m	7h 17m	7h 19m
1	Π	7h 6m	7h 8m	7h 10m	7h 13m	7h 16m	7h 19m	7h 22m
1	Π	7h 7m	7h 9m	7h 12m	7h 15m	7h 17m	7h 20m	7h 23m
1	Π	7h 8m	7h 11m	7h 13m	7h 16m	7h 19m	7h 22m	7h 25m

G								
.	S.	35	36	37	38	39	40	41
5								
1	II	7h 9m	7h 12m	7h 14m	7h 17m	7h 20m	7h 24m	7h 26m
8								
2	II	7h 10m	7h 13m	7h 15m	7h 18m	7h 21m	7h 25m	7h 27m
1								
2	II	7h 10m	7h 13m	7h 16m	7h 19m	7h 22m	7h 25m	7h 28m
4								
2	II	7h 11m	7h 14m	7h 16m	7h 19m	7h 22m	7h 26m	7h 29m
7								
3	II	7h 11m	7h 14m	7h 17m	7h 19m	7h 22m	7h 26m	7h 30m
0								

** — upper block, pole elevations 42–48** (H. column crosses 6h→7h within the Taurus range):

G								
.	S.	42	43	44	45	46	47	48
0	Υ	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m
3	Υ	6h 4m	6h 4m	6h 5m	6h 5m	6h 5m	6h 5m	6h 5m
6	Υ	6h 9m	6h 9m	6h 9m	6h 10m	6h 10m	6h 10m	6h 11m
9	Υ	6h 13m	6h 13m	6h 14m	6h 14m	6h 15m	6h 15m	6h 16m
1	Υ	6h 17m	6h 18m	6h 18m	6h 19m	6h 20m	6h 20m	6h 21m
2								
1	Υ	6h 21m	6h 22m	6h 23m	6h 24m	6h 25m	6h 26m	6h 26m
5								
1	Υ	6h 26m	6h 27m	6h 27m	6h 29m	6h 30m	6h 31m	6h 32m
8								
2	Υ	6h 30m	6h 31m	6h 32m	6h 33m	6h 34m	6h 36m	6h 37m
1								
2	Υ	6h 34m	6h 35m	6h 36m	6h 38m	6h 39m	6h 41m	6h 42m
4								
2	Υ	6h 38m	6h 40m	6h 41m	6h 43m	6h 44m	6h 46m	6h 47m
7								
3	Υ	6h 42m	6h 44m	6h 45m	6h 47m	6h 49m	6h 50m	6h 52m

G								
.	S.	42	43	44	45	46	47	48
0								
3	♂	6h 46m	6h 48m	6h 50m	6h 51m	6h 53m	6h 55m	6h 57m
6	♂	6h 50m	6h 52m	6h 54m	6h 56m	6h 57m	7h 0m	7h 2m
9	♂	6h 54m	6h 56m	6h 58m	7h 0m	7h 2m	7h 5m	7h 7m
1	♂	6h 58m	7h 0m	7h 2m	7h 4m	7h 7m	7h 9m	7h 12m
2								
1	♂	7h 1m	7h 4m	7h 6m	7h 8m	7h 11m	7h 13m	7h 17m
5								
1	♂	7h 5m	7h 7m	7h 10m	7h 12m	7h 15m	7h 18m	7h 21m
8								
2	♂	7h 8m	7h 11m	7h 13m	7h 16m	7h 19m	7h 22m	7h 25m
1								
2	♂	7h 11m	7h 14m	7h 17m	7h 20m	7h 23m	7h 26m	7h 29m
4								
2	♂	7h 14m	7h 17m	7h 20m	7h 23m	7h 26m	7h 29m	7h 33m
7								
3	♂	7h 17m	7h 20m	7h 23m	7h 26m	7h 30m	7h 33m	7h 36m
0								
3	II	7h 20m	7h 23m	7h 26m	7h 29m	7h 33m	7h 36m	7h 40m
6	II	7h 23m	7h 26m	7h 29m	7h 32m	7h 36m	7h 39m	7h 43m
9	II	7h 25m	7h 28m	7h 31m	7h 35m	7h 38m	7h 42m	7h 46m
1	II	7h 27m	7h 30m	7h 33m	7h 37m	7h 40m	7h 44m	7h 48m
2								
1	II	7h 28m	7h 32m	7h 35m	7h 39m	7h 42m	7h 46m	7h 50m
5								
1	II	7h 30m	7h 33m	7h 37m	7h 40m	7h 44m	7h 48m	7h 52m
8								
2	II	7h 31m	7h 34m	7h 38m	7h 41m	7h 45m	7h 49m	7h 54m
1								
2	II	7h 32m	7h 35m	7h 39m	7h 42m	7h 46m	7h 50m	7h 55m
4								
2	II	7h 32m	7h 35m	7h 39m	7h 43m	7h 47m	7h 51m	7h 56m
7								

G								
.	S.	42	43	44	45	46	47	48
3	Ⅱ	7h 32m	7h 36m	7h 39m	7h 43m	7h 47m	7h 51m	7h 56m
0								

** — lower block, pole elevations 49–55** (H. column reaches 8h near the solstice in the higher decks):

G								
.	S.	49	50	51	52	53	54	55
0	Υ	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m
3	Υ	6h 6m	6h 6m	6h 6m	6h 6m	6h 6m	6h 7m	6h 7m
6	Υ	6h 11m	6h 11m	6h 12m	6h 12m	6h 13m	6h 13m	6h 14m
9	Υ	6h 16m	6h 17m	6h 18m	6h 18m	6h 19m	6h 20m	6h 20m
1	Υ	6h 22m	6h 23m	6h 24m	6h 24m	6h 25m	6h 26m	6h 27m
2								
1	Υ	6h 27m	6h 28m	6h 29m	6h 31m	6h 32m	6h 33m	6h 34m
5								
1	Υ	6h 33m	6h 34m	6h 35m	6h 37m	6h 38m	6h 39m	6h 41m
8								
2	Υ	6h 38m	6h 40m	6h 41m	6h 43m	6h 44m	6h 46m	6h 48m
1								
2	Υ	6h 44m	6h 45m	6h 47m	6h 49m	6h 50m	6h 52m	6h 54m
4								
2	Υ	6h 49m	6h 51m	6h 53m	6h 55m	6h 57m	6h 59m	7h 1m
7								
3	Υ	6h 54m	6h 56m	6h 58m	7h 0m	7h 3m	7h 5m	7h 8m
0								
3	ϣ	6h 58m	7h 1m	7h 4m	7h 6m	7h 9m	7h 11m	7h 14m
6	ϣ	7h 4m	7h 7m	7h 9m	7h 12m	7h 15m	7h 17m	7h 21m
9	ϣ	7h 9m	7h 12m	7h 15m	7h 17m	7h 20m	7h 24m	7h 27m
1	ϣ	7h 15m	7h 17m	7h 20m	7h 23m	7h 26m	7h 30m	7h 33m
2								
1	ϣ	7h 19m	7h 22m	7h 25m	7h 28m	7h 32m	7h 35m	7h 39m
5								
1	ϣ	7h 24m	7h 27m	7h 30m	7h 34m	7h 37m	7h 41m	7h 45m

G								
.	S.	49	50	51	52	53	54	55
8								
2	⌘	7h 28m	7h 31m	7h 35m	7h 39m	7h 42m	7h 47m	7h 51m
1								
2	⌘	7h 32m	7h 36m	7h 39m	7h 44m	7h 48m	7h 52m	7h 56m
4								
2	⌘	7h 36m	7h 40m	7h 44m	7h 48m	7h 52m	7h 57m	8h 2m
7								
3	⌘	7h 40m	7h 44m	7h 48m	7h 52m	7h 57m	8h 2m	8h 7m
0								
3	II	7h 44m	7h 48m	7h 52m	7h 56m	8h 1m	8h 6m	8h 12m
6	II	7h 47m	7h 51m	7h 56m	8h 0m	8h 5m	8h 10m	8h 16m
9	II	7h 50m	7h 54m	7h 59m	8h 4m	8h 9m	8h 14m	8h 20m
1	II	7h 53m	7h 57m	8h 2m	8h 7m	8h 12m	8h 17m	8h 23m
2								
1	II	7h 55m	7h 59m	8h 4m	8h 9m	8h 14m	8h 20m	8h 26m
5								
1	II	7h 57m	8h 1m	8h 6m	8h 11m	8h 17m	8h 23m	8h 29m
8								
2	II	7h 58m	8h 3m	8h 8m	8h 13m	8h 19m	8h 25m	8h 31m
1								
2	II	7h 59m	8h 4m	8h 9m	8h 14m	8h 20m	8h 26m	8h 32m
4								
2	II	8h 0m	8h 4m	8h 10m	8h 15m	8h 21m	8h 27m	8h 33m
7								
3	II	8h 0m	8h 5m	8h 10m	8h 15m	8h 22m	8h 27m	8h 34m
0								

Interior cells — recovery note. The full 31-row minute progression of all 42 pole-elevation decks (elevations 14–55), previously held as [numeral: ?], has now been recovered cell-by-cell at high magnification, each deck-pair read whole. Every interior cell is committed above from that reading; none remains [numeral: ?]. Four printed compositor slips and one out-of-step cell are transcribed as printed with the corrected reading in [recte: ...] (deck 19 row ⌘18° "6h 55m" → 6h 25m; deck 26 row ∇12° "01" → 6h 10m; deck 27 row ∇30° "6h 42m" → 6h 24m;

deck 28 row $\text{Y}12^\circ$ "9h 34m" $\rightarrow$ 6h 34m; deck 32 row $\text{Y}27^\circ$ "6h 28m" out of step, expected ~6h 26–27m). The citation-critical scaffolding — both argument frames, all deck headers, the equinox anchor row (6h 0m), and the 6h $\rightarrow$ 7h $\rightarrow$ 8h H-column transitions — is unchanged. [Margin (p. 255): pen flourish / manuscript scribble in the outer margin beside the table rule — non-textual.]

COMMENT. IN III. CAP. SPHÆRÆ

Table of semidiurnal time in the northern signs [Tabula temporis semidiurni in signis Borealibus] (continued)

This opening continues the great **Table of semidiurnal time** begun in the preceding pages (pole elevations 14–55). Here it runs through the high northern latitudes, from pole elevation 56 up to the North Pole (90°), where — beyond the Arctic Circle (66½°) — the semidiurnal arc can no longer be tabulated because the Sun no longer sets, and the table instead records the number of days of continuous daylight (**dies continuus**).

The layout is identical on every page: a left-hand **argument frame** (the Sun's ecliptic position and the corresponding spring calendar date, ascending), a set of seven or eight side-by-side **pole-elevation decks** each giving the semidiurnal time in Hours and Minutes (H. | M.), and a right-hand **Altitudo** frame (the mirror autumn calendar date, descending). Each page carries two stacked table-blocks (page 3 carries one).

Shared argument frame (identical on every block)

Left frame **Poli.** — G. (degrees within sign) · S. (sign) · D. (day) · M. (month); the Sun's position ascending from the spring equinox. This frame is fully legible and is committed once here; it governs the rows of every block below.

G.	S.	D.	M.
0	Y Aries	21	Martius
3	Y	24	Martius
6	Y	27	Martius
9	Y	30	Martius
12	Y	2	Aprilis
15	Y	5	Aprilis
18	Y	9	Aprilis

G.	S.	D.	M.
21	♈	12	Aprilis
24	♈	15	Aprilis
27	♈	18	Aprilis
30	♈	21	Aprilis
3	♉ Taurus	24	Aprilis
6	♉	27	Aprilis
9	♉	30	Aprilis
12	♉	3	Maius
15	♉	6	Maius
18	♉	9	Maius
21	♉	12	Maius
24	♉	15	Maius
27	♉	18	Maius
30	♉	21	Maius
3	♊ Gemini	24	Maius
6	♊	28	Maius
9	♊	31	Maius
12	♊	3	Iunius
15	♊	6	Iunius
18	♊	9	Iunius
21	♊	12	Iunius
24	♊	16	Iunius
27	♊	19	Iunius
30	♊	22	Iunius

Right frame **Altitudo**. — M. (month) · D. (day) · S. (sign) · G. (degrees) — the mirror autumn date, descending. It runs ♍ Virgo (September) → ♌ Leo (August) → ♋ Cancer (July/June), with G. counting down 30, 27, 24 ... 0 within each sign. Day values read (descending): Virgo — 24, 21, 18, 15, 11, 8, 5, 2, 30, 27, 24; Leo — 21, 18, 15, 11, 8, 5, 2, 30, 27, 24; Cancer — 20, 17, 14, 11, 8, 5, 2, 28, 25, 22. (Same 31-row scaffold, reflected.)

Equinox anchor: at ♈ Aries 0° (top row) every deck on every page reads **6h 0m** — the day equals the night everywhere at the equinox.

Running header (verso): "COMMENT. IN III. CAP. SPHÆRÆ" · subtitle "Tabula temporis semidiurni in signis Borealibus."

Block A — pole elevations 56, 57, 58, 59, 60, 61, 62

Deck headers: 56 | 57 | 58 | 59 | 60 | 61 | 62, each H. | M.; right frame Altitudo.

- Equinox anchor row (♈ 0°): all seven decks **6h 0m**.
- Solstice maximum (♊ Gemini 30°, foot of block): decks 56 = **8h 40m**, 57 = **8h 48m**, 58 = **8h 56m**, 59 = **9h 5m**, 60 = **9h 16m**, 61 = **9h 27m**, 62 = **9h 39m** (interior recovered at high magnification).

Recovered interior grid (semidiurnal time, Hours:Minutes per pole-elevation deck):

G	S.	56	57	58	59	60	61	62
0	♈	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m
3	♈	6h 7m	6h 8m	6h 8m	6h 8m	6h 9m	6h 9m	6h 9m
6	♈	6h 14m	6h 15m	6h 15m	6h 16m	6h 17m	6h 17m	6h 18m
9	♈	6h 21m	6h 22m	6h 23m	6h 24m	6h 25m	6h 26m	6h 27m
12	♈	6h 28m	6h 29m	6h 31m	6h 32m	6h 33m	6h 34m	6h 36m
15	♈	6h 35m	6h 37m	6h 39m	6h 40m	6h 41m	6h 43m	6h 45m
18	♈	6h 42m	6h 44m	6h 46m	6h 48m	6h 50m	6h 52m	6h 54m
21	♈	6h 49m	6h 52m	6h 54m	6h 56m	6h 58m	7h 0m	7h 3m
24	♈	6h 56m	6h 59m	7h 1m	7h 4m	7h 6m	7h 9m	7h 12m
27	♈	7h 3m	7h 6m	7h 9m	7h 12m	7h 15m	7h 18m	7h 21m
30	♈	7h 10m	7h 13m	7h 16m	7h 19m	7h 23m	7h 26m	7h 30m

G								
.	S.	56	57	58	59	60	61	62
3	☾	7h 17m	7h 20m	7h 24m	7h 27m	7h 31m	7h 35m	7h 39m
6	☾	7h 24m	7h 27m	7h 31m	7h 35m	7h 39m	7h 43m	7h 48m
9	☾	7h 30m	7h 34m	7h 38m	7h 43m	7h 47m	7h 52m	7h 57m
1	☾	7h 37m	7h 41m	7h 45m	7h 50m	7h 55m	8h 0m	8h 5m
2								
1	☾	7h 43m	7h 48m	7h 52m	7h 57m	8h 3m	8h 8m	8h 14m
5								
1	☾	7h 49m	7h 54m	7h 59m	8h 4m	8h 10m	8h 16m	8h 23m
8								
2	☾	7h 55m	8h 1m	8h 6m	8h 11m	8h 18m	8h 24m	8h 31m
1								
2	☾	8h 1m	8h 7m	8h 12m	8h 18m	8h 25m	8h 32m	8h 39m
4								
2	☾	8h 7m	8h 13m	8h 18m	8h 25m	8h 32m	8h 39m	8h 47m
7								
3	☾	8h 12m	8h 18m	8h 24m	8h 31m	8h 38m	8h 46m	8h 55m
0								
3	☿	8h 17m	8h 23m	8h 30m	8h 37m	8h 45m	8h 53m	9h 3m
6	☿	8h 22m	8h 28m	8h 35m	8h 42m	8h 51m	9h 0m	9h 9m
9	☿	8h 26m	8h 33m	8h 40m	8h 47m	8h 56m	9h 6m	9h 16m
1	☿	8h 30m	8h 37m	8h 44m	8h 52m	9h 1m	9h 11m	9h 22m
2								
1	☿	8h 33m	8h 40m	8h 48m	8h 56m	9h 5m	9h 15m	9h 27m
5								
1	☿	8h 36m	8h 43m	8h 51m	8h 59m	9h 9m	9h 19m	9h 31m
8								
2	☿	8h 38m	8h 45m	8h 53m	9h 2m	9h 12m	9h 22m	9h 35m
1								
2	☿	8h 39m	8h 47m	8h 55m	9h 4m	9h 14m	9h 25m	9h 37m
4								
2	☿	8h 40m	8h 48m	8h 56m	9h 5m	9h 15m	9h 26m	9h 38m
7								
3	☿	8h 40m	8h 48m	8h 56m	9h 5m	9h 16m	9h 27m	9h 39m

G								
.	S.	56	57	58	59	60	61	62
0								

Block B — pole elevations 63, 64, 65, 66, 66½, 67, 68

Deck headers: 63 | 64 | 65 | 66 | 66½ | 67 | 68, each H. | M.; right frame
Altitudo. (Deck 66½ = the Arctic Circle, 66°30'.)

- Equinox anchor row (Υ 0°): all seven decks **6h 0m**.
- Solstice maxima (Π Gemini 30°, foot of block) — these read cleanly at the column feet:

Pole	Semidiurnal max at Π 30°
63	9h 54m
64	10h 12m
65	10h 35m
66	11h 10m
66½	12h 0m (<i>Arctic Circle — at the solstice the Sun just grazes the horizon, so the semidiurnal arc = 12h = a 24-hour day</i>)
67	<i>dies continuus</i> — continuous day of 22 days
68	<i>dies continuus</i> — continuous day of 42 days

- For decks 67 and 68 the column terminates partway down (the Sun ceases to set); beneath, the print reads "**Dies continuus**" spanning, and "**Dierum 22**" / "**Dierum 42**" respectively.
- Recovered interior grid (semidiurnal time, Hours:Minutes per deck; *dc* = *dies continuus* — the column has terminated, see the Dierum count):

G								
.	S.	63	64	65	66	66½	67	68
0	Υ	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m
3	Υ	6h 10m	6h 10m	6h 10m	6h 11m	6h 11m	6h 11m	6h 12m
6	Υ	6h 19m	6h 20m	6h 20m	6h 21m	6h 22m	6h 23m	6h 24m
9	Υ	6h 28m	6h 29m	6h 31m	6h 32m	6h 33m	6h 34m	6h 36m
1	Υ	6h	6h 39m	6h 41m	6h 43m	6h 44m	6h 46m	6h 47m

G								
.	S.	63	64	65	66	66½	67	68
2		37m						
1	Υ	6h	6h 49m	6h 51m	6h 54m	6h 56m	6h 57m	6h 59m
5		47m						
1	Υ	6h	6h 59m	7h 2m	7h 5m	7h 7m	7h 8m	7h 12m
8		56m						
2	Υ	7h 6m	7h 9m	7h 12m	7h 16m	7h 18m	7h 20m	7h 24m
1								
2	Υ	7h	7h 19m	7h 23m	7h 27m	7h 29m	7h 31m	7h 36m
4		16m						
2	Υ	7h	7h 29m	7h 33m	7h 38m	7h 40m	7h 43m	7h 48m
7		25m						
3	Υ	7h	7h 39m	7h 43m	7h 49m	7h 52m	7h 55m	8h 1m
0		34m						
3	ϣ	7h	7h 49m	7h 54m	8h 0m	8h 3m	8h 7m	8h 14m
		44m						
6	ϣ	7h	7h 58m	8h 4m	8h 11m	8h 15m	8h 18m	8h 26m
		53m						
9	ϣ	8h 3m	8h 8m	8h 14m	8h 22m	8h 27m	8h 31m	8h 40m
1	ϣ	8h	8h 18m	8h 26m	8h 34m	8h 38m	8h 43m	8h 53m
2		12m						
1	ϣ	8h	8h 28m	8h 36m	8h 45m	8h 50m	8h 55m	9h 7m
5		21m						
1	ϣ	8h	8h 38m	8h 47m	8h 57m	9h 2m	9h 8m	9h 21m
8		30m						
2	ϣ	8h	8h 48m	8h 58m	9h 8m	9h 14m	9h 21m	9h 35m
1		39m						
2	ϣ	8h	8h 57m	9h 8m	9h 20m	9h 26m	9h 34m	9h 50m
4		48m						
2	ϣ	8h	9h 7m	9h 18m	9h 31m	9h 39m	9h 47m	10h 6m
7		57m						
3	ϣ	9h 5m	9h 16m	9h 28m	9h 43m	9h 51m	10h 0m	10h 22m
0								
3	Π	9h	9h 25m	9h 40m	9h 54m	10h 4m	10h 14m	10h 41m

G		S.	63	64	65	66	66½	67	68
			13m						
6	II		9h 21m	9h 33m	9h 48m	10h 6m	10h 17m	10h 29m	11h 2m
9	II		9h 28m	9h 41m	9h 57m	10h 17m	10h 30m	10h 43m	11h 32m
1 2	II		9h 34m	9h 49m	10h 6m	10h 28m	10h 42m	11h 0m	<i>dc</i>
1 5	II		9h 40m	9h 55m	10h 14m	10h 38m	10h 55m	11h 18m	<i>dc</i>
1 8	II		9h 45m	10h 1m	10h 21m	10h 49m	11h 8m	11h 47m	<i>dc</i>
2 1	II		9h 49m	10h 6m	10h 27m	10h 57m	11h 21m	<i>dc</i>	<i>dc</i>
2 4	II		9h 52m	10h 9m	10h 32m	11h 4m	11h 34m	<i>dc</i>	<i>dc</i>
2 7	II		9h 53m	10h 11m	10h 34m	11h 7m	11h 47m	<i>dc</i>	<i>dc</i>
3 0	II		9h 54m	10h 12m	10h 35m	11h 10m	12h 0m	<i>dc</i>	<i>dc</i>

Terminated columns: deck 67 → *dies continuus*, **Dierum 22**; deck 68 → *dies continuus*, **Dierum 42**.

Running header (recto): "IOAN. DE SACRO BOSCO." · subtitle "Tabula temporis semidiurni in signis Borealibus." · page-signature at foot: ``

Block A — pole elevations 69, 70, 71, 72, 73, 74, 75

Deck headers: 69 | 70 | 71 | 72 | 73 | 74 | 75, each H. | M.; right frame Altitudo.

- Equinox anchor row (Υ 0°): all seven decks **6h 0m**.
- Every deck reaches continuous day before the solstice; beneath the terminated columns the print reads "**Dies continuus**" and, at the foot, "**Dierum**" with the count of continuous-daylight days:

Pole	Continuous day (Dierum)
69	55
70	65
71	74
72	82
73	89
74	97
75	104

- Recovered interior grid (semidiurnal time, Hours:Minutes per deck; *dc* = *dies continuus* — the column has terminated, see the Dierum count). Every deck reaches continuous day before the solstice, so rows below Π 3° are all in *dies continuus*:

G	S.	69	70	71	72	73	74	75
0	Υ	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m
3	Υ	6h 13m	6h 13m	6h 14m	6h 15m	6h 16m	6h 17m	6h 18m
6	Υ	6h 25m	6h 26m	6h 28m	6h 29m	6h 31m	6h 33m	6h 36m
9	Υ	6h 34m	6h 40m	6h 42m	6h 44m	6h 47m	6h 50m	6h 54m
12	Υ	6h 50m	6h 53m	6h 56m	6h 59m	7h 3m	7h 7m	7h 12m
15	Υ	7h 3m	7h 6m	7h 10m	7h 14m	7h 19m	7h 25m	7h 31m
18	Υ	7h 16m	7h 20m	7h 25m	7h 30m	7h 36m	7h 44m	7h 51m
21	Υ	7h 28m	7h 33m	7h 39m	7h 46m	7h 53m	8h 1m	8h 10m
24	Υ	7h 41m	7h 47m	7h 54m	8h 2m	8h 10m	8h 20m	8h 31m
27	Υ	7h 55m	8h 2m	8h 9m	8h 18m	8h 28m	8h 40m	8h 54m
30	Υ	8h 8m	8h 16m	8h 25m	8h 35m	8h 47m	9h 1m	9h 18m
33	ϝ	8h 22m	8h 31m	8h 41m	8h 53m	9h 7m	9h 24m	9h 45m
36	ϝ	8h 36m	8h 46m	8h 58m	9h 12m	9h 28m	9h 49m	10h 16m

G		S.	69	70	71	72	73	74	75
9	☾		8h 50m	9h 2m	9h 15m	9h 32m	9h 52m	10h 19m	11h 1m
1	☾		9h 4m	9h 18m	9h 34m	9h 53m	10h 19m	10h 59m	<i>dc</i>
2									
1	☾		9h 20m	9h 36m	9h 51m	10h 22m	10h 56m	<i>dc</i>	<i>dc</i>
5									
1	☾		9h 36m	9h 54m	10h 17m	10h 53m	<i>dc</i>	<i>dc</i>	<i>dc</i>
8									
2	☾		9h 54m	10h 14m	10h 45m	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>
1									
2	☾		10h 10m	10h 38m	11h 27m	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>
4									
2	☾		10h 30m	11h 8m	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>
7									
3	☾		10h 54m	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>
0									
3	☿		11h 28m	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>

Terminated columns (Dierum totals as tabled above): 69 = 55, 70 = 65, 71 = 74, 72 = 82, 73 = 89, 74 = 97, 75 = 104.

Block B — pole elevations 76, 77, 78, 79, 80, 81, 82

Deck headers: 76 | 77 | 78 | 79 | 80 | 81 | 82, each H. | M.; right frame Altitudo.

- Equinox anchor row (☾ 0°): all seven decks **6h 0m**.
- Continuous-day counts at the foot (**Dies continuus · Dierum**):

Pole	Continuous day (Dierum)
76	110
77	117
78	123
79	127
80	134
81	140
82	145

- Recovered interior grid (semidiurnal time, Hours:Minutes per deck; *dc* = *dies continuus* — the column has terminated, see the Dierum count). Rows below $\propto 6^\circ$ are all in *dies continuus*:

G		S.	76	77	78	79	80	81	82
0	Υ		6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m
3	Υ		6h 19m	6h 21m	6h 23m	6h 25m	6h 27m	6h 30m	6h 34m
6	Υ		6h 38m	6h 42m	6h 45m	6h 49m	6h 55m	7h 1m	7h 10m
9	Υ		6h 58m	7h 3m	7h 9m	7h 15m	7h 24m	7h 33m	7h 46m
12	Υ		7h 18m	7h 24m	7h 32m	7h 41m	7h 52m	8h 7m	8h 27m
15	Υ		7h 38m	7h 47m	7h 57m	8h 9m	8h 24m	8h 43m	9h 10m
18	Υ		8h 0m	8h 10m	8h 23m	8h 39m	8h 49m	9h 27m	10h 9m
21	Υ		8h 22m	8h 35m	8h 51m	9h 8m	9h 44m	10h 23m	<i>dc</i>
24	Υ		8h 45m	9h 2m	9h 23m	9h 51m	10h 35m	<i>dc</i>	<i>dc</i>
27	Υ		9h 10m	9h 32m	10h 0m	10h 45m	<i>dc</i>	<i>dc</i>	<i>dc</i>
30	Υ		9h 39m	10h 7m	10h 53m	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>
33	$\propto$		10h 13m	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>
36	$\propto$		11h 1m	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>

Terminated columns (Dierum totals as tabled above): 76 = 110, 77 = 117, 78 = 123, 79 = 127, 80 = 134, 81 = 140, 82 = 145.

Running header (verso): "COMMENT. IN III. CAP. SPHÆRÆ" · subtitle "Tabula temporis semidiurni in signis Borealibus."

Block (single) — pole elevations 83, 84, 85, 86, 87, 88, 89, 90

Deck headers: 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90, each H. | M.; right frame Altitudo. (Deck 90 = the North Pole; its H. | M. cells are effectively empty — the Sun is above the horizon for the whole northern half of the ecliptic.)

- Equinox anchor row (Υ 0°): every deck **6h 0m**.
- At these extreme latitudes each column terminates after only a few degrees of ecliptic longitude; the semidiurnal time climbs very steeply (e.g. deck 83 reads 6h 0m → 6h 39m → 7h 19m → 8h 3m → 8h 50m → 9h 50m before continuous day). These first-rows are recovered at high magnification (full grid below).
- Continuous-day counts at the foot (**Dies continuus · Dierum**):

Pole	Continuous day (Dierum)
83	151
84	156
85	161
86	166
87	172
88	176
89	182
90	187

- The lower two-thirds of page 3 is blank (the northern-signs table ends here). The southern-signs counterpart ("in signis Australibus") does **not** begin within this opening; it would start on the following page.
- Recovered interior grid (semidiurnal time, Hours:Minutes per deck; *dc* = *dies continuus* — the column has terminated, see the Dierum count). At these extreme latitudes each column terminates after only a few degrees, so rows below Υ 15° are all in *dies continuus*:

G									
.	S.	83	84	85	86	87	88	89	90
0	Υ	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m	6h 0m [?]
3	Υ	6h 39m	6h 46m	6h 55m	7h 10m [?]	7h 34m	8h 27m	<i>dc</i>	<i>dc</i>
6	Υ	7h 19m	7h 33m	7h 54m	8h 29m	9h 30m	<i>dc</i>	<i>dc</i>	<i>dc</i>
9	Υ	8h 3m	8h 26m	9h 3m	10h 14m	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>
1	Υ	8h	9h 29m	10h	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>
2		50m		47m					

G									
.	S.	83	84	85	86	87	88	89	90
1	Υ	9h	11h	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>	<i>dc</i>
5		50m	22m						

Uncertain cells (per the reprocess, transcribed as read): deck **86**, row Υ 3° — leading hour digit ambiguous (6/7), read **7h 10m** on the column-monotone constraint; deck **90**, row Υ 0° — faint, read **6h 0m**. Terminated columns (Dierum totals as tabled above): 83 = 151, 84 = 156, 85 = 161, 86 = 166, 87 = 172, 88 = 176, 89 = 182, 90 = 187.

CAPVT TERTIVM (continued)

Table of semidiurnal time in the southern signs [Tabula temporis semidiurni in signis Australibus]

This batch is occupied entirely by the southern-signs counterpart of the table begun earlier — the *Tabula temporis semidiurni in signis Australibus*. No commentary prose resumes on any of the three pages; the table continues past the elevation reached here into the following pages.

Layout. Each page carries the table in two stacked half-page bands. Every band has:

- a **left argument frame** headed *Pol.i.* with sub-columns G. | S. | D. | M. (degree of the sign · sign · day · month) — the Sun's ecliptic longitude in the descending southern signs and the corresponding calendar date;
- a group of **pole-elevation decks**, each headed by a whole number (the elevation of the pole in degrees) over sub-columns H. | M. (hours · minutes of semidiurnal time);
- a **right argument frame** headed *Altitudo.* with sub-columns M. | D. | S. | G. (month · day · sign · degree) — the mirror argument in the ascending signs (Pisces → Capricorn), paired row-by-row with the left frame by the symmetry about the winter solstice.

The seven decks per band and their distribution across the three pages:

- **p. 259 (printed 247):** upper band decks **0–6**, lower band decks **7–13**
- **p. 260 (printed 248):** upper band decks **14–20**, lower band decks **21–27**
- **p. 261 (printed 249):** upper band decks **28–34**, lower band decks **35–41**

The left argument frame and the right Altitudo frame are **identical on every band and every page**; they are transcribed once below and apply throughout.

Left argument frame (Pol i. → G. | S. | D. | M.) — identical for all bands

The solar-longitude / calendar-date argument runs from 0° Libra (autumnal equinox) down to 30° Sagittarius (= 0° Capricorn, winter solstice).

G.	S. (sign)	D.	M. (month)
0	♎ Libra	24	Sept.
3		27	Sept.
6		30	Sept.
9		3	Octob.
12		6	Octob.
15		9	Octob.
18		12	Octob.
21		15	Octob.
24		18	Octob.
27		21	Octob.
30		24	Octob.
3	♏ Scorpius	27	Octob.
6		30	Octob.
9		2	Nouemb.
12		5	Nouemb.
15		8	Nouemb.
18		11	Nouemb.
21		14	Nouemb.
24		17	Nouemb.
27		20	Nouemb.
30		23	Nouemb.
3	♐ Sagittarius	26	Nouemb.
6		29	Nouemb.
9		2	Decemb.
12		5	Decemb.

G.	S. (sign)	D.	M. (month)
15		8	Decemb.
18		11	Decemb.
21		14	Decemb.
24		17	Decemb.
27		19	Decemb.
30		22	Decemb.

Right argument frame (Altitude. → M. | D. | S. | G.) — identical for all bands

Runs 30° Pisces down to 0° Capricorn; paired row-for-row with the left frame.

M. (month)	D.	S. (sign)	G.
Martius	21	♋ Pisces	30
	18		27
	15		24
	12		21
	9		18
	6		15
	3		12
Februar.	28		9
	25		6
	22		3
	19		0
Februar.	16	♊ Aquarius	27
	13		24
	10		21
	7		18
	4		15
	1		12
Ianuar.	29		9
	26		6
	23		3
	21		0

Arg (G° / sign)	0	1	2	3	4	5	6
3° $\underline{\circ}$	6 00	6 00	6 00	6 00	6 00	5 59	6 00
6° $\underline{\circ}$	6 00	6 00	6 00	5 59	5 59	5 59	5 59
9° $\underline{\circ}$	6 00	6 00	5 59	5 59	5 59	5 59	5 58
12° $\underline{\circ}$	6 00	6 00	5 59	5 59	5 59	5 58	5 58
15° $\underline{\circ}$	6 00	6 00	5 59	5 59	5 58	5 58	5 57
18° $\underline{\circ}$	6 00	6 00	5 59	5 59	5 58	5 58	5 57
21° $\underline{\circ}$	6 00	6 00	5 59	5 58	5 58	5 57	5 57
24° $\underline{\circ}$	6 00	6 00	5 59	5 58	5 58	5 57	5 56
27° $\underline{\circ}$	6 00	6 00	5 58	5 58	5 57	5 56	5 56
30° $\underline{\circ}$	6 00	5 59	5 58	5 58	5 57	5 56	5 55
3° $\mathbb{M}$	6 00	5 59	5 58	5 58	5 57	5 56	5 55
6° $\mathbb{M}$	6 00	5 59	5 58	5 58	5 57	5 56	5 54
9° $\mathbb{M}$	6 00	5 59	5 58	5 57	5 56	5 55	5 54
12° $\mathbb{M}$	6 00	5 59	5 58	5 57	5 56	5 55	5 53
15° $\mathbb{M}$	6 00	5 59	5 58	5 57	5 56	5 55	5 53
18° $\mathbb{M}$	6 00	5 59	5 58	5 57	5 55	5 54	5 53
21° $\mathbb{M}$	6 00	5 59	5 58	5 57	5 55	5 54	5 52
24° $\mathbb{M}$	6 00	5 59	5 58	5 56	5 55	5 54	5 52
27° $\mathbb{M}$	6 00	5 59	5 57	5 56	5 55	5 54	5 52
30° $\mathbb{M}$	6 00	5 59	5 57	5 56	5 54	5 53	5 51
3° $\times$	6 00	5 59	5 57	5 56	5 54	5 53	5 51
6° $\times$	6 00	5 59	5 57	5 56	5 54	5 53	5 51
9° $\times$	6 00	5 58	5 57	5 56	5 54	5 52	5 51
12° $\times$	6 00	5 58	5 57	5 56	5 54	5 52	5 50
15° $\times$	6 00	5 58	5 56	5 55	5 53	5 52	5 50
18° $\times$	6 00	5 58	5 56	5 55	5 53	5 51	5 50
21° $\times$	6 00	5 58	5 56	5 55	5 53	5 51	5 50
24° $\times$	6 00	5 58	5 56	5 55	5 53	5 51	5 50
27° $\times$	6 00	5 58	5 56	5 55	5 53	5 51	5 50
30° $\times$	6 00	5 58	5 56	5 55	5 53	5 51	5 50

p. 259 (printed 247) — Block B, decks 7–13

Decks 9 and 13 lost the Sagittarius tail (rows 22–31) at the band seam; those cells are [numeral: ?].

Arg (G° / sign)	7	8	9	10	11	12	13
0° ♌	6 00	6 00	6 00	6 00	6 00	6 00	6 00
3° ♌	6 00	6 00	6 00	6 00	5 59	5 59	5 59
6° ♌	5 59	5 59	5 59	5 59	5 58	5 58	5 57
9° ♌	5 58	5 58	5 58	5 58	5 57	5 57	5 55
12° ♌	5 58	5 58	5 58	5 57	5 56	5 56	5 55
15° ♌	5 57	5 57	5 57	5 56	5 55	5 55	5 54
18° ♌	5 57	5 56	5 56	5 55	5 54	5 54	5 53
21° ♌	5 56	5 56	5 56	5 55	5 53	5 53	5 52
24° ♌	5 55	5 55	5 55	5 54	5 53	5 52	5 51
27° ♌	5 55	5 54	5 54	5 53	5 52	5 51	5 50
30° ♌	5 54	5 53	5 53	5 53	5 51	5 50	5 48
3° ♍	5 54	5 53	5 53	5 52	5 51	5 49	5 48
6° ♍	5 53	5 52	5 52	5 51	5 50	5 48	5 48
9° ♍	5 53	5 52	5 52	5 50	5 49	5 47	5 47
12° ♍	5 52	5 52	5 51	5 50	5 49	5 47	5 47

Arg (G° / sign)	7	8	9	10	11	12	13
15° ℳ	5 52	5 51	5 51	5 49	5 48	5 46	5 46
18° ℳ	5 52	5 51	5 50	5 49	5 48	5 46	5 46
21° ℳ	5 51	5 50	5 50	5 48	5 47	5 45	5 45
24° ℳ	5 51	5 50	5 49	5 48	5 46	5 44	5 44
27° ℳ	5 51	5 49	5 48	5 47	5 46	5 43	5 43
30° ℳ	5 50	5 48	5 47	5 47	5 45	5 42	5 42
3° ↗	5 50	5 48	[numeral: ?]	5 46	5 45	5 42	[numeral: ?]
6° ↗	5 50	5 48	[numeral: ?]	5 45	5 44	5 41	[numeral: ?]
9° ↗	5 49	5 47	[numeral: ?]	5 45	5 44	5 41	[numeral: ?]
12° ↗	5 48	5 47	[numeral: ?]	5 44	5 43	5 40	[numeral: ?]
15° ↗	5 48	5 47	[numeral: ?]	5 44	5 42	5 40	[numeral: ?]
18° ↗	5 47	5 46	[numeral: ?]	5 43	5 42	5 40	[numeral: ?]
21° ↗	5 47	5 46	[numeral: ?]	5 42	5 41	5 39	[numeral: ?]
24° ↗	5 47	5 46	[numeral: ?]	5 42	5 41	5 39	[numeral: ?]
27° ↗	5 46	5 46	[numeral: ?]	5 41	5 41	5 39	[numeral: ?]
30° ↗	5 46	5 46	[numeral: ?]	5 41	5 41	5 39	[numeral: ?]

p. 260 (printed 248) — Block A, decks 14–20

Deck 20 row 9 (24° $\underline{\circ}$) prints 5 55, breaking the monotone fall of the column and row; both passes confirm the print reads 55, a source error for 45 — transcribed as printed with [recte: 5 45]. Deck 15 row 8 (21° $\underline{\circ}$) reads 5 50, skipping 51 where decks 14 and 16 step through it — read as printed (a possible 1-minute source slip; not corrected).

Arg (G° / sign)	14	15	16	17	18	19	20
0° $\underline{\circ}$	6 00	6 00	6 00	6 00	6 00	6 00	6 00
3° $\underline{\circ}$	5 59	5 59	5 58	5 58	5 58	5 58	5 58
6° $\underline{\circ}$	5 57	5 57	5 56	5 56	5 56	5 56	5 56
9° $\underline{\circ}$	5 56	5 56	5 55	5 55	5 55	5 54	5 54
12° $\underline{\circ}$	5 55	5 55	5 54	5 54	5 54	5 53	5 52
15° $\underline{\circ}$	5 54	5 54	5 53	5 53	5 53	5 52	5 51
18° $\underline{\circ}$	5 52	5 52	5 51	5 51	5 51	5 50	5 49
21° $\underline{\circ}$	5 51	5 50	5 50	5 49	5 49	5 48	5 47
24° $\underline{\circ}$	5 50	5 49	5 49	5 48	5 48	5 46	5 55 [recte: 5 45]
27° $\underline{\circ}$	5 49	5 48	5 48	5 47	5 47	5 45	5 44
30° $\underline{\circ}$	5 48	5 47	5 47	5 46	5 46	5 44	5 43
3° $\mathfrak{M}$	5 47	5 46	5 45	5 44	5 44	5 42	5 41
6° $\mathfrak{M}$	5 46	5 45	5 44	5 43	5 43	5 41	5 39
9° $\mathfrak{M}$	5 45	5 44	5 43	5 42	5 41	5 40	5 37

Arg (G° / sign)	14	15	16	17	18	19	20
12° ™	5 44	5 43	5 42	5 41	5 40	5 39	5 36
15° ™	5 43	5 42	5 41	5 40	5 39	5 38	5 35
18° ™	5 42	5 41	5 40	5 39	5 38	5 37	5 34
21° ™	5 41	5 40	5 39	5 38	5 37	5 36	5 33
24° ™	5 40	5 39	5 38	5 37	5 36	5 35	5 32
27° ™	5 39	5 38	5 37	5 36	5 35	5 34	5 31
30° ™	5 39	5 37	5 36	5 35	5 34	5 33	5 30
3° ↗	5 38	5 36	5 35	5 34	5 33	5 32	5 28
6° ↗	5 37	5 35	5 34	5 33	5 32	5 31	5 27
9° ↗	5 37	5 34	5 33	5 32	5 31	5 30	5 26
12° ↗	5 36	5 33	5 32	5 31	5 30	5 29	5 25
15° ↗	5 36	5 33	5 31	5 30	5 29	5 28	5 25
18° ↗	5 35	5 32	5 31	5 29	5 29	5 27	5 25
21° ↗	5 35	5 31	5 30	5 28	5 28	5 26	5 24
24° ↗	5 35	5 31	5 30	5 28	5 28	5 26	5 24
27° ↗	5 35	5 30	5 30	5 28	5 28	5 26	5 24
30° ↗	5 35	5 30	5 30	5 28	5 28	5 26	5 24

p. 260 (printed 248) — Block B, decks 21–27

Deck 25 row 2 (3° ♀) prints 5 68 — minutes > 59 is impossible; both passes confirm the tens digit is a closed 6, a source error for 58 — transcribed as printed with [recte: 5 58]. Decks 26 and 27 lost the Sagittarius tail (rows 22–31) at the band seam; those cells are [numeral: ?].

Arg (G° / sign)	21	22	23	24	25	26	27
0° ♀	6 00	6 00	6 00	6 00	6 00	6 00	6 00
3° ♀	5 58	5 58	5 58	5 58	5 68 [recte: 5 58]	5 58	5 58
6° ♀	5 56	5 56	5 56	5 55	5 55	5 55	5 55
9° ♀	5 54	5 54	5 54	5 53	5 53	5 52	5 52
12° ♀	5 52	5 52	5 52	5 51	5 51	5 50	5 50
15° ♀	5 51	5 51	5 50	5 49	5 49	5 48	5 48
18° ♀	5 49	5 49	5 48	5 47	5 47	5 46	5 46
21° ♀	5 47	5 47	5 46	5 45	5 44	5 43	5 43
24° ♀	5 45	5 45	5 44	5 43	5 42	5 41	5 40
27° ♀	5 43	5 43	5 42	5 41	5 40	5 39	5 38
30° ♀	5 42	5 42	5 40	5 39	5 38	5 37	5 36
3° ♂	5 40	5 40	5 38	5 37	5 36	5 35	5 34
6° ♂	5 38	5 38	5 36	5 35	5 34	5 33	5 32
9° ♂	5 36	5 37	5 34	5 33	5 32	5 31	5 30
12° ♂	5	5	5	5	5 30	5 29	5 28

Arg (G° / sign)	21	22	23	24	25	26	27
	35	36	32	31			
15° ℳ	5	5	5	5	5 28	5 27	5 26
	34	35	31	30			
18° ℳ	5	5	5	5	5 26	5 25	5 24
	32	34	29	28			
21° ℳ	5	5	5	5	5 25	5 23	5 22
	31	33	27	26			
24° ℳ	5	5	5	5	5 23	5 22	5 20
	30	33	26	25			
27° ℳ	5	5	5	5	5 22	5 20	5 18
	29	32	25	23			
30° ℳ	5	5	5	5	5 21	5 19	5 17
	28	31	23	22			
3° ↗	5	5	5	5	[numeral: ?]	[numeral: ?]	[numeral: ?]
	27	30	22	20			
6° ↗	5	5	5	5	5 17	[numeral: ?]	[numeral: ?]
	26	29	21	18			
9° ↗	5	5	5	5	5 16	[numeral: ?]	[numeral: ?]
	25	28	20	17			
12° ↗	5	5	5	5	5 15	[numeral: ?]	[numeral: ?]
	24	27	19	16			
15° ↗	5	5	5	5	5 14	[numeral: ?]	[numeral: ?]
	23	27	18	15			
18° ↗	5	5	5	5	5 13	[numeral: ?]	[numeral: ?]
	23	26	17	14			
21° ↗	5	5	5	5	5 12	[numeral: ?]	[numeral: ?]
	22	26	17	13			
24° ↗	5	5	5	5	5 11	[numeral: ?]	[numeral: ?]
	22	26	17	13			
27° ↗	5	5	5	5	5 11	[numeral: ?]	[numeral: ?]
	22	26	17	13			
30° ↗	5	5	5	5	5 11	[numeral: ?]	[numeral: ?]
	22	26	17	13			

p. 261 (printed 249) — Block A, decks 28–34

Deck 28's Sagittarius tail is the un-isolated leftmost (c1-resident) column and is held [numeral: ?] below row 13; scattered ink-heavy tail cells of decks 31–34 are likewise [numeral: ?]. The hour digit begins to fall to 4 in decks 33–34 near the foot (5 03 → 4 xx), but those units did not separate and are held [numeral: ?].

Arg (G° / sign)	28	2 9	3 0	31	32	33	34
0° ♎	6 00	6 0 0	6 0 0	6 00	6 00	6 00	6 00
3° ♎	5 57	5 5 7	5 5 7	5 57	5 57	5 57	5 57
6° ♎	5 54	5 5 4	5 5 4	5 54	5 54	5 54	5 54
9° ♎	5 52	5 5 2	5 5 2	5 51	5 51	5 51	5 51
12° ♎	5 49	5 4 9	5 4 9	5 49	5 48	5 48	5 47
15° ♎	5 47	5 4 7	5 4 7	5 46	5 45	5 45	5 44
18° ♎	5 44	5 4 4	5 4 4	5 43	5 42	5 42	5 41
21° ♎	5 42	5 4 2	5 4 1	5 40	5 39	5 39	5 38
24° ♎	5 39	5 3 9	5 3 8	5 37	5 36	5 36	5 35
27° ♎	5 37	5 3 3	5 3 3	5 34	5 33	5 33	5 32

Arg (G° / sign)	28	2 9	3 0	31	32	33	34
		7	6				
30° $\underline{\alpha}$	5 35	5 3 5	5 3 4	5 31	[numeral : ?]	5 30	5 29
3° $\mathfrak{M}$	5 32	5 3 2	5 3 1	5 29	5 29	5 27	5 26
6° $\mathfrak{M}$	5 30	5 3 0	5 2 9	5 27	5 27	5 25	5 23
9° $\mathfrak{M}$	[numeral : ?]	5 2 8	5 2 7	5 25	5 24	5 22	5 20
12° $\mathfrak{M}$	[numeral : ?]	5 2 6	5 2 5	5 23	[numeral : ?]	5 20	[numeral : ?]
15° $\mathfrak{M}$	[numeral : ?]	5 2 4	5 2 3	5 21	[numeral : ?]	5 18	[numeral : ?]
18° $\mathfrak{M}$	[numeral : ?]	5 2 2	5 2 0	5 19	[numeral : ?]	5 16	[numeral : ?]
21° $\mathfrak{M}$	[numeral : ?]	5 2 0	5 1 8	5 17	[numeral : ?]	5 14	[numeral : ?]
24° $\mathfrak{M}$	[numeral : ?]	5 1 8	5 1 6	5 15	[numeral : ?]	5 13	[numeral : ?]
27° $\mathfrak{M}$	[numeral : ?]	5 1 6	5 1 4	5 13	[numeral : ?]	5 11	[numeral : ?]
30° $\mathfrak{M}$	[numeral : ?]	5 1 5	5 1 3	5 11	[numeral : ?]	5 09	[numeral : ?]

Arg (G° / sign)	28	2 9	3 0	31	32	33	34
3° ✕	[numeral : ?]	5 1 3	5 1 1	5 09	[numeral : ?]	5 07	[numeral : ?]
6° ✕	[numeral : ?]	5 1 2	5 1 0	5 08	[numeral : ?]	5 05	[numeral : ?]
9° ✕	[numeral : ?]	5 1 1	5 0 9	5 07	[numeral : ?]	5 04	[numeral : ?]
12° ✕	[numeral : ?]	5 1 0	5 0 8	5 06	[numeral : ?]	5 03	[numeral : ?]
15° ✕	[numeral : ?]	5 0 9	5 0 7	5 05	[numeral : ?]	[numeral : ?]	[numeral : ?]
18° ✕	[numeral : ?]	5 0 8	5 0 6	5 04	[numeral : ?]	[numeral : ?]	[numeral : ?]
21° ✕	[numeral : ?]	5 0 8	5 0 5	5 03	[numeral : ?]	[numeral : ?]	[numeral : ?]
24° ✕	[numeral : ?]	5 0 7	5 0 4	[numeral : ?]	[numeral : ?]	[numeral : ?]	[numeral : ?]
27° ✕	[numeral : ?]	5 0 7	5 0 4	[numeral : ?]	[numeral : ?]	[numeral : ?]	[numeral : ?]
30° ✕	[numeral : ?]	5 0 7	5 0 4	[numeral : ?]	[numeral : ?]	[numeral : ?]	[numeral : ?]

p. 261 (printed 249) — Block B, decks 35–41

These are the tightest, high-elevation cells; the hour digit crosses from 5 to 4 in the Sagittarius tail, where the closer per-deck pass resolved most units. Residual band-edge and ink-heavy units are [numeral: ?].

Arg (G° / sign)	3 5	36	37	38	39	40	41
0° $\underline{\text{N}}$	6 0 0	6 00	6 00	6 00	6 00	6 00	6 00
3° $\underline{\text{N}}$	5 5 7	5 57	5 57	5 56	5 56	5 56	5 56
6° $\underline{\text{N}}$	5 5 4	5 53	5 53	5 53	5 52	5 52	5 52
9° $\underline{\text{N}}$	5 5 1	5 50	5 50	5 49	5 49	5 48	5 48
12° $\underline{\text{N}}$	5 4 7	5 46	5 46	5 46	5 45	5 44	5 44
15° $\underline{\text{N}}$	5 4 4	5 43	5 43	5 42	5 41	5 40	5 40
18° $\underline{\text{N}}$	5 4 1	5 39	5 39	5 38	5 38	5 36	5 36
21° $\underline{\text{N}}$	5 3 8	5 36	5 36	5 35	5 34	5 32	5 32
24° $\underline{\text{N}}$	5 3 4	5 33	5 33	5 32	5 30	5 28	5 28
27° $\underline{\text{N}}$	5 3 1	5 29	5 29	5 28	5 27	5 24	5 24
30° $\underline{\text{N}}$	5 2 8	5 26	5 26	5 25	5 23	5 21	5 21
3° $\underline{\text{M}}$	5	5 23	5 23	5 21	5 20	[numera	5 17

Arg (G° / sign)	3 5	36	37	38	39	40	41
	2					l: ?]	
	5						
6° ™	5	5 20	5 20	5 18	5 17	5 13	5 13
	2						
	1						
9° ™	5	5 17	5 17	5 15	5 13	[numera l: ?]	5 09
	1						
	8						
12° ™	5	5 14	5 14	5 12	5 10	[numera l: ?]	5 06
	1						
	5						
15° ™	5	5 11	5 11	5 09	5 07	5 02	5 02
	1						
	2						
18° ™	5	[numera l: ?]	5 08	5 06	5 04	4 59	[numera l: ?]
	1						
	0						
21° ™	5	5 06	5 05	5 03	5 01	4 56	4 54
	0						
	7						
24° ™	5	5 03	5 03	5 00	4 59	4 53	4 51
	0						
	4						
27° ™	5	5 00	5 00	4 58	4 58	4 51	4 48
	0						
	2						
30° ™	5	4 58	4 58	4 56	[numera l: ?]	4 48	4 46
	0						
	0						
3° ↗	4	4 56	4 56	4 53	4 51	4 46	4 43
	5						
	8						
6° ↗	4	4 54	4 53	4 51	4 48	4 44	4 41
	5						

Arg (G° / sign)	3 5	36	37	38	39	40	41
	6						
9° ✕	4	4 53	4 51	4 48	4 46	4 43	4 40
	5						
	4						
12° ✕	4	4 51	[numera l: ?]	4 46	4 44	4 41	4 38
	5						
	3						
15° ✕	4	4 50	[numera l: ?]	4 44	4 43	4 40	4 36
	5						
	2						
18° ✕	4	4 48	[numera l: ?]	4 43	4 41	4 39	4 35
	5						
	1						
21° ✕	4	4 47	[numera l: ?]	[numera l: ?]	4 40	4 38	4 35
	5						
	0						
24° ✕	4	4 46	[numera l: ?]	4 41	4 39	4 38	4 34
	5						
	0						
27° ✕	4	4 44	[numera l: ?]	4 41	4 38	4 38	4 34
	4						
	9						
30° ✕	4	4 43	[numera l: ?]	4 41	4 38	4 38	[numera l: ?]
	4						
	9						

Commentary on the *Sphere* of Sacrobosco — Chapter III (continued)

Running header (verso, p. 262 / p. 264): COMMENT. IN III. CAP. SPHÆRÆ

Running header (recto, p. 263): IOAN. DE SACRO BOSCO.

Table caption (repeated at the head of each page): *Tabula temporis semidiurni in signis Australibus* — Table of semidiurnal time in the southern signs.

This entire batch is a single continuation of the southern-signs table of semidiurnal time (the length, in hours **H.** and minutes **M.**, of the half-day for the descending zodiacal signs Libra–Scorpius–Sagittarius at successively higher pole elevations). It carries the pole-elevation decks **42° through 82°** (plus a **66½°** deck for the Arctic Circle). The table does **not** end within this batch and no commentary prose resumes here: at the foot the ruled grid continues, and the high-latitude columns have entered the *Nox continua* (continuous polar night) region, so more decks and then the *De climatibus* prose lie in the following batch.

Shared argument frame (identical for every deck band)

Each deck band is read against the same double argument. **Left frame ("Poli.")** gives the calendar entry for the descending northern-hemisphere reading; **right frame ("Altitudo.")** gives the mirrored entry (Pisces/Aquarius/Capricorn) for the complementary reading. G. = degree within the sign; S. = sign; D. = day; M. = month.

Poli: G.	Poli: S.	Poli: D.	Poli: M.		Alt: M.	Alt: D.	Alt: S.	Alt: G.
0	♎ Libra	24	Sept.		Marti us	21	♊ Pisces	30
3		27				18		27
6		30				15		24
9		3	Octob.			12		21
12		6				9		18
15		9				6		15
18		12				3		12
21		15			Febru ar.	28		9
24		18				25		6
27		21				22		3
30		24				19		0
3	♏ Scorpius	27				16	♐ Aquarius	27
6		30				13		24
9		2	Novem ber.			10		21
12		5				7		18

Poli: G.	Poli: S.	Poli: D.	Poli: M.		Alt: M.	Alt: D.	Alt: S.	Alt: G.
15		8				4		15
18		11				1		12
21		14			Ianua r.	29		9
24		17				26		6
27		20				23		3
30		23				21 [?]		0
3	♐ Sagittarius	26				18	♑ Capricornus	27
6		29				15		24
9		2	Dece mb.			12		21
12		5				9		18
15		8				6		15
18		11				3		12
21		14			Dece mb.	31		9
24		17				28		6
27		19				25		3
30		22				22		0

Band A — pole elevations 42°–48°

Semidiurnal time (H. hours, M. minutes) per 3° of solar longitude. This band is fully committed (cross-validated across overlapping tiles, and the 48° column independently checked against its printed minutes column).

Poli G.	42	43	44	45	46	47	48
♌ 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0
3	5 56	5 56	5 55	5 55	5 55	5 55	5 55
6	5 51	5 51	5 51	5 50	5 50	5 50	5 49
9	5 47	5 47	5 46	5 46	5 45	5 45	5 44
12	5 43	5 42	5 42	5 41	5 40	5 40	5 39

Poli G.	42	43	44	45	46	47	48
15	5 39	5 38	5 37	5 36	5 35	5 34	5 34
18	5 34	5 33	5 33	5 31	5 30	5 29	5 28
21	5 30	5 29	5 28	5 27	5 26	5 24	5 23
24	5 26	5 25	5 24	5 22	5 21	5 19	5 18
27	5 22	5 20	5 19	5 17	5 16	5 14	5 13
30	5 18	5 16	5 15	5 13	5 11	5 10	5 8
ℳ 3	5 14	5 12	5 10	5 9	5 7	5 5	5 3
6	5 10	5 8	5 6	5 4	5 3	5 0	4 58
9	5 6	5 4	5 2	5 0	4 58	4 55	4 53
12	5 2	5 0	4 58	4 56	4 53	4 51	4 48
15	4 59	4 56	4 54	4 52	4 49	4 47	4 43
18	4 55	4 53	4 50	4 48	4 45	4 42	4 39
21	4 52	4 49	4 47	4 44	4 41	4 38	4 35
24	4 49	4 46	4 43	4 40	4 37	4 34	4 31
27	4 46	4 43	4 40	4 37	4 34	4 31	4 27
30	4 43	4 40	4 37	4 34	4 30	4 27	4 24
↗ 3	4 40	4 37	4 34	4 31	4 27	4 24	4 20
6	4 37	4 34	4 31	4 28	4 24	4 21	4 17
9	4 35	4 32	4 29	4 25	4 22	4 18	4 14
12	4 33	4 30	4 27	4 23	4 20	4 16	4 12
15	4 32	4 28	4 25	4 21	4 18	4 14	4 10
18	4 30	4 27	4 23	4 20	4 16	4 12	4 8
21	4 29	4 26	4 22	4 19	4 15	4 11	4 6
24	4 28	4 25	4 21	4 18	4 14	4 10	4 5
27	4 28	4 25	4 21	4 17	4 13	4 9	4 4
30	4 28	4 24	4 21	4 17	4 13	4 9	4 4

Band B — pole elevations 49°–55°

Fully committed (deck 51 cross-validated across the two overlapping column tiles).

Poli G.	49	50	51	52	53	54	55
$\underline{\alpha} 0$	6 0	6 0	6 0	6 0	6 0	6 0	6 0
3	5 54	5 54	5 54	5 54	5 54	5 53	5 53
6	5 49	5 49	5 48	5 48	5 47	5 47	5 46
9	5 44	5 43	5 42	5 42	5 41	5 40	5 40
12	5 38	5 37	5 36	5 36	5 35	5 34	5 33
15	5 33	5 32	5 31	5 29	5 28	5 27	5 26
18	5 27	5 26	5 25	5 23	5 22	5 21	5 19
21	5 22	5 20	5 19	5 17	5 16	5 14	5 12
24	5 16	5 15	5 13	5 11	5 10	5 8	5 6
27	5 11	5 9	5 7	5 5	5 3	5 1	4 59
30	5 6	5 4	5 2	5 0	4 57	4 55	4 52
$\mathfrak{M} 3$	5 2	4 59	4 56	4 54	4 51	4 49	4 45
6	4 56	4 53	4 51	4 48	4 45	4 43	4 39
9	4 51	4 48	4 45	4 43	4 40	4 36	4 33
12	4 45	4 43	4 40	4 37	4 34	4 30	4 27
15	4 41	4 38	4 35	4 32	4 28	4 25	4 21
18	4 36	4 33	4 30	4 26	4 23	4 19	4 15
21	4 32	4 29	4 25	4 21	4 18	4 13	4 9
24	4 28	4 24	4 21	4 16	4 12	4 8	4 4
27	4 24	4 20	4 16	4 12	4 8	4 3	3 58
30	4 20	4 16	4 12	4 8	4 3	3 58	3 53
$\times 3$	4 16	4 12	4 8	4 4	3 59	3 54	3 48
6	4 13	4 9	4 4	4 0	3 55	3 50	3 44
9	4 10	4 6	4 1	3 56	3 51	3 46	3 40
12	4 7	4 3	3 58	3 53	3 48	3 43	3 37
15	4 5	4 1	3 56	3 51	3 45	3 40	3 34
18	4 3	3 59	3 54	3 49	3 43	3 37	3 31
21	4 2	3 57	3 52	3 47	3 41	3 35	3 29
24	4 1	3 56	3 51	3 46	3 40	3 34	3 28
27	4 0	3 56	3 50	3 45	3 39	3 33	3 27
30	4 0	3 55	3 50	3 45	3 38	3 33	3 26

Band C — pole elevations 56°–62°

Decks 56–59 committed. Decks 60–62 committed for the summer half (Libra 0° through Scorpius 6°); their winter-half interior cannot be separated deck-by-deck at the 0–1 min resolution and is held.

Poli G.	56	57	58	59	60	61	62
♌ 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0
3	5 53	5 52	5 52	5 52	5 51	5 51	5 51
6	5 46	5 45	5 45	5 44	5 43	5 43	5 42
9	5 39	5 38	5 37	5 36	5 35	5 34	5 33
12	5 32	5 31	5 29	5 28	5 27	5 26	5 24
15	5 25	5 23	5 21	5 20	5 19	5 17	5 15
18	5 18	5 16	5 14	5 12	5 10	5 8	5 6
21	5 11	5 8	5 6	5 4	5 2	5 0	4 57
24	5 4	5 1	4 59	4 56	4 54	4 51	4 48
27	4 57	4 54	4 51	4 48	4 45	4 42	4 39
30	4 50	4 47	4 44	4 41	4 37	4 34	4 30
♍ 3	4 43	4 40	4 36	4 33	4 29	4 25	4 21
6	4 36	4 33	4 29	4 25	4 21	4 17	4 12
9	4 30	4 26	4 22	4 17	4 13	4 8	4 3
12	4 23	4 19	4 15	4 10	4 5	4 0	3 55
15	4 17	4 12	4 8	4 3	3 57	3 52	3 46
18	4 11	4 6	4 1	3 56	3 50	3 44	3 37
21	4 5	3 59	3 54	3 49	3 42	3 36	3 29
24	3 59	3 53	3 48	3 42	3 35	3 28	3 21
27	3 53	3 47	3 42	3 35	3 28	3 21	3 13
30	3 48	3 42	3 36	3 29	3 22	3 14	3 5
♎ 3	3 43	3 37	3 30	3 23	3 15	3 7	2 57
6	3 38	3 32	3 25	3 18	3 9	3 0	2 51
9	3 34	3 27	3 20	3 13	3 4	2 54	2 44
12	3 30	3 23	3 16	3 8	2 59	2 49	2 38
15	3 27	3 20	3 12	3 4	2 55	2 45	2 33
18	3 24	3 17	3 9	3 1	2 51	2 41	2 29

Poli G.	56	57	58	59	60	61	62
21	3 22	3 15	3 7	2 58	2 48	2 38	2 25
24	3 21	3 13	3 5	2 56	2 46	2 35	2 23
27	3 20	3 12	3 4	2 55	2 45	2 34	2 22
30	3 20	3 12	3 4	2 55	2 44	2 33	2 21

Band D — pole elevations 63°–68° (with 66½° = the Arctic Circle)

Decks 63–68 are recovered in full and independently checked. The winter-half minutes fall toward the polar-night threshold; the (*nox continua*) boundary was corrected where the print shows small positive values just above the threshold (deck 66½ Sgr 21°/24°/27° = 0 39 / 0 26 / 0 13; deck 67 Sgr 18° = 0 [?], positive but minute uncertain).

Poli G.	63	64	65	66	66½	67	68
± 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0
3	5 50	5 50	5 50	5 49	5 49	5 49	5 48
6	5 41	5 40	5 40	5 39	5 38	5 37	5 36
9	5 32	5 31	5 29	5 28	5 27	5 26	5 24
12	5 23	5 21	5 19	5 17	5 16	5 16	5 13
15	5 13	5 11	5 8	5 6	5 4	5 3	5 1
18	5 4	5 1	4 58	4 55	4 53	4 52	4 48
21	4 54	4 51	4 48	4 44	4 42	4 40	4 36
24	4 44	4 41	4 37	4 33	4 31	4 29	4 24
27	4 35	4 31	4 27	4 22	4 20	4 17	4 12
30	4 26	4 21	4 17	4 11	4 8	4 5	3 59

Poli G.	63	64	65	66	66½	67	68
ℳ 3	4 16	4 11	4 6	4 0	3 57	3 53	3 46
6	4 7	4 2	3 56	3 49	3 45	3 42	3 34
9	3 57	3 52	3 46	3 38	3 33	3 29	3 20
12	3 48	3 42	3 34	3 26	3 22	3 17	3 7
15	3 39	3 32	3 24	3 15	3 10	3 5	2 53
18	3 30	3 22	3 13	3 3	2 58	2 52	2 39
21	3 21	3 12	3 2	2 52	2 46	2 39	2 25
24	3 12	3 3	2 52	2 40	2 34	2 26	2 10
27	3 3	2 53	2 42	2 29	2 21	2 13	1 54
30	2 55	2 44	2 32	2 17	2 9	2 0	1 38
↗ 3	2 47	2 35	2 20	2 6	1 56	1 46	1 19
6	2 39	2 27	2 12	1 54	1 43	1 31	0 58
9	2 32	2 19	2 3	1 43	1 30	1 17	0 28
12	2 26	2 11	1 54	1 32	1 18	1 0	(nox continua)
15	2 20	2 5	1 46	1 22	1 5	0 42	(nox continua)
18	2 15	1 59	1 39	1 11	0 52	0 [?]	(nox continua)
21	2 11	1 54	1 33	1 3	0 39	(nox continua)	(nox continua)

Poli G.	63	64	65	66	66½	67	68
24	2 8	1 51	1 28	0 56	0 26	(nox continua)	(nox continua)
27	2 7	1 49	1 26	0 53	0 13	(nox continua)	(nox continua)
30	2 6	1 48	1 25	0 50	(nox continua)	(nox continua)	(nox continua)

Band E — pole elevations 69°–75°

Decks 69–71 and the summer halves of 72–75 committed; the winter halves enter *Nox continua* and the intervening minutes straddle the seam, so they are held.

Pol i G.	69	70	71	72	73	74	75
± 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0
3	5 47	5 47	5 46	5 45	5 44	5 43	5 42
6	5 35	5 34	5 32	5 31	5 29	5 27	5 24
9	5 26	5 20	5 18	5 16	5 13	5 10	5 6
12	5 10	5 7	5 4	5 1	4 57	4 53	4 48
15	4 57	4 54	4 50	4 46	4 41	4 35	4 29
18	4 44	4 40	4 35	4 30	4 24	4 16	4 9
21	4 32	4 27	4 21	4 14	4 7	3 59	3 50
24	4 19	4 13	4 6	3 58	3 50	3 40	3 29
27	4 5	3 58	3 51	3 42	3 31	3 20	3 6
30	3 52	3 44	3 35	3 25	3 13	2 59	2 42
ℳ 3	3 38	3 29	3 19	3 7	2 53	2 36	2 15
6	3 24	3 14	3 2	2 48	2 32	2 11	1 44
9	3 10	2 58	2 45	2 28	2 8	1 41	0 59
12	2 56	2 42	2 26	2 7	1 41	1 1	(nox continu a)
15	2 40	2 24	2 9	1 38	1 4	(nox continu a)	(nox continu a)
18	2 24	2 6	1 43	1 7	(nox	(nox	(nox

Pol i G.	69	70	71	72	73	74	75
					<i>continua</i>	<i>continua</i>	<i>continua</i>
21	2 6	1 46	1 15	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)
24	1 50	1 22	0 34	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)
27	1 30	0 52	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)
30	1 6	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)
♂ 3	0 32	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)
6	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)	(<i>nox continua</i>)
9 – 30	(<i>nox continua</i> — <i>sun does not rise</i>)						

Band F — pole elevations 76°–82°

Summer-half rows committed where legible; the winter half is almost entirely *Nox continua*.

Poli G.	76	77	78	79	80	81	82
± 0	6 0	6 0	6 0	6 0	6 0	6 0	6 0
3	5 41	5 39	5 37	5 35	5 33	5 30	5 26
6	5 22	5 18	5 15	5 11	5 5	4 59	4 50
9	5 12	4 57	4 51	4 45	4 36	4 27	4 14

Poli G.	76	77	78	79	80	81	82
12	4 42	4 36	4 28	4 19	4 8	3 53	3 33
15	4 22	4 13	4 3	3 51	3 36	3 17	2 50
18	4 0	3 50	3 37	3 41 [recte: 3 21]	3 1	2 33	1 51
21	3 38	3 25	3 9	2 52	2 16	1 37	(nox contin ua)
24	3 15	2 58	2 37	2 9	1 25	(nox contin ua)	(nox contin ua)
27	2 50	2 28	2 0	1 15	(nox contin ua)	(nox contin ua)	(nox contin ua)
30	2 21	1 53	1 7	(nox continua)	(nox contin ua)	(nox contin ua)	(nox contin ua)
М 3	1 47	(nox contin ua)	(nox contin ua)	(nox continua)	(nox contin ua)	(nox contin ua)	(nox contin ua)
6	0 59	(nox contin ua)	(nox contin ua)	(nox continua)	(nox contin ua)	(nox contin ua)	(nox contin ua)
9 – x ⁷ 30	(nox continua — sun does not rise)						

Commentary on Chapter III of the *Sphere* of Johannes de Sacrobosco

Table of Semidiurnal Time in the Southern Signs (*Tabula Temporis Semidiurni in signis Australibus*) — continued and concluded

[This is the final pole-elevation block of the southern-signs table, covering elevations of the pole 83°–90°. The table runs across the top (printed p. 253) and its frame closes at the foot of that page; the table ends here.]

Deck (semidiurnal time, hours H / minutes M) by elevation of the pole (heading *Poli*): 83 · 84 · 85 · 86 · 87 · 88 · 89 · 90. The left argument frame carries the descending autumn signs — ♎ Libra, ♏ Scorpius, ♐ Sagittarius — with their degrees and the months September, October, November, December; the right argument frame (heading *Altitudo*) carries the mirror signs ♋ Pisces, ♊ Aquarius, ♏ Capricornus with their degrees and the months March, February, January, December. Because these are the winter (southern-declination) dates, only the rows nearest the equinox carry any semidiurnal value; the whole remainder of the frame — from ♎ 18° downward through all three southern signs — falls in the wide **Nox continua** (continuous night) region and is blank.

Argument anchored at the autumnal equinox (♎ 0°); each step is 3° of ecliptic longitude:

Ecliptic argument (♎ from equinox)	83	84	85	86	87	88	89	90
♎ 0° (equinox)	6 ^h 0 ^m	6 ^h 0 ^m	6 ^h 0 ^m	6 ^h 0 ^m	6 ^h 0 ^m	6 ^h 0 ^m	6 ^h 0 ^m	—
♎ 3°	5 ^h 21 ^m	5 ^h 14 ^m	5 ^h 5 ^m	4 ^h 50 ^m	4 ^h 26 ^m	3 ^h 33 ^m	(blank)	—
♎ 6°	4 ^h 41 ^m	4 ^h 27 ^m	4 ^h 6 ^m	3 ^h 31 ^m	2 ^h 30 ^m	(blank)		
♎ 9°	3 ^h 57 ^m	3 ^h 34 ^m	2 ^h 57 ^m	1 ^h 46 ^m	(blank)			
♎ 12°	3 ^h 10 ^m	2 ^h 31 ^m	1 ^h 13 ^m	(blank)				

Ecliptic argument (♌ from equinox)	83	84	85	86	87	88	89	9
$\text{♌} 15^\circ$	2 ^h 10 ^m	0 ^h 38 ^m	(bla nk)					0
$\text{♌} 18^\circ \rightarrow \text{♐} 30^\circ$	Nox continua (continuous night — all cells blank)							

The column headed **90** shows only a single **M** sub-column and is blank throughout this block (no semidiurnal value entered even at the equinox row). Column **89** carries only the equinox value 6^h 0^m.

Left argument frame (verbatim, descending): ♌ Libra, degrees 0, 3, 6 ... 30 (September–October); ♏ Scorpius, 3, 6 ... 30 (November); ♐ Sagittarius, 3, 6 ... 30 (November–December). Right argument frame (*Altitudo*, verbatim, mirror): ♓ Pisces, degrees 30, 27 ... 0 (March); ♒ Aquarius, 27 ... 0 (February); ♑ Capricornus, 27 ... 0 (January–December).

Quantity of the Day and Night in the Northern Hemisphere (*Quantitas diei, et noctis in hemisphærio Boreali*)

With the Sun located in the northern signs (*Sole existente in signis Borealibus*)

The **semidiurnal arc** — that is, half the length of the day — is found in the common cell, namely under the given elevation of the pole and opposite the given degree of the zodiac (or the given day).

The **seminocturnal arc** — that is, half the length of the night — is what remains once the semidiurnal arc has been subtracted from 12 hours.

The **semidiurnal arc doubled** makes up the whole diurnal arc, that is, the entire length of the day.

The **seminocturnal arc doubled** constitutes the whole nocturnal arc, that is, the entire length of the night.

Example (Exemplum)

With the Sun located in the 12th degree of Taurus, or in the 18th degree of Leo — that is, on 3 May, or on 11 August — at an elevation of the arctic pole of 42°:

- The **semidiurnal arc** is found in the common cell to be 6^h 58^m.
- The **seminocturnal arc** is 5^h 2^m, which remains when the semidiurnal arc 6^h 58^m is subtracted from 12 hours.
- The **diurnal arc** contains 13^h 56^m, namely double the semidiurnal arc of 6^h 58^m.
- The **nocturnal arc** comprises 10^h 4^m, namely double the seminocturnal arc of 5^h 2^m.

The Time of the Sun's Rising and Setting in the Northern Hemisphere (*Tempus ortus, et occasus Solis in hemisphærio Boreali*)

The Sun's **rising after midnight**, in the manner of astronomers, is indicated by the seminocturnal arc.

The Sun's **setting after noon**, likewise in the manner of astronomers, is expressed by the semidiurnal arc.

The Sun's **rising after sunset**, in the Italian manner, is shown by the nocturnal arc.

The Sun's **setting after sunrise**, finally, in the Babylonian manner, is expressed by the diurnal arc.

Example (Exemplum)

On 3 May, or on 11 August, at an elevation of the arctic pole of 42°:

- The Sun **rises** 5^h 2^m after midnight, because such is the seminocturnal arc.
- The Sun **sets** 6^h 58^m after noon, because such is the semidiurnal arc.
- The Sun **rises** again 10^h 4^m after sunset, because such is the nocturnal arc.
- The Sun **sets**, finally, 13^h 56^m after sunrise, because such is the diurnal arc.

The Time of Noon and Midnight in the Northern Hemisphere (*Tempus meridiei, et mediæ noctis in hemisphærio Boreali*)

Noon after sunset is given by the semidiurnal arc subtracted from 24 hours.

Midnight after sunset is exhibited by the seminocturnal arc.

Noon after sunrise is shown by the semidiurnal arc.

Midnight after sunrise, finally, is what remains of the seminocturnal arc subtracted from 24 hours.

Example (Exemplum)

On 5 November, or on 7 February, at an elevation of the arctic pole of 42°:

- **Noon** falls at 18^h 58^m after sunset, which is what remains when the semidiurnal arc 5^h 2^m is subtracted from 24 hours.
- **Midnight** falls at 6^h 58^m after sunset, because such is the seminocturnal arc.
- **Noon** again, after sunrise, falls at 5^h 2^m, because such is the semidiurnal arc.
- **Midnight**, finally, after sunrise falls at 17^h 2^m, which is what remains when the seminocturnal arc 6^h 58^m is subtracted from 24 hours.

The Beginning and End of Continuous Days and Nights in the Northern Hemisphere (*Dierum et noctium continuarum initium ac finis in hemisphærio Boreali*)

The **beginning of any continuous day** occurs as many days before 22 June as there are in half the number of the whole continuous day.

The **end**, however, occurs the same number of days after 22 June.

The **beginning of any continuous night** occurs as many days before 22 December as are contained in half the number of the whole continuous day. For continuous nights are very nearly equal to continuous days.

The **end**, however, occurs the same number of days after 22 December.

Example (Exemplum)

At an elevation of the arctic pole of 68°:

- The **beginning of the continuous day**, which lasts 42 days, falls roughly on the 21st day before 22 June — that is, roughly on 1 June.
- The **end**, however, falls on the 21st day after 22 June — that is, roughly on 13 July.

- The **beginning of the continuous night**, likewise of 42 days, falls on the 21st day before 22 December — that is, roughly on 1 December.
- The **end**, however, falls on the 21st day after 22 December — that is, roughly on 12 January.

Quantity of Day and Night; Time of the Sun's Rising and Setting; Time of Noon and Midnight; and Finally the Beginning and End of Continuous Days and Nights, in the Southern Hemisphere (*Quantitas diei, ac noctis: Tempus ortus & occasus Solis: Tempus meridiei & mediæ noctis: Dierum denique & noctium continuarum initium ac finis, in hemisphærio Australi*)

All of these are drawn from the same table, just as in the northern hemisphere — provided that whatever we have said about the northern signs is understood as said of the southern, and whatever was set down about the southern signs is transferred to the northern.

Example (Exemplum)

The **semidiurnal arc**, at an elevation of the antarctic pole of 42° , with the Sun located in the 12th degree of Scorpio or in the 18th degree of Aquarius, is found in the table to contain $6^h 58^m$ — just as in the northern hemisphere, with the Sun located in the 12th degree of Taurus or in the 18th degree of Leo.

The **seminocturnal arc**, likewise, at the same elevation of the antarctic pole of 42° , with the Sun located in the 12th degree of Taurus or in the 18th degree of Leo, contains $6^h 58^m$ — just as in the northern hemisphere, with the Sun located in the 12th degree of Scorpio or in the 18th degree of Aquarius.

The **continuous day**, of 42 days, has its beginning on 1 December and its end on 12 January, where the antarctic pole is elevated 68° — just as we said of the continuous night in the northern hemisphere.

The **continuous night**, there likewise, begins on 1 June and ends on 13 July — just as was said of the continuous day in the northern hemisphere.

Which Hour from Sunrise or Sunset Corresponds to a Given Hour from Noon or Midnight, and Conversely; Likewise Which Hour from Sunset Corresponds to a Given Hour from Sunrise, and Conversely (*Quota hora ab ortu vel occasu datæ horæ à Meridie vel media nocte respondeat, & contra. Item quæ hora ab Occasu datæ horæ ab Ortū respondeat, & contra*)

Let the **seminocturnal arc be subtracted from the given hour reckoned from midnight** — 24 hours being added first, if the subtraction cannot otherwise be made. Or let the same seminocturnal arc be subtracted from the hour reckoned from noon — 12 hours being added first. The number left over will give the hour counted from sunrise.

Conversely, if the seminocturnal arc is added to the given hour reckoned from noon or from midnight — with a further 12 hours added if the given hour is reckoned from noon — the result will be the hour counted from sunset.

Example (Exemplum)

Suppose the seminocturnal arc contains 5 hours. Let the given hour be 8 from midnight: subtract 5 from 8, and there remains hour 3 from sunrise. Again, let the given hour be 3 from midnight; adding 24 (because 5 cannot be subtracted from 3) gives 27, from which, if 5 is taken away, the remainder will be hour 22 from sunrise. Finally, let the given hour be 6 from noon; adding 12 gives 18, from which, if 5 is taken away, there remains hour 13 from sunrise.

Conversely, let the given hour be 8 from midnight: add the seminocturnal arc of 5 hours, and it becomes hour 13 from sunset. Likewise, let the given hour be 6 from noon; adding 12 gives 18, to which, if the seminocturnal arc of 5 hours is added, it comes to hour 23 from sunset.

In turn, if the seminocturnal arc is added to the hour reckoned from sunrise, the hour reckoned from midnight will emerge — 24 being cast off first, if they can be cast off; or the hour reckoned from noon, 12 being cast off, if they can be cast off. Thus, if it is hour 4 from sunrise, let 5 (that is, the seminocturnal arc) be added: it becomes hour 9 from midnight. Likewise, let it be hour 22 from sunrise; adding 5 gives 27, and casting off 24 leaves hour 3 from midnight. Finally, let it be hour 10 from sunrise; adding 5 gives 15, and casting off 12 leaves hour 3 from noon.

Likewise, if the seminocturnal arc is subtracted from the hour reckoned from sunset — 24 being added first, if the subtraction cannot be made — the remainder becomes the hour reckoned from midnight; or, if 12 can be cast off from the residue, then casting off 12 gives the hour reckoned from noon. Thus, if it is hour 16 from sunset, subtracting 5 leaves hour 11 from midnight. Likewise, let it be hour 23 from sunset; subtracting 5 leaves 18, and taking away 12 leaves hour 6 from noon. Finally, let it be hour 3 from sunset; subtracting 5 — with 24 added first — leaves 22, and casting off 12 leaves hour 10 from noon.

Finally, if the nocturnal arc is added to the hour reckoned from sunrise, the hour reckoned from sunset results — 24 being subtracted from the sum, if they can be subtracted. Thus, if it is hour 19 from sunrise, adding ten gives 29, and taking away 24 leaves hour 5 from sunset. Likewise, let it be hour 8 from sunrise; adding 10 gives hour 18 from sunset.

The **same nocturnal arc**, subtracted from the hour reckoned from sunset — 24 being added first, if it cannot be subtracted — leaves the hour reckoned from sunrise. Thus, if it is hour 20 from sunset, subtracting 10 leaves hour 10 from sunrise. Likewise, if it is hour 9 from sunset, subtracting 10 — with 24 added first, so that they become 33 — leaves hour 23 from sunrise.

DIGRESSIO GEOMETRICA DE CREPVSCVLIS

A GEOMETRIC DIGRESSION ON THE TWILIGHTS

[Margin: Why the twilights are treated here]

Since, together with John of Sacrobosco [Sacrobosco], the author of the *Sphere*, we have discussed at length the natural and artificial days throughout the whole globe, and on that subject have set out a table of semidiurnal arcs for every elevation of the pole, reason and orderly arrangement seem to require that in this place we say something also about the twilights — namely, in what manner they behave in the various regions. This is all the more fitting because the author himself, a little later, when he treats of those who live beneath the pole, briefly explains how many degrees the Sun stands below the horizon, according to certain writers, at the beginning of morning twilight or the end of evening twilight. I trust this discussion will not be unwelcome to students, since it contains subtle and penetrating demonstrations, and very few writers have handled the subject at all. It will also satisfy not a few among the common sort, who are accustomed to call "artificial days" the time from dawn to the end of

evening twilight; for if they add both twilights, morning and evening, to any diurnal arc, they will make up the length of the whole artificial day (as they choose to define it).

[Margin: Pedro Nunes]

Pedro Nunes [Pedro Nunes] the Portuguese, a celebrated mathematician of our own age, sixty-four years ago published a learned and elegant book on the twilights, in which he demonstrated with great acuteness many things that are not unpleasant to know, and which would seem paradoxical were they not secured by the firmest demonstrations. This book — both for the outstanding and excellent sharpness of mind that shines out in it, and for the manifold usefulness to be drawn from it, but most of all because it contains nearly everything that can be said about the twilights — this book, I say, I have resolved to reduce to a compendium in the present digression, for the benefit of students. I do so, however, altering some of the demonstrations and adding others, so that the whole matter may become clearer; and omitting as well not a few of the propositions demonstrated in that book by Pedro Nunes, because these have been treated more openly and plainly by us elsewhere, and do not pertain to the matter of the twilights. This whole subject we shall comprise in twenty-four propositions, beginning here.

PROPOSITIO I.

In the same circle, or in two equal circles, if two equal arcs are taken — whether continuous or not continuous, and whether the one lies wholly outside the other, or they have a common part — then, if perpendiculars are dropped from their endpoints to a diameter or to diameters, the segments of that diameter (or of those diameters) will be unequal, unless the equal arcs stand equally distant from another diameter (or diameters) that cut the first diameter(s) at right angles; and the greater segment will be the one nearer to this latter diameter.

We demonstrated this proposition in our treatment of the Sines, in the case where the equal arcs lie continuous within the same quadrant. Here, however, we shall demonstrate it more universally, exactly as it has been set out by us in this theorem.

Diagram. A circle $ABCD$ with center E , having two diameters AC (vertical) and BD (horizontal) that cut each other at right angles. Various perpendiculars are dropped from points on the circumference to the horizontal diameter, with several auxiliary circles constructed about right triangles near the upper arc.

- A (top of circle): apex of the vertical diameter
- B (far left, on horizontal diameter): left end of diameter BD
- C (bottom of circle): lower end of the vertical diameter
- D (far right, on horizontal diameter): right end of diameter BD
- E (center): center of the circle
- On the horizontal diameter, from left to right: N, M, L, K, E, a, T, [Z?], h — the feet of the several perpendiculars
- Upper arc labels: G, H (upper left); F, X, V (near apex A); S, i (upper right)
- Interior auxiliary points: Q, R (left of center), and small letters d, f, c near the upper middle
- Lower arc labels: P, O (near the bottom, left of C), b (right of C)

Let there be, then, in the circle $ABCD$ whose center is E , two diameters AC , BD cutting each other at right angles; and first let there be two equal, non-continuous arcs FG , HI , and let the perpendiculars FK , GL , HM , IN be dropped. I say that KL is greater than MN . For, the chords FG , IH being joined, and the perpendiculars GQ , IR being drawn, let FK , HM be extended as far as O , P . Let circles also be described about the right triangles FGQ , HIR , which will be equal [Margin: a — scholium, III.32], since their diameters are FG , HI [Margin: b — III.29], which are equal, because the arcs FG , HI are equal. And because arc GBO is greater than arc IBP , angle F will be greater than angle H [Margin: c — scholium, III.27]; and therefore, since (as angle F is to angle H) so is arc GdQ to arc IeR [Margin: d — VI.33], that arc too will be greater by so much. Since, then, these arcs are less than a semicircle — because $GdQF$, $IeRH$ are semicircles — the straight line GQ will be greater than the straight line IR [Margin: e — scholium, III.29]. But GQ is equal to LK , and IR to NM [Margin: f — I.34]. Therefore LK will also be greater than NM ; which was to be proved.

Next, let there be two equal, continuous arcs FG , FS , of which the former lies wholly in quadrant AB , while the latter lies partly in the same quadrant and partly in quadrant AD . The chord FS being joined, let the perpendicular ST be dropped. I say that here too the straight line TK is greater than the straight line KL . For since arc SDO is greater than arc GBO — which will be established if arc AV is taken equal to arc AS , and arc AX equal to arc AF ; because, when Xab is dropped perpendicular to BD , arc VBb will be equal to arc SDO , and this arc VBb is greater than arc GBO — [Margin: g — scholium, III.27] the angle OFS will be greater than the angle OFG . Therefore, the perpendicular Sf being drawn, Sf will be, as before, greater than the chord GQ of the arc of the circle described about the right triangle SFf , and so on. The demonstration will be the same when the

continuous equal arcs lie within the same quadrant — which, moreover, we have also shown otherwise in the treatment of the Sines.

Finally, let there be two equal arcs FS , XY that share a common part. I say again that, the perpendiculars Xab , YZ being dropped, the straight line TK is greater than the straight line Za . For, the chord XY being joined [Margin: h — scholium, III.27], the angle OFS will again be greater than the angle bXY , because arc SDO is greater than arc YDb ; and so on.

The proposition will be demonstrated no differently if there are two equal circles, as is evident, provided an arc of the one circle is transferred to the other. Therefore, in the same circle, and so on; which was to be demonstrated.

COROLLARIUM.

It is gathered from this that, if perpendiculars are raised on a diameter from two equal straight lines, the arcs cut off from the circle are unequal, and the greater is the one which lies farther from the other diameter, or whose straight line stands farther from the center. For let the straight lines Zi , TK be equal, and let the perpendiculars ZY , aX , TS , KF be raised. I say that arc XY is greater than arc FS . For if the arcs XY , FS be said to be equal, then, as has been demonstrated, the straight line TK will be greater than Za — which is contrary to the hypothesis. But if XY be believed less than FS , then, if arc Xi be taken equal to arc FS and the perpendicular ih be dropped, it will again be, as has been demonstrated, that the straight line TK is greater than ha , and therefore much greater than Za — which contends with the hypothesis. A fortiori, then, the greater straight line, standing farther from the center, will cut off a greater arc than the lesser straight line nearer to the center.

PROPOSITIO II.

If two equal segments are taken on the diameters of unequal circles, and perpendicular lines are raised to the diameters, these will intercept unequal arcs; and the arc of the smaller circle will be greater than the arc similar to that of the larger — whether the segments taken on the diameters stand equally distant from the centers, or whether the segment in the smaller circle stands farther from the center.

Diagram. Two unequal concentric circles, the outer $ABCD$ and the inner $EFGH$, described about the common center K , with perpendiculars raised at points L , M on a diameter and various chords and intersecting lines drawn.

- A (top of outer circle); N (top, at the apex where lines meet, beside A)
- O (upper right, on the outer circle)
- E, I (upper center, on the inner circle)
- Z (left, inner circle); F (left, on the horizontal diameter, inner circle)
- B (far left, outer circle, on the diameter); D (far right, outer circle, on the diameter)
- X (left of center); V, K (center — the common center); L (just below K); T (above L)
- M (right of center, on the diameter); H (lower right, near D)
- R (mid-lower right); G, P (lower left); b (lower center)
- S (bottom); C, Q (very bottom)

Let there be two unequal circles $ABCD$, $EFGH$, described about the same center K ; and from the points L , M let the perpendiculars LN , MO be raised. Then the segment LM will be, as it were, two equal parts equally distant from the centers. I say that arc Ia is greater than the arc similar to NO . For let NL , OM be produced to cut the circles at P , Q , R , S ; and let the straight lines IR , NS be joined, cutting each other at T [Margin: a — I.16]. Then the angle PIR will be greater than the angle QNS ; and accordingly, if the angle PIb be made equal to the angle QNS [Margin: b — scholium, III.22], the arcs Pb , QS will be similar; and therefore PR will be greater than the arc similar to QS . But arc PR is equal to arc Ia , and arc QS to arc NO [Margin: c — scholium, III.27]. Therefore arc Ia also is greater than the arc similar to NO ; which was proposed.

But if the segment VX be equal to the segment LM and farther from the center, then arc YZ will be greater than arc Ia [Margin: d — corollary 1 of this book]. Since, then, Ia has been shown to be greater than the arc similar to NO , YZ will be much greater than the arc similar to NO . Therefore, if there be taken, and so on; which was to be shown.

PROPOSITIO III.

To disclose the cause of the twilights.

[Margin: What twilight is]

Twilight is the doubtful light before the rising of the Sun and after its setting: the former is called morning twilight, the latter evening. [Margin: Morning twilight] Morning twilight begins when the Sun, in the vertical circle passing through it, stands eighteen degrees below the eastern horizon; [Margin: Evening twilight] evening twilight ends when the Sun stands as many degrees below the western

horizon. So, at least, the astronomers commonly teach, though some place that distance from the horizon a little greater, and others a little less. In what manner this distance is to be examined, we shall demonstrate below in proposition 24. [Margin: What the cause of twilight is] The cause of both twilights is as follows. When a portion of the air, condensed by vapors and illuminated by the Sun, can be seen by us, twilight comes to be; and this first happens, before the rising of the Sun, when the Sun is depressed about eighteen degrees below the horizon. For when it stands more degrees from the horizon, it does indeed illuminate the finer and purer air; but because there are no vapors in it, the light of the Sun is not reflected to us, and darkness occupies our horizon — for the reason that the denser air, though illuminated by the Sun, cannot be seen by us. By the same reasoning, after the setting of the Sun, the denser air illuminated by the Sun can in no way be discerned by us, so long as the Sun stands more than eighteen degrees, or thereabouts, below the horizon. That this may be made plainer, let *ABC* be the greatest circle on the earth, concentric with the vertical circle passing through the Sun as it moves below the horizon, about which [continues below]

Diagram. The twilight figure: two concentric circles (the earth *ABC* and, about it, the circle *KLM* enclosing the vapor-bearing air), with the sensible horizon *DE* tangent at *A*, the eye placed at *A*, the center *N*, and the solar rays *CF*, *BF* and *GI*, *HI* drawn to form the shadow-cone.

- *I* (upper-left apex); *F* (upper-right apex) — meeting points of the drawn rays
- *D* (left, on the horizontal line *DE*); *A* (center, on the line — the eye and point of tangency)
- *K* (right of center, on the line); *E* (right end of the line); *L* (just below *E*)
- *B* (left, below *D*); *c* (center, below *A*); *G* (right, below *K*)
- *H* (far left); *M* (lower left); *C* (lower right)
- *N* (center of the circles)

Diagram. The twilight figure repeated (identical to fig-p269-2): the two concentric circles of the earth and of the vapor-bearing air, with the horizon *DE* tangent at the eye *A*, center *N*, and the solar rays forming the shadow-cone *CFB* and the rays *GI*, *HI*.

- *I* (upper-left apex); *F* (upper-right apex)
- *D*, *A*, *K*, *E* (on the horizontal line, left to right); *L* (just below *E*)
- *B* (left, below *D*); *c* (center); *G* (right); *H* (far left); *M* (lower left); *C* (lower right)

- N (center) about which let there be another circle *KLM*, enclosing that part of the air to which the vapors can rise. Let the eye also be placed at *A*, and the sensible horizon *DE*, tangent to the earth at *A*. It is now certain that an eye placed at *A* can see nothing of the air, however condensed, below *AE* [Margin: a — III.16], for the reason that no straight line can be intercepted between the tangent *DE* and the circle *AC*. Accordingly, the rays of the Sun touching the earth, *CF*, *BF*, being posited, so that the cone of shadow is *CFB*, no air will be able to be seen beyond *E*, because there it is purer and free of vapors; nor short of *E*, even if the point *K* lies in denser air, on account of the shadow *CEB*, to which the solar rays do not come. But as soon as the rays of the Sun touching the earth, the Sun having moved toward the horizon, have become *GI*, *HI* — of which *GI* runs through *K*, the intersection of the line of sight with the outermost circle of the denser air — the light of the Sun will begin to be reflected to the eye placed at *A*, and the beginning of morning twilight will come to be, and it will last until the rising of the Sun, when the solar ray coincides with the straight line *DE*. So too the evening twilight will last as long as, after the setting of the Sun, the ray of the Sun touching the earth cuts the segment *AK*; but as soon as the ray of the Sun coincides with *GI*, the end of evening twilight will be at hand. The astronomers therefore commonly affirm that the ray of the Sun passes through *K* only at the moment when it lies hidden eighteen degrees below the horizon — although, as I said above, some posit more degrees, and some fewer.

[Margin: The Sun's distance below the horizon at the beginning of morning twilight and the end of evening twilight is variable]

But this distance cannot be fixed; rather it is variable, according as the vapors in the air stand higher or lower. [Margin: In what sense the greatest height of the vapors is to be taken] For when the vapors ascend beyond the point *K*, it is evident that the Sun stands farther from the horizon at the beginning of morning twilight, or at the end of evening, than when they are raised only as far as *K*; and nearer, when the topmost vapors do not reach the point *K*. Moreover, the greatest distance of the vapors from the earth is taken along the semidiameter *NO*, produced as far as *K*, so that the magnitude of this distance is *OK* — which we shall investigate a little later, in proposition 6, the Sun being placed at the beginning of morning twilight, or at the end of evening, eighteen degrees below the horizon. We have therefore disclosed the cause of the twilights; which was to be done.

SCHOLIUM.

And so, if below the horizon there be conceived a circle parallel to the horizon, cutting off from all the verticals eighteen degrees, or more or fewer — according as the Sun's distance below the horizon is set greater or less than eighteen degrees at the beginning of morning twilight, or at the end of evening (for it has been demonstrated by Theodosius [Theodosius] that all the arcs of the verticals between the horizon and its parallel are equal [Margin: b — Theodosius, *Spherics* II.10]) — then, as often as the Sun by its diurnal motion has reached that parallel in the morning, morning twilight will take its beginning, in whatever parallel the Sun may be; and the evening twilight will cease when the Sun, after setting, has come to the same parallel. The magnitude of the arc of each parallel between the horizon and its parallel will be the measure of the twilight, so that the twilight is as long — that is, lasts as long — as the time in which the Sun traverses that arc. But because the arcs of the parallels intercepted between the horizon and the said parallel of it are not similar (for only the greatest circles, which pass through the poles of the parallels, or touch one and the same parallel, cut off similar arcs from the parallels, as is established from Theodosius [Margin: c — Theodosius, *Spherics* II.13 & 16]), the twilights of all the parallels cannot be equal, since the Sun traverses the arcs of the twilights in unequal times. [Margin: Why the twilights of the whole year are not equal] Only the morning and evening twilight of one and the same parallel are equal to each other. For since the plane of the Sun's parallel, cutting the horizon and its parallel [Margin: d — XI.16], makes common sections parallel in the very plane of the Sun's parallel, the arcs of the parallel placed between those parallel lines will be equal [Margin: e — scholium, III.27]; and therefore the Sun will traverse them in equal times, and will make the twilights equal. [Margin: Morning and evening twilight are equal] [Margin: The parallel of the twilights] Henceforth we shall call that parallel of the horizon, in what follows, the "parallel of the twilights."

PROPOSITIO IV.

A luminous sphere illuminates the half of an equal opaque sphere; but more than the half of a smaller opaque sphere; and, finally, less than the half of a larger opaque sphere.

Diagram. First figure for Proposition IV: two equal circles stacked vertically between two parallel vertical tangent lines. The upper circle (luminous *ABCD*) has center *I*; the lower circle (opaque *EFGH*) has center *K*.

- Upper circle: B (top), A (left), I (center), C (right), D (bottom)

- Lower circle: F (top), E (left), K (center), G (right), H (bottom)
- The tangent lines AE (left) and CG (right) join the corresponding points

Diagram. Second figure for Proposition IV: a larger circle and a smaller circle inscribed within a triangle (cone) whose apex is at G below. The larger circle is above, the smaller below, with the two tangent sides of the triangle meeting at G .

- Larger (upper) circle: H (top), A (left), C (right), B (bottom)
- Smaller (lower) circle: I (top/center), D (left), F (right), K (bottom)
- E (on the axis, between B and I); G (apex at the bottom)

Let there first be two equal spheres, the luminous $ABCD$ and the opaque $EFGH$, whose centers I, K let be joined by the straight line IK . Let both be cut by a plane drawn through the straight line IK [Margin: a — Theodosius, *Spherics* I.6], making the greatest circles, in which let the diameters AC, EG be raised perpendicular to IK ; and let the straight lines AE, CG be joined [Margin: b — I.28]. And since AC, EG are parallel, on account of the right angles I, K , and since AI, EK are equal, as also CI, GK , the semidiameters of equal circles, then AE, CG also will be parallel and equal [Margin: c — I.33]. The angles A, E, C, G are therefore right [Margin: d — I.29]; and accordingly the straight lines AE, CG will touch the circles [Margin: e — corollary 1 of III.16], and will be the outermost rays that can fall from the sphere $ABCD$ onto the sphere $EFGH$. Therefore the luminous sphere $ABCD$ illuminates EFG , the half of the opaque sphere; which is the first point.

Let there be next a larger luminous sphere ABC , and a smaller opaque [sphere...]

PROPOSITIO IV [conclusion]

[continued from the preceding]...the smaller circle DEF , whose centres are H and I . Let both be cut through their centres, so that they become great circles, [e] which the rays AD, CF touch. When straight lines HA, HC, ID, IF are drawn from the centres to the points of contact, [f] the angles A, C, D, F will be right angles.

Now since the straight lines AG, CG are equal, as are DG, FG , [g] the straight line GH , when drawn, will bisect the angle G ; and in the same way the straight line GI will also bisect that same angle, [h] so that GI does not differ from GH . Therefore, since in the triangles AGH and CGH the two angles A and C are equal to the two angles D and F , [i] the remaining angles at H will also be equal; [k] and consequently the arcs AB and CB will be equal as well. In no other way [l] the arcs DK and FK are shown to be equal. And the angles AHG, CHG , as well as DIG, FIG , are acute; [m] therefore the arcs BA, BC , and likewise KD, KF , will be less than a

quadrant. Hence the arcs ABC and DKF will be less than a semicircle, while DEF will be greater than a semicircle.

Since, then, AD and CF are the extreme rays of the sphere ABC falling upon the sphere DEF, it is clear that the larger sphere illuminates more than half of the smaller sphere — which is the second point.

But if the smaller sphere DEF is supposed to be the luminous one, and the larger ABC the opaque one, we shall show by the same reasoning that the arc ABC is less than a semicircle. Therefore the extreme rays DA, FC illuminate a part of the larger sphere less than a half — which is the third point. Therefore a luminous sphere, and so on. Which was to be demonstrated.

[Margin, keyed references for Prop. IV: e = scholium on III.17; f = III.18; g = corollary 2 of III.36; h = scholium on III.37; i = I.32; k = III.25; l = I.17; m = scholium on III.27.]

PROPOSITIO V.

To estimate how great the arc of a great circle on the earth is that is illuminated by the Sun.

Since the Sun is larger than the earth, [a] more than half of the earth will be illuminated. We shall determine this quantity as follows. Let a great circle in the Sun be AB, and in the same plane a great circle on the earth DEF, whose centre is L. Let the two extreme rays that are tangent to both spheres be AD and BF. When the centres C and L are joined, let the semidiameters CA, CB, LD, LF be drawn along the straight line C–L to the points of contact; [b] and the angles A, B, D, F will be right angles.

Next, taking AG and BH equal to DL and FL themselves, let the straight lines GL and HL be drawn, cutting the arc DEF at I and K. [c] Now since AG, DL and BH, FL are parallel and equal, [d] GL will also be parallel and equal to AD, and HL parallel and equal to BF; [e] and therefore the angles G, DLG, H, FLH will likewise be right angles. [f] Consequently DI and FK will be quadrants; and IE will be the excess by which the illuminated part on one side surpasses a quadrant, and KE the excess by which the illuminated part on the other side surpasses a quadrant. Thus the part of the earth illuminated throughout its whole circuit contains, beyond a quadrant, as many degrees as are contained in the arc IE, or KE.

Now since, according to Ptolemy [Claudius Ptolemy], the distance of the centre of the earth from the centre of the Sun at mean distances — that is, the straight line

LC — contains about 1168 semidiameters of the earth, while the semidiameter of the Sun AC is $5\frac{1}{2}$ of the parts of which the semidiameter of the earth DL, or AG, is 1, and consequently CG is $4\frac{1}{2}$ of such parts: then, with CL set as the whole sine, the straight line CG is the sine of the angle CLG, as we showed in the treatise on Sines. If it be worked out:

As CL, 1168, is to CG, $4\frac{1}{2}$, so is CL — the whole sine, 100000 — to another quantity.

The sine CG will be found to be 385, to which in the table of Sines corresponds 13 minutes (a little more). And so the Sun still illuminates, over the whole compass of the earth, 13 minutes beyond a half — namely 90 degrees 13 minutes — over the circuit of the whole earth. For such is the angle CLD, or CLF. Therefore how great the arc of a circle is, and so on, we have estimated. Which was to be done.

Diagram. A geometric figure for Proposition V, showing two circles (the Sun and the earth) with their common envelope of tangent lines.

- Upper small circle (the earth, centre L): D (left end of the horizontal diameter), L (centre/top), F (right end), E (lowest point, in the row "I E K")
- Middle row on the earth's lower arc: I (left), E (centre), K (right)
- Lower large circle (the Sun, centre C): A (left end of the horizontal diameter), G (between A and C), C (centre), H (between C and B), B (right end)
- The two outer straight lines are the extreme tangent rays AD (on the left, from A up to D) and BF (on the right, from B up to F), drawn extended and crossing above the figure.

[Margin, keyed references for Prop. V: a = 4 of this [digression]; b = III.18; c = I.28; d = I.33; e = I.29; f = scholium on III.27.]

[Worked proportion, displayed on the page:] As CL, 1168 — to CG, $4\frac{1}{2}$ — so CL, the whole sine, 100000 — to another quantity.

SCHOLIVM.

By this same technique we shall investigate what part of the smaller sphere the larger one illuminates, once the distance between the two centres and the ratio of the semidiameters are known.

PROPOSITIO VI.

To investigate at how great a distance from the earth the topmost vapours stand that condense the air.

Let a plane be drawn through D, the centre of the Sun as it stands at the beginning of the morning twilight, and through the centre of the earth H, producing in the sky the great circle of the Sun ASC, in the Sun the great circle ABC, and on the earth the great circle EFG, about which let the arc of a circle MKN be described, enclosing the topmost vapours. Let the section of the true Horizon be OP, and of the sensible Horizon QR, touching the earth at F, so that the arc PD is understood to be 18 degrees — the distance of the Sun below the Horizon at the beginning of twilight, as it is commonly set down. For since the semidiameter of the earth is of insensible magnitude with respect to the heavens, the point R does not sensibly differ from P; and consequently the arcs RD and PD do not sensibly differ from one another. Let the extreme ray of the Sun that touches the earth at I be CI, cutting the sensible Horizon and the arc MKN at K, where the Sun's light is first reflected in the condensed air to the sight placed at F. Moreover, from H through F let the straight line HFS be extended, [a] which will be perpendicular to QR, [b] and therefore S will be the pole of the Horizon. And finally let the straight lines HI, HLK be joined; so that... Since the straight lines KF, KI are tangent to the circle EFG, [c] the straight line HK will bisect the angle FKI.

[Margin, keyed references for the opening of Prop. VI: a = III.18; b = I.29; c = scholium on III.37.]

PROPOSITIO VI [continued]

the straight line HK will bisect the angle FKI. [d] And since the angle HIK is a right angle, the two angles F, K of the triangle FHK will be equal to the two angles I, K of the triangle IHK; [e] and therefore the remaining angles at H will be equal as well. And so, if to the right angle SHP, of 90 degrees, the angle DHP of 18 degrees is added (for the arc PD is commonly reckoned to be that much by the authorities when the Sun is at the beginning of the morning twilight, [f] as we said above), the whole angle DHS will become 108 degrees.

From this, if the angle DHI is subtracted — which in the preceding [proposition] we found to be 90 degrees 13 minutes (for the arc TI, made up of a quadrant and 13 minutes, is composed thus, as is clear from the preceding proposition) — the remaining angle FHI will be 17 degrees 47 minutes. And therefore its half FHK will be about 8 degrees 54 minutes, and its complement FKH will be 81 degrees 6 minutes. Now since, if HK is set as the whole sine, the semidiameter of the earth FH — which Ptolemy makes 3579 miles — is the sine of the angle FKH, if it be worked out:

As FH, the sine 98796 of the angle FKH, is to HK, the whole sine 100000, so is FH, 3579 (the semidiameter), to another quantity.

HK will be found to be about $3622\frac{2}{3}$ miles; from which, when the semidiameter HL of 3579 miles is subtracted, there will remain KL, the elevation of the topmost vapours, of about $43\frac{2}{3}$ miles. Therefore at how great an interval, and so on, we have investigated. Which was to be done.

Diagram. The twilight figure for Proposition VI: a large circle (the vertical plane through the Sun and the earth's centre) enclosing the small circle of the earth EFG, with the vapour arc, the true and sensible horizons, and the rays. The corners of the woodcut are filled with decorative foliate scrollwork.

- S: at the top of the large circle (pole of the Horizon)
- Q (upper) and O (lower): on the left edge, the left ends of the two horizon lines
- R (upper) and P (lower): on the right edge, the right ends of the two horizon lines (sensible Horizon QR and true Horizon OP)
- H: centre of the small earth-circle (left of centre)
- M: on the horizon line to the left of H; E just below; the earth-circle points E, F, G with F, L, I, N and K in the central cluster
- Central cluster (small circle EFG, centre H, and the vapour arc MKN): K (upper), F, L, I, N, and G (at the foot)
- T: at the foot of the large circle
- A, D, C, B: at the lower part of the large circle (D the centre of the Sun's great circle ABC), with A to the left, C and B to the right

[Margin, keyed references (top of p. 260): c = scholium on III.37; d = III.18; e = I.32; f = 32 of this [digression].]

[Worked proportion, displayed on the page:] As FH, the sine 98796 of the angle FKH — to HK, the whole sine 100000 — so FH, 3579, the semidiameter — to another quantity.

SCHOLIVM.

It is moreover evident that, if the distance of the Sun from the centre of the earth is supposed greater than Ptolemy establishes; likewise if the ratio of the Sun's semidiameter to the earth's semidiameter is different from the ratio $5\frac{1}{2}$ to 1, as Ptolemy will have it — then the angle DHI, by the preceding proposition, will be found not to be 90 degrees 13 minutes, but either greater or less. Likewise, if the

Sun's distance below the Horizon at the beginning of twilight is set greater or less than 18 degrees, as others prefer, the elevation of the topmost vapours will be found not to be $43\frac{2}{3}$ miles, but either more or fewer; especially if the Sun's distance from the earth, and the earth's semidiameter, are established differently from what we have supposed. And this is perhaps the reason why Alhazen [Ibn al-Haytham] and Vitellio [Witelo] found the elevation of the topmost vapours to be about 52 miles.

If the angle DHI, by the preceding [proposition], were found to be only 90 degrees 12 minutes, and the distance of the Sun below the Horizon at the beginning of twilight were 19 degrees 30 minutes, then the angle DHS would be 109 degrees 30 minutes, and the angle FHI 19 degrees 18 minutes, and FHK 9 degrees 39 minutes, and FKH 80 degrees 21 minutes. And thus HK would be found to be 3630, and, the semidiameter HL of 3579 being subtracted, the remaining elevation of the vapours LK would be 51[?] miles, a little more. But whatever be the case about this variation, our demonstration will not change: it is enough that we have prescribed the method by which the elevation of the topmost vapours can be explored, provided the Sun's distance from the Horizon at the beginning of twilight, and the part of the earth illuminated by the Sun, are settled, and so on.

PROPOSITIO VII.

From the given height of some lofty mountain, to find the arc of the vertical circle by which those who dwell on the mountain's summit see the Sun rise earlier than those at its foot; and moreover to determine the interval of time between the two sunrises.

Let there be a great circle on the earth ABC, whose centre is D. Let the height of the mountain be BE, its base toward the east [being at F] [?]; and let the vertical circle drawn through the centre of the Sun and the summit of the mountain be GHI, in whose plane let the circle of the earth DBEH lie, [and let] the line

[Margin, keyed reference for the start of Prop. VII: on p. 260 the paragraph shares the reference apparatus of Prop. VI above.]

PROPOSITIO VII [continued]

[and let] the line drawn from the centre of the earth through the summit of the mountain, falling at H, [be] the vertex of those dwelling on the mountain's summit. Their true Horizon is GI, their sensible Horizon KL, touching the earth at B. Let the straight line DFP be drawn from the centre through the vertex of those

dwelling at the mountain's base F, falling at their Zenith P; and the difference between H and P will be insensible.

Let there also be drawn, from E, the summit of the mountain, the straight line MN, touching the earth at O, to which let QR be drawn parallel, through the centre; and QR will be the true Horizon of those dwelling at O, and MN the sensible one, and S their Zenith. And since the earth is of insensible magnitude with respect to the heavens, LI and NR will be of insensible quantity: so that, when the Sun's rising is discerned at I by those situated at F, it is supposed to rise at L. By like reasoning, when the Sun rises for those inhabiting the point O, it will be supposed to rise at N.

But the rising Sun is seen along the same straight line MN both by those dwelling at O and by those situated at E, the summit of the mountain. Now since it rises earlier for those who dwell at O than for those who are at F, the mountain's base, the Sun will also seem to rise earlier from the summit E than from the base F; and the difference of their risings on the vertical circle will be the arc IR, between the true Horizons, which we shall determine thus.

Let us suppose that BE, the height of the mountain, comprises 3 miles, or 24 stadia. And since the semidiameter of the earth DB, according to Ptolemy, contains 28636 stadia, the whole DE will comprise 28660 stadia. Since therefore, with DE set as the whole sine, the semidiameter DO is the sine of the angle DEO [a] — because the angle O is a right angle in the triangle DEO — if it be worked out:

As DE, of 28660 stadia, is to DO, of 28636 stadia, so is DE, the whole sine, 100000, to another quantity.

DO will be produced as the sine 99916, to which in the table of Sines corresponds 87 degrees 39 minutes, for the angle DEO. Therefore its complement, 2 degrees 21 minutes, will give the angle EDO, or the arc HS, to which the sought arc IR is equal — because, if from the quadrants HI and SR the common arc SI is subtracted, the equal arcs HS and IR remain. So when the Sun rises for those who are on the mountain's summit, it is depressed for those dwelling at F, the mountain's base, by 2 degrees 21 minutes.

The time between the two risings will be obtained if, by Proposition 14, the twilight is computed for the given day and the given latitude of the place, the Sun's distance below the Horizon at the beginning of twilight being set at 2 degrees 21 minutes; which, for latitude 38 degrees, on the fifth of August, is found to comprise 0 hours 13 minutes. And by so much time will the Sun rise earlier on

the mountain's summit than at its base. Therefore, from the given height of some lofty mountain, and so on, we determine. Which was to be done.

Diagram. The mountain-sunrise figure for Proposition VII: a large circle (the vertical plane) with the small earth-circle near the centre, the mountain, and the horizons and rays of the two observers (summit and base). Several peripheral labels are small and hard to resolve

- H (top) and P (just to its right, top): the vertical falling-point and the Zenith of the base-dwellers
- S: upper right (Zenith of the O-dwellers)
- M: upper left
- Q: left, below M
- K: left (mid); below it a letter read as G [?]
- E: near the centre (mountain summit)
- B: near the centre (mountain base on the earth-circle)
- F, O: central cluster (F the base, O the tangent point); D: centre (of the earth-circle)
- A (lower-left of centre) and C (lower-right of centre)
- L and I: on the right edge (L upper, I below)
- N: lower right
- R: at the foot of the ray, lower right

[Margin, keyed reference for Prop. VII: a = III.18.]

[Worked proportions, displayed on the page:] As DE, of 28660 stadia — to DO, of 28636 stadia — so DE, the whole sine, 100000 — to another quantity.

SCHOLIVM.

With these things thus premised, let us approach the demonstrations of the twilights. Here I wish the reader to be warned that we shall treat only of the twilights of those parallels that are cut by the Horizon; and indeed of those which have one morning twilight before the Sun's rising and another evening one after the Sun's setting.

[Margin: Which parallels' twilights are dealt with in this place.]

PROPOSITIO VIII.

When the Sun is at two points equally distant from either solstice, the twilights are equal.

For since through those two points one and the same parallel runs — as we demonstrated in Book I of the Astrolabe, Lemma 49, Number 1 — the same arc of that parallel will always be intercepted between the Horizon and the parallel of the Twilights; and therefore the same twilight will result. Therefore, when the Sun is [at such points], and so on. Which was to be shown.

PROPOSITIO IX.

For two points on either side, equally distant from one or the other equinox, the corresponding twilights are unequal; and that one will be the greater which inclines toward the visible pole.

Let the Meridian be ABCD, its centre E. Let the diameter of the Equator be AC; the diameter of the parallel toward the visible pole D be FG; that of the other, toward the other pole, be HI; and let the declinations AF, AH be equal. Let the diameter of the oblique Horizon be KL; the diameter of the parallel of the twilights be MN; and let the axis of the world DB [a] cut the diameters FG, HI in half at Y and Z, because it makes right angles with the said diameters. Now with semicircles FSG, HVI described about those same diameters from X [Y?] [?] and Z, let perpendiculars OS, PT, QV, RX be raised to those same diameters.

Since both the Horizon and the parallel FST are at right angles to the Meridian, [b] their common section will be a straight line at right angles to it; and therefore it will also be perpendicular to the diameter FG. Therefore the perpendicular OS will be the common section of the Horizon and the parallel; and so the Sun will rise at S, when it describes that parallel. In the same way PT will be shown to be the common section of the Horizon and the same parallel; and therefore the Sun, at the beginning of the morning twilight, will stand at T, and the arc ST will be the length of the twilight on that

Diagram. The figure for Proposition IX: the Meridian circle ABCD with centre E, crossed by the diameters of the Equator, the two symmetric parallels, the oblique Horizon, and the parallel of the twilights, with the erected semicircles and perpendiculars. Two small stars mark Sun positions. Several peripheral labels are small and crowded

- Circle (Meridian) with points A, B, C, D on it and centre E

- F (upper left), O with a star [Sun mark] (top), S (upper right)
- A, H (left, below F)
- Y (upper centre)
- I (right, upper); T (right, below S)
- K, M (left edge)
- Q, Z, O (central row); P (centre-right)
- G [ς?] [?] and C (lower right)
- V (lower left), B with a star [Sun mark] (lower centre)
- X (bottom left), I (bottom right)

[Margin, keyed references for Prop. IX: a = III.3; b = 19 [of this? — abbreviation unclear] [?].]

in that parallel. Likewise the arc VX will be the length of the twilight in the parallel HVI. I say, therefore, that the twilight ST is greater than the twilight VX. For just as OP and QR are equal on the diameters of equal parallels, and OP is farther from the centre Y than QR is from the centre Z, so the arc ST will be greater than the arc VX — which was the claim. Therefore, for two points, and so on — which was to be demonstrated.

[Margin: a — Elements I.34; b — Corollary 1 of this [treatise].]

COROLLARIUM

It follows from these results that in the northern region the twilights of the northern points of the ecliptic are greater than those of the corresponding southern points; but in the southern region they are smaller.

[Margin: Where the twilights become longer.]

PROPOSITIO X

When the Sun is traversing the northern signs, the twilight in the northern region becomes longer the closer it is to the beginning of Cancer — provided always that the Sun's parallel cuts both the horizon and the parallel of the twilights.

Diagram. A circle representing the meridian ABCD, with the equator, the northern parallels, the oblique horizon, and the parallel of the twilights drawn as chords, and several perpendiculars raised to the diameters. Sun-positions are marked with small stars.

- A (upper left, star): endpoint of the equator diameter

- F, H (top): parallel endpoints
- D (upper right, star): pole / solstitial Sun mark
- K (left), M (left): horizon / twilight-parallel endpoints
- B (lower left, star): pole mark
- Y, E, T (centre): centres of parallels and of the sphere (E)
- O, V, P, N (right of centre): feet of the perpendiculars OP, TV
- L (right): horizon endpoint
- C, I, G (lower right): equator / parallel endpoints; zodiac glyphs (♌, ♍) appear beside the lower-right labels
- [2 further peripheral labels illegible]

Let the meridian be ABCD about the centre E; the diameter of the equator, AC; the diameters of the northern parallels, FG and HI; the diameter of the horizon, KL; the diameter of the parallel of the twilights, MN. If from O, P, and TV perpendiculars are raised to the diameters, these will cut off, on the parallels described about the diameters, the arcs of the twilights, as we showed in the preceding proposition. I say that the arc of the twilight owed to the straight line OP is greater than the arc of the twilight that corresponds to the straight line TV. For since OP recedes farther from the centre X than TV does from the centre Y — (a) and since OP and TV are equal — (b) there will correspond a greater arc to the straight line OP in its own parallel than is owed to an arc similar to the one that TV subtends in its own parallel. Consequently the twilight will be greater when the Sun traverses the parallel of diameter FG than the twilight of the parallel of diameter HI. Therefore, when the Sun traverses the northern signs, a longer twilight occurs in the northern region, and so on — which was to be demonstrated.

[Margin: a — Elements I.34; b — Proposition 2 of this [treatise].]

SCHOLIVM

The contrary happens in the southern region; for there the twilight becomes greater the less the Sun, when in the southern signs, is distant from the beginning of Capricorn. This is clear in the same figure if D is understood to be the south pole, and B the north.

COROLLARIVM

Hence it follows that in the northern region the greatest twilight occurs — the Sun being in the southern region, that is, the Sun being at the beginning of Capricorn (♑).

That in the northern region the greatest twilight should occur when the Sun is at the beginning of Capricorn, just as the longest day does — yet not in the first degree of Capricorn, but in some parallel between the tropic of Capricorn (♑) and the equator, as we shall demonstrate in Propositions 19, 20, 21, and 22 — although the day is then shortest: this seems scarcely credible.

[Margin: c — Corollary 9 of this [treatise].]

Diagram. A second circle (the meridian) with equator, parallels, horizon, and twilight-parallel drawn as intersecting chords, several perpendiculars, and a small Sun-star. It illustrates where the longest twilight falls.

- F, A, H (upper left)
- D (top, star): Sun/solstice mark
- K, M (left): horizon / twilight-parallel endpoints
- V, B (lower left, star), X (bottom left)
- Q, E, Z, R (centre): centres and foot of perpendiculars
- O, Y (centre), P, T (centre-right)
- S (upper right), I, N (right)
- C (lower right)
- [1–2 peripheral labels illegible]

[Margin: Where the longest twilight occurs.]

PROPOSITIO XI

Wherever the Sun may be, the twilights become longer in the more northerly places than where the Sun's parallel cuts both the horizon and the parallel of the twilights.

Diagram. A large circle (the meridian ABCD) crowded with chords and perpendiculars: the equator AC, northern parallel FG, southern parallel HI, the axis of the world BD, the oblique horizon KL, the twilight-parallel MN, the more-northerly horizon OP, the twilight-parallel QR, and the perpendiculars SY and VZ. Sun-positions are marked with stars at B and D.

- A, F, H (upper left)
- O, Q, R (upper-left interior)
- D (upper right, star)
- S (upper-right interior)
- K, M (left)
- E (centre)
- V, Y, Z, X, P, d (centre and lower interior)
- B (lower left, star)
- I, C, G (bottom / lower right)
- L, N (right)
- [several interior labels crowded and partly illegible]

[Margin: a — Elements I.29.]

Let the meridian be ABCD, whose centre is E; the diameter of the equator, AC; the diameter of the northern parallel, FG; of the southern, HI; let D be the north pole, B the south; the axis of the world, BD; the diameter of the oblique horizon, KL; the diameter of the parallel of the twilights, MN; the diameter of the more-northerly horizon, OP; the diameter of the twilight-parallel QR; and so on. I say that the twilight owed to the straight line VX in the northern parallel is greater than the twilight owed to the straight line ST; and greater than the twilight of the straight line Ed at the equator, [which is owed to] the twilight of the straight line Eb; and greater than the twilight of the straight line ef in the southern parallel, [owed to] the twilight of the straight line ig.

For, the perpendiculars SY and VZ having been dropped to the parallels of the twilights: (a) since the angle AEK is equal to the interior angle AbM, and this to the interior angle FTM, the angle FTM will be equal to the angle AER, the complement of the pole's altitude in the less northerly region. (b) Likewise, since the angle AEO is equal to the interior angle AdQ, and this to the interior angle FXQ, the angle FXQ will be equal to the angle AEO, the complement of the pole's altitude in the more northerly region. But the angle AEK is greater than the angle AEO. Therefore the angle FTM will also be greater than the angle FXQ — (c) which is likewise evident from the fact that in the triangle TXh the former is exterior and the latter the interior opposite angle. Hence we shall conclude that the straight line XZ is greater than the straight line TY. For if it were equal — since the perpendiculars SY and VZ are also equal, being the sines of 18° — (d) the angles STY and VXZ would be equal, the opposite of which we have shown—

[Margin: b — Elements I.29; c — Elements I.16; d — Elements I.4.]

—demonstrated. But if XZ be supposed less than TY, let Za be made equal to TY itself; (e) and in the same way, the straight line Va having been drawn, the angle VaZ will be equal to the angle STY. (f) Since, then, the angle VXZ is greater than the angle VaZ, the angle VXZ will also be greater than the angle STY — though this one was shown to be the greater. Therefore XZ is greater than TY; and consequently the two squares on XZ and ZV will be greater than the two squares on TY and YS. (g) That is, the square of the straight line VX will be greater than the square of the straight line ST, and therefore the straight line VX greater than the straight line ST.

[Margin: e — Elements I.4; f — Elements I.16; g — Elements I.47.]

But perhaps we shall show more easily that VX is greater than ST in this way. The angle TSY is [equal to] the angle of the pole's altitude above the horizon KL, (h) because STY is equal to the exterior angle FSK, that is, to the angle AEK, the altitude of the equator. And in the same way XVZ will be the angle of the pole's altitude above the horizon OP, (i) since VXZ is equal to the exterior angle FVO, that is, to the angle AEO, the altitude of the equator. Therefore the angle XVZ will be greater than the angle TSY, because the altitude of the pole above the horizon OP is set greater than above KL. Hence, if VZ be imagined to coincide with SY, and the straight line ZX with the straight line YT, VX will fall beyond ST, (k) and VX will be greater than ST, since it is set opposite the obtuse angle STN.

[Margin: h — Elements I.29; i — Elements I.29; k — Elements I.19.]

Since, then, VX is farther from the centre of the parallel than ST, (l) there will correspond to the straight line VX a greater arc of twilight than to the straight line ST in the northern parallel — which was the claim.

[Margin: l — Corollary 1 of this [treatise].]

Next, (m) since on the diameter of the equator the straight line Ed is equal to the straight line VX, and the straight line Eb to the straight line ST, Ed will also be greater than Eb — that is, the point d will lie beyond b. (n) Therefore a greater twilight corresponds to the straight line Ed at the equator than to the straight line Eb — which was the claim.

[Margin: m — Elements I.34; n — Corollary 1 of this [treatise].]

Finally, (o) since on HI, the diameter of the southern parallel, the straight line ef is equal to the straight line VX, and the straight line ig to the straight line ST, ef will also be greater than ig, and ef is farther from the centre of the parallel than ig. (p) Therefore a longer twilight is owed to the straight line ef than to the

straight line *ig* — which was the claim. And so, wherever the Sun may be, and so on — which was to be demonstrated.

[Margin: o — Elements I.34; p — Corollary 1 of this [treatise].]

PROPOSITIO XII

When the Sun occupies points of the ecliptic equally distant on either side from either of the equinoctial points, then for those living under the equator — that is, in the right sphere — the twilights are equal. But when the Sun occupies two points unequally distant from either of the equinoctial points, the twilights become unequal, the greater indeed at the more remote point, the smaller at the nearer; so much so that at the tropics the twilights become longest, while, the Sun occupying the equinoctial points themselves, the shortest twilights are produced.

Diagram. A circle (the meridian ABCD) crossed by four vertical chords (the parallels seen edge-on in the right sphere) and by the horizontal equator diameter BD and the line KL.

- H, A, F, Q (top, left to right)
- B (left end of the horizontal diameter), D (right end)
- N, E, M, S (upper-middle row)
- K (lower left), P, V, O, T (lower-middle row), L (lower right)
- I, C, G, R (bottom, left to right)

Let the meridian be ABCD about the centre E; the diameter of the equator, AC; the diameters of the two parallels equally distant from the equator, FG and HI; and QR, the diameter of the parallel more distant from the equator. The horizon is the perpendicular BD, and its parallel of the twilights KL. I say that in the parallels FG and HI equal twilights occur, and so on. For since MO and NP (a) are equal to one another and are equally distant from the centres M and N of the equal parallels, (b) the arcs of the twilights owed to them will be equal — which is the first point.

Next, (c) since MO and ST are likewise equal and recede equally from the centres M and S of unequal parallels, (d) the perpendiculars from S and T, erected in the plane of the parallel toward the plane of the meridian, will cut off a greater arc than would be similar to the arc cut off by the perpendiculars erected from M and O; and therefore the twilight owed to the straight line ST will be longer than the twilight corresponding to the straight line MO. From which it follows that, since the tropics recede most from the equinoctial, the greatest twilights occur there — which is the second point.

Finally, since the equator is the greatest of all the parallels, (e) the perpendiculars from E and V, erected in the plane of the equator toward the plane of the meridian, will cut off a smaller arc of twilight than would be similar to the arcs of the twilights in the other parallels. Therefore the shortest twilight occurs when the Sun, at the equinoctial point, occupies points of the ecliptic, and so on — which is the third point. Therefore, the Sun [occupying points equally distant, the twilights are equal], and so on — which was to be demonstrated.

[Margin: a — Elements I.34; b — Proposition 1 of this [treatise]; c — Elements I.34; d — Proposition 2 of this [treatise]; e — Proposition 2 of this [treatise].]

PROPOSITIO XIII

To compute the length of the twilight in the right horizon.

Let the figure of the preceding proposition be repeated, in which the diameter [of the equator is AC, of the parallels FG and] HI, of the right horizon BD, and of the parallel of the twilights KL. When the Sun is at the equator, EV will be the right sine of the arc of 18° , by which the Sun is hidden below the horizon at the beginning of the morning twilight, or at the end of the evening; and accordingly the arc BK will be equal to the arc of the twilight at the equator. Therefore the twilight comprises the 18° of the Sun's concealment below the horizon — that is, 1 hour 12 minutes.

But when the Sun is in any parallel HI, since the semidiameter [HN is the sine of the complement] of the parallel's declination, and NP the sine of the twilight in the same parallel, as is established, and as we have shown in Book 1 of the *Gnomonics* [propos. 35] — (for if about HI a semicircle of the parallel be conceived, [erect to the meridian ABCD,] and from N and P perpendiculars be erected to the same meridian, these will cut off from the parallel the arc of the twilight whose sine is NP) — if it be made:

As HN, the sine of the complement of the declination, is to NP, the sine of the Sun's concealment below the horizon, so HN, the whole sine in the parallel, is to another —

there will result [i.e. "prodibit"]

NP, the sine of the twilight, in parts of the whole sine in the parallel. Therefore from the table of sines the twilight will be known. At Cancer (♋) and Capricorn (♎) is found the twilight that is greatest, [namely] $19^\circ 41'$ — that is, 1 hour 19

minutes. And so in the right horizon, and so on, we have computed it — which was to be done.

PROPOSITIO XIV

To find the length of the twilight in any oblique horizon whatever.

Let the meridian be ABCD about the centre E; the diameter of the equator, AC; the parallel FG (northern in the first figure, but southern in the second); the axis of the world, BD; the diameter of the horizon, HI, and of its parallel of the twilights, KL. Then FH will be the meridian altitude of the parallel FG, and IG its meridian depression. From F let the perpendicular FP be dropped to HI, meeting, when produced, in Q, with GQ parallel to HI itself. Then FP will be the sine of the meridian altitude, and PQ of the depression. Let NO also be drawn parallel to GQ; and since (a) FG is bisected in N, (b) FQ too will be bisected in O. And when from N the parallel FZG has been described, in each figure, and MZ and XY drawn perpendicular to FG, and likewise Vd perpendicular to AC, then Dd, on the meridian, will be the arc of the twilight at the equator, and ZY the arc of the twilight in the parallel.

[Margin: a — Elements III.3; b — Elements VI.2.]

Although both in the *Gnomonics* Book 1, proposition 35, and in the *Astrolabe* Book 3, in the scholium to Canon 10, we have handed down the finding of the twilights by several methods, still it is desirable to show here at least one way as well — and indeed the most expeditious — both because we are treating of twilights in this little book, and also because this finding is necessary for what follows. We shall therefore learn the twilight Dd at the equator thus. Let it be made:

Diagram, first (upper) figure — northern parallel. A circle (meridian ABCD) densely filled with chords and perpendiculars: equator AC, northern parallel FG, axis BD, horizon HI, twilight-parallel KL, the described parallel FZG, and the perpendiculars FP, GQ, NO, MZ, XY, Vd. Sun-positions marked with stars.

- A (upper left, star), F (top), D (upper right, star): with a zodiac glyph near the top
- H (left), K (lower left)
- O, R, E (interior, E at centre)
- N, M, P, S, V, X, T, Q (interior points)
- B (bottom, star)
- I, L, Y, z (right side)

- C, G, and small d, q, e (lower right / interior)
- [several crowded interior labels illegible at this resolution]

As AR, the sine of the altitude of the equator (or of the complement of the pole's altitude), is to RS, the sine of 18° , so the whole sine AE is to another.

For the number produced will give EV, the sine of the twilight Dd, (c) because the ratio of AR to RS is the same as that of AE to EV. We shall find the same by another method as follows. Since, in the right-angled triangle ETV, (d) the angle EVT is equal to the angle AEH, the complement of the pole's altitude, ETV will be the angle of the pole's altitude. (e) If, then, it be made:

As ET, the whole sine, is to EV, the secant of the angle TEV of the pole's altitude, so ET, the sine of 18° , is to another,

the same sine EV of the twilight Dd at the equator will be produced.

[Margin: c — Elements VI.2; d — Elements I.29; e — Problem 5 of [the treatise on] rectilinear triangles.]

Diagram, second (lower) figure — southern parallel. A circle (meridian ABCD) similarly crowded with chords and perpendiculars for the case where the parallel FG is southern; the described parallel FZG and perpendiculars MZ, XY, Vd are drawn, with Sun-positions marked.

- A, F (upper left/top), D (upper right, star)
- H, R, M, P (upper-left interior)
- E (centre), N (below centre)
- R, S, X, T, V (lower interior)
- B (lower interior, star)
- Z (left), C (lower right), L, I (right)
- f, Y, Q, q (bottom)
- [several crowded interior labels illegible at this resolution]

But the twilight ZY in the parallel we shall find thus. Since (f) as FO, half the sum of FP (the sine of the meridian altitude) and PQ (the sine of the meridian depression), is to Fe, the straight line composed of FP (the sine of the meridian altitude) and Pe (the sine of 18°), so the whole sine FN is to FX — if it be made:

As half the sum of the sine of the meridian altitude and the meridian depression, is to the sum of the sine of the meridian altitude and the sine of 18° , so the whole sine is to another,

there will be generated FX, the versed sine of the arc FY, which is composed of the semidiurnal arc FZ and the arc of the twilight ZY; and accordingly the arc FY will be known. From this, if the semidiurnal arc FZ be subtracted, the twilight ZY will be left known. And so in any oblique horizon whatever, and so on, we have found it — which was to be done.

[Margin: f — Elements VI.2 or 4.]

SCHOLIVM

The arc corresponding to a given versed sine is easily derived from it, as we taught in the treatment of sines, by this method. When the versed sine is greater than the whole sine — for example, 184493 — the first figure on the left, 1, which is equivalent to the whole sine 100000, being set aside, let the remaining sine 84493 be taken: the arc [of it] is $57^{\circ} 40'$. For this, added to the quadrant, will make the arc sought, $147^{\circ} 40'$. But when the versed sine is less than the whole sine, subtracting it from the whole sine, let the sine of the arc of the remainder be taken; for this, subtracted from the quadrant, will make the arc that is sought. Thus, if the versed sine be, say, 79104: this being subtracted from the whole sine 100000, the remaining sine becomes 20896, whose arc is $12^{\circ} 4'$; and this, subtracted from the quadrant of 90° , leaves the arc of $77^{\circ} 56'$ corresponding to the versed sine, which we seek.

[Margin: How the arc corresponding to a versed sine is to be derived.]

Although we posit that the arc of the Sun's concealment below the horizon at the beginning of the morning twilight and at the end of the evening comprises, by the common opinion of astronomers, 18° , still the twilights will be computed in the same way if that concealment be set greater or less, as is evident.

This too must not be left unknown: that in the northern signs there is twilight all night long at that elevation of the pole where the meridian depression of the proposed parallel is either equal to the arc of the Sun's concealment below the horizon (which we posit to contain 18°) or less, as is evident from the figure of this proposition. And this will happen when the versed sine of the arc formed from the semidiurnal arc and the arc of the twilight is found to be 200000 or greater.

[Margin: When there is twilight through the whole night.]

Consult also proposition 35 of book 1 of the *Gnomonics*, and its scholium, so that the whole variety of the twilights may be more clearly grasped.

The semidiurnal arc, moreover — which is to be subtracted from the arc compounded of the semidiurnal arc and the twilight arc — is to be obtained either from the first of the tables which we published, with Notes, in the *New Description of Sundials*, or is to be sought by means of the following proposition 16.

PROPOSITIO XV.

Given the distance of any point of the ecliptic from either of the equinoctial points, to investigate its declination; and conversely, from a given declination, to find the corresponding point on the ecliptic.

Let AB be an arc of the equator, and AC an arc of the ecliptic, such that A is the beginning of Aries or of Libra. Let CB be an arc of a great circle drawn through the poles of the world and through the given point C on the ecliptic, so that CB is the arc of its declination, which is proposed for investigation. Since in the right-angled spherical triangle ABC the base AC is known — namely the distance of the given point C from the nearest equinoctial point A — and the angle A of the maximum declination, which is opposite the sought arc CB of declination, is also known, then, by the first method of problem 8 of spherical triangles (I mean the problems of spherical triangles which we demonstrated at the end of Lemma 53, book 1, of the *Astrolabe*), let it be arranged:

As the whole sine is to the sine of the base AC — the distance of the point C from the equinoctial point A — so is the sine of the angle A, of the maximum declination, to another [number];

and the sine of the arc of declination CB, which is sought, will be produced.

Diagram. A small figure of two intersecting great-circle arcs forming the spherical triangle of the proposition.

- A (upper left): vertex A, the equinoctial point
- C (right): vertex C, the given point on the ecliptic
- B (lower center): vertex B, the foot of the declination arc

But if the declination of the point C is known, we shall find its distance from the equinoctial point A in the following way. Let it be arranged:

As the sine of the angle A, of the maximum declination, is to the sine of the arc of declination CB, known — so is the whole sine to another;

for the sine of the base AC, the distance of the point C from point A, will be generated, since — as we said — as the sine of the angle A of maximum declination is to the sine of the arc CB, so is the whole sine to the sine of the base AC.

Or in this way. Since in the same right-angled triangle ABC the angle C[D?] of declination is known, together with the angle A of maximum declination which is opposite the given arc CB, then, by the first method of problem 14 of spherical triangles, let it be arranged:

As the whole sine is to the secant of the complement of the given angle A, of the maximum declination — so is the sine of the declination CB to another;

the sine of the base AC will be found, as before. For example, if the declination CB is given as 14 degrees 40 minutes, there will be found — by either method — a sine of about 63496, to which corresponds 39 degrees 25 minutes. Therefore, if the given point stands off from the beginning of Aries following the succession of the signs, the point C will be at 9 degrees 25 minutes of Taurus; but if against the succession of the signs, it will be at 20 degrees 35 minutes of Aquarius. And if it recedes from the beginning of Libra following the succession of the signs, the point C will be at 9 degrees 25 minutes of Scorpio; but at 20 degrees 35 minutes of Leo if it recedes from Libra against the succession of the signs. We have therefore investigated the declination of any point, etc. Which was to be done.

PROPOSITIO XVI.

To compute the semidiurnal arc of any point of the ecliptic, whose declination is given, for any latitude of place; and conversely, from a given semidiurnal arc, to search out the corresponding point of the ecliptic.

In the figure of proposition 14, since it holds that as FO is to FP, so is FN to FM, let it be arranged:

As TO — the half of the sum of the sines of the meridian altitude and of the depression — is to FP, the sine of the meridian altitude — so is FN, the whole sine, to another;

there will result FM, the versed sine of the semidiurnal arc FZ, from which, as we explained in the scholium of proposition 14, the semidiurnal arc itself will be extracted.

[Margin: 2 of the sixth (Euclid VI.2).] Or, if it be arranged:

As FO — the half of the aforesaid sum — is to OP, the difference between that half and the meridian altitude — so is FN, the whole sine, to another;

MN will be produced, the sine of the arc Zf, by which the semidiurnal arc FZ differs from the quadrant Ff. Therefore the arc ZF will be known, which, added to the quadrant Ff in the northern signs, and subtracted from the same quadrant in the southern, makes up or leaves the semidiurnal arc FZ.

Or if it be arranged, as we showed in problem 2 of the *New Description of Sundials*:

As the whole sine is to the tangent of the pole's altitude — so is the tangent of the declination to another;

there will arise the same sine of the arc by which the semidiurnal arc differs from the quadrant.

Diagram. The upper of two nearly identical large circular figures (the reused "figure of proposition 14"): a circle representing the meridian, crossed by numerous chords and diameters forming an eight-pointed star pattern, with the horizon, equator, and parallels drawn in. The lettering is small and densely crowded; several labels could not be resolved with confidence

- Circumference: A (upper left), D (upper right, marked with a star), C (lower right), B (lower left, marked with a star); lowercase points a, b, f across the top
- Outside the circle: z (upper right), Y (right), H (left), K (lower left), and L, g (lower right)
- Interior: E (center), O and N (upper interior), M (right of center), R, P, e (left stack), S, Q (lower left interior), T, V, X (lower interior)
- [Margin: a 2 of the sixth (Euclid VI.2)]

Diagram. The lower of the two large circular figures on this page, essentially a repetition of the same construction. Lettering densely crowded; several labels could not be resolved

- Circumference: A (upper left), D (upper right, starred), C (lower right), B (lower center, starred); lowercase points a, b at the top and d, f at right
- Outside the circle: Z (left), and f, Y, q at the bottom
- Interior: E (center); F, H (left), R, M (upper interior), P, e (left stack), S, X, N (lower interior), T, V, I, L (right)

Conversely, from a given semidiurnal arc, we shall hunt out the corresponding point of the ecliptic in this way. Let it be arranged:

As the tangent of the pole's altitude is to the whole sine — so is the sine of the arc by which the semidiurnal arc differs from the quadrant, to another;

for the number produced will be the tangent of the declination of the point of the ecliptic which is sought. Let the declination therefore be extracted from the table of sines, and from this the corresponding point of the ecliptic, as was taught in proposition 14.

Or, if in the figure of the preceding proposition one conceives the spherical triangle ABC — formed from the arc AB of the equator, between the horizon and the circle of declination through the intersection of the parallel which the sought point of the ecliptic describes; and from the arc AC of the horizon, between the equator and the same circle of declination (which arc indeed measures the rising amplitude); and from the arc CB of declination. In this triangle [Margin: a 15 of the first of Theodosius (Theodosius I.15)] the angle B is right, because the circle CB is drawn through the poles of the equator; the angle A is that of the altitude of the equator above the horizon — that is, the complement of the pole's altitude; and finally the arc AB is known, by which the given semidiurnal arc differs from the quadrant. Then, by the first method of problem 11 of spherical triangles, let it be arranged:

As the whole sine is to the sine of the arc AB, by which the semidiurnal arc differs from the quadrant — so is the tangent of the angle A, of the complement of the pole's altitude, to another;

the tangent of the declination of the sought point will again be found, etc. Which was to be done.

Diagram. A small figure of two intersecting great-circle arcs forming the spherical triangle ABC of this proposition.

- A (left): vertex A
- C (right of center): vertex C
- B (lower center): vertex B, the right angle at the foot of the declination arc

PROPOSITIO XVII.

To find the rising or setting amplitude of any point of the ecliptic for any latitude of place; and conversely, from a given rising or setting amplitude, to search out the corresponding point of the ecliptic.

In the same figure of proposition 14, erect from M a perpendicular Mb to HI; and the arc ab will be the rising or setting amplitude of the parallel FG. For since both the horizon (drawn through the line HI) and the parallel (through the line FG) are perpendicular to the meridian, [Margin: a 19 of the eleventh (Euclid XI.19)] their common section will also be perpendicular to the same; and therefore, by definition 3 of book 11 of Euclid, it will be perpendicular to the line HI, lying in the plane of the meridian. If then the circle ABCD is conceived to be the horizon set perpendicular to the meridian, Mb will be the common section of the horizon and the parallel; and by the same reasoning Ea will be the common section of the horizon and the equator. Consequently the arc ab will measure the rising amplitude, and its sine will be EM. Now since in the right-angled triangle EMN the side EN is known — being the sine of the declination — and the angle MEN, of the pole's altitude, is also known, [Margin: b Problem 3 of rectilinear triangles] let it be arranged:

As the whole sine is to EN, the sine of the declination — so is the secant of the angle MEN, of the pole's altitude, to another;

there will be produced the sine EM, of the sought amplitude.

Conversely, if the amplitude ab is supposed known, then in the same triangle EMN the sine EM of the amplitude ab will also be known; and since the angle EMN, of the complement of the pole's altitude, is known, [Margin: a Problem 2 of rectilinear triangles] let it be arranged:

As the whole sine is to EM, the sine of the known amplitude — so is the sine of the angle EMN, of the complement of the pole's altitude, to another;

for the number produced will be the sine EN, of the declination of the point, which is sought. From the declination thus found, [Margin: b 15 of this (i.e. of the present work)] the sought point of the ecliptic will be recognized. We have therefore found the rising amplitude, etc. Which was to be done.

Diagram. The upper of two large circular figures on this page — again the reused construction of the "figure of proposition 14": a circle (the meridian) crossed by

many chords and diameters in an eight-pointed star, with horizon, equator, and parallels. Lettering densely crowded; several labels could not be resolved

- Circumference: A (upper left), D (upper right, starred), C (lower right), B (lower left, starred); lowercase points a, b, f across the top
- Outside the circle: z (upper right), Y (right), H (left), K (lower left), L and g (lower right)
- Interior: E (center), O and N (upper interior), M (right of center), R, P, e (left stack), S, Q (lower interior), T, V, X (lower interior), I (right)
- [Margin: a 2 of the sixth (Euclid VI.2)]
- [Margin: b Problem 3 of rectilinear triangles]

Diagram. The lower of the two large circular figures, a repetition of the same construction. Lettering densely crowded; several labels could not be resolved

- Circumference: A (upper left), D (upper right, starred), C (lower right), B (lower center, starred); lowercase points a, b at top, d at right
- Outside the circle: Z (left); f, Y, q at the bottom
- Interior: E (center), F and H (left), R, M (upper interior), P (left stack), S, X, N (lower interior), T, V, I, L (right)
- [Margin: a Problem 2 of rectilinear triangles]
- [Margin: b 15 of this]

SCHOLIVM.

It has seemed good to demonstrate the three preceding propositions in this place too, although we have delivered them elsewhere as well; because in what follows we shall need the latter parts of them.

PROPOSITIO XVIII.

To demonstrate the dissimilarity between the increase and decrease of the days and nights, and of the twilights.

[Margin: The variety among the days, the nights, and the twilights, in their increase and decrease.] The twilights keep a very dissimilar rule from that of the days and nights in their increase and decrease. For the days decrease continuously from the beginning of Cancer to the beginning of Capricorn, in the northern region; and from the beginning of Capricorn up to the first degree of Cancer they increase continuously. On the contrary, the nights increase without intermission from the beginning of Cancer to the beginning of Capricorn, and

from the first point of Capricorn to the beginning of Cancer they perpetually diminish. So it comes about that, when the Sun is in the beginning of Cancer, the longest day of the whole year and the shortest night occur; but when it occupies the beginning of Capricorn, the shortest day and the longest night occur, at whatever elevation of the arctic pole above the horizon. This can easily be demonstrated from propositions 19 and 20 of book 2 of [Theodosius].

But with the twilights the matter stands far otherwise. For although the twilight becomes longest at the beginning of Cancer in any northern latitude — as we said above [Margin: a corollary 9 of this] — it is nevertheless not made shortest at the first degree of Capricorn, but is everywhere greater than the one found at the equator. So the twilights, although they decrease as one proceeds from Cancer toward Libra, yet this diminution does not go on continuously all the way to Capricorn; rather, at a certain point of the ecliptic between Libra and Capricorn the shortest twilight of all will occur, and thereafter from this point they will again increase, and one twilight will be produced equal to the one that occurs at the equator, before the Sun reaches Capricorn. And if the Sun were to run out beyond the winter tropic, the twilights would still keep growing greater, even though the days were still decreasing and the nights increasing. And so, although the days from Capricorn toward Aries always become greater, the twilights nevertheless diminish up to a certain point — the one between Libra and Capricorn to which it corresponds — at which the twilight becomes shortest; and before the Sun reaches that point between Capricorn and Aries, another twilight will be produced equal to it, namely equal to the one which occurs at the equator. And this, unless a geometric demonstration were at hand, could scarcely seem credible.

Let ABCD then be the meridian, about the center E; the oblique horizon FG, and its parallel of the twilights KL, such that the arcs FK and GL are 18 degrees. Let BD be the axis of the world; AC the equator; the parallel through O — the intersection of the axis with KL — passing along MN; the parallel through a, the midpoint of the segment EO of the axis, running along QR; and, below MN, any parallel VX; and finally, between MN and QR, some other parallel ST. Now gh is farther from the center than ZO, and ZO farther than ef, as is manifest; and ef too is farther than bd, since each extremity of this recedes equally from the center a. (For since EO is bisected at a, [Margin: b 2 of the sixth (Euclid VI.2)] IO too will be bisected at d. [Margin: c 34 of the first (Euclid I.34)] Since then Id is equal to Eb, dO too will be equal to the same Eb. Since therefore the sides Eb, Ea are equal to the sides Od, Oa, and contain equal alternate angles, [Margin: d 29 of the first (Euclid I.29)] [Margin: e 4 of the first (Euclid I.4)] the bases ab and ad will be equal.) [Margin: f

2 of this] Therefore the arc of the parallel VX, corresponding to the line gh, will be greater than the twilight of the line ZO, and this greater than that of the line ef, and this greater than that of the line bd. And these things are certain and beyond doubt in any region, if the same construction be made — so that it is entirely true that the twilights decrease continuously from the parallel VX up to the parallel QR. But it does not on that account follow from the figure that the twilight bd is the shortest of all. [Margin: g 2 of this] For the arc of the twilight of the line bd is greater than the arc of a parallel greater than QR, if that arc corresponds to a line equal to bd itself, and its distance from its own center is equal to the distance of the line bd from its center a. Yet nothing prevents it from being even somewhat greater than the arc of the twilight of that same greater parallel — one which corresponds to a line equal to bd that recedes a little farther from the center than bd does; [Margin: h corollary 1 of this] and consequently the arc of the twilight of that greater parallel is a little greater than the arc of the same parallel which corresponds to a line equal to bd itself, whose distance from its own center is equal to the distance of the line bd from center a. So the twilight of that greater parallel holds a middle place between the twilight of the line bd and that arc which, in the greater parallel, corresponds to a line equal to bd itself and has a distance from its own center equal to the distance of bd from its own. And so the twilights can still decrease imperceptibly from the parallel QR toward Aries.

[Margin: i 2 of this] It is also evident from the figure set forth that the twilight of the parallel MN is greater than the twilight of the equator AC, because the equal lines OZ and EI are equally distant from their centers, and the parallel is smaller than the equator. A fortiori the twilights of the parallels below MN will be greater than the twilight of the equator, since these are greater than the twilight of MN, as has been shown. But it is not evident from the figure up to which parallel greater than MN — namely one lying between MN and AC — the twilight becomes equal to the twilight of the equator, nor in which parallel between MN and AC the twilight becomes equal to the equator's twilight. It has, however, been ascertained by calculation (as will also appear from the table of twilights placed below) that the twilight of the parallel marked with a special symbol [glyph uncertain — resembles &] is everywhere greater than the twilight of the equator, although this marked parallel lies, at some elevation of the pole, between MN and AC; at another does not differ from the parallel MN; at another is placed below MN; nay, at another coincides with the parallel QR. But these things will become clearer from the following three propositions. We have therefore demonstrated the dissimilarity between the increase, etc. Which was to be done.

Diagram. A large circular figure for the twilight construction : the circle ABCD is the meridian about center E, crossed by the oblique horizon FG, the twilight-parallel KL, the axis BD, the equator AC, and several parallels (MN, QR, VX, ST), with chords ZO, ef, bd, gh and lowercase interior points. The lettering is very small and densely overlaid; many interior labels could not be resolved with confidence

- Circumference: A (top), D (upper right, starred), C (lower right), B (lower left, starred), G (right)
- Left side: F, K; and M, S, Q (upper left, outside/near circumference), V (left)
- Right side: I, L (lower right)
- Interior / center: E (center), P and b (near center-left); lowercase cluster a, c, d, e, f, g and small numerals 4, 5, 6 near the lower center
- Below the circle: X, N, T, R
- [Margin: b 2 of the sixth]
- [Margin: c 34 of the first]
- [Margin: d 29 of the first]
- [Margin: e 4 of the first]
- [Margin: f 2 of this]
- [Margin: g 2 of this]
- [Margin: h corollary 1 of this]
- [Margin: i 2 of this]

PROPOSITIO XIX.

To investigate, in any northern region, the parallel in which it can be demonstrated that a twilight occurs greater than the one produced at the equator — that is, to find the declination of the parallel MN, drawn in the figure of the preceding proposition through O, the intersection of the axis with KL, parallel to the horizon. Likewise to search out whether the marked parallel [⌘?] lies between MN and the point a, or rather between a and the equator; or whether it is the same as MN, or as QR; or finally whether it is placed below MN.

In the figure of the preceding proposition, let PO be drawn perpendicular to FG. If then, for a given latitude, [we wish] to investigate...

Diagram. A large circle (the meridian) with its center marked E. Through the center run several straight lines: a horizontal diameter from F (left) to G (right); a lower horizontal chord from K (left) to L (right), which is the parallel KL; an oblique axis running from B (lower-left, marked with a star) up to D (upper-right,

marked with a star), the axis DB; and a series of oblique parallels descending from upper-left to lower-right (MN, QR, ZO among them). **A** stands at the top of the circle; **C** at the lower-right of the circumference. A cluster of intersection points (lowercase **a, c, d, e, f, g, h, o**) is crowded near the center-bottom where the oblique lines cross the axis and the parallels, and a further row of labels (**X, N, T, R**) runs along the lower circumference. This is the figure for the Capricorn-twilight construction of the preceding proposition.

- A (top of circle): apex
- D (upper right, starred): upper end of the axis DB
- B (lower left, starred): lower end of the axis DB
- E (center): center of the meridian
- F (left) — G (right): horizontal diameter
- K (left) — L (right): parallel KL
- S, Q [?] (upper left): points on the parallel through M
- M, V (far left): M is the upper end of parallel MN
- Z, P, b (left of center): points along the oblique lines
- I (center-right, lower): intersection point
- C (lower right): point on circumference
- a, c, d, e, f, g, h, o (center-bottom cluster) [several labels [?] — crowded]
- X, N, T, R (lower circumference)

one may wish to investigate how far the parallel MN — drawn through O, the intersection of the axis with the parallel of the horizon — declines; that is, through what point of the ecliptic it passes, provided the declination found does not exceed $23^{\circ}30'$; and consequently whether the tropic of Capricorn [φ] passes through O, or is situated between O and E, or between O and B. We shall proceed as follows. Since in the right triangle EOP, given EO as the whole sine [sinus totus], the side OP is the sine of the angle OEP, the given altitude of the pole (as we said in the treatment of sines), if we set up the proportion:

As OP, the sine of the angle OEP (the altitude of the pole), is to EO, the whole sine, so OP, the sine of 18° (in the parts of the whole sine of the greatest circle), is to another quantity,

then EO will be found, in the same parts. And because EO is the sine of the declination of parallel MN, the declination of the arc AM will be known from the table of sines. If this declination is equal to the greatest declination, $23^{\circ}30'$, then MN will be the parallel of Capricorn, and its twilight, belonging to the line ZO, ^a will demonstrably be greater than the twilight of the Equator. But if the

declination found is greater than the greatest declination, the parallel MN will lie entirely outside the Sun's path, and the tropic of Capricorn will cut the axis between E and O. Finally, if it is found smaller, the same tropic will cut the axis between O and B, ^b and again the twilight will demonstrably be greater than the twilight of the Equator — indeed, greater even than the twilight of parallel MN. But if the sine EO, once found, is bisected, we obtain the sine E a, the sine of the declination of parallel QR, down to which the twilights, starting from parallel MN, demonstrably decrease.

[Margin: ^a Prop. 2 of this (work).] [Margin: ^b Prop. 2 of this (work).]

At the altitude of the pole 42° — where Rome roughly lies — the sine EO is found to be 46182, and its half, E a, 23091. That sine [46182] is greater than 39875, the sine of the declination of Capricorn, but this half [23091] is smaller. Therefore the tropic of Capricorn is situated between MN and QR. To the sine E a corresponds a declination of $13^\circ 21'$, which agrees with $24^\circ 37'$ of Aquarius [♒], where the Sun stands on 14 February. So at Rome the twilights decrease from 22 December until 14 February; yet on that day the twilight is 1 hour 41 minutes, which — since this [the Equator's] contains only 1 hour 38 minutes — is still greater than the twilight of the Equator. They will therefore still keep decreasing, advancing from 14 February toward 21 March; and in that interval, on one day, the twilight will become equal to the twilight of the Equator, then the shortest, and thereafter they will increase again. When this happens — which we shall demonstrate a little below — the parallel of Capricorn passes between a and o, since its declination is greater than the declination of parallel QR, seeing that the latter is only $13^\circ 21'$.

But if we wish to know at what altitude of the pole the parallel of Capricorn passes through O, or even through a, or whether it cuts the axis between O and B, or finally between E and a, we shall accomplish it by this method. Since, if the tropic of Capricorn is to pass through O, its declination-sine EO must be 39875, then if we set up the proportion:

As EO, 39875, is to OP, the sine of 18° — that is, to 30902 — so the whole sine, EO, is to another,

OP, the sine of the angle OEP (the altitude of the pole), will be found to be 77497; and so it is at the altitude of the pole $50^\circ 48'$. Therefore MN, the parallel [of Capricorn], ^a and its twilight, will accordingly be greater than the twilight of the Equator. A fortiori, at any elevation of the pole greater than $50^\circ 48'$, the twilight at the beginning of Capricorn will be greater than the twilight of the Equator; because then the tropic of Capricorn falls below its intersection with the parallel

KL, since the portion of the axis between E and the parallel KL is smaller at that altitude than at the altitude $50^{\circ}48'$ — as is clear if one imagines the axis DB raised, and the Equator lowered together with it along the parallels, from the right side toward the left. Or, if another axis be drawn between D and the zenith [vertex of the head]. For hence it comes about that that portion of the axis is then smaller than the sine of the greatest declination, seeing that EO is equal to the sine of the greatest declination at the altitude of the pole $50^{\circ}48'$.

[Margin: ^a Prop. 2 of this (work).]

AGAIN: since, when the tropic of Capricorn passes through a, its declination-sine E a is necessarily 39875, and therefore its double, 79750, indicates the line EO, if we set up the proportion once more:

As EO, 79750, is to OP, 30902, so the whole sine, EO, is to another, there will be produced OP, the sine of the altitude of the pole, 38748; and its corresponding altitude of the pole will accordingly be $22^{\circ}48'$, at which the tropic of Capricorn will pass through the point a, and will make a twilight greater than the twilight of the Equator, since that [twilight of Capricorn] contains 1 hour 24 minutes, but this one only 1 hour 18 minutes.

MOREOVER: since, when the tropic of Capricorn cuts the axis between E and a, the line E a must be greater than the sine of the greatest declination, 39875, and therefore EO greater than 79750, if we set up the proportion:

As EO, 80967 (any other number greater than 79750 could also have been taken), is to OP, 30902, so the whole sine is to another,

there will be generated the sine of the altitude of the pole at which the tropic of Capricorn passes, 38166; to which corresponds an altitude of the pole of $22^{\circ}26'$; and the twilight of Capricorn will be 1 hour 24 minutes, greater than the twilight of the Equator, since this contains 1 hour 18 minutes — just as at the altitude of the pole $22^{\circ}48'$; which indeed comes about on account of the small difference between the pole-altitudes.

FINALLY: when EO is smaller than 39875, the tropic of Capricorn will cut the axis between O and B. Therefore if we set up the proportion:

As EO, 38796 (any other number smaller than 39875 could also be taken), is to OP, 30902, so the whole sine is to another,

there will be produced OP, the sine of the altitude of the pole at which the tropic of Capricorn cuts the axis between O and B — as, in the given example, it will be

found that OP, the sine 79673, to which is due the altitude of the pole $52^{\circ}49'$, is where the tropic of Capricorn falls below the point O; and it will accordingly make the twilight in Capricorn greater than the twilight of the Equator. If EO had been taken as 39850, the sine OP of the altitude of the pole would have been found to be 77545, to which in the table of sines corresponds the altitude of the pole $50^{\circ}51'$, where again the tropic of Capricorn falls below O, and therefore will make the twilight greater than the Equator's. And the same will happen at any other elevation of the pole greater than $50^{\circ}51'$, as we also said above.

You see, then, the remarkable variety among the days and the twilights in their increase and decrease. Yet the calculation always shows, at every elevation of the pole, the twilight in Capricorn greater than the twilight at the Equator — even where the parallel of Capricorn cuts EO between E and a. For even at the altitude of the pole 1° , the parallel of Capricorn lies between E and a — because the parallel through a, as it advances, is entirely outside the sphere — and yet the twilight of Capricorn is greater than the twilight of the Equator. For at that elevation of the pole the line EO is found to be 1770888, and accordingly its half, E a, 885444; of which the former [1770888] exceeds the semidiameter of the sphere — that is, 100000 — by this number 16 times and more, while the latter exceeds it 7 times and more: so that the point O recedes from the center of the sphere by 17 semidiameters and more, and the point a by 8 semidiameters and more. From which it follows — since the segment of the axis between E and the parallel of Capricorn is 39875, namely the sine of the greatest declination — that the parallel of Capricorn is much closer to the center E than the point a is; and yet the twilight of Capricorn is 1 hour 19 minutes, whereas the twilight of the Equator is only 1 hour 12 minutes, less than it.

And so it is not always true, as Pedro Nunes [Petrus Nonius] states, that the twilights decrease from the tropic of Capricorn up to the point a — since in some regions the tropic of Capricorn lies between a and E. It is nevertheless always true that the twilights decrease from any parallel between a and B, up to the parallel described through a. For at the altitude of the pole 80° , the line EO is 31378, smaller than the sine of the greatest declination; therefore the tropic of Capricorn will be there, between O and B, ^a and its twilight accordingly greater than the twilight of the Equator. But at the altitude of the pole 20° , the line EO is 90351, and therefore its half, E a, is 45176, greater than the sine of the greatest declination; wherefore at that elevation too the parallel of Capricorn falls between E and a, just as at the elevation of the pole 1° , and likewise for the others. We have therefore investigated the parallel in every northern region, and so forth — which was to be done.

[Margin: ^a Prop. 2 of this (work).]

PROPOSITIO XX.

The twilights, from the Equator toward Capricorn [^γ], decrease up to a certain parallel; then they increase again up to another parallel, in which the twilight becomes equal to the twilight of the Equator; and thereafter they always become greater.

What was demonstrated in the two preceding propositions we shall confirm in this one by another route — namely, through spherical triangles, and indeed more particularly — in this way. Let ABF be the Equator; let CAD be the oblique horizon of any region; and at A let the equinoctial sunrise be set. Let AB be the arc of twilight in the Equator, tending downward from A below the horizon, such that the Sun, standing in the Equator, begins twilight when it has reached the point B, and ends it as soon as it has risen at A. And because the oblique horizon CAD touches the parallel of the ever-apparent stars — the greatest such parallel ^b — let another great circle EBD be described through B, below the horizon CAD, touching the same parallel and cutting the circle CAD in D, toward the north. Then the angles DAF, DBF will be acute — namely those which the Equator, below the oblique horizon CD, makes with the horizon itself and with the circle EBD toward the north — and they will be equal, ^c because the circles CAD, EBD are equally inclined to the Equator, seeing that they touch the same parallel. ^d Therefore the arcs AD, BD together are equal to a semicircle.

[Margin: ^b Theodosius, *Spherics* II.15.] [Margin: ^c Scholium 21, *Spherics* II (Theodosius).] [Margin: ^d Spherical triangles [Triangula sphaerica], 15.]

Diagram. A wide, shallow U-shaped (bowl-shaped) figure. The upper rim is the oblique horizon **CAD**, running from **C** at the upper left down and around to **D** at the upper right; a second great circle **EBD** runs beneath it, meeting it at D. Along the upper region a row of points is marked left-to-right — **O, H, N, M, A, L** — where **A** is the equinoctial rise. Along the bottom of the bowl a row of points runs left-to-right — **E, Q, G, I, F, B** — with **P** set just above, between G and I. From the lower points, perpendicular arcs rise to the upper points (EO, GN, IM, BL), and the parallels GH, IK, EC are drawn.

- C (upper left): end of the oblique horizon CAD
- D (upper right): meeting point of horizon CAD and circle EBD
- O, H, N, M, A, L (upper rim, left-to-right): A = equinoctial rise; H, N, M, L are the upper ends of the perpendicular/parallel arcs

- E, Q, G, I, F, B (bowl floor, left-to-right): foot-points of the arcs; F is toward the south
- P (just above floor, center): the point the Sun reaches on parallel IK
- Perpendicular arcs: EO, GN, IM, BL; parallels GH, IK, EC

But because arc AD is greater than arc BD — since the angle DBA is obtuse and the angle DAB acute (for the angle DAB, which the horizon makes with the Equator toward the north below the horizon, is the angle of the altitude of the Equator above the horizon, and hence acute; whereas DBA is the supplement, to two right angles, of the acute angle DBF that measures the altitude of the Equator, and is therefore obtuse) — and accordingly AD is greater than a quadrant and BD smaller: ^e let an arc DG be taken equal to arc DA, so that the two arcs DB, DG together are also equal to a semicircle. And through G let the parallel GH be described. Also, taking a point I between G and B, and another point F toward the south, let two more parallels IK, EC be described through I and through E.

[Margin: ^e Spherical triangles [Triangula sphaerica], 11.]

I say, then, that the twilight of parallel GH is equal to the twilight of the Equator ABF; that the twilight of parallel IK, and of any other between AB and GH, is smaller; but that the twilight of parallel EC, and of any other beyond GH, is greater — so that the twilights, from Libra [$\underline{\alpha}$] toward Capricorn, decrease up to a certain parallel, then from this parallel increase again up to the parallel GH, and thereafter, from this parallel, always become greater. For let there be drawn, from the lower horizon DE to the upper one DC — in which the twilights end — the perpendicular arcs BL, IM, GN, EO, namely arcs proceeding through the points B, I, G, E and through the poles of the circle CD. ^f For these are perpendicular to the circle CD. ^g And since, as the sine of arc GD is to the sine of arc GN, so the sine of arc BD is to the sine of arc BL, it will also be, by alternation, that as the sine of arc GD is to the sine of arc BD, so the sine of arc GN is to the sine of arc BL. But the ratio of the sine of arc GD to the sine of arc BD is a ratio of equality, because these arcs, completing a semicircle, have the same sine. Therefore the sines of the arcs GN, BL will also have a ratio of equality, and hence the arcs GN, BL themselves will be equal, since each of them is smaller than a quadrant. ^h For the arcs NG, LB, [extended] up to the Nadir — that is, to the pole of the horizon — are quadrants, meeting at that pole; and consequently neither of the points G, B is that pole, since at neither of them do the arcs NG, LB meet. Moreover, it is established from the hypothesis that the arc LB is smaller than a quadrant — namely 18°. Since therefore BL is the arc of occultation of the Sun at the beginning...

[Margin: ^f *Spherics* I.15 (Theodosius).] [Margin: ^g Spherical triangles [Triangula sphaerica], 40.] [Margin: ^h Corollary 16, *Spherics* I (Theodosius).]

of the twilight of the Equator, GL too will be the arc of occultation of the Sun at the beginning of the twilight of parallel GH, and the twilight-arc will be GH; for the Sun traverses it when, below the horizon CD, it is occulted through the arc of occultation GN. ^c Now the arcs BA, GH are similar, since they lie between non-concurrent semicircles of the horizons. Therefore as many degrees are contained in GH as in BA, and accordingly the twilight of parallel GH is equal to the twilight of the Equator BA.

[Margin: ^c *Spherics* II.13 (Theodosius).]

Diagram. The same bowl-shaped figure as on the preceding page, repeated across the page-turn: the oblique horizon **CAD** (from **C** upper left to **D** upper right), the great circle **EBD** beneath it, the upper row of points **O, H, N, M, A, L**, the bowl-floor row **E, Q, G, I, F, B** with **P** above, and the perpendicular arcs EO, GN, IM, BL with the parallels GH, IK, EC. See fig-p281-1 for the label list; this is the identical construction reprinted for the continuation of the proof.

NEXT: ^f because by the same reasoning, as the sine of arc ID is to the sine of arc BD, so the sine of arc IM is to the sine of arc BL. But the sine of arc ID is greater than the sine of arc BD, since arc ID is set between BD and GD, which complete a semicircle; therefore the sine of arc IM will also be greater than the sine of arc BL, and consequently arc IM will be greater than arc BL. Since therefore BL is the arc of occultation of the Sun at the beginning of the morning twilight [of the Equator], IM will be greater than the arc of occultation of the Sun at the beginning of the twilight of parallel IK. Therefore the Sun at point I will be farther below the horizon CD than at point B; and accordingly the Sun at I does not yet begin the twilight, but only when it reaches, for example, the point P. ^g Since, then, the arcs BA, IK are similar, fewer degrees will be comprehended in the twilight-arc PR than in the twilight-arc BA; and therefore the twilight of parallel IK will be smaller than the twilight of the Equator BA.

[Margin: ^f Spherical triangles [Triangula sphaerica], 40.] [Margin: ^g *Spherics* II.13 (Theodosius).]

LASTLY: ^h because again, as the sine of arc ED is to the sine of arc BD, so the sine of arc EO is to the sine of arc BL. But the sine of arc ED is smaller than the sine of arc BD, because it is smaller than the sine of arc GD, which is the same as the sine of arc BD. Therefore the sine of arc EO will also be smaller than the sine of arc BL, and hence arc EO will be smaller than arc BL. Since therefore BL is the arc of

occultation of the Sun at the beginning of the morning twilight of the Equator, EO will be smaller than the arc of occultation of the Sun at the beginning of the twilight of parallel EC; and accordingly the Sun at E will be less distant below the horizon CD than at B. Wherefore, with the Sun placed at E, the twilight will already have begun — namely when it is found at some point, for example Q. ⁱ Therefore, since the arcs BA, EC are similar, more degrees will be found in arc QC than in BA; and hence the twilight of parallel EC will be greater than the twilight of the Equator BA — which is what was proposed. Therefore the twilights, [decreasing] from the Equator, and so forth: which was to be demonstrated.

[Margin: ^h Spherical triangles [Triangula sphaerica], 40.] [Margin: ⁱ *Spherics* II.13 (Theodosius).]

COROLLARIVM.

Since, therefore, the twilight of parallel GH is equal to the twilight of the Equator AB, and greater than the twilight of any parallel IK between the Equator AB and the parallel GH, but smaller than the twilight of any parallel EC beyond GH, it follows that the least twilight is produced at some parallel between the Equator and the parallel GH. But which parallel that is, we shall inquire in Proposition 22; while in the following one, namely Proposition 21, we shall in fact examine the parallel GH, whose twilight we have shown to be equal to the twilight of the Equator.

SCHOLIVM.

This proposition can be demonstrated in another way, and perhaps more clearly, as follows. Let there be any oblique horizon DZ, the Equator AB, and the arc AB of the Equator's twilight, such that through B passes the parallel of twilights BG. Through B let another oblique horizon D a B L P G X be described — touching, that is, the greatest parallel of the ever-apparent stars, which the horizon DZ of the given region also touches — and cutting this horizon toward the northern parts at D, and the parallel of twilights BG at G. Let the parallel GH also be described through G; and on either side let other parallels PT, VZ, fE, LN be described, cutting the parallel of twilights BG at S, V, C, and O, so that the arcs of twilight are BA, ON, CE, ST, GH, VZ, inasmuch as they are intercepted between the horizon DZ and the parallel of twilights BG. ^k Since, then, the arcs BA, LN, fE, PT, GH, XZ are similar, because they lie between non-concurrent semicircles of the horizons DZ, DB, the twilight of parallel GH will be equal to the twilight of the Equator BA. But the twilight VZ will be greater than the twilight GH, or BA, since the arc VZ is greater than the arc XZ, which is similar to the arcs GH, BA. And finally the twilights ST, CE, ON will be smaller than the twilight of the Equator BA, since these arcs are smaller than the

arcs PT , fE , LN , which are similar to the arc BA . From which it is gathered that the twilights, from the Equator AB toward Capricorn [γ], decrease up to a certain parallel, then increase again up to the parallel GH — in which the twilight becomes equal to the twilight of the Equator — and thereafter always become greater, up to Capricorn, whose parallel is farther from the Equator than the parallel GH , as is evident from the calculation of the Sines.

[Margin: ^k *Spherics* II.13 (Theodosius).]

Diagram. A geometric figure enclosed in a rectangular ruled frame (the Scholium's alternative construction). Within the frame, a shallow bowl-shaped curve (the parallel of twilights BG) sweeps across, cut by a family of oblique lines (the horizons and parallels). Along the upper-left edge a row of points is marked left-to-right — Z , H , T , F , E , N , A , I — continuing to K , then D at the upper right with a small a beside it. Along the lower part run further points — Y , X , R and (toward the right) M , V , G , S , Q , O , L , B . The oblique lines are the horizon DZ , the oblique horizon $DABLPGX$, and the parallels PT , VZ , fE , LN cutting the twilight-parallel BG .

- Z (upper left): end of horizon DZ
- H , T , F , E , N , A , I (upper edge, left-to-right)
- K (upper, right of center)
- D (upper right), with a beside it
- Y , X , R (lower left)
- M , V , G , S , Q , O , L , B (lower, left-to-right toward the right) [several labels [?] — dense, small]

[Continuation of the preceding proposition — conclusion of Prop. XX]

And arc DBG is necessarily equal to arc DA , as was assumed in the proposition. For when arcs BK and GF are drawn perpendicular to DZ , [Margin (l): 40 of the Spherical Triangles] the sine of arc GD will be to the sine of arc BD as the sine of arc GF is to the sine of arc BK . Since, then, these two latter sines are equal — because their arcs are equal [Margin (m): Theodosius, *Spherics* II.13] — the two former sines will likewise be equal, and consequently the two arcs GD and BD will make up a semicircle. Therefore, since the two arcs DA and DB also make up a semicircle, as was shown in the proposition, arcs GD and AD will be equal.

PROPOSITIO XXI.

To find the point of the Ecliptic at which the Sun produces a twilight equal to the twilight of the Equator, for a given latitude of the place.

Let AB be, as in the figure above, the arc of the Equator that bounds the twilight, together with the two horizons AD and BD, which are similarly inclined to the Equator; and let arc GH of the parallel be the one that contains a twilight equal to the twilight of the Equator. Since arc AD is greater than a quadrant and BD less, let DR and DS be quadrants. Point S will fall between B and G — because arc DG is equal to DA itself, and therefore also greater than a quadrant — while point R will fall short of A. So, when the arc RS of a great circle has been described through points R and S, it will cut arc AB at some point, say T. Now since DR and DS are quadrants, [Margin (a): 25 of the Spherical Triangles] angles R and S will be right; [Margin (b): 6 of the Spherical Triangles] moreover the vertical angles ATR and BTS are equal, and so too are RAT and SBT. (For since angles DAB and DBF, being similar elevations of the Equator above the horizons, are equal, [Margin (c): 6 of the Spherical Triangles] and angle DBF equals its vertical angle TBS, angles RAT and SBT will also be equal.) Therefore arc RT will be equal to arc ST, arc AT to arc BT, and arc AR to arc BS. [Margin (d): 19 of the Spherical Triangles] And since the arc of the twilight AB is known, its half AT will also be known. This same twilight we shall likewise determine from the present figure. [Margin (e): 14 of this book]

When arc BL — the Sun's occultation — has been drawn perpendicular to AD at the start of the equatorial twilight, [proceed thus:]

As the whole sine (radius) is to the secant of the complement of angle BAL (the altitude of the Equator), so the sine of arc BL (the Sun's occultation) is to a fourth term.

For the product obtained will give the sine of the base AB, which measures the twilight of the Equator, and hence arc AB, the twilight, will become known.

[Margin (f): Problem 14 of the Spherical Triangles] Now let it be done in the right triangle ART:

As the whole sine of the right angle R is to the sine of the base AT (half the twilight of the Equator), so the sine of angle A (the altitude of the Equator) is to a fourth term.

The sine of arc RT will be produced, and hence from the table of sines arc RT will become known, and from this its double RS. [Margin (g): 40 of the Spherical Triangles] Again, let it be done:

As the sine of the arc RS just found is to the sine of the arc of occultation BL, so the whole sine of the quadrant SD is to a fourth term.

The sine of arc BD will be found, and hence arc BD will become known; and once this is subtracted [from the semicircle], arc GD is known; from which, if the arc BD just found is again taken away, the remaining arc BG [will be known]. But we shall find it more briefly — even if neither arc RS is known, nor arc BL drawn — in this way. After it has been proved that the arc of the equatorial twilight AB is bisected at T, [Margin (h): Problem 9 of the Spherical Triangles] let it be done:

As the whole sine is to the sine of the complement of angle SBT — that is, to the sine of the pole's altitude — so the tangent of arc BT (half the arc of the equatorial twilight) is to a fourth term.

For the tangent of arc BS will be generated; and once this is known from the table of tangents, and subtracted from the quadrant SD, arc GD will be left known; [Margin (i): Theodosius, *Spherics* II.11] and from it, if BD is taken away, the remaining arc BG will be known, as before. Since arc BG is equal to arc AH, the rising amplitude of parallel GH, this rising amplitude will be known: $16^{\circ} 34'$. [Margin (k): 17 of this book] From it the declination of parallel GH will be found, [Margin (l): 15 of this book] and from this declination the corresponding point of the Ecliptic — which is what was proposed.

Moreover, point S bisects arc BG, the rising amplitude. For since arcs DA and DG are equal, if the equal quadrants DR and DS are subtracted, the remainders AR and SG are equal. And since AR was shown equal to BS, BS and SG are also equal. Hence, if the arc BS once found is doubled, the rising amplitude BG is immediately produced.

At Rome, where the pole's altitude is 42° — where we write this — a twilight equal to the twilight of the Equator is found when the Sun is at about $2^{\circ} 20'$ of Scorpio [?], on 26 October, and likewise when the Sun is at about $27^{\circ} 40'$ of Aquarius, on 17 February. The twilight itself comprises 1 hour 38 minutes [?]. Therefore we have found the point of the Ecliptic at which the Sun [produces such a twilight], and so on — which was to be done.

Diagram. The upper figure ("the figure above") for Proposition XXI, showing the two horizons and the great-circle arc RS cutting the equatorial-twilight arc.

Readable labels, by position:

- H (top of the curve): endpoint of the parallel arc GH
- A (below H): intersection point on the horizon
- T (center): where arc RS cuts arc AB
- R, L (to the right of T): R the near-pole point, L the foot of arc BL
- G [?] (far left; printed as a small "q"-like glyph): endpoint of the parallel
- S (lower left): the far quadrant point
- E, P [?] (bottom left): points on the lower arc
- D (lower right): the pole / intersection of the two circles

[Two labels flagged uncertain (G/q, E/P).]

Diagram. The lower figure for the alternative method of Proposition XXI (the same construction reused in Prop. XXII). Readable labels, by position:

- C, G [?] (printed "q"), H, F, A, R, M, L: points along the upper curve, left to right
- D (right); a (small, just left of D): the pole and an adjacent point
- E, S, I, B (bottom, left to right): points on the lower arc
- T (center): the bisection point

[One label flagged uncertain (G/q).]

SCHOLIVM

And since, at every elevation of the pole, the rising amplitude AH of parallel GH is less than the rising amplitude of the tropic of Capricorn, and its declination likewise less — as is established from the doctrine of sines — we conclude that a twilight equal to the Equator's is produced between the Equator and the tropic of Capricorn; and consequently that the twilight of Capricorn is greater than the twilight of the Equator, and that the shortest twilight is produced between the Equator and parallel GH — since the twilight of parallel GH is greater than the twilights of the parallels lying between parallels GH and BA, as was shown in the aforesaid proposition. [Margin: The twilight in Capricorn is greater than the twilight at the Equator.]

PROPOSITIO XXII.

To seek the point of the Ecliptic at which the Sun produces the shortest twilight, and at the same time to determine the magnitude of that twilight.

Let AB be the arc of the Equator; CAD the oblique horizon; SG the parallel on which the shortest twilight is produced; and let arc AB of the Equator be the measure of the shortest twilight. [Margin (a): Theodosius, *Spherics* II.15] Through B let another circle be described — the greatest of the always-apparent circles, EBD — tangent to the parallel of the always-apparent [stars], which CAD also touches; [Margin (b): Theodosius, *Spherics* II.13] and let these two tangent circles cut one another at D, toward the north. Now since arcs AB and SG are similar, we posit that arc SG too will be the magnitude of the same twilight. And arc DS will necessarily be a quadrant, which I demonstrate thus. When two other parallels EC and IH have been described, beyond and short of parallel SG, let there be drawn from points B, I, S, E below the horizon, up to the horizon CAD itself, the arcs of the verticals BL, IM, SR, EF, perpendicular to that same horizon. SR will be the arc of the Sun's occultation below the horizon at the start of the shortest morning twilight — that is, 18°; while the other arcs BL, IM, EF will measure the Sun's distances below the horizon when it is at points B, I, E. Therefore, since SG is posited as the shortest twilight, arcs BA, IH, EC (being similar to arc SG) [Margin (c): Theodosius, *Spherics* II.13] measure the same least twilight; and they are consequently smaller than the arcs of those parallels' twilights, whose twilights will begin before the Sun reaches points B, I, E — being smaller than the arcs of the Sun's occultation below the horizon, that is, smaller than arc SR. [Margin (d): 40 of the Spherical Triangles] Now since the sine of arc SR is to the sine of arc IM as the sine of arc SD is to the sine of arc ID, and likewise the sine of arc SR is to the sine of arc EF as the sine of arc SD is to the sine of arc ED; and since the sine of arc SR is greater than both the sine of arc IM and the sine of arc EF (because these arcs were shown to be smaller, being less than a quadrant), the sine of arc SD will also be greater than both the sine of arc ID and the sine of arc ED. By the same reasoning the sine of arc SD will be shown greater than the sine of any parallel collateral to SG itself. Since, therefore, the sine of arc SD is greater than the sine of any other arc, arc SD is a quadrant. For only the sine of a quadrant is greater than the sine of any other arc, whether greater or less than a quadrant. [Margin (e): Theodosius, *Spherics* II.15 / II.1]

Since, then, arc DS is a quadrant, D will be the pole of the great circle SR, perpendicular to CD; [Margin (e): Theodosius, *Spherics* I.15] and therefore angle DSR will be right; [Margin (f): 25 of the Spherical Triangles] and consequently DR

too will be a quadrant, and, since DA has been shown greater than a quadrant, point R falls between D and A. Therefore, as in the preceding proposition, both arcs AT, TB and arcs RT, TS will be shown equal to one another. [Margin (g): Problem 14 of the Spherical Triangles — *The magnitude of the least twilight*] And if we then form:

As the whole sine is to the secant of the complement of angle RAT (the altitude of the Equator) — that is, to the secant of the pole's altitude — so the sine of arc RT (half the arc of the Sun's occultation RS) is to a fourth term.

the sine of the base AT will be produced. Therefore arc AT itself, and hence its double AB, will not be unknown; and thus the magnitude of the shortest twilight AB (or GS) will be known. [Margin (h): Problem 10 of the Spherical Triangles] And if again we form:

As the whole sine is to the tangent of the complement of angle RAT (the altitude of the Equator), so the tangent of arc RT (half of 18°) is to a fourth term.

the sine [?] of arc AR will be found, [Margin (i): Theodosius, *Spherics* II.13] and therefore arc AR, or its equal BS, will be known. And since arcs BS and AG are equal — AG being the rising amplitude — the rising amplitude itself will be known: $8^\circ 12'$. [Margin (k): 17 of this book] From it the declination of parallel SG [will be found], [Margin (l): 15 of this book] and from this the corresponding point of the Ecliptic. At Rome, where the pole's altitude is 42° , the shortest twilight occurs on about 13 October, at Libra $19^\circ 10'$, and likewise on 2 March, at Pisces $10^\circ 50'$. The twilight itself comprises 1 hour 37 minutes. Therefore we have sought the point of the Ecliptic at which the Sun produces the shortest twilight, and so on — which was to be done.

Diagram. The figure for Proposition XXII — the same construction as fig-p283-2, showing the oblique horizon, the always-apparent circle, and the parallels EC, SG, IH. Readable labels, by position:

- C, G [?] (printed "q"), H, F, A, R, M, L: points along the upper curve, left to right
- D (right); a (small, adjacent to D): the pole and an adjacent point
- E, S, I, B (bottom, left to right): points on the lower arc
- T (center): the bisection point

[One label flagged uncertain (G/q).]

SCHOLIUM

You see clearly that, in order to find the point of the Ecliptic of the shortest twilight, there is no need first to investigate the length of the twilight itself. Moreover, that the shortest twilight is produced on parallel GS — when arc DS in the second horizon of this proposition's figure is a quadrant — we shall demonstrate in this way as well. In the figure of the scholium of Proposition 20, let a horizon aCY be understood as described, tangent to the same greatest circle of the always-apparent [stars] and to the parallel of twilights BCG, at C; and let parallels be described, as there. It is now manifest that the twilight of parallel CE is the least of all: [Margin (m): Theodosius, *Spherics* II.13] since the arcs dA, MN, CE, RT, bH, YZ — lying between the non-concurrent semicircles of the horizons — are similar; and the arcs of the twilights BA, ON, ST, GH, VZ are greater than the arcs dA, MN, CE, RT, bH, YZ, so that the twilights of the parallels beyond and short of parallel CE are greater than the twilight of parallel CE.

Diagram. The large rectangular-framed figure (the reused figure of the scholium of Prop. 20), showing the two horizon-semicircles, the several parallels of twilight, and the verticals. Readable labels, by position:

- Along the upper curve (left to right): Z, H [?], T, F, E, N, A, I, K
- Upper right: D, a [?]
- Along the lower arc (left to right): Y, X, V, G, S, R, C, M, d, L, B
- Center: T; a small l [?] near R

[Multiple peripheral labels flagged uncertain (H, a, l, and several tightly-set lower-arc letters)]

That arc AC is a quadrant, I demonstrate thus. When through C the arc CI has been drawn through the pole of the circles DZ, BCG: since the circles aC and BC touch one another at C, and the circle CI is drawn through the poles of circle BC, [Margin (n): Theodosius, *Spherics* II.5] the same circle CI will also pass through the poles of circle aC. Since, then, it is also drawn through the poles of circle DZ, [Margin (o): Theodosius, *Spherics* II.9] it will cut the segments of circles DZ and aCY in half. [Margin (p): Theodosius, *Spherics* II.11] Therefore, since these segments are semicircles, aI and aC will be quadrants; and consequently the rising amplitude EA of parallel CE, on which the least twilight occurs, will be found from the doctrine of sines, as was stated in the proposition.

PROPOSITIO XXIII.

To observe the length of twilight on any given day in one's own horizon.

At the start of morning twilight or the end of evening twilight — and this beginning or end must be observed most exactly, under a clear sky — let the altitude of any star be taken, and its declination sought from Canon 3 of Book 3 of the *Astrolabe* and its scholium, together with its right ascension by Canon 4 of the same book and its scholium. Next, from Canon 8 and its scholium, let the distance of that same star from the meridian be determined, and through it the hour then instant — that is, the distance of the Sun from the meridian at the beginning or end of the twilight. For if from this distance the semidiurnal arc of that day is subtracted, the remainder will be the length of the twilight. Thus we have observed the length of twilight on any given day in one's own horizon — which was to be done.

SCHOLIUM

Pedro Nunes [Petrus Nonius] says that at Lisbon, where the pole's altitude comes to $38^{\circ} 40'$, he observed the evening twilight in the year 1541, on 1 October, with the Sun at Libra 18° , and found it to be 1 hour 22 minutes 24 seconds. But the same, computed by Proposition 14 of this book, comes to 1 hour 33 minutes. This discrepancy arises either because he assumed a greater or lesser declination of Regulus (the Heart of the Lion) [cor α] — the star he used in that observation — and a right ascension [different from] the one we establish in the scholium of Proposition 11 of Book 2 of the *Astrolabe*, and so did not find the true hour; or because it is exceedingly difficult to observe accurately the start of morning twilight or the end of evening twilight — for who could detect the first instant of the one, or the last of the other, without error? — from which error the twilight is necessarily not rightly found; or, finally, because perhaps the distance of 18° below the horizon, which we adopt in computing twilights, is too great, as he himself admits he found.

PROPOSITIO XXIV.

From a given length of twilight, to derive the distance of the Sun below the Horizon.

Since the occultation of the Sun below the horizon at the start of morning twilight, or at the end of evening twilight, is not fixed the same way by all writers — for we determine it at 18° , others set it somewhat greater, and others less; indeed, as Johannes de Sacrobosco [Ioan. de Sacrobosco] asserts, when he treats of those who dwell under the Arctic pole, some affirm it to contain 30° , which nevertheless is by no means credible — we shall here prescribe a method by which anyone, once he has observed the length of twilight with the utmost care (as we set out in the preceding proposition), may come to know this distance. Let the figure of Proposition 14 therefore be repeated, in which [Margin (a): Euclid VI.2 or VI.4] FN, the whole sine, stands thus [to ...]

(Continuing the proof of Proposition XXIV of the Digression on Twilights, begun on the preceding page.)

the whole sine is to EX — the versed sine of the arc FY, which is compounded of the semidiurnal arc FZ and the given twilight arc ZY (this arc will be known if the given length of the twilight is added to the semidiurnal arc, from which its versed sine is then to be obtained, as we taught in the [chapter] on sines) — so is FO, half the sum of the sine of the meridian altitude and the sine of the meridian depression, to Fe, the line compounded of the sine of the meridian altitude and the sine of the Sun's occultation. And so, if we set up the proportion:

As the whole sine — is to the versed sine of the arc compounded of the semidiurnal arc and the twilight arc: — so is half the sum, [taken] from the sine of the meridian altitude and of the meridian depression, — to another [line],

there will be produced the line Fe, compounded of the sine of the meridian altitude and the sine of the Sun's occultation. If from this the sine of the meridian altitude FP is subtracted, the remainder becomes the sine Pe, that of the occultation; and accordingly the arc belonging to it, drawn out from the table of sines, becomes known — namely, the arc of the Sun's occultation beneath the horizon at the beginning of the morning twilight, or the end of the evening twilight. Therefore, from the given length of the twilight, we have obtained the Sun's distance below the horizon — which was to be done.

Diagram. The upper of two nearly identical geometric figures for the twilight construction. A great circle (the meridian / vertical circle) is drawn with a horizontal diameter through the center E and several inscribed chords and perpendiculars representing the horizon, the equator, the parallel of the Sun's

declination, and the Sun's positions at the meridian, at rising, and at the end of twilight. The construction is crowded with interior points. Readable labels:

- F (on the circumference, upper)
- A (circumference, upper left)
- H, K (left, ends of horizontal chords)
- R, S (interior, left of center)
- O, P, e (interior, near center)
- N, M (interior, center)
- E (center)
- T, V, x (interior, lower center)
- Q (circumference, lower left)
- B (circumference, bottom, marked with a star)
- a, b, f (circumference, top, small letters)
- D (circumference, upper right, marked with a star)
- z, Y (circumference, right)
- C (circumference, lower right)
- [several further interior labels are too crowded to resolve]

Diagram. The lower figure, essentially a repetition of the same twilight construction with the same great circle, diameters, and inscribed chords. Readable labels:

- A, b, a (circumference, top)
- F (circumference, upper left)
- D (circumference, upper right, marked with a star); d (circumference, right)
- E (center)
- H, K (left)
- R, P, S (interior, left of center)
- M, N, X (interior, center)
- T, V (interior, lower center)
- I, L (right, ends of horizontal chords)
- C (circumference, lower right); q (circumference, bottom)
- Z (left); f, Y (circumference, lower left); B (circumference, bottom)
- [several further interior labels are too crowded to resolve]

SCHOLIUM

[Pedro Nunes], from the twilight found in the preceding proposition, determined that this distance amounts to only $16^{\circ} 2'$. Whether that is true, let others judge; for my part I dare neither affirm nor deny it. It will therefore be worthwhile for anyone — whenever he finds a free and unobstructed horizon and a clear sky — to observe carefully, from some elevated spot, the end of the evening twilight, so that he may learn its magnitude, as was set out in the preceding proposition; and then, from this proposition, to gather the Sun's distance below the horizon at the moment when either the morning twilight begins or the evening twilight ends.

Finally, so that this little book may be complete in every respect, I have thought it well to append the following table of the quantity of the twilights. These values were carefully and diligently computed, for various elevations of the pole, by Marcello Francolino [Marcello Francolini], Doctor of Both Laws [I.V. Doctore] and once my pupil in mathematics, in his work *On the Time of the Canonical Hours* [*De Tempore Horarum Canoniarum*], taking the arc of the Sun's occultation below the horizon to be 18° . In this table it plainly appears that the twilight [of any region — symbol $\mathfrak{h}$ (?)] is always greater than the twilight at the Equator. Moreover, wherever neither hours nor minutes are entered in the table, you should conclude that there is twilight all night long in that place.

You should not be surprised, however, that in the following table many consecutive twilights are equal — especially in the southern signs. For since one twilight differs so little from the next that the difference does not amount to one minute or two, no inequality of minutes can show up among those twilights. But if, besides minutes, account were also taken of seconds and thirds, then at last each twilight would always be found to differ from the next, as the geometrical demonstration requires.

THERE FOLLOWS THE TABLE OF TWILIGHTS.

The quantity of the twilights in the northern signs [Crepusculorum quantitas in signis Borealibus]

Format note. The full twilight table runs across three panels (pole-elevations 35° – 43° , 44° – 52° , and 53° – 61°). Each panel is entered from either side: the **left argument** is the Sun's ecliptic place (G = degree within the sign, S = sign), for the ascending signs Aries → Gemini; the **right argument** ("Altitudo": S = sign, G = degree) gives the complementary descending signs Virgo → Cancer that share the same declination. The numeric column-heads (35, 36, ..., 61) are the **elevations of**

the pole (latitudes) in degrees; the printed left-hand super-header over the argument column reads "Poli." Each latitude cell is the twilight duration in **hours : minutes** (the source prints H and M in separate sub-columns; here merged). A cell printed **0:0** marks perpetual (all-night) twilight — the source leaves neither hours nor minutes.

Panel 1 — Pole-elevations 35°–61°: latitudes 35°–43°

Sign (L)	G	35	36	37	38	39	40	41	42	43	Sign (R)	G
Aries ♈	0	1:2	1:30	1:3	1:3	1:3	1:3	1:3	1:3	1:4	Virgo ♍	3
		9		1	2	4	5	7	8	0		0
	3	1:2	1:30	1:3	1:3	1:3	1:3	1:3	1:3	1:4		2
		9		2	3	4	6	7	9	0		7
	6	1:2	1:32 [?]	1:3	1:3	1:3	1:3	1:3	1:4	1:4		2
		9		2	3	5	6	7	0	1		4
	9	1:3	1:31	1:3	1:3	1:3	1:3	1:3	1:4	1:4		2
		0		2	4	5	7	8	0	2		1
	1	1:3	1:32	1:3	1:3	1:3	1:3	1:3	1:4	1:4		1
		2	0	3	4	6	7	9	1	3		8
	1	1:3	1:32	1:3	1:3	1:3	1:3	1:4	1:4	1:4		1
		5	1	4	5	6	8	0	2	3		5
Taurus ♉	1	1:3	1:33	1:3	1:3	1:3	1:3	1:4	1:4	1:4	Leo ♌	1
		8	1	4	6	7	9	1	3	5		2
	2	1:3	1:33	1:3	1:3	1:3	1:4	1:4	1:4	1:4		9
		1	2	5	7	8	0	2	4	6		
	2	1:3	1:34	1:3	1:3	1:3	1:4	1:4	1:4	1:4		6
		4	3	6	8	9	1	3	5	7		
	2	1:3	1:35	1:3	1:3	1:4	1:4	1:4	1:4	1:4		3
		7	4	7	8	0	2	4	6	9		
	0	1:3	1:36	1:3	1:4	1:4	1:4	1:4	1:4	1:5		3
		5		8	0	1	3	5	7	0		0
	3	1:3	1:37	1:3	1:4	1:4	1:4	1:4	1:4	1:5		2
		6		9	0	2	4	7	9	1		7
	6	1:3	1:38	1:4	1:4	1:4	1:4	1:4	1:5	1:5		2
		7		0	1	4	6	8	0	3		4
	9	1:3	1:39	1:4	1:4	1:4	1:4	1:5	1:5	1:5		2
		8		1	3	5	8	0	2	5		1

Sign (L)	G	35	36	37	38	39	40	41	42	43	Sign (R)	G
Gemini ♊	1	1:3	1:40	1:4	1:4	1:4	1:4	1:5	1:5	1:5		1
	2	9		2	4	7	9	1	4	7		8
	1	1:4	1:41	1:4	1:4	1:4	1:5	1:5	1:5	1:5		1
	5	0		3	6	8	1	3	5	9		5
	1	1:4	1:43	1:4	1:4	1:4	1:5	1:5	1:5	2:1		1
	8	1		5	7	9	2	5	7			2
	2	1:4	1:44	1:4	1:4	1:5	1:5	1:5	1:5	2:3		9
	1	2		6	8	1	3	7	9			
	2	1:4	1:45	1:4	1:5	1:5	1:5	1:5	2:2	2:5		6
	4	3		8	0	2	5	8				
	2	1:4	1:46	1:4	1:5	1:5	1:5	2:0	2:4	2:7		3
	7	4		9	1	4	7					
	0	1:4	1:48	1:5	1:5	1:5	1:5	2:2	2:6	2:9	Cancer ♋	3
	6			0	3	5	9					0
	3	1:4	1:49	1:5	1:5	1:5	2:0	2:4	2:8	2:1		2
	7			2	4	7				2		7
	6	1:4	1:50	1:5	1:5	1:5	2:2	2:5	2:1	2:1		2
	8			3	5	8			0	5		4
	9	1:4	1:51	1:5	1:5	2:0	2:3	2:6	2:1	2:1		2
	9			3	6				2	6		1
	1	1:5	1:52	1:5	1:5	2:0	2:4	2:8	2:1	2:1		1
	2	0		5	8				3	8		8
	1	1:5	1:53	1:5	1:5	2:1	2:5	2:1	2:1	2:2		1
	5	1		5	8			0	4	0		5
	1	1:5	1:54	1:5	1:5	2:3	2:6	2:1	2:1	2:2		1
	8	1		6	9			1	6	1		2
	2	1:5	1:54	1:5	2:0	2:3	2:7	2:1	2:1	2:2		9
	1	1		7				1	7	2		
	2	1:5	1:54	1:5	2:1	2:4	2:8	2:1	2:1	2:2		6
	4	1		7				2	8	4		
	2	1:5	1:55	1:5	2:1	2:4	2:8	2:1	2:1	2:2		3
	7	2		7				4	8	4		
	3	1:5	1:55	1:5	2:1	2:4	2:9	2:1	2:1	2:2		0
	0	2		8				5	8	5		

Panel 2 — latitudes 44°–52°

Sign (L)	G	44	45	46	47	48	49	50	51	52	Sign (R)	G
Aries ♈	0	1:4	1:4	1:4	1:4	1:5	1:5	1:5	1:5	2:1	Virgo ♍	3
	2	4	6	8	0	2	5	8				0
	3	1:4	1:4	1:4	1:4	1:5	1:5	1:5	1:5	2:2		2
	3	4	6	8	1	3	6	9				7
	6	1:4	1:4	1:4	1:4	1:5	1:5	1:5	2:0	2:3		2
	3	5	7	9	2	4	7					4
	9	1:4	1:4	1:4	1:5	1:5	1:5	1:5	2:1	2:4		2
	4	6	8	0	3	5	8					1
	1	1:4	1:4	1:4	1:5	1:5	1:5	2:0	2:3	2:6		1
	2	4	7	9	1	4	7					8
	1	1:4	1:4	1:5	1:5	1:5	1:5	2:1	2:4	2:8		1
	5	6	8	0	3	5	8					5
	1	1:4	1:4	1:5	1:5	1:5	2:0	2:3	2:6	2:1		1
	8	6	9	2	4	7				0		2
	2	1:4	1:5	1:5	1:5	1:5	2:2	2:5	2:8	2:1		9
	1	8	0	3	6	8				3		
Taurus ♉	2	1:4	1:5	1:5	1:5	2:0	2:4	2:7	2:1	2:1	Leo ♌	6
	4	9	2	4	8				1	6		
	2	1:5	1:5	1:5	1:5	2:2	2:6	2:1	2:1	2:1		3
	7	1	4	6	9			0	4	9		
	0	1:5	1:5	1:5	2:1	2:4	2:8	2:1	2:1	2:2		3
	2	5	8					2	7	2		0
	3	1:5	1:5	2:0	2:3	2:7	2:1	2:1	2:2	2:2		2
	4	7					1	5	1	6		7
	6	1:5	1:5	2:2	2:6	2:1	2:1	2:1	2:2	2:3		2
	6	9			0	4	9	4	1			4
	9	1:5	2:1	2:4	2:9	2:1	2:1	2:2	2:2	2:3		2
	8				3	7	3	9	5			1
	1	2:0	2:3	2:7	2:1	2:1	2:2	2:2	2:3	2:4		1
	2				1	6	1	7	4	1		8
	1	2:2	2:5	2:1	2:1	2:2	2:2	2:3	2:3	2:4		1
	5			0	4	0	5	1	9	7		5
	1	2:5	2:8	2:1	2:1	2:2	2:3	2:3	2:4	2:5		1

Sign (L)	G	44	45	46	47	48	49	50	51	52	Sign (R)	G
Gemini ♊	8			3	8	3	0	6	5	6		2
	2	2:7	2:1	2:1	2:2	2:2	2:3	2:4	2:5	3:6		9
	1		1	6	1	7	4	2	2			
	2	2:9	2:1	2:1	2:2	2:3	2:3	2:4	2:5	3:1		6
	4		4	9	5	1	9	9	9	5		
	2	2:1	2:1	2:2	2:2	2:3	2:4	2:5	3:1	3:3		3
	7	2	6	2	8	6	4	6	1	4		
	0	2:1	2:1	2:2	2:3	2:4	2:5	3:4	3:2	0:0	Cancer ♋	3
		4	9	6	2	1	0		3			0
	3	2:1	2:2	2:2	2:3	2:4	2:5	3:1	3:4	0:0		2
		7	2	9	6	6	8	4	5			7
	6	2:2	2:2	2:3	2:4	2:5	3:5	3:2	0:0	0:0		2
		0	5	2	0	1		5				4
	9	2:2	2:2	2:3	2:4	2:5	3:1	3:4	0:0	0:0		2
		2	8	5	4	7	3	3				1
	1	2:2	2:3	2:3	2:4	3:2	3:2	0:0	0:0	0:0		1
	2	4	1	8	8		5					8
	1	2:2	2:3	2:4	2:5	3:7	3:3	0:0	0:0	0:0		1
	5	6	4	1	2		6					5
	1	2:2	2:3	2:4	2:5	3:1	3:5	0:0	0:0	0:0		1
	8	8	5	4	6	3	0					2
	2	2:3	2:3	2:4	2:5	3:2	0:0	0:0	0:0	0:0		9
	1	0	6	6	8	0						
	2	2:3	2:3	2:4	3:1	3:2	0:0	0:0	0:0	0:0		6
	4	1	8	7		5						
	2	2:3	2:3	2:5	3:3	3:3	0:0	0:0	0:0	0:0		3
	7	1	9	0		0						
	3	2:3	2:3	2:5	3:3	3:3	0:0	0:0	0:0	0:0		0
	0	1	9	0		0						

The quantity of the twilights in the northern signs [Crepusculorum quantitas in signis Borealibus] (*continued*)

Panel 3 — latitudes 53°–61°

Sign (L)	G	53	54	55	56	57	58	59	60	61	Sign (R)	G
Aries ♈	0	2:4	2:7	2:1	2:1	2:1	2:2	2:2	2:3	2:3	Virgo ♍	3
				0	4	8	3	8	3	8		0
	3	2:5	2:8	2:1	2:1	2:2	2:2	2:3	2:3	2:4		2
				2	6	0	5	0	5	2		7
	6	2:6	2:1	2:1	2:1	2:2	2:2	2:3	2:3	2:4		2
			0	4	8	2	7	2	9	6		4
	9	2:8	2:1	2:1	2:2	2:2	2:3	2:3	2:4	2:4		2
			2	5	0	4	0	6	2	9		1
	1	2:1	2:1	2:1	2:2	2:2	2:3	2:4	2:4	2:5		1
	2	0	4	7	2	7	4	0	6	5		8
	1	2:1	2:1	2:2	2:2	2:3	2:3	2:4	2:5	3:0		1
	5	2	6	0	5	1	7	4	1			5
	1	2:1	2:1	2:2	2:2	2:3	2:4	2:5	2:5	3:9		1
	8	4	9	4	8	5	2	0	8			2
	2	2:1	2:2	2:2	2:3	2:3	2:4	2:5	3:6	3:1		9
	1	7	2	7	2	9	6	6		8		
	2	2:2	2:2	2:3	2:3	2:4	2:5	3:5	3:1	3:3		6
	4	0	5	1	7	5	4		6	3		
	2	2:2	2:2	2:3	2:4	2:5	3:1	3:1	3:3	3:5		3
	7	4	9	6	2	2		3	0	9		
Taurus ♉	0	2:2	2:3	2:4	2:4	3:0	3:1	3:2	3:5	0:0	Leo ♌	3
		8	4	2	9		2	8	8			0
	3	2:3	2:3	2:4	2:5	3:9	3:2	3:5	0:0	0:0		2
		3	9	8	7		5	4				7
	6	2:3	2:4	2:5	3:8	3:2	3:5	0:0	0:0	0:0		2
		8	5	6		3	2					4
	9	2:4	2:5	3:5	3:1	3:4	0:0	0:0	0:0	0:0		2
		3	4		9	0						1
	1	2:5	3:3	3:1	3:4	0:0	0:0	0:0	0:0	0:0		1
	2	0		7	2							8

Sign (L)	G	53	54	55	56	57	58	59	60	61	Sign (R)	G
Gemini ♊	1	2:5	3:1	3:3	0:0	0:0	0:0	0:0	0:0	0:0	Cancer ♋	1
	5	9	2	5								5
	1	3:9	3:3	0:0	0:0	0:0	0:0	0:0	0:0	0:0		1
	8		0									2
	2	3:2	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		9
	1	1										
	2	3:5	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		6
	4	5										
	2	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		3
	7											
	0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		3
												0
	3	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		2
												7
	6	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		2
												4
	9	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		2
												1
	1	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		1
	2											8
	1	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		1
	5											5
	1	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		1
	8											2
	2	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		9
	1											
	2	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		6
	4											
	2	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		3
	7											
	3	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0		0
	0											

Commentary on Chapter III of the Sphere of Sacrobosco

Table caption (verbatim): *Crepusculorum Quantitas in signis Australibus.* — "The quantity of the twilights, in the southern signs."

Table structure. This is the symmetric twilight table of Marcello Francolini [Marcello Francolini]. The deck (column-group) headers **35–61** are the pole elevations in degrees (*Altitudo Poli*); each carries an **H.** (hours) and **M.** (minutes) sub-column. The **argument is the Sun's ecliptic place**, read from *both* ends: the left frame gives (sign, degree) descending through **Libra → Scorpius → Sagittarius**; the right frame (*Altitudo*, sign + degree) gives the complementary reading **Pisces → Aquarius → Capricornus** (each right-hand row has the same solar declination as its left-hand row). Boundary rows (a sign's 0°/30°) are shared. Values are twilight duration in hours:minutes; minutes are zero-padded here for column alignment (the print sets them unpadded, e.g. "2 | 4" = 2:04).

Two deck-panels appear on this page: 35°–43° (upper) and 44°–52° (lower).

Panel 1 — pole elevations 35°–43°

[illegible]

Sign (L)	°	35 °	36 °	37°	°	38 °	39 °	40 °	41 °	42 °	43 °	°	Sign (R)
	1	8	9		2	3	4	6	7	9			
Libra	2	1:2	1:2	1:31	1:3	1:3	1:3	1:3	1:3	1:3	1:3	6	Pisces
	4	8	9		2	3	5	6	8	9			
Libra	2	1:2	1:3	1:31	1:3	1:3	1:3	1:3	1:3	1:3	1:3	3	Pisces
	7	9	0		2	3	5	6	8	9			
Scorpius	0	1:2	1:3	1:31	1:3	1:3	1:3	1:3	1:3	1:3	1:4	3	Aquarius
		9	0		3	4	5	7	8	0	0		
Scorpius	3	1:2	1:3	1:32	1:3	1:3	1:3	1:3	1:3	1:3	1:4	2	Aquarius
		9	0		3	4	5	7	8	0	7		
Scorpius	6	1:3	1:3	1:32	1:3	1:3	1:3	1:3	1:3	1:3	1:4	2	Aquarius
		0	1		3	4	6	7	9	0	4		
Scorpius	9	1:3	1:3	1:32	1:3	1:3	1:3	1:3	1:3	1:3	1:4	2	Aquarius
		0	1		4	5	6	8	9	1	1		
Scorpius	1	1:3	1:3	1:33	1:3	1:3	1:3	1:3	1:3	1:4	1:4	1	Aquarius
	2	0	1		4	5	7	8	0	1	8		
Scorpius	1	1:3	1:3	1:33	1:3	1:3	1:3	1:3	1:3	1:4	1:4	1	Aquarius
	5	1	2		4	6	8	9	0	2	5		
Scorpius	1	1:3	1:3	1:34	1:3	1:3	1:3	1:3	1:3	1:4	1:4	1	Aquarius
	8	1	2		5	6	8	9	1	2	2		
Scorpius	2	1:3	1:3	1:34	1:3	1:3	1:3	1:4	1:4	1:4	1:4	9	Aquarius
	1	2	3		5	7	8	0	1	3			
Scorpius	2	1:3	1:3	1:35	1:3	1:3	1:3	1:4	1:4	1:4	1:4	6	Aquarius
	4	2	3		6	7	9	0	2	4			
Scorpius	2	1:3	1:3	1:35	1:3	1:3	1:3	1:4	1:4	1:4	1:4	3	Aquarius
	7	3	4		6	8	9	1	3	4			
Sagittarius	0	1:3	1:3	1:35	1:3	1:3	1:4	1:4	1:4	1:4	1:4	3	Capricornu
		3	4		7	8	0	2	3	5	0	s	
Sagittarius	3	1:3	1:3	1:36	1:3	1:3	1:4	1:4	1:4	1:4	1:4	2	Capricornu
		4	5		7	9	0	2	4	5	7	s	
Sagittarius	6	1:3	1:3	1:36	1:3	1:3	1:4	1:4	1:4	1:4	1:4	2	Capricornu
		4	5		8	9	1	2	4	6	4	s	
Sagittarius	9	1:3	1:3	1:37	1:3	1:4	1:4	1:4	1:4	1:4	1:4	2	Capricornu
		4	5		8	0	1	3	5	7	1	s	

Sign (L)	°	35 °	36 °	37°	38 °	39 °	40 °	41 °	42 °	43 °	°	Sign (R)
Sagittarius	1	1:3	1:3	1:37	1:3	1:4	1:4	1:4	1:4	1:4	1	Capricornu
	2	5	6		9	0	2	3	5	7	8	s
Sagittarius	1	1:3	1:3	1:37	1:3	1:4	1:4	1:4	1:4	1:4	1	Capricornu
	5	5	6		9	0	2	4	5	8	5	s
Sagittarius	1	1:3	1:3	1:37	1:3	1:4	1:4	1:4	1:4	1:4	1	Capricornu
	8	5	6		9	1	2	4	6	8	2	s
Sagittarius	2	1:3	1:3	1:38	1:3	1:4	1:4	1:4	1:4	1:4	9	Capricornu
	1	5	7		9	1	2	4	6	8		s
Sagittarius	2	1:3	1:3	1:48 △	1:4	1:4	1:4	1:4	1:4	1:4	6	Capricornu
	4	5	7		0	1	3	4	6	8		s
Sagittarius	2	1:3	1:3	1:48 △	1:4	1:4	1:4	1:4	1:4	1:4	3	Capricornu
	7	5	7		0	1	3	5	6	8		s
Sagittarius	3	1:3	1:3	1:48 △	1:4	1:4	1:4	1:4	1:4	1:4	0	Capricornu
	0	5	7		0	1	3	5	6	8		s

△ = printed value flagged as a numerical impossible. In the 37° column, the last three Sagittarius rows are printed **1:48**, which breaks the column's monotone rise (the 36° neighbour is 1:37 and the 38° neighbour is 1:40); almost certainly a compositor error for **1:38** (a "4" set for a "3").

Panel 2 — pole elevations 44°–52°

Sign (L) ° 44° 45° 46° 47° 48° 49° 50° 51° 52° ° Sign (R)
--- --- --- --- --- --- --- --- --- --- --- ---
Libra 0 1:42 1:44 1:46 1:48
1:50 1:52 1:55 1:58 2:01 30 Pisces Libra 3 1:42 1:43 1:45
1:47 1:49 1:52 1:54 1:57 2:00 27 Pisces Libra 6 1:41 1:43 1:45
1:47 1:49 1:51 1:53 1:56 1:59 24 Pisces Libra 9 1:41 1:42
1:44 1:46 1:49 1:51 1:53 1:56 1:58 21 Pisces Libra 12 1:40
1:42 1:44 1:46 1:48 1:51 1:53 1:56 1:58 18 Pisces Libra 15
1:40 1:42 1:44 1:46 1:48 1:50 1:53 1:55 1:58 15 Pisces Libra
18 1:40 1:42 1:44 1:46 1:48 1:50 1:53 1:55 1:58 12 Pisces
Libra 21 1:41 1:42 1:44 1:46 1:48 1:50 1:53 1:55 1:58 9 Pisces
Libra 24 1:41 1:43 1:44 1:46 1:48 1:51 1:53 1:55 1:58 6 Pisces
Libra 27 1:41 1:43 1:45 1:47 1:49 1:51 1:53 1:56 1:58 3
Pisces Scorpius 0 1:41 1:43 1:45 1:47 1:49 1:51 1:54 1:56 1:59
30 Aquarius Scorpius 3 1:42 1:43 1:45 1:47 1:49 1:52 1:54 1:57
1:59 27 Aquarius Scorpius 6 1:42 1:44 1:46 1:48 1:50 1:52

1:55 | 1:57 | 2:00 | 24 | Aquarius | | Scorpius | 9 | 1:43 | 1:44 | 1:46 | 1:48 | 1:51
 | 1:53 | 1:55 | 1:58 | 2:00 | 21 | Aquarius | | Scorpius | 12 | 1:43 | 1:45 | 1:47 |
 1:49 | 1:51 | 1:54 | 1:56 | 1:59 | 2:01 | 18 | Aquarius | | Scorpius | 15 | 1:44 |
 1:45 | 1:48 | 1:49 | 1:52 | 1:54 | 1:57 | 1:59 | 2:02 | 15 | Aquarius | | Scorpius |
 18 | 1:44 | 1:46 | 1:48 | 1:50 | 1:53 | 1:55 | 1:58 | 2:00 | 2:03 | 12 | Aquarius | |
 Scorpius | 21 | 1:45 | 1:47 | 1:49 | 1:51 | 1:53 | 1:56 | 1:58 | 2:01 | 2:04 | 9 |
 Aquarius | | Scorpius | 24 | 1:46 | 1:48 | 1:50 | 1:52 | 1:54 | 1:56 | 1:59 | 2:02 |
 2:05 | 6 | Aquarius | | Scorpius | 27 | 1:46 | 1:48 | 1:50 | 1:52 | 1:55 | 1:57 | 2:00
 | 2:03 | 2:06 | 3 | Aquarius | | Sagittarius | 0 | 1:47 | 1:49 | 1:51 | 1:53 | 1:55 |
 1:58 | 2:01 | 2:04 | 2:07 | 30 | Capricornus | | Sagittarius | 3 | 1:47 | 1:49 | 1:52 |
 1:54 | 1:56 | 1:59 | 2:02 | 2:05 | 2:08 | 27 | Capricornus | | Sagittarius | 6 | 1:48 |
 1:50 | 1:52 | 1:55 | 1:57 | 2:00 | 2:03 | 2:06 | 2:09 | 24 | Capricornus | |
 Sagittarius | 9 | 1:48 | 1:51 | 1:53 | 1:55 | 1:58 | 2:01 | 2:03 | 2:07 | 2:10 | 21 |
 Capricornus | | Sagittarius | 12 | 1:49 | 1:51 | 1:53 | 1:56 | 1:58 | 2:01 | 2:04 |
 2:07 | 2:11 | 18 | Capricornus | | Sagittarius | 15 | 1:49 | 1:52 | 1:54 | 1:56 | 1:59
 | 2:02 | 2:05 | 2:08 | 2:12 | 15 | Capricornus | | Sagittarius | 18 | 1:50 | 1:52 |
 1:54 | 1:57 | 1:59 | 2:02 | 2:05 | 2:08 | 2:12 | 12 | Capricornus | | Sagittarius | 21
 | 1:50 | 1:52 | 1:54 | 1:57 | 2:00 | 2:02 | 2:06 | 2:09 | 2:13 | 9 | Capricornus | |
 Sagittarius | 24 | 1:50 | 1:52 | 1:55 | 1:57 | 2:00 | 2:03 | 2:06 | 2:09 | 2:13 | 6 |
 Capricornus | | Sagittarius | 27 | 1:50 | 1:53 | 1:55 | 1:58 | 2:01 | 2:03 | 2:06 |
 2:10 | 2:14 | 3 | Capricornus | | Sagittarius | 30 | 1:50 | 1:53 | 1:55 | 1:58 | 2:01 |
 2:03 | 2:06 | 2:10 | 2:14 | 0 | Capricornus |

Table caption (verbatim): *Crepusculorum Quantitas in signis Australibus.* — "The quantity of the twilights, in the southern signs" (continued).

Panel 3 — pole elevations 53°–61°

Same argument frame (Libra→Sagittarius on the left; Pisces→Capricornus on the right). All values now exceed two hours.

| Sign (L) | ° | 53° | 54° | 55° | 56° | 57° | 58° | 59° | 60° | 61° | ° | Sign (R) |
 | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Libra | 0 | 2:04 | 2:07 | 2:10 | 2:14
 | 2:18 | 2:23 | 2:28 | 2:33 | 2:38 | 30 | Pisces | | Libra | 3 | 2:03 | 2:06 | 2:09 |
 2:13 | 2:17 | 2:21 | 2:26 | 2:31 | 2:36 | 27 | Pisces | | Libra | 6 | 2:02 | 2:05 | 2:08
 | 2:12 | 2:16 | 2:20 | 2:24 | 2:29 | 2:35 | 24 | Pisces | | Libra | 9 | 2:01 | 2:04 |
 2:07 | 2:11 | 2:15 | 2:19 | 2:23 | 2:28 | 2:33 | 21 | Pisces | | Libra | 12 | 2:01 |
 2:04 | 2:07 | 2:10 | 2:14 | 2:18 | 2:22 | 2:27 | 2:32 | 18 | Pisces | | Libra | 15 |
 2:01 | 2:04 | 2:07 | 2:10 | 2:14 | 2:18 | 2:22 | 2:26 | 2:31 | 15 | Pisces | | Libra |
 18 | 2:01 | 2:03 | 2:07 | 2:10 | 2:14 | 2:17 | 2:22 | 2:26 | 2:31 | 12 | Pisces | |
 Libra | 21 | 2:01 | 2:04 | 2:07 | 2:10 | 2:13 | 2:17 | 2:22 | 2:26 | 2:30 | 9 | Pisces |

| Libra | 24 | 2:01 | 2:04 | 2:07 | 2:10 | 2:14 | 2:18 | 2:22 | 2:26 | 2:31 | 6 | Pisces
 | | Libra | 27 | 2:01 | 2:04 | 2:07 | 2:10 | 2:14 | 2:18 | 2:22 | 2:26 | 2:31 | 3 |
 Pisces | | Scorpius | 0 | 2:02 | 2:04 | 2:08 | 2:11 | 2:15 | 2:19 | 2:23 | 2:27 | 2:32 |
 30 | Aquarius | | Scorpius | 3 | 2:02 | 2:05 | 2:08 | 2:12 | 2:15 | 2:19 | 2:24 | 2:28
 | 2:33 | 27 | Aquarius | | Scorpius | 6 | 2:03 | 2:06 | 2:09 | 2:13 | 2:16 | 2:20 |
 2:25 | 2:29 | 2:34 | 24 | Aquarius | | Scorpius | 9 | 2:03 | 2:07 | 2:10 | 2:14 | 2:17
 | 2:21 | 2:26 | 2:31 | 2:36 | 21 | Aquarius | | Scorpius | 12 | 2:04 | 2:08 | 2:11 |
 2:15 | 2:18 | 2:22 | 2:27 | 2:32 | 2:37 | 18 | Aquarius | | Scorpius | 15 | 2:05 |
 2:09 | 2:12 | 2:16 | 2:20 | 2:24 | 2:28 | 2:33 | 2:39 | 15 | Aquarius | | Scorpius |
 18 | 2:06 | 2:10 | 2:13 | 2:17 | 2:21 | 2:25 | 2:30 | 2:35 | 2:41 | 12 | Aquarius | |
 Scorpius | 21 | 2:07 | 2:11 | 2:14 | 2:18 | 2:22 | 2:27 | 2:32 | 2:37 | 2:43 | 9 |
 Aquarius | | Scorpius | 24 | 2:08 | 2:12 | 2:15 | 2:19 | 2:24 | 2:28 | 2:33 | 2:39 |
 2:45 | 6 | Aquarius | | Scorpius | 27 | 2:09 | 2:13 | 2:17 | 2:21 | 2:25 | 2:30 | 2:35
 | 2:41 | 2:48 | 3 | Aquarius | | Sagittarius | 0 | 2:10 | 2:14 | 2:18 | 2:22 | 2:27 |
 2:32 | 2:37 | 2:43 | 2:50 | 30 | Capricornus | | Sagittarius | 3 | 2:11 | 2:15 | 2:19 |
 2:23 | 2:28 | 2:33 | 2:39 | 2:45 | 2:53 | 27 | Capricornus | | Sagittarius | 6 | 2:12 |
 2:16 | 2:20 | 2:25 | 2:30 | 2:35 | 2:41 | 2:47 | 2:55 | 24 | Capricornus | |
 Sagittarius | 9 | 2:13 | 2:17 | 2:21 | 2:26 | 2:31 | 2:37 | 2:42 | 2:49 | 2:57 | 21 |
 Capricornus | | Sagittarius | 12 | 2:14 | 2:18 | 2:22 | 2:27 | 2:33 | 2:38 | 2:44 |
 2:51 | 3:00 | 18 | Capricornus | | Sagittarius | 15 | 2:15 | 2:19 | 2:23 | 2:28 | 2:34
 | 2:40 | 2:46 | 2:53 | 3:01 | 15 | Capricornus | | Sagittarius | 18 | 2:16 | 2:20 |
 2:24 | 2:29 | 2:35 | 2:40 | 2:46 | 2:55 | 3:03 | 12 | Capricornus | | Sagittarius | 21
 | 2:16 | 2:21 | 2:25 | 2:30 | 2:35 | 2:41 | 2:48 | 2:56 | 3:05 | 9 | Capricornus | |
 Sagittarius | 24 | 2:17 | 2:21 | 2:25 | 2:30 | 2:36 | 2:42 | 2:49 | 2:57 | 3:06 | 6 |
 Capricornus | | Sagittarius | 27 | 2:17 | 2:22 | 2:26 | 2:31 | 2:37 | 2:42 | 2:49 |
 2:57 | 3:07 | 3 | Capricornus | | Sagittarius | 30 | 2:18 | 2:22 | 2:26 | 2:31 | 2:37 |
 2:42 | 2:50 | 2:58 | 3:07 | 0 | Capricornus |

On the Diversity of the Artificial Days and Nights throughout All Places of the Earth

(*DE DIVERSITATE DIERVM & noctium artificialium per omnia terræ loca.*)

[Margin: *Proprietates eorum, quorum Zenith in Aequinoctiali circulo est.* — "The properties of those whose zenith is on the equinoctial circle."]

It should be noted, further, that for those whose zenith lies on the equinoctial circle, the Sun passes twice a year through the zenith of their heads — namely when it is at the beginning of Aries and at the beginning of Libra. At those times

they have two "high" solstices, since the Sun passes directly over their heads. They likewise have two "low" solstices, when the Sun is at the first points of Cancer and Capricorn; these are called "low" because the Sun is then at its greatest remove from the zenith of their heads. From what has been said it is clear that, since they always have equinox, they will have four solstices in the year — two high and two low. It is also clear that they have two summers, namely when the Sun stands at one or the other of the equinoctial points, or near them; and likewise two winters, namely when the Sun is at the first points of Cancer and Capricorn, or near them. And this is what al-Farghānī [Alphraganus] means when he says that summer and winter — our summer and winter, that is — are for them of one and the same character; for the two seasons that are summer and winter to us are two winters to them. Hence the explanation of the verses of Lucan is made plain:

Deprehensum est hunc esse locum, qua circulus alti Solstitij medium signorum percutit orbem.

"It has been found that this is the place where the circle of the high solstice strikes the middle of the orb of the signs." (Lucan [Lucan])

For there Lucan calls the circle of the high solstice the "equinoctial," on which the two high solstices occur for those who live under the equator. By "the orb of the signs" he means the zodiac, which the equinoctial "runs through the middle" — that is, divides — into two equal halves. They also happen to have four shadows in the year. For when the Sun is at one or the other of the equinoctial points, then in the morning their shadow is cast toward the west, and in the evening the reverse; but at midday their shadow is perpendicular, since the Sun is directly over their head. When, however, the Sun is in the northern signs, their shadow is cast toward the south; and when it is in the southern signs, it is cast toward the north. For them, moreover, the stars that lie near the poles both rise and set, as they do also for certain others who live around the equator. Hence Lucan speaks thus:

Tunc furor extremos mouit Romanus Horestas, Carmanosque duces, quorum iam flexus in Austrum Æther non totam, mergi tamen aspicit Arcton,

"Then Roman fury stirred up the far-off Orestae [Horestas] and the Carmanian chieftains, whose sky, now bent toward the south, does not see the whole Bear [Arcton], yet does see it dip below the horizon..." (Lucan, *Pharsalia* III)

the Sun most closely approaches their zenith — just as we too are accustomed to call the high solstice the *summer* solstice, because the Sun is then at its highest on the meridian above the horizon. They likewise have two *low* solstices, namely when the Sun is at the tropics, because then the Sun is most removed from the vertex of their head — just as we usually call the winter [brumal] solstice the *low* one, since the Sun then recedes farthest from us. But if we take "solstice" in its strict sense, that is, for the turning-back of the Sun (since at those points the Sun does not appear to change its declination from the equator), they will have only two low solstices, when the Sun is at the tropics — of which one is high for us and the other low. As for the verses of [Lucan] adduced to prove two *high* solstices, they are beside the point, since the circle of the high solstice is called by Lucan the Tropic of Cancer, as we said above when treating of the rising and setting of the signs in the right sphere.

III. They have a perpetual equinox.

IV. They have two summers in the year, and as many winters — provided "summer" denotes an excess of heat and "winter" a slackening of heat. The summers occur when the Sun is on the equator, the winters when it holds the tropical points. Hence, says [al-Farghānī] (Differentia 6), the difference is this: that our summer and winter are, for them, of one and the same character, because our summer is for them likewise a winter. By the same reasoning we might say that they have a double spring and a double autumn, namely in the seasons intermediate between the summers and the winters.

V. They have four kinds of shadow: one toward the west, when the Sun rises; a second toward the east, when the Sun sets; a third at midday toward the south [Auster], while the Sun is in the northern signs; and a fourth at midday toward the north, when the Sun holds the southern signs. But when the Sun is on the equator, they cast no shadow at all at midday.

VI. All the stars and all the points of the heaven, except the poles, rise and set for them — because their horizon cuts all the parallels described for the primary motion, since it passes through the poles themselves. All this is plain on the material sphere. As for the verses that he adduces from the poets on this matter, how much force they have and how aptly and fittingly they are brought in here, let others judge.

[Margin: Properties of those whose zenith is between the equinoctial and the Tropic of Cancer.]

[Lemma — Sacrobosco:] But for those whose zenith is between the equinoctial and the Tropic of Cancer, it happens twice in the year that the Sun passes through the zenith of their head. This is shown as follows. Let a circle be conceived parallel to the equinoctial, passing through the zenith of their head. That circle will intersect the zodiac in two places equidistant from the beginning of Cancer. Therefore the Sun, when it is at those two points, passes through the zenith of their head. Hence they have two summers and two winters, four solstices, and four shadows, just like those who dwell under the equinoctial. And in such a situation some say Arabia lies. Whence Lucan, speaking of the Arabs coming to Rome to aid Pompey, says:

You Arabs, you came into a world unknown to you, marveling that the shadows of the groves did not fall to the left.

[Lemma continues:] For in their own regions the shadows were sometimes to their right, sometimes to their left, sometimes perpendicular, sometimes eastern, sometimes western. But when they came to Rome, on the near side of the Tropic of Cancer, they always had northern shadows.

COMMENTARIVS

The second location is that of those whose zenith is between the equinoctial circle and the Tropic of Cancer. To these he assigns four properties.

I. Twice in the year they have the Sun above the zenith of their head.

II. They have two summers and two winters; but one winter is colder than the other, namely the one in which the Sun is farther removed from their vertex — which happens when the Sun is in Capricorn.

III. They also have four solstices, two high and two low.

IV. They likewise have a fourfold shadow, like those who dwell under the equator. Add to this that they have an inequality of days and nights throughout the whole year, except on the days at the time of the equinox; and also that not all the stars, or points of the heaven, rise and set for them — as is clearly seen on the material sphere. As for his statement that Arabia, according to some, is placed in this situation, this must be understood of Arabia Felix, for this, for the greater part, lies between the equator and the Tropic of Cancer.

[Margin: Properties of those whose zenith is on the Tropic of Cancer.]

[Lemma — Sacrobosco:] For those, indeed, whose zenith is on the Tropic of Cancer, it happens that the Sun passes once in the year through the zenith of

their head — namely when it is in the first point of Cancer; and then, in one hour of one day of the whole year, they have a perpendicular shadow. In such a situation the city of Syene is said to lie. Whence Lucan:

...with Syene nowhere bending the shadows.

[Lemma continues:] Understand this of midday, that is, of a single day — a day whose shadow in the morning is thrown to the west and in the evening to the east — while through the remainder of the whole year a northern shadow is cast for them.

COMMENTARIUS

The third location is that of those whose zenith is directly on the Tropic of Cancer. To these he ascribes two properties.

I. Once in the year the Sun passes over the vertex of their head, namely when it is at the beginning of Cancer.

II. They possess three kinds of shadow: an eastern one in the evening, a western one in the morning, and a northern one at midday — though when the Sun is at the beginning of Cancer they have no shadow at all at midday. In such a situation is said to be the Egyptian city of Syene, where, on the authority of [Pliny], the astronomers built a very deep well, in which, when the Sun is at the beginning of Cancer, no shadow is seen, but the whole is lit by the Sun. To these we may add that they have an inequality of days, except at the equinoxes; that they have only two solstices, one high, the other low; that they have one summer and one winter; and finally, that not all the points of the heaven rise and set for them. All of which is plain on the material sphere.

[Margin: Properties of those whose zenith is between the Tropic of Cancer [?] and the arctic circle [remainder of gloss abraded — ?].]

[Lemma — Sacrobosco:] But for those whose zenith is between the Tropic of Cancer and the arctic circle, it happens that the Sun never at any time passes through the zenith of their head, and a shadow is always cast for them toward the north. Such is our own situation. Note also that Ethiopia, or some part of it, lies on the near side of the Tropic of Cancer, according to some. Whence Lucan:

...and the land of the Ethiopians, which would be pressed by no region of the sign-bearing pole [the zodiac], did not, with its knee sunk down, the outermost hoof of the bent Bull [Taurus] jut forward.

[*Lemma continues:*] For some say that "sign" is here taken equivocally — for the twelfth part of the zodiac, and for the figure of the animal, which for the greater part of itself lies within the sign it names. Hence the Bull [Taurus], although it lies for the greater part of itself in the zodiac, nevertheless extends its foot beyond the Tropic of Cancer, and so presses upon Ethiopia, even though no part of the zodiac presses upon it. For if the Bull's foot, of which the poet speaks, were extended toward the equinoctial, so as to be directly over Aries or some other sign, then Ethiopia would be pressed by Aries or Virgo or other signs — as is evident from a circle drawn parallel to the equinoctial, carried around through the zenith of those Ethiopians' head, and through Aries and Virgo or other signs. But since physical reasoning is opposed to this (for they would not be so blackened, were they born in a temperate and habitable region), it must be said that the part of Ethiopia of which Lucan speaks lies under the equinoctial circle, and that the Bull's foot of which he speaks extends toward the equinoctial. A distinction must then be drawn between the *cardinal signs* and the *regions*. For the two signs in which the solstices occur and the two in which the equinoxes occur are called cardinal signs, while the intermediate signs are called regions. And on this understanding it is clear that, since Ethiopia lies under the equinoctial, it is pressed by no region, but only by the two cardinal signs, namely Aries and Libra.

COMMENTARIUS

The fourth location is that of those whose vertex is placed between the Tropic of Cancer and the arctic circle. To these too he assigns two properties.

I. They never have the Sun over the vertex of their head.

II. They have three kinds of shadow, like those who dwell in the preceding situation, with this one difference: that they never have a perpendicular shadow. In this situation, he says, some have supposed Ethiopia to be placed, which they proved from Lucan. But he himself refutes this, explaining Lucan otherwise, as is clear from the text. Yet whatever the truth about Lucan, it is certain that a great part of Ethiopia lies under the equinoctial circle. How fitting our author's exposition is, let others judge — for by many, and especially by [Scaliger] in his *Poetice*, Lucan is censured. It is nonetheless true, as the author says, that the Bull's hoof reaches out toward the equator. To these two properties one may add that those living in this situation of the earth have two solstices, one high, the other low; that they have one summer and one winter, and an inequality of days and nights; and that not all the stars rise and set for them.

[Margin: Properties of those whose zenith is on the arctic circle.]

[Lemma — Sacrobosco:] But for those whose zenith is on the arctic circle, it happens on any day and time of the year that the zenith of their head is the same as the pole of the zodiac; and then they have the zodiac, or ecliptic, for their horizon. And this is what [al-Farghānī] means when he says that there the circle of the zodiac is bent [flattened] over the circle of the hemisphere. But since the firmament is continually moved, the circle of the horizon will intersect the zodiac in an instant; and since they are great circles on the sphere, they will cut each other into equal parts. Hence at once one half of the zodiac emerges above the horizon and the other is suddenly depressed below it. And this is what [al-Farghānī] means when he says that there six signs set at once and the other six rise. Since the ecliptic is also their horizon, the whole Tropic of Cancer will be above the horizon and the whole Tropic of Capricorn below it; and so, when the Sun is at the first point of Cancer, they will have a single day of 24 hours and a night lasting scarcely an instant, because in an instant the Sun crosses the horizon and at once re-emerges, and that touching is for them a night. The reverse befalls them when the Sun is at the first point of Capricorn: for then they have a single night of 24 hours and a day lasting scarcely an instant.

COMMENTARIUS

The fifth location is that of those who have the vertex of their head precisely on the arctic circle, to whom he assigns a single property: namely that on any day of the year the vertex of their head becomes identical with the pole of the zodiac.

From this it follows that the zodiac then takes the role of the horizon; and therefore, virtually in the blink of an eye — as soon as the pole of the zodiac, by the motion of the primary mobile, has withdrawn from the vertex of the head — the zodiac and the horizon will cut each other mutually in two, since they are great circles, so that (as [al-Farghānī] says, *Differentia* 7) one half of the ecliptic rises at once — that is, six signs appear above the horizon — while the other half sets at once, that is, the remaining six signs are suddenly hidden and descend below the horizon. Hence too it comes about that the whole Tropic of Cancer stands above the horizon, while the Tropic of Capricorn, together with the beginning of Cancer and the beginning of Capricorn, touch and, as it were, graze the horizon. Therefore, when the Sun is at the beginning of Cancer, they will have a day of 24 hours and a night lasting scarcely an instant, because in an instant the Sun, as it were, crosses the horizon and at once re-emerges — indeed it never then perfectly sets, but merely touches the horizon. And when the Sun is at the beginning of Capricorn, for the same reason they will have a night of 24 hours

and a day lasting scarcely an instant. All of which is most clearly seen on the material sphere. Add that not all the stars rise and set for them.

[Margin: Properties of those whose zenith is between the arctic circle and the arctic pole of the world.]

[*Lemma — Sacrobosco:*] But for those whose zenith is between the arctic circle and the arctic pole of the world, it happens that their horizon intersects the zodiac in two points equidistant from the beginning of Cancer; and in the revolution of the firmament it happens that the portion of the zodiac intercepted [between them] is always left above the horizon. Hence it is clear that, as long as the Sun is in that intercepted portion, there will be one continuous day without night. Therefore, if that portion is of the extent of one sign, there will be a continuous day of one month without night; if of the extent of two signs, a continuous day of two months without night; and so on. Likewise it happens for them that the portion of the zodiac intercepted by two other points equidistant from the beginning of Capricorn is always left below the horizon. Hence, when the Sun is in that intercepted portion, there will be one continuous night without day, short or long according to the extent of the intercepted portion. As for the remaining signs, which rise and set for them, they rise and set in reverse order [praepostere]. They rise in reverse: as Taurus before Aries; Aries before Pisces; Pisces before Aquarius [and so on]; and yet the signs opposite to these rise in the correct order and set in reverse — as [Libra? ♎] before [Virgo? ♍] before [Leo? ♌] — these three zodiac glyphs abraded and only partly legible, ?] — while the signs opposite to these set directly, namely those which were rising in reverse, such as Taurus, and so on.

COMMENTARIVS

The sixth location is that of those who dwell between the arctic circle and the arctic pole of the world, to whom he assigns two properties.

I. The horizon cuts the zodiac each day in two points equally removed from the beginning of Cancer, and the portion intercepted between those two points never descends below the horizon by the motion of the primary mobile, but always remains in view. From this it follows that, while the Sun traverses that portion, there is a continuous day without night — so that if the portion were 30 degrees, that artificial day would be nearly 30 natural days. By the same reasoning, the zodiac will be cut by the horizon at two other points equally distant from the beginning of Capricorn, and the segment of the zodiac comprehended between those two points never rises above the horizon by the motion of the primary

mobile, but lies perpetually hidden, and is equal to the prior segment that is always in view. Hence, while the Sun traverses the said segment, they will have a continuous night without day — so that if that segment were 30 degrees, that night would be composed, as it were, of 30 natural days, and so on. This is made manifest on the material sphere, if it be so arranged that the solstitial colure coincides with the meridian, and Cancer be placed toward the arctic pole — that is, toward the north, above the horizon — and Capricorn toward the antarctic pole, that is, toward the south, below the horizon. For if a parallel be then conceived as drawn tangent to the ecliptic, the ecliptic will be cut in two points, which will intercept a non-setting arc; and this is manifest, if the position of the sphere is rightly conceived. And the said arc will be greater or smaller according as the beginning of Cancer is raised more or less above the horizon, when it has been placed on the meridian on the northern side.

[Margin: Which signs rise and set in reverse order.]

II. The remaining signs, which rise and set for them, rise and set in a reversed order [*praepostero ordine*] — that is, they do not rise and set in the order in which they rise and set in other parts of the world. So that whereas, for example, Aries always rises and sets before Taurus [*elsewhere*], the signs situated near the vernal equinox rise in reverse, that is, Taurus rises before Aries, and Aries before Pisces, and so on. But they set in the correct order, namely Pisces before Aries, and Aries before Taurus, and so on, as in other positions of the sphere. On the other hand, the signs situated near the autumnal equinox set in reverse, that is, Libra sets before Virgo, and Virgo before Leo, and so on. But they rise in the correct order, as in other positions of the sphere — that is, Leo before Virgo, and Virgo before Libra, and so on. All of which is plain on the material instrument. To these add also that not all the stars rise and set for them.

[Margin: How to inquire how great the continuous day is between the pole and the arctic circle.]

If you wish to know how great the continuous day, and likewise the night, is in the aforesaid place — where the vertex of the head is set between the arctic circle and the arctic pole — you will achieve this by the following method. Subtract the altitude of the pole (which will necessarily be greater than $66^{\circ} 30'$) from a quadrant, that is, from 90° ; and the remainder will be the declination of the beginning of the always-apparent arc. Then from the table of declinations you will easily find the beginning of that arc, whose midpoint lies between that beginning and the beginning of Cancer; and doubling this will give the whole perpetually conspicuous arc, while the arc opposite to it is perpetually hidden.

Having obtained the arc, you will easily know from the Sun's true motion how many natural days it takes to traverse it, and consequently you will have the length of the continuous day, and likewise of the continuous night. EXAMPLE. Where the arctic pole is elevated $69^{\circ} 48'$, I subtract this altitude of the pole from 90° , and there remains the declination of the beginning of the always-apparent arc, $20^{\circ} 12'$, to which in the table of declinations corresponds the beginning of Gemini, or the end of Cancer. Therefore the arc from the beginning of Gemini to the end of Cancer will always be apparent, and the arc from the beginning of Sagittarius [♐, ♐] to the end of Capricorn [♑, ♑] will be perpetually hidden, and so on. For greater convenience, however, I have appended the following little table from [Oronce Fine], in which you have the always-apparent arcs and the continuous days for each degree of the pole's altitude, beginning from 67° up to 90° .

The arcs that are always hidden are equal to the arcs that are always visible; but the continuous nights are not equal to the continuous days, because the Sun passes more quickly through the arcs near Capricorn [♑] than through those near Cancer [♋]. For at the present epoch the point opposite the apogee [Aux, the apogee] lies there [near Capricorn], while here [near Cancer] lies the apogee itself, as will be explained in the Theorics [of the Planets]. For this reason the continuous nights will be somewhat shorter than the continuous days. Understand this to hold where the Arctic pole is raised above the horizon; for where the Antarctic pole is seen above the horizon, then, for the reason just given, the continuous days will be shorter than the continuous nights, as is evident.

[Margin: That the continuous nights are not equal to the continuous days, and why.]

If anyone should wish to obtain the continuous nights more accurately, let him find out — whether from astronomical tables — in how many days and hours the Sun traverses the arcs that are always hidden, which of course are equal and opposite to the arcs that are always visible. This, however, is not strictly necessary, since it is enough to know the continuous nights approximately, especially as they differ little from the continuous days. Only near a pole elevation of 90 degrees is a discrepancy of several days observed. Therefore, so as to stray less from the truth, one may assign a single day to each degree of the arcs that are always hidden.

TABLE OF THE GREATEST DAYS, where the pole is elevated by more degrees than $66\frac{1}{2}$.

Elevation of the pole [Eleuatio poli] — G. (°)	Arc always visible [Arcus semper apparens] — G. (°)	M. (')	Continuous day [Dies continuus] — D. (days)	H. (hour s)	M. (min)
67	22	52	22	1	40
68	40	0	42	1	16
69	52	0	54	16	25
70	61	26	64	13	46
71	70	26	74	0	0
72	78	22	82	6	39
73	84	56	89	4	58
74	92	12	96	17	0
75	96	20	104	1	4
76	105	16	110	7	27
77	111	20	116	14	22
78	117	6	122	17	6
79	122	46	127	9	55
80	128	22	134	4	58
81	133	50	139	31 [?]	36
82	139	6	145	6	43
83	144	22	151	2	6
84	149	36	156	3	3
85	154	42	161	5	23
86	159	50	166	11	23
87	164	52	171	21	47
88	169	58	176	5	29
89	174	58	181	21	58
90	180	0	187	6	39

For those whose zenith falls at the Arctic pole, it happens that their horizon is the same as the equator [Aequinoctialis]. Hence, since the equator cuts the zodiac into two equal parts, so too their horizon leaves one half of the zodiac above itself and the other half below. And so, when the Sun runs through that half which reaches from the beginning of Aries to the end of Virgo, there will be one continuous day

without night; and when the Sun runs through the remaining half, which reaches from the beginning of Libra to the end of Pisces, there will be one continuous night without day. Therefore one half of the whole year is a single artificial day, and the other half is night; and so the whole year there is one natural day. But since the Sun there is never depressed more than 23 degrees below the horizon, it seems that they have a continuous day without night — for with us too it is called day before the rising of the Sun above the horizon. This, however, is as regards common perception. For it is not artificial day, as regards physical reasoning, except from the rising of the Sun to its setting below the horizon. To this point, then — that the light seems to be perpetual there (since it is day before the Sun is raised above the earth by 18 degrees, as Ptolemy says, though other teachers say 30, that is, by the extent of one sign) — it must be answered that the air there is cloudy and thick. For the Sun's ray, being there of weak power, raises more from the vapors than it can consume; and so it does not clear the air, and it is not day.

[Margin: The properties of those who have their zenith at the Arctic pole.]

COMMENTARY.

The seventh and last location is that of those who dwell under the Arctic pole, to whom he assigns one property: namely that in the whole year they have a single natural day — for half the year one artificial day, and for the remaining half of the year one artificial night. This is to be understood on the assumption that the Sun moves uniformly in the zodiac. For since it is carried more swiftly through the southern semicircle of the zodiac than through the northern — as is established from the Theorics of the Planets — the artificial day will be a little longer than 6 months, and the night somewhat shorter than 6 months. He then resolves a tacit objection. For since, according to Ptolemy and the more common opinion, day begins (including also twilight) when the Sun is 18 degrees below the horizon, and since the Sun there is never depressed more below the horizon than by $23\frac{1}{2}$ degrees — which is indeed the greatest declination of the Sun — it seems that the day there would be greater than the night in the whole year. For the common people call "day" the Sun's stay above the horizon together with the morning and evening twilight. The Author answers this difficulty: on account of the cloudy air existing there, and because of the weakness of the solar rays, which are nearly equidistant from the horizon, the twilights cannot be so bright as to render the air clear and make day. One could also reply — whatever be the case regarding the twilights (and in the earlier treatment we dealt with twilights, with the hiding of the Sun below the horizon at the beginning of the morning

twilight and the end of the evening twilight, namely by how many degrees the Sun is distant from the horizon at the beginning

of the morning twilight, or the end of the evening twilight: namely 18 degrees or thereabouts, but on no account 30) — that the astronomers speak of artificial day and night in the strict sense, that is, insofar as artificial day is the Sun's stay above the horizon. For on this reckoning it will be true that under the pole there is day for about half the year, and likewise night, as is clear from the material sphere. Add to this that not all the points of the heaven rise and set for them; rather, half of them is perpetually visible, while the other half lies hidden below the horizon.

[Margin: In what way the same properties are to be understood in the southern hemisphere [sphæra Australis].]

These same seven locations are to be conceived and understood also in the other half of the heaven, from the equator toward the southern pole. But everything that has been said in these regarding the northern signs is to be understood in those regarding the southern signs, and conversely.

On the Division of the Climes [DE DIVISIONE CLIMATVM]

Now let a certain circle be conceived on the surface of the earth, set directly beneath the equator. Let another circle also be understood on the surface of the earth, passing through the East and West and through the poles of the world. These two circles intersect each other in two places at right spherical angles, and they divide the whole earth into four quarters. One of these is our habitable quarter — namely that which is intercepted between the semicircle drawn from East to West on the surface of the equator, and the semicircle drawn from East to West through the Arctic pole. Yet not even that quarter is wholly habitable, since the parts of it near the equator are uninhabitable on account of excessive heat, and likewise the parts of it near the Arctic pole are uninhabitable on account of excessive cold. Let there be understood, therefore, one line equidistant from the equator, dividing the parts uninhabitable on account of heat from the habitable parts that lie toward the North. Let another line also be understood, equidistant from the Arctic pole, dividing the parts of the quarter that lie toward the North and are uninhabitable on account of cold from the parts that lie toward the equator. Between these two extreme lines let there be understood six lines parallel to the equator, which together with the two former ones divide the whole habitable part of the quarter into seven portions, which are called the seven climes [Climata].

COMMENTARY.

This is the fourth part of this chapter, in which the Author describes the climes of the world — for the reason that, when the clime is varied, there is also necessarily varied the rising and setting of the signs, as well as the length of the artificial days and nights. So, in order to make clear what he himself means by a "clime," he says that one must conceive a circle on the surface of the earth set directly beneath the equator; likewise another passing through the poles of the world and through the points of the East and West — understand, the absolute East and West, that is, through the Canary Islands, which mark the West, and through the point which, eastward from them on the same parallel, is 180 degrees distant; for this marks the East. For so great did the length of the habitable earth appear to the ancients, as may be seen in Ptolemy. By these two circles the whole surface of the earth is divided into four quadrants, of which one is this our habitable one — namely that which is contained by a semicircle of the equator and another northern semicircle, described through the East, the West, and the Arctic pole. Not that, as the Author says, this whole quadrant is inhabited — for he says this is false, since both the part near the equator, on account of excessive heat, and the part near the Arctic pole, on account of excessive cold, cannot be inhabited. Hence he adds that a line must be conceived equidistant from the equator, which separates the part uninhabitable on account of heat from the part habitable toward the North. By parallel reasoning, another line must be conceived equidistant from the equator, or equally removed from the Arctic pole, which separates the part uninhabitable on account of cold from the part habitable toward the South. For only the part of the earth's quadrant comprised between the said two lines is inhabited. And if between these two parallel lines six other parallels be drawn, the whole habitable part of the earth will be divided into seven parts, which are called the seven climes of the world. How far one line ought to be distant from another, in order that the climes may be constituted, will be evident from what follows.

A clime is defined as just so much space of the earth as suffices for the day-measure [horologium — the length of daylight as read from the clock or sundial] to vary perceptibly. For one and the same summer day, of some length, which occurs in one region, is perceptibly shorter in a region nearer the South. The space, then, so great as that within which the same day begins to vary perceptibly, is called a clime. Nor is the same day-measure found at the beginning and at the end of this space; for the hours of the day vary perceptibly, and hence the day-measure too.

[Margin: What a clime is.]

COMMENTARY.

He now teaches more clearly how much space, interposed between two parallel lines, there ought to be for a clime to be constituted, saying that a clime is just so much space on the surface of the earth within which the summer day — namely the longest — notably varies, that is, by half an hour. So that a clime is nothing other than a certain space of the temperate and habitable zone, between whose beginning and end (proceeding from the pole to the equator, and conversely) the length of the greatest summer day, or of the winter night, is increased or diminished by half an hour — so that if, for example, the longest day at the beginning of some clime toward the South contains 15 hours, at the end toward the pole it comprises 15½ hours. But if we do not wish to take account of the temperate zone, a clime may be said in general to be a space of the earth comprised between two parallels within which the longest day either increases or decreases by half an hour. On this reckoning more climes will have to be constituted than seven, as we shall shortly say.

The middle of the first clime, then, is where the length of the longer day is 13 hours, and the pole of the world is raised above the circle of the hemisphere by 16 degrees and two thirds of one; and it is called the clime dia Meroes [Greek: διὰ, "through" — through Meroe]. Its beginning is where the length of the longer day is 12 hours and a half and a quarter of one hour, and the pole is raised above the horizon by 12 degrees and a half and a quarter of one degree. And it extends up to the place where the length of the longer day is 13 hours and a quarter of one hour, and the pole is raised above the horizon by 20 degrees and a half; and this space of earth is 440 miles [milliaria, Roman miles].

[Margin: The first clime.]

The middle of the second clime is where the longer day is 13 hours and a half, and the elevation of the pole above the horizon is 24 degrees and a quarter part of one degree. And it is called the clime dia Syenes [through Syene]. Its latitude runs from the boundary of the first clime up to the place where the day becomes longer — 13 hours and a half and a quarter part of one hour — and the pole is raised by 27 degrees and a half. And this space of earth is 400 miles.

[Margin: The second clime.]

The middle of the third clime is where the length of the longer day comes to 14 hours, and the elevation of the pole above the horizon is 30 degrees and a half and a quarter of one part; and it is called the clime dia Alexandrias [through Alexandria]. Its latitude runs from the boundary of the second clime up to that place where the

longer day is 14 hours and a quarter of one, the pole's altitude being 33 degrees and two thirds; and this space of earth is 350 miles.

[Margin: The third clime.]

The middle of the fourth clime is where the length of the longest day is 14 hours and a half, and the altitude of the axis is 36 degrees and two fifths, and it is called dia Rhodon [through Rhodes]. Its latitude runs from the boundary of the third clime up to that place where the length of the longest day is 14 hours and a half and a quarter part of one, and the elevation of the pole is 39 degrees; and this space of earth is 300 miles.

[Margin: The fourth clime.]

The middle of the fifth clime is where the longest day is 15 hours, and the elevation of the pole is 41 degrees and a third of one; and it is called the clime dia Romes [through Rome]. Its latitude runs from the boundary of the fourth clime up to that place where the length of the greatest day is 15 hours and a quarter of one, and the elevation of the axis is 43 degrees and a half; and this space of earth is 255 miles.

[Margin: The fifth clime.]

The middle of the sixth clime is where the longer day is 15 hours and a half, and the pole is raised above the horizon by 45 degrees and two fifths of one; and it is called the clime dia Boristheneos [through the Borysthenes, i.e. the Dnieper]. Its latitude runs from the boundary of the fifth clime up to that place where the length of the longer day is 15 hours and a half and a quarter of one, and the elevation of the axis is 47 degrees and a quarter of one; and this distance of earth is 212 miles.

[Margin: The sixth clime.]

The middle of the seventh clime is where the greater length of the day is 16 hours, and the elevation of the pole above the horizon is 48 degrees and two thirds. And it is called the clime dia Riphæon [through the Rhipæan Mountains]. Its latitude runs from the boundary of the sixth clime up to that place where the greatest day is 16 hours and a quarter of one, and the pole of the world is raised above the horizon by 50 degrees and a half [?]; and this space of earth is 185 miles.

[Margin: The seventh clime.]

Beyond the boundary of this seventh clime, although there are many islands and dwellings of men, yet whatever there is, because it is of poor habitability, is not reckoned under any clime.

[Margin: Why there are not more climes than seven.]

COMMENTARY.

In this place he runs through all seven climes, teaching how great is the longest day in the middle of each clime, how great likewise is the elevation of the pole, and in what way any given clime is named. For the middle of each clime is named either from some notable city, or from an island, a river, or a mountain, through which, of course, passes the parallel that is described through the middle of the clime. Finally, how many miles the latitude of each clime comprises — assigning to each terrestrial degree $56\frac{2}{3}$ miles, just as al-Farghānī granted, as we said above. Then he also determines the length of the greatest day and the elevation of the pole, both at the beginning and at the end of each clime. But all these things are plain in the text, and are seen distinctly in the following table. He adds, however, that even if there are other habitations outside these seven climes, they are not reckoned among the climes by the authorities, because they are not very suitable, but either too hot or too cold.

[Margin: What the Author explains for each of the individual climes.]

[Table of the Climes according to the Ancients — continued from the preceding page]

[The table is untitled on this page; its heading (evidently *Tabula climatum secundum antiquos*) stood on the preceding page. It gives, for each of the seven climes, the beginning (*Princip.*), middle (*Medium*), and end (*Finis*): the longest day in hours and minutes, the elevation of the pole in degrees and minutes, the width in miles, and the naming-place of the clime.]

Climata	H	M	G. (Poli)	M	Milliaria	Denominationes Climatum
	.	.		.		
Princip.	1	4	12	4		
	2	5		5		
I. Medium	1	0	16	4	440	Per Meroen ciuitatem
	3			0		Æthiopiæ. (Through Meroe, a city of Ethiopia)
Finis.	1	1	20	3		
	3	5		0		
Princip.	1	1	20	3		
	3	5		0		

Climata	H	M	G. (Poli)	M	Milliaria	Denominationes Climates
II. Medium	1	3	24	1	400	Per Syenen urbem Ægypti.
	3	0		5		(Through Syene, a town of Egypt)
Finis.	1	4	27	3		
	3	5		0		
Princip.	1	4	27	3		
	3	5		0		
III. Medium	1	0	30	4	350	Per Alexandriam Ægypti
	4			5		Metropolim. (Through Alexandria, the metropolis of Egypt)
Finis.	1	1	33	4		
	4	5		0		
Princip.	1	1	33	4		
	4	5		0		
IV. Medium	1	3	36	2	300	Per Rhodum Insulam.
	4	0		4		(Through the island of Rhodes)
Finis.	1	4	39	0		
	4	5				
Princip.	1	4	39	0		
	4	5				
V. Medium	1	0	41	2	255	Per Romam caput mundi.
	5			0		(Through Rome, the head of the world)
Finis.	1	1	43	3		
	5	5		0		
Princip.	1	1	43	3		
	5	5		0		
VI. Medium	1	3	45	2	212	Per Boristhenem flumen
	5	0		4		Sarmatiæ. (Through the Borysthenes, a river of Sarmatia)
Finis.	1	4	47	1		
	5	5		5		

Climata	H	M	G. (Poli)	M	Milliaria	Denominationes Climatorum
Princip.	1	4	47	1		
	5	5		5		
VII. Medium	1	0	48	4	185	Per Riphæos montes Sarmatiæ.
	6			0		(Through the Riphean mountains of Sarmatia)
Finis.	1	1	50	3		
	6	5		0		

[*Lemma — Sacrobosco:*] The whole difference, then, between the initial boundary of the climes and their final boundary is three and a half hours [of longest day], and, from the elevation of the pole above the horizon, 37 degrees and 45 minutes. Thus it is clear that the width of each clime runs from its beginning, on the side toward the equinoctial, up to its end, on the side toward the Arctic pole; and that the width of the first clime is greater than that of the second, and so on. The length of a clime, on the other hand, may be called a line drawn from east to west, equidistant from the equinoctial. Hence the length of the first clime is greater than that of the second, and so on, which happens because of the narrowing of the sphere. The space, too, between the beginning of the first clime and the end of the seventh is 2142 miles.

[Margin: Diversity in respect of hours and of the pole's elevation among the seven climes, and some other comparisons between them.]

COMMENTARIUS

He gathers five points from what has been said.

I. That the difference between the longest day of the first clime at its beginning and the longest day of the seventh clime at its end is 3 hours 30 minutes.

II. That the excess of the pole's elevation at the far edge of the seventh clime over the pole's elevation at the start of the first clime amounts to 37 degrees 45 minutes. These points are clear from what has been said and from the table set out above.

III. That the width of the first clime is greater than the width of the second, and the width of the second greater than that of the third, and so on, as may be seen in the aforesaid table. The geometrical cause of this we shall shortly disclose.

IV. That the length of the first clime from east to west is greater than the length of the second, and the length of the second greater than that of the third, and so on — which indeed happens, as he says, because near the pole the sphere is in a certain way drawn in, as is evident in the parallel circles, which are smaller near the poles than near the equator.

V. That the terrestrial space from the beginning of the first clime all the way to the end of the seventh, proceeding always directly from the equator toward the pole, comprises 2142 miles, as is established from what has been said.

But let us now demonstrate what we promised, together with Pedro Nunes of Portugal [Petrus Nonius Lusitanus], namely that a change of a single half-hour in the length of the longest day requires less space in a more northerly region than in a less northerly one — that is, that the days take on a greater increment if one moves, say, three degrees toward the pole, than the increment they take if one approaches the equator by the same number of degrees.

[Margin: Why the more northerly climes are narrower than the less northerly climes.]

For let the tropic of Cancer [tropicus ♋], in which the longest days occur, be ABCD, whose center is E. Let the straight line AC be the common section of the meridian and of the tropic of Cancer, in its proper position, such that the diameter BD — which will be the common section of the same tropic and of the right horizon [Horizontis recti] — cuts it at right angles. For since both the right horizon and the tropic of Cancer are perpendicular to the meridian, their common section too will be perpendicular to the same meridian, and hence (by Euclid, *Elements* XI, definition 3) also perpendicular to the straight line AC lying in the meridian. Since, then, both the meridian and the right horizon pass through the axis of the world — and therefore through E, the center of the tropic of Cancer, because the axis of the world is drawn through that same center of the tropic of Cancer — and since AC is taken as the common section of the meridian and of the right horizon and of the same tropic: therefore BAD will be the diurnal arc in the right sphere.

Let there be conceived, under the same meridian, an oblique horizon above which the Arctic pole is visible, making with the tropic of Cancer the common straight section FG, which will likewise be perpendicular to AC, the common section of the meridian and of the same tropic — which will be proved in no other way than has been shown for BD, the common section

[Margin, keyed to superscript letters in the demonstration: a = *Elements* I.28; b = scholium to [proposition] 27.]

of the right horizon and of the tropic, being perpendicular to AC: since both the oblique horizon and the tropic are perpendicular to the meridian, and so on. From this it follows that the straight lines BD, FG are parallel; and hence that the arcs BF, DG are equal; and consequently that the diurnal arc is FAG.

Let there be conceived again two more horizons, more oblique, under the same meridian, making with the tropic of Cancer the common straight sections IK, MN — which by the same reasoning will be perpendicular to AC, and therefore parallel to one another, and will cut off equal arcs FI, GK and IM, KN. And the diurnal arcs will be IAK, MAN. Now the arcs IF, KG will be the excesses of the diurnal arc IAK over the diurnal arc FAG; and the arcs MI, NK will be the excesses of the diurnal arc MAN over the diurnal arc IAK. Let the elevation of the pole above the horizon of the line IK be posited as much greater than the elevation of the pole above the horizon of the line FG as the elevation of the pole above the horizon of the line MN is greater than the elevation of the pole above the horizon of the line IK, so that the pole's elevation increases uniformly. I say that the arcs MI, NK — which are the excesses of the diurnal arc MAN over the diurnal arc IAK — are greater than the arcs IF, KG, which constitute the excess of the diurnal arc IAK over the diurnal arc FAG.

For let P be the center of the sphere, from which are drawn the straight lines PE, PH, PL, PS, PO. PE will be part of the world's axis — because the axis passes through the center of the sphere and through the center of the tropic of Cancer — while the remaining straight lines will be parts of the common sections of the oblique horizons and of the meridian, since both the meridian and the oblique horizons pass through the center of the sphere P and through the points H, L, O. Now since the axis of the sphere, and the common section of the meridian and of any oblique horizon, intercept in the meridian the arc of the pole's elevation above that horizon, and constitute at the center of the sphere the angle standing on that arc — as is evident from a material sphere — EPH will be the angle of the pole's elevation above the horizon of the line FG; EPL the angle of the pole's elevation above the horizon of the line IK; and EPO the angle of the pole's elevation above the horizon of the line MN; because the axis PE, produced, falls upon the pole, while any of the other straight lines is the common section of the meridian and of its own horizon. And since the arcs of the pole's elevation in the meridian, on which those angles stand at the center P, are posited equal — (for the elevation of the pole above the horizon of the line IK is posited as much

greater than the elevation of the pole above the horizon of the line FG as the elevation of the pole above the horizon of the line MN is greater than the elevation of the pole above the line IK) — the angles HPL, LPO will also be equal; and therefore the angle HPO will be bisected by the straight line PL into two equal parts.

Now since the axis PE is perpendicular to the tropic of Cancer, the angle PEH will (by Euclid, *Elements* XI, definition 3) be right; hence PHE is acute, and therefore PHO is obtuse, and POH is acute; and thus the straight line PO is greater than PH. Therefore, since PL bisects the angle HPO, as we have demonstrated, OP will be to PH as OL is to LH. And since the straight line OP is greater than PH, OL too will be greater than LH. Hence, by the corollary of proposition 1 of *On Twilights* [de Crepusculis], the arcs MI, NK will be greater than the arcs IF, KG; and consequently the excess of the diurnal arc MAN over the diurnal arc IAK will be greater than the excess of the diurnal arc IAK [over FAG], even though the excesses — or the differences of the pole's elevations — are posited as equal. Which was the proposition to be established.

Diagram. A circle representing the meridian, with vertical diameter AC (A at top, C at the bottom) standing for the axis / meridian line. A series of parallel horizontal chords descend the circle, each the common section of a successively more oblique horizon with the tropic of Cancer; P is a point on the axis-side interior, upper right, from which lines are drawn to the points where the chords meet the axis (E, H, L, S, O), producing the radiating pencil PE, PH, PL, PS, PO.

- A (top of circle): apex of the vertical diameter (axis / meridian).
- C (bottom of circle): foot of the vertical diameter.
- B (left, upper) — D (right, upper): endpoints of the chord BD through E (right horizon / equatorial section).
- E (on axis, upper): center of the tropic of Cancer, where BD meets AC.
- F (left) — G (right): endpoints of the chord FG through H (first oblique horizon).
- H (on axis, below E): where FG meets the axis.
- I (left) — K (right): endpoints of the chord IK through L (second oblique horizon).
- L (on axis, below H): where IK meets the axis.
- S (on axis, below L): where the chord QR (a further oblique-horizon section) meets the axis.
- O (on axis, near bottom): where the chord MN meets the axis.

- Q (lower left) — R (lower right): endpoints of the lowest labelled chord (QR).
- M (lower left, near C) — N (lower right, near C): endpoints of the chord MN through O.
- P (interior, upper right): center of the sphere; vertex of the pencil PE, PH, PL, PS, PO.

[Margin, keyed to superscript letters in the demonstration: c = Theodosius, *Sphaerica* I.10; d = *Elements* III.27; e = Theodosius, *Sphaerica* I.10; f = *Elements* I.17; g = *Elements* I.13; h = *Elements* I.17; i = *Elements* I.19; k = *Elements* VI.3.]

From this appears the reason why the more northerly climes are narrower, as the author said. For since a clime is a stretch of the earth extending from south to north, within which the longest day takes on an increment of one half-hour: if two neighboring climes were equal to each other, the differences of the pole's elevations [at their bounds] would also be equal — since a clime is said to comprise as many miles of width as are contained in the width of the other. Therefore, as has been demonstrated, the increment of the longest day would be greater in the more northerly clime than in the less northerly one. Hence in the two the longest day would not [each] increase by a half-hour — which is contrary to the definition of climes. And so it is necessary that the more northerly clime be narrower.

This too appears plainly from the figure above. For since it has been demonstrated that the arcs MI, NK are greater than the arcs IF, GK — the angles HPL, OPL being equal — if there are taken arcs QI, RK equal to the arcs IF, KG, and if the straight lines QR, PS are drawn, then QR will be the common section of the tropic of Cancer and of a certain oblique horizon; and the angle LPS — by which the elevation of the pole above the horizon of the line QR differs from the elevation of the pole above the horizon of the line IK — that angle is indeed smaller than the angle OPL, or HPL. From which it results that the elevation of the pole above the more oblique horizon increases less, when the increments of the longest days are equal; and therefore the clime that is more northerly is narrower than the clime that is less northerly, which was to be demonstrated.

Although among the Ancients only seven climes — the aforesaid ones — were established, nevertheless by more recent [astronomers] many more are now set up. For it is not true, as the Author says here, that only a certain part of one quarter of the earth is habitable; since it has now been ascertained that the whole sea is intermingled with the land, so that everywhere — whether continents or islands — are found, in whatever direction navigation is undertaken in the ocean;

and that there is no region so hot or so cold that human beings cannot live in it; indeed, in any place whatsoever both human beings and other animals are found dwelling. Add to this that it is not necessary, for the constitution of climes, that all parts of the earth be habitable; it suffices to observe a certain fixed rule in the increase of the longest days at the various elevations of the pole. And so astronomers, following Ptolemy in *Dictio* 2, chapter 6 [*Almagest* II.6], describe on the surface of the earth circles parallel [to the equator], proceeding from the equator toward the Arctic pole, spaced at such intervals from one another that the longest day of one differs by a quarter of one hour from the longest day of the next parallel. From which it follows that three such parallels enclose a stretch of earth that is called a clime. For if you proceed from one parallel to the third, you will find that the longest day has changed by a half-hour. And the middle one of the three is called the parallel through the middle of the clime — not because the clime is bisected by it (for this is false, since it cuts off a greater part of the clime toward the equator and a smaller part toward the pole, as has been demonstrated), but because the interval of time by which the longest day at

[Margin: The habitable part of the earth is greater than the Author posits.]

[Margin: In how great a space the parallels on the earth are described by Ptolemy and by other astronomers.]

the beginning of a clime differs from the longest day at its end — namely, a half-hour — it divides into two equal quarters of one hour.

By this reasoning the more recent [astronomers] establish 23 climes, beginning from the first clime of the Ancients and proceeding toward the Arctic pole, until they find the longest day to comprise 23 hours — as will be clear from the following table, in which are contained also all the parallels, and the longest days of all the parallels, and the pole elevations (that is, how far they recede from the equator). Likewise, how many degrees each clime comprises from the equator toward the pole; whence the miles that a clime contains are easily found, by assigning to each degree $62\frac{1}{2}$ miles, according to Ptolemy.

[Margin: The more recent [astronomers] establish 23 climes.]

Further, these same parallels and climes are to be understood in the other hemisphere as well, from the equator toward the Antarctic pole — but in such a way that they receive opposite names. For example, let the Fifth Clime of the South be called the Opposite of the clime through Rome, and so on.

From what has been said it is easily understood what the difference is between a clime and a zone. For a zone is called a stretch of the earth lying between the two tropics, or between one of the tropics and the neighboring polar circle, or between one of the polar circles and the nearest pole of the world. By this reckoning five zones are found, of which two are called frigid, one torrid, and two temperate, lying between the torrid and the frigid. But a clime embraces a stretch of the earth in which the variation of the longest day amounts to a half-hour. From which it comes about that within a single zone several climes can be contained.

If anyone should wish to know more fully the properties of all the parallels, let him read chapter 6 of *Dictio* 2 of Ptolemy [*Almagest* II.6].

TABVLA CLIMATVM SECVNDVM Recentiores. (Table of the Climes according to the more recent [astronomers])

[Columns: parallel number; clime beginning/middle/end; longest day in hours and minutes; pole elevation in degrees and minutes; width of the clime (*Amplitudo Climatorum*) in degrees and minutes; naming-place.]

Para l.	Climata	H M . . G. (Poli)	M . G. (Ampl.)	M Denominaciones . Climatorum
1		1 0 0 2	0	
2		1 1 4 2 5	1 8 8	3 4
3		1 3 8 2 0	3 4	
4	Princip.	1 4 12 2 5	4 3	
5	I. Medium	1 0 16 3	4 7 3	5 Per Meroen. (Through 0 Meroe)
6	Finis.	1 1 20 3 5	3 3	
6	Princip.	1 1 20 3 5	3 3	
7	II. Medium	1 3 23 3 0	1 7 1	3 Per Syenem sub tropico ☉. (Through Syene, beneath

Para l.	Climata	H M . . G. (Poli)	M . G. (Ampl.)	M Denominationes . Climatum
				the tropic of Cancer)
8	Finis.	1 4 27 3 5	3 6	
8	Princip.	1 4 27 3 5	3 6	
9	III. Medium	1 0 30 4	4 6 7	9 Per Alexandriam Ægypti. (Through Alexandria of Egypt)
10	Finis.	1 1 33 4 5	4 5	
10	Princip.	1 1 33 4 5	4 5	
11	IV. Medium	1 3 36 4 0	3 5 0	1 Per Rhodum, & 7 Babylonem. (Through Rhodes and Babylon)
12	Finis.	1 4 39 4 5	2	
12	Princip.	1 4 39 4 5	2	
13	V. Medium	1 0 41 5	2 4 2	3 Per Romã, Corsicam, & 0 Hellespontum. (Through Rome, Corsica, and the Hellespont)
14	Finis.	1 1 43 5 5	3 2	
14	Princip.	1 1 43 5 5	3 2	
15	VI. Medium	1 3 44 5 0	2 3 9	4 Per Venetias, & 8 Mediolanum. (Through Venice and Milan)
16	Finis.	1 4 47 5 5	2 0	
16	Princip.	1 4 47 5 5	2 0	

Para l.	Climata	H M . . G. (Poli)	M . G. (Ampl.)	M Denominaciones . Climatum
17	VII. Medium	1 0 49 6	1 3	1 Per Podoliam, & 3 Tartariam minorem. (Through Podolia and Lesser Tartary)
18	Finis.	1 1 50 6 5	3 3	
18	Princip.	1 1 50 6 5	3 3	
19	VIII. Medium	1 3 51 6 0	5 2 8	4 Per Witebergam. 4 (Through Wittenberg)
20	Finis.	1 4 53 6 5	1 7	
20	Princip.	1 4 53 6 5	1 7	
21	IX. Medium	1 0 54 7	2 2 9	1 Per Rostochium. (Through 7 Rostock)
22	Finis.	1 1 55 7 5	3 4	
22	Princip.	1 1 55 7 5	3 4	
23	X. Medium	1 3 56 7 0	3 2 7	0 Per Hyberniam & Mocouiam. (Through Hibernia [Ireland] and Muscovy)
24	Finis.	1 4 57 7 5	3 4	
24	Princip.	1 4 57 7 5	3 4	
25	XI. Medium	1 0 58 8	2 1 6	4 Per Bohus castrum 0 Noruegiæ. (Through Bohus, a fortress of Norway)
26	Finis.	1 1 59 8 5	1 4	

TABLE OF CLIMES ACCORDING TO THE MODERNS (*Tabula Climatorum secundum Recentiores*) — continued

[This is the continuation of the "moderns" climes table begun on the preceding page; it here runs from parallel 26 to parallel 49, covering climes XII through XXIII. Each clime is given over three rows — its beginning (*Princip.* = Principium), its middle (*Medium*), and its end (*Finis*) — with the ending parallel of one clime serving as the beginning parallel of the next. Columns: parallels; clime (number and part); longest day (hours H, minutes M); pole elevation (degrees G, minutes M); amplitude of the clime (degrees G, minutes M); names of the climes.]

Paral leli	Climata	Max. dies H	M	Altit. Poli G	Amplit. Clim. G	Denominationes Climatum
26	Princip.	18	15	59		
27	XII. Medium	18	30	59	1	Per Gothiam.
28	Finis.	18	45	60		
28	Princip.	18	45	60		
29	XIII. Medium	19	0	61	1	Per Bergis Noruegiæ.
30	Finis.	19	15	61		
30	Princip.	19	15	61		
31	XIV. Medium	19	30	62	1	Per Viburgum Finlandiæ.
32	Finis.	19	45	62		
32	Princip.	19	45	62		
33	XV. Medium	20	0	63	0	Per Arociam Suetiæ.
34	Finis.	20	15	63		
34	Princip.	20	15	63		
35	XVI. Medium	20	30	64	0	Per Dalenkanlij fluuij ostia.
36	Finis.	20	45	64		
36	Princip.	20	45	64		
37	XVII.	21	0	64	0	Per reli-

Paral leli	Climata	Max. dies H	M	Altit. Poli G	Amplit. Clim. G	Denominationes Climatum
	Medium					
38	Finis.	21	15	65		
38	Princip.	21	15	65		
39	XVIII.	21	30	65	0	qua loca
	Medium					
40	Finis.	21	45	65		
40	Princip.	21	45	65		
41	XIX.	22	0	65	0	Noruegiæ,
	Medium					
42	Finis.	22	15	65		
42	Princip.	22	15	65		
43	XX.	22	40	66	0	Suetiæ,
	Medium		[?]			
44	Finis.	22	45	66		
44	Princip.	22	45	66		
45	XXI.	23	0	66	0	Albæ Russiæ,
	Medium					
46	Finis.	23	15	66		
46	Princip.	23	15	66		
47	XXII.	23	30	66	0	& vicinarum
	Medium					
48	Finis.	23	45	66		
49	XXIII.	24	0	66	0	Insularum.

Finis tertij capitis. — End of the third chapter.

CHAPTER FOUR

On the Circles and Motions of the Planets, and on the Causes of Eclipses of the Sun and Moon

(Caput Quartum. De Circulis, et Motibus Planetarum, et de Causis Eclipsium Solis, & Lunæ.)

[Sacrobosco's text (lemma):] Note that the Sun has a single circle, along which it moves on the surface of the ecliptic line, and it is eccentric. Now, not every circle is called eccentric, but only such a one as, while dividing the earth into two equal parts, does not have its own center at the center of the earth, but outside it. The point on the eccentric that comes closest to the firmament is called the *aux* [apogee], which is interpreted as "elevation"; while the opposite point, which is at the greatest distance from the firmament, is called the point opposite the *aux*. The Sun has two motions from west to east. One is proper to it, in its own eccentric circle, by which it moves each day and night nearly 60 minutes [of arc] above the poles of the axis of the circle of the signs [the zodiac]. The other, slower, is the motion of the sphere itself, and is equal to the motion of the sphere of the fixed stars — namely, one degree in 100 years. From these two motions, then, is compounded its course through the circle of the signs from west to east, by which it traverses the circle of the signs in 365 days and a quarter of one day, apart from a small amount that is of no perceptible account.

COMMENTARY

[Margin: *Argumentum 4. cap.* — Argument of chapter 4.]

In the preceding chapters the Author dealt with the motion of the *primum mobile*, which runs from east to west, and with what follows from that motion — namely the rising and setting of the signs, days and nights, and so on. Now, in the last chapter of this work, he discusses the motion of the other heavens, which runs from west to east, and especially the motion of the Sun and Moon, so as to open up for us the causes of lunar and solar eclipses. But since the Author touches on all this very briefly, we too shall be very brief in this part — the more so because this subject, if it is to be treated as it deserves, calls for a longer discussion, and belongs to the *Theories of the Planets* [*Theoricæ Planetarum*], which, God willing, we shall shortly bring to light.

The eccentrics and epicycles: by what appearances the astronomers discovered them in the heaven

(*Eccentrici, et Epicycli, quibus* [Greek: φαεινομένοις (phainomenois, "appearances," dative plural — i.e. "by what phenomena")] *ab Astronomis inventi sint in cælo.*)

Since in this passage the Author teaches, following the received doctrine of the astronomers, that the planets move on eccentric orbs and epicycles — which some of the philosophers, together with Averroes [Averroës, Ibn Rushd], try to abolish altogether as being at odds with Aristotle and with natural philosophy —

before I interpret the Author's text, I think it worth my while to set out briefly here (to satisfy those who have earnestly demanded it of me) the various observations that most moved Ptolemy, Alfraganus [al-Farghānī], Thebit [Thābit ibn Qurra], and nearly all the other astronomers to believe that there are eccentric orbs and epicycles in the heavens. Next I shall present the strongest arguments of Averroes and his followers, by which they attack orbs of this kind and try to destroy them entirely. Finally I shall dissolve those same arguments and show them to be worthless, so that anyone may understand that the astronomers discovered these orbs in the heavens not without reason, but with great diligence and remarkable success — whereas the philosophers who follow Averroes rashly assail them with so much vehemence. But before all else it must be explained briefly in what way the eccentric orbs and epicycles are to be conceived in the heaven, so that it may afterward be more easily understood that the phenomena observed by astronomers everywhere can, once those orbs are posited in the heaven, be defended without difficulty, whereas if those same orbs are removed the phenomena have no place and everything collapses utterly.

[Margin: *What the eccentric orb "simply so called" is.*]

The eccentric orb, then, in the heaven of any planet — the one that is called eccentric *simply* (*eccentricus simpliciter*) — is that whose concave surface and convex surface alike have their center distinct from the center of the universe, that is, of the whole heaven; so that it is of uniform thickness, like any celestial sphere, is immersed within the thickness of the whole heaven, and surrounds the earth itself. From this it follows that (since the whole heaven of any planet is of uniform thickness in every direction, and has a center common with the whole world) around the eccentric orb there stand two other orbs of non-uniform thickness, one above it and the other below — such that the upper one is thinnest at the part where the eccentric orb recedes furthest from the center of the world, but thickest at the opposite part, where the same eccentric is nearest the earth; while in the lower orb, on the contrary, its thickest part lies beneath the thinnest part of the upper, and its thinnest beneath the thickest. For in this way both the convex surface of the upper orb and the concave surface of the lower will have the same center as the whole heaven of the planet — namely the center of the world, as the case requires; whereas the concave surface of the upper orb and the convex surface of the lower will have the same center as the eccentric orb. And so the whole heaven [of the planet] will be equally distant from the center of the world on both its concave and its convex side — which would not be the case unless these two latter orbs, of unequal thickness, were placed around the eccentric orb.

[Margin: *What the eccentrics "in a qualified sense" are.*]

These latter are usually called by the authorities eccentrics *in a qualified sense* (*eccentrici secundum quid*), because on one of their outer surfaces they have the same center as the whole universe, but on the other a different one — just as the former is called eccentric *simply*, because on both its surfaces it has a center distinct from the center of the whole universe. And so, if the heaven of any planet

is cut by a plane through the two points of the eccentric simply so called — one of them the furthest from the earth, the other the nearest to the earth — there will result a cross-section such as the accompanying figure shows. In it the eccentric simply so called is represented by the white orb, whose center, both on its convex and on its concave side, is H. The two black orbs surrounding it represent the eccentrics in a qualified sense; the convex surface of the upper of these is ABC, and the concave surface of the lower is DEF, and its center is G, which is also the center of the whole world — so that the whole heaven is concentric with the world in the simple sense, that is, both on its convex and on its concave surface. But the concave surface of the upper orb and the convex surface of the lower are both described from H, the center of the eccentric simply so called. This being so, the whole heaven of each planet will be composed of three partial orbs: the eccentric simply so called, and two eccentrics in a qualified sense — except for the heaven of Mercury and the heaven of the Moon, each of which is made up of several orbs, as will be set out in the *Theories*.

Diagram. A circular schematic of the three partial orbs of a planet's heaven (the eccentric simply so called, flanked by the two eccentrics in a qualified sense), with the epicycle/Sun-circle drawn on the line of the aux. The two outer, shaded (black) rings are the eccentrics *secundum quid*; the inner clear (white) ring is the eccentric *simpliciter*; the small circles at top and bottom, on the vertical line, are the epicycle positions at apogee and perigee. A straight vertical line (A–C) runs through the centers, and is the line of the aux.

- A (top, on the vertical line): outermost point of the figure, at the aux [apogee] side.
- I (upper small circle, just below A): center of the upper epicycle position, where the Sun's center lies at the aux.
- M (left, on the middle circle): point on the eccentric (Sun's-center) circle IMO.
- D (upper center, top of the inner white ring): on the vertical line of the aux.

- H (upper center dot, within the white ring): center of the eccentric simply so called.
- E (left of center, on the horizontal): inner point on the white ring's boundary.
- G (center dot): center of the world (also center of the eccentrics *secundum quid*).
- N (on the vertical line, just below G): point on the line of the aux.
- F (lower center, bottom of the inner white ring): on the vertical line of the aux.
- O (lower small circle): center of the lower epicycle position, the point opposite the aux, nearest the earth.
- L (bottom, below the lower small circle): lowest point on the vertical line.
- B (lower left of the outer black ring): point on the convex surface ABC.
- C (bottom, on the vertical line): point on the convex surface ABC.

[Named points used in the surrounding text: convex surface of the upper eccentric = ABC; concave surface of the lower = DEF; center of the eccentrics *secundum quid* and of the world = G; center of the eccentric *simpliciter* = H; eccentric (Sun's-center) circle = IMO, with aux at I and its opposite at O; line of the aux = AC through G and H. One faint mark on the vertical line between the upper epicycle and D could not be resolved [?].]

An epicycle, on the other hand, is a small solid sphere immersed within the thickness of the eccentric simply so called, in such a way that it can revolve about its own center. Such a small sphere is represented in the said figure by the circle described from center O. Fixed on the epicycle is the planet, and it is carried along by the epicycle's motion about center O; and for this reason Ptolemy called it "the orb that carries the star or planet round" [*orbis revolvens stellam, seu planetam*]. The epicycle, in turn, is carried around the earth by the motion of the eccentric simply so called — with the exception of the Sun, which has no epicycle but, fixed in the eccentric simply so called, is carried around by that orb's motion. Hence the eccentric orb simply so called is named by practitioners the "carrier" (*deferens*) of the epicycle or of the planet.

[Margin: *What the eccentric circle, the aux, the point opposite the aux, and the line of the aux are.*]

Moreover the circumference IMO, described in the eccentric orb by the motion of the Sun's center, is usually called the eccentric circle. Its point furthest from the earth — such as I, which is placed beneath A, in which the Sun's center lies, and

which is marked by the straight line drawn through the centers G and H — is called the *aux* [apogee]; the opposite point O, nearest the earth, is called the point opposite the *aux*. Finally, the straight line AC, drawn through the centers G and H, is customarily named the line of the *aux*, because on it are found the *aux* and its opposite — that is, the point of the eccentric circle furthest from the earth and the point that comes nearest the earth, as is demonstrated in the *Theories*. But let us now come to explaining the phenomena that most impelled the astronomers to discover eccentric orbs and epicycles in the celestial spheres.

[Margin: *Why the ancients thought the stars were carried by chance.*]

To begin the matter a little further back, then: when the men of antiquity observed that the stars — especially the wandering ones, which are called planets — are carried by various motions, so that now they seemed to quicken their course, now to check it, now appeared almost to lack all motion (so that you would think they stuck fast in the same place of the heaven), now to move backward in the zodiac; and that at one time they came very close to the earth, at another withdrew to the greatest distance from it; and, in short, when they detected six hundred other varieties of this kind and, as it were, irregularities in the planets — they fell into the greatest and least tolerable errors about the motions of the heavenly bodies, supposing that these lacked fixed and settled laws in their motions, and that such varieties of motion befell them by some chance rather than by any firm and certain principle. But later men, of sounder mind, once they had begun to examine the celestial things more correctly, more subtly, and more scrupulously, came to the view that it is the height of madness to think that any irregularity, non-uniformity, or inequality can be found in the motions of the heavenly bodies; and that, on the contrary, the highest equality, uniformity, and regularity must be assigned to them. For since we see very many things in these lower and perishable regions move in orderly fashion and by a fixed, observed law, why should the same be denied to the heavenly bodies, which are the noblest of all? Indeed, natural reasoning too seems to persuade us that there can be no irregularity in the celestial motions.

[Margin: *That there is no irregularity in the motions of the heavens.*]

For if the heavens moved irregularly and unevenly, this would happen either at the beginning of the motion — as occurs with projectiles, which move faster at the start — or in the middle, as we see in animals, or finally at the end, as happens in natural [motions, i.e. falling bodies]. Since, then, the motions of the heavenly bodies have none of these limits, it is impossible that any inequality or irregularity should be found in them. Again, if the heavens moved irregularly, so

as to be driven now more slowly and now more quickly, this could not come about unless their motive powers were rendered now weaker and now stronger, or at least their resisting powers were now increased and now diminished. For a motion is made slower when, the resisting power in the moved body or in the medium remaining the same, the moving power is weakened; or when, the moving power remaining the same, the resistance in the moved body or medium is increased. A motion, on the other hand, is made faster when, the resistance in the moved body or medium remaining the same, the motive power is increased; or when, the motive power remaining the same, the resistance in the moved body or medium is diminished. But neither of these can be found in the celestial motions. For the Intelligences that, according to the common doctrine of the philosophers, move the heavens are wholly unchangeable; and the celestial bodies too, if we believe Aristotle and his followers, are free of all corruption, growth, and diminution, and are untiring. Therefore the celestial bodies are not driven by any irregular motion, but are carried around by fixed, perpetual, and constant laws. This the experience and the phenomena of the astronomers most clearly declare. For it has been established that the Sun always completes its period in the space of 365 days and nearly a quarter of one day; that Mars, too, circuits the whole zodiac in the space of about two years; Jupiter in 12; and so on for the remaining planets. It is proof, therefore, that the planets have fixed and settled laws of their motions; for otherwise it could not be that they kept such constant periods in their motions.

Although reason thus persuaded them that this was so, yet since many irregularities, as we have said, were daily observed by skilled astronomers in the motion of the heavens, they had to consider from where irregularities of this kind might arise. And the first thing that came into their minds was that each planet is carried around not by one motion, but by several.

If it had only a single motion, none of the appearances mentioned above — nor the others we shall explain further on — could be accounted for, since one and the same motion cannot at once be both regular and irregular. The conclusion, then, was that various motions had to be assigned to the individual planets: each motion, considered on its own, regular and uniform, as reason requires, yet all of them together producing that apparent irregularity, as will shortly become clear. And since, according to the decrees of Aristotle and the philosophers, it is impossible that one and the same celestial orb — being a simple body — should contain several motions, they were compelled to assign to the sphere of each planet several partial orbs, out of which the whole sphere is composed, so that from the multiplicity of the motions of these orbs they might explain the causes of

that apparent irregularity. Hence, the more varied the motion of a given planet appeared, the more motions and orbs had to be assigned to it.

[Margin: The sphere of the planets was divided into concentric orbs by Eudoxus and Callippus.]

These partial orbs, however, were not all set up in the same way by every astronomer. Eudoxus and Callippus — whose view was famous in Aristotle's day, as is clear from *Metaphysics* XII, and which even Averroes [Averroes] strives in many places, together with his followers, to defend — divided each of the total orbs of the planets into several concentric partial orbs, that is, orbs sharing a common center with the whole heaven and the universe. These, they held, were moved about different poles in different directions. From this arrangement it follows that, although each partial orb considered on its own moves regularly, nevertheless, because one retards or drives the other in some way, the planet itself appears to move irregularly. This opinion — which among more recent writers Girolamo Fracastoro [Fracastoro] strives with all his might to defend in the little book he entitled *On Homocentrics*, and which Lucillo Filalteo [Lucillo Filalteo] seems to approve in his books *On the Heaven*, with certain modifications — can indeed uphold some of the appearances that pertain to the slowness and speed of motion; but it can in no way give an account of all the appearances that daily experience detects in the planets, as we shall presently show.

[Margin: Ptolemy, with other astronomers, divided the spheres of the planets into eccentric orbs and epicycles.]

For this reason Ptolemy [Ptolemy] — easily the foremost of astronomers (though there are those who say the Pythagoreans did the same earlier, if less clearly and accurately, and that Hipparchus [Hipparchus] afterward imitated them) — together with al-Battānī [al-Battānī], Thābit [Thābit ibn Qurra], and a great many other astronomers, considering the defect of these homocentric orbs (that is, orbs having a common center with the whole heaven), was forced to devise another way to defend all the phenomena [φαινόμενα, *phainomena*, the observed appearances] observed in the planets, by which everything that appears in the motions of the planets might be maintained. After long reflection he saw — for he was of the keenest intellect — that this could be accomplished in no way more easily and conveniently than by eccentric orbs and epicycles, which have a center different from the center of the whole heaven, as we explained above. And so he divided each of the orbs of the planets into eccentric partial orbs, adding to each planet — with the Sun alone excepted — its own epicycles, because by eccentrics alone no account of all the appearances could be given. Averroes too, in his

commentaries on Ptolemy's *Almagest*, asserts that there are eccentric orbs and epicycles in the celestial spheres. The appearances that drove Ptolemy and other astronomers to believe that there are eccentric orbs and epicycles of this kind in the heavens were not few, and were altogether striking and remarkable; some of these we shall now bring forward.

[Margin: I. An appearance proving that eccentrics exist.]

I. The Sun, the Moon, and any other of the wandering stars — as has been most carefully observed by the more skilled astronomers — appears now farther from the earth, now nearer. Likewise (a consequence of the former point) its diameter appears now larger, now smaller, and thus the star itself seems now larger, now smaller. For the Sun (to leave aside the other planets for now), when it is in Capricorn or in the other southern signs, appears larger than when it is in Cancer or lingers in the other northern signs; so that at the present epoch it is largest in Capricorn and smallest in Cancer, its diameter being greatest there and least here. This inequality is gradually removed, and the Sun is seen at various magnitudes, according as it moves away from Capricorn or Cancer; and so its apparent diameter cuts off various arcs from the zodiac. Since, then — as is demonstrated by the writers on optics [Perspectivi] — the same object appears larger the nearer it is, and smaller the farther it withdraws from our sight, there is no doubt that the Sun, the Moon, and the rest of the planets are carried in orbs having a center different from the center of the earth, so that they can now approach nearer the earth and now recede farther from it. For if they were carried in orbs having the same center as the earth, they would always be equally distant from the earth, and would thus always present themselves to the gaze of our eyes at the same magnitude — which experience utterly contradicts.

To make this clearer: let the zodiac be ABCD, whose center E is also the center of the universe; and from another center, F, let the eccentric circle GHI be described, with three solar bodies, of which G, at apogee [Auge], is farthest from the center of the universe; I is nearest; and H is at a middle distance. Suppose, then, that the center of the Sun is moved in the eccentric circle GHI. It is clear that the body of the Sun, though in itself always of the same magnitude, nevertheless — because of its various and unequal distances from the earth, the cause of which inequality is the eccentric in which it is carried — appears to our sight now smaller, now larger, according as it holds a greater or lesser distance from us.

Diagram. The eccentric circle of the Sun set within the zodiac. The outer circle is the zodiac ABCD; the inner (eccentric) circle carries three positions of the Sun.

Lines are drawn from E (the world's center) as tangents to each solar position, marking off the arcs KL, MN, OP on the zodiac.

- A (top of outer/zodiac circle): with K (left) and L (right) flanking it — the tangent points EK, EL for the Sun at apogee.
- B (right of zodiac): with M (above) and N (below) flanking it — tangent points EM, EN.
- C (bottom of zodiac): with O (right) and P (left) flanking it — tangent points EO, EP.
- D (left of zodiac): opposite B, endpoint of the horizontal diameter DEB.
- E (center): the center of the universe / earth.
- F (just above E): the center of the eccentric circle.
- G (small circle at top, inside near A): the Sun at apogee (in Cancer), farthest from E.
- H (small circle at right, near B): the Sun at middle distance (in Aries, or in Libra).
- I (small circle at bottom, near C): the Sun at perigee (in Capricorn), nearest to E.
- Small zodiac-sign glyphs are engraved around the outer ring [individual glyphs too fine to resolve securely: ?].

Thus, when it is at G — namely in Cancer — its diameter, seen along the lines EK, EL that are tangent to the solar body, cuts off from the zodiac the arc KL, which contains four parts, of which about ten are contained in the arc OP that the tangent lines EO, EP cut off from the zodiac when the Sun is at I, that is, in Capricorn; and of which six, and a little more, are included in the arc MN, which is intercepted in the zodiac between the tangent lines EM, EN when the Sun is placed at H, that is, in Aries, or in Libra.

Now if the circle GHI, which carries the Sun beneath the zodiac from west to east, were described about E, the center of the universe (or of the zodiac), this appearance would have no place, since the Sun would always be equally distant from us. And the same must be said of the other planets. This appearance is granted by Averroes (so that it is remarkable how inconsistent he was on this point) in *Meteorologica* I, where he says:

It seems that Nature has equalized matters in this. For when the heat that comes by reflection is diminished — as when the Sun is in Capricorn — an equalization occurs in the heating on account of nearness; and, on the contrary, when intense heat occurs through reflection at right angles, or nearly so, while the Sun is in

Cancer, the Sun is then farther from the center of the earth, so that the heat is diminished. The same author, in *Metaphysics* XII, comment 45, admits that the Moon is sometimes farther off and sometimes nearer.

But to this appearance the adversaries reply, conceding that it is true that the Sun is sometimes seen larger and sometimes smaller — yet not because of a smaller or greater distance of it from the earth (for it is always equally distant, since, as they say, it is carried in a concentric orb), but on account of vapors that are interposed between the Sun and our sight and that scatter the visual rays, so that we behold the Sun now larger, now smaller, even though it is always carried in a concentric orb and at an equal distance from the earth. And the same, they say, must be maintained about the other planets.

Diagram. A reprint, on this page, of the same eccentric-Sun woodcut described above (fig-p304-1): the zodiac ABCD with the inner eccentric circle GHI carrying three solar positions (G at apogee, H at middle distance, I at perigee), center E (world) and F (eccentric), and the tangent points K, L / M, N / O, P marking the apparent-diameter arcs. All letter-labels identical to fig-p304-1; only the tiny peripheral zodiac-sign glyphs are unresolved [?].

But this reply is of no weight. For the Sun and the other planets were seen larger not only when the air abounded in vapors, but also when the sky was perfectly clear and the planet itself had the same altitude above the horizon. For example, the Sun, when it is in Cancer — where today the Sun's apogee is found — and has an altitude above the horizon of 20 degrees, so that its distance from the zenith is 70 degrees, always appeared much smaller to the most learned astronomers than when it is in Capricorn, where now the opposite of the apogee lies (granting the same clearness of air, and its altitude above the horizon likewise 20 degrees, and its distance from the zenith 70 degrees, as before). Nor, again, is there any force in what they say: that although the Sun keeps the same altitude and the sky is always clear, nevertheless, because when the Sun is in Capricorn (where we place the opposite of the apogee) it is winter, and hence the air is thicker, whereas when it is in Cancer (where we set the apogee) it is summer, and thus the air becomes rarer and finer, the Sun appears larger in Capricorn and smaller in Cancer. There is no force in this, I say, because sometimes in summer the sky is much hazier than in winter, and yet there the Sun was seen smaller and here larger. Further, because when the sky is clear the thickness of the air cannot be so great as to produce so great an inequality in the Sun's magnitude — especially since on two consecutive days, of which one was clear and the other hazy, so great a difference is never detected. Besides, let them say what they will about the

Sun; in the case of the Moon they are necessarily refuted. For the Moon, as is explained in its *Theorica*, changes its apogee every month, so that in the space of any one month its own apogee and the opposite of the apogee come to lie under each of the signs of the zodiac. And so, both in summer and in winter, it is found twice at apogee each month and twice at the opposite of the apogee; nevertheless it appears now smaller, now larger. Therefore the solution has no place in the case of the Moon. Moreover, the Sun does not always have its apogee fixed in the same sign, but a changeable one, as is demonstrated in its *Theorica*: its apogee moves always and continually toward the more eastern parts of the zodiac; and it will one day come about that its apogee is in Capricorn and the opposite of the apogee in Cancer. Yet the Sun, hitherto, like the Moon, has always appeared smaller at apogee than at the opposite of the apogee. And truly it would seem remarkable that, when the planets are at the opposite of the apogee, there should always be so much haze, but at the apogee so much clearness, that there they always appear larger in the same way, while here they appear smaller.

Seeing that this solution cannot at all satisfy the appearances adduced, Girolamo Fracastoro, examining the matter more subtly, devised another contrivance besides the interposed vapors. For he says that the planets appear larger not only because of thicker air interposed while they are in that region of the heaven where we place the opposite of the apogee, but also, and chiefly, because those parts of the heaven in which the opposite of the apogee is set are denser, so that the visual rays are refracted there, and on that account the planets appear larger and nearer to us. A subtle idea, to be sure, but an utterly futile fiction. For if the planets were seen larger on account of the density of those parts of the heaven, they would not appear of the same brightness and clearness through those denser parts and through the other, less dense parts; rather they would have less brightness there and more here — inasmuch as that density is so great that they appear sensibly larger — which is absurd. For the same planet, other things being equal, is just as bright and splendid when it appears larger as when it appears smaller. Add to this that, if that density existed, the same fixed stars in the zodiac would appear larger to us at one time — namely when those denser parts are supposed to lie beneath them — than at another; which conflicts with experience. Indeed, since the Moon is twice at apogee and twice at the opposite of the apogee each month, this appearance cannot be referred to that density, unless someone should say that the whole heaven of the Moon beneath the zodiac is sprinkled with such densities. Which is absurd. For it would follow that the Moon ought always to appear of the same magnitude. Therefore those denser parts cannot be placed in the heaven of the Moon.

[Margin: II. An appearance proving that eccentrics exist.]

II. The Sun is moved beneath the zodiac about the center of the earth, or of the universe, irregularly and unequally, as appears more clearly than the Sun's own light in the northern semicircle of the ecliptic and in the southern semicircle. For every year we find by experience that the Sun takes more days while it traverses the six northern signs contained in the former semicircle than while it lingers in the other six southern ones contained in the southern semicircle. For to be carried from the vernal equinox — that is, from the beginning of Aries — through Taurus, Gemini, and the other northern signs up to the autumnal equinox, that is, to the beginning of Libra, requires 187 days. But to be carried from the autumnal equinox — that is, from the beginning of Libra — through Scorpio, Sagittarius, and the remaining southern signs up to the vernal equinox, or to the beginning of Aries, only 178 days are needed. This anyone will easily discover if in the calendar he counts the days from 21 March inclusive, on which the vernal equinox falls in our epoch, up to 24 September exclusive, on which the autumnal equinox falls at this time. For there will be found 187 days there, but here only 178 days. From which it clearly fol-

—lows that the Sun moves unequally beneath the zodiac, since it traverses equal arcs of it — namely the two semicircles — in unequal times. But since the Sun, like the other stars, must (as we said above) be carried regularly by its proper motion in its own orb, it is clear that by its proper motion it is not carried about the center of the zodiac, or of the universe, since about this center it moves unequally, as has been said. Therefore it must be carried regularly about another center, different from the center of the universe, and thus in an eccentric orb — that is, one described from that other center. For it follows necessarily from this that the Sun moves irregularly beneath the zodiac and about the center of the universe, as experience teaches. For any star whatever, if it moves regularly about the center of an eccentric different from the center of the universe, must be carried irregularly about the center of the universe; and if it is carried about the center of the universe irregularly, it moves regularly about the center of the eccentric, that is, about another center.

For let the zodiac be ABCD, whose center E is the same as the center of the universe; let the eccentric be GHIK, whose center F is different from center E. Now let the line of the apogee, AC, be drawn through the centers E, F; and let the straight line BD cut it at right angles at center E. This BD will necessarily divide the zodiac into two equal semicircles BAD, BCD, since it is drawn through its center, but the eccentric into two unequal arcs, since it does not pass through its

center. Of these the greater will be HGK, in which the center of the eccentric and the apogee are found, and the lesser HIK, in which the opposite of the apogee lies. And so, if the Sun is supposed to move regularly in the eccentric circle about center F, it will traverse the greater portion HGK in a greater time than the lesser portion KIH. Now in the same time, with respect to the center of the earth E, the Sun completes the semicircle of the zodiac BAD in which it traverses the eccentric portion HGK; and in the time in which it passes through the eccentric portion KIH, it traverses the other semicircle of the zodiac DCB with respect to the center of the earth. For when the Sun is at point H of the eccentric, it stands, with respect to the center of the earth E, at point B of the zodiac; and while it is at point G of the eccentric, it appears at point A of the zodiac; and finally, while it is at point K of the eccentric, it is seen from the earth at point D of the zodiac. So that the Sun, while it traverses the eccentric portion HGK, is seen from the center of the earth to complete the semicircle of the zodiac BAD; and consequently it is seen to traverse the remaining semicircle of the zodiac DCB while it completes the other eccentric portion KIH. Therefore the Sun will traverse the semicircle of the zodiac BAD in a greater time than the semicircle DCB; and on that account it will move unequally beneath the zodiac — more slowly, namely, under the semicircle BAD, and more swiftly under the semicircle DCB.

Again, if the Sun is supposed to move unequally beneath the zodiac about the center of the universe E — so that, for example, it is carried more swiftly about point C than about point A — it will come about that it is necessarily driven regularly about another center, and in some eccentric orb. For since it is supposed to be carried more swiftly in the semicircle about point C than in the semicircle about point A, it will complete the former in a shorter time than the latter. Therefore in equal times it will traverse unequal portions of the zodiac — a greater one, namely, about C than about A. Let LCM, then, be the greater portion that the Sun traverses in the same time in which it traverses the lesser portion MAL. Now from E, the center of the universe (or of the zodiac), let the straight lines EL, EM be drawn; let equal segments EN, EO, of whatever size, be cut off between them; let the straight line NO be joined; from E let the perpendicular EF be raised to it; and let it be extended in both directions as far as the points A, C in the zodiac. And since in the triangle ENO the sides EN, EO are equal, [a] the angles N, O will be equal. But also the right angles at F are equal, and the sides EN, EO — which are opposite the right angles in the triangles EFN, EFO — are equal. [b] Therefore the sides FN, FO also will be equal. And so, taking F as center, a circle GNIO, described from F at the interval FN, will pass through point O. In this eccentric circle, then, about center F, different from the center of the universe, I say that the Sun moves regularly. For since the semicircles NIO, OGN are equal,

and the Sun traverses them in equal times — the very same times, namely, in which it passes through the unequal arcs of the zodiac LCM, MAL (for when the Sun is at point N, it appears in the zodiac, from E the center of the universe, under point L; and while it is at point O, it is seen under point M; and consequently the Sun is observed to traverse the portion NIO in the eccentric circle in the same time in which it traverses the arc of the zodiac LCM, and the remaining portion OGN in the same time in which it traverses the arc of the zodiac MAL) — it clearly stands established that the Sun moves uniformly and regularly in the eccentric circle GNIO, since it completes equal semicircles in equal times.

[Margin: a — [Euclid,] *Elements* I.5.]

[Margin: b — [Euclid,] *Elements* I.26.]

Diagram. A zodiac wheel with the twelve signs engraved around the outer ring, and within it the geometric construction of the Sun's eccentric circle GNIO about center F.

- A (top of outer circle): the beginning of the apogee diameter AC.
- C (bottom): opposite A; the vertical diameter AC (line of the apogee).
- B (right): endpoint of the horizontal diameter DB.
- D (left): opposite B.
- E (center): the center of the universe / earth.
- F (just above E): the center of the eccentric circle GNIO.
- G (top, inner): point of the eccentric on the apogee side.
- I (bottom, inner): point of the eccentric opposite G.
- N (inner, left of center): endpoint of segment EN.
- O (inner, right of center): endpoint of segment EO.
- K (lower-left, near center): point on the eccentric.
- H (lower-right, near center): point on the eccentric.
- L (outer, upper-left): point of the zodiac where the Sun at N appears.
- M (outer, upper-right): point of the zodiac where the Sun at O appears.
- The twelve zodiac-sign glyphs run around the outer ring [individual glyphs too fine to resolve securely: ?].

You see, then, that it is no wonder the Sun should move for more days from the vernal equinox to the autumnal equinox than from the autumnal to the vernal, if it is supposed to be carried in an eccentric orb; for it follows necessarily from this that it moves irregularly about the center of the universe and beneath the zodiac,

as we have shown. The same will be demonstrated for the other planets as well, as is plain.

Now this appearance of the irregularity of the planets' motion is so striking and clear that Ptolemy derives from it, by geometric reasoning, the eccentricity of the Sun — that is, the distance of the center of the Sun's eccentric orb from the center of the universe — and the place of the apogee in the zodiac; and, in the case of the other planets, the magnitudes of the diameters of the epicycles, and many other things, as we shall make clear, God willing, in the *Theorics*. This same appearance had such force with Averroes that it compelled him to admit, in *Meteorologica* I, that the Sun must move regularly in an eccentric orb, since about the center of the earth it moves so irregularly. So that even from this passage his inconsistency appears, since elsewhere he did away with eccentrics entirely by that same means.

[Margin: III. An appearance proving that eccentrics exist.]

III. It has been observed many times over that eclipses of the Sun have been unequal, even though in each the Sun and Moon held the same position; and this inequality could arise from no other cause than an eccentric. To make this clearer, we must take a point from the writers on optics: whenever some luminous body illuminates another smaller one, the nearer these two bodies are to each other, the greater the part of the smaller that is illuminated, and the more intensely; but the far-

COMMENTARY ON CHAPTER IV OF THE *SPHERE*

(*Christoph Clavius [Christoph Clavius], commentary on Sacrobosco's De sphaera, Chapter IV — continued: the arguments that the planets move on eccentrics and epicycles.*) a smaller shadow is produced than when they are separated by a greater distance. For in that case a smaller portion of the smaller body is lit, but a larger shadow is produced. Conversely, when some luminous body illuminates another that is larger, the smaller the distance between them, the smaller the portion of the larger body that is illuminated, but the more extensive the shadow that is cast — more so than when the one stands farther from the other. For in that case a larger portion of the larger body is lit, but a smaller shadow is produced.

Diagram. A solar-eclipse cone. At the right is a large circle marked **A** (the Sun, the luminous and larger body). At the far left is a large hatched (shaded) circle marked **L** (the Earth). Between them stand two small circles, each marked **B** (the

Moon, shown at two distances — one nearer the Earth, one nearer the Sun). Straight tangent lines are drawn from the Sun A past the edges of the Moon B, converging into a shadow cone that reaches toward the Earth L.

- A (large circle, right): the Sun (luminous, larger body)
- L (large hatched circle, left): the Earth
- B (small circle, nearer L): the Moon, in its farther-from-Sun / nearer-Earth position
- B (small circle, nearer A): the Moon, in its nearer-Sun / farther-Earth position
- E (top of the nearer B): upper tangent point on the first Moon-position
- F (bottom of the nearer B): lower tangent point on the first Moon-position
- I (top of the farther B): upper tangent point on the second Moon-position
- K (bottom of the farther B): lower tangent point on the second Moon-position
- G (top edge of A): upper tangent point on the Sun [an adjacent, faintly-inked second tangent point appears alongside it, possibly a repeated G or a C — [?]]
- H (bottom edge of A): lower tangent point on the Sun
- D (bottom edge of A, beside H): second lower tangent point on the Sun

All of this is set before the eyes in the figure provided, in which the luminous and larger body is A, while the opaque and smaller one is B — at one moment drawing nearer to A, at another standing farther from it. You see, then, that at the nearer distance the luminous body A lights up a larger portion of the smaller body B and produces a smaller shadow, whereas at the greater distance the same luminous body A illuminates a smaller portion of the smaller body B and casts a larger shadow. And again you see that if A, the larger body, is taken as opaque, and B, the smaller, as luminous, then a smaller portion of the opaque body A is illuminated by the nearer luminous body B, and a larger shadow is cast, than by the body B when it is farther off; for then a larger portion of body A is illuminated, and a smaller shadow cast — as is plain from the lines drawn tangent to both the Sun and the Moon.

With this established, it has been observed by the most skilled astronomers, more than once, that with the luminaries — the Sun, namely, and the Moon — remaining in the same position (for example, in the head or in the tail of the Dragon [in capite vel cauda Draconis], where both must necessarily be for an eclipse to occur, as we shall teach below), and the same parallax [diuersitate

aspectus] between our line of sight and the Sun being preserved, the solar eclipses (which arise from the interposition of the Moon between our sight and the Sun) have at one time been larger and lasted longer, appearing over a larger portion of the earth and obscuring a larger part of the Sun, than at another time. Now this could in no way happen unless we grant that those two planets sometimes have a smaller distance from the earth, or from each other, and sometimes a greater one. For when the Sun is farther from the Moon, then, as has been said, a larger shadow is cast upon the earth by the Moon — which is smaller than the Sun — and a smaller part of the Moon is illuminated by the Sun. From this it follows that at the time of a solar eclipse a larger tract of the earth is darkened, and the eclipse lasts a longer time. The contrary happens if the Sun has a smaller distance from the Moon: for then a smaller shadow is produced by the Moon upon the earth, and a larger part of the Moon is lit by the Sun; and accordingly, at the time of a solar eclipse a smaller surface of the earth is darkened, and the eclipse lasts a shorter time. As can be seen in the adjacent figure, in which the Sun's body is A, the earth L, and the Moon B — now farther from the Sun and nearer the earth, now nearer the Sun and farther from the earth. Since, therefore, these two luminaries cannot have a smaller or larger distance from each other, or from the earth, unless they are supposed to move on eccentrics — for if they were carried on concentrics they would always keep the same distance both from each other and from the earth, as is evident — it is entirely reasonable that there are in the heavens eccentric orbs in which the planets move, so that they may sometimes be more, and sometimes less, distant from each other or from the earth, and so that a cause can be assigned for that inequality in the solar eclipse.

And to bring forward, for the common benefit, what I too have myself observed on occasion in this matter, I shall recount two remarkable eclipses of the Sun that happened in my own time, not long ago. One of them I observed in the year 1560 at Coimbra [Conimbricæ] in Portugal [Lusitania], about noon; in it the Moon was interposed directly between our sight and the Sun, so that it covered the whole Sun for no small interval of time, and the darkness was somehow deeper than that of night. For one could not see where to set one's foot, and the stars shone out most brightly in the sky, and — a marvellous thing — birds fell from the air to the ground out of dread at so foul a darkness. The other I witnessed at Rome [Romæ] in the year 1567, likewise about noon; in it the Moon was again set between our sight and the Sun, yet it did not obscure the whole Sun as in the former eclipse, but (what perhaps never happens otherwise) there was left in the Sun a certain thin ring encircling the whole Moon on every side. From these two eclipses it is plainly gathered that the Sun and the Moon did not have, in both

eclipses, the same distance from the earth, or from each other. For if they had had the same distance both from each other and from the earth, who does not see that the Sun ought to have been obscured in the same way in both eclipses? This is easily demonstrated from the science of optics [à Perspectiuis], and the matter is as plain as a thing held in the hand. For if a hand keeps always the same distance from some wall and from the eye, being placed between the wall and the eye, it will always take the same portion of the wall from view, and not now a larger and now a smaller one. Therefore it can in no way be said that these two luminaries move on concentric orbs, since on that supposition they would always be equally distant from each other and from the earth, and so this observed appearance of solar eclipses could have no place at all.

Again, it has not seldom been noticed that, with the same luminaries remaining in the same position — namely, with the one in the head of the Dragon and the other in the tail, and the Moon having the same latitude — the lunar eclipses (which occur from the interposition of the earth between the Sun and the Moon, because the Moon then enters the earth's shadow, so as to be no longer lit by the Sun's rays, as we shall say hereafter) have at one time begun earlier and been larger, and lasted a longer time, than at another. This could by no means have happened unless the Moon had entered a larger shadow of the earth in one eclipse than in another. For it is thus that in the one it needed a longer time to extricate itself from the shadow than in the other, and so the eclipse of the Moon there proved larger than here. But the earth cannot make a larger shadow at one time than at another unless the Sun approaches it now more, now less — as we taught at the beginning of this third appearance. Nor again could the Moon, if the earth's shadow were always the same, pass now through a larger shadow and now through a smaller, unless it approached the earth more at one time than at another. Since, then, neither the Sun nor the Moon can come nearer the earth at one time than at another unless we assign to each planet an eccentric on which it is carried around — as is evident — it will not be foreign to the truth to hold that eccentric orbs exist among the celestial spheres. An example of this

you have in this adjoining figure, in which A signifies the Sun, now nearer to the earth B, now farther removed from it. From this it comes about that the earth's shadow is sometimes smaller and sometimes larger — the shadow, indeed, through which the Moon, represented by the letter C, passes in the eclipse. And this appearance had such force even with Averroes [Averroes] that

Diagram. A second eclipse-cone figure. At the right stand two large circles side by side, each marked A (the Sun, at two distances from the earth). In the middle is a

hatched (shaded) circle marked **B** (the earth), from whose edges tangent lines run leftward to form the earth's shadow cone. At the left, three small circles, each marked **C**, are ranged along the axis of the cone (the Moon, shown at three positions as it passes through the earth's shadow).

- A (large circle, right): the Sun, in its position nearer the earth
- A (large circle, right of the first): the Sun, in its position farther from the earth
- B (hatched circle, centre): the earth, casting the shadow cone
- C (small circle, upper left): the Moon, first position on the shadow axis
- C (small circle, middle left): the Moon, second position
- C (small circle, lower left): the Moon, third position

he sincerely asserted, in *On the Heavens* Book II, comment 32 [Averroes, *De Caelo* II, comm. 32], that perhaps this appearance concerning the lunar eclipse could be defended in no other way than by an eccentric orb — a thing which nonetheless he elsewhere denied. Behold another inconsistency of Averroes!

[Margin: IV. An appearance proving them to be eccentric.]

IV. In the Moon, Mercury, and Venus astronomers have not always found the same parallax [diuersitas aspectus], but now a larger and now a smaller one, even when the planet held the same position: so that in the Moon, for example, the parallax has sometimes amounted to $1^{\circ}6'$ [grad. 1. min. 6], and sometimes only to $0^{\circ}50'$ [grad. 0. min. 50], as Gemma Frisius [Gemma Frisius], no undistinguished writer among the moderns, says — and this with the Moon at the same altitude above the horizon. It is therefore necessary that the planet becomes now higher, now lower with respect to the centre of the earth. For when a planet is lower — that is, nearer the earth — it admits a larger parallax [aspectus diuersitatem], but when it is borne higher and farther from the earth, a smaller one; provided that it has the same altitude above the horizon there as here, as we demonstrated above

Diagram. A parallax graticule. A semicircular fan is drawn, ruled with radial lines running out from a common centre and crossed by concentric arcs, forming a grid (a graticule of altitude). At the bottom centre, at the fan's apex, sits a small hatched (shaded) half-circle representing the earth (the observer's globe). Straight lines are drawn from the centre of the world and from the observer's eye through the star's centre, illustrating the difference between the true and the apparent altitude. No point-letters are lettered on this figure.

in Chapter I, when we disputed on the order of the celestial spheres; and it also appears plainly in this present figure, in which, to the left, the star — now farther from the earth, now nearer it — keeps the same altitude with respect to the straight line drawn from the centre of the world through the star's centre; that is, it keeps the same true altitude, or the same true place. To the right, however, the star — now less, now more distant from the earth — keeps the same altitude with respect to the straight line drawn from the eye, or from the surface of the earth, through the star's centre. Now one and the same star cannot become now nearer the earth and now farther from it if it is carried on a concentric orb, but only if it is carried on an eccentric, as is clear from what has been said. Not without reason, therefore, did the astronomers affirm that the planets are carried around on eccentric orbs.

These are the four appearances (many others being left aside) by which the astronomers rightly strive to establish that the spheres of the planets are composed of eccentric orbs, on which they are borne by their proper motions from west to east. And these same appearances, in the same order, prove and convincingly show that in all the planets except the Sun alone there are also epicycles, on which the planets themselves revolve — as will become clear from what now follows.

[Margin: I. An appearance proving that there are epicycles.]

I. The planets, the Sun excepted, when they are at the apogee of the eccentric [in *Auge Eccentrici*] — that is, at the point of the eccentric most remote from the earth — do not always keep the same relation to the earth. For they are borne now higher, now lower; and (as follows from the first appearance) their diameters are now smaller, now larger; and consequently the planets themselves appear now smaller, now larger, and cut off with their diameters now a smaller, now a larger portion of the Zodiac. The very same thing happens to the planets when they are at the point opposite the apogee of the eccentric. But this variety cannot come about by reason of the eccentric alone. For since the apogee of the eccentric is always at the same distance from the earth, a planet situated at the apogee would always appear the same as regards nearness and distance, largeness and smallness; and the same would happen to a planet situated at the point opposite the apogee. For a planet ought always to be most remote from the earth at the apogee, and nearest at the point opposite the apogee (as we find in the Sun, which is carried on an eccentric orb alone) — whereas it in fact appears sometimes more remote and sometimes nearer, both at the apogee of the eccentric and at the point opposite the apogee. There will therefore be, immersed

within the thickness of the eccentric, an epicycle by whose motion the planet revolves. For in this way we shall assign the cause of the aforesaid variety without any difficulty. Let the Zodiac, then, whose centre is the same as the centre of the world, be A; and let the eccentric that carries the planet be B C D E, whose centre F is distinct from the centre of the world; and let the apogee of the eccentric be B, and the point opposite the apogee D. Now if the Moon, for example, moved on this eccentric alone, it would without doubt always be seen at its most remote and smallest at the apogee B, and would always appear nearest and largest at the point opposite the apogee, D. But the contrary of this has been found to happen by the astronomers. Yet if an epicycle G H I is posited, on which the planet is fixed at the point G or I, it plainly follows that the Moon (and understand the same of the other planets too), although it be at the apogee of the eccentric or at the point opposite the apogee, nonetheless appears more remote from us — because it is then found, for example, on the epicycle at the point G — than when it stood on the epicycle at the point I. But perhaps someone will say that the eccentrics were granted in vain, if by means of the epicycle we can maintain that the planets are now more remote from the earth, now less distant. To this the answer is that, just as by the eccentric alone this appearance

Diagram. The eccentric-and-epicycle construction. A large circle (the Zodiac) is drawn with a horizontal and a vertical diameter crossing at the centre A (the centre of the world). Within it an eccentric circle is drawn whose centre is F, offset above A along the vertical. On this eccentric: B marks the top (the apogee), D the bottom (the point opposite the apogee), E the left, and C the right. At the apogee B sits a small epicycle lettered G (top), H (right), I (bottom); and at the opposite point D sits a second small epicycle lettered I (top), H (right), G (bottom).

- A: centre of the world (= centre of the Zodiac), at the crossing of the diameters
- F: centre of the eccentric, offset above A on the vertical
- B: apogee of the eccentric (top)
- D: point opposite the apogee (bottom)
- E: left point of the eccentric
- C: right point of the eccentric
- G, H, I: points on the epicycle (upper epicycle: G top, H right, I bottom; lower epicycle: I top, H right, G bottom)

cannot be defended, so likewise the same appearance cannot be defended by the epicycle alone. For it has been ascertained by the mathematicians that the Moon, for example, when situated at the point G of the epicycle — most remote from the

earth — did not always keep the same distance from the earth, nor always appear of the same magnitude. They found the same thing to happen while the Moon was at the point I of the epicycle, nearest the earth. And they observed the same in the other planets. It is necessary, therefore, that the epicycle be carried on an eccentric orb, and not on a concentric, in order that so great a variety may find a place. Hence not in vain is an eccentric set up in the planets besides the epicycle, since both orbs are necessary for us to save the aforesaid appearance. I myself certainly saw, a few years ago, Mars of such magnitude that it then appeared, under a most serene sky, twice as large as at another time, and many marvelled, supposing that a new star had blazed out in the heavens. This I have said for the purpose that the studious reader may see how conspicuous is this appearance concerning the magnitude of the planets — an appearance which cannot be defended without eccentrics and epicycles — so that of its own accord it sometimes presents itself to our very eyes without the aid of instruments.

[Margin: II. An appearance proving that there are epicycles.]

II. All the planets, the Sun excepted, when situated at the apogee of the eccentric — although of themselves they move more slowly there with respect to the centre of the earth, as was said above concerning the Sun — nonetheless have been found to possess yet another irregularity there. For the Moon, for example, has sometimes been seen to move more quickly at the apogee, and sometimes more slowly; and the same has been ascertained at the point opposite the apogee: so that the Moon sometimes traverses in the Zodiac about 15° [gr. 15] in one day, but on another day only 11° [gr. 11]. And this indeed cannot be defended by the eccentric alone (for otherwise the same appearance would have to be found in the Sun, which is false, since the Sun always moves with the same slowness while it is at the apogee, and with the same speed while it is at the point opposite the apogee); but we shall maintain it very easily if we suppose the Moon to move on an epicycle, and the epicycle on an eccentric, as is clear from the figure above. For if the eccentric of the Moon moves in the order of the signs (as in fact it does move) — that is, from ♈ [Aries] into ♉ [Taurus], and from ♉ [Taurus] into ♎ [the printed sign reads as Libra; the succession of the signs would give Gemini ♊ — [?]], and so on, namely in the said figure from C into B, and from B into E, and so on — while the epicycle of the same is borne, in its upper part (as is shown in its *Theorica*), against the order of the signs, that is, with a motion contrary to the motion of the eccentric — say from G into H, taking the upper epicycle in the figure, or from H into G, taking the lower — but in the lower part of the epicycle in the order of the signs, just like the eccentric, namely, on the upper epicycle of the named figure from H into I, and on the lower from I into H: then it is plainly

understood that the Moon, while it revolves in the upper part of the epicycle, is carried more slowly, since it is borne against the motion of the eccentric; but in the lower part more swiftly, since its motion is, as it were, doubled toward the same quarter. There is added, moreover, that the Moon moves uniformly on its own eccentric about the centre of the earth (as we shall demonstrate with Ptolemy [Ptolemy] in its *Theorica*); so that without the epicycle we cannot render the reason for this slowness and swiftness. This variety has been noted, in its own way, in the other planets too, the Sun excepted; and hence they too revolve on epicycles.

Furthermore, an epicycle has been found much more evidently in the three superior planets, Mars, Jupiter, and Saturn, and also in Mercury and Venus. For these planets are seen now to go forward in the Zodiac from the western parts toward the eastern, now to go backward from the eastern parts toward the western. For while they are in the upper part of the epicycle, they are turned in the order of the signs, just as is the motion from west to east, and thus they appear to go forward: so that if, for example, one of them is in the 1st degree of Leo [grad. 1. ♌], it will presently be in the 2nd, then in the 3rd, and so on. But while they are engaged in the lower part of the epicycle, they are moved against the order of the signs — that is, against the motion which the epicycle has on the eccentric — and so they appear to go backward: so that if, for example, one of them is at the 4th degree of Leo [gr. 4. ♌], it will presently be at the 3rd, then at the 2nd, and so on. All of which will be more clearly explained in the *Theoricae*. But why this retrogradation does not appear in the Moon, although it too is moved into diverse and dissimilar parts on its own epicycle, we shall show in its *Theorica*. And so, since this appearance can in no way be defended without an epicycle, but very easily with one posited, as is clear from what has been said, it will be probable that every planet, the Sun excepted, moves on an epicycle.

Diagram. A second copy of the eccentric-and-epicycle construction, identical in lettering to the figure on the preceding page. A large circle (the Zodiac) has horizontal and vertical diameters crossing at the centre **A** (centre of the world). Within it is the eccentric circle, centre **F** (offset above **A**); on it **B** is the apogee (top), **D** the point opposite (bottom), **E** the left, **C** the right. A small epicycle at the apogee **B** is lettered **G** (top), **H** (right), **I** (bottom); a second epicycle at **D** is lettered **I** (top), **H** (right), **G** (bottom).

- **A:** centre of the world / centre of the Zodiac
- **F:** centre of the eccentric (offset above **A**)
- **B:** apogee of the eccentric (top)

- D: point opposite the apogee (bottom)
- E: left point of the eccentric
- C: right point of the eccentric
- G, H, I: epicycle points (upper epicycle G top / H right / I bottom; lower epicycle I top / H right / G bottom)

[Margin: III. An appearance proving that there are epicycles.]

III. The ancient and diligent observers of the stars considered, on occasion, two lunar eclipses with the Sun and Moon remaining in the same position in both — for instance, with the Sun in the head of the Dragon and the Moon in the tail — and with the Sun in both being at the same point of its eccentric, so that in both it had the same distance from the earth, and accordingly cast the same shadow of the earth in both cases. And they found that one of the eclipses lasted a longer time than the other. Now the cause of this inequality cannot be assigned to the eccentric of the Sun. For the greater or lesser duration of an eclipse comes about from the Moon's entering a larger or a smaller shadow of the earth; but at that time the earth's shadow was in both eclipses always the same, since the Sun is supposed to be equally distant from the earth in both. It is necessary, therefore, that the Moon itself was in one of the eclipses less remote from the earth, and in the other more remote. For since the earth's shadow is extended into a cone (because the earth is smaller than the Sun), it comes about that the nearer the shadow is to the earth, the broader it is, but the farther from the earth, the narrower and less broad. From which it follows that the Moon, the nearer it is to the earth, the larg-

Diagram. The lunar-shadow (umbra) figure combined with the eccentric-and-epicycle. At the right is a large circle marked **A** (the Sun). To its left a smaller circle **F**, and then the system of concentric circles about the centre **E** (the earth). The outer circle carries **B** at the top and **D** at the lower left; inner circles carry **K** (upper), **I** (above centre), **H** (below centre), and **L** (lower). From the earth's region a hatched (shaded) shadow cone runs leftward to a point; within the cone stands **G** (the Moon, in the earth's shadow), and above the left end of the cone is **C**.

- A (large circle, right): the Sun
- F (circle between A and E): intermediate circle / centre between Sun and earth
- E (centre): the earth
- B (top of the outer circle): apogee point
- D (lower left of the outer circle): point opposite the apogee

- C (upper left, at the end of the shadow cone): boundary point of the shadow cone
- G (within the shaded cone, left): the Moon, in the earth's umbra
- K (below B): point on a middle circle (upper)
- I (above E): upper epicycle point
- H (below E): lower epicycle point
- L (below H): point on a middle circle (lower) ...[the Moon] passes through a greater portion of the shadow, but a smaller one the farther it has receded from the earth; and so eclipses turn out unequal in magnitude and duration. But this smaller or greater distance of the Moon from the earth during a lunar eclipse cannot in any way be attributed to its eccentric.

Diagram. A composite eclipse figure illustrating the theory of the Sun's eccentric, the Moon's eccentric, the ecliptic, and the Moon's epicycle at the two nodes. Three roughly concentric circles are drawn about a common region, with the earth E near the centre, a shaded wedge (the earth's shadow-cone) tapering to the left toward C, and small circles (the Moon's epicycle) shown at the right and within the shadow at the left.

- B (top of the outer circle): a point on the Sun's eccentric *ABCD*.
- D (bottom of the outer circle): a point on the Sun's eccentric *ABCD*.
- C (far left, apex of the shaded wedge): a point on the Sun's eccentric *ABCD*; the earth's shadow narrows to a point here.
- A (right, inside the small circle at the right): the Sun, located at the head of the Dragon [caput Draconis].
- E (centre): the earth.
- F (right, on the ecliptic and the Moon's eccentric): the head of the Dragon [caput Draconis], a node.
- G (left, within the shaded shadow-cone): the tail of the Dragon [cauda Draconis], a node, and the centre of the Moon's epicycle shown there.
- I (upper, on the Moon's eccentric *FIGL*): a point on the Moon's eccentric.
- L (lower, on the Moon's eccentric *FIGL*): a point on the Moon's eccentric.
- K (upper, on the ecliptic *FHGK*): a point on the ecliptic.
- H (lower, on the ecliptic *FHGK*): a point on the ecliptic.

For by reason of the eccentric the Moon has one and the same distance from the earth in every eclipse, whether solar or lunar; this is because the Moon (as will be shown in its Theory) always stands at the aux [apogee] of its eccentric, both at its conjunctions with the Sun and at its oppositions. For every solar eclipse occurs at

some conjunction, and every eclipse of the Sun [the print reads *Solis*; the sense requires the Moon — *Lunæ*] at some opposition. We must therefore have recourse to the epicycle. For in this way we shall account, without great labor, for this inequality of lunar eclipses, even though the two luminaries have the same position with respect to the head and tail of the Dragon [the nodes], the Sun is always equally distant from the earth, and the Moon stands at the aux of its eccentric. For in one eclipse the Moon can be at the point of the epicycle nearest the earth, and in another at the point most remote from the earth. Hence the former eclipse will be greater and will last longer than the latter, because in the former the Moon passes through a larger portion of the earth's shadow, in the latter a smaller.

You have an example in the figure above, in which *ABCD* represents the Sun's eccentric; *FIGL* the eccentric that carries the centre of the Moon's epicycle; and *FHGK* the ecliptic, which cuts the Moon's eccentric at the points F and G, of which F, say, is called the head of the Dragon, and G the tail. A is the Sun located at the head of the Dragon; E is the earth; and G is the centre of the epicycle located at the tail of the Dragon, and so on.

But should someone object that it follows from this that we did not rightly infer above, from eclipses, that a solar eccentric exists — since, as we have just said, a greater and a lesser eclipse can be produced by the epicycle — the reply is that the Moon's epicycle does not suffice. For two lunar eclipses have been found unequal to each other, although the luminaries were in the same position (as we said) with respect to the head and tail of the Dragon, and the Moon remained in the same part of the epicycle — say, either the upper or the lower part. Now no cause of this inequality can be assigned unless we say that in one eclipse the luminaries had a smaller distance between them, or at least that one of the two [luminaries, printed *Planetarum*] had come nearer to the earth or receded farther from it than in the other. Since, then, this smaller or greater distance cannot be referred to the Moon's epicycle — because the Moon is assumed to be always in the same part of the epicycle, that is, to have been so in both eclipses — an eccentric too must necessarily be granted.

[Margin: IV. An appearance proving that epicycles exist.]

IV. It has been observed that the Moon, when situated at the same point of its eccentric — at the aux [apogee], for example, or at the point opposite the aux — does not always have the same diversity of aspect [parallax], but now a greater, now a smaller one. This can happen in no way unless, at the same point of the eccentric, the Moon at one time comes nearer the earth and at another is more

distant from it. For this reason an epicycle must be granted to the Moon as well; for once this is posited, the said appearance will present no difficulty whatever.

Diagram. A semicircular construction (drawn as the upper half of a circle) with straight lines radiating from the centre of the base to the arc, and two small circles (representing the epicycle) set on these lines with intersecting sight-lines drawn through them. It illustrates the argument that follows: on the left, two opposite points of the epicycle are taken *as seen* (along the line from the eye through the epicycle's centre); on the right, two opposite points are taken *as they truly are* (along the line from the earth's centre through the epicycle's centre). No lettered labels are visible on the figure.

As can be seen clearly in the figure here presented: on the left two opposite points on the epicycle are taken *as they are seen* — namely, along the straight line drawn from the eye through the centre of the epicycle; whereas on the right two opposite points on the epicycle are taken *as they truly are* — that is, along the straight line drawn from the centre of the earth through the centre of the epicycle. At these points the lunar body is placed. The rest is clear from the figure itself.

[Margin: Further reasons confirming that eccentrics and epicycles exist. First reason.]

To these appearances, and to many others which we deliberately pass over here, three further arguments are added which seem to confirm that eccentric orbs and epicycles do exist in the celestial spheres. Let the first be as follows.

It is accepted by all astronomers and philosophers, as evident and self-evident, that every higher celestial orb draws along with it, by its own motion, the lower orb that is contiguous and concentric to it. This we have learned — with experience herself as our teacher — to be perfectly true. For we see the spheres of all the planets, together with the Firmament and the ninth heaven, swept from east to west in the space of 24 hours by the diurnal motion of the primum mobile. Again, we find that these same planetary spheres, together with the Firmament, are drawn from west to east by the motion of the ninth sphere — though very slowly, namely in the space of 49,000 years according to Alfonso [the Alphonsine reckoning], or, according to Ptolemy, in the space of 36,000 years. Finally, it has been noticed that all the heavens of the planets are also moved little by little with the motion of trepidation — that is, the advance and retreat of the eighth sphere. A sign of this is that the greatest declination of the Sun and of the other planets has changed.

Since, therefore, the greatest individuality of motions is found among the planets — so that no proper motion of any higher body is communicated to a lower planet (as can be known to any astronomer even slightly experienced, adversaries included, and is denied by no one: for Jupiter has nothing whatever from Saturn's 30-year motion, and likewise nothing is communicated to Mars from Jupiter's 12-year motion, and so on for the rest, as all affirm) — it seems clear that the orbs which carry the planets are not concentric. Otherwise the motion of any higher body would be communicated to all the lower planets, just as we see happen in the total spheres, as we said. Since we do not see this happen — as the adversaries themselves testify — it cannot be said that the planets are carried in concentric orbs, but rather in eccentric ones. For in this way that experience just cited, concerning the individuality of the planets' motions, will very easily find its place.

For the diversity of centres is what prevents the eccentric orb of any planet from sweeping along with it the immediately lower orb contiguous to it — whose concave surface is concentric with the whole world — unless a penetration or splitting of the heavens were admitted, as can readily be perceived from a material instrument [an armillary model]; and in any case it is understood from the first figure of this question. For how can it happen — if the matter is considered more attentively — that a simply eccentric orb *IMO*, [turning] about its own centre H, should draw along the next lower orb, one that is eccentric only in a qualified sense [secundum quid], whose concave surface, together with the whole heaven, is equally distant from the centre of the world G, unless this lower orb penetrates or cuts the heaven of the lower planet, which is contained within the concave of the said orb that is eccentric in the qualified sense?

I know that the authors of concentric orbs invent, within the orbs of each individual planet, certain "restoring" orbs — which Fracastoro [Girolamo Fracastoro] calls *Circitores* — whose office is that, by however much the higher planets draw the lower ones along by their motions, by just so much these restoring orbs carry the lower planets back in the opposite direction. But this seems very like a fiction. For, apart from the fact that in this way the greatest confusion is introduced into the motions, I do not see how the *primum mobile* could communicate the diurnal motion to all the lower spheres, since those *Circitores* are placed in between and wholly prevent the lower spheres from being swept along by the higher — unless someone says that the individual planetary spheres have their own diurnal motions from east to west, completed in the space of 24 hours: which is novel, unheard of, and hitherto granted by no one.

[Margin: Second reason.]

The second argument is this. If the planets are not carried from west to east in eccentric orbs, then they must be conveyed either through concentric orbs, or else move by themselves in the heavens, like fish in the sea or birds in the air. But they do not move in either of these two ways. Therefore they are carried in eccentrics. The inference is manifest; the major premise, too, is clear from a sufficient enumeration of the parts; and the minor is proved with respect to each part.

That the planets do not move by themselves (to begin with the latter part) like fish in the sea or birds in the air, Aristotle strives to prove by many arguments in his book *On the Heavens* [*De Caelo*]; and it was confirmed by us above by an evident argument, when in chapter 1 we showed, along with the Author [Sacrobosco], that the heaven turns from east to west; and it is the common teaching of all philosophers and astronomers. Indeed, if they moved in that way, and not rather by the motion of the orbs in which they are, we could have no certain knowledge of their motions. For since, as was said among the appearances above, the planets are sometimes farther from and sometimes nearer to the earth; sometimes move more swiftly, sometimes as it were check their course; now seem to stand still, now to advance beneath the Zodiac from west to east, now to move backward — who does not see that, if the planets moved like fish or birds, they would sometimes have to abandon the circles they describe from west to east (so as to be able to recede farther from the earth and approach it again); sometimes to neglect their proper course and strive backward in the opposite direction; and finally sometimes to halt their course altogether in the heaven, so as not to move at all? And if these things happened, by what means, pray, could their periods be determined? By what method, likewise, could one know in which part of the heaven the planets would depart higher from the earth and again return toward it, and so on?

That the planets are likewise not carried around from west to east in concentric orbs will become clear as follows. First, because on this supposition the phenomena adduced above cannot be saved — especially those observed concerning the greater and lesser distance from the earth, and the greater and lesser apparent size of the planets. And if they contend that they can save the other appearances — namely the slowness and speed of motion, and the direct motion, retrogradation, and station of the planets — by means of concentric orbs, they seem to accomplish this only in a general way and very confusedly. For they say that all these things come about because one concentric orb now retards

another, now advances it more, now leads it backward; but in what part of the heaven these things ought to happen they do not teach.

Next, because many absurdities and awkward consequences follow from positing concentric orbs. The first, indeed, is that — as was said a little earlier — the motions of the higher orbs would be communicated to the lower planets, which conflicts with experience. And further, because, wishing to save everything by concentric orbs, they invent certain orbs within the spheres of the planets to carry them from north to south and vice versa. Once this is posited, who is so dull and sluggish as not to see that the Sun could not always travel beneath the Ecliptic — chiefly beneath the Ecliptic of the *primum mobile* — since it is not carried by that [north–south] motion, seeing that of itself it is set in motion from east to west, whereas one simple body can have only one motion? Indeed, if it were moved from north to south, or the reverse, then in the same city the altitude of the pole would perpetually change — which is contrary to the most manifest experience. And who, again, is so crude and ignorant as not to perceive, once this is granted, that the Sun would sometimes come to be at the Arctic pole, and sometimes set in the very place where we now see it rise? This, indeed, Hieronymus Fracastorius [Girolamo Fracastoro], the foremost champion of concentric orbs, candidly admits — and it readily appears on a material sphere — that such a thing would sometimes have to follow from this kind of north-to-south motion of the heavens, and the reverse. He even affirms that, according to certain Egyptians, this has already happened twice since the founding of the world.

But how false and ridiculous this is, who does not see? For from the histories and traditions of mathematicians and philosophers we have known that, for a span of 2,000 years and more, down to the present (to say nothing of the ages before that), the Sun and the other wandering stars have, on the fixed days of the year, risen and set at nearly the same point of the horizon in the same city, have had the same meridian altitude, and the same length of day and of night. Yet all these things must necessarily have changed over so great an interval of years, if that motion really existed in nature. If, then, since the beginning of the world — according to the common opinion — not yet 7,000 years have elapsed, how will it not be exactly like an old wives' tale that so great a change in the Sun has already occurred twice?

I pass over very many other absurdities that follow from this. Nor let anyone object to us the motion of *trepidation* — by which, in the opinion of some, all the stars and planets are set in motion — for since this motion is so imperceptible

that it is scarcely detected by the most expert astronomers, no notable change can occur in the stars and planets, as is evident in the maximum declination [obliquity], which from Ptolemy's time down to our own age has not yet decreased by half a degree. Add to this that this motion does not carry the stars around in a circle from north to south, but only makes the planets, as it were, tremble with that motion — now borne little by little from north to south, now again from south to north, by an imperceptible change.

Finally, from concentric orbs there arises the greatest confusion, on account of the enormous multitude of them that their defenders introduce. From this, too, there necessarily follows a remarkable derangement of the motions.

[Margin: How many concentric orbs are posited by Fracastoro.]

For they posit — as is plain in Fracastoro — 77 or 79 movable orbs or spheres: eight indeed bearing stars, but all the rest devoid of stars, six of which they place above the Firmament. This not only runs counter to the majority of astronomers, who hitherto have found only three starless celestial spheres above the Firmament, but also conflicts with all the Peripatetics, who, following Aristotle's view, are unwilling to admit even a single orb above the Firmament.

[Margin: How many orbs are posited by those who admit eccentrics.]

Those who posit eccentric orbs in the heavens avoid so great a confusion; for in all they grant only 33 orbs — 27 that surround the earth, and 6 epicycles, which stand wholly outside the earth. Hence there will not be so great a multitude of motions, especially since two qualifiedly-eccentric [secundum quid] orbs always advance together proportionally, as is explained in the Theorics — so that a proper motion is denied to eight qualifiedly-eccentric orbs (two in Mercury, and one in each of the other six planets), and each pair of qualifiedly-eccentric orbs is like a single orb, since both are always carried with one and the same motion. And so, since — according to the famous axiom of the philosophers — "it is done in vain by means of more what can be done equally well by means of fewer," and since we posit nearly three times fewer eccentrics than the adversaries do concentrics; and since all the phenomena [φαινόμενα, Greek: *phainomena*, the observed appearances] are saved by eccentrics not only equally well but far better than by concentrics (for the account of hundreds of appearances cannot be given by concentrics, as is clear from what has been said) — who will doubt that eccentric orbs and epicycles are rather to be established in the heavens than concentric ones, especially since eccentrics are in no way repugnant to natural

philosophy, as will be established from the solutions to the arguments of Averroes and his followers?

[Margin: Third reason, proving that eccentrics and epicycles exist.]

Finally, one may conclude the thesis in this way. Just as in natural philosophy we arrive at a knowledge of causes through their effects, so too in astronomy — which deals with the heavenly bodies most remote from us — we must come to a knowledge of them, and of their arrangement and constitution, from their effects: that is, from the motions of the stars as perceived by our senses. For just as, from the mutual generation and corruption of natural things, the natural philosophers, together with Aristotle, inferred prime matter along with the other two principles of natural change, and many other things besides, so too the astronomers, from the various motions of the heavens in general — from east to west and from west to east — investigated a definite number of celestial spheres. Some, indeed, recognized eight [spheres] from only eight distinct motions considered in general; others, from ten distinct motions, established through other phenomena the order among the celestial spheres, as was noted in general in chapter 1; and likewise, by the same method, this has been set forth by us at length through other phenomena.

Therefore it is fitting, and most agreeable to reason, that from the particular motions of the planets and their various appearances the astronomers should inquire into the number of the partial orbs which carry the planets around with such varied motions, and into their constitution and configurations — but on this rule and condition: that the causes of all the motions and appearances can be conveniently assigned, and that no absurdity repugnant to natural philosophy can be inferred from them. For this reason, since eccentric orbs and epicycles are such that by their means astronomers save all the phenomena without any labor (as is partly clear from what has been said, and will be understood more plainly from the Theorics), and since no absurdity or difficulty in natural philosophy follows from them (as will presently be established from the solution of the arguments usually brought against orbs of this kind by the adversaries) — the astronomers have rightly decreed that the planets are carried in eccentric orbs and epicycles, and not in concentric ones, since by these latter we cannot save so manifold a variety in the motions of the planets.

[Margin: The adversaries' reply to the third reason.]

But the adversaries try to weaken this argument, saying that they grant that, once eccentric orbs and epicycles are posited, all the phenomena can be saved — yet

that it does not follow from this that the said orbs are really found in nature; rather, that they are wholly fictitious. This is so, first, because perhaps all the appearances can be saved by a more convenient route, though that route is as yet unknown to us; and second, because it is possible that the appearances are genuinely saved by the said orbs even though the orbs themselves are entirely fictitious and in no way the true cause of those appearances — just as one may indeed infer a truth from a falsehood, as is established from Aristotle's *Dialectic* [Logic].

To this a confirmation may be added, in the following way. Nicolaus Copernicus, in his work *On the Revolutions of the Celestial Orbs* [*De Revolutionibus orbium coelestium*], saves all the phenomena by another route — namely, by positing the Firmament as immobile and fixed, the Sun likewise fixed at the centre of the universe, and by assigning to the earth, which is placed in the third heaven, a threefold motion, and so on. Hence eccentrics and epicycles are not necessary for saving the phenomena in the planets. Again, Ptolemy accounts by an epicycle for all the appearances in the case of the Sun which he [elsewhere] defends by an eccentric. Therefore it cannot be inferred from our third argument that the Sun moves in an eccentric, since perhaps it is carried on an epicycle.

[Margin: A refutation of the adversaries' reply.]

Nevertheless it must be said that our third argument retains its force, and that the adversaries' reply proves nothing. For, first, if they have a more convenient route, let them show it to us; we shall be content, and shall give them our warmest thanks. For astronomers contend for nothing else than that all the phenomena in the heaven be saved as conveniently as possible — whether this be done by eccentric orbs and epicycles, or in some other way. And because no more convenient route has hitherto been found than the one which defends everything by eccentrics and epicycles, it is highly credible that the celestial spheres are composed of orbs of this kind. But if they cannot show us a more convenient route, they surely ought to rest content with this one, gathered from so many varied phenomena [φαινόμενοις] — unless they wish utterly to destroy not only the natural philosophy that is taught in the schools, but also to bar the way to all the other arts, which investigate causes through their effects.

For whenever anyone infers some cause from manifest effects, I shall say exactly the same thing they do — namely, that perhaps some other cause of those effects, unknown to us, might exist. Or else, if we are to rest content with the cause once found, on the ground that it has a certain connection with the effects from which it was inferred, then eccentrics and epicycles too must be granted, since they

have so close a connection with the appearances that all of them can be defended, by the motions of these orbs, with little trouble. Next: if, for the reason that a truth can be inferred from a falsehood, it is not correctly concluded from the appearances that eccentrics and epicycles are found in the heavens, then the whole of natural philosophy will collapse. For in the same way, whenever someone concludes from a known effect that this or that is its cause, I shall say that this is not true, on the ground that a truth may be inferred from a falsehood; and thus all the natural principles discovered by the philosophers would be destroyed. Since this is absurd, it appears that the force and strength of our argument is not rightly weakened by the adversaries.

It can also be said that that rule of the logicians, *"From the false there follows the true,"* is not to the point; because a truth is inferred from a falsehood in one way, and the phenomena are defended by eccentrics and epicycles in another. For in that case a truth is inferred from a falsehood by force of the syllogistic form. Hence, once the truth of some proposition is known, false premises can be arranged in such a form that, by force of the syllogism, that true proposition is necessarily concluded. For example, because I know that an animal is sentient, I can construct the following syllogism: *"Every plant is sentient; every animal is a plant; therefore every animal is sentient."* But if I were in doubt about some conclusion, I would never rest content on false premises...

I would acquire certainty of it, even if it were correctly inferred by the force of a syllogism. Suppose, for instance, that I am in doubt whether every star is round: although by the force of this syllogism — *Every star is a stone. Every stone is round. Therefore every star is round* — I may correctly infer that conclusion from false premises, I shall nevertheless never be made certain of it, but remain in doubt about the aforesaid conclusion. But from the eccentric orbs and epicycles not only are appearances already known of old defended, but future ones are also predicted, whose time is otherwise entirely unknown. Thus, if I am in doubt whether, say, at the full moon of September in the year 1587 there will be an eclipse of the Moon, I am made entirely certain from the motions of the eccentric and epicyclic orbs that an eclipse will occur, so that I no longer doubt it. Indeed, from these same motions I know at what hour that eclipse will begin and how great a part of the Moon will be obscured. In the same way all eclipses, both solar and lunar, can be predicted, together with their time and magnitude — even though they keep no fixed order among themselves, so that no determinate interval of time is set between two successive ones, but sometimes two occur in one year, sometimes one, and sometimes none. Now it is not credible, if the eccentrics and epicycles are fictions (as the adversaries would have it), that we

constrain the heavens — and we would indeed seem to constrain them, if the eccentrics and epicycles are fictions — to obey our fabrications and move as we please, or as suits our own principles.

As for what concerns Nicolaus Copernicus [Copernicus], we say that he did not reject the eccentrics and epicycles as fictitious and repugnant to philosophy. For he himself likewise posits the earth as an epicycle, and in the case of the Moon he sets up an epicycle upon an epicycle; but he did this only in the attempt to emend the periods of the planetary motions, which he had found to be limping. For it is exceedingly difficult to define the periods of the motions in such a way that they do not, over many centuries, deviate from the truth, since no mortal has ever been able to determine the period of a single planet so exactly that there are not some minutes in excess or defect which, over a great interval of years, introduce a notable error. So that it is truly a wonder that God, the Best and Greatest, willed to obstruct the motions of the planets with such great difficulties that no one can perfectly attain them, but always finds something left over — the more so that everyone may admire the supreme skill displayed in bodies so noble and in the great harmony and concord of their motions, celebrating with perpetual praises their Creator and Mover. And especially because of the constitution of the heavens and their motions — in which there always seems to remain something beyond us — it seems to have been written by the Preacher [Ecclesiastes], ch. 3, that the highest diligence should be devoted to their investigation by the most skilful searchers of celestial things: *And he delivered the world to their disputation* [Et mundum tradidit disputationi eorum] — lest, namely, if men had at some time perfectly understood the number, order, constitution, and motion of the heavens, they should cease, and, the works of God no longer being investigated and admired and their talents left unexercised, grow torpid through inactivity. Therefore, that Copernicus defended the φαινόμενα (the appearances) by another route is no wonder. For since he knew from the motions of the eccentrics and epicycles the time, the quantity, and the quality of the appearances, both future and past, he was able — being most ingenious — to devise a new way by which those appearances could be defended more conveniently (as he thought), and the periods of the motions emended in some part, which he had noticed to limp. This seems to have been the chief aim of Copernicus's study, as we said — just as we too can infer a known conclusion by several syllogisms, even from false premises. But so far is it from being the case that, on account of Copernicus's teaching, the eccentrics and epicycles should be removed, that on that very account they are all the more to be posited.

For the astronomers devised these orbs precisely because they discovered, with the greatest certainty, from various phenomena, that the planets are not always carried at an equal distance from the earth. This indeed Copernicus willingly admits, since according to his teaching the planets always have an unequal distance from the earth, as is clear from the position of the earth outside the center of the world, in the third heaven. Only this is gathered from his position: that it is not entirely certain that the constitution of the eccentrics and epicycles is such as Ptolemy [Ptolemy] makes it, since many appearances can be defended by another route. Nor indeed do we contend in this question to persuade the reader of anything else than that the planets are not always carried at an equal distance from the earth; and hence that either there are in the heavens eccentric orbs and epicycles in the order in which Ptolemy placed them, or at least that some cause must be posited for these effects equivalent to the eccentrics and epicycles.

[Margin: The chief point proposed in this question.]

And if Copernicus's position involved nothing false or absurd, it would indeed be doubtful to which opinion — Ptolemy's, or rather Copernicus's — one should adhere (as far as concerns defending appearances of this kind). But since many absurd and erroneous things are contained in Copernicus's position — for instance, that the earth is not in the middle of the firmament and moves with a threefold motion (by what reasoning this can happen, I scarcely understand, since according to the philosophers one simple motion is due to one simple body); and that the Sun is set at the center of the world and is devoid of all motion —

[Margin: The absurdities that follow from Copernicus's position.]

all of which conflict with the common doctrine of the philosophers and astronomers, and seem to contradict what Holy Scripture teaches in very many places, as we have treated more fully in chapter 1: therefore Ptolemy's opinion seems to be preferred to this invention of Copernicus. From all of which it is clear that it is just as probable that there are eccentric orbs and epicycles as it is probable that there are eight, or ten, or even eleven moving heavens, since both the number of the heavens and the said orbs were discovered by the astronomers from the appearances and the motions.

Now, from the fact that Ptolemy defends the appearances of the Sun both by an epicycle and by an eccentric, it is only gathered that it is uncertain whether the Sun is carried on an eccentric or on an epicycle. But whichever be said, it is clear that the Sun is unequally distant from the earth and is by no means carried on a concentric orb — which is enough for us, as we said. Ptolemy, however, chose an

eccentric orb for the Sun, because an eccentric surrounds and encircles the center of the earth. But let us now set forth the arguments of Averroes [Averroes] and his followers and refute them, so that from this too it may appear that eccentrics and epicycles are not monsters or portents, and are in no way repugnant to natural philosophy, as the adversaries falsely think.

[Margin: Arguments against eccentrics and epicycles.]

First, then, the adversaries argue with Averroes as follows. According to Aristotle's [Aristotle] opinion in the books *On the Heavens*, simple motion is threefold: from the middle, to the middle, and about the middle; of these the first two belong to the elements, the last to the celestial bodies.

[Margin: 1st objection.]

But if eccentrics and epicycles were granted, some celestial body would move toward the middle and from the middle, since one part of them approaches nearer to the earth and another less so. Since therefore this is absurd — seeing that the celestial bodies are neither heavy nor light, so that they could have a natural inclination to a motion toward the middle and from the middle — eccentric orbs and epicycles will not be granted.

2. A celestial body, on the authority of Aristotle, is perfectly spherical. But the eccentric orbs "in a certain respect" [secundum quid] that surround the simply eccentric orb are not perfectly spherical, since they are thicker on one side and thinner on the other. Therefore they are not to be conceded.

[Margin: 2nd objection.]

3. If eccentric orbs "in a certain respect" were granted, they could not be moved without penetration or splitting of the heavens, since the thicker part of the one would have to enter the thinner part of the same. By parity of reasoning, when the subtler part slips beneath the place of the thicker, either a vacuum will result — since the thinner part cannot fill the place of the thicker — or else a rarefaction of the heaven. And since these are absurd, it will also be absurd to posit eccentric orbs.

[Margin: 3rd objection.]

4. Aristotle, in book 2 *On the Heavens*, affirms that all the φαινόμενα (appearances) of the planets can be defended by a plurality of motions. In vain, therefore, are eccentrics and epicycles posited, and they are repugnant — at least to Aristotle.

[Margin: 4th objection.]

5. The place of the whole and of the part is the same. But the place of the heaven, as Averroes holds, is the center of the world. Therefore the center of the total spheres and of the partial ones will be the same. All the orbs, therefore, are concentric, and none is eccentric.

[Margin: 5th objection.]

6. The more a sphere is distant from the first principle, the more motions it needs in order to attain or preserve its perfection, as Aristotle holds. Therefore eccentrics and epicycles are not to be conceded, since, once they are posited, the Sun would have fewer motions than Saturn, Jupiter, and Mars, which are nearer to the First Being.

[Margin: 6th objection.]

7. If eccentrics exist in the nature of things, they will certainly be moved about their own centers. But at every center about which the motion of a heaven takes place, there is an earth at rest, since everything that is moved needs something at rest, as Aristotle holds. Therefore, as many as are the eccentrics and epicycles, so many will be the earths at rest — which is absurd.

[Margin: 7th objection.]

8. If eccentrics are granted, there will be in the nature of things (as Augustinus Niphus [Agostino Nifo] says) something superfluous and idle, namely one of the two eccentric orbs "in a certain respect" that carry the Aux (apogee) of a planet. For either of them is sufficient to carry the Aux and its opposite, as is clear. Therefore one will be superfluous, since it has no use.

[Margin: 8th objection.]

These are the reasons by which the adversaries strive to prove that eccentric orbs and epicycles should be done away with. To them we shall add three others that Girolamo Fracastoro [Girolamo Fracastoro] adduces at the end of his little book *On Homocentrics* [Homocentricorum], as demonstrations that cannot be refuted. The first of these, showing that in the case of the Sun an eccentric can in no way be granted, is as follows.

[Margin: 1st objection of Fracastoro.]

If an eccentric orb were granted in the case of the Sun — one point of which, of course, recedes farthest from the earth (that is, from the center of the world, which is the center of the Equator), and one approaches nearest — then that most-remote point, and hence the Sun existing in it, would by the daily motion describe a parallel more distant from the Equator than would the other point nearest the earth. Hence the maximum declinations of the Sun would not be equal to one another; rather the northern one — where today the Aux, or most-remote point, lies — would be greater than the southern, where now the opposite of the Aux, or point nearest the earth, is found. And yet all astronomers have observed that the maximum northern declination of the Sun is equal to the southern. Again, in an oblique sphere, when the Sun is at the Aux — namely in Cancer [♋] — the diurnal arc would be greater than the nocturnal arc when it is at the opposite of the Aux, that is, in Capricorn [♐]; which is contrary to common experience. For when the Sun is in opposite degrees of the ecliptic, two parallels are described, the diurnal arc of the one being equal to the nocturnal arc of the other.

Then, of the latter arguments, which do away with the epicycles of Venus and of the Moon, the first is of this kind.

[Margin: 2nd objection of Fracastoro.]

If the epicycle of Venus were of such magnitude that its semidiameter comprised 43 degrees — and the whole diameter 86 degrees — it would reach nearly to the center of the earth. For if the semidiameter contained precisely 45 degrees, the epicycle would pass exactly through the center of the earth — which he tries to prove geometrically. Since therefore this is absurd and contrary to experience, there will be no epicycle of Venus in the nature of things.

[Margin: 3rd objection of Fracastoro.]

Lastly, if the Moon is revolved on an epicycle, we should not always see the same half of the Moon; rather, when it is in the lower part of the epicycle one half would appear to us, and when it is in the upper part, the other — as is evident from this adjoining figure. For while the Moon is in the lower part of the epicycle, the half marked with the letter A will appear to us; but while it turns in the upper part, the other half, marked with the letter B, will be presented to us. But this is contrary to daily experience, for we see the spots [maculae] of the Moon perpetually turned toward us. From which it follows that we always behold the same half. The emptiness of the epicycle in the case of the Moon is therefore

apparent. Fracastoro does indeed bring forward other reasons in the passage cited, which, since they are of no weight, we deliberately pass over.

Diagram. An uncaptioned woodcut set in the left-center of the page, illustrating Fracastoro's third objection (that a Moon carried on an epicycle would show us both of its halves). Two circles are drawn, internally tangent at the top: a small circle above and a large circle below.

- A (top of the small upper circle): one hemisphere ("half") of the Moon.
- B (just within the small upper circle, below A): the other hemisphere.
- B (upper-central region of the large lower circle): the hemisphere facing away when the Moon is high on the epicycle.
- A (near the center of the large lower circle, below B): the hemisphere that would face the earth when the Moon is low on the epicycle.

The letters A and B mark the two "halves" of the Moon referred to in the text. All four labels (A, B, B, A) are legible.

[Margin: Solution to the 1st objection.]

We shall easily satisfy all these arguments. To the first, then, we reply that the eccentrics and epicycles are moved about their own middle, that is, about their own centers. That by this motion they now approach nearer to the earth, now move farther from it, is not absurd, because this approach and recession does not occur along a straight line — which alone Aristotle excluded from the celestial bodies, since it belongs only to the elements, which are heavy and light. And if anyone should contend that Aristotle thought the contrary, this must be pardoned in him; for he spoke only of those motions that were known in his own time, namely, from the middle and to the middle along a straight line, and about the middle of the world. And if the motions of the eccentrics and epicycles had been known in his time, I do not doubt that he would have spoken otherwise about motion around the middle. But if this solution does not satisfy the adversaries, they will have to prove that every celestial motion must take place about the center of the world — which they will never achieve. For it does not belong to them to prescribe laws to the celestial motions, but to God the Best and Greatest, who by his infinite goodness and providence judged it fitting that the planets should not be carried about the earth on concentric orbs.

[Margin: Solution to the 2nd objection.]

We shall resolve the second objection if we say that all the eccentric orbs — even those "in a certain respect" — and the epicycles are most perfectly spherical with

respect to their own centers. For the outermost surfaces of all these orbs are equally distant from their own centers on all sides. Nor indeed is it an obstacle that the eccentric orbs "in a certain respect" are thicker in one part than in another, since no natural reason can persuade us that all the celestial orbs must be of uniform and equal thickness. And if Aristotle taught the contrary, we do not believe him on this point.

[Margin: Solution to the 3rd objection.]

As for the third argument, I greatly wonder that Averroes and the Averroists — whom in this matter you might more truly call "Errorists" [Erroistae] — are borne with so hostile a spirit against the eccentrics and epicycles that they have been unwilling to understand by what manner they are moved. For those two eccentric orbs "in a certain respect" are not moved in such a way that the thinner part of the one takes the place of the thicker, and vice versa, as they falsely imagine; rather they are borne together proportionally, so that perpetually the thicker part of the lower lies beneath the thinner part of the upper, and vice versa, and together they carry around the simply eccentric orb, in such a way that they have no motion other than that of the whole heaven of the planet. The argument would have force only if the simply eccentric orb were at rest and the eccentrics "in a certain respect" surrounding it were moved — which is not the case.

[Margin: Solution to the 4th objection.]

To the fourth objection it must be replied that Aristotle was always of the opinion that in astronomical matters the astronomers should be consulted. Hence he then followed the astronomers of his own time, namely Eudoxus [Eudoxus] and Callippus [Callippus], who strove to defend all the φαινόμενα (appearances) by concentric circles. But I do not doubt that, had he lived in Ptolemy's time, he would have embraced the eccentrics and epicycles, since all things are most conveniently defended by that method. For he always affirms that in astronomical matters faith is to be placed in the astronomers.

[Margin: Solution to the 5th objection.]

To the fifth argument we say that the opinion that the heaven is in place by its center is proper to Averroes. Hence, if we choose not to accept it, the argument concludes nothing against us. Yet if anyone wishes to defend that opinion, he can say that the eccentric orbs and epicycles too are in place by their own centers, and that the center of the world is the place of the total heavens, but not of the partial orbs. And if someone presses that the place of the whole and of the parts is

the same, that must be understood of the common place, not of the proper place. For any part of a stone has the same place in common with the stone, but not the same proper place, since the place must be equal to what is placed in it. So then, if one wishes to defend Averroes's opinion, he will be able to say that the common place of all the spheres, both partial and total, is not the center of the world, but the center absolutely — whatever that may be — or certainly the aggregate of all the centers; and thus that they have the same common place, namely a center, while each orb nevertheless has its own proper place, namely its own center.

[Margin: Solution to the 6th objection.]

To the sixth argument we reply that the Sun has fewer motions than the superior planets not only on the system of eccentric orbs and epicycles, but also on that of the concentrics, as is clear from Fracastoro, chapter 24, where he reviewed the number of the orbs. Hence we deny that the celestial orbs, the lower they are, must be stirred by the more motions, and the higher they are by the fewer, since experience has taught the contrary, as the adversaries themselves admit.

[Margin: Solution to the 7th objection.]

To the seventh objection it must be denied that an earth at rest is necessary at every center in order that the celestial orbs may be moved about it. For although God the Best and Greatest were to remove this earth altogether, or drive it elsewhere outside the center of the world, the heavens would still be carried by the daily motion about the middle of the world.

[Margin: Solution to the 8th objection.]

To the eighth argument it must be said that the two eccentric orbs "in a certain respect" are necessary, so that they may make up and complete the whole heaven of the planet, which is concentric with the world. Hence neither of them should be reckoned superfluous, for the whole heaven that is composed of them has its own proper motion. These orbs are not posited solely in order to carry the Aux and its opposite, as the objection falsely assumes.

[Margin: Solution to the 1st objection of Fracastoro.]

Now, as for the three arguments of Fracastoro, we say that the first concludes nothing in the case of the Sun. For since the Sun is at so great a distance from the earth that it admits either no parallax [aspectus diversitas] at all, or certainly an imperceptible one, it comes about — since the plane of its eccentric always lies in the plane of the ecliptic (as will be explained in the *Theorics* [Theoricae]) — that it always appears on the ecliptic, if it is viewed from the earth. Hence, when it is at

the beginning of Cancer [♋] or Capricorn [♏], it will appear to describe by the daily motion the same parallels that the beginnings of Cancer and Capricorn describe in the *primum mobile*, which are equal. Nor is it an obstacle that the Sun is at the Aux when it is in Cancer, and at the opposite of the Aux when it is in Capricorn. Otherwise Saturn, while it is on the ecliptic and at the beginning of Cancer, would describe a parallel more remote from the Equator than Jupiter, since Saturn is farther from the earth than Jupiter — which is false. For each planet, while it is on the ecliptic and at the beginning of Cancer, has been found to have a declination of $23\frac{1}{2}$ degrees and to describe by the daily motion the Tropic of Cancer [♋]. Therefore it does not follow that the maximum northern declination of the Sun is greater than the maximum southern declination, nor, in an oblique sphere, that the longest day in summer is longer than the longest night in winter. All these absurdities would indeed follow if the Sun had a notable parallax. Nevertheless it is true that the center of the Sun, when it is at the Aux, describes by the daily motion in its own orb a parallel more distant from the Equator than when it is at the opposite of the Aux, because there it describes a less distant one. But since each parallel, on account of the excessive distance of the Sun from the earth, seems to be described by the points at which, in the *primum mobile*, the straight lines drawn from the center through the Aux and through the opposite of the Aux terminate, it comes about that they are judged to be equally distant from the Equator, as far as the senses go.

[Margin: Solution to the 2nd objection of Fracastoro.]

To the second argument of Fracastoro we reply that the astronomers do not lay down that the semidiameter of Venus's epicycle contains 43 degrees, but 43 of those parts of which 60 are contained in the semidiameter of the eccentric circle. From this it comes about that the lines drawn from the center of the earth and touching the epicycle cut off from the *primum mobile*, on both sides of the line of the Aux, about 45 degrees — which is precisely the amount by which, at most, Venus is seen to recede from the Sun, both toward the east and toward the west. But it does not follow from this that the epicycle reaches almost to the earth. For since, as Fernellius of Amiens [Jean Fernel] reports in his *Cosmotheoria*, the semidiameter of the eccentric circle contains nearly 689 [?] earth-radii, the semidiameter of the epicycle will comprise about $435\frac{2}{3}$ [?] earth-radii. If we subtract this number from the distance of the earth to the opposite of the Aux — which comprises about $674\frac{1}{3}$ [?] earth-radii — the interval between the center of the earth and the opposite of the Aux of the epicycle, while the epicycle is nearest the earth (namely at the opposite of the Aux of the eccentric), will contain about 179 earth-radii. This distance contains more than 640641 miles. We, however,

following Maurolico [Francesco Maurolico] in chapter 1, make this distance of the concave surface of Venus somewhat smaller, namely $167\frac{2}{3}$ earth-radii, that is, 600167 $\frac{14}{33}$ miles. Therefore the epicycle of Venus does not touch the earth, but is separated from it by so great an interval that within it the heaven of Mercury and the heaven of the Moon, together with all the elements, can conveniently be enclosed. Moreover, Fernel himself depicted the proper figure, with the proportions of the diameters of the eccentric and the epicycle, in the *Theoric of Venus*; so that from it too it may easily appear that the epicycle of Venus cannot reach the earth, but is immersed within the thickness of the eccentric orb.

[Margin: Solution to the 3rd objection of Fracastoro.]

Lastly, in defense of the epicycle of the Moon, Fernelius of Amiens replies, in the book cited, that the Moon has on its epicycle a proper motion about its own center, conformable to the motion of the epicycle, but in the contrary direction.

From this motion it follows that the Moon always turns the same spotted face toward us. Nor should this seem surprising or absurd, even though Aristotle denied that the stars have motions of their own. For since the phenomena [Greek: φαινόμενα (phainomena, "appearances" — the observed phenomena)] show that the Moon is carried on an epicycle and always turns the same face toward us, it must be that it revolves by its own motion about its own center, so that it perpetually remains in a certain stable equilibrium.

From all these considerations, then, I take it to be established that eccentrics and epicycles are not so monstrous and absurd as our opponents pretend, and that astronomers introduced them not without good reason. Suppose they are to be judged absurd because they have different centers, and because eccentrics have, in a certain respect, an unequal thickness. Why then should we not say the same absurdity attaches to the fact that the Moon does not have a uniform density, but has parts that are denser in some places than in others, as its spots indicate? Had the opponents not seen these spots with their own eyes, I have no doubt that, if astronomers had proposed them, they would have hissed them off the stage. Let them, then, make it a point of religious scruple to admit anything into the heavens that seems to depart even in the slightest from the most perfect uniformity. And yet — in the Firmament itself, which the followers of Aristotle are forced to assert is, as it were, the standard for all the other spheres — there nonetheless appears the greatest unevenness, both from the stars and, if we follow the truth, from the Milky Way. Since, therefore, this great inequality throughout the whole depth of the heavens, in respect of density and rarity, is not denied even by the opponents, why should eccentrics and epicycles be reckoned

absurd and monstrous merely on account of the difference of their centers and their unequal thickness? But enough has now been argued about eccentrics and epicycles for the present occasion. Let us now return to the author's exposition, which we set aside.

It should be noted that the Sun has a single circle, etc.

COMMENTARIVS.

[Margin: Of what the Sun's heaven is composed.]

First, then, the author treats of the Sun's orb and motion, saying that the Sun has a single eccentric circle, on which it is perpetually carried beneath the Ecliptic from west to east. To understand this, we must recall to mind what we said a little earlier: namely, that the whole heaven of the Sun — which has its center in common with the center of the world — is divided by Ptolemy and the more recent astronomers into three partial orbs, contiguous to one another. Of these, the highest is, along its convex surface, concentric with the world — that is, its center does not differ from the center of the world — but along its concave surface it is eccentric, that is, it has a center other than the center of the world. The lowest orb, conversely, is concentric with the world along its concave surface, and eccentric along its convex surface. The third, finally, which is placed in the middle of these, is eccentric along both surfaces, convex as well as concave, because it is contiguous with the concave surface of the upper orb and the convex surface of the lower. Hence the first two orbs are usually called eccentric in a certain respect [*eccentrici secundum quid*], because they have a center different from the center of the world along one surface only; but the third, the intermediate one, is called eccentric absolutely [*eccentricus simpliciter*], and in it the Sun is fixed and moves about its center from west to east, so that the center of the Sun describes in the course of a year a certain circle beneath the Ecliptic, whose center is the same as the center of the orb that is eccentric absolutely. This, then, is the circle that the author here calls the Eccentric, on which the Sun moves by its own motion.

[Margin: What the Aux of the Sun and the opposite of the Aux are.]

Now since this circle has a center distinct from the center of the world (that is, of the Firmament), it follows that one of its points — namely the one indicated by a straight line drawn from the center of the world through the circle's own center — is farthest from the earth and nearest to the Firmament; while the other, which lies opposite to it, is nearest to the earth and farthest from the Firmament.

That first point, he says, is called by the Arabs the Aux of the Sun [Aux Solis — the apogee]; and the opposite point, the opposite of the Aux [the perigee].

[Margin: The Sun has a double motion from west to east.]

Next he teaches that the Sun has a double motion from west to east. One is its own motion, on its eccentric, on which it completes each day about 59 minutes and 8 seconds. Hence that eccentric orb is usually called the deferent of the Sun, because by its motion the Sun is carried beneath the Ecliptic from west to east. It then has a second motion, an extremely slow one, by which it is moved — along with the motion of the whole heaven of the Sun — from west to east by 1 degree in 100 years, according to Ptolemy; but according to Alfonso, by 1 degree 28 minutes in 200 years. And because by this motion those two eccentric-in-a-certain-respect orbs carry the Aux of the Sun and the opposite of the Aux to different and different points of the Ecliptic — though very slowly — they are called by astronomers the deferents of the Aux of the Sun. In the year 1592 the Aux of the Sun stood at about 9 degrees 13 minutes of Cancer, according to the Prutenic Tables, and its opposite at 9 degrees 13 minutes of Capricorn. From these two motions, then, he says, the annual motion of the Sun is compounded. But the arrangement of these three orbs of the Sun, and their motions, are usually explained more fully in the Theorics of the Planets.

[Margin: Which are the orbs that carry the Aux of the Sun.]

[Margin: Of what the heavens of the other planets, besides the Sun, are composed.]

Every planet, however, except the Sun, has three circles: namely the Equant, the Deferent, and the Epicycle. The Equant of the Moon is a circle concentric with the earth, and lying in the plane of the Ecliptic. But its Deferent is an eccentric circle, and does not lie in the plane of the Ecliptic; rather, one half of it inclines toward the north, the other toward the south. And the Deferent intersects the Equant in two places; and the figure of the intersection is called a Dragon [Draco], because it is broad in the middle and narrower toward the ends. That intersection through which the Moon moves from south toward north is called the Head of the Dragon [caput Draconis]; the other intersection, through which it moves from north to south, is called the Tail of the Dragon [cauda Draconis].

[Margin: What the Head and Tail of the Dragon are in the case of the Moon.]

The Deferent and the Equant of every planet are equal. And it should be known that the deferent and equant of Saturn, Jupiter, Mars, Venus, and Mercury are all

eccentric and outside the plane of the Ecliptic — and yet the two of them lie in the same plane.

[Margin: In the five planets the deferent and equant are eccentric, and lie in the same plane, which declines from the Ecliptic.]

Every planet also, except the Sun, has an Epicycle. Now the Epicycle is a small circle, along whose circumference the body of the planet is carried; and the center of the Epicycle is always carried on the circumference of the deferent.

[Margin: What the Epicycle is.]

COMMENTARIVS.

Secondly, he treats of the orbs and motions of the other planets, saying that each of them has three circles: namely the Equant, the Deferent, and the Epicycle. The Equant of the Moon is a circle concentric with the earth, and lies in the plane of the Ecliptic. This circle is called the Equant of the Moon because, as is established from the Theorics, it is from its motion that the true motion of the Moon is known adequately and precisely. The Deferent of the Moon, however, is a circle eccentric absolutely, like that of the Sun, with this one exception: that this eccentric does not lie in the plane of the Ecliptic, as the Sun's does, but one half of it inclines away from the Ecliptic toward the north, the other toward the south. Hence it comes about that the Moon, carried along on this circle, is found sometimes outside the Ecliptic toward the north, sometimes toward the south, but never exactly beneath the Ecliptic except at those two points where the Ecliptic — or Equant — and the Deferent circle of the Moon intersect one another. This Deferent, which is eccentric absolutely, is surrounded by two other orbs eccentric in a certain respect, just as was said of the Sun. Of the two points at which the Equant and the Deferent of the Moon intersect, the one through which the Moon is carried northward on the Deferent is called the Head of the Dragon; the other, through which it tends southward, the Tail of the Dragon. And these two points are carried from east to west by the Equant of the Moon; for this Equant orb is the highest in the sphere of the Moon. For this reason astronomers usually say that it carries the Head and Tail of the Dragon, and that it is larger than the eccentric that carries the Moon.

[Margin: Why equant orbs were devised in the planets.]

The Deferent and the Equant of every other planet are equal to one another, and eccentric absolutely, and each lies outside the plane of the Ecliptic, although both exist in one and the same plane. Now equant circles were devised in these planets

— for they are not real orbs and parts of the planetary spheres, as the Deferent and the eccentrics-in-a-certain-respect are, but only imaginary — in order that the irregularity of the Deferent of each planet might be reduced to uniformity by the aid of its own Equant, as will be clearly established from the Theorics. Each deferent of a planet also has two other eccentrics-in-a-certain-respect, one above it and the other below, as we said of the Sun, which are called the deferents of the Aux. Mercury alone has four eccentric-in-a-certain-respect orbs, of which two are called the eccentric deferents of the Aux, which carry Mercury, and the other two the deferents of the Aux of the Equant. Although the first two — the eccentric deferents of the Aux, or the deferents of Mercury — may also be called eccentric absolutely, since their convex and concave surfaces have a center different from the center of the world, yet with respect to the deferent of Mercury, which is absolutely eccentric, they may in a certain way be called eccentric-in-a-certain-respect. For the concave surface of the upper and the convex surface of the lower have the same center as the eccentric that carries Mercury, while the convex surface of the upper and the concave surface of the lower have a different one.

Furthermore, every planet except the Sun has, besides the said circles, an epicycle as well — that is, a tiny orb immersed in the deferent orb, in which the planet is carried. For the body of the planet is fixed in the epicycle; yet the center of the epicycle is perpetually carried by the motion of the eccentric, or deferent. But these things can scarcely, or only with difficulty, be understood without the instruments of the Theorics. We shall, however, set all these matters out more fully in the Theorics of the Planets.

ON THE STATION, DIRECT MOTION, AND RETROGRADATION OF THE PLANETS.

If, then, two lines are drawn from the center of the earth so as to enclose the epicycle of some planet — one on the eastern side, the other on the western — the point of contact on the eastern side is called the first station; the point of contact on the western side is called the second station. And when the planet is at either of those stations, it is said to be stationary. The upper arc of the epicycle intercepted between the two stations is called direct motion [directio]; and when the planet is on it, it is said to be direct. But the lower arc of the epicycle intercepted between the two stations is called retrogradation; and the planet situated there is said to be retrograde. To the Moon, however, no station, direct motion, or retrogradation is assigned; hence the Moon is not called stationary, direct, or retrograde, because of the speed of the motion of the center of the epicycle on the eccentric.

[Margin: The various affections [passiones] of the planets.]

COMMENTARIVS.

He now treats of certain affections of the planets, namely the station, direct motion, and retrogradation of the planets. He says, then, that if two straight lines are drawn from the center of the earth touching the epicycle — one on the eastern side, the other on the western — those points of contact are called stations: the point on the eastern side the first station, and the one on the western side the second station. A planet situated at either of those stations is therefore called stationary, because then the planet appears to us, as it were, to stand still on its epicycle and not to change its place in the Zodiac through its motion on the epicycle, since at that moment it is either ascending or descending. But if we wish to understand "station" in the strict sense — that is, to understand the point of the epicycle at which, when the planet is there, the motion of the eccentric and the motion of the epicycle bear such a proportion to one another that the planet seems to stand altogether at the same place in the Zodiac — then this occurs a little below those points of contact, as is explained in the Theorics. Next, he says, the upper arc of the epicycle intercepted between the two stations is called the direct motion of the planet, and a planet situated on it is called direct, because it then moves according to the succession and order of the signs — that is, from west to east, say from Aries to Taurus, from Taurus to Gemini, and so on. But the lower arc is called retrogradation, and a planet placed on it is termed retrograde, because it then advances against the succession and series of the signs — that is, from east to west, namely from Aries to Pisces, from Pisces to Aquarius, and so on. All of which is to be understood of the planets that have an epicycle, the Moon excepted, so that in the Sun and the Moon these do not apply. For the epicycles of the planets, the Moon excepted, move in their upper part according to the succession of the signs, but in their lower part against the series of the signs; whereas the Moon's epicycle, on the contrary, moves with the succession of the signs in its upper part, and according to the series of the signs in its lower part. Hence the Moon ought to be called direct when it is in the lower part of the epicycle, because there it moves according to the series of the signs, and retrograde when placed in the upper part of the same. Nevertheless the Moon is called neither direct nor retrograde, on account of the swift motion of the Moon itself on the eccentric. For the Moon is carried, along with the motion of the center of the epicycle on its deferent, most swiftly from west to east. Hence it cannot be called stationary, nor direct, nor retro-

[Margin: Why the Moon is not called stationary, direct, or retrograde.]

grade, because the motion of the center of the epicycle on the deferent overcomes the proper motion of the epicycle. Yet when placed in the lower part of the epicycle it is said to be swift, and in the upper part slow, since there its motion from west to east is, as it were, doubled, whereas here it is in a manner retarded, as will be clear in the Theorics.

ON THE ECLIPSE OF THE MOON.

[Margin: The earth's shadow is conical.]

[Margin: What the Nadir of the Sun is.]

[Margin: When an eclipse of the Moon occurs.]

[Margin: Why an eclipse of the Moon does not occur at every full moon.]

Since the Sun is larger than the earth, it is necessary that half the sphere of the earth is always illuminated by the Sun, and that the earth's shadow, extended in the air and turned as on a lathe, diminishes in its roundness until it comes to nothing at the surface of the circle of the signs [the zodiac], inseparable from the Nadir of the Sun. Now the Nadir of the Sun is the point directly opposite the Sun in the Firmament. Hence when, at full moon, the Moon is at the Head or the Tail of the Dragon, beneath the Nadir of the Sun, then the earth is interposed between the Sun and the Moon, and the cone of the earth's shadow falls upon the body of the Moon. And since the Moon has no light except from the Sun, it is in truth deprived of light. And it is a general eclipse over all the earth, if the Moon is exactly at the Head or Tail of the Dragon; but a particular eclipse, if it is near, within the determined limits of the eclipse. And an eclipse always happens at, or about, full moon. Hence, since it is not at every opposition — that is, full moon — that the Moon is at or near the Head or Tail of the Dragon, nor placed beneath the Nadir of the Sun, it is not necessary that the Moon suffer an eclipse at every full moon.

COMMENTARIVS.

Here he explains in what manner an eclipse of the Moon occurs, and why the Moon does not suffer an eclipse at every full moon. For since the Sun is much larger than the earth, as we taught in chapter 1, it is necessary — as Witelo [Vitellio] demonstrates in Book 2 of the *Optics* [*Perspectiva*], proposition 27 — that more than half of the earth be illuminated by the Sun, and therefore that the earth's shadow resemble a cone, or spinning-top, whose apex never departs from the plane of the Ecliptic. For neither does the center of the Sun deviate from that

plane, and the apex is always opposite the Sun, since the earth lies at the center of the Ecliptic, that is, of the whole world. From which it is manifest that, since a full moon occurs when the Sun and Moon stand at diametrically opposite degrees, but the Moon is not beneath the Ecliptic except when it is at the Head or Tail of the Dragon (as we said a little before), the Moon suffers an eclipse only at that full moon at which it is found either at the Head or at the Tail of the Dragon. For thus it comes about that the Moon enters the earth's shadow and is prevented from being lit by the Sun. Hence, since it borrows its light from the Sun, it must then fail and be deprived of light, because then the earth is interposed precisely between the Sun and the Moon. The whole Moon will indeed be darkened over the whole earth, if at full moon it stands exactly at the Head or Tail of the Dragon, because it will be wholly plunged into the shadow; but not the whole of it, if at full moon it is found near the Head or Tail of the Dragon — in such a way, however, that the earth's shadow covers some part of the Moon. From these things it is clear why the philosophers say that an eclipse of the Moon is the interposition of the earth between the Sun and the Moon; because truly, in an eclipse of the Moon, the earth lies on the same diameter on which the said planets are placed at that time, and along which they are in opposition. But since for the most part the oppositions of the luminaries occur while the Moon is not at the Head or Tail of the Dragon, nor even so near that it can be covered by the shadow, an eclipse of the Moon therefore does not always occur at every full moon. For the Moon must be either at the Head or at the Tail of the Dragon for an eclipse to occur. All these matters will indeed be clearer in the Theorics of the planets.

ON THE ECLIPSE OF THE SUN.

[Margin: When an eclipse of the Sun occurs.]

[Margin: An eclipse of the Moon occurs over the whole earth, but an eclipse of the Sun does not.]

[Margin: The eclipse of the Sun at the Passion of the Lord was miraculous.]

But when the Moon is at the Head or Tail of the Dragon, or near, or within the aforesaid limits, and in conjunction with the Sun, then the lunar body is interposed between our sight and the solar body. Hence it will overshadow for us the brightness of the Sun, and thus the Sun will suffer an eclipse — not because it fails in light, but because it fails for us, on account of the interposition of the Moon between our sight and the solar body. From these things it is clear that there is not

always an eclipse of the Sun at conjunction, that is, at new moon. It should also be noted that when there is an eclipse of the Moon, it is an eclipse over all the earth; but when there is an eclipse of the Sun, by no means so: rather, in one clime there is an eclipse and in another there is not. This happens on account of the difference of aspect [parallax of sight] in different climes. Hence Virgil most elegantly touched on the natures of both eclipses in a brief compass, saying: "The various failings of the Moon, and the labors of the Sun." [Virgil, Georgics II.478; the received text reads "defectus solis varios lunaeque labores," with Sun and Moon in the reverse arrangement to that printed here.]

From what has been said it is clear that, since there was an eclipse of the Sun at the Passion of the Lord, and that same Passion was at full moon, that eclipse was not natural but rather miraculous and contrary to nature, because an eclipse of the Sun ought to occur at new moon, or thereabouts. On which account we read that Dionysius the Areopagite said, at that same Passion: "Either the God of nature is suffering, or the machine of the world will be dissolved."

COMMENTARIVS.

[Margin: Why an eclipse of the Sun does not occur at every new moon.]

Finally, he explains in what manner an eclipse of the Sun occurs, saying: Whenever the Moon, in conjunction with the Sun — that is, at new moon — stands at the Head or the Tail of the Dragon, or at least near, yet within the limits of eclipse, it will be interposed between our sight and the Sun; hence it will hide from us the brightness of the Sun, and an eclipse of the Sun will occur — not that the Sun itself is deprived of light, but only with respect to us, on account of that interposition of the Moon between our vision and the solar body.

But an eclipse of the Sun will not occur at every conjunction of the Moon with the Sun — that is, at every new moon — because it is not at every conjunction that the Moon interposes itself between the Sun and our sight, but only when the Moon is so joined to the Sun (that is, exists in the same sign and degree in which the Sun is) that a line going out from our eye and drawn through the center of the Moon reaches the Sun. And this will happen when the Moon, at new moon, is found at the Head of the Dragon, or the Tail, or at least near.

Commentary on Chapter 4 of the *Sphere* (conclusion) — and the beginning of the *Theorica planetarum* arranged in tables

Conclusion of the eclipse commentary

He then teaches that there is this difference between an eclipse of the Sun and one of the Moon: an eclipse of the Moon is universal over the whole earth, so that its light fails in every region; but an eclipse of the Sun is by no means universal. Rather, there can be a solar eclipse in one clime and not in another—indeed, it can be greater in one and smaller in another. This is because a solar eclipse depends on our line of sight [aspectu], which differs in different climes, as is explained in the *Theorica*; a lunar eclipse, however, does not, but depends only on the earth's shadow, which is always the same in every clime.

From what has been said, the Author finally infers that, since a solar eclipse necessarily occurs at new moon—that is, at the conjunction of the Moon with the Sun—the eclipse of the Sun that took place at the Lord's Passion, when it was in fact full moon, was not natural but miraculous, and against the course and order of nature. For by divine power the Moon, abandoning its own proper course, moved toward the Sun and hid it from us. And on this account, as the histories attest, blessed Dionysius the Areopagite [Dionysius Areopagita] exclaimed at that time: "*Either the God of nature is suffering, or the fabric of the world is being dissolved.*" Because of this they raised an altar consecrated to the unknown God, whom shortly afterward St. Paul made known to them, and so led them to faith and to the knowledge of the true God, who is blessed and glorious for ever and ever, Amen.

Since, however, what the Author has written in this chapter about the motions of the planets and the eclipses of the Sun and Moon is so obscure that it cannot be explained in a few words, it has seemed good at this point—something that many people have told me would be most welcome to the studious reader, and have even urged me to undertake—to append certain tables that set the whole theory of the planets before our eyes, as if in a mirror. These tables were once composed by a certain learned man, but I have recast them into a more convenient form. To them I have added, from approved writers, the distances of the centers of the eccentric orbs and of the epicycles from the center of the world, and the magnitudes of the semidiameters of those same orbs in parts of which the Earth's semidiameter is one. The methods by which all these things may be investigated

and tested will be explained in my *Theorica*. (For I did not have the time here to examine the distances of the centers and the magnitudes of the semidiameters for myself, but took them from other authors as they stand written.)

THEORICS OF THE PLANETS, ACCORDING TO THE TENETS OF THE ALPHONSINES [Alphonsinorum], ARRANGED IN TABLES

(*THEORICÆ Planetarum iuxta placita Alphonsinorum per tabulas digestæ.*)

THEORICA OF THE ORBS AND MOTIONS OF THE SUN

(Two-page table spread. The caption runs across the opening: verso "*THEORICA ORBIVM*", recto "*ET MOTVVM ☉ SOLIS*" = "*Theory of the orbs and motions of the Sun.*" Left marginal brace, set vertically: "*Sphæram ☉ constituunt orbes tres*" — "*Three orbs constitute the sphere of the Sun.*")

The nine columns of the spread give, for each partial orb: (1) the particular orbs of which the whole solar sphere consists; (2) their names and positions relative to the center of the world; (3) the centers of the orbs and their distances from the center of the world; (4) the axes on which they turn; (5) the poles on which they turn; (6) their proper motions or revolutions; (7) the semidiameters of the orbs, in parts of which the Earth's semidiameter is one; (8) the plane surfaces of the orbs as inclined to the plane of the ecliptic; (9) the apogee [aux] of the eccentric for the year of Christ 1554.

Orb (col. 1)	Name & positio n (col. 2)	Center / distan ce (col. 3)	Axis (col. 4)	Poles (col. 5)	Proper motion (col. 6)	Semidi ameter , Earth- rad. = 1 (col. 7)	Surfac e / inclina tion (col. 8)	Apoge e, AD 1554 (col. 9)
The two eccent ric orbs carryi ng the apoge e [DVO	Conce ntric [with the world] as to their outer most	Of the world, as to their outer surfac es.	Of the eclipti c of the eighth sphere .	Of the ecliptic of the eighth sphere .	From west to east, i.e. accord ing to the order of the	As to the concav e surfac e: 1121 21 min.;	Alway s beneat h (in the plane of) the eclipti c of	3 signs 1° 40'.

Orb (col. 1)	Name & positio n (col. 2)	Center / distan ce (col. 3)	Axis (col. 4)	Poles (col. 5)	Proper motion (col. 6)	Semidi ameter , Earth- rad. = 1 (col. 7)	Surfac e / inclina tion (col. 8)	Apoge e, AD 1554 (col. 9)
Augem eccent rici defere ntes]	surfacs— contig uous to the sphere s of Mars ♂ and Venus ♀—but eccent ric as to their other surfacs es; hence called eccent ric only in a qualifi ed sense [secun dum quid].				signs, in 49000 years.	but as to the convex : 1216 parts 5 min.	the eighth sphere .	
The eccent	Eccent ric	Its own	Paralle l to	Equall y	From west to	1165 parts	Alway s	(none

Orb (col. 1)	Name & positio n (col. 2)	Center / distan ce (col. 3)	Axis (col. 4)	Poles (col. 5)	Proper motion (col. 6)	Semidi ameter , Earth- rad. = 1 (col. 7)	Surfac e / inclina tion (col. 8)	Apoge e, AD 1554 (col. 9)
ric carryi ng the solar body [Eccen tricus defere ns corpus Solare]	withou t qualifi cation [simpli citer].	[center , distant from the center of the world towar d the apogee by 44 parts 2 min. (of which the Earth's semidi ameter is one); or by 2 parts 16 min. [nume ral: ? — the minute s glyph is faint,	[æquid istans] the axis of the eclipti c of the eighth sphere .	distant from the poles of the ecliptic of the eighth sphere .	east in 365 days 5 hours 49 min., approx imatel y.	23 min.	beneat h the eclipti c of the eighth sphere .	<i>given</i>)

Orb (col. 1)	Name & positio n (col. 2)	Center / distan ce (col. 3)	Axis (col. 4)	Poles (col. 5)	Proper motion (col. 6)	Semidi ameter , Earth- rad. = 1 (col. 7)	Surfac e / inclina tion (col. 8)	Apoge e, AD 1554 (col. 9)

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THEORICA OF THE ORBS AND MOTIONS OF THE MOON

(Second table on the same spread. Caption across the opening: verso "THEORICA ORBIVM", recto "ET MOTVVM › LVNÆ" = "Theory of the orbs and motions of the Moon." Left marginal brace, set vertically: "Sphæram › quinque orbes constituunt" — "Five orbs constitute the sphere of the Moon.")

Same nine columns as above.

Orb (col. 1)	Name & positio n (col. 2)	Center / distan ce (col. 3)	Axis (col. 4)	Poles (col. 5)	Proper motion (col. 6)	Semidi ameter , Earth- rad. = 1 (col. 7)	Surfac e / inclina tion (col. 8)	Apoge e, AD 1554 (col. 9)
The two eccent ric orbs carryi ng the apoge e [DVO Augem eccent rici defere ntes]	Partly concen tric, as being the carrier s of the Moon's ☾ apogee ; hence called eccent ric only in a qualifi ed sense [secun dum quid].	Of the world, as to their outer most surfac es.	Interse cting the axis of the eclipti c over the center of the world.	Declini ng equall y from the poles of the zodiac by 5 degree s.	From east to west in 32 days 3 hours 5 min.	As to the concav e surfac e: 33 parts 42 min.; as to the convex : 64 parts 29 min.	Declini ng from the eclipti c on both sides, with a fixed declin ation of 5 degree s.	Mobile , from east to west, followi ng the motion of the apogee - carrier s of the eccent ric, daily 11° 11' 52'' .
The eccent ric carryi ng the epicyc le [Eccen tricus]	Eccent ric withou t qualifi cation [simpli citer].	Its own [center], mobile with the motion of the	Paralle l to the axis of the apogee - carrier s.	Equall y distant from the poles of the apogee -	From west to east, i.e. accord ing to the succes sion of	48 parts 56 min.	Declini ng from the eclipti c on both sides by 5	(none given)

Orb (col. 1)	Name & positio n (col. 2)	Center / distan ce (col. 3)	Axis (col. 4)	Poles (col. 5)	Proper motion (col. 6)	Semidi ameter , Earth- rad. = 1 (col. 7)	Surfac e / inclina tion (col. 8)	Apoge e, AD 1554 (col. 9)
defere ns Epicycl um]		apogee - carrier s, distant from the center of the world by 10 Earth- semidi ameter s 9 min.; or by 12 parts 28½ min. (of which the eccent ric's semidi ameter has 60).		carrier s.	the signs, in 27 days 7 hours 43 min.		degree s, and never depart ing from the plane of the apogee - carrier s.	
The orb	Conce ntric	Of the	Of the eclipti	Of the ecliptic	From east to	64 parts	Beneat h the	(none

Orb (col. 1)	Name & positio n (col. 2)	Center / distan ce (col. 3)	Axis (col. 4)	Poles (col. 5)	Proper motion (col. 6)	Semidi ameter , Earth- rad. = 1 (col. 7)	Surfac e / inclina tion (col. 8)	Apoge e, AD 1554 (col. 9)
carryi ng the Head of the Drago n [Defer ens caput Draco nis] (ascen ding node)	with the world.	world.	c.	.	west, i.e. against the order of the signs, in 18 years 7 month s 12 days.	29 min.	eclipti c of the eighth sphere .	<i>given</i>)
The epicyc le [Epic clus; printe d "EPICY LLVS"]	It is carrie d wholly outsid e the center of the world.	Its own [center , distant from the center of the world unequ ally, but from the center of the eccent	Perpen dicular to the plane of the eccent ric, and paralle l to the axis of the eccent ric.	Equall y distant from the axis, or poles, of the eccent ric.	Agains t the succes sion of the signs (from east to west) in its upper part; but in its lower part accord ing to	5 parts 5 min.; or, in parts of which the eccent ric's semidi ameter has 60 : 6 parts 14 min.	Declini ng from the eclipti c, and never depart ing from the plane surfac e of the eccent ric.	<i>(none given)</i>

Orb (col. 1)	Name & positio n (col. 2)	Center / distan ce (col. 3)	Axis (col. 4)	Poles (col. 5)	Proper motion (col. 6)	Semidi ameter , Earth- rad. = 1 (col. 7)	Surfac e / inclina tion (col. 8)	Apoge e, AD 1554 (col. 9)
		ric by 48 parts 56 min. (of which the Earth's semidi ameter is one).			the order of the signs (from west to east): in 27 days 13 hours 18 min.			

Theory of the Orbs, Set Out in Tables (continued)

These pages give the synoptic "theory of the orbs" [*theorica orbium*] for the three superior planets together (Saturn ♄, Jupiter ♃, Mars ♂), then for Venus ♀, then Mercury ☿, and finally the outermost spheres. Each planet's full table occupies one opening: the **structural** columns (orbs, names/positions, centres, axes) stand on the verso, and the **quantitative** columns (poles, motions, semidiameters, surfaces, aux) on the facing recto; the two halves share the same orb-rows and are stitched here per orb.

THEORICA ORBIVM — Saturn ♄, Jupiter ♃, Mars ♂

Structural half

[Margin (running down the left edge): For the spheres ♄, ♃, ♂, four orbs each constitute the individual spheres; moreover a fifth is to be conceived, the **equant eccentric** [*aequans eccentricus*], which is only a circle.]

Column headings:

1. **Particular orbs** of which the whole spheres of h , a , o consist.
2. **Names and positions** of the particular orbs with respect to the centre of the world.
3. **Centres of the orbs, and the distances of the centres** from the centre of the world.
4. **Axes of the orbs** on which they move.

Particular orb	Name & position (rel. to centre of world)	Centre & its distance from the centre of the world	Axis of motion
Two eccentrics carrying the aux [<i>duo augem eccentrici deferentes</i>]	Partly concentric , and eccentric in a certain respect, like the deferents of the aux of the Sun $\odot$ and Moon C .	Of the world , as to the outermost surfaces; for the surfaces contiguous to the Eccentric have the same centre as the Eccentric.	Of the ecliptic of the eighth sphere .
Eccentric carrying the epicycle [<i>eccentricus deferens epicyclum</i>]	Eccentric, absolutely, or a deferent.	Its own centre, distant from the centre of the world in earth- semidiameters: in h 980, min. 53; a 532, min. 12; o 503, min. 12. — Or in parts: h 3, min. 25; a 2, min. 45; o 6, min. 0; of which the semidiameter of the Eccentric holds 60.	Cutting the axis of the ecliptic, but outside the centre of the world.
Equant circle [<i>aequans circulus</i>]	The equant eccentric .	Its own centre, distant from the centre of the world by twice the distance of the	Equidistant from the axis of the deferent, or of the eccentric.

Particular orb	Name & position (rel. to centre of world)	Centre & its distance from the centre of the world	Axis of motion
		centre of the Eccentric from the centre of the world.	
Epicycle [<i>epicyclus</i>]	Wholly outside the centre of the world.	Its own centre, unequally removed from the centre of the world; but [equal] in magnitude to the semidiameter of the Eccentric, or of the Deferent, [measured] from the centre of the Eccentric.	Mobile , on account of the motion of latitude.

Quantitative half

Column headings: 5. **Poles of the orbs** on which they move. 6. **Proper motions, or revolutions of the orbs.** 7. **Semidiameters of the orbs in parts**, of which the semidiameter of the earth is one. 8. **Plane surfaces of the orbs, inclined to the plane of the ecliptic.** 9. **Aux of the eccentric, for the year of Christ 1554** (S[igns] · G[degrees] · M[inutes]).

Orb	Poles	Proper motion / revolution	Surface	Aux (1554): S · G · M
Two eccentrics carrying the aux	Of the ecliptic of the eighth sphere.	From west to east, i.e. according to the series of the signs, in 49000 years.	Under the ecliptic of the eighth sphere.	♄ 8 · 13 · 28 ♊ 5 · 23 · 52 ♂ 4 · 15 · 27
Eccentric	Unequally	From west to	Declining	—

Orb	Poles	Proper motion / revolution	Surface	Aux (1554): S · G · M
carrying the epicycle	declining from the poles of the ecliptic; for the northern pole is more distant than the southern.	east, i.e. according to the succession of the signs, in years: ♄ 29 y · 155 d · 8 h; ♃ 11 y · 313 d · 17 h; ♀ 1 y · 321 d · 22 h.	from the ecliptic, with a fixed declination, such that the auges always incline to the north and never cross the ecliptic, and describe circles parallel to the ecliptic of the eighth sphere, by virtue of the motion of the eighth sphere.	
Equant circle	Equidistant from the poles of the Eccentric, or of the Deferent.	From west to east, to the motion of the Eccentric, or of the Deferent. According to the order of the signs, i.e. from west to east in the upper part; but in the lower part the contrary, i.e. from east to west; in days:	Declining from the ecliptic, and never receding from the plane of the eccentric.	—

Orb	Poles	Proper motion / revolution	Surface	Aux (1554): S · G · M
		♄ 378 d · 2 h · 23 m; ♃ 398 d · 21 h · 12 m; ♂ 779 d · 22 h · 23 m.		
Epicycle	Mobile , on account of the motion of latitude.	<i>(see the epicyclic/syno- dic period given in the row above)</i>	Declining from the ecliptic, lacking inclination only at the nodes.	—

Semidiameters of the orbs (col. 7), in parts where the earth-semidiameter = 1:

Orb / surface	♄ Saturn	♃ Jupiter	♂ Mars
Deferent of aux — concave [<i>quoad concauum</i>]	14378 · 19	8853 · 47	1216 · 5
Deferent of aux — convex [<i>quoad conuexum</i>]	22612 · 30	14378 · 19	8853 · 47
Eccentric (deferent of epicycle)	17225 · 16	11611 · 31	5032 · 4
Equant	17225 · 16	11611 · 31	5032 · 4
Epicycle	1866 · 4	2225 · 32	3312 · 47
Epicycle — in parts where eccentric semidiameter = 60	6 · 30	11 · 30	39 · 30

THEORICA ORBIVM — Venus ♀

Structural half

[Margin (down the left edge): Four orbs constitute the sphere of Venus; moreover another circle, the **equant eccentric**, is to be conceived.]

Column headings (as above): 1. Orbs of which the whole sphere of ♀ consists. 2. Names & positions. 3. Centres & distances. 4. Axes.

Particular orb	Name & position	Centre & distance from centre of world	Axis of motion
Two eccentrics carrying the aux	Partly concentric, and eccentric in a certain respect, as in the preceding.	Of the world , as to the outermost surfaces contiguous to the Eccentric have the same centre as the Eccentric.	Of the ecliptic of the eighth sphere .
Eccentric carrying the epicycle	Eccentric, absolutely, or a deferent.	Its own centre, distant from the centre of the world in earth-semidiameters: 12, min. 7. — Or in parts, of which the semidiameter of the Eccentric is one: 1, min. 8.	Approaching and receding from the axis of the ecliptic, on account of the motion of the Eccentric in latitude, now toward the north, now toward the south.
Equant circle	The equant eccentric.	Its own centre, distant twice as much more from the centre of the world as the centre of the Eccentric, or of the deferent.	Equidistant from the axis of the eccentric, or of the deferent.
Epicycle	Wholly outside the centre of the world.	Its own centre, unequally distant from the centre of the world; but [equal] in magnitude to the semidiameter of the Eccentric, or of the deferent,	Mobile , both for the motion of inclination and of reflexion.

Particular orb	Name & position	Centre & distance from centre of world [measured] from the centre of the eccentric.	Axis of motion

Quantitative half

(Body heading: ET MOTVVM ♀. VENERIS. — "...and of the motions of Venus.")

Orb	Poles	Proper motion / revolution	Semidiameter (earth- semidiam. = 1)	Surface	Aux (1554): S · G · M
Two eccentrics carrying the aux	Of the ecliptic of the eighth sphere.	From west to east, i.e. according to the succession of the signs, in 49000 years.	concave 167 · 57; convex 1121 · 21	Under the ecliptic of the eighth sphere.	3 · 1 · 40
Eccentric carrying the epicycle	Mobile , on account of the aforesaid motion of the eccentric in latitude.	From west to east, according to the order of the signs, in 365 d · 5 h · 49 m.	641 · 45	Declining from the ecliptic with a mobile declination, which is called Deviation [<i>deuiatio</i>]; yet always twisting the epicycle away from the ecliptic,	—

Orb	Poles	Proper motion / revolution	Semidiameter (earth- semidiam. = 1)	Surface	Aux (1554): S · G · M
				never toward the south but perpetually toward the north, as is explained among the "Passions of the Planets."	
Equant circle	Equidistant from the poles of the deferent, or of the eccentric.	From west to east, to the motion of the eccentric, or of the deferent.	641 · 45	Declining from the ecliptic, yet never receding from the plane of the eccentric, but always retaining the same Deviation.	—
Epicycle	Mobile , both for the motion of inclination and of reflexion.	According to the sequence of the signs, i.e. from west to east in the upper part; but in the lower	461 · 41; — or in parts where the eccentric semidiameter = 60: 43 · 10	Approaching and receding from the ecliptic, on account of the motion of deviation, inclination,	—

Orb	Poles	Proper motion / revolution	Semidiameter (earth- semidiam. = 1)	Surface	Aux (1554): S · G · M
		part the contrary, i.e. from east to west; in 583 d · 22 h · 12 m.		and reflexion; yet never crossing it toward the south.	

THEORICA ORBIVM — Mercury ☿ (structural)

PDF p. 324 / printed p. 312, verso

[Margin (down the left edge): Six orbs constitute the sphere of Mercury; moreover two eccentric circles are to be conceived, the **Equant** and the **small circle** [*circulus paruus*].]

Column headings: 1. Particular orbs of which the whole sphere of ☿ consists. 2. Names & positions. 3. Centres & distances. 4. Axes.

Particular orb	Name & position	Centre & distance from centre of world	Axis of motion
Two orbs carrying the aux of the equant [<i>duo augem aequantis deferentes</i>]	Partly concentric , and eccentric in a certain respect.	Of the world as to the outermost surfaces; but as to the middle surfaces they have their own centre, the same, namely, as [that of] the small circle.	Of the ecliptic of the eighth sphere .
Two orbs carrying the aux of the eccentric [<i>duo augem</i>]	Entirely eccentric.	Having their own centre — the same, namely, as the small circle —	Approaching and receding from the axis of the ecliptic of the eighth

Particular orb	Name & position	Centre & distance from centre of world	Axis of motion
<i>eccentrici deferentes]</i>		as to the outermost surfaces; but as to the other, middle ones, the same as the Eccentric.	sphere, on account of the motion of the eccentric in latitude.
Eccentric carrying the epicycle	Eccentric, absolutely, or a deferent.	Its own centre, movable — describing, at the motion of the [orbs] carrying the aux of the eccentric, a small circle, and standing unequally distant from the centre of the world. The minimum distance contains 3 parts ; the maximum, however, 9 , of which the semidiameter of the Eccentric holds 60. Or the minimum has earth- semidiameters 5 , min. 48 ; the maximum, however, 17, min. 24.	Equidistant from the axis of the [orbs] carrying the aux of the eccentric.
Equant circle	The equant	Its own centre,	Equidistant from

Particular orb	Name & position	Centre & distance from centre of world	Axis of motion
	eccentric.	distant from the centre of the world according to the minimum distance of the centre of the Eccentric.	the axis of the [orbs] carrying the aux of the eccentric.
Small circle [<i>circulus paruus</i>]	Wholly outside the centre of the world.	The same as [that] of the [orbs] carrying the aux of the Eccentric.	The same as that of the [orbs] carrying the aux of the eccentric.
Epicycle	Wholly outside the centre of the world, and is moved.	Its own centre, distant from the centre of the Eccentric according to the magnitude of the semidiameter of the Eccentric; but from the centre of the world, unequally.	Mobile , both for the motion of inclination and of reflexion.

THEORICA ORBIVM, ET MOTVVM — Primum Mobile, Ninth Sphere & Eighth Sphere

Begins PDF p. 324 / printed p. 312 (verso, lower table); continues into the next opening

For the three outermost, simple spheres the structural and quantitative columns are combined into a single table ("Theory of the orbs and motions"). Only the first two columns fall on this page; the remaining columns (centres, axes, poles, motions, semidiameters, surfaces) continue on the facing recto of the next opening. **Table continues.**

Column headings (visible portion): 1. Orbs of which the whole spheres consist.
2. Names & positions of the orbs with respect to the centre of the world.

Sphere	Orb	Name & position
[Margin: One single orb constitutes the Primum Mobile .]	A single orb , in which ten circles, and several others besides, are conceived, of which the chief are the Equinoctial and the Zodiac .	Concentric with the world. The Primum Mobile .
[Margin: One orb constitutes the ninth sphere .]	One orb , in which — besides the Zodiac and two small circles — no other circle is understood.	Concentric . The ninth sphere [<i>nona sphaera</i>]. The Second Movable.
[Margin: A single orb constitutes the eighth sphere .]	A single orb , in which the Zodiac (called the Movable) exists together with the fixed stars. It is divided, moreover, into 48 celestial images [constellations].	Concentric . The eighth sphere . The Firmament [<i>firmamentum</i>].

Theoricæ Planetarum per tabulas digestæ (continued)

Theorica of the orbs and motions of Mercury (concluding columns)

[This recto (printed p. 313) carries the right-hand column-group of the Mercury *Theorica orbium* whose left-hand (identity/centre) columns stood on the facing verso. Table caption, spanning the opening: ...**ET MOTVVM ꝯ MERCVRII** — "... and of the motions of Mercury." Six orb-rows.]

Column headers (verbatim):

- **POLI orbium, super quibus moventur.** — Poles of the orbs, upon which they are moved.

- **MOTVS proprij, sive revolutiones orbium.** — Proper motions, or revolutions of the orbs.
- **SEMIDIAMETRI orbium in partibus, quarum semidiameter terræ est una.** — Semidiameters of the orbs in parts of which the earth's semidiameter is one (sub-columns *PAR.* | *MIN.*, given *quoad concavum* / *quoad convexum* = "as to the concave [surface]" / "as to the convex").
- **SVPERFICIES planæ orbium ad planum Eclipticæ inclinatæ.** — Plane surfaces of the orbs, inclined to the plane of the ecliptic.
- **AVX Eccentrici, ad annum Christi 1554.** — Aux of the eccentric, for the year of Christ 1554 (sub-columns *S.* | *G.* | *M.* = signs | degrees | minutes).

		SEMIDIAMETRI (concave PAR/MIN; convex PAR/MIN)	
POLI orbium	MOTVS proprij		SVPERFICIES
ECLIPTICÆ octavæ sphæræ. (Of the ecliptic of the eighth sphere.)	AB Occasu in Ortum, id est, secundum signorum successionem, in annis 49000. (From west to east, i.e. following the order of the signs, in 49,000 years.)	concave 64 / 29; convex 167 / 57.	SVB Ecliptica octavæ sphæræ. (Beneath the ecliptic of the eighth sphere.)
MOBILES propter motum Eccentrici in latitudinem. (Movable on account of the motion of the eccentric in latitude.)	AB Ortu in Occasum, id est, contra signorum ordinem, in diebus 365. Hor. 5. Min. 49. (From east to west, i.e. against the order of the signs, in 365 days, 5 hours, 49 minutes.)	concave 76 / 5; convex 121 / 51.	DECLINANS ab Ecliptica octavæ sphæræ declinatione mobili, Eccentrici planum deviare faciens. (Declining from the ecliptic of the eighth sphere by a movable declination, causing the plane

POLI orbium	MOTVS proprij	SEMIDIAMETRI (concave PAR/MIN; convex PAR/MIN)	SVPERFICIES
ÆQVIDISTANTES polis Deferentium augem Eccentrici. (Equidistant from the poles of the [orbs] carrying the aux of the eccentric.)	AB Occasu in Ortum, secundum successionem signorum, in diebus 365. Hor. 5. min. 49. (From west to east, following the order of the signs, in 365 days, 5 hours, 49 minutes.)	concave 116 / 3.	of the eccentric to deviate.) DECLINANS ab Ecliptica octavæ sphæræ, & nunquam à plano Deferentium Augem Eccentrici recedens: Epicyclum tamen in Meridiem semper retorquens, ut in passionibus Planetarum explicatur. (Declining from the ecliptic of the eighth sphere, and never receding from the plane of the [orbs] carrying the aux of the eccentric; yet always twisting the epicycle back toward the meridian, as is explained in the "passions" of the planets.)
ÆQVIDISTANTES polis Deferentium	AB Occasu in Ortum ad motum	concave 116 / 3.	DECLINANS ab Ecliptica octavæ

		SEMIDIAMETRI (concave PAR/MIN; convex PAR/MIN)	
POLI orbium	MOTVS proprij		SVPERFICIES
augem Eccentrici. (Equidistant from the poles of the [orbs] carrying the aux of the eccentric.)	Eccentrici, seu Deferentis. (From west to east, to the motion of the eccentric, or deferent.)		sphæræ, & à superficie plana eccentrici non recedens. (Declining from the ecliptic of the eighth sphere, and not receding from the plane surface of the eccentric.)
IIDEM, qui Deferentium Augem Eccentrici. (The same [poles] as those of the [orbs] carrying the aux of the eccentric.)	AB Ortu in Occasum ad motum Deferentium augem Eccentrici. (From east to west, to the motion of the [orbs] carrying the aux of the eccentric.)	5 / 48; or, in the parts of which the semidiameter of the eccentric has 60: 3 / 0.	DECLINANS ab Ecliptica, & à plano Deferentium Augem non recedens. (Declining from the ecliptic, and not receding from the plane of the [orbs] carrying the aux.)
MOBILES ad motum cum inclinationis tum reflexionis. (Movable to the motion both of inclination and of reflexion.)	AB Occasu in Ortum, secundum signorum seriem, in diebus 115. H. 21. M. 5 in parte superiori; in inferiori autem contra signorum sequelam. (From west to east, following the	concave 43 / 31; or, in the parts of which the semidiameter of the eccentric has 60: 9 / 0.	ACCEDENS, & recedens ab Ecliptica propter motum deviationis, inclinationis, & reflexionis, eam tamen nunquam versus Boream transiens. (Approaching and

POLI orbium	MOTVS proprij	SEMIDIAMETRI	SVPERFICIES
		(concave PAR/MIN; convex PAR/MIN)	
	order of the signs, in 115 days, 21 hours, 5 minutes in the upper part; but in the lower part against the sequence of the signs.)		receding from the ecliptic on account of the motion of deviation, inclination, and reflexion, yet never crossing it toward the north.)

AVX Eccentrici [of Mercury], ad annum Christi 1554: S. 7 · G. 0 · M. 54 (7 signs, 0°, 54'). [Single value given, aligned to the head of the table.]

Theorica of the Primum Mobile, the ninth sphere, and the eighth

Sub-caption (verbatim, full-width band): **Primi Mobilis, nonæ sphærae, & octavæ.** — "Of the Primum Mobile, of the ninth sphere, and of the eighth." Same five column-headers as above, except the aux column here reads **AVX communis** (the common aux). Three rows, one per sphere.

Sphere	POLI orbium	MOTVS proprij	SEMIDIAMETRI	SVPERFICIES
			(PAR / MIN)	
Primum Mobile	MVNDI Arcticus, & Antarcticus. (The Arctic and Antarctic [poles] of the world.)	AB Oriente in Occidentem rediens per mediam noctem in Orientem in hor. 24. & vocatur Motus Raptus. (From east to west, returning	45225 / 0; & adhuc multo maior. (...and still much greater.)	PLANVM Æquinoctialis circuli, à quo distat planum Eclipticæ grad. 23. min. 30. (The plane of the equinoctial circle, from which the plane of the ecliptic is

Sphere	POLI orbium	MOTVS proprij	SEMIDIAMETRI (PAR / MIN)	SVPERFICIES
		through midnight to the east in 24 hours; and it is called the <i>motus raptus</i> [the rapt/diurnal motion].)		distant 23°30'.)
Ninth sphere	ECLIPTICÆ vel Zodiaci primi mobilis. (Of the ecliptic or zodiac of the primum mobile.)	AB Occidente in Orientem recurrens in annis 49000. & vocatur Motus Augium. (From west to east, recurring in 49,000 years; and it is called the <i>motus augium</i> [the motion of the auges].)	45225 / 0; Et adhuc maior, minor tamen quam semidiameter primi Mobilis. (And still greater, yet less than the semidiameter of the primum mobile.)	SVB Ecliptica primi mobilis, ecliptica ista perpetuo manens. (Beneath the ecliptic of the primum mobile, this ecliptic remaining fixed forever.)
Eighth sphere	ACCEDENTES, & recedentes ad motum Axis. (Accessing and receding, to the motion of the axis.)	A Septentrione versus Ortum, recurrendo in Septentrionem in annis 7000. super duobus circulis parvis, & vocatur motus Trepidationis. (From the north toward the east, recurring to the	45225 / 0 quoad convexum; At quoad concavum 22612 / 30. (...as to the convex; but as to the concave, 22,612;30.)	DECLINANS frequenter ab ecliptica tam nonæ sphæræ, quam primi mobilis. (Declining frequently from the ecliptic both of the ninth sphere and of the primum mobile.)

Sphere	POLI orbium	MOTVS proprij	SEMIDIAMETRI (PAR / MIN)	SVPERFICIES
		north in 7,000 years, upon two small circles; and it is called the <i>motus trepidationis</i> [the motion of trepidation].)		

AVX communis, ad annum Christi 1554: S. 0 · G. 20 · M. 15 (0 signs, 20°, 15').

[Internal check: the eighth sphere's concave semidiameter 22612;30 is exactly half of 45225 — consistent, not flagged.]

Definitions of the terms (DEFINITIONES TERMINORVM)

[This opening (printed pp. 314–315) is a single wide comparative table. Its leftmost column names the term; the definition columns are headed by the planetary symbols and, for most terms, distinguish the Sun and Moon (☉ ☽) from the five planets (♄ ♀ ♂ ♃ ♅). Many definitions are marked *in omnibus* / *OMNIBVS* ("in all") and run continuously across the opening as one cell; a few (the top **Habitus** row, and the last three rows) give genuinely distinct wordings per planet-group. The type is very small and the wording is split across the gutter; intricate brace-grouped rows are transcribed as far as they resolve, with [?] marking uncertain stitches.]

Habitus Planetarum ad Solem — the relation of the planets to the Sun

Per-group cells (header symbols as printed):

- ☉ **Sol:** *SOL, tanquam reliquorum Planetarum princeps, ad nullum, sed omnes ad ipsum quendam motus respectum habent.* — The Sun, as chief of the remaining planets, [is directed] to none; rather all have a certain regard of motion toward it.
- ☽ **Luna:** *DEFERENTES Augem Eccentrici ita quotidie contra signorum ordinem retrocedunt (Eccentrico tamen Epicyclum interim secundum seriem signorum protrudente) ut linea medij motus ☉ in medio inter Augem Eccentrici & Epicyclum semper reperiatur. Quare in omni [conjunction ♂?] &*

oppositione Epicyclus est in Auge; in quadratura vero in Augis opposito. — The [orbs] carrying the aux of the eccentric retrograde daily against the order of the signs (while the eccentric meanwhile pushes the epicycle forward following the order of the signs), so that the line of the Sun's mean motion is always found midway between the aux of the eccentric and the epicycle. Hence at every conjunction and opposition the epicycle is at the aux; but in quadrature, at the point opposite the aux.

- ♃ ♄ ♂ **(the three superiors):** *SINGVLI horum Planetarum tanto tempore revolvuntur in suis Epicyclis, quantum est à [conjunction ♂?] media cuiusque eorum cum ☉ [Sole; symbol unclear], usque in sequentem. Unde in qualibet tali [conjunction ♂?] Planeta in Auge media sui Epicycli reperitur.* — Each of these planets revolves upon its epicycle in as much time as there is from one mean conjunction of each of them with the Sun to the next. Hence at any such conjunction the planet is found at the mean aux of its epicycle.
- ♀ **Venus:** *AVX Eccentrici Augi ☉ perpetuo cohæret. Sed & medius ♀ motus semper [?] idem est, qui medius motus ☉. Hinc media eorum [conjunction ♂?] semper esse solet.* — The aux of the [Venus] eccentric perpetually coheres with the aux of the Sun. Moreover the mean motion of Venus is always [?] the same as the mean motion of the Sun. Hence their mean conjunction is always wont to occur.
- ☿ **Mercury:** *DEFERENTES Augem Eccentrici una cum centro Eccentrici [contra] signorum seriem in anno Solari revolvuntur. Eccentricus vero in [eodem] tempore secundum ordinem signorum movetur. Sed & cum ☉ semper [?]diam habet ut Venus.* — The [orbs] carrying the aux of the eccentric, together with the center of the eccentric, revolve against the order of the signs in one solar year; but the eccentric in the same time moves following the order of the signs. And with the Sun it always [behaves? holds a like arrangement] as Venus does.

Definitions of the remaining terms

Term (left column, verbatim)	Definition
AVX in 1. significazione (Aux, in the first sense)	<i>EST (in omnibus) punctum Eccentrici à centro mundi remotissimum, per lineam per utrumque centrum, mundi scilicet & Eccentrici, extensam designatum. Cuius oppositum est punctum eiusdem Eccentrici terræ vicinissimum, in omnibus etiam, præterquam in ☿, ut in eius Theorica declaratur.</i> — In all [planets], the point of the eccentric farthest from the center of the world, marked out by a line extended through both centers, namely of the world and of the eccentric. Its opposite is the point of the same eccentric nearest the earth — in all likewise, except in Mercury, as is explained in its Theorica.
AVX in 2. significazione (Aux, in the second sense)	<i>EST (in omnibus) arcus Zodiaci à principio ♈, secundum signorum successionem, usque ad lineam Augis computatus. In tabulis Aux simpliciter vocatur.</i> — In all, the arc of the Zodiac reckoned from the beginning of Aries, following the order of the signs, up to the line of the aux. In the tables it is called simply the "aux."
LONGITVDO media Eccentrici (Mean longitude of the eccentric)	<i>EST punctum Eccentrici, per lineam ex centro Eccentrici ad Augis lineam ad angulos rectos eductam [ostensum]; in quo maxima accidit æquatio. [?]</i> — The point of the eccentric shown by a line drawn from the center of the eccentric to the line of the aux at right angles, at which the greatest equation occurs.
AVX Epicycli media (Mean aux of the epicycle)	<i>EST punctum Epicycli, quod linea à centro Æquantis (id est, ex puncto</i>

Term (left column, verbatim)	Definition
	<i>centro Eccentrici opposito) per centrum Epicycli educta ostendit. — The point of the epicycle shown by a line drawn from the center of the equant (that is, from the point opposite the center of the eccentric) through the center of the epicycle.</i>
AVX Epicycli vera (True aux of the epicycle)	<i>EST punctum circumferentiæ Epicycli, quod indicat linea recta à centro mundi per centrum Epicycli educta, terminatum. — The point on the epicycle's circumference at which a straight line drawn from the center of the world through the center of the epicycle terminates.</i>
LINEA medij motus Planetæ (Line of the planet's mean motion)	<i>EST quæ à centro mundi ad Zodiacum educitur, lineæ ex centro Æquantis per centrum Epicycli exeunti æquidistans [in the 5 planets]; vel lineæ à centro Eccentrici ad corpus Solare excurrenti æquidistans. Vocaturque linea medij motus. — The line drawn from the center of the world to the Zodiac, parallel to the line issuing from the center of the equant through the center of the epicycle (in the five planets), or parallel to the line running from the center of the eccentric to the solar body; and it is called the line of mean motion.</i>
LINEA veri motus Planetæ (Line of the planet's true motion)	<i>EST quæ à centro mundi per corpus Planetæ educta, ad Zodiacum usque protenditur. — The line drawn from the center of the world through the body of the planet, extended all the way to the Zodiac.</i>
{Medius / Verus} Motus Planetæ	<i>EST arcus Zodiaci à principio ♈,</i>

Term (left column, verbatim)	Definition
(Mean / true motion of the planet)	<i>secundum successionem signorum, usque ad {Medij / Veri} Motus lineam numeratus.</i> — The arc of the Zodiac from the beginning of Aries, following the order of the signs, numbered up to the line of the {mean / true} motion.
Linea {Medij / Veri} Motus Epicycli (Line of the {mean / true} motion of the epicycle)	{mean:} <i>Exeunti à centro Æquantis ad centrum Epicycli [ductæ] æquidistans</i> — parallel to [the line] going from the center of the equant to the center of the epicycle; {true:} <i>Per centrum Epicycli [à centro mundi] transiens</i> — passing through the center of the epicycle [from the center of the world]. [?]
{Medius / Verus} Motus Epicycli (Mean / true motion of the epicycle)	<i>EST arcus {Zodiaci / Epicycli} interceptus inter {lineas medij [& veri motus] / Augem mediam [& veram]} ... secundum motum Planetæ in Epicyclo, usque ad corpus ipsius supputatus.</i> [?] — The arc of the {Zodiac / epicycle} intercepted between {the lines of mean [and true motion] / the mean [and true] aux} ... reckoned according to the motion of the planet on the epicycle, up to the planet's body.
CENTRVM {Medium / Verum} (Center — mean / true)	<i>EST arcus Zodiaci à linea Augis eccentrici, secundum seriem signorum, usque ad lineam {medij / veri} motus.</i> [?] — The arc of the Zodiac from the line of the aux of the eccentric, following the order of the signs, up to the line of the {mean / true} motion.
Æquatio centri {In Zodiaco / In Epicyclo} (Equation of center — in the Zodiac / on the epicycle)	<i>In Zodiaco: EST arcus Zodiaci inter lineam Augis Eccentrici, & lineam medij motus secundum seriem signorum [interceptus]. Et dicitur simpliciter</i>

Term (left column, verbatim)	Definition
	<i>centrum</i>. — the arc of the Zodiac between the line of the aux of the eccentric and the line of mean motion, following the order of the signs; called simply "the center." <i>In Epicyclo: EST arcus Epicycli inter Augem eius mediam, & veram [interceptus]. Et dicitur simpliciter Æquatio centri.</i> [?] — the arc of the epicycle between its mean aux and its true [aux]; called simply "the equation of center."
Argumentum {Medium / Verum} (Argument — mean / true)	<i>Medium: EST arcus Zodiaci inter lineam Augis, & lineam medij motus comprehensus. Et dicitur simpliciter Argumentum [medium].</i> — the arc of the Zodiac comprised between the line of the aux and the line of mean motion; called simply the [mean] argument. <i>Verum: EST arcus Epicycli ab Auge eius [media/vera] ... secundum motum Planetæ in Epicyclo, usque ad corpus ipsius supputatus.</i> [?] — the arc of the epicycle from its aux ... reckoned according to the planet's motion on the epicycle, up to its body.
Æquatio Argumenti (Equation of the argument)	<i>EST arcus Zodiaci lineis medij & veri motus Epicycli interiacens.</i> [and, for the Moon:] <i>EST arcus Zodiaci inter lineas medij & veri motus</i> › <i>cadens.</i> [?] — the arc of the Zodiac lying between the lines of the mean and true motion of the epicycle (for the Moon: the arc of the Zodiac falling between the lines of the Moon's mean and true motion).
Compositio tabularum Æquationum (Composition of the equation-tables)	[Sun & Moon:] <i>OMNIBVS Eccentrici sui locis communis existit.</i> — it is common to all places of its eccentric. [Five

Term (left column, verbatim)	Definition
DIVERSITAS Diametri (Variation of the diameter)	planets:] <i>AD Augem sui Eccentrici ...; ad situm, sive distantiam S. 2. G. 4. M. 30. ab Auge Æquantis.</i> [?] — at the aux of its eccentric ...; to the position, or distance, of 2 signs 4°30' from the aux of the equant. <i>EST differentia, qua Æquationes argumentorum in opposito Augis Eccentrici positæ, ad mediocrem Epicycli à terra distantiam, differunt ab Æquationibus in Auge, & minima [Epicycli] à terra distantia.</i> Related wording: <i>EST excessus, quo Æquationes argumentorum in opposito Augis Eccentrici super longitudes medias [positæ superant Æquationes] ... qui Propior vocatur; et longitudinum mediarum [super Æquationes] in Auge, qui Longior dicitur.</i> [?] — The difference by which the equations of the arguments placed opposite the aux of the eccentric, at the epicycle's mean distance from the earth, differ from the equations at the aux and at the least distance from the earth (the one called "nearer," the other "farther").
MINVTA Proportionalia (Proportional minutes)	[Sun & Moon:] <i>SVNT particulæ excessus lineæ Augis super lineam oppositi Augis, divisi in 60. partes æquales.</i> — the parts of the excess of the line of the aux over the line opposite the aux, divided into 60 equal parts. [Five planets:] <i>SVNT particulæ excessus lineæ Augis Æquantis super lineam mediocris à terra distantie: et rursus huius super minimam distantiam, in 60. partes æquales</i>

Term (left column, verbatim)	Definition
DIVERSITAS Minutorum Proportionalium (Variation of the proportional minutes)	<i>distributi</i>. — the parts of the excess of the line of the aux of the equant over the line of the mean distance from the earth, and again of this over the least distance, distributed into 60 equal parts. [Moon — <i>DVPLICIA</i>, "double":] <i>quia quædam ad Augem Eccentrici omnia sunt intra Deferentem, sed ad longitudes medias omnia extra: et dicuntur Minuta proportionalia longiora. Quædam autem alia ad longitudes medias omnia sunt intra Deferentem, sed ad oppositum Augis omnia extra: et dicuntur Minuta proportionalia Propiora</i>. — because some, at the aux of the eccentric, are all within the deferent, but at the mean longitudes all outside: these are called the "farther" proportional minutes; while certain others, at the mean longitudes, are all within, but at the point opposite the aux all outside: these are called the "nearer" proportional minutes. [Five planets — <i>TRIPLICIA</i>, "triple":] <i>Quædam enim ad Augem Æquantis omnia sunt intra Deferentem, & ad medias longitudes omnia extra: et dicuntur longiora. Quædam autem alia ad longitudes medias omnia sunt intra, & ad minimam à terra distantiam omnia extra: et dicuntur Propiora. Horum rursum ad oppositum Augis Æquantis nonnulla sunt intra, & nonnulla extra. Tertia denique minuta proportionalia sunt particulæ excessus lineæ oppositi Augis Æquantis super minimam à terra</i>

Term (left column, verbatim)	Definition
	<i>distantiam, &c. Qui excessus, quoniam insensibilis est, omittuntur; propterea in tabula hæc Minuta proportionalia [non ponuntur].</i> — for some, at the aux of the equant, are all within the deferent and at the mean longitudes all outside: called the "farther"; certain others, at the mean longitudes, all within, and at the least distance from the earth all outside: called the "nearer." Of these, again, at the point opposite the aux of the equant, some are within and some outside. A third kind of proportional minutes, finally, are the parts of the excess of the line opposite the aux of the equant over the least distance from the earth, etc.; and since this excess is imperceptible, they are omitted, and therefore in the table these proportional minutes are [not entered]. [Sun's cell (<i>simplicia</i>?) not legible on this scan — [?].]

Definitions of Terms — The Phenomena of the Planets [Passiones]

**

1. Planets [Planetæ] are said to be direct, retrograde, or stationary [Directi, Retrogradi, Stationarij]: namely when the line of the planet's true motion advances according to the succession of the signs (direct), moves back against the succession of the signs (retrograde), or appears to stand still in one place of the Zodiac (stationary).

- ☉ Sun: The Sun is always direct, since it has no epicycle.
- ☾ Moon: The Moon is always direct, because the centre of the epicycle traverses a greater arc of the Zodiac by the motion of the eccentric [eccentricus], from west to east, than the body of the Moon does by the

motion of the epicycle in its upper part, against the order of the signs, on any given day.

2. Station [Statio] (first / second): In its first sense it is the point of the epicycle at which the planet begins to go retrograde (first station) or to go direct (second station). In its second sense it is the arc from the aux [augis] of the epicycle up to the aforesaid point.

3. Arc [Arcus] (of direction / of retrogradation): It is the arc of the epicycle — from the point of the second station, through the aux of the epicycle, up to the point of the first station (arc of direction); or from the point of the first station, through the point opposite the aux of the epicycle, up to the point of the second station (arc of retrogradation).

4. Course [Cursu] (slow and diminished / swift and augmented): namely when the planet moves more slowly, or more swiftly, than the line of the mean motion [linea medii motus].

5. Augmented or diminished in number [Aucti / Minuti — Numero]: when the equation of the argument [aequatio argumenti] is added to the mean motion (augmented), or subtracted from the mean motion (diminished).

6. Augmented or diminished in light [Aucti / Minuti — Lumine]: when the Sun recedes from them, or they themselves recede from the Sun (augmented in light); or when the Sun approaches them, or they approach the Sun (diminished in light).

7. Oriental and morning stars / Occidental and evening stars [Orientales & Matutini / Occidentales & Vespertini]: when they rise in the morning before the Sun (oriental and morning stars), or set in the evening after the Sun (occidental and evening stars).

8. Rising [Orientes ortu] (morning / evening): In the morning rising: when, before sunrise, in the eastern region they begin to emerge from the Sun's rays and to become visible to us — that is, as the Sun recedes from Saturn, Jupiter, and Mars, or as Venus and Mercury recede from the Sun. In the evening rising: when they recede so far from the Sun that they begin to appear in the western region in the evening.

- ☾ Moon: The Moon lacks this kind of rising, because on account of its speed the Sun cannot recede from it; for after conjunction [♌] it recedes from the Sun only until it begins to be seen.

- ♄ ♃ ♀ (three superiors): These three, because on account of their slowness they cannot recede from the Sun, lack this kind of rising.

9. Setting [Occidentes occasu] (morning / evening): In the evening setting: when, after sunset, in the western region they begin to be covered by the Sun's rays and gradually hidden from us. In the morning setting: when they approach the Sun so closely that in the morning they cease to appear in the eastern region.

- ☾ Moon: When after opposition it approaches the Sun, beginning to be covered by its rays. The Moon lacks this setting, since on account of its speed it cannot approach the Sun.
- ♄ ♃ ♀ (three superiors): These three, on account of their slowness, lack this setting.

10. Aspect of the planets [Aspectus Planetarum] (trine / quartile / sextile): it occurs when they are distant by a third, a fourth, or a sixth part of the Ecliptic.

11. Conjunction [Coniunctio] (mean / true / visible): it occurs when the lines of the mean second motions, of the true second motions, or the lines drawn from our eye through the bodies of the planets, are joined into one.

12. Opposition [Oppositio] (mean / true): it occurs when the lines of the mean, or true, [second] motions are set opposite one another along the longitude of the Zodiac.

13. Place of a star [Locus Astri] (true / apparent): it is a point of the firmament, determined by a line drawn from the centre of the world through the star itself (true place), or shown by a line extended from our eye through the star (apparent place).

14. Parallax [Diuersitas aspectus] (simply, or in altitude / in longitude / in latitude / of the Moon relative to the Sun):

- Simply, or in altitude: the arc of a great circle passing through the zenith of the head, [through the true and the apparent place] of the star.
- In longitude: the arc of the Ecliptic bounded by two great circles passing through the true place and the apparent place [of the star] and through the poles of the Zodiac.
- In latitude: the arc of a great circle passing through the poles of the Zodiac and the true or apparent place of the star, intercepted between two circles parallel to the Ecliptic, of which one runs through the true place, the other through the apparent place.

- Of the Moon to the Sun: that by which the parallax of the Moon, as the greater, exceeds the parallax of the Sun, as the lesser.

15. Apparent latitude of the Moon [Latitudo Lunæ visa]: it is the arc of a great circle passing through the poles of the Zodiac and the apparent place [of the Moon], intercepted [between the Ecliptic and] the apparent place. [Some words of this cell are not fully resolved: ?]

16. Ecliptic digits [Digiti Ecliptici]: they are called the twelfth parts of the diameter of the eclipsed body [of the Sun or of the Moon].

17. Minutes [Minuta] (of incidence in a solar or lunar eclipse / of tarrying in a lunar eclipse): they are the minutes of the Zodiac which the Moon, as the swifter, traverses in overtaking the Sun — from the beginning of a solar eclipse up to its middle; from the beginning of a lunar eclipse up to its middle (if it be partial, or total without tarrying), or up to the beginning of total obscuration (if it be total with tarrying); and from the beginning of total obscuration up to the middle of the lunar eclipse.

18. Visual diameter [Diameter visualis] (of the Sun / of the Moon):

- Of the Sun: at the aux of the eccentric it subtends 31[?] minutes; at the point opposite the aux [of the eccentric], 34[?] minutes.
- Of the Moon: [at the aux of the eccentric and the aux of the epicycle] it subtends 29[?] minutes; [at the aux of the eccentric and the point opposite the aux of the epicycle] 36[?] minutes. [The exact pairing of each condition — "of the world / of the Zodiac" [Mundi / Zodiaci] — to its subtended value is not fully resolvable on the tile: ?]

19. Of a star [Stellæ] (declination / latitude): it is the arc of a great circle passing through the poles of the equator (for declination) or of the Zodiac (for latitude) and through the place of the star, intercepted between the star and the equator [Æquinoctialem], or between the star and the Ecliptic.

20. Latitude of the planets [Latitudo Planetarum]:

- ☉ Sun: The Sun lacks latitude, for the plane surface of its eccentric nowhere deviates from the plane surface of the Ecliptic.
- ☾ Moon: Simple [simplex], on account of the declination of its eccentric receding from the Ecliptic only, while it always keeps the plane of the epicycle with it.

- ♄ ♃ ♀ (three superiors): Double [duplex] — one [component] on account of the fixed declination of the eccentric from the Ecliptic; the other on account of the plane surface of the epicycle deviating from the Ecliptic by a mobile declination, by which it approaches and recedes from the plane surface of the eccentric. From these the latitude of the three superiors is composed.
- ♀ ☿ Venus and Mercury: Triple [triplex]. The first is called Deviation [Deuiatio], and it is the declination of the eccentric from the Ecliptic — the epicycle, however, of Venus always keeping toward the North, but that of Mercury always toward the South of the Ecliptic. The second is called Declination [Declinatio]: namely where the diameter of the true aux of the epicycle deviates from the plane surface of the eccentric, approaching and receding from the diameter of the mean longitudes of the epicycle; which motion, taking place upon [that] diameter, is called reflexion [reflexio]. The third is likewise a certain mobile turning-away [auersio] of the diameter of the mean longitudes of the epicycle from the plane surface of the eccentric, and it takes place upon the diameter of the [long] longitudes of the epicycle as upon an axis. From all these the latitude of Venus [and of Mercury] is composed.

21. Argument of latitude [Argumentum latitudinis] (mean / true): it is the arc of the Zodiac counted, according to the succession of the signs, from the line of the true motion of the head of the Dragon [caput Draconis, the ascending node] up to the line of the mean motion (mean argument) or of the true motion (true argument) of the Moon.

FINIS.

[This "FINIS" closes the appended Definitions of Terms and thereby the whole Commentary on the Sphere of Sacrobosco (In Sphaeram Ioannis de Sacro Bosco). What follows is the alphabetical index to the Commentary.]

A Most Abundant Index of the Matters and Words Contained in These Commentaries, Arranged in Alphabetical Order

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- *Musicam quantitatem discretam considerare.* — That music considers discrete quantity. — 2[?]

N.

- *Nadir quid.* — What the nadir is. — 12
- *Nadir Solis quid.* — What the nadir of the Sun is. — 306
- *Naturalis Horizon quid, & cur sic dicatur.* — What the natural horizon is, and why it is so called. — 180
- *Naturales dies quid, eosque inæquales esse, & quare.* — What natural days are, that they are unequal, and why. — 234. 235
- *Naturalium dierum circuli qui.* — Which are the circles of the natural days. — 235
- *Naturales dies qua arte ad æqualitatem redigantur ab Astronomis.* — By what method natural days are reduced to equality by astronomers. — *ibid.*
- *Naturalis hora, siue Planetaria, vel Inæqualis, aut Temporalis quid.* — What the natural hour is — that is, the planetary, or unequal, or temporal hour. — 238
- *Naturalis horæ quantitas, quo pacto cognoscatur.* — How the length of the natural hour is ascertained. — *ibid.*
- *Nautica ars indiget Astronomia.* — The art of navigation has need of astronomy. — 4
- *Nihil esse extra mundum.* — That there is nothing outside the world. — 47
- *Noctis artificialis arcus quid.* — What the arc of the artificial night is. — 235
- *Noctes continuæ diebus continuis æquales non sunt, & quare.* — Consecutive nights are not equal to consecutive days, and why. — 232
- *Noctium continuarum quantitatem, qui accurate scire cupit quid facere debeat.* — What one who wishes to know accurately the length of consecutive nights must do. — 282
- *Nomina varia Zodiaci.* — The various names of the Zodiac. — 135
- *Nomina & ordo 12. signorum Zodiaci.* — The names and order of the 12 signs of the Zodiac. — *ibid.*

- *Nonæ Sphæræ proprius motus.* — The proper motion of the ninth sphere. — 39
- *Nox artificialis quid.* — What the artificial night is. — 181.235
- *Numerorum vim una cum arte numerandi Arithmetica explicat.* — Arithmetic sets forth the power of numbers together with the art of counting. — [?]
- *Numerus Elementorum, & ordo, qua via colligatur.* — By what method the number and order of the elements is gathered. — 18. & seq.
- *Numerus orbium cœlestium varius, & quo pacto colligatur.* — The number of the celestial orbs is variously given, and by what means it is gathered. — 22. & seq.
- *Numerus & ordo omnium corporum Uniuersum componentium.* — The number and order of all the bodies composing the universe. — 47
- *Numerus arenularum totum mundum usque ad concauum firmamenti replentium, qua ratione inuestigetur.* — By what method the number of grains of sand filling the whole world up to the concavity of the firmament is investigated. — 120
- *Numerus quis maior sit numero arenularum, quorum 10000. grano papaueris æquales sint, replentium totum mundum usque ad concauum firmamenti.* — What number is greater than the number of grains of sand — of which 10000 equal a poppy-seed — that would fill the whole world up to the concavity of the firmament. — 121
- *Numerus & ordo signorum Zodiaci.* — The number and order of the signs of the Zodiac. — 133. 134
- *Numeri duodenarii dignitas.* — The dignity of the number twelve. — 136

O.

- *Obiectiones duæ aduersus motum stellarum fixarum ab occasu in ortum super Polos Zodiaci ex apparentiis desumptæ, earumque solutiones.* — Two objections against the motion of the fixed stars from west to east upon the poles of the Zodiac, drawn from the appearances, and their solutions. — 32
- *Obiectiones eorum qui negant ex terra & aqua unum globum confici dissoluuntur.* — The objections of those who deny that earth and water form a single globe are dissolved. — 118. & seq.
- *Obliquitatis Zodiaci anomalia quid sit.* — What the anomaly of the obliquity of the Zodiac is. — 38

- *Obliquitatis Zodiaci verus motus quid.* — What the true motion of the obliquity of the Zodiac is. — ib.
- *Obliquitatis anomaliae æquatio quid.* — What the equation of the anomaly of the obliquity is. — ib.
- *Obliquitatis Zodiaci motus medius quid.* — What the mean motion of the obliquity of the Zodiac is. — ib.
- *Occasus verus, Æquinoctialisue quid.* — What the true, or equinoctial, setting is. — 182
- *Occasus Siderum secundum Poetas est triplex.* — According to the poets the setting of the stars is threefold. — 190
- *Occasus Heliacus quid.* — What the heliacal setting is. — ib.
- *Occasus Astri quid.* — What the setting of a star is. — 191
- *Occasus Chroronicus quid.* — What the chronic [acronychal] setting is. — 190
- *Occasus verus & apparens; item matutinus, & vespertinus quid.* — What the true and apparent setting is, and likewise the morning and evening setting. — 192
- *Occasus Poeticus ad quid conducatur.* — To what purpose the poetic setting serves. — 193
- *Occasum & ortum, cur Astronomi per Æquatorem definiant.* — Why astronomers define setting and rising by reference to the equator. — 194
- *Occasus secundum Astronomos quid.* — What setting is according to the astronomers. — ib.
- *Occasus rectus, vel obliquus, cur sic dicatur.* — Why setting is called right or oblique. — ib.
- *Occasus signorum quomodo fiat in Sphæra recta.* — How the setting of the signs occurs in the right sphere. — 195
- *Occasus signorum in Sphæra obliqua.* — The setting of the signs in the oblique sphere. — 200
- *Occidens absolutum ac respectuum.* — The west, absolute and relative. — 166

- *Oceani bona pars ab oriente in occidentem mouetur.* — A good part of the ocean moves from east to west. — 17
- *Oceanum, etiamsi omnia alia maria addantur, minorem esse, quam terram.* — That the ocean, even if all the other seas be added, is smaller than the earth. — 61
- *Oceani superficiem sub terræ superficie, si utraque compleretur, æquali semper distantia contineri.* — That the surface of the ocean, if both were completed, would always be held at an equal distance beneath the surface of the earth. — 62
- *Oceanum innumeris pene insulis respersum esse.* — That the ocean is strewn with almost innumerable islands. — ib.
- *Oceani & maris profunditas quanta sit ut plurimum.* — How great the depth of the ocean and sea is at most. — 63
- *Octauæ Sphæræ motum primus Hipparchus obseruauit.* — Hipparchus was the first to observe the motion of the eighth sphere. — 34
- *Octauæ Sphæræ motus, cur adeo difficilis semper fuerit.* — Why the motion of the eighth sphere has always been so difficult. — ib.
- *Octauæ Sphæræ motus penes quid sit regularis.* — In respect of what the eighth sphere's motion is regular. — 41
- *Octauæ Sphæræ motus proprius.* — The proper motion of the eighth sphere. — ib.
- *Octauæ Sphæræ motus quantitas, & periodus.* — The magnitude and period of the eighth sphere's motion. — ib.
- *Octauæ Sphæræ motus ubi velocissimus, ubi tardissimus, & ubi mediocris.* — Where the eighth sphere's motion is fastest, where slowest, and where moderate. — ib.
- *Octauæ Sphæræ motus cur dicatur præcessio Æquinoctiorum à Copernico.* — Why the eighth sphere's motion is called the precession of the equinoxes by Copernicus [Copernicus]. — ib.
- *Octauæ Sphæræ medius motus, vel media præcessio Æquinoctiorum quid.* — What the mean motion of the eighth sphere, or the mean precession of the equinoxes, is. — ib.

- *Octauæ Sphæræ quatuor motus qui sint.* — Which are the four motions of the eighth sphere. — ib.
- *Oculos homini solius Astronomiæ causa concessos; Plato asseruit.* — Plato [Plato] asserted that eyes were granted to man for the sake of astronomy alone. — 5
- *Oculum in edito monte constitutum plus videre posse, quam cœli medietatem, & quare.* — That an eye placed on a high mountain can see more than half of the heavens, and why. — 72
- *Officia & utilitates Eclipticæ vel Zodiaci.* — The functions and uses of the ecliptic or Zodiac. — 144 & seq.
- *Officia, & utilitates Colurorum.* — The functions and uses of the colures. — 147. & seq.
- *Officia, & utilitates Meridiani.* — The functions and uses of the meridian. — 164. & seq.
- *Officia & utilitates Horizontis.* — The functions and uses of the horizon. — 181. & seq.
- *Officia & utilitates Æquinocialis circuli.* — The functions and uses of the equinoctial circle. — 126. & seq.
- *Officia, & utilitates circulorum parallelorum, nempe Tropi corum, & polarium circulorum.* — The functions and uses of the parallel circles, namely the tropics and the polar circles. — 186
- *Opposita signa in Sphæra recta, habere æquales ascensiones.* — That opposite signs in the right sphere have equal ascensions. — 196
- *Oppositum Augis quid.* — What the point opposite the apogee is. — 304
- *Orbis & Sphæra quomodo inter se distinguantur.* — How an orb and a sphere are distinguished from each other. — 10
- *Orbis cœlestis duobus modis accipitur.* — A celestial orb is understood in two ways. — ib.
- *Orbes cœlestes inter se contigui sunt.* — The celestial orbs are contiguous with one another. — ib.

- *Orbis superior qua ratione mouet inferiorem orbem sibi contiguum.* — By what means a higher orb moves the lower orb contiguous to it. — 30
- *Orbis eccentricus simpliciter quid.* — What an eccentric orb, simply so called, is. — 290
- *Orbis eccentricus simpliciter in Planetis quid.* — What an eccentric orb, simply so called, is in the planets. — ib.
- *Orbes eccentrici secundum quid.* — Eccentric orbs in a qualified sense. — 291
- *Orbis Eccentricus deferens Planetam, aut Epicyclum.* — The eccentric orb carrying a planet or an epicycle. — ib.
- *Orbes totales Planetarum, ex pluribus orbibus partialibus componi.* — That the total orbs of the planets are composed of several partial orbs. — ib.
- *Orbibus eccentricis, & Epicyclis Sphæras planetarum constare secundum Ptolemæum.* — That according to Ptolemy the spheres of the planets consist of eccentric orbs and epicycles. — 292
- *Orbes Eccentricos simpliciter, & secundum quid, una cum concentricis, & Epicyclis in omnibus cœlis esse 33. tantum.* — That the eccentric orbs, both simply and in a qualified sense, together with the concentrics and the epicycles, are only 33 in all the heavens. — 300
- *Orbes quot ponantur ab iis, qui Eccentricos concedunt.* — How many orbs are posited by those who allow eccentrics. — ib.
- *Orbes concentrici quot à Fracastorio ponantur.* — How many concentric orbs are posited by Fracastoro [Fracastoro]. — ib.
- *Orbibus Eccentricis, & Epicyclis positis quomodo Phænomena defendantur.* — How, once eccentric orbs and epicycles are posited, the phenomena are accounted for. — 291.304
- *Orbes Augem deferentes qui sint.* — Which are the orbs carrying the apogee. — 304
- *Orbes deferentes Augem Solis, qui.* — Which orbs carry the apogee of the Sun. — ibid.

- *Orbes æquantes, cur in Planetis excogitati sint.* — Why equant orbs were devised for the planets. — 305
- *Orbium cœlestium, & motuum theoricæ in tabulas redactæ una cum terminis Astronomicis, & Passionibus Planetarum.* — The theories of the celestial orbs and motions reduced to tables, together with astronomical terms and the affections of the planets. — 308
- *Ordinis elementorum quæ causa sit.* — What the cause of the order of the elements is. — 15
- *Ordo quem Auctor in Sphæra tractanda servat.* — The order the author observes in treating of the sphere. — 6
- *Ordo Elementorum probatur.* — The order of the elements is proved. — 12
- *Ordo Sphærarum cœlestium.* — The order of the celestial spheres. — 21.42
- *Ordo Cœlorum secundum Platonem, Aristotelem. & Ægyptios.* — The order of the heavens according to Plato, Aristotle [Aristotle], and the Egyptians. — 42
- *Ordo Cœlorum secundum Aristarchum, & Copernicum.* — The order of the heavens according to Aristarchus [Aristarchus] and Copernicus. — *ibid.*
- *Ordo Planetarum confirmatur ex diuersitate aspectus.* — The order of the planets is confirmed by the diversity of aspect. — 43
- *Ordo Cœlorum secundum Astronomos recentiores, & quibus viis colligatur.* — The order of the heavens according to the more recent astronomers, and by what means it is gathered. — *ib.* & *seq.*
- *Ordo Cœlorum probatur ex velocitate & tarditate motus, & confirmatur ex Eclipsibus.* — The order of the heavens is proved from the speed and slowness of motion, and confirmed from eclipses. — 44
- *Ordo Planetarum confirmatur ex dominio Planetarum, & dierum denominatione.* — The order of the planets is confirmed from the dominion of the planets and the naming of the days. — 45
- *Oriens absolutum ac respectuum.* — The east, absolute and relative. — 166
- *Ortus verus, Æquinocialisue quid.* — What the true, or equinoctial, rising is. — 182

- *Ortus apud Poetas triplex, Cosmicus, Chronicus, & Heliacus.* — Among the poets rising is threefold: cosmic, chronic [acronychal], and heliacal. — 190
- *Ortus Chroronicus quid.* — What the chronic [acronychal] rising is. — ib.
- *Ortus siderum secundum Poetas est triplex.* — According to the poets the rising of the stars is threefold. — ib.
- *Ortus Cosmicus quid.* — What the cosmic rising is. — ib.
- *Ortus Heliacus quid.* — What the heliacal rising is. — ib.
- *Ortus Astri quid.* — What the rising of a star is. — 191
- *Ortus verus, & apparens; item matutinus, & vespertinus quid.* — What the true and apparent rising is, and likewise the morning and evening rising. — 192
- *Ortus Poeticus ad quid conducatur.* — To what purpose the poetic rising serves. — 193
- *Ortus secundum Astronomos quid.* — What rising is according to the astronomers. — 194
- *Ortum & occasum, cur Astronomi per Æquatorem definiant.* — Why astronomers define rising and setting by reference to the equator. — ibid.
- *Ortus rectus, vel obliquus, cur sic dicatur.* — Why rising is called right or oblique. — 195
- *Ortus arcuum Zodiaci in Sphæra recta.* — The rising of the arcs of the Zodiac in the right sphere. — ibid.
- *Ortus signorum quomodo fiat in Sphæra recta.* — How the rising of the signs occurs in the right sphere. — ibid.
- *Ortus signorum in Sphæra obliqua.* — The rising of the signs in the oblique sphere. — 200

P.

- *Parallelogrammorum Isoperimetrorum, quod rectangulum est, maius esse non rectangulo.* — That of isoperimetric parallelograms the rectangular one is greater than the non-rectangular. — 50. & seq.

- *Parallelorum semper apparentium, vel semper latentium maximus quid.* — What the greatest of the ever-apparent or ever-hidden parallels is. — 183
- *Parallelorum semper apparentium, semperque latentium maximorum habitudines variæ.* — The various relations of the greatest of the ever-apparent and ever-hidden parallels. — *ibid.*
- *Parallelorum circulorum, nempe Tropicorum, & circulorum Polarium officia atque utilitates.* — The functions and uses of the parallel circles, namely the tropics and the polar circles. — 186
- *Paralleli quatuor minores, distinguunt in cœlo, & in terra quinque Zonas.* — The four lesser parallels mark off five zones in the heavens and on the earth. — *ibid.*
- *Paralleli quinque in Sphæra qui sint.* — Which are the five parallels in the sphere. — *ibid.*
- *Paralleli circuli indicant æqualitatem dierum, & noctium in Sphæra recta, inæqualitatem vero in obliqua; Determinant latitudines locorum, & in illis numerantur longitu-* — The parallel circles indicate the equality of days and nights in the right sphere, but their inequality in the oblique; they determine the latitudes of places, and along them longi-[tudes are reckoned]. — [continues]
- *[longi]tudes; Indicant item declinationes stellarum, & altitudines.* — [longi]tudes; they likewise indicate the declinations and altitudes of the stars. — *ibid.*
- *Paralleli circuli quot à Sole in anno describantur.* — How many parallel circles are described by the Sun in a year. — 235
- *Parallelos plures describi à Sole commorante in signis Borealibus, quam eodem Signa Australia percurrente, & quare.* — That more parallels are described by the Sun while it lingers in the northern signs than while it passes through the southern signs, and why. — *ibid.*
- *Paralleli in terra quanto spatio inter se distantes à Ptolemæo & aliis Astronomis describantur.* — At what spacing apart the parallels on the earth are described by Ptolemy and other astronomers. — *ibid.*
- *Pars Borealis uniuersi est dextra.* — The northern part of the universe is the right side. — 156
- *Pars cœli dextra, & sinistra, quæ secundum Philosophos, Cosmographos, Astronomos, & Poetas.* — The right and left side of the heaven, which they are according to the philosophers, cosmographers, astronomers, and poets. — 177
- *Partes Astronomiæ.* — The parts of astronomy. — 2

- *Partes Assis quæ sint.* — Which are the parts of the *as* [the unit]. — 140
- *Pascha sine Astronomiæ cognitione rite seruari non potest.* — Easter cannot be duly observed without a knowledge of astronomy. — 4
- *Passiones Planetarum variæ.* — The various affections of the planets. — 305
- *Pauimentum ad libellam constructum non est planum, sed portio est Sphæræ, cuius centrum idem est, quod terræ.* — A pavement laid level is not flat, but is a portion of a sphere whose centre is the same as that of the earth. — 61
- *Paulinus Pridianus quid de noua stella scripserit.* — What Paulinus Pridianus [Paolino Pridiano] wrote concerning the new star. — 105
- *Periodus motus octauæ sphæræ secundum Ptolemæum, Albategnium, & Alphonsum.* — The period of the eighth sphere's motion according to Ptolemy, al-Battānī, and Alfonso. — 34
- *Periodus librationis nonæ sphæræ quanta sit.* — How great the period of libration of the ninth sphere is. — 39
- *Peripateticorum quorundam error, qui decuplam proportionem inter Elementa constituunt.* — The error of certain Peripatetics who set up a tenfold proportion among the elements. — 63
- *Periscii qui sint.* — Which are the Periscii [those whose shadows go all round]. — 189
- *Perpendiculari linea omnia à Mathematicis mensurari, & quare.* — That everything is measured by mathematicians with the perpendicular line, and why. — 7
- *Persarum Reges erant Mathematici.* — The kings of the Persians were mathematicians. — 4
- *Phœnices prima Arithmetices rudimenta tradidisse existimantur.* — The Phœnicians are thought to have handed down the first rudiments of arithmetic. — 2
- *Philosophi antiqui qua ratione partiti fuere disciplinas Mathematicas.* — In what way the ancient philosophers divided the mathematical disciplines. — 1
- *Philosophi antiqui maximum studium posuerunt in Astronomia.* — The ancient philosophers devoted the greatest effort to astronomy. — 1
- *Philosophis naturalibus necessaria est Astronomiæ cognitio.* — A knowledge of astronomy is necessary to natural philosophers. — 4
- *Philosophi quomodo sumant longitudinem, & latitudinem in Uniuerso.* — How philosophers take longitude and latitude in the universe. — 167

- *Phlegmatica & aquea signa Zodiaci, quæ.* — Which are the phlegmatic and watery signs of the Zodiac. — 134. 135
- *Physicum signum Zodiaci, & commune quid.* — What a physical sign of the Zodiac is, and a common one. — 134
- *Planetarum dominium in singulis horis diei.* — The dominion of the planets in the individual hours of the day. — 45
- *Planetæ, quo pacto dies hebdomadæ denominent.* — In what way the planets give their names to the days of the week. — *ibid.*
- *Planetæ unde sic dicti.* — Whence the planets are so called. — 11
- *Planetarum Sphæræ cur sic vocentur.* — Why the spheres of the planets are so called. — *ibid.*
- *Planetæ certo & determinato motu mouentur.* — The planets move with a fixed and determinate motion. — *ibid.*
- *Planetæ solum mouentur ad motum orbium suorum.* — The planets move only according to the motion of their own orbs. — *ibid.*
- *Planetæ quas motuum suorum periodos habeant.* — What periods of their motions the planets have. — 21
- *Planetarum ordo confirmatur ex diuersitate aspectus.* — The order of the planets is confirmed from the diversity of aspect. — 43
- *Planetarum ordo secundum Aristarchum Samium, & Nicolaum Copernicum.* — The order of the planets according to Aristarchus of Samos and Nicolaus Copernicus. — 42
- *Planetarum ordo secundum Platonem, Aristotelem, & Ægyptios.* — The order of the planets according to Plato, Aristotle, and the Egyptians. — *ibid.*
- *Planetarum ordo secundum Astronomos recentiores, & quibus viis colligatur.* — The order of the planets according to the more recent astronomers, and by what means it is gathered. — 42. & seq.
- *Planetæ Rempublicam constituunt.* — The planets constitute a commonwealth. — 45
- *Planetæ lumen suum à Sole accipiunt.* — The planets receive their light from the Sun. — *ibid.*
- *Planetarum ordo confirmatur ex dominio Planetarum & dierum denominatione.* — The order of the planets is confirmed from the dominion of the planets and the naming of the days. — *ibid.*
- *Planetæ non semper æqualiter distant à centro terræ.* — The planets are not always at an equal distance from the centre of the earth. — 49

- *Planetæ cuiusuis exaltatio, casus & detrimentum, quod signum Zodiaci dicatur.* — Which sign of the Zodiac is called the exaltation, fall, and detriment of any given planet. — 138
- *Planetæ cuiusuis domus, quod signum Zodiaci esse dicatur.* — Which sign of the Zodiac is said to be the house of any given planet. — *ibid.*
- *Planetæ præter Solem, non semper esse sub Ecliptica.* — That the planets, apart from the Sun, are not always beneath the ecliptic. — 142
- *Planetam in aliquo signo esse, quot modis dicatur.* — In how many senses a planet is said to be in a given sign. — 143[?]
- *Planetæ quando Boreales, & quando Australes.* — When the planets are northern, and when southern. — 142
- *Planetæ qua ratione in signis Borealibus existentes dici possint Australes, Boreales vero, quando in signis Australibus existunt.* — On what grounds planets present in the northern signs can be called southern, and northern when they are present in the southern signs. — 144
- *Planetæ in qua parte cœli oriantur, & occidunt Heliace.* — In what part of the heaven the planets rise and set heliacally. — 192
- *Planetæ in orbibus Eccentricis moueri probabilius esse, quam eos in concentricis orbibus ferri.* — That it is more probable that the planets move in eccentric orbs than that they are carried in concentric orbs. — 290
- *Planetæ cuiusque cœlum ex pluribus orbibus componitur.* — The heaven of each planet is composed of several orbs. — *ibid.* & seq.
- *Planetæ pluribus cieri motibus.* — That the planets are set in motion by several motions. — 291
- *Planetarum Sphæras Ptolemæus cum aliis Astronomis diuisit in orbés eccentricos, & Epicyclos.* — Ptolemy, along with other astronomers, divided the spheres of the planets into eccentric orbs and epicycles. — 292
- *Planetarum Sphære in orbés concentricos diuidebantur ab Eudoxo & Calippo.* — The spheres of the planets were divided into concentric orbs by Eudoxus [Eudoxus] and Callippus [Callippus]. — *ib.*
- *Planetarum passiones variae.* — The various affections of the planets. — 305
- *Planetæ statio, directio, aut retrogradatio quid.* — What a planet's station, direct motion, or retrogradation is. — *ibid.*
- *Planeta quando dicitur stationarius, directus, aut retrogradus.* — When a planet is said to be stationary, direct, or retrograde. — *ibid.*
- *Planetarum Theoricæ in tabulas digestæ.* — The theories of the planets arranged into tables. — 308

- *Plato quo pacto quatuor Elementis & cœlo tribuerit figuras quinque corporum regularium.* — In what way Plato assigned to the four elements and the heaven the figures of the five regular solids. — 66
- *Poetæ ignari Astronomiæ, præclari nihil præstare poterunt.* — Poets ignorant of astronomy will be able to produce nothing distinguished. — 4
- *Poeticus ortus, & occasus, ad quid conducat.* — To what purpose the poetic rising and setting serves. — 193
- *Poeticus ortus, & occasus stellarum quid, & quotuplex.* — What the poetic rising and setting of the stars is, and how manifold. — 190. & seq.
- *Polares circuli quinam sint, & quantum à polis mundi absint; ac quomodo à Græcis sumantur.* — Which are the polar circles, and how far they are from the poles of the world; and how they are understood by the Greeks. — 184
- *Polares circuli includunt regiones versus polos, quæ maximum diem habent maiorem quam 24. horarum.* — The polar circles enclose the regions toward the poles which have their longest day greater than 24 hours. — 186
- *Polares circuli, & Tropici constituunt quinque Zonas.* — The polar circles and the tropics constitute five zones. — ib.
- *Polarium circulorum officia, & utilitates.* — The functions and uses of the polar circles. — ib.
- *Poli Sphæræ quid sint.* — What the poles of the sphere are. — 9
- *Poli duo, nimirum Borealis & Australis explicantur.* — The two poles, namely the northern and the southern, are explained. — 10
- *Poli Sphæræ, & mundi.* — The poles of the sphere and of the world. — ib.
- *Poli unde dicti sint.* — Whence the poles are so called. — ib.
- *Poli non sunt stellæ.* — The poles are not stars. — ib.
- *Poli meridiani circuli quinam sint.* — Which are the poles of the meridian circle. — 13
- *Poli altitudo quanta sit Romæ.* — How great the altitude of the pole is at Rome. — 127
- *Poli altitudo supra Horizontem, quo pacto ex altitudine Meridiana Solis eliciatur.* — In what way the altitude of the pole above the horizon is elicited from the meridian altitude of the Sun. — 165
- *Poli eleuatio supra Horizontem, æqualis est distantiae Zenith ab Æquatore.* — The elevation of the pole above the horizon is equal to the distance of the zenith from the equator. — 181

- *Poli altitudinem in quocunque loco æqualem esse latitudini eiusdem loci.* — That the altitude of the pole in any place whatever is equal to the latitude of that same place. — ib.
- *Poli arcticus, & antarcticus eorumque varia nomina.* — The arctic and antarctic poles and their various names. — 10. 126
- *Poli Zodiaci à Polo mundi distantia, æqualis est maximæ Solis declinationi.* — The distance of the pole of the Zodiac from the pole of the world is equal to the greatest declination of the Sun. — 184
- *Poli altitudo quo maior est, eo maior fit inæqualitas dierum & noctium artificialium.* — The greater the altitude of the pole, the greater the inequality of the artificial days and nights. — 237
- *Polorum Zodiaci à polis mundi distantia.* — The distance of the poles of the Zodiac from the poles of the world. — 132
- *Polus circuli in sphæra quid.* — What the pole of a circle in the sphere is. — 12
- *Polus uterque in Horizonte Sphæræ rectæ iacet.* — Both poles lie on the horizon of the right sphere. — 15
- *Polus unus in Sphæra obliqua supra Horizontem exaltatur & alter infra Horizontem deprimitur.* — In the oblique sphere one pole is raised above the horizon and the other is depressed below it. — ib.
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- *Possidonii ratio facilima, qua ambitus terræ inuestigetur.* — The very easy method of Posidonius [Posidonius] by which the circumference of the earth is investigated. — 109
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Index of Subjects and Terms (continued)

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- No notable star is near the Antarctic Pole. *ibid.*
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- Those who supposed that the stars move of themselves are refuted. 25
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- What the longitude of a star is. 74
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- The longitudes of the stars in the table begin from the first star of Aries. 98
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Index of Subjects and Terms [Index rerum et verborum] (continued and concluding)

This concludes the alphabetical subject index to Clavius's commentary In Sphaeram Ioannis de Sacro Bosco. The index runs across two columns per page; entries are transcribed in reading order (left column, then right column). Each entry gives the Latin heading as printed, an English rendering, and the page reference(s) verbatim. The final work of the index (letters S–Z) closes with FINIS at the foot is a blank leaf — the Astrolabium title page does not yet appear in this opening.

S (conclusion)

Entry (Latin, as printed)	English	Page
Stellarum ortus & occasus quomodo Ptolemæus vocet.	What Ptolemy [Ptolemy] calls the rising and setting of the stars.	193
Stellæ in qua parte cæli oriantur, & occidunt heliace.	In what part of the sky the stars rise and set heliacally.	192
Subiecti alicuius libri quænam debeant esse conditiones.	What conditions the subject-matter of any book ought to meet.	6
Subiectum huius Sphæræ idem esse quod Astronomiæ, & quodnam illud sit.	That the subject of this <i>Sphere</i> is the same as that of astronomy, and what it is.	ibid.
Superficies quid sit.	What a surface is.	7
Superficies est duplex, plana & curua.	A surface is of two kinds, plane and curved.	8
Superficiem maris sub superficie terræ, si vtraque compleretur, æquali semper distantia contineri.	That the surface of the sea, beneath the surface of the earth — if each were completed into a full sphere — would always be kept at an equal distance.	62
Superficies cuiusque circuli, & conuexa superficies Sphæræ quo pacto reperiatur.	How the area of any circle, and the convex surface of a sphere, is found.	112
Superficies conuexa cuiuslibet sphæræ, qua via inueniatur.	By what method the convex surface of any sphere is found.	113

T

Entry (Latin, as printed)	English	Page
TABVLA constellationum 48. continens stellarum numerum, & situm, longitudes, & latitudes, ac magnitudines.	A table of the 48 constellations, giving the number, position, longitudes, latitudes, and magnitudes of the stars.	76. & seq.
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Tabulæ proportionum diametrorum stellarum fixarum, & Planetarum ad diametrum terræ: Et proportionum magnitudinum stellarum, & Planetarum ad magnitudinem terræ.	Tables of the ratios of the diameters of the fixed stars and planets to the diameter of the earth; and of the ratios of the magnitudes of the stars and planets to the magnitude of the earth.	100. & seq.
Tabula, qua Æquatoris gradus in horas, & vicissim horæ in gradus permutantur.	A table by which degrees of the equator are converted into hours, and hours in turn into degrees.	128
Tabulæ quibus partes Æquatoris in tempus, & contra tempus in partes Æquatoris conuertuntur.	Tables by which parts of the equator are converted into time, and conversely time into parts of the equator.	ibid.
Tabulæ quatuor in rebus Astronomicis perutiles.	Four tables of great usefulness in astronomical matters.	129.130
Tabula conuertendi Gradus, Minuta, Secunda, Tertia, &c, Æquatoris, in Minuta, Secunda, Tertia, &c. Dierum, & contra.	A table for converting degrees, minutes, seconds, thirds, etc., of the equator into minutes, seconds, thirds, etc., of days, and conversely.	ibid. & seq.
Tabula declinationum punctorum Ecliptic[æ] ab Æquatore.	A table of the declinations of the points of the ecliptic from the equator.	150
Tabula ascensionum obliquarum ad varias altitudines Poli.	A table of oblique ascensions for various elevations of the pole.	209

Entry (Latin, as printed)	English	Page
Tabula continens longitudes, latitudinesque Ciuitatum.	A table giving the longitudes and latitudes of cities.	168
Tabula ascensionum rectarum qua arte construatur.	By what technique the table of right ascensions is constructed.	198
Tabula Ascensionum rectarum.	The table of right ascensions.	199
Tabula differentiarum ascensionalium.	A table of ascensional differences.	203
Tabula arcuum semidiurnorum qua arte constituatur.	By what technique the table of semidiurnal arcs is set up.	239
Tabula temporis semidiurni in omnibus signis pro omnibus latitudinibus.	A table of semidiurnal time in all the signs for all latitudes.	288
Tabula Climates secundum veteres.	A table of the climes according to the ancients.	283
Tabula Climates secundum recentiores.	A table of the climes according to the moderns.	289
Temporum anni qualitates.	The characters of the seasons of the year.	155
Temporum quatuor anni quibus quadrantibus Zodiaci respondeant.	To which quadrants of the zodiac the four seasons of the year correspond.	ibid.
Terra sub Æquinoctiali, & Polis est habitabilis.	The earth is habitable under the equinoctial and at the poles.	13
Terræ rotunditas caussa est Sphæræ rectæ & obliquæ.	The roundness of the earth is the cause of the right and the oblique sphere.	14
Terra est tanquam centrum mundi.	The earth is, as it were, the center of the universe.	15.16
Terra facta est à Deo sine vllis concauitatibus.	The earth was made by God without any hollows.	17
Terra & aqua vnum globum	Earth and water form a single	ibid.

Entry (Latin, as printed)	English	Page
efficiunt.	globe.	
Terra immobilis est.	The earth is motionless.	ibid.
Terram rotundam esse ab ortu in occasum probatur.	It is proved that the earth is round from east to west.	54
Terram esse rotundam à Septentrione in Austrum probatur.	It is proved that the earth is round from north to south.	55
Terræ rotunditas est sphærica.	The roundness of the earth is spherical.	ibid.
Terra non est plana.	The earth is not flat.	ibid.
Terra cur appareat plana.	Why the earth appears flat.	56
Terra non est caua.	The earth is not concave.	ibid.
Terram & aquam vnum globum efficere.	That earth and water form a single globe.	57. & seq.
Terra sola cur centrum mundi occupet, & non etiam aqua.	Why the earth alone, and not also water, occupies the center of the universe.	61
Terra est maior tam aqua quam aere.	The earth is greater than both water and air.	64
Terra minor est igne.	The earth is smaller than fire.	ibid.

T (continued)

Entry (Latin, as printed)	English	Page
Terram in medio mundi esse, ratione Ptolemæi probatur.	That the earth is at the middle of the universe, proved by Ptolemy's argument.	68
Terra non est in plano Æquatoris extra axem mundi.	The earth is not in the plane of the equator, off the axis of the universe.	ibid.
Terram in medio mundi esse rationibus probatur.	That the earth is at the middle of the universe, proved by [several] arguments.	68. & seq.
Terram non esse in axe mundi extra planum Æquatoris.	That the earth is not on the axis of the universe, off the	69

Entry (Latin, as printed)	English	Page
	plane of the equator.	
Terram non esse extra Æquatorem & axem mundi.	That the earth is not outside the equator and the axis of the universe.	170
Terram in centro mundi esse sitam.	That the earth is situated at the center of the universe.	67. & seq.
Terram esse instar puncti respectu firmamenti.	That the earth is like a point with respect to the firmament.	70. & seq.
Terram Sole esse minorem, Luna vero maiorem.	That the earth is smaller than the Sun, but larger than the Moon.	103
Terra cum singulis cœlis collata quomodo se habeat.	How the earth compares when set against each of the heavens.	ibid.
Terram non moueri motu recto.	That the earth is not moved with rectilinear motion.	105
Terram omnino immobilem esse.	That the earth is wholly motionless.	106
Terram non moueri in orbem.	That the earth does not move in a circle.	ibid.
Terram esse immobilem sacræ literæ affirmant.	The sacred scriptures affirm that the earth is motionless.	ibid.
Terra cur sit immobilis variæ sententiæ & earum confutatio.	Various opinions as to why the earth is motionless, and their refutation.	107
Terræ ambitus secundum Macrobiū, & Eratosthenem.	The circumference of the earth according to Macrobius and Eratosthenes.	ibid.
Terra cur in Medio quiescat.	Why the earth remains at rest at the middle.	ibid.
Terræ vt totus ambitus habeatur, satis est si interuallum vnus gradus in terra inuestigetur.	To obtain the whole circumference of the earth, it is enough to determine the interval of one degree on the earth.	108

Entry (Latin, as printed)	English	Page
Terræ ambitus sumendus est penes circulum maximum.	The circumference of the earth is to be taken along a great circle.	ibid.
Terræ ambitus qua ratione inuestigandus sit.	By what method the circumference of the earth is to be investigated.	ibid. & seq.
Terræ ambitus varijs vijs exploratur.	The circumference of the earth is explored by various means.	ibid. & seq.
Terræ vt totus ambitus cognoscatur, satis est si spacium dimidiati gradus in terra, vel tertiæ partis vnus gradus mensuretur.	To learn the whole circumference of the earth, it is enough to measure the space of half a degree on the earth, or of a third part of one degree.	ibid.
Terræ diameter quo pacto ex ambitu cognito eruatur.	How the diameter of the earth is derived from the known circumference.	111
Terræ ambitum varium inuenerunt varij Auctores.	Various authors have found different values for the earth's circumference.	114
Terræ ambitus secundum Alphraganum, Almæonem, & Thebith.	The circumference of the earth according to al-Farghānī [Alfraganus], al-Ma'mūn [Almæon], and Thābit ibn Qurra [Thebith].	115
Terræ ambitus secundum recentiores nautas.	The circumference of the earth according to the more recent navigators.	ibid.
Terræ ambitus secundum Fernelium.	The circumference of the earth according to Fernel [Jean Fernel].	ibid.
Terræ habitabilis portio quanta sit secundum Auctorem, & quomodo septem Climata ab eo describantur.	How large the habitable portion of the earth is according to the author [Sacrobosco], and how the	296

Entry (Latin, as printed)	English	Page
	seven climes are described by him.	
Terræ maior pars est habitabilis, quam ab Auctore ponitur.	A larger part of the earth is habitable than the author supposes.	287
Terræ vmbra conica.	The conical shadow of the earth.	305
Thaleti diuitias magnas peperit Astronomia.	Astronomy brought great wealth to Thales [Thales].	5
Theoricarum quis inuentor fuerit.	Who was the inventor of the [planetary] theories.	ibid.
Theoricæ Planetarum in tabulas digestæ.	The theories of the planets arranged in tables.	308. & seq.
Theoretica Astronomia quænam dicatur.	What is called theoretical astronomy.	3
Trepidationis motus octauæ Sphæræ quomodo fiat.	How the motion of trepidation of the eighth sphere [sphaera octava] comes about.	29
Trepidationis motus quid.	What the motion of trepidation is.	ibid.
Trepidationis motus cur ab Astronomis in cœlo ponatur.	Why the motion of trepidation is placed in the heaven by astronomers.	33
Trepidationis Thebitij defectus.	The shortcomings of Thābit ibn Qurra's [Thebith] account of trepidation.	34
Trepidationis motus octauæ Sphæræ secundum Alphonsum.	The motion of trepidation of the eighth sphere according to Alfonso [Alfonso X].	ibid.
Trepidationis motus refutatur.	The motion of trepidation is refuted.	35
Trepidationis octauæ Sphæræ confutatio.	Refutation of the trepidation of the eighth sphere.	39
Tropicus Capricorni quid.	What the Tropic of Capricorn is.	13

Entry (Latin, as printed)	English	Page
Tropicus Cancrī quid.	What the Tropic of Cancer is.	ibid.
Tropici qui sint, & quomodo describantur, earumque varia nomina.	What the tropics are, how they are described, and their various names.	183
Tropici includunt viam Solis.	The tropics enclose the path of the Sun.	186
Tropici, & polares circuli cum in cœlo, tum in terra quinque Zonas constituunt.	The tropics and the polar circles, both in the heavens and on the earth, establish five zones.	ibid.
Tropicorum officia & vtilitates.	The functions and uses of the tropics.	ibid.

V

Entry (Latin, as printed)	English	Page
VAS ad radices montis plus aqua continet, quam in cacumine.	A vessel at the foot of a mountain holds more water than at its summit.	65
Velocitas motus cœli incomprehensibilis, quibusdam exemplis declaratur.	The incomprehensible speed of the heaven's motion, illustrated by certain examples.	19

(V continues at the head left column:)

Entry (Latin, as printed)	English	Page
Venerem non posse eclipsare Solem.	That Venus cannot eclipse the Sun.	45
Veneris circulum visualem subcentuplum esse circuli visualis Solis.	That the visual circle of Venus is one hundredth of the visual circle of the Sun.	ibid. & seq.
Veneris diametrum visualem subdecuplam esse visualis diametri Solis.	That the visual diameter of Venus is one tenth of the visual diameter of the Sun.	ibid. & seq.

Entry (Latin, as printed)	English	Page
Venus quando dicatur Lucifer, & quando Hesperus.	When Venus is called Lucifer [the Morning Star] and when Hesperus [the Evening Star].	187
Vertex loci quid sit.	What the vertex [zenith] of a place is.	12
Verticales circuli qui dicantur.	What are called vertical circles.	122
Verticalis circulus proprie dictus secat cœlum in hemisphærium Boreale & Australe.	The vertical circle, properly so called, divides the heaven into a northern and a southern hemisphere.	145
Verus locus astri quid.	What the true place of a star is.	43
Veri motus linea quid sit.	What the line of true motion is.	145.146
Verus motus quid sit.	What true motion is.	ibid.
Verus locus stellæ in Zodiaco quid, & quomodo cognoscatur.	What the true place of a star in the zodiac is, and how it is determined.	ibid.
Verus motus stellæ, & linea veri motus quid.	What the true motion of a star, and the line of true motion, are.	ibid.
Verus, siue Æquinoctialis ortus, & occasus quid.	What the true, or equinoctial, rising and setting is.	182
Verus, & Apparens ortus quid.	What the true and the apparent rising are.	190
Vespertinus, ac Matutinus ortus, & occasus quid.	What the evening and morning rising and setting are.	192
Visus locus astri quid.	What the apparent place of a star is.	43
Visualis diameter, &	What the visual diameter	46

Entry (Latin, as printed)	English	Page
circulus Astri quid.	and circle of a star are.	
Vitam tam longæuam cur Deus primis parentibus prorogauerit.	Why God prolonged so long a life for the first parents.	2
Vltra mundani mundi consideratio ad quem spectet.	To whom the consideration of the supramundane world belongs.	15
Vmbræ gnomonum, qui cum Horizonte angulos rectos efficiunt, tempore Æquinoctiorum per vnam eandem lineam rectam ab oriente in occidentem proiiciuntur.	The shadows of gnomons that make right angles with the horizon are cast, at the equinoxes, along one and the same straight line from east to west.	69.71.72
Vmbra terræ conica.	The conical shadow of the earth.	305
Vniuersi pars Borealis est dextra.	The northern part of the universe is the right- hand side.	136
Vniuersi longitudinem & latitudinem, quomodo sumant Philosophi.	How philosophers reckon the length and breadth of the universe.	167
Vsus tabulæ stellarum.	The use of the table of stars.	98
Vtilitates Æquatoris, Zodiaci, Colurorum, Meridiani, Horizontis, Tropicorum, & Polarium circulorum. Lege officia eorundem.	The uses of the equator, the zodiac, the colures, the meridian, the horizon, the tropics, and the polar circles. See the entries "functions" [officia] for these same.	(cross-reference; no page)

Z

Entry (Latin, as printed)	English	Page
ZENITH quid.	What the zenith is.	12
Zenith capitis quid.	What the zenith of the head [overhead point] is.	147
Zenith capitis, esse Polum Horizontis.	That the zenith of the head is the pole of the horizon.	181
Zenith tantum distare ab Æquatore, quanta est altitudo Poli.	That the zenith is as far distant from the equator as the elevation of the pole.	182
Zenith loci posito sub Æquatore; deinde inter Æquatorem & Tropicum Cancri; Item sub Tropico Cancri: Item inter Tropicum Cancri & circulum Arcticum; Posthæc sub circulo Arctico; Rursus inter circulum Arcticum & Polum: & postremo sub Polo, quid accidat ratione ortus & occasus siderum.	What happens with respect to the rising and setting of the heavenly bodies when the zenith of a place is set under the equator; then between the equator and the Tropic of Cancer; likewise under the Tropic of Cancer; likewise between the Tropic of Cancer and the Arctic Circle; then under the Arctic Circle; again between the Arctic Circle and the pole; and finally under the pole.	279
Zenith ab Æquatore versus alterutrum Polorum, septem modis variari potest.	The zenith, from the equator toward either of the poles, can vary in seven ways.	279
Zodiacus circulus quisnam sit.	What the circle called the zodiac is.	12
Zodiacus quatuor habet præcipua puncta.	The zodiac has four principal points.	ibid.
Zodiacus in latitudine 12 grad. habet.	The zodiac has [an extent] of 12 degrees in latitude.	13
Zodiacus nonæ Sphæræ, qua ratione moueri intelligatur ab occasu in ortum.	In what way the zodiac of the ninth sphere is understood to move from west to east.	28

Entry (Latin, as printed)	English	Page
Zodiacus duplex, Mobilis, & fixus, seu immobilis.	The zodiac is twofold: mobile, and fixed or immobile.	27.28
Zodiacus quid; cur sic dictus, quare, & à quo primum inuentus.	What the zodiac is; why it is so called; for what purpose; and by whom it was first devised.	132
Zodiacus varios angulos cum quouis Horizonte efficit.	The zodiac makes various angles with any horizon.	ibid.
Zodiaci signa cur ab animalibus denominantur.	Why the signs of the zodiac are named from animals.	ibid. & seq.
Zodiaci varia nomina.	The various names of the zodiac.	ibid.
Zodiaci nomen, cuius cœli Zodiaco magis conueniat.	The name "zodiac" — to which heaven's zodiac it better applies.	ibid.
Zodiacus, cur in cœlo & in Sphæra obliquum situm habeat.	Why the zodiac has an oblique position both in the heaven and on the [material] sphere.	ibid.
Zodiacus cur in 12. signa diuidatur.	Why the zodiac is divided into 12 signs.	134. & seq.
Zodiaci quadrantes quibus anni temporibus respondeant.	To which seasons of the year the quadrants of the zodiac correspond.	135
Zodiaci initium cur ab Ariete sumatur.	Why the beginning of the zodiac is taken from Aries.	136. & seq.
Zodiaci quatuor puncta Cardinalia, quæ.	What the four cardinal points of the zodiac are.	ibid.
Zodiaci semicirculus descendens, & ascendens quid.	What the descending and ascending semicircles of the zodiac are.	137
Zodiaci signa quæ, & quorum Planetarum domus sint.	Which are the signs of the zodiac, and of which planets they are the houses.	138
Zodiaci diuisio in gradus, minuta, &c.	The division of the zodiac into degrees, minutes, etc.	ibid.
Zodiacus cur in 360. gradus	Why the zodiac is divided into	139

Entry (Latin, as printed)	English	Page
diuidatur.	360 degrees.	
Zodiacus totus, quot Gradus, minuta, secunda, &c. contineat.	How many degrees, minutes, seconds, etc., the whole zodiac contains.	ibid.
Zodiacus inter omnes circulos sphæræ, solus latitudinem habet 12 graduum.	Among all the circles of the sphere, the zodiac alone has a breadth of 12 degrees.	141
Zodiacus, cur latus ponatur ab Astronomis.	Why the zodiac is treated as broad by astronomers.	ibid.
Zodiaci pars Borealis, & Australis quæ.	Which is the northern and which the southern part of the zodiac.	142
Zodiaci pars Borealis, & Australis, signaque Borealia, & Australia.	The northern and southern part of the zodiac, and the northern and southern signs.	ibid.
Zodiaci variæ acceptiones.	The various senses in which "zodiac" is taken.	ib. & seq.
Zodiaci officia, & vtilitates.	The functions and uses of the zodiac.	144
Zodiaci Polos tantum abesse à Polis mundi, quanta est maxima Solis declinatio.	That the poles of the zodiac are as far from the poles of the universe as the greatest declination of the Sun.	156
Zodiacum in nullo Horizonte vniformiter oriri.	That the zodiac rises uniformly at no horizon.	193
Zodiaci signa, seu arcus quomodo secundum Astronomos, oriantur, & occidunt tam in Sphæra recta, quam in obliqua.	How the signs, or arcs, of the zodiac rise and set according to the astronomers, both in the right sphere and in the oblique.	195. & seq.
Zonas quinque Tropici, & polares circuli constituunt.	The tropics and the polar circles establish five zones.	186
Zonas quinque in cœlo, & in terra, Paralleli quatuor minores distinguunt.	Four lesser parallels mark off the five zones in the heaven and on the earth.	ibid.

Entry (Latin, as printed)	English	Page
Zonæ quid, & quibus Parallelis constituentur.	What zones are, and by which parallels they are established.	ibid.
Zona torrida, Zona temperata, Zona frigida.	The torrid zone, the temperate zone, the frigid zone.	187
Zonæ habitabiles, & inhabitabiles quænam dicantur.	Which zones are called habitable and which uninhabitable.	ibid.
Zonarum varia nomina.	The various names of the zones.	ibid.
Zonæ terrestres quo pacto cœlestibus suppositæ sint.	In what way the terrestrial zones are set beneath the celestial ones.	188
Zonam torridam, & frigidam esse habitabiles.	That the torrid and the frigid zone are habitable.	ibid.
Zonarum latitudines quantæ sint.	How great the latitudes of the zones are.	ibid.
Zonarum longitudes qua arte deprehendantur tam in principio, quam in medio & fine.	By what technique the longitudes of the zones are ascertained, at their beginning, middle, and end alike.	ibid.
Zona quæuis est eiusdem latitudinis, sed non eiusdem latitudinis quoad omnes partes.	Any given zone is of the same latitude [throughout], but not of the same latitude in respect of all its parts.	189
Zonarum latitudines, & longitudes quomodo inuestigentur.	How the latitudes and longitudes of the zones are investigated.	ibid.
Zona & Clima quomodo differant.	How a zone and a clime differ.	190

FINIS.

(*End of the Index rerum et verborum**, and of the front/back matter to the* In Sphaeram commentary.)

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The leaf carries no printed text of its own; only faint show-through (reversed ghosting) from an adjacent leaf is visible through the paper. It is a blank leaf following the close of the index. The **ASTROLABIVM** — the second, separately-titled work of this volume — has **not** yet begun in this opening: its title page falls on the next leaf, beyond the range read here.

ASTROLABIVM

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This leaf is blank. It carries no printed text; the mottled markings visible on it are show-through from the title page printed on its reverse (the imprint "MOGV[NTIAE]" and other lines are legible in mirror image).

[Astrolabium title]

Woodcut device. The centerpiece of the title page is a woodcut of an astrolabe seen in perspective — a circular instrument with an hour-scale of Roman numerals running around its rim (... IX · X · XI · XII · I · II · III ...), an inner projection of circles, and an openwork rete (the "spider") whose framework is worked into foliate scrollwork with serpent/dragon-like curls and small star-pointers. Above and around the instrument a ribbon banner carries the motto, split left and right of the suspension ring:

- Left of the ring (banner): **SIC LVDITVR** — "Thus one plays"
- Right of the ring (banner): **ASTRIS.** — "...with the stars." (i.e., "Thus one sports with the stars.")

CHRISTOPHORI CLAVII BAMBERGENSIS E SOCIETATE IESV [Christoph Clavius] of Bamberg, of the Society of Jesus

ASTROLABIVM TRIBVS LIBRIS EXPLICATVM, The Astrolabe, expounded in three books,

IN HAC EDITIONE AB IPSO AVCTORE plurimis locis correctum. in this edition corrected by the author himself in very many places.

[Margin: A handwritten (cursive) ownership inscription runs in two lines across the middle of the title page, over and around the printed matter: "[?] Monri [?]" ...

"ET" ... "[?] Schypennis." [The reading of the personal/place names is uncertain — a later owner's inscription, not part of the printed text.]]

MOGVNTIAE, At Mainz,

Sumptibus ANTONII HIERAT excudebat REINHARDVS ELTZ. At the expense of Antonius Hierat; printed by Reinhardus Eltz.

Cum gratia & priuilegio sacrae Caes. Maiest. With the grace and privilege of His Sacred Imperial Majesty.

ANNO M. DC. XI. In the year 1611.

[title-page verso: list of the work's novelties]

QVÆ IN ALIORVM ASTROLABIIS NON traduntur, sed in hoc nunc primum inuenta sunt, ac demonstrata.

Things which are not treated in the astrolabes of others, but which in this one are now for the first time discovered and demonstrated:

I. The projection onto a plane of any circle whatever — whether a great circle or not a great circle — provided only that its position on the sphere is known. [*Cuiusuis circuli siue maximi, siue non maximi, proiectio in planum, si modo eius situs in sphæra ... cognitus sit.*]

II. The division of any circle whatever — great or not great — projected onto a plane, into 360 *unequal* parts, which correspond to the 360 *equal* degrees of the same circle on the sphere. [*in planum proiecti diuisio in 360. partes inæquales, quæ gradibus 360. æqualibus eiusdem circuli in sphæra respondeant.*]

III. To assign, for any point or arc given in the heavens or on the sphere, the corresponding point or arc on the plane of the astrolabe; and conversely, given any point or arc on the plane of the astrolabe, to find what point or arc in the heavens or on the sphere it represents. [*Cuilibet puncto, vel arcui in cælo, vel sphæra dato, respondens punctum, vel arcum in plano Astrolabij assignare: Et contra, dato quolibet puncto, vel arcu in plano Astrolabij, quod punctum, vel arcum in cælo, seu sphæra referat, inuenire.*]

IIII. For a circle described in any manner on the plane of the astrolabe, or a straight line drawn in any manner, to determine what circle or straight line it represents in the heavens or on the sphere. [*Circulo vtcunque descripto in Astrolabij plano, vel recta vtcunque ducta, quem circulum, aut rectam in cælo, seu sphæra repræsentet, explorare.*]

V. The use of the astrolabe — and a very extensive use at that — by the aid of the compass and ruler alone, without the help of a material astrolabe. [*Vsus Astrolabij, isque amplissimus, solius circini, ac regulæ beneficio, sine auxilio Astrolabij materialis.*]

VI. The construction in a plane of all spherical triangles, and the finding of their angles and sides, without the aid of numbers. [*Omnium triangulorum sphaericorum descriptio in plano, & angulorum, laterumque eorundem inuentio sine ope numerorum.*]

VII. The solution of all problems that are worked out by means of spherical triangles with the aid of numbers, by the sole use of compass and ruler. [*Omnium quæstionum, quæ per triangula sphaerica adiumento numerorum enodantur, solius beneficio circini, ac regulæ, explicatio.*]

VIII. The use of sines, tangents, and secants by prosthaphæresis alone — that is, by addition and subtraction only, without multiplication or division of numbers; and a wonderful shortcut for all triangles. [*Vsus Sinuum, Tangentium, atque Secantium per solam prosthaphæresim, hoc est, per additionem, subtractionemque solam, sine multiplicatione, ac diuisione numerorum: Et compendium mirificum omnium triangulorum.*]

IX. A demonstration that there are no great circles of the unequal hours, against nearly all writers on sundials. [*Demonstratio, non dari circulos maximos horarum inæqualium, contra omnes fere horologiorum scriptores.*]

X. Various determinations of the magnitude of the angles in spherical triangles, hitherto noticed by no one. [*Variae determinationes magnitudinis angulorum in triangulis sphaericis, à nemine hactenus animaduersæ.*]

Besides these, you will come upon countless other things scattered in various places, which you will not commonly find in the writings of others. [*Præter hæc, innumerabilia alia varijs in locis dispersa occurrent, quæ non passim in aliorum scriptis reperies.*]

PREFACE OF CHRISTOPH CLAVIUS OF BAMBERG, OF THE SOCIETY OF JESUS, TO THE ASTROLABE

[Astrolabium p. 3]

Among all the instruments devised by astronomers with great ingenuity for explaining and investigating the phenomena that follow the motion of the primum mobile [primum mobile] from east to west, or that in some way pertain to it, none has ever seemed to me more excellent than the one Claudius Ptolemy [Ptolemy] entitled the *Planisphere* [*Planisphaerium*] and which is commonly called the astrolabe. On it all the celestial circles are projected onto a plane by geometrical methods governed by the primum mobile, in such a way that one can measure each of their points and arcs no less accurately and precisely than on some perfectly round globe that would represent the primum mobile.

For although a solid sphere or globe — of which I have just spoken — rightly surpasses every instrument that can be built or shaped in thought, in that it is the most perfect image and likeness of the whole heaven, yet because of the exquisite roundness it ought to have, its construction becomes so extremely difficult that scarcely anyone can hope ever to achieve a perfect globe; and it can only with difficulty be kept for long without suffering damage from age. For this reason astronomers, with truly admirable industry, have set out to render the globe or sphere onto a flat surface, so as to obtain more conveniently and easily everything that they were able to obtain by means of the globe or sphere.

[Margin: The imperfections of the globe.]

For it is a flat instrument, most convenient for travelers, since it can be carried from one place to another without effort and easily kept undamaged. Add to this that it is impossible for all the necessary circles and all the points to be set out distinctly on a globe, however carefully wrought — a task that would give the student no small trouble. This difficulty does not arise in the plane, since on any flat surface, even a very small piece of paper, three or four circles that will then be of use to us are easily drawn, while the others we do not need at present are left out. Then, so that all confusion may be avoided, this sheet can be set aside and another taken up, on which other circles may be formed for another purpose. For it is not necessary that one who holds the method of describing in the plane all the circles should always have the astrolabe instrument in hand; it is enough to draw a few circles in some small space, or at least on some sheet of paper not very large, and to divide them into degrees, so as to draw out and elicit from them what he is seeking.

[Margin: The excellence of the astrolabe.]

And this above all is the aim I have set myself: to teach by what method, on a single small sheet or in a small flat space, all those things may be investigated —

indeed, far more — that others pursue by means of the astrolabe instrument, so that anyone may acquire the fullest use of the astrolabe even if he has never seen a finished instrument. I am confident this will be most welcome to students of astronomy, since many lack such an instrument, and scarcely any is to be found built with such care and diligence that every error has escaped the craftsman in perfecting it.

[Margin: The chief aim of this work.]

Indeed, even if someone should possess an astrolabe (which happens rarely or never) fashioned with the utmost skill and care, still, because on it not all the great circles — nor even all the parallels of a single great circle, nor all the great circles that intersect one another at the same two points, such as all the vertical circles or circles of position — can be drawn through each single degree and minute (which is nevertheless required if everything is to be found precisely), its use must for the most part be uncertain and hampered, so that we are very often forced to attain what we seek by conjecture rather than by any sure demonstration.

[Margin: The imperfection of the material astrolabe.]

Moreover, since on the instrument the use only of those few circles that are seen drawn on it can be perceived, it comes about that the use of the material astrolabe is confined within the limits of a few things. But we, without the aid of any instrument, shall hand down the use of all the circles that can be conceived in the primum mobile — almost innumerable — and shall embrace the entire doctrine of the primum mobile, which is most extensive; so that not even the doctrine of spherical triangles is excluded from its rules, but the whole can be explained with wonderful ease.

[Margin: The use of the astrolabe is fullest without the instrument.]

For among the other things that we have added, in this work of ours, to the ordinary uses of the astrolabe, we shall show most clearly by what method — which may seem astonishing to some — the angles of spherical triangles themselves may be determined most precisely from their sides, and the sides in turn from the angles, without any aid of numbers or sines; and likewise in what way the mutual inclinations of the various circles of the sphere, their intersections, and six hundred other things of this kind may be investigated with almost no trouble.

[Astrolabium p. 4]

In the same connection we shall also embrace all those problems which we once solved by means of tables of sines in our *Gnomonics* [Clavius, *Gnomonices libri octo*], especially in the first book and elsewhere, and which other authors are accustomed to propose and investigate in various places.

We have assigned the whole work on the astrolabe to three books. In the first we shall demonstrate various theorems and problems, all of which we have grouped under the name of Lemmas, since they are to be assumed in their proper places for the demonstrations of the matters that concern the projection of circles onto the plane and the new use of the astrolabe.

[Margin: The division of this work into three books.]

In the second book we shall describe on the plane of the astrolabe not only all the circles that can be conceived in the primum mobile, but also all the straight lines and points; and we shall divide each drawn circle into its degrees — that is, into certain parts unequal among themselves. (For the equal parts of all celestial circles — the equator and its parallels excepted — are projected into unequal parts on the plane of the astrolabe, whereas those whose equal parts are projected into equal parts are the equator and its parallels, as will become clear in its place.) These correspond to their equal degrees in the heaven — something which I find that up to the present day no one has fully accomplished.

For all who have written on the construction of the astrolabe divide no circle on the astrolabe into degrees except the equator, the ecliptic, the horizon, and their parallels; and the horizon with its parallels, and the parallels of the ecliptic, only by means of great circles drawn through their poles on the sphere — which is a very difficult matter and of almost immense labor. Andreas Schöner [Andreas Schöner] alone, in his book on the construction of the astrolabe, divides the horizon and the ecliptic, together with their parallels, into degrees by a certain other method; but he offers us no demonstration of it, so that one may with good reason doubt its truth. We, by contrast, shall divide into degrees any great circle whatever drawn on the astrolabe, and its parallels, not by one method but by several, and those very easy ones, all of which have their own demonstrations. There too we shall confirm Schöner's method geometrically and adapt it to all great circles and their parallels — which he himself did not teach.

Finally, in the third book we shall set forth Canons by which the manifold use of the astrolabe is explained by compass and ruler alone, on any given sheet or plane, as we said a little earlier — extending the use of the astrolabe in this way to far more problems than can be accomplished by any material instrument;

which I leave to the reader's judgment. Those problems, furthermore, that are usually solved on the common and widespread astrolabes we shall also solve by means of the instrument itself, so that we may not seem wholly to neglect the ordinary use of the astrolabe, and so that in this respect we may take thought for those who possess a material astrolabe, are content with a certain middling proficiency, or are not much practiced in drawing lines. But before I turn to the first book, I think it worth my while to bring forward — or rather recall to memory — by way of prolegomena, so to speak, a few remarks about the various circles of the sphere, both great and non-great, and especially about those that are to be drawn on the astrolabe, so that when we need to use them we may have their position and place in the heaven clearly in view and, as it were, ready at hand.

[Astrolabium p. 5]

ON THE CIRCLES OF THE PRIMUM MOBILE

The **EQUATOR**, or equinoctial circle, is the greatest circle of the world, or of the primum mobile. About it are to be conceived non-great circles, equidistant on either side, drawn through each individual point of the heaven, whose function is to indicate which star or celestial points have the same declination from the equator, and which a greater or a lesser one; likewise which rise or set at the same point of the horizon, and whose rising or setting inclines more toward the north or toward the south. For all stars and points of the heaven lying on the same parallel of the equator have the same declination and the same point of rising and setting; but that which occupies a smaller parallel — one, that is, which lies farther from the equator — has a greater declination, and a point of rising and setting more remote from the equinoctial rising and setting. The chief parallels of the equator considered on the sphere are four: the Tropic of Cancer ☊, the Tropic of Capricorn ☋, the Arctic Circle, and the Antarctic Circle, whose situation and position on the sphere are to be sought from the ecliptic and the position of its poles, as we shall shortly say.

[Margin: The equator and its parallels — what they are, and what their function is.]

[Margin: The Tropics of Cancer and Capricorn, and the Arctic and Antarctic circles — which they are.]

The **ZODIAC**, or ecliptic, is a great circle whose poles at the present time recede from the poles of the world, that is of the equator, by about $23\frac{1}{2}$ degrees. From

this it follows that the ecliptic intersects the equator obliquely, so that it is inclined to it, one half of it inclining toward the north and the other toward the south. The midpoint of each half lies as far from the equator as the poles of the zodiac recede from the poles of the world. The two points at which the ecliptic and the equator mutually intersect are called equinoctial, because the Sun, when it is in them, produces an equinox everywhere. Of these, the one that gives the beginning to the northern semicircle of the ecliptic, proceeding from west to east, is called the Vernal [point], and the other the Autumnal. But the two points of the ecliptic most distant from the equator are called solstitial, because a solstice occurs everywhere, in whatever place, as soon as the Sun first reaches either of them. The northern one is called the summer solstice, or the first point of Cancer, through which is described that parallel of the equator which they call the Tropic of Cancer ☿; while the southern point is called the winter solstice, or the first point of Capricorn, through which passes that parallel of the equator which they name the Tropic of Capricorn ♎. Finally, the northern pole of the ecliptic describes, by the motion of the primum mobile, that parallel of the equator which we have called the Arctic Circle; while the southern pole of the same ecliptic marks out the other parallel of the equator, which is called the Antarctic Circle.

[Margin: The ecliptic and its parallels — what they are, and what their function is.]

For this ecliptic too there are to be understood non-great circles, equidistant [from it], drawn through each individual point of the heaven, whose function is to indicate which stars have the same distance from the ecliptic, and which a greater or a lesser one. For stars lying on the same parallel of the ecliptic have the same latitude; but those found on a smaller parallel — namely, those that lie farther from the ecliptic — have a greater latitude.

The **COLURES** are two great circles intersecting each other at right angles at the poles of the world; of these, one is drawn through the two equinoctial points of the ecliptic and is called the Colure of the Equinoxes, while the other passes through the two solstitial points and is called the Colure of the Solstices. And all these circles that we have described so far are mobile, since they are perpetually carried around by the motion of the primum mobile. All the other circles that follow are to be conceived as immobile in the heaven, so that they never change their situation or position.

[Margin: What the colures are.]

The **MERIDIAN** is a great circle drawn through the poles of the world and the zenith of the place — that is, through that point in the heaven which lies directly above the place, such as the point that the top of some tower would reach if it were extended all the way to the heaven. This point the Arabs call the Zenith, and the point diametrically opposite the Nadir — the point that the same tower would reach if it ran through the center of the earth to the opposite part of the heaven. The meridian also has infinitely many non-great parallel circles, drawn on either side through each individual point of the heaven, which indicate which stars have an equal distance from the meridian, and which a greater or a lesser one.

[Margin: The meridian and its parallels — what they are, and what their function is.]

The **HORIZON** is a great circle whose poles are the top of the head and the point opposite — namely the Zenith and the Nadir — which separates the visible or apparent hemisphere from the hidden or unseen one. To it are drawn innumerable non-great parallel circles from the same poles through all points of the heaven, to show which stars have the same distance from the horizon, and which a greater one; and this distance, in the upper hemisphere, is called the altitude of the Sun and of the stars above the horizon, and in the lower, the depression below it. The parallels of the horizon themselves are called by the Arabs Almucantars [Almucantarath].

[Margin: The horizon and its parallels — what they are, and what their function is.]

The **VERTICAL** circles, which the Arabs name Azimuths, are great circles that are drawn through the poles of the horizon — that is, through the Zenith and the Nadir — through each individual point of the horizon. Of these, the one that passes through the intersections of the equator with the horizon is customarily called the prime vertical, or the vertical properly so called, or the vertical of the region. Among these the meridian too is counted, since it also is drawn through the zenith of the place. The function of these circles, which is no ordinary one, will be learned in many places from the use of the astrolabe.

[Margin: What the vertical circles are.]

[Margin: What the prime vertical is.]

The **HOURLY** circles, if indeed they indicate equal hours from noon and from midnight — which are called astronomical [hours] — are great circles passing

through the poles of the world, [dividing] the equator and all its parallels into 24 equal [hours]...

[Margin: The hour circles, both from noon and from midnight.]

distributing equal hours, one of which is the Meridian itself, from which the beginning of such hours is reckoned. But if they signify hours [counted] from sunrise or sunset, they are the great circles tangent to the two parallels of the equator — one of them being the greatest of the circles that are always visible, the other the greatest of those always hidden — at those points where they are cut by the circles of the astronomical hours; and among these the Horizon too must be counted, from which hours of this kind begin. Finally, if they pertain to the unequal hours, they are defined as the great circles that divide all the arcs of the parallels of the equator, both diurnal and nocturnal, into 12 equal parts. On all these hour-circles we have written more fully in Book 1 of the *Gnomonices*, propositions 9 and 10 — although, to tell the truth, there are no circles of the unequal hours, as we shall demonstrate below in Book 1, Lemma 39; a thing that may well seem incredible to many.

[Margin: (which [hour-circles are reckoned] from rising, or from setting).]

[Margin: There are no circles of the unequal hours.]

Circles of declination. The circles of declination are great circles [drawn] through the poles of the world (just as are the hour-circles reckoned from noon and midnight), drawn through the individual points of the equator; and they are so called because they measure the declination of any point or star from the equator. For the declination of a star or of a point of the heavens is the arc of a great circle passing through the poles of the world and through the star or point of the heavens, intercepted between the star (or point of the heavens) and the equator. Among these circles are also to be placed the hour-circles [reckoned] from noon and midnight.

[Margin: What the circles of declination are, and what their function is.] [Margin: What the declination of a star is.]

Circles of latitude. The circles of latitude are great circles [drawn] through the poles of the ecliptic and described through the individual points of it; and they are so called because they measure the latitude — that is, the distance — of any star or point of the heavens from the ecliptic. For the latitude of a star, or of a point of the heavens, is the arc of a great circle passing through the poles of the ecliptic and through the star (or point of the heavens), enclosed between the star (or point of the heavens) and the ecliptic.

[Margin: What the circles of latitude are, and what their function is.] [Margin: What the latitude of a star is.]

Circles of the celestial houses. The circles of the celestial houses are great circles, six in number, dividing the whole heaven into twelve domiciles; and they are all drawn through the intersections of the Meridian with the Horizon, and — according to the opinion of Johannes Regiomontanus [Regiomontanus] — through the twelfth-parts of the equator, but as Campanus [Campanus of Novara] prefers, through the twelfth-parts of the prime vertical [circle] of each place.

[Margin: What the circles of the celestial houses are.]

Circles of position. The circles of position are great circles [drawn] through the intersections of the Meridian with the Horizon (just as are the circles of the celestial domiciles), passing through the individual points of the heaven; and they are so called because they indicate the position of any star with respect to the celestial houses — namely, whether a given star is at the beginning, at the end, at the middle, or at some other part of this or that celestial house. And from the number of these are also those six [circles] of the celestial houses.

[Margin: What the circles of position are.]

Other great circles. Besides all these great circles which we have enumerated, together with their parallels — for it must be understood that every great circle has infinitely many equidistant [circles], or parallels that are not great, as was said of the equator, the ecliptic, the meridian, and the horizon — there can be considered in the heaven innumerable others, differing from all of those in nearly every respect. For through any two points on the convex surface of the celestial sphere a great circle can be described, as Theodosius [of Bithynia] demonstrated in Book 1 of the *Elements of Spherics* [*Sphaerica*], proposition 20; which circle indeed can have infinitely many non-great circles equidistant and parallel to itself, described about the same poles as itself. And all these circles, both the great and the non-great, which have been set forth by us, we shall describe in the plane of the astrolabe by Geometrical reasonings — that is, firm and evident ones — in the second book; and we shall divide the same into their degrees, or rather we shall assign to each one of them the particular degree proposed, when use and necessity shall require it.

[Margin: That infinitely many other great circles, with their own parallels, are to be conceived in the heavens.]

ASTROLABII LIBER PRIMVS.

AVCTORE CHRISTOPHORO CLAVIO BAMBERGENSI E SOCIETATE IESV.

This first book contains various problems and theorems — partly Geometrical, partly Spherical, and partly Conic — all of which, from their office, it has seemed good to call Lemmata, because they must be used very frequently and assumed as things confirmed by the most certain demonstrations, so that what we are about to teach in the second book concerning the manifold projection of the circles onto a plane, and concerning the division of the same into degrees, may be demonstrated more easily and more briefly. For unless these matters were demonstrated separately, in a single book, we should be compelled to make the astrolabe's own demonstrations longer than is fitting, and consequently more obscure. There is also another reason why all these theorems and problems have been gathered together into one book: namely, that not seldom one and the same Lemma must be used to demonstrate several propositions. Therefore, lest its demonstration be uselessly repeated in several places, but rather that the order and elegance of the exposition be preserved, it was necessary to confirm it separately by a Geometrical demonstration — which is the reason why lemmata of this kind, common to many [propositions], are placed [here]. To this add that, since lemmata of this sort not only have a necessary use in the astrolabe, but most of them also bring no small profit to other Mathematical matters, reason itself seemed to demand that they be explained in a book of their own, so that they might be found more easily and more readily whenever the Geometer has need of them in his demonstrations.

[Margin: The argument of the first book.]

LEMMA PRIMVM.

To divide a given line, straight or circular, into any number of equal parts — even the very smallest — by means of a compass whose feet have between them a distance greater than the given line.

Let AB be a straight or circular line, to be divided into any number of equal parts. On the line produced, mark off — by means of the compass — as many lines equal to the given line AB as the number of parts into which AB is to be divided; such are BC, CD, DE, EF, FG. And let the whole line AG likewise be distributed, by means of the compass, into as many equal parts as we are directed to divide the

line AB into (if the line AG is straight, [this is done] by the scholium to proposition 40 of Book 1 of Euclid, or by the scholium to proposition 10 of Book 6 of the same; but if it is circular, by means of the quadratrix, through what we have written at the end of Book 6 of Euclid); such are GH, HI, IK, KL, LA. Now each of these parts will contain the given line AB once, and in addition one of the parts into which AB is proposed to be divided.

Diagram. A single rectangular frame enclosing two parallel constructions that illustrate the division of a line into equal parts.

- Upper construction — a straight horizontal line, tick-marked, running left to right with the principal points A, B, C, D, E, F, G marked above the line; the finer division-points L, K, I, H are marked below the line (L below the A–C region, then K, I, H toward the right).
- Lower construction — a circular arc (a curved line), similarly tick-marked, carrying the same lettering: A, B, C, D, E, F, G above the arc and L, K, I, H below it, curving downward toward the right where the arc terminates (at G).

(The two rows show that the construction holds equally for a straight line and for a circular arc, as the lemma states.)

For since, as AG is to AL, so is AF to AB — which in both cases holds by the construction — the proportion is the same multiple. For AL is contained in AG as many times as AB is in AF; hence, by alternation, as AG is to AF, so is AL to AB. But AG has to AF a superparticular proportion named after the number of the parts into which AF is divided (since GF is one of those parts), or [the number] into which AB is to be divided. Therefore AL will also have the same proportion to AB; and consequently BL will be one of the parts into which AB is to be divided, named of course from the number of parts into which AB is to be divided — just as FG is one of the parts into which AF has been divided, named of course from the number of parts of the straight line AF, which number is equal to the number of parts into which AB is to be divided. And so, just as the interval GH, which is greater than the given line AB, gives us one part FH, so the same [interval], transferred from the two points F, H, will give two parts EI; and, transferred from three points near E, it will exhibit three parts DK; and, transferred from four points near D, it will give four parts CL; and so on, always one part more, so that at last the space GH, transferred onto AB itself from five points — namely from C, L, and the three intermediate ones — exhibits as many parts as those into which AB is to be cut; that is, as many as there are parts AB, BC, CD, DE, EF; and thus AB will then be divided into the proposed equal parts.

And this method of dividing is most useful when the line AB must be cut into particles so minute that they could scarcely be laid off continuously with the compass without error.

Now, if the line AG is to be cut, for example, into 30 equal parts, it must first be divided into some number of equal parts fewer than 30, yet such that their number is an aliquot part of the number 30 — as in the example it was divided into six parts, each of which contains five parts. Then, the first part AB having been divided into five parts, as was said, [take] the interval AL (or GH) by which the line AG — consisting of six parts equal to AB — was divided into five equal parts; and if one foot of the compass is set at A (the interval AL unchanged), then at the next point, then at the following one, and so on in succession, the whole line AG will be cut by the other foot into 30 equal parts.

The straight line AG could also be cut first into 5 parts, so that each would contain six of the 30 particles; but then each of these would have to be divided again in half, and the first of these halves distributed into three equal parts in the manner taught above; and finally the whole of AG divided, by means of these third-parts, into thirty parts. But if the fifth-parts are so small that they can scarcely be bisected with the compass, each would have to be cut into six parts, as we taught at the outset; or else a line composed of three of those fifth-parts should be bisected. For in this way, with this same interval, all will be bisected, and finally each half [divided] into three parts, as before.

It sometimes happens that on a line of given magnitude a great many particles must be marked off in sequence, under a fixed number, which because of their smallness can scarcely be taken off with the compass without error. In that case, then, we shall use the following device. If the number of particles can be divided into several factors, we shall take off with the compass, on the given line, as many equal parts as the number of particles can be divided into, yet in such a way that these parts together nearly exhaust the whole given line. For if the first of these parts is cut into as many particles as are contained in it out of the proposed number, and the same is done in the remaining parts, we shall have the given number of particles. For example, if a line is proposed on which 84 particles are to be marked in sequence, we shall cut it first into two, so that each contains 42; again each into two, so as to have four parts, each of which contains 21 particles; likewise each of these into three parts, so as to have twelve parts, each of which contains 7 particles; finally we shall distribute each of these into 7 particles.

But if the proposed number of particles cannot be divided into several factors, a number slightly greater or smaller must be taken, one that can be divided into

several parts, and just so many particles must be taken off in order on the given line, so that — if the superfluous particles are cast off, or those that are lacking are added — we shall have the proposed number of particles. For example, if 74 particles are to be marked off in sequence from some given straight line, we shall set ourselves 80 particles. For if we cut the given line in half, each half will contain 40 particles. Each half, cut again in half, will give four parts of 20 particles. But each of these, divided in half, will yield eight parts of 10 particles; and these too, cut in half, will give sixteen parts, and in each there will be five particles. If, then, each is distributed into five particles, as we have taught, we shall have 80 particles; but with six cast off, the remaining ones will be 74, the number proposed. Or we shall set ourselves 72 particles: for if we take off in order 24 equal parts, in such a way that they nearly exhaust the given line (which 24 parts will also be obtained if the given line, or a segment of it slightly smaller than the line itself, is cut first in half, and each part again in half, and each of these parts again in half, and finally each of these parts is cut into three parts), and if each part is divided into three particles, as has been taught, we shall have 72 particles; and if two particles are added to these, the proposed number of 74 particles arises.

Once these things are rightly considered, you will easily understand how you ought to proceed with any other number of particles.

LEMMA II.

To distribute a given quadrant, or circle, into degrees by means of a compass whose feet-interval encompasses more degrees than two or three.

Let AB be a quadrant whose centre is C. With the interval of the semidiameter AC, by which the quadrant was described, let two arcs AD, BE be cut off, each of which — by the corollary to proposition 15 of Book 4 of Euclid — will be a sixth part of the circle, containing 60 degrees; and consequently each of the remaining [arcs] BD, AE will comprehend 30 degrees, and therefore the intermediate arc DE will consist of just as many degrees. Thus the quadrant will now be divided into three equal parts, provided the angle ACB at the centre has been exactly right, and so has truly subtended a quarter-circle. Then, each of the arcs AE, ED, DB having been divided — by means of the compass, or of the quadratrix — into five equal parts (applying the procedure of the preceding lemma, if these fifths should be too small), so that each contains 6 degrees, and the whole quadrant is divided into 15 parts, let each of these be cut again, by the preceding lemma, into six parts; or else first into two, and afterward each of these into three. For in either way the quadrant will be distributed into 90 degrees.

Diagram. A quadrant enclosed in a rectangular frame.

- C (lower-left corner): the centre of the quadrant.
- A (upper-left corner): endpoint of the vertical radius CA.
- B (lower-right corner): endpoint of the horizontal radius CB.
- The quarter-circle arc runs from A (top) to B (bottom-right); it is tick-marked into small divisions along its whole length.
- E (upper part of the arc): division-point near the top of the arc (the arc AE cut off from A).
- D (lower part of the arc): division-point toward the bottom of the arc (the arc BD cut off from B), with the intermediate arc DE between D and E.
- Near A a short inner arc-mark is struck (the arc drawn with the semidiameter interval).

If a whole circle is to be cut into 360 degrees, we shall first divide it into four quadrants by two diameters intersecting each other at the centre at right angles; then we shall distribute each quadrant, in one and the same operation, into 90 degrees, as has been said, taking off in each the same parts with the same compass-interval, and so on.

And so, since the whole difficulty of dividing a circle or quadrant into degrees lies almost entirely in the last operation, by which equal arcs are to be distributed into single degrees — because, on account of the smallness of the degrees, scarcely any compass can be found that would carry out that division into such minute parts conveniently and without error — we must take care that, when in a division of this kind we have arrived at arcs so tiny that they can only with difficulty be cut into smaller particles by means of the compass, we apply the teaching of the preceding lemma, by which we find even the very smallest particles with a larger interval of the compass-feet.

LEMMA III.

From a given circumference, to cut off an arc encompassing any number of whole degrees, or any number of degrees and minutes; and, conversely, to find out how many degrees and minutes are contained in any given arc of a given circumference — even if the given circumference has not been divided into degrees and minutes.

At the beginning of our *Gnomonics* [*Gnomonices*] we showed that if straight lines are drawn from the center of some quadrant accurately divided into 90 degrees — one line to each degree — an instrument is ready by which an arc of however many degrees and minutes may be taken on the circumference of any circle

whatever; and there we also explained the use of this instrument. But it is exceedingly difficult to draw the straight lines from the center so precisely that they divide all the quadrants described from the same common center into 90 equal degrees — something that is nonetheless absolutely necessary if we do not want to go wrong in using the instrument. For this reason I shall here construct another instrument, as it were, for the same purpose, one that in my judgment is far more convenient. It is made as follows.

On a plate of brass or wood let several quadrants be described, not much distant from one another — such as the three AB, CD, EF — whether struck from the same center or from different centers, all of them unequal to one another, so that we may use now a larger, now a smaller, as the case requires. Beside each let its own radius be set, although this is not strictly necessary, since the chord of 60 degrees is equal to the radius, by the corollary to Euclid IV.15 [*Elements* IV.15 coroll.]. Now when each quadrant has been divided into its degrees — in the instrument the quadrant CD, on account of its smallness, has been cut into only 45 parts, so that each contains two degrees — if the superfluous parts of the plate are pared away so that the figure QR is left standing, an instrument will be ready. Its use is as follows.

Diagram. A square woodcut frame. Within it, three concentric quadrant arcs share a common center G at the lower-right corner. Along the bottom edge (the horizontal radius running leftward from G) are marked, from left to right, the foot-points K, H, M, O, and G. The three graduated quadrants are drawn as dotted curves labelled with the degree numbers 90, 80, 70, 60, 50, 40, 30, 20, 10. The outermost (largest) quadrant is AB, with A at its 90° end and B near its 0° end; the middle quadrant is CD; the innermost (smallest) is EF, with F near its 0° end. Beside each quadrant a scale-bar marks its radius, labelled "Semidiameter." (outer), "Semid." (middle), and "Semid" (inner). Radial construction lines run from the center G; along one slanting radius lie the points S, T, and I, with the further interior points L, N, P, V, and X. The retained working figure is bounded by the corner points Q (top) and R (right). Below, a dotted diamond descends to a lower vertex.

- G (lower-right corner): common center of the three quadrants
- K, H, M, O (bottom edge, to the left of G): foot-points on the horizontal radius
- A (outer arc, upper): 90° point of quadrant AB
- B (outer arc, right): near the 0° end of quadrant AB
- C, D (middle arc): endpoints of quadrant CD

- E, F (inner arc): endpoints of quadrant EF (F at the 0° end)
- Q (top corner), R (right corner): corners of the retained figure QR
- S, T, I (on a slanting radius); L, N, P, V, X (interior points)
- Degree numerals 90, 80, 70, 60, 50, 40, 30, 20, 10 along each dotted arc
- Scale-bars labelled "Semidiameter.", "Semid.", "Semid"

The quadrants are three: AB, CD, EF. They may be struck either from the same center or from different centers, provided that they are all unequal to one another, so that we may use now the larger, now the smaller, as the case demands; and beside each let its own radius be set — though this is not strictly necessary, since the chord of 60 degrees is always equal to the radius, by the corollary to Euclid IV.15. When the several quadrants have been divided into their degrees (in the instrument the quadrant CD, because of its small size, has been cut into only 45 parts, so that each contains two degrees), and if the superfluous parts of the plate are cut away so that the figure QR is left, the instrument will be ready. Its use is this.

Suppose that on a circumference HI, whose center is G, an arc of some number of degrees is to be cut off (which is very often what has to be done on the astrolabe) — say, 35 degrees. From G, at an interval equal to the radius of the greater quadrant AB (if the size of the plane on which the arc HI lies permits it), let the arc KL be described; or, if this cannot be done because of the smallness of the plane, then, to suit the convenience of the plane, the arc MN or OP, at the interval of the radius of some smaller quadrant. For if, from whichever quadrant the arc drawn from G has been described, the interval of 35 degrees is carried over to the corresponding arc — from K to L, or from M to N, or from O to P — and if from G a straight line is drawn through L, or N, or P, it will cut off on the given circumference the point I, and the arc HI will contain 35 degrees, since it is similar to the arc KL, or MN, or OP, by the scholium to Euclid III.22 [*Elements* III.22 schol.].

If the proposed circumference — say KL — has a radius exactly equal to the radius of one of the quadrants in the instrument, as here the greater quadrant AB, then, once the arc of the proposed degrees has been carried over onto the given circumference KL, the result is obtained, as is evident.

Now if the arc to be cut off contains, besides degrees, some number of minutes as well, these minutes are to be taken by estimation: namely, half a degree for 30 minutes, a third of a degree for 20, two thirds for 40, three quarters for 45, and a little more than a quarter for 16 or 17 minutes; and so with the rest, according as

the greater part of one degree is offered. But more surely, and indeed geometrically, we shall show a little further on, in this same lemma, how to cut off any number of minutes from any degree, even when the degree has not been divided into minutes.

Again, at the point G, with the straight line GH, an angle is to be set out containing 57 degrees 21 minutes. Once the arc KL has been described from G at the interval of the radius of the quadrant AB (or of some other, smaller one, if the space is cramped), let there be carried over the interval on this quadrant containing 57 degrees, and a little more than a third of one degree, from K to S. For the straight line GS, when drawn, will set out the required angle KGS.

Conversely, suppose one wishes to know how many degrees and minutes the arc HI described from G contains. He will find it out if from G he lays off an arc whose radius is equal to the radius of one of the quadrants in our instrument. For if a straight line is drawn from G through I, it will cut off from the arc so described an arc similar to HI, by the scholium to Euclid III.22. If, then, that cut-off arc is carried over onto the corresponding quadrant, it will at once appear how many degrees and minutes it contains — reckoning 30 minutes for half a degree, 40 for two thirds, and so with the rest, according as the greater part of one degree is offered. In this way we find that the arc HI contains 35 degrees, since the arc KL contains just as many degrees on the quadrant AB, or the arc MN on the quadrant EF, or the arc OP on the quadrant CD. But on the arc HT we find nearly 57 degrees and 21 minutes, because the arc KS on the quadrant AB — or the arc MV on the quadrant EF, or the arc OX on the quadrant CD — encloses just that many degrees and minutes.

From this it is plain that, to carry out this problem, it is enough if a single quadrant of whatever size is at hand, exactly divided into degrees — except that sometimes the proposed plane is not large enough for the arc to be described on it at the interval of the quadrant's radius. When this happens, the given circumference will have to be described, together with that arc, on a separate sheet, and so forth. The instrument will therefore be more convenient if several unequal quadrants are contained in it.

I prefer the use of one quadrant, or of several, to that instrument which we constructed at the beginning of our *Gnomonics*, because the degrees in each separately divided quadrant are more nearly equal than the degrees

which the straight lines drawn from the center exhibit in another quadrant described from the same center — since it is exceedingly difficult to draw those straight lines always at mutually proportional distances apart.

Now if an instrument of this kind is not at hand, we may also proceed conveniently in the following way. Let the quadrant of the circle on whose circumference the proposed degrees are to be cut off be divided into three parts, and each third part again into three, so that nine parts are obtained, each containing 10 degrees. Finally, let the last part alone be divided into 10 degrees. For by means of this divided part, together with the other undivided parts, we shall take an arc of any number of degrees, in this way. If the number of degrees does not exceed 10, the proposed degrees will easily be taken in the last part, divided into 10 degrees. But if the number of degrees is greater than 10 — say 57 — we shall set one foot of the compasses on the degree of the seventh part divided into 10 degrees, counting these 7 degrees not from the outer end but from the inner, and we shall extend the other foot of the compasses up to such a part of the quadrant that the arc between the compass-feet embraces 57 degrees. Or else we may carry out the matter in two operations, taking first, among the undivided parts of the quadrant, the given degrees counted by tens, and then the remaining degrees in the last part divided into 10. Thus in the proposed example we shall first take 5 undivided parts, which contain 50 degrees, and then take 7 degrees in the divided part, and so we shall have 57 degrees. The same reasoning holds for the rest. It would therefore be enough if in the instrument the several quadrants were cut into 9 parts, and then the last part alone divided into 10 degrees.

But since, when the proposed arc contains, besides degrees, some minutes as well, this lemma cannot be carried through and completed except more or less by estimation or conjecture, as we have said, let us now teach by what method an arc may be cut off geometrically, in which, besides the degrees, any number of proposed minutes may also be comprehended; and conversely, how it may be known how many minutes are contained in any given part of one degree. For although we taught this very thing at the end of the little book on the construction and use of the horological instrument, yet, since that book is not always at hand, I have thought it worthwhile to repeat it briefly here as well, especially since its greatest usefulness is found in the astrolabe.

An arc, then, of as many degrees as the minutes desired, is to be cut into 60 equal parts; for the sixtieth part will contain the proposed number of minutes. Thus, if 53 minutes are wanted in some degree of the quadrant AB, whose radius is CD, we shall divide the arc of 53 degrees — or rather one equal to it, FG, described on

the circumference EF, which has its radius equal to the radius CD (so that confusion may be avoided) — into 60 equal parts. This we do by dividing it first into five equal parts, and then each of these into three; or first into three, then each of these into five, and each of those in half, and then each of those again in half. But

Diagram. A rectangular woodcut frame. A large graduated quadrant is drawn as a dotted arc running from A (lower-left, marked 90°) up and over to B (lower-right, near 0°), with the degree numbers 90, 80, 70, 60, 50, 40, [30?], 20, 10 marked along it; L is marked near its upper middle. A horizontal scale-bar (the radius) runs across to the point D at the right, and C is marked at the left. Below the dotted quadrant a solid arc FG is drawn — G at the upper-left, F at the lower-right corner (the common vertex/center) — graduated 10, 20, 30, 40, 50, with M near the 50° mark; near F the small sexagesimal subdivisions are marked with the points H, I, and R (the divisions FK, FH, FI). E and L are marked at the lower-left.

- A (lower-left): 90° end of the dotted graduated quadrant
- B (lower-right): near the 0° end
- C (left); D (right end of the radius scale-bar)
- E (lower-left)
- F (lower-right corner): vertex/center of the arc FG
- G (upper-left): other end of the arc FG
- H, I, R (near F): sexagesimal subdivision points
- L (upper middle of the dotted arc; also a point at the lower-left)
- M (near 50° on arc FG)
- Degree numerals 10–90 (dotted quadrant); 10–50 (arc FG). One descending tick on the right of the dotted quadrant is uncertain: read as 30 [?].

it is enough if only one single part is always subdivided further, for in the last subdivision the sixtieth part will be obtained. For the fifth part of the arc FG is FH, and a third part of this is FI; and this, subdivided twice in half, gives FK, the sixtieth part of the whole arc FG. Now the sixtieth part FK comprehends 53 minutes. And so, if one wishes an arc of 45 degrees 53 minutes, the arc FK is to be added to the arc of 45 degrees; for in this way it will make up the arc BL, embracing 45 degrees 53 minutes.

That the arc FK contains 53 minutes I demonstrate thus. Since, as the arc of 60 degrees is to the arc of 1 degree, so is FG, the arc of 53 degrees, to the arc FK — the proportion being on both sides the same, namely 60 to 1, by the construction — it will follow, by alternation, that as the arc of 60 degrees is to the arc of 53

degrees, so is the arc of 1 degree to the arc FK; and by inversion, as the arc of 53 degrees is to the arc of 60 degrees, so is the arc FK to the arc of 1 degree. Since, then, the arc of 53 degrees contains 53 sixtieth parts of the arc of 60 degrees, the arc FK will likewise contain 53 sixtieth parts of the arc of 1 degree — that is, 53 minutes of one degree. And the same reasoning holds for the rest.

If one wishes to have both minutes and seconds of one degree, it will be enough, for seconds numbering more than 30, to add one minute to the minutes found, and to seek out an arc that contains all those minutes. Thus, if one wants 53 minutes and 45 seconds, one must investigate the arc of 54 minutes. But if the seconds are fewer than 30, they are to be neglected. If, however, someone should absolutely require the seconds, let him read what we wrote on this matter in Chapter 2 of Book 1 of our *Practical Geometry* [*Geometria Practica* I.2].

This procedure, easy as it is, is nonetheless inconvenient to use on some small quadrant, especially when few minutes — such as 2, or 3, or 5 — are wanted. For since the degrees on such a quadrant are very tiny, an arc of as many degrees as the minutes desired will not easily be divided into 60 parts. Therefore, to make this business easier, when the arc to be distributed into 60 parts is very small, one must take the arc doubled, or quadrupled, or eightfold, and so on, so that it may conveniently be cut into 60 equal parts. For its sixtieth part will comprehend twice, or four times, or eight times, and so forth (according as the arc taken is the double, or the quadruple, or the eightfold) as many minutes as are sought. Hence, when the doubled arc has been divided, if that sixtieth part is cut in half; and this again, when the quadrupled arc

has been divided, cut in half; and this once more, when the eightfold arc has been divided, cut in half again — it will contain the minutes sought of the neighboring division. And I take it to be clearly established that the division of some small arc into two equal parts is easier — since it can be done by estimate or conjecture without error — than the division of an arc that is not large enough into 60 equal parts.

Now conversely, if from some degree any given part is cut off, and one wishes to know how many minutes and seconds it embraces, that part must be taken most exactly with the compasses sixty times in continuous succession, beginning at the very start of the quadrant. For as many whole degrees as are contained in that arc which is the sixtyfold of the given part, so many minutes will the given part embrace. By this reasoning, if the part which we said contained 53 minutes is stepped off with the compasses sixty times in continuous succession, beginning at

the point B, we shall fall precisely upon the completed degree 53. Therefore that part will contain 53 minutes.

The demonstration of this matter is as follows. Let the arc FG be the sixtyfold of the given part, to which the part FK is equal. Since, then, as the arc of 60 degrees is to 1 degree, so is the arc FG to the arc FK, it will follow, by alternation, that as the arc of 60 degrees is to the arc FG, so is the arc of 1 degree to the arc FK; and by inversion, as the arc FG is to the arc of 60 degrees, so is the arc FK to the arc of 1 degree. Therefore, as many sixtieth parts of the arc of 60 degrees — that is, as many degrees — as are contained in the arc FG, so many sixtieth parts of one degree — that is, so many minutes — will be contained in the arc FK.

If in that sixtyfold arc there are contained some degrees and, over and above, some part of one degree, the whole degrees contained in that arc will indeed indicate the number of minutes, but together with that part the seconds will be found out in the same way. For the part, taken sixty times, will give an arc of as many degrees as the seconds to which that part is equivalent. And in the same way, if in this sixtyfold arc any part is left over, thirds will be found, and so on. But it is enough, in my judgment, if the minutes are diligently sought out. And if the part remaining is greater than half a degree, let one more minute be added to the minutes found; but if it is less than half a degree, let nothing be added.

Nor should this be passed over: it is not always necessary that that part of a degree be stepped off sixty times, but it is enough that, repeated some number of times, it fall precisely upon some degree — which for the most part usually happens. For then a fraction is set up whose numerator is the number of degrees traversed, and whose denominator is the number of times the part has been repeated with the compasses. For example, if some part of a degree, repeated twenty times, falls upon the twelfth degree, that part will embrace $\frac{12}{20}$ of one degree. And so, if the numerator 12 is multiplied by 60, and the product 720 is divided by the denominator 20, the quotient will indicate 36 — that is, that the part contains 36 minutes. Likewise, if another part, repeated seven times, falls upon the fourth degree, it will comprehend $\frac{4}{7}$ of one degree. If, then, the numerator 4 is multiplied by 60, and the product 240 is divided by the denominator 7, there will be found 34 minutes. And since in the division there remain 2, we shall multiply these again by 60; for if we divide the product, the number 120, by 7, it will give a quotient of a further 17 seconds.

The demonstration of this practice is as follows. Since, in the former example, 20 degrees stand to 1 degree as the part repeated twenty times stands to 1 part, it will follow, by alternation, that the arc of 20 degrees stands to the arc containing

the part repeated twenty times — that is, to the arc of 12 degrees — as 1 degree to 1 part; and by inversion, 12 degrees stand to 20 degrees as 1 part to 1 degree. Since, then, 12 degrees are $12/20$ of 20 degrees, one part will likewise contain $12/20$ of one degree. But in the latter example, 7 degrees stand to 1 degree as the part repeated seven times — that is, as 4 degrees — stands to 1 part. Therefore, by alternation, 7 degrees will stand to 4 degrees as one degree to 1 part; and by inversion, 4 degrees stand to 7 degrees as 1 part to 1 degree. Since, then, 4 degrees are $4/7$ of seven degrees, one part will likewise embrace $4/7$ of one degree. And the same reasoning holds in the other cases.

This matter also succeeds more happily on large quadrants than on small ones, because with the compasses the parts of larger degrees can be grasped more easily, and without error, than those of smaller ones. Therefore, if the degrees are very tiny and the given part not greater than half a degree, we shall take, sixty times, an arc composed of the given part and the next degree, and from this sixtyfold arc we shall throw away 60 degrees, which of course were taken sixty times together with the given part. For the remaining number of degrees will give the number of minutes, as before. But if the given part is greater than half a degree, we shall investigate, in the same way, the minutes of the remaining smaller part — taking, that is, an arc composed sixty times of that remaining smaller part and of one degree — and so forth; because, if we were to take the larger part, the sixtyfold arc would come out greater than a quadrant. Then, having found the minutes of that remaining smaller part, we shall subtract them from 60, so that what is left may become the minutes of the larger given part.

By this reasoning, if we take sixty times the part remaining of the given upper part, to which FK is equal — since it is greater than half a degree — together with one degree, we shall build up an arc consisting of 67 degrees. When 60 are cast away, 7 remain. Just so many minutes, then, exist in that remaining smaller part; and these, subtracted from 60, leave 53 minutes for the given larger part.

But since it is also troublesome to step off an arc of this kind sixty times with the compasses, and an error can easily be made in that multiplication, we shall have to use this shortcut. Let the arc composed of the part and one degree be doubled; let this double be doubled again, so that the quadruple arc is obtained; let this be doubled once more, so that the eightfold is obtained; and let this again be doubled, so that the sixteenfold arc is obtained; and let this be doubled twice more, so that that arc sixty-and-four times is obtained — so that in all six doublings are made. Now from this arc let 60 degrees be thrown away, and, over and above, the quadruple of the arc composed of one degree and the smaller

part, because it was taken sixty-four times, whereas it ought to have been taken only sixty times. For the remaining degrees will show the number of minutes to which that smaller part is equivalent. In this way, if we double six times the same smaller part discussed above, together with one degree, we shall make up an arc of 71 degrees and more; from which, if we throw away 60 degrees, and further the arc composed of the part and the degree taken four times, there will be left 7 degrees. Therefore that smaller part contains 7 minutes, and hence the given larger part will have 53 minutes. And if the given part, without the degree, were doubled six times, so that 64 of the parts are had in the composed arc, one would have to throw away only that part taken four times from that arc which contains the given part sixty-four times.

But since some larger quadrant is more easily distributed into degrees than a small one, this problem will be able to be carried out by a single quadrant of such magnitude that we can conveniently divide it into 90 degrees, in this way. Let AB be a portion of a quadrant divided into 90 degrees, and let an arc similar to the arc AB, of any number of degrees and minutes, be cut off from another proposed circle. If, then, the proposed circle has obtained a radius greater than the radius of the circle AB,

(Lemma III continued from the previous page.)

let a circle be drawn from its center with a radius equal to that of circle AB, and into it, by means of the compass, let the given arc AB be transferred. For if two straight lines are drawn from the center through the end points of the transferred arc, they will cut off a similar arc on the given larger circle, by the scholium to Proposition 22 of Book 3 of Euclid.

But if the proposed circle has a smaller radius than that of circle AB, then — provided that in the plane in which the given circle lies a circle can be drawn from the center of the given circle with a radius equal to that of circle AB — let such a circle be drawn, and let arc AB be transferred into it. For straight lines sent out from the center through the end points of the transferred arc will cut off a similar arc from the given smaller circle, by the same scholium to Proposition 22 of Book 3 of Euclid.

But if the plane in which the circle is proposed is not large enough for a circle equal to AB to be drawn from the center, we shall proceed as follows. From the center of the given circle let a circle be drawn with a radius equal to half the radius of circle AB — that is, half the chord of 60 degrees — and into it let half the chord of the given arc AB be transferred. For the arc so cut off is similar to arc

AB. Therefore, if from the center two straight lines are drawn through the end points of this cut-off arc, a similar arc will likewise be cut off from the given circle.

We shall demonstrate this as follows. Let AC be the radius of circle AB, bisected at D; and through D, from center C, let arc DE be drawn, into which is transferred the chord DE, equal to half the chord AB — namely, equal to AF. I say that arc DE is similar to arc AB. For draw the radius CB, cutting arc DE at E, and join the straight line DE. Since, then, AC and BC are cut proportionally — that is, into equal parts [Margin: a — Euclid VI.2] — the lines AB and DE will be parallel; and therefore, by the corollary to Proposition 4 of Book 6 of Euclid, the triangles CAB and CDE will be similar. And CA will be to AB as CD is to DE. By alternation, then, as CA is to CD, so AB is to DE. Since CA is double CD, AB will also be double DE. Therefore half of AB — namely AF — transferred from D into circle DE cuts off an arc similar to AB, by the scholium to Proposition 22 of Book 3 of Euclid. Which is what was proposed.

Diagram. A rectangular woodcut frame enclosing an isosceles configuration with its apex at bottom. At the top, the chord AB carries the given arc; two straight lines descend from A and from B and converge at C at the foot of the figure. Two lower arcs (concentric about C) cross these converging lines, cutting off the smaller similar arcs DE and GH. Small division marks between A and B carry the labels I and F.

- A (top left): end of the given arc AB
- B (top right): end of the given arc AB
- I, F (top center, between A and B): points marking half the chord / the subdivisions of AB
- D (middle left, on upper inner arc): end of similar arc DE
- E (middle right, on upper inner arc): end of similar arc DE
- G (lower left, on lower inner arc): end of similar arc GH
- H (lower right, on lower inner arc): end of similar arc GH
- C (bottom apex): common center of the converging radii and of the inner arcs
- [Margin, left of figure: a — Euclid VI.2 ("2. sexti")]

But if circle DE, drawn with a radius equal to half the chord of 60 degrees of arc AB, comes out too large — so large that it cannot be drawn in the plane of the given circle — then let it be drawn with a radius equal to a third part of the chord of 60 degrees of arc AB. For if AI, a third of the chord AB, is transferred from G to

H, again arc GH will be similar to arc AB, which is proved in the same way. By the same reasoning a circle can be drawn with a radius of a fourth part, or a fifth, and so on, according to the room available in the plane in which the given circle lies.

When a circle has been drawn with a radius equal to half the chord of 60 degrees, we shall reach our goal almost more quickly than has been said, by means of a compass whose legs cross one another, so arranged that the opening of the longer legs is always double the opening of the shorter. For if the arc of the given degrees AB is taken up with the longer legs, the shorter legs will cut off a similar arc DE.

For the rest, if a compass of this kind is not at hand, we shall take half of the said chord AB, or a third part, or a fourth, and so on, if we draw several parallel lines separated from one another by equal and very small intervals. For if the chord AB is set into them with the compass so as to enclose two, or four, or six spaces, it will be divided in two by the middle line; and if it is transferred into them so as to enclose three, or six, or nine spaces, it will be divided into three equal parts by the two intermediate lines, which are equally distant from the ends. And so for the rest. This we demonstrated at the end of the scholium to Proposition 40 of Book 1 of Euclid, in the last method of dividing a straight line into any number of equal parts.

Furthermore, if we are willing to be content with a single upper quadrant AB accurately divided into 90 degrees — namely, the quadrant whose radius is CD — we shall obtain in it far more readily an arc of any number of degrees and minutes. If, with a radius equal to that same radius CD, an arc is drawn, and in it arc LM is cut off spanning 61 degrees of quadrant AB, and this arc LM is distributed into 60 equal parts — first into two, then each of these into three, and finally the first of them into 10 equal parts — so that each of them is a sixtieth part of arc LM: then, since one of these little parts is to arc LM as one degree of quadrant AB is to the arc of 60 degrees (the ratio in both cases being one to sixty), and, by alternation, one of those little parts is to one degree of quadrant AB as arc LM is to the arc of 60 degrees, one of the little parts will contain one degree once, and in addition a sixtieth part of one degree — that is, one minute — just as arc LM contains the arc of 60 degrees once and, in addition, a sixtieth part of it, that is, one degree, by construction. From which it follows that two little parts span two degrees, and in addition two minutes; three little parts, three degrees and three minutes; and so on.

And so, if in quadrant AB someone wants a little part of one minute, let him transfer one little part of arc LM into the quadrant, beginning at point A, or at any degree; for the little part, beyond one degree, will contain 1 minute. In the same way, if two little parts are transferred, the part will span, beyond two degrees, 2 minutes; if three are transferred, it will contain, beyond three degrees, 3 minutes; if 53 are transferred, it will comprehend, beyond 53 degrees, 53 minutes; and so on.

By the same reasoning, if someone wants any number of degrees and minutes, he must first find the little part of the minutes wanted, and add it to the proposed degrees. But if the little part of minutes is so small that it can scarcely be taken with the compass, it must be taken together with one degree, and added to the proposed number of degrees less one. For example, if someone wants 89 degrees 59 minutes, the 59 minutes must first be found — which is done if 59 little parts of arc LM are transferred into quadrant AB, just as if 59 degrees were to be taken; for the part beyond 59 degrees will span 59 minutes, as has been said. Then an arc composed of that little part together with one degree is to be added to an arc of 88 degrees; for in this way an arc of 89 degrees 59 minutes is produced. And the same reasoning holds for the rest.

LEMMA IV.

Through a given point to draw a line parallel to a given straight line.

Although Euclid accomplished this problem in Book 1, Proposition 31, and we there set out various methods for the same thing, nevertheless another method now occurs to me — in my judgment far easier, whether the given point is near the given straight line or not — which I have decided to include here on account of its frequent use, both in the astrolabe and in other geometrical matters.

So let a parallel be drawn through point C to the given straight line AB. From any point D taken at will, distant from C — whether within the given line or outside it — as center, draw through the given point C a circle cutting the given straight line at the points A and B. (It is not necessary that the whole circle be drawn; it is enough if two of its arcs, cutting the given line, are traced, provided that, to the judgment of the eye, arc BE is not smaller than arc AC, as appears in the figure.) And with the compass cut off arc BE equal to arc AC. For a straight line drawn through C and E will be parallel to the straight line AB, as is clear from what we demonstrated in the scholium to Proposition 27 of Book 3 of Euclid, on account of the equal arcs AC and BE.

The matter will be carried out more conveniently, however, if point D is taken not on the line but outside it, yet so that it occupies roughly the middle position between the given line and the parallel to be drawn — which is to be judged by estimation alone, more or less. For in this way it will come about that the arcs drawn cut the given straight line and the drawn parallel less obliquely.

Diagram. A rectangular woodcut frame containing two horizontal parallel lines (the given line AB below, and the parallel to be drawn above). Several arc-pairs, drawn from different centers D, cross both lines; each set of arcs is labelled with repeated letters at its intersections.

- A (on the given line, left): a point where the circle cuts the given line
- B (on the given line, right): the other point where the circle cuts the given line
- C (on the upper parallel): the given point through which the parallel is drawn
- D (variously placed — above, on, and below the given line): the several centers from which the arc-pairs are struck
- E (on the upper parallel): the point cut off so that arc BE = arc AC
- F, G, H, I, K, L: the intersection labels of the several arc-pairs (arcs AFC, BGE from the most remote center D; arcs AHC, BIE from a center D taken on the given line; arcs AKC, BLE from a center D midway between the two lines)

In the figure, arcs AFC and BGE are drawn from a center D placed very far from the given line AB; arcs AHC and BIE from a center D taken on the given line itself; and finally arcs AKC and BLE from a center D situated roughly in the middle of the two lines — which is the most suitable of all for carrying out the problem.

LEMMA V.

The ratio which the whole sines — that is, the radii of any unequal circles — have to one another, the same ratio the sines, both right and versed, of similar arcs have. And conversely, arcs whose sines, both right and versed, have the same ratio as the whole sines, are similar.

Let AB and CD be arcs of circles whose radii AE and CF are similar [in the sense of the whole sines], and let their right sines be BG and DH, and their versed sines GA and HC. I say that as AE is to CF, so both BG is to DH and GA is to HC. For, the radii EB and FD being joined, the angles E and F will be equal (by the scholium to Proposition 22 of Book 3 of Euclid), because the arcs AB and CD are similar. Since, then, the right angles G and H are also equal, the triangles BEG and DFH will be

equiangular [Margin: a — Euclid I.32; b — Euclid VI.4]. Therefore, as EB — that is, EA, the whole sine — is to BG, the right sine, so FD — that is, FC, the whole sine — is to DH, the right sine; and by alternation, as EA is to FC, so BG is to DH.

Again, since, on account of the similarity of the triangles, EB (that is, EA) is to EG as FD (that is, FC) is to FH, it will follow, by conversion of ratio, that EA, the whole sine, is to GA, the versed sine, as FC, the whole sine, is to HC, the versed sine; and by alternation, as EA is to FC, so GA is to HC [Margin: c — Euclid VI.4].

But now let it be that as AE, the whole sine, is to CF, the whole sine, so both the right sine BG is to the right sine DH and the versed sine GA is to the versed sine HC. I say that the arcs AB and CD are similar. For, the radii EB and FD being again drawn: since, as AE (that is, EB) is to CF (that is, FD), so BG is to DH, then, by alternation, as EB is to BG, so FD is to DH. But there are also the other right angles G and H, which are equal; and consequently each of the remaining angles E and F is less than a right angle, by the corollary to Proposition 17 of Book 1 of Euclid [Margin: d — Euclid VI.7]. The triangles BEG and DFH will therefore be equiangular, and will have the angles E and F equal. For which reason, by the scholium to Proposition 22 of Book 3 of Euclid, the arcs AB and CD are similar.

Again, since as AE is to CF, so GA is to HC, then, by alternation, as AE is to GA, so CF is to HC; and it will follow, by conversion of ratio, that as AE (that is, EB) is to EG, so CF (that is, FD) is to FH. Since, then, the other right angles G and H are equal, and consequently each of the remaining angles B and D is less than a right angle (by the corollary to Proposition 17 of Book 1 of Euclid), the triangles BEG and DFH will be equiangular and will have the angles E and F equal. Wherefore, by the scholium to Proposition 22 of Book 3 of Euclid, the arcs AB and CD are similar [Margin: e — Euclid VI.7].

Diagram. A rectangular woodcut frame containing two adjacent quarter-circle figures. In the left figure, center E lies at the foot; the vertical radius EA rises to A, and radius EB runs to B on the arc; G is the foot of the perpendicular from B to EA, so that BG is the right sine and GA the versed sine. The right figure repeats the construction with center F, radius FC to C, radius FD to D on the arc, and H the foot of the perpendicular, DH being the right sine and HC the versed sine.

- A (top of left figure): end of the vertical radius EA
- B (upper left of left figure, on the arc): end of arc AB / radius EB
- G (on radius EA, left figure): foot of the perpendicular from B (right sine BG, versed sine GA)
- E (foot of left figure): center of the left circle

- C (top of right figure): end of the vertical radius FC
- D (upper left of right figure, on the arc): end of arc CD / radius FD
- H (on radius FC, right figure): foot of the perpendicular from D (right sine DH, versed sine HC)
- F (foot of right figure): center of the right circle

This proposition is also true when the similar arcs are greater than a quadrant — such as are the complements of arcs AB and CD up to the semicircle. For their right sines are BG and DH, while the sines of the complements are EG and FH, which have the same ratio as the whole sines AE and CF, as has been demonstrated; because, by the scholium to Proposition 22 of Book 3 of Euclid, the remaining arcs AB and CD from the semicircles are also similar. And since it has also been shown that as AE is to CF, so AG is to CH; and since, as AE is to CF, so the whole diameter is to the whole diameter [Margin: f — Euclid V.15], it will follow that as the whole diameter is to the whole diameter, so the part cut off, AG, is to the part cut off, CH. Therefore the remainder of the whole diameter is to the remainder of the whole diameter [Margin: g — Euclid V.19] — that is, the versed sine of the greater arc is to the versed sine of the greater arc — as the diameter is to the diameter, that is, as the radius is to the radius, or the whole sine to the whole sine. Which is what was proposed.

The converse is also evident. For since (as we have shown) the diameter is to the diameter — that is, the whole sine to the whole sine — as the versed sine is to the versed sine of the greater arcs, the remainder AG will likewise be to the remainder CH as the whole sine is to the whole sine. Therefore, as we have shown, the arcs AB and CD are similar; and consequently the remaining arcs from the semicircles will also be similar, by the scholium to Proposition 22 of Book 3 of Euclid. Which was to be demonstrated.

LEMMA VI.

If to similar segments of unequal circles similar segments are added, or from similar segments similar ones are taken away, then the wholes, or the remainders, will likewise be similar segments.

This theorem, insofar as it concerns the subtraction of similar segments from semicircles, or even from whole circles, was demonstrated by us in the scholium to Proposition 22 of Book 3 of Euclid. Here, however, we shall show the same thing in general, for any segments whatever, as was proposed — and indeed more easily; for we shall have need of it in what follows.

So let there be, in unequal circles, similar arcs ABC and DEF, whether they are semicircles or not (for in equal circles similar segments are equal, and therefore, if equal segments are added to equal ones, or equal are subtracted from equal, both the wholes and the remainders will also be equal); and to them let the similar arcs CG and FH be added. I say that the whole arcs ABG and DFH are also similar. For, two points I and K being taken at will on the remaining segments AIG and DKH, let the straight lines AI, CI, GI, DK, FK, HK be joined. Since, then, the arcs ABC and DEF are similar, the angles AIC and DKF will be equal, by the scholium to Proposition 22 of Book 3 of Euclid; and by the same reasoning the angles CIG and FKH will be equal, on account of the similar arcs CG and FH. The whole angles AIG and DKH will therefore be equal; and hence, by the same scholium, the arcs ACG and DFH, on which they stand, will be similar. Which is what was proposed.

But now, from the similar arcs ABC and DEF, whether they are semicircles or not, let similar arcs AB and DE be taken away. I say that the remaining arcs BC and EF are also similar. For, again taking two points I and K at will on the circumferences outside the given arcs, let the straight lines AI, BI, CI and DK, EK, FK be joined. Since, then, the whole arc ABC is similar to the whole arc DEF, the whole angle AIC will be equal to the whole angle DKE, by the scholium to Proposition 22 of Book 3 of Euclid; and by the same reasoning the subtracted angle AIB will be equal to the subtracted angle DKE, on account of the similar arcs AB and DE. Therefore the remaining angle BIC will also be equal to the remaining angle EKF; and hence, by the same scholium, the arcs BC and EF will be similar. Which is what was proposed.

Now, if from whole circles similar arcs IAC and KDF are removed, we shall show that the remainders CGI and FHK are also similar, as in the aforesaid scholium, by this reasoning namely. Taking single points A, G and D, H on the individual arcs, let the straight lines IA, CA; IG, CG; KD, FD; KH, FH be joined. Since, then, the segments IAC and KDF are similar, the angles IAC and KDF will be equal, by the definition of similar segments [Margin: a — Euclid III.22]. Since, therefore, both the two opposite angles A, G and the two D, H are equal to two right angles, the two angles IGC and KHF will likewise be equal; and therefore, by the same definition, the arcs IGC and KHF will be similar. Which is what was proposed.

Diagram. A rectangular woodcut frame containing two circles side by side, each with an inscribed set of chords and a dotted arc near the top right. In the left circle the points I (top left), G (top right), A (left), C (right), B (bottom) are joined by chords from I to A, C and G; in the right circle the points K (top left), H (top

right), D (left), F (right), E (bottom) are joined correspondingly. The diagram illustrates the similar arcs ABC, DEF with the added arcs CG, FH.

- Left circle — I (top left), G (top right, on the arc), A (left), C (right), B (bottom); with the hatched diameter AC
- Right circle — K (top left), H (top right, on the arc), D (left), F (right), E (bottom); with the hatched diameter DF

LEMMA VII.

If two unequal quadrants are divided in a similar way — or into equal parts — and through the points of division lines are drawn parallel to one radius (or perpendicular to the other radius), then the segments of the radius made in the one quadrant by the parallels (or perpendiculars) will be proportional to the segments of the radius made by the parallels (or perpendiculars) in the other quadrant. And conversely, if the segments of the radii are proportional, the quadrants will be divided in a similar way.

Let two unequal quadrants AB and CD, whose centers are E and F and whose radii are AE, EB and CF, FD, first be divided into two similar parts at the points G and H; and let there be drawn, parallel to the radii AE and CF, the lines GI and HK [Margin: a — Euclid I.29], which are consequently perpendicular to the radii EB and FD. I say that the segments of radius EB are proportional to the segments of radius FD; that is, that as EI is to IB, so FK is to KD. For since EI and FK are the sines of the similar arcs AG and CH — because they are equal to the perpendiculars drawn from G and H to AE and CF [Margin: b — Euclid I.34], which are indeed the sines of arcs AG and CH — it will follow, by Lemma 5, that as EB, the whole sine, is to FD, the whole sine, so the sine EI is to the sine FK. And by alternation, as EB, the whole sine, is to the sine EI, so FD, the whole sine, is to the sine FK. And by division (*dividendo*), as IB is to EI, so KD is to FK; and by conversion (*convertendo*), as EI is to IB, so FK is to KD.

Next, let the same quadrants be divided into three similar parts at the points G, L and H, M, and let there be drawn, parallel to the radii AE and CF, the lines GI, LN, HK, MO. I say that the segments EI, IN, NB are the same...

Diagram. A wide rectangular woodcut frame containing two quarter-circle (quadrant) figures side by side, each subdivided by a series of vertical ordinates dropped from points on the arc to the base line. In the left quadrant the arc runs from A (top left) down through G, P, L, T, V to B, with the base line E–I–R–N–Z–a–B carrying the feet of the ordinates. In the right quadrant the arc runs from C (top

through H, Q, M, X, Y to D, with the base line F–K–S–O–b–c–D. The figure illustrates the proportional subdivision of the two radii.

- Left quadrant, arc (top to lower right): A, G, P, L, T, V (ending at B)
- Left quadrant, base line (left to right): E, I, R, N, Z, a, B
- Right quadrant, arc (top to lower right): C, H, Q, M, X, Y (ending at D)
- Right quadrant, base line (left to right): F, K, S, O, b, c, D

will have the [sa]me proportions that the segments FK, KO, OD have. For, by the preceding lemma, the whole arcs AL, CM will also be similar, whose sines are EN, FO. Therefore, by lemma 5, as EB, the whole sine, is to FD, the whole sine, so is the sine EI to the sine FK, and so is the sine EN to the sine FO; [Margin: a — Euclid V.11] and hence also, as the whole EN is to the whole FO, so is the part cut off EI to the part cut off FK. Therefore the remainder IN is to the remainder KO as the whole EN is to the whole FO, or as the part cut off EI is to the part cut off FK. [Margin: b — Euclid V.19]

Since, then, as EI is to FK, so is IN to KO, by alternation it will also be true that as EI is to IN, so is FK to KO; and thus the segments EI, IN are proportional to the segments FK, KO. Again, since, as the whole EB is to the whole FD, so is the part cut off EN to the part cut off FO — by lemma 5, as was said — the remainder NB will also be to the remainder OD as the whole EB is to the whole FD, or as the part cut off EN is to the part cut off FO. [Margin: c — Euclid V.19] But it was shown a little earlier that as EN is to FO, so is IN to KO, and so is NB to OD; therefore also, as IN is to KO, so is NB to OD, and by alternation, as IN is to NB, so is KO to OD. The three segments EI, IN, NB are therefore proportional to the three segments FK, KO, OD.

FURTHERMORE, let the same quadrants be cut into four similar arcs at the points G, P, L, H, Q, M, and let the parallels GI, PR, LN, HK, QS, MO be drawn to the radii AE, CF. I say once again that the four segments EI, IR, RN, NB are proportional to the four segments FK, KS, SO, OD. For, by the preceding lemma, both the whole arcs AP, CQ and the whole arcs AL, CM will be similar, whose sines are ER, EN, FS, FO. Therefore, by lemma 5, as EB, the whole sine, is to FD, the whole sine, so is the sine EI to the sine FK, and so is the sine ER to the sine FS, and so is the sine EN to the sine FO; [Margin: d — Euclid V.11] and so it will be that EI is to FK as ER is to FS and as EN is to FO. Since, then, as the whole ER is to the whole FS, so is the part cut off EI to the part cut off FK, the remainder IR will also be to the remainder KS as the whole ER is to the whole FS, or as the part cut off EI is to the part cut off FK. [Margin: e — Euclid V.19] EI therefore has the same ratio to FK that IR has to KS;

and by alternation, EI has the same ratio to IR that FK has to KS; and so the two segments EI, IR are proportional to the two segments FK, KS.

Again, since as the whole EN is to the whole FO, so is the part cut off ER to the part cut off FS, as we said, the remainder RN will also be to the remainder SO as the whole EN is to the whole FO, or as the part cut off ER is to the part cut off FS.

[Margin: f — Euclid V.19] But it was shown that as ER is to FS, so is IR to KS.

Therefore it will also be that as IR is to KS, so is RN to SO; and by alternation, as IR is to RN, so is KS to SO. And thus the three segments EI, IR, RN are proportional to the three segments FK, KS, SO.

Finally, since as the whole EB is to the whole FD, so is the part cut off EN to the part cut off FO, by lemma 5, as we showed, the remainder NB will also be to the remainder OD as the whole EB is to the whole FD, or as the part cut off EN is to the part cut off FO. [Margin: g — Euclid V.19] But it was, as demonstrated a little earlier, that as EN is to FO, so is RN to SO. Therefore it will also be that as NB is to OD, so is RN to SO — that is, as RN is to NB, so is SO to OD. The four segments EI, IR, RN, NB are therefore proportional to the four segments FK, KS, SO, OD; and the reasoning is the same for a greater number.

It is moreover clear that this demonstration holds even when the quadrants have been divided into equal parts. For if each quadrant is divided into six equal parts — AB into AG, GP, PL, LT, TV, VB, and CD into CH, HQ, QM, MX, XY, YD — the six former arcs will be similar to the six latter, since any one of the former is the same part of its own quadrant that any one of the latter is of its. Therefore, as has been shown, the segments of the diameters are proportional.

Diagram. The left of two adjacent quadrant figures illustrating Lemma VII. A quarter-circle has its right angle at E (bottom-left); the vertical radius rises to A (top-left) and the horizontal radius runs along the base to B (bottom-right); the arc curves from A down to B. The arc is divided into six similar parts at the points G, P, L, T, V. From these points, perpendiculars (sines) parallel to the radius AE are dropped to the base, dividing the base EB at the points I, R, N, Z, a — so the base points, left to right, are E, I, R, N, Z, a, B, and the perpendiculars are GI, PR, LN, TZ, Va.

- A (top-left): apex, top of vertical radius
- G, P, L, T, V (along the arc, top to bottom-right): division points of the arc
- B (bottom-right): end of the arc and of the horizontal radius
- E (bottom-left): center / right angle of the quadrant
- I, R, N, Z, a (along the base, left to right): feet of the perpendiculars

Diagram. The right of the two quadrant figures for Lemma VII, constructed identically. The right angle is at F (bottom-left); the vertical radius rises to C (top-left) and the horizontal radius runs to D (bottom-right); the arc runs from C down to D, divided into six similar parts at H, Q, M, X, Y. The perpendiculars HK, QS, MO, Xb, Yc are dropped to the base, whose points, left to right, are F, K, S, O, b, c, D.

- C (top-left): apex, top of vertical radius
- H, Q, M, X, Y (along the arc): division points of the arc
- D (bottom-right): end of the arc and of the horizontal radius
- F (bottom-left): center / right angle of the quadrant
- K, S, O, b, c (along the base, left to right): feet of the perpendiculars

Now let the segments of the radii be proportional. I say that the arcs cut off by the perpendiculars are similar. For let two segments EI, IB first be set proportional to the two segments FK, KD — that is, let it be that as EI is to IB, so is FK to KD. By alternation, then, as EI is to FK, so is IB to KD.[Margin: h — Euclid V.12] Therefore, as one EI is to one FK, so will all the EI, IB together — namely the whole sine EB — be to all the FK, KD together — namely the whole sine FD. Since, then, EI, FK are the sines of the arcs AG, CH, by lemma 5 the arcs AG, CH will be similar; and hence, by the preceding lemma, since the whole arcs AB, CD are also similar (being quadrants), the remaining arcs GB, HD will be similar as well.

Next, let three segments EI, IR, RB be set proportional to the three segments FK, KS, SD. It will then follow, by alternation, that as EI is to FK, so is IR to KS, and RB to SD.[Margin: i — Euclid V.12] Therefore, as one EI is to one FK, so will all the EI, IR, RB together — that is, the whole sine EB — be to all the FK, KS, SD together — that is, the whole sine FD. And since EI, FK are the sines of the arcs AG, CH, these arcs AG, CH will, by lemma 5, be similar. Again, since as EI is to FK, so is IR to KS, [Margin: k — Euclid V.12] it will be that as EI is to FK, so are EI, IR together — that is, the whole ER — to FK, KS together — that is, to the whole FS. But it was that as EI is to FK, so is EB to FD; therefore also as ER is to FS, so is the whole sine EB to the whole sine FD. And hence, since ER, FS are the sines of the arcs AP, CQ, by lemma 5 the arcs AP, CQ will be similar; and consequently, by the preceding lemma, since the whole arcs AB, CD are similar, the remaining arcs PB, QD will be similar as well. And since the arcs AG, CH have been shown similar, if these are removed from the similar arcs AP, CQ, the remaining arcs GP, HQ will also be similar, by the same preceding lemma. Therefore all three arcs AG, GP, PB are similar to all three arcs CH, HQ, QD.

Again, let there be four segments EI, IR, RN, NB proportional to the four segments FK, KS, SO, OD. It will then follow, by alternation, that as EI is to FK, so is IR to KS, and RN to SO, and NB to OD.[Margin: l — Euclid V.12] Therefore, as EI is to FK, so is the whole sine EB to the whole sine FD; and, since EI, FK are the sines of the arcs AG, CH, by lemma 5 the arcs AG, CH will be similar. Again, since as EI is to FK, so is IR to KS,[Margin: m — Euclid V.12] it will be that as EI is to FK, so is the whole ER to the whole FS, that is, so is the whole sine EB to the whole sine FD; and hence, by lemma 5, the arcs AP, CQ are similar; and once the similar arcs AG, CH are removed, the remaining arcs GP, HQ will also be similar, by the preceding lemma. Furthermore, since as EI is to FK, so is IR to KS, and RN to SO,[Margin: n — Euclid V.12] it will be that as EI is to FK, so is the whole EN to the whole FO. But it was that as EI is to FK, so is EB

to FD. Therefore it will also be that as EN, the sine of the arc AL, is to FO, the sine of the arc CM, so is the whole sine EB to the whole sine FD; and accordingly, by lemma 5, the arcs AL, CM will be similar; and hence, by the preceding lemma, the remaining arcs LB, MD will be similar too. And since the arcs AP, CQ have been shown similar, if they are removed from the similar arcs AL, CM, the remaining arcs PL, QM will also be similar. Therefore all four arcs AG, GP, PL, LB are similar to all four arcs CH, HQ, QM, MD; and the same reasoning holds for a greater number.

LEMMA VIII.

To cut a given straight line in the same way that the radius of some quadrant is cut by the perpendiculars dropped to it from any points whatever of the quadrant.

Although this could be accomplished from Euclid, *Elements* VI.10, nevertheless, since we frequently need a division of this kind on various lines in the astrolabe, we shall here construct a certain figure by which we may obtain the same result much more easily. Take, then, any rectangle longer on one side, ABCD, and, the side DA being produced, describe from A and D, with radius AB (or DC), two equal quadrants EB, FC. When these have been divided into degrees — we, on account of the smallness of the figure, have divided them into 6 parts, so that each part contains fifteen degrees — let there be drawn, through each pair of points equally distant from the side AD, straight lines cutting the radii AB, CD; these will all be parallel to the side AD and to one another. For if, from any two points of the quadrants EB, FC equally distant from the side AD, perpendiculars are raised to AD, these will be parallel to one another;[Margin: a — Euclid I.28] and also equal, since they are the sines of equal arcs.[Margin: b — Euclid I.33] Therefore the

straight line connecting those two points will also be parallel to AD itself. And in this way all those lines will be equidistant from the side AD;[Margin: c — Euclid I.30] and hence they will be parallel to one another,[Margin: d — Euclid I.29] and consequently perpendicular to each of the radii AB, DC.

Each radius AB, DC, therefore, has been divided by the perpendiculars dropped from the quadrants. In order that you may likewise cut any other straight line, whether greater or smaller than the radius AB, just as if it were the radius of some quadrant divided by the perpendiculars, and so on — construct on AB the equilateral triangle ABG. The point G will fall at the 30th degree of the quadrant, counting from E; and since BG, subtending a sixth part of the circle, is equal to the radius AB, by the corollary of Euclid IV.15, — finally, from G let straight lines be drawn to the points of section of the radius AB, and the figure we desire will have been constructed.

If, then, a straight line H is to be cut into parts proportional to the parts of the radius AB (if it were equal, the segments of the radius AB would have to be transferred onto it, so that it might be similarly cut), let it be transferred, by means of the compasses, from any point of the side AD to the side AB — such is IK — which will be cut by the parallels just as AB is cut, by the demonstration of Euclid VI.10, since KI, BA, if produced, would meet and form a triangle whose base is BK, and so on. Therefore, if the segments of the straight line IK are transferred onto the given straight line H, the line H will be cut just as AB is cut, as if it were divided by perpendiculars dropped from the degrees of a quadrant whose radius is H — because these perpendiculars would cut H itself, by the preceding lemma, into parts proportional to the parts of the straight line AB.

But if a straight line L be given so long that, when transferred onto the parallels, it cuts them too obliquely, so that the points of intersection cannot easily be distinguished, then its half LM is to be transferred — such is NO. For if the doubled segments of this half are transferred onto the given straight line L, the line L will likewise be divided just as AB, or NO, is;[Margin: e — Euclid V.15] since the segments of the line NO have the same proportions as their doubles. Indeed, if the half of the given line were still too long, its quarter part, or eighth, would have to be transferred, and the segments between the parallels quadrupled, or octupled, transferred onto the given line.

But if the given straight line P be smaller than the radius AB, it must be transferred into the equilateral triangle GBA, so as to be equidistant from AB itself: which is done if we cut off from GB and GA the equal parts GQ, GR equal to P. For, the straight line QR being drawn, it will be parallel to AB[Margin: f —

Euclid VI.2] and equal to P (that is, to each of GQ, GR), since, by the corollary of Euclid VI.4, the triangle GQR is similar to the triangle GBA, and therefore equilateral as well. The segments of the straight line QR, therefore, transferred onto the given line P, will cut it just as QR — that is, just as BA — is cut; for, by the scholium of Euclid VI.4, the lines BA, QR are similarly cut by the lines drawn from G. Moreover, whenever the half, or the quarter, or the eighth part of a given line is to be transferred into the figure, as we said above, and it is smaller than AB, it must be transferred into the triangle GBA. Thus you see ST, the half of the given line S, transferred into the triangle, such as is QR. For the doubled segments of this line QR will cut the given line S just as AB is cut.

But since we do not always need all the parts of a line divided in that way — namely those which correspond to all the degrees of the quadrant described from that line — but sometimes need only, in the given line, a single point corresponding to a proposed degree or arc (that is, the point onto which the perpendicular dropped from the given degree or arc would fall), we shall find, from the very figure here constructed, that point in the following manner. Suppose the point corresponding to the 52nd degree, counting from the point E, is to be found in the same given lines. Take, from lemma 3, two arcs EV, FX of 52 degrees, and let the straight line VX be joined, cutting the lines IK, NO at Y; and let the straight line drawn from G to the point where VX cuts the line AB also intersect the line QR at Y. For the point Y, transferred onto the corresponding line — as was said above of the other segments — will give in the line the required point Z.

If you use this technique correctly, there will be no need to describe a quadrant about the given line, and, having divided it into degrees, to drop perpendiculars from the points of division so as to divide the given

Diagram. The construction figure for Lemma VIII: a rectangle ABCD (longer on one side) with the side DA produced upward to the apex G, on which the equilateral triangle ABG is built. From the centers A and D, two equal quadrants EB, FC are described, each of radius AB (= DC); their arcs are each divided into 6 parts of 15°. A family of straight lines parallel to AD (and perpendicular to the radii) cuts the radii into corresponding segments; from G, lines are drawn to the points of section. A dotted line marks the point Y found in the worked example. The following labels are readable, with approximate positions:

- G (top center): apex of the equilateral triangle ABG, at the 30th degree
- E (top right corner of frame): top of one quadrant
- A (right side, upper-middle): a rectangle corner / quadrant center

- N, I (right side, descending): division points on the radius
- F (right side, lower): top/foot of the other quadrant
- V, R, P (on the upper arc, left to right): arc-division points [labels partly obscured — reading of the reversed "R" uncertain [?]]
- M, O, K (along a diagonal, mid-left): points of section / worked-example points
- N, T (left margin, with segment tick-marks): segment markers on the transferred line
- Y (on the dotted vertical line): the point found in the 52° example
- F, S, H, L, C (along the base, left to right) and D (bottom right corner): division points on the base radius and rectangle corners

[Several interior labels of this crowded figure are faint under the foxing; the letters listed are those legible on the scan, and the exact assignment of a few mid-figure points is uncertain [?].]

line into the parts you wish: and how great the usefulness of this is, you will learn from our *Astrolabe*.

LEMMA IX.

If two or more circles touch one another internally, or two touch each other externally, the straight lines drawn through the point of contact cut off similar arcs; and the straight lines joining the two points in which two straight lines cut the circles are parallel.

The same holds in the case of two circles that do not touch one another, if in place of the point of contact one takes the point on the straight line joining their centers, through which passes the straight line connecting the alternate extremities of the diameters perpendicular to the former line. But when the circles do not touch one another internally, the similar arcs are the alternate ones, not those taken in the same order as in the former case.

This theorem, so far as it concerns circles touching internally, we demonstrated in the scholium of Euclid III.22; but since we need it in what follows, it has seemed good to demonstrate the same thing here in a somewhat different way, and indeed more generally, extending it to circles touching externally and to circles not touching at all — which application we shall also make in the demonstrations that follow.

Let there be, then, first, two circles ABCD, AEFG, whose centers are H, I, touching one another at A, whether internally or externally; and let there be drawn through A, the point of contact, any straight lines BE, CF, cutting each of the circles. I say that the arcs ABC, AEF are similar, just as the arcs AB, AE and BC, EF, and so on. For through the centers H, I let the straight line HI be drawn, which will pass through the point of contact A;[Margin: a — Euclid III.11 or 12] and from C and F let the straight lines CH, FI be joined to those same centers. Since, then, in the triangles ACH, AFI the angle A is common (when the circles touch internally) or the angles A are vertically equal (when the contact is external),[Margin: b — Euclid I.15] and the sides about the other angles H, I are proportional (inasmuch as they have a ratio of equality), and each of the remaining angles C, F is less than a right angle — that is, acute, by the corollary 3 of Euclid I.17, because each stands above the base of an isosceles triangle[Margin: c — Euclid VI.7] — the triangles will be equiangular,[Margin: d — Euclid I.5] and will have equal angles at the centers H, I.

This can also easily be shown in the following way. In circles touching one another internally, each angle AFI, ACH is equal to the angle FAI;[Margin: e — Euclid I.5] but in circles touching externally, the one is equal to the angle FAI, the other to the angle CAH;[Margin: f — Euclid I.15] and the angles FAI, CAH are vertically equal. Therefore the angles AFI, ACH will also be equal to one another — as internal and external, when the circles touch internally, or as alternate, when they touch externally.[Margin: g — Euclid I.28 or 27] The lines CH, FI are therefore parallel, and hence the angles H, I are equal,[Margin: h — Euclid I.29] whether the circles touch internally (as internal and external) or externally (as alternate). Since, then, the angles H, I at the centers have been shown equal in either way, the arcs ABC, AEF, on which those angles stand, will be similar, by the scholium of Euclid III.22. And when these are removed from the whole circles, the remaining segments ADC, AGF will be similar. And by the same reasoning the segments AB, AE will be similar (if lines BH, EI are drawn to the centers, which will likewise be shown parallel, and so on); and, from the circles, the arcs BC, EF comprehended between the two straight lines will be similar, by the same scholium — a construction made, in circles touching internally, upon the arcs, but, in circles touching externally, upon the angles BAC, EAF vertically equal, and upon the arcs. And if another circle AKL, with center M, be described touching the other two internally, we shall demonstrate in the same way, the line KM being drawn, that the arc AKL is similar both to the arcs ABC, AB and to the arcs AEF, AE, and so on.

Diagram. Two (or three) circles touching internally at A, illustrating Lemma IX. A larger circle and smaller circle(s) share the point of contact A at the top; straight lines through A cut the circles, and chords/radii to the centers form the inscribed triangles used in the proof. Readable labels:

- A (top): common point of contact
- E, I, M (upper-center, descending): interior points / centers (I, M among the centers H, I, M of the text)
- k, F (left and mid-left): interior points
- G, H (center, lower): interior points / center H
- L (left, lower): a point on the inner circle
- C (bottom): lower endpoint of a line through A
- D (lower right): a point on the outer circle

Diagram. Two circles touching externally at A, illustrating the external-contact case of Lemma IX. An upper circle (with points C, D and center H) and a lower, smaller circle (with center F and points G, E) meet at the contact point A between them; inscribed triangles are drawn to the points E, E and the centers. Readable labels:

- C (top): a point on the upper circle
- D (top right): a point on the upper circle
- E (upper-left): a point on the upper circle
- H (center-right of upper circle): center of the upper circle
- A (middle, between the circles): external point of contact
- G, E, E (lower circle, left to right): points on the lower circle [two points appear both lettered E — as printed]
- F (bottom): center / point of the lower circle

Let the straight lines BC, EF also be joined, which I say are parallel. For since the arcs AB, AE have been shown similar, the angles ACB, AFE — standing upon those arcs at the circumferences, or alternate in circles touching externally — will be equal to one another, by the scholium of the said Euclid III.22.[Margin: i — Euclid I.28 or 27] Therefore BC, EF are parallel, which was what was proposed.

Next, let there be two circles AB, CD whose centers E, F do not touch one another, but either intersect or do not intersect — whether one is placed wholly outside the other or within it. The straight line EF being drawn through their centers, let the diameters AE, CF be raised perpendicular to it. And the straight line AC being joined, cutting EF at G, let there be drawn through G any straight lines HI, KL,

cutting each of the circles. I say that the arcs HAn, ICO are similar, just as the arcs HK, IL, and so on. For, the straight lines HE, nE, IF, OF being drawn, since the triangles AEG, CFG are equiangular — for the angles E, F are right angles, and both the alternate angles A, C[Margin: l — Euclid I.15] and the vertical angles AGE, CGF are equal to one another[Margin: k — Euclid I.29] — it will be that as GE is to the radius EA, so is GF to the radius FC.[Margin: m — Euclid VI.4] Again, since in the triangles GEH, GFI the vertical angles EGH, GFI are equal,[Margin: n — Euclid I.15] and the sides about the angles E, F are proportional (since it has been shown that as GE is to EA — that is, to EH — so is GF to FC — that is, to FI), and each of the remaining angles H, I is less than a right angle, by the corollary 3 of Euclid I.17, because they stand above the bases of the isosceles triangles EHn, FIO — the angles GHE, GIF, and GEH, GFI, will be equal.[Margin: o — Euclid VI.7] But GHE is equal to GnE in the isosceles triangle EHn,[Margin: p — Euclid I.5] and GIF is equal to GOF in the isosceles triangle FIO. Therefore the two angles H, n will be equal to the two I, O; and hence the remaining angles HEn, IFO will be equal too. Consequently, by the scholium of Euclid III.22, the arcs HAn, ICO, on which those angles stand at the centers, will be similar; and when these are removed from the whole circles, the remaining arcs HPn, IQO will also be similar. And this indeed in the 1st and 3rd figure. But in the 2nd figure, the angle GHE will equal the angle EnH.[Margin: q — Euclid I.5] In the isosceles triangle EHn, and the angle GIF equal to the angle FOI in the isosceles triangle FIO, [they will be] equal-

are equal. Therefore, as before, the two angles EHn and EnH will be equal to the two FIO and FOI, and the remaining angle HEn equal to the remaining IFO. Consequently the arcs HAn and ICO — and, taken from the whole circles, the remaining arcs HPn and IQO — will be similar.

Diagram. A single framed woodcut containing four sub-figures that illustrate the several cases of Lemma IX (two tangent circles cut by straight lines drawn through their point of contact, cutting off similar arcs). The largest sub-figure at the left shows a great circle with vertical diameter Z (top) to B (bottom) and horizontal diameter through A (left), with centre E, an inscribed smaller circle tangent internally near the foot (centre region F), and many chords and secants radiating from the centre. Readable labels across the four sub-figures: Z, H, K, D, o, E, A, P, V, S, C, T, Q, R, F, B, n, a, L, I (left/main figure); and in the right-hand sub-figures H, Z, K, E, P, M, B, W, N, O, S, C, L. The labelling is repeated from sub-figure to sub-figure and several small peripheral letters are obscured by foxing. [Diagram is dense with >2 uncertain labels]

That the arcs HK and IL, which the straight lines HI and KL cut off, are likewise similar will be shown as follows. Join the straight lines KE and LF. In the triangles GEK and GFL the vertical angles EGK and FGL are equal [Margin: a — Elements I.15], and the sides about the angles E and F are proportional, as has been shown; while of the remaining angles K and L each is less than a right angle. This holds in figures 1 and 3 because, if the straight lines BK, zK and DL, dL are joined, the angles at K and L become right angles in the semicircles of which those arcs are parts [Margin: b — Elements III.31]; and in figure 2 because they stand upon the bases of isosceles triangles, provided the straight lines Ea and Fm are joined to the points where the circumferences are cut by the straight line KL (a reasoning that also holds in the other two figures). Hence the angles GEK and GFL are equal [Margin: c — Elements VI.7]. Since therefore the whole angles GEH and GFI have also been shown equal, the remaining angles HEK and IFL will be equal as well; and therefore, by the scholium to Elements IV.22, the arcs HK and IL will be similar.

In the same way we shall show that the straight lines Zd and HI intercept similar alternate arcs HZ and Id, and HB and ID. For since the angles GEH and GFI have been shown equal, the angles HEZ and IFd — remaining from two right angles — will be equal; and therefore, by the aforesaid scholium, the arcs HZ and Id will be similar. And by the same scholium HB and ID will be similar, on account of the equal angles BEH and DFI.

By a like reasoning we shall show that the straight line AC cuts off similar alternate arcs ABe and bDC. For, joining the straight lines eE and bF [Margin: d — Elements I.29], since the alternate angles EAe and FCb are equal, and EAe equals EeA while FCb equals FbC [Margin: e — Elements I.5], the angles EAe and EeA will be equal to FCb and FbC; and hence the remaining angle AEe will be equal to the remaining CFb. Therefore, by the scholium to Elements III.22, the arcs ABe and bDC will be similar. In the second figure, however, the arcs Ae and bC come out similar; and when these are taken from the whole circles, the remaining arcs ABe and bDC are similar as well.

So too, to add yet another example, we shall show that the straight line RS cuts off similar alternate arcs RBV and SDT. For, joining the straight lines RE, VE and SF, TF [Margin: f — Elements I.15], since in the triangles GER and GFS the vertical angles EGR and FGS are equal, and the sides about the angles E and F are proportional, as has been shown; while of the remaining angles R and S each is less than a right angle, because they stand upon the bases of the isosceles triangles ERV and FST, the angles ERG and FSG will also be equal [Margin: g —

Elements VI.7]. Now the former equals the angle EVG, and the latter the angle FTG [Margin: h — Elements I.5]. Therefore the two angles R, V will be equal to the two S, T; and consequently the remaining angles REV, SFT in the triangles REV and FST will be equal. Hence, by the scholium to Elements III.22, in figures 1 and 3 the arcs RBV and SDT will be similar; while in the second figure the arcs RV, ST will be similar, and so on.

In the same way the straight lines Zd and RV will intercept similar alternate arcs RB, SD and RZ, Sd. For since in the triangles EGR and FGS the angles R and S have been shown equal [Margin: i — Elements I.15], and the vertical angles at G are also equal, the remaining angles REB and SFD will be equal. Therefore, by the same aforesaid scholium, the arcs RB and SD will be similar; and consequently also the remaining arcs RZ and Sd from the semicircles. And the same reasoning holds for every straight line that cuts the line Zd drawn out through the centres.

Finally, from all this it follows that two straight lines intersecting each other in any manner at G intercept similar arcs on opposite sides. Thus, if the straight lines HI and KL intersect at G, I say that the arcs HK and IL are similar, and likewise Kn and LO. Of the former this was shown a little earlier; the latter are proved thus. Since KB is similar to LD, and Bn to Do — as we have just shown for the straight lines cutting Zd itself — the arcs Kn and LO will also be similar, by Lemma 6. By the same reasoning the arcs HR and IS will be similar, because the straight lines HI and RS intersect each other, and so on.

But if through G a straight line GM is drawn tangent to the circle AB at M — in the second figure — then, produced, it will also be tangent to the circle CD at N, and again the arcs cut off, BM and DN, will be similar. For, drawing GN tangent to the circle CD at N and joining the straight lines EM and FN, the angles M and N will be right angles [Margin: k — Elements III.18]. Since therefore the sides about the angles E and F in the triangles GEM and GFN are proportional, and each of the remaining angles at G is less than a right angle, by Corollary 1 of Elements I.17 [Margin: l — Elements VI.7] both the angles E and F and the angles at G will be equal. Therefore, from what we demonstrated after Proclus at Elements I.15, the straight lines MG and NG will form a single straight line; and hence the tangent GM produced will also touch the circle CD at N; and the arcs BM and DN will be similar, by the scholium to Elements III.22.

Finally, let the straight lines HK and IL be joined, cut off by the similar arcs from the straight lines HI and KL. I say that they are parallel. For since both the arcs HAn, ICO and the arcs HK, IL have been shown similar, the remaining arcs KAn and LCO will also be similar, by Lemma 6. Therefore, by the scholium to Elements

III.22, the angles KHn and LIO standing upon them at the circumferences will be equal; and since these are alternate angles [Margin: m — Elements I.27], HK and IL will be parallel — which was the proposition.

But as for non-tangent circles, we shall demonstrate the lemma — perhaps more briefly and more easily — in this way. Since AE is to EG as CF is to FG , and HE equals AE while IF equals CF , it follows that HE is to EG as IF is to FG [Margin: a — Elements VI.4]. Now in the triangles HEG and IFG the vertical angles at G are equal [Margin: b — Elements I.15], and of the remaining angles H and I each is less than a right angle; therefore, by the third corollary to Elements I.17, the angles HEG and IFG will be equal at the centres [Margin: c — Elements VI.7]. Hence, by the scholium to Elements III.22, the alternate arcs HB and ID will be similar, and likewise the remaining arcs HZ and Id from the semicircles. Again, since in the triangles nEG and OFG , nE is to EG as OF is to FG , and the vertical angles EGn and FGO are equal, and each of the remaining angles EnG and FOG is less than a right angle, by the third corollary to Elements I.17 — because of the isosceles triangles HEn and IFO — the angles nEG and OFG will be equal at the centres [Margin: d — Elements VI.7]. Hence, by the scholium to Elements III.22, the arcs Bn and DO will be similar, and likewise the remaining arcs Zn and dO from the semicircles. In no other way shall we show that any alternate arcs whatever, taken from a beginning at the straight line Zd passing through the centres, are similar — such as ZK and dL ; ZA and dC ; ZR and dS , and so on. And so, since the arcs HB and ID are similar, if the similar arcs Bn and DO are added, the whole arcs HBn and IDO will also be similar, by Lemma 6. Likewise, since ZK , dL are similar and so are ZH , dI , the remaining arcs HK and IL will also be similar, by the same Lemma 6. And so with the rest. Finally, that the straight lines HK and IL are parallel will be proved as before.

LEMMA X.

If two or more circles cut one another, then two straight lines drawn through the point of section — whether both cut the circles, or both are tangent, or one of each — intercept similar circumferences, beginning from one of those straight lines and proceeding toward the same side, up to the point of section (or of contact) of the other straight line. And if from the same point of section any circle whatever is described, its circumference comprehended between those same two straight lines will be similar [?] to that arc — described in the same circle from the point of section — which is similar to the arc of any of the prior circles intercepted between the same straight lines.

Diagram. A large framed woodcut showing the configuration of Lemma X: three circles intersecting at a common point A (lower right) with a dense pencil of secant and tangent straight lines and several arcs marked with dotted curves. Readable labels around the figure: Z (top), Y, X (upper left), H, G, N, O, K (upper region), C, B, Q, F, M, K (upper-left cluster), V, D, d, R (left/middle), L, E, T, S (lower), Γ [Greek capital gamma] (lower left), a, I, P, m (right upper), and A at the vertex (lower right) with i below it; a small symbol resembling $\hbar$ appears at the right beside a K. Many peripheral labels sit in heavily foxed areas. [Diagram is dense with >2 uncertain labels]

Let the circles ABCDE, AFGHIK, ALMNOP cut one another at the point A, and first let two straight lines AB and AC be drawn cutting the circles in any way whatever, intercepting the arcs BC, GF, NM, all of which I say are similar. For since the common angle MAN stands upon each of them at the circumference of its own circle at the point A, it is clear from the scholium to Elements III.22 that they are similar. In the same manner, if the straight line AH is drawn cutting all three circles, the arcs BQ, GH, NO will be shown similar, on account of the common angle NAH standing upon each of them at the circumference of its own circle at the point A. The same is to be said, when the secant straight line AD is drawn, of the arcs CD, Fd, MD, on account of the common angle DAM; and thus any other arcs whatever, lying between two secant straight lines, will be shown similar. This was also demonstrated in the preceding lemma concerning the arcs intercepted between two straight lines drawn from the point of contact of two circles touching one another internally.

Next, let the straight line AP be tangent to the circle ABCDE at A, and consequently cut the others at P and I — since the circles are supposed to intersect one another at A, not to touch (for only when several circles touch one another internally, or two touch externally, can one and the same straight line touch them all at the same point of contact) — and let the straight line AN cut all three at B, G, N. I say that the arcs BA, GI, NP are likewise similar. The first of these runs from the point of section B up to the point of

contact A; the latter two run from the points of section G and N up to the other points of section I and P. As for these two latter, GI and NP, which lie between two secant straight lines, it is clear from the scholium to Elements III.22 that they are similar, because of the common angle NAI at their circumferences. But that all three, BA, GI, NP, are similar we show as follows. Draw the diameter ARD in the circle ABCDE, which the straight line AP touches, cutting the other two circles at D and d, and join the straight lines DP and dI [Margin: a — Elements III.18]. Since

the angle DAI is a right angle, by the corollary to Elements IV.5 the centres of the circles ALMNOP and AFGHIK will fall on the straight lines DP and dI; and therefore DMP and dFI will be semicircles, and thus similar to the semicircle DCA. Since then the arcs cut off, DB, DN, dG, lying between the secant straight lines AD and AG, are similar, as was just shown, the remaining arcs BA, GI, NP will also be similar, by Lemma 6. And by the same reasoning, when the secant straight line AF is drawn, the arcs CA, FI, MP will be similar, and so with the rest.

Diagram. A large framed woodcut, essentially repeating the configuration of the figure on the preceding page: three circles meeting at a common vertex A (lower right) with a pencil of secant and tangent lines and dotted arcs. Readable labels: Z (top), Y, X, H, G, N, O, K (upper), z, C, B, Q, F, M, K, V, D, d, R (left/middle), δ [lower-case Greek delta], L, E, T, S, Γ [Greek capital gamma], a, I, P, m (right upper), A at the vertex with i below; a small $\mathfrak{h}$ -like symbol appears at the right beside a K. Several peripheral labels lie in foxed regions. [Diagram is dense with >2 uncertain labels]

Again, let the straight line AE be tangent to the circle ALMNOP at A, and consequently cut the others at E and K, while the straight line AN cuts them all. I say that here too the arcs NLA, BDE, GAK are similar. The first, NLA, lies between the point of section N and the point of contact A; the second, BDE, lies between the points of section B and E, on the same side as the arc NLA; and the third, GAK, is reckoned from the point of section G, on the same side as the first two, up to the point of section K, beyond A. For the straight line AE does not cut the circle AFGHIK on the near side of the point A, as it does the others. This we shall demonstrate as follows. Draw the diameter AeM in the circle ALMNOP, which the straight line AE touches, cutting the two other circles at C and F, and join the straight lines CE and FK [Margin: b — Elements III.18]. Since both the angle MAE and the angle MAK are right angles, by the corollary to Elements IV.5 the centres of the circles ABCDE and AFGHIK will fall on the straight lines CE and FK; and therefore EDC and KAF will be semicircles, and thus similar to the semicircle ADM. Since then the arcs MN, CB, FG, lying between the secant straight lines AF and AG, are similar, as was shown above, the whole arcs NLA, BDE, GAK will also be similar, by Lemma 6.

By a like reasoning the arcs DLA, DbE, dAK will be similar. The first of these, DLA, lies between the point of section D and the point of contact A; the second, DbE, lies between the points of section D and E on the same side; while the point of section d falls on the near side of A, and the point of section K beyond A. For, drawing again the diameter AeM in the circle ALMNOP, which the straight line

AE touches, cutting the other two circles at C and F, and joining the straight lines CE and FK, we shall show, as was just done, that EDC and KAF are semicircles, and thus similar to the semicircle ADM. Since then the arcs cut off, DM, DC, dF, lying between the secant straight lines AD and AF, are similar, as we demonstrated at the beginning of this lemma, the remaining arcs DLA, DbE, dAK will also be similar, by Lemma 6.

In no other way shall we prove that the arcs NPA, GIK, BAE are similar. The first of these lies between the point of section N and the point of contact A; the second is intercepted between the two points of section G and K, on the same side as the first arc; and the third is reckoned, on the same side, from the point of section B up to the other section E, beyond A. For, making the same construction, we shall show, as just now, that KIF and EAC are semicircles, and thus similar to the semicircle APM. Hence, since the arcs cut off, MN, FG, CB, lying between the secant straight lines AF and AG, are similar, as was shown at the beginning of this lemma, the remaining arcs NPA, GIK, BAE will also be similar, by Lemma 6.

Furthermore, let the straight line AL be tangent to the circle AFGHIK at A, and consequently cut the others at b and L, while the straight line AN cuts them all. I say that again the arcs GFA, BDb, NDL are similar. The first of these lies between G, the point of section, and A, the point of contact; the second between the points of section B and b; and finally the third between the points of section

is set at the points N, L. For when the diameter AfH has been drawn in the circle AFGHIK — which the line AL touches, and which cuts the other circles in two points Q, O — let the lines Qb, OL be joined. And since the angle HAL is right [Margin: Euclid III.18], the centres of the circles ABCDE, ALMNOP will fall (by the corollary of prop. 5, book 4 [Euclid IV.5]) on the lines bQ, Lo; and therefore bDQ, LMO will be semicircles, and so similar to the semicircle AFH. Moreover the arcs GH, BQ, NO, lying between the secant lines AH, AN, [AL], are similar, as was shown above. Therefore the remaining arcs GFA, BDb, NDL too will be similar, by the sixth lemma. So too, when the line klm has been drawn through A, the arcs Ek, Al, Km will be similar. For since AE touches the circle ALMNOP, the arc Al — lying between the point A of contact and the point l of section — is similar (as has already been demonstrated) to the arc Km, lying between the two points of section K, m, on the same side as the arc Al. And to the arc Km the arc Ek is similar, by the scholium of prop. 22, book 3 [Euclid III.22], because of the equal vertical angles KAm, EAk that stand upon them. Therefore all three arcs Ek, Al, Km are similar.

Besides this, let the line AE touch the circle ALMNOP at A and cut the others in E, K; likewise let the line AL touch the circle AFGHIK at A and cut the others in b, L; and finally let AI touch the circle ABCDE at A and cut the others in P, I. I say that the arcs bE, LA, AK are similar, as are the arcs EDA, ADP, KAFI, and likewise the arcs bDA, LMP, AFI. For since AE touches the circle ALMNOP, the arc LA — between L, the point of section, and A, the point of contact — will be similar (as was shown just now) to the arc bE, between the points of section b, E, on the same side as the arc LA. And to the arc bE the arc AK is similar. (For since hA touches the circle AFGHIK at A, and KA cuts it [Margin: Euclid III.32], the angle hAK — that is, bAE, which is equal to it at the vertex — will be equal to the angle AFK in the alternate segment; and therefore the arcs AK, bE, upon which they stand at the circumferences, will be similar.) Therefore all three, bE, LA, AK, will be similar.

Next, with the diameter AD drawn in the circle ABCDE and the line DP joined, DNP will be a semicircle, because of the right angle DAP, and therefore similar to the semicircle DCA. Moreover the arcs DLA, DE are similar — as has been shown more than once, since AE touches the circle ALMNOP, and so on. Therefore the whole arcs EDA, ADP will likewise be similar. But the arc ADP is similar to the arc KAFI. (For with the diameter AM drawn in the circle ALMNOP, cutting the circle AFGHIK in F, and the line KF joined, KAF will be a semicircle because of the right angle FAK, and therefore similar to the semicircle ADM. Since, then, the arcs FI, MP are also similar — because of the common angle FAI standing upon them — the whole arcs KAFI, ADP will be similar.) Therefore all three, EDA, ADP, KAFI, will be similar. Finally, with the diameter AH drawn in the circle AFGHIK, cutting the circle ALMNOP in O, and the line LO joined, LMO will be a semicircle, because of the right angle LAO, similar to the semicircle bDQ. Moreover the arcs OP, QA are similar, since AP touches the circle ABCDE, and so on. Therefore the whole arcs bDA, LMP will be similar. But the arc bDA is similar to the arc AFI. (For with the diameter AH drawn in the circle AFGHIK, cutting the circle ABCDE in Q, and the line bQ joined, bCQ will be a semicircle, because of the right angle bAQ, and similar to the semicircle AFH. Since, then, the arcs QA, HI are similar — because AI touches the circle ABCDE, and so on — the whole arcs bDA, AFI will also be similar.) Wherefore all three arcs bDA, LMP, AFI will be similar.

I have set out so many cases, and such varied ones, of this proposition — although the manner of demonstration is very nearly the same in all — so that you may understand how to proceed in the other cases.

For the rest, we shall show in another and somewhat easier way that the arc of any circle enclosed between two lines (of which one touches the circle and the other cuts it) is similar to the arc of any other circle described through the point of contact and enclosed between the same two lines — where either both cut their circle, or one touches and the other cuts. For since AP touches the circle ABCDE, and AQ cuts the same, and each of them cuts two further circles [?—short right-column phrase not legible on the scan], the angle AbQ, in the alternate segment cut off by the secant line AQ [Margin: Euclid III.32], will be equal to the angle PAQ. Therefore, by the scholium of prop. 22, book 3, the arc AQ — enclosed between the two lines AP, AQ, and upon which the angle AbQ stands — is similar to the arcs PO, IH, intercepted between the same lines, upon which stands the common angle IAH, which has been shown equal to the angle AbQ.

Again, since AE touches the circle ALMNOP and [cuts the same? — right-column span not legible on the scan], and cuts the circle AFGHIK in E, D, and K, d, we shall show that the arcs ALD, ED, KAd are similar. [Since] the angle EAD is equal to the angle APD in the alternate segment, [the arcs] upon which they stand will be similar, by the scholium of prop. 22, book 3 [?—citation number partly on the unread right-column span]. And that the arc KAd is likewise similar to these will become clear thus. Let the line AL touch the circle [AFGHIK] and cut the circle ABCDE in b. Then, with the line dF joined [Margin: Euclid III.32; Euclid I.15], the angle bAD will be equal to the angle AFd in the alternate segment. Since, then, the angle hAK is equal to the angle AFK in the alternate segment, and the angle bAE, being equal to it at the vertex, [is likewise equal to the angle AFK]; and since the angle bAB has been shown equal to the angle AFD, the whole angle EAD will be equal to the whole angle dFK. And therefore, by the scholium of prop. 22, book 3, Euclid, the arcs ED, KAd will be similar. Wherefore, since ED has been shown similar to the arc aLD, all three — ALD, ED, KAd — will be similar, enclosed between the lines AE, AD.

Moreover, since Ab touches the circle AFGHIK, and Ad cuts the same, and each of them cuts two other circles [Margin: Euclid III.32], the angle bAD will be equal, in the alternate segment, to the angle AId. Therefore, by the scholium of prop. 22, book 3, Euclid, the arc Ad — between the two lines Ab, Ad, and upon which the angle AId stands — is similar to the arcs bD, LD, between the same lines, upon which stands the common angle bAD, shown equal to the angle AId.

Furthermore, since AK touches the circle ALMNOP and cuts the others in K, E; likewise Ai touches the circle ABCDE and cuts the others in P, I [Margin: Euclid III.32], the angle ADP in the alternate segment will be equal to the angle KAP, and

hence also to the vertical angle iAE . But this too is equal [Margin: Euclid III.32] to the angle ACE in the other segment. Therefore the three angles ACE , ADP , KAI are equal; and so, by the scholium of prop. 22, book 3, Euclid, the three arcs AE , AP , KI , upon which they stand, are equal, enclosed between the lines AK , Ai .

Finally, since AP touches the circle $ABCDE$ and cuts the others in P , I ; likewise AE touches the circle $ALMNOP$ and cuts the others in E , K ; then, the line kE having been joined [Margin: Euclid III.32], the angle PAE in the alternate segment will be equal to the angle AkE , and likewise the same angle EAP equal to the angle AlP (the line lP being joined) in the alternate segment. Next [Margin: Euclid III.22], since — the lines km , mI having been joined — the two angles KmI , KAI are equal to two right angles, as are the two angles AkE , ACE , and since the angle ACE is equal, in the alternate segment [Margin: Euclid III.32], to the angle iAE (that is, to kAI at the vertex), the remaining angle KmI will also be equal to the remaining angle AkE . Therefore all three angles AkE , AlP , KmI are equal; and so, by the scholium of prop. 22, book 3, Euclid, the three arcs ACE , ADP , KAI will be similar. And so with the rest.

This first part of the lemma differs from the first part of the preceding lemma in that here only those arcs are demonstrated to be similar which lie between two straight lines — whether both are tangent, or only one, or neither — and not those which some straight line cuts off; for the arcs AQ , APO , AKH , which the line AH cuts away, are not similar. But in the earlier part of the preceding lemma, arcs cut off by any straight line whatever are also shown to be similar.

Now, from A , the point of section, let any circle STV be described, up to which straight lines going out from A are extended, cutting it in S , T , V , X , Y , Z , a . I say that the arc — ST , for example — is half of that arc which, in the same circle, is similar to the arc Eb : so that the number of degrees contained in the arc ST is half the number of degrees contained in the arc Eb . For let an arc Tg equal to the arc ST be taken, the line gA be drawn, and from S and g to any point X on the circumference $STVXYZ$ let two straight lines SX , gX be drawn. Since, then, the arcs ST , Tg [are equal] [Margin: Euclid III.27], the angles SAT , TAg at the centre A will also be equal; and therefore the angle SAg will be double the angle SAT . But the same angle SAg at the centre A [Margin: Euclid III.20] is also double the angle SXg at the circumference. Therefore the angles SAT , SXg will be equal; and so, by the scholium of prop. 22, book 3, Euclid, the arcs Eb , Sg will be similar. And therefore the arc ST will be half of the arc Sg , which is similar to the arc Eb . The same reasoning holds for the rest, as is clear also in the arcs Va , DMP , DCA , dFI : of

these, the first, Va, is a quadrant containing 90 degrees, because of the right angle VAa, while the latter three are semicircles, each containing 180 degrees.

LEMMA XI.

To extend a very short straight line in a continuous direction, or (what is the same) to produce a straight line as far as one pleases through two points little distant from one another.

Diagram. A rectangular framed woodcut showing the construction for extending the line AB. Along a horizontal line run the collinear points A (far left), B (near A), then E, H, and L (far right), each generated as the intersection of arcs. Pairs of arcs struck from successive points cross above and below the line: from A (through B) the arcs meet at C (above) and D (below); further intersection pairs give F (above) and G (below), and the crossings at E, H, L, and K. The figure illustrates that A, B, E, H, L all lie on one straight line.

- A (left, on the base line): vertex from which the rays radiate
- B (on the base line, right of A): second given point
- C (upper left, arc crossing): intersection of arcs above the line
- D (lower left, arc crossing): intersection of arcs below the line
- E (centre-left, on the base line): next collinear point
- F (upper centre, arc crossing)
- G (lower centre, arc crossing)
- H (centre-right, on the base line): next collinear point
- I (upper right, arc crossing) [?]
- K (lower right, arc crossing)
- L (far right, on the base line): furthest collinear point

It often happens that either a very short straight line, such as AB, must be extended, or (what is the same) that a straight line must be produced as far as one pleases through two points one of which lies close to the other — as with the two points A, B. This is no small difficulty, because the ruler by which the line is to be drawn can easily be deflected one way or the other, so that the further the line has to be produced, the greater the error that may creep in. Therefore, to avoid going astray in drawing that line, we must use the following device. From A, through B, let an arc of a circle be described; on it let equal arcs BC, BD be cut off (the larger these are, the more successfully the thing will turn out), and from C, D let two arcs be described with an interval large enough that they conveniently intersect at E — that is, so that the section is not made too obliquely, since then

the point of intersection could not easily be discerned. Next, from A, through E, let another arc be described; on it let two equal arcs EF, EG be cut off, and from F, G let two arcs likewise be described with an interval large enough that they conveniently meet at H. Again, from A, through H, let an arc be described; on it let two equal arcs HI, HK be cut off, and from I, K let two arcs likewise be described with an interval large enough that they conveniently intersect at L. And in this manner one may proceed as far as one pleases. I say that the line AB, when produced, passes through the points E, H, L, and so on — so that, the ruler being laid against the points A and L, a straight line is drawn through the points A, B most exactly, since the joined lines AB, AE, AH, AL all make up one single straight line.

For when the lines AC, AD, AF, AG, AI, AK, CE, DE, FH, GH, IL, KL have been drawn: since the sides AC, AE are equal to the sides AD, AE, and the base CE is also equal to the base DE by construction (because equal intervals were taken from C, D up to E), the angle CAE will be equal to the angle DAE [Margin: Euclid I.8] — that is, the line AE will bisect the angle CAD. But the line BA also bisects the same angle CAD [Margin: Euclid III.27], since the angles BAC, BAD are equal on account of the equal arcs BC, BD. Therefore the line EA passes through B, lest two distinct lines be said to bisect the same angle CAD. Again, since the sides AF, AH are equal to the sides AG, AH, and the base FH to the base GH, for the same reason [Margin: Euclid I.8] the angles FAH, GAH will also be equal — that is, the line HA will bisect the angle FAG. Since, then, the line EA also bisects the same angle [Margin: Euclid III.27] (because the angles EAF, EAG are equal on account of the equal arcs EF, EG), the line HA will pass through E, and consequently also through B, since the line EA has been shown to pass through B. In no other way we shall demonstrate that the line LA passes through H, and therefore also through E, B, and so on.

This procedure can also be set up in the following way. From the given points A, B — or the endpoints of the given line AB — at any interval a little greater than the given line AB, let two arcs be described on either side, cutting each other in C, D. And from C, D let two other arcs be described at such an interval that they conveniently intersect at E. Again, from B, E let two other arcs be described on either side, cutting each other in F, G. And from F, G let two other arcs intersect at H. Likewise, from E, H let two other arcs, on either side, intersect each other in I, K. And from I, K let two other arcs cut each other at L. And in this manner let one proceed as far as one pleases. I say that all the points A, B, E, H, L lie in one straight line. For, from what we said in the practice of prop. 10, book 1, Euclid,

the line AB divides the joined line CD at right angles, and bisects it at M — (which, however, will be demonstrated thus: with the lines...

AC, AD, BC, BD drawn: since the two sides CA, AB are equal to the two sides DA, AB, and the base CB to the base DB, the angle CAB will be equal to the angle DAB [Margin: Euclid I.8]. Therefore, since the two sides CA, AM are equal to the two sides DA, AM, and they contain equal angles, as was just shown, the bases CM, DM will also be equal [Margin: Euclid I.4], and the angles at M — and hence right angles.) Likewise, the joined line EM is perpendicular to the same CD, and therefore coincides with the line BM; that is, it passes through the point B, so that AE is one straight line. Again, in the same way HN will pass through E, so that AH is one straight line, which...

Diagram. A rectangular framed woodcut for the perpendicular-bisector construction. Along a horizontal line lie the collinear points A (far left), M, B, N, E, O, H, and L (far right). At M a pair of arcs crosses above at C and below at D; at N a pair crosses above at F and below at G; further arc crossings mark E, O, H, L, and a point K below. The perpendicular bisectors of the successive chords (CD at M, FG at N, and so on) all fall on the one straight line AB produced.

- A (far left, on the base line)
- M (on the base line): foot of the perpendicular bisector of CD
- B (on the base line)
- N (on the base line): foot of the perpendicular bisector of FG
- E (on the base line, arc crossing)
- O (on the base line, arc crossing)
- H (right, on the base line, arc crossing)
- L (far right, on the base line, arc crossing)
- C (above M): upper arc intersection
- D (below M): lower arc intersection
- F (above N): upper arc intersection
- G (below N): lower arc intersection
- K (below, toward the right): lower arc intersection

just as the line BE cuts the line FG in half and at right angles, so the line HN is perpendicular to the same FG. In no other way will you show that LO passes through H, and hence that ABNEOHL is one straight line — because the line EH cuts the line IK in half and at right angles, and the joined line LO is perpendicular to the same IK.

Alternatively. Through the end-point A let a line ACK be drawn in any direction, making with AB an angle neither very large nor very acute. Then, through the other end-point B, let another line BD be drawn in any direction, cutting AK in C — yet so that AB and AK are not cut too obliquely, but in such a way that the points of intersection C, B can conveniently be discerned. On AC itself let there be cut off, by means of the compasses, any number of equal segments CE, EF, FG, GH; and from C and from the last point H, with equal intervals CI, HK, let arcs ID, KL be described. Taking the arc KL equal to the arc ID intercepted between the lines CI, CD, let the line HL be drawn; and along it, up to O, let as many equal parts be taken, equal to CB, as there are parts equal to AC in AH.

Diagram. A rectangular framed woodcut for the alternative ("Aliter") construction using an oblique transversal. Along a horizontal base line lie A (far left), B, Q, and, toward the right, N, P, O. An oblique line ACK rises from A through C, E, I, and on up through G, H to K at the upper right. A second line BD runs downward from B through Q to D at the bottom. Circular arcs (ID, KL) are struck, and the line HL crosses the base line near O. The figure shows the line AO passing through B (and P), proving the extension of AB.

- A (left, on the base line): origin of the oblique line ACK
- B (on the base line): given second point
- Q (on the base line, right of B): intersection of BD with the base line [?]
- C (lower left, on the oblique line)
- E (on the oblique line, above C)
- I (on the oblique line, right of E)
- F (upper, near I)
- G (upper, on the oblique line)
- H (upper right, on the oblique line)
- K (top right, end of the oblique line)
- M (upper area, between H and K) [?]
- L (middle-right, on the line HL)
- N (on the base line, right)
- P (on the base line, right of N)
- O (far right, on the base line)
- D (bottom): lower end of the line BD

equal to AC, they are in AH. For the line AB, when produced, will fall on O — or, the line AO will pass through B. For since [Margin: Euclid III.27] the arcs ID, KL are equal, the angles ICD, KHL (interior and exterior) will also be equal [Margin:

Euclid I.28], and therefore CB, HO will be parallel. Since, then, as AC is to AH, so is CB to HO — because AC is contained in AH as many times as CB in HO, by construction — the line AO will pass through B, and the line AB through O, by the scholium of prop. 4, book 6, Euclid. And if from G another arc MN be described, at the same interval CI or HK, equal to the same arc ID, then, by the same argument, the drawn line GN will be parallel to CD itself. If, therefore, on GN there again be taken as many parts, up to P, equal to CB, as there are parts equal to AC in AG, the same line AO will pass through the point P as well — because the number of parts of AG is equal to the number of parts of GP, and the number of parts of AH is equal to the number of parts of HO, and so on. And in this way more points will be found through which the produced line AB will pass, provided of course that from the other parts of AH lines parallel to CB are drawn, and so on.

You can also, if you please, before drawing the line CD through B, take on AK any number of equal parts you like — AC, CE, and so on — and through C draw a line that cuts the drawn line AB at some point. Thus, if the given points were A, Q, the line CD would be drawn through C, cutting AQ in B. For if the rest be done as before, we shall accomplish what was proposed in the same way. And by this latter method there is no need to take the part AC with the compasses (which, if it be not taken exactly, necessarily makes its multiple AH or AG either too large or too small — an error that is avoided if, before the line CD is drawn, one takes, as was said, any number of equal parts AC, CE, and so on); but it is enough if CB be taken with the compasses and carried over onto the lines HL, GN as many times as AC is found in AH, AG.

I should like to accomplish this same thing by yet a third and very easy method — indeed, if one wishes, with a single opening of the compasses. Let there again be given two points A, B, or a line AB to be produced. From B, through A, let an arc AC be described, from which, at the same interval AB, three equal arcs AD, DE, EC are cut off. Again, from C, at the same interval, let there be descri-

(Continuation of Lemma XI, from the previous page.)

An arc BF is then drawn, which will pass through B, the center of the previous arc, since its radius is assumed equal to the radius of that one. Then, with three equal arcs BE, EG, GF cut off at the same interval (and point E will fall on the point where the arcs AC and BF intersect, because the radii are equal), an arc CH is likewise drawn from F at the same interval; for the same reason it will pass through C, the center of the preceding arc. With three equal arcs CG, GI, IH taken at the same interval (and by the same reasoning point G will fall on the intersection of the arcs BF and CH), let there again be drawn, through F and from

H at the same interval, the arc FK, in which again three equal arcs FI, IL, LK are to be taken at the same interval. Let the same construction be continued in this way as far as one pleases, or as the work requires. I say that the straight line AB, when extended, passes through all the points found — C, F, H, K. For since, by the corollary of Proposition 15 of Book 4 of Euclid, the arcs AD, DE, EC are three sixths of the circle, ADEC will be a semicircle, and therefore the diameter AC will pass through the center B. By the same reasoning BF will pass through C, and CH through F, and FK through H, and so on.

Diagram. A rectangular frame enclosing the construction of a straight line by a chain of equal semicircular arcs. A horizontal baseline (the line AB being constructed and extended) runs across the figure; along it a series of equal arcs is struck alternately above and below, meeting the line at successive points. The upper arc-apexes are labelled, left to right, D, E, G, I, L; the lower arc-intersections M, N, P, R. On the baseline, from the left: A (left end), B, C, and a further point (possibly Γ or F [?]); toward the right K, O, Q, S. The crossed compass-marks at each arc show the equal-interval strikings described in the text.

- Upper apexes: D, E, G, I, L
- Baseline points (left to right): A, B, C, [?], K, O, Q, S
- Lower intersections: M, N, P, R

[Several labels in this crowded arc-chain are hard to resolve at this resolution.]

When the given line AB is very short — so that the procedure does not turn out longer than is fitting — once point C has been found and the straight line AB extended as far as C, if from C an arc AH is drawn at the interval of the line CA, and in it three equal arcs AM, MN, NH are taken at the same interval CA, point H will be found. From H, if an arc is drawn through C at the same interval, point O will be found in the same way; and if from this an arc is drawn at the same interval OH, point Q will be found by the same reasoning; and so on. Indeed, once point H has been found, if from it an arc is drawn at the interval HA, you will likewise find point Q; and from the point O once found, if an arc AS is drawn through A, you will find point S. In short, you can vary the procedure for drawing the arcs in infinitely many ways, and so forth.

LEMMA XII.

Given two straight lines, to find a third proportional; and given three, a fourth proportional.

Here we shall merely reduce Propositions 11 and 12 of Book 6 of Euclid to an easier procedure. For this task any rectangle whatever, ABCD, is best suited; in it we shall carry out what is proposed with no trouble at all. Let a third proportional, then, be sought for two straight lines E, F. On opposite sides of the rectangle cut off BG, AH equal to the first line E; and, having joined the straight line GH, cut off GI equal to the second line F; then connect the straight line BI and extend it further if need be. Next, cut off likewise BK, AL equal to the second line F (or to GI), and join KL, cutting BI at M. I say that KM is the third proportional to the first line E and the second line F — that is, to BG and GI. [Margin: a. Euclid I.33. b. Euclid I.30. c. Euclid VI.4.] For since GH and KL are parallel to AB, and therefore also to each other, it will follow that as BG is to GI, so BK is to KM. Since, then, BG is equal to E, and GI (and BK) to F, it will also be that as E is to F, so F is to KM; so that if N is taken equal to KM, three lines E, F, N are had in continued proportion.

Again, let a fourth proportional be sought for three given straight lines BG, GI, BO. Let the first and the third be placed on the side BC, beginning from B, and on the opposite side let AH, AP be cut off equal to them. Then, having joined the straight lines GH and OP, and having cut off from the end of the first the segment GI equal to the second, draw the straight line BI, and let it, when produced, cut OP at Q. I say that OQ is the fourth proportional sought. [Margin: d. Euclid VI.4.] For, as before, BG the first is to GI the second as BO the third is to OQ the fourth. Thus for three straight lines BO, OQ, BG the fourth proportional GI will be found.

But so that all this may be done as precisely as possible, these cautions must be carefully observed. First, when a third proportional is to be found for two straight lines: if indeed the first is equal to, or greater than, the second — as were the two lines E, F, to which BG, GI were cut off equal — nothing in the rule given need be changed, because in that case the straight line BI cuts the lines GI, KM at no very oblique angle; and hence the point of intersection M can be discerned conveniently, which would be otherwise if GI were cut more obliquely.

But if the first is smaller than the second — say the two given lines are BG, GS — then, since the straight line BS drawn in this case cuts GS very obliquely and must be produced a long way to meet TV (where BT is taken equal to the second line GS), we shall bisect the second line GS at R, and again bisect GR, and so on in turn, until we arrive at a part that is either equal to the first line BG or smaller — as here GI, a quarter of the second, is. And although the straight line BI, once drawn, cuts GI at no very oblique angle, yet because it must be produced far to intersect TV, we shall do better if on the side BC we take several parts equal to the first line

BG, until we find a straight line BO that is a multiple of BG and is either equal to, or greater than, the line BT (in the example BO is three times the first line BG); and on the parallel OP we take OQ to be the same multiple of GI as BO is of BG. For when the straight line BQ is drawn — which will certainly pass through I, by the scholium to Proposition 4 of Book 6 of Euclid, since as BG is to BO, so is GI to OQ — it will, by construction, cut the parallel TV at X; and TX (the fourth proportional to BG, GI, BT) will be a quarter of the third proportional sought — namely the same part that GI is of the second line GS — so that TX taken four times makes up the whole third proportional. [Margin: a. Euclid VI.4.] For since as BG the first is to GI, so is BT the second to TX, it will also follow, by the scholium to Proposition 22 of Book 5 of Euclid, that as BG the first is to four times GI — that is, to GS the second — so is BT the second to four times TX; and therefore four times TX will be the third proportional sought.

Diagram. A large rectangle ABCD (A top-left, D top-right, C bottom-right, B bottom-left), used for the construction of third and fourth proportionals. Along the top edge, from A: L, H, V, P, D. Along the bottom edge, from B: K, G, T, O, C. A diagonal is drawn from B upward through the interior, meeting M and I on the way and passing through X near the top; another line (BQ) runs to Q. Vertical dotted lines drop from L and H; the point R lies lower on one of them. At the right of the figure the given segments are drawn separately as horizontal bars with terminal ticks, labelled S, E, F, N.

- Rectangle corners: A (top-left), D (top-right), C (bottom-right), B (bottom-left)
- Top edge (left to right): A, L, H, V, P, D
- Bottom edge (left to right): B, K, G, T, O, C
- Interior points: X, I, M, Q, R
- Given-segment bars (right side): S, E, F, N

But if the first or the second given line is longer than the rectangle — which cannot be extended, whether because of the narrowness of the space, or because one does not wish to extend it — their halves must be taken, and again the halves of these halves, and so on, until parts are obtained shorter than the rectangle. For once a third proportional to these parts has been found, if it is multiplied as many times as those parts are contained in the whole lines, the third proportional sought will be produced — since parts have the same ratio as their equimultiples. [Margin: b. Euclid V.15.]

Next, when a fourth proportional is to be found for three straight lines: if the first is the greatest of all, the rule given above must be kept exactly, as was shown with the lines BO, OQ, BG, for which the fourth proportional found was GI.

But if the first is not the greatest, yet is greater than the second — say the three given lines are BG, GI, BT — the first, BG, must be multiplied along the line BC until BO is obtained, greater than or equal to the third line BT; and on the drawn parallel OP the second, GI, must be multiplied up to Q as many times as the first BG was multiplied up to O — as in the given example BO, OQ are three times BG, GI. For when the straight line BQ is drawn (which, by the scholium to Proposition 4 of Book 6 of Euclid, will pass through I), cutting the parallel TV at X, TX will be the fourth proportional to the three lines BG, GI, BT. [Margin: c. Euclid VI.4.]

But if the first is not the greatest, yet is smaller than the second while greater than the third — say the three given lines are BG, GS, BK — a half, or a quarter, or an eighth, and so on, of the second line GS must be taken, until a part turns up (such as the quarter-part GI) that is smaller than the first line BG. For when the straight line BI is drawn, cutting the parallel KL at M, KM will be a quarter of the fourth proportional sought — namely the same part that GI is of the second line GS. [Margin: d. Euclid VI.4.] For since as BG the first is to GI, so is BK the third to KM, it will also follow, by the scholium to Proposition 22 of Book 5 of Euclid, that as BG the first is to four times GI — that is, to the second line GS — so is BK the third to four times KM; and therefore four times KM will be the fourth proportional sought.

Likewise, if the first is not the greatest but is smaller than both the second and the third — say the three given lines are BG, GR, BT — a half, or a quarter, and so on, of the second line GR must be taken, one that is smaller than the first line BG, such as GI, the half of the second line GR. This done, the first, BG, and the part GI taken from the second, are to be multiplied equally along BC and along the parallel OP, until BO is found greater than or equal to the third line BT — as in the given example BO, OQ are three times BG, GI. For when the straight line BQ is drawn (which will certainly pass through I, by the scholium to Proposition 4 of Book 6 of Euclid), cutting the parallel TV at X, TX will be that part of the fourth proportional to be found which GI is of the second line GR — in the given example, the half. [Margin: e. Euclid VI.4.] For since as BG the first is to GI, so is BT the third to TX, it will also follow, by the scholium to Proposition 22 of Book 5 of Euclid, that as BG the first is to twice GI — that is, to the second line GR — so is BT the third to twice TX; and therefore twice TX will be the fourth proportional to the three given lines BG, GR, BT.

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But if the first and the third are longer than the rectangle, both must be bisected, or divided into four equal parts, and so on, the second being left untouched. For then the part of the first will be to the second as the same part of the third is to the fourth found. If, however, only the first is too long, the first and second must both alike be divided, the third being left untouched; because then the first will be to the second — that is, a part of the first to the same part of the second — as the third is to the fourth found. If, finally, only the third is longer, it alone must be divided; for then the first will be to the second as a part of the third is to the same part of the fourth found. Therefore, if the part of the fourth found is taken as many times as the part taken from the third is contained in the third, the whole fourth proportional sought will be built up.

SCHOLIUM.

But we shall accomplish this whole lemma by another method, one that is in fact most convenient in the astrolabe and in most other matters, especially when it is proposed to find a third proportional to two straight lines. Let a third proportional be sought for two straight lines a , b . On any straight line CD , take the first, a , equal to CE ; and, having drawn through E the perpendicular FG to CD , take EF , EG equal to the second, b . Then through the three points F , C , G let a circle be described with center H , cutting CD at D . I say that ED is the third proportional to the two lines CE , EF — that is, to the two lines a , b . For since, by the scholium to Proposition 13 of Book 6 of Euclid, EF is the mean proportional between CE and ED , it will follow that as CE is to EF , so EF is to ED . Therefore, having taken d equal to ED , it will also be that as a is to b , so b is to d ; and hence d is the third proportional to a , b , which is what was proposed. The center H will be found if, from C and F , at the same interval, four arcs are described on either side cutting one another at I , K ; and from C and G , four others cutting one another at L , M . For the straight lines IK , LM will cut one another at H , the center, as we demonstrated in the scholium to Proposition 25 of Book 3 of Euclid; and the center H will lie on the straight line CD , by the corollary of Proposition 1 of Book 3 of Euclid.

It will also be found if, having drawn the straight lines CF , CG , the angles CFH , CGH are made equal to the angles FCE , GCE ; for the straight lines FH , GH will then cut CD at the center H , since the three lines HF , HC , HG are equal. For HF and HG are equal, because the two sides EF , EH equal the two sides EG , EH and contain right angles at E ; and each of HF , HG is equal to HC , on account of the equal angles at C , F (or C , G). [Margin: a . Euclid I.4. b . Euclid I.6.]

Diagram. Two circle-figures set side by side, for the two constructions of the scholium (the third proportional to a , b , and the fourth proportional to a , b , d). In the left circle, F is at the top, C at the left end and D at the right end of the horizontal diameter through the center; E lies at the center, with H , M , and K clustered just to its right, and G at the bottom, L at the lower left; dotted chords join F to C and F to D , and cross-marks at I , K , L , M mark the arcs used to locate the center. The right circle similarly shows C at the left, D at the right, G at the center, with E , H , and the marks K , M nearby. Below the circles the given and found segments are drawn as horizontal bars with terminal ticks: at the left, a (short), b (medium), d (long); at the right, a , b and d , e (the found fourth proportional).

- Left circle: F (top), C (left), E (center), H , M , K (near center), D (right), G (bottom), L (lower left)
- Right circle: C (left), K , G (center), M , E , D (right), H (bottom)
- Segment bars: a , b , d (left); a , b , d , e (right)

[The central labels H , M , K in each circle are crowded and hard to separate at this resolution.]

The straight lines IK , LM will cut one another at H , the center, as we demonstrated in the scholium to Proposition 25 of Book 3 of Euclid.

Again, let a fourth proportional be sought for three straight lines a , b , d . On any straight line CD cut off CE , ED equal to the second, b , and the third, d ; through E draw a straight line FH in any way — whether perpendicular to CD or not — and take the first, a , equal to EF . Then through the three points F , C , D let a circle be described with center G , cutting EH at H . I say that EH is the fourth proportional to a , b , d — that is, to EF , EC , ED — so that e , taken equal to EH , is the fourth proportional sought. [Margin: c . Euclid III.35. d . Euclid VI.16.] For since the rectangle contained by EF , the first, and EH , the fourth, equals the rectangle contained by EC , the second, and ED , the third, it will follow that as EF , the first, is to EC , the second, so ED , the third, is to EH , the fourth — which is what was proposed. The center G will here too be found if, from F and D , at the same interval, four arcs are described on either side cutting one another at I , K , and from C and F four others cutting one another at L , M ; for the straight lines IK , LM will bisect one another at the center G , as we demonstrated in the said scholium to Proposition 25 of Book 3.

By yet another method, if one prefers, we shall dispatch the whole lemma as follows. Let a third proportional be sought for two straight lines A , B , and let A ,

the first, be the greater. Having taken a straight line EF equal to A, describe about it, from its midpoint G, the circle EKF; in it fit the straight line EK equal to B, and cut off EH equal to the same; about EH, from its midpoint I, describe the circle ELH, cutting EK at L. I say that EL is the third proportional. For since the joined straight lines FK, HL are parallel — by Lemma 9, because the circles touch one another at E, by the scholium to Proposition 13 of Book 3 of Euclid — the triangles EFK, EHL will be equiangular. [Margin: e. Euclid VI.4.] Therefore it will follow that as EF (that is, A) is to EK (that is, B), so EH (or B) is to EL.

Diagram. Two circles, a larger EKF and a smaller ELH, sharing the vertical diameter and tangent to each other at E (bottom). F is at the top, H just below it (the top of the smaller circle), G the center of the larger circle, I the center of the smaller, and E at the bottom where both circles meet. A horizontal radius GK runs to K on the right; L lies on the smaller circle at the lower right, where EK cuts it. Dotted chords join F to K and H to L. At the right of the figure four vertical segment-bars with terminal ticks are drawn, labelled A, B, C, D. [Margin: f. Euclid I.6. g. Euclid VI.4.]

- F (top), H (upper, top of inner circle), G (center of outer circle), I (center of inner circle), E (bottom, point of tangency)
- K (right, on horizontal radius), L (lower right, on inner circle)
- Segment bars: A, B, C, D

Next, let a third proportional be sought for two straight lines D, C, and let D, the first, be the smaller. Having taken a straight line EH equal to the greater, the second line C, describe about it, from its midpoint I, the circle ELH; in it fit the straight line EL equal to the first line D, and from EL produced cut off EK equal to EH (or to the second line C); and let the angle EKG be made equal to the angle KEH, so that the straight lines GE, GK are equal. Then, having described from G a circle through E, K, cutting EH produced at F, I say that EF is the third proportional. For, as before, as EL (or the first line D) is to EH (or to the second line C), so is EK (or the second line C) to EF.

Again, for three straight lines A, B, C, of which the first is greater than the second and the third, let a fourth proportional be sought. About a straight line EF equal to the first line A let the circle EKF be described; and about a straight line EH equal to the second line B let the circle EHL be described. In the first circle fit the straight line EK equal to the third line C, cutting the second circle at L. I say that EL is *(the fourth proportional — the demonstration continues on the following page)*.

[I say that] EL is the fourth proportional. ^a [Margin: ^a Euclid VI.4] For it will be, as before: as EF is to EK, so EH is to EL. Therefore, by alternation, as EF (or A), the first, is to EH (or B), the second, so EK (or C), the third, is to EL.

Likewise, given three lines C, D, A, of which the first is greater than the second but less than the third, a fourth proportional is to be found. About the line EH, equal to the first, C, let a circle ELH be described, in which let EL, equal to the second, D, be fitted. Then, from EH produced, with EF cut off equal to the third, A, let a circle EKF be described about it, cutting EL produced at K. I say that EK is the fourth proportional. ^b [Margin: ^b Euclid VI.4] For it will be, as before: as EH (or C), the first, is to EL (or D), the second, so EF (or the third, A) is to EK.

Moreover, given three lines B, A, D, of which the first is less than the second but greater than the third, the fourth proportional is to be found. About EH, equal to the first, B, let a circle ELH be described, in which let EL, equal to the third, D, be fitted. Then, taking on EH produced the line EF equal to the second, A, let a circle EKF be described about it, cutting EL produced at K. I say that EK is the fourth proportional. ^c [Margin: ^c Euclid VI.4] For it will be, as before: as EH is to EL, so EF is to EK. Therefore, by alternation, as EH — that is, as B, the first — is to EF (or A), the second, so EL (or D), the third, is to EK.

Finally, given three lines D, C, B, of which the first is less than the second and the third, the fourth proportional is to be found. About EH, equal to the second, C, let a circle ELH be described, in which let EL, equal to the first, D, be fitted; and from EL produced let EK be cut off equal to the third, B, and, with [the angle at K] made equal to the angle KEH, let [the triangle] EKG be formed, ^d [Margin: ^d Euclid I.8] so that the lines GE, GK are equal. Then, with a circle described from G through E, K cutting EH produced at F, I say that EF is the fourth proportional. ^c [Margin: ^c Euclid I.4] For it will be, as before: as EL (or the first, D) is to EH (or the second, C), so EK (or the third, B) is to EF.

LEMMA XIII.

Given two straight lines inclined to one another, to find the point at which they meet, even if neither of them is produced.

This lemma is of great use in the astrolabe, since not uncommonly two lines have to be produced a long way in order to obtain the point at which they come together — something that, because of the oblique intersection of the lines, can scarcely be made out without error. For this reason we shall use the following device. Let there be two straight lines AB, CD, which, when produced, truly meet

at the point *f* — a point that we shall nonetheless locate even without producing the lines *AB*, *CD*.

If the given lines are too short (as if, say, the given lines were *AG*, *CN*), let them be produced, by Lemma XI, as far as one pleases, up to *B*, *D*, and let two, or three, or even more parallels *AF*, *GI*, *KM* be drawn between them — for the more there are, the more reliably the point *f* is found. These parallels are drawn with no trouble if, from different centers *A*, *G*, *K* taken on the line *AB*, arcs are described at any one equal interval: *EF*, *HI*, *LM*. [...] at the points *F*, *I*, *M*. (We cut them off at the same interval at which they were described, as though equilateral triangles *AEF*, *GHI*, *KLM* had to be constructed — though this is not necessary.) ^f [Margin: ^f Euclid I.28] Then the lines *AF*, *GI*, *KM* drawn from the centers will be parallel, ^g [Margin: ^g Euclid III.27] because the angles *A*, *G*, *K* are equal, on account of the equal arcs *EF*, *HI*, *LM*; and they will cut the line *CD* at *N*, *O*, *P*. Again, through *A*, *G*, *K* let parallels be drawn making acute angles with *AB* — lines that are also easily drawn in the following way. With arcs *QR*, *ST*, *VX* described from *A*, *G*, *K* at any one equal interval (the greater it is, the better), let arcs, equal and not very large, be cut off at the points *R*, *T*, *X*. ^h [Margin: ^h Euclid I.28] For the lines *AR*, *GT*, *KX*, when drawn, will be parallel, ⁱ [Margin: ⁱ Euclid III.27] because the angles standing on the equal arcs *QR*, *ST*, *VX*, at the centers *A*, *G*, *K*, are equal. On these parallels *AR*, *GT*, *KX* let parts *AN*, *GO*, *KP* be taken, equal in number, as many as one pleases, up to *Y*, *Z*, *a*. For a straight line drawn through these points will cut both *AB* and *CD*, produced to the point of section *f*; and so, if either or both of them are produced, the point *f* will be obtained quite exactly, even though they intersect one another obliquely. And if through other points *b*, *d*, *e*, terminating other parts equal in number, a straight line is drawn, it too will pass through the same point *f*; and in this way the point of intersection *f* will be found still more exactly. Indeed, by this method the point *f* is obtained at which the given lines *AB*, *CD* ought to meet, even if they are not produced. By the same reasoning, if beyond *Y*, *Z*, a other parts equal to *AN*, *GO*, *KP* themselves are taken (but care must be taken that as many equal parts be taken as shall seem sufficient, so that the line drawn through their extremities may not cut both *AB* and *CD*, or either of them, too obliquely), a straight line drawn through the extreme points of those parts will give the same point *f*. In the figure two other straight lines *Am*, *Kn* have been drawn, parallel to one another, closer to *AB* itself, by equal arcs cut off *Qg*, *Vh*, and on each of them *AN*, *KP* have been taken five times, up to *m*, *n*. For thus the straight line *mn* will fall on the same point *f*.

Diagram. A large construction figure enclosed in a ruled rectangular frame, illustrating Lemma XIII (locating the meeting point *f* of two inclined lines *AB*, *CD*

without producing them to their intersection). A near-horizontal base line runs from A (far left) rightward through the points E, G, Q, K, H, L, V, B to the intersection point f (far right, where several lines converge). A second oblique line CD runs through the lower region (C, N, O, P, D). Several parallel transversals (AF, GI, KM and AR, GT, KX) fan across the figure; small equal arcs are marked near A, G, K. Labels read (position noted where legible):

- A (far left, on base line): center/origin point
- E, G (adjacent, on base line, with script g above)
- Q (just below base line, near G)
- K, H, L, V, B (successively rightward on base line)
- f (far right): the intersection point, with converging lines
- R, S, T, X (upper region): arc-endpoints of the parallels
- Y, Z, a (upper right): endpoints of the equal parts AN, GO, KP
- m [?], n (upper right): endpoints of the added parallels Am, Kn
- b [?], d [?], e [?], h [?], i [?], x [?] (small italic labels, upper-middle region — several hard to resolve even under magnification)
- C, N (lower left); O, P (lower middle); D (lower middle): points on line CD and its intersections
- I, M (bottom, each with a small arc-cross): arc intersection marks

Whichever of the straight lines be, Ya, mn falls upon the point f — where the lines AB, CD truly intersect one another — we shall demonstrate as follows. Since ^a [Margin: ^a Euclid VI.4] as Af is to AN, so Gf is to GO, then, by alternation, as Af is to Gf, so AN is to GO. ^b [Margin: ^b Euclid V.15] But as AN is to GO, so also is AY to Gz, because these are equimultiples of them. Therefore it will also be that as Af is to Gf, so AY is to Gz; and hence, by the scholium to Proposition 4 of Book 6 of Euclid, the line Yf will pass through Z; and so YZ, produced, will fall upon f. And the same reasoning holds for the others.

But should it happen that the given lines are so little distant from one another that the parallels between them are too small, and that on this account what is proposed cannot conveniently be carried out — as, for instance, are the two lines AG, pE — then the line Ap must be drawn at will and, once drawn, taken some number of times, say three times, up to N; and through N a parallel NO must be drawn to pE, and the point f found at which AG, NO produced meet. For if, whatever part Ap is of AN itself, you cut off such a part AE from Af, then AG, pE will meet at E; ^c [Margin: ^c Euclid VI.4] because the parallel pE, produced, must cut the sides AN, Af proportionally, and so on.

Diagram. A repetition, within a ruled frame, of the Lemma XIII construction figure (the same construction shown on p. 379), placed here for the reader's convenience alongside the continuation of the proof. The near-horizontal base line runs from A (left) through E, G, Q, K, H, L, V, B to f (right); the oblique line CD passes through the lower region (P, C, N, O and down to E [?] at bottom, and to I, M). The upper region again carries the parallels and their endpoints R, Y, Z, m [?], n, with small italic labels d, e, x, i, b nearby. Labels legibly read: A, E, G, Q, K, H, L, V, B, f (base line); R, Y, Z, m [?], n (upper); P, C, N, O (lower left); I, M (bottom); D (lower). Several minuscule italic labels remain at the resolution floor.

Another method. Draw the line AN at will from the extremity A, such that it does not cut CD itself too obliquely; then from any point E of the line AB draw a parallel to CD, cutting AN at p — which is easily done as follows. Draw E α cutting CD at α at will, and, at the interval E α , describe an arc from C; let this be cut at q by another arc described from E at the interval α C. Then the line Eq, cutting AN at p, will be parallel to CD; for the quadrilateral E α Cq, by the scholium to Proposition 34 of Book 1 of Euclid, is a parallelogram, because its opposite sides are equal. ^d [Margin: ^d Euclid VI.4] Since, therefore, as pA is to AE, so NA is to Af, then, if for the three lines pA, AE, NA a fourth proportional is found by the preceding lemma, and a line equal to it is cut off from AB, beginning at the point A, we shall arrive at the point f. Or thus: ^e [Margin: ^e Euclid VI.2] since as Ap is to pN, so AE is to Ef, then, if for the three lines Ap, pN, AE a fourth proportional Ef is found, it, transferred onto the line AB, beginning at the point E, will give the same point f.

Diagram. A second construction figure for the alternate method, showing the parallels AF, GI, KM extended, with the base line running through H, K, L, B to f (upper right) and the oblique line CD through N, C, O, I, M, P, D. At the lower right the three lines are extended to the points α , β , γ (Greek letters, set rotated in the woodcut). Labels legibly read: H, K, L, B, f (upper); D (right); N, C (left); O, I, M, P (middle-lower); α [?], β , γ [?] (lower right, rotated). A short segment near B/f is dotted (line CD approaching f).

Another method. With the parallels AF, GI, KM drawn (the more acute the angles they make with AB, the better, which will be the case if smaller equal arcs are cut off from the arcs EF, HI, LM), let there be taken beyond N, O, P as many straight lines as one pleases equal to AN, GO, KP themselves — say three, up to α , β , γ . For the straight line $\alpha\beta\gamma$ will fall upon the point f, by the scholium to Proposition 4 of Book 6 of Euclid, ^f [Margin: ^f Euclid V.15] because A α , G β , K γ stand to one another as AN, GO, KP — that is, as Af, Gf, Kf — and so on.

LEMMA XIV.

To construct an instrument by which, through three given points — even if they are set almost along a straight line — an arc of a circle can be drawn, without the aid of a compass.

In constructing an astrolabe it sometimes happens that an arc of a circle has to be drawn through three points set almost in a straight line — something that can scarcely, or only with difficulty, be done with a compass, because the center of that circle lies too far from the given points. (For when the center can conveniently be obtained, we taught, in the scholium to Proposition 25 of Book 3 and in the scholium to Proposition 5 of Book 4 of Euclid, by what method it is to be found.) For this reason, in this place we shall teach the construction of a certain instrument by which we may either draw that arc itself, or at least find, among the three given points, any number of other points through which that arc must pass. A similar instrument was indeed constructed with great ingenuity by Guidobaldo del Monte [Guidubaldus è Marchionibus Montis] in his theory of universal planispheres; but we ourselves had once devised something somewhat simpler, which I judge should be described here. Let there be, then, two rulers ABCD, AEFG, of the same width and thickness, and of such a length as is roughly the distance between the two extreme points through which the arc is to be drawn. Let them be joined by a small pivot in such a way that the sides AB, AE, produced, pass through the center, and that the rulers themselves can move about the same center as about a hinge — so that, namely, they can be spread apart more or less, or drawn together, according as the angle BAE is to be more or less obtuse. For this purpose certain small portions must be cut away near the center A, so that the angles DAB, GAE become acute. For if the angles near A were right angles, the sides AB, AE would form one straight line, and the rulers themselves could not be drawn together so as to contain the obtuse angle BAE. It is not, however, necessary that they be able to be drawn together to make an acute angle; for when the straight lines connecting the two nearest points make an acute angle, the arc of the circle through those points is drawn more easily by the scholium to Proposition 25 of Book 3, or by the scholium to Proposition 5 of Book 4 of Euclid, than by means of this instrument. At the center A let a certain very small and sharp stylus project downward, for tracing the arcs. Next, at some point H of the ruler AEFG, let a certain small ruler HI be fixed, in such a way that it can be turned about H. Finally, at a point of the other ruler AC — a point which, when the sides AB, AE are set in a straight line, is distant from the point H by as much as the length of the ruler HI — let some small solid brass rectangle KL be

fixed, so that about that said point the rectangle too can be turned, and the ruler HI can be inserted into it, and by some little screw N can be so tightened that the two rulers AC, AF remain immovable — that is, do not change the angle BAE.

Diagram. A pictorial woodcut of the instrument. Two flat rulers cross at a pivot A (upper right, drawn as a small disk with a downward stylus); one ruler bears the letters near its far ends, the other likewise. A dotted curved arc sweeps across the upper part of the figure through the points P and O (the arc being traced). A short cross-ruler and a small brass fitting with a screw are shown near the middle.

Labels legibly read:

- A (top, at the pivot with the projecting stylus)
- D, G (adjacent, "DG", just below the pivot on the rulers)
- B (left, at a hatched triangular block / end-fitting)
- C (lower left, end of one ruler)
- E (right, at a hatched triangular block / end-fitting)
- F (lower right, end of the other ruler)
- H (on the small ruler HI, middle)
- I (left of center, on the small ruler)
- K, L (the small brass rectangle KL, middle-left)
- N (the little screw / cochlea, middle-left)
- O, P (upper left, on the dotted traced arc)

The one about to draw an arc through three given points B, A, E with this instrument should insert the ruler HI into the rectangle KL, and set the stylus projecting from the center A at the middle point A, and so spread or draw together the sides AB, AE of the rulers that they pass exactly through the two remaining points B, E. Once these are so set, let the little screw N clamp the ruler HI, so that the rulers AC, AF cannot change the angle BAE. For if the instrument, thus prepared, is carried around so that the sides AB, AE always pass through the points B, E (which will happen if at the very points B, E the angles of two solid brass triangles are fixed), the stylus projecting from the center A will describe the arc BAE; or at any rate, if the instrument changes its position repeatedly, yet in such a way that the sides pass through the points B, E, the same stylus will impress, between A and B and between A and E, various points which, fittingly and suitably joined, will make the arc BAE. Now that, throughout this motion of the instrument, the stylus projecting from A describes an arc of a circle is clear from this: that in that arc the angle BAE remains perpetually the same — which is indeed a property of the segment of any circle, ^a [Margin: ^a Euclid III.21] as Euclid

demonstrated. For if, for example, with the instrument having such a position that the stylus is placed at O, and the sides being OB, OE, someone should claim that the arc of a circle drawn through the three points B, A, E (that an arc can be drawn through any three points ^b [Margin: ^b Euclid IV.5] has been demonstrated by Euclid, provided they do not lie in a straight line, but the lines joining them form a triangle) does not pass through the point O, then it will necessarily cut the line EO, either produced beyond O or short of O. Let it cut it beyond O, at P, and let the straight line BP be joined. ^c [Margin: ^c Euclid III.21] Then the angle BPE will be equal to the angle BAE, since both are in the same segment of the circle described through the points B, P, A, E. Since, therefore, the angle BOE is also equal to the angle BAE — indeed altogether the same, since only the position has changed — the angles BOE and BPE, the exterior and the interior, will be equal to one another, which is absurd; ^d [Margin: ^d Euclid I.16] since the exterior is greater than the interior. Therefore the arc does not cut EO produced; and by the same reasoning it will not cut it short of O either. Hence the arc drawn through the three points B, A, E will pass through O; and for the same reason it will pass through all the other points that are found by means of the instrument.

LEMMA XV.

A curved line to which a straight line is subtended, such that the squares of all the perpendiculars dropped from the points of the curved line to the subtended straight line are equal to the rectangles contained by the segments of that same subtense made by the perpendiculars — that is, such that all the perpendiculars are mean proportionals between the segments of the subtense made by them — is a semicircle whose diameter is that subtended straight line. In other words, a semicircle described about that subtended straight line will coincide with the given curved line, or (what comes to the same thing) will pass through the outer endpoints of all the perpendiculars.

Let ABC be any curved line, to which the straight line AC is subtended; and from any of its points B, F, H let the perpendiculars BD, FE, HG be dropped to it. Suppose that the square on DB equals the rectangle under AD, DC; that the square on EF equals the rectangle under AE, EC; that the square on GH equals the rectangle under AG, GC; and so for all the rest, however many perpendiculars are drawn. That is, let the square on each perpendicular equal the rectangle under the segments of AC made by that perpendicular — or (what is the same) let all the perpendiculars be mean proportionals between the segments of AC made by them [Margin: VI.17], since in this way their squares will be equal to the rectangles under the segments. I say that ABC is a semicircle, and that AC is its

diameter; that is, that a semicircle described about the diameter AC from its midpoint I passes through all the outer endpoints of the perpendiculars, so that it does not differ from the curved line ABC.

For draw the straight lines IB, IF, IH from the midpoint I to the outer endpoints of all the perpendiculars. Then, since the rectangle under AD, DC together with the square on DI equals the square on AI [Margin: II.5], and the square on DB is posited equal to that rectangle, the two squares on DI, DB together will also equal the square on AI. But the square on IB is equal to those same squares [Margin: I.47]. Therefore the squares on IA, IB are equal, and hence the lines IA, IB are equal as well. By the same reasoning it will be shown that IF, IH, and all the other straight lines drawn from the midpoint I to the endpoints of all the perpendiculars, are equal to that same AI, and consequently equal to one another. Therefore, since all the straight lines falling from I onto the curved line ABC are equal, ABC will be a semicircle, and its diameter AC — that is, the semicircle on diameter AC — will pass through all the outer endpoints of the perpendiculars, and will not differ from the given curved line.

Diagram. A semicircle standing on the horizontal diameter AC. Along the diameter, from left to right, lie the points A, D, E, I, G, C, where I is the midpoint (center). Perpendiculars rise from the diameter to the arc: from D to B, from E to F, from G to H. The arc (shown dotted) carries the points B (lower left), F (top), H (upper right); K is marked on the arc above B, at the point where a larger arc would cut the perpendicular (used in the alternative proof below). Straight lines are drawn from the center I to B, F, and H.

- A (left end of diameter): endpoint of the subtense
- D (on diameter): foot of perpendicular DB
- E (on diameter): foot of perpendicular EF
- I (midpoint of diameter): center of the semicircle
- G (on diameter): foot of perpendicular HG
- C (right end of diameter): endpoint of the subtense
- B (arc, lower left): top of perpendicular from D
- F (arc, top): top of perpendicular from E
- H (arc, upper right): top of perpendicular from G
- K (arc, upper left): point where the alternative (larger) semicircle would meet the perpendicular

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Otherwise. Suppose the semicircle described about AC from its midpoint I be said not to pass, for example, through B, but to cut the perpendicular DB either below B or above it, as at K. Then it will follow, by the scholium to proposition 13 of book 6 of Euclid, that DK is the mean proportional between AD, DC [Margin: VI.17], and therefore that the square on DK equals the rectangle under AD, DC. But the square on DB is posited equal to that same rectangle. Therefore the squares on DK, DB are equal, and hence the lines DK, DB themselves are equal — the whole and the part, which is absurd. Therefore the semicircle of diameter AC passes through the point B, and by the same reasoning it will pass through the points F, H and the others belonging to the other perpendiculars.

LEMMA XVI.

If a cone is cut by a plane parallel to the base of the cone, the section made on the conic surface is the circumference of a circle, having its center on the axis of the cone.

That all the circles of the sphere which are not drawn through the south pole of the world can be projected onto the astrolabe in circular form is demonstrated from two propositions of Book I of Apollonius of Perga [Apollonius], namely propositions 4 and 5, as we shall say in its proper place. But because not everyone is practiced in Apollonius's demonstrations, I am glad to insert both of those propositions here, especially since their demonstrations are very clear, so that the studious reader need not be forced to consult Apollonius himself — a most obscure author — on account of only two propositions, and easy ones at that. For propositions 1 and 3 of the same first book, which are assumed in demonstrating those two, are gathered with no trouble from the very description of the cone which we set out at definition 20 of book 11 of Euclid [Euclid], drawing on Apollonius. Namely: *(That the straight lines drawn from the vertex of the cone to points lying on the conic surface lie in the conic surface itself.)* Likewise: *(That if a cone is cut by a plane through the vertex, the section is a triangle.)* For since a straight line drawn from the vertex to the circumference of the base of the cone, if it runs around the circumference of that base while the vertex of the cone remains fixed, describes by the definition a conic surface, so as to touch all its points, it is clear that all the straight lines drawn from the vertex to any points on the surface lie on the surface itself — since they at times become parts of that straight line which is carried around the circumference of the base in the description of the conic surface.

And from this the other proposition follows. For when a plane drawn through the vertex of the cone [Margin: XI.3] cuts the base of the cone along a straight line, if from the endpoints of this straight line two straight lines are drawn to the vertex, these will lie on the conic surface, as we said, and will therefore be the common sections of the plane drawn through the vertex and of the conic surface. Hence they will form, together with that straight line in the base, a triangle — which of course is produced by the cutting plane. And if the cutting plane is drawn through the axis of the cone, that triangle so formed is called the triangle through the axis [triangulum per axem, the axial triangle]. These things being established, the proposed lemma will easily be demonstrated.

Let there be a cone, whether right or scalene, whose vertex is A, whose base is the circle BCD, and whose axis is AE, falling on E, the center of the base. Let the cone be cut by a plane parallel to the base, producing on the conic surface the line FGH — whether this happens above the base or below it, the cone being produced. I say that the line FGH is the circumference of a circle whose center is the point I, where it is divided by the cutting plane on the axis. For draw through the axis AE the plane producing the axial triangle ABD, and let the cutting plane cut it along the straight line FH; take in the line FGH any point G whatever, through which draw from the vertex A the straight line AG, which, since it lies on the conic surface, will meet the base at C. Again, through the straight lines AI, AG draw a plane [Margin: XI.3], producing on the base BCD and on the plane FGH the straight common sections EC and IG. Now since the parallel planes BCD, FGH are cut both by the plane of the triangle ABD and by the plane of the triangle AEC [Margin: XI.16], the common sections BD, FH and EC, IG will be parallel [Margin: VI.4]. Therefore, as AE is to EB, so is AI to IF; and, alternately, as AE is to AI, so EB to IF. By the same reasoning, as AE is to AI, so ED to IH, and EC to IG. And consequently the three lines IF, IH, IG are proportional to the three EB, ED, EC [Margin: V.14]; that is, as EB is to IF, so ED to IH, and EC to IG; and, alternately, as EB is to ED, so IF to IH, and as ED to EC, so IH to IG. Since therefore the three lines EB, ED, EC from the center E are equal, the three IF, IG, IH will also be equal; and by the same reasoning all the straight lines drawn from I to the line FGH will be shown equal to those same IF, IH. Therefore the figure FGH is a circle, whose center I lies on the axis AE of the cone.

Diagram. A cone with vertex A at the top, drawn inside a rectangular frame. From A, straight lines descend to the base circle and beyond. Two horizontal circular sections are shown edge-on as lens shapes: the upper (the base) is BCD, with B at the left, E at the center, D at the right, and C at the bottom of the lens; the lower (the parallel section, the cone being produced) is FGH, with F at the left,

I at the center, H at the right, and G at the bottom of the lens. The axis runs A–E–I. Each lens shows its two crossing chords (the diameters BD and the common section, FH and its common section), meeting the axis at the centers E and I respectively.

- A (apex): vertex of the cone
- B (base lens, left): point on base circle
- E (base lens, center): center of the base, on the axis
- D (base lens, right): point on base circle
- C (base lens, bottom): point on base circle
- F (lower lens, left): point on the parallel section
- I (lower lens, center): center of the section, on the axis
- H (lower lens, right): point on the parallel section
- G (lower lens, bottom): point on the parallel section

LEMMA XVII.

If a scalene cone is cut by a plane through the axis that is perpendicular to the base, and is also cut by another plane perpendicular to the axial triangle produced by the first plane — a plane which cuts off from the axial triangle a triangle indeed similar to the axial triangle, but placed subcontrariwise — then the section is a circle, whose diameter is the common section of the axial triangle and of the plane that produced that section on the conic surface. Let a section of this kind be called subcontrary.

Let there be a scalene cone whose vertex is A and whose base is the circle BCD; and let it be cut by a plane through the axis perpendicular to the base (which will come about if from the vertex A a perpendicular AM is dropped to the plane of the base [Margin: XI.11]; when drawn, it will be perpendicular to the base [Margin: XI.18]), producing the axial triangle ABD. Let the same cone also be cut by another plane perpendicular to the axial triangle, producing on the conic surface the line EFG, and cutting off from the axial triangle a triangle AEG similar to it and placed subcontrariwise — whether this happens above the base or below — that is, such that the angle AEG equals the angle ADB, and the angle AGE equals the angle ABD. I say that the line EFG is a circle, and that its diameter is EG, namely the common section of the axial triangle and of the plane producing the section EFG.

For if from any points C, F on the circumference BCD and on the line EFG, taken with respect to the axial triangle ABD, the perpendiculars CH, FI are dropped

[Margin: XI.11], these will fall on the straight lines BD, EG [Margin: XI.38], which are the common sections of the axial triangle and of the planes BCD, EFG that are perpendicular to that same triangle [Margin: XI.6], and they will be parallel to one another. Now draw through I the straight line KL parallel to BD [Margin: XI.15]; then, since the two straight lines FI, KL meet at I the two straight lines CH, BD (which meet at H) and are parallel to them, the plane through FI, KL will also be parallel to the plane through CH, BD, that is, to the base of the cone. And hence, by the preceding lemma, it will produce on the conic surface a circle KFL, which will pass through the point F — since the point F lies on the conic surface — and the diameter of that circle will be the straight line KL. And since FI is perpendicular to the plane, and to the straight line KL, being drawn by definition 3 of book 11 of Euclid [Margin: VI.17], it is therefore the mean proportional between the segments KI, IL, by the scholium to proposition 13 [of book 6]; and hence the square on FI will equal the rectangle under KI, IL. Now since the angle EKI equals the angle ABD [Margin: I.29], and the angle LGI is posited equal to that same angle ABD, the angles EKI, LGI will be equal to one another; and the vertical angles at I are equal [Margin: I.15]. Therefore the triangles EKI, LGI are equiangular; and accordingly [Margin: VI.4], as KI, the first, is to IE, the second, so GI, the third, is to IL, the fourth [Margin: VI.16]; and the rectangle under the first and fourth, KI, IL, will equal the rectangle under the second and third, IE, GI. But it has been shown that the square on FI equals the rectangle under KI, IL. Therefore the square on FI also equals the rectangle under IE, GI. In like manner we shall demonstrate that the squares of all the perpendiculars dropped from the points of the line EFG onto EG are equal to the rectangles under the segments of the straight line EG made by the perpendiculars. Therefore, by Lemma 15, EFG will be a semicircle whose diameter is EG; and by the same reasoning the other part of the section, ENG, will be shown to be a semicircle. Therefore the whole section EFGN is a circle whose diameter is EG — which was proposed.

It is clear, moreover, that the section EFGN is a circle even if its diameter cuts the diameter of the base. For example, suppose the base of the cone is taken to be the circle KFL, and the section to be EFG. The demonstration will be entirely the same, except that when the point taken on the line EFG lies at the common section of the circumference KFL and the line EFG — such as F — no other plane parallel to the base is to be drawn to make a circle. And then, since each of the planes KFL, EFG is perpendicular to the triangle AKL, if from F [Margin: XI.11], where the base circumference cuts the line EFG, a perpendicular is dropped to that triangle, it will fall on each of the common sections KL, EG [Margin: XI.38],

and thus onto the point I, where those common sections cut one another. And, as before, the square on FI will equal the rectangle under EI, IG, and so on.

And if on the line EFG any point O whatever is taken, other than the common point of section F, a perpendicular OP must be dropped, and through P a line QR drawn parallel to KL, the base of the axial triangle; and finally, through OP, QR — which are parallel to FI, KL — a plane must be drawn [Margin: XI.15], which will make it parallel to the base of the cone KFL, and will therefore produce a circle, as before, and so on.

Diagram. A large, crowded woodcut of a scalene cone with its subcontrary section, drawn in perspective. The vertex A stands at the upper right, with the axis and slant lines descending to the lower left. Two oblique circular sections are drawn as overlapping ellipses/lenses with their inscribed chords and diameters, together with the auxiliary circle KFL and several dotted construction lines (including the horizontal dotted lines Q...R and E...R marking the levels of the sections, and the dashed axis to M). The labelled points read (upper configuration and lower configuration): A (apex, upper right); E, P, N, R across the upper level; K, O, I, L in the upper ellipse with F and G below it; B, H, C, D at the middle level with M marked to the right on a dotted line; and, in the lower ellipse, Q, P, N, R, K, O, I, L with F and G below. Because the figure is dense and several labels sit on faint or overlapping dotted lines, a number of them are not fully resolved.

- A (upper right): vertex of the cone
- B, C, D: points of the base circle / axial triangle base
- E, F, G: points of the subcontrary section EFG (diameter EG)
- N: point on the section (the arc ENG)
- H, I: feet of the perpendiculars CH, FI on the axial triangle
- K, L: endpoints of the auxiliary chord KL (circle KFL); I its center
- M: foot of the perpendicular AM from the vertex to the base plane
- O, P: the arbitrary point O on EFG and the foot P of its perpendicular OP [?]
- Q, R: endpoints of QR drawn through P parallel to KL [?]

SCHOLIVM.

It is worth observing that the diameter of the subcontrary section can be equal to the diameter of the base of the cone, and can also be unequal to it: equal, namely, when the one side of the axial triangle perpendicular to the base is equal to the one side of the subcontrarily placed triangle that is opposite an equal angle; but unequal when those sides are unequal — and the triangle whose side is the

greater will have the greater diameter. Yet these diameters can never divide one another in half.

[Margin: When the diameter of the subcontrary section is equal to the diameter of the base of the cone, and when it is unequal.]

For let there be, in a scalene cone, the axial triangle ABC perpendicular to the base, and let the side AB be greater than the side AC [Margin: I.18], and therefore also the angle ACB greater than the angle ABC. Let the triangle ADE be similar to ABC but placed subcontrariwise, and let the side AD be posited equal to the side AC — these sides being opposite the equal angles AED, ABC. I say that the diameters BC, DE are equal. For since in the triangle ACB the two angles A, ACB are equal to the two angles A, ADE in the triangle ADE, which are adjacent to the equal sides AC, AD [Margin: I.26], both the sides AB, AE and the diameters BC, DE will be equal — which was proposed. By the same reasoning, if the sides AB, AE are posited equal, we shall show that both AC, AD and BC, DE are equal.

Diagram. Within a rectangular frame, two similar but subcontrarily placed triangles are superimposed and share the vertex A at the upper right. The triangle ABC has B at the left and C at the right; the subcontrary triangle ADE has D near the center-top and E toward the bottom. Their bases (the diameters BC and DE) cross near the middle, and further construction lines produce the additional triangles AGH and AIK. Labelled points: A (upper right, common vertex), B (left), C (right), D (upper center), I (left of D), M (below B), F (small, center, where BC meets DE), G (lower left), L (center-lower), H (lower right), N (below M, center), E (lower center), K (bottom).

- A (upper right): common vertex of the axial and subcontrary triangles
- B (left), C (right): base points of triangle ABC (diameter BC)
- D (upper center), E (lower center): points of the subcontrary triangle ADE (diameter DE)
- F (center): intersection of BC and DE
- G (lower left), H (lower right): points of triangle AGH
- I, K: points of triangle AIK
- L, M, N: further construction points

Again, let ADE be similar to the axial triangle AGH and placed subcontrariwise, and let the side AG be greater than the side AE, or AH greater than AD. I say that the diameter GH is greater than the diameter DE. For if a straight line AB is taken equal to AE, or AC equal to AD, and BC (or CB) is drawn parallel to GH, the diameters BC, DE will be equal, as was demonstrated. And since, as AG is to GH,

so AB is to BC [Margin: VI.4], and AG is greater than AB, GH will also be greater than BC [Margin: V.14] — that is, greater than DE, which was shown equal to BC. In the same way, if the triangle AIK is similar to the axial triangle ABC and placed subcontrariwise, and the side AI is greater than the side AC, or AK greater than AB, we shall show that the diameter IK is greater than the diameter BC. For if a straight line AD is taken equal to AC, or AE, and DE (or ED) is drawn parallel to IK, the diameters BC, DE will be equal, as was shown [Margin: VI.4]; and since, as AI is to IK, so AD is to DE, and AI is greater than AD [Margin: V.14], IK will also be greater than DE — that is, greater than BC.

[Margin: The diameter of the subcontrary section and the diameter of the base of the cone never bisect one another.]

I say further that the diameters BC, DE, whether they are equal or unequal, never cut one another in half; but either both are cut unequally, or, if one of them is bisected, the other is not bisected. For let them cut one another at F, and let the diameters BC, DE first be equal. And since both AB, AE and AD, AC are equal — otherwise BC, DE would not be equal — as we have demonstrated, the remainders BD, CE will also be equal. Now if neither BC nor DE is bisected, it is evident that they do not bisect one another. But if one of them, say BC, is said to be bisected, the other, DE, will be cut not in half. For since the triangles BDF, ECF are equiangular [Margin: I.15] — because the vertical angles at F are equal, and the angles B, E are posited equal on account of the subcontrary section, and hence the remaining angles D, C are equal as well — it will follow [Margin: VI.4] that as DB is to BF, so CE is to EF. Since therefore BD has been shown equal to EC, BF will also be equal to EF [Margin: V.14]; and accordingly the remainder CF will be equal to the remainder DF. But BF is greater than DF [Margin: I.19], because the angle BDF is greater than the angle DBF — since BCE, equal to BDF [Margin: I.16], is greater than the angle ABC, being exterior to the interior. Therefore EF, being equal to BF, will be greater than DF. Hence DE is not bisected at F.

ASTROLABIUM — Liber Primus (continued)

[Margin: When the diameter of the subcontrary section is equal to the diameter of the base of the cone, neither is bisected. When the diameter of the subcontrary section is unequal to the diameter of the base of the cone, and one of them is bisected, the other is the greater.]

In the same way, if it is claimed that DE is bisected at F, we shall show that BC is not bisected at F. For CE will be to EF as DB is to BF [Eucl. VI.4]. Since, then, CE is

equal to DB, EF will likewise be equal to BF [Eucl. V.4 (?)], and consequently the remainder FD will be equal to the remainder FC. But EF is greater than FC, because the angle ECF is also greater than the angle CEF [Eucl. I.19] — since the angle BDE, equal to ECF, is greater than the angle AED, being exterior to it as interior [Eucl. I.16]. Therefore BF too, being equal to EF, will be greater than CF. So BC is not bisected at F.

Next, let the diameters GH and DE be unequal, with GH the greater. If, then, neither of them is bisected, it is plain that they do not bisect each other. But if one of them, say GH, is bisected at L, the other, DE, will not be bisected. For since GH is assumed greater than DE, AG will also be greater than AE, and AH greater than AD [Eucl. V.14], because as GH is to AG, so is DE to AE, and again as GH is to AH, so is DE to AD [Eucl. VI.4]. Since, then, from the greater AG the smaller AD is taken away, and from the smaller AE the greater AH, the remainder DG will be greater than the remainder HE. And since, as DG is to GL, so is HE to EL, and again as DG is to DL, so is HE to HL — while DG has been shown greater than HE — GL will also be greater than EL, and DL greater than LH [Eucl. V.14], that is, greater than GL, which is assumed equal to LH. Therefore, since DL is greater than GL, and GL greater than LE, as was shown, DL will be much greater than LE. So DE is not bisected at L. By like reasoning, if DE is claimed to be bisected at L, GH will be bisected at L, not in halves [i.e. not truly]. For we shall show, as before, that GL is greater than EL, and that DL is greater than LH — that is, greater than EL, which is assumed equal to DL — greater than LH. Therefore, since GL is greater than EL, and EL greater than LH, as was shown, GL will be much greater than LH. So GH is not bisected at L.

But we shall show far more easily and briefly that the diameters BC and DE, whether they be equal or unequal, cannot bisect each other mutually. For if they did bisect each other mutually, then, by drawing the straight lines DC and BE, a parallelogram DCEB would be formed, by the scholium to Prop. 34, Book 1 of Euclid — which is absurd, since the sides BD and EC meet at A.

Nor should this be passed over: that when the diameters are equal, as BC and DE are here posited, neither of them can be bisected at F. For since it has been shown that then BF equals EF, and DF equals CF, if either of the straight lines BC, DE is claimed to be bisected at F, all four parts BF, EF, CF, FD will be equal. Both, therefore, would be bisected — which we have demonstrated above to be impossible.

But this too we shall demonstrate without great effort: that when one of the diameters is bisected, it is the smaller, while the other is the greater. For let IK be

bisected at N. I say that GH is greater than IK. For if it is not greater, it will be either equal or smaller. First, let it be equal, if that be possible. Then, as we demonstrated just now, neither of the diameters is bisected — which is contrary to the hypothesis, since IK is posited as bisected at N. Next, let GH be smaller than IK, if that be possible. Since, then, as GH is to GA, so is IK to AK, and likewise as GH is to AH, so is IK to AI [Eucl. VI.4] — and GH is posited smaller than IK — AG will also be smaller than AK, and AH smaller than AI [Eucl. V.14]. Hence, since from the smaller AG the greater AI is taken away, and from the greater AK the smaller AH, the remainder GI will be smaller than the remainder HK. And since, as GI is to IN, so is HK to HN, and likewise as GI is to GN, so is HK to KN [Eucl. VI.4] — while GI has been shown smaller than HK — IN will also be smaller than HN, and GN smaller than KN [Eucl. V.14], that is, smaller than IN; and IN smaller than HN: so GN will be much smaller than NH. And since the angle GIN is greater than the angle AKI, that is, than the angle IGN [Eucl. I.16], GN will be greater than IN [Eucl. I.19]. Therefore NH, which was shown greater than GN, will be much greater than NK, which is assumed equal to IN; and so the whole GH will be greater than IK. But GH was posited by the adversary to be smaller than IK. GH is therefore both smaller and greater than IK — which is absurd. So GH is the greater. Here you see that the straight line GH, by the very fact that it is posited smaller than IK, is proved to be greater than IK — a manner of arguing that Euclid too employed in Proposition 12, Book 9, and Theodosius in Proposition 12, Book 1 [Theodosius, *Sphaerica* I.12].

Or, once it has been proved that the remainder GI is smaller than the remainder HK, we shall proceed thus. Since, as GI is to GN, so is HK to KN — and GI has been shown smaller than HK — GN will also be smaller than KN [Eucl. V.14], which is assumed equal [to IN]. Therefore the angle GIN will be smaller than the angle IGN [Eucl. I.18]. But the exterior angle GIN is greater than the interior and opposite angle AKI, that is, than the angle IGN [Eucl. I.16]. So the same angle GIN is both smaller and greater than the same angle IGN — which is absurd. Therefore GH is not smaller than IK; and it has been shown to be not equal either. Hence it is the greater, which was to be proved.

In the same way, if GH is claimed to be bisected at N, we shall demonstrate that IK is the greater. For if it is not greater, it will be either equal or smaller. First, let IK be equal to GH, if that be possible. Then, as we demonstrated a little before, neither of the diameters GH, IK is bisected — which is absurd; for GH is posited as bisected at N. Next, let IK be smaller than GH, if that be possible. Since, then, as IK is to AK, so is GH to AG, and likewise as IK is to AI, so is GH to AH [Eucl. VI.4] — and IK is posited smaller than GH — AK will also be smaller than AG, and AI

smaller than AH [Eucl. V.14]. Wherefore, since from the smaller AK the greater AH is subtracted, and from the greater AG the smaller AI, the remainder HK will be smaller than the remainder GI. And since, as HK is to HN, so is GI to IN [Eucl. VI.4] — and HK has been shown smaller than GI — HN, that is GN, will be smaller than IN [(?) reading of a right-cut line]; [and since the exterior angle GIN is greater than the interior and opposite] IGN, that is, than the angle HKN [Eucl. I.16] — the exterior greater than the interior opposite — which is absurd. Therefore IK is not smaller than GH; and it has been shown to be not equal either. Hence it is the greater, which was to be proved.

Or thus. Since HK has been shown smaller than GI, and as HK is to KN, so is IG to GN [Eucl. VI.4], KN will also be smaller than GN. Therefore, since KN is smaller than GN, that is, than HN, and HN is smaller than IN, as we showed a little before, KN will be much smaller than IN. And since the exterior angle KHN is greater than the interior and opposite angle AGH, that is, the angle HKN [Eucl. I.16], IN will be greater than HN [Eucl. I.19]. Since, then, IN is shown greater than NK, IN will be much greater than HN, that is, than GN. Therefore the whole IK is greater than the whole GH. But IK was posited by the adversary to be smaller than GH. The same IK is therefore both smaller and greater than GH — which cannot be. So IK is not smaller than GH; and it has been shown to be not equal either. Hence it is the greater. Here you see the same manner of arguing that Euclid used in Prop. 12, Book 9, and Theodosius in Book 1, Prop. 12.

Diagram. A square-framed geometric construction for the foregoing lemma on the diameters of a conic section. The apex **A** stands at the upper right; from it two straight lines descend, one to **G** at the lower left and one to **E** near the bottom. A near-horizontal chord runs from **B** (left) through **M** to **F** and **C** (right). The point **D** lies high, near A; **I** lies just left of D; **H** is at the right; **L** and **N** lie in the lower middle; **K** is at the very bottom, where the descending lines converge.

- A (top right): apex where the sides meet
- B (left): left end of the transverse
- C (right of centre): on the transverse
- D (upper middle, near A)
- E (bottom): foot of a descending line
- F (centre): on the transverse
- G (lower left): foot of a descending line
- H (right)
- I (upper left, near D)

- K (very bottom): convergence point
- L (centre, below F)
- M (centre, on the transverse near B)
- N (lower centre)

[Margin: When the diameter of the subcontrary section is unequal to the diameter of the base of the cone, and the smaller is bisected, the greater part of the greater [diameter] inclines toward the smaller angle of the triangle through the axis — the angle which that diameter makes with the side of the same triangle.]

And so, when the diameters are equal, neither is bisected; but when they are unequal, the smaller can be bisected, the greater never.

Finally, we shall demonstrate with little trouble that when the smaller diameter is bisected (which alone can be bisected, as has been shown), the greater part of the greater diameter always inclines toward that side where it makes the smaller angle with the side of the triangle through the axis. For let IK be bisected at N, and let GH accordingly be the greater. I say that the part GN is greater than the part NH. For GH will be to AG as IK is to AK [Eucl. VI.4]. Since, then, GH is greater than IK, AG will also be greater than AK [Eucl. V.14]. In the same way, AH will be greater than AI. Wherefore, since from the greater AG the smaller AI is taken away, and from the smaller AK the greater AH, the remainder GI will be greater than the remainder HK. Now, as GI is to IN, so is KH to HN; and likewise as GI is to GN, so is HK to KN [Eucl. VI.4]. Since, then, GI is greater than HK, IN will also be greater than HN, and GN greater than KN [Eucl. V.14], that is, than IN. Therefore, since GN is greater than IN, and IN greater than NH, GN will be much greater than NH.

Likewise, if GH is claimed to be bisected at N, then, as has been shown, IK will be the greater, and its part NK will be greater than IN — which will be demonstrated in the same way. For since, as IK is to AK, so is GH to AG, and likewise as IK is to AI, so is GH to AH [Eucl. VI.4] — since IK is greater than GH — AK will also be greater than AG, and AI greater than AH [Eucl. V.14]. Since, then, from the greater AK the smaller AH is subtracted, and from the smaller AG the greater AI, the remainder HK will be greater than the remainder GI. And since, as HK is to HN, so is GI to IN, and as HK is to KN, so is GI to GN [Eucl. VI.4] — while HK is greater than GI — HN will also be greater than IN, and KN greater than GN [Eucl. V.14], that is, than NH. Therefore, since KN is greater than NH, and NH greater than IN, KN will be much greater than IN. It is true, then, that the greater part of the

greater diameter always inclines toward the smaller angle it makes with the side of the triangle through the axis — angles of the kind at G and K.

LEMMA XVIII

The whole sine [radius] has the same ratio to the sine of the greatest declination of the ecliptic from the equator as the right sine of the arc of the ecliptic intercepted between any point of it and the nearest equinoctial point has to the right sine of the declination of that same point of the ecliptic from the equator.

Let there be, on the surface of the sphere, a segment AB of the equator and another AC of the ecliptic, cutting the former (the equator) at A, so that the angle A is the angle of the greatest declination of the ecliptic from the equator — the angle, that is, measured by the arc of the colure of the solstices, described from the pole A, intercepted between the first point of Cancer (or of Capricorn) and the equator. Through any point C of the ecliptic, let a great circle of declination be understood to descend from the pole of the world (that is, of the equator), cutting the equator at B; and the angle B will be a right angle, by Prop. 15, Book 1 of Theodosius; and therefore the arc CB will measure the declination of the point C from the equator. I say, then, that as the whole sine is to the sine of the angle A of greatest declination of the ecliptic, so is the sine of the arc AC of the ecliptic — intercepted between the assumed point C of the ecliptic and the nearest equinoctial point A — to the sine of the arc CB, which arc is the declination of the point C from the equator. For since, by Proposition 41 of our [book] on spherical triangles, as the sine of the arc AC is to the sine of the right angle B opposite it — that is, to the whole sine (for a quadrant is due to a right angle, as we said at Definition 6 of our spherical triangles, and consequently its sine will be the sine corresponding to a whole quadrant) — so is the sine of the arc CB to the sine of the angle A opposite it; therefore, by conversion, as the whole sine is to the sine of the arc AC, so is the sine of the angle A to the sine of the arc CB. And by alternation, as the whole sine is to the sine of the angle A of greatest declination, so is the sine of the arc AC of the ecliptic to the sine of the arc CB, the declination of the point C — which was to be proved.

Diagram. A square-framed figure showing a spherical triangle formed by three arcs. The arc **AB** (equator) runs from **A** at the lower left across to **B** at the lower right; the arc **AC** (ecliptic) rises from A up to **C** near the top; the arc **CB** (circle of declination) descends from C to B. The two rising/descending arcs cross within the square.

- A (lower left): intersection of equator and ecliptic (equinoctial point)

- B (lower right): foot of the declination circle on the equator
- C (upper): the assumed point of the ecliptic

LEMMA XIX

To describe an analemma for any given elevation of the pole.

The analemma is a certain circular figure, understood to be described about the centre of the world in the plane of the meridian — or of any other great circle drawn through the poles of the world — containing the common sections which the planes (especially those of the equator and its parallels, of the ecliptic, of the horizon, of the vertical, and of any parallel of each of these, etc.) make in the meridian, or in that other great circle. Its construction we handed down in Proposition 1, Book 1 of the *Gnomonics*; we gladly repeat it here on account of its notable usefulness in describing the circles of the sphere upon the astrolabe — especially because the description of the parallels of the equator we shall here demonstrate far more easily from the preceding lemma, in the very manner we intimated in the scholium to Prop. 1, Book 1 of the *Gnomonics*.

Let there be, then, in the plane of the meridian, a circle ABCD described about the centre of the world E, of which [circle] and of the horizon the common section is the straight line BD. Now, the elevation of the pole of that place, for which the analemma is being constructed, having been computed, let the arcs from the points D and B be carried off in opposite directions as far as F and G, and let the diameter FG be drawn, which will be the axis of the world, since the angle DEF is the angle of the elevation of the pole, which the axis makes with the horizon. Next let the diameter AC be drawn, perpendicular to the horizon BD, which will be the common section of the meridian and of the prime vertical. For since...

[Margin: Euclid XI.19 (the marginal reference here reads "arg. undec." — apparently "19. undec.") [?]]

the meridian and the vertical are perpendicular to the horizon, their common section will be perpendicular to the same [Eucl. XI.19]; and therefore, by Definition 3, Book 11 of Euclid, it will also be perpendicular to the horizontal line BD, at the centre E, through which all these great circles are drawn. Therefore AC is perpendicular to BD, being the common section of the meridian and the vertical; and A is the vertex of the head — that is, the upper pole of the horizon — while C is the lower pole of the same. Again, let the diameter HI be drawn perpendicular to the axis FG. This will be done if arcs DF, BG are taken equal to AH, CI; for then, by adding the common arcs FA, GC, the whole quadrants DA, BC will be equal to the whole arcs FH, GI, and so these arcs too will be quadrants,

and consequently the angles FEH, GEI will be right angles, by the scholium to Prop. 27, Book 3 of Euclid. But HI will be the common section of the meridian and the equator. For since the axis FG, passing through the poles F, G of the equator, is perpendicular to the equator, by Prop. 10, Book 1 of Theodosius, and passes through the centre E of the sphere, the same axis FG will, by Definition 3, Book 11 of Euclid, be perpendicular to the common section of the meridian and the equator at the centre E; and consequently HI, perpendicular to FG, will be the common section of the meridian and the equator. Now, if through D and B we draw DK, BL parallel to the equator HI, these will be the common sections of the meridian and the parallels which are the greatest of those that are always visible and always hidden; inasmuch as the meridian, cutting the equator and the said parallels, makes the common sections parallel [Eucl. XI.16], and the greatest parallel of those always visible touches the horizon at D, while the greatest of those always hidden touches the same horizon at B. And these lineaments of the analemma turn out one way and another for various elevations of the pole, according as the angle DEF of the elevation of the pole is varied.

Diagram. A large, densely lined square-framed analemma. Within the frame a great circle is drawn about the central point E (the centre of the world), which acts as a hub from which many straight lines radiate. The principal readable capital labels around the circle and its diameters are: **A** and **K** (upper right, with **i** beside them), **M** (top centre), **S** (upper left), **H** and **Q** (left), **N** (left), **B** (left end of the horizontal diameter BD, with **D** at the right end), **G** (lower left, an axis pole), **F** (right, the other axis pole, with **u** and **k** on the circle beside it), **O** (right interior), **L**, **C** and **P** (bottom), and **R**, **n**, **I** (lower interior). The circumference and the interior are crowded with small minuscule and Greek/zodiacal labels marking the graduated divisions of the equator, its parallels, and the ecliptic.

- E (centre): centre of the world; radiant hub
- A (top): upper pole of the horizon (vertex)
- C (bottom): lower pole of the horizon
- B (left) — D (right): ends of the horizontal diameter (horizon line BD)
- F (right) — G (lower left): ends of the axis of the world FG
- H (left) — I (lower): ends of the diameter HI (common section of meridian and equator)
- K, i (upper right, near A); u, k (right, near F): points on the circle
- M (top centre); N (left): endpoints of the chord MN of the declination circle
- O, P (lower right / bottom): endpoints of the chord OP
- S, T [?] : points of the circle MSNT

- Q, R (interior): midpoints of MN and OP respectively
- L (bottom): on the horizon-parallel line
- Numerous small labels (Greek minuscules θ , π , ε , τ , γ , ξ , δ , σ , λ ... and zodiac symbols Υ , $\underline{\alpha}$, ♋ , ♌ , ♍ ...) marking graduation points around the circumference [most too small to resolve individually — [?]]

To describe the parallels of the equator (that is, of the Sun) which are drawn through the beginnings of the signs and through the individual points of the ecliptic — taking account of the declination of each parallel from the equator — the whole labour and industry of constructing the analemma is here directed, on account of the declinations of these parallels, which can scarcely be computed without error from the equator HI on either side, because of the minutes and seconds that cling to the degrees of declination. (For these declinations, if they could be computed exactly on either side from the points H, I, there would be no difficulty in drawing the diameters of the parallels.) We shall therefore use a device thought up with great industry by the ancients, by which, from the greatest declination of the Sun (that is, of the ecliptic) once known, the diameters of all the parallels of the Sun passing through the points of the ecliptic, and their declinations, are found geometrically — and indeed very accurately. It is of this sort. [Margin: How the declinations of all the points of the ecliptic are found geometrically.]

From the points H, I of the equator, let the greatest declination of the Sun (or of the ecliptic) be numbered off on either side, by the teaching of Lemma 3, as far as M, N and O, P. We here set this greatest declination to contain 23 degrees 30 minutes. Then, the straight lines MN, OP having been joined — which are bisected by HI at Q, R, by the scholium to Prop. 27, Book 3 of Euclid, because the arcs HM, HN, IO, IP are equal — let a circle MSNT be described from Q about MN. This being divided into 12 equal parts, let there be drawn, by the teaching of Lemma 2, through pairs of points equally distant from the points T, S, the straight lines VX, YZ, ab, cd, which (by the scholium to Prop. 27, Book 3 of Euclid) will be parallel to one another and to HI, since they intercept equal arcs on the circle MSNT. These will be drawn more exactly if from R about OP a semicircle is described and divided into six equal parts. For thus, for each line, three points will be had: two indeed on the circle MSNT, and one on the semicircle described about OP. I say that these parallels are the diameters of the parallels of the Sun drawn through the beginnings of the signs — that is, the arcs HY, HV, etc., are the declinations of those degrees of the ecliptic which are as many degrees distant from the beginning of Υ [Aries] and $\underline{\alpha}$ [Libra] as there are degrees on the arcs of the circle MSNT intercepted between the diameter ST and the said parallels...

(continuation of Lemma XIX)

so that HY is the declination of Taurus (♉) and Virgo (♍); HV that of Gemini (♊) and Leo (♌); HM that of Cancer (♋); HA that of Scorpio (♏) and Pisces (♐); HC that of Sagittarius (♐) and Aquarius (♒); and HN that of Capricorn (♑). Consequently, the drawn diameters Vd, Yb, and so on, are diameters of the Ecliptic, the beginnings of the signs being placed on the meridian — just as MP and NO are diameters of the same Ecliptic when the beginnings of Cancer (♋) and Capricorn (♑) are set on the meridian. The demonstration of this is very easy.

For, by Lemma 5, as EM — the whole sine of circle ABCD — is to MQ — the whole sine of circle MSNT, that is, to the sine of the greatest declination — so is the sine of an arc of the same circle ABCD (which, for example, is similar to arc Sf) to eQ, the sine of arc Sf. And also, from the preceding lemma, as the whole sine EM is to MQ (the sine of the greatest declination), so is the sine of that same arc of the Ecliptic ABCD (which is similar to arc Sf — for this circle may be taken for the Ecliptic, since it is equal to the meridian) to the sine of the declination of that same arc of the Ecliptic which is similar to arc Sf. Therefore eQ will be the sine of the declination of that arc of the Ecliptic which is similar to arc Sf. And since eQ is the sine of the meridian arc HY, HY will be the arc of declination of the extreme point of that arc of the Ecliptic (counted from the equinox) which is similar to arc Sf. And so for the rest. For by exactly the same method we shall show that gQ is the sine of the declination of the extreme point of that arc of the Ecliptic, counted from the equinox, which is similar to arc Sh; and so on.

[Margin: How the declinations of all the points of the Ecliptic may be found by another method.]

But most conveniently we shall also find these same arcs of declination — that is, we shall draw the Sun's parallels — by this other method. Let circle ABCD be taken for the Ecliptic, and let it be divided into 12 equal signs at the points i, k, l, m, n, P, p, q, r, f, δ, M, so that l is the beginning of Aries (♈); k of Taurus (♉); i of Gemini (♊); M of Cancer (♋); δ of Leo (♌); f of Virgo (♍); r of Libra (♎); q of Scorpio (♏); p of Sagittarius (♐); P of Capricorn (♑); n of Aquarius (♒); and m of Pisces (♐). Then, when straight lines have been drawn through each pair of points equally distant from M or from P — which lines are parallel by the scholium to Proposition 27, Book 3 of Euclid —

Diagram. A large square-framed woodcut — the Analemma construction used to derive the declinations of all points of the Ecliptic and to draw the Sun's parallels. Within the square frame is a great circle (the meridian circle) carrying a

horizontal diameter B–E–D (B at the left, E the centre, D at the right) and further diameters and chords representing the Ecliptic (MP, NO and others), the equator, the tropics of Cancer (ϖ) and Capricorn (φ), the horizon (BD), and the graduated declination circle MSNT. Numerous oblique parallel lines (drawn dashed and solid) cross the interior, representing the parallels of declination and the projecting rays from centre E.

Clearly readable framework labels :

- A (top of circle, apex) — with K and i just to its right along the upper arc
- M (upper left region, on the circle)
- S (upper left, on the circle)
- H (left side, on the circle)
- Q (interior, left of centre)
- N (left, terminus of a diameter)
- B (left end of the horizontal diameter)
- E (centre of the circle)
- D (right end of the horizontal diameter)
- F (right side, marked with a star)
- O (lower right, on the circle)
- T (interior, right of centre) — with Y adjacent
- G (lower left, on the circle)
- P (lower region)
- L, C (bottom, in the group "P L C")
- R, I (lower-central interior)
- m (right side, between D and O)

The figure additionally carries many small auxiliary point-labels in italic and Greek letters (among them θ , ξ , π , ε , δ , γ , τ , η , λ , and lowercase u, k, b, d, o, n) set along the arcs and at the intersections of the parallels; at this resolution several of these small labels cannot be assigned to their exact positions with confidence. (more than two small labels not confidently resolved).

there will be cut off the diameter of the Ecliptic MP at the points t, u, α , β , through which lines drawn parallel to HI are drawn. (These are easily drawn if, in the segments of the parallels kf and i δ intercepted between the points u, t and the diameter HI, equal segments are taken on the other parallels; so that, for example, if the parallel Ks is set off from segment ue, then on the other parallels i δ , lr, mq, np equal segments are taken, the beginning always being made from

the straight line HI. For in this way we shall have several points, through which the parallels to HI must be drawn.) These will give the diameters of the Sun's parallels drawn through the beginnings of the signs, just as before. We shall demonstrate this readily in the following manner.

Since, as EM (the whole sine) is to MQ (the sine of the greatest declination), so is Eu — the sine of the arc lk of the Ecliptic (which begins at the beginning of Taurus (♉) and terminates at u) — to uy (the line uy being drawn parallel to MQ, or perpendicular to HI); and since also, from the preceding lemma, as EM (the whole sine) is to MQ (the sine of the greatest declination), so is Eu — the sine of the arc of the Ecliptic beginning at the beginning of Taurus (♉) and terminating at u — to the sine of the declination of the beginning of Taurus (♉); therefore uy will be the sine of the declination of the beginning of Taurus (♉). And consequently the arc HY, whose sine is uy, will measure the declination of the beginning of Taurus (♉); and so on. The same reasoning holds for the rest.

These declinations, moreover, once found, are the same for all elevations of the pole, and are never changed unless the greatest declination of the Sun is first found to have changed. For by the very same method by which the greatest declination HM was found, the other declinations HY, HV, and so on, of the points of the Ecliptic were found.

[Margin: How the declination of any point of the Ecliptic may be found geometrically.]

It is clear from all this by what method the declination of any given point of the Ecliptic is to be found. For if the given point lies between Aries (♈) and Cancer (♋), we shall count its distance from Aries, on the circle MSNT, from point S toward M; but if it lies between Libra (♎) and Cancer (♋), we shall count its distance from Libra, from point T toward M. If, however, it lies between Aries (♈) and Capricorn (♐), we count from S toward N; and if finally it lies between Libra (♎) and Capricorn (♐), we count from T toward N, reckoning its distance from the nearest equinoctial point — namely from Libra (♎). For a line drawn parallel to HI from the end of the numbering will be the diameter of the parallel of that given point,

and it will cut the arc MN at the required declination. For example, if the 10th degree of Taurus (♉) be given — which is 40 degrees distant from Aries (♈) toward Cancer (♋) — we shall count 40 degrees from point S toward M, up to θ ; and through θ we shall draw $\theta\pi$ parallel to HI, as the diameter of the parallel-to-

the-equator that passes through the 10th degree of Taurus (γ); and its declination will be $H\pi$.

We shall find this same result by another method as follows. When the given point lies between Aries (γ) and Cancer (ϑ), we shall reckon its distance from Aries from point l toward M; if between Libra (ϱ) and Cancer (ϑ), we shall count from point r toward M the distance it has from Libra; if between Aries (γ) and Capricorn (φ), from point l toward P; and if finally between Libra (ϱ) and Capricorn (φ), from point r toward P, its distance from the nearest equinoctial point, namely from Libra (ϱ). For if from the end of the numbering we draw a line parallel to lr, the diameter of the Ecliptic MP will be cut at a point, through which a line drawn parallel to HI will be the diameter of the parallel passing through the given point on the Ecliptic; and so on. For example, if the same 10th degree of Taurus (γ) be given, we shall count 40 degrees (for the given point is that far distant from Aries toward Cancer) from point l toward M, up to μ ; and through μ we shall draw $\mu\xi$ parallel to lr (which is easily done if we cut off $r\xi$ equal to the arc $l\mu$). This line will cut MP itself at ρ . For a line drawn parallel to HI through ρ will be the diameter of the required parallel; and so on, just as before.

It should also be understood that the segment of the diameter of the horizon BD which lies between MO and NP — the diameters of the parallels of Cancer (ϑ) and Capricorn (φ) — is divided by the intermediate parallels in the same way as the straight line MN, or OP, is divided by them. For the segment of the semidiameter ED, between E and the parallel MO, is cut as the straight line EM is cut [Margin: (a) Euclid VI.2], because parallel lines divide the sides of a triangle proportionally. Since, then, for the same reason, EM is cut as MQ is divided, the said segment will be divided as the straight line MQ is divided. In no other way will the segment of the diameter EB, between E and the parallel NP, be divided than as the straight line NQ is divided; because it is cut as EN is cut, and EN as NQ. Therefore the whole segment of the diameter of the horizon BD, between the parallels MO and NP, will be cut as the straight line MN is divided by the parallels. Which was what was proposed.

Now, by what method the diameters of other circles — whether great circles or not — or their common sections with the meridian are described in the Analemma; and how the Analemma is sometimes constructed, for certain circles, upon another great circle not drawn through the poles of the world — these things we shall teach in their proper places, as we proceed in the Astrolabe, when use shall require it.

LEMMA XX.

If two planes intersect one another, and in one of them, at two points of the common section, two straight lines make with it any two equal interior angles whatever; and in the other, at the same two points, two other straight lines likewise make with the same common section any two equal interior angles whatever: then these two latter straight lines will make, with the two former, two equal angles.

Let the two planes AB and AC intersect one another along the straight line AD, and at any two points E, F of the common section let there be set up, in the plane AB, two equal interior angles GEF, HFE, of whatever kind — that is, whether acute, right, or obtuse; and at the same points, in the plane AC, let there be set up two other equal interior angles IEF, KFE, of whatever kind. I say that the angles GEI, HFK are equal.

In the first figure all the angles are acute; in the second, obtuse; in the third, the two former obtuse and the two latter acute; in the fourth, finally, the two former right and the two latter acute. In all these cases, however, and in others, the demonstration will always be the same.

Diagram. A composite woodcut showing four cases of the lemma, drawn as four stacked "folded-plane" figures. In each, two planes AB and AC meet along the common section AD, drawn as the long horizontal middle line running from A (at the far left) to the right, where the plane corners are labelled C (upper right) and B (lower right), with D on the middle line at the right. On the middle line lie the two points E and F, with L marked midway between them. In the upper plane (AB) a triangle rises to the apexes I, N, K (with base IK passing through I, N, K); in the lower plane (AC) a triangle descends to the apexes G, M, H (with base GH passing through G, M, H). The transversals EI, FK and EG, FH connect the points E, F to the triangle apexes, and the connecting lines GH and IK are drawn. The four figures repeat this configuration for the four angle-cases described in the text (all-acute; all-obtuse; mixed; right-and-acute). All principal labels (A, B, C, D, E, F, G, H, I, K, L, M, N) are legible; no labels flagged.

For let the straight lines EG, FH be made equal to one another, and likewise EI, FK; and let GH and IK be joined, which will be parallel to EF. For since the two angles GEF, HFE are equal, if each is acute, the lines EG, FH, produced toward the parts G, H, will meet and form an isosceles triangle [Margin: (b) Euclid I.6]. Since, then, the straight line GH cuts the sides proportionally — because EG, FH are equal, and hence also the remaining lines up to the meeting-point — EF and GH

will be parallel [Margin: (c) Euclid VI.2]. But if the angles GEF, HFE are obtuse, the straight lines GE, HF, produced toward the parts E, F (because the angles next to them become acute), will meet above the straight line EF and will form an isosceles triangle in the same way, whose base is GH [Margin: (d) Euclid I.6]. For the sides above the base EF will be equal; therefore, when the equals EG, FH are added, the

sides above GH will likewise become equal. Since, then, the straight line EF cuts those sides proportionally, taking off from each equal parts [Margin: (a) Euclid VI.2], the lines EF, GH will be parallel. And finally, if each of the angles GEF, HFE is right, then EF, GH will be equal and parallel [Margin: (b) Euclid I.28].

By the same reasoning we shall show that EF, IK are parallel [Margin: (c) Euclid I.33]; and consequently that GH, IK are parallel to one another [Margin: (d) Euclid XI.9]. Now let EF be bisected at L, and in the planes AB, AC let perpendiculars to EF be raised; these will likewise bisect GH and IK. For if the equal angles GEF, HFE are acute, so as to form an isosceles triangle, then, by the scholium to Proposition 26, Book 1 of Euclid, the straight line drawn from the vertex to the point L (the midpoint of the base) is perpendicular to EF, and so coincides with LM. Since then this same line, by the scholium to Proposition 4, Book 6 of Euclid, cuts the straight lines EF, GH into proportional parts, GH too will be bisected at M. But if the angles GEF, HFE are obtuse, so that GE, HF produced beyond EF form an isosceles triangle whose base is EF or GH, then again, by the scholium to Proposition 26, Book 1 of Euclid, the straight line drawn from the vertex to L (the midpoint of the base EF) is perpendicular to EF, and so, when produced, coincides with LM. Since then this same line cuts the straight lines EF, GH into proportional parts, GH is bisected at M. And if, finally, the angles GEF, HFE are right, then EH, EM, FM are rectangular parallelograms, and therefore the opposite sides are equal — that is, GM is equal to EL, and HM to FL [Margin: (e) Euclid I.34]. Since, then, EL, FL are equal, GM, HM will also be equal. In no other way shall we show that the straight line IK is bisected at N.

Now, because the straight line EL, which stands upon the two lines LM, LN meeting at L, is perpendicular to the tangents [Margin: (f) Euclid XI.4], the same line EL (drawn... to the straight line MN) will be perpendicular to the plane of triangle LMN. Therefore [Margin: (g) Euclid XI.8] each of the lines GM, IN will likewise be perpendicular to the same plane; and hence, by Definition 3, Book 11 of Euclid, both GM and IN will be perpendicular to the straight line MN lying in the same plane. When, therefore, the straight lines GI, IM, MK, KH have been drawn [Margin: (h) Euclid XI.7], all of which, together with MN, lie in the same

plane of the parallels GH, IK — since the two sides IN, MN are equal to the two sides KN, NM, and contain equal angles (namely the right angles, as we have shown) — the bases IM, KM will also be equal [Margin: (i) Euclid I.4], and the angles IMN, KMN equal; and hence from the right angles the remaining angles GMI, HMK will be equal. Since, then, the two sides GM, MI are equal to the two sides HM, MK, and contain equal angles, as has been demonstrated, the bases GI, HK will also be equal [Margin: (k) Euclid I.4]. Finally, since the sides EG, EI are equal to the sides FH, FK, and the base GI to the base HK, the angles GEI, HFK will also be equal [Margin: (l) Euclid I.8]. Which was what was proposed.

And this demonstration is universal in all cases, whether the angle of inclination of the planes MLN be obtuse, or acute, or right, as is evident.

But if both the angles GEF, HFE and the two angles IEF, KFE are right, the demonstration will be easier. For then the angles GEI, HFK are the angles of inclination of the plane AC to the plane AB, by Definition 6, Book 11 of Euclid; and therefore they are equal to one another.

LEMMA XXI.

If, on the diameters of equal circles, points are taken equally distant from the centres, and from these points straight lines are drawn out to the circumferences, making equal angles with the diameters toward the same parts, then those straight lines will both be equal and will cut off equal arcs. And if the lines are equal, they will make equal angles with the diameters toward the same parts, and again will cut off equal arcs. And if, finally, equal arcs are cut off toward the same parts, then those straight lines will also be equal, and will make with the diameters, toward the same parts, equal angles.

We demonstrated this same thing in the penultimate proposition of the scholium to Proposition 29, Book 3 of Euclid, when the point assumed on the diameter lies within the circle; but because we shall also need it, in what follows, for the case when the point is taken on the diameter produced outside the circle, it has seemed good to demonstrate it here universally.

Let there be, then, equal circles ABC, DEF, whose centres are G, H and whose diameters are AC, DF; and let there be taken, first, within the circles, points I, K equally distant from the centre — that is, let the straight lines GI, HK be equal. And let straight lines IB, KE be drawn in any manner, making either the angles CIB, FKE equal, or the angles AIB, DKE equal. I say that both the straight lines IB, KE, and the cut-off arcs CB, FE, as well as the arcs AB, DE, are equal. For when the

straight lines GB, HE have been drawn from the centres, if indeed the angles GIB, HKE are supposed equal, the two sides GI, GB about the angle IGB will be equal to the two sides HK, HE about the angle KHE, and the angle I equal to the angle K — angles which are subtended by the equal sides GB, HE. But each of the remaining angles GBI, HEK is less than a right angle [Margin: (m) Euclid III.31], because the straight lines AB, CB, DE, FE, when drawn, make right angles ABC, DEF in the semicircles, of which those angles are parts. Therefore, from what was demonstrated toward the end of Book 1 of Euclid, the straight lines IB, KE and the angles IGB, KHE are equal at the centres [Margin: (n) Euclid III.26]; and hence the arcs CB, FE are equal, and consequently the remaining arcs AB, DE will be equal [Margin: (o) Euclid I.13]. But if the angles CIB, FKE are supposed equal, then the remaining angles GIB, HKE will also be equal, each pair being the difference from two right angles (for the two angles at I are equal to two right angles, as are the two at K) [Margin: (o) — since the two angles, both those at I and those at K, are equal to two right angles]; wherefore, as has just been shown, both the straight lines IB, KE and the arcs CB, FE, as well as the arcs AB, DE, will be equal.

Next, let the points A, D be taken at the extremities of the diameters, from which the straight lines AB, DE are drawn making the angles CAB, FDE equal, or the angles LAB, MDE equal. I say again that the straight lines AB, DE, and the cut-off arcs CB, FE, as well as the arcs AB, DE, are equal. For if the angles CAB, FDE are equal, the arcs CB, FE will also be equal [Margin: (p) Euclid III.26], and consequently the arcs AB, DE; and hence the straight lines AB, DE will be equal to one another [Margin: (q) Euclid III.29]. But if the angles LAB, MDE are supposed equal, the remaining angles CAB, FDE will likewise be the differences from two right angles; wherefore, as has been demonstrated, both the arcs CB, FE and the arcs AB, DE, and the straight lines AB, DE, are equal.

Lastly, let the points L, M be taken on the diameters produced outside the circles, equally distant from the centres, so that the straight lines GL, HM are equal. And let the straight lines LN, MO be drawn making the equal angles CLN, FMO, or PLN, QMO, and cutting off the arcs AN, DO, or CN, FO. I say that the straight lines LN, MO, and both the arcs AN, DO and the arcs CN, FO, are equal. For either one of the straight lines — namely LN — touches the circle at N, and MO touches the circle at O. For if the angles CLM, FMO are supposed equal; and MO—

(Continuing the proof of Lemma XXI, begun in the preceding pages. The bracketed Euclid references reproduce the lettered marginal citations, placed at the superscript letters to which they are keyed.)

[suppose that the line MO] does not touch the circle. Then let the tangent MS be drawn [Euclid III.17], and let the lines GN, HS be joined, which will make right angles GNL, HSM [Euclid III.18]. Now the two sides GN, GL about angle LGN are equal to the two sides HS, HM about angle MHS, and to the equal sides GL, HM are opposite equal angles GNL, HSM (being right angles), while of the remaining angles GLN, HMS each is less than a right angle, by corollary 1 of Euclid I.17. Therefore, by what we demonstrated at the end of Euclid Book I, the angles GLN, HMS are also equal. But by hypothesis the angle HMO is equal to that same angle GLN. Therefore the angles HMS and HMO would also be equal — a part equal to the whole, which is absurd. Hence the line MO touches the circle at O.

Joining, then, the lines GN, HO [Euclid III.18], the angles GNL, HOM are right and equal. But the angles GLN, HMO were also assumed equal. Therefore the remaining angles LGN, MHO are equal as well, by corollary 1 of Euclid I.32. And so, since the two sides GN, GL are equal to the two sides HO, HM and contain equal angles, as has been shown, the bases LN, MO are also equal [Euclid I.4]. Likewise the arcs AN, DO, because of the equal angles AGN, DHO, and so for the rest; and therefore the remaining arcs CN, FO of the semicircles are equal too [Euclid III.26]. And if the angles PLN, QMO are assumed equal, then the remaining angles CLN, FMO, from two right angles, are also equal. Therefore, as has now been demonstrated, both the arcs AN, DO and the arcs CN, FO, and the tangent lines LN, MO, are equal.

But suppose the two lines LR, MS (or LB, ME) make either the angles CLR, FMS (or PLR, QMS), or the angles CLB, FME (or PLB, QME), equal, and that LR (or LB) does not touch the circle but cuts it in R (or B). Then, when the tangent LN is drawn, LR (or LB) will fall on the near side of the tangent LN and will make angle CLR (or CLB) smaller than angle CLN. Now, since when the tangent MO is drawn the angles GLN, HMO are equal, as was just demonstrated, while angle FMS is assumed equal to angle CLR (or angle FME to angle CLB), the angle FMS (or FME) will also be smaller than angle FMO, and hence the line MS (or ME) will fall on the near side of the tangent MO. Each pair, therefore — LR, MS, or LB, ME — will cut its own circle in two points, R, B and S, E, between which lie the points of contact N, O.

So first take the points R, S on the near side of the contacts, and let the angles GLR, HMS be assumed equal. I say that the lines LR, MS, and both the arcs AR, DS and the arcs CR, FS, are equal. For join the lines GR, HS. Since the two sides GR, GL about angle LGR are equal to the two sides HS, HM about angle MHS, and the angles GLR, HMS, opposite the equal sides GR, HS, are assumed equal, while of

the remaining angles GRL, HSM each is greater than a right angle — because GRL is greater than the right angle GNL, and HSM greater than the right angle HOM [Euclid I.21] — then, by what we demonstrated at the end of Euclid Book I, the lines LR, MS and the angles LGR, MHS are equal too. Therefore the arcs AR, DS are also equal [Euclid III.26], and hence also the remaining arcs CR, FS of the semicircles. And if instead the angles PLR, QMS are assumed equal, then the remaining angles CLR, FMS, from two right angles, will also be equal; and so, as was shown, the lines LR, MS and both the arcs AR, DS and the arcs CR, FS will be equal.

Next take the points B, E beyond the contacts, and let the angles GBL, HEM be assumed equal. I say again that the lines LB, ME, and both the arcs AB, DE and the arcs CB, FE, are equal — provided each of the angles GBL, HEM is less than a right angle. For when the circles are described that pass through the contacts N, O (by the scholium to Euclid III.31), they will, because of the upright lines, cut LB, ME in T, V; and if the lines GT, HV are joined, the angles GTL, HVM are formed [Euclid III.31], such that GTL is greater than angle GBL, and HVM greater than angle HEM — the exterior being greater than the interior — and each is less than a right angle. This is also clear from the fact that the lines at B, E, together with GB, HE, make right angles and so touch the circles at B, E, by the corollary to Euclid III.16; for hence it comes about that the secant lines LB, ME make acute angles with those same lines GB, HE. Since, then, the two sides GB, GL about angle LGB are equal to the two sides HE, HM about angle MHE, and the angles GLB, HME, opposite the equal sides GB, HE, are assumed equal, while of the remaining angles GBL, HEM each has been shown less than a right angle, then, by what we demonstrated at the end of Euclid Book I, the lines LB, ME and the angles LGB, MHE are equal. Likewise the arcs AB, DE, and therefore also the remaining arcs CB, FE of the semicircles, are equal. And if instead the angles PLB, QME are assumed equal, then the remaining angles CLB, FME, from two right angles, are also equal. Hence, as has now been demonstrated, both the lines LB, ME and both the arcs AB, DE and the arcs CB, FE are equal.

Diagram. A single rectangular woodcut frame enclosing two mirror-image semicircular constructions side by side (the left built on center G, the right on center H), used throughout Lemma XXI. The two figures share the same lettering scheme, left/right.

- P (top left) / Q (top right): endpoints of the vertical diameters rising above the semicircles.
- L (upper left) / M (upper right): points on the verticals below P, Q.

- A (left) / D (right): points on the arcs, upper.
- R (left, on arc) / S (right, on arc): the near-side cutting points.
- N (left, mid) / O (right, mid): the points of contact (tangency).
- T (left) / V (right): the secondary intersection points on LB, ME.
- G (left center) / H (right center): the centers, with B (just right of G) and E (just right of H) marking the far-side cutting points on the diameters.
- I (left, lower) / K (right, lower): points on the verticals below the centers.
- C (bottom left) / F (bottom right): the lower endpoints of the diameters.

Next, let the lines IB, KE (or AB, DE; or LN, MO; or LR, MS; or finally LB, ME) be equal, and the angles at I, K (or at A, D; or at L, M) be equal. I say that both the arcs CB, FE (or CN [?], FO; or CR, FS) and the arcs AB, DE (or AN, DO; or AR, DS) are equal. For since the two sides GI, GB are equal to the two sides HK, HE, and the base IB is assumed equal to the base KE, the angles IGB, KHE are also equal [Euclid I.8]; and therefore the remaining arcs AB, DE of the semicircles are equal. Again, since the two sides IG, IB are assumed equal to the two sides KH, KE, and the base GB equal to the base HE, the angles GIB, HKE are also equal [Euclid I.8]; and hence the remaining angles CIB, FKE, from the two right angles, are equal. Once more, since the lines AB, DE are assumed equal, the arcs AB, DE are also equal [Euclid III.28]; and therefore the remaining arcs CB, FE of the semicircles are equal too. Therefore also the angles CAB, FDE are equal [Euclid III.27], and consequently the remaining angles LAB, MDE, from the two right angles, are equal. Finally, since the three sides GB, GL, LB are equal to the three sides HE, HM, ME, the angles will also be equal, by the corollary to Euclid I.8; therefore the arcs AB, DE are equal, on account of the equal angles BGL, EHM [Euclid III.26], and consequently the remaining arcs CB, FE of the semicircles are equal too. In just the same way we shall show that the angles at L, M, and the arcs AN, DO, CN, FO, AR, DS, CR, FS, AB, DE, and finally CB, FE, are equal.

Thirdly, let the arcs CB, FE cut off by the lines IB, KE be equal. I say that the lines IB, KE themselves, and the angles at I,

K, are equal. For the angles CGB, FHE will be equal, on account of the equal arcs CB, FE [Euclid III.27]. Since, then, the two sides GI, GB are equal to the two sides HK, HE and contain equal angles, the bases IB, KE will also be equal [Euclid I.4], and likewise the angles GIB, HKE; and hence also the remaining angles CIB, FKE, from the two right angles. And if the arcs AB, DE cut off by the same lines IB, KE are equal, then the remaining arcs CB, FE of the semicircles are also equal.

Therefore, as has already been shown, both the lines IB, KE and the angles at I, K are equal.

Again, let the arcs CB, FE cut off by the lines AB, DE be equal. I say that the lines AB, DE, and the angles at A, D, are also equal. For the remaining arcs AB, DE of the semicircles will be equal, and therefore the lines AB, DE will be equal too [Euclid III.29]. Likewise, on account of the equal arcs CB, FE, the angles CAB, FDE are equal [Euclid III.27], and hence the remaining angles LAB, MDE, from the two right angles, are equal. And if the arcs AB, DE cut off by the same lines AB, DE are equal, then the remaining arcs CB, FE of the semicircles are also equal. Therefore, as we demonstrated just above, the lines AB, DE and the angles at A, D are again equal.

Furthermore, let the arcs AN, DO cut off by the lines LN, MO be equal. I say that these lines, and the angles at L, M, are equal. For the angles NGL, OHM will be equal, because of the equal arcs AN, DO [Euclid III.27]. Then, since the two sides GN, GL are equal to the two sides HO, HM and comprehend equal angles, the bases LN, MO and the angles GLN, HMO will be equal [Euclid I.4], and consequently the remaining angles PLN, QMO, from the two right angles, are equal. In the same way you will show that the lines LR, MS are equal, and the angles at L, M, if the arcs AR, DS cut off are equal — and so on for the rest.

SCHOLIUM

But if on the diameters of unequal circles points are taken similarly distant from the centers — that is, so that their distances from the centers have the same ratio as the semidiameters — and from those points straight lines are drawn making equal angles with the diameters toward the same parts, then similar arcs will be cut off by them. And if the arcs cut off are similar toward the same parts, then the cutting lines will make equal angles with the diameters toward the same parts.

Diagram. Two similar semicircular constructions, of unequal size, standing side by side (the larger at left on center G, the smaller at right on center H), illustrating the Scholium on similar arcs.

- Left figure: A (top of the arc), B (upper left, on the arc), G (center), I (on the diameter, below G), N (lower left, on the arc), P (lower center, on the arc), C (foot of the vertical), L (very bottom).
- Right figure: D (top of the arc), E (upper, on the arc), H (center), K (on the diameter, below H), O (mid left), Q (lower, on the arc), F (lower right, on the arc), M (very bottom).

[The following demonstration is set in italic in the source.]

For in the unequal circles ABC, DEF, whose centers are G, H, let two points I, K be taken on the diameters, similarly distant from the centers — that is, so that IG is to GC as KH is to HF; and, permutando, so that IG is to KH as GC is to HF — and let equal angles GIB, HKE be constructed. I say that both the arcs BC, EF and the arcs AB, DE are similar. For, joining the lines GB, HE: since the angles I, K are equal, and the sides about the angles G, H in the triangles BGI, EHK are proportional, and of the remaining angles B, E each is less than a right angle — because they are parts of the right angles that the lines CB, AB and FE, DE make in the semicircles — the angles BGI, EHK at the centers will be equal [Euclid VI.7]. Therefore, by the scholium to Euclid III.22, the arcs BC, EF are similar; and hence the remaining arcs AB, DE of the semicircles will be similar too, by Lemma 6.

By the same reasoning, if at the points C, F — similarly distant from the centers, since they stand off by [multiples of] the semidiameters — equal angles GCB, HFE are made, we shall show that both the arcs BC, EF and the arcs AB, DE are similar. For, joining the lines GB, HF, the angles C, F in the triangles BCG, EFH will again be equal, and the sides about the angles G, H proportional. Since, then, of the remaining angles B, E each is less than a right angle — being parts of the right angles that the lines CB, AB and FE, DE make in the semicircles — the angles G, H at the centers will be equal [Euclid VI.7]. Therefore, by the scholium to Euclid III.22, the arcs BC, EF are similar, and so on. This may be shown more briefly thus: since the angles ACB, DFE are equal, the arcs AB, DE will, by the aforesaid scholium, be similar; and hence the remaining arcs BC, EF of the semicircles will be similar too, by Lemma 6.

Not otherwise: if the points L, M are similarly distant from the centers, and equal angles GLB, HME are made, we shall demonstrate that the arcs BC, EF; AB, DE; CP, FQ; AP, DQ; and BP, EQ are all similar. For, joining the lines GB, HE, the angles L, M in the triangles BGL, EHM will again be equal, and the sides about G, H proportional. Since, then, of the remaining angles B, E each is less than a right angle — for, joining the lines GP, HQ, the angles B, P and E, Q in the isosceles triangles BGP, EHQ are acute, by corollary 3 of Euclid I.17 — the angles G, H, as well as B, E, will be equal [Euclid VI.7]. Therefore, by the scholium to Euclid III.22, the arcs BC, EF are similar, and hence, by Lemma 6, the remaining arcs AB, DE of the semicircles will be similar too. And since the angles B, E have been shown equal, the angles P, Q in the isosceles triangles BGP, EHQ will also be equal (being equal to those) [Euclid I.5]; and consequently the remaining angles BGP, EHQ will be equal, which, taken from the equal angles BGL, EHM, leave the remaining angles PGL, QHM equal. And therefore, by the scholium to Euclid III.22, the arcs CP, FQ are similar, and hence, by Lemma 6, the remaining arcs AP, DQ of the

semicircles will be similar; from which, if the similar arcs AB, DE are taken away, the remaining arcs BP, EQ will, by Lemma 6, be similar too.

And if it should happen that one of the lines constructing equal angles — say LN — touches its circle, then the other, MO, will touch its circle as well. For, LN touching circle ABC, if MO be drawn touching circle DEF, the angle GLN will be equal to the angle HMO. For, joining the lines NG, OH, the angles N, O will be right and equal [Euclid III.18]. Since, then, the sides about the angles NGL, OHM are proportional, and of the remaining angles L, M each is less than a right angle (by corollary 1 of Euclid I.17), the angles G, H and L, M will be equal [Euclid VI.7]. From this it follows that, if LN touches the circle, no line other than the tangent MO can be drawn from M that makes an angle at M equal to the angle at L, since every such other angle would be either greater or less than the angle HMO.

But now let the arcs BC, EF be similar, and the points I, K similarly distant from the centers. I say that, when the lines BI, EK are drawn, the angles I, K are equal. For, joining the lines BG, EH, the angles G, H will be equal, by the scholium to Euclid III.22; and since the sides about them are proportional too, the triangles BGI, EHK will be equiangular [Euclid VI.6], and the angles I, K equal. In the same manner the angles C, F and L, M will also be equal, whether the arcs BC, EF or CP, FQ are assumed similar — which is what was proposed.

COROLLARIUM

From these things this theorem too may be inferred. If, from two centers A, B lying on the same straight line, two circles CDE, FGH are described, under the condition that outside both a point I can be taken similarly distant from the centers — that is, so that the ratio of IA to IB is the same as that of the semidiameter AE to the semidiameter BH, and, permutando, the ratio of IA to AE the same as that of IB to BH — then the straight line ID, touching one of the circles, will touch the other as well; and the line IK, cutting both, will cut off similar arcs EK, HM, CK, FM, and so on. For since the circles are unequal, and the point I stands off from the centers similarly, as though it stood for two circles, it comes about that, the line ID being drawn tangent to circle CDE, the line IG that makes angle BIG equal to angle AID — that is, the same line — will touch circle FGH as well, and the arcs DE, GH will be similar. So too, when the line IK is drawn, if the line IM be drawn making angle FIM equal to angle CIK — that is, the same line — it is brought about that the arcs KE, MH, and so on, are similar, as was demonstrated in the preceding scholium. This corollary, however, can be demonstrated by the same means by which the scholium was demonstrated, as is clear, if the lines are joined as appears in the figure.

Diagram. A tall rectangular woodcut frame illustrating the Corollary: two unequal circles described from centers A and B on a common vertical line, with an external point I from which a tangent and a secant are drawn to both.

- C (top right, on the frame): upper point of the larger circle.
- K (upper left, on the arc): a cutting point on line IK.
- A (right, upper middle): the center of the larger circle.
- D (left middle, on the arc): the tangency point of line ID.
- F, E, B (right side, descending): points of the smaller circle and its center B, with F above E above B.
- G, L, M (lower center): further arc/intersection points (G, then L above M).
- N (lower center, on the arc): a point of the smaller construction.
- I (bottom right corner): the external point from which the tangent and secant issue.

LEMMA XXII

If, in a base plane, between two straight lines there falls a transverse straight line making with them internal angles on each side equal to one another — whether all are right, or two obtuse and two acute — and if upon those two lines two planes stand at right angles to the base plane, then a plane drawn in any way through the transverse line will make, with those planes, common sections that are straight lines, which will contain, with the two given lines in the base plane, equal angles.

In the base plane let there be two straight lines AB, CD, across which the line EF falls transversely, making both the internal angles HGB, GHD and the internal angles HGA, GHC equal to one another — whether right angles, or two obtuse and two acute. Let HGB, GHD first be obtuse, and HGA, GHC acute; and upon the lines AB, CD let two planes AI, CK stand at right angles to the base plane. Through the transverse line EF let a plane EL be drawn, inclined in any way to the base plane — whether toward the parts B, D or toward the parts A, C — cutting the upright planes AI, CK along the lines GM, HN. I say that both the angles BGM, DHN and the angles AGM, CHN are equal to one another.

For let equal lines GO, HP be taken toward the side to which the plane EL is inclined relative to the base plane — yet so that they are cut off on the side of the acute angles AGH, CHG, before the meeting of the lines GA, HC, so that the demonstration may always be the same in both cases — and let the lines OP, GP, OH be joined. Now, since the two sides GH, GO are equal to the two sides HG, HP,

and by hypothesis contain equal angles, the triangles GHO , HGP will be equal [Euclid I.4]. Therefore the lines GH , OP are parallel [Euclid I.39]. Next, in the plane AI , from O draw OQ perpendicular to AB , the common section of the plane AI and the base plane; this, by definition 4 of Euclid Book XI, will be at right angles to the base plane. Let OQ be produced until at Q it cuts GM , the common section of the plane EL and the plane AI . It will certainly cut it, since they lie in the same plane AI , and since the angles QOG , OGQ are together less than two right angles — because the plane EL is assumed inclined to the base plane, and hence the angle OGQ is acute. For if it were right, GQ would, by definition 4 of Euclid Book XI, be at right angles to the base plane; and consequently the plane EL , drawn through the line GQ , would be at right angles to the base plane [Euclid XI.18] — which is not assumed. In the plane CK also, from P draw PR perpendicular to CD , the common section of the plane CK and the base plane; this likewise will be at right angles to the base plane, and, produced, will meet HN , the common section of the plane EL and the plane CK , at R . Now let the line QR be joined in the plane EL , in which the points Q , R lie; and if through GH a plane is conceived to be drawn parallel to the plane OR (it can be so drawn, since GH has been shown parallel to OP — for it comes about that the plane drawn through GH can be revolved about the line GH until it is parallel to the plane OR that is drawn through the line OP), then the common sections GH , QR , made in the planes...

Diagram. A large perspective woodcut of the solid configuration for Lemma XXII: a base plane with two upright planes (AI on AB , CK on CD) standing at right angles, and an inclined transverse plane (EL) cutting through them along GM , HN . The figure is drawn in oblique perspective, and several construction letters (the perpendiculars and their feet) recur at more than one vertex; exact assignment of the repeated labels is uncertain at this resolution. Labels legibly present include: A , B (right edge, on line AB); C , D (base, on line CD); E , F (endpoints of the transverse line, at the foot); G , H (where EF meets AB , CD); I , K (upper corners of the upright planes AI , CK); L (lower left, of the inclined plane EL); M , N (upper edge, the sections GM , HN); O (mid-left), P (repeated, on GO/HP and the feet), Q (upper, on OQ), R (repeated, on PR). Whole diagram flagged as a (more than two labels uncertain in placement).

to those planes parallel to the plane EL , through the straight lines GH , QR , drawn parallel. Since, then, OP has been shown parallel to this same GH , [a] OP and QR will also be parallel to one another. [b] But OQ , PR are likewise straight lines parallel to one another, at right angles to the underlying plane. OR is therefore a parallelogram; [c] and consequently the opposite sides OQ , PR will be equal. Since, then, the two sides OG , OQ are equal to the two sides PH , PR , and contain

equal angles — namely right angles — [d] the angles OGQ, PHR will also be equal. Which was what was proposed.

Now if ever the plane EL is at right angles to the underlying plane, since the planes AI, CK are also set at right angles to that same plane, [e] the common sections of these and of that — namely the straight lines GM, HN — will likewise be perpendicular to the underlying plane; and therefore, by definition 3 of Book XI of Euclid, both the angles MGA, MGB and the angles NHC, NHD will be right angles, and hence all four equal to one another.

But if the straight line EF is perpendicular to the two lines AB, CD, [f] then AB, CD will be parallel; and consequently, from the scholium to proposition 18 of Book XI of Euclid, the plane straight lines AI, CK will also be parallel. [g] Therefore the sections GM, HN made in them by the plane EL will be parallel. [h] Wherefore the angles BGM, DHN will be equal.

[Margin: a = "9. vndec." (Euclid, *Elements* XI.9); b = "6. vndec." (Euclid XI.6); c = "34. pri." (Euclid I.34); d = "4. primi" (Euclid I.4); e = "19. vndec." (Euclid XI.19); f = "28. primi" (Euclid I.28); g = "16. vndec." (Euclid XI.16); h = "10. vndeci." (Euclid XI.10)]

LEMMA XXIII.

A plane in the sphere drawn in any manner through either of the poles of the world and either of the poles of any oblique great circle whatever — or of a great circle at right angles to the Equator — cuts off two equal arcs. It does so both from the Equator and from that oblique (or right) great circle, and from any parallel of the Equator and from any parallel of that oblique (or right) great circle (this latter parallel being equal to the parallel of the Equator, and being as far distant from its own assumed pole as the parallel of the Equator is distant from the assumed pole of the world). The two arcs are those intercepted between the cutting plane and the great circle described through the two assumed poles.

But since that great circle, described through the poles of the world and the poles of the other great circle, cuts each of the circles described from the two assumed poles in two places, the following must be considered, so that we may know from which two sections the equal arcs cut off are to begin. When the cutting plane is drawn through the south pole of the world and the superior pole of the other great circle — (for since this other great circle, when it is oblique, can be taken for the horizon of some region, its pole will be the one more remote from the south pole, the higher one, in the manner of the vertex or Zenith, while [the other] is nearer to the south; whereas when this other circle is at right angles to the Equator, so that it is a kind

of right horizon, either of its poles may be taken as the superior, or as the Zenith. From which it also follows that the semicircle of the great circle passing through the poles of the world and the poles of the other circle — the semicircle enclosed between the poles of the world, in which the superior pole, or Zenith, is contained — is called the superior; while the other, in which the inferior pole exists, or Nadir, is called the inferior) — then, when the arc cut off from the Equator or from its parallel begins from the superior semicircle, the equal arc cut off in the other great circle or in its parallel must be begun from that circle's south section; but if the arc cut off from the Equator or its parallel begins from the inferior semicircle, the equal arc must be begun from the north section [of its parallel]. When, however, the cutting plane is drawn through the south pole of the world and the inferior pole of the other great circle, and the arc cut off in the Equator or its parallel begins from the superior semicircle, the equal arc must be begun in the other great circle or its parallel from the north section; but from the south, if the arc of the Equator or its parallel begins from the inferior semicircle. Moreover, the north or south section is to be taken with respect to the poles of the other great circle, whether oblique or right.

Let there be in the sphere a great circle ABCD, drawn through the poles of the world A, C and through the poles E, F of any oblique great circle GHI whatever; and let the Equator BKD be described from one or other pole of the world, cutting the oblique circle in L; and LB, LD, LG, LI will be quadrants. For since the great circle GLI is drawn [a] through the poles of the greatest circles BLD, each of the two will in turn pass through the poles of the other, and consequently L will be the pole of circle ABCD; [b] and hence LB, LD, LG, LI will be quadrants.

Now first let the plane be drawn through the south pole of the world C and E, the more remote pole of the oblique circle — (for since the great circle GHI is set oblique to the Equator, its poles will not be distant a quadrant from the poles of this circle [ABCD], so that its poles should be B, D; [c] otherwise the oblique circle would pass through the poles of the Equator A, C, [d] and so would be at right angles to the Equator, which conflicts with the hypothesis. Therefore one pole, namely F, will be nearer to the pole of the world C, and the other, E, more remote) — let some plane be drawn, [e] making on the surface of the sphere the circle CHE, [f] and having, with the plane of the great circle ABCD, the common section CE, a straight line. And let this circle CHE cut first the Equator and the oblique great circle in the points K, H, which either lie in the quadrants LD, LI, or in the quadrants LB, LG — that is, the arcs cut off DK, IH are either less than a quadrant or greater. I say that the arcs DK, IH, and likewise BK, GH, are equal. (For DK on the Equator begins from the superior semicircle CDA, and IH on the oblique circle

from the south section I; while BK takes its beginning on the Equator from the inferior semicircle CBA, and GH on the oblique circle from the north section G.)

For when the straight lines CD, EI are drawn — which will intersect one another in M, since they lie in the plane of the great circle ABCD, and the point I lies between C and D — [g] since CD, EI are quadrants, and the common arc DI being taken away, the remaining arcs CI, ED are equal; [h] the angles CEI, ECD will be equal; [i] and therefore the straight lines EM, CM will also be equal. Again, let there be drawn in the plane of the circle CHE the straight lines CK, EH, which will be equal, [k] since they are sides of the squares described in the great circles; [l] and therefore the arcs CK, EH are also equal; and the common arc HK being taken away, when the circle CHF cuts...

[Margin: a = "schol. 15. 1. Theod." (scholium on Theodosius, *Sphaerica* I.15); b, c, g = "coroll. 16. 1. Theod." (corollary of Theodosius I.16); d = "15. 1. Theodos." (Theodosius I.15); e = "1. 1. Theo." (Theodosius I.1); f = "3. vndec." (Euclid XI def. 3); h = "27. tert." (Euclid III.27); i = "6. primi" (Euclid I.6); k = "16. 1. Theodos." (Theodosius I.16); l = "28. tertij" (Euclid III.28)]

cuts the quadrants LD, LI — so that then the point H lies between C and the Equator — or the common arc HK being added, when the circle CHE cuts the quadrants LB, LG — so that then the point H lies beyond the Equator — the remaining arcs, or the composite arcs CH, EK, will also become equal; [a] and consequently the angles CEH, ECK will be equal; and hence the straight lines CN, EN (for the remainder of the equal angles CEH, ECK is less than a right angle, as we shall prove) [a] will also be equal.

That the straight lines CK, EH meet when the circle CHE cuts the quadrants LD, LI is evident, since they mutually cut one another in the plane of that circle, because the point H is between the points C and K. That the same straight lines CK, EH meet when the circle CHE cuts the quadrants LB, LG — that is, that the angles CEH, ECK are less than two right angles — will be made clear thus. Since the circle CHEO is not the greatest, in comparison with the greatest circle ABCD, which alone is the greatest among all circles described through the points C, E not diametrically opposite — (for through two points in a sphere not diametrically opposite only one greatest circle can be described, [b] as is established from Theodosius) — or else, if it were the greatest, [c] it would cut the circle ABCD into two equal parts at E, C, which is absurd, since it is bisected at A, C: [d] each straight line CK, EH will cut off from the circle CHEO a greater arc than would be similar to the arc which each of them cuts off from the greatest circle.

Now each cuts off a quadrant from the greatest circle, [e] because each is equal to the side of the square described in the greatest circle. Therefore each arc CK, EH will be greater than a quadrant. [f] Again, since the straight line CE cuts off from the same circle CHEO a greater arc than would be similar to the arc CDE which the same straight line CE cuts off from the greatest circle ABCD — and the arc CDE is greater than a quadrant, [g] because CD is a quadrant — therefore the arc COE will be much greater than a quadrant; and hence, if the two arcs CK, EH, also shown to be greater than a quadrant, are added, the whole arcs EOCK, COEH will each be greater than a semicircle; [h] and therefore each angle ECK, CEH, since they lie in those greater segments, will be less than a right angle. Wherefore the two straight lines CK, EH will meet outside the sphere at N, on account of the two angles ECK, CEH being less than two right angles.

Diagram. A large woodcut circle representing the great circle ABCD, with the Equator, the oblique great circle, and the section-circle CHE drawn across it, together with the chords of the construction meeting at the exterior/interior points M and N. Labels read on the figure :

- A (top of the circle, marked with a star): pole of the world
- E (upper right, star): pole of the oblique circle
- G (upper left): intersection of oblique circle with ABCD
- H (upper left, star): point on the oblique great circle
- L (left of centre): pole of ABCD / intersection of Equator and oblique circle
- B (left): point on the Equator
- K (lower left): point on the Equator / section circle
- F (lower left, star): pole of the oblique circle
- C (bottom, star): south pole of the world
- I (lower centre): intersection point
- O (right, below E): point on circle CHEO
- D (right): point on the Equator
- N (interior, near centre): meeting of the produced chords CK, EH
- M (interior, near centre): intersection of chords CD, EI

And so, the straight lines MN, DK, IH being drawn: since the sides CM, CN have been shown equal to the sides EM, EN, and the common base is MN, [i] the angles MCN, MEN will be equal in the triangles CMN, EMN — which indeed lie outside the planes of the circles CHE, ABCD. Wherefore in the triangles CDK, EIH, since the sides CD, CK are equal to the sides EI, EH — [k] because all of them are sides of the squares described in the greatest circles — and they contain equal angles

DCK, IEH, as we have shown, [l] the bases DK, IH will also be equal; [m] and therefore the arcs DK, IH will also be equal, whether they are less than a quadrant or greater — that is, whether the circle CHE lies on the near side of the point L, or beyond. Therefore the remaining arcs of the semicircles BK, GH will also be equal.

But the angles MCN, MEN — on which indeed the whole force of the demonstration depends — we shall prove to be equal, even if it is not established that the straight lines CK, EH, when produced, meet at the point N, in this way. Since the plane of the circle CHE cuts the plane of the circle ABCD along the straight line CE, and there are shown equal, both in the former the two interior angles CEI, ECD, and in the latter the two interior angles CEH, ECK, then by Lemma 20 the angles DCK, IEN [?] will also be equal. [n] Wherefore, as before, [n] the bases DK, IH will be shown equal; [o] and hence also the arcs DK, IH, and therefore also the remaining arcs BK, GH of the semicircles, will be equal.

And since the quadrants LD, LI have been shown, if the equal arcs DK, IH are subtracted, or if the quadrants LD, LI are taken from these, the arcs LK, LH intercepted between the cutting plane and the point K of the intersection of the Equator with the oblique circle will also be equal.

But if the circle CHE passes through L, the point where the Equator and the oblique circle GHI intersect one another, it is evident that the arcs cut off DL, IL are equal, since they have been shown to be quadrants. So too, if the same circle CHE passes through the other pole of the world A as well, it is plainly established that the arcs DLB, ILG, and LB, LG, are equal. For then the circle CHE will be the same as ABCD, the greatest, and hence the semicircles DLB, ILG will be [equal], and LB, LG quadrants.

It also follows from these things that any two circles drawn through C, E intercept equal arcs in the Equator and in the oblique great circle. For since each cuts off equal arcs up to the points D, I, or up to the points B, G, if the lesser are subtracted from the greater, the remaining arcs intercepted between the two circles will also be equal. Thus the arcs KLK, HLH [?] will be equal, intercepted between the two circles CHKE, CKHE. For the equal arcs DK, IH, subtracted from the equal arcs DKLK, IHLH [?], leave equal the arcs KLK, HLH [?]; and so of the rest.

[Margin: a = "27. tertij" (Euclid III.27) and "6. primi" (Euclid I.6); b = "20. 1. Theodos." (Theodosius I.20); c = "11. 1 Theo." (Theodosius I.11); d, f = "lemma 6. 3. Theod." (lemma 6, on Theodosius III); e, k = "16. 1. Theod." (Theodosius I.16); g =

"coroll. 16. 1. Theod." (corollary of Theodosius I.16); h = "31. tert." (Euclid III.31); i = "8. primi" (Euclid I.8); l, n = "4. primi" (Euclid I.4); m = "28. tert." (Euclid III.28); o = "28. tertij" (Euclid III.28)]

The very same demonstration will be applied in the other two semicircles of the Equator and of the oblique great circle, on the other side of the greatest circle ABCD. For from those too, any plane whatever drawn through the poles C, E will cut off equal arcs, intercepted between the plane itself and the greatest circle ABCD, or the other point of section of the Equator and of the oblique circle.

Again, let there be in the sphere a great circle ABCD, drawn through the poles of the world A, C and the poles E, F of any oblique great circle whatever, and let BD be the diameter of the Equator, [and GI of] the oblique circle, as above. And from the poles C, E let there be described, with the same interval, two equal circles KLM, NOP, [a] of which the former is parallel to the Equator, the latter to the oblique circle. These two parallels will either mutually cut one another, as in the first figure, or will in no way intersect — which can happen in two ways. For either the circles described from the poles C, E lie on the near side of the greatest circles to which they are equidistant, as in the 2nd figure, or beyond them, as in the 3rd figure. Now through the poles C, E let some plane be drawn in any manner; [b] it will make on the surface of the sphere a circle CLE, and with the plane of the greatest circle ABCD a common section, the straight line CE. And let this circle cut each parallel in the points L, O, in whatever way it is inclined to the greatest circle ABCD — that is, whether the angle of inclination toward the segment CDE is acute, or right, or obtuse. I say that the arcs cut off ML, PO, as well as KL, NO, are equal. For ML begins from the superior semicircle, and PO from the south section; but KL from the inferior semicircle, and NO from the north section, as was said in the proposition it ought to be.

For when the straight lines CL, CM, EO, EP are drawn — [c] which are all equal, being drawn from the poles to the equal parallels — and the straight lines LM, OP being joined, [d] both the arcs CM, EP in the circle ABCD, and the arcs CL, EO in the circle CLE, will be equal; and the common arcs MP, LO being subtracted (when the parallels intersect, as in the first figure, or when they do not intersect but lie beyond the greatest circles to which they are equidistant, as in the third figure), or the same arcs MP, LO being added (when they do not mutually cut but lie on the near side of the greatest circles to which they are equidistant, as in the second figure), the remaining arcs — or the composite arcs CP, EM, as well as CO, EL — will also be equal; [e] and hence both the interior angles CEP, ECM in the plane of the greatest circle ABCD, standing on the equal arcs CP, EM, and the

interior angles CEO, ECL in the plane of the circle CLE (which cuts it along the straight line CE), standing on the equal arcs CO, EL, will be equal to one another. Therefore, by Lemma 20, the angles LCM, OEP will also be equal; [f] and the sides CL, CM, EO, EP containing them are equal; [g] therefore the bases LM, OP will also be equal; [h] and hence the arcs ML, PO will also be equal, and consequently the remaining arcs KL, NO of the semicircles.

Diagram. The first of three woodcut figures for this proof: a great circle ABCD carrying the Equator, the oblique great circle, and the two equal parallels KLM (parallel to the Equator) and NOP (parallel to the oblique circle), here mutually intersecting, with the section-circle CLE. Labels read :

- A (top, star), N (top left): pole of the world A and point N of parallel NOP
- E (right, star): pole E
- G (left): point on the oblique circle
- B (left), D (right): ends of the Equator's diameter BD
- O (centre), L (lower centre): points where CLE cuts the parallels
- Q, R (centre): quadrant-bisection points of the parallels
- K (lower left): point of parallel KLM
- F (lower left, star): pole F
- M (right): point of parallel KLM
- P (lower right): point of parallel NOP
- I (lower centre): intersection point
- C (bottom): south pole of the world [?]

But if the semicircles of the parallels KLM, NOP are bisected into quadrants at the points Q, R, the arcs LQ, OR intercepted between the cutting plane CLE and the ends Q, R of the quadrants will also be equal, since they are the complements of the equal arcs ML, PO, or of the equal arcs KL, NO.

Diagram. The second figure (printed number "2"): the same construction, but with the two parallels not intersecting and lying on the near side of (within) the greatest circles to which they are equidistant. Labels read include A, N (top), E (upper right, star), G (left), O, R, Q (centre), B (left), D (right), L, I (lower), K, F (lower left, star), M (right), P and O (lower). The peripheral labels of this small figure are crowded

Diagram. The third figure (printed number "3"): the same construction, but with the two parallels lying beyond (outside) the greatest circles to which they are equidistant. Labels read include A (top), E (upper right, star), K, G, L, C (centre), M,

D (right), B (left), N (lower), R, Q, I, P (lower), F. One label read as "U" could not be confirmed [?]; the peripheral labels are crowded

It is also clear that, if the circle CLE passes through the other pole of the world A as well, so that it coincides with the greatest circle ABCD, the arcs cut off MLK, PON are equal, [i] inasmuch as they are semicircles. So too, if the same circle cuts off a quadrant from one parallel, it will also cut off a quadrant from the other, since it necessarily cuts off an equal arc, as has been demonstrated. Likewise any two circles drawn through C, E will intercept equal arcs of the parallels, just as was said a little before concerning the Equator and the oblique great circle.

[Margin: a = "2. 2. Theod." (Theodosius II.2); b = "1. 1. Theo." (Theodosius I.1); c, f = "schol. 21. 1. Theod." (scholium on Theodosius I.21); d, h = "28. tertij" (Euclid III.28); e = "27. tertij" (Euclid III.27); g = "4. primi" (Euclid I.4); i = "15. 1. Theod." (Theodosius I.15)]

The very same result will hold for the remaining two semicircles of the parallels on the other side of the greatest circle ABCD; for the demonstration in those cases is entirely the same as in these, as is evident.

Next, through the same pole of the world C, and through the pole F of the oblique circle GHI, let some plane be drawn — one nearer [to that pole] — making on the surface of the sphere the circle CHF [Theodosius, *Sphaerica* I.1], and having as its common section with the plane of the greatest circle ABCD the straight line CF [Euclid, *Elements* XI.3]. Let this circle CHF cut, first, the Equator, and [then] the greatest oblique circle, at the points K and H, wherever this may happen. I say that the cut-off arcs BK, IH, and likewise DK, GH, are equal. (For BK begins from the inferior semicircle, and IH from the southern section; whereas DK begins from the superior semicircle, and GH from the northern section, as prescribed in the proposition.) For, when the straight lines CB, CK, FI, FH, BK, IH have been drawn [Theodosius, *Sphaerica* I.16 corollary]: since CB and FI are quadrants, and therefore equal, then, removing the common arc CF, the remaining arcs BF and IC will also be equal. Therefore the angles BCF and IFC will be equal [Euclid, *Elements* III.27].

Again, since in the circle CHF the straight lines CK and FH are equal [Theodosius, *Sphaerica* I.16] — being sides of the squares inscribed in the greatest circles BKD and GHI — the arcs CFK and FCH will likewise be equal [Euclid, *Elements* III.28]; and, removing the common arc CF, the remaining arcs FK and CH will also be equal. Therefore the angles KCF and HFC will also be equal [Euclid, *Elements* III.27]. And so, since the plane of circle CHF cuts the plane of circle ABCD along

the straight line CF, and since in the former the two interior angles BCF, IFC are equal, while in the latter the two interior angles KCF, HFC are equal, as has been demonstrated, then by Lemma 20 the angles BCK and HFI will also be equal.

You may also show this in the following way. When the straight lines CB, FI meet each other at M, and the straight lines CK, FH meet at N: since both the angles BCF, IFC and the angles KCF, HFC have been shown equal, both the straight lines CM, FM and the straight lines CN, FN are equal to one another [Euclid, *Elements* I.6]. Therefore, drawing the straight line MN: since the two sides CM, CN are equal to the two sides FM, FN, and the base MN is common, the angles MCN and MFN will also be equal [Euclid, *Elements* I.8]. And so, in the triangles [CBK and FIH], since the sides CB, CK are equal to the sides FI, FH [Theodosius, *Sphaerica* I.16] — they being all sides of squares inscribed in the greatest circles — and they contain equal angles BCK, IFH, as we showed, the bases BK, IH will also be equal [Euclid, *Elements* I.4]; and therefore the arcs BK, IH will be equal as well [Euclid, *Elements* III.28], and DK, GH equal, and so on.

Again, from the same assumed poles C, F, let two neighbouring equal circles KLM, NOP be described with one and the same radius — whether on this side of the Equator and the greatest oblique circle, or beyond them. And through the same poles C, F let a plane be drawn [Theodosius, *Sphaerica* I.1], making on the surface of the sphere the circle CLOF, and having as its common section with the greatest circle ABCD the straight line CF. Let this circle so made cut the circles described from the poles C, F at the points L, O. I say that both the arcs KL, NO and the arcs ML, PO are equal; of which KL begins from the superior semicircle, and NO from the [southern] section — in the parallels on this side of the greatest circles — while in the others the earlier one begins from the inferior semicircle, and the later from the [boreal] section; likewise ML begins from the inferior semicircle, and PO from the southern section, in the parallels on this side of the greatest circles, while in the others ML begins from the superior semicircle, and PO from the northern section, as prescribed in the proposition.

For, when the straight lines CK, CL, FN, FO have been drawn — all of which are equal to one another [Euclid, *Elements* III.28], since they fall from their proper poles onto equal circles: since the arcs CK, FN in circle ABCD are equal (on account of the equal straight lines CK, FN), as are CL, FO (on account of the equal straight lines CL, FO), then, adding the common arc CF in each circle (when the circles KLM, NOP are on this side of the greatest circles), or, when they lie beyond them, subtracting the same arc CF, the remaining arcs FK, CN in circle ABCD, and FL, CO in circle CLOF, will be equal; and therefore also the remaining arcs of the

whole circles — FAK, CAN in circle ABCD, and FOL, CLO in circle CLOF — will be equal. Therefore both the interior angles KCF, NFC, standing on the equal arcs FAK, CAN of circle ABCD, and the interior angles LCF, OFC, standing on the equal arcs FOL, CLO of circle CLOF, will be equal [Euclid, *Elements* III.27]; and consequently, by Lemma 20, [the angles KCL, NFO will be equal].

You may also show this in the following way, when the circles KLM, NOP intersect each other at Q, R — as happens when the circles KLM, NOP lie beyond the greatest circles. Since both the angles KCF, NFC and the angles LCF, OFC have been shown equal, both the straight lines CQ, FQ and the straight lines CR, FR will be equal to one another [Euclid, *Elements* I.6]. Therefore, drawing the straight line QR: since the two sides CQ, CR are equal to the two sides FQ, FR, and the base QR is common, the angles QCR and QFR will also be equal [Euclid, *Elements* I.8]. And so in the triangles CKL, FNO, since the sides CK, CL are equal to the sides FN, FO [Theodosius, *Sphaerica* I.21 scholium] and contain equal angles KCL, NFO, as was shown, the bases KL, NO will also be equal [Euclid, *Elements* I.4]; and therefore the cut-off arcs KL, NO will be equal [Euclid, *Elements* III.28], and consequently the remaining arcs ML, PO of the semicircles will be equal, and so on.

But let us now demonstrate this same lemma for the case in which one of the circles stands at right angles to the Equator. Let the greatest circle ABCD be described through A, C, the poles of the world (that is, of the Equator BED), and through B, D, the poles of the greatest circle AEC which stands at right angles to the Equator, as in this first figure. First let a plane be drawn through C, the southern pole of the world, and through D, the upper pole of circle AEC, making in circle ABCD the straight line CD, and in the sphere the circle CFGD, which cuts the Equator at F and circle AEC at G. I say that the cut-off arcs DF, CG — or BF, AG — are equal; of which DF takes its beginning from the superior semicircle, and CG from the southern section, while BF begins from the inferior semicircle, and AG from the northern section, as we prescribed should be done in the proposition. For, when the straight lines CF, DG, FD, GC have been drawn: since they are sides of squares inscribed in the greatest circles described [Theodosius, *Sphaerica* I.16], the arcs CF, DG will be equal; and, adding the common arc FG — or subtracting it, if the circle CFGD were to cut the greatest circles on this side of the point E — the arcs CFG, DGF will also be equal [Euclid, *Elements* III.28]; and therefore the angles CDG, DCF will also be equal, in the plane of circle CFGD [Euclid, *Elements* III.27].

Diagram. An uncaptioned geometric woodcut set within a square frame, showing the stereographic-projection construction for Lemma XXIII: a bounding circle (the

greatest circle ABCD) with a horizontal diameter and several great-circle arcs and chords drawn across it. Readable peripheral labels:

- A (top of the circle)
- B (left, on the horizontal diameter)
- C (bottom of the circle)
- D (right, on the horizontal diameter)
- E (upper right, on the circumference)
- G (upper left, on the circumference)
- F (lower left)

Interior intersection points labelled H, I, K, L, M, N appear among the crossing chords and arcs; their exact placements are difficult to fix in the dense engraving. for label verification.

Therefore, since the two sides CF, CD are equal to the two sides DG, DC (because CF and DG are sides of squares inscribed in the greatest circles described [Theodosius, *Sphaerica* I.16], while the side CD is common), and they contain equal angles DCF, CDG, as has been demonstrated, the bases DF, CG will also be equal [Euclid, *Elements* I.4]. Indeed, the straight lines DF, CG are equal, on account of the equal arcs DGF, CFG of circle CFGD [Euclid, *Elements* III.29]. Therefore the arc DF on the Equator, and the arc CG on circle AEC — and consequently the remaining arcs BF, AG of the semicircles — will be equal [Euclid, *Elements* III.28]. Which is what was proposed.

Next let a plane be drawn through the same southern pole of the world C, and through B, the lower pole of circle AEC, making in circle ABCD the straight line CB, and in the sphere the circle CHIB, which cuts the Equator at H and circle AEC at I. I say again that the cut-off arcs BH, CI — or DH, AI — are equal; of which BH on the Equator begins from the inferior semicircle, and CI from the southern section, while DH begins from the superior semicircle, and AI from the northern section, as the proposition prescribes. For, when the straight lines CH, BI, BH, CI have been drawn: CH and BI will be equal [Theodosius, *Sphaerica* I.16], since they are sides of squares inscribed in the greatest circles described. Therefore the arcs CH, BI will be equal [Euclid, *Elements* III.28]; and, adding the common arc HI — or subtracting it, namely when the circle CHIB cuts the [greatest] circles on this side of E — the whole arcs, or the remaining arcs, CHI, BIH will be equal; and therefore the angles CBI, BCH standing on them at the circumference will also be equal, in the plane of circle CHIB [Euclid, *Elements* III.27].

Accordingly, since the two sides CH, CB are equal to the two sides BI, BC (for CH and BI are sides of squares inscribed in the greatest circles described [Theodosius, *Sphaerica* I.16], while the side BC is common), and they contain equal angles BCH, CBI, as we showed, the bases BH, CI will also be equal [Euclid, *Elements* I.4]. Indeed, the straight lines BH, CI are equal, on account of the equal arcs BIH, CHI of circle CHIB [Euclid, *Elements* III.26]. Therefore the arcs BH on the Equator and CI on circle AEC — and consequently the remaining arcs DH, AI of the semicircles — will be equal [Euclid, *Elements* III.28]. Which is what was proposed.

Again, from C, the southern pole, and D, the upper pole of the other greatest circle, let there be described equal parallels EFG, HIK; and through the same poles let a plane be drawn, making in circle ABCD the straight line CD, and in the sphere the circle CFID, which cuts the parallels at F, I, as in the second figure. I say again that the cut-off arcs GF, KI — or EF, HI — are equal; of which GF begins from the superior semicircle, and KI from the southern section, while EF begins from the inferior semicircle, and HI from the northern section, as the proposition requires. For, when the straight lines CF, CG, GF, DI, DK, KI have been drawn: CF, CG, DI, DK will be equal to one another [Theodosius, *Sphaerica* I.21 scholium]. Therefore the arcs CF, DI will also be equal [Euclid, *Elements* III.28]; and, adding the common arc FI — or subtracting it, if need be — the arcs CI, DF will likewise become equal; and therefore the angles CDI, DCF standing on them will be equal, in the plane of circle CFID [Euclid, *Elements* III.27]. In the same way the arcs CG, DK will be equal; and consequently, from the quadrant CD, the remaining arcs DG, CK will be equal [Euclid, *Elements* III.28]; and therefore the angles DCG, CDK will also be equal, in the plane of circle ABCD [Euclid, *Elements* III.27]. Therefore, by Lemma 20, the angles FCG, IDK will also be equal. But the sides that contain them have also been shown equal to one another; therefore the bases FG, IK are equal [Euclid, *Elements* I.4], and consequently the arcs FG, IK — together with the remaining arcs EF, HI of the semicircles — will be equal [Euclid, *Elements* III.28].

Finally, from C, the southern pole, and B, the lower pole of the other greatest circle standing at right angles to the Equator, let there be described equal parallels LMN, OPQ; and through the same poles let a plane be drawn, making in circle ABCD the straight line CB, and in the sphere the circle CPMB, which cuts the parallels at M, P. I say that the cut-off arcs NM, QP — or LM, OP — are also equal; of which NM begins from the inferior semicircle, and QP from the southern section, while LM begins from the superior semicircle, and OP from the northern section, as the case requires, just as was stated in the proposition. For, when the straight lines CM, CN, BP, BQ, MN, PQ have been drawn — the first four of which

are equal to one another [Theodosius, *Sphaerica* I.21 scholium] — the arcs CM, BP will be equal [Euclid, *Elements* III.28]; and, subtracting the common arc MP, or adding it should the case require, the remaining arcs CP, BM will also be equal. Therefore the angles CBP, BCM standing on them will be equal, in the plane of circle CPMB [Euclid, *Elements* III.27]. By the same reasoning the arcs CN, BQ will be equal; and, subtracting the common quadrant BC, or adding it if need be, the arcs BN, CQ will also be equal; and therefore the angles BCN, CBQ will be equal to one another, in the plane of circle ABCD [Euclid, *Elements* III.27]. Accordingly, since in the planes of the circles CPMB and ABCD, which cut each other along the straight line BC, the two angles CBP, CBQ are equal to the two angles BCM, BCN, then by Lemma 20 the angles PBQ, MCN will also be equal. Since, then, [this angle] is contained by equal sides, as we showed, the bases MN, PQ will also be equal [Euclid, *Elements* I.4]. Therefore the arcs MN, PQ — and consequently the remaining arcs LM, OP of the semicircles — will be equal [Euclid, *Elements* III.28]. Which is what was proposed.

Diagram. The "first figure" repeated at the head of the page (the same construction as fig-p45-1): a bounding circle ABCD within a square frame, with a horizontal diameter and crossing great-circle arcs. Readable labels: A (top), B (left), D (right), C (bottom), G (upper interior), I (upper right), with further interior points E, F, H clustered in the lower half. for label verification.

Diagram. The "second figure" (posterior figura): a bounding circle within a square frame with a dense web of chords and projected arcs for the right-angle case of the lemma. Readable peripheral and interior labels:

- A (top)
- N (upper left), O (upper right)
- L (upper right, below O)
- M (upper interior), P (interior, beside M)
- H (right, upper), I (right, interior)
- B (left), D (right)
- K (lower right)
- E (lower left), F (bottom, center), G (bottom, right), Q (lower right)
- C (bottom)

Several interior labels sit among overlapping arcs and are only approximately placed. for label verification.

SCHOLIUM.

[Margin: Another demonstration of the whole lemma.]

Since, moreover, this lemma is one of the principal ones — having a wonderful use in dividing the circles of the astrolabe into degrees — I should like to demonstrate it by another method, so that its truth may become more evident. Let there again be, in the sphere, a greatest circle ABCD, described through A, C, the poles of the world (or of the Equator PKD), and through E, F, the poles of any oblique greatest circle GKI whatever. Let H be the center of the sphere and of all the greatest circles; AC the axis of the Equator; and EF the axis of the oblique circle. Since these axes are perpendicular to their own circles, they will [also] be perpendicular to the diameters of their proper circles BD, GI [Theodosius, *Sphaerica* I.10; Euclid, *Elements* XI, definition 3]; so that, by the scholium to *Elements* III.27, AB, BC, CD, DA and EG, GF, FI, IE are quadrants.

Let there also be described, from the poles C, F, four parallels — two from each pole — LMN, OPQ, RMS, TPV, which are to be equal. First, let a plane be understood to be drawn through C, the pole of the Equator, and through E, the pole of the oblique circle that is the more remote from C, making in circle ABCD the common section, the straight line CE, and on the surface of the sphere the circle C-a-b-E when it inclines toward the parts D, I, or the circle C-b-d-E when it inclines toward the parts B, G. Let the first circle cut the Equator and the greatest circle GKI at z and a, and the parallels LMN, TPV at g and h; and let the second circle cut those same circles at b and d, and at i and k, and the parallels OPQ, SMR at n and o, and at p and q. I say that the cut-off arcs Dz, Ia and Bz, Ga are equal; of which Dz begins from the superior semicircle, and Ia from the southern section, while Bz begins from the inferior semicircle, and Ga from the northern section. Likewise, for the same reason, the arcs Db, Id — or Bb, Gd — on the Equator and on the greatest oblique circle are equal. For a like reason I say that in the parallels LMN, TPV the arcs Ng, Vh — or Lg, Th — are equal; and likewise Ni, Vk — or Li, Tk; and finally, in the parallels OPQ, SMR, the arcs Qn, RO — or On, SO; and likewise Qp, Rq — or Op, Sq.

For, when the straight line DI has been joined: since the quadrants EI, CD are equal, then, taking away the common arc DI, the remaining arcs DE, IC will also be equal. Therefore, by the scholium to *Elements* III.27, CE and ID will be parallel; and consequently the angles HXY, HXZ will be equal to the angles HID, HDI, as exterior to interior [Euclid, *Elements* I.29]. Since, then, these latter are equal — in the isosceles triangle HDI — the former will be equal as well [Euclid, *Elements* I.5]; and therefore the straight lines XH, YH will be equal [Euclid, *Elements* I.6],

that is, the points Y, X will be equally distant from the center H. Let the planes of the circles C-a-b-E and C-b-d-E also make, on the Equator, the straight sections YZ, Yb; on the greatest oblique circle GKI, the straight sections Xa, Xd; and, on the parallels LMN, TPV, OPQ, SMR, the straight sections eg, ei; fh, fk; and rnp, soq [?].

And so, since the straight line CE falls upon the straight lines BD, GI lying in the plane of circle ABCD, making equal angles HXY, HX, and since upon the straight lines BD, GI stand the planes of the circles BKD, GKI, which are at right angles to the plane of circle ABCD [Theodosius, *Sphaerica* I.15]: the common sections YZ, Xa and Yb, Xd — of the planes C-a-b-E, C-b-d-E (drawn through CE) with the Equator and with the greatest oblique circle — will make with the diameters BD, GI, at the points Y, X, equal angles DYZ, IXa and DYb, IXd, by the preceding Lemma 22. Since, then, the points Y, X are equally distant from the center H, as has been shown, these same common sections YZ, Xa and Yb, Xd will, by Lemma 21, cut off from the circles BKD, GKI equal arcs DZ, Ia and Db, Id; and likewise BZ, Ga and Bb, Gd.

Again, when the straight line LT has been joined: since the straight lines drawn from the poles C, E to the points L, T of equal circles are equal [Theodosius, *Sphaerica* I.21 scholium], the arcs CL, ET will be equal [Euclid, *Elements* III.28]; and therefore, by the scholium to *Elements* III.27, TL and CE will be parallel; and so the angles Nef, Vfe will be equal to the angles NLT, VTL, as exterior to interior [Euclid, *Elements* I.29]. But the angles NLT, VTL are equal [Euclid, *Elements* III.27], because the arcs NT, LV on which they stand are equal. (For since the arcs TV, LN, which the diameters TV, LN of equal circles subtend, are equal [Euclid, *Elements* III.28], then, adding the common arc NV, the whole arcs NT, LV will become equal.) Therefore the angles Nef, Vfe will also be equal to one another. Moreover, since in the triangles Elf, Cme the angles E, C are equal [Euclid, *Elements* I.5], on account of the isosceles triangle CHE, and the angles l, m are right (because the axes EF, CA are perpendicular to their circles, and therefore also to the diameters of those same circles [Theodosius, *Sphaerica* I.10; Euclid, *Elements* XI, definition 3]), and since the straight lines El, Cm — the versed sines of the equal arcs ET, CL — are equal, as we demonstrated in treating the definitions of the sines, the straight lines lf, me will also be equal [Euclid, *Elements* I.26]; and therefore the points f, e will be equally distant from the centers l, m.

And so, since the straight line CE falls upon the straight lines LN, TV lying in the plane of circle ABCD, making equal angles Nef, Vfe, and since upon the straight lines LN, TV stand the planes of the circles LMN, TPV, which are at right angles to the plane of circle ABCD [Theodosius, *Sphaerica* I.15]: the common sections eg, fh

and ei, fk — of the planes C-a-b-E, C-b-d-E (drawn through CE) with the parallels LMN, TPV — make with the diameters LN, TV, at the points e, f, equal angles Neg, Vfh and Nei, Vfk, by the preceding Lemma 22. Since, then, the points e, f are equally distant from the centers m, l, as has been shown, those common sections eg, fh and ei, fk will, by Lemma 21, cut off equal arcs Ng, Vh and Ni, Vk; and likewise Lg, Th and Li, Tk.

Finally, when the straight line QR has been joined: since the whole arcs AE, FC are equal — because the angles AHE, FHC at the center are equal, being vertical angles [Euclid, *Elements* III.28] — and since the arcs AQ, FR that are cut off are likewise equal [Euclid, *Elements* III.28] — because the straight lines AQ, FR, falling from the poles A, F onto equal circles at Q, R, are equal [Theodosius, *Sphaerica* I.21 scholium] — the remaining arcs EQ, CR will also be equal; and therefore, by the scholium to *Elements* III.27, CE and QR will be parallel. Therefore the straight lines OQ, SR, produced, since they cut QR itself at Q, R, will also cut its parallel CE, produced, at r, S; and the angles OQR, SRQ will be equal to the angles Orf, Sfr, as exterior to interior [Euclid, *Elements* I.29]. But the angles OQR, SRQ are equal [Euclid, *Elements* III.27], because the arcs OR, SQ on which they stand are equal. (For since the arcs RS, QO, which the diameters RS, QO of equal circles subtend, are equal [Euclid, *Elements* III.28], then, adding the common arc OS, the whole arcs OR, SQ will become equal.) Therefore the angles Orf, Sfr will also be equal. Moreover, since in the triangles rtC, fuE the angles r, f have been shown equal, and the angles t, u are right (because the axes AC, FE are perpendicular to their circles, and therefore also to the diameters of those same circles [Theodosius, *Sphaerica* I.10; Euclid, *Elements* XI, definition 3]), and since the sides Ct, Eu are also equal — for, since (as we demonstrated in treating the definitions of the sines) the versed sines At, Fu of the equal arcs AQ, FR are equal, the remaining parts Ct, Eu of the diameters AC, FE will also be equal — the straight lines rt, fu will likewise be equal [Euclid, *Elements* I.26]; and therefore the points r, f will be equally distant from the centers t, u.

And so, since the straight line rf — that is, CE produced — falls upon the straight lines Or, Sf lying in the plane of circle ABCD, making equal angles Orf, Sfr, and since upon the straight lines Or, Sf stand the planes of the circles OPQ, SMR, which are at right angles to the plane of circle ABCD [Theodosius, *Sphaerica* I.15]: the common sections rnp, soq — of the plane C-b-d-E (drawn through CE) with the planes of the circles OPQ, SMR — will make, by the preceding Lemma 22, with the diameters OQ, SR (produced to the points r, f) equal angles Orn, Sfo, or Orp, Sfq. Since, then, the points r, f are equally distant from the centers t, u, as we have

shown, those common sections rn , so will, by Lemma 21, cut off from the circles OPQ , SMR equal arcs Qn , Ro and Qp , Rq ; and likewise On , So and Op , Sq .

Diagram. A large, very densely engraved geometric woodcut within a square frame — the figure for the alternative demonstration (Scholium). A bounding circle (the greatest circle $ABCD$) carries the Equator PKD and an oblique greatest circle GKI , four parallels (LMN , OPQ , RMS , TPV), the two projecting circles C - a - b - E and C - b - d - E , and their many chords and sections, drawn in solid and dotted lines. Readable capital labels around and within the figure: A and T (top), G , O (upper left), Q (upper), E (right), S (left), B (left), Z (right), D (right), V (right), F (lower left), L (lower left), M and X (center), N (lower right), R (bottom left), C (bottom), I (bottom). Numerous lowercase interior points (a , b , d , e , f , g , h , i , k , m , n , o , p , q , r , s , t , u , z , and others) mark the intersection points named in the text, but they are small, overlapping, and in several places abraded or ambiguous. — many labels (>2) cannot be fixed with confidence at this resolution.

[Scholium to Lemma XXIII, continued]

But if it should ever happen that the common section rn — which the plane drawn through CE makes with the circle OPQ — is tangent to the circle OPQ , then the other common section so will likewise be tangent to the circle SMR , as we demonstrated in Lemma 21. Therefore that plane drawn through CE will be tangent to both of the circles OPQ and SMR . The points of contact will be found if circles are described about the diameters OQ and SR , and tangent lines are drawn to them from r and s .

Next, imagine a plane drawn through C , the pole of the equator, and F , the nearer pole of the oblique circle. In the circle $ABCD$ it makes the common section, the straight line CF ; but on the surface of the sphere it makes the circle $CabdZF$, which cuts the equator at Z and b , the greatest oblique circle GKI at a and d , the parallel LMN at f , the circle SMR at h , the parallel OPQ at i and k , and finally the parallel TPV at l and m . I say that the arcs cut off — reckoning always, on the equator and its parallels, from the upper semicircle, and, on the greatest oblique circle and its parallels, from the northern intersection; or, on the former, from the lower semicircle and, on the latter, from the southern intersection, just as the proposition prescribed the construction to be made — namely the arcs BZ , Ia , Bb , Id ; DZ , Ga ; Db , Gd on the equator and on the greatest oblique circle GKI ; likewise Lf , Rh , Nf , Sh on the parallels LMN and SMR ; and finally Oi , Vl ; Ok , Vm ; Qi , Tl ; Qk , Tm on the parallels OPQ and TPV — are equal to one another.

Diagram. A large sphere set in a square frame, giving the full stereographic construction for this lemma: the equatorial great circle, the greatest oblique circle GKI, several parallels (LMN, SMR, OPQ, TPV), the poles of the equator and of the oblique circle, and dashed projection rays running from the projecting pole out to the points Y, X (and n, p) where the produced diameters meet the equatorial plane. The grid of circles and projection lines is extremely crowded and many peripheral labels are small.

- A, T (top, with G adjacent): pole / top of the sphere and neighbouring points
- S, k, P, m (upper right quadrant): parallel- and oblique-circle points
- h, n (left, with dashed rays running left to O): section points and projection rays
- O, K, u , H (centre): oblique-circle and centre labels
- Y, E, F, e (left of centre): projection point Y, centre E, pole F
- L, q , M (lower left): parallel LMN endpoints and centre q
- a, f (lower right): oblique-circle points
- R, C (bottom centre): pole/section
- X, P (bottom right, with dashed ray): projection point X and pole P
- [Several additional small lowercase labels along the parallels are not individually resolvable]

For, when the straight line BI has been joined, since the quadrants BC and FI are equal, then — the common arc CF being subtracted — the remaining arcs BF and CI will also be equal. Therefore, by the scholium to Proposition 27 of Euclid Book 3, the lines BI and CF will be parallel; and consequently the straight lines HB and HI, which cut BI, when produced will also cut its parallel CF, produced, at Y and X. [Margin: a = Euclid I.29] And therefore the angles HBI and HIB will be equal to the angles HYX and HXY, as exterior to interior. [Margin: b = Euclid I.5] But HBI and HIB are equal, being in the isosceles triangle HBI. [Margin: c = Euclid I.6] Therefore HYX and HXY will also be equal; and for that reason the straight lines HY and HX will also be equal — that is, the points Y and X will be equally distant from the centre H.

Let the plane of the circle $CabdZF$ also make: in the equator, the common section, the straight line Yzb ; in the circle GKI, the straight line Xad ; in the parallels LMN and SMR, the straight lines ef and gh ; and in the parallels OPQ and TPV, the straight lines nik and plm .

And so, since the straight line XY — that is, CF produced — falling upon the straight lines DY and GX, which lie in the plane of the circle ABCD, makes the

equal angles HYX and HXY ; and since upon the straight lines DY and GX stand the planes of the circles BKD and GKI , [Margin: $d =$ Theodosius, *Sphaerics* I.15] which are perpendicular to the plane of the circle ABCD : the common sections Yzb and Xad — of the plane of the circle CabdzF , drawn through CF , with the planes of the circles BKD and GKI — will make with the diameters DB and GI , produced at the points Y and X , equal angles DYb and GXd , by the preceding Lemma 22.

Therefore, since the points Y and X are equally distant from the centre H , as we have shown, the same common sections Yzb and Xad will, by Lemma 21, cut off from the circles BKD and GKI equal arcs Bz , Ia , Bb , Id ; likewise Dz , Ga ; Db , Gd .

Again, when the straight line LR has been joined [Margin: $e =$ scholium to Theodosius I.21]: since the straight lines falling from the poles C and F to the points L and R of equal circles are equal, [Margin: $f =$ Euclid III.28] the arcs CL and FR will also be equal; and, the common arc LR being subtracted, the remaining arcs CR and FL will be equal. Therefore, by the scholium to Proposition 27 of Euclid Book 3, the lines CF and RL will be parallel; [Margin: $g =$ Euclid I.29] and consequently the angles Neg and Sge will be equal to the angles NLR and SRL , as exterior to interior. [Margin: $h =$ Euclid III.27] But NLR and SRL are equal, because the arcs NR and SL on which they stand are equal. ([Margin: $i =$ Euclid III.28] For since the arcs NL and SR , which the diameters NL and SR subtend, are equal, then, the common arc LR being taken away, the remaining arcs NR and SL will also be equal.) Therefore the angles Neg and Sge will also be equal to one another. Moreover, since in the triangles eqC and grF the angles at q and r are right angles ([Margin: $k =$ Theodosius I.10] because the axes CA and FE are perpendicular to their own circles, and therefore also to the diameters of those circles, by Definition 3 of Euclid Book 11), and since the angles at e and g have been shown to be equal, and the straight lines Cq and Fr — the versed sines of the equal arcs CL and FR — are also equal, as we demonstrated when treating the definitions of sines: [Margin: $l =$ Euclid I.26] the straight lines eq and gr will also be equal; and therefore the points e and g will be equally distant from the centres q and r .

And so, since the straight line CF , falling upon the straight lines LN and RS , which lie in the plane of the circle ABCD , makes the equal angles geN and egS ; and since upon the straight lines LN and RS stand the planes of the circles LMN and RMS , [Margin: $m =$ Theodosius I.15] which are perpendicular to the plane of the circle ABCD : the common sections ef and gh — which the plane of the circle CabdzF , drawn through CF , makes in the planes of the circles LMN and RMS — will make with the diameters LN and RS , at the points e and g , equal angles feN and hgS , by

the preceding Lemma 22. Therefore, since the points e and g are equally distant from the centres q and r , as we have shown, the same common sections ef and gh will, by Lemma 21, cut off from the circles LMN and RMS equal arcs Lf and Rh ; likewise Nf and Sh .

Finally, when the straight line OV has been joined: since the quadrants CDFG are equal, and the arcs cut off, DV and GO, are likewise equal [Margin: n = Euclid III.28] ...

[The foot of p. 48 — the middle of this last ("Denique") demonstration, from roughly "*Nam arcus EV, AO, toti æquales sunt...*" through "*...erunt quoque rectæ nt, pu, æquales; ideoque puncta n, p, à centris t, u,*" — falls below the tiled region and was not captured; it is not transcribed here. Its marginal citations, printed alongside, are o = scholium to Theodosius I.21; p = Euclid III.27; q = Euclid I.29; r = Euclid III.27; s = Euclid III.28; t = Theodosius I.10; u = Euclid I.26; x = Theodosius I.15. The demonstration's conclusion resumes at the top of p. 49.]

have been demonstrated to be equally distant from the centres t and u ; the same common sections nik and plm will, by Lemma 21, cut off from the circles OPQ and TPV the equal arcs $Oi, Vl; ok, Vm$; likewise $Qi, Tl; Qk, Tm$.

But if it should ever happen that the common section Yzb — which the plane drawn through CF makes with the equator — is tangent to the equator BKD, then the other common section Xad will likewise be tangent to the oblique circle GKI, as we demonstrated in Lemma 21. Therefore in that case the plane drawn through CF will be tangent to both of the greatest circles BKD and GKI. The points of contact will be found if circles are described about the diameters BD and GI, and tangent lines to them are drawn from Y and X. By the same reasoning, if the common section nik — which the same plane drawn through CF makes with the circle OPQ — is tangent to the circle OPQ, then the other common section plm will likewise be tangent to the circle TPV, as was shown in Lemma 21. So in that case the plane drawn through CF will be tangent to both of the circles OPQ and TPV. And the points of contact will be found in the same way, if circles are described about the diameters OQ and TV, and straight lines are drawn from the points n and p to touch them.

This latter demonstration, moreover, can easily be adapted, if one wishes, to a great circle that is perpendicular to the equator, and to its parallels as well. But for the sake of brevity let us be content with the earlier demonstration, which also holds for circles perpendicular to the equator, as has been shown.

LEMMA XXIV.

If in a sphere there is an oblique circle, whether a great circle or not a great circle, and if through any point of its diameter — the diameter which a great circle drawn through its own poles and through the poles of the world produces — a straight line is drawn perpendicular to that diameter: then the plane drawn through either of the poles of the world and through that perpendicular will make, in the plane of the equator, a common section — namely a straight line perpendicular to the diameter of the equator, the diameter which that same great circle, drawn through the said poles, produces.

In the sphere ABCD, whose centre is E, let there be any oblique circle FGHI — that is, one not drawn through the poles of the world — whether a great circle or not a great circle. Through A and C, the poles of the world, and through O and P, the poles of the oblique circle, let the great circle ABCD be drawn. [Margin: *a* = Theodosius I.15] Since this circle cuts the oblique circle in half and at right angles, it will make as their common section the diameter of the oblique circle, FH; and to this, through any point K, let the perpendicular GKI be drawn. Through this perpendicular and through C, the pole of the world, let a plane be drawn, making on the surface of the sphere the circle CGQI, and in the plane of the equator BLDM — produced beyond the sphere too, if need be — the straight line LM, which cuts the diameter BD of the equator (also produced, if necessary, by the same great circle ABCD) at N. I say that LM is perpendicular to BD, produced if need be, at N.

[Margin: *b* = Theodosius I.15 (marginal digits at the resolution floor)] For since the oblique circle FGHI is perpendicular to the circle ABCD, then, by Definition 4 of Euclid Book 11, the straight line GKI — which is drawn perpendicular to FH, the common section of these circles — will be perpendicular to the plane of that same circle ABCD. [Margin: *c* = Euclid XI.18] Therefore the plane in which the circle CGQI lies, drawn through GI, will also be perpendicular to that same circle ABCD. [Margin: *d* = Theodosius I.15] Since, then, the plane of the equator BLDM is perpendicular to the plane of the circle ABCD — because it is drawn through the latter's poles (for since ABCD is drawn through A and C, the poles of the equator, the equator will in turn pass through the poles of ABCD, by the scholium to Proposition 15 of Theodosius Book 1) — and since it has also been shown that the plane of the circle CGQI is perpendicular to the plane of that same circle ABCD: [Margin: *e* = Euclid XI.19] then LM too, the common section of the plane of the equator and the plane of the circle CGQI, will be perpendicular to the plane of that same circle ABCD; and therefore, by Definition 3 of Euclid Book 11, that same

straight line LM will be perpendicular to the diameter of the equator BD — produced too, if need be — at N. Which is what was proposed.

Diagram. A sphere in a square frame illustrating Lemma XXIV: the great circle ABCD through the poles of the world A (top) and C (bottom), the oblique circle FGHI, its poles O and P, the great circle CGQI drawn through the pole C and the perpendicular GKI, and the equatorial diameter BD produced to meet the section LM at N (with M carried out to the right of the frame). The figure is densely ruled with intersecting great circles and projection chords.

- A (top): pole of the world
- Q, F, I, K, M, G (upper-left cluster): oblique circle FGHI and section points
- N, E (centre): centre E of the sphere and intersection N
- L, P (lower left): section endpoint and pole P
- B (left): equatorial diameter endpoint
- D, O (right of centre): equatorial diameter endpoint and pole O
- N (right, on the produced diameter), M (far right): where LM meets BD produced, carried outside the frame
- C (bottom): pole of the world
- [A small G / K / H / H cluster at the foot of the figure is crowded]

LEMMA XXV.

If in a sphere a great circle is drawn through the poles of the world and through the poles of any greatest oblique circle and of its parallels; and if in that great circle, from either pole of the world, a line is drawn parallel to the diameter of the oblique circle; and if through this line a plane is extended in any way whatever: then the two arcs — both of the greatest oblique circle and of any of its parallels — intercepted between the great circle drawn through the poles of the world and of the oblique circle, and the cutting plane, will be equal to one another.

In a sphere let the great circle ABCD be drawn through A and C, the poles of the world, and through E and F, the poles of the greatest oblique circle GHIK and of any parallel of it, GHIK; [Margin: f = Theodosius I.15] and accordingly cutting each of them in half, so that in each the semicircle is GHIK, and the diameter is GK. Parallel to this diameter, in the same great circle, let CL be drawn through C, the pole of the world; and through CL let a plane CLHI be drawn in any way, cutting either the greatest oblique circle or its parallel along the straight line HI. I say that, in the one as in the other, the arcs GH and KI, intercepted between the

cutting plane and the great circle ABCD, are equal. For if through the straight line CL a plane is imagined drawn parallel to the circle GHIK, [continues on p. 50]

parallel [to the circle GHIK]: [Margin: a = Euclid XI.10] then the sections made by the plane CLHI — namely the straight lines CL and HI — will be parallel. But the diameter GK is also assumed parallel to that same CL. [Margin: b = Euclid XI.9] Therefore GK and HI will be parallel to one another as well; and consequently, by the scholium to Proposition 27 of Euclid Book 3, the intercepted arcs GH and KI will be equal.

From this it follows that the arcs intercepted between any two planes drawn through CL are likewise equal. For each such plane cuts off equal arcs intercepted between itself and the great circle ABCD. Therefore, if the smaller are subtracted from the greater, the arcs remaining, intercepted between the two planes, will be equal.

This same demonstration fits the remaining semicircles on the other side of the great circle ABCD as well, as is evident.

Diagram. A circle in a square frame for Lemma XXV: the greatest oblique circle GHIK and a parallel of it (both lettered with G, H, I, K), the diameter GK, the chord/section HI, and the line CL, showing the equal intercepted arcs GH and KI.

- A, G (top): pole and oblique-circle point
- H, E, K (upper right): section and parallel points
- G (upper left): oblique-circle point
- I, B (left of centre): section point and frame point
- L (lower left): endpoint of CL
- D (right): frame point
- I, K (lower right): section and parallel points
- F (lower left), C (bottom): pole/endpoint labels

LEMMA XXVI.

If a circle in a sphere passes through either of the poles of the world, then its diameter drawn from that pole will be perpendicular to the common section of the plane of that circle and the plane of the equator.

In a sphere let AB be the equator, whose poles are C and D, and let CE be any circle drawn through the pole C, whose plane makes in the plane of the equator the common section, the straight line FG (for it will meet the equator, since it is not parallel to it); and from the pole C let the diameter of the circle CE be drawn,

meeting the common section FG at F. I say that CF is perpendicular to FG. [Margin: *c* = Theodosius I.20] For through H, the pole of the circle CE, and through C, the pole of the equator, let the great circle CHEADB be drawn; [Margin: *d* = Theodosius I.15] this will cut each of them in half and at right angles, and accordingly will pass through the diameter CE — that is, through the straight line CF. Therefore each plane, both that of the circle CE and that of the equator, will in turn be perpendicular to the plane of the great circle CHEADB; [Margin: *e* = Euclid XI.19] and consequently their common section FG will also be perpendicular to that same plane — that is, by Definition 3 of Euclid Book 11, perpendicular to the straight line CF. Which is what was proposed.

When the circle drawn through the pole C is a great circle, such as ABCD is, it is evident that its diameter CD is perpendicular to AB, the common section of the given circle and the equator. For since the diameter CD of a great circle drawn through the poles is the axis, [Margin: *f* = Theodosius I.10] and the axis is perpendicular to the equator and passes through I, the centre of the sphere, then, by Definition 3 of Euclid Book 11, that same diameter CD will be perpendicular to AB, the common section of the circle CADB and the equator (for this section is a diameter of the equator, [Margin: *g* = Theodosius I.11] since great circles bisect one another).

Diagram. A circle in a square frame for Lemma XXVI: the equator AB drawn horizontally, the circle CE through the pole C, the vertical diameter CD, the oblique line FG (the common section), and the centre I.

- D (top): pole/diameter endpoint
- G (upper left, on a line running out of the circle): endpoint of the section FG
- F (left, on the horizontal), A (just right of F): section endpoint and equator endpoint
- I (centre): centre of the sphere
- B (right): equator endpoint
- E (lower centre-left), H (lower left): circle-CE point and its pole H
- C (bottom): pole of the world

LEMMA XXVII.

In a right cone, all the straight lines drawn from the vertex to the circumference of the base are equal to one another. But in a scalene cone they are unequal: the least is the one drawn to the extremity of the base of the triangle-through-the-axis (that base being perpendicular to the base of the cone) on the side of the angle of

inclination of the axis; the greatest is the one drawn to the other extremity of the base of the same triangle through the axis; and the nearer any line is to the least, the smaller it always is. Only two will be equal, one on each side of the least, or of the greatest.

First, let ABC be a right cone, whose base is the circle BDCE, and whose axis AF, perpendicular to the base, meets it at the centre F; and let any number of straight lines be drawn from the vertex A to the circumference of the base: AB, AC, AD, AE. I say that they are all equal. For when the straight lines FB, FC, FD, FE have been drawn from the centre F: since the sides AF, FB are equal to the sides AF, FD, and contain equal angles — because all the angles at F which the axis AF makes are right angles, by Definition 3 of Euclid Book 11 — [Margin: h = Euclid I.4] the bases AB and AD will also be equal. In no other way will AD, or AB, be shown equal to AC, or AE. And the reasoning is the same for the rest.

Next, let ABC be a scalene cone, whose base is the circle BDECF, whose axis AG is oblique to the base toward B, and let the triangle ABC through the axis be perpendicular to the base; and from the vertex A let the perpendicular HA be dropped, which will fall on BC — that is, either on the point B, or between B and G, or outside the base. [Margin: i = Euclid XI.38 (first digit uncertain)] Now from the vertex A let any number of straight lines AB, AD, AE, AC be dropped, of which AB and AC fall on the extremities B and C of BC, the diameter of the base. I say that of all of them AB is the least and AC the greatest, and that AD is less than AF, and so on. For when the straight lines HD, HE have been joined, all the angles which the perpendicular AH makes with the straight lines HD, HE, HC, and with the others drawn through H, will be right angles, by Definition 3 of Euclid Book 11. In the first figure, then, it is evident that the perpendicular AH — or AB — is the least of all the lines drawn from A to the circumference of the base, [Margin: k = Euclid I.19] since it is less than AD, than AE, than AC, and than any other whatever — inasmuch as in right triangles it is subtended by acute angles, while the others are subtended by the right angle. But in the other two figures, [Margin: l = Euclid III.7 & 8] since HB is the least of the straight lines drawn from H to the circum[ference ...]

Diagram. A cone in a square frame for Lemma XXVII (the right-cone case): apex A at the top, the elliptical base BDCE with centre F, the axis AF, and the lines AB, AC, AD, AE drawn from the apex to the base circumference; chords FB, FC, FD, FE are drawn to the centre.

- A (apex, top): vertex of the cone
- D (on the left-hand slant, meeting the base): base-circumference point

- B (left of the base): base endpoint
- F (centre of the base): centre / foot of the axis
- C (right of the base): base endpoint
- E (lower right of the base): base-circumference point

falling on the circumference, the two squares on the lines HB, HA will be less than the two squares both of the lines HD, HA, of the lines HE, HA, and of the lines HC, HA. [Margin: ^a 47. pri. — Euclid, *Elements* I.47] For the square on the line AB equals the two squares on the lines HB, HA; the square on the line AD equals the two squares on the lines HD, HA; the square on the line AE equals the two squares on the lines HE, HA; and the square on the line AC equals the two squares on the lines HC, HA. Therefore the square on the line AB will also be less than the square on AD, than the square on AE, and than the square on AC; and consequently the line AB will be less than any of the lines AD, AE, AC, and so on for the rest. **AB is therefore the least of all.**

Next, [Margin: ^b 15. vel 7. vel 8. tert. — Euclid, *Elements* III.15 (or 7, or 8)] since in all the figures the line HC is the greatest of all those falling from H onto the circumference, the two squares on the lines HC, HA will be greater than the two squares both of the lines HE, HA and of the lines HD, HA. [Margin: ^c 47. pri. — *Elements* I.47] Now the square on the line AC equals the two squares on the lines HC, HA; the square on the line AE equals the two squares on the lines HE, HA; and the square on the line AD equals the two squares on the lines HD, HA. Therefore the square on the line AC will be greater than both the square on AE and the square on AD; and consequently the line AC will be greater than AE and than AD. And since it is greater even than AB — for AB has been shown to be the least of all — **AC is therefore the greatest of all.**

Again, [Margin: ^d 15. vel 7. vel 8. tert. — *Elements* III.15 (or 7, or 8)] since HD is less than HE, the two squares on the lines HD, HA will be less than the two squares on the lines HE, HA. [Margin: ^e 47. pri. — *Elements* I.47] Now the square on the line AD equals the two squares on the lines HD, HA, and the square on the line AE equals the two squares on the lines HE, HA. Therefore the square on the line AD will be less than the square on the line AE; and so the line AD, being nearer to the least line AB, will be less than the more distant AE, and so on for the rest.

Finally, let the arc BF be taken equal to the arc BD, and let the line HF be joined, which will be equal to the line HD — in the first figure by *Elements* III.29, in the second by the last proposition of the scholium of the same proposition, or by lemma 21 demonstrated above, and in the third likewise by the same lemma 21. Then, the line AF having been drawn, since the sides AH, HF are equal to the sides

AH, HD and contain right angles (by *Elements* XI, def. 3), [Margin: ^f 4. primi — *Elements* I.4] the bases AF, AD will also be equal. That no other line among these can be equal to them is clear, since every line drawn from A between D and C, or between F and C, is greater than AD or AF, whereas one drawn between B and D, or between B and F, is less, as has been demonstrated.

Diagram. Three cones drawn side by side, illustrating the equal and unequal slant-lines of Lemma XXVII. Each figure shows an apex A above an elliptical base, with several slant lines drawn from A down to labelled points on the base circle (the lines HB, HD, HE, HF, HC of the proof).

- A (apex, top of each figure): vertex of the cone
- B, H (left end of the base): foot of the base diameter and the foot of the perpendicular from A
- C (right end of the base): other end of the base diameter
- F (upper part of the base circle): point on the circumference
- G (centre of the base): centre of the base circle
- D (lower-left of the base): point on the circumference
- E (bottom of the base): point on the circumference

(The middle and right figures repeat the same lettering — A apex; B, H, C on the base diameter; F, G, D, E on the circumference — for the successive cases of the lemma.)

LEMMA XXVIII.

If in a cone there is a circle equidistant from the base, straight lines drawn from the vertex along the conic surface will cut off similar arcs from the base and from the equidistant circle.

In the cone ABC, whether right or scalene, let the circle EF be equidistant from the base BC, and from the vertex A let two lines AH, AK be drawn in any way to the circumference of the base, cutting the circumference of the circle EF at I, L. I say that the arcs HK, IL are similar. For, the axis AD having been drawn, cutting the plane of the circle EF at the point G — which by lemma 16 will be the centre of the circle EF — let a plane be drawn through the lines AD, AH cutting the parallel circles BC, EF along the lines DH, GI; likewise, through the lines AD, AK let another plane be drawn cutting the same circles along the lines DK, GL. [Margin: ^g 16. unde — *Elements* XI.16] Then the lines DH, DK will be parallel to the lines GI, GL. [Margin: ^h 10. unde — *Elements* XI.10] Therefore the angles HDK, IGL

at the centres will be equal; and so, by the scholium of *Elements* III.22, the arcs HK, IL will be similar. By the same reasoning the arcs BH, EI will likewise be similar, as also the arcs CK, FL, [Margin: ⁱ 16. unde — *Elements* XI.16] because the lines DB, DH are parallel to GE, GI, and the lines DC, DK to GF, GL; [Margin: ^k 10. unde — *Elements* XI.10] and consequently the angles BDH, EGI, as well as CDK, FGL, at the centres are equal.

The same follows if the circle EF is set as the base of the cone and, below it, the circle BC parallel to it, as is clear from the demonstration.

Therefore, if either of the circles EF, BC is divided into equal parts, and from the vertex A straight lines are sent out through the points of division, the other circle also will be cut into equal parts.

Diagram. A cone with vertex A and an upper circle EF parallel to (equidistant from) the lower base circle BC; straight lines are drawn from A through both circles, cutting off corresponding arcs.

- A (apex, top): vertex of the cone
- Upper (equidistant) circle: E (left), G (top centre — centre of the circle), F (right), I (left, on the near circumference), L (centre-lower)
- Lower (base) circle: B (left), D (centre — centre of the base), C (right), H (lower-left, on the circumference), K (bottom, on the circumference)

LEMMA XXIX.

If two straight lines touch one another at one point, and from any point outside them several straight lines are drawn in the same plane that cut them, then the segments of the more remote line — proceeding from the assumed point toward the point of section of the given lines — will have a greater ratio than the segments of the nearer line.

Let the two lines AB, AC touch or cut one another at A, and from the point D let any number of lines be drawn — DA, DE, DF, DG, DB — cutting both. I say that the ratio of BG to GF is greater than that of CK to KI; that the ratio of GF to FE is greater than that of KI to IH; and that the ratio of FE to EA is greater than that of IH to HA. For, through I let IM be drawn parallel to AB, cutting the lines DB, DG at M, N; and through M let ML be drawn parallel to DG, which will cut the line AC produced at L. For since MD and ND meet at A, ML will fall as parallel to ND outside the triangle DMN. Since, then, as BG is to GF, so is MN to NI (by the scholium of *Elements* VI.4), [Margin: ^a 2. sexti — *Elements* VI.2] and as MN is to NI,

so is LK to KI, it will also be that as BG is to GF, so is LK to KI. [Margin: ^b 8. quin — *Elements* V.8] Now LK has a greater ratio to KI than CK has to KI. Therefore BG will also have a greater ratio to GF than CK to KI.

In the same way, if through H a line HP is drawn parallel to AB, cutting DG, DF at P, Q, and through P a line PO is drawn parallel to DF, cutting AK produced at O, then as GF is to FE, so will PQ be to QH (by the scholium of *Elements* VI.4); [Margin: ^c 2. sexti — *Elements* VI.2] and as PQ is to QH, so is OI to IH. Therefore it will also be that as GF is to FE, so is OI to IH. [Margin: ^d 8. quin — *Elements* V.8] Now OI has a greater ratio to IH than KI has to IH. Therefore GF will also have a greater ratio to FE than KI to IH.

And so one must proceed with the remaining segments, if there are more, until one arrives at the last two, FE, EA. For then the line must not be drawn through A parallel to AB, but only through F must FR be drawn parallel to DE, cutting AI produced at R. [Margin: ^e 2. sexti — *Elements* VI.2] For again, as FE is to EA, so will RH be to HA; [Margin: ^f 8. quinti — *Elements* V.8] and RH has a greater ratio to HA than IH has to HA. Therefore FE will also have a greater ratio to EA than IH to HA. Which was to be proved.

Diagram. The figure for Lemma XXIX. Along a horizontal top line lie the points A, E, F, G, B (left to right); the point D lies at the lower left, from which several straight lines fan out and cut the two lines meeting at A. A network of the auxiliary parallels (IM, ML, HP, PO, FR) crosses the figure.

- A (top left, on the top line): meeting-point of the two given lines
- E, F, G (along the top line): successive section points
- B (top right, on the top line): endpoint
- D (lower left): the assumed external point from which the transversals are drawn
- H, I, K, L (middle band): section points on the nearer line and their images
- M, N (middle): points where IM cuts DB, DG
- P, Q (middle): points where HP cuts DG, DF
- O (mid-right): where PO cuts AK produced
- R, S (mid-centre) [S?]: auxiliary points

LEMMA XXX.

If two isosceles triangles have equal bases, but the sides of one are greater than the sides of the other, then the smaller sides will contain the greater angle. And if

the sides of one are greater than the sides of the other and contain a greater angle, then the base of the former will be greater than the base of the latter.

Let two isosceles triangles ABC, DEF have equal bases BC, EF, but let the sides DE, DF be greater than the sides AB, AC. I say that the angle A is greater than the angle D. [Margin: § 23. primi — *Elements* I.23] For let there be described on the base EF a triangle EGF, equilateral and equiangular with the triangle ABC; and the point G will fall inside the triangle DEF. For if it fell outside, either the lines EG, FG would enclose the lines ED, FD, [Margin: ^h 21. primi — *Elements* I.21] and thus the sides GE, GF — that is, AB, AC — would be greater than the sides DE, DF, which is contrary to the hypothesis; or one of them would cut the other of the lines DE, DF, and thus one of the angles GEF, GFE would be greater than one of the angles DEF, DFE, and the other less. [Margin: ⁱ 5. primi — *Elements* I.5] Since, then, DEF, DFE are equal, the angles GEF, GFE would be unequal, which is absurd, [Margin: ^k 5. primi — *Elements* I.5] since they are equal to one another. The same would follow if the point G were said to fall on either of the lines DE, DF. Nor indeed can it be said to fall on D, for then the sides DE, DF would be equal to the sides AB, AC, which conflicts with the hypothesis. [Margin: ^l 21. primi — *Elements* I.21] The point G therefore falls inside the triangle DEF; and so the angle G — that is, the angle A — will be greater than the angle D. Which was to be proved.

Diagram. Two isosceles triangles for the first part of Lemma XXX. The left triangle is ABC (apex A, base B–C); the right figure shows the triangle DEF (outer apex D) with the equilateral–equiangular triangle EGF (inner apex G) described on its base E–F.

- Left triangle: A (apex), B (lower left), C (lower right)
- Right figure: D (outer apex), G (inner apex, described triangle), E (lower left), F (lower right)

Diagram. Figures for the second part (converse) of Lemma XXX. A large isosceles triangle ABC (apex A, base B–C) has the segment GH drawn parallel to the base (G on side AB, H on side AC); a smaller isosceles triangle DEF (apex D, base E–F) stands to the right.

- Large triangle: A (apex), G (on side AB), H (on side AC), B (lower left), C (lower right)
- Small triangle: D (apex), E (lower left), F (lower right)

Again, let the two sides AB, AC of the isosceles triangle ABC be greater than the two sides DE, DF, and let the angle A be greater than the angle D. I say that the base BC is greater than the base EF. For, the lines AG, AH having been cut off

equal to DE, DF, [Margin: ^m 2. sexti — *Elements* VI.2] the line GH drawn will be parallel to BC. [Margin: ⁿ 4. sexti — *Elements* VI.4] Therefore, as AB is to BC, so is AG to GH. Now AB is greater than AG. [Margin: ^o 14. quin — *Elements* V.14] Therefore BC will also be greater than GH. Likewise, since the sides AG, AH are equal to the sides DE, DF, and the angle A is greater than the angle D, [Margin: ^p 24. pri — *Elements* I.24] the base GH will be greater than the base EF. But BC has been shown to be greater than GH. Therefore BC is much greater than EF. Which was to be proved.

LEMMA XXXI.

If in a scalene cone a circle is set subcontrariwise to the base, then straight lines drawn from the vertex along the conic surface — one of which is the side of the axial triangle that stands perpendicular to the base — will cut off dissimilar arcs from the base and from that circle. And if on one of them two opposite equal arcs are cut off, on the other two unequal arcs will be cut off: the greater indeed toward the smaller angle of the axial triangle, the lesser toward the greater angle.

In the scalene cone ABC, let the axial triangle be ABC, at right angles to the base BC, and let the circle of the subcontrary section be DE. Its diameter DE having been bisected at F, let GH be drawn through F parallel to the base BC, and through GH let a plane be drawn at right angles to the axial triangle (or parallel to the base of the cone), producing, by lemma 17, the circle GIHK, which cuts the circle of the subcontrary section at I, K. Let there first be drawn two lines AL, AM through I, K, the common sections of the circles DIE, GIH, cutting the base at L, M. I say that the arcs BL, DI, as well as BM, DK, and also CL, EI, and CM, EK, are dissimilar. For let the planes of the circles DE, GH cut one another along the line IK. And since each circle is at right angles to the triangle ABC, [Margin: ^a 19. unde — *Elements* XI.19] their common section IK will also be perpendicular to that same triangle; [Margin: ^b 38. unde — *Elements* XI.38] and therefore it will fall both on DE, the common section of the circle DIEK and of the triangle ABC, and on GH, the common section of the circle GIHK and of the same triangle ABC, and so will pass through the point F, where these common sections divide one another; and it will make (by *Elements* XI, def. 3) the angles DFI, GFI right angles. And since the diameter DE is bisected at F, the diameter GH will be greater, and its greater part FG will incline toward the smaller angle AGH, as we demonstrated in the scholium of lemma 17; and therefore the centre of the circle GIHK will lie on the line FG. Let it be N. Then the segment IGK will be greater than a semicircle; but IDK is a semicircle, because F is the centre of the circle DIEK. Therefore both the

arcs IGK, IDK and the arcs IHK, IEK are dissimilar; and IGK is too great to be similar to the arc IDK, while IHK is too small to be similar to the arc IEK. And since the semicircles IDK, IEK are bisected at D, E — because, by the next-to-last proposition of the scholium of *Elements* III.27, on account of the right angles at F, the four arcs DI, IE, EK, KD are quadrants — likewise the arcs IGK, IHK are bisected at G, H. For the line NF, dividing the line IK from the centre N at right angles, [Margin: ^c 3. tertij — *Elements* III.3] cuts it in half. Therefore the arc IHK will also be bisected, by the last proposition of the scholium of *Elements* III.27; and consequently the remaining arcs GI, GK, taken from the semicircles, will be equal. Therefore the arcs GI, GK — the halves of the arc IGK — are too great to be similar to the arcs DI, DK, which are the halves of the arc IDK; while HI, HK — the halves of the arc IHK — are too small to be similar to the arcs EI, EK, which are the halves of the arc IEK. And since the arcs BL, BM, CL, CM are similar to the arcs GI, GK, HI, HK (by lemma 28), the arcs BL, BM, CL, CM will in the same way be dissimilar to the arcs DI, DK, EI, EK.

Diagram. A scalene cone with vertex A (upper right) shown with its base circle and an inner circle set subcontrariwise (the subcontrary section); many straight lines are drawn from A through both circles, together with the chords and common sections used in the proof.

- A (apex, upper right): vertex of the cone
- Inner (subcontrary / GIHK) circle: D (upper left), K, L (upper), G (left), N (centre), I (small, centre), H (right), T (lower left), E (lower right), M (lower centre)
- Base circle: O (upper left), B (left), C (right), P (bottom left), L (bottom centre), Z (bottom right-centre)

Next let another line AP be drawn, cutting the circumference of the base and the subcontrary section at R, and the circle GH at T; and from R let a perpendicular RY be dropped to the diameter DE, which, when produced, cuts the circumference on the other side at S; and from A through S let the line AS be drawn, cutting the circumference of the base at O and the circle GH at V. I say that the arcs BP, BO are likewise dissimilar to the arcs DR, DS, and the arcs CP, CO to the arcs ER, ES. For since RS is perpendicular to the triangle ABC (by *Elements* XI, def. 4), inasmuch as it is a perpendicular drawn to DE, the common section of the triangle ABC and the circle DRE — which circle is at right angles to that triangle — [Margin: ^d 18. unde — *Elements* XI.18] the triangle ARS, drawn through RS, will likewise be at right angles to the same triangle ABC, and will make in the circle GH the common section TV, cutting the diameter GH at X. Since, then, both

the plane of the circle GH and the triangle ARS are at right angles to the triangle ABC, [Margin: ^e 19. unde — *Elements* XI.19] their common section TXV will also be perpendicular to it; [Margin: ^f 3. tertij — *Elements* III.3] and so, by *Elements* XI, def. 3, the angles at X will be right angles; and thus each of RS, TV will be bisected at Y, and therefore each arc RDS, TGV, by the last proposition of the scholium of *Elements* III.27, will also be bisected; and consequently both the remaining arcs ER, ES and HT, HV will be equal, from the semicircles.

Now, if a line is drawn from A to X, it will pass through Y. For since that line, lying in the plane of the triangle ABC, cuts the line DE, which lies in that same triangle, and, lying also in the triangle ATV, cuts the line RS, which lies in that same triangle — while the only point of the line RS that lies in the triangle ABC is Y (because RS is perpendicular to that triangle) — it will pass entirely through the point Y. Therefore, the line AN having been drawn to N, the centre of the circle GH, cutting the semidiameter DF at i, the ratio of GX to XN will be greater, by lemma 29, than that of DY to Yi; [Margin: ^g 8. quin — *Elements* V.8] and DY has a greater ratio to Yi than to YF. Therefore GX will have a much greater ratio to XN than DY to YF. [Margin: ^h 10. sexti — *Elements* VI.10] If, then, GN is cut at Q so that GQ is to QN as DY is to YF, the point Q will fall between G and X. For if it fell beyond X, the ratio of GQ to QN would be much greater than that of GX to XN, since then GQ would be greater than GX, and QN less than XN. And since, by lemma 7, if through Q a perpendicular were drawn to GH, or a parallel to TV, it would cut off an arc similar to the arc RDS, the arc TGV will be too great to be similar to the arc RDS; and so the halves GT, GV are too great to be similar to the halves DR, DS, and therefore the remaining arcs HT, HV, from the semicircles, will be too small to be similar to the remaining arcs ER, ES, from the semicircles. And since, by lemma 28, the arcs BP, BO, CP, CO are similar to the arcs GT, GV, HT, HV, the arcs BP, BO, CP, CO will in the same way be dissimilar to the arcs DR, DS,...

ASTROLABIUM — Liber Primus (continued)

Lemma XXXI (conclusion)

DR, DS, ER, ES are dissimilar. In the same way we shall show that, wherever the perpendicular TV cuts the semidiameter GN, and the perpendicular RS cuts the straight line Di, the arc cut off by the perpendicular TV is greater than would make it similar to the arc which the perpendicular RS then cuts off, and so on.

But if the perpendicular TV passes through the center N — and accordingly the perpendicular RS through the point i — it is clear that the arc cut off by the

former is greater than would make it similar to the arc cut off by the latter, since the former is a semicircle whereas the latter is less than a semicircle. And by the same reasoning, if the perpendicular TV cuts GF beyond the center N but short of F — and therefore the perpendicular RS cuts the semidiameter DF beyond i but short of F — then from the circle GH an arc greater than a semicircle will be cut off, and from the circle DE one less than a semicircle; and so the former will be greater than would make it similar to the latter.

The opposite will happen if, on the side of the other semicircle IEK, any straight line Ab is drawn from the vertex A, cutting the circle GH in d, and a perpendicular bg is dropped to DE, cutting the circumference on the other side at the point c, through which a straight line is drawn from the vertex A cutting the circle GH in e. [Margin: Euclid XI.18] For this triangle Abc will be perpendicular to the triangle ABC, since it is drawn through the straight line bg, which is perpendicular to the triangle ABC; and with the circle GH it will make a straight section de, which cuts GH in f. Therefore, since both the plane of the circle GH and that of the triangle Abc are perpendicular to the triangle ABC, [Margin: Euclid XI.19] their common section de will also be perpendicular to the triangle ABC; and hence, by definition 3 of Euclid's Book XI, it is perpendicular to the straight line GH at f as well. [Margin: Euclid III.2] Each of bc and de will therefore be bisected at g and f; and hence, by the last proposition of the scholium on proposition 27 of Euclid's Book III, each arc bEc and dHe will be bisected at E and H; and the straight line Ag, when drawn, will pass through the point f. For the demonstration here is exactly the same as that in the triangle ARS: since the straight line Ag, lying in both planes — that of the triangle ABC and that of the circle GH — cuts each of the straight lines GH and de that lie in those planes, and therefore passes through their common section f; for f is the only point of the line de that, the line being perpendicular to the triangle ABC, lies in the triangle ABC.

Therefore, by Lemma 29, the ratio of Eg to gF will be greater than that of Hf to fF. [Margin: Euclid V.8] But the ratio of Hf to fF is greater than that of Hf to fN. Much greater, then, will be the ratio of Eg to gF than that of Hf to fN; and accordingly the arc bEc will be greater than would make it similar to the arc dHe — which will be shown just as it was proved that the arc TGV is greater than would make it similar to the arc RDS, because the ratio of GX to XN was greater than that of DY to Yf. Therefore the halves Eb, Ec too will be greater than would make them similar to the halves Hd, He; and so the remaining arcs Db, Dc, drawn from the semicircles, will be less than would make them similar to the remaining arcs. And since, when the straight lines Ab, Ac are produced to the base, the arcs Cz, Ca, Bz,

Ba are similar to the arcs Hd, He, Gd, Ge by Lemma 28, in the same manner the arcs Eb, Ec, Db, Dc will be dissimilar.

CAETERUM (Moreover). That, on the side of the semicircle IEK, larger arcs are cut off by the straight lines drawn from the vertex A than would make them similar to the arcs cut off from the base BC — that is, to the arcs cut off from the circle GH, since these are similar to the arcs of the base by Lemma 28 — this too we shall easily demonstrate in the following way. Let a straight line bc be drawn anywhere perpendicular to the diameter DE, and from the vertex A let the straight lines Ab, Ac be dropped, cutting the circle GH in d, e, and let the straight line d,e be joined. [Margin: Euclid I.28] And since IK and bc are parallel, on account of the right angles at F, g, two parallel planes can be drawn through them. Let a plane, then, be conceived as drawn through IK parallel to the triangle Abc; [Margin: Euclid XI.16] and in these parallel planes the plane of the circle GIHK will make the parallel sections IK, de. Since, then, bc has been shown parallel to that same IK, [Margin: Euclid XI.9] bc and de will also be parallel. Therefore, by the corollary of proposition 4 of Euclid's Book VI, the triangle Ade will be similar to the triangle Abc. [Margin: Euclid VI.4] Hence Ab will be to bc as Ad is to de. Since, then, Ab is greater than Ad, [Margin: Euclid V.14] bc will also be greater than de. And so, since the circle DE is smaller than the circle GH — because the diameter DE has been shown to be smaller than the diameter GH — bc, the greater line, will cut off from the smaller circle DE a greater arc bEc than de cuts off from the greater circle GH; from those things which are proved in the lemma of proposition 6, Book [III?], the halves Eb, Ec will be greater than would make them similar to the halves Hd, He. [Margin: Euclid I.29] Since [each arc bEc, dHe] is bisected at E, H by the last proposition of the scholium on proposition 27 of Euclid's Book III, at right angles by the construction; likewise the diameter GH cuts de at right angles; and IK is cut at right angles by GH, as we showed above; and because DE, GH are perpendicular to the triangle ABC of right angles, [their common section] is perpendicular to that same triangle, by definition 3 of Euclid's Book XI; [Margin: Euclid III.3] and accordingly each of bc, de is likewise bisected. So, since the arcs Cz, Ca are similar by Lemma 28, the arcs Eb, Ec will also be [greater than would make them similar to] Ca, and the remaining Db, Dc, drawn from the semicircles, less than would make them equal to the remaining arcs.

Diagram. A scalene cone in a rectangular frame, with a subcontrary section. The apex is A (top right); the base is a large ellipse at the bottom with B at the lower left and C at the lower right, and the base-points P, L, Z along the lower rim. A pencil of straight lines runs from the apex A to points on the base. An oblique circle (the subcontrary section) crosses the cone in the upper-middle region,

drawn as an ellipse with G at its left and H at its right and its center N; within it lie the section-ellipse ends D (upper) and E (lower right), with interior points T, K, X and the foot F. Interior base-line points O, M and i are marked. This is the scalene-cone construction of Lemma XXXI.

- A (top right): apex of the cone
- B (lower left): left end of the base
- C (lower right): right end of the base
- G (left, oblique circle): left end of the section-circle GH
- H (right, oblique circle): right end of the section-circle GH
- N (center of oblique circle): center of GH
- D (upper, on the oblique section): upper end of the inner section
- E (lower right, on the oblique section): lower end of the inner section
- T (below G): point on the section-circle
- K (upper, near H): point on the section-circle
- X (interior of GH): section point
- O (base, interior left): point on the base
- M (interior, center): point on the base line
- i (interior, near base): foot on the base
- F (near E, lower): foot of the perpendicular
- P (base, lower left): base point
- L (base, lower center): base point
- Z (base, lower right): base point

EX HIS OMNIBUS (From all this). From all this it is established that any arc of either circle intercepted between the side of the triangle through the axis and any straight line dropped from the vertex is greater than would make it similar to the arc of the other circle intercepted between the same straight lines, taken as far as the end of the semicircle. For it has been demonstrated in this way that the arcs BP, BL, BZ are greater than would make them similar to the arcs DR, DI, Db; likewise the arcs Eb, EI, ER are greater than would make them similar to the arcs CZ, CL, CP; and the same holds of the rest. And so, if the arc of the semicircle BLC responding to one degree be cut off, [it will be greater than one degree]; and the arc responding to two degrees will be greater than two degrees; and so on continually to the end of both semicircles DB, in the arcs. So too, if the semicircle CLB be cut into its own degrees, the individual arcs of the semicircle EID, the starting point always made from the points E, C, will be greater than 1, 2, 3, 4, 5, 6, and so on, degrees.

POSTREMO (Finally). Finally, let the opposite arcs DR, E[c] be equal, and let the straight lines ARP, Aca be drawn, cutting the circle GH in...

GH, in T, e. I say that the arcs BP, Ca are unequal — BP being the greater, Ca the smaller. For, two other arcs DS, Eb being taken equal to DR, Ec themselves, let the straight lines RS, bc be joined, and through S, b let two straight lines AS, Ab be drawn, cutting the base in O, Z and the circle GH in V, d, and let the straight lines TV, de be joined. Then, as we demonstrated a little before, bc and de will be parallel. For since the arcs Eb, Ec are equal, the remaining arcs bI, cK will also be equal, out of the quadrants. Therefore, by the scholium on proposition 27 of Euclid's Book III, IK and bc are parallel. And so, if a plane parallel to the triangle Abc — drawn through bc — be conceived as passing through IK, [Margin: Euclid XI.16] it will make in these parallel planes, on the plane of the circle GH, the parallel sections IK, de. Since, then, bc has been shown parallel to that same IK, [Margin: Euclid XI.9] bc and de will also be parallel. In the same way RS, TV will be parallel; and accordingly both the triangles Abc, Ade and the triangles ARS, ATV will be similar, by the corollary of proposition 4 of Euclid's Book VI. But Abc and ARS are isosceles, because, by Lemma 27, both Ab, Ac and AR, AS, being equally distant from the least line AD, are equal. Therefore Ade and ATV are isosceles as well, [being equally distant from the greatest line AE]. And since the sides AR, AS are smaller than the sides Ab, Ac, by Lemma 27, [Margin: Euclid III.29] while the base RS is equal to the base bc (on account of the equal arcs RDS, bEc), the angle RAS will, by the preceding Lemma 30, be greater than the angle bAc. Since, then, by Lemma 27, the sides Ad, Ae are smaller than the sides Ab, Ac, the base TV will, by the preceding Lemma 30, be greater than the base de; and therefore, by the scholium on proposition 28 of Euclid's Book III, the arc TGV will be greater than the arc dHe. But since TV has been shown parallel to IK, and GH cuts IK at right angles, [Margin: Euclid I.29] TV too will be cut at right angles, and in half at X; and accordingly, by the last proposition of the scholium on proposition 27 of Euclid's Book III, the arc TGV too will be bisected at G. And by the same reasoning the arc dHe will be bisected at H. Since, then, the arc TGV has been shown greater than the arc dHe, the halves GT, GV will also be greater than the halves Hd, He. But similar to these four arcs, by Lemma 28, are the four arcs BP, BO, CZ, Ca. Therefore BP, BO too will be greater than Cz, Ca. By the same reasoning, if the arcs BP, Ca be assumed equal, we shall show that Ec is greater than DR. For, the same construction being made, the angle dAe will be greater than the angle TAV, and the base Bc greater than the base RS, and so on.

IMMO (Indeed). Indeed, this latter part, once the former is demonstrated, can easily be deduced in the following way. Let the opposite arcs DR, EC again be

equal. I say that the arc GT is greater than the arc He. For since GT is greater than would make it similar to the arc DR, the same arc GT will also be greater than would make it similar to the arc EC — that is, more degrees are contained in GT than in Ec. But more degrees are contained in Ec than in He, since Ec is greater than would make it similar to the arc He. Therefore far more degrees will be contained in GT than in He, and accordingly, in the same circle, the arc GT will be greater than the arc He. And since GT, He are similar to the arcs BP, Ca by Lemma 28, BP too will be greater than Ca.

SIT RURSUS (Again). Again, let GT be equal to He. I say that Ec is greater than the arc DR. For in the same way there will be more degrees in Ec than in He — that is, than in GT. But there are still more in GT than in DR. Therefore there will be far more in Ec than in DR; and so Ec, in the same circle, will be greater than DR, and so on.

ITAQUE (And so). And so the individual arcs of the semicircle BLC, from B up to L — which point corresponds to the point I on the quadrant DI — are greater than the individual equal arcs corresponding to them from C up to L. For the arcs of the circumference CL are equal to the arcs of the circumference CM, which are opposite to the arcs of the circumference BL and have been shown to be smaller than the arcs of the circumference BL. In the same way, the individual arcs of the semicircle EID, from E up to the point that corresponds to the midpoint of the semicircle CLB, are greater than the individual equal corresponding arcs from D up to the same point that corresponds to the midpoint of the semicircle CB.

LEMMA XXXII.

If, on the diameter of a circle, some point other than the center is taken, and straight lines are drawn from it so as to intercept two equal arcs on the circumference of the circle, then the angles they contain will be unequal, and the greater will be the one whose lines lie farther from the center. And if the straight lines drawn contain equal angles, the intercepted arcs will be unequal, and the greater will be the one whose lines lie nearer to the center.

In the circle ABC, whose center is D, on the diameter AC, let three straight lines EC, EF, EB first go out from a point E other than the center, intercepting two continuous equal arcs CF, FB — whether their starting point C be at the end of the diameter or not. I say that the angle CEF is greater than the angle FEB. For, the chord CB being drawn, [Margin: Euclid IV.5] let a circle be circumscribed about the triangle BCE, which will cut the circle ABC in B, C, [Margin: Euclid III.13] since it cannot be tangent to it at those two points. Now let the straight line DF be

drawn and produced until it cuts the circle BCE in G; since the arc BFC is bisected at F, the straight line BC will also be bisected, at right angles, by the scholium on proposition 3 of Book III; and accordingly, by the corollary of proposition 1 of Book III, [DF produced] passes through the center of the circle BGC, and by the same scholium the arc BGC will be bisected at G. Therefore, the straight line EF being produced until it cuts the arc BGC in H, the arc BG — that is, CG — will be greater than the arc BH. Much greater, then, will be the arc CH than the arc BH. Therefore, by the scholium on proposition 27 of Euclid's Book III, the angle CEH will be greater than the angle BEH. Which was to be proved.

Diagram. The first figure of Lemma XXXII, in a rectangular frame: two overlapping circles. The larger is the circle ABC with vertical diameter AC (A at top, C below), center D, and the point E on the diameter below D; the smaller circle is described about the triangle BCE and passes through B, C, E, meeting the larger circle at B and C. From E straight lines run to the circumference; the construction points F, G, H mark the equal-arc bisection (chord CB, the line DF produced to G, EF produced to H). The letters F, G, H appear at two positions, illustrating the construction for both the continuous and the alternative case.

- A (top): end of the diameter
- B (upper-left, on circle ABC): endpoint of the intercepted arcs
- C (mid-left / lower, on circle ABC): starting point on the diameter
- D (center): center of circle ABC
- E (center, below D): the chosen point on the diameter
- F (mid-left and lower): bisection point of arc BC / CF
- G (lower-left, two positions): point where DF meets circle BGC
- H (upper-left and lower-left): point where EF produced meets the arc

Diagram. A single circle in a rectangular frame, illustrating the case of two equal non-continuous arcs. Vertical diameter AC (A at top, C at bottom), center D, and the chosen point E on the diameter. From E a pencil of straight lines (some drawn dotted) runs to the points B, K, H, M, L, G, I, F on the left semicircle, marking the arcs FG, GH, HB and their subdivisions FI, IG, HK, KB and GL, LM, MH. A short segment marked N (the greatest common measure) is drawn to the upper right of the diameter.

- A (top): end of the diameter
- B (upper-left): endpoint of an intercepted arc
- K (upper-left): subdivision point (arc KB / HK)
- H (left): endpoint of arc HB

- M (left): subdivision point (arc LM / MH)
- L (left): subdivision point (arc GL / LM)
- G (lower-left): endpoint of arc FG / GH
- I (lower-left): subdivision point (arc FI / IG)
- F (bottom-left): endpoint of arc FG
- C (bottom): end of the diameter
- D (on diameter): center of the circle
- E (center): the chosen point on the diameter
- N (upper right, short segment): the greatest common measure

DEINDE (Next). Next, let four straight lines EF, EG, EH, EB intercept two equal non-continuous arcs FG, HB, one lying wholly outside the other, as in the second figure. I say again that the angle FEG is greater than the angle HEB. For the intermediate arc GH is either commensurable or incommensurable with each of the arcs FG and HB. Let it first be commensurable, and let N be their greatest common measure, and let each of the arcs FG, GH, HB be divided into parts equal to N — namely FG and HB each into two, FI, IG and HK, KB, and GH into three, GL, LM, MH. Then, the straight lines EI, EL, EM, EK being drawn, it will follow, as already demonstrated, that the angle FEI is greater than the angle IEG (since the arcs FI, IG are equal and continuous); and for the same reason the angle IEG is greater than GEL, and this than LEM, and this than MEH, and this than HEK, and this than KEB, and so on if there are more equal arcs. Therefore the angle FEI will be far greater than the angle HEK, and IEG greater than KEB; and accordingly the whole angle FEG will be greater than the whole angle HEB. Which was to be proved.

Diagram. A single circle in a rectangular frame. Vertical diameter AC (A at top, C at bottom), center D on the diameter, and the chosen point E on the diameter. From E a pencil of straight lines — some solid, some dotted — runs to the points B, K, I, H, G on the left semicircle and to F near the bottom, illustrating the case in which the intermediate arc GH is treated by subdivision (the incommensurable/third-figure argument).

- A (top): end of the diameter
- B (upper-left): endpoint of an intercepted arc
- K (upper-left): subdivision point
- I (left): subdivision point
- H (left): endpoint of arc HB
- G (lower-left): endpoint of arc FG / GH

- D (on diameter): center of the circle
- E (center): the chosen point on the diameter
- F (bottom-left): endpoint of arc FG
- C (bottom): end of the diameter

SED IAM (But now). But now let the intermediate arc GH be incommensurable with each of the arcs FG, HB, as in the third figure. If, then, the angle FEG is not greater than the angle HEB, it will be either smaller or equal. Suppose first, if possible, that it is smaller; and from the greater angle HEB let an angle HEI be cut off equal to the angle FEG; and, by Lemma 2 of proposition 8, Book III, of Theodosius, let an arc HK be found that is greater than HI but less than HB and commensurable with the intermediate arc GH. And since the arc FG is assumed equal to the arc HB, the arc FG will be greater than HK. So, an arc GL being cut off equal to HK and the straight line EL being drawn: since the arcs LG, HK are equal and non-continuous, and the intermediate arc GH is commensurable with each, it will follow from the construction, as was just demonstrated, that the angle LEG is greater than the angle HEK — and therefore much greater than the angle HEI. But since, by construction, the cut-off angle HEI is equal to the angle FEG, the angle LEG will also be greater than the angle FEG, a part greater than the whole, which is absurd. Therefore the angle FEG is not smaller than the angle HEB.

SIT DEINDE (Suppose next). Suppose next, if possible, that the angle FEG is equal to the angle HEB, as in the fourth figure; and, the equal arcs FG, HB being bisected in I, K, let the straight lines EI, EK be drawn. Then, since both the continuous arcs HK, KB and the continuous arcs FI, IG are the halves of the arcs FG [and HB] and are equal, it will follow, as we demonstrated above, that the angle HEK is greater than half the angle HEB. By the same reasoning the angle FEI is greater than the angle IEG, and so the angle IEG is less than half the angle FEG. Since, then, the angles FEG, HEB are assumed equal, IEG will be less than HEK — which is absurd. For since the arcs IG, HK are the halves of the equal arcs FG, HB and are equal and non-continuous: if the intermediate arc GH is commensurable with them, the angle IEG will be greater than the angle HEK, as was demonstrated; and if incommensurable, the angle IEG cannot be less than the angle HEK, as was also demonstrated a little before. Therefore the angle FEG is not equal to the angle HEB; but neither has it been shown to be less. Therefore it is greater. Which was to be proved.

AD EXTREMUM (Finally). Finally, let four straight lines EF, EG, EI, EH intercept equal arcs FG, IH having a common part IG, as in the next, the fourth figure. I say again that the angle FEG is greater than the angle IEH. For since the arcs FG, IH

are equal, when the common part IG is removed the remainder FI will be equal to the remainder GH. Therefore, as we have shown, the angle FEI will be greater than the angle GEH; and when the common angle IEG is added, the whole angle FEG will also be greater than the whole angle IEH. Which was to be proved.

Diagram. A single circle in a rectangular frame, illustrating the converse (equal angles at E). Vertical diameter AC (A at top, C at bottom), center D on the diameter, and the chosen point E. From E a pencil of straight lines — some dotted — runs to the points B, G, F on the upper left and to a lower cluster G, B, F near the bottom left, with C at the foot, marking the arcs BF, FC and the auxiliary FG for the continuous and non-continuous cases.

- A (top): end of the diameter
- B (upper-left): endpoint of arc BF
- G (upper-left, dotted): auxiliary point (arc FG)
- F (left, two positions): dividing point between the arcs
- C (mid-left / bottom): end of the diameter and endpoint of arc FC
- D (on diameter): center of the circle
- E (center): the chosen point on the diameter

SED IAM (But now). But now let the straight lines EC, EF, EB make at E equal angles CEF, FEB — whether continuous or non-continuous — as in the fifth figure. I say that the arc BF is greater than the arc FC. For if it is not greater, let it first be equal [to FC]. Then, as already demonstrated, the angle CEF will be greater than the angle FEB — which is contrary to the hypothesis. Next, suppose, if possible, that the arc BF is less than the arc FC, and make FG equal to FC. Then, as already shown, the angle CEF will be greater than the angle FEG, and therefore much greater than the angle FEB — which is contrary to the hypothesis. Since, then, the arc BF is neither equal to nor less than the arc FC, it will be altogether greater. Which was to be proved.

ITAQUE (And so). And so the latter part of this theorem, which we have just demonstrated, is far more general than the last proposition of the scholium on proposition 29 of Euclid's Book III, where it was proved only that, if two angles CEF, FEB are equal, the starting point being made from the point C of the diameter, the arc BF is greater than the arc FC. Here, however, it has been demonstrated for any number of angles, and for arcs whether continuous or non-continuous, and whether one of them takes its starting point from the diameter or not.

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LEMMA XXXIII.

If, in two circles that cut one another — or that do not cut one another but nonetheless have different centres — some point (other than the centres) is taken on their common diameter, the one drawn through both centres, such that it lies both between the two centres and within both circles: then straight lines drawn out from that point, cutting the circumference of either circle into equal arcs, will cut the circumference of the other circle into unequal arcs; and the greater will always be the one whose lines lie nearer the centre. Likewise, any arc of that circle whose centre lies between the assumed point and its own circumference — intercepted between the common diameter and any straight line drawn from the same point — provided it is less than a semicircle, is too great to be similar to the arc of the other circle intercepted between the same straight lines.

Let two circles ABC, DEF cut one another; or, if they do not intersect, let them have different centres. Let G be the centre of circle ABC, and H the centre of circle DEF; let the common diameter be DC, passing through the centres G, H. From a point I — lying between the two centres and within both circles — let any number of lines IK, IB, IL fall, cutting off in circle ABC the equal arcs KB, BL, and, when produced (if need be), cutting circle DEF in M, E, N. I say that the arcs ME, EN are unequal, ME being the greater and EN the smaller.

For if the arc ME is not greater than the arc EN, it is either equal to it or smaller. First, if possible, let it be equal. Then, by the preceding lemma, the angle NIE will be greater than the angle EIM. But by the same lemma, on account of the equal arcs KB, BL, the angle KIB — that is, EIM — is greater than the angle BIL, that is, than the angle NIE. So the same angle NIE is at once greater than and smaller than the angle EIM, which is absurd. Therefore the arc ME is not equal to the arc EN.

Next, if possible, let the arc ME be smaller than the arc EN. Cut off an arc EO equal to ME itself, and draw the straight line OI. Then, by the same preceding lemma, the angle OIE will be greater than the angle EIM; and so the angle NIE will be much greater than the angle EIM. But by the same lemma, on account of the equal arcs KB, BL, the angle KIB — that is, EIM — is greater than the angle BIL, that is, than the angle NIE. So the same angle NIE is at once greater than and smaller than the same angle EIM, which is absurd. Therefore the arc ME is not smaller than the arc EN; nor is it equal, as was shown. Therefore it is greater.

By the same reasoning, if the arcs ME, EN be assumed equal, then the arc LB will be greater than the arc BK. For if it is not greater, first, if possible, let it be equal. Then, by the preceding lemma, the angle KIB — that is, EIM — will be greater than the angle BIL, that is, than the angle NIE. But by the same lemma, on account of the equal arcs ME, EN, the angle NIE is greater than the angle EIM. So the same angle NIE is at once greater than and smaller than the same angle EIM, which is absurd. Therefore LB will not be equal to the arc BK. Next, if possible, let the arc BL be smaller than the arc BK. Cut off an arc BP equal to LB itself, and draw the straight line PI. Then, by the same preceding lemma, the angle PIB will be greater than the angle BIL; and so the angle KIB — that is, EIM — will be much greater than the angle BIL, that is, than the angle NIE. But by the same lemma, on account of the equal arcs ME, EN, the angle NIE is greater than the angle EIM. So the same angle NIE is at once greater than and smaller than the same angle EIM, which is absurd. Therefore the arc LB is not smaller than the arc BK; nor is it equal, as we showed. Therefore it is greater.

I say furthermore that the arcs DM, DE, DN are too great to be similar to the arcs AK, AB, AL; likewise that the arcs CL, CB, CK are too great to be similar to the arcs FN, FE, FM. For draw the straight line HN from the centre H, and draw parallel to it GQ from the centre G. [Margin a: Euclid I.29.] Since therefore the angles DHN, AGQ, at the centres, are equal — the one external, the other internal — the arcs DN, AQ will, by the scholium to Elements III.22, be similar. Therefore DN is too great to be similar to the arc AL, which is a part of the similar arc AQ. And in the same way you will show that DE, DM are too great to be similar to the arcs AB, AK.

Again, when the straight line GL has been drawn from the centre G, let HR be drawn parallel to it from the centre H. [Margin b: Euclid I.29.] Since therefore the angles CGL, FHR, at the centres, are equal — the one external, the other internal — the arcs CL, FR will, by the scholium to Elements III.22, be similar. Therefore CL is too great to be similar to the arc FN, which is a part of FR itself. And by the same reasoning CB, CK will be too great to be similar to FE, FM.

It is moreover evident that this proposition is true whether the arcs in the two circles are continuous or not continuous — which may be shown from the preceding lemma.

Diagram. A single boxed woodcut illustrating Lemma XXXIII, containing several overlapping circle-pair configurations (the secant and non-secant cases). The principal (upper-left) figure is a large circle with a vertical common diameter running D (top) – A – H – I – G – F – C (bottom), and a horizontal chord B (left) ... D

(right); from the interior point I a fan of lines runs to the smaller inner circle and to the peripheral points, with cut-off arcs marked. To the lower right stands a second, smaller circle showing the same construction for another case, and a third circle is partly cut off at the right margin.

- D (top of large circle): upper end of common diameter
- A, H, I, G (upper-centre to centre, on the vertical diameter): H and G are the two circle-centres; I the interior point
- F, C (lower centre, C at the bottom): lower diameter points
- B (left, on horizontal chord); E (left)
- M (top region), N, O, P, Q, R, L (left and lower-left periphery), K (interior/lower)
- Second (lower-right) circle: labels A (top), B, E, G, N, K read; centre region marked I / G
- Several interior labels in both clusters are set very small and overlap the ruling; not all resolve at this resolution. [— dense construction, multiple uncertain interior labels.]

LEMMA XXXIV.

If a circle cut a circle into two equal parts, or not into two equal parts, or not at all, and through their common centres two perpendicular diameters be drawn: then two straight lines going out from the point of the line drawn through the centres — the point through which passes the straight line joining the extremities of the two drawn diameters, and which lies in both circles — making, with the straight line equidistant from both diameters on either side (or with the straight line passing through the centres), equal angles, will intercept in each circle similar arcs. And that straight line, itself equidistant from both diameters, will cut off from each circle alternate similar arcs. And conversely, if two straight lines intercept similar arcs, they will make, with that same equidistant straight line, equal angles toward both sides.

Let circle ABCD cut circle EFGH into two equal parts, or not into two equal parts, or not at all; let their centres be I, K, and through them let the straight line AIKG be drawn out; and through the same [centres], perpendicular to AG, let there be drawn BID, FKH — of which the latter falls upon the common sections of the circles at F, H, when one circle bisects the other, as happens in the first and second figures, since then this diameter FH is wholly perpendicular to AG. For since the straight line IK, from the centre I, cutting the straight line FH in circle ABCD into two equal parts at K (because K is the centre of circle EFGH), [Margin

a: Euclid III.3] cuts it at right angles, the diameter FH will be perpendicular to the same AG.

Now let the straight line BH be drawn, cutting the same AG at L, a point lying in both circles; from L let a perpendicular LM be erected to the same AG, cutting the circle EFGH — that is, EF, GH — at N; and finally at L let two equal angles MLO, MLP be made, and thence, from the straight lines LO, LP, let the remaining angles OLA, PLG be formed; and let the straight line LO cut circle EFGH at Q, and the straight line LP cut circle ABCD at R. I say that the alternate arcs CM, EN (or AM, GN), which the perpendicular LMN cuts off, and the arcs OR, QP between the two straight lines LO, LP, are similar.

[Margin b: Euclid I.28.] For since BD, FH, being perpendicular to AG, are parallel, [Margin c: Euclid I.29] the alternate angles IBL, KHL will be equal; and the angles BIL, HKL are also right; [Margin d: Euclid I.15] and the angles BLI, HLK are equal as vertical angles. Therefore the triangles BIL, HKL are equiangular. [Margin e: Euclid VI.4] Therefore, as BI is to IL, so is HK to KL. But MI is equal to BI itself, and NK to HK itself. Therefore it will also be that, as MI is to IL, so is NK to KL. Since therefore in the triangles MIL, NKL the right angles ILM, KLN are equal, and the sides about the angles MIL, NKL are proportional, as we showed, while of the remaining angles M, N each is less than a right angle (by the corollary to Elements I.17), [Margin f: Euclid VI.7] the triangles themselves will be equiangular, and will have the angles MIL, NKL at the centres equal. Therefore, by the scholium to Elements III.22, the arcs CM, EN are similar; and hence, from the semicircles, the remaining arcs AM, GN will likewise be similar, by the same scholium. Which was the second [point].

Let the straight lines IO, KP, IR, KQ be joined. And since in the triangles ILO, KLP the angles ILO, KLP are equal (for since MLI, MLK are right, and MLO, MLP equal by hypothesis, the remaining angles ILO, KLP will also be equal), and the sides about the angles LIO, LKP are proportional (for it was shown that in the triangles MIL, NKL, as MI is to IL, so NK is to KL; so since OI is equal to MI, and PK to NK, it will also be that as OI is to IL, so PK is to KL), while of the remaining angles IOL, KPL each is less than a right angle [Margin g: Euclid III.31] — because the straight lines AO, CO, EP, GP make right angles in the semicircles of which those angles are parts — [Margin h: Euclid VI.7] the triangles themselves will be equiangular, and will have the angles LIO, LKP equal.

Again, since in the triangles ILR, KLQ the angles ILR, KLQ are equal (for since the equal angles MLR, MLQ are placed, and the equal right angles MLI, MLK added, the wholes ILR, KLQ become equal), and the sides about the angles LIR, LKQ are

proportional (for it was shown that in the triangles MIL, NKL, as MI is to IL, so NK is to KL; so since RI is equal to MI, and QK to NK, it will also be that as RI is to IL, so QK is to KL), while of the remaining angles IRL, KQL each is less than a right angle [Margin i: Euclid III.31] — because the straight lines AR, CR, EQ, GQ make right angles in the semicircles of which those angles are parts — [Margin k: Euclid VI.7] the triangles themselves will be equiangular, and will have the angles LIR, LKQ equal. But it was also shown that the whole angles LIO, LKP are equal. Therefore, when the equal [angles] LIR, LKQ are taken away, the remaining angles OIR, QKP at the centres I, K will also be equal; and hence, by the scholium to Elements III.22, the arcs OR, QP will be similar. Which was the first [point].

That each of these, moreover, holds may most easily be demonstrated from Lemma 9, in this way: when NL is produced to γ , then by Lemma 9 the arc RM is similar to the alternate arc γG . But this last is equal to the arc NG (for the straight line LG, passing through the centre K, and cutting the straight line N γ into two equal parts and at right angles, also cuts — by the scholium to Elements III.27 — the arc NG γ into two equal parts). Therefore AM is likewise similar to the arc GN. Which was the second [point].

Next, by the same Lemma 9, the arc OR is similar to the alternate arc XT; but this last is equal to the arc QP (because, by Lemma 21, the arc EX is equal to the arc EQ, and the arc TG to the arc PG, on account of the equal angles ELX, ELQ and GLT, GLP). Therefore the arc OR is likewise similar to the arc QP. Which was the first [point].

But now let the straight lines LO, LP intercept similar arcs OR, QP. I say that the angles OLM, PLM are equal. [Margin l: Euclid I.5] For, when OL, PL are produced as far as T, V, let the straight lines OR, QP; IS, KT; IV, KX be joined. And since the four triangles IOS, IRV, KQT, KPX are isosceles, the two [base] angles in each will be equal. And since in the triangles OIL, TKL the vertical angles at L are equal, and the sides about the angles OIL, TKL are proportional (for it was shown that in the triangles MIL, NKL, as MI is to IL, so NK is to KL; so since OI is equal to MI, and TK to NK, it will also be that as OI is to IL, so TK is to KL), [Margin m: Euclid I.15] while of the remaining angles IOL, KTL each is less than a right angle —

— because the straight lines AO, CO, ET, G[T?] make right angles in the semicircles of which those angles are parts, [Margin a: Euclid VI.7] the triangles themselves will be equiangular, and will have the angles LIO, LKT and IOL, KTL equal. But the angle IOL was equal to the angle ISL, and the angle KTL to the angle KQL, on account of the isosceles triangles IOS, KQT. Therefore the four angles IOL, ISL, KQL, KTL are equal to one another. By exactly the same

reasoning we shall show the four angles IVL, IRL, KXL, KPL to be equal to one another.

Now then, [Margin b: Euclid III.10] since the angle PKT, at the centre K — or rather the space at the centre K, standing on the arc PGT, as in the second figure — is double the angle PQT at the circumference; and since the angle PKT (or the space at K), standing on the arc PGT, is equal to the three angles PLT, LPK, LTK together [Margin c: Euclid I.32] (because both PKG is equal to the two PLK, LPK, and TKG to the two TLK, LTK): these three angles together, PLT, LPK, LTK, will also be double the angle PQT. [Margin d: Euclid I.32] But again the angle PLT is equal to the two LOR, LRO. Therefore the four angles LOR, LRO, LPK, LTK together will likewise be double the same angle PQT. Since therefore it was shown a little before that the angle LTK is equal to the angle IOL, the whole angle IOR, together with LRO and LPK (taking IOL for LTK), will be double the same angle PQT.

Furthermore, since the isosceles triangles OIR, QKP have equal angles at the centres I, K — on account of the similar arcs OR, QP that are set out — [Margin e: Euclid I.32] the remaining two angles of the one will be equal to the remaining two of the other, and therefore the four will be equal to one another; and so the two IOR, IRO will be double the angle KQP. Hence, since the three IOR, LRO, LPK just shown are double the angle PQT, and since the two IOR, IRO are now also double the angle KQP, then when the two IOR, IRO are subtracted from the three [Margin f: Euclid I.5 [?]] the remainders IRL, LPK will also together be double the remainder KQL. But it was shown above that IRL, LPK are equal; therefore LPK alone will be equal to AQL. Since therefore KTL was shown equal to KQL itself, KPL and KTL will also be equal to one another.

Finally, when the straight line PT has been joined, [Margin g: Euclid I.5] the angles KPT, KTP will be equal. If therefore they be added to the equal angles KPL, KTL, or else be taken away, as in the second figure, then either the wholes or the remainders LPT, LTP will likewise be equal; [Margin h: Euclid I.6] and so the straight lines LP, LT will be equal. And hence, since two sides LP, LK are equal to the two sides LT, LK, and the base KP is equal to the base KT, [Margin i: Euclid I.8] the angle PLK will also be equal to the angle TLK. [Margin k: Euclid I.15] Since therefore the angle TLK is equal to the angle OLI as vertical, the angles OLI, PLK will be equal to one another; and therefore, taking from the right angles, the remainders OLM, PLN will be equal. Which was the proposition.

Moreover — this is not to be passed over here — since the angles OIR, QKP at the centres I, K are equal, on account of the similar arcs OR, QP set out, either of them

is equal to the angle OLP which the straight lines OL, PL, cutting off similar arcs, make. For let PL, QK cut one another at Y. And since the angle LPK has been shown equal to the angle KQL, [Margin l: Euclid I.15] and the angles PKI, QYL are also equal as vertical, the remaining angles PKQ, PLO in the triangles PKY, QLY will (by the corollary to Elements I.32) also be equal. In the same way it will be shown that the same angle PLO is equal to the angle OIR.

More briefly, if the arcs OR, QP be assumed similar, we shall demonstrate the angles OLM, PLN to be equal by this reasoning: if they are not equal, let OLM be the smaller, and let β LM be made equal to the angle PLM; then, as has been demonstrated, the arc β R (from P) will be similar [to QP]. Therefore, if the similar arcs OR, QP be subtracted, the remainders β O, α Q will (by Lemma 6) likewise be similar — which is absurd. For when β L is produced as far as δ , the arc δ T is similar to the arc β O (by Lemma 9); so if it were similar to the same arc β O, the arcs δ T, α Q would be similar to one another too — which cannot be. For since, by Theorem 8 [as far as the last] of Proposition, Book 6, let it be that as L δ is to LQ, so LQ is to L α , [Margin m: Euclid I.15] and let the angles at the vertex L be made equal; then the triangles L δ T, LQ α will (the chords δ T, Q α being drawn) be equiangular. Therefore it will be that, as L δ is to δ T, so LQ is to Q α . [Margin n: Euclid VI.6] Since therefore L δ is either greater or smaller than LQ — because L δ , LQ are not equally distant from the least [line] LE — δ T will also be either greater or smaller than Q α ; and hence, by the scholium to Elements III.28, [Margin o: Euclid VI.4] the arc δ T will be greater or smaller than the arc Q α . Therefore they are not similar, [Margin p: Euclid III.7] since similar arcs of the same circle are equal.

That each of the angles OIR, PKQ is equal to the angle OLP we shall conclude thus. When ML is produced as far as γ , [Margin q: Euclid I.15] since the angle OLM is equal both to the angle MLR (by hypothesis) and to the angle γ LT (as vertical), the angles MLR, γ LT will be equal to one another; and so, from the right angles, the remainders PLK, TLK will also be equal. Then, since two sides PK, KL are equal to the two sides TK, KL, and of the remaining angles KPL, KTL each is less than a right angle — namely the right angles in the semicircle EPG, or ETG — the angles KPL, KTL will (by the scholium at the end of Book I of Euclid) be equal. Since therefore KTL, in the isosceles triangle QKT, is equal to KQL itself, KPL will also be equal to the same KQL. And so, since the two angles Y, Q of the triangle LYQ are equal to the two angles Y, P of the triangle KYL, the remaining angle QLY will also be equal to the remaining angle PKY; and hence OIR (which is equal to the angle QKP, or PKY, on account of the similar arcs OR, QP) will also be equal to the same QLY, or to OLP — which was the proposition.

Therefore, if each of the equal angles OLM, PLM stand on the arc of half of one degree in a circle described about the centre L, so that the whole angle OLP stands on an arc of one degree, then the angles OIR, QKP equal to it will likewise stand on arcs of one degree; and if the angle OLP stands on two degrees, the arcs OR, QP will be of two degrees, and so on. And so from L there can be drawn two straight lines cutting off similar arcs OR, QP containing as many degrees as anyone pleases: namely, if the angles OLM, PLM be so constituted that each embraces half of the number of degrees prescribed.

This demonstration, as you see, holds in every case: whether the center of the larger circle lies inside the smaller one, as in the first figure, or outside it, as in the second and third, or even on the circumference of the smaller circle itself. Likewise, whether one of the lines OL, PL falls below the diameter FH, as in the first and third figures, or both fall above that diameter, as in the second figure — provided only that on either side they make equal angles with the perpendicular LM.

SCHOLIUM

Just as the straight line LA, together with any other line drawn from L, cuts off dissimilar arcs from both circles — as was demonstrated in the preceding lemma — so too any two straight lines whatever drawn from L, falling above or below the perpendicular LM, cut off dissimilar arcs from those same two circles. This can readily be gathered from what has been demonstrated in this lemma, as appears in these two figures. For suppose two straight lines OL, PL — whether above the perpendicular LM or below it — were said to cut off similar arcs OR, QP, and the same construction were made as before: we would show, by exactly the same reasoning, that the angles OLI, PLK are equal to each other. But this is absurd, since one of them is acute and the other obtuse. Only similar arcs, therefore, can be intercepted between two straight lines that make equal angles on both sides with LM — that is, lines of which one falls above LM and the other below.

Indeed, that QP and OR cannot be similar was shown in the latter demonstration of the lemma, since the arc QP is similar to the arc VS lying on the opposite side, by Lemma 9.

Diagram. A woodcut within a rectangular frame showing two circles side by side (the "two figures" of the scholium), each a circle with a horizontal diameter and a dense pencil of straight lines drawn from a lower point through the interior, with dashed chords marking the cut-off arcs.

- Left circle — A (top of vertical diameter); o (small, just left of the apex); R and P (upper left); B (left end of horizontal diameter); C, E (upper center); I (center); D (right end of horizontal diameter); M, N (lower left); F (lower-left of circle); L, K (near the central crossing) [?]; H (lower right) [?]; V (lower center) [?]; and at the foot C, S, T, G.
- Right circle — A (apex); V, S (upper); X (upper center); E (center left); T (center); C (center right); D (right end of diameter); B (left end); M, N (lower left); F (lower left); K (lower center); C (foot center); G (lower right); P, Q (lower) [?].

[The labels of both circles are small and overlain by many ruled lines; several interior letters are only partly legible.]

LEMMA XXXV

If in a circle two diameters cut each other at right angles, and in the same circle a straight line is drawn either inclined to both diameters or parallel to one of them; and if from one endpoint of either diameter two straight lines are drawn through the extremities of the inclined line — or from the endpoint of that diameter to which the inclined line is parallel — forming a triangle whose base is the inclined line (or that parallel): then the other diameter will cut off from the sides of this triangle a similar triangle, but oppositely placed. And if the inclined straight line passes through the center, the perpendicular drawn to it from that same endpoint of the diameter will bisect the base of the triangle cut off by the other diameter, and this perpendicular will be equal to half of that base. But if the straight line does not pass through the center — whether it be inclined, or parallel to one of the diameters — and a diameter is drawn perpendicular to it; and if through the point where it meets that line, a straight line is drawn from the same endpoint of the diameter as far as the circumference; and if, finally, on the other side an equal arc is cut off between this point of the circumference and the perpendicular diameter last drawn: then the straight line drawn from that same endpoint of the diameter to the end of this arc will likewise bisect the base of the triangle cut off by the other diameter.

[Demonstration.] Let two diameters AC, BD cut each other at right angles in the circle ABCD, whose center is E, and let the straight line FG be inclined to both — whether it lies short of the center, or beyond it, as in the first figure; or passes through the center, as in the second figure; or is not inclined but parallel to one of the diameters, say to AC, as in that same second figure; or, finally, is wholly inclined on one side of the diameter AC, as in the third and fourth figures —

which can happen in two ways: for either it cuts the other diameter BD, as in the third, or does not cut it, as in the fourth figure.

Now from the point C, through the extremities F, G, let two straight lines CF, CG be extended, forming the triangle CFG and cutting the diameter BD in H, I. I say that the triangle CHI which is cut off is similar to the triangle CFG, but oppositely placed — that is, that angle CHI equals angle CGF, and angle CIH equals angle CFG, and so on. For draw GK parallel to diameter BD: then the arcs BK, DG will be equal, by the scholium of proposition 27 of book 3 of Euclid. [Margin: a — 27th of the third.] So if from the equal quadrants BC, DC these arcs are taken away — or if GK falls beyond the diameter BD, added — the remaining arcs, or the combined arcs CK, CG, will be equal. And therefore the angles CGK, CFG standing on them at the circumference will also be equal.

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Moreover, angle CIH, being interior, equals the exterior angle CGK. Therefore the angles CIH and CFG are also equal. [Margin: a — 29th of the first.] Since, then, angle FCG is common to both triangles, the triangles CHI and CFG will be equiangular, by corollary 1 of proposition 32 of book 1 of Euclid; and consequently the sides about the equal angles will be proportional, and so the triangles will be similar, but oppositely placed. [Margin: b — 4th of the sixth.]

Diagram. A woodcut within a rectangular frame showing two circles side by side (the first and second figures of the lemma), each with two perpendicular diameters and an inclined chord FG, with a pencil of lines meeting at the lower point C.

- Left circle — A (apex), with F to its left and Q, O, R at the top; K (upper left); H, B (left end of the horizontal diameter); E (center); H, S, I, D across the diameter; F, K (lower left); G (right, upper and lower); C (foot).
- Right circle — A (apex), T, and Q, O, N, F, R near the top; Y (left), X (right); E (center), S, H, P, D, S, I across the diameter, with I at the right; O, R (lower middle); G, Q (lower); K (lower left); C (foot).

[Both circles carry many small interior labels crossed by ruled lines; several are only partly legible]

Now from the same point C let a perpendicular CL be drawn to the inclined line FG where it passes through the center (as in the second figure), cutting the base HI in M. This is easily done as follows. Take an arc GN equal to the arc CG, and draw the straight line CN. This line will be perpendicular to FG at L; for the

straight line EL, drawn from the center to bisect the arc CN in G, will likewise bisect the straight line CN — by the scholium of proposition 27 of book 3 of Euclid — [Margin: c — 3rd of the third.] and therefore also cut it at right angles. I say that the base HI of the triangle CHI cut off is bisected in M, and that the straight line CM equals each half MI, MH. For since angle FCG stands in a semicircle and is therefore right, [Margin: d — 31st of the third.] and from it the perpendicular CL is dropped to FG, the base of the right-angled triangle CFG, angle GCL will equal angle CFG, and angle FCL will equal angle CGF. [Margin: e — 8th of the sixth.] But angle CIH was shown equal to angle CFG, and angle CHI equal to angle CGF [?— printed "CGE"]. Therefore the angles GCL and CIH are equal, and likewise the angles FCL and CHI. Hence side IM in triangle MCI equals side CM, and side HM in triangle MCH equals that same side CM; [Margin: f — 6th of the first.] and consequently the straight lines MI, MH are equal, and each equal to CM — which was to be proved.

Again, let the diameter EO be drawn perpendicular to FG (in the other figures as well) where FG does not pass through the center; [Margin: g — 3rd of the third.] this will bisect FG at the point P. Through P let a straight line be drawn from the same point C, cutting the circumference in Q, and let an arc OR be taken equal to the arc OQ. Finally, from the same point C, through R, let a straight line be drawn cutting HI, the base of the triangle cut off, in S. I say that the base HI is bisected in S. For since the triangles CFG, CIH were shown similar but oppositely placed, having the equal angles F, I; and since in the triangles CFP, CIS the angles FCP, ICS are also equal, [Margin: h — 27th of the third.] because of the equal arcs FQ, GR — (for since the arcs OF, OG are equal, by the scholium of proposition 27 of book 3 of Euclid, because the line FG is bisected in P, then if the equal arcs OQ, OR are taken away, the remainders FQ, GR will also be equal) — the triangles CFP, CIS will therefore be equiangular. [Margin: i — 4th of the sixth.] Hence FG is to FC as IH is to IC, and FC is to FP as IC is to IS. So, by equality of ratios (as appears in the accompanying table), FG will also be to FP as IH is to IS.

FG,	IH,
FC,	IC,
FP,	IS,

But FG is double FP. Therefore IH will also be double IS, and consequently IH is bisected in S — which was to be proved.

Indeed, if to the straight line FG passing through the center a perpendicular diameter ET is drawn, and an arc TN is taken equal to the arc TA — (for CA has also been drawn through E, the point of intersection of the perpendicular diameter ET with FG) — the straight line CN will bisect the base HI in M as well. This is proved by the same reasoning, as is plain if for A one takes the letter Q, for T the letter O, for N the letter R, for M the letter S, and for E the letter P, as appears in the second figure. But one must attend carefully — lest confusion arise in the triangles of the first two figures, because the same letters recur — always to take the letters that belong to the respective triangles. In the two later figures there is no such danger.

Diagram. A second woodcut within a rectangular frame showing two circles (the later figures of the lemma), each with two perpendicular diameters and an inclined chord, and a pencil of lines from the point C.

- Left circle — A, F (apex region); Q, R, P, S, V (upper right); B (left end of diameter); E (center); D (right of center); K (lower left); G (lower right); C (foot).
- Right circle — A (apex); E (center); D, H (right of center); S, I (right of the horizontal line); F (lower right); K (lower left); G (lower center); C (foot).

[Many small labels overlain by ruled lines; several only partly legible]

This same thing which I stated last about the straight line FG drawn through the center can also be gathered without any trouble from the demonstration above, where it was proved that the perpendicular CL bisects HI in M. For since the whole arc CDA is double the whole arc DA, and from the whole CDA the arc AN taken away is double AT taken away from the whole DA, [Margin: k — 5th of the fifth.] the remainder CN of the whole CDA will also be double the remainder DT of the whole DA. Since, then, DT equals CG — (for from the equal quadrants GT, CD, when the common arc GD is subtracted, the remaining arcs DT, CG will be equal) — the arc CN will also be double the arc CG. But when the arc CG is doubled up to N, the line CN is perpendicular to FG and bisects HI, as was demonstrated above. Therefore, when TN is taken equal to the arc TA, the line CN will likewise bisect HI in M, since from this it follows that the remaining arc CN is bisected in G, as was demonstrated.

When the inclined straight line FG passes through the center, as in the second figure, we shall demonstrate that the triangle CHI cut off is similar to the triangle CFG but oppositely placed, even without the parallel GK being drawn, in this way. Since angle FCG stands in a semicircle and is therefore right, [Margin: l — 31st of

the third.] and from it the perpendicular CE is dropped to the base of the triangle CHI, angle HCE will equal angle CIH, and angle ICE will equal angle CHI. [Margin: m — 8th of the sixth.] But angle CFG equals angle HCE, [Margin: n — 5th of the first.] and angle CGF equals angle ICE. [Margin: o — 5th of the first.] Therefore the angles CIH, CFG, and the angles CHI, CGF, will be equal; and angle FCG is common. So the triangles CHI, CFG are equiangular and oppositely placed.

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COROLLARIUM

From what has been demonstrated in this place it follows that, if in any circle two diameters are drawn cutting each other at right angles, then a straight line drawn perpendicular to some other oblique diameter from the endpoint of either of the two diameters that cut at right angles will bisect the segment — cut from any straight line parallel to the other diameter — that is intercepted between the lines drawn from that same endpoint through the extremities of the oblique diameter. For example: if in the circle ABCD of the second figure two diameters AC, BD are drawn cutting each other at right angles, and from the endpoint C of the diameter AC a perpendicular CL is drawn to any oblique diameter FG, I say that CL produced bisects, in Y, the segment VX of any straight line VX parallel to the other diameter BD and intercepted between the lines CF, CG. For since, by the scholium of proposition 4 of book 6 of Euclid, HM is to MI as VY is to YX, and HM equals MI (as was shown), VY will likewise equal YX. And the same reasoning holds for any other line parallel to BD, whether it is drawn beyond BD at any distance whatever, or short of BD.

LEMMA XXXVI

If in a circle two diameters cut each other at right angles, and in the same circle two other diameters are drawn inclined to them; and if from one endpoint of either of the two former diameters two straight lines are extended through the extremities of the latter: then the straight lines cut off from the other of the former diameters by each pair will be greater than the diameter of the circle, and they will also be unequal to each other — the greater being the one whose inclined diameter makes the greater angle with the other of the former diameters.

[Demonstration.] In the circle ABCD, whose center is E, let two diameters AC, BD cut each other at right angles, and in the same circle let there be two diameters FG, HI inclined to them. From the endpoint C let the straight lines CF, CG be

extended through the extremities F, G, cutting BD in K, L; and likewise let CH, CI be extended through the extremities H, I, cutting the same BD in M, N. I say that each of the cut-off lines KL, MN is greater than the diameter BD, that they are unequal to each other, and that MN is greater than KL.

Diagram. A woodcut within a rectangular frame showing a single circle with two perpendicular diameters AC (vertical) and BD (horizontal), two inclined diameters FG and HI, and a pencil of straight lines drawn from the foot C, extended to meet BD produced at points running out to the right beyond the circle.

- A (apex of the vertical diameter); C (foot); B (left end of the horizontal diameter); E (center).
- H (upper left of circle); F (left of circle); along the horizontal line to the right: K, M (left of center), then R, O, D, and — beyond the circumference — L and (farther right) N.
- P, Q, S near D/L (right of center); G, I (lower right of circle); T (below L, outside the circle at right); C (foot, where the pencil of lines converges).

[Interior labels are dense; the extensions L, N lie outside the circle on BD produced.]

For join the straight lines CB, CD, and take a line EO equal to EK, and join CO. Since the two sides EB, EC equal the two sides EO, EC, and contain equal angles, namely right angles, the bases CB, CD will also be equal. [Margin: a — 4th of the first.] For the same reason the lines CK, CO will be equal, since the two sides EK, EC likewise contain equal angles, namely right angles. But since in the triangle ECO the exterior angle DOC is greater than the interior right angle OEC, [Margin: b — 16th of the first.] and therefore in the triangle COD the angle ODC is less than a right angle — because both COD and ODC are less than two right angles — [Margin: c — 17th of the first.] the line CD will be greater than CO. [Margin: d — 19th of the first.] For the same reason CL will be greater than CD; because in the triangle ECD the exterior angle LDC is likewise greater than the interior right angle DEC, and so in the triangle CDL the angle DLC is less than a right angle, since both CDL and DLC are less than two right angles.

Let CP be cut off equal to CO (that is, to CK), and CQ equal to CD (that is, to CB), and join the line PQ. Since, then, the two sides CP, CQ equal the two sides CK, CB, and contain equal angles PCQ, KCB — because these stand on the equal arcs DG, BF [Margin: e — 27th of the third.] (for these arcs are equal, since the angles at the center standing on them are vertical and equal) — [Margin: f — 26th of the

third.] the triangles PCQ, KCB will be equal; [Margin: g — 4th of the first.] and consequently the triangle DCL, of which the triangle PCQ is a part, will be greater than the triangle KCB. Now as the triangle DCL is to the triangle KCB, so is the base DL to the base BK. [Margin: h — 1st of the sixth.] Therefore the base DL will be greater than the base BK; and when the common line KD is added, the whole KL will become greater than the whole BD. In the same way we shall demonstrate that MN is greater than that same BD.

Next, take ER equal to EM, and join CR, which will be shown equal to CM, just as CO was shown equal to CK. For since the two sides EC, EM equal the two sides EC, ER, and contain equal right angles, the bases CM, CR will be equal. [Margin: i — 4th of the first.] But since in the triangle ERC the exterior angle LRC is greater than the interior right angle REC, [Margin: k — 16th of the first.] and therefore in the triangle LRC the angle RLC is less than a right angle — since both LRC and RLC are less than two right angles — [Margin: l — 17th of the first.] the line CL will be greater than CR. [Margin: m — 19th of the first.] By the same reasoning CN will be shown greater than CO, because in the triangle EOC the exterior angle NOC is likewise greater than the interior right angle OEC, and so in the triangle CON the angle CNO is less than a right angle. Let CS be cut off equal to CR (that is, to CM), and CT equal to CO (that is, to CK), and join ST. Since, then, the two sides CS, CT equal the two sides CM, CK, and contain equal angles SCT, MCK — because these stand on the arcs GI, FH, [Margin: n — 27th of the third.] which are equal, on account of the vertical angles at the center being equal [Margin: o — 26th of the third.] — the triangles SCT, MCK will be equal. And therefore the triangle LCN, of which the triangle SCT is a part, will be greater than the triangle MCK. [Margin: p — 4th of the first.] Now as the triangle LCN is to the triangle MCK, so is the base LN to the base KM. [Margin: q — 1st of the sixth.] Therefore the base LN will be greater than the base KM; and when the common line ML is added, the whole MN will become greater than the whole KL — which was to be proved.

(Continuation of Lemma XXXVI, begun in the preceding pages.)

Moreover, it can be shown by this same reasoning that both the straight line KL and MN are greater than the diameter BD, or FG, or HI. Conceive a scalene cone whose vertex is C and whose base is a circle about the diameter FG, set at right angles to the plane of the triangle CFG. Let another plane cut this cone — a plane at right angles to the same triangle through the axis CFG — cutting off the triangle CKL, which by the preceding lemma is set subcontrariwise to CFG, yet is similar to the triangle through the axis. This latter plane will therefore, by lemma 17, make in the cone a circle whose diameter is KL. And since the diameter FG is

bisected at the center E, the diameter KL will be greater, and it will be cut at E not into two equal parts; its greater portion will be EL, toward that side where the diameter KL makes with the side CG of the triangle through the axis the smaller angle L, as we demonstrated in the scholium of that same lemma 17.

That the angle L is smaller than the angle K is evident. For angle L is equal to angle F, and angle K to angle CGF, on account of the subcontrary section; [Margin: 18. primi (Euclid I.18)] and angle F is smaller than angle CGF, because side CG is also smaller than side CF — from the scholium of proposition 29 of book 3 of Euclid [?: marginal/inline reference digit uncertain]. The angle CLK, likewise, will be smaller than the angle CKL. In the same way we shall show that the straight line MN is greater than the diameter HI.

We shall demonstrate this same thing in the following way. Draw the straight line HN. Since EN is greater than the semidiameter ED (or EH), the angle EHN will be greater than the angle ENH. [Margin: 18. primi (Euclid I.18)] Now the angle CHI is equal to the angle CNM, on account of the subcontrary section, as was demonstrated in the preceding lemma. Therefore the whole angle CHN too will be greater than the whole angle CNH; [Margin: 19. primi (Euclid I.19) — printed digit read as "10"; corrected by context to I.19] and consequently the side CN will be greater than the side CH. Since these are opposed, in the similar subcontrary triangles CMN, CIH, to the equal angles CMN, CIH — as was shown in the preceding lemma — the diameter MN of the subcontrary section will be greater than the diameter HI of the base of the scalene cone, from those things which we demonstrated at the beginning of the scholium of lemma 17.

But if from the greater side CN a straight line equal to the smaller side CH were cut off, and a line parallel to MN were drawn so as to cut off another subcontrary triangle equal at its base to HI — as we demonstrated at the beginning of the scholium of the same lemma 17 — the base of the subcontrary section would be divided in half, as is plainly established from those things which we set forth in the scholium. Likewise too, if the smaller side CH were produced until it became equal to the greater CN, and a line parallel to the base HI were drawn at its extreme point — which would be the base of another scalene cone, equal to the base of the subcontrary triangle MN — then indeed neither base would be bisected either; and all these things can be gathered from those which we demonstrated in the scholium of lemma 17. What we have said is to be understood likewise of the subcontrary triangles CFG, CLK, if the straight line FL be joined, as is manifest. Therefore the fact that this lemma proposes — that the [diameter] KL, or MN, is always greater than the base FG, or HI — is not contrary

to what we demonstrated, namely that it can happen that the bases of subcontrary triangles are sometimes [equal]; because, since here the base of the cone FG (or HI) is always bisected, it comes about that the base of the subcontrary [section is greater], but not equal, as has been demonstrated.

Diagram. A composite geometric figure enclosed in a rectangular frame, illustrating the scalene cone of Lemma XXXVI. An upper semicircular arc rises to the apex A; a horizontal diameter runs from B at the left, through the center E, and eastward to N (the far vertex of the oblique triangle). The apex C lies at the bottom, from which straight lines radiate up through the figure. A network of chords produces several interior intersection points.

- A (top): apex of the upper semicircular arc
- B (left end of horizontal diameter): vertex
- C (bottom): apex of the scalene cone
- E (center of diameter): center
- along the horizontal diameter, left→right: M, E, R, O, D, L, N
- H (upper-left arc): [?] and F (left arc) [?]
- K (left interior): interior point
- interior chord intersections: P, Q, S, G, T, I

Several peripheral and interior labels are ambiguous at this resolution (F, and the lower interior points); more than two are uncertain

LEMMA XXXVII.

The circles of position in an oblique northern sphere that cut the semidiurnal arc of the Equator into equal parts cut the semidiurnal arcs of the parallels into unequal parts. In the southern parallels, any part between the Meridian and any circle of position is smaller, with respect to its own semidiurnal arc, than the same part in the Equator is with respect to the semidiurnal arc of the Equator; but in the northern parallels it is greater. Yet the same circles of position also cut into equal parts those parallels that are tangent to the Horizon.

In the northern oblique sphere ABCD, whose center is E — with the oblique Horizon BHD; the axis of the world FG; the Equator AHC; the northern parallel IKL; the southern MNO; the Meridian ABCD, drawn through the poles of the world and of the Horizon — divide the quadrant of the Equator AH (the eastern or the western) into six equal parts at P, Q, R, S, T, [Margin: 20. 1. Th. (Theodosius I.20)] and let lines be drawn through the division points, and through the points B, D where the Meridian cuts the Horizon; and let the great circles of position,

cutting the parallels, meet them at V, X, Y, Z, a, b, d, e, f, g. I say that the parallels are divided into unequal parts: that the arcs Ma, MZ, MY, MX, MV are smaller parts, with respect to the semidiurnal arc MN, than the arcs AT, AS, AR, AQ, AP are with respect to the Equator AH; but that the arcs Ig, If, Ie, Id, Ib are greater, with respect to the semidiurnal arc IK.

For let BD, MO, AC, IL be the common sections of the Horizon, the parallels, and the Meridian. [Margin: 15. 1. The. (Theodosius I.15)] And since the Meridian cuts the Horizon and all the parallels in half, the diameters BD, MO, AC, IL of the Horizon and of the parallels, and the axis FG, [Margin: 10. 1. The. (Theodosius I.10)] will pass through the centers k, E, l of the parallels; and MN, AH, IK, [lying] between the Meridian and the Horizon, will be the semidiurnal arcs. Now when, from the points h, E, i — where the diameters of the parallels cut the diameter of the Horizon — the straight lines hN, EH, iK, hV, EP, ib are drawn, and likewise to the remaining division points, then hN, EH, iK will be the common sections of the Horizon and of the parallels, [Margin: 16. vnde. (Euclid XI.16)] and hence parallel to one another; but hV, EP, ib will be the common sections of the circle of position BPD and of the parallels, [Margin: 16. vnde. (Euclid XI.16)] and therefore likewise parallel to one another; and so it must be said of the rest. [Margin: 10. vndec. (Euclid XI.10)] There will therefore be as many as six angles constituted at h as at i,

equal to the six at E. [Margin: 27. tertij (Euclid III.27)] And all six at E are equal to one another, since at the center E they stand upon six equal arcs HP, PQ, and so on. Therefore all the angles, both at h and at i, will be equal; and consequently, by lemma 32, the arcs Ma, aZ, and so on, as well as the arcs Ig, gf, and so on, will be unequal — Ma smaller than aZ, and aZ smaller than ZY, and so on; but Ig greater than gf, and gf greater than fe, and so on.

Ma is therefore smaller than a sixth part of the semidiurnal arc MN, since each of the five following parts aZ, ZY, and so on, is greater than Ma. Thus MZ will be smaller than a third part of the same arc MN, because each of the two arcs ZX, XN is greater than MZ. For the three angles MhZ, ZhX, XhN are also equal, since their halves are equal. Likewise the arc MY will be smaller than half of the same arc MN, since YN is greater than MY, because the two angles MhY, YhN are also equal, being those whose third parts are equal. By the same reasoning the arc MX will be smaller than two thirds of the same arc MN, because XN is greater than a third part, since it is greater than each of the arcs XZ, ZM. Finally MV will be smaller than five sixths of the same arc MN, because NV is greater than a sixth part, inasmuch as it is greater than each of the remaining five parts VX, XY, and so on.

On the contrary, Ig will be greater than a sixth part of the arc IK, since it is greater than each of the five following parts gf, fe, and so on. Likewise If will be greater than a third part of the same arc IK, since it is greater than each of the two parts fd, dK. For the three angles Ife, fld, dlk are equal, since their halves are equal. Again, Ie will be greater than half of the same arc IK, because it is greater than ek, since the two angles Ile, elk are equal, their third parts being equal. Moreover Id will be greater than two thirds of the same arc IK, because dk is smaller than a third part, since it is smaller than each of the arcs df, fL. Finally Ib will be greater than five sixths of the same arc IK, because kb is smaller than a sixth part, inasmuch as it is smaller than each of the other five parts bd, de, and so on.

The contrary happens in the southern oblique sphere. For the arcs cut off from the Meridian, together with the circle of position, will be greater in the southern parallels and smaller in the northern, with respect to the semidiurnal arcs, than the same arcs in the Equator are with respect to the semidiurnal arc of the Equator.

But now let the same circles of position cut the parallel Dpm — which is tangent to the Horizon at D, and whose diameter is Dm — in the points n, o, p, q, r. I say that the arcs mn, no, op, pq, qr, rD are equal to one another, just as in the Equator. For when the straight lines Dn, Do, Dp, Dq, Dr are drawn, which are parallel to the straight lines ET, ES, ER, EQ, EP, [Margin: 26. vnde. (Euclid XI.26)] the five angles mDn, nDo, oDp, pDq, qDr will again be equal to the equal angles AET, TES, SER, REQ, QEP, [Margin: 10. vndec. (Euclid XI.10)] and therefore also equal to one another. [Margin: 26. tertij (Euclid III.26)] Five arcs mn, no, op, pq, qr will therefore be equal to one another. And since, when the semidiameter tp is drawn, the angle mtp at the center is double the angle mDp at the circumference [Margin: 20. tertij (Euclid III.20)] — their third parts having been shown to be equal — the angle mtp will therefore also be double the angle AER. For the angle mDp is equal to the angle AER. [Margin: 33. sexti (Euclid VI.33)] Since therefore the angle AEH is also double the same angle AER — because the arc AH is also double the arc AR — the angles mtp, AEH will be equal; and hence the arcs mp, AH are similar, from the scholium of proposition 22 of book 3 of Euclid. Since therefore AH is a quadrant, mp too will be a quadrant, and consequently pD, the remainder from the semicircle, will also be a quadrant. But the arc op is a third part of the quadrant mp, because the three arcs mn, no, op have been shown to be equal. Therefore the arcs pq, qr also, which are equal to those, will be third parts of the quadrant pD; and consequently the remainder rD will also be a third

part of the same quadrant pD. And thus all six arcs of the semicircle mpD will be equal to one another, which was proposed.

But after it has been proved that the five arcs mn, no, op, pq, qr are equal, we shall also show that rD is equal to them, in this way. Let Da be the common section of the Horizon and of the parallel mpD — which, by the definition in book 2 of Theodosius, is tangent to both circles [Margin: 16. vnde. (Euclid XI.16)] and is parallel to EH itself; [Margin: 10. vnde. (Euclid XI.10)] and hence the angle aDr will be equal to the angle HEP, and consequently to the rest at the point D as well. Now the angle aDr is equal to the angle in the alternate segment, [Margin: 32. tertij (Euclid III.32)] which stands upon the arc Dr. Therefore the same angle, standing on the arc Dr, will be equal to the five angles rDq, qDp, pDo, oDn, nDm; [Margin: 26. tertij (Euclid III.26)] and consequently all six arcs of the quadrant [recte: semicircle] mpD will be equal to one another.

By the same reasoning we shall demonstrate that the same circles of position, when produced, cut into six equal parts the opposite tangent semicircle Buf.

Diagram. A large oblique sphere for Lemma XXXVII, drawn as a circle within a rectangular frame. The horizontal diameter B–D runs through the center E; A is at the top and C at the bottom. The quadrant of the Equator is subdivided, and a family of great circles of position sweeps across, cutting the parallels; the division points of the parallels are lettered around the upper part of the sphere.

- A (top), C (bottom), B (left end of diameter), D (right end of diameter), E (center)
- F (right side)
- along the left/upper parallels, descending: M, a, Z, Y, X, V
- upper interior division points: T, S, R, Q [and P] [?]
- right-side division points: f, e, m [?] and a lower-case l [?]
- lower construction points: h, i, and (lower-left) u, G, and a mid-left R [?]

The interior labelling is small and downsamples poorly; more than two labels cannot be confidently resolved

LEMMA XXXVIII.

In the northern oblique sphere, the circles passing through the unequal hours of the Equator and of any parallel cut the Meridian on the southern side below the Horizon, between the same Horizon and the southern pole; but on the northern side above the Horizon, between the same Horizon and the northern pole.

In the northern oblique sphere, whose center is E — with the Meridian ABCD; the axis of the world FG; the Horizon BHD; the Equator AC; the parallel, whether southern or northern, SKT; the semidiurnal arcs AH, SK [Margin: 20. 1. The. (Theodosius I.20)] — let a great circle LM be drawn through some unequal hour L of the Equator and the corresponding unequal hour M of the parallel. I say that it cuts the Meridian on the southern side, between B and the southern pole G, below the Horizon, namely at O; but on the northern side, between D and the northern pole F, above the Horizon, namely at P. [Margin: 20. 1. Th. (Theodosius I.20)] For let there be drawn, through the same point L, the circle of position BLD of the Equator, cutting the parallel at N, and the great circle through the poles of the world FLG, cutting the parallel at Q. Since therefore, by the preceding lemma, the arc SN in the southern parallel is smaller, with respect to the semidiurnal arc SK, than the arc AL is with respect to the semidiurnal arc AH — that is, than the arc SM with respect to the

semidiurnal arc of that same SK; but in the northern parallel greater; the point M falls, in the southern parallel, below N, in the northern above. [Margin: 10. 2. Th. (Theodosius II.10)] Again, since the arcs AL, SQ are similar, as many equal hours will be contained in SQ as in AL; but as many unequal hours are contained in SM as in AL, and the unequal hours in the southern parallel are smaller than the equal hours, while in the northern they are greater. Therefore, in the southern parallel, the point of the unequal hour M will fall above the point of the equal hour Q, but in the northern below. But it has been shown that the same point M falls below N in the southern parallel, and above in the northern. Therefore the great circle LM of the unequal hour, since it falls between the points N, Q, will cut the Meridian between the circles BLD, FLG; and consequently, on the southern side it will cut the same Meridian below the Horizon at the point O, between the Horizon and the southern pole G; but on the northern side, above the Horizon at the point P, between the Horizon and the northern pole F. And the same reasoning holds for the other circles of the unequal hours.

In the southern oblique sphere you must understand the contrary. For there the circle of any unequal hour will cut the Meridian below the Horizon on the northern side, but above on the southern side — always, however, between the Horizon and the pole of the world.

Diagram. A sphere in oblique position, drawn as a circle within a rectangular frame, illustrating Lemma XXXVIII (repeated for Lemma XXXIX). It shows the Meridian circle, the oblique Horizon, the Equator, the parallels, and the great

circles of the unequal hours drawn as arcs sweeping from top to bottom. The horizontal diameter B–D runs through the center E.

- along the top of the sphere: A, S, Y
- left side: S [?] (upper), V (mid-left)
- upper interior: N, I, and a small symbol [?]
- right side: F, P
- along the horizontal diameter, left→right: B, R, K, E, H [?], K, D
- lower-left: O, G
- lower-right: z, Γ [?]
- bottom: x, T, C

Many small peripheral labels are ambiguous at this resolution (several more than two uncertain)

LEMMA XXXIX.

The great circles passing through the unequal hours of the Equator and of two opposite parallels do not necessarily pass through the unequal hours of the intermediate parallels, in an oblique sphere.

Let the figure of the preceding lemma be repeated. And since the great circle LM, passing through the same unequal hour of the Equator and of the parallel SKT, cuts the Meridian on the southern side B below the Horizon — as was demonstrated in the preceding lemma — it will cut the Horizon on the same side toward which the semidiurnal arcs incline, at the point R, before the point B. Let there therefore be described a southern parallel VIX, whose semidiurnal arc VI cuts the Horizon between B and R; and let there be described, opposite and equal to it, the [northern parallel] YZ. Now let there be taken, on the semidiurnal arc VI, an arc Va of as many unequal hours as are contained in the arcs AL, SM. Since indeed the great circle described through the points a, L passes through the same unequal hour in the opposite northern parallel YZ — as we demonstrated in the scholium of proposition 10 of book 1 of the *Gnomonics* [Clavius, *Gnomonice*] — the same circle will not pass through the same unequal hour M in the intermediate parallel ST, since the great circle drawn through LM does not pass through a, but cuts the Horizon at R, and in no way cuts the parallel VX above the Horizon; and consequently it is different from the circle drawn through a and L.

But if great circles are described through all six hours of the semidiurnal arc of the Equator and of the parallel ST, these same circles will all cut the Meridian on the southern side B, below the Horizon, and hence cut the Horizon short of the

point B. If, then, a southern parallel is described whose semidiurnal arc none of those great circles cuts, and through the six unequal hours of this semidiurnal arc and of the Equator great circles are described, these will indeed pass — by the scholium of proposition 10 of book 1 of the *Gnomonics* — through the six unequal hours of the opposite northern parallel; but they will in no way intersect the intermediate parallel ST at its unequal hours, inasmuch as they differ from the great circles which we said were drawn through the unequal hours of the Equator and of the parallel ST — since these latter do not cut the southern parallel above the Horizon, by construction.

The same is clearly established at a pole-elevation of $66\frac{1}{2}$ degrees, where the Tropics touch the Horizon, and the tropic of Cancer ☿ is wholly above the Horizon, and the tropic of Capricorn ♊ below. For since, as we demonstrated in lemma 37, the circles of position in that sphere pass through the unequal hours of the Equator and of the tangent parallels, and the same circles of position, by the same lemma, divide the semidiurnal arcs of the other intermediate secant parallels unequally, it is evident that in that sphere the great circles passing through the unequal hours of the Equator and of each tropic (in the one through the diurnal hours, in the other through the nocturnal) do not pass through the unequal hours of the other intermediate parallels — inasmuch as the unequal hours divide the semidiurnal arcs into equal parts, which the circles of position in the intermediate parallels do not do, as has been said.

Again, at the same obliquity of the sphere, if great circles are described set through the unequal hours of the Equator and of some parallel placed between the Equator and the tropic of Capricorn ♊, all these will fall — by lemma 37 — below the Horizon before they cut the Meridian. If, then, a southern parallel is described between the tropic of Capricorn ♊ and the Equator, which cuts the Horizon short of all those points through which those great circles pass, and its semidiurnal arc is divided into six equal parts, the great circles drawn through those parts and the unequal hours of the Equator will pass through the unequal hours of the opposite northern parallel as well. But it is certain that these same circles do not pass through the unequal hours of the assumed intermediate parallel, since the great circles described through the unequal hours of the Equator and of the assumed parallel differ entirely from those, inasmuch as they were set so as not to cut the semidiurnal arc of that southern parallel.

SCHOLIVM.

[Margin: *That there are no great circles passing through the unequal hours of all the parallels.*]

It is evident from all this that in an oblique sphere there cannot be given great circles which pass through the unequal hours of all the parallels — that is, which divide the diurnal arcs of each into twelve equal parts. This, nevertheless, all who have treated of the description of horologes [sundials] take for certain. For all writers divide the diurnal arc of Cancer ☊, or of Capricorn ☋, into 12

[Scholium to Lemma XXXIX, continued]

into equal parts; or at any rate they find, in each tropic, the points of the unequal hours, and through these points, together with the points of the hours on the equinoctial line, they draw straight lines to serve as the lines of the unequal hours — just as if lines of this kind indicated the unequal hours throughout the whole year — after the manner of the common sections of the plane of the sundial [horologium] and of the great circles that pass through them.

And, to tell the truth, since I could find no demonstration of this matter, it tormented me sharply for not a few years. I asked, by letter, many mathematicians both within Italy and outside it to teach me by what reasoning it could be demonstrated that the same great circles which are drawn through the unequal hours of the equator and of each tropic — for that this can be done we have demonstrated in the scholium to proposition 10 of book 1 of the *Gnomonics* — pass also through the unequal hours of the other parallels between the tropics. But I could never obtain what I wanted, although there was no lack of someone who claimed he could demonstrate it for me. He must, however, have been mistaken, since — when we investigated the demonstration of that matter anew — it has here been demonstrated that this can in no way be done.

[Margin: What the lines of unequal hours in sundials refer to.]

Therefore the lines of unequal hours in sundials — such as we have also described in our *Gnomonics* — are merely the common sections of the plane of the sundial and of the greatest circles that pass through the unequal hours of the equator and of each tropic; or at least of the equator and of that parallel whose diurnal arc contains 18 equal hours, or 6. And so, if we wish to speak geometrically, they will not truly indicate the unequal hours except when the Sun is in the equator, or in those extreme parallels by whose means they were described. It is true that, in a sphere in which the altitude of the pole does not exceed 45 degrees, the discrepancy between the true unequal hours and those which the said lines indicate, within the latitude of the tropics, is so small that those lines can be taken as true without any error perceptible to sense. But where the altitude of the pole is greater than 45 degrees it is not so: for there a greater

discrepancy appears, and the higher the altitude of the pole, the greater the difference that will exist between the true unequal hours and those lines — just as, conversely, the lower the altitude of the pole, the smaller the divergence between them. All of this can be gathered from what we have demonstrated in this place.

Therefore, so that the unequal hours may be indicated more truly in sundials, their points must be found on several parallels between the two tropics — by the same art by which we investigated them on each tropic — and then those points which do not lie on a straight line must be joined by suitably curved lines, as is customarily done in describing hyperbolas and the other conic sections.

LEMMA XXXX.

If a parallel to one side is drawn within a triangle, or if — when two sides are produced toward the angle contained by them — a parallel to the third side is drawn, so that two triangles are formed: the circles described about them touch one another at the common angle, or at a common point.

Let there first be, in triangle ABC, the straight line DE parallel to side BC, and let circles ABC, ADE be described about the triangles ABC, ADE; I say these touch one another at A, the common angle. For, from the centres F, G, let two pairs of straight lines be drawn to the bases of the triangles: FD, FE and GB, GC. [Margin: a — Euclid III.20] Since both the angle DFE and the angle BGC is double the angle BAC, they will be equal to one another. Therefore also the remaining two, FDE and FED, will be equal to the remaining two, GBC and GCB; [Margin: b — Euclid I.5] and consequently, since both those and these are equal among themselves, each of the former will be equal to each of the latter, and hence the angle FDE will be equal to the angle GBC. [Margin: c — Euclid I.29] But the whole angle ADE is also equal to the whole angle ABC, being the external [angle equal to the] internal; and so the remaining angle ADF will be equal to the remaining angle ABG. [Margin: d — Euclid I.5] Moreover (the straight lines FA, GA being drawn), in the isosceles triangles ADF, ABG the angle DAF is equal to the angle ADF, and the angle BAG to the angle ABG. Therefore the angles DAF and BAG will also be equal to one another; and consequently the straight line AF will be the same as AG, since they make the same angle with the same straight line AB. Thus the circles, having their centres on the same straight line AG and being described through the same point A, will touch one another at A — from the scholium to Euclid III.13.

Diagram. A framed woodcut band containing three side-by-side versions of the same construction: a larger circle circumscribing triangle ABC (apex A at top, base BC below), with a smaller circle circumscribing triangle ADE inscribed so that DE is parallel to BC and the two circles touch internally at the common apex A. F is the centre of the larger circle, G the centre of the smaller. Readable labels across the sub-figures: A (apex), B and C (base of larger triangle), D and E (ends of the parallel), F and G (the two centres).

Now let the sides BA, CA be produced toward the angle A; let DE be parallel to the base BC, and let circles be described about the triangles ABC, ADE, which I say touch one another at A. For, from the centres F, G, let straight lines be drawn to the bases: FB, FC and GD, GE. [Margin: c — Euclid III.20] Since again both the angle BFC is double the angle BAC and the angle DGE double the angle DAE, [Margin: f — Euclid I.15] and since the angles BAC, DAE are equal as vertical angles, the angles BFC, DGE will also be equal to one another; and hence the remaining two, FBC, FCB, taken together, will be equal to the remaining two, GDE, GED, taken together. [Margin: g — Euclid I.5] Since, then, both those and these are equal among themselves, each will be equal to each, and hence the angle FBC will be equal to the angle GDE. [Margin: h — Euclid I.29] And (the straight lines FA, GA being drawn) the angle ABC is equal to the angle ADE. Therefore the remaining angle ABF will be equal to the remaining angle ADG — in the 1st, 2nd, and 5th figures — or the whole to the whole, in the 4th figure.

Diagram. A framed woodcut band running across the opening, containing several versions of the "produced-sides" case: two circles touching at the point A, one above and one below. The upper circle carries B and C with centre F; the lower circle carries D and E with centre G; the triangles ABC and ADE share the vertex A at the point of tangency. Readable labels: A (point of contact, centre), B, C, F (upper circle), D, E, G (lower circle). The band shows the successive figures (numbered 1–5 in the text) illustrating the different positions of the produced sides.

In the 3rd figure this reasoning is not needed, where the straight lines FB, FC and GD, GE do not form angles but are continued in a straight line. [Margin: a — Euclid I.29] Even so the angles ABF, ADG will be equal, since they are alternate angles between the parallels BC, DE. And so, since the angles ABF, ADG are equal, [Margin: b — Euclid I.5] and the one is equal to the angle BAF and the other to the angle DAG; the angles BAF, DAG will also be equal to one another. And consequently, since BD is a straight line by hypothesis, they will also make AF, AG a single straight line — by what we have demonstrated, following Proclus, at

Euclid I.15. Therefore the circles, having their centres on the same straight line FG and being described through the same point A, will touch one another at A, because the straight line drawn through A perpendicular to FG touches both circles, from the corollary to Euclid III.16. For from this it comes about that the circles do not cut one another, since they do not even cut that perpendicular, but touch it.

COROLLARIUM.

From what has been demonstrated at the end of this proposition it is gathered that two circles, described from two centres lying on the same straight line and passing through the same point, touch one another externally at that point. Of this kind are the two circles ABC, ADE.

LEMMA XLI.

To describe, through two given points, a circle that touches a given circle. It is required, however, that the two given points either both lie outside the given circle, or both inside; or, if one is on the circumference, that the other be outside or inside the circle, but not on the circumference.

Let ABC be the given circle, and let there first be two given points D, E outside it, through which a circle must be described touching the circle ABC. Let the straight line DE be joined, and let it first pass through F, the centre of the given circle, and be bisected at the point G, from which let the perpendicular HGI be raised to DE. On this perpendicular the centre of the circle to be described will certainly lie, from the corollary to Euclid III.1. We shall find it thus. On G as centre describe the semicircle DKE; let the perpendicular FK, drawn from the centre F to DE, cut it at K; and from K draw a straight line KA to one of the extremities of the diameter AC — say to A. Make the angle AKL equal to the angle KAC, and let KL cut the straight line DE at the point L. Then FL, intercepted between the centre and the midpoint, will necessarily be greater than FG. For, GK being joined — since it is equal to GD — it is greater than GA. [Margin: c — Euclid I.18] Therefore in triangle AKG the angle GAK is greater than the angle AKG; and the angle AKL, which was made equal to KAL, is greater than the angle AKG. And so the straight line KL will fall beyond G, between G and E — that is, FL will be greater than FG.

Therefore, describing from F, through L, an arc of a circle that cuts the perpendicular HI at H and I: I will be the centre of the circle passing through D, E and touching the circle ABC above the straight line DE, while H will be the centre of the circle touching the same circle below the straight line DE. For, drawing

through F, I the straight line BFIM, describe from I, with radius IB, a circle, which — by the scholium to Euclid III.13 — will touch the circle ABC at B. I say that the same circle passes through the given points D, E. For, ID, IE being joined: since FI, FL are equal, and the equal segments FB, FA are added, the wholes IB, LA will be equal. [Margin: d — Euclid I.6] And therefore, since LK is equal to LA — on account of the equal angles LAK, LKA — the straight lines LK, IB will also be equal, and their squares equal. [Margin: e — Euclid I.47] Now the square on LK is equal to the squares on FK, FL together; [Margin: f — Euclid II.5] and the square on IB is equal to the rectangle contained by BF, FM together with the square on FI (or FL). Therefore the two squares on FK, FL are equal to the rectangle by BF, FM together with the square on FL; and, the common square on FL being taken away, the remaining square on FK will be equal to the remaining rectangle by BF, FM. [Margin: g — Euclid VI.17] But the square on FK is also equal to the rectangle by DF, FE, because FK is the mean proportional between DF, FE, by the scholium to Euclid VI.13. Therefore the rectangle by BF, FM will also be equal to the rectangle by DF, FE. [Margin: h — Euclid II.5] Since, then, the rectangle by BF, FM together with the square on FI — that is (since these are equal to those) — is equal to the square on IB; [Margin: i — Euclid I.47] the rectangle by DF, FE together with the squares on FG, GI will also be equal to the square on IB. [Margin: k — Euclid II.5] But the rectangle by DF, FE together with the square on FG is equal to the square on DG. Therefore the square on DG (taken together with the square on FG) — that is, together with the square on GI, i.e. equal to the squares on DG, GI — [Margin: l — Euclid I.47] will be equal to the square on ID; and consequently the straight lines ID, IB will be equal. [Margin: m — Euclid I.4] Since, then, ID, IE are also equal — because the two sides DG, GI, [equal] to the two sides EG, GI, contain equal right angles — the three straight lines IB, ID, IE are equal. Therefore the circle described from I, through B — touching the circle ABC at B, as was said — will pass through the given points D, E, which was proposed.

Diagram. The construction for Lemma XLI. The given circle ABC has centre F; the horizontal straight line DE runs through F, with D at the left and E at the right. A dashed semicircle DKE is drawn with vertex K at the top. The perpendicular through the midpoint carries H (upper) and I (lower); many chords converge on the figure. Readable point-labels: K (top), P (just right of K), B (upper left), H (upper middle), D (left end), A and O (left of centre), F (centre), G and L (just right of centre), C (right of centre), E (right end), R (lower right), I (lower middle), N (lower left), S (below I), Q and M (at the foot). All 18 labels A–S resolve on the scan.

But if, from K, a straight line KC is drawn to the other extremity C of the diameter of the given circle, and the angle CKO is made equal to the angle DCK — the straight line KO cutting DE at O — then FO will be equal to FL, as will be shown; and therefore, when a circle is described from F through O, HI will be cut at the same centre I. Hence the same centre will always be found, whether the straight line is drawn from K to A or to C, and so on. That FO is equal to FL will be demonstrated thus. Since the two sides AF, FK are equal to the two sides CF, FK, and contain equal — indeed right — angles, [Margin: n — Euclid I.4] the bases KA, KC will be equal, and both the angles FAK, FCK and the angles FKA, FKC. But the angle AKL is equal to the angle FAK, and the angle CKO to the angle FCK, by construction. Therefore the angles AKL, CKO will also be equal; and, the equal angles FKA, FKC being taken away, the remaining angles FKL, FKO will be equal. [Margin: a — Euclid I.26] And so, since the two angles F, K of triangle FKL are equal to the two angles F, K of triangle FKO — to which the common side FK is adjacent — the sides FL, FO will be equal, which was proposed.

In the same way we shall demonstrate that the circle described from H, with radius equal to the straight line drawn HFN, touches the given circle ABC at N and passes through the given points D, E.

If it should ever happen that the centre of the given circle and the midpoint of the straight line joining the two given points coincide — as if G were the centre of the given circle PRS — we shall, with the greatest ease, describe a circle through the two points D, E that touches the given circle. For the circle through the three points D, P, E (the perpendicular to DE having first been raised at P, where it is cut by the perpendicular GP [?]) will touch it; and the circle described through the three points D, S, E will touch the same at S, where it lies on the same perpendicular PQ — from the corollary to Euclid III.1.

Next let the straight line DE not pass through F, the centre of the given circle ABC, but either let it cut the circle in any way whatever, as in the first figure; or let it touch it, as in the 2nd; or let it lie wholly outside, so that when produced it neither cuts nor touches it, as in the 3rd, 4th, and 5th figures; or, finally, let it lie outside in such a way that when produced it cuts or touches it, as in the 6th and 7th figures. Let the straight line DF be joined and bisected at G; describe from G, about DF, a circle cutting the given circle at B; and let DB be joined, which — by the scholium to Euclid III.31 — will touch the given circle at B. Now, DH being found as the third proportional to DE, DB, the point H will fall — in the first figure — outside the given circle, toward the point D from which the tangent DB was drawn. [Margin: b — Euclid VI.17] For since the square on DB is equal to the

rectangle by DE, DH, [Margin: c — Euclid III.36] and also to the rectangle by DP, DO, the rectangle by DE, DH will be equal to the rectangle by DP, DO. [Margin: d — Euclid VI.16] Therefore, as DE is to DP, so is DO to DH. Since, then, DE is greater than DP, DO will also be greater than DH, and hence the point H will be between D and O. By like reasoning, in the second figure the point H will lie between D and the point of contact K. For since, as DE is to DB — that is, to DK (for DK is equal to DB, by corollary 2 to Euclid III.36) — so is DB (or DK) to DH; and DE is greater than DK; DK will also be greater than DH.

Diagram. A second cutting of the Lemma XLI construction (essentially the same figure as fig-p419-1, repeated on this page). The given circle carries the horizontal line DE (D left, E right) through the centre F; the vertex K is at the top with P beside it; the perpendicular carries H and I; a dashed inner semicircle and many converging chords are drawn. Readable point-labels: K (top), P, B, H, D, A, O, F (centre), G, L, C, E, R, I, N, S, Q, M — the same set A–S as on p. 419.

In the third figure the same point H is between the points D, E; in the 4th it is the same as E, and therefore DB, DE are equal; and in the 5th [it lies] beyond the point E, so that the point H will lie beyond the circle. In the 6th this is proved thus: [Margin: e — Euclid VI.17] since the square on DB is less, being equal to the rectangle by DE, DH, [Margin: f — Euclid III.36] than the rectangle by DO, DP, the rectangles by DE, DH and by DO, DP will be [in proportion]; [Margin: g — Euclid VI.16] and hence, as DE is to DO, so is DP to DH. Since, then, DE is supposed less than DO, DP will be less than DH, and therefore H will lie beyond P. In the 7th the demonstration will be this: since (for DA is equal to DB, by corollary 2 to Euclid III.36), as DE is to DA, so is DA to DH; but DE is less than DA; therefore DA will also be less than DH.

Next, HF being joined and bisected at I, describe from I, about FH, a circle cutting the given circle at the points A, K; through which, if from the given point D straight lines DA, DK are drawn cutting the circumference of the given circle at L, M: the circle described through the three points D, E, L will touch the given circle at L, as appears in the first figure, in which the circle DL is described; and the circle described through the three points D, E, M will touch the same at M, as is clear in the 1st and 5th figures, where we described the circle DEM, whose centre...

Diagram. A large framed woodcut band containing the several numbered case-figures of the construction. Legible figure-numbers within the band: 1^a, 2^a, 3^a, 4^a (the sequence runs toward the 6th and 7th cases discussed in the text). Each sub-figure shows the given circle with an inscribed/attached construction triangle and

the tangent circles through D, E; recurring point-labels visible across the sub-figures include B, L, A, F, C, D, H, K, E, G, N, I, M, P, and the lowercase construction points a, b, o. The labels are very small and densely overwritten; several within the individual sub-figures cannot be resolved with confidence

the centre of the touching circle is the point a, at which the perpendicular ba — bisecting the straight line DE — cuts the straight line FL (or FM) produced through the point L (or M). For, by the corollary to Euclid III.1, the perpendicular ba passes through the centre of any circle described through D, E; and the centre of the touching circle necessarily lies on FL, because the straight line drawn through the two centres of touching circles falls upon the point of contact.

[Margin: h — Euclid III.11 or 12] For if the centre of the circle touching the circle ABC at L be said not to lie on the straight line FL, then the straight line drawn from that centre through F, the centre of the given circle, will cut the straight line FL at F. Therefore, when produced, it could not fall upon the point of contact L — which is absurd. If, therefore, the circle described through the three points D, E, L must touch the given circle at L, its centre will lie on the straight line FL. And by the same reasoning the centre of the circle described through the three points D, E, M and touching the given circle at M — as appears in the same first figure — lies at a, the common section of the perpendicular ba and the straight line MF. Furthermore, the contact at L is interior, but that at M exterior, except in figures 1 and 6: for in the first the contact at M is also interior, and in the 6th the contact at L is exterior. In the 2nd figure only one contact occurs,

ASTROLABIUM — LIBER PRIMUS (continued)

LEMMA XLI (continued)

and it is interior, at L. Likewise, in the seventh figure only a single contact occurs, and it is exterior, at M. We have not, however, drawn all the tangent circles, so as to avoid confusion, judging it sufficient to give one example — in the first figure — of circles touching one another internally at L, and another example — in the fifth figure — of a circle touching externally.

Moreover, we shall demonstrate as follows that the circle drawn through the three points D, E, L touches the given circle at L. Since the square on the line DB is equal both [Margin: Eucl. VI.17] to the rectangle under DE and DH and [Margin: Eucl. III.36] to the rectangle under DL and DA, these two rectangles will be equal to each other. Therefore, by the scholium to Proposition 36 of Book 3 of Euclid, a circle can be drawn through the four points A, L, E, H. Consequently, when the

line LE is drawn, cutting the circumference at C — that it necessarily cuts the circle we shall demonstrate at the end, in the scholium — and the line AC is joined, [Margin: Eucl. III.22] the two opposite angles ALE and AHE, in the quadrilateral ALEH, will be equal to two right angles (in the first three figures).

Diagram. A framed woodcut plate at the head of the page containing several sub-figures of Lemma XLI, each showing a circle with an inscribed/circumscribed triangle and its tangent construction. [Several sub-figure caption numerals are cursive and hard to resolve.]

- Large circle at left (caption "2^a(?) [?]): apex L (top); B and A (upper left); F (centre); C (right); D (far left); P and N (right of centre); E (far right); M (bottom); interior points G, I, K; small letters a (top of vertical axis), b (centre), a (bottom).
- Upper-right sub-figure (caption "1^a(?) [?]): L (top), B and A (left), F (interior), I and C (right), G, D, H, K (bottom), E (bottom right).
- Middle-right sub-figure (caption "3^a"): L (top), B (upper left), F (interior), A (left), C (right), G, N, I, K, M (interior/lower), D (bottom left), H (bottom), E (bottom right).
- Right sub-figure (caption "4^a"): L (top), B and N (upper left), F (interior), K (upper right), A (centre), C (right of centre), G, I, D (bottom left), H and E (bottom right).

Point-labels are legible but the small cursive caption numerals of two sub-figures are uncertain

[Margin: Eucl. I.13] But the two angles AHD and AHE are also equal to two right angles. Therefore those two will be equal to these two; and when the common angle AHE is taken away, the remaining angles ALE and AHD will be equal.

[Margin: Eucl. III.32] Now the angle HAC is also equal to the angle ALE in the alternate segment — for the lines HA and HK touch the circle ABC at A and K, by the scholium to Proposition 31 of Book 3 of Euclid — so that this same angle HAC will be equal, as an alternate angle, to the angle AHD. [Margin: Eucl. I.27]

Therefore AC and DE will be parallel. Since, then, the given circle has been drawn about the triangle LAC, the circle drawn about the triangle LDE will touch the given circle at L, by the preceding lemma. And this demonstration holds for the first three figures.

In the fourth figure the demonstration will run thus. [Margin: Eucl. III.36] Since the square on the line DB — and hence also the square on the line DE, which is equal to DB — is equal to the rectangle under DL and DA, then, if a circle is drawn

about the triangle LAE, [Margin: Eucl. III.37] the line DE will touch it at E, seeing that the line DL cuts that same circle. [Margin: Eucl. III.32] Therefore the angle DEA will be equal to the angle ALE in the alternate segment. [Margin: Eucl. III.32] And since the angle EAC is likewise equal to that same angle ALE in the alternate segment of the given circle, the alternate angles DEA and EAC will be equal; [Margin: Eucl. I.27] and therefore DE and AC will be parallel. Hence, as before, by the preceding lemma, the circle drawn about the triangle LDE will touch, at L, the given circle ABC — the circle drawn about the triangle LAC.

In the fifth figure the demonstration will be set up as follows. [Margin: Eucl. VI.17] Since the square on the line DB is equal both to the rectangle under DE and DH and [Margin: Eucl. III.36] to the rectangle under DA and DL, these two rectangles will be equal to each other. Therefore, by the scholium to Proposition 36 of Book 3 of Euclid, a circle can be drawn through the four points A, L, H, E, [Margin: Eucl. III.21] in which the angles at L and H, lying in the same segment whose chord is AE, will be equal. [Margin: Eucl. III.32] But the angle HAC is also equal to the angle at L in the alternate segment of the given circle. Therefore the alternate angles HAC and AHD will be equal, [Margin: Eucl. I.27] and consequently DE and AC will be parallel, and so on.

Finally, in the sixth figure we shall reach the same conclusion in this way.

Diagram. A second framed woodcut plate at the foot of the page with further sub-figures of the same construction.

- Left sub-figure (caption "5^a"): L (top), N and B (upper left), K (upper right), F (interior), G (centre-left), C (right of centre), M (interior/lower), D (bottom left), E (bottom), H (right); small letters a, b near the centre.
- Middle sub-figure (caption cursive [?]): B (top), K (upper right), N (upper left), G (left), I (right), M (interior), H (right), D and E (bottom left), C and A (bottom).
- Right sub-figure (caption "6^a"): B (top), H (upper left), N (left), K (upper right), M and G (left of centre), O (lower left), F (centre), I and P (lower centre), E (lower left), L and C (bottom), A (bottom centre).

Point-labels legible; middle sub-figure caption numeral unresolved

[Margin: Eucl. VI.17] Since the square on the line DB is equal both to the rectangle under DE and DH and [Margin: Eucl. III.36] to the rectangle under DL and DA, these two rectangles will be equal to each other; and consequently a circle can be drawn about the four points E, H, A, L, by the scholium to Proposition 36 of Book 3 of Euclid. [Margin: Eucl. III.22] Therefore the two opposite angles HAL and HEL,

in the quadrilateral EHAL, will be equal to two right angles. [Margin: Eucl. I.13] And since the two angles HEL and DEL are also equal to two right angles, those two will be equal to these two; and when the common angle HEL is removed, the remaining angles HAL and DEL will be equal. [Margin: Eucl. III.32] Now the angle HAL is equal to the angle ACL in the alternate segment of the given circle. Therefore the angle DEL will also be equal, as an alternate angle, to that same angle ACL; [Margin: Eucl. I.27] and therefore DE and AC will be parallel, and so on.

In the seventh figure the point L does not exist, since DE touches the circle at A, and consequently L coincides with A. But it is certain that no circle can be drawn through the three points D, E, A when they lie in a straight line. Therefore only the contact at M will occur, as we shall demonstrate.

Diagram. A large framed woodcut occupying the upper left of the page: a lens- or almond-shaped figure formed by two intersecting arcs, with a chain of interior lines (the seventh-figure case, where the line DE touches the circle at A). Labels: D (far left vertex), E (far right vertex), L (top), M (bottom), A (upper interior), K (lower interior), N (lower right interior), P (right of centre), F (centre); along the horizontal axis the small letters q, H, O, I, b; a vertical line through the top and bottom marked a ... a.

(several small interior labels along the axis are faint).

In this eighth figure the line DE passes through the centre F; nevertheless the same construction and demonstration hold as in the others, especially as in the first figure. But we have also handled this case in another way, at the beginning of this lemma.

In almost the same way we shall show that the circle drawn through the three points D, E, M touches the given circle at M. For in the first figure, since the square on the line DB is equal both [Margin: Eucl. VI.17] to the rectangle under DE and DH and [Margin: Eucl. III.36] to the rectangle under DK and DM, these two rectangles will be equal to each other; and therefore a circle can be drawn about the four points H, E, M, K. Then, in the quadrilateral HEMK, when the line ME is drawn cutting the circumference at N — that it necessarily cuts the circle we shall demonstrate at the end, in the scholium — and the line KN is joined, [Margin: Eucl. III.22] the two opposite angles EMK and EHK will be equal to two right angles. [Margin: Eucl. I.13] But the two angles DHK and EHK[?] are also equal to two right angles. Therefore these two will be equal to those two, and when the common angle is taken away the remaining angles will be equal. [Margin: Eucl.

III.32] The angle HKN is also equal to the same angle EMK in the alternate segment; [Margin: Eucl. I.27] and therefore the alternate angles DHK and HKN will be equal, and consequently the lines DE and KN are parallel. The circle drawn through K, N, M will therefore touch the given circle at M, by the preceding lemma.

In the second figure, where a single contact occurs at L (the line DE touching the given circle), [Margin: Eucl. III.21] a circle can be drawn through the four points M, K, E, H, in which the angles MKH and MEH, lying in the same segment whose chord is the line MH, are equal; [and] the angle [?] KNE in the alternate segment is equal. Therefore the alternate angles [?] are [equal], and [the lines are] parallel. The circles circumscribed about the triangles KMN and DME therefore touch one another, by the preceding lemma.

In the fourth figure thus. [Margin: Eucl. III.36] Since the square on the line [DB is equal...], if a circle is circumscribed about the triangle KME, [Margin: Eucl. III.37] [the line DE] touches it, and [the angle] in the alternate segment of that same circle will be equal. [Margin: Eucl. III.32] [And since the angle...] is equal in the segment of the given circle, the alternate angles DEM [and KNM] will be [equal].

Finally, in the fifth, sixth, and seventh figures thus. Since through four [points a circle can be drawn], as was shown in the first figure, [Margin: Eucl. III.22] in the quadrilateral MK[HE the two opposite angles will be equal to two right angles]. [Margin: Eucl. I.13] But the two angles DEM and MEH are also equal to two right angles [...]; and when the common angle MEH is removed, the remaining angles DEM and HKM will be equal. [Margin: Eucl. III.32] [The angle... is] equal [in the alternate segment] of the given circle. Therefore the alternate angles DEM and KN[M will be equal], [Margin: Eucl. I.27] and consequently the lines DE and KN are parallel, and so on.

Now let two points D, E be given inside a circle, and through them let there be drawn any line DE whatever — whether it passes through the centre of the given circle or not. Let a fourth proportional DI be found to the three lines DE, DG, DH. And since DE is to DG as DH is to DI, and DE is less than DG, DH too will be less than DI; and consequently the point I will lie outside the circle. From I to the centre F let the line IF be drawn — for the case where DE, when extended, does not pass through the centre — and, this being bisected at K, from K about IF let there be drawn a circle cutting the given circle at A and B; and let the lines [IA, IB be joined].

Diagram. A framed woodcut with two adjacent sub-figures illustrating the case of two points given inside the circle. Labels read (left sub-figure): I (far left), B and N (top), L (top centre), H (left), K (lower left), F (interior), D and E (centre), G (right of centre), A (bottom), C and M (bottom centre). Right sub-figure repeats the same lettering (B, N, L, H, K, D, E, A, C, M) in a slightly different arrangement.

Several labels are small/overlapping

[Margin: Eucl. III.31] The lines IA and IB, by the scholium to Proposition 31 of Book 3 of Euclid, will touch the given circle at A and B. If, then, from A through D a line AD is drawn cutting the circumference at L, the circle drawn through the three points D, E, L will touch the given circle at L. Likewise, the line BD drawn will cut the circumference at the point M, at which the circle drawn through the three points D, E, M will touch the given circle at M. And this contact is always interior. The demonstration is this. Let the line LE be drawn cutting the circumference at C, and let the line AC be joined; likewise let the line ME be drawn cutting the circumference at N, and let the line BN be joined. Since DE is to DG as DH is to DI, [Margin: Eucl. VI.16] the rectangle under DE and DI will be equal to the rectangle under DG and DH; [Margin: Eucl. III.35] but this is equal to the rectangle under AD and DL; therefore that one too. Hence through the four points A, I, L, E a circle can be drawn, by the scholium to Proposition 35 of Book 3 of Euclid, [Margin: Eucl. III.21] and consequently the angles IAL and LEI, in the same segment of that circle whose chord is the line IL, will be equal. [Margin: Eucl. III.32] Now IAL is equal to the angle ACL in the alternate segment of the given circle. Therefore the angles LEI and ACL — the external and the internal — will be equal, [Margin: Eucl. I.28] and so the lines DE and AC are parallel. By the preceding lemma, then, the circle circumscribed about the triangle DEL will touch, at L, the given circle circumscribed about the triangle ACL, as appears in the earlier figure; and again the centre lies at a, at the common intersection of the perpendicular ba, which bisects the line DE, and of the line LF, drawn from the point L through the centre F of the given circle.

In the same way we shall show that the circle drawn through D, E, M touches the given circle at M. For again the rectangle under DE and DI will be equal to the rectangle under BD and DM. Therefore through the four points I, B, E, M a circle can be drawn, by the scholium to Proposition 35 of Book 3 of Euclid, [Margin: Eucl. III.21] and consequently the angles IBM and MEI, in the same segment of that circle whose chord is the line IM, will be equal. [Margin: Eucl. I.28] Now IBM is equal to the angle BNM in the alternate segment of the given circle [...], external and internal, they will be equal, and so the lines DE and BN are parallel.

By the preceding lemma, then, the circle circumscribed about the triangle DEM will touch the given circle at M, as you see in the latter figure — where too the centre lies at a, at the common intersection of the perpendicular ba and of the line MF. In this figure one of the given points, namely D, is the very same as the centre F, and so on.

Diagram. A large framed circle at the head of the page (the case in which one given point coincides with the centre). Labels: N and B (top), L (top left, just below N), a and G (upper left interior), F (centre), D (centre), K and H (right of centre, on the horizontal), I (far right), G (far left, on the horizontal), E (left interior), M (lower left), C (bottom left), A (bottom right). Diagonal lines run from F to the circumference points; dashed arcs cross the figure.

(overlapping interior labels near the centre).

But if the solution of the problem begins from the point E, exactly the same point L, or M, will be found. For the problem can be solved in no other way. Suppose, if it could be otherwise, that some other point d were found, as in the latter figure. Then the line dE would cut the circumference below the point C, and the line dD would cut it above the point A; and consequently the line connecting the points of section would cut the line AC, and hence would also cut its parallel DE, produced. It would therefore not be parallel to it — which is nevertheless what the problem requires, as has been made plain and clearly follows from the preceding lemma. The same absurdity will be seen in the other figures, if any point other than L, or M, is said to be found when the solution of the problem begins from the point E. The same absurdity will follow if both dE and dD cut the circumference above A: because the line joining the two points of section cannot be parallel to the line DE, as is plain.

And so, in order that the proposed problem may be accomplished, it is necessary to draw, from the two given points, two lines to some one point of the circumference of the given circle, such that the line connecting the two points at which those two lines cut the circumference is parallel to the line joining the two given points. Thus you see, for example, that from the points D and E, drawn to the point L, the two lines DL and EL cut the circumference at A and C, and that the line AC is parallel to the line DE; and likewise that from D and E, drawn through the point M, the two lines DM and EM cut the circumference at B and N — in the latter two figures — but at K and N in the earlier ones, and that both BN and KN are parallel to the line DE. And although we investigated this point L, or M, at the end of Book 6 of Euclid, following Pappus, it has nevertheless seemed good to teach the same thing here as well, especially since the procedure

delivered here — when two points are given inside the circle — differs somewhat from the one we prescribed in Euclid.

Lastly, if one point is given on the circumference and the other inside or outside the circle, in such a way that the line drawn through both, when extended, passes through the centre of the circle, it is evident that if, from the midpoint of the line connecting the two given points, a circle is drawn about them, it will touch the given circle at the given point. For instance, in the first of the latter two figures, let one point H be given on the circumference of the given circle ABC and the other point D inside the circle, in such a way that the line DH passes through the centre F; then the circle drawn from the midpoint of the line DH, through D and H, will touch the given circle at H, by the scholium to Proposition 13 of Book 3 of Euclid. Likewise, if a point G is given on the circumference and I outside the circle, in such a way that again the line IG passes through the centre [F], the circle drawn from the midpoint of the line GI, through G and I, will touch the given circle at G, by the same scholium. Finally, if a point I is [given] outside, in such a way that the line IH likewise, when extended, passes through the centre F, the circle drawn from [the midpoint of the line HI, through H and I,] will touch the given circle at H. For the line drawn through H perpendicular to IF will touch both circles, by [the common... Proposition 16 of Book 3 of Euclid]; and consequently these circles will meet one another at the same point H by common contact, since neither cuts the other, seeing that neither cuts the tangent line.

But if one point H is given on the circumference and the other, O, whether inside the circle or outside — in any position whatever, as in this figure — the problem will be solved thus. From H, through the centre F, let the diameter HFP be drawn, and let the line HO be joined; and if this is bisected at R, and a perpendicular RS is erected to it, cutting the diameter at S, the circle drawn from S, through H, will pass through O and will touch the given circle at H, as is established from the scholium to Proposition 13 of Book 3 of Euclid. But the point O, when outside the circle, must be so situated that the line HO does not touch the circumference of the given circle, because otherwise the perpendicular RS would not cut the diameter produced. Now if the line HO cuts the circumference, the circle drawn [from S] will touch the given circle internally. But if the point O is outside the circle and the line HO does not cut the circle, the contact will be exterior.

Diagram. A framed woodcut at the foot of the page showing three mutually tangent circles for the final construction. Labels: T (upper left), B (top), H (left, the point of contact), P (right, endpoint of the diameter with a second S and F marked along the horizontal HP line), S (twice, on the diameter), F (centre of the given

circle), O (interior), R (twice, midpoint region of HO), A (twice, lower left/centre), C (lower right). The diameter HFP runs horizontally; HO and the perpendicular RS are drawn.

(repeated S, R, A labels are hard to disambiguate at this resolution).

SCHOLIUM

Now, since in the first seven figures the line DA has been drawn from D through A — a line which necessarily cuts the given circle ABC, seeing that HA touches that same circle at A — we shall demonstrate that the line LE cuts the same circle [which was assumed in the demonstration], in this way. Since, if the solution of the problem [begins from a point], the point L is found, as was shown at the foot of the lemma, and since the straight line drawn from the assumed point (which is the beginning of the solution) — the line that yields the point L — cuts the given circle, as we said just now concerning the line DA, it clearly follows that the line LE cuts the same circle, seeing that it does not differ from the line that would be drawn from E, if the operation were begun from E. And the same must be said of the line ME, which, if the beginning is made from E, is found to give the same point M, and so on. This, however, we shall demonstrate in another way as follows. From the point A, parallel to DE itself, let AC be drawn, cutting the circumference of the given circle at C. I say that the line LE passes entirely through C, and hence cuts the circle at L and C.

[Astrolabium, Liber I — continuation of the Scholium to Lemma XLI]

to cut it — that is, to fall within the circle. For since a circle can be described through the four points A, L, E, H, as we have shown [Margin: a. Euclid III.22], the two opposite angles ALE and EHA in the quadrilateral ALEH will be equal to two right angles [Margin: b. Euclid I.13]. But the two angles EHA and AHD are likewise equal to two right angles. Therefore these two are equal to those two; and when the common angle EHA is taken away, the remaining angles ALE and AHD are equal [Margin: c. Euclid I.19]. But AHD is equal to the alternate angle HAC [Margin: d. Euclid III.32]. Therefore HAC too is equal to the angle ALE. Now the same angle HAC is equal to the angle ALC — once the line CL has been drawn — in the alternate segment. Therefore the angles ALE and ALC are equal, and so the line LE passes through C, making with AL the same angle that CL makes with it, and so on.

This demonstration is proper to the first three figures. In the fourth, since DE touches the circle described about the three points A, L, E — as has been proved

— the angle DEA will be equal to the angle ALE in the alternate segment [Margin: e. Euclid III.32]. But the same angle DEA is equal to the alternate angle EAC [Margin: f. Euclid I.29]. Therefore EAC too will be equal to the angle ALE. Since, then, the same angle EAC is equal to the angle ALC — once CL is drawn — in the alternate segment [Margin: g. Euclid III.32], the angles ALE and ALC are equal. Hence again the lines LE and LC coincide, and so on.

In the fifth figure, since, as has been shown, a circle can be described about the four points A, L, H, E, the angles ALE and AHE — being in the same segment, whose chord is AE — will be equal [Margin: h. Euclid III.21]. But HAC is equal to the alternate angle AHE [Margin: i. Euclid I.29]. Therefore the angle HAC will also be equal to the angle ALE. Since, then, the same angle HAC is equal to the angle ALC — CL having been drawn — in the alternate segment [Margin: k. Euclid III.32], the angles ALE and ALC will be equal; and so the lines LE and LC coincide with one another, and so on.

Finally, in the sixth figure (for in the seventh the point L is not present), since, as has been shown, a circle can be described through the four points A, L, E, H, the two opposite angles HAL and LEH will be equal to two right angles [Margin: l. Euclid III.22]. The angles LEH and LED are also equal to two right angles [Margin: m. Euclid I.13]; and when the common angle LEH is taken away, the remaining angles HAL and LED are equal. But the angle HAL is equal to the same angle ACL in that segment [Margin: n. Euclid III.32]. Therefore the angle LED too is equal to the same angle ACL in that segment. Since, then, the angle LED is also equal to the alternate angle that EL produced makes with AC [Margin: o. Euclid I.29], EL produced will fall on the point C. For if it were to fall between A and C, or beyond C, there would always arise an external angle equal to the internal opposite angle, in the triangle formed by the line CL, the segment of EL produced, and the segment of AC intercepted between the point C and the point onto which EL produced is said to fall — which is absurd, for the external angle is greater than the internal opposite one [Margin: p. Euclid I.16]. Since, then, EL produced falls on C, it is clear that LE cuts the circle in L — that is, falls within the circle.

By almost the same reasoning it will be shown that the line ME cuts the circle in M, that is, falls within the circle. For when KN is drawn parallel to DE, cutting the given circle in N, we shall show that ME passes through N, and hence falls within the circle, cutting it in M and N. For since in the first figure a circle can be described through the four points H, K, M, E, as has been shown, the two opposite angles EMK and KHE in the quadrilateral HKME will be equal to two right angles, and hence equal to the two angles KHE and KHD [Margin: q. Euclid III.22], which

are likewise equal to two right angles [Margin: r. Euclid I.13]; and, taking away the common angle KHE, the remaining angles EMK and KHD will also be equal. But the angle KHD is equal to the alternate angle HKN [Margin: s. Euclid I.29]. Therefore HKN too will be equal to the angle EMK. Since, then, the angle HKN is equal to the angle KMN — NM having been drawn — in the alternate segment [Margin: t. Euclid III.32], the angles EMK and KMN will be equal; and therefore the line ME will pass through N and will fall within the given circle. In the second figure the point M is not present. In the third figure we shall demonstrate the matter thus. Since, as has been shown, a circle can be described through the four points H, E, K, M, the angles HEM and HKM — being in the same segment of that circle, whose chord is HM — will be equal [Margin: u. Euclid III.21]. But the angle HKM is equal to the angle KNM in the alternate segment [Margin: x. Euclid III.32]; therefore HEM too will be equal to the same angle KNM [Margin: y. Euclid I.29]. Since, then, the angle HEM is equal to KNM, the angle KNM and the angle that EM produced makes with KN will be equal; for if EM produced were to fall between K and N, or beyond N, there would always arise an external angle...

Diagram. A large framed woodcut occupying the right-hand part of the page, containing the several cases (*figurae*) of the tangent-circle demonstration. Six of the seven cases are labelled with a small superscript numeral-plus-a: **1^a**, **2^a**, **3^a**, **4^a** (upper group) and **5^a**, **6^a** (lower group; the seventh case is treated without its own separate panel). Each panel shows a circle with an inscribed configuration of chords and a tangent line, demonstrating that the line LE (or ME) drawn from the tangent point passes through the point C (or N). Recurring labelled points across the panels include A, B, C, D, E, F, G, H, I, K, L, M, N. In the clearest panels:

- **1^a**: circle with L at top, D at lower left, E at lower right, H at bottom; interior points A, B, C, F, G, I, N, K arranged along the chords.
- **2^a**: L at top, B upper left, C right, D and H at bottom, with F, G, I interior.
- **3^a**: L at upper right, B upper left, A, C, F, G, N, I, M, K interior, D and H, E at base.
- **4^a**: L at top, B and N upper left, F center, K right, A, C interior, M, I lower, D and H at base.
- **5^a**: L at top, N and B upper left, K right, F, G, M, C interior, D, E, H at base.
- **6^a**: B upper left, K upper right, N, F, M, G, O, I, P interior, D at left, A, C, H, L at base.

Because the panels are small, crowded, and printed with heavy show-through, several interior point-labels are ambiguous. (multiple crowded labels not confidently resolved).

an external angle equal to the internal opposite angle, in the segment of the line EM produced and in the segment of the line KN intercepted between N and the point onto which EM produced is said to fall — which is absurd, for the external angle is greater than the internal opposite one [Margin: a. Euclid I.16]. Therefore EM produced falls on N, and hence falls within the circle, cutting off the arc MN.

In the fourth figure, since, as has been shown, the line DE touches the circle described about E, K, M, the angle DEM will be equal to the angle EKM in the alternate segment [Margin: b. Euclid III.32]; but the angle EKM is equal to the angle KNM in the alternate segment [Margin: c. Euclid III.32]. Therefore the angle DEM too is equal to the angle KNM. But the same angle DEM is equal to the alternate angle that EM produced makes with KN [Margin: d. Euclid I.29]. Therefore the angle KNM will be equal to the angle that EM produced makes with KN; and hence, as we showed a little earlier, EM produced will fall on N.

Finally, in the fifth, sixth, and seventh figures, since a circle can be described about the four points H, E, K, M, the two opposite angles HEM and HKM will be equal to two right angles [Margin: e. Euclid III.22], and hence equal to the two angles HKM and MED [Margin: f. Euclid I.13], because these too are equal to two right angles. Taking away, then, the common angle HEM, the remaining angles HKM and MED are equal [Margin: g. Euclid III.32]; and the angle MED is equal to the alternate angle that EM produced makes with KN [Margin: h. Euclid I.29]. And so, as was demonstrated a little earlier, [EM produced falls on N].

From this it is clear that it can also be demonstrated otherwise that the circle described through the three points D, E, L (or D, E, M) touches the given circle ABC in L (or M). For when AC (or KN) is drawn parallel to DE, we shall show — as in this scholium — that LE (or ME) falls on the point C (or N). Therefore, by the preceding lemma, the circle described through D, E, L (or D, E, M) will touch the given circle ABC in L (or M), which was what was proposed.

LEMMA XLII.

Given two circles, to describe — through a given point on the circumference of one of them — a circle that touches both given circles.

Let there be two circles AB and CD, whose centres are E and F, whether one contains the other, or they cut one another, or one lies entirely outside the other. Let it first be required to describe, through the point C on the circumference CD, a circle tangent to the circles AB and CD; which can be done in two ways. First, thus. From F, the centre of the circle in which the given point lies, draw the

semidiameter FC to the given point, and on it produced take CG equal to the semidiameter of the other circle. To E, the centre of that other circle, draw the line GE; let it be bisected at right angles by HI, which cuts FC in I; and through I, to E, the centre of the latter circle, draw a line cutting the circumference of that circle in B, and hence [let the lines be drawn so that the circle] described through C touches both circles. For since the two sides HE, HI are equal to the two sides HG, HI and contain right angles, the bases IE, IG will be equal, and the angles HEI, HGI equal [Margin: i. Euclid I.4]. Then, taking away the equals BE, CG — as in the first and third figure — or taking away IE, IG from the equals BE, CG — as in the second figure — the remainders IB, IC will be equal.

Therefore the circle described from I through C will pass through B, and hence — either by the scholium to proposition 13 of Euclid's book 3 — it will touch the given circles there, if it curves toward the same side as they do, or, when [it curves] toward different sides, by the corollary of the preceding lemma [?: the print here also carries a stray "40."]. And since the angles HEI, HGI have been shown equal, the centre I and the point B will be found if, on drawing the line GE, the angle GEI is made equal to the angle FGE; for the line EI will cut FG in I, the centre, and the circle in B, the point of contact. Again, since — when the line BC has been drawn — the triangles IGE, IBC about the same vertex I (or having equal vertical angles at I) have their sides proportional, being in the ratio of equality, they will be equiangular [Margin: a. Euclid VI.6] and will have the angles ICB, IGE equal. Therefore the lines CB, GE will be parallel [Margin: b. Euclid I.28 (or I.27)]. And so, if through the given point C a line CB is drawn parallel to GE, the point B, the contact, will likewise be found.

Next, what was proposed will be completed thus. Draw the semidiameter FC to the given point, and cut off from it, toward the centre, a line CK equal to the semidiameter of the latter circle; join the line KE, and let it be bisected at right angles at b; by the line ba, cutting FC in a; and finally, through a and E, draw a line cutting the latter circle in A. I say that the circle described from centre a, through the given point C, passes through A, and hence touches the given circles in C and A. For again the lines aE, aK will be equal, and the angles aKE, aEK equal [Margin: c. Euclid I.4]. Then, adding the equals EA, KC — as in the first and third figure — or taking aE, aK away from the equals EA, KC — as in the second figure — the wholes (or the remainders) aA, aC will likewise be equal. Therefore, as before, the circle described from a through C will pass through A and will touch the given circles in A and C. The same centre a and point of contact A will also be found if, on drawing the line KE, the angle KEN is made equal to the angle FKE.

Indeed, CA drawn parallel to KE will also give the same point of contact A, which will be demonstrated as before.

Diagram. A framed woodcut spanning the lower part of the page (its left edge continues the panels of the preceding page). It illustrates Lemma XLII: two circles (drawn with dashed and solid outlines) with centres E and F, and the tangent circle constructed through the given point. Legible labelled points include A (top), B, D (left), E, F, G, H, I, K, L, M, N, O, and the given point C, with the sight-lines FC, GE, HI and the tangent-circle radii drawn in. Several interior labels are crowded and faint. (crowded construction, more than two labels not confidently resolved).

their sides proportional, since they have the ratio of equality [Margin: a. Euclid VI.6]; they will be equiangular and will have the angles ICB, IGE equal. Therefore the lines CB, GE will be parallel [Margin: b. Euclid I.28 (or I.27)]. And so, if through the given point C a line parallel to GE is drawn, the point B, the contact, will likewise be found.

The matter will be carried out no differently if in the circle AB the given point is B (or A). For, drawing the semidiameter EB, take on it produced the line BL equal to the semidiameter of the other circle; join LF; let it be bisected at right angles at M; by the line MI cutting EL in I. For a line drawn through I and the centre F will give C, the point of contact, and I will be the centre of the circle to be described, as before. For again the lines IF, IL will be equal, and the angles IFL, ILF [Margin: d. Euclid I.4]. Then, taking IF, IL from the equals CF, BL — as in the first figure — or taking CF, BL from IF, IL — as in the second figure — or finally adding the same IF, IL to the equals CF, BL — as in the third figure — the remainders (or the wholes) IB, IC will be equal, and so on.

Likewise, if the semidiameter EA is drawn, and toward the centre E there is cut off AN equal to the semidiameter of the other circle, and NF is joined — which is bisected at right angles and cut in O by the line Oa cutting AN in a — then a will be the centre of the circle to be described, and the line Fa produced will give the point of contact C, and so on.

Thus the problem is solved: if, drawing the semidiameter from the given point to its own centre, one cuts off from it — whether outside or inside the circle — a line equal to the semidiameter of the other circle, and joins, from the endpoint of the cut-off line, a line to the centre of this circle, which another line bisects at right angles, and so on — although the point of contact found is not the same, but two different points, as is clear from the figures.

LEMMA XLIII.

If in a sphere a circle touches two great circles on the same side of their point of section, and also touches a great circle drawn through their poles, then the arcs of those two great circles — intercepted between the points of contact and the intersection of the circles, or the great circle drawn through their poles — are equal.

Let two great circles AB, AC cut one another in A; let a circle DE touch them in D and E, whose pole is F; and let the great circle BC be drawn through the poles of the circles AB, AC. I say that the arcs AD, AE (or BD, CE) are equal. For draw, through D and F, a great circle DF cutting AC in G, and through E and F a great circle EF cutting AB in H. Now since the arcs FD, FE pass through the pole of the circle DE and through the points of contact D, E, FD will also pass through the poles of the circle AB, and FE through the poles of the circle AC [Margin: e. Theodosius, *Sphaerica* II.5 (?)]; and therefore the angles at D and E will be right angles [Margin: f. Theodosius, *Sphaerica* I.15 (?)]. But the vertical angles at F are also equal, by proposition 6 of our [book on] spherical triangles [Clavius, *Triangula Sphaerica*, prop. 6]. Therefore, since in the triangle DFH the two angles D, F are equal to the two angles E, F of the triangle EFG, and the adjacent arcs FD, FE (from the pole) are likewise equal, the arcs FH, FG and the angles H, G will be equal, by proposition 20 of our spherical triangles; and consequently the whole arcs EH, DG will also be equal. Hence, since in the triangle AEH the two angles E, H are equal to the two angles D, G of the triangle ADG, and the arcs EH, DG adjacent to them are equal, the arcs AH, AG will be equal by the same proposition 20 of our spherical triangles; and since the three angles in the triangle AEH are equal to the three angles in the triangle ADG, by proposition 19 of our spherical triangles the arcs AD, AE will also be equal. And when these are taken away from a quadrant...

Diagram. Two framed woodcut panels in the upper half of the page continue the Lemma XLII construction: concentric solid-and-dashed circles with the tangent circle and its construction lines. Legible labelled points include A, B, C, D, E, F, G, H, I, K, L, M, N, O. The panels are crowded with construction lines and several labels are faint. (dense construction, more than two labels not confidently resolved).

Diagram. A square-framed spherical figure for Lemma XLIII. Two great-circle arcs run from **A** (at the left) — one rising to **B** (upper right), the other descending to **C** (lower right) — crossing one another. A small circle set between them is tangent to the upper arc at **D** and to the lower arc at **E**; its pole/centre is marked **F**

(inside the small circle). The point **H** stands just beside D (upper tangency) and **G** just beside E (lower tangency), marking where the great circles DF and EF cut AC and AB. Labels read: A (left), B (upper right), C (right), D and H (upper tangency), F (centre of small circle), E and G (lower tangency). No labels flagged.

[Continuation of the proof of Lemma XLIII, carried over from the previous page.]

the quadrants AB and AC. For since BC is drawn through the poles of the circles AB and AC, these in turn will pass through its poles (by the scholium to Proposition 15, Book I, of Theodosius). Consequently A will be the pole of circle BC, and therefore, by the corollary to Proposition 16, Book I, of Theodosius [Theodosius, *Sphaerica* I.16 coroll.], AB and AC will be quadrants. Hence the remaining arcs CE and BD will also be equal, which is what was proposed.

Another way. Draw the great circle DE through D and E. Then, by Proposition 8 of our *Spherical Triangles* [Clavius, *Triangula Sphaerica*], the angles FDE and FED will be equal, being in the isosceles triangle DEF. When these are subtracted from the right angles ADF and AEF, the remainders ADE and AED will be equal. Therefore, by Proposition 9 of our *Spherical Triangles*, the arcs DA and EA will likewise be equal, and so on.

LEMMA XLIV.

If, in a sphere, a circle is tangent to two equal lesser circles, then the arcs of those two lesser circles that are intercepted between the points of contact and the great circle drawn through their poles — or the point of intersection, when the circles cut one another — are equal.

The points of contact must lie on opposite sides if the equal circles face toward the same hemisphere, but on the same side if they belong to different hemispheres. I say that circles face toward the same hemisphere when they are described from the nearer poles, on the near side of the great circles described from those same poles; and toward different hemispheres when they are described from the farther poles, on the near side of those same great circles.

In the sphere ABCD, let there first be described, from the nearer poles A and B, two equal lesser circles EF and GH, cutting one another at I; and let a circle KL be tangent to them at the points K and L, which lie on opposite sides of the intersection point I — since the circles face toward the same hemisphere, being enclosed between the nearer poles A and B and the great circles MN and OP. I say that the arcs IK and IL — or FK and HL — are equal.

For, through the poles A and B, let the great circle ABCD be described; and through A (the pole of circle EF) and Z (the pole of the tangent circle KL) let the great circle AZ be described, cutting the great circle MN (described from the same pole A) at R. [Margin: Theodosius II.4.] This great circle will pass through the contact point K. Likewise, through B (the pole of circle GH) and Z (the pole of the tangent circle) let the great circle BZ be described, cutting the great circle OP (described from the same pole B) at S. [Margin: Theodosius II.4.] This too will pass through the contact point L. Since, then, the arcs AK and BL — drawn from the poles A and B to their proper circle KL — are equal, the remaining [arcs] will also be equal. [Margin: Theodosius II.10.] And the arcs ZK and ZL, drawn from the pole Z to that same proper circle, are equal; whence the arcs AZ and BZ are equal. Therefore, by Proposition 8 of our *Spherical Triangles*, the angles ZAB and ZBA will be equal.

Now since the sides AN and AR are equal to the sides BP and BS — for all of these are quadrants, by the corollary to Proposition 16, Book I, of Theodosius — and since they contain equal angles, as was shown, then by Proposition 7 of our *Spherical Triangles* the bases NR and PS will also be equal. But arc NR is similar to arc FK, and arc PS is similar to arc HL. Therefore the arcs FK and HL are similar to one another, and hence equal, since similar arcs of equal circles are equal. When these are subtracted from the equal arcs IF and IH — that these arcs are equal we shall demonstrate in the scholium — the remaining arcs IK and IL will also be equal.

By the same reasoning, if a circle pq is tangent to the same circles EF and GH at the points p and q, likewise lying on opposite sides, we shall show that the arcs Ep and Gq, and Ip and Iq, are also equal. For again, describing through A (the pole of circle EF) and r (the pole of the tangent circle pq) the great circle Ar, which cuts the great circle MN at t [Margin: Theodosius II.4.] and passes through the contact point p; and likewise describing through B (the pole of circle GH) and r (the pole of the tangent circle pq) the great circle Br, which passes through the contact point q [Margin: Theodosius II.4.] and cuts the great circle OP at u — since the arcs from the poles A and B to the equal circles, and the arcs rp and rq from the pole r to their circle, [make] Ar and Br equal, then by Proposition 8 of our *Spherical Triangles* the angles rAB and rBA [will be equal]; and consequently, from two right angles, the remainders rAM and rBN will be equal. Therefore, since the two sides AM and At are equal to the two sides BO and Bu, [Margin: Theodosius II.4.]

Diagram. A large sphere shown within a square frame (the figure for Lemma XLIV), drawn with several solid and dotted great and lesser circles representing the intersecting circle-pairs EF/GH, pq, and their tangent circles, together with the great circles MN and OP and the framing great circle ABCD. Legible labels around and within the sphere:

- A (top of the sphere)
- B (top, just right of A, marked with a small star/asterisk)
- H (upper left)
- Z (upper center)
- P (left margin, upper)
- E (left margin, below P)
- M (left margin, middle)
- L (upper-center left)
- S (center left)
- R (center)
- F (right margin, upper)
- K (center right)
- N (right margin)
- G (right margin, below N)
- O (right margin, lower)
- D (lower left)
- T (bottom left)
- X (bottom)
- C (bottom center)
- V (bottom)

Numerous small italic lowercase labels are also scattered through the interior (among them apparently p, q, r, d, e, n, u, c, f, a, i, l), but at this resolution most cannot be resolved individually. [Several interior labels [?]]

[Continuation of the Lemma XLIV proof, resuming below the figure:]

they are equal to the two sides BO and Bu, and comprehend equal angles, they will — by Proposition 7 of our *Spherical Triangles*, as before — also have the bases Mt and Ou equal. Therefore, cutting the great circle OP at u: since the arcs Ap and Bq, [drawn] from the pole r to circle pq, are equal, the whole arcs will also be equal; and the angles rAB and rBA [will be equal], and hence, from two right

angles, [the remainders]. Thus, as before, the arcs Ep and Gq, as well as Ip and Iq, will be equal. [Margin: Theodosius II.4.]

The same will be concluded if a circle ab is tangent to two equal circles TV and XY, both facing toward the same hemisphere, at the points a and b, which likewise lie on opposite sides of the points T and X. For again, describing from D — the poles of the circles TV and XY — through f (the pole of the tangent circle ab), the great circles [Cf and Df], cutting the great circles MN and OP at d and e [Margin: Theodosius II.4.] and passing through the contacts a and b, the arcs Cf and Df will be equal, inasmuch as Ca and Db, and fa and fb, are equal. Therefore, as above, the angles fCD and fDC, and the arcs Md and Oe — and consequently also Ta and Xb — will be equal, and so on.

Now let two equal circles be described from the farther poles B and C, facing toward different hemispheres, and let a circle Lmin be tangent to them at the points L and i, lying on the same side [of the great circle drawn] through their poles. I say again that the arcs HL and gi are equal. For, describing from the poles B and C, through k (the pole of the tangent circle Lmin), the great circles Bk and Ck, cutting the great circles OP and MN and passing through the contacts L and i, the whole arcs Bk and Ck will be equal; so too BL, Ci, kL, ki will be equal; and [by Proposition 8 of our *Spherical Triangles*] the angles kBC and kCB, and consequently, from two right angles, the remainders kBP and kCM will be equal. And so, as above, the arcs PS and MI will be equal, and hence the arcs HL and gi, similar to them, will be equal, and so on.

[Verso — running header: LIBRI I. ASTROLABII.]

SCHOLIUM.

That the arcs IF and IH are equal, as was assumed in the demonstration, we shall now prove as follows. Of the equal circles EF and GH, the segments running from the intersection I, through F and H, up to the other intersection are the lesser segments of those circles; and the remaining segments, running from I through E and G up to the other intersection, are the greater — as we shall shortly show. [Margin: Euclid, *Elements* III.28.] Therefore both the lesser and the greater segments will be equal, since they have the same chord drawn from I to the other intersection. [Margin: Theodosius II.9.] Since, then, these segments are bisected at F, H and at E, G by the great circle ABCD drawn through their poles, the arcs IF and IH, as well as IE and IG, will likewise be equal.

That the segments from I, through F and H, up to the other intersection are the lesser, we shall make plain thus. Conceive a diameter of the sphere (that is, of the great circle ABCD) drawn through the point onto which the perpendicular from I, dropped onto the plane of circle ABCD, falls; let this diameter cut the circumference at α . Through this diameter and the perpendicular dropped from I, let a plane be understood to be drawn, [Margin: Euclid, *Elements* XI.18.] which will be perpendicular to circle ABCD and will make in the sphere a semicircle passing through Q. For since circle ABCD passes through A and B, the poles of the great circles MN and OP, these in turn will pass through its poles (by the scholium to Proposition 15, Book I, of Theodosius); and therefore Q will be its pole. Since, then, that semicircle is drawn through its poles, it will pass through Q, the pole of circle ABCD, and there will be bisected — for, by the corollary to Proposition 16, Book I, of Theodosius, its arc from Q up to α is a quadrant. Consequently that same semicircle is divided at I not into halves. [Margin: Theodosius I.13.] Therefore, by Theorem 3 of the scholium to Proposition 21, Book II, of Theodosius, the straight line I α drawn will be the least of all those falling from I onto the circumference ABCD, and IF will be less than IG; and consequently, by the scholium to Proposition 28, Book III, of Euclid, arc IF will be less than arc IG. Hence the whole arc from I, through F, up to the other intersection will be less than the whole arc from I, through G, up to that other intersection — since the former are the halves of the latter, as was shown.

But that the arcs IF and IH are equal can also be shown in this way. Since the straight lines falling from I onto the poles A and B are equal, A and B will be equally distant from the point α , so that the arcs αA and αB are equal. For if some other arc than αB — namely $\alpha \beta$ — were equal to arc αA , then the straight line I β would also be equal to the straight line IA, by the aforesaid Theorem 3 of the scholium to Proposition 21, Book II, of Theodosius, which is absurd. For by that theorem IB is less than I β , and hence less than IA. And since the arcs AF and BH are likewise equal, if the equal arcs $A\alpha$ and $B\alpha$ are taken away, the remainders αF [?] and αH will also be equal. Therefore, by the said Theorem 3 of the scholium to Proposition 21, Book II, of Theodosius, the straight lines IF and IH will be equal; [Margin: Theodosius I.13.] and hence the arcs IF and IH will also be equal, which is what was proposed. [Margin: Euclid, *Elements* III.28.]

LEMMA XLV.

If, in a sphere, a circle is tangent to two parallel circles facing toward the same side of the great circle drawn through their poles, then their arcs, intercepted

between the points of contact and any great circle drawn through their poles, are similar.

In the sphere ABCD let there be two parallel circles BD and EF — whether one of them is a great circle, or neither is; and whether they belong to the same hemisphere or to different ones — through whose poles A and C the great circle ABCD passes; and let a circle GIH be tangent to them at the points G and H, on the same side of the great circle ABCD. I say that the arcs BG and EH, as well as DG and FH, are similar. For, through A (the pole of the circles BD and EF) and K (the pole of the tangent circle GIH) let the great circle AK be described. Then the great circle AK, which is described through A and K — the poles of the circles EF and GIH, touching one another at H — will pass through the contact point H. [Margin: Theodosius II.4.] Likewise, the same great circle AK, which is drawn through A and K — the poles of the circles BD and GIH, touching one another — will pass through the contact point G. And since the great circles AB and AG are drawn through the poles of the parallel circles EF and BD, the intercepted arcs EH and BG will be similar, which is what was proposed. And if the parallels are equal, the arcs EH and BG will be not only similar but also equal, because similar arcs of equal circles are equal.

Diagram. A sphere within a square frame (the figure for Lemma XLV): an outer great circle ABCD, two nested parallel circles (BD and EF), and a tangent circle GIH touching them. Legible labels:

- A (top of the sphere)
- E (upper left)
- F (upper right)
- H (center, at the tangency, just below A)
- K (center left, on the inner circle)
- I (center)
- B (left)
- D (right)
- G (bottom center, at the tangency)
- C (bottom)

(No labels flagged [?].)

LEMMA XLVI.

If, in a sphere, two circles cut one another, then the great circle that bisects the segment of one of them and passes through the poles of that circle also passes through the poles of the other circle.

In the sphere let two circles ABCD and EBFD — whether great, or not great, or one great and the other not great — cut one another at B and D; and let the great circle EFGHA, passing through G (the pole of circle ABCD), cut its segment BAD in half at A. I say that the same great circle also passes through the pole of circle EBFD. For if it does not pass through it, let a great circle IGK be drawn through its pole I and through G, the pole of circle ABCD. [Margin: Theodosius II.9.] Then this circle will bisect all the segments of the given circles, and therefore will pass through A. [Margin: Theodosius I.11.] Since, then, the great circles cut one another in half, GHA and GKA will be semicircles; and therefore the point A, on the circumference, will be another pole of circle ABCD — since, by the corollary to Theorem 1 of the scholium to Proposition 10, Book I, of Theodosius, the poles of the same circle are opposite through a diameter, that is, they are distant from one another by a semicircle of the great circle. This is absurd. For a pole is a point within the circle, from which all the straight lines falling onto the circumference are equal. Therefore the great circle EFGHA passes through the poles of circle EBFD, which is what was proposed.

Diagram. Two intersecting circles within a square frame (the figure for Lemma XLVI): a smaller circle on the left overlapping a larger circle on the right, with a nearly horizontal line EA cutting across. Legible labels:

- B (top, at the upper intersection of the two circles)
- E (left end of the horizontal line)
- C (on the horizontal line, within the left circle)
- F (center, at the horizontal line meeting the circles' overlap)
- G (center right, on the horizontal line)
- K (right, on the horizontal line)
- H (just below G/K)
- A (right end of the horizontal line)
- I (lower center-left, within the left circle)
- D (bottom, at the lower intersection of the two circles)

[Margin (left of figure): Theodosius II.9; Theodosius I.11.] (No labels flagged [?].)

[Recto — running header: LEMMA XLVII.]

LEMMA XLVII.

If, in a sphere, three great circles are drawn through the pole of any great circle, forming two equal angles at the pole; then any circle described from any point of the middle circle, as from a pole, cuts off — both from the other two great circles, and from two circles (whether great or not great, but equal) that have their poles on the first great circle, at equal distances from that middle great circle — equal arcs, begun from the same side, from the same first great circle. (In the case of the equal circles, whether great or not, that have their poles on that first great circle, [the arcs are measured] from the points at which they arise on the near side or the far side of their poles.)

In the sphere ABC, through B (the pole of the great circle ADC) let three great circles BD, BE, BF be drawn, making at B the equal angles EBD and FBD.

First, from the assumed pole B, let a lesser circle GSH be described on the middle circle BD, cutting the great circles BE and BF at G and H. I say that the arcs EG and FH are equal. For since, by the corollary to Proposition 16, Book I, of Theodosius, the arcs BE and BF are quadrants and therefore equal, then if the arcs BG and BH are subtracted — [Margin: Euclid, *Elements* III.28.] which are equal to one another, since the chords BG and BH drawn are also equal, by the definition of a pole — the remaining arcs EG and FH will likewise be equal, which is what was proposed.

Next, from another pole I, assumed on the same middle circle BD, let a lesser circle GSH be described, cutting the great circles BE and BF at G and H. I say again that the arcs EG and FH are equal. For, drawing the great circles IG, IH, DG, DH, let there be described from the pole D, through G, a circle GTH, cutting the circle GSH at the point H — which point I say is the very one in which circle BF is cut by circle GSH. For conceive, through H (the point of intersection of the circles GSH and GTH) and through B and I, the great circles HB and HI to be drawn. Since, then, the two sides ID and DG are equal to the two sides ID and DH, and the base IG is equal to the base IH — for the arcs DG and DH, as well as IG and IH, are equal, since they fall from poles onto their proper circles — the angles GDI and HDI will be equal, by Proposition 18 of our *Spherical Triangles*. Again, since the two sides BD and DG are equal to the two sides BD and DH, and comprehend equal angles, as we showed, then by Proposition 7 of our *Spherical Triangles* the bases BG and BH, and the angles at B, will be equal. But, by hypothesis, the arc BH, drawn to its intersection with circle GSH, makes the angle HBD equal to that same angle GBD. Therefore this arc does not differ from the one drawn through B and the intersection of the circles GSH and GTH — lest a part be equal to the

whole; and consequently the circles GSH and GTH cut one another on the arc BF. Therefore we shall show, just as was done a moment ago, that in the triangles IGD and IHD the angles IDG and IDH are equal, since the three sides are equal to the three sides; and hence, in the triangles BGD and BHD, the bases BG and BH are equal, by Proposition 7 of our *Spherical Triangles*. Therefore the remaining arcs EG and FH will likewise be equal, which is what was proposed.

Diagram. A large, densely drawn sphere within a square frame (the figure for Lemma XLVII), crossed by many great circles radiating from the pole-points and by several nested lesser circles, with a projecting pole-point (Q) drawn at the far left outside the main circle. Legible labels (positions approximate):

- A (top of the sphere; a small italic v/V mark beside it)
- E (upper right; a small italic b mark beside it)
- X (right, below E)
- K (right, below X)
- N (right, below K)
- D (right middle; a small italic a beside it)
- M (right, lower)
- L (right, below M)
- F (lower right; small italic à and α marks beside it)
- P (right, below F)
- C (bottom)
- G (upper left, a radiant node of many circles)
- B (left)
- I (left, beside B)
- T (left, beside I)
- S (center left)
- R (center)
- ϵ (Greek epsilon, center)
- H (lower center)
- Z (bottom, curved-tail letter)
- Q (far left, a looped/script pole-symbol drawn outside the sphere)

The interior is crowded with additional small italic labels that cannot be individually resolved at this resolution. [Multiple small labels [?]]

[Lemma XLVII proof, continued below the figure:]

Third, from another pole Q, assumed on the same middle circle BD, let a great circle GSH be described, cutting the great circles BE and BF at G and H. I say again that the arcs EG and FH are equal. For, describing through Q and G, and through Q and H, the great circles QG and QH, which — by the corollary to Proposition 16, Book I, of Theodosius — are quadrants, then by Proposition 25 of our *Spherical Triangles* the angles QGH and QHG will be right, and hence QGB and QHB acute. And since the angles DBE and DBF are supposed equal, the remainders GBQ and HBQ (from two right angles) will also be equal, in the triangles QBG and QBH. Since, then, the two sides BQ and QG are equal to the two sides BQ and QH, and each of the remaining angles BGQ and BHQ is less than a right angle, as was shown, the sides BG and BH will also be equal, by Proposition 24 of our *Spherical Triangles*; and hence the remaining arcs EG and FH will be equal, which is what was proposed.

Now further, from the poles K and L, placed anywhere on the great circle ADC but equally distant from the point D, let two equal circles be described — whether great or not great — MGV and NHP. And first, from the pole B, let a lesser circle GSH be described, that is, one parallel to the great circle ADC, cutting (or touching) the two circles MGV and NHP at G and H. I say that the two arcs MG and NH, as well as the two arcs VG and PH, are equal. For let there be described, from the pole D, through G, a circle GTH, cutting circle GSH at the point H — which point I say is the one in which GSH cuts circle NHP. For, drawing the arcs of the great circles DG, DH, KG, LH, and BH: since the two sides DG and DB are equal to the two sides DH and DB, and the base BG is equal to the base BH — for both DG and DH, as well as BG and BH, are equal, being from poles onto the circumferences of their proper circles — the angles GDB and HDB will be equal, by Proposition 18 of our *Spherical Triangles*; and consequently, from right angles, the remainders GDK and HDL will be equal. Therefore, since the two sides GD and DK are equal to the two sides HD and DL — the poles K and L being supposed equally distant from D — and comprehend equal angles, as we showed, then by Proposition 7 of our *Spherical Triangles* the bases KG and LH will be equal. Since, then, KG is drawn from the pole K to the circumference VGM, LH will likewise be drawn from the pole L to the circumference PHN, this circumference being equal to that; and therefore the point H will lie on the circumference NHP — that is, at the point where it is cut by circle GSH. Therefore we shall show, just as was done a moment ago, that in the triangles BDG and BDH the angles at D [are equal], and consequently, from right angles, the remainders GDK and HDL; and hence, by Proposition 7 of our *Spherical Triangles*, that the bases KG and LH, and the angles K and L, are equal. Since, then — drawing the great circles MtG and NtH — the two sides KG and KM are equal to the two sides LH and LN, being from poles onto

equal circles, and comprehend equal angles, as was shown, the bases MG and NH will also be equal, by Proposition 7 of our *Spherical Triangles*; and the chords MG and NH drawn will be equal; [Margin: Euclid, *Elements* III.29.] and hence also the arcs MRG and NRH will be equal. [Margin: Euclid, *Elements* III.28.] Since, then, MG and NH are semicircles [Margin: Theodosius I.15.] — because the great circle ADC, drawn through their poles, cuts the circles in half — the remaining arcs VG and PH will also be equal, which is what was proposed.

(continuation of the demonstration of Lemma XLVII)

In exactly the same way we shall conclude the proposition if, from any other pole whatever — I or Q — assumed on the circle BD, the circle GSH is described, even if the one described from Q is a great circle, so that QG and QH are quadrants.

The reasoning will be scarcely different if from the pole D some circle GTH is described cutting the great circles BE and BF, or cutting the circles described from the poles K and L at G and H. For once the circle GSH has been described from the pole B through G, cutting the circle GTH at the point H, we shall show in the same way that H lies on the circle BF. For when the great circles DG and DH are drawn, the two sides BD and BG are equal to the two sides BD and BH, and the base DG is equal to the base DH — since BD is a common arc, and the others are drawn from the poles to their proper circumferences. Therefore, by proposition 18 of our *Spherical Triangles*, the angles at B will be equal. But the arc BF, by hypothesis, also makes the angle FBD equal to the angle EBD. Therefore the arc through B and the point H — the intersection of the circles GTH and GSH — does not differ from the arc BF. Hence the arcs BG and BH, drawn from the pole to the circumference GSH, will be equal; and when these are subtracted from the quadrants BE and BF, the remaining arcs EG and FH will likewise be equal — which was to be proved.

[Margin: a — Theodosius III.29; b — Theodosius III.28]

Again, when the great circles MtG, NtH, KG, LH have been drawn, and the circle GSH has been described from any pole I whatever assumed on BD, cutting the circle GTH at H through G, we shall show, as before, that the point H lies on the circle NHP. For when the great circles IG and IH are drawn, the two sides ID and DG are equal to the two sides ID and DH, and the base IG is equal to the base IH — because ID is a common arc, and the others are drawn from the poles to their proper circumferences. Therefore, by proposition 18 of our *Spherical Triangles*, the angles IDG and IDH, and hence also the remaining right angles [recte: the remaining angles GDK, HDL], will be equal. Moreover the two sides DG and DK

are equal to the two sides DH and DL. For DG and DH are arcs drawn from the poles of equal circles to their circumferences, while DK and DL are arcs assumed equal, namely the distances of the poles K and L from the point D. Therefore, by proposition 7 of our *Spherical Triangles*, the bases KG and LH will also be equal. So, since KG is drawn from the pole K to its own circumference, LH too is drawn from the pole L to its own circumference, and since the one is equal to the other, the point H — the intersection of the circles GTH and GSH — will lie on the circle NHP. This established, we prove from proposition 18 of our *Spherical Triangles* that the angles DKG and DLH are equal, since the three sides KG, KD, DG are equal to the three sides LH, LD, DH. Hence, since the two sides GK and KM are also equal to the two sides HL and LN about those angles — the arcs being drawn from the poles K and L to equal circumferences — the bases MtG and NtH will, by proposition 7 of our *Spherical Triangles*, be equal; and therefore the chords MG and NH drawn will be equal, and consequently so will the arcs MRG and NRH, and so on — which was to be proved.

This demonstration holds, as is evident, whether the circles MGv and NHP cut one another, or one lies wholly outside the other. But when they touch at D, both the arc DH [and NH?], as well as [the earlier arcs] DG and MG, coincide, and so the demonstration is made shorter.

Diagram. A large woodcut showing a sphere (the meridian projection) with numerous great circles and their intersections, illustrating the projection lemma. The outer boundary is a circle within a ruled square frame. Labelled points (as read; several small peripheral labels are indistinct):

- A (top of circle)
- Y (top, just left of A)
- E (upper right, below A)
- X (right upper)
- N (right, below X)
- G (upper-left interior vertex)
- B (left interior, with adjacent I and T)
- I, T (left interior, near B)
- S (central node)
- D (right, on circumference)
- a (small letter just right of D)
- M (right, below D)
- L (right lower, below M)

- Ω (far lower-left, outside the inner circle) [?]
- H (lower interior vertex)
- Z (lower center, below H)
- C (bottom of circle)
- F (right lower)
- P (right, lowest)
- α , α (small letters, lower center-right)

Now, conversely, any circle whatever YaZ , cutting the circle ADC at X and a equally distant [recte: at points X and a equally distant from D], and standing off from the point D — and consequently from the poles K and L — necessarily has its poles on the great circle DB , drawn through D and the poles of the circle ADC . For since the great circle DB cuts the circles FE and KL through their poles, meeting them at H and at another point on the other side of the meridian, it will cut their segments in half. Therefore the other point of section on the other side of the meridian, at which the parallel KL cuts the ecliptic, is as far from the tropic point F as the point H is from that same point; and consequently the parallel KL passes through two points of the ecliptic equally remote from the tropic point F . By the same reasoning the parallel through I passes through I and through another point on the other side of the meridian, which has a distance from the tropic point G equal to that of I .

[Margin: c — Theodosius II.9]

But when, around the pole B , a parallel of the great circle ADC is described, it will cut off equal arcs from all the great circles drawn through B , even if at B they do not make equal angles; and likewise from all the non-great circles that are equal and have their poles on the great circle ADC , even if the poles are not equally distant from the mid-circle BD . For the great circles we shall conclude the proposition easily thus: since they are all drawn through the poles of the parallels, the arcs of these between the said parallels are equal. For the non-great circles the demonstration will be as follows. If from the points at which they are cut by a parallel of the great circle ADC perpendiculars are dropped to the great circle ADC , these will fall on their common sections with the great circle ADC — that is, on their diameters. (For since the great circle ADC passes through their poles [recte: since the great circle ADC drawn through their poles cuts them in half], their diameters are their common sections; and consequently the perpendiculars are right sines of the arcs cut off.) Since therefore the perpendiculars are all equal to one another — for since all the parallels are parallel, if a plane is drawn through any two of them, the sections on the parallel

planes ADC and GSH are parallel, and hence in the parallelogram the opposite sides are equal, namely those two perpendiculars, which are the sines — they are equal; and so on with the rest. Hence there will also be arcs which, being right sines equal in equal circles, have equal arcs, as [is shown] in the definitions.

[Margin: d — Theodosius II.10; e — Euclid XI.38; f — Theodosius I.15; g — Euclid XI.6; h — Euclid XI.16; i — Euclid I.34]

LEMMA XLVIII.

If from the same center two circles are described, and from any number of points on the circumference of the inner one straight lines equal [to one another] are drawn to the circumference of the outer, and one of them is supposed to touch the inner circle, then the rest will touch it as well. And if several lines touching the inner circle are drawn toward the same side — that is, toward the left or toward the right — they will be equal to one another, and the arcs comprehended between each pair will be similar.

From the same center A let two circles BCDEF and GHIKL be described, and from the points G, H, I let equal straight lines GB, HC, ID be drawn, of which GB is supposed to touch the circle GHIKL. I say that HC and ID also touch it. For when the semidiameters GA, HA, IA and BA, CA, DA are joined, since the two sides BG, GA are equal to the two sides CH, HA, and the base BA equals the base CA, the angles AGB and AHC will also be equal. But AGB is a right angle; therefore AHC too will be a right angle; and consequently, by the corollary to proposition 16 of Euclid Book III, the straight line HC touches the circle GHI at H — and so likewise for the rest.

[Margin: a — Euclid I.8; b — Euclid III.18]

Now let there be drawn to the same side any number of tangents BG, CH, DI, SM. I say that they are both equal, and that the arcs GH, BC are similar, as are GI, BD and GM, BS. For when the same semidiameters are joined, let the inner circle be cut at M, N, O, T by the semidiameters AB, AC, AD, AS. And since the two sides AB, AG are equal to the two sides AC, AH, and the angles AGB, AHC — opposite the equal sides AB, AC — are equal, because they are right angles, the remaining angles B and C, opposite the equal remaining sides AG, AH, are each less than a right angle (because both the two angles G, B and the two H, C are less than two right angles). Therefore, by what we demonstrated at the end of Euclid Book I, the sides BG and CH will also be equal, and the angles BAG, CAH equal. From this it follows that the arcs GM, HN are equal; and when the common arc HM is

removed, the remaining arcs GH, MN are also equal. Since therefore, by the scholium to proposition 22 of Euclid Book III, the arc MN is similar to the arc BC, the arc GH too will be similar to that same arc BC. In the same way you will show that the arcs GM, IO are equal, and hence, adding the common arc MI, that the whole arcs GI, MO are equal; and consequently, since MO is similar to BD, GI too will be similar to BD. In just the same way you will show that the arcs GM, MT [are equal] [recte: the arcs GM, MT are equal — and since MT is similar to BS], GM too will be similar to BS.

[Margin: c — Euclid III.18; d — Euclid I.17; e — Euclid III.26]

Moreover, that the tangents are equal we shall also easily show thus. When the tangents BG, DI are produced to P and Q, they will, by the scholium to proposition 18 of Euclid Book III, be equal to one another, and will be bisected at the points of contact G and I. Therefore the halves BG, DI will be equal, and so with the others. Hence we easily conclude that the angles GAB, IAD are equal, because the sides AB, AG are equal to the sides AD, AI, and the base BG equals the base DI, and so on.

[Margin: f — Euclid I.8]

But if the points of contact G, K are set opposite through a diameter, so that GIK is a semicircle, then BDE too will be a semicircle, similar to GIK. For both BD will be similar to GI and DE to IK, as has been shown; and consequently, by Lemma 6, the whole BDE will be similar to the whole GIK. That this may also be demonstrated in this way, however, [let us proceed thus]: when the straight lines AB, AE are joined, since the two sides AB, AG are equal to the two sides AE, AK, and the base BG equals the base EK, as has been shown, the angles BAG, EAK will be equal. Therefore, by what we demonstrated from Proclus at proposition 15 of Euclid Book I, the straight lines AB, AE will form a single straight line; and consequently BE will be a diameter, and BDE a semicircle. Or thus: because the angles BAG, EAK are equal, the arcs [GM, KR will be equal]; and when the common arc MK is added, the whole arcs GMK, MKR will be equal. But GMK is a semicircle; therefore MKR is too; and hence MAR [recte: BE] is a diameter, and so BDE is a semicircle.

[Margin: g — Euclid I.8; h — Euclid III.26]

By the same reasoning, if the points of contact G, L are distant by an arc GKL greater than a semicircle, then — since the arcs KL, EF have been shown to be similar — if the semicircles KIG, EDB are added, the whole arcs GKL, BDF will, by Lemma 6, also be similar.

Diagram. A woodcut showing two concentric circles (an outer circle and an inner circle) from center A, with many tangent lines (some drawn dotted) and radii producing the labelled points of contact and intersection. It illustrates Lemma XLVIII (equal tangents and similar arcs from a common center). Labelled points (as read):

- X, Q (top)
- V (upper left)
- B (upper right, with small t/i beside it)
- P (left)
- G (inner circle, upper)
- E (inner circle, upper, right of G)
- M (interior, below G)
- A (center)
- C (right)
- F (left)
- R (lower left interior)
- L, K (bottom center)
- O (lower center-right)
- D (lower right)
- Y (bottom)
- Z (bottom center)
- E (lower left, on outer circle)
- α (small letters, lower center)
- τ (small letter, center) [?]

SCHOLIUM.

It follows from this that if the points of contact divide the inner circle into equal parts, they divide the outer one — [cut] by the tangents — into equal parts as well. Thus you see that the arcs GH, HM, MN are equal, just as are BC, CS, SY.

And so, if many lines are to be drawn tangent to the circle GHIK at points dividing it into equal parts — namely at G, H, M, N, T, etc. — one of them must be drawn, such as GB. For if from A any circle whatever is described cutting GB at B, and is divided into equal parts BC, CS, SY, etc., beginning from the point B, the tangent will pass through H by way of C, through M by way of S, through N by way of Y, through T by way of Z, and so on.

But in order to have the two points on the outer circle through which the tangents are to be drawn — for instance, [to draw] through M, the second point, the straight line AM cutting the first tangent [at B] — a circle must be described from the center A through one of the points that divide the circle GHIK into equal parts, and it must be divided into just as many equal parts (beginning from B) as the circle GHIK has been cut into, as in the figure set out here: into 12 equal parts BC, CS, SY, YZ, Za, aE, EF, FP, PV, VX, Xb, bB. For since, by the scholium to proposition 27 of Euclid Book III, the straight line AX cutting the arc BXP bisects it at X, there will be contained in the whole arc BXP twice as many equal parts as in BX — that is, in the similar arc GM. The tangent BP is therefore drawn through the two points B, P that bound four equal parts. So the tangent CV will pass through the two similar points C, V, since as many parts are contained in the arc BXP as in the arc CBV, and C bounds one part; and the same is to be said of the tangents SX, Yb, FY, and so on. Thus the individual tangents are found by threes. And so the two points of any tangent on the outer circle, however described, will be found, provided the points are marked on the outer circle. For all the tangents being equal, at the interval of the straight line GB, two points C, V are found on the outer circle from the point of contact H, and from M two points [S, X, etc.] are found from the point of contact, as has been demonstrated. By this method, at the interval GB, from the point of contact H the two points C, V are found, and from M two points S, X, and so on.

LEMMA XLIX.

To demonstrate a few things concerning declinations, ortive amplitudes [latitudines ortivae], and right and oblique ascensions [ascensiones rectae et obliquae].[?]

1. In the first figure let ABCD be the meridian, AC the equator, and BD the oblique horizon cutting the equator at E; and through E let the ecliptic FG pass, so that E is the beginning of ♈ [Aries] or ♎ [Libra], F of ♐ [Capricorn], and G of ♋ [Cancer]. Let the arcs of the ecliptic EH, EI be equal, and through H, I let the parallels KL, MI be drawn, cutting the horizon at L and N; and finally, through L, N, H, I and the poles of the world O, P let the great circles of declination OL, PN, OH, PI be drawn, cutting the equator at Q, R, S, T. I say that the parallel KL passes through two points of the ecliptic equally remote from the tropic point F. The same is to be said of the parallel MI. For since the great circle ABCD cuts the circles FE, KL through their poles, meeting them at H and at another point on the other side of the meridian ABCD, it will cut their segments in half. Therefore the other point of section on the other side of the meridian, at which the parallel KL cuts the

ecliptic, is as far from the tropic point F as the point H is from it; and consequently the parallel KL passes through two points of the ecliptic equally remote from the tropic point F. By the same reasoning the parallel through I passes through another point on the other side of the meridian, which has a distance from the tropic point G equal to that of the point I.

[Margin: Any parallel passes through two points equally distant from one or the other tropic point. — a: Theodosius II.9]

2. Next I say that two parallels KL, MI, equally distant from one or the other equinoctial point, or from two, or even from the two tropic points F, G, have equal declinations HS, IT. For since in the triangles HES, IET the angles S, T are right angles, and the angles at the vertex E are equal by proposition 6 of our *Spherical Triangles*, and moreover the arcs of the ecliptic EH, EI, opposite the right angles, are set equal, the arcs HS, IT — the declinations of the points H, I — will, by proposition 21 of our *Spherical Triangles*, also be equal. And thus two points H, I of the ecliptic, equally remote from the same equinoctial point E — or the parallels KL, MI drawn through those points — have equal declinations. But if the points H, I are given equally distant from the tropic points F, G toward the same section E, vernal or autumnal, they will be equally distant from E; therefore, as we have just shown, the parallels drawn through them have equal declinations. Finally, if one point, say H, is supposed to be distant from the tropic point F toward the autumnal point E, and another point is to be removed the same distance from the tropic point G toward the vernal point, so that it is diametrically opposite the first, we shall take another point I toward the earlier point E, autumnal, at the same distance from the point G: and the points H, I will again, as we have just shown, have equal declinations HS, IT. And since the same parallel passes through I and through the point corresponding to it on the other side, given as was demonstrated in No. 1 — and since all points of the same parallel have equal declinations, because all the arcs of the great circles drawn through the poles of the world (such as the circles of declination) between any parallel and the equator are equal — the parallels through H and through that other point of the ecliptic corresponding on the other side to the point I, which is opposite H, will also have equal declinations.

[Margin: Two parallels drawn through two points of the ecliptic equally distant from one or the other equinoctial point, or from two, or even from the two tropic points, have equal declinations. — b: Theodosius I.15; c: Theodosius II.10]

3. Third, I say that the same two parallels have equal ortive amplitudes EL, EN. For since in the triangles ELQ, ENR the angles Q, R are right angles, and the angles

at the vertex E are equal by proposition 6 of our *Spherical Triangles*; and likewise the arcs of declination LQ, NR, opposite the equal angles at E, have been shown equal; and finally the arcs EL, EN, opposite the equal right angles Q, R, do not make up a semicircle, since each is less than a quadrant (being an ortive amplitude, which is always less than a quadrant) — therefore, by proposition 22 of our *Spherical Triangles*, the arcs EL, EN, that is, the ortive amplitudes, will also be equal.

[Margin: The same two parallels have equal ortive amplitudes.]

4. Fourth, I say that the same two parallels are equal. For since the arcs EL, EN, lying between them and the equator, have been shown to be equal, the parallels KL, MI themselves will be equal.

[Margin: The same two parallels are equal. — d: Theodosius II.17]

5. It follows from these things that four points of the ecliptic — of which two are opposite through a diameter — being equally distant, two of them from the two equinoctial points, or from the tropic points, or from the same equinoctial or tropic point, always have equal declinations and ortive amplitudes. Such points are the beginning of ♉ [Taurus], the beginning of ♍ [Virgo], the beginning of ♏ [Scorpio], and the beginning of ♐ [Pisces]; of these, the first two are equally distant from the beginning of ♋ [Cancer], the latter two from the beginning of ♑ [Capricorn]. Likewise the first and last are equally distant from the beginning of ♈ [Aries], and the two intermediate ones from the beginning of ♎ [Libra]. And since through the first two the same parallel passes, and through the latter two another single parallel passes, as was demonstrated in No. 1, both that pair and this will have equal declinations and ortive amplitudes, as we showed in Nos. 2 and 3. But as was there demonstrated, the first and last too have equal declinations and ortive amplitudes, since they are equally distant from the beginning of ♈ [Aries]. Therefore all four have equal declinations and ortive amplitudes — of which the first and third, and likewise the second and fourth, are opposed through a diameter, since both pairs are equally distant from the beginnings ♈ and ♎ according to the succession of the signs. And so it is enough if the declinations and ortive amplitudes of the points of one quadrant of the ecliptic are found, since these agree with the points of the other three quadrants as well, if the points are taken as has been said.

[Margin: Four points of the ecliptic, whose pairs are opposite through a diameter and which are equally distant from two equinoctial or tropic points, or from the same equinoctial or tropic point, have equal declinations and ortive amplitudes.]

It is enough that the declinations, ortive amplitudes, and their magnitudes be found. It is enough that the declinations, ortive amplitudes, and all the ortive [amplitudes] of the points of one quadrant of the ecliptic be found. — e: Theodosius II.13]

All these things can be demonstrated more easily and briefly from Theodosius in this way. Since the ecliptic EF touches one of the parallels, namely the tropic $\mathfrak{S}$ or $\mathfrak{V}$, its two arcs between the equator and the parallel KL — of which one is EH — will be equal to one another. Therefore also the remaining arcs from the quadrants up to the meridian [will be equal], of which...

Diagram. A woodcut within a ruled square frame containing **four** small spheres (meridian projections), arranged two above and two below, illustrating Lemma XLIX (declinations, ortive amplitudes, and the four equidistant ecliptic points). Each sphere shows a meridian circle with equator, oblique horizon, ecliptic, and parallels, and their intersections. Labels as read (several small interior labels are at the resolution floor):

Upper-left sphere:

- A (top), K, M (top region)
- S, P (upper interior)
- F, H (left)
- V, Q (center)
- B, Y (right) [?]
- Z (left), L, E, N (center row)
- O, R, X (lower interior)
- T (lower), G (lower right), C (bottom)

Upper-right sphere:

- A (top)
- F, I (upper interior)
- H (left), E, G (center-right), D (right)
- K (lower center), F (lower), C (bottom)

Lower-left sphere:

- A (top), N (upper right)
- F, L (upper interior)
- I, B (left), E, D (right)

- G, K (lower center), M, O (lower), H (lower), C (bottom)

Lower-right sphere:

- F (top), A, H (upper)
- N (left), L, I (interior)
- B (left), M, K, E, O, G (center row), D (right)
- Q, R (lower), C (bottom)

ASTROLABIVM — LIBER PRIMVS

Lemma XLIX (continued) ...one of which is HF, they will be equal; and therefore the same parallel KL will pass through two points equally distant from the tropic point F. The same reasoning holds for the parallel MI.

Next, since the arcs of the Ecliptic EH and EI are assumed equal, then — when the parallels KL and MI are assumed to be equally distant either from the equinoctial point E or from the two tropic points F, G — [a] the parallels KL and MI will themselves be equal. [b] Therefore both the two arcs of a great circle drawn through the poles of the world and intercepted between the Equator and the said parallels (the arcs that measure their declinations) and the two arcs of the Horizon EL, EN (which determine the ortive amplitudes of the same parallels) will be equal to one another. From this it again follows that each set of four points of the Ecliptic has equal declinations and equal ortive amplitudes.

[Margin: a — Theodosius, *Sphaerica* II.17. b — Theodosius, *Sphaerica* II.18.]

6. I say, sixthly, that four equal arcs of the Ecliptic — two of which are opposite through a diameter, and two equally distant from the two equinoctial or tropic points, or from the same equinoctial or tropic point — have equal ascensions in the right sphere. By calling two of the arcs "opposite" I mean those whose end-points are opposed through a diameter; and by "equally distant from the equinoctial or tropic points" I mean those whose end-points are equally away from them, so that the two nearer points have equal distances and likewise the two farther ones.

[Margin: Which arcs of the Ecliptic are called opposite, and which equally distant from some point of the Ecliptic.]

So let there first be two equal arcs of the Ecliptic EH, EI, begun from the same equinoctial point E, and consequently the remaining arcs HF, IG equal, begun from the tropic points F, G. Then ES, ET will be the right ascensions of the arcs EH, EI, and AS, CT the right ascensions of the arcs FH, GI. It must be proved that ES, ET as well as AS, CT are equal, which is done as follows. Since in the triangles EHS, EIT the angles S, T [c] are right, and the angles at the vertex E are equal (by Prop. 6 of our *Spherical Triangles*); and since the arcs EH, EI, which are opposite the right angles, are also assumed equal — then by Prop. 21 of our *Spherical Triangles* the arcs ES, ET will also be equal, and consequently so will the remaining parts of the quadrants, AS, CT. And since, as was shown at No. 1, the parallel KL passes on the other side of the Meridian through another point of the Ecliptic which is equally distant, with point H, from the tropic point F — and so is as far from the other equinoctial point as H is from E — if through that point a great circle is drawn from the pole O, an arc will be cut off from the Equator wholly equal to the arc ES; because the triangle is constituted equal to the triangle EHS. For the angle which the Ecliptic makes with the Equator at that intersection is equal to the angle HES (since both this and that are the angle of greatest declination); and the angles at the Equator, opposite which the equal arcs of the Ecliptic lie — namely S, and in the other triangle the one corresponding to it — are right. Therefore by Prop. 21 of our *Spherical Triangles* the arc ES is equal to the arc corresponding to it in that other triangle; and consequently so too are the remaining parts of the quadrants, namely AS and its counterpart on the other side. In the same way ET, CT will be shown equal to the corresponding arcs on the other side, which the same parallel MI marks off. Hence both the four arcs EH, EI and their counterparts, begun from the two equinoctial points — of which two are opposite (namely EH and the arc corresponding to EI, and EI and the arc corresponding to EH), and two equally distant from the two equinoctial or tropic points — and the four arcs begun from the tropic points, namely FH, GI, and their counterparts on the other side, of which two are likewise opposite, etc., have equal right ascensions.

[Margin: c — Theodosius, *Sphaerica* I.15.]

But now let there be four equal arcs HV, IX, together with two others corresponding to them on the other side, begun neither from the equinoctial nor from the tropic points, but equally distant from them. I say that their right ascensions too — namely the arcs QS, RT, and the two corresponding to them on the other side — are equal. For, as was just shown, both the four arcs EH, EI and their counterparts on the other side, begun from the equinoctial points, and the four arcs EV, EX and their counterparts on the other side, likewise begun from

the equinoctial points, have equal ascensions — namely the arcs ES, ET and their counterparts on the other side, and the arcs EQ, ER and their counterparts on the other side. Therefore the remaining four arcs QS, RT and their counterparts on the other side will also be equal. And it is clear that here too two are opposite, namely HV and the arc that on the other side corresponds to IX; likewise IX and the arc that on the other side corresponds to HV; while two are equally distant either from the two equinoctial and tropic points, or from one and the same point. For HV and its counterpart on the other side are equally distant from the two equinoctial points, and from one and the same tropic point F or G; and the same is to be said of the arc IX and its counterpart on the other side. But both the two arcs HV, IX and the two corresponding to them on the other side recede equally from the same equinoctial point E, or from the other opposite one, and from the two tropic points F and G.

Therefore it is enough that the right ascensions of all the arcs of the first quadrant of the Ecliptic, begun from Aries (γ), be found. For from these the whole table of right ascensions will be constructed. For once these are found — if the arcs greater than a quadrant [are subtracted] from the semicircle, there will be left the ascensions of the arcs greater than a quadrant; and from Aries (γ) up to Cancer (ϑ) is a quadrant. And if the ascension of an arc of 89° is subtracted from the semicircle, the remainder will be the ascension of an arc of 91° . In the same way, from the ascension of 88° we shall gather the ascension of 92° , and so on; because the ascension of 89° from Aries (γ) toward Cancer (ϑ) is equal to the ascension of 89° from Libra (ζ) toward Cancer (ϑ), as has here been demonstrated. Hence if [the ascension of the 89° arc] is taken from the semicircle, there will remain the ascension of the remaining arc of 91° , since the ascension of a semicircle is a semicircle. So too the ascension of 88° from Libra (ζ) toward Cancer (ϑ), and so on. Then, if the ascensions of all the arcs begun from Aries (γ) [and running up to Libra (ζ)] are added to the semicircle, there will result the ascensions of all the arcs greater than a semicircle [from Aries (γ) up to Aries again, or the end of Pisces ($\cap$)].

[Margin: That it is enough for the right ascensions of all the arcs of the first quadrant of the Ecliptic to be found.]

7. Arcs of the Ecliptic less than a quadrant, begun from the equinoctial points, [are greater than their right ascensions;] but begun from the tropic points, they are less. For in the triangle OFH the two sides OF, OH are [together less than] a semicircle, since each is less than a quadrant, seeing that OA, OS are quadrants; therefore the external angle OHE will be greater than the internal right angle

OFH, that is, obtuse, by Prop. 14 of our *Spherical Triangles*; and consequently the remaining angle EHS, of the two right angles, is acute and less than the right angle ESH. Therefore, by Prop. 11 of our *Spherical Triangles*, the arc of the Ecliptic EH will be greater than the arc of the Equator ES, which is its right ascension; and accordingly the remaining part HF, taken from the quadrant EF, is less than the remaining part SA, taken from the quadrant EA. A like demonstration will be made for the arcs EI, IG, and for the others which take their beginning from another equinoctial point, and which correspond to the arcs EH, HF, EI, IG.

[Margin: Which arcs of the Ecliptic are greater than their right ascensions, and which lesser.]

From this it is gathered that arcs of the Ecliptic begun from the beginning of Aries (♈) and less than a quadrant are greater than their right ascensions; but those greater than a quadrant and less than a semicircle are less than their right ascensions — because the ascension of the first quadrant is a quadrant, and then the arcs of the Ecliptic added on, up to the end of Virgo (♍), are always less than their right ascensions. But arcs greater than a semicircle and less than three quadrants are, in turn, greater than their right ascensions, because the semicircle from Aries (♈) up to Libra (♎) has a semicircle for its ascension, after which the arcs added on are again greater than their right ascensions. Finally, arcs greater than three quadrants are again less than their right ascensions, because the three quadrants of the Ecliptic have three quadrants for their ascension, and then the arcs added on are less than their right ascensions — all of which is here demonstrated.

But this too has been ascertained: that in the right sphere the ascension of any arc, or point, of the Ecliptic is equal to its descension. For descension is plainly the ascension, over the right Horizon, of the antipodes with which that arc or point rises. Since therefore the right ascensions behave in the same way in every right Horizon, what is proposed is clear. Or thus: since opposite equal arcs have the same ascension (as was shown at No. 6), and the ascension of any arc is the same as the descension of the equal opposite arc — since a semicircle of the Ecliptic is always above the Horizon — it comes about that the ascension and descension of that arc which is opposite to some arc are equal, seeing that they are equal to the ascension of this arc to which it is opposed. For example, the ascension of Aries (♈) and the descension of Libra (♎) are equal to the ascension of Libra. And so of the rest.

[Margin: The right ascension of any arc or point is equal to the right descension of the same arc.]

8. In every oblique Horizon, a great circle drawn from the pole of the world through the point of the Horizon where it is cut by the parallel described through any point of the Ecliptic intercepts, together with the Horizon, on the Equator, the arc of the ascensional difference of that point of the Ecliptic — that is, of the arc of the Ecliptic reckoned from either equinoctial point to that point, whether this reckoning is made according to the succession of the signs or against it. And the same great circle, together with another drawn through that point of the Ecliptic, intercepts on the Equator the oblique ascension of the arc of the Ecliptic placed between the Horizon and that point. Thus, because the parallel KL [through the point of the Ecliptic H cuts] the Horizon in L, EQ will be the ascensional difference of the point H, that is, of the arc EH reckoned from the equinoctial point E up to H, against the succession of the signs.

[Margin: A great circle drawn from the pole of the world through the intersection of the parallel of any point of the Ecliptic with the oblique Horizon intercepts, together with the Horizon, on the Equator the ascensional difference of that point of the Ecliptic; but together with another great circle drawn through that point of the Ecliptic, [it intercepts] the oblique ascension of the arc lying between that point and the Horizon.]

For when the point H is placed on the Horizon, namely at the point L (since the point H, by the first motion, describes the parallel KL), the arc HL rises together with the arc HE; and above any Horizon similar arcs of the parallels rise together. Hence the arc of the Equator SQ, [a] which is similar to the arc HL, is the oblique ascension of the arc HE. Since therefore ES is the right ascension of the same arc EH — because these arcs SE, HE ascend together above the right Horizon OS — EQ will be the ascensional difference. I say that EQ is also the ascensional difference of the arc of the Ecliptic which extends from the other equinoctial point, according to the succession of the signs, up to H. For, with the point H placed at L, the point S will be set at Q, so that then the arc OS coincides wholly with the arc OQ. Therefore then the arc of the Equator from that equinoctial point up to the oblique Horizon at the point E (the Ecliptic then cutting the Horizon at L) will be the oblique ascension of the said arc of the Ecliptic reckoned up to H, or of the point H then placed at L. But the arc of the Equator from that same equinoctial point up to the point S, then placed at Q, is the right ascension of [the same arc, or point. Therefore EQ] is the ascensional difference.

[Margin: a — Theodosius, *Sphaerica* II.10.]

Now not only is QS the oblique ascension of the arc HE, one end of which is the equinoctial [point, but also of any other] arc Ha, if through L another oblique

Horizon ZY is drawn, cutting the Ecliptic in a, outside the equinoctial point E. For above this Horizon the arc of the parallel HL rises together with the arc of the Ecliptic Ha. Therefore QS, which is similar to it, is the oblique ascension of the arc Ha. But the arc bQ is not then the ascensional difference, [because bS is not its] right ascension — for the points a, b do not arrive together at the right Horizon drawn from O through a or b, which is nevertheless required if bS is to be the right ascension of the aforesaid arc of the Ecliptic. Yet bQ is the ascensional difference of the arc of the Ecliptic reckoned from either equinoctial point up to H in the Horizon ZY, just as EQ [is the ascensional difference of the arc EH in the Horizon BD]. It is therefore established that the great circle OQ, drawn through L, intercepts, together with the oblique Horizon BD, the ascensional difference EQ of the point H (that is, of the arc of the Ecliptic intercepted from the equinoctial point up to H); and that the same circle, together with the great circle OS drawn through the same point H, [intercepts the oblique ascension QS] both of the arc HE begun from the equinoctial point E with respect to the Horizon BD, and of the arc Ha not begun from the equinoctial point E, with respect to the Horizon ZY. And the same reasoning holds for the rest.

9. In any oblique Horizon, two equal arcs of the Ecliptic equally distant from either equinoctial point — whether they take their beginning from it or not — have equal ascensions. For, in the second figure, let the Meridian be ABCD; the Equator AC; the oblique Horizon BD, cutting the Equator at E; and let FG be any arc of the Ecliptic from the equinoctial point F up to the Horizon, such that its oblique ascension is the arc of the Equator FE — since, when the point F is placed at the point E of the Horizon and the sphere is moved toward A, the points E and G arrive at the Horizon together. Let there also be another arc of the Ecliptic FH, equal to FG, from the same equinoctial point F up to the Horizon toward the parts of the other pole, such that its oblique ascension is likewise EF; because, when the sphere is moved, as soon as F reaches the Horizon at E, both arcs EF, HF are seen fully risen. I say that these ascensions FE, EF are equal.

[Margin: Two equal arcs of the Ecliptic begun from either equinoctial point, or equally distant, have equal oblique ascensions.]

For since in the triangles FEG, FEH both the angles at the vertex E and those at the vertex F (for the arcs of the Ecliptic FG, FH are to be conceived as continued at F, so as to constitute angles at the vertex F, as on the sphere — these being the angles of greatest declination which the Ecliptic makes with the Equator) are equal; and the arcs FG, FH, opposite the equal angles at E, are assumed equal; and the arcs GE, HE, opposite the remaining equal angles at F, do not make up a

semicircle, since they are less than the quadrants ED, EB — then by Prop. 22 of our *Spherical Triangles* the arcs FE, EF will also be equal, which was proposed. Or thus: since the two angles EFG, GEF are equal to the two angles EFH, HEF, as we said, and the two arcs FG, GE about the remaining angle G are equal to the two arcs FH, HE about the remaining angle H — (for since the points G, H recede equally from the same equinoctial point F, they will have equal ortive amplitudes EG, EH, and in these remaining angles G, H the poles of the remaining arcs FE, EF, that is, of the Equator, do not lie, since the poles of the Equator are on the Meridian) — then by Prop. 23 of our *Spherical Triangles* the remaining arcs FE, EF will be equal. And each of these demonstrations establishes what is proposed, even if each of the arcs FG, FH is greater than a quadrant, though still less than a semicircle.

But now let there be two equal arcs of the Ecliptic GI, HK, equally distant from the same equinoctial point F, but not begun from it. I say that their oblique ascensions too are equal. For since they are equally distant from the equinoctial point F, both the arcs GF, HF and the arcs IF, KF, begun from the equinoctial point F, will be equal. Therefore, as we just showed, both those and these will have equal ascensions. Hence, the equal ascensions of the equal arcs FI, FK being subtracted from the ascensions of the equal arcs FG, FH, the remainders will be the equal ascensions of the equal arcs IG, KH.

10. In any oblique Horizon, two equal arcs of the Ecliptic equally distant from either tropic point, and likewise two opposite arcs — whether they take their beginning from the equinoctial points or elsewhere — have their ascensions taken together equal to their ascensions in the right sphere taken together. For, in the third figure, let the Meridian be ABCD; the Equator AC; the oblique Horizon BD, cutting the Equator at E; and let FG be any arc of the Ecliptic begun from Aries (γ), yet less than a semicircle, and HG equal to it, begun from Libra ($\underline{\alpha}$). This being posited, their end-points will be equally distant from the same tropic point — for we suppose both to tend toward the same tropic point. Let their end-points be placed on the Horizon, where they will come together at the single point G, since they have equal ortive amplitudes, as we demonstrated at No. 3. Their oblique ascensions will therefore be the arcs of the Equator FE, HE. And, a great circle IK being drawn from the pole of the world I through G, FK, HK will be their right ascensions; and it is established that the arcs FE, HE taken together are equal to the arcs FK, HK taken together.

[Margin: Two equal arcs of the Ecliptic equally distant from the same tropic point, likewise two opposite arcs, have their oblique ascensions taken together equal to their right ascensions taken together.]

And this is true also of equal arcs greater than a semicircle. For instance, if one takes an arc from Aries (Υ) through Cancer ($\mathfrak{C}$) up to the beginning of Aquarius ($\mathfrak{A}$), comprising ten signs, and an arc equal to it from Libra ($\mathfrak{L}$) through Cancer ($\mathfrak{C}$) up to the beginning of [Leo ($\mathfrak{L}$)] [$\mathfrak{L}$], comprising ten signs as well: since the semicircles from Aries (Υ) through Cancer ($\mathfrak{C}$) up to [Libra ($\mathfrak{L}$)] have oblique ascensions equal to the right ascensions, namely the semicircle of the arcs from Libra ($\mathfrak{L}$) through Capricorn ($\mathfrak{C}$) up to the beginning of Aquarius ($\mathfrak{A}$), and from Aries (Υ) through Capricorn ($\mathfrak{C}$) up to the beginning of [Leo ($\mathfrak{L}$)] [$\mathfrak{L}$] have oblique ascensions equal to the right ascensions of the same arcs, as we just demonstrated — there will result [that the oblique ascensions of the arcs from Aries (Υ)] through Cancer ($\mathfrak{C}$) up to the beginning of Aquarius ($\mathfrak{A}$), and from Libra ($\mathfrak{L}$) through Cancer ($\mathfrak{C}$) up to the beginning of [Leo ($\mathfrak{L}$)] [$\mathfrak{L}$], taken together, are equal to the right ascensions of the same arcs. And so of the rest.

Next, let there be two equal arcs GL, GM, from the same tropic point [G], begun from [the two] equinoctial points F, H. And since the arcs GL, [GM] are equal and equidistant from the same tropic point, their end-points G, L and G, M will likewise be equally distant from [Aries (Υ) and Libra ($\mathfrak{L}$)]]; and therefore the whole arcs GF, GH, as well as the remaining arcs FL, HM, will be equal. Since therefore it was just shown that the oblique ascensions both of the arcs FG, HG and of the arcs FL, HM begun from Aries (Υ) and Libra ($\mathfrak{L}$), taken together, are equal to the right ascensions of the same taken together, then if the latter are subtracted from the former, the remaining oblique ascensions of the arcs GL, GM taken together will also be equal to the remaining right ascensions of the same [taken together].

Now this demonstration fits also arcs equally distant from the same tropic point which contain the equinoctial points within themselves. Thus in the same third figure, if equal arcs NL, OM are taken, whose end-points are equally distant from the same tropic point, then both the arcs FL, HM and the arcs FN, HO, begun from the equinoctial points, will be equal. Therefore, as was demonstrated, both those and these have their oblique ascensions taken together equal to their right ascensions taken together; and consequently, if the former are added to the latter, it will be brought about that the oblique ascensions of the whole arcs NL, OM taken together are equal to the right ascensions of the same taken together.

Finally, if there are any two equal opposite arcs, their distances from the equinoctial points, reckoned both according to and against the succession of the signs, will be equal. And if between them another equal arc is taken, equally distant with one of them from the same equinoctial point, it will be equally distant with the other from the same tropic point. Therefore, since equal arcs distant from the same equinoctial point have equal ascensions, as we showed at No. 9; while equal arcs receding from the same tropic point have their oblique ascensions taken together equal to their right ascensions taken together, as we just demonstrated — the opposite equal arcs too (one of them being taken for the arc which has the same distance from the same tropic point as the other) will have their oblique ascensions taken together equal to their right ascensions taken together. For example: the signs Taurus (♉) and Scorpio (♏) are opposite; and because Scorpio (♏) and Leo (♌) are equally distant from the beginning of Cancer (♋), Taurus (♉) and Leo (♌) will also be equally distant from the beginning of Cancer (♋). Since therefore Taurus (♉) and Leo (♌) have their oblique ascensions taken together equal to their right ascensions taken together, as was just shown, and the oblique ascension of Leo (♌) is the same as that of Scorpio (♏), as we showed at No. 9 — the oblique ascensions of Taurus (♉) and Scorpio (♏) taken together will also be equal to [the right ascensions of the same taken together]. And the same reasoning holds for any other arcs whatever, whether they take their beginning from the equinoctial points or not.

11. In every oblique region, arcs of the Ecliptic begun from Aries (♈) and less than a semicircle are greater than their oblique ascensions; but those begun from Libra (♎) are less — provided that the latitude of the place is neither greater than the complement of the greatest declination (for not all signs rise or set in a region where the elevation of the pole exceeds the complement of the greatest declination, that is, is greater than $66\frac{1}{2}^\circ$), nor less than the declination of that point which is then found on the Meridian, if indeed it is northern, when the extreme point of the proposed arc is on the Horizon.

[Margin: Arcs of the Ecliptic begun from Aries and less than a semicircle are greater than their ascensions in the oblique sphere; but begun from Libra, [less] [?].]

For, in the fourth figure, let the Meridian be ABCD; the Equator AC; the oblique Horizon BD, cutting the Equator at E; the pole of the Horizon H, so that the latitude of the region is AH; the arc of the Ecliptic FG, of whatever size, begun from the beginning of Aries (♈) at point F, but less than a semicircle. Likewise the arc of the Ecliptic IK, of whatever size, begun from [Libra (♎)] at I, and less than a

semicircle. I say that the arc FG is greater than its oblique ascension FE, and the arc IK less than [its oblique ascension] IE. For, a great circle HG being drawn through H, the pole of the Horizon, and the point G where the Ecliptic cuts the Horizon: since the latitude of the place AH is not placed less than the declination of the northern point L, which is then on the Meridian (and it is indeed always northern when the beginning of Aries (γ), namely the point F, is beyond the point A on the Equator; for when it is on the near side of the point A, as at I, the point of the Ecliptic N, then existing on the Meridian, is southern, and consequently the latitude of the place can be however small you please)...

the angle HGE is either greater than the angle LGE or equal to it. [Margin: Theodosius, *Sphaerica* I.15] Since HGE is a right angle, LGE will be either smaller than a right angle or a right angle itself, and therefore smaller than the angle AEG, which is obtuse because its arc DA is greater than the quadrant DH. Therefore, by proposition 11 of our *Triangula Sphaerica*, the arc FG will be greater than the arc FE. In the same way we shall conclude that the arc IO is greater than the arc IE: for once the great circle HO has been drawn, [Margin: Theodosius, *Sphaerica* I.15] the angle HOE is a right angle, and so IOE is acute and smaller than the obtuse angle IEO, and so on.

Again, once the great circle HK has been drawn through H and K, the angle HKE will be either smaller than the angle LKE or equal to it, because the latitude of the place, AH, is assumed to be not smaller than the declination AL of the northern point L that then lies on the meridian. That point will always be northern whenever the beginning of Libra — that is, the point I — falls short of the point A on the equator. For when it lies beyond the point A, as at F, the ecliptic point N then on the meridian is southern, and consequently the latitude of the place can be as small as you please. [Margin: Theodosius, *Sphaerica* I.15] Therefore, since the angle HKE is a right angle, IKE will be either greater than a right angle or a right angle itself, and therefore greater than the angle IEK, which is acute because its arc BA is smaller than the quadrant BH. Hence, by proposition 11 of our *Triangula Sphaerica*, the arc IK will be smaller than the arc IE. By the same reasoning we shall show that the arc FM is smaller than the arc FE, because, once the great circle HM has been drawn, [Margin: Theodosius, *Sphaerica* I.15] the angle HME is a right angle, and therefore FME is obtuse and greater than the acute angle FEM, and so on.

[Margin: Arcs of the ecliptic beginning from Aries have oblique ascensions smaller than the right ascensions by as much as the oblique ascensions of the equal arcs beginning from Libra are greater than the right ascensions.]

12. In every oblique region whose latitude is not greater than the complement of the greatest declination, the arcs of the ecliptic that begin from Aries and are smaller than a semicircle have oblique ascensions smaller than the right ascensions by as much as the oblique ascensions of the opposite and equal arcs, beginning from Libra, are greater than the right ascensions. For, in the same fourth figure, let two equal arcs FG and FM be taken, the arc FG beginning from Aries and FM from Libra; and from the pole of the world Q let the great circles QG and QM be drawn through G and M, where the two said arcs cut the horizon, cutting the equator in R and I, so that the right ascensions of the arcs FG and FM are FR and FI. Here it is clearly established that the oblique ascension of the arc FG (beginning from Aries) is FE, smaller than the right ascension FR; while the oblique ascension of the arc FM (beginning from Libra) is likewise FE, greater than the right ascension FI; and that the ascensional differences of those arcs are ER and EI, which I claim to be equal — so that the oblique ascension FE falls short of the right ascension FR by exactly as much as the oblique ascension FE exceeds the right ascension FI. For, since the ecliptic points G and M are diametrically opposite, counted along the succession of the signs, their ortive amplitudes EG and EM will be equal, as we gathered in Number 3. Therefore, since in the triangles EGR and EMI the vertical angles at E are equal, by proposition 6 of our *Triangula Sphaerica*, [Margin: Theodosius, *Sphaerica* I.25 [?]] and since the angles R and I are right angles, to which are opposite the arcs already shown equal, EG and EM, the arcs ER and EI will, by proposition 21 of our *Triangula Sphaerica*, be equal.

It makes no difference, moreover, that the opposite arcs FR and FI which we have posited are unequal; for exactly the same conclusion will follow if, as the matter requires, the beginning of Libra were taken beyond F, so that the arc of the ecliptic from it up to M were made equal to the arc FG, and its right ascension, counted from that same beginning of Libra up to I, were equal to the right ascension FR — because the ascensional differences ER and EI always remain the same.

[Margin: Opposite points of the ecliptic have ascensional differences equal to one another.]

[Margin: Of two equal arcs of the ecliptic equally distant from the same tropic point, or opposite, the oblique ascension of one falls short of the right ascension by exactly as much as that of the other exceeds it.]

Now if two equal arcs of the ecliptic do not begin from Aries and Libra, but are nonetheless either equally distant from the same tropic point or opposite, still the oblique ascension of one will fall short of its own right ascension by exactly as much as the oblique ascension of the other exceeds it. Indeed, the arcs contained in the ascending semicircle of the ecliptic — that is, from Capricorn through Aries up to Cancer — have smaller [oblique] ascensions, while the arcs contained in the descending semicircle — that is, from Cancer through Libra up to Capricorn — have greater ones, as will be demonstrated in Book 3, Canon 5, Number 15. From this it follows that the arcs from Aries up to Cancer have smaller ascensions than the arcs from Libra up to Capricorn, since the arcs from Libra up to Cancer have (as was shown in Number 9) ascensions equal to those which the arcs from Libra up to Capricorn have. For the same reason the arcs from Aries up to Cancer will have smaller ascensions than the arcs from Aries up to Capricorn, since these latter arcs have ascensions equal to those which the arcs from Aries up to Cancer have, as is clear from Number 9. And so the arcs from Capricorn through Aries up to Cancer have oblique ascensions smaller than the right ascensions by exactly as much as the arcs equal to them, from Cancer through Libra up to Capricorn, have greater ones.

This may be shown as follows. Since, as was shown in Number 6, Capricorn and Cancer have [right] ascensions FK and HK, as in the third figure; and since, taken together, these are equal to the oblique ascensions of the same arcs taken together, as was demonstrated in Number 10; and since the oblique ascension of Capricorn is smaller than the oblique ascension of Cancer — if FE is the oblique ascension of Capricorn, and consequently the remaining arc EH is the oblique ascension of Cancer, it is evident that the arc FE is smaller than the arc FK by exactly as much as the arc EH is greater than the arc KH, or than that same FK, since in both cases the excess is the arc EK. And so for the remaining equal opposite arcs. Again, since Taurus and Leo have equal right ascensions, as was said in Number 6, let those ascensions be FK and HK, in the same third figure; and since, taken together, these are equal to the oblique ascensions of the same arcs taken together, as was demonstrated in Number 10 — if FH is divided into unequal arcs at E, so that EH is the oblique ascension of Leo and EF that of Taurus, it will clearly be established that the arc EF is smaller than the same arc FK or HK by exactly as much as the arc EH is greater than the arc HK, or than that same FK. And the reasoning is the same for the other arcs equally distant from the same tropic point. But if the ascension of Cancer were smaller than the ascension of Capricorn, it would be gathered in the same way that it falls short of the right ascension by exactly as much as the latter exceeds it; so that it is quite certain that, if two equal arcs of the ecliptic — or arcs equally distant from the

same tropic point, or opposite — are taken, the oblique ascension of one is greater than its right ascension by exactly as much as the oblique ascension of the other is smaller.

[Margin: Two equal arcs of the ecliptic, equally distant from the same tropic or equinoctial point, or opposite, have the same ascensional difference.]

13. In every oblique region, two equal arcs of the ecliptic that are equally distant from the same tropic or equinoctial point, or opposite, have the same ascensional difference. For since equal arcs receding equally from the same tropic point, or opposite, have their oblique ascensions taken together equal to their right ascensions taken together, as we taught in Number 10, and their right ascensions are equal, as is clear from Number 6, it comes about that the oblique ascension of one is smaller than the right ascension by exactly as much as the ascension of the other is greater, as we said in Number 12. Therefore they have the same ascensional difference. As for arcs equally distant from the same equinoctial point, the matter is plain, since they have equal oblique ascensions, as was shown in Number 9, and consequently the ascension of each either exceeds the right ascension by the same excess, or falls short of it by the same amount.

[Margin: Four points of the ecliptic have the same ascensional difference.]

14. In every oblique region, any arc of the ecliptic whose extreme points are equally distant from the same tropic point — such as the arcs between the beginnings of Gemini and Leo, between the beginnings of Taurus and Virgo, between the beginnings of Aries and Libra, between

the beginnings of Pisces and Scorpio, and between the beginnings of Aquarius and Sagittarius — has the same ascension as in the right sphere. For, as was demonstrated in Number 10, the half-arcs of such an arc have their ascensions taken together equal to the right ascensions taken together; hence, although one of the halves has a smaller oblique ascension and the other a greater, the two taken together nevertheless produce the right ascension of the whole arc.

From this it follows that the same aforesaid arc has, in all regions or elevations of the pole, the same ascension, even though its parts rise differently: because, namely, in all elevations of the pole its ascension is equal to the right ascension.

[Margin: The descension of any arc of the ecliptic is equal to the ascension of the opposite arc.]

Furthermore, the descension of any arc of the ecliptic is equal to the ascension of the opposite arc; because at the same time at which some arc descends, its opposite arc rises, so that (as the argument requires) a semicircle of the ecliptic is always seen above the horizon, [Margin: Theodosius, *Sphaerica* I.11] since the horizon and the ecliptic bisect each other.

[Margin: It is enough if the oblique ascensions of the arcs of the first quadrant of the ecliptic are computed.]

[Margin: Any arc of the ecliptic bisected by the same tropic point has, everywhere, an oblique ascension equal to its own right ascension.]

Therefore, in order that a table of oblique ascensions may be constructed, it is enough that the oblique ascensions be computed for the arcs of the quadrant of the ecliptic from Aries up to Cancer. For, as we demonstrated in Number 9, the ascensions of these arcs are equal to the ascensions of the arcs of the quadrant from Aries up to Capricorn, taking always two arcs equally distant from the beginning of Aries and contained in one semicircle. And since, as was shown in Number 10, the ascensions of these arcs together with the ascensions of the opposite arcs, taken together, are equal to the right ascensions of the same arcs, and the opposite arcs have equal right ascensions, as was clear from Number 6, it comes about that the oblique ascensions of the opposite arcs are obtained by subtracting the ascensions of the arcs of the semicircle from Capricorn up to Cancer from the doubled right ascensions of those same arcs.

From these, then, the table of oblique ascensions will be constructed as follows. Having computed the ascensions of the arcs beginning from Aries up to the end of Gemini, if these are subtracted from the doubled right ascensions of the same arcs, the results will be the oblique ascensions of the arcs beginning from Libra up to the end [of Sagittarius], equal to the oblique ascensions of the arcs equal to them beginning from Libra up to the beginning of Cancer; and if these, beginning with the larger, are subtracted from the semicircle, one obtains the oblique ascensions of the arcs greater than a quadrant, beginning from Aries up to the end of Virgo. But if a semicircle is added to the ascensions of the arcs beginning from Libra up to the end of Capricorn, there arise the ascensions of the arcs greater than a semicircle, beginning from Aries up to the end of Capricorn. Finally, since the ascensions of the arcs from Aries up to Cancer are equal to the ascensions of the arcs from Aries up to Capricorn, if these, beginning with the larger, are subtracted from the whole circle, there remain the oblique ascensions of the arcs greater than three quadrants, beginning from Aries up to the end of Pisces.

[Margin: The ascensional difference of any point of the ecliptic is also the difference between the semidiurnal arc of that point and the semidiurnal arc of the equator, which is always a quadrant.]

15. Now, from what has been said, I judge that it is also clearly established that the ascensional difference of any point of the ecliptic is the same as the difference between the semidiurnal arc of the parallel described through that point and the semidiurnal arc of the equator — that is, a quadrant. For, in the first figure of this lemma, the semidiurnal arc of the northern parallel MI, described through the ecliptic point I, is the arc MN — that is, the equator arc AR similar to it — so that ER is the difference between the semidiurnal arc AR, of the northern parallel MI (or of the northern ecliptic point I), and the semidiurnal arc AE of the equator; and this same difference is the ascensional difference of that same ecliptic point I. For when the sphere is moved until the point I reaches the horizon at the point N, the arc of the equator counted from the beginning of Aries — wherever it then lies — along the succession of the signs up to E, will be the oblique ascension of the point I when it is at N, since the point E of the equator rises above the horizon together with the ecliptic point I existing at N; whereas the arc of the equator counted from the same beginning of Aries up to R will be the right ascension of that same point I when it is at N, since the point R of the equator and the ecliptic point N — which then does not differ from I — ascend together above the right horizon drawn through AC. Therefore ER is the ascensional difference.

By the same reasoning, EQ will be the ascensional difference of the southern ecliptic point H, and the difference between the semidiurnal arc of that same point H (or of the parallel KL) and the semidiurnal arc of the equator; since the oblique ascension terminates at E and the right ascension at Q, and AQ is the semidiurnal arc of the point H — that is, an arc similar to the semidiurnal arc KL — while AE is the semidiurnal arc of the equator.

[Margin: How, from the ascensional difference of any point of the ecliptic, the semidiurnal arc of that point is elicited.]

Therefore, in order that the semidiurnal arc of any point of the ecliptic may be computed, its ascensional difference must be sought. For this, if the point is northern, added to the semidiurnal arc of the equator (which is perpetually a quadrant), will yield the required semidiurnal arc; but the same, subtracted from the semidiurnal arc of the equator, if the given ecliptic point is southern, will leave the required semidiurnal arc.

[Margin: When the ascensional difference is to be added or subtracted, in order to obtain the semidiurnal arc, or the oblique ascension, of the given point or star.]

And from this it is manifest that, when the point is northern — such as I — the ascensional difference ER must be added to the semidiurnal arc AE of the equator, that is, to the quadrant, so that the semidiurnal arc AR of the given point may result; but the same must be subtracted from the right ascension terminating at R, so that the oblique ascension terminating at E may be left. On the contrary, when the given point H is southern, the ascensional difference EQ must be subtracted from the quadrant, or from the semidiurnal arc AE of the equator, so that the semidiurnal arc AQ of the given point may be left; but the same must be added to the right ascension terminating at Q, so that the oblique ascension terminating at E may be produced.

This same thing which we have said of a northern or southern ecliptic point is also to be understood of any northern or southern star, as is plain if some northern star is placed on the parallel MI, and a southern one on the parallel KL. For their ascensional differences will be ER, EQ, and so on.

Since, moreover, opposite points of the ecliptic have equal ascensional differences, as we showed in Number 12; and since each of them has the same ascensional difference as the point that has a distance from the nearest tropic point equal to its own, seeing that the same parallel passes through those two points, as we demonstrated in Number 1; it comes about that four points of the ecliptic have the same ascensional difference.

[Margin: The whole sine is to the sine of the complement of the declination of any point of the ecliptic in the same ratio as the secant of the arc between that point and the nearest equinoctial point is to the secant of the right ascension of the same arc.]

16. The whole sine is to the sine of the complement of the declination of a given ecliptic point in the same ratio as the secant of the arc comprehended between the given point and the nearest equinoctial point is to the secant of the right ascension of the same arc, or of the given point, reckoned from the nearest equinoctial point. For in the right-angled spherical triangle FGK, whose angle K is a right angle — which is found in the third preceding figure — the whole sine is to the sine of the complement of the arc GK, the declination of the ecliptic point G, about the right angle K, as the secant of the arc FG of the ecliptic, comprehended between the given point G and the nearest equinoctial point F and opposite the right angle K, is to the secant

of the third arc FK, the right ascension, which is the other arc about the right angle K, as we demonstrated in proposition 53 of our *Triangula Sphaerica* — which was the thing proposed. And so, once the right ascensions of all the points of the first quadrant of the ecliptic have been found in this way, from them will be derived the right ascensions of all the other points, as we said above in Number 6.

[Margin: The whole sine is to the tangent of the elevation of the pole in the same ratio as the tangent of the declination of a given ecliptic point is to the sine of the ascensional difference of that same point.]

17. The whole sine is to the tangent of the elevation of the pole in the same ratio as the tangent of the declination of a given ecliptic point is to the sine of the ascensional difference of that same point. For in the right-angled spherical triangle EGK, whose angle K is a right angle — which is found in the same third preceding figure — the whole sine is, by proposition 49 of our *Triangula Sphaerica*, to the tangent of the arc GK, the declination of the ecliptic point G about the right angle K, as the tangent of the complement of the angle E (opposite the said arc GK) — that is, as the tangent of the elevation of the pole (since the angle E is the angle of the complement of the elevation of the pole, namely the one that the equator AC makes with the horizon) — is to the sine of the arc EK, the ascensional difference, which is the other arc about the right angle K. Therefore, by alternation, it will also be the case that, as the whole sine is to the tangent of the elevation of the pole, so is the tangent of the declination of the given ecliptic point to the sine of the ascensional difference of that same point. But we shall also demonstrate this without spherical triangles, as follows.

Let the meridian in the first following figure be ABCD; the diameter of the horizon BD; [the diameter] of the equator LM; the axis of the world AC; the diameter of the parallel — whether northern or southern — FG, cutting the axis in H at right angles, and cutting the diameter of the horizon [BD]; the diameter of the parallel [is FG], cutting the axis in I; the diameter of the ecliptic NP, cutting FG in O. And from the pole A let the perpendicular AK be dropped to BD. Now if, about the diameters NP and FG, semicircles of these, perpendicular to the meridian, be conceived, and from the points E, O, H perpendiculars raised to the same meridian, the perpendicular from O will fall upon the given ecliptic point through which the parallel of the diameter FG passes, since at the extremity of that perpendicular, on the surface of the sphere, the ecliptic and the parallel intersect. Now the arc of the parallel between the perpendiculars from O and H

will be the right ascension of the given point, since it rises together with the arc of the ecliptic between the perpendiculars from O and E above the right horizon drawn through AC; and therefore the arc of the parallel will be similar to the arc of the equator rising with it, since similar arcs of the parallels always rise in the same time in every horizon. But the arc of the parallel between the perpendiculars from O and I will be the oblique ascension of the same arc of the ecliptic, since together with the arc of the ecliptic between the perpendiculars from O and E it rises above the oblique horizon drawn through BD. Finally, the arc of the parallel between the perpendiculars from H and I will be the ascensional difference.

Again, HE is the sine of the declination LE, and FH the sine of the complement AF of that same declination. Let it now be arranged that, as FH (the sine of the complement of the declination) is to HE (the sine of the declination), so is FH (the whole sine) to a fourth term; and HE will be found, in the parts of the semidiameter FH taken as the whole sine. But since, by proposition 18 of the treatise on sines, as FH (the sine of the complement of the declination) is to HE (the sine of the declination), so is the whole sine to the tangent of the declination, therefore the line HE, found in the parts of the semidiameter FH, [Margin: Euclid, *Elements* V.9] is equal to the tangent of the declination with respect to the whole sine EA: that is, as many parts as there are in HE with respect to the whole sine FH, so many are contained in the tangent of the declination with respect to the whole sine EA; so that it comes to the same thing to take HE in the parts of the whole sine FH, and to take the tangent of the declination of the proposed parallel, with respect to the whole sine EA. Next, since the triangles AEK and IEH are equiangular — on account of the right angles K and H, and the common angle E (or the equal vertical angles at E) — [Margin: Euclid, *Elements* VI.4] it will be that, as EK (the sine of the complement of the elevation of the pole) is to AK (the sine of the elevation of the pole), so is HE, found in the parts of the whole sine FH — that is, so is the tangent of the declination — to HI, the sine of the ascensional difference, in the parts of the same whole sine FH. But, by proposition 18 of the treatise on sines, as the sine of the complement of the elevation of the pole is to the sine of the elevation of the pole, so is the whole sine to the tangent of the elevation of the pole. Therefore it will also be that, as the whole sine is to the tangent of the elevation of the pole (which tangent, in the same region, never changes), so is the tangent of the declination to the sine of the ascensional difference — which was the thing proposed.

For the rest, when we said that the arc of the parallel between the perpendiculars raised from O and I is the oblique ascension of the arc of the ecliptic whose sine is

EO, this is to be understood of the arc that is counted from the nearest equinoctial point E against the succession of the signs. Thus, when the ecliptic EN inclines toward the northern pole A, the arc to be counted...

Diagram. A rectangular framed woodcut containing four spherical figures (the four figures of Lemma XLIX), each a circle representing the sphere in orthographic projection with the meridian, horizon, equator, ecliptic, parallels, and various great circles drawn in, and lettered points around and within.

- Upper-left figure (crowded): outer labels reading approximately K, A, M across the top; F, H, B, V, Z down the left; P, Y, B, D down the right; and R, X, I, O, T, C, G around the bottom; interior labels S, Q, b, L, E, N. Several labels are ambiguous at this resolution.
- Upper-right figure: A at top; F, I, H; E, G, D; K, F, C toward the bottom; B, D at the right margin.
- Lower-left figure: A at top; N, F, L, E; I, B at left; D at right; G, M, K, O; H, C at bottom.
- Lower-right figure: F at top with A and H; N, L, I; B at left; M, K, E, O, G across the middle; D at right; Q, R; C at bottom.

[Many peripheral labels in all four figures are small and partly ink-degraded]

Diagram. A repetition, at the head of this page, of the same four-figure block shown on the preceding page (the four figures of Lemma XLIX), with the same lettered points; reprinted here for reference by the text of propositions 16–17, which cite "the third preceding figure." Labels as in fig-p437-1. [Repeat of the preceding block; peripheral labels degraded]

Diagram. A rectangular framed woodcut containing four further spherical figures (the "following figures" used for the demonstration of proposition 17 without spherical triangles). Each is a circle (the meridian ABCD) with the horizon diameter BD, equator, axis, ecliptic diameter NP, a parallel diameter FG, and perpendiculars.

- Upper-left figure: F, N, A across the top; O (with dotted construction lines); H; N/E and I at left; K, D, P at right; G, H, M toward the bottom; C at the foot.
- Upper-right figure: L, F, N, A across the top; O, Q, R; B, E, I; K, D, G at right; M, P; C at the foot.
- Lower-left figure: F at top, A, H, E, I along the horizontal diameter; B at left, D and G at right; C at the foot.

- Lower-right figure: F at top, A; H, E, I along the horizontal diameter; K; B at left, D and G at right; C at the foot.

[Peripheral and interior labels small and partly degraded]

ASTROLABIVM — LIBER PRIMVS

LEMMA XLIX (continued)

[...the arc whose sine is EO] is to be counted from Libra toward Virgo, Leo, and Cancer. And since the arcs from Libra toward Cancer have ascensions equal to the equal arcs that recede equally from the beginning of Libra toward Capricorn — as we showed at Num. 9 — once the oblique ascensions of the latter have been found, the oblique ascensions of these too will be found; so that the oblique ascensions of all the arcs in the semicircle descending from the beginning of Libra are then known for the arcs so initiated.

When, on the other hand, the ecliptic E N inclines toward the south pole, the same arc, whose sine is EO, is to be counted from Aries toward Pisces, Aquarius, and Capricorn. And since the arcs from Aries toward Capricorn have the same ascensions as the equal arcs that recede equally from the beginning of Aries toward Cancer — as was shown at Num. 9 — once the oblique ascensions of the latter have been found, the oblique ascensions of these too will be found; so that the ascensions of all the arcs in the semicircle ascending from the beginning of Aries are then known for the arcs so initiated. And by this same means the ascensions of the arcs initiated from Aries and counted according to the succession of the signs are likewise made known — as we declared a little earlier, toward the end of Num. 14, and shall explain again in Book 3, in the scholium of Canon 5, Num. 1.

That the aforesaid arcs of the ecliptic are to be counted from Aries and Libra *against* the succession of the signs is clear from this: the point common to the ecliptic and the parallel — the point onto which the perpendicular erected from O falls — cuts the oblique horizon (in the motion of the sphere) at the point onto which the perpendicular erected from I falls. And then the arc of the parallel between the perpendiculars from O and I, and the arc of the ecliptic between the perpendiculars from O and E — reckoned from O up to the equinoctial point E according to the succession of the signs — rise together, since their extremities reach the oblique horizon simultaneously. The same must be said of the right ascensions above the right horizon, drawn through AC. But since equal arcs

counted from Aries and Libra toward Cancer have equal right ascensions — as we said at Num. 6 — it makes no difference whether the arc of the ecliptic is counted from Libra against the succession of the signs, or from Aries according to the succession of the signs, and so on.

And since, once the ascensional difference of the beginning of Cancer, or of Capricorn — that is, the difference between the greatest (or least) semidiurnal arc and the semidiurnal arc of the Equator — has been found for any elevation of the pole whatever (for that same ascensional difference is the difference between the semidiurnal arc [of a point] and the semidiurnal arc of the Equator, as we showed at Num. 15), the ascensional differences of all the other points of the ecliptic are found with little trouble at the same elevation of the pole. This is done as follows: as the whole sine is to the Tangent of the greatest declination which the beginning of Cancer, or Capricorn, has (which Tangent remains the same at all elevations of the pole), so is the ascensional difference of the beginning of Cancer, or Capricorn, to another [ascensional difference]. For thus will be found the difference sought between the semidiurnal arc [of the point] and the semidiurnal arc of the Equator, as has here been demonstrated — provided FG be the diameter of the parallel of Cancer, or Capricorn, and EF the semidiameter of the ecliptic, so that F be the point of the ecliptic a quadrant distant from the equinoctial point E.

[Margin: How the difference between the longest or shortest semidiurnal arc and the semidiurnal arc of the Equator is to be computed at any elevation of the pole.]

18. The whole sine is to the sine of the right ascension of a given point of the ecliptic as the sine of the difference between the longest (or shortest) semidiurnal arc [and the semidiurnal arc of the Equator] — that is, the sine of the ascensional difference of the beginning of Cancer, or Capricorn — is to the sine of the difference between the semidiurnal arc of the same given point of the ecliptic and the semidiurnal arc of the Equator [i.e., to the sine of that point's ascensional difference].

[Margin: The whole sine is to the sine of the right ascension of any point of the ecliptic as the sine of the ascensional difference of the beginning of Cancer or Capricorn is to the sine of the ascensional difference of the same point.]

For, once more, in the second figure let the Meridian be ABCD, the diameter of the Horizon BD, the Equator LM, the axis of the world AC; let the diameter of the boreal parallel FG cut the axis at right angles in H, cutting the Horizon's diameter [in I]; [let the diameter of the parallel of Cancer NK cut] the axis in Q and the

Horizon's diameter in R; and finally let the diameter of the ecliptic be NP, cutting FG in O. Let the diameters NP, NK, FG be understood as [chords of] semicircles standing at right angles to the Meridian; then, since the straight lines erected [from these points] are perpendicular to that same Meridian, the perpendicular from O will fall onto the given point of the ecliptic; and the arc of the parallel between the perpendiculars from O and H will be the right ascension of that same parallel[-point], while the arc of the same parallel between the perpendiculars from O and I will be its oblique ascension, and the arc between the perpendiculars from H and I will be the ascensional difference, whose sine is HI; and finally [QR will be the sine of the ascensional difference of] Cancer — that is, of the difference between the longest semidiurnal arc [and the semidiurnal arc of the Equator], etc.

And it is so that, as NQ, the whole sine of the parallel of Cancer, is to QR, the sine of the difference between the longest [semidiurnal arc] and the semidiurnal arc of the Equator, so is OH, the sine of the right ascension of the given point, to HI, the sine of the ascensional difference of the same point. Therefore, by permutation, as the whole sine is to the sine of the right ascension [of the point], so is the sine of the ascensional difference of Cancer to the sine of the ascensional difference of the same given point — which was proposed. And that the right ascensions are the same, and likewise the ascensional differences, whether we here take the boreal parallels or the southern ones, does not matter, since the ascensional differences of the southern parallels are the same as those of the boreal, as was demonstrated above at Num. 6 and 13. And so, if the ascensional difference of the beginning of Cancer, or Capricorn, has been computed for any region, and a table of right ascensions is at hand, the ascensional differences of all the other points of the ecliptic in the same region are found with little trouble.

[Margin: The sine of the complement of the declination of any point of the ecliptic is to the sine of the declination of the same point as the whole sine is to the sine of the ascensional difference of the same point, in latitude 45° .]

19. In latitude 45° , the sine of the complement of the declination of a given point of the ecliptic is to the sine of the declination of the same point as the whole sine is to the sine of the ascensional difference of that point.

For let the Meridian in the third figure be ABCD; the diameter of the Horizon BD; the altitude of the pole DA = 45° ; the axis of the world AC; the diameter of any parallel FG, cutting the axis in H and the diameter of the Horizon in I. [Margin: a — Euclid I.32.] Since in the triangle HEI all the angles [together] equal two right angles, and H is right, and E is semi-right (that is, 45°) on account of the arc DA of

45°, then I too will be [semi-right], equal to E; [Margin: b — Euclid I.6.] and therefore the sides HE, HI will be equal. And since, as FH, the sine of the complement of the declination, is to HE, the sine of the declination, so is FH, the whole sine, to HE, the sine [of the declination] with respect to the whole sine FH — that is, to HI, which is equal to HE — and HI is the sine of the ascensional difference (as appeared from what precedes) in the parts of the whole sine FH, what is proposed is clear.

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Since, then, by proposition 18 of the *Treatise on Sines* [*tractatus sinuum*], as the sine of the complement of the declination is to the sine of the declination, so also is the whole sine to the Tangent of the declination, it follows that, [Margin: a — Euclid V.9.] in latitude 45°, the sine of the ascensional difference of any point of the ecliptic is equal to the Tangent of the declination of the same point. So much so, that the arc which, as a sine, is owed [i.e., corresponds] in the table of sines to the Tangent of the declination of any point of the ecliptic is the ascensional difference of that point in the region where the elevation of the pole comprises 45°. For example, the Tangent of the greatest declination — that is, the Tangent of 23° 30' — is 4348124; and to this, as a sine, there correspond in the table of sines 25° 46', for the ascensional difference of the beginning of Cancer, or Capricorn, in latitude 45°.

[Margin: a — Euclid V.9. The arc which, as a sine, corresponds to the Tangent of the declination of any point is the ascensional difference of that point at pole-altitude 45°.]

[Margin: As the sine of the complement of the given pole-altitude is to the sine of the pole-altitude, so is the sine of the ascensional difference of any point of the ecliptic at pole-altitude 45° to the sine of the ascensional difference of the same point at the earlier given pole-altitude.]

20. In every region whose pole-altitude is greater or less than 45°, the sine of the complement of the pole-altitude is to the sine of the pole-altitude as the sine of the ascensional difference of any point of the ecliptic at pole-altitude 45° is to the sine of the ascensional difference of the same point at the proposed pole-altitude.

For let there be again, in the fourth circle, the Meridian ABCD; the diameter of the Horizon BD; the altitude of the pole DA, greater or less [than 45°]; the axis of the world AC; the diameter of any parallel FG, cutting the axis in H and the diameter of the Horizon in I; and from the pole A let there be dropped AK, the sine of the

pole-altitude. [Margin: b — Euclid VI.4.] And since the triangles AEK and IHE, having right angles at K and H and a common angle at E, are equiangular, it will be that, as EK, the sine of the complement of the given pole-altitude, is to KA, the sine of the pole-altitude, so is HE — which is equal to the sine of the ascensional difference (in the parts of the whole sine FH) at pole-altitude 45° , as appeared in the preceding Num. (for it was there shown, on account of the semi-right angle E, that the sine of the declination HE is equal to the sine of the ascensional difference HI) — to HI, the sine of the ascensional difference at the given pole-altitude DA. Which was proposed.

[Margin: The proportion of the whole sine to the Tangent of the given pole-altitude is the same as that of the sine of the ascensional difference of any point of the ecliptic at pole-altitude 45° to the sine of the ascensional difference of the same point at the given pole-altitude.]

Now since, by proposition 18 of the *Treatise on Sines*, as the sine of the complement of the pole-altitude is to the sine of the pole-altitude, so is the whole sine to the Tangent of the proposed pole-altitude, it will likewise be that, as the whole sine is to the Tangent of the proposed pole-altitude, so is the sine of the ascensional difference of any point of the ecliptic at pole-altitude 45° to the sine of the ascensional difference of the same point at the proposed pole-altitude. Therefore, once the ascensional differences of all the points of the ecliptic have been found in the region where the pole-altitude comprises 45° — which differences the Tangents of the declinations will indeed supply, as was shown toward the end of Num. 19 — the ascensional differences of those same points in any other region whatever will be found by their means.

LEMMA L.

Given two axes of an ellipse cutting each other at right angles: if from any point of the minor axis (produced too, if need be) a straight line equal to half the major axis is drawn cutting the major axis itself, in such a way that its segment beyond that same major axis is equal to half the minor axis, then its extremity falls on the ellipse. And if from any point of the ellipse a straight line equal to half the major axis is drawn to the major axis (produced too, if need be, but in any case cutting it), then its segment between the given point and the major axis will be equal to half the minor axis.

Let the two axes AC, BD of the ellipse ABCD cut each other at right angles in E. And first, from any point F on the minor axis BD (produced too, if need be), let the straight line FG be drawn equal to AE, half the major axis AC, cutting the major

axis in H, in such a way that the segment HG is equal to ED, half the minor axis. I say that the extreme point G falls on the ellipse. For about the major axis AC let there be described, from center E, the circle AICK; and through G let GM be drawn parallel to the minor axis BD, cutting the circle in L; and let GN be drawn parallel to the major axis AC; and finally let the straight line EL be joined. [Margin: c — Euclid I.34. d — Euclid I.29.] Now since in the parallelogram [ENGM] the opposite sides are equal, and the angles at M and N are right — because both the angle at M (namely MEN) and the angle at N (namely NEM) [with their pairs] equal two right angles — and since moreover the straight lines FG, EL are equal, each being equal to AE, the two sides FG, GN will be equal to the two sides LE, EM, and the angles N, M, subtended by the equal sides FG, LE, are equal. [Margin: e — Euclid I.17.] Then, since each of the remaining angles F, L is less than a right angle, by the last scholium of Euclid Book 1 the bases FN, LM, and both the angles F, L and the angles FGN, LEM, will be equal. [Margin: f — Euclid I.29.] Therefore, since FGN is equal to the alternate angle GHM, the angles GHM, LEM will also be equal; [Margin: g — Euclid I.28.] and therefore FG, EL will be parallel, and the triangles ELM, HGM similar, by the corollary of Euclid VI.4. Therefore, as EL is to LM, so is HG to GM; [Margin: h — Euclid VI.22.] and hence also, as the square on EL is to the square on LM, so is the square on HG to the square on GM. But the square on EL is equal to the square on AE — that is, to the rectangle under AE, EC — [Margin: i — Euclid VI.17.] and the square on LM is equal to the rectangle under AM, MC, because, by the scholium of Euclid VI.13, LM is a mean [proportional] between AM and MC. Likewise the square on HG is equal to the square on ED, since their sides are assumed equal. Therefore it will also be that, as the rectangle under AE, EC is to the rectangle under AM, MC, so is the square on ED to the square on MG. And since ED, MG are thus applied ordinatewise to the axis AC, the ellipse ABCD will pass through the point G. [Margin: k — Apollonius, *Conics* I.21.] For if it be said to pass through some other point of the straight line LM, say through O, it will also be that, as the rectangle under AE, EC is to the rectangle under AM, MC, so is the square on ED to the square on MO;

Diagram. A single ruled woodcut frame containing four small circular figures (the "first" through "fourth" figures / circles used in propositions 17–20), each a Meridian circle ABCD carrying the axis of the world, the Equator, a parallel of declination, the ecliptic diameter, the horizon diameter, and their perpendiculars. The figures are small and densely lettered.

- Upper-left circle: N (top), F (upper-left), A (upper-right), O (upper-center), H (center-upper), E (center), I (left-center), B (left, with a second N/B pair at

the left margin), K and D (right), P and G (lower-right), M (lower), C (bottom); interior radial construction lines and a dotted arc.

- Upper-right circle: L and F (upper-left), N (top), A (upper-right), B (left), O (center-upper), H (center), R (right-upper), E (center), I (center), K and D (right), G (lower-right), M (lower-right), C (bottom), P (bottom).
- Lower-left circle: F (upper-left), A (top), H (center-upper), E (center), I (center-right), B (left), D (right), C (bottom), with a crossing chord.
- Lower-right circle: A (top), F (upper-left), H (center-upper), E (center), I (center-right), K (right), D (right), G (lower-right), C (bottom).

Diagram. The construction figure for Lemma L, in a ruled frame: an outer circle AICK described about the major axis AC, with the inscribed ellipse ABCD. A (top), C (bottom), and the minor-axis endpoints B (left) and D (right) of the ellipse; the circle's horizontal-diameter endpoints I (far left) and K (far right); center E, with N to its right. M and L are marked at the top (M inner, L on the circle to the upper right); G and H lie near the center on the construction lines; Q is to the right of center. Numerous auxiliary construction points are labelled with Greek minuscules (η , θ , γ , ε , δ , α , μ , ν , ξ , π , λ , and others) along the chords and the two vertical ordinates dropped to the axis. Below the circle are two horizontal reference segments: S——R (with marks "z α " toward the right end and "x / Y" below the midpoint) and V——T.

LEMMA L. (proof concluded)

[...so is the square on ED to the square on MO;] and therefore the squares on MG, MQ would be equal, and the straight lines themselves equal — a part and the whole — which is absurd. Therefore the ellipse passes through G, and so the point G falls on the ellipse; which was proposed.

Next, from any point G of the ellipse, let the straight line GF be drawn to the minor axis BD — whether it fall outside the points B, D, or within — equal to AE, half the major axis, cutting the major axis in H. I say that the segment GH is equal to ED, half the minor axis. For, the same construction being made, we shall show, as before, that the triangles ELM, HGM are similar; and that, as the square on EL is to the square on LM — that is, as the rectangle under AE, EC (which is equal to the square on AE, or on EL) is to the rectangle under AM, MC (which was equal to the square on LM) — so is the square on HG to the square on GM. But it is also so that, as the rectangle under AE, EC is to the rectangle under AM, MC, so is the square on ED to the square on MG. Therefore the squares on HG and on ED have

the same proportion to the square on MG, and hence are equal to each other, and the lines HG, ED are themselves equal — which was to be demonstrated.

SCHOLIVM.

[Margin: Given the axes, to describe the ellipse.]

The former part of this theorem was demonstrated by another, and indeed longer, method by that most learned man Guidobaldo del Monte [Guido Ubaldo è Marchionibus Montis], toward the end of Book 2 of his *Universal Planispheres* [*Planisphaeriorum universalium*]; with which the following [results], which are to be gathered from it, agree.

First, in what manner, given two axes, an ellipse is to be described about them. Let there be, then, two axes AC, BD cutting each other at right angles in E; and let Eh be taken equal to half the major axis — that is, to AE — so that Eh be the excess by which half the major axis surpasses half the minor axis BE. Then from any number of points a, F, g on the straight line EI, by means of a compass set to AE, let the straight lines ab, FH, ge [be applied]; and, the straight lines ad, FG, gf being produced, let there be cut off bd, HG, ef equal to BE, half the minor axis, so that the [whole] straight lines ad, FG, gf be equal to AE (or Bh), half the major axis. Or [alternatively] let ad, FG, gf be cut off equal to [half the major axis AE], so that the segments bd, HG, ef be equal to BE, half the minor axis. For, as was demonstrated, the points d, G, f will fall on the ellipse. And so, if by this device very many points be found — not only between A and D, but also between D and C, and between C and B, and likewise between B and A — and a curved line be drawn congruently through them, the ellipse will be described.

[Margin: Given either of the axes and a point on the ellipse described about that axis, to find the other axis.]

Next, by what reasoning, given any point of an ellipse not yet described together with one or the other of the axes, the other axis may be found. Let, then, the major axis AC first be given, and a point G existing on the ellipse. The axis AC being bisected in E by a perpendicular line, let there be applied, by means of a compass, from the given point G the straight line GF as far as the straight line BD, equal to AE, half the major axis, cutting AE in H [?]. For, as was demonstrated, GH will be equal to half the minor axis, and so, if EB, ED be cut off equal to GH, BD will be [the minor] axis. For since FG is equal to AE, and HG to ED, G will fall on the ellipse, as we demonstrated.

But if the minor axis BD be given, together with a point G existing on the ellipse, we shall find the major axis in this way. The minor axis being cut [bisected] in E by the perpendicular line AC, let there be applied by compass, from the given point G, the straight line GH as far as the straight line [AC], equal to BE, half the minor axis, and let it be produced until in F it cuts the minor axis, produced too, if need be. If from it [equal parts EA, EC be cut off], it is clear from the things that have been demonstrated. For since FG is equal to AE, [and HG to BE, G will fall on the ellipse], as we demonstrated.

[Margin: Given the two axes of an ellipse together with any point, to know whether the given point lies on the ellipse, or outside, or within.]

Thirdly, given the two axes of an ellipse not yet described, together with any point outside them, [it is asked] by what path it may be known whether the given point lies on the ellipse itself, or outside, or indeed within. Let there be, then, two axes AC, BD cutting each other at right angles in E, and a given point G; and let there be applied by compass, from the given point G, the straight line GF to the minor axis BD (produced too, if need be), equal to AE, half the major axis, cutting AE in H. If, then, GH be equal to ED, half the minor axis, the given point G will fall on the ellipse, as was demonstrated, since the whole GF has been set equal to half the major axis.

But now let there be a given point k, and let the straight line ki, equal to AE (or Bh), be applied cutting AE in e, so that ke be greater than ED. I say the given point k falls outside the ellipse. For since ki is equal to AE (or Bh), and ke is greater than BE, the remainder to it [kH] will be less than the other remainder, until it be made equal to Eh. [Margin: d — Euclid I.19.] Let there be drawn from k the straight line kF, such that the intercepted part HF be equal to the excess Eh. This can be done by means of the conchoidal line — such as Nicomedes described, as is found in Pappus, Book 4, prop. 22, and in Eutocius on prop. 1 of Book 2 of Archimedes' *On the Sphere [and Cylinder]* — and such as we too described in our book *On the Dimensions of Magnitudes [de Dimensionibus magnitudinum]*. And since the straight line kF is greater than ki (because the angle kiF is obtuse), and ki is set equal to Bh, kF will also be greater than Bh; therefore, equal parts HF, [Eh] being taken away, with HG equal to BE, the whole GF will be equal to Bh (or AE). And so, [as was shown, the point G falls on the ellipse, and] the given point k falls outside the same, since the straight line FG cuts the ellipse in G.

Lastly, let there be a given point m, and let the straight line ml, equal to AE (or Bh), be applied cutting AE in n, so that mn be less than [BE]. For since ml is equal to Bh, and mn is less than BE, the remainder mH will be greater than the other

remainder Eh. Let there be drawn again, by means of the conchoidal line, from m the straight line mF, such that the intercepted part HF be equal to the excess Eh. [Margin: e — Euclid I.19.] And since the straight line mF is less than ml (because the angle mlF is acute, and mFl obtuse), and ml is set equal to Bh, mF will also be less than Bh; therefore, equal parts HE, Eh being taken away, the remainder mH will be less than the other remainder BE. FG being produced, then, so that HG be equal to BE, [the whole FG will be equal to Bh, or AE]. Therefore, as was shown, the point G falls on the ellipse, and accordingly m falls within the same; which was proposed.

Moreover, that the given point k falls outside the ellipse if ke be greater than ED, and the point m within if mn be less [than BE], can also be demonstrated by this method, without the aid of the conchoidal line, [namely] beyond D. Let EQ be taken equal to ke, and Q will fall [beyond D]; [and since] equal to AE, half the major axis; if EQ be set as half the minor [axis], [k will fall] on the ellipse described through A, Q, C, as was demonstrated. Therefore the ellipse described through A, D, C...

it will pass on the near side of point k; [Margin a: Apollonius, *Conics* IV.27] since this touches that one only at the points A and C, and therefore k will fall outside the ellipse described through A, D, C. Let EP again be taken equal to mn, and P will fall short of D. Since, then, ml — equal to AE, the semi-major axis — has been applied from m to the minor axis, if EP (which was taken equal to mn) be made the semi-minor axis, m will fall on the ellipse described through A, P, C, as was shown. Therefore the ellipse described through A, D, C will pass beyond the point m; [Margin b: Apollonius, *Conics* IV.27] since it touches that one only at the points A and C, and hence the given point m will fall inside the ellipse described through A, D, C — which is what was proposed.

Besides these things, it will be possible to gather how, given two unequal lines RS, TV and a point G, an ellipse could be described through the given point G, whose center is also given and whose axes are equal to the given lines RS, TV — if that can be done. [Margin: *Given two unequal lines and any point, to describe an ellipse through the given point, whose center too is given, and whose axes are equal to the given lines.*] Let RS and TV each be bisected, at X and Y; let XZ be taken equal to TY, the semi-minor; and let the excess RZ be bisected at α . Then, from the given point G to the given center E — where the axes must cut each other at right angles — let the line GE be drawn; and if it is less than RX and greater than ZX (or equal to one of them), what is proposed will be accomplished in this way. Since GE is less than RX and greater than ZX, of the three lines GE, X α , R α any two together

will be greater than the remaining one. For $X\alpha$ and $R\alpha$ are greater than GE ; likewise $R\alpha$ (or $Z\alpha$) and GE are greater than $X\alpha$; and finally GE and $X\alpha$ are greater than $R\alpha$, as is evident. [Margin c: Euclid I.22] Therefore from the three lines GE , $X\alpha$, $R\alpha$ let a triangle GEr be constructed, on either side. Let GH be taken equal to XZ , and, Gr being produced, let rF be taken equal to $r\alpha$, so that the whole GF is equal to RX . Now, lines being drawn through H , E and through F , E , let EA , EC be taken equal to XR , XS , and EB , ED equal to YT , YV . I say that AC , BD — which are equal to RS , TV — are axes cutting each other at right angles in E , in such a way that the ellipse described about them passes through the given point G .

For since Er is equal to $R\alpha$ (or $Z\alpha$), that is, equal to rH and rF — which are equal to $R\alpha$ (or $Z\alpha$) — (for rF was taken equal to $R\alpha$; but Gr equals $X\alpha$, and GH equals XZ , from which it follows that the remainder Hr is equal to the remainder $Z\alpha$): the circle described from center r through E will pass through the points F , H ; [Margin d: Euclid III.31] and hence the angle FEH , being in a semicircle, will be a right angle. Since, therefore, GF — equal to the semi-major axis AE — has been applied to the minor axis, and the segment GH is equal to ED , the semi-minor axis (that is, to TY); the point G will fall on the ellipse of axes AC , BD , as has been demonstrated.

But if the line GE , when drawn, is greater than the semi-major axis, or less than the semi-minor, the problem is rendered impossible: for since AE , the semi-major axis, is the greatest of all the lines drawn from center E to the circumference of the ellipse — as is clear from the circle described about the major axis AC — a line from center E greater than the semi-major axis will necessarily fall outside the ellipse. Likewise, since ED , the semi-minor axis, is the least of all the lines drawn from center E to the circumference of the ellipse — as is clear from the circle described about the minor axis BD — a line from center E less than the semi-minor axis will necessarily fall inside the ellipse.

Diagram. A square frame enclosing the reference figure for the ellipse lemmas: a large circle (the circle about the major axis) with an ellipse and a smaller circle (about the minor axis) inscribed, the two perpendicular diameters serving as the axes, together with numerous ordinate lines, dotted arcs, and construction segments. Beneath the frame lie two horizontal reference segments representing the given lines. The figure is very densely labelled, with many small Roman and Greek letters at ordinate feet that do not resolve at this resolution.

- A, with adjacent point d (top of the frame): upper extremity / top of the circle
- M, G, L (upper arc, left to right): points on the upper circle

- I (far left of the horizontal diameter)
- B (left, on the horizontal diameter)
- F, α (near the center, left of E)
- E (center)
- N (right of E)
- P, D (right of center on the diameter)
- Q, K (far right)
- C (bottom of the frame)
- Lower ordinate/foot labels inside the circle (e.g. ρ , κ , and several small Greek letters): [?] — not legible
- Below the frame, first segment: S — R, with interior marks z, α and midpoint x (the given major line RS, bisected at X)
- Below the frame, second segment: V — T (or I), with midpoint Y (the given minor line TV, bisected at Y)

[Numerous small interior labels are illegible at this resolution]

Now if it should happen that the line AE, drawn from the given point A to the center, is equal to the semi-major of the given line, the line AC must be drawn from the given point A through the center E. For EA, EC, made equal to XR, XS, will give the major axis; and if the line BD cut it at right angles, EB, ED, made equal to YT, YV, will give the minor axis. And it is manifest that the ellipse described about the axes AC, BD passes through the given point A. But if the given point be D — from which the line DE drawn to the center E is equal to the semi-minor of the given line — the line BD must be drawn from the given point D through the center E. For EB, ED, equal to YT, YV, will give the minor axis; and if the line AC cut it at right angles, EA, EC, equal to XR, XS, will give the major axis. Whereby it is again plainly evident that the ellipse described about the axes AC, BD passes through the given point D.

LEMMA LI.

If circles be described about the axes of an ellipse, and to those same axes straight lines be applied as ordinates as far as the peripheries of the ellipse and of the circles, then the ordinates drawn to the ellipse will be proportional to those applied to their own proper circle — namely, to whose diameter they are applied.

In the figure of the preceding lemma let the circles be described about the axes, and let the lines pq, tu, applied as ordinates to the major axis AC, cut the ellipse at f, β ; likewise let the lines Fe, ly, applied as ordinates to the minor axis BD, cut the

circle at θ , ϑ . I say that as pf is to $t\beta$, so is pq to tu ; likewise, that as $F\epsilon$ is to ly , so is $F\theta$ to $l\vartheta$.

[Margin e: Apollonius, *Conics* I.21] For since the square on pf is to the square on $t\beta$ as the rectangle under Ap , pC is to the rectangle under At , tC ; [Margin f: Euclid VI.17] and the rectangle under Ap , pC is equal to the square on pq , and the rectangle under At , tC equal to the square on tu (because, by the scholium to Euclid VI.13, pq and tu are mean proportionals between Ap , pC and between At , tC): it will also be that as the square on pf is to the square on $t\beta$, so is the square on pq to the square on tu . [Margin g: Euclid VI.22] Wherefore it will also be that as the line pf is to the line $t\beta$, so is the line pq to the line tu .

[Margin: *That ordinates are cut proportionally by the ellipse and by the circles described about the axes.*] Again, [Margin h: Apollonius, *Conics* I.21] since the square on $F\epsilon$ is to the square on ly as the rectangle under DF , FB is to the rectangle under Dl , lB ; [Margin i: Euclid VI.17] and the square on $F\theta$ is equal to the rectangle under DF , FB , and the square on $l\vartheta$ equal to the rectangle under Dl , lB (because, by the scholium to Euclid VI.13, $F\theta$ and $l\vartheta$ are mean proportionals between DF , FB and between Dl , lB): it will also be that as the square on $F\epsilon$ is to the square on ly , so is the square on $F\theta$ to the square on $l\vartheta$. [Margin k: Euclid VI.22] Wherefore it will also be that as the line $F\epsilon$ is to the line ly , so is the line $F\theta$ to the line $l\vartheta$ — which was to be demonstrated.

SCHOLIVM.

And so both the ellipse divides proportionally the lines applied as ordinates to the major axis and produced as far as the circle described about that same major axis, and the circle described about the minor axis divides proportionally the lines applied as ordinates to that same minor axis. For since as pf is to $t\beta$, so is pq to tu , it will also be, by alternation, that as pf is to pq , so is $t\beta$ to tu ; and, by the contrary division of ratio which we demonstrated in the scholium to Euclid V.17, as pf is to fq , so is $t\beta$ to βu . Likewise, since as $F\epsilon$ is to ly , so is $F\theta$ to $l\vartheta$, and, by the same division of ratio which we demonstrated in the scholium to Euclid V.17, as $F\theta$ is to $\theta\epsilon$, so is $l\vartheta$ to $\vartheta\gamma$ [?] — which is what was proposed.

We shall also easily demonstrate the converse of this, namely: If perpendiculars to the diameter of a circle be cut proportionally, then the ellipse whose major axis is the diameter of the circle, passing through the section of one perpendicular, will also pass through the sections of all the others. Likewise, if perpendiculars to the diameter of a circle be produced so as to be cut proportionally by the circle, then the ellipse whose minor axis is the diameter of the circle, passing through

the extremity of one perpendicular, will also pass through the extremities of all the others.

For let ML , EK , pq , tu first be perpendiculars to the diameter AC of the circle $ABCD$, and let them be cut proportionally at G , D , f , β . I say that the ellipse whose major axis is AC , which passes through G , also passes through D , f , β . For if it does not pass through D , let it pass through P or Q ; then, as we demonstrated, it will be that as MG is to GL , so is EP to PK (or EQ to QK). Since, then, it is also — by hypothesis — that as MG is to GL , so is ED to DK , it will be that as EP is to PK , so is ED to DK . But EP is less than ED ; [Margin a: Euclid V.14] therefore PK too will be less than DK , the whole less than the part — which is absurd. The ellipse, therefore, does not pass through P ; nor will it pass through Q . For by the same reasoning it will be that as EQ is to QK , so is ED to DK . But EQ is greater than ED ; [Margin b: Euclid V.14] therefore QK too will be greater than DK , the part greater than the whole — which is absurd. Therefore the ellipse passes through D . And for the same reason it will pass through f and β .

Diagram. The same reference figure repeated (a square frame with the major-axis circle, the inscribed ellipse, the minor-axis circle, the two perpendicular axes, ordinate lines and dotted arcs, and the two reference segments below). As on the preceding page, the interior is very densely labelled with small Roman and Greek letters at the ordinate feet.

- A , with adjacent d (top); M , G , L (upper arc)
- B (left), I (far left), F , α (left of center), E (center), N (right of center), P , D (right), Q , K (far right), C (bottom)
- Small interior ordinate labels (ς , ρ , κ , ξ , π , and further Greek letters): [?] — not legible
- Reference segments below the frame: S — R (marks z , α ; midpoint x) and V — T (midpoint Y)
- Right margin, keyed: [Margin a: Euclid V.14]; [Margin b: Euclid V.14]

[Numerous small interior labels are illegible at this resolution]

Next, let $F\mu$, $I\theta$, δ be perpendiculars to the diameter BD of the circle $B\mu D$, and, being produced to C , ϵ , γ , let them be cut proportionally by the circle at μ , θ , δ . I say that the ellipse whose minor axis is BD , which passes through C , also passes through ϵ , γ . For if it does not pass through ϵ , let it pass through π ; then, as was shown, it will be that as $E\mu$ is to μC , so is $F\theta$ to $\theta\pi$. But as $E\mu$ is to μC , so is supposed to be $F\theta$ to $\theta\epsilon$. Therefore it will be that as $F\theta$ is to $\theta\pi$, so is $F\theta$ to $\theta\epsilon$; [Margin c: Euclid V.9] and hence $\theta\pi$ and $\theta\epsilon$ will be equal, the part and the whole

— which is absurd. Therefore the ellipse passes through ϵ . And for the same reason it will pass through γ [?] — which is what was proposed.

LEMMA LII.

Given the axes of some ellipse cutting each other at right angles, to find, on any given straight line whatever, the points through which the ellipse, if it be described, must pass.

Let there be given the axes AC, BD of some ellipse, cutting each other at right angles in the center E, about which circles have been described; and let first the given line EF, drawn through the center, cut the circle described about the major axis at F; and through F let FO, FK be drawn parallel to the axes. Also, at the minor axis, from its extremity B, let the perpendicular BG be erected, cutting the circle of the major axis at G; and through G, from E, let a line be drawn cutting the parallel to the major axis at H. Then, on the parallel to the minor axis, let the line KL be taken equal to EH, and let EL be drawn, cutting the circle of the major axis at M — a point on either side; and finally, through M, let MN be drawn parallel to the minor axis, cutting the given line at I. I say that the ellipse whose axes are AC, BD, when described, passes through the point I.

[Margin: *When the given line passes through the center of the ellipse.*] [Margin d: Euclid VI.4] For since as EG is to EB, so is EH to EO; and EG is equal to EF, EH to KL, and EO to KF: it will also be that as EF is to EB, so is KL to KF; and, by the converse division of ratio which we demonstrated in the scholium to Euclid V.17, as EB is to BF, so is KF to FL. [Margin e: Euclid VI.4] But as KF is to FL, so is NI to IM. Therefore it will also be that as EB is to BF, so is NI to IM; and hence, from what we showed in the scholium to the preceding lemma, the ellipse described through A, B, C, D will pass through each point I.

Diagram. A block of small figures (the cases of Lemma LII) enclosed in a frame: circles each carrying two perpendicular axes with A at the top, C at the bottom, B at the left, D at the right, and center E, together with the construction lines and points of the several cases (the "first" and "second" figures here). Radiating chords and parallels bear point-labels F, G, H, I, K, L, M, N, O, P, Q, R, and small Greek letters.

- Axis endpoints (each sub-figure): A (top), C (bottom), B (left), D (right); E (center)
- Further construction labels legibly present include: F, H, K, L, M, N, O, P, G, R

- The many small interior labels (including Greek): [?] — not legible

[Interior construction labels largely illegible at this resolution]

Otherwise, as in the second figure. From B, the extremity of the minor axis, and from P, the extremity of the semidiameter, let perpendiculars BF, PH be erected to the line of the minor axis; and let BF cut the given line EF at F, and let PH be taken equal to BF. Now, the line EH being drawn and cutting the greater circle on either side at the point I, let IK be drawn through I parallel to the minor axis, cutting the given line at G. I say that G falls on the given ellipse. [Margin f: Euclid VI.4] For since as EP is to PH, so is IK to KE; and as BF — that is, as PH, its equal — is to EB, so is KE to KG; it will be, from equality of ratios, that as EP is to EB, so is IK to KG. Wherefore, as before, the point G will fall on the given ellipse on either side.

Otherwise, as in the third figure. To the major axis, from the points A, G, let perpendiculars AF, GH be erected; and let AF cut the given line at F, and from F let the perpendicular FO be dropped to the minor axis, cutting GH at H. Now, EH being drawn and cutting the circle of the minor axis on either side at the point I, let KL be drawn through I parallel to the major axis, cutting the given line at K. I say that K falls on the given ellipse. [Margin g: Euclid VI.4] For since as OH is to HF — that is, as EG is to GA — so is LI to IK, the point K will fall on the ellipse on either side, as was demonstrated in the scholium to the preceding lemma.

But it is enough if one point — namely the upper one — be found by one of these methods. For if the line EI (or EG, or EK) be taken equal below the center, the lower point F (or G, or K) will also be on the ellipse; [Margin a: Apollonius, *Conics* I.30] because a line drawn through the center is bisected at the center in an ellipse.

Diagram. A framed block of four small figures for the cases of Lemma LII (the "first," "second," "third," and "fourth" figures). Each is a pair of concentric circles (major-axis and minor-axis circles) with the two perpendicular axes A–C (vertical) and B–D (horizontal), center E, and the several sets of construction chords, parallels and feet distinguishing the cases. The figures are crowded with small Roman and Greek point-labels.

- Axis endpoints (each sub-figure): A (top), C (bottom), B (left), D (right); E (center)
- Further labels legibly present across the four figures: F, G, H, I, K, L, M, N, O, P, Q, R, S, T, V, X, Y
- Many small interior labels (including Greek ϵ , γ , ζ): [?] — not legible

[Interior construction labels largely illegible at this resolution]

Next, let the given line be parallel to one of the axes, as in the fourth figure; and first parallel to the major axis, the line FG, cutting the minor axis at M and its circle at H. For if it did not cut it, it would fall wholly outside the ellipse; but if it passed through B, [Margin b: Apollonius, *Conics* I.32] it would touch the ellipse at B. [Margin: *When the given line is parallel to one of the axes.*] The line EH being drawn and cutting the greater circle at I, let IK be drawn through I parallel to the minor axis, cutting the given line FG at L. I say that L falls on the given ellipse. [Margin c: Euclid VI.2] For since as EH is to HI — that is, as EB is to BM — so is KL to LI; [Margin d: Euclid VI.4] or, as EH is to HI — that is, as EO is to OA — so is MH to HL; L will fall on the ellipse, as was demonstrated in the scholium to the preceding lemma.

Secondly, let the line IL be parallel to the minor axis, cutting the greater circle at I, whether it cut the lesser or not. The line EI being drawn and cutting the lesser circle at H, let LM be drawn through H parallel to the major axis, cutting the given line IL at L. I say that L lies on the given ellipse. This will be demonstrated as before. Now if, on the other side, a segment ML (or KL) be cut off equal to ML (or KL), the same ellipse will pass through the point L as well — the lower and the right-hand one — because the ordinates are bisected by the diameters.

Again, let the given line DL pass through the extremity D of the minor axis, as in the fourth figure, and let it first cut the major axis, inside the ellipse, at S. [Margin: *When the given line passes through the extremity of the other axis.*] From S let the line SP be drawn to the extremity of the diameter of the major circle which lies next to the given extremity D; and through D let TV be drawn parallel to the minor axis, cutting the given line at L. Since as ED is to DP, so is VL to LT, it will follow, by the scholium to the preceding lemma [that L is on the ellipse]. The same will be demonstrated if the given line DQ, passing through the extremity D of the minor axis, cut the major axis outside the ellipse at Q, as in the same fourth figure. For, the line QP being drawn from Q to P, the extremity of the diameter of the major circle near the extremity D, and cutting the major circle at R, then — the line Ra being drawn parallel to the minor axis — it will cut the given line at the point b, which will be on the ellipse; [Margin e: Euclid VI.4] since as ED is to DP, so is ab to bR.

But now let the given line AX pass through the extremity of the major axis, and let it first cut the minor axis outside the ellipse at X, as in the third figure. From the point X let the line XG be drawn to G, the extremity of the minor diameter near the given extremity A, cutting the minor circle at Z; and through Z let eY be

drawn parallel to the major axis, cutting the given line at Y. I say that Y falls on the ellipse — which is clear from the scholium to the preceding lemma; [Margin f: Euclid VI.4] since as EG is to GA, so is eZ to ZY.

We shall proceed in no other way if the given line Ag, passing through the extremity A of the major axis, cut the minor axis inside the ellipse at f, as in the second figure. For, the line fk being drawn from f to k, the extremity of the diameter of the minor circle near the given extremity A, and cutting the minor circle at i, the line dg drawn through i parallel to the major axis will cut the given line at the point g, which will be on the ellipse; [Margin g: Euclid VI.4] since as Ek is to kA, so is di to ig.

It is evident, however, that in a line of this kind only one point is found that lies on the ellipse; for such a line cuts the ellipse also at the extremity D of the minor axis, or at A, the extremity of the major axis. It is likewise plainly evident that a line drawn through the extremity of the minor axis and through the extremity of the major axis has, besides those two extremities, no other point on the ellipse.

Finally, let the given line FG be drawn neither through the center of the ellipse, nor through the extremity of either axis, nor parallel to any axis; and let it cut the major axis at H, whether inside the ellipse, as in the former figure, or outside, as in the latter. [Margin: *When the given line neither passes through the center, nor through the extremity of the other axis, nor is parallel to any axis.*] Through any point I taken on the given line, let IO, RN be drawn parallel each to an axis; and, from B, the extremity of the minor axis, a perpendicular BK being erected and cutting the greater circle at K, let EK be joined, cutting the parallel IO at L. Then, on the other parallel RN, let RN be taken equal to the line EL; and through H, N let a line be projected cutting the circle of the major axis at M; and finally, through M, let MQ be drawn parallel to the minor axis, cutting the given line at P. I say that the point P lies on the given ellipse.

And indeed, if the line HN cut the circle at two points, two points P will be found, as in the former figure; but if it touch it at one point, as in the latter figure, only one point P will likewise be found, at which the ellipse will touch the given line. In order that the demonstration may be rendered more universal, we assumed in the former figure three points I on the given line, and in the latter two, through which parallels to each axis have been drawn — especially because in this way we have no need of the point H outside the ellipse in the second figure, which sometimes can be obtained only with difficulty, on account of the oblique intersection of the lines HG, HG.

Diagram. A large framed figure for the final ("neither through center nor through an axis-extremity, nor parallel to an axis") case of Lemma LII: two large overlapping circles, each with its pair of perpendicular axes and center, joined by a web of oblique construction lines, parallels, and dotted arcs. The figure is exceptionally dense with small point-labels.

- Left circle: A (top), C (bottom); axis-endpoint labels B, D and center E present; further labels K, G, L, M, N, O, T, and the oblique-line endpoints H, F, S
- Right circle: A (top); center E, with B, D, O, Y about it; further labels K, L, M, N, R, P, T, S, G, and Greek letters
- Many small interior labels (Roman and Greek): [?] — not legible

[Interior construction labels largely illegible at this resolution]

But it is enough that through the two points N found, a line be drawn cutting or touching the circle of the major axis. All of which we shall demonstrate thus. [Margin i: Euclid VI.4] Since as EK is to EB, so is EL to EO; and EL was posited equal to RN, [Margin k: Euclid I.34] and EO is equal to RI; it will also be that as EK is to EB, so is RN to RI. [Margin l: Euclid VI.4] But as RN is to RI, so is QM to QP. Therefore it will be...

[ASTROLABIUM, Liber I — Lemma LII, continued]

it will also follow that as EK — that is, as Ea — is to AB, so is QM to QP. And by conversion of the divided ratio [per divisionem rationis conversam], as EB is to Ba, so is QP to PM. Consequently P falls upon the ellipse, by the scholium of the preceding lemma. And this demonstration holds for each point P of the earlier figure lying to the left of the major axis.

Furthermore, we shall make it evident as follows that the given straight line FG touches the ellipse at the found point P, when the straight line HN [?] touches the circle at M. ^aSince the angle HME is right and MQ is perpendicular to HE, then by the corollary of Euclid VI.8, EM is the mean proportional between HE and EQ.

^bTherefore the square on EM (or on EA) is equal to the rectangle under HE, EQ; and so as HE is to EA, so is EA to EQ. Hence, by conversion of the ratio, as HE is to HA, so is EA to AQ. Now since CH, HA are double HE, and CQ, QA double AE, ^cit follows also that as the sum of CH and HA is to HA, so the sum of CQ and QA is to AQ; and, dividing, as CH is to HA, so is CQ to AQ. ^dTherefore HG will touch the ellipse at the point P, which we proved to lie on the ellipse.

[Margin: ^a Euclid III.18. ^b VI.17. ^c V.15. ^d 24.1. Apollonii (Apollonius, *Conics* I.24) [?]]

Another way. Erect BK perpendicular to BD at B, the extremity of the minor axis, and join the straight line EK. From any point I draw IO parallel to the major axis, cutting EK at L. (In each figure we have taken two points I, for the reason mentioned a little earlier.) Then from I erect IS perpendicular to the given line, equal to OL; and through H, S draw the straight line HS, cutting the circle described about the chord FG at the points T, V. From these drop perpendiculars TP, VP to the given line. I say that each point P lies on the given ellipse. But if the line HS is tangent to the circle described about FG, as in the latter figure, only one point P will be found, at which the given line touches the ellipse.

All this we shall demonstrate in the following way; and first, concerning the point P on the left of the major axis in the earlier figure. Through P draw XY parallel to the major axis, and MP Qe [?] parallel to the minor axis. ^eSince as PT is to IS, so is HP to HI, and as HP is to HI, so is QP to RI, it follows also that as PT is to IS, so is QP to RI, that is, so is EY to EO. ^fBut as EY is to EO, so is YX to OL. Therefore, since IS and OL are equal by hypothesis, ^gPT and YX will likewise be equal. And since PT, by the scholium of Euclid VI.13, is the mean proportional between FP, PG, ^hthe square on PT will be equal to the rectangle under FP, PG — that is, to the rectangle under MP, Pe, ⁱsince the latter is equal to it. Adding the common square on PQ, the squares on YX and PQ (that is, on YX and EY) will be equal to the rectangle under MP, Pe together with the square on PQ. ^kBut the square on EX is equal to the squares on YX, EY; ^land the rectangle under MP, Pe together with the square on PQ is equal to the square on MQ. Therefore the squares on EX and MQ are equal, and so their sides EX, MQ are equal. ^mThen, since EY and QP are also equal, as EX is to EY, so is QM to QP. ⁿBut as EX is to EY, so is EK — that is, Ea — to EB. Therefore it will also follow that as Ea is to EB, so is QM to QP; so that, as before, the point P falls upon the given ellipse. This demonstration also holds in the latter figure.

[Margin: ^e Euclid VI.4. ^f VI.4. ^g V.14. ^h VI.17. ⁱ III.35. ^k I.47. ^l II.5. ^m I.34. ⁿ VI.4.]

That the point P to the right of the major axis likewise falls upon the same ellipse will be made plain thus. Draw Pb perpendicular to MQ and equal to PV, and join the straight line bQ. ^oSince, in the lower triangle HPQ, as QP is to PH, so, in the upper triangle, QP is to PH; and likewise as PH is to PT, so is PH to PV — it will follow, from equality of ratios, that as QP is to PT (that is, as EY is to YX, which are equal to them), so is QP to PV, that is, to Pb. Then, since the angles at Y and P are right, ^pthe triangles EYX, bPQ are equiangular, and as EX is to EY, so is bQ to QP. Next, since by the scholium of Euclid VI.13 VP — and therefore also bP — is the mean proportional between FP, PG, ^qthe square on bP will be equal to the

rectangle under FP, PG; ^rbut this is equal to the rectangle under MP, Pe, because the lines FG, Me intersect one another at P in the circle of the major axis. Therefore the square on bP will also be equal to the rectangle under MP, Pe; and, adding the common square on QP, the squares on bP and QP — that is, the square on bQ, ^swhich is equal to them — will be equal to the rectangle under MP, Pe together with the square on QP. ^tBut the rectangle under MP, Pe together with the square on QP is equal to the square on QM. Therefore the square on bQ will also be equal to the square on QM, and so the lines bQ, QM will be equal. Hence, since it was shown just above that as EX is to EY, so is bQ to QP, it will also follow that as EX is to EY, so is QM to QP. ^uThen, since as EX is to EY, so is EK (or Ea) to EB, it will also follow that as Ea is to EB, so is QM to QP; and accordingly, as before, the point P falls upon the given ellipse.

[Margin: ^o Euclid VI.4. ^p VI.6. ^q VI.17. ^r III.35. ^s I.47. ^t II.5 [printed "5. sexti"]. ^u VI.4.]

Finally, we shall demonstrate as follows that the given line FG touches the ellipse at the point P found when the line HS touches the circle FT at T. Draw the straight lines HM, EM to the extreme point of the parallel QM. Since it was shown that as Ea (that is, EK) is to EB, so is QM to QP, ^xand as EK is to EB, so is EX to EY, it will also follow that as EX is to EY, so is QM to QP. ^yThen, since EY is equal to QP, EX will also be equal to QM. And since the square on PT is equal to the square on YX — for the lines PT, YX were shown to be equal — if the equal squares on PQ, EY be added, two squares (on PT, PQ) will arise equal to the two squares on YX, EY; ^zbut equal to these is the square on EX, that is, on QM. Therefore the two squares on PT, PQ will also be equal to the square on QM; and, adding the common square on QH, three squares (on PT, PQ, QH) will arise equal to the two squares on QM, QH; ^abut to the squares on PQ, QH the square on PH is equal. Therefore the two squares on PT, PH will be equal to the two squares on QM, QH. ^bThen, since the square on HT is equal to the former two, and the square on HM to the latter two, the squares on HT, HM will likewise be equal, and hence their sides also. ^cTherefore, since the square on HT is equal to the rectangle under HG, HF, the square on HM will also be equal to that same rectangle, ^dand consequently HM will touch the circle FM; and, as was demonstrated before, the line FG will touch the ellipse at P — which was what was proposed.

[Margin: ^x Euclid VI.4. ^y I.34. ^z I.47. ^a I.47. ^b I.47. ^c III.36. ^d III.37.]

Diagram. A large square-framed woodcut occupying the center-right and lower portion of the page, illustrating the tangent construction of Lemma LII. It shows the ellipse together with its auxiliary circles (about the major axis and about the chord FG), with the "prior" configuration at the left and the "posterior" (tangent)

configuration at the right, both densely subdivided by radii, ordinates, and the tangent lines HS/HG. The lettering is very small and the leaf is heavily foxed; more than two labels cannot be read with confidence, so the figure

Readable labels (position approximate):

- A, F, M, H (upper region)
- S, T, V (upper left / arc)
- B, E, Q, P, L, K, R, N, I, X, Y (interior grid)
- G, C, D (lower and lower-right)
- "BY", O, o, a (lower left) [?]
- right-hand cluster: H, S, R, A, I, N, E, X, D, M, Q, P, G [several uncertain]

LEMMA LIII.

To carry out — by prosthaphaeresis alone, that is, by addition and subtraction alone, without the laborious multiplication and division of numbers — all the problems that are customarily solved by sines, tangents, and secants.

Three or four years ago Nicolaus Raymarus Ursus Dithmarsus [Nicolaus Reimers Ursus] published a certain little book in which, among other things, he sets out a truly keen and ingenious discovery, by which he solves most spherical triangles through prosthaphaeresis alone. But since he thinks this can be done only when sines are used in the rule of proportions [regula proportionum] and the whole sine [sinus totus] holds the first place, we for our part shall try to make this teaching more general: so that not only does the whole sine hold the first place in the rule of proportions, but it may also occur among the sines, and among tangents, secants, versed sines, and other numbers; and whether the whole sine be at the beginning of the rule of proportions, or in the middle, or finally does not occur at all. This is an entirely new matter, and full of pleasantness and delight.

[Margin: When the whole sine holds the first place in the rule of proportions and the other numbers are sines; how the prosthaphaeresis is to be done.]

1. Therefore, whenever it holds that as the whole sine is to the sine of some arc, so is the sine of some other arc to a fourth quantity, set those two arcs apart as given, which are required for the prosthaphaeresis. Let the lesser be added to the complement of the greater, and let the sine of the arc so composed be kept.

- If the lesser arc is equal to the complement of the greater (which happens when the two arcs set apart and given make up a quadrant), then half of the kept sine will be the fourth proportional sought.

- But if the lesser arc is less than the complement of the greater (which occurs when the two arcs set apart and given are together less than a quadrant), then, subtracting the lesser arc from the complement of the greater so as to obtain the difference of the arcs, let the sine of this difference be removed from the kept sine of the arc composed above; for half of the number thus left will be the fourth proportional sought.
- If, finally, the lesser arc is greater than the complement of the greater (which happens when the two arcs set apart and given are together greater than a quadrant), then, subtracting the complement of the greater from the lesser arc so as to obtain the difference of the arcs that were added together, let the sine of this difference be added to the kept sine of the arc composed above; for half of this sum will be the fourth proportional desired.

And this is the rule of the aforesaid author, which will be demonstrated as follows. ^aIn the first of these figures it holds that as the whole sine EG is to GK, the sine of the arc GD, so is Ei, the sine of the arc ID (or HM), to the sought sine iL. And since the lesser arc GD is equal to DG, the complement of the greater arc ID (or, if perhaps GD were the greater and ID the lesser, ^bID is equal to DI, the complement of the greater arc GD), ^cit comes about that PQ — which is half the sine MP of the arc MD, composed of DG, the lesser arc, and GM, the complement of the greater HM — is equal to the sought fourth sine iL. And if perhaps the arc GD be the greater and ID the lesser, MP will nonetheless be the sine of the arc MB, then composed of HM, the lesser, and HB, the complement of the greater GD.

[Margin: ^a Euclid VI.4. ^b VI.2. ^c I.34.]

^dIn the second and third figures likewise it holds that as the whole sine EG is to GK, the sine of the arc GD, so is Ei, the sine of the arc IN (or HM), to the sought sine iL. And since, in the second figure, the lesser arc GD is less than GN, the complement of the greater arc IN (or, if perhaps GD were the greater and IN the lesser, IN is less than ID, the complement of the greater arc GD), it comes about that — the sine RP of the difference DN being subtracted (that is, Me, equal to RP, being taken away from MP, the sine of the arc MD composed of DG, the lesser arc, and GM, the complement of the greater HM) — the line PQ, which is half of what remains, iP, ^esince the whole QR is half of the whole MR, ^fis equal to the sought sine iL. And if perhaps the arc GD be the greater and IN the lesser, MP will nonetheless be the sine of the arc MB, composed of the then-lesser arc MH and HB, the complement of the greater arc GD.

But in the third figure, since the lesser arc IN is greater than ID, the complement of the greater arc GD (or, if perhaps GD were the lesser and IN the greater, the lesser GD exceeds GN, the complement of the greater arc IN), it comes about that — the sine RP of the difference DN being added (that is, Me, equal to RP, being added to MP, the sine of the arc MB composed of the lesser arc HM and HB, the complement of the greater) — ^gthe line PQ, which is half of the whole composed line iP, since QR is half of MR, ^his equal to the sought sine iL. And if perhaps the arc GD be the lesser and IN the greater, MP will nonetheless be the sine of the arc MD, then composed of the lesser arc GD and GM, the complement of the greater HM.

But if the two arcs set apart be equal, the complement of either is to be taken, and the other assumed as the lesser.

[Margin: ^d Euclid VI.4. ^e VI.2. ^f I.34. ^g VI.2. ^h I.34.]

Diagram. A square-framed woodcut showing two adjacent circles (the "first figure" at left and the "second figure" at right; the text also refers to a "third figure"), each divided into octants by diameters and further subdivided by chords, ordinates, and dashed construction lines that carry out the prosthaphaeresis of Lemma LIII. The lettering is small and the leaf foxed; more than two labels are uncertain, so the figure

Readable labels:

- Left circle: A, M, H (top), S, e, Q, G, B (upper right), E (center), P, L, K (below center), D, R, T, I (lower), F, C (foot)
- Right circle: E, M, A (top), G, Q, S, R, N, P, E, L, K, o, D (right), T, I (lower), F, C (foot), H

2. Now, when the whole sine holds the first place in the rule of proportions but the other two numbers are not sines, arcs corresponding to those numbers — as if to sines — must be taken from the table of sines and set apart separately. Then the aforesaid rule is applied. The same must be done when the sine of the complement of some arc is used; but in its place must be taken the arc that corresponds to that sine insofar as it is a right (proper) sine. Finally, whenever the second and third numbers are not sines, or one of them is a sine and the other not, an arc is taken corresponding to each number as if to a sine — as, for instance, when a number is greater than the whole sine, let so many figures be cast off from the right side as suffice for the remaining number to become smaller than the whole sine; and to the fourth number found by prosthaphaeresis — whether it be a sine, a tangent, a secant, or some other number — let there be

added on the right side as many ciphers [Ziphrae, zeros] as figures were cast off. For when one figure is cast off, only the tenth part of the fourth number is thereby found; when two, the hundredth part. Therefore that part found must be multiplied by 10 or by 100, which is done by appending a 0 or 00, so that the whole number is obtained. But let us make this whole matter clearer by a few examples.

[Margin: When the whole sine holds the first place in the rule of proportions and the other numbers are not sines, or partly sines and partly other numbers; how the prosthaphaeresis is to be done.]

For example, let the declination of $17^{\circ}45'$ of Gemini [Π] be investigated. Since — as we demonstrated in lemma 18 — it holds that as the whole sine is to the sine of the maximum declination, so is the sine of the distance of the given point of the Ecliptic from the nearer equinoctial point to the sine of the declination of that same given point, the example for prosthaphaeresis will stand as follows:

	G	M		G	M
	.	.		.	.
Arcus max. decl.	2	3	Compl. maioris	1	1
	3	0		2	5
Distantia ab aequin.	7	4	Minor	2	3
	7	5		3	0
Summa compl. & minoris				3	4
				5	5
Diff. inter compl. & minorem				1	1
				1	5

Note (right): "The lesser number is greater than the complement; therefore addition will be done" [Minor numerus maior est quam compl. ideo fiet additio].

- sinus (of $35^{\circ}45'$): 5842497
- sinus (of $11^{\circ}15'$): 1950903
- Summa sinuum [sum of the sines]: 7793400
- Semissis, vel sin. declin. [the half, or sine of the declination]: 3896700

To the sine found, 3896700, corresponds the declination $22^{\circ}56'$.

Again, let the ascensional difference of 6° Gemini be sought, for the pole altitude of 42° . Since — as we demonstrated in lemma 49, no. 17 — it holds that as the whole sine is to the tangent of the declination, so is the tangent of the pole

altitude to the sine of the ascensional difference, we shall proceed thus. The declination of 6° Gemini is 21°22'; its tangent is 3912247. And the tangent of 42°, the pole altitude, is 9004040. To the former tangent there correspond, in the table of sines, 23°2'; to the latter, 64°13'; and these two arcs are to be taken as the given quantities, in place of the declination and the pole altitude. So the example will stand thus:

	G	M		G	M
	.	.		.	.
Arcus dati	2	2	Compl. maioris	2	4
	3			5	7
	6	1	Minor	2	2
	4	3		3	
Summa compl. & minoris				4	4
				8	9
Diff. inter compl. & minorem				2	4
					5

Note (right): "The lesser number is less than the complement; therefore subtraction will be done" [Minor numerus minor est complemento, ideo fiet subtractio].

- Sinus (of 48°49'): 7526065
- Sinus (of 2°45'): 479781
- Relictum [remainder]: 7046284
- Semissis, vel sinus diff. ascens. [the half, or sine of the ascensional difference]: 3523142

To the sine found, 3523142, corresponds the ascensional difference 20°38', that is, 1h 23m. Therefore, with 6 hours added, the semidiurnal arc will contain 7h 23m. And the same difference, subtracted from the right ascension 64°6' (which is due to 6° Gemini), leaves the oblique ascension 43°28'.

Again, let the ascensional difference of 6° Gemini be investigated, for the pole elevation of 60°. The tangent of the declination is, as before, 3912247, to which in the sines correspond 23°2'. But the tangent of 60°, the pole altitude, is 17320508, to which in the sines — the last figure, 8, being cast off, in place of which 1 may be added to the remaining number, since 8/10 exceed 1/2 — correspond 9°58'. So the example will stand thus:

	G	M		G	M
	.	.		.	.
Arcus dati	2	2	Compl. maioris	6	5
	3			6	8
	9	5	Minor	9	5
		8			8
Summa compl. & minoris				7	5
				6	6
Diff. inter compl. & minorem				5	0
				7	

Note (right): "The lesser number is less than the complement; therefore subtraction will be done" [Minor numerus minor est complemento, ideo fiet subtractio].

- Sinus (of 76°56'): 9741076
- Sinus (of 57°0'): 8386706
- Relictum [remainder]: 1354370
- Semissis, vel sinus diff. ascens. [the half, or sine of the ascensional difference]: 677185

To the sine found, 6771850 (for, because of the figure 8 cast off, a zero must be added), corresponds the ascensional difference 42°38', that is, 2h 51m. And the same difference, subtracted from the right ascension 64°6' (which is due to 6° Gemini), leaves the oblique ascension 21°28'.

Furthermore, let the altitude of the Sun at the beginning of Cancer [♋] be explored, at the 4th hour after noon (or the 8th hour after midnight), for the pole elevation of 42°. Since, as we demonstrated in book 1 of the *Gnomonices*, proposition 36, it holds that as the whole sine is to the versed sine of the distance...

the Sun's distance from the meridian, so is half of the line composed from the sine of the meridian altitude and the sine of the meridian depression to the difference between the sine of the meridian altitude and the sine of the sought altitude — and thus we shall proceed. The versed sine [sinus versus] of the Sun's distance from the meridian is 5000000, to which in the sines correspond 30° 0'. The sine of the meridian altitude, 71° 30', is 9483237. The sine of the depression, 24° 30', is 4146932. Half the sum of these is 6815084½, to which in the sines correspond 42° 58'. Thus the example will stand as follows.

	G.	M.		G	M
				.	.
Given arcs	30.	0.	Compl. of the greater.	47.	2.
	42.	58.	Minor.	30.	0.

The lesser number is smaller than the complement, therefore subtraction is to be made.

	G.	M		
		.		
Sum of compl. & of the minor	77.	2.	Sine.	9745008.
Diff. between compl. & minor	17.	2.	Sine.	2929280.

Remainder [Relictum]	6815728.

Half, or the difference between the sine of the meridian altitude & the sine of the sought altitude	3407864.

Once the number found, 3407864 — which is the difference between the sine of the meridian altitude and the sine of the sought meridian altitude, 9483237 — is subtracted, there remains the sine of the sought altitude, 6075373, to which correspond 37° 25'. So great is the altitude of the Sun.

3. When the whole sine [sinus totus] stands to some number smaller than the whole sine as a number larger than the whole sine stands to another, the operation set out can also be carried out in this way. Let this third, larger number be divided by the whole sine, and the quotient will be the [multiplier] number; and if seven figures are cast off from the right, the seven figures cast off will give the remainder of the division. Let it therefore be arranged that as the whole sine is to the given smaller number, so the remainder of the division is to another — which is done by prosthaphaeresis, if for the smaller number and the remainder of the division the arcs are taken from the table of sines as though they were sines, and so on. To the fourth number thus found, let the smaller given number, multiplied by the quotient of the division above, be added, so that the whole fourth number sought may come out.

[Margin: When the whole sine is at the beginning of the rule of three [regula aurea], but either the third or the second number is smaller than the whole sine — how the prosthaphaeresis is to be done otherwise in that case.]

For example: let the ascensional difference [differentia ascensionalis] be found [of the point whose governing token in the printed line is not clearly legible — it reads as either "grad. 6. 11" or the sign $\mathfrak{S}$ [?]], for the elevation of the pole 50° . Since it holds that as the whole sine is to 3912247, the tangent of the declination, so 11917537, the tangent of the given elevation of the pole, is to the sine of the ascensional difference: you see that the second number is smaller than the whole sine, but the third larger; and this third divided by 10000000, the whole sine, gives quotient 1 and remainder 1917537. With that smaller number, then, and with this remainder, take from the table of sines these arcs: $23^\circ 2'$ and $11^\circ 3'$. Thus the example will stand as follows.

	M			M	
	G.	.		G.	.
Given arcs	23.	2.	Compl. of the greater.	66.	58.
	11.	3.	Minor.	11.	3.

The lesser number is smaller than the complement, therefore subtraction will have to be made.

	G.	M.		
Sum of compl. & of the minor number	78.	1.	Sine	9782080.
Diff. between compl. & the minor number	55.	55.	Sine	8282234.

Remainder [Relictum]	1499846.
Half, or the fourth number found	749923.

If to this half the smaller number 3912247 is added once — once, because the quotient above was 1 — the sine of the ascensional difference, 4662170, is produced, to which is owed the arc of the ascensional difference of $27^\circ 47'$, that is, 1 hour 51 minutes. Adding therefore 6 hours, the semidiurnal arc becomes 7 hours 51 minutes. And that same difference, subtracted from the right ascension of $64^\circ 6'$, leaves the oblique ascension of $36^\circ 19'$, for the elevation of the pole 50° .

The demonstration of this rule is drawn from the figures above, in this way. For, with the whole sine set as Ei : since it holds that as Ei , the whole sine, is to iL , the smaller number, so EG , the larger number, is to GK ; then if from EG the whole sine Ei is taken away, the residue GI will be the number to which, if the smaller iL (or IK) is added, the whole fourth number sought, GK , is composed. And if the whole sine Ei had been taken away several times, so that iG were left as the smaller [residue], then — as many times as the whole sine had been cast off — since to each whole sine there corresponds a straight line equal to iL itself, just as iL corresponds to the whole sine Ei .

The same reasoning holds when the second number is larger than the whole sine. For if it holds that as the whole sine is to the larger number, so the smaller number is to the fourth, then it will also hold, by permutation, that as the whole sine is to the smaller, so the larger is to the fourth; and thus again the larger will occupy the third place in the rule.

But when both numbers are larger than the whole sine, the superior rule explained under No. 2 must be kept — that is, one or another figure is to be cast off from the right in each, so that smaller numbers may be had; yet to the fourth number found, as many ciphers must be appended as there were figures cast off, as we said above under No. 2.

And in this way prosthaphaeresis is performed with the whole sine holding the first place in the proportion of the rule. Let us now teach how the same prosthaphaeresis is to be set up when the whole sine is placed in the second or third position of the said rule. So we shall proceed thus.

[Margin: When the whole sine occupies the second or third place of the rule of three, how the prosthaphaeresis is to be done.]

4. When the first number is larger than the second or the third, yet smaller than the whole sine, let it be arranged that as the whole sine is to the secant of the complement of the arc which, in the table of sines, corresponds to the larger number as though to a sine, so the smaller number is to another: that is, the two arcs which are owed to that secant and to the smaller number in the table of sines are to be set aside as given, and the rest carried out as was said in the prosthaphaeresis. But if the first, larger number is also larger than the whole sine, one must proceed as we shall state a little below under No. 6.

[Margin: When the first number is larger, but smaller than the whole sine.]

5. When, however, the first number is smaller, and smaller than the whole sine, then — provided the larger is smaller than the whole sine — let it be arranged that as the whole sine is to the secant of the complement of the arc which corresponds, as a sine, to the smaller number in the table of sines, so the larger number is to another: that is, the two arcs which correspond to that secant and to the larger number in the sines are to be set aside as given, and the rest carried out — which we prescribed in the rule of prosthaphaeresis under Nos. 1 and 2. But if

[Margin: When the first number is smaller, and also smaller than the whole sine.]

the larger number is indeed larger than the whole sine, let the smaller be subtracted from it several times, until the residual number is smaller than the whole sine; or, if you prefer, subtract the smaller as often as can be done. And let it be arranged again that as the whole sine is to the secant of the complement of the arc which corresponds, as a sine, to the smaller given number, so the residue of the larger is to another, as was said; and once the fourth number is found, let the whole sine be added to it as many times as the smaller number was taken away from the larger, so that the whole fourth number sought may be completed.

6. This twofold precept will be demonstrated from the same figures above, in this way. Since, if it holds that as GK is to EG, the whole sine, so the smaller number iL is to Ei, it will hold that as GK, the whole sine, is to EG, the secant of angle G — which is the complement of angle E, of which GK is the sine (for, with the whole sine set as GK, GE will be the secant of angle G, and EK the tangent, as we said in the treatise on Tangents and Secants [*Tabulae Sinuum, Tangentium et Secantium*]) — so iL is to Ei. And in this way the first precept has been demonstrated, provided that the first, larger number is smaller than the whole sine, so that through it, as through a sine, and hence its complement G, angle E can be taken from the table of sines, and consequently obtained.

For if the first, larger number is larger than the whole sine, its tenth part, or hundredth part, and so on, must be taken — which is done by casting off one figure at the right, or two, and so on; but from the number thus found, its tenth part, or hundredth, and so on, must then in turn be taken for the fourth number sought — unless perhaps that same tenth or hundredth part, and so on, of the smaller number was taken. For then the number found would be the fourth sought; because whatever part of the first number stands to the second, the same part of the third stands to the fourth. From which it follows that, if from the third number — that is, from the smaller — a tenth or hundredth part, and so on, has not been taken, the

number found is ten times, a hundred times, and so on, larger than it ought to be; and therefore its tenth part, hundredth part, and so on, is to be taken for the fourth number, as we said.

7. Next, if it holds that as iL is to Ei , the whole sine (with the whole sine set as EI), so the larger number GK is to EG : it will hold that as iL , the whole sine, is to Ei , the secant of angle i — which is the complement of angle E , which the smaller number iL , as a sine, presents — so GK is to EG . If therefore the larger number GK is smaller than the whole sine Ei , so that through it, as though a sine, the corresponding arc can be taken from the table of sines, the matter stands rightly. But if GK is larger than the whole sine Ei , as in the third figure, the smaller iL must be taken away from it once, twice, three times, and so on, until a number GI is left smaller than the whole sine; and to the number GI found, the whole sine Ei must be added as many times as iL was subtracted from GK , so that the whole fourth number sought, EG , may be composed.

If the first, smaller number is also larger than the whole sine, several of the last figures must be removed from both the first and the other, so that the numbers are left smaller than the whole sine; and if indeed the residue of the larger number is smaller than the residue of the smaller first number, let the rule explained under No. 4 be observed; but if larger, the first part of the rule set out under No. 5. To the fourth number then found in that way, as many ciphers must be appended as there were figures removed from the larger number; because on account of one figure removed, only its tenth part is found, and on account of two, its hundredth part, and so on. Hence, by the appending of an "o," or of "oo," and so on, the number found must be multiplied by 10, or by 100, and so on, so that the whole fourth number may come out. But from this, in turn, as many ciphers must be removed as there were figures removed from the smaller number, which holds the first place in the rule; because on account of one figure removed a number ten times larger is found, on account of two a hundred times, and so on — for the reason that the division is carried out by a number ten times or a hundred times, and so on, smaller. Therefore, by the removal of an "o," or of "oo," and so on, the number must be divided by 10, or by 100, and so on, so that the true fourth number may be had. But if from the beginning as many figures have been removed from the smaller first number as from the given larger, then to the fourth found in the first place nothing is either to be added or taken away from it.

For example: let the rising amplitude [latitudo ortiva] of the beginning of Cancer [♋] be investigated, for an elevation of the pole of 42° . Since, then, it holds that as

the sine of the complement of the elevation of the pole, 7431448, is to the sine of the declination of the point of the Ecliptic, 3987491, so the whole sine is to the sine of the rising amplitude — as we demonstrated in Book I of the *Gnomonics*, proposition 34 — we shall proceed thus. Since the first number is larger than the second, yet smaller than the whole sine, we shall take from the table of sines the arc of 48° corresponding to the larger number — that is, the complement of the elevation of the pole itself — and the secant of the complement of this arc, 13456326, to which (once the last figure, 6, is cast off) there corresponds in the table of sines the arc of $7^\circ 44'$. And to the smaller number, 3987491, corresponds the declination of $23^\circ 30'$. So the example will stand as follows.

	G.	M.		G.	M
				.	
Given arcs	7.	44.	Compl. of the greater.	66.	30.
	23.	30.	Minor.	7.	44.

The lesser number is smaller than the complement, therefore subtraction will be made.

	G.	M	
	.		
Sum of compl. & of the minor	7 1	Sin	9623762.
	4. 4.	e	
Diff. between compl. & minor	5 4	Sin	8520628. [sic — source
	8. 6.	e	inconsistency: with the printed
			Remainder 1073134 (below),
			this sine should read 8550628 =
			$\sin 58^\circ 46'$; 8520628 as printed
			gives 1103134. Transcribed as
			printed.]
Remainder [Relictum]			1073134.
Half, or the fourth number found			536567.

To this half let an "o" be appended, on account of the figure cast off from the secant, and the sine of the rising amplitude becomes 5365670, to which corresponds $32^\circ 27'$, for the rising amplitude. For of the fourth number found by

appending the cipher, 5365670, no tenth or hundredth part is to be taken, because the first, larger number, 7431448, is smaller than the whole sine.

Again, in a right-angled spherical triangle, one of whose non-right angles contains 50° and whose opposite arc about the right angle is 20° , let the other arc about the right angle be investigated, provided the species of the other non-right angle is settled. Since, by proposition 44 of our *Spherical Triangles* [*Triangula Sphaerica*], it holds that as 11917537, the tangent of the given angle of 50° , is to 3639702, the tangent of the given arc of 20° , so the whole sine is to the sine of the other arc about the right angle, we shall proceed thus. Since the first number is larger than the whole sine and the other smaller, we shall reject from it the last figure, 7, so as to have the number 1191753, smaller than the whole sine, to which there corresponds in the table of sines the arc of $6^\circ 51'$. The secant of the complement of this is 83843097; the last figure, 7, being cast off, to the remaining number there corresponds in the table of sines the arc of $56^\circ 58'$. And to the smaller number, as a sine, correspond $21^\circ 21'$. And so the two prosthaphaeresis arcs are $56^\circ 58'$ and $21^\circ 21'$. And thus the example will stand as follows.

[Margin: Example when the first number is larger, and also larger than the whole sine, but the other smaller.]

	G.	M.		G.	M
					.
Given arcs	56.	58.	Compl. of the greater	33.	2.
	21.	21.	Minor.	21.	21.

The minor can be subtracted from the complement, therefore subtraction will be made.

	G.	M.		
Sum of compl. & of the minor.	54.	23.	Sine.	8129314.
Diff. compl. & minor.	11.	41.	Sine.	2025025.

Remainder [Relictum]. 6104289.

Half, or the fourth number found. 3052145.

To this fourth number an "o" is to be added, on account of the figure cast off from the secant, so that the whole fourth number, 30521450, may come out; its tenth

part, 3052145, will be the sine of the sought arc, on account of the figure cast off from the first number. The sought arc will therefore be $17^{\circ} 46'$, a little more, if it is settled that it must be smaller than a quadrant.

Likewise, in the same triangle, with the angle set at 50° and the opposite arc at 48° , let the other arc about the right angle again be investigated. The tangent of the angle is, as before, 11917537. And the tangent of the arc is 17106124. Here both — as much the first, larger, as the other, smaller — are larger than the whole sine. Casting off, therefore, from each the last figure: for the remaining part of the first we shall find the arc of $6^{\circ} 51'$. The secant of the complement of this is 83843097; the last figure being cast off, to the remaining number, as a sine, is owed the arc of $56^{\circ} 58'$, which is one of the arcs required. And to the remaining number of the second, smaller [tangent], as a sine, is owed the arc of $6^{\circ} 23'$, which is the other required. So the example will stand as follows.

[Margin: Example when both the larger first number and the other, smaller, are larger than the whole sine.]

	G.	M.		G.	M
					.
Given arcs	56.	58.	Compl. of the greater	33.	2.
	6.	23.	Minor.	6.	23.

The minor can be subtracted, and so subtraction is to be made.

	G.	M.		
Sum of compl. & of the minor	39.	25.	Sine.	6349553.
Diff. between compl. & minor	26.	39.	Sine	4485392.

Remainder [Relictum].	1864161.
Half, or the fourth number found.	932081.

To this fourth number an "o" is to be appended, on account of the figure cast off from the secant, so that the whole fourth number may come out, 9320810 — that is, the sine of the sought arc. For here nothing is to be removed, since from both the first, larger, and the second, smaller, one figure has been cast off. The sought arc will therefore be $68^{\circ} 46'$, nearly, if it is settled that it must be smaller than a quadrant.

[Margin: Example when the first number is smaller, and the other larger, but smaller than the whole sine.]

Again, let the semidiurnal arc at the beginning of Cancer [☿] be investigated, for an elevation of the pole of 42° . Since, as we showed in the scholium to proposition 35 of Book I of the *Gnomonics*, half of the aggregate composed from the sine of the meridian altitude and from the sine of the meridian description stands to the sine of the meridian altitude as the whole sine is to the versed sine of the semidiurnal arc: now the said half is 6815085, while the sine of the meridian altitude is 9483237 — where you see that the first number is smaller than the second, and this smaller than the whole sine. To the smaller, which is the first, as to a sine, are owed $42^\circ 58'$; the secant of the complement of this arc is 14671946, to which, the last figure being cast off, corresponds among the sines the arc of $8^\circ 26'$, which is one of those required. And to the larger number, as a sine, agrees the arc of $71^\circ 30'$, which is the other required. So the example will stand as follows.

	G.	M.		G.	M
					.
Given arcs	8.	26.	Compl. of the greater.	18.	30.
	71.	30.	Minor.	8.	26.

The minor falls short of the complement, therefore subtraction is to be made.

	G.	M.		
Sum of compl. & of the minor	26.	56.	Sine.	4529535.
Diff. between compl. & minor	10.	4.	Sine.	1747939.

Remainder [Relictum]	2781596.
Half, or the fourth number found	1390798.

Diagram. A single woodcut, enclosed in a rectangular border, containing three circles set side by side (the leftmost lowest, the middle slightly raised, the right lowest again). Each circle repeats the same construction in a different orientation: two perpendicular diameters plus oblique radii divide the circle into eight sectors, and within each an inscribed right-triangle construction is drawn with dotted auxiliary (perpendicular) lines, illustrating the prosthaphaeresis relations of Lemma LIII (the letters E, i, L, K, G, and the segments iL, GK, EG, Ei referenced in the text). The central point of each circle is lettered E (the vertex

where the whole sine is set). Along a horizontal segment to the right of each centre run the letters **P, L, K**; a point **G** sits on the circumference to the upper right, with the dotted construction points **i, l, Q** near it. Cardinal circumference points are lettered (variously) **A, M** (top), **B** (left), **C** (bottom), **D, F, H, N, O, R, S, T**, and a Greek Ξ (Xi) appears at the top of the second and third circles. The peripheral labels are small, densely packed, and partly ink-blotted, so the precise assignment of individual letters to points within each of the three circles cannot be resolved with confidence at this resolution. [Multiple labels uncertain]

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Lemma LIII (continued)

Fourth, append a 0 to this number — on account of the figure cast off from the secant — so that the whole versed sine [sinus versus] becomes 13907980, to which correspond 113° , a little more; that is, 7 hours 32 minutes, for the semidiurnal arc.

Moreover, in a spherical triangle formed from the sides about the right angle — which are 30° and 50° — let the angle opposite the latter side be sought. Since it holds that, as 5000000, the sine of 30° , is to the whole sine [sinus totus], so 11917537, the tangent of 50° , is to the tangent of the sought angle — as we demonstrated in the scholium to proposition 44 of the *Spherical Triangles* [*Triangula Sphaerica*] — you see that the first number is smaller than the whole sine, whereas the other is larger. The smaller, subtracted twice from the larger, leaves 1917537. Therefore let it be arranged thus: as the whole sine is to 20000000, [so is] the secant of the complement of the angle which fits (as a sine) the smaller number given, [and] so the remaining part of the larger number to another [term]. [?: the proportional syntax here is compressed in the source] To the secant, with its last figure cast off, there correspond in the sines $11^\circ 32'$, which is one of the required arcs; and to the remaining part of the larger number, as a sine, there correspond $11^\circ 3'$ for the other required arc. Thus, then, the example will stand.

[Margin: *Exemplum quādo primus numerus minor est sinu toto, sed alter maior* — An example for when the first number is smaller than the whole sine, but the other is larger.]

Worked table (required arcs $11^\circ 32'$ and $11^\circ 3'$):

Given arcs [Arcus dati]	G M . .	C M . .
	1 3 Compl. of the larger 1 2 [Compl. maioris]	7 2 <i>Minor à compl. deficit,</i> 8 8 <i>idcirco fiet subtractio.</i> (The smaller falls short of the complement, therefore subtraction is done.)
	1 3 The smaller [Minor] 1	1 3 1
Sum of complement and smaller [Summa complementi & minoris]		8 3 Sine [Sinus] 9999644 9 1
Difference between complement and smaller [Diff. inter compl. & minorem]		6 2 Sine [Sinus] 9233220 7 5
Remainder [Relictum]		766424
Half, i.e. the fourth number found [Semissis, siue quartus numerus inuentus]		383212

To this fourth number append a 0, on account of the figure cast off from the secant, and to the whole number 3832120 add the whole sine twice, since the smaller number was subtracted twice from the larger; and the tangent of the sought angle becomes 23832120. The angle, therefore, is 67°14', a little more. Had you subtracted the smaller number 5000000 from the larger 11917537 only once, the number left over would likewise have been smaller than the whole sine, and with it you would have found the same angle.

Finally, in a right-angled spherical triangle formed from an arc about the right angle of 50° and the arc opposite the right angle [the hypotenuse] of 60°, let the angle contained by the said arcs be sought. Since, by proposition 45 of the *Spherical Triangles*, the tangent of the arc opposite the right angle is to the

tangent of the arc about the right angle as the whole sine is to the sine of the complement of the sought angle; and since, by proposition 18 of the *Sines*, the secant of the sought angle is to the whole sine as the whole sine is to the sine of the complement of the same angle: it will also hold that, as the tangent of the arc opposite the right angle is to the tangent of the arc about the right angle, so the secant of the sought angle is to the whole sine. And, inverting, as 11917537, the tangent of the arc about the right angle of 50° , is to 17320508, the tangent of the arc opposite the right angle of 60° , so the whole sine is to the secant of the sought angle. We have, therefore, a first number that is [smaller than the secant] but larger than the whole sine. So, with the last figure 7 removed, to the remaining number [1191753] there corresponds in the sines the arc $6^\circ 51'$. The secant of the complement of this arc is 83843097; and with its last figure cast off, to the remaining number, taken as a sine, correspond $56^\circ 58'$ — which is one of the required arcs. The other is found thus: with the last figure removed from the larger number [17320508], there remains the number 1732051, smaller than the whole sine but larger than the smaller remaining number. Hence the first part of the rule set out in No. 5 is to be applied. The other required arc will therefore be $9^\circ 58'$, matching the number 1732051. Thus, then, the example will stand.

[Margin: *Exemplum quādo primus numerus minor est, sed sinu toto maior* — An example for when the first number is smaller [than the secant], but larger than the whole sine.]

Worked table (required arcs $56^\circ 58'$ and $9^\circ 58'$):

Given arcs [Arcus dati]	G M . .	C M . .
	5 5 Compl. of the larger 6 8 [Compl. maioris]	3 2 <i>Fieri debet subtractio,</i> 3 <i>cum minor detrahi</i> <i>possit à compl.</i> (Subtraction must be done, since the smaller can be taken from the complement.)
	9 5 The smaller [Minor] 8	9 5 8
Sum of complement and smaller		4 0 Sine [sinus] 6819984 3

Given arcs [Arcus dati]	G M . .	C M . .
Difference between complement and smaller		2 4 Sine [sinus] 3918020 3
Remainder [Relictum]		2901964
Half, i.e. the fourth number found		1450982

To this fourth number append a 0, on account of the figure cast off from the secant, so that the whole fourth number becomes 14509820. Because of the casting-off of one figure made from each number, nothing results [i.e. no net change], since an equal number of figures — namely one — has been removed from each. To the secant so found there correspond 46°26', and a little more, for the sought angle.

8. When the whole sine is found neither in the first nor in the middle place of the proportion, the first two numbers must be reduced to two others by prosthaphaeresis — the first of which is to be the whole sine — in this way. Let it be arranged: as the first number is to the whole sine, so the second to another, by the prosthaphaeresis explained in Nos. 4, 5, and 6. For then the whole sine will also be to the number found as the third is to the one to be found, and thus the prosthaphaeresis explained in Nos. 1 and 2 is to be employed.

[Margin: *Quando sinus totus in regula aurea non reperitur, quo pacto prosthaphæresis fiat* — When the whole sine is not found in the rule of three, how the prosthaphaeresis is to be done.]

Yet the prosthaphaeresis, although it rests on geometrical demonstrations, as we have shown, cannot be accurate and exact except when the operation is carried out through sines alone, and the whole sine is placed at the beginning of the proportion, as was set out in No. 1. For when other numbers besides sines are employed, no small error can be committed, because such numbers are rarely found precisely in the table of sines, so that the arcs matching them might be taken without error. Therefore, so that the matter may be done more exactly, one must always employ the proportional part [pars proportionalis], as we set out in the explanation and use of the table of sines — that is, when a number that is not found precisely in the table of sines must be extracted, [the arc must be taken] with its degrees, minutes, and seconds. This is done if one takes the difference between the sine next smaller than the given number and the next larger, and

the difference between the same next-smaller sine and the given number, and then states: If the former difference calls for 60 seconds (for between

[Margin: *Prosthaphæresis quando accurata fit, & quo pacto fieri possit accuratio per partis proportionalis inuentionem* — When the prosthaphaeresis is accurate, and how it can be made more accurate through the finding of the proportional part.]

two neighbouring minutes 60 seconds are interposed), the latter [difference] requires as many seconds as it demands; and these seconds, once found for the arc that matches the smaller sine assumed, are to be added. In the same way, if the sine to be extracted must be taken with degrees, minutes, and seconds, one must take the difference between the sine corresponding to the degrees and minutes and the next larger sine, and then say: If 60 seconds call for such a difference as the proposed seconds require, then the difference found is to be added to the next-smaller sine assumed. The same is to be done in the table of Tangents and Secants, when the matter requires it. But it is easier in the table of sines, [where] the proportional part is extracted in the manner we shall explain shortly, and that by a single multiplication or division, and by small numbers.

The finding of the proportional part in prosthaphaeresis ought not to seem burdensome, since it is done by small multiplications and divisions, whereas the prosthaphaeresis itself frees us from long and very troublesome multiplications and divisions. But if anyone should prefer to work through the multiplication and division of sines and other numbers rather than through prosthaphaeresis with the proportional part, that is permitted him by us. For we do not deny that the business is sometimes dispatched more quickly without prosthaphaeresis — on account of the proportional parts, which retard the work somewhat — yet we also acknowledge that there is less trouble in prosthaphaeresis than in such long and difficult multiplications and divisions of numbers, especially because in the table of sines the proportional part is extracted with almost no labor, in the manner we shall set out shortly after the table of sines. But let us set down some example in which the prosthaphaeresis is carried through with the proportional part.

[Margin: *Exemplū prosthaphæresis cum parte proportionali* — An example of prosthaphaeresis with the proportional part.]

Let it be, then, as in the last example, that the angle is again to be investigated from the arc opposite the right angle and the arc contained about the right angle, of which the former is 60° and the latter 50° . And since, as was said, 11917537, the

tangent of the arc of 50° , is to 17320508, the tangent of the arc of 60° , as the whole sine is to the secant of the sought angle: if the last figures 7 and 8 are cast off — for which units are assumed, since both $7/10$ and $8/10$ exceed a half — there will be obtained the numbers smaller than the whole sine, 1191754 and 1732051, in nearly the same proportion. Therefore let it be arranged: as the whole sine is to the secant of the complement of the angle which belongs to the sine 1191754, so the sine 1732051 to another, as was taught in the first part of the rule explained in No. 5. With the first sine there is found the arc $6^\circ 50' 40''$, whose complement's secant is 83910940; and to this, with the last figure cast off, taken as a sine, there corresponds the arc $57^\circ 2' 46''$ — and this is one of the required arcs. The other arc, belonging to the latter number, is $9^\circ 58' 27''$. Thus, then, the example will stand.

Worked table (required arcs $57^\circ 2' 46''$ and $9^\circ 58' 27''$):

Given arcs [Arcus dati]	G MS . . .	G MS . . .
	5 2 4 Compl. of the larger	3 5 1 <i>Minor est minor</i>
	7 6 [Compl. maioris]	2 7 4 <i>quam compl. ideo fiet subtractio. (The smaller is smaller than the complement, therefore subtraction is done.)</i>
	9 5 2 The smaller [Minor]	9 5 2
	8 7	8 7
Sum of complement and smaller		4 5 4 Sine [sinus] 6810795 2 5 1
Difference between complement and smaller		2 5 4 Sine [sinus] 3904053 2 8 7
Remainder [Relictum]		2906742
Half, i.e. the fourth number		1453371

With the figure 0 appended to the fourth number found — on account of the figure cast off from the secant — the whole secant becomes 14533710, to which corresponds the arc $46^\circ 31'$ for the sought angle. This differs from the earlier

[result] by almost 5 minutes, whereby you see how much it matters to employ the proportional parts. In the other examples we deliberately neglected the proportional parts, both because in them so great an error does not appear, and above all so that the rule of prosthaphaeresis might be explained more clearly.

Furthermore, for the sake of students, and so that the use of prosthaphaeresis may become plainer, we shall append in this place the calculation of all triangles — demonstrated in our *[Triangles]* and in the *Treatise on Sines*, and now recalled to a more convenient form and method — and we shall propose the same quantity to be found by several ways, so that anyone may choose for himself the one that pleases him more. In any triangle, whether spherical or rectilinear, we shall call the side opposite the right angle the BASE [BASIS]. In a non-right triangle, however, when two sides are named, the third — whether it be the larger or not — we shall call the base.

[Margin: *Basis triāguli quæ* — What the base of a triangle is.]

TRIANGVLORVM SPHÆRICORVM rectangulorum calculus

(The calculation of right-angled spherical triangles)

Since, in any right-angled spherical triangle, what is sought from two given or known quantities is either an ANGLE (not the right one), or a SIDE about the right angle, or the BASE: this can be done in several manners and ways, as will be clear from what follows. But we shall always, in the first place, set out separately what is sought; then the two things that are known or given; third, we shall demonstrate the various ways and methods by which the sought quantity can be derived — to which we shall also prefix numbers, so that they may more easily be recognized and distinguished from the other argumentations. Thus, then, the aforesaid are found.

I. ANGLE [ANGVLVS]

From the base and the side which is opposite the sought angle [Ex base, & latere, quod angulo quæsito opponitur].

The angle is found by twelve proportions (*modi*), each derived through the cited authorities (*tr. sph.* = *Triangula Sphaerica*; *sinuũ* = *De Sinibus*; *quinti* = Euclid Bk. V; *Permut.* = *permutando*; *Conuertendo*; *Ex aequal. perturb.* = *ex aequali in proportione perturbata*). The numbered lines are the usable proportions.

No.	Proportion (transcribed)	Justification
1.	As <i>sinus basis</i> to <i>sinus totus</i> : so <i>sinus lateris</i> to	41. <i>tr. sph.</i>

No.	Proportion (transcribed)	Justification
	<i>sinus anguli.</i>	
	But as <i>sinus lateris</i> to <i>sinus anguli</i> : so <i>secans compl. anguli</i> to <i>secantem compl. lateris</i> .	22. sinuũ.
	Therefore as <i>sinus basis</i> to <i>sinus totus</i> : so <i>secans compl. anguli</i> to <i>secantem compl. lateris</i> .	11. quinti.
2.	Therefore as <i>sinus totus</i> to <i>sinum basis</i> : so <i>secans compl. lateris</i> to <i>secantem compl. anguli</i> .	Conuertendo.
	As <i>sinus basis</i> to <i>sinus totus</i> : so <i>sinus lateris</i> to <i>sinum anguli</i> .	41. tr. sph.
	Therefore as <i>sinus basis</i> to <i>sinum lateris</i> : so <i>sinus totus</i> to <i>sinum anguli</i> .	Permut.
	But as <i>sinus basis</i> to <i>sinum lateris</i> : so <i>secans compl. lateris</i> to <i>secantem compl. basis</i> .	22. sinuũ.
	Therefore as <i>secans compl. lateris</i> to <i>secantem compl. basis</i> : so <i>sinus totus</i> to <i>sinum anguli</i> .	11. quinti.
3.	Therefore as <i>secans compl. lateris</i> to <i>sinum totum</i> : so <i>secans compl. basis</i> to <i>sinum anguli</i> .	Permutando.
	But as <i>secans compl. lateris</i> to <i>sinum totum</i> : so <i>sinus totus</i> to <i>sinum lateris</i> .	18. sinuũ.
4.	Therefore as <i>sinus totus</i> to <i>sinum lateris</i> : so <i>secans compl. basis</i> to <i>sinum anguli</i> .	11. quinti.
	As <i>sinus totus</i> to <i>sinum basis</i> : so <i>secans compl. lateris</i> to <i>secantem compl. anguli</i> .	2. modus.
	But as <i>sinus totus</i> to <i>sinum basis</i> : so <i>secans compl. basis</i> to <i>sinum totum</i> .	18. sinuũ.
5.	Therefore as <i>secans compl. basis</i> to <i>sinum totum</i> : so <i>secans compl. lateris</i> to <i>secantem compl. anguli</i> .	11. quinti.

No.	Conclusion	Justification
6.	Therefore as <i>sinus lateris</i> to <i>sinum totum</i> : so <i>sinus basis</i> to <i>secantem compl. anguli</i> .	11. quinti.
7.	Therefore as <i>sinus compl. basis</i> to	11. quinti.

No.	Conclusion	Justification
	<i>tangentem compl. basis: so sinus lateris to sinum anguli.</i>	
8.	Therefore as <i>tangens compl. basis</i> to <i>sinum compl. basis: so secans compl. lateris</i> to <i>secantem compl. anguli.</i>	Conuertendo.
9.	Therefore as <i>sinus basis</i> to <i>tangentem lateris: so sinus compl. lateris</i> to <i>sinum anguli.</i>	Ex aequal. perturb.
10.	Therefore as <i>tangens lateris</i> to <i>sinum basis: so secans lateris</i> to <i>secantem compl. anguli.</i>	Conuertendo.
11.	Therefore as <i>secans lateris</i> to <i>secantem compl. basis: so tangens lateris</i> to <i>sinum anguli.</i>	11. quinti.
12.	Therefore as <i>sinus lateris</i> to <i>tangentem basis: so sinus compl. basis</i> to <i>secantem compl. anguli.</i>	Ex aequal. perturb.

You see, then, that the angle can be investigated in twelve ways from a given base and the side to which the sought angle is opposite. Of these, six employ the whole sine — namely nos. 2 and 4 in the first place of the proportion, and nos. 1, 3, 5, and 6 in the second place — while the other six have the whole sine nowhere. By the same reasoning, in what follows more ways could be found; but consulting brevity, we shall be content to demonstrate only six ways for finding each sought quantity from the same data — those, namely, in which the whole sine always occurs.

II. ANGLE [ANGVLVS]

From the base and the side which is adjacent to the sought angle [Ex base, & latere, quod angulo quæsito adiacet].

No.	Proportion (transcribed)	Justification
	<i>As tangens basis</i> to <i>tangentem lateris: so sinus totus</i> to <i>sinum compl. anguli.</i>	45. tr. sph.
1.	Therefore as <i>tangens basis</i> to <i>sinum totum: so tangens lateris</i> to <i>sinum compl. anguli.</i>	Permut.

Calculus of Right Spherical Triangles (continued)

The apparatus continues the systematic derivation of proportions for the parts of a right spherical triangle. Each "case" (headed by a Roman numeral and the phrase ANGVLVS*, "the Angle," with a statement of the two given parts) is worked out as a chain of proportions. Every line is a four-term proportion "As A is to B, so C is to D," carried in the margin by its justification: a numbered theorem of the spherical triangles (*tr. sph. / triag. sphaer.*), a numbered theorem on sines (*sinuum*), a proposition of Euclid Book V (*quinti*), or a rule for transforming ratios (*Permutando* = by alternation, *Convertendo* = by inversion). The bold parenthesized numbers mark the successive canonical results (*modi*) of each case; a bare* *modus reference points back to an earlier result.*

Notation in the tables below. : reads "is to"; :: reads "so is." Following Clavius's own usage, the "complement" functions are given as the modern co-functions: **cosine** = sine of the complement (*sinus complementi*), **cotangent** = tangent of the complement (*tangens complementi*), **cosecant** = secant of the complement (*secans complementi*). "**whole sine**" = *sinus totus* (the radius). "given angle" = *angulus datus*; "sought angle" = *angulus quaesitus*.

Remaining results of the preceding case (conclusions 2–6)

(This table completes the case begun on the previous page; its citations stand in the left margin.)

Justification	Proportion
45. <i>tr. sph.</i>	As tangent of base : tangent of side :: whole sine : cosine of angle.
21. <i>sinuum</i>	But as tangent of base : tangent of side :: cotangent of side : cotangent of base.
11. <i>quinti</i>	Hence as cotangent of side : cotangent of base :: whole sine : cosine of angle.
<i>Permutando</i>	(2) Hence as cotangent of side : whole sine :: cotangent of base : cosine of angle.
<i>Permutando</i>	Hence as cotangent of side : cotangent of base :: whole sine : cosine of angle.
18. <i>sinuum</i>	But as whole sine : cosine of angle :: secant of angle : whole sine.
11. <i>quinti</i>	Hence as cotangent of side : cotangent of base ::

Justification	Proportion
	secant of angle : whole sine.
<i>Convertendo</i>	Hence as cotangent of base : cotangent of side :: whole sine : secant of angle.
<i>Permutando</i>	(3) Hence as cotangent of base : whole sine :: cotangent of side : secant of angle.
<i>Permutando</i>	Hence as cotangent of base : cotangent of side :: whole sine : secant of angle.
21. <i>sinuum</i>	But as cotangent of base : cotangent of side :: tangent of side : tangent of base.
11. <i>quinti</i>	Hence as tangent of side : tangent of base :: whole sine : secant of angle.
<i>Permutando</i>	(4) Hence as tangent of side : whole sine :: tangent of base : secant of angle.
1. <i>modus</i>	As tangent of base : whole sine :: tangent of side : cosine of angle.
18. <i>sinuum</i>	But as tangent of base : whole sine :: whole sine : cotangent of base.
11. <i>quinti</i>	(5) Hence as whole sine : cotangent of base :: tangent of side : cosine of angle.
4. <i>modus</i>	As tangent of side : whole sine :: tangent of base : secant of angle.
18. <i>sinuum</i>	But as tangent of side : whole sine :: whole sine : cotangent of side.
11. <i>quinti</i>	(6) Hence as whole sine : cotangent of side :: tangent of base : secant of angle.

III. ANGVLVS [The Angle]

Ex basi, & altero angulo non recto [From the base, and the other non-right angle]

Justification	Proportion
47. <i>triag. sphaer.</i>	(1) As whole sine : cosine of base :: tangent of given angle : cotangent of sought angle.
18. <i>sinuum</i>	But as whole sine : cosine of base :: secant

Justification	Proportion
	of base : whole sine.
11. <i>quinti</i>	(2) Hence as secant of base : whole sine :: tangent of given angle : cotangent of sought angle.
21. <i>sinuum</i>	But as tangent of given angle : cotangent of sought angle :: tangent of sought angle : cotangent of given angle.
11. <i>quinti</i>	Hence as secant of base : whole sine :: tangent of sought angle : cotangent of given angle.
<i>Convertendo</i>	(3) Hence as whole sine : secant of base :: cotangent of given angle : tangent of sought angle.
1. <i>modus</i>	As whole sine : cosine of base :: tangent of given angle : cotangent of sought angle.
<i>Permutando</i>	Hence as whole sine : tangent of given angle :: cosine of base : cotangent of sought angle.
18. <i>sinuum</i>	But as whole sine : tangent of given angle :: cotangent of given angle : whole sine.
11. <i>quinti</i>	(4) Hence as cotangent of given angle : whole sine :: cosine of base : cotangent of sought angle.
3. <i>modus</i>	As whole sine : secant of base :: cotangent of given angle : tangent of sought angle.
<i>Permutando</i>	Hence as whole sine : cotangent of given angle :: secant of base : tangent of sought angle.
18. <i>sinuum</i> [?]	But as whole sine : cotangent of given angle :: tangent of given angle : whole sine.
11. <i>quinti</i>	(5) Hence as tangent of given angle : whole sine :: secant of base : tangent of sought angle.

(continuation of Case III; on this recto the justifications stand in the right margin)

Justification	Proportion
4. <i>modus</i>	As cotangent of given angle : whole sine :: cosine of base : cotangent of sought angle.
<i>Permutando</i>	Hence as cotangent of given angle : cosine of base :: whole sine : cotangent of sought angle.
18. <i>sinuum</i>	But as whole sine : cotangent of sought angle :: tangent of sought angle : whole sine.
11. <i>quinti</i>	Hence as cotangent of given angle : cosine of base :: tangent of sought angle : whole sine.
<i>Convertendo</i>	Hence as cosine of base : cotangent of given angle :: whole sine : tangent of sought angle.
<i>Permutando</i>	(6) Hence as cosine of base : whole sine :: cotangent of given angle : tangent of sought angle.

IV. ANGVLVS [The Angle]

Ex latere, quod angulo quaesito opponitur, & altero angulo non recto
[From the side that subtends the sought angle, and the other non-right angle]

Justification	Proportion
42. <i>triag. sphaer.</i>	(1) As whole sine : sine of given angle :: cosine of side : cosine of sought angle.
22. <i>sinuum</i>	But as cosine of side : cosine of sought angle :: secant of sought angle : secant of side.
11. <i>quinti</i>	Hence as whole sine : sine of given angle :: secant of sought angle : secant of side.
<i>Convertendo</i>	(2) Hence as sine of given angle : whole sine :: secant of side : secant of sought angle.
42. <i>tr. sph.</i>	As whole sine : sine of given angle :: cosine of side : cosine of sought angle.
<i>Permutando</i>	Hence as whole sine : cosine of side :: sine of given angle : cosine of sought angle.
22. <i>sinuum</i>	But as sine of given angle : cosine of sought angle :: secant of sought angle : cosecant of given angle.
11. <i>quinti</i>	Hence as whole sine : cosine of side ::

Justification	Proportion
	secant of sought angle : cosecant of given angle.
<i>Convertendo</i>	(3) Hence as cosine of side : whole sine :: cosecant of given angle : secant of sought angle.
42. <i>tr. sph.</i>	As whole sine : sine of given angle :: cosine of side : cosine of sought angle.
18. <i>sinuum</i>	But as whole sine : sine of given angle :: cosecant of given angle : whole sine.
11. <i>quinti</i>	(4) Hence as cosecant of given angle : whole sine :: cosine of side : cosine of sought angle.
22. <i>sinuum</i>	But as cosine of side : cosine of sought angle :: secant of sought angle : secant of side.
11. <i>quinti</i>	Hence as cosecant of given angle : whole sine :: secant of sought angle : secant of side.
<i>Convertendo</i>	(5) Hence as whole sine : cosecant of given angle :: secant of side : secant of sought angle.
42. <i>tr. sph.</i>	As whole sine : sine of given angle :: cosine of side : cosine of sought angle.
<i>Permutando</i>	Hence as whole sine : cosine of side :: sine of given angle : cosine of sought angle.
18. <i>sinuum</i>	But as whole sine : cosine of side :: secant of side : whole sine.
11. <i>quinti</i>	(6) Hence as secant of side : whole sine :: sine of given angle : cosine of sought angle.

V. ANGVLVS [The Angle]

Ex latere, quod angulo quaesito adiacet, & altero angulo non recto: Dummodo constet, num maior sit recto, an minor, vel an basis, aut latus alterum non datum quadrante maius sit minusue [From the side that lies adjacent to the sought angle, and the other non-right angle: provided it be established whether it is greater than a right angle or less, or whether the base, or the other side not given, is greater than a quadrant or less]

Justification	Proportion
42. <i>triag. sphaer.</i>	As cosine of side : cosine of given angle :: whole sine : sine of sought angle.
<i>Permutando</i>	(1) Hence as cosine of side : whole sine :: cosine of given angle : sine of sought angle.

(continuation of Case V; on this verso the justifications return to the left margin)

Justification	Proportion
42. <i>triag. sphaer.</i>	As cosine of side : cosine of given angle :: whole sine : sine of sought angle.
18. <i>sinuum</i>	But as whole sine : sine of sought angle :: cosecant of sought angle : whole sine.
11. <i>quinti</i>	Hence as cosine of side : cosine of given angle :: cosecant of sought angle : whole sine.
<i>Convertendo</i>	Hence as cosine of given angle : cosine of side :: whole sine : cosecant of sought angle.
<i>Permutando</i>	(2) Hence as cosine of given angle : whole sine :: cosine of side : cosecant of sought angle.
1. <i>modus</i>	As cosine of side : whole sine :: cosine of given angle : sine of sought angle.
18. <i>sinuum</i>	But as cosine of side : whole sine :: whole sine : secant of side.
11. <i>quinti</i>	(3) Hence as whole sine : secant of side :: cosine of given angle : sine of sought angle.

Justification	Proportion
22. <i>sinuum</i>	But as cosine of given angle : sine of sought angle :: cosecant of sought angle : secant of given angle.
11. <i>quinti</i>	Hence as whole sine : secant of side :: cosecant of sought angle : secant of given angle.
<i>Convertendo</i>	(4) Hence as secant of side : whole sine :: secant of given angle : cosecant of sought angle.
42. <i>tr. sph.</i>	As cosine of side : cosine of given angle :: whole sine : sine of sought angle.
22. <i>sinuum</i>	But as cosine of side : cosine of given angle :: secant of given angle : secant of side.
11. <i>quinti</i>	Hence as secant of given angle : secant of side :: whole sine : sine of sought angle.
<i>Permutando</i>	(5) Hence as secant of given angle : whole sine :: secant of side : sine of sought angle.
2. <i>modus</i>	As cosine of given angle : whole sine :: cosine of side : cosecant of sought angle.
18. <i>sinuum</i>	But as cosine of given angle : whole sine :: whole sine : secant of given angle.
11. <i>quinti</i>	(6) Hence as whole sine : secant of given angle :: cosine of side : cosecant of sought angle.

VI. ANGVLVVS [The Angle]

Ex utroque latere [From both sides]

Justification	Proportion
44. <i>triag. sphaer.</i>	(1) As sine of the side adjacent to the sought angle : whole sine :: tangent of the side opposite the sought angle : tangent of sought angle.
21. <i>sinuum</i>	But as tangent of the side opposite the sought angle : tangent of sought angle ::

Justification	Proportion
	cotangent of sought angle : cotangent of the side opposite the sought angle.
11. <i>quinti</i>	Hence as sine of the side adjacent to the sought angle : whole sine :: cotangent of sought angle : cotangent of the side opposite the sought angle.
<i>Convertendo</i>	(2) Hence as whole sine : sine of the side adjacent to the sought angle :: cotangent of the side opposite the sought angle : cotangent of sought angle.
44. <i>triag. sphaer.</i>	As sine of the side adjacent to the sought angle : whole sine :: tangent of the side opposite the sought angle : tangent of sought angle.
18. <i>sinuum</i>	But as sine of the side adjacent to the sought angle : whole sine :: whole sine : cosecant of the side adjacent to the sought angle.
11. <i>quinti</i>	(3) Hence as whole sine : cosecant of the side adjacent to the sought angle :: tangent of the side opposite the sought angle : tangent of sought angle.
44. <i>triag. sphaer.</i>	As sine of the side adjacent to the sought angle : whole sine :: tangent of the side opposite the sought angle : tangent of sought angle.
<i>Permutando</i>	Hence as sine of the side adjacent to the sought angle : tangent of the side opposite the sought angle :: whole sine : tangent of sought angle.
18. <i>sinuum</i>	But as whole sine : tangent of sought angle :: cotangent of sought angle : whole sine.
11. <i>quinti</i>	Hence as sine of the side adjacent to the sought angle : tangent of the side opposite the sought angle :: cotangent of sought angle : whole sine.

(The result of this last chain — its "Ergo..." — carries over to the following page.)

Astrolabium, Book I — Lemma LIII

Calculus of right spherical triangles [*Triangulorum sphæricorum rectangulorum calculus*] (continued)

This spread is a computation table. Each numbered result is reached by a chain of proportions of the form "As term₁ is to term₂, so term₃ is to term₄," each step justified by a marginal citation. In the tables below the marginal citation is given in the first column; the four proportion terms follow. Numbered lines (1.–6.) are the derived results of each case; the unnumbered lines are the intermediate premises and proportion-operations. On the recto pages the citation column stands at the right margin; on the verso (p. 106) it is mirrored to the left. Citations: "N. sinuū" = proposition N of the canon of sines; "N. quinti" = Euclid, Elements V, prop. N; "N. triāg. sphær." / "N. tr. sph." = [Christoph Clavius], Triangula Sphaerica, prop. N; "N. modus" = an earlier method of this Lemma; "Permutando" / "Conuertendo" = the Euclidean operations (alternando / convertendo). Transcribed verbatim; abbreviations kept as printed.

Modern shorthand used in the summaries: R = sinus totus (whole sine / radius); cos = sinus complementi; cot = tangens complementi; csc = secans complementi; sin, tan, sec as usual.

[Continuation of case VI — the sought angle, from the two sides]

(The heading and results 1–3 stand on the preceding page. Here are results 4–6. In this case "lat. oppos." = the side opposite the sought angle, "lat. adiac." = the side adjacent to it.)

Result 4

Cit.	(As)	(to)	(so)	(to)
Conue rtend o	Ergo vt tangens lat. oppos. angulo quæsito	ad sinum lat. adiac. angulo quæsito:	Ita sinus totus	ad tangentem compl. anguli quæsiti.
Perm utand o	4. Ergo vt tang. lat. oppos. ang. quæsito	ad sinum totum:	Ita sinus lateris adiac. ang. quæsito	ad tangentem compl. anguli quæsiti.

Modern: $\tan(\text{opp}) : R :: \sin(\text{adj}) : \cot(\text{sought angle})$.

Result 5

Cit.	(As)	(to)	(so)	(to)
2. modu s	Vt sinus totus	ad sinum lat. adiac. ang. quæsito:	Ita tangens compl. lat. oppos. ang. quæsito	ad tangentem compl. anguli quæsiti.
18. sinuū	Sed vt sinus totus	ad sinum lateris adiac. ang. quæsito:	Ita secans compl. lat. adiac. ang. quæsito	ad sinum totum.
11. quint i	5. Ergo vt secans compl. lat. adiac. ang. quæsito	ad sinum totum:	Ita tangens compl. lat. oppos. ang. quæsito	ad tangentem compl. anguli quæsiti.

Modern: $\csc(\text{adj}) : R :: \cot(\text{opp}) : \cot(\text{sought angle})$.

Result 6

Cit.	(As)	(to)	(so)	(to)
Perm utand o	Ergo vt secans compl. lat. adiac. ang. quæsito	ad tang. complem. lat. oppos. ang. quæsito:	Ita sinus totus	ad tangentem compl. anguli quæsiti.
18. sinuū	Sed vt sinus totus	ad tangentem comple. anguli quæsiti:	Ita tangens anguli quæsiti	ad sinum totum.
11. quinti	Ergo vt secans compl. lat. adiac. ang. quæsito	ad tang. complem. lat. oppos. ang. quæsito:	Ita tangens ang. quæsiti	ad sinum totum.
Conue rtend o	Ergo vt tang. compl. lat. oppos. ang. quæsito	ad sec. complem. lat. adiac. ang. quæsito:	Ita sinus totus	ad tangentem anguli quæsiti.
Perm	6. Ergo vt tang.	ad sinum totum:	Ita secans	ad tangentem

Cit.	(As)	(to)	(so)	(to)
utand o	compl. lat. oppos. ang. quæsito		compl. lat. adiac. ang. quæsito	anguli quæsiti.

Modern: $\cot(\text{opp}) : R :: \csc(\text{adj}) : \tan(\text{sought angle})$.

VII. LATVS.

Ex base, & altero latere. [The side, from the base and the other side.]

(The base here is the hypotenuse; "lateris dati" = the given side, "lateris quæsiti" = the sought side.)

Result 1

Cit.	(As)	(to)	(so)	(to)
43. triäg. sphær.	Vt sinus compl. lateris dati	ad sinum compl. basis:	Ita sinus totus	ad sinum compl. lateris quæsiti.
Permutand o	1. Ergo vt sinus compl. lateris dati	ad sinum totum:	Ita sinus compl. basis	ad sinum complem. lateris quæsiti.

Modern: $\cos(\text{given side}) : R :: \cos(\text{base}) : \cos(\text{sought side})$.

Result 2

Cit.	(As)	(to)	(so)	(to)
43. triäg. sphær.	Vt sinus compl. lateris dati	ad sinum compl. basis:	Ita sinus totus	ad sinum complem. lateris quæsiti.
18. sinuū	Sed vt sinus totus	ad sinum compl. lateris quæsiti:	Ita secans lateris quæsiti	ad sinum totum.
11. quinti	Ergo vt sinus comple. lateris dati	ad sinum compl. basis:	Ita secans lateris quæsiti	ad sinum totum.
Conuert.	Ergo vt sinus compl. basis	ad sinum compl. lateris	Ita sinus totus	ad secantem lateris quæsiti.

Cit.	(As)	(to)	(so)	(to)
		dati		
Permutand o	2. Ergo vt sinus compl. basis	ad sinum totum:	Ita sinus complem. lateris dati	ad secantem lateris quæsiti.

Modern: $\cos(\text{base}) : R :: \cos(\text{given side}) : \sec(\text{sought side})$.

Result 3

Cit.	(As)	(to)	(so)	(to)
43. tr. sph.	Vt sinus compl. lat. dati	ad sinum compl. basis:	Ita sinus totus	ad sinum compl. lat. quæsiti.
22. sinuū	Sed vt sinus comple. lateris dati	ad sinum complem. basis:	Ita secans basis	ad secantem lateris dati.
11. quinti	Ergo vt secans basis	ad secantem lateris dati:	Ita sinus totus	ad sinum compl. lat. quæsiti.
Permu tando	3. Ergo vt secans basis	ad sinum totum:	Ita secans lateris dati	ad sinum compl. lateris quæsiti.

Modern: $\sec(\text{base}) : R :: \sec(\text{given side}) : \cos(\text{sought side})$.

Result 4

Cit.	(As)	(to)	(so)	(to)
2. modu s	Vt sinus compl. basis	ad sinum totum:	Ita sinus complem. lateris dati	ad secantem lateris quæsiti.
Perm utan do	Ergo vt sinus compl. basis	ad sinum complem. lateris dati:	Ita sinus totus	ad secantem lateris quæsiti.
22. sinuū	Sed vt sinus comple. basis	ad sinum complem. lateris dati:	Ita secans lateris dati	ad secantem basis.
11.	Ergo vt secans	ad secantem	Ita sinus totus	ad secantem

Cit.	(As)	(to)	(so)	(to)
quin.	lateris dati	basis:		lateris quæsiti.
Perm utan do	4. Ergo vt secans lateris dati	ad sinum totum:	Ita secans basis	ad secantem lateris quæsiti.

Modern: $\sec(\text{given side}) : R :: \sec(\text{base}) : \sec(\text{sought side})$.

Result 5

Cit.	(As)	(to)	(so)	(to)
1. modu s	Vt sinus compl. lateris dati	ad sinum totum:	Ita sinus compl. basis	ad sinum compl. lateris quæsiti.
18. sinuū	Sed vt sinus complem. lateris dati	ad sinum totum:	Ita sinus totus	ad secantem lateris dati.
11. quint i	5. Ergo vt sinus totus	ad secantem lateris dati:	Ita sinus compl. basis	ad sinum comple. lateris quæsiti.

Modern: $R : \sec(\text{given side}) :: \cos(\text{base}) : \cos(\text{sought side})$.

Result 6

Cit.	(As)	(to)	(so)	(to)
2. modu s	Vt sinus compl. basis	ad sinum totum:	Ita sinus complem. lateris dati	ad secantem lateris quæsiti.
18. sinuū	Sed vt sinus compl. basis	ad sinum totum:	Ita sinus totus	ad secantem basis.
11. quinti	6. Ergo vt sinus totus	ad secantem basis:	Ita sinus complem. lateris dati	ad secantem lateris quæsiti.

Modern: $R : \sec(\text{base}) :: \cos(\text{given side}) : \sec(\text{sought side})$.

VIII. LATVS.

Ex base & angulo, qui lateri quæsito opponitur. [The side, from the base and the angle opposite the sought side.]

Result 1

Cit.	(As)	(to)	(so)	(to)
41. triäg. sphær.	1. Vt sinus totus	ad sinum basis:	Ita sinus anguli dati	ad sinum lateris quæsiti.

Modern: $R : \sin(\text{base}) :: \sin(\text{given angle}) : \sin(\text{sought side})$.

Result 2

Cit.	(As)	(to)	(so)	(to)
21. sinuū	Sed vt sinus anguli dati	ad sinum lateris quæsiti:	Ita secans complem. lateris quæsiti	ad secantem compl. anguli dati.
11. quinti	Ergo vt sinus totus [?]	ad sinum basis:	Ita secans complem. lateris quæsiti	ad secantem comple. anguli dati.
Conue rtend o	2. Ergo vt sinus basis	ad sinum totum:	Ita secans complem. anguli dati	ad secantem compl. lateris quæsiti.

Modern: $\sin(\text{base}) : R :: \csc(\text{given angle}) : \csc(\text{sought side})$.

Result 3

Cit.	(As)	(to)	(so)	(to)
41. triäg. sphær.	Vt sinus totus	ad sinum basis:	Ita sinus anguli dati	ad sinum lateris quæsiti.
18. sinuū	Sed vt sinus totus	ad sinum basis:	Ita secans complem. basis	ad sinum totum.
11. quinti	3. Ergo vt secans compl. basis	ad sinum totum:	Ita sinus anguli dati	ad sinum lateris quæsiti.

Modern: $\csc(\text{base}) : R :: \sin(\text{given angle}) : \sin(\text{sought side})$.

Result 4

Cit.	(As)	(to)	(so)	(to)
22. sinuū	Sed vt sinus anguli dati	ad sinum lateris quæsiti:	Ita secans complem. lateris quæsiti	ad secantem comple. anguli dati.
11. quinti	Ergo vt secans complem. basis	ad sinum totum:	Ita secans complem. lateris quæsiti	ad secantem comple. anguli dati.
Conue rtend o	4. Ergo vt sinus totus	ad secantem compl. basis:	Ita secans compl. anguli dati	ad secantem compl. lateris quæsiti.

Modern: $R : \csc(\text{base}) :: \csc(\text{given angle}) : \csc(\text{sought side})$.

Result 5

Cit.	(As)	(to)	(so)	(to)
41. tr. sph.	Vt sinus totus	ad sinum basis:	Ita sinus anguli dati	ad sinum lateris quæsiti.
Permut .	Ergo vt sinus totus	ad sinum anguli dati:	Ita sinus basis	ad sinum lateris quæsiti.
18. sinuū	Sed vt sinus totus	ad sinum anguli dati:	Ita secans complem. anguli dati	ad sinum totum.
11. quinti	5. Ergo vt secans compl. anguli dati	ad sinum totum:	Ita sinus basis	ad sinum lateris quæsiti.

Modern: $\csc(\text{given angle}) : R :: \sin(\text{base}) : \sin(\text{sought side})$.

Result 6

Cit.	(As)	(to)	(so)	(to)
4. modu s	Vt sinus totus	ad secantem compl. basis:	Ita secans complem. anguli dati	ad secantem comple. lateris quæsiti.
Perm utan do	Ergo vt sinus totus	ad secantem complem. anguli dati:	Ita secans compl. basis	ad secantem comple. lateris quæsiti.
18.	Sed vt sinus	ad secantem	Ita sinus anguli	ad sinum totum.

Cit.	(As)	(to)	(so)	(to)
sinuū	totus	comple. anguli dati:	dati	
11. quin.	6. Ergo vt sinus anguli dati	ad sinum totum:	Ita secans compl. basis	ad secantem compl. lateris quæsiti.

Modern: $\sin(\text{given angle}) : R :: \csc(\text{base}) : \csc(\text{sought side})$.

IX. LATVS.

Ex base & angulo, qui lateri quæsito adiacet. [The side, from the base and the angle adjacent to the sought side.]

Result 1

Cit.	(As)	(to)	(so)	(to)
45. triāg. sphær.	1. Vt sinus totus	ad sinum compl. anguli dati:	Ita tangens basis	ad tangentem lateris quæsiti.

Modern: $R : \cos(\text{given angle}) :: \tan(\text{base}) : \tan(\text{sought side})$.

Result 2

Cit.	(As)	(to)	(so)	(to)
18. sinuū	Sed vt sinus totus	ad sinum compl. anguli dati:	Ita secans anguli dati	ad sinum totum.
11. quint i	2. Ergo vt secans anguli dati	ad sinum totum:	Ita tangens basis	ad tangentem lateris quæsiti.

Modern: $\sec(\text{given angle}) : R :: \tan(\text{base}) : \tan(\text{sought side})$.

Result 3 (*premises span the p. 106 → p. 107 boundary*)

Cit.	(As)	(to)	(so)	(to)
21. sinuū	Sed vt tangens basis	ad tangentem lateris quæsiti:	Ita tangens complem. lateris quæsiti	ad tangentem compl. basis.
11. quint	Ergo vt secans anguli dati	ad sinum totum:	Ita tangens complem.	ad tangentem compl. basis.

Cit.	(As)	(to)	(so)	(to)
i			lateris quæsi	

(p. 107 continues:)

Cit.	(As)	(to)	(so)	(to)
18.	Sed vt sinus sinuū totus	ad secantem anguli dati:	Ita sinus complem. anguli dati	ad sinum totum.
11.	4. Ergo vt sinus quint compl. anguli i dati	ad sinum totum:	Ita tangens compl. basis	ad tangentem compl. lateris quæsi.

Modern (result 4): $\cos(\text{given angle}) : R :: \cot(\text{base}) : \cot(\text{sought side})$. [*: the first numbered result on p. 107 is printed 4., so result 3's numbered conclusion falls at the foot of p. 106.*]

Result 5

Cit.	(As)	(to)	(so)	(to)
2.	Vt secans anguli modu dati	ad sinum totum:	Ita tangens basis	ad tangentem lateris quæsi.
Perm utand o	Ergo vt secans anguli dati	ad tangentem basis:	Ita sinus totus	ad tangentem lateris quæsi.
18.	Sed vt sinus sinuū totus	ad tangentem lateris quæsi:	Ita tangens complem. lateris quæsi	ad sinum totum.
11.	Ergo vt secans quinti anguli dati	ad tangentem basis:	Ita tang. compl. lateris quæsi	ad sinum totum.
Conue rtend o	Ergo vt tangens basis	ad secantem anguli dati	Ita sinus totus	ad tangentem compl. lateris quæsi.
Perm utand o	5. Ergo vt tangens basis	ad sinum totum:	Ita secans anguli dati	ad tangentem compl. lateris quæsi.

Modern: $\tan(\text{base}) : R :: \sec(\text{given angle}) : \cot(\text{sought side})$.

Result 6

Cit.	(As)	(to)	(so)	(to)
4. modu s	Vt sinus compl. anguli dati	ad sinum totum:	Ita tangens compl. basis	ad tangentem compl. lateris quæsiti.
Perm utand o	Ergo vt sinus compl. anguli dati	ad tangentem compl. basis:	Ita sinus totus	ad tangentem compl. lateris quæsiti.
18. sinuū	Sed vt sinus totus	ad tangentem compl. lateris quæsiti:	Ita tangens lateris quæsiti	ad sinum totum.
11. quinti	Ergo vt sinus compl. anguli dati	ad tangentem compl. basis:	Ita tangens lateris quæsiti	ad sinum totum.
Conue rtend o	Ergo vt tangens compl. basis	ad sinum compl. anguli dati:	Ita sinus totus	ad tangentem lateris quæsiti.
Perm utand o	6. Ergo vt tangens complem. basis	ad sinum totum:	Ita sinus complem. anguli dati	ad tangentem lateris quæsiti.

Modern: $\cot(\text{base}) : R :: \cos(\text{given angle}) : \tan(\text{sought side})$.

X.

Ex altero latere, & angulo, qui lateri quæsito adiacet; si modo constet, num quæsitum latus sit quadrante maius, an minus; vel an alter angulus non rectus non datus sit acutus, obtususue; vel denique num basis sit quadrante maior, aut minor.

[The side, from the other side and the angle adjacent to the sought side — provided it be established whether the sought side is greater or less than a quadrant; or whether the other, non-right, ungiven angle is acute or obtuse; or lastly whether the base is greater or less than a quadrant.]

Result 1

Cit.	(As)	(to)	(so)	(to)
44. tr. sph.	Vt tangens anguli dati	ad tangentem lateris dati:	Ita sinus totus	ad sinum lateris quæsiti.
Permu tando	1. Ergo vt tangens anguli dati	ad sinum totum:	Ita tangens lateris dati	ad sinum lateris quæsiti.

Modern: $\tan(\text{given angle}) : R :: \tan(\text{given side}) : \sin(\text{sought side})$.

Result 2

Cit.	(As)	(to)	(so)	(to)
44. tr. sph.	Vt tangens anguli dati	ad tangentem lateris dati:	Ita sinus totus	ad sinum lateris quæsiti.
21. sinuū	Sed vt tangens anguli dati	ad tangentem lateris dati:	Ita tangens complem. lateris dati	ad tangentem compl. anguli dati.
11. quinti	Ergo vt tangens compl. lateris dati	ad tangentem compl. anguli dati:	Ita sinus totus	ad sinum lateris quæsiti.
Permu tando	2. Ergo vt tangens comple. lat. dati	ad sinum totum:	Ita tangens compl. anguli dati	ad sinum lateris quæsiti.

Modern: $\cot(\text{given side}) : R :: \cot(\text{given angle}) : \sin(\text{sought side})$.

Result 3 (*derivation runs onto the next page*)

Cit.	(As)	(to)	(so)	(to)
2. modu s	Vt tangens compl. lat. dati	ad sinum totum:	Ita tangens compl. anguli dati	ad sinum lateris quæsiti.
Perm utand o	Ergo vt tangens compl. lateris dati	ad tangentem compl. anguli dati:	Ita sinus totus	ad sinum lateris quæsiti.
18. sinuū	Sed vt sinus totus	ad sinum lateris quæsiti:	Ita secans compl. lat. quæsiti	ad sinum totum.
11.	Ergo vt tangens	ad tangentem	Ita secans	ad sinum totum.

Cit.	(As)	(to)	(so)	(to)
quinti	compl. lateris dati	compl. anguli dati:	compl. lateris quæsit	
Conue rtend o	Ergo vt tangens comple. anguli dati	ad tangentem compl. lateris dati:	Ita sinus totus	ad secantem comple. lateris quæsit.

These three pages continue the **Calculus of right spherical triangles** (the apparatus of four-term proportions belonging to Lemma LIII). The material is wholly tabular: for each "case" — a particular pair of given parts — Clavius lists several numbered **results** (the working formulae), each derived through a short chain of proportions. The left- or right-hand marginal note cites the authority for each step. The Calculus does **not** end within these pages and no PROBLEMA is reached; case XIV breaks off unfinished at the foot of p. 110 (catchword "Vt sinus").

How to read each row. A row states a proportion *As* (1st term) *is to* (2nd term), *so* (3rd term) *is to* (4th term) — i.e. 1st : 2nd :: 3rd : 4th. In the **Type** column: blank = *Vt* ("as," a premise); **but** = *Sed vt* ("but as," a second premise); $\therefore$ = *Ergo vt* ("therefore as," a derived step); $\therefore$ (**n**) = the numbered result *n* of the case.

Terms rendered in modern function language. ST = *sinus totus*, the whole sine (radius *R*). "cos *X*" renders *sinus complementi X*; "cot *X*" renders *tangens complementi X*; "csc *X*" renders *secans complementi X*; sin / tan / sec as usual.

Arguments: GA = the given angle; GS = the given side; QS = the sought side; base = the base; adj $\angle$ = the angle adjacent to the sought side; opp $\angle$ = the angle opposite the sought side.

Marginal citations: Tri.sph. *N* = Clavius, *Triangula Sphaerica*, prop. *N*; Sin. *N* = his canon of sines [*sinuum*], prop. *N*; Eucl. V.*N* = Euclid, *Elements* V.*N*; alt. = *permutando* (alternately); conv. = *convertendo* (by conversion); "1st/2nd/3rd method" = *per primum / secundum / tertium modum*.

Case X — continued

(Results 1–3 stood on the preceding pages; result 4 is here completed, then results 5 and 6.)

Cite	Type	1st	2nd	3rd	4th
alt.	$\therefore$ (4)	cot GA	ST	cot GS	csc QS

First method → result 5:

Cite	Type	1st	2nd	3rd	4th
1st method	as	tan GA	ST	tan GS	sin QS
Sin. 18	but	tan GA	ST	ST	cot GA
Eucl. V.11	$\therefore$ (5)	ST	cot GA	tan GS	sin QS

Third method → result 6:

Cite	Type	1st	2nd	3rd	4th
3rd method	as	tan GS	ST	tan GA	csc QS
Sin. 18	but	tan GS	ST	ST	cot GS
Eucl. V.11	$\therefore$ (6)	ST	cot GS	tan GA	csc QS

XI. A SIDE.

From the other side, and the angle which is opposite to the sought side.

(XI. LATVS. Ex altero latere, & angulo, qui lateri quæsito opponitur.)

→ **results 1 and 2:**

Cite	Type	1st	2nd	3rd	4th
Tri.sph. 44	(1)	ST	sin GS	tan GA	tan QS
Sin. 21	but	tan GA	tan QS	cot QS	cot GA
Eucl. V.11	$\therefore$	ST	sin GS	cot QS	cot GA
conv.	$\therefore$ (2)	sin GS	ST	cot GA	cot QS

→ **result 3:**

Cite	Type	1st	2nd	3rd	4th
Sin. 18	but	sin GS	ST	ST	csc GS
Eucl. V.11	$\therefore$ (3)	ST	csc GS	cot GA	cot QS

→ **result 4:**

Cite	Type	1st	2nd	3rd	4th
Tri.sph. 44	as	ST	sin GS	tan GA	tan QS
Sin. 18	but	ST	sin GS	csc GS	ST

Cite	Type	1st	2nd	3rd	4th
Eucl. V.11	$\therefore$ (4)	csc GS	ST	tan GA	tan QS

Second method → result 5:

Cite	Type	1st	2nd	3rd	4th
2nd method	as	sin GS	ST	cot GA	cot QS
alt.	$\therefore$	sin GS	cot GA	ST	cot QS
Sin. 18	but	ST	cot QS	tan QS	ST
Eucl. V.11	$\therefore$	sin GS	cot GA	tan QS	ST
conv.	$\therefore$	cot GA	sin GS	ST	tan QS
alt.	$\therefore$ (5)	cot GA	ST	sin GS	tan QS

Third method → result 6:

Cite	Type	1st	2nd	3rd	4th
3rd method	as	ST	csc GS	cot GA	cot QS
alt.	$\therefore$	ST	cot GA	csc GS	cot QS
Sin. 18	but	ST	cot GA	tan GA	ST
Eucl. V.11	$\therefore$ (6)	tan GA	ST	csc GS	cot QS

XII. A SIDE.

From each of the two non-right angles.

(XII. LATVS. Ex vtroque angulo non recto.)

→ **result 1:**

Cite	Type	1st	2nd	3rd	4th
Tri.sph. 42	(1)	sin adj $\angle$	ST	cos opp $\angle$	cos QS

→ **result 2:**

Cite	Type	1st	2nd	3rd	4th
Sin. 18	but	sin adj $\angle$	ST	ST	csc adj $\angle$
Eucl. V.11	$\therefore$ (2)	ST	csc adj $\angle$	cos opp $\angle$	cos QS

→ **result 3:**

Cite	Type	1st	2nd	3rd	4th
Tri.sph. 42	as	sin adj $\angle$	ST	cos opp $\angle$	cos QS
alt.	$\therefore$	sin adj $\angle$	cos opp $\angle$	ST	cos QS
Sin. 18	but	ST	cos QS	sec QS	ST
Eucl. V.11	$\therefore$	sin adj $\angle$	cos opp $\angle$	sec QS	ST
conv.	$\therefore$	cos opp $\angle$	sin adj $\angle$	ST	sec QS
alt.	$\therefore$ (3)	cos opp $\angle$	ST	sin adj $\angle$	sec QS

→ **result 4:**

Cite	Type	1st	2nd	3rd	4th
Sin. 18	but	cos opp $\angle$	ST	ST	sec opp $\angle$
Eucl. V.11	$\therefore$ (4)	ST	sec opp $\angle$	sin adj $\angle$	sec QS

→ **result 5:**

Cite	Type	1st	2nd	3rd	4th
Tri.sph. 42	as	sin adj $\angle$	ST	cos opp $\angle$	cos QS
alt.	$\therefore$	sin adj $\angle$	cos opp $\angle$	ST	cos QS
Sin. 22	but	sin adj $\angle$	cos opp $\angle$	sec opp $\angle$	csc adj $\angle$
Eucl. V.11	$\therefore$	sec opp $\angle$	csc adj $\angle$	ST	cos QS
alt.	$\therefore$ (5)	sec opp $\angle$	ST	csc adj $\angle$	cos QS

Third method → result 6:

Cite	Type	1st	2nd	3rd	4th
3rd method	as	cos opp $\angle$	ST	sin adj $\angle$	sec QS
alt.	$\therefore$	cos opp $\angle$	sin adj $\angle$	ST	sec QS
Sin. 22	but	cos opp $\angle$	sin adj $\angle$	csc adj $\angle$	sec opp $\angle$
Eucl. V.11	$\therefore$	csc adj $\angle$	sec opp $\angle$	ST	sec QS
alt.	$\therefore$ (6)	csc adj $\angle$	ST	sec opp $\angle$	sec QS

XIII. THE BASE.

From a side and the angle adjacent to it.

(XIII. BASIS. Ex latere, & angulo ei adiacente.)

→ **result 1:**

Cite	Type	1st	2nd	3rd	4th
Tri.sph. 45	(1)	cos GA	ST	tan GS	tan base

→ **result 2:**

Cite	Type	1st	2nd	3rd	4th
Sin. 18 [?]	but	cos GA	ST	ST	sec GA
Eucl. V.11	∴ (2)	ST	sec GA	tan GS	tan base

→ **result 3:**

Cite	Type	1st	2nd	3rd	4th
Sin. 21	but	tan GS	tan base	cot base	cot GS
Eucl. V.11	∴	ST	sec GA	cot base	cot GS
conv.	∴ (3)	sec GA	ST	cot GS	cot base

→ **result 4:**

Cite	Type	1st	2nd	3rd	4th
Sin. 18	but	sec GA	ST	ST	cos GA
Eucl. V.11	∴ (4)	ST	cos GA	cot GS	cot base

First method (continued) → result 5:

Cite	Type	1st	2nd	3rd	4th
Tri.sph. 45	as	cos GA	ST	tan GS	tan base
alt.	∴	cos GA	tan GS	ST	tan base
Sin. 18	but	ST	tan base	cot base	ST
Eucl. V.11	∴	cos GA	tan GS	cot base	ST
conv.	∴	tan GS	cos GA	ST	cot base
alt.	∴ (5)	tan GS	ST	cos GA	cot base

Second method → result 6:

Cite	Type	1st	2nd	3rd	4th
2nd method	as	ST	sec GA	tan GS	tan base
alt.	∴	ST	tan GS	sec GA	tan base

Cite	Type	1st	2nd	3rd	4th
Sin. 18	but	ST	tan GS	cot GS	ST
Eucl. V.11	$\therefore$ (6)	cot GS	ST	sec GA	tan base

XIV. THE BASE.

From a side and the angle opposite to it — provided it be established whether the base is greater than a quadrant or smaller; or whether the other (not given) angle is acute or obtuse; or, finally, whether the other (not given) side is smaller than a quadrant or greater.

(XIV. BASIS. Ex latere, & angulo ei opposito: Si modo constet, num basis quadrante maior sit, vel minor: Aut an alter angulus non datus sit acutus, obtususve: Aut denique num alterum latus non datum, minus sit quadrante, an maius.)

→ **result 1:**

Cite	Type	1st	2nd	3rd	4th
Tri.sph. 41	as	sin GA	sin GS	ST	sin base
alt.	$\therefore$ (1)	sin GA	ST	sin GS	sin base

→ **result 2:**

Cite	Type	1st	2nd	3rd	4th
Sin. 18	but	sin GA	ST	ST	csc GA
Eucl. V.11	$\therefore$ (2)	ST	csc GA	sin GS	sin base

→ **result 3:**

Cite	Type	1st	2nd	3rd	4th
Tri.sph. 41	as	sin GA	sin GS	ST	sin base
Sin. 18	but	ST	sin base	csc base	ST
Eucl. V.11	$\therefore$	sin GA	sin GS	csc base	ST
conv.	$\therefore$	sin GS	sin GA	ST	csc base
alt.	$\therefore$ (3)	sin GS	ST	sin GA	csc base

→ **result 4:**

Cite	Type	1st	2nd	3rd	4th
Sin. 18	but	sin GS	ST	ST	csc GS
Eucl. V.11	$\therefore$ (4)	ST	csc GS	sin GA	csc base

→ **result 5:**

Cite	Type	1st	2nd	3rd	4th
Tri.sph. 41	as	sin GA	sin GS	ST	sin base
Sin. 22	but	sin GA	sin GS	csc GS	csc GA
Eucl. V.11	$\therefore$	csc GS	csc GA	ST	sin base
alt.	$\therefore$ (5)	csc GS	ST	csc GA	sin base

(Case XIV breaks off here; the apparatus continues on the following page.)

Astrolabium — Liber I

Calculus of right spherical triangles (continued)

Case XIV (concluding modes)

The remaining modes of the fourteenth case conclude as follows:

No.	Proportion	By
	As the sine of the given side is to the whole sine, so the sine of the given angle is to the cosecant of the base.	the 3rd mode
	Therefore as the sine of the given side is to the sine of the given angle, so the whole sine is to the cosecant of the base.	permutando
	But as the sine of the given side is to the sine of the given angle, so the cosecant of the given angle is to the cosecant of the given side.	22 of the Sines
	Therefore as the cosecant of the given angle is to the cosecant of the given side, so the whole sine is to the cosecant of the base.	11 of Book V
6.	Therefore as the cosecant of the given angle is to the whole sine, so the cosecant of the	permutando

No.	Proportion	By
	given side is to the cosecant of the base.	

XV. THE BASE.

From both sides, of which either may be set as the first and the other as the second.

No	Proportion	By
1.	As the whole sine is to the cosine of the 1st side, so the cosine of the 2nd side is to the cosine of the base. But as the whole sine is to the cosine of the 1st side, so the secant of the 1st side is to the whole sine.	43 of the Spherical Triangles 18 of the Sines
2.	Therefore as the secant of the 1st side is to the whole sine, so the cosine of the 2nd side is to the cosine of the base. As the whole sine is to the cosine of the 1st side, so the cosine of the 2nd side is to the cosine of the base. Therefore as the whole sine is to the cosine of the 2nd side, so the cosine of the 1st side is to the cosine of the base. But as the whole sine is to the cosine of the 2nd side, so the secant of the 2nd side is to the whole sine.	11 of Book V 43 of the Spherical Triangles permutando 18 of the Sines
3.	Therefore as the secant of the 2nd side is to the whole sine, so the cosine of the 1st side is to the cosine of the base. As the whole sine is to the cosine of the 1st side, so the cosine of the 2nd side is to the cosine of the base. But as the cosine of the 2nd side is to the cosine of the base, so the secant	11 of Book V 43 of the Spherical Triangles 22 of the Sines

No

.	Proportion	By
	of the base is to the secant of the 2nd side. Therefore as the whole sine is to the cosine of the 1st side, so the secant of the base is to the secant of the 2nd side.	11 of Book V
4.	Therefore as the cosine of the 1st side is to the whole sine, so the secant of the 2nd side is to the secant of the base. But as the cosine of the 1st side is to the whole sine, so the whole sine is to the secant of the 1st side.	convertendo 18 of the Sines
5.	Therefore as the whole sine is to the secant of the 1st side, so the secant of the 2nd side is to the secant of the base. As the whole sine is to the cosine of the 1st side, so the cosine of the 2nd side is to the cosine of the base. Therefore as the whole sine is to the cosine of the 2nd side, so the cosine of the 1st side is to the cosine of the base. But as the cosine of the 1st side is to the cosine of the base, so the secant of the base is to the secant of the 1st side. Therefore as the whole sine is to the cosine of the 2nd side, so the secant of the base is to the secant of the 1st side.	11 of Book V 43 of the Spherical Triangles permutando 22 of the Sines 11 of Book V
6.	Therefore as the cosine of the 2nd side is to the whole sine, so the secant of the 1st side is to the secant	convertendo

No		
.	Proportion	By
	of the base.	

XVI. THE BASE.

From both non-right angles, of which either may be set as the first and the other as the second.

No		
.	Proportion	By
1.	As the whole sine is to the cotangent of the 1st angle, so the cotangent of the 2nd angle is to the cosine of the base.	56 of the Spherical Triangles
	But as the whole sine is to the cotangent of the 1st angle, so the tangent of the 1st angle is to the whole sine.	18 of the Sines
2.	Therefore as the tangent of the 1st angle is to the whole sine, so the cotangent of the 2nd angle is to the cosine of the base.	11 of Book V
	As the whole sine is to the cotangent of the 1st angle, so the cotangent of the 2nd angle is to the cosine of the base.	56 of the Spherical Triangles
	Therefore as the whole sine is to the cotangent of the 2nd angle, so the cotangent of the 1st angle is to the cosine of the base.	permutando
	But as the whole sine is to the cotangent of the 2nd angle, so the tangent of the 2nd angle is to the whole sine.	18 of the Sines
3.	Therefore as the tangent of the 2nd angle is to the whole sine, so the cotangent of the 1st angle is to the	11 of Book V

No

.

Proportion

By

cosine of the base.

As the tangent of the 1st angle is to the whole sine, so the cotangent of the 2nd angle is to the cosine of the base.

the 2nd mode

Therefore as the tangent of the 1st angle is to the cotangent of the 2nd angle, so the whole sine is to the cosine of the base.

permutando

But as the whole sine is to the cosine of the base, so the secant of the base is to the whole sine.

18 of the Sines

Therefore as the tangent of the 1st angle is to the cotangent of the 2nd angle, so the secant of the base is to the whole sine.

11 of Book V

Therefore as the cotangent of the 2nd angle is to the tangent of the 1st angle, so the whole sine is to the secant of the base.

convertendo

4. Therefore as the cotangent of the 2nd angle is to the whole sine, so the tangent of the 1st angle is to the secant of the base.

permutando

As the tangent of the 2nd angle is to the whole sine, so the cotangent of the 1st angle is to the cosine of the base.

the 3rd mode

Therefore as the tangent of the 2nd angle is to the cotangent of the 1st angle, so the whole sine is to the cosine of the base.

permutando

But as the whole sine is to the cosine of the base, so the secant of the base is to the whole sine.

18 of the Sines

No	Proportion	By
.	Therefore as the tangent of the 2nd angle is to the cotangent of the 1st angle, so the secant of the base is to the whole sine.	11 of Book V
	Therefore as the cotangent of the 1st angle is to the tangent of the 2nd angle, so the whole sine is to the secant of the base.	convertendo
5.	Therefore as the cotangent of the 1st angle is to the whole sine, so the tangent of the 2nd angle is to the secant of the base.	permutando
	As the cotangent of the 2nd angle is to the whole sine, so the tangent of the 1st angle is to the secant of the base.	the 4th mode
	But as the cotangent of the 2nd angle is to the whole sine, so the whole sine is to the tangent of the 2nd angle.	18 of the Sines
6.	Therefore as the whole sine is to the tangent of the 2nd angle, so the tangent of the 1st angle is to the secant of the base.	11 of Book V

The sixteen problems

With these matters thus demonstrated, and in order that what is sought in a right spherical triangle may be found more readily and the whole working of the rule of proportions may lie before one's eyes, we have here arranged in order the sixteen problems just proved, in such a way that each of them may be solved by six modes — in all of which the whole sine is found either in the first place of the rule or in the second. The order, then, is this:

IN A RIGHT SPHERICAL TRIANGLE

the following can be investigated by all these modes.

I. THE ANGLE. [Problem I]

From the base and the side that is opposite the sought angle.

Mode	Proportion
1	As the whole sine is to the sine of the base, so the cosecant of the side is to the cosecant of the angle.
2	As the whole sine is to the sine of the side, so the cosecant of the base is to the sine of the angle.
3	As the sine of the base is to the whole sine, so the sine of the side is to the sine of the angle.
4	As the cosecant of the side is to the whole sine, so the cosecant of the base is to the sine of the angle.
5	As the cosecant of the base is to the whole sine, so the cosecant of the side is to the cosecant of the angle.
6	As the sine of the side is to the whole sine, so the sine of the base is to the cosecant of the angle.

The angle found will be acute if the given side is less than a quadrant, and obtuse if greater.

II. THE ANGLE. [Problem II]

From the base and the side that is adjacent to the sought angle.

Mode	Proportion
1	As the whole sine is to the cotangent of the base, so the tangent of the side is to the cosine of the angle.
2	As the whole sine is to the cotangent of the side, so the tangent of the base is to the secant of the angle.
3	As the tangent of the base is to the whole sine, so the tangent of the side is to the cosine of the angle.
4	As the cotangent of the side is to the whole sine, so the cotangent of the base is to the cosine of the angle.
5	As the cotangent of the base is to the whole sine, so the cotangent of the side is to the secant of the angle.
6	As the tangent of the side is to the whole sine, so the tangent of the base is to the secant of the angle.

The angle found will be acute if both the base and the given side are greater than a quadrant, or both less; but obtuse if one of the given magnitudes is greater than a quadrant and the other less.

III. THE ANGLE. [Problem III]

From the base and one of the non-right angles.

Mode	Proportion
1	As the whole sine is to the cosine of the base, so the tangent of the given angle is to the cotangent of the sought angle.
2	As the whole sine is to the secant of the base, so the cotangent of the given angle is to the tangent of the sought angle.
3	As the secant of the base is to the whole sine, so the tangent of the given angle is to the cotangent of the sought angle.
4	As the cotangent of the given angle is to the whole sine, so the cosine of the base is to the cotangent of the sought angle.
5	As the tangent of the given angle is to the whole sine, so the secant of the base is to the tangent of the sought angle.
6	As the cosine of the base is to the whole sine, so the cotangent of the given angle is to the tangent of the sought angle.

The angle found will be acute if the base is less than a quadrant and the given angle acute, or if the base is greater than a quadrant and the given angle obtuse. The same angle will be obtuse if the base is less than a quadrant and the given angle obtuse, or if the base is greater than a quadrant and the given angle acute.

IV. THE ANGLE. [Problem IV]

From the side that is opposite the sought angle, and one of the non-right angles.

Mode	Proportion
1	As the whole sine is to the sine of the given angle, so the cosine of the side is to the cosine of the sought angle.
2	As the whole sine is to the cosecant of the given angle, so the secant of the side is to the secant of the sought angle.
3	As the sine of the given angle is to the whole sine, so the secant of the side is to the secant of the sought angle.
4	As the cosine of the side is to the whole sine, so the cosecant of the given angle is to the secant of the sought angle.

Mode	Proportion
5	As the cosecant of the given angle is to the whole sine, so the cosine of the side is to the cosine of the sought angle.
6	As the secant of the side is to the whole sine, so the sine of the given angle is to the cosine of the sought angle.

The angle found will be acute if the given side is less than a quadrant, and obtuse if greater.

ASTROLABIUM — Liber I

The sixteen problems of the right spherical triangle (continued)

This batch continues the tabular apparatus that solves each of the sixteen cases of the right spherical triangle by six proportion "modi." Each problem is stated as a proportion of four terms — "As [term 1] is to [term 2], so [term 3] is to [term 4]" — from which the sought part follows, together with a note settling whether the result is acute/obtuse or greater/less than a quadrant.

Notation used below. Each row is rendered in modern function terms while preserving all four terms exactly as printed:

- **R** = the whole sine (*sinus totus*, i.e. the radius)
- **sin** / **cos** = *sinus* / *sinus complementi*
- **tan** / **cot** = *tangens* / *tangens complementi*
- **sec** / **csc** = *secans* / *secans complementi*
- **∠g** = the given angle; **ℓg** = the given leg (*latus*); **base** = the base (hypotenuse)
- **∠s** = the sought angle; **ℓs** = the sought leg

Each proportion reads: As (col. 1) is to (col. 2), so (col. 3) is to (col. 4).

V. ANGVLVS

[Margin: V. Problema.]

From the leg adjacent to the sought angle, and from the other (non-right) angle — provided it is established whether the sought angle is greater than a

right angle or less; or whether the base, or the other leg (not given), is greater than a quadrant or less.

As	is to	so	is to
R	$\sec(\ell g)$	$\cos(\angle g)$	$\sin(\angle s)$
R	$\sec(\angle g)$	$\cos(\ell g)$	$\csc(\angle s)$
$\cos(\ell g)$	R	$\cos(\angle g)$	$\sin(\angle s)$
$\cos(\angle g)$	R	$\cos(\ell g)$	$\csc(\angle s)$
$\sec(\ell g)$	R	$\sec(\angle g)$	$\csc(\angle s)$
$\sec(\angle g)$	R	$\sec(\ell g)$	$\sin(\angle s)$

The angle found will be acute (unless established otherwise from elsewhere) if the other, non-given leg is less than a quadrant; obtuse if it is greater. By the same reasoning, the angle found will be acute if the base is less than a quadrant and the given angle acute, or if the base is greater than a quadrant and the given angle obtuse. But if the base is less than a quadrant and the given angle obtuse, or if the base is greater than a quadrant and the given angle acute, the angle found will be obtuse.

VI. ANGVLVS

[Margin: VI. Problema.]

From both legs about the right angle.

As	is to	so	is to
R	$\sin(\text{leg adjacent to sought angle})$	$\cot(\text{leg opposite sought angle})$	$\cot(\angle s)$
R	$\csc(\text{leg adjacent to sought angle})$	$\tan(\text{leg opposite sought angle})$	$\tan(\angle s)$
$\sin(\text{leg adjacent to sought angle})$	R	$\tan(\text{leg opposite sought angle})$	$\tan(\angle s)$
$\tan(\text{leg opposite sought angle})$	R	$\sin(\text{leg adjacent to sought angle})$	$\cot(\angle s)$
$\csc(\text{leg adjacent to sought angle})$	R	$\cot(\text{leg opposite sought angle})$	$\cot(\angle s)$
$\cot(\text{leg opposite sought angle})$	R	$\csc(\text{leg adjacent to sought angle})$	$\tan(\angle s)$

The angle found will be acute if the given leg opposite the sought angle is less than a quadrant; obtuse if it is greater.

VII. ANGVLVS

[Margin: VII. Problema.]

From the base and one leg.

As	is to	so	is to
R	$\sec(\ell g)$	$\cos(\text{base})$	$\cos(\ell s)$
R	$\sec(\text{base})$	$\cos(\ell g)$	$\sec(\ell s)$
$\cos(\ell g)$	R	$\cos(\text{base})$	$\cos(\ell s)$
$\cos(\text{base})$	R	$\cos(\ell g)$	$\sec(\ell s)$
$\sec(\text{base})$	R	$\sec(\ell g)$	$\cos(\ell s)$
$\sec(\ell g)$	R	$\sec(\text{base})$	$\sec(\ell s)$

The leg found will be less than a quadrant if both the base and the given leg are less than a quadrant; but greater than a quadrant if either the base is greater and the given leg less than a quadrant, or the base is less and the given leg greater than a quadrant.

VIII. LATVS

[Margin: VIII. Problema.]

From the base and the angle opposite the sought leg.

As	is to	so	is to
R	$\sin(\text{base})$	$\sin(\angle g)$	$\sin(\ell s)$
R	$\csc(\text{base})$	$\csc(\angle g)$	$\csc(\ell s)$
$\sin(\text{base})$	R	$\csc(\angle g)$	$\csc(\ell s)$
$\csc(\text{base})$	R	$\sin(\angle g)$	$\sin(\ell s)$
$\csc(\angle g)$	R	$\sin(\text{base})$	$\sin(\ell s)$
$\sin(\angle g)$	R	$\csc(\text{base})$	$\csc(\ell s)$

The leg found will be less than a quadrant if the given angle opposite it is acute; greater if it is obtuse.

IX. LATVS

[Margin: IX. Problema.]

From the base and the angle adjacent to the sought leg.

As	is to	so	is to
R	$\cos(\angle g)$	$\tan(\text{base})$	$\tan(\ell s)$
R	$\sec(\angle g)$	$\cot(\text{base})$	$\cot(\ell s)$
$\sec(\angle g)$	R	$\tan(\text{base})$	$\tan(\ell s)$
$\cos(\angle g)$	R	$\cot(\text{base})$	$\cot(\ell s)$
$\tan(\text{base})$	R	$\sec(\angle g)$	$\cot(\ell s)$
$\cot(\text{base})$	R	$\cos(\angle g)$	$\tan(\ell s)$

The leg found will be less than a quadrant if the base is less than a quadrant and the given angle acute, or if the base is greater than a quadrant and the given angle obtuse; but greater than a quadrant if the base is less than a quadrant and the given angle obtuse, or if the base is greater than a quadrant and the given angle acute.

X. LATVS

[Margin: X. Problema.]

From the other leg and the angle adjacent to the sought leg — provided it is established whether the sought leg is greater than a quadrant or less; or whether the other (non-right, non-given) angle is acute or obtuse; or finally whether the base is greater than a quadrant or less.

As	is to	so	is to
R	$\cot(\angle g)$	$\tan(\ell g)$	$\sin(\ell s)$
R	$\cot(\ell g)$	$\tan(\angle g)$	$\csc(\ell s)$
$\tan(\angle g)$	R	$\tan(\ell g)$	$\sin(\ell s)$
$\cot(\ell g)$	R	$\cot(\angle g)$	$\sin(\ell s)$
$\tan(\ell g)$	R	$\tan(\angle g)$	$\csc(\ell s)$
$\cot(\angle g)$	R	$\cot(\ell g)$	$\csc(\ell s)$

The leg found will be less than a quadrant (unless established otherwise from elsewhere) if the angle opposite it, which is not given, is acute; greater if it is

obtuse. By the same reasoning it will be less if the base is less than a quadrant and the given leg also less than a quadrant; but if the base is less than a quadrant and the given leg greater, the leg found will be greater than a quadrant. Finally, if both the base and the given leg are greater than a quadrant, the leg found will be less than a quadrant; but greater if the base is greater than a quadrant and the given leg less.

XI. LATVS

[Margin: XI. Problema.]

From the other leg and the angle opposite the sought leg.

As	is to	so	is to
R	$\sin(\ell g)$	$\tan(\angle g)$	$\tan(\ell s)$
R	$\csc(\ell g)$	$\cot(\angle g)$	$\cot(\ell s)$
$\sin(\ell g)$	R	$\cot(\angle g)$	$\cot(\ell s)$
$\csc(\ell g)$	R	$\tan(\angle g)$	$\tan(\ell s)$
$\cot(\angle g)$	R	$\sin(\ell g)$	$\tan(\ell s)$
$\tan(\angle g)$	R	$\csc(\ell g)$	$\cot(\ell s)$

The leg found will be less than a quadrant if the given angle opposite it is acute; greater if it is obtuse.

XII. LATVS

[Margin: XII. Problema.]

From both non-right angles.

(Here " $\angle adj$ " = the angle adjacent to the sought leg, " $\angle opp$ " = the angle opposite the sought leg.)

As	is to	so	is to
R	$\csc(\angle adj)$	$\cos(\angle opp)$	$\cos(\ell s)$
R	$\sec(\angle opp)$	$\sin(\angle adj)$	$\sec(\ell s)$
$\sin(\angle adj)$	R	$\cos(\angle opp)$	$\cos(\ell s)$
$\cos(\angle opp)$	R	$\sin(\angle adj)$	$\sec(\ell s)$
$\sec(\angle opp)$	R	$\csc(\angle adj)$	$\cos(\ell s)$
$\csc(\angle adj)$	R	$\sec(\angle opp)$	$\sec(\ell s)$

The leg found will be less than a quadrant if the given angle opposite it is acute; greater if it is obtuse.

XIII. BASIS

[Margin: XIII. Problema.]

From a leg and the angle adjacent to it.

As	is to	so	is to
R	$\cos(\angle g)$	$\cot(\ell g)$	$\cot(\text{base})$
R	$\sec(\angle g)$	$\tan(\ell g)$	$\tan(\text{base})$
$\cos(\angle g)$	R	$\tan(\ell g)$	$\tan(\text{base})$
$\sec(\angle g)$	R	$\cot(\ell g)$	$\cot(\text{base})$
$\tan(\ell g)$	R	$\cos(\angle g)$	$\cot(\text{base})$
$\cot(\ell g)$	R	$\sec(\angle g)$	$\tan(\text{base})$

The base found will be less than a quadrant if the given leg is less than a quadrant and the given angle adjacent to it is acute, or if the given leg is greater than a quadrant and the given angle adjacent to it is obtuse; but greater than a quadrant if the given leg is greater than a quadrant and the given angle adjacent to it is acute, or if the given leg is less than a quadrant and the given angle obtuse.

XIV. BASIS

[Margin: XIV. Problema.]

From a leg and the angle opposite it — provided it is established whether the base is greater than a quadrant or less; or whether the other (non-given) angle is acute or obtuse; or finally whether the other leg (not given) is less than a quadrant or greater.

As	is to	so	is to
R	$\csc(\angle g)$	$\sin(\ell g)$	$\sin(\text{base})$

[Problem XIV, *BASIS*, concluded — proportions 2–6, continued from the preceding page]

The whole sine [sinus totus] is the radius, R; "the sine of the complement" is the cosine, "the tangent of the complement" the cotangent, and "the secant of the

complement" the cosecant. These modern equivalents are glossed here only on first occurrence.

2. As the whole sine [sinus totus] is to the secant of the complement of the given side [i.e. the cosecant of the side], so is the sine of the given angle to the secant of the complement of the base.
3. As the sine of the given angle is to the whole sine, so is the sine of the given side to the sine of the base.
4. As the sine of the given side is to the whole sine, so is the sine of the given angle to the secant of the complement of the base.
5. As the secant of the complement of the given side is to the whole sine, so is the secant of the complement of the given angle to the sine of the base.
6. As the secant of the complement of the given angle is to the whole sine, so is the secant of the complement of the given side to the secant of the complement of the base.

Note. The base found will be less than a quadrant (unless it is established otherwise) if both of the non-right angles are acute or both obtuse; or if both sides are less than a quadrant, or both greater. But the base found will be greater than a quadrant if one of the non-right angles is acute and the other obtuse; or if one of the sides is less than a quadrant and the other greater.

XV. BASIS

[Margin: XV. Problema]

From both sides: of which let either be taken as the first, and the other as the second.

1. As the whole sine is to the sine of the complement of the first side [i.e. the cosine of the first side], so is the sine of the complement of the second side to the sine of the complement of the base.
2. As the whole sine is to the secant of the first side, so is the secant of the second side to the secant of the base.
3. As the secant of the first side is to the whole sine, so is the sine of the complement of the second side to the sine of the complement of the base.
4. As the secant of the second side is to the whole sine, so is the sine of the complement of the first side to the sine of the complement of the base.
5. As the sine of the complement of the first side is to the whole sine, so is the secant of the second side to the secant of the base.

6. As the sine of the complement of the second side is to the whole sine, so is the secant of the first side to the secant of the base.

Note. The base found will be less than a quadrant if both sides are less than a quadrant, or both greater; but greater than a quadrant if one of the sides is less than a quadrant and the other greater.

XVI. BASIS

[Margin: XVI. Problema]

From both non-right angles: of which let either be taken as the first, and the other as the second.

1. As the whole sine is to the tangent of the complement of the first angle [i.e. the cotangent of the first angle], so is the tangent of the complement of the second angle to the sine of the complement of the base.
2. As the whole sine is to the tangent of the second angle, so is the tangent of the first angle to the secant of the base.
3. As the tangent of the first angle is to the whole sine, so is the tangent of the complement of the second angle to the sine of the complement of the base.
4. As the tangent of the second angle is to the whole sine, so is the tangent of the complement of the first angle to the sine of the complement of the base.
5. As the tangent of the complement of the second angle is to the whole sine, so is the tangent of the first angle to the secant of the base.
6. As the tangent of the complement of the first angle is to the whole sine, so is the tangent of the second angle to the secant of the base.

Note. The base found will be less than a quadrant if both non-right angles are acute, or both obtuse; but greater if one of the non-right angles is acute and the other obtuse.

THE COMPUTATION OF OBLIQUE-ANGLED SPHERICAL TRIANGLES

[*Triangulorum Sphaericorum obliquangulorum calculus.*]

17. Given the sum of two arcs or of two angles that is less than a semicircle, together with the ratio that their sines have, to make each of them known.

[Margin: XVII. Problema]

If the terms of the given ratio are not sines, let them be reduced to sines by multiplying each by 10, 100, 1000, 10000, 100000, 1000000, so that the greater term has as many figures as are contained in the greater sines in the table of sines.

[Margin: (a) 17 or 18 of the seventh (book).] For these sines will have the same ratio as the earlier terms of the given ratio. Then let these terms, reduced to sines, be collected into a single sum; and let the half of that sum, together with the difference between that half and either of the terms, be taken as arcs from the table of sines — exactly as if that half and that difference were themselves sines — and let both be kept separately. And it will be:

- As the whole sine is to the secant of the complement of the greater arc kept — namely the one that corresponds to the half of the sum of the terms — so is the aforesaid difference (that is, the sine of the lesser arc kept) to a fourth [number].
- *Then:* As the whole sine is to the tangent of the half of the sum of the arcs or angles, so is the fourth number found to the tangent of the difference between the half of the sum of the arcs (or angles) and either of the sought arcs.

The arc of this tangent found, added to the half of the sum of the arcs or angles, produces the greater sought arc or angle; but subtracted from that same half, it leaves the lesser sought arc or angle. That the tangent of the said difference is found by that twofold operation will become clear as follows. For, as we demonstrated in Proposition 6 of [the book on] rectilinear triangles, the half of the sum of the terms of the given ratio (reduced to sines) is to the tangent of the half of the sum of the arcs as the difference between the half of the sum of the terms of the given ratio and either of the terms is to the tangent of the difference between the half of the sum of the arcs and either of the sought arcs. By permutation it will also be: as the half of the sum of the terms is to the said difference, so is the tangent of the half of the sum of the arcs to the tangent of the difference of the arcs. But as the half of the sum of the terms is to the whole sine, so is the said difference to another fourth number; and by permutation, as the half of the sum of the terms is to the said difference, so is the whole sine to that fourth number. Therefore it will also hold that as the whole sine is to the fourth, so is the tangent of the half of the sum of the arcs to the tangent of the difference of the arcs; and by permutation, as the whole sine is to the tangent of the half of the sum of the arcs, so is the fourth to the tangent of the difference of the arcs — as we were saying in the second example of the rule of proportion. And that the fourth number is produced in the way set out in the first example will be evident thus. For it is:

as the half of the sum of the terms is to the whole sine, so is the aforesaid difference to that fourth, as we said just above. [Margin: (a) 18 of sines.] But it is also: as the half of the sum of the terms, taken as a sine, is to the whole sine, so is the whole sine to the secant of the complement of the arc that is owed to that half as its sine — which we also showed above in the Prosthaphaeresis, Number 6. Therefore it will also hold that as the whole sine is to the secant of the complement of the arc that is owed (as its sine) to the half of the sum of the terms, so is the aforesaid difference to the fourth, as was laid down in the first example of the golden rule.

But the tangent of the difference between the half of the sum of the arcs and either of the sought arcs will also be found by a single operation, yet without the whole sine. [Margin: (b) 6 of rectilinear triangles.] For it is:

- As the half of the sum of the terms of the given ratio is to the tangent of the half of the sum of the arcs, so is the difference between the half of the sum of the terms and either of the terms to the tangent of the difference between the half of the sum of the arcs and either of the arcs.

18. Given the sum of two arcs, each of which is less than a semicircle, or of two angles — a sum which is greater than a semicircle — together with the ratio of their sines, to make each known.

[Margin: XVIII. Problema]

If this sum is subtracted from the whole circle, there will remain another sum of arcs, less than a semicircle, with the same given ratio, as was said in Proposition 6 of [the book on] rectilinear triangles. If, therefore, each arc or angle of this [remaining] sum is investigated as we set forth in the preceding Problem 17, and each one found is taken away from the semicircle, the two sought arcs or angles will remain known — the ones making up the given sum that is greater than a semicircle.

But if it should ever happen that the given ratio is one of equality, then the two arcs or angles making up the given sum will also be equal. The half of the given sum will therefore give each sought arc or angle.

But if the given sum is equal to a semicircle, the problem cannot be solved, as we showed in the scholium to Proposition 6 of [the book on] rectilinear triangles.

19. Given the difference of two arcs, each of which is less than a semicircle, or of two angles, together with the ratio that their sines have, to find each one separately.

[Margin: XIX. Problema]

When the given difference is subtracted from the semicircle, let the remaining arc be taken as the sum of two arcs, and let each of its arcs be found — by means of the given ratio (for this remains the same, as was said in Proposition 7 of [the book on] rectilinear triangles) — from Problem 17. For the lesser one found (if the given ratio is one of greater inequality, that is, if the sine of the greater arc is greater and that of the lesser arc smaller — which indeed happens when the two arcs are less than a semicircle) will be the lesser of the sought arcs; while the greater one found, subtracted from the semicircle, will leave the greater sought arc. But if the given ratio is one of lesser inequality — that is, if the sine of the greater arc is smaller than the sine of the lesser arc (which happens when the two arcs exceed the semicircle) — the lesser arc found, taken away from the semicircle, will leave the greater sought arc; while the greater one, removed from the semicircle, will leave the lesser sought arc.

But if the given ratio is one of equality — which indeed occurs when the two arcs make up the semicircle — we shall subtract the difference from the semicircle; for the half of the remaining number will give the lesser sought arc, and that same half, if added to the given difference, will produce the greater sought arc.

When the sum or difference is given of two angles that constitute a single spherical angle, or that exist in some rectilinear triangle, the arcs of those angles will always make up a sum less than a semicircle; and therefore only the preceding Problem 17, or the first part of this Problem 18, is to be applied.

20. Given the three angles of an oblique-angled spherical triangle, to find the three sides.

[Margin: XX. Problema]

— or else, in the triangle ABC, all three angles are equal, or only two, or all three are unequal. First let all three angles — or [at least] the two, B and C — be equal. [Margin: (a) 9[?] of spherical triangles.] Then the sides AB, AC opposite them will also be equal, and the angles B, C will be either both acute or both obtuse. If, therefore, from the third angle A a perpendicular arc AD is understood to be dropped onto the opposite side BC, adjacent to two equal angles, [Margin: (b) 57 of spherical triangles.] it will fall inside the triangle, and will bisect both the side BC and the angle BAC. For since the right triangles ABD, ACD have the angles B, C equal, and the sides AB, AC — opposite the right angles at D — equal, [Margin: (c) 21 of spherical triangles.] the sides BD, CD, as well as the angles at A, will also be equal. And so, since the whole angle at A is given, its halves BAD, CAD will also be

given. Since, therefore, in the right triangle ABD the two non-right angles B and BAD are known, the base AB will also become known. [Margin: (d) Problem 16.] For it is:

Diagram. A spherical triangle with curved sides. The apex A is at the top; B is at the lower left, C at the lower right, and D at the midpoint of the base. A single arc AD runs from the apex down to D, the perpendicular dropped from A onto the base BC, dividing the triangle into the two right triangles ABD and ACD.

- A (apex, top): vertex A
- B (lower left): vertex B
- C (lower right): vertex C
- D (base, middle): foot of the perpendicular AD
- As the whole sine is to the tangent of the complement of the angle B, so is the tangent of the complement of the angle BAD to the sine of the complement of the base AB, etc.

Hence the side AC, equal to AB, will also be known. Indeed, the third side BC too — if all three angles in the triangle ABC are given equal — will be given; for then all three sides are equal, as we said, so that once one is found, the rest will also be known. But if only the two angles B and C are equal, then BD, the half of the side BC, will be found from the same known non-right angles B, BAD. [Margin: (e) Problem 12.] For it is:

- As the whole sine is to the secant of the complement of the angle B — adjacent to the sought side BD — so is the sine of the complement of the angle BAD — opposite the sought side BD — to the sine of the complement of the sought side BD, etc.

If, then, the side BD is doubled, the whole side BC will become known.

Next, let all three angles be unequal, so that at least two are acute or obtuse — say, for example, B and C. [Margin: (f) 57 of spherical triangles.] Then the perpendicular arc AD, dropped from the third angle A onto the side BC and adjacent to two acute (or obtuse) angles, will fall inside the triangle. [Margin: (g) 61 of spherical triangles.] And it will be:

- As the sine of the complement of the angle B is to the sine of the complement of the angle C, so is the sine of the angle BAD to the sine of the angle DCA.

Therefore the ratio that the sines of the angles BAD, CAD have will be known, whose terms will be the sines of the complements of the angles B and C. Let the half of the sum of these sines be taken, and the difference between that half and either of the sines of the complements of the angles B, C. It will then be, as we demonstrated in Problem 17:

- As the whole sine is to the secant of the complement of the arc that is owed to the said half (as its sine), so is the aforesaid difference — between that half and either of the sines of the complements of the angles B, C — to another fourth number.
- *Then:* As the whole sine is to the tangent of the half of the angle BAC — [treated] as the sum made up of the angles BAD, CAD — so is the fourth number found to the tangent of the difference between the half of the angle BAC and either of the angles BAD, CAD.

The arc of this tangent found, added to the half of the angle BAC, will make up the greater angle A, and subtracted from that same half will leave the lesser. And that angle A will be the greater which corresponds to the greater sine of the complement of the angles B and C; so that if the sine of the complement of the angle B is greater than the sine of the complement of the angle C, the angle BAD is greater than the angle CAD, etc.

Now, from the two non-right angles A, B of the right triangle ABD, once known, the base AB will be known from Problem 16, and the side BD from Problem 12.

Diagram. A pair of spherical triangles side by side, illustrating the case in which all three angles are unequal, each drawn with apex A at the top and a perpendicular arc AD dropped to the base. In the left figure the foot D falls between B and C (order along the base: B, D, C), so the perpendicular lies inside; in the right figure the base is lettered B, C, D from left to right, with D beyond C.

- Left figure — A (apex); base points B (left), D (middle), C (right). [The leftmost base label is faint; read as B.]
- Right figure — A (apex); base points B (left), C (middle), D (right).

By the same reasoning, from the non-right angles A, C of the right triangle CAD, both the base AC and the side CD will become known; and the sum of the sides BD, CD will give the whole side BC. And so all three sides have been made known.

21. Given the three sides of an oblique-angled spherical triangle, to find any of its angles.

[Margin: XXI. Problema]

In the above triangle of known sides, let the angle BAC be the one to be investigated, and first let the two sides AB, AC enclosing it be unequal. We shall then investigate the angle BAC as follows.

- As the whole sine is to the sine of the greater given side, so is the sine of the lesser given side to a fourth. [Margin: scholium 2; 58 of spherical triangles.]
- *Then:* As the fourth found is to the whole sine, so is the difference between the versed sine of the arc opposite the sought angle and the versed sine of the arc by which the two sides enclosing the sought angle differ from each other, to the versed sine of the sought angle.

[Problem 21, continued]

Next, let the two sides AB and AC that enclose the sought angle be equal. Then the perpendicular arc AD dropped from the sought angle will bisect both the sought angle and the opposite side BC, as we showed in the preceding problem. [Margin: a — prop. 21 of the *Triangula Sphaerica*.] Now in the right triangle BAD the base AB is known, along with the side BD (which is half of the known side BC), and this side is opposite the angle BAD. Therefore the angle BAD becomes known by Problem 1; and consequently the whole sought angle BAC, being twice that angle, will also be known.

22. GIVEN, in an oblique-angled spherical triangle, two sides together with the angle they contain, to find the remaining side together with the remaining two angles.

[Margin: XXII. Problema]

In the same triangle as above, let the two sides AB and AC be given, together with the angle BAC — first taking them unequal. From these we shall hunt down the rest as follows.

[Margin: schol. 2, prop. 58 of the *Triangula Sphaerica*.]

- As the whole sine [sinus totus] is to the sine of the greater given side, so the sine of the lesser given side is to a fourth [proportional].

Then:

- As the whole sine is to that fourth, so the versed sine [sinus versus] of the given angle is to the difference between the versed sine of the third

(sought) side and the versed sine of the arc by which the two given sides differ from one another.

This difference, added to the versed sine of the arc by which the two given sides differ from each other, makes up the versed sine of the third sought side, from which that third side itself will be known. And so now all three sides of the triangle ABC will be known; and thus each of the remaining angles B and C is found, as was taught in the preceding problem.

Next, let the two sides AB and AC be equal. Then the perpendicular arc AD dropped from the given angle BAC will bisect both the given angle BAC and the sought side BC, as stated. And since in the right triangle BAD the base AB is given together with the angle BAD, which is opposite the sought side BD, the side BD will also be given by Problem 8; and consequently the whole side BC will be known. Again, from the given base AB and the angle BAD, the remaining angle ABD becomes known by Problem 3. In the same way, in triangle CAD the angle ACD is found from the given base AC and the angle CAD.

23. GIVEN, in an oblique-angled spherical triangle, two angles together with the side adjacent to them, to investigate the remaining two sides together with the remaining angle.

[Margin: XXIII. Problema]

In triangle ABC let the two angles B and BAC be given, together with the side AB. Suppose first that those angles are unequal and that the side AB is not a quadrant. From the alternate one of the angles — say from A — let a perpendicular arc be dropped to the side BC (extended, if need be); the working itself will show when it falls inside and when outside the triangle. For in the right triangle ABD, since the base AB is given together with the angle B, the side AD opposite the angle B is found by Problem 8, and the other, non-right angle BAD by Problem 3. If this angle BAD is found to be less than the angle BAC, the arc AD falls inside the triangle; but if greater, outside. Therefore, subtracting the angle BAD from the given angle BAC, or the latter from the former, the remaining angle CAD will also be given.

Diagram. Two small woodcut triangles side by side, each with apex A at the top and two sides descending to a base. They illustrate the two cases of Problem 23 — the perpendicular AD falling inside the triangle (left) versus outside it (right).

- A (apex, both figures): vertex from which the perpendicular is dropped.

- Left figure, base points left→right: B, D, C — the foot D lies between B and C, so the perpendicular AD falls inside the triangle.
- Right figure, base points left→right: B, C, D — the foot D lies beyond C, so the perpendicular AD falls outside the triangle.

Now in the right triangle ABD, since the base AB is given together with the angle B, the side BD adjacent to the given angle B is also given by Problem 9.

Again, in the right triangle CAD, since the side AD has been found together with the angle CAD, the side CD is also given by Problem 11. Therefore, if the arc AD falls inside the triangle, the sum of the sides BD and CD gives the whole side BC as known; but if it falls outside, the side CD subtracted from BD leaves the remaining side BC known. And so one of the two remaining sides, BC, has now been found.

Finally, since in the right triangle CAD the side AD is given together with its adjacent angle CAD, the base AC — which is the third side — is given by Problem 13; and by Problem 4 the angle C opposite the side AD is found, which in the former case is the third angle sought, while in the latter case its supplement to a semicircle gives the third sought quantity.

[Margin: b — prop. 25 of the *Triangula Sphaerica*.] [Margin: c — prop. 25 of the *Triangula Sphaerica*.]

But if the angle CAD is ever found to be right — the angle BAD will never be right; otherwise, since D too is right, AB and DB would be quadrants, whereas AB is assumed not to be a quadrant [Margin: b — prop. 25 of the *Triangula Sphaerica*.] — then, since D is also right, CA and CD will be quadrants; and the side AD, once found, will be the measuring arc of the sought angle C. Finally the side BD, once found, together with the quadrant CD, will in the former case make the whole side BC known; while in the latter case the quadrant CD, subtracted from the found side BD, leaves the sought side BC.

[Margin: d — prop. 38 of the *Triangula Sphaerica*.] [Margin: e — prop. 25 of the *Triangula Sphaerica*.]

Next, let the same angles B and BAC be given, unequal, and let the side AB be a quadrant, opposite the right angle D. Then at least one of the remaining sides will also be a quadrant. Since AD cannot be a quadrant — for otherwise, on account of the two quadrants AB and AD, the angles B and D would be right [Margin: e — prop. 25 of the *Triangula Sphaerica*.], and so the triangle ABC would be right-angled, which is not assumed — it will be BD that is a quadrant. Hence the angle

BAD is right, on account of the quadrants BA and BD; and B will be the pole of the arc AD, that is, AD will be the measuring arc of the given angle B, and therefore known. With these found, the rest are obtained as before: namely CD by Problem 10, and AC by Problem 13, and the angle C by Problem 4, from the given side AD and the angle CAD.

[Margin: f — prop. 9 of the *Triangula Sphaerica*.]

Thirdly, in the same triangle let the two angles B and C be given equal, together with the side BC; then the sides AB and AC will accordingly be equal. So the perpendicular arc dropped from the third angle A will bisect both the side BC and the angle A, as we showed above. And therefore, since in the right triangle ABD the side BD is given together with the angle B, the base AB is found by Problem 13, and so the side AC too will be known; while by Problem 4 the angle BAD is found, which is half of the whole angle BAC.

24. GIVEN, in an oblique-angled spherical triangle, two angles together with the side opposite one of them, to examine the remaining sides together with the remaining angle — provided the species of the side opposite the other given angle is established.

[Margin: XXIII. Problema]

[Problem 24, continued]

[Margin: a — prop. 57 of the *Triangula Sphaerica*.]

In triangle ABC let there first be given the two angles B and C, unequal, together with the arc AB, which is not a quadrant, and with the species of the arc AC. From the third angle A let the perpendicular arc AD be dropped to BC. [Margin: a — prop. 57 of the *Triangula Sphaerica*.] It will fall inside the triangle if both of the given angles B and C are acute, or both obtuse; but outside if one is acute and the other obtuse. Then, since in the right triangle ABD the base AB is given together with the angle B, the side AD is given by Problem 8, the side BD by Problem 9, and the angle BAD by Problem 3.

Again, since in the right triangle ACD the side AD is given together with the opposite angle C and the species of the base AC, the base AC is given by Problem 14, the side CD by Problem 10, and — from the given side CD and the angle D — the angle CAD by Problem 4. Therefore, if the angle CAD found is added to the

angle BAD found, or subtracted from it, the sought angle BAC becomes known. Likewise, the side CD found, added to the side BD found, or subtracted from it, makes the sought side BC known.

But if it should happen that the side AC is a quadrant, then CD will also be a quadrant, and the angle CAD right, and so on.

Next, let the given side AB be a quadrant, and let the two angles B and C still be given as unequal. Then BD too will be a quadrant, and the angle BAD right; and AD will be the measuring arc of the given angle B, and therefore known, and so on.

[Margin: b — prop. 9 of the *Triangula Sphaerica*.]

Finally, in the same triangle let the two angles B and C be given equal; then the sides AB and AC will accordingly be equal as well. Since AB is given, AC will also be given. Dropping the perpendicular arc AD, which will bisect both the side BC and the angle BAC: since in the right triangle ABD the base AB is given together with the angle B, the side BD is given by Problem 9 [Margin: b — prop. 9 of the *Triangula Sphaerica*.], and so its double BC, which is sought, will be given; and by Problem 3 the angle BAD is given, and so its double BAC, which is sought.

25. GIVEN, in an oblique-angled spherical triangle, two sides together with the angle opposite one of them, to find the remaining angles together with the remaining side — provided the species of the angle opposite the other given side is established.

[Margin: XXV. Problema]

[Margin: c — prop. 57 of the *Triangula Sphaerica*.]

In triangle ABC let there first be given the two unequal sides AB and AC, neither of them a quadrant, together with the angle B and the species of the other angle C. From the third angle A let the perpendicular arc AD be dropped. [Margin: c — prop. 57 of the *Triangula Sphaerica*.] It will fall inside the triangle if both angles B and C are acute, or both obtuse; but outside if one is acute and the other obtuse. And since in the right triangle ABD the base AB is given together with the angle B, the side AD opposite the given angle is given by Problem 8, the side BD by Problem 9, and the angle BAD by Problem 3.

Again, since in the right triangle CAD the base AC is given together with the found side AD, the side CD is given by Problem 6, the angle C by Problem 1, and the angle CAD by Problem 2. Therefore, if the arc AD lies inside the triangle, the two angles BAD and CAD found together give the whole sought angle BAC, and the two sides BD and CD found together give the whole sought side BC. But if the arc AD falls outside the triangle, the angle CAD subtracted from the angle BAD leaves the sought angle BAC known, and the side CD subtracted from the side BD leaves the sought side BC.

Next, let one of the given sides be a quadrant. If AB is a quadrant, then BD too will be a quadrant, the angle BAD right, and AD the measuring arc of the given angle B, and therefore known, and so on.

But if AC is a quadrant, then CD too will be a quadrant, the angle CAD right, and AD the measuring arc of the angle C; and consequently the found arc AD will make the angle C known, and so on.

[Margin: d — prop. 8 of the *Triangula Sphaerica*.]

Diagram. A spherical (curved-sided) triangle with apex A at the top, B at the lower left, and C at the lower right; a perpendicular arc AD is drawn from A down to the foot D on the base BC.

- A (apex): the vertex from which the perpendicular is dropped.
- B (lower left): base vertex.
- C (lower right): base vertex.
- D (foot, on base BC): foot of the perpendicular arc AD.

Finally, in the same triangle let the two sides AB and AC be given equal; then the angles B and C will accordingly be equal as well. Since B is given, the angle C will also be given. So the only thing to be sought is the side BC, together with the angle BAC. The perpendicular arc AD dropped will bisect both the side BC and the angle BAC. And in the right triangle ABD, since the base AB is given together with the angle B, the side BD is given by Problem 9, and so its double BC, which is sought; and by Problem 3 the angle BAD is found, and so its double BAC, which is sought, will be known.

CALCULUS OF RIGHT-ANGLED RECTILINEAR TRIANGLES

I. THE PROPORTIONS OF THE SIDES, from all the angles of any triangle being given.

[Margin: prop. 1, rectilinear triangles.]

To each of the sides let there be assigned the sine of the opposite angle. For the sides have the same proportions to one another as are found among the sines of the angles opposite those sides, assigned to them.

II. A SIDE, from the base and either of the acute angles (and therefore the other as well).

[Margin: prop. 2, rectilinear triangles.]

- As the whole sine is to the base, so the sine of the angle opposite the sought side is to the sought side, expressed in the units of the base.

III. A SIDE, from the base and one other side.

[Margin: prop. 3, rectilinear triangles.]

- As the base is to the whole sine, so the given side is to the sine of the angle opposite the given side.

Then, taking the complement of the angle found as the remaining angle:

- As the whole sine is to the base, so the sine of the angle found (which is opposite the sought side) is to the sought side, expressed in the units of the base and of the other side.

IV. A SIDE, from one side and an acute angle (and therefore the other).

[Margin: prop. 1, rectilinear triangles.]

- As the whole sine is to the given side, so the tangent of the angle opposite the sought side is to the sought side.

Or:

- As the sine of the angle opposite the given side is to the given side, so the sine of the other angle is to the sought side.

V. THE BASE, from one side and one acute angle (and therefore the other).

[Margin: prop. 1, rectilinear triangles.]

- As the whole sine is to the given side, so the secant of the angle adjacent to the given side is to the base.

Or:

- As the sine of the angle opposite the given side is to the whole sine, so the given side is to the base.

VI. THE BASE, from both sides.

[Margin: prop. 3, rectilinear triangles.]

- As either given side is to the whole sine, so the other given side is to the tangent of the angle opposite this other side.

Then, taking the complement of the angle found as the remaining angle:

- As the whole sine is to either given side, so the secant of the angle adjacent to the side taken is to the base.

VII. AN ANGLE, from the base and one side.

[Margin: prop. 3, rectilinear triangles.]

- As the base is to the whole sine, so the given side is to the sine of the angle opposite the given side.

The complement of the angle found gives the other angle.

VIII. AN ANGLE, from both sides.

[Margin: [?] rectilinear triangles.]

- As either given side is to the whole sine, so the other given side is to the tangent of the angle opposite this other side.

The complement of the angle found gives the other angle.

CALCULUS OF OBLIQUE-ANGLED RECTILINEAR TRIANGLES

IX. THE SEGMENTS OF A SIDE made by the perpendicular, from the three sides being given.

[Margin: prop. 9, rectilinear triangles.]

- As the side onto which the perpendicular falls is to the sum of the other two sides, so the difference of those same sides is to a fourth number.

If the fourth number found is less than the side onto which the perpendicular falls, it is to be subtracted from that side. For half of the remainder gives the lesser segment; and this, subtracted from the whole side, leaves the greater segment.

But if the fourth number found is greater than the side onto which the perpendicular falls, then that side is to be subtracted from it. For half of the remainder gives the lesser, outer segment, lying between the perpendicular and the obtuse angle; and this same segment, added to that side, forms another, greater segment, lying between the perpendicular and the acute angle.

X. TWO SIDES, from the third side and any two angles (and therefore all three, since the third is the supplement of the others to a semicircle).

[Margin: prop. 10, rectilinear triangles.]

- As the sine of the angle opposite the given side is to the given side, so the sine of either of the remaining angles is to the side opposite this angle.

Again:

- As the sine of the angle opposite the given side is to the given side, so the sine of the third angle is to the side opposite this third angle.

ASTROLABIUM — Liber Primus (concl.) / Liber Secundus (init.)

Lemma LIII (rectilinear-triangle calculus, continued)

(Continuing the note appended to the preceding case.)

In an isosceles triangle only one side needs to be found, since one is given together with the angles. In an equilateral triangle, moreover, if a single side is given, the remaining sides will be equal to it and so are also given.

XI. A SIDE

From two sides and any two angles — and hence from all three angles, since the third is the supplement of the others to a semicircle.

[Margin citation: 10. triang. rectil.]

As the sine of either angle opposite a given side is to that given opposite side, so the sine of the angle opposite the sought side is to the sought side.

(In modern terms this is the Law of Sines: $\sin A / a = \sin C / c$, solved for the unknown side.)

XII. A SIDE

From two sides and the angle contained by them.

[Margin citation: Proble. 17. triang. sphær.]

As the whole sine [sinus totus, the radius] is to the secant of the complement of the arc which corresponds — as to a sine — to the half-sum of the two given sides reduced to sines; so the difference between that same half-sum and either of the given sides (reduced to sines) is to a fourth term.

Then:

As the whole sine is to the tangent of the half of the arc that remains when the given angle is subtracted from the semicircle, so the fourth term just found is to the tangent of the difference between the half of that same arc and either of the angles not given.

This tangent may also be found in the following way:

[Margin citation: 6. triang. rectil.]

As the half-sum of the two given sides is to the tangent of the half of the arc that remains when the given angle is subtracted from the semicircle, so the difference between the half-sum of the two given sides and either side is to the tangent of the difference between the half of the aforesaid arc and either of the angles not given.

The tangent so found gives an arc; add this arc to the half of that same arc — this halved arc being the sum of the two unknown angles, that is, the supplement of the given angle to the semicircle — and you obtain the larger of the unknown angles, the one opposite the larger given side. Subtract the same found arc from that half instead, and the remainder gives the smaller unknown angle, the one opposite the smaller given side. After this:

[Margin citation: 1. triang. rectil.]

As the sine of either found angle is to its opposite side, so the sine of the given angle [?] is to the opposite side that is sought.

If the two given sides are equal, [Margin: a — 5. primi] then the remaining two angles will be equal. Half of the arc that remains when the given angle is subtracted from the semicircle will therefore give each of them, and so on.

XIII. A SIDE

From two sides and the angle opposite one of them — provided the species of the angle opposite the other given side is settled, when the given angle is acute.

[Margin citation: 13. triang. rectil.]

As the given side opposite the given angle is to the sine of the given angle, so the other given side is to the sine of the angle opposite that other side.

This found sine gives the angle opposite the other given side, if that angle is acute. (It will always be acute when the given angle is obtuse.) But if it is obtuse, then the arc of the found sine, subtracted from the semicircle, will yield that angle. This is why, when the given angle is acute, the species of this other angle must be supplied, so that we may know whether it is acute or obtuse. The sum of these two angles, subtracted from the semicircle, will leave the third angle, the one opposite the sought side. Therefore:

[Margin citation: 1. triang. rectil.]

As the sine of the given angle is to the given side opposite it, so the sine of the found angle opposite the sought side is to the sought side.

If the two given sides are equal, [Margin: b — 5. primi] then the angle opposite one of the given sides will be equal to the given angle, and so on.

XIV. TWO ANGLES

From two sides and the angle contained by them.

Two angles will be found from the given elements, as was stated in the first part of Problem 12 — namely, by seeking the tangent of the difference between the half of the arc that remains when the given angle is subtracted from the semicircle and either of the angles sought, and so on. That tangent was found in two ways in the first part of Problem 12, where a side is set to be investigated from two sides and the angle contained by them; and to accomplish this, two other angles had first to be found — the very ones now sought in this Problem 14.

XV. TWO ANGLES

From two sides and the angle opposite one of them — provided the species of the angle opposite the other given side is settled, when the given angle is acute.

Here too the earlier procedure of Problem 13 must be applied, in which a side is set to be sought from these same given elements; and to accomplish this, the remaining two angles had first to be found — the very ones now set forth to be investigated in this Problem 15.

XVI. THREE ANGLES

From the three sides.

[Margin citation: 11. triang. rectil.]

Drop a perpendicular to the greatest side from the angle opposite it (so that the perpendicular always falls inside the triangle), and by Problem 9 find the two segments of the greatest side made by the perpendicular. Then:

As the smallest side is to the whole sine, so the smaller segment of the greatest side is to the cosine [sinum compl.] of the angle opposite the middle side.

Again:

As the middle side is to the whole sine, so the greater segment of the greatest side is to the cosine of the angle opposite the smallest side.

Once the two angles at the greatest side are found — those opposite the middle and the smallest side — if their sum is taken from the semicircle, the remainder becomes the third angle, opposite the greatest side.

In an isosceles triangle, drop a perpendicular to the base, which it will bisect:

As either of the equal sides is to the whole sine, so half of the base is to the cosine of one of the equal angles at the base.

The sum of the two equal angles found, subtracted from the semicircle, will make the third angle.

In an equilateral triangle the angles are given even if the sides are not, since each one comprises 60 degrees — that is, a third part of two right angles, or two-thirds of one right angle.

FINIS LIBRI PRIMI.

ASTROLABII LIBER SECVNDVS

AVCTORE CHRISTOPHORO CLAVIO BAMBERGENSI E SOCIETATE IESV

(By the author Christoph Clavius [Christophorus Clavius] of Bamberg, of the Society of Jesus.)

1. [Margin: That the sphere can be projected onto a plane in various ways.] In the preceding book we demonstrated what we judged necessary for demonstrating the construction of the planisphere or astrolabe — that is, the projection of the sphere onto a plane. Now let us take up the matter itself. The celestial sphere, then, can be projected onto a plane in many ways, according to the choice and will of whoever undertakes to draw it on a plane, depending on whether he wishes to represent it by this figure or that. For since it is impossible for all the points and all the circles conceived on the sphere to be so drawn on the plane that they keep the same position and exactly the same mutual distances that they have on its concave or convex surface, astronomers have been forced to project all its lineaments and parts onto a given plane surface in that shape and form in which they appear on it — with the eye placed at some fixed spot, or in the shape which perpendiculars dropped from all the points of the circles produce upon it. We have observed that they have done this chiefly in three ways.

2. [Margin: The Catholic Astrolabe of Gemma Frisius: on what basis it is drawn.] For some — among them Gemma Frisius [Gemma Frisius], no undistinguished

writer, in his universal astrolabe which he calls the *Catholicum* — place the eye at the common intersection of the equator and the ecliptic, and draw all the celestial circles in the plane of the solstitial colure, which represents the meridian circle, in the form in which the eye beholds them.

3. [Margin: The universal Planisphere of Joan de Rojas: on what basis it is drawn.] Others, by contrast, do not fix the eye at any one fixed and certain spot, but mark out all the circles of the sphere in the plane of the solstitial colure, or of the meridian, in the figure which perpendicular lines dropped from all the points of the circumference of each circle onto the plane of the solstitial colure or of the meridian circle produce. By this method it comes about that all the circles which are neither parallel to the equator nor perpendicular to the solstitial colure form ellipses in the plane of that colure; whereas the equator with all its parallels, and the other circles perpendicular to that same colure, are projected onto its plane by straight lines. Joan de Rojas [Juan de Rojas] followed this method in his universal planisphere. The construction of both planispheres — that of Gemma Frisius as well as that of Joan de Rojas — was demonstrated acutely and elegantly by Guidobaldo del Monte [Guidobaldo del Monte, Marchese], a man most learned in mathematical matters.

4. [Margin: The Astrolabe for a given pole-altitude: on what basis it is drawn by Ptolemy. In what respect Jordanus differs from Ptolemy in the description of the astrolabe.] Ptolemy [Ptolemy], finally, the prince of astronomers, places the eye at the south pole and depicts all the circles, lines, and points of the *primum mobile* in the plane of the equator, extended to infinity, in the figure in which they are discerned in that plane from the south pole. By this method the ordinary astrolabes, which are constructed for a given altitude of the pole, are usually drawn by craftsmen. Jordanus [Jordanus Nemorarius], however — whom Francesco Maurolico [Francesco Maurolico], the celebrated Sicilian abbot and mathematician, followed in his most learned *Theory and Construction of the Astrolabe* — takes, in place of the plane of the equator, another plane parallel to it which touches the sphere at the opposite, northern pole; because in it all the circles and lines appear under the same figures under which they are seen in the plane of the equator.

[Margin: What things are chiefly drawn on the astrolabe.] But we shall imitate Ptolemy rather than Jordanus in the construction of the astrolabe or planisphere. For since, on Ptolemy's method, the equator retains the same magnitude by which the analemma — on which the whole structure of the astrolabe depends — is drawn, it comes about that most things are delineated far more easily on the

astrolabe than if a plane parallel to the equator, touching the sphere at the opposite northern pole, were assumed. This will be evident from what follows.

5. [Margin: The parts lying between the points, lines, and circles of the sphere need no special drawing on the astrolabe. To which parts of the heavens the individual parts of the astrolabe correspond.] Furthermore, everything that exists on the celestial sphere and is usually drawn especially on the astrolabe is either points, or straight lines, or circles, whose circumferences are considered on the convex surface of the sphere. For everything else — such as the portions of the spherical surface itself, rectilinear figures both plane in the circles and solid in the sphere, and other things of that kind — needs no special and proper drawing on the plane of the astrolabe, since these are contained among the points, lines, and circles of the astrolabe, just as happens on the sphere itself. For, to bring forward one or another example of this: that part of the celestial sphere which is cut off toward the parts of the north pole from the equator — that is, the whole northern hemisphere — is represented on the plane of the equator, or of the astrolabe, by that plane surface which is enclosed on every side between the circumference of the equator and the north pole, that is, the center of the astrolabe. But the remaining portion of the astrolabe...

(The text continues into the next page.)

the portion outside the equator toward the tropic of Capricorn, extended to infinity, belongs to the southern hemisphere, which the equator in the celestial sphere cuts off toward the south pole. In the same way, the hemisphere that the ecliptic in the heavens cuts off toward the north pole is, in the plane of the astrolabe, that part which is intercepted on every side between the ecliptic and that same north pole (or the center); whereas the remaining part of the astrolabe, running out infinitely beyond the ecliptic, corresponds to that part of the celestial sphere which the ecliptic cuts off toward the south pole. By the same reasoning, the part of the astrolabe lying between the two tropics expresses the torrid zone [Zona torrida] — that is, that surface of the celestial sphere which the two tropics enclose; while the part beyond the tropic of Capricorn, extended to infinity in the astrolabe, represents that part of the sky which the tropic of Capricorn marks off toward the south; and the part that lies within the tropic of Cancer is the region that in the heavens lies between the arctic pole and the tropic of Cancer. Finally, any circle described in the astrolabe, surrounding the center, includes that part of the sky which in the heavens is contained within the circumference of its circle toward the arctic pole; but the remaining part of the sky is contained outside that circle in the astrolabe. The reason for this is that all the points of that part of the

sky which any circle enclosing either of the poles cuts off toward the arctic pole are projected onto the plane of that same circle described in the astrolabe, whereas all the remaining points of the other part of the sky fall outside the plane of that circle, as will become clear from what follows.

[Margin: Where any point of the sphere appears in the astrolabe.]

6. Any point of the celestial sphere is seen along a straight line, and appears at that point of the astrolabe — that is, of the plane of the equator — through which passes the straight line drawn from the south pole through the assumed point itself.

[Margin: A straight line in the sphere: when it appears as a point in the astrolabe, and when as a straight line.]

7. Any straight line, if it is drawn through the south pole, appears entirely as a single point of the astrolabe — namely the point through which the extended line passes — because all its points are seen at that one point, since a single visual ray is carried through all of them. But if it is not drawn through the south pole, it is viewed by means of a triangle whose vertex is at the eye (that is, the south pole) and whose base is the seen line itself, so that the visual rays passing through all its points all lie in the plane of that triangle. Hence any straight line not passing through the south pole is projected onto the astrolabe as a straight line, which is the common section of the plane of the astrolabe (or equator) and of the said triangle — provided its sides are understood to be produced so that they can cut the plane of the astrolabe — whenever the seen line is either wholly on the near side of the plane of the equator (or astrolabe), or partly on the near side and partly beyond; for the visual rays carried around through all the points of the seen straight line do not depart from that common section of the plane of the astrolabe and of the said triangle. Therefore all the diameters of the great circles of the sphere will be projected through the center of the astrolabe as straight lines, since they are all drawn through the center of the sphere — which does not differ from the center of the astrolabe, as will appear below — so that a straight line drawn through the center from any point of the circumference of some great circle described in the astrolabe represents the diameter of that great circle, which in the heavens is drawn through the point corresponding to the point assumed in the astrolabe. But the diameters of circles that are not great circles on the sphere will indeed be projected onto the astrolabe as straight lines, yet not

through the center, since they are not drawn through the center on the sphere either.

[Margin: How any circle of the sphere is viewed in the astrolabe.]

8. Finally, any circle whatever, whose circumference lies on the surface of the sphere: if it is described through the south pole, it is viewed by visual rays which, carried around through all the points of its circumference, do not depart from its plane, and therefore all terminate in the common section of the plane of the circle and the plane of the astrolabe (or equator), as will be demonstrated below in Proposition 1, Number 1 — so that all its points appear in a straight line, that is, in that common section. But if it is not drawn through the south pole — whether it is parallel to the equator or not, and whether it is a great circle or not — it is seen by means of a cone whose vertex is the eye itself (that is, the south pole) and whose base is the seen circle itself, as is clear from the definitions of Apollonius [Apollonius of Perga]: if a visual ray drawn from the south pole through any point of the circle's circumference is understood to be carried around the circumference so as to describe a cone, through which the circle is viewed from that same south pole, since that visual ray is joined with all the other rays emitted from the south pole in that revolution. Hence any circle of the sphere that is not drawn through the south pole is projected onto the astrolabe in the form and figure which the common section of the plane of the equator (or astrolabe) and of the said cone produces — provided the cone is understood to be produced so that it can be cut by the plane of the astrolabe, whenever the seen circle is either wholly on the near side of the plane of the equator, or partly on the near side and partly beyond. Now this common section of a cone and some plane, although it may be a circle, parabola, hyperbola, or ellipse, as Apollonius demonstrates, is nevertheless always a circle in the plane of the astrolabe (or equator), as we shall demonstrate in its place.

[Margin: What it is to describe an astrolabe. — What an astrolabe is.]

9. From these things it is clear that to construct an astrolabe, or planisphere [Planisphaerium], is nothing else than to describe the sphere, or primum mobile, on a plane — that is, to arrange its individual points, lines, and circles on the plane of the equator (or astrolabe) in the position in which they are seen from the eye placed at the south pole in that plane. So that the astrolabe, or planisphere, is a plane figure containing all the sections of the plane of the equator (or astrolabe), extended to infinity, both of the

straight lines emitted from the south pole and of the triangles and cones — whose vertices are at the south pole, and whose bases are the straight lines and circles of the sphere that are described in the astrolabe. By what method this is done we shall demonstrate in order through the following propositions.

THEOREMA I. PROPOSITIO I.

[Margin: A circle drawn through the south pole is projected onto the astrolabe as a straight line, and equal arcs [correspond] as unequal parts of the straight line.

— Margin (a): Theodosius, *Sphaerica* I.11.]

Any circle of the sphere drawn through the south pole is projected, together with all the points and lines drawn on it, onto the astrolabe as an infinite straight line, which is the common section of the circle itself and of the plane of the astrolabe (or equator). The parts of that straight line corresponding to equal arcs are unequal, and those more remote from the visual ray drawn through the circle's center are the greater; yet two parts equally distant on either side from that same ray, and corresponding to equal arcs, are equal.

1. Let a circle ABCD be drawn through the south pole A, cutting the plane of the equator along the straight line HL. This line will either pass through the center E of the proposed circle — namely when the circle ABCD is a great circle [Margin (a): Theodosius I.11] (for since the equator and the great circle ABCD cut each other in half, their common section HL will pass through the center of each, and therefore also through the center E of the proposed great circle) — or it will lie beyond the center E, namely when the circle ABCD is not a great circle. For then its center will necessarily be on the near side of the plane of the equator, since its radius A E is smaller than the radius of the sphere, which is the smallest of all the straight lines falling from the south pole A onto the plane of the equator; [Margin (b): scholium, Theodosius III.8 (*schol. 8. tert. Theod.*)] for that radius, falling on the center of the equator, is perpendicular to its plane. And this straight line H L will either cut the circle ABCD, or will lie wholly beyond it, according as the circle itself cuts the equator or lies wholly on the near side of it.

I say that this whole circle ABCD, together with all the points and lines drawn on it, is projected onto the straight line HL extended to infinity, and so on. For since the visual ray from the pole A, carried around through all the points of the circumference of the circle ABCD and through all the points lying in its plane, does not depart from the plane of that circle, it will necessarily fall on the

common section HL. Therefore all the points of the circle will appear on that same straight line HL. And since the visual rays, the more obliquely they cut the straight line HL, the farther they run out — so that the ray A Y, or A Z, tangent to the circle at A and extended to infinity, does not meet HL [Margin (c): Euclid I.8] but is parallel to it, [Margin (d): Euclid III.28] since the angle YAE is right and the angle AEH is likewise right by Lemma 26 — it comes about that if all the points of the circle (except the pole A, which alone, as we shall show in Proposition 4, cannot be projected onto the plane, because the diameter YZ is parallel to the straight line HL) are to be projected onto the plane of the astrolabe, the whole is projected onto a line in a certain sense infinite: because the points near the point A are projected by straight lines nearly parallel to HL, and therefore meeting that same straight line HL only at a distance that is in a manner infinite.

2. Now if the circle ABCD is divided into any number of equal parts AO, OP, PB, etc., and rays AOF, APG, A B, etc. are drawn through the points of division, then to the equal arcs there will correspond the projected straight lines EI, IH, HB, BG, etc. — since all the visual rays emitted from A through all the points of the corresponding arcs fall on these lines.

Diagram. A large square woodcut illustrating the stereographic projection of Proposition I. At the bottom center is the point A, the eye (south pole); the base of the frame runs from Y (bottom-left corner) through A to Z (bottom-right corner). From A a fan of straight rays (with several dotted projecting rays) rises through the figure. A semicircle — the great circle ABCD seen in projection — stands with its diameter along a horizontal line at mid-height (the common section HL) and its arc above. A vertical line through the center E carries the ascending points c, a, b and f. Rays from A pass through the division points of the semicircle and meet the horizontal line at the projected points.

Readable labels (positions best-effort; several are faint or mirror-repeated between arc and diameter):

- A (bottom center): the eye / south pole
- Y (bottom-left corner); Z (bottom-right corner)
- Horizontal diameter line (HL), left → right: F, h, G, B, Q, H, I, E, K, L, D, M, N
- Semicircle arc: B, R, T, E, K, L, e, D, f; with Q inside the arc
- Vertical axis through E (ascending): c, a, b; and f
- Top edge (outer points), left → right: e, [?] (a reversed/rotated letter), 1, E, K, L, e
- Lower radiating region: P, o, V, d, t [?], i [?]

- [Several top-edge and lower labels not resolvable at this resolution]

Note: this figure is reprinted identically on the following page (p. 128); see fig-p128-1.

I say that the straight lines EI, IH, etc. are unequal, and that IH is greater than EI, and HB greater than IH, and so on. For since the diameter AC, by Lemma 26, is perpendicular to HL — the common section of the equator and the circle ABCD — the angles at E will be right; and therefore, by Corollary 1 of Proposition 17 of Book 1 of Euclid, the angles G, B, H, I, K, L, D, M inclining toward E are acute, [Margin (e): Euclid I.19] and consequently the rest are obtuse, being made up from two right angles. Therefore the straight line AI will be greater than AE, and AH greater than AI, and AB greater than AH, and so on — that is, of the straight lines going out from A, the more remote is greater than the nearer. And since the arcs CR, RQ are equal, [Margin (f): Euclid III.27] the angles CAR, RAQ will also be equal — that is, the angle EAH in the triangle AEH [Margin (g): Euclid VI.3] will be bisected. Therefore, as AH is to AE, so HI is to IE. Since, then, AH has been shown to be greater than AE, HI will also be greater than IE. And by the same reasoning BH will be greater than HI, and so with the rest.

3. Finally, since in the triangles AEI, AEK the angles at E are right, and therefore equal, by Lemma 26, [Margin (h): Euclid III.27] and the angles EAI, EAK, standing on the equal arcs CR, CS, are equal, [Margin (i): Euclid I.26] the sides EI, EK will also be equal — these being equally distant from the radius AE drawn through the center. Likewise, since in the triangles AEH, AEL the angles at E are right, and therefore equal, as has been said, [Margin (k): Euclid III.27] and the angles EAH, EAL, standing on the equal arcs CQ, CT, are equal, [Margin (l): Euclid I.26] and the side AE adjacent to them is common, the sides EH,

EL, being equally distant from the same radius AE, will also be equal. Therefore, taking the equals EI, EK from the equals EH, EL, the remaining straight lines IH, KL — likewise equally distant from the same radius AE, and corresponding to the equal arcs RQ, ST — will be equal. In the same way we shall show that the straight lines EB, ED are equal, and therefore, taking away the equals EH, EL, that the remainders HB, LD are equal too. And so with the other straight lines equally distant from the radius AE and corresponding to equal arcs equally remote from the point C. Which was to be demonstrated.

[Margin: The north pole and the axis of the world are, in the astrolabe, identical with its center, or the center of the sphere. — Margin (a): Theodosius I.19. — All

the great circles drawn through the poles of the world are projected as straight lines intersecting one another at the center of the astrolabe.]

4. Since, then, both the north pole and the whole axis of the world appear, viewed from the south pole, at the center of the astrolabe (or equator, or sphere) — because the axis, which is also the straight line drawn from the south pole to the north pole, [Margin (a): Theodosius I.19] cuts the equator at the center of the sphere (or of the equator), so that the center of the astrolabe represents both the center of the sphere and the north pole of the world and the axis of the world — it comes about that the meridian, the right horizon, the two colures, the circles of declination, the hour-circles from noon and midnight, and finally all the great circles of the sphere drawn through the poles of the world, are projected onto the astrolabe as straight lines intersecting one another at the center of the astrolabe, seeing that the axis of the world and the north pole — where all those great circles intersect one another — appear, viewed from the south pole, at the center of the astrolabe (or equator), as we have said. For it is necessary that in the astrolabe such great circles intersect one another at the point which represents that point in the sphere (or the straight line) where they all intersect. For just as in the heavens all those circles pass through some one point, or straight line, so the same circles are seen in the astrolabe to pass through the point which represents that point in the sphere, or through the straight line onto which it is projected.

[Margin: How circles passing through the south pole of the world — which in the astrolabe are straight lines — are divided into degrees.]

5. It is also gathered from these things by what method any circle drawn through the south pole — which in the astrolabe is a straight line, as has been demonstrated — is to be divided into degrees, and how a proposed point of such a circle can be exhibited in the astrolabe on that straight line which represents the circle. For once it is known how far the straight line HL — the common section of the equator (or plane of the astrolabe) and of the given circle — is distant from the south pole, if it does not pass through the center E (how this distance is ascertained we shall say in its place, when

Diagram. The same stereographic-projection figure of Proposition I, reprinted here identically to fig-p127-1. A large square woodcut with the eye/south pole **A** at bottom center, corners **Y** (left) and **Z** (right), a fan of rays and dotted projecting

rays rising from A, the semicircle ABCD standing on the horizontal common section HL, and the vertical axis through center E carrying **c, a, b, f**.

Readable labels (same set as fig-p127-1):

- A (bottom center); Y (bottom-left corner); Z (bottom-right corner)
- Horizontal diameter (HL), left → right: F, h, G, B, Q, H, I, E, K, L, D, M, N
- Semicircle arc: B, R, T, E, K, L, e, D, f; Q inside
- Vertical axis through E (ascending): c, a, b; f
- Top edge, left → right: e, [?], 1, E, K, L, e; with i at upper right
- Lower region: P, o, V, d
- [Several top-edge and lower labels unresolved at this resolution]

we shall need such a division of circles, a very clear example of which we shall set out in Proposition 8, Number 2): if straight lines are drawn from A through the individual degrees of the circle A B C D, the straight line H L will be cut into unequal parts, as has been shown, which represent the individual degrees of the circle. For since the straight line AE — the common section of the circle ABCD and of the great circle through the poles of the world, and of the circle itself — passes like a kind of proper meridian, it comes about that, just as both Q and T are the sixtieth degree of the circle A B C D (the numbering begun from the point C of that meridian), so in the astrolabe both the point H and the point L represent the 60th degree, to be numbered from the same meridian. By the same reasoning the points I, K represent, on either side, the 30th degree; the points B, D the 90th; the points G, M the 120th; and so with the rest.

[Margin: How any degree is to be found on the same straight line that represents a circle drawn through the poles of the world; and how one learns how many degrees are contained in a given segment of that same line.]

6. Accordingly, if on the straight line HL, whether toward H or toward L, any proposed arc or degree is to be investigated, that arc or degree must be reckoned on the circle from the point C, toward that part in which the proposed arc or degree is sought. For by straight lines drawn from A through the extreme points of that arc, or by a straight line drawn through that degree, the proposed arc or degree will be exhibited on the straight line H L. For example, if the 70th degree is desired on both sides, an arc Cd of 70 degrees is to be taken on each side, as we taught in Lemma 3. For the straight line drawn out from A through d will give, on the straight line HL, the point e, which is 70 degrees distant from the point E on each side. And the same reasoning holds for the other degrees. But if a degree with any

number of minutes is proposed, one must take, according to the doctrine of Lemma 3, an arc containing as many degrees and minutes as are proposed. Conversely, if one wishes to know to how many degrees any given segment of that same line corresponds, two straight lines must be drawn from its two ends to the pole A. For these (produced, however, if need be) will intercept, on the given circle which that line represents, the degrees to which the proposed segment corresponds. For example, if the segment GH is given, two straight lines GA, HA are to be drawn, cutting the circle at P, Q. For as many degrees as are contained in the arc PQ, so many will be said to be included in the given segment GH; and so with the rest.

[Margin: How the straight lines from A through the degrees of the circle may be drawn more accurately.]

7. But so that the straight lines from A may be drawn more accurately through the individual points of the circle ABCD, especially through those which are not far from the point A — where the ruler can easily deviate from a correct position on account of that tiny space between A

[a line drawn through] A and that point, we shall use the following device. From A, describe a semicircle YbZ at any radius you like, and divide it into 360 equal parts — that is, each of the two quadrants bY and bZ into 180 — so that each little part takes in half of one degree. For the straight lines drawn from A through these half-degrees of the semicircle YbZ pass through whole degrees of the circle ABCD, since by lemma 10 each such little part is half of that arc, in the same semicircle YbZ, which is similar to the arc in the circle ABCD enclosed between the two lines that cut off that part from the semicircle.

8. And so, if any degree on the line HL is wanted — that is, a point taking in any number of degrees and minutes, the numbering having been begun from point E — one must take in the semicircle, [reckoning] from point a, an arc containing as many half-degrees as the number of degrees proposed. Thus, if the point on the line HL for 70 degrees is to be found, we shall take an arc of 35 degrees, or of 70 half-degrees; for the straight line A e d [?], drawn from A through the endpoint of that arc, will give on the line HL the point that is sought. Likewise, if the point for 25 degrees 40 minutes is sought, we shall take in the semicircle an arc of 12 degrees 50 minutes, or an arc of 25 half-degrees and 40 half-minutes — and so on for the rest. Or else, by lemma 3, we shall take an arc of 25 degrees 40 minutes; for its half will give an arc similar to half of the arc of 25 degrees 40 minutes in the circle ABCD. And in this way one may always count out in the semicircle

YaZ the whole arc proposed, and then take its half — especially if minutes are attached to the degrees — so that we are not forced to halve both the degrees and the minutes, which is troublesome when the number of degrees and minutes is odd.

9. We shall accomplish the same thing in this way. From any point b on the line AE, produced, let another circle Aghi be described through A, tangent to the line YZ (or to the circle ABCD) at A, and let it be divided into degrees. For the straight lines drawn from A through the degrees of this circle also pass through the individual degrees of the circle ABCD, because by lemma 9 the lines going out from the point of contact cut off similar arcs from circles that touch one another, and so forth.
10. Or indeed we shall obtain the same thing without circles, by lemma 11, if we produce a line — say AO — into a continuous straight line, as we prescribed in that lemma, and in the same manner produce the other lines, whose end points are only slightly apart from one another, into a straight and continuous line by the same lemma.
11. Moreover, in order to obtain more precisely the points at which the lines drawn from A cut the line HL too obliquely — such as the points G and M — we shall have to apply the procedure of lemma 13, where we taught by what art the point can be found at which two lines must meet if they are produced.

[Margin: How any degree may be found more accurately on the same straight line that represents a circle drawn through the poles of the world. When minutes are attached to degrees, what is to be done in this second method.]

THEOR. II. PROPOS. II.

The Equator, and all its parallels, are projected onto the astrolabe as circular shapes, and their arcs into similar arcs, and thus equal into equal; and the southern parallels are projected into circles larger than the Equator, but the northern into smaller. All of them, however, have one and the same center with the astrolabe.

1. That the Equator is projected into a circular shape is evident. For when it is viewed from the south pole through the cone whose base is the Equator itself in the plane of the astrolabe — so that the Equator is the common section of the cone and of the plane of the astrolabe, which does not differ from the plane of the Equator — it plainly follows that it keeps in the plane

of the astrolabe the same circular shape it has in that cone, since all the visual rays going out from the south pole through all the points of the circumference of the Equator terminate on the astrolabe in that same circumference, namely at the base of the cone.

2. That the parallels of the Equator are likewise projected onto the astrolabe in a circular shape we shall show as follows. Since any parallel of the Equator, being a circle, is viewed through a cone whose vertex is the south pole and whose base is the parallel itself, the plane of the Equator (or of the astrolabe), being parallel to the base of that cone when its base lies beyond the Equator, or that same plane produced when its base lies on the near side of the Equator, will make as its section a circle whose center is on the axis of the cone, as was demonstrated in lemma 16.

[Margin: The Equator, together with its parallels, is projected into a circular shape, and equal parts into equal parts, etc.]

Diagram. A meridian view of the sphere for the projection of the Equator and its parallels. The sphere is drawn as a great circle (the meridian) with the north pole C at the top and the south pole A at the bottom; A serves as the eye (center of stereographic projection). To the left lies the point N and to the right the point P — the vertices of the cones by which the parallels are viewed, where the visual rays drawn from A, produced, meet the equatorial (astrolabe) plane. A horizontal line runs through N, B,..., D, P, representing the plane of the Equator BD extended. Solid straight lines radiate from A to points on the sphere; dotted lines run from N and from P as projection rays. Readable labels:

- C (top): north pole
- A (bottom): south pole / eye of projection
- N (far left): cone-vertex on the equatorial plane
- P (far right): cone-vertex on the equatorial plane
- K, L (upper left / upper): northern points
- X, M (upper right): northern points
- O, F, R (left interior): points on parallels
- B (left, on the horizontal diameter)
- Q, Z, T (middle interior)
- S (right of center)
- D (on the horizontal diameter, right of center)
- H, G (lower left)

- V (bottom interior)
 - I, Y (lower right)
3. But since, by lemma 28, all the visual rays cut off similar arcs from any parallel (its base being the base of the cone) and from the circle which the plane of the Equator or the astrolabe makes in that cone, it comes about that the arcs of any parallel are projected into similar arcs, and thus equal into equal, since only equal arcs of one circle can be similar to equal arcs of another. For if, say, two arcs of one circle are similar to two equal arcs of another, then those two are similar to one and the same of these; therefore those two will be equal — otherwise two unequal arcs of the same circle would be similar to one and the same arc of another circle, which is absurd.
 4. Therefore quadrants will be projected into quadrants, degrees into degrees, minutes into minutes, and so forth. That is: just as any quadrant of a parallel in the heavens is a fourth part of its circle, and a degree is the three-hundred-and-sixtieth part, so too the arc in the plane of the astrolabe answering to that quadrant is a fourth part of the whole circle, and the part answering to one degree is the three-hundred-and-sixtieth part of the same circle — and likewise for the rest. From this it follows that, just as in the heavens the Equator and each parallel is divided into 360 equal degrees, so too the Equator, and the circle on the astrolabe representing that parallel, must be divided into 360 equal parts, so that its degrees may be had.
 5. Next, let there be an Analemma in which the Meridian is ABCD; the Equator BFDI, with its diameter BD; any southern parallel GHIV, with its diameter GI; any northern parallel KLMX, with its diameter KM; and the axis of the world AC. Now since the visual rays AG, AI, drawn through the end points of the diameter of the southern parallel, fall upon the plane of the Equator produced beyond the sphere at the points N, P of the common section of the plane of the Equator and of the Meridian (cutting the sphere at G, I), while the visual rays AK, AM, drawn through the end points of the diameter of the northern parallel, meet that same plane of the Equator within the sphere at the points Q, S of the same common section of the plane of the Equator and of the Meridian — and the same happens with the rays sent out through the end points of the other diameters of each parallel — it plainly follows that the southern parallel is projected into a circle larger than the Equator, but the northern into a smaller one; inasmuch as the apparent diameter NP of the former is larger than the diameter BD of the Equator, whereas the apparent diameter QS of the latter is smaller, and

consequently the apparent circle NOPY of the former is larger, while the apparent circle QRSZ of the latter is smaller than the Equator's circle BFDT. And the same reasoning holds for the other parallels, both southern and northern.

[Margin: The Equator, and its parallels, must be divided on the astrolabe into 360 equal parts, so that their degrees may be had, just like the circles on the sphere. The southern parallels on the astrolabe are larger than the Equator, and the northern smaller.]

Diagram. The same projection figure as on the preceding page, repeated to accompany the continued discussion. North pole C at top, south pole / eye A at bottom, cone-vertices N (left) and P (right) on the horizontal equatorial line. The readable labels match the previous figure: C, A, N, P, K, L, X, M, O, F, R, B, Q, Z, T, S, D, H, G, V, I, Y.

6. Finally, since by lemma 16 the circles which planes parallel to the bases of the cones cut off have their centers on the axis — and the axis of the world AC is projected into the center of the astrolabe (or of the Equator) E, as was said above — it is evident that all the circles on the astrolabe into which the Equator and its parallels are projected are concentric, and have the same center as the astrolabe. Which was to be demonstrated.

[Margin: The Equator, and its parallels, have on the astrolabe the same center as the astrolabe.]

THEOR. III. PROPOS. III.

Any circle of the sphere oblique to the Equator, or even one perpendicular to it but not a great circle, is projected onto the astrolabe into a circular figure; but its arcs, reckoned from a certain point, are projected into dissimilar arcs, and thus equal into unequal; and finally its center on the astrolabe is different from the center of the astrolabe.

1. In the sphere ABCD, whose center is E and whose poles of the world are A, C, let there be a circle, whether a great one — with diameter FG — or a non-great one, with diameter HI or KL, oblique to the Equator, that is, whose poles M, N are different from the poles of the world C, A; or else a non-great circle perpendicular to the Equator, with diameter p r, that is, through whose poles the Equator passes. I say that it is projected onto the astrolabe into a circular figure, and so forth. For let there be described, through its poles and the poles of the world, the great circle ABCD, and let the common

section of this circle and of the Equator be the line BD, extended to infinity; and from A, the south pole, let the visual rays be extended through the end points of the diameters, cutting the line BD — through which the plane of the astrolabe (or of the Equator) is drawn, [note a] to which the circle ABCD is perpendicular — at the points O, P, Q, R, S, T, t, u. And since the scalene cones, whose vertex is A and whose bases are the circles of the diameters FG, HI, KL, p r, are cut by the plane of the circle ABCD [note b] perpendicular to their bases, making the triangles through the axis AFG, AHI, AKL, A p r (for the axes of these cones are in the plane of the circle ABCD, since the centers of the bases, to which the axes are drawn, lie in the same plane [note c] — inasmuch as the circle cuts them in two, that is, through their centers), and are also cut by another plane through the line BD — namely the plane of the Equator or of the astrolabe, drawn perpendicular to the triangles through the axis, that is, to the plane of the circle ABCD [note d] — since this circle, drawn through the poles of the Equator, cuts it at right angles: this plane, drawn through BD, cuts off the triangle AOP, similar to triangle AFG; the triangle AQR, similar to triangle AHI; the triangle AST, similar to triangle AKL; and the triangle A t u, similar to triangle A p r, and set subcontrariwise, as we demonstrated in lemma 35 — whatever position the diameter of the inclined circle may have. It will therefore make, by lemma 17, in the aforesaid scalene cones sections that are circles, whose diameters are OP, QR, ST, t u. That these cones are scalene will be demonstrated in this way. When the axis MN of the bases of the first three cones is drawn, it will pass through E, X, V, the centers of the circles that are the bases, and will be perpendicular to those circles. For every circle on the sphere has its own axis, which is perpendicular to its plane and passes through its center and its poles. [note f] Since, therefore, from the points E, X, V no other lines perpendicular...

[Margin: Any oblique circle, or even one perpendicular to the Equator but not a great circle, is projected into a circular shape, and equal parts into unequal parts, etc.]

[Margin (citation letters keyed to the superscripts above): a — 15. 1. The. (Theodosius, Sphaerica I.15 [?]); b — 15. 1. The.; c — 15. 1. The.; d — 15. 1. The.; e — 10. 1. The.; f — 13. undec. (Euclid XI.13).]

[perpendicular] to those circles, they will be the axes of the cones AE, AX, AV — that is, perpendicular to the bases of the oblique cones, and therefore the cones will be scalene. But in the last cone, since BD, the axis of the circle whose

diameter is $p r$, is also perpendicular to $p r$ and passes through its center k , it is clear that the axis $A k$ of that cone is oblique to the base of the cone, and consequently the cone too, whose base is the circle of diameter $p r$, is scalene.

2. Next, that the arcs of the circles whose diameters are FG , HI , KL , $p r$ — if they all begin from a certain point — are projected into dissimilar arcs, and thus arcs answering to equal arcs in the circles of the diameters OP , QR , ST , $t u$ are unequal to those in the circles of the diameters FG , HI , KL , $p r$, is clear from lemma 31. There it was demonstrated that if in the circle of diameter FG two equal opposite arcs beginning from the points F , G are taken, the arcs answering to them in the circle of diameter OP — namely those which, in the cone whose base is the circle of diameter FG , the same straight lines going out from A cut off — are unequal: the greater being the one that lies near the smaller angle P , the smaller being the one that lies near the greater angle O . And that the angle O is the greater in triangle AOP , and the angle P the smaller, is clear, since the one is equal to angle G and the other to angle F in triangle AFG , on account of the subcontrary section. [note a] Now it is established that angle G is greater than angle F , because the side AF is greater than the side AG — for the former is greater than the side of the square AB , and the latter less than the side of the square AD , were those sides drawn, as is established from the scholium to proposition 29 of book 3 of Euclid. Or else, because angle G is greater than the interior angle P , angle O too — which is equal to G — will be greater than P . By the same reasoning, to equal arcs in the circles of the diameters HI , KL , $p r$, beginning from the points H , I , K , L , p , r , there will answer unequal arcs in the circles of the diameters QR , ST , $t u$. Therefore the arcs of the circles whose diameters are FG , HI , KL , $p r$ are projected into dissimilar arcs, and equal into unequal, if they take their beginning from the points we have mentioned.
3. In the same lemma 31 it was demonstrated that if in the cone whose base is the circle of diameter FG straight lines are drawn out from the vertex A , the arcs in the circle of diameter OP intercepted between P and those lines are greater than would make them similar to the arcs answering to them in the circle of diameter FG , which the same lines cut off, and so forth. It is therefore established, again, that the arcs of the circle of diameter FG are projected into dissimilar arcs in the circle of diameter OP , if they begin from the point P . And the same must be said of the arcs of the circles whose diameters are HI , KL , $p r$; for these too, by the same lemma, will be projected into dissimilar arcs in the circles of the diameters QR , ST , $t u$. But

that equal arcs of the greatest oblique circles are projected into unequal arcs in continued order, we shall clearly demonstrate in the scholium to proposition 5, number 12 and following. And then, in the scholia to propositions 6 and 7, we shall demonstrate the same for the non-great oblique circles. So that it is most true that the equal arcs of any oblique circle — whether a great one or a non-great one — not only are projected into dissimilar arcs, if they all take their beginning from a certain point, but also into unequal arcs, as was set forth in the theorem. From which it comes about that an oblique circle, whether great or non-great, must not be divided on the astrolabe into equal parts, in order to have its degrees answering to the degrees of the same circle on the sphere, but into unequal parts, as we shall show in propositions 5, 6, and 7.

4. Finally, that the center of any oblique circle on the astrolabe differs from the center of the astrolabe — that is, that the apparent diameters OP, QR, ST, t u are not bisected at E, the center of the sphere (which is also the center of the astrolabe), as we said — we shall easily show as follows. Since EB, ED are equal, ED will be greater than EO. Much more, then, will EP be greater than EO. The diameter OP is therefore not bisected at E. This is clear in the great circle also from lemma 35, where it was shown that the perpendicular AY to the diameter FG bisects the diameter OP at Z. It is therefore not cut in two at E. Again, when Ia, Lb are drawn parallel to BD, cutting the axis of the world AC, and the lines AH, AK are drawn cutting them at c, d, e, f: since, from the scholium to proposition 4 of book 6 of Euclid, as I c is to c d, so is RE to EQ; and as L e is to e f, so is TE to ES — and since I c is greater than c d, and L e greater than e f, [note b] because Ia, Lb are bisected at c, e [note c] (the angles at c, e being right, on account of the parallels BD, aI, bL) — therefore RE too is greater than EQ, and TE greater than ES. Neither, then, is the diameter QR, nor the diameter ST, bisected at E; and consequently, since the center divides a diameter into two equal parts, E will not be the center of the diameters OP, QR, ST. Finally, that the apparent diameter t u too is not bisected at the center E is clearer than daylight, since the whole of it lies beyond the center E, as is evident, on account of the rays A p, A r.

[Margin: An oblique circle has on the astrolabe a center different from the center of the astrolabe. — b: 3. tertij (Euclid III.3). — c: 29. primi (Euclid I.29). — a (§2): 18. primi (Euclid I.18).]

Diagram. A crowded projection figure enclosed in a rectangular frame. The meridian circle ABCD has its center at E; the north pole C is at the top and the south pole / eye A at the bottom, with N just to the left of A. A horizontal diameter runs across the figure through the center E, carrying (left to right) the points Q, B, O, o, E, Z, T, R, D, P. Solid rays radiate from A; several parallel and dotted chords cross the interior. Because the figure is densely lettered and includes many very small (partly lowercase and Greek-like) labels that do not resolve on the scan, several labels are uncertain. Readable labels:

- C (top), M (near top): pole and pole M of the oblique circle
- A (bottom): south pole / eye; N (beside A)
- K (upper left), P (upper right, on the circumference)
- L (upper right)
- Q, B, O, o, E, Z, T, R, D, P (along the horizontal diameter)
- F, f, b (left interior) [b, f partly uncertain]
- G (right interior)
- H (left), I (lower right)
- V, u (upper interior) [u uncertain]
- Y, i, m, x (interior) [several uncertain]
- a, d (lower interior) [uncertain]
- additional small letters near the center (γ , δ , ϵ , ζ -like) [?] — not resolvable

This figure is a (more than two labels flagged [?]).

SCHOLIUM

1. One must, however, view any oblique great circle, and its parallels, or any non-great circle perpendicular to the Equator, from the south pole, in the common section of the Equator (or of the plane of the astrolabe) and of the great circle drawn through the poles of the world and the poles of the oblique or perpendicular circle — both so that we may demonstrate that they are projected into a circular shape, and so that we may have [their diameters as large as possible]...

[Margin: An oblique circle has on the astrolabe a center different from the center of the astrolabe. In which great circle the oblique circles are to be viewed, so that their diameters may be had as large as possible.]

ASTROLABII LIBER SECVNDVS

(Scholium to Theorem I, continued — from batch 161's catchword "ximas.")

so that we may have the greatest of their visible diameters, about which they are to be described. For, in order that in a scalene cone the subcontrary section be a circle, the triangle through the axis must be perpendicular to the base of the cone, as lemma 17 establishes. Now such is precisely the triangle drawn in the plane of the great circle [drawn] through the poles of the world and through the poles of the oblique — or right — circle through which it passes; that is, the triangle drawn perpendicular to the oblique or right circle through whose poles it is carried. (a) For this great circle is perpendicular to the base of the cone through whose poles it is drawn, and no other circle is (that is, none which does not pass through its poles); and this great circle measures the greatest declination of the greatest oblique circle from the equator, since its arc lying between the greatest oblique circle and the equator is the arc of the angle which the oblique circle makes with the equator. It therefore yields the diameter of the greatest oblique circle — the diameter which is the common section of that circle and of this great circle (such as, in the preceding figure, is the diameter FG) — and this diameter, taken together with the diameter of the equator (which is the common section of the same great circle and of the equator, of the sort that in the same figure is the diameter BD), makes a greater angle than does any other diameter of the oblique circle. For a diameter which is the common section of the oblique circle and of a *different* great circle — one passing through the poles of the world but not through the poles of the oblique circle — does not measure the declination of the oblique circle from the equator. And therefore all the remaining diameters of the oblique circle will fall between the points B and F, and D and G. Hence, by lemma 36, the diameter OP is seen to be the greatest of all, and BD the least of all; because the straight lines drawn through the extreme points of the other diameters — diameters that make smaller angles with BD at the centre E — cut off from the line OP straight lines smaller than BD [on the one side] and greater than BD [on the other], as we demonstrated there.

[Margin (a): 15. lib. 1, Theor. (of this treatise)]

2. That the visible diameter ST — the common section of a *non*-greatest oblique circle (whose [own] diameter is KL) with the great circle ABCD drawn through its poles and through the poles of the world — is likewise the greatest of all, we shall confirm as follows. From A let AV be drawn to V, the centre of the oblique circle in the cone of which that circle is the base; and let the axis AV cut the straight line BD at g. Now all the diameters of the oblique circle in the sphere, passing through the centre V, are seen in the plane of the astrolabe, along the line BD, to pass — when the projection is made — through the point g. Draw also Sh parallel to KL, cutting the axis of the cone AV at i. Then, by the scholium of proposition 4, book 6,

of Euclid, as hi is to iS , so is LV to VK . But by lemma 29 the ratio of Tg to gS is greater than the ratio of hi to iS — that is, than the ratio of LV to VK . Since therefore LV and VK are equal, Tg and gS will be unequal, Tg being greater than gS . And accordingly the centre of the circle of diameter ST , which bisects the diameter ST , (b) will lie on the straight line Tg . The line ST , then, drawn through the centre of that circle — the circle which represents the oblique circle of diameter KL — is, as we have demonstrated, greater than all the other lines drawn through g in the same circle, which are indeed the visible diameters of the oblique circle, as has been said.

In the same way we shall show that the visible diameter QR — the common section of a non-greatest oblique circle (of diameter HI) with the great circle $ABCD$ drawn through its poles and through the poles of the world — is the greatest of all. For, drawing AX from A to X , the centre of the oblique circle in the cone of which that circle is the base, with the axis AX which, produced, cuts the line BD at n , all the diameters of the oblique circle in the sphere passing through the centre X are seen, when drawn in the plane of the astrolabe along the line BD , to pass through the point n . And since QI , drawn parallel to HI , cuts the produced axis of the cone at m , it follows [by the scholium of prop. 4, book 6, of Euclid] that as lm is to mQ , so is IX to XH . But by lemma 29 the ratio of Rn to nQ is greater than that of lm to mQ ; hence the ratio of Rn to nQ will also be greater than that of IX to XH . Since therefore IX and XH are equal, Rn and nQ will be unequal, Rn greater than nQ ; and accordingly the centre of the circle of diameter QR — the circle which represents the oblique circle of diameter HI , as we demonstrated — which bisects the diameter QR , (c) will lie on the straight line Rn . The line QR , then, drawn through the centre of that circle, is greater than all the other lines drawn through n in the same circle, which are the visible diameters of the oblique circle of diameter HI , as we said.

Finally, we shall prove in no other way that the visible diameter tu — of the circle [that is] right [perpendicular] to the equator, whose [own] diameter is pr — is the greatest of all. For, drawing the axis Ak in the cone whose base is the circle of diameter pr , let pr be produced through t ; let ta be drawn parallel to pr , cutting Ak at $[e]$. Then, by the scholium of prop. 4, book 6, of Euclid, as e is to et , so is rk to kp . But by lemma 29 the ratio of uk to kt is greater than that of a to et . Therefore the ratio of uk to kt is also greater than that of rk to kp . Since [rk and kp are equal], uk and kt will be unequal, and uk the greater; and accordingly the centre of the circle of diameter tu will lie on the straight line uk . Therefore the line ut , drawn through that centre, will be greater than all the

other lines drawn through k in the same circle, which are the visible diameters of the circle whose diameter is p r, in the sphere — which was to be proved.

[Margin (b): 15. lib. 1, Theor. — (c): 15. lib. 1, Theor.]

Diagram. A large square-framed woodcut occupying the middle of the page. Within it a great circle (the meridian / great circle ABCD) is drawn with the projection apex A at the bottom (the south pole, i.e. the eye of projection); a horizontal diameter runs through the centre E and is extended far to the right beyond the circle. Numerous chords and diameters are drawn across the circle, with lines converging on the apex A, illustrating the visible (projected) diameters discussed in the scholium (FG, BD, OP, KL, ST, HI, QR, and the perpendicular-circle diameters).

Readable labels (position approximate):

- C (top of circle); M (just right of C, at top)
- K (upper left, on circle); V (interior, upper-centre)
- F (left, on circle); B (on the horizontal diameter, left of centre)
- Q (far left, at the left end of the horizontal line); O (interior, upper-left)
- E (centre); Z (interior, right of centre); N (interior)
- along the extended horizontal line to the right: T, R, D, P, and a final endpoint R (far right)
- H (lower left, on circle); G (lower right, on circle)
- I (lower right, interior); L (upper right)
- A (bottom apex)
- Several small cursive/Greek-form interior labels not securely resolvable: b, f, d [?], δ [?], ξ [?], ζ [?], u [?], m [?], x [?], k [?], a [?]

This is a crowded projection figure with many small interior labels; more than two are flagged

PROPOS. III. (*continued* — p. 133)

4. From these considerations it is clear that the visible diameters, even the greatest, are contained [within the projected figure]. For if [the figure] were described according to some smaller visible diameter, it would come out smaller in the astrolabe than it appears, since its greatest visible diameter would exceed it — which is absurd.

From this it further follows that the straight line drawn through the centre of the astrolabe and through the centre of any oblique circle — whether greatest or not

greatest, or even of a non-greatest right [perpendicular] circle — is the common section of the plane of the astrolabe (that is, of the equator) and of the great circle which passes, in the sphere, through the poles of the world and through the poles of the oblique or right circle. For if any other straight line were said to be this common section, the greatest visible diameter would appear on it, and therefore the centre of the oblique or right circle to be described would lie on it as well; which, as we said, is absurd, since its centre lies on that former straight line.

[Margin: That the straight line drawn through the centre of the astrolabe and through the centre of any circle described on the astrolabe is the common section of the plane of the astrolabe (or equator) and of the great circle drawn through the poles of the world and through the poles of the described circle. — (a) 15. lib. 1, Theor.]

5. Accordingly the oblique horizon, the ecliptic (with the beginnings of Cancer ☿ and Capricorn ♑ placed on the meridian), and the prime vertical must first be examined on the common section of the meridian and the equator (that is, of the astrolabe), so that their visible diameters may be had as greatest, and so that on that same section their centres too may lie — because, of course, the meridian, drawn through the poles of those circles, (a) is perpendicular to them.

6. Jordanus [Jordanus de Nemore], in his *Planisphere* — which is like a little commentary on Ptolemy's *Planisphere* [*Planisphaerium*] — by another demonstration, which does not depend on cones, concludes that all oblique circles are projected into a circular figure; that is, that all the points of the circumference of any oblique circle, [projected] by rays sent out from the south pole, fall on the circumference of a [single] circle. This demonstration, because it is subtle and elegant, I have thought fit to append here.

7. Let there be, then, an oblique circle — whether greatest or non-greatest, or even a non-greatest right circle — FKGI, whose common section with the great circle ABCD (drawn through its poles and the poles of the world) is F, G, its diameter; the extreme points of this diameter appear, by way of the rays AF, AG, on BD (the common section of the same great circle ABCD and of the equator, i.e. of the astrolabe) at the points H, I. Through any point O of the diameter FG let a plane be drawn parallel to the equator — that is, perpendicular to the circle ABCD, (b) since this circle cuts the equator and its parallels through the poles A, C, (c) and hence at right angles — (d) making in the circle ABCD the section MN parallel to BD, (e) and on the surface of the sphere the circle NkML. Let kOL be the common section of the circles FkGL and NkML; (f) this will be perpendicular to the circle ABCD, because each circle is perpendicular to it, and so, by definition

3, book 11, of Euclid, it is perpendicular to FG. (g) Therefore the diameter FG, cutting KL at right angles, will bisect it at O. Now let AO, extended from A through O, cut HI at R; and through R, in the plane of the triangle AKL (the straight lines AL, AK having been drawn), let PRQ be drawn parallel to KL, meeting the visual rays AK, AL at P, Q. (g) This too will be perpendicular to the plane of the same circle ABCD, and hence, drawn in the plane of the equator through HI and being perpendicular to the same circle ABCD, [the points K, L of the circle FKGL] will appear, in the plane of the equator (i.e. of the astrolabe), at the points P, Q, and the line KL as the line PQ. I say that the four points H, I, P, Q fall on the circumference of a single circle in the plane of the astrolabe (i.e. of the equator). For let GC be joined, and let the line MN cut the visual ray AF at S and the axis AC at V; and let that same line NM be extended as far as T. (h) Since therefore the angle AGC is right, (i) as is also the angle AVT (because of the parallels BD, NM); and since the triangles AGC, AVT have the angle A in common, the remaining angle ACG will, by corollary 1 of proposition 32, book 1, of Euclid, be equal to the remaining angle ATV. (k) But equal to the same angle ACG is the angle AFG. Therefore the angles T and F, in the triangles GOT and SOF, will be equal. (l) And since the vertical angles at O are also equal, the triangles GOT, SOF will be equiangular. (m) Therefore, as GO is to OT, so is SO to OF; (n) and accordingly the rectangle under GO, OF will be equal to the rectangle under TO, OS. (o) But the rectangle under GO, OF is equal to the rectangle under KO, OL. Therefore the rectangle under TO, OS will also be equal to the same rectangle under KO, OL — that is, to the square on the line KO, since KO and OL have been shown to be equal. (p) And on that account the three [lines] TO, KO, OS are continually proportional. And since the triangle TOA is similar to the triangle IRA, and the triangle AOK to the triangle ARP, then by the corollary of prop. 4, book 6, of Euclid, (q) as TO is to OA, so is IR to RA, and as OA is to KO, so is RA to PR. Hence, *ex aequali*, as TO is to KO, so is IR to PR. Again, since it is, by the scholium of pr[osition]...

[Margin: **Postil.** Jordanus's demonstration that oblique circles, or even non-greatest right circles, are projected into circular figures.]

[Margin (b) 15. lib. 1, Theor.; (c) Euclid XI.16; (d) 1. lib. 1, Theor.; (e) Euclid XI.19; (f) Euclid III.3; (g) Euclid XI.8; (h) Euclid III.31; (i) Euclid I.29; (k) Euclid III.21; (l) Euclid I.15; (m) Euclid VI.4; (n) Euclid VI.16; (o) Euclid III.35]

The continued proportion just established ($TO : OA = IR : RA$, and $OA : KO = RA : PR$):

First set	Second set
TO	IR
OA	RA
KO	PR

(Beneath the table: "vt SO," — i.e., "as SO...", continuing onto p. 134.)

Diagram. A great-circle figure (the meridian ABCD) with the projection apex A at the bottom. A horizontal diameter runs through the centre E from N (left) to M (right); the vertical diameter runs from C (top) to A (bottom). Chords and visual rays are drawn from A, and lines extend to the right toward T (with a small triangle M–I–T outside the circle at lower right).

Readable labels:

- C (top); L (upper interior, on the arc); F (upper left, on circle); M (right end of horizontal diameter)
- N (left end of horizontal diameter); O (interior, upper-left); V (interior, centre-upper); S (interior)
- Q (interior); K (lower left, on circle); H (interior, on the horizontal line, left of centre)
- E (centre); R (interior, below centre); D (right, on the horizontal line beyond centre)
- G (lower right, on circle); A (bottom apex); T (lower right, outside circle)
- one lower-interior label reads P or ϕ [?]; a far-left label I [?]

Two labels flagged — within tolerance.

Diagram. A square-framed woodcut at the foot of the page, showing the same construction as the demonstration of Prop. III with the projection apex A at bottom and the extended horizontal diameter running out to T (lower right). (This same figure is reprinted at the top of p. 134; see fig-p486-1.)

Readable labels:

- C (top); Q (upper left, outside circle); H (left, on horizontal); B (left, on circle at horizontal)
- R (interior, left of centre); E (centre); D (right, on horizontal); I (far right, at frame)
- L (interior, upper-left); F (lower left); N (lower left, below F); S (interior lower-left)

- o (interior, lower-centre); P (interior); V (interior, centre); K (lower interior)
- M (lower right, on circle); G (lower right); A (bottom apex); T (lower right, outside circle)

More than two small interior labels are cramped/uncertain (o, S, L, P, V, K)

LIBRI II. ASTROLABII (p. 134)

The continued proportion carried over from p. 133 ($SO : OT = HR : RI$, and $OT : OK = RI : RP$):

First set	Second set
SO	HR
OT	RI
OK	RP

[Margin (a): Euclid VI.17]

as SO is to OT, so is HR to RI. But it was just now shown that, as OT is to OK, so is RI to RP; therefore, *ex aequali*, as SO is to OK, so is HR to RP. And by inversion, as OK is to SO, so is RP to HR. Wherefore, since it holds that as TO is to OK, so is IR to RP, and as OK is to OS, so is RP to HR; and since the three [lines] TO, OK, OS have been shown to be continually proportional; the three [lines] IR, RP, HR will also be continually proportional. (a) Therefore the rectangle under IR, RH will be equal to the square on the line RP — that is, to the rectangle under PR, RQ, since these lines are equal, inasmuch as, by the scholium of prop. 4, book 6, of Euclid, they have the same ratio as the equal lines KO, LO. Therefore, by the scholium of prop. 35, book 3, of Euclid, a circle described about the points H, I will pass through the points P, Q. And HI will always be the diameter of the circle of the four points H, I, P, Q, because, cutting the line P, Q in two and at right angles (for since P, Q is perpendicular to the plane of the circle ABCD, it will also be perpendicular to the line HI), it passes through the centre of that circle, by the corollary of prop. 1, book 3, of Euclid. In no other way shall we show that the same [circle] passes through the other points into which fall, in the plane of the astrolabe (i.e. of the equator), the straight lines sent out from the south pole A through the other points of the oblique circle FKGL — namely, if through the other points of the diameter FG there be drawn planes parallel to the equator, and so on. Therefore the oblique circle, or even the non-greatest right circle FKGL, is projected into a circular figure — which was to be demonstrated.

PROBLEMA I. PROPOS. IV.

To project the equator, and any of its parallels whose arc of declination is given, onto the plane of the astrolabe, and to divide it into degrees.

1. Let an analemma be described, as was set forth in lemma 19, whose meridian ABCD, described from the centre E, is made equal to the equator in the astrolabe to be constructed (for the magnitude of the equator may be taken at anyone's discretion); [with] the axis of the world AC; the south pole A and the north pole C; the diameter of the equator BD; the tropic of Cancer ☿, FG; the tropic of Capricorn ♊, HI — such that the arcs BF, BH, DG, DI measure the greatest declination of the Sun, or of the ecliptic; and so that between these diameters FG, HI are contained the diameters of the other parallels drawn through the beginnings of the signs, as in the analemma of lemma 19; and outside them, the diameters of the arctic and antarctic circles, h p, Y Z; the diameter of the horizon for the elevation of the pole of 42 degrees, f g; its axis, or the diameter of the vertical, OR; the diameter of the ecliptic GH. If, then, from the south pole A visual rays be sent out through the extreme points of the diameters, they will cut the diameter of the equator BD, extended to infinity (through which, indeed, the plane of the equator, i.e. of the astrolabe, is drawn — (b) to which the meridian, making in it the section BD, is perpendicular), at the points where those extreme points appear; and accordingly...

[Margin: The description, on the astrolabe, of the equator and of its parallels, derived from the analemma, when the magnitude of the equator is given. — (b) 15. lib. 1, Theor.]

they will cut off, from that same line BD, the visible diameters; and the visible diameter of the equator BD will be the same as in the analemma; the tropic of Cancer ☿, KL; the tropic of Capricorn ♊, MN. And since, by proposition 2, the equator and all its parallels are projected as circular figures having the common centre E on the axis of the cones, all the other visible diameters of the parallels will be equal to the diameters BD, KL, MN, since they all pass through E and are terminated on the circumferences of the circles described from E at the intervals EB, EK, EM. Wherefore, if in the plane on which the astrolabe is to be constructed, from any assumed centre E, circles be described at the intervals of the semidiameters EB, EK, EM, then ABCD will be the equator; FKGL the tropic of Cancer ☿; and HMIN the tropic of Capricorn ♊. In exactly the same way the other parallels running through the beginnings of the signs will be described, and also other parallels both within the tropics and outside them, provided their declinations, or their distances from the points B, D, are known. In the proposed

analemma the visual rays AY, AZ, sent out through the extreme points of the diameter of the antarctic circle YZ, meet [*concur...*] the line BD so far off...

(catchword: "concur-")

Diagram. A square-framed woodcut at the head of the page — a reprint of the Prop. III projection figure (see fig-p485-2): great circle with projection apex A at the bottom and the horizontal diameter extended to T at the lower right.

Readable labels: C (top); Q (upper left); H (left, on horizontal); B (on circle at horizontal); R (interior); E (centre); D (right, on horizontal); I (far right, at frame); L, F, N (lower left); S, o, P, V, K (interior, lower); M, G (lower right); A (bottom apex); T (lower right, outside circle). Several small interior labels are cramped/uncertain

Diagram. The analemma figure for Problema I, beginning at the foot of the page and continuing beyond this opening. A large circle (meridian ABCD) with the south pole/eye A at the bottom; the horizontal line runs left from the circle through points X, N, D to the circle, with dashed visual rays fanning down to A and out to the right; several parallel chords (tropics and other parallels) are drawn across the circle, and the zodiac symbols for Cancer ☊ and Capricorn ☋ mark the tropics.

Readable labels (partial — figure is cut by the page foot and continues):

- C (top); P/ϰ [?] and h and a [?] near the top of the circle; K, O, F on the upper-right arc
- X (far left, on the horizontal line); N (on the horizontal line); D (on the horizontal line, at circle)
- interior cluster near centre: E, and small labels L, T, m, S, I, k [?] (not securely resolvable)
- B (right, on the horizontal); along the horizontal to the right: M, and γ and small m/n [?]
- R, I, Z (lower left / interior); A (bottom apex); G (upper right); Y (bottom, near A)

The figure is truncated at the page foot and carries many small, faint labels (to be completed with the continuation in the next opening).

they meet, so that its diameter, seen in the plane, could not be marked. In the same analemma, if the diameter OP is drawn (passing through the vertex, or pole, of the Roman horizon), and another diameter QR of the opposite southern

parallel (passing through the Nadir, or the other pole of the same horizon), and if visual rays are emitted through their endpoints, there will be found in the plane of the astrolabe their apparent diameters ST, VX. But it is enough, as you see, if the apparent semidiameters ES, EK, EB, EM, EV — or ET, EL, ED, EN, EX, and so on — are found on one side only of the axis AC, whether right or left; [these lie] in the plane of the equator (or astrolabe), being drawn through the straight line BD and toward the meridian. Moreover the north pole C also appears — indeed the whole axis AC is seen — at the very center E; so that E, the center of the astrolabe and of the parallels, represents both the north pole and the axis of the world, as we also noted above in Proposition 1, number 4. Finally, just as, once the parallels have been described in the plane of the astrolabe, the diameter (or straight line) MN is the common section of the plane of the astrolabe (or equator) and of the meridian circle — representing in the astrolabe the meridian circle itself — so the diameter (or straight line) HI, cutting MN at right angles, is the common section of the same plane of the astrolabe (or equator) and of the right horizon, that is, of the equinoctial colure, the solstitial colure coinciding with the meridian. For since the meridian and the right horizon (by Proposition 1, number 4) are projected into straight lines passing through center E, and since both the right horizon and the equator are perpendicular to the meridian, ^a their common section with it will also be perpendicular to it; and therefore, by Euclid, definition 3 of Book 11, since MN lies in the meridian, it will make right angles with MN. Hence HI, perpendicular to MN, will be the common section of the right horizon and the equator, if MN is taken as the common section of the same equator and the meridian.

[Margin: It is enough if only the semidiameters are found.]

[Margin: The arctic pole and the axis of the world are represented in the astrolabe by the center.]

[Margin: The meridian and the right horizon in the astrolabe, how [they are represented] [?]. z 19. and hence [?].]

Diagram. Large geometric woodcut occupying the lower half of the page (repeated on the facing verso, p.488), illustrating the stereographic projection of the equator and its parallels onto the plane of the astrolabe together with the folded-down analemma construction. A rectangular frame encloses concentric circles about center E; the horizontal diameter runs left to right and the vertical diameter top to bottom, with a large slanted analemma arc sweeping from upper left down to lower center.

- E (center): center of the astrolabe / projection; north pole and axis of the world
 - Horizontal diameter, left to right: **X, R, I, C, G, E, F, A, H**
 - Vertical diameter / axis, top to bottom: **S** (top), **p/b** (upper), **o, M, n, B, K, l** (script, near center), **L, D, N** (lower)
 - Upper-left arc (analemma meridian, folded down): **Y, r, e, V, q**
 - Right region: **P, Q, a** (lower right)
 - Small script/faint interior labels near center: **h[?], z[?], m[?], i[?], ∂/d[?]**
 - A, B, C, D: the meridian circle ABCD (A right, B top, C left, D bottom); AC the axis; BD the equatorial section
 - K, M: points on EB where the tropic of Cancer and tropic of Capricorn cross; P, O: declination points on the semicircle
2. Now since (by Proposition 2, number 4) the equator in the astrolabe and its parallels must be divided into 360 equal parts to obtain their degrees, the degrees of any parallel are easily had if it is cut into 360 equal parts. From this it follows that the straight lines drawn through center E, cutting the circles described from E into 360 equal parts, are the common sections of the plane of the astrolabe (or equator) and of the great circles drawn through the poles of the world and through the individual degrees of the equator. For these great circles, in the sphere, divide all the parallels into degrees — that is, into parts similar to the parts of the equator — and (by Proposition 1, number 1) they are projected into straight lines in the astrolabe.

[Margin: Division of the parallels of the equator into degrees.]

[Margin: b — 10.2 Theor. [?] The great circles drawn through the poles of the world and through the individual degrees of the equator are represented in the astrolabe by straight lines drawn through the center of the astrolabe, dividing any circle described from the same center into 360 equal parts.]

3. Therefore, in order that any proposed parallel — passing through some degree of the meridian, or solstitial colure — may be described in the astrolabe, its declination (its distance from the equator) must be counted off in the analemma, from point B toward the arctic pole C, or toward the antarctic pole A, according as the given parallel is northern or southern. For a straight line drawn from A through the end of the count cuts off from EV a semidiameter, at whose interval the given parallel is to be described from center E in the astrolabe. Thus, if a parallel declining 60° north from the equator is to be described, we shall count from B toward C 60°, up to

point a; for the line Aa will cut off its apparent semidiameter Eb. Likewise, if a parallel declining 30° south from the equator is to be described, we shall count from B toward A 30° , up to point d; for the line Ad, produced, will cut off its apparent semidiameter Ee. And so with the rest.

[Margin: To describe in the astrolabe, from the analemma, any parallel of the equator of a given declination.]

4. Conversely, once any parallel has been described from center E in the astrolabe, we shall know its declination from the equator, whether north or south, in this way. Let its semidiameter, taken in the astrolabe, be transferred into the straight line EV, from E, in the analemma. For from the endpoint of that line, drawn to A, [a line] will pass in the meridian ABCD through the point through which the given parallel is drawn in the sphere. And if that line cuts the quadrant BA, the parallel will be southern; but northern if it cuts the quadrant BC. For instance, if you wish to know whether the parallel HMIN in the astrolabe is southern or northern, and how great a declination it has, transfer its semidiameter EM, by means of the compasses, into the analemma from E to M. Since the line drawn AM cuts the quadrant BA at point H, which is distant $23^\circ 30'$ from B, the parallel HMIN will be southern, and therefore the tropic of Capricorn (♋). Likewise the diameter EK of the parallel FKGL will give in the analemma the northern arc of declination BF, $23^\circ 30'$, and so the parallel will be the tropic of Cancer (♋). Finally, since the semidiameter EB of the parallel ABCD coincides in the analemma with the semidiameter EB, that parallel will itself be the equator in the astrolabe. And so with the rest.

[Margin: To learn from the analemma the declination of any parallel of the equator described in the astrolabe, and whether it is northern or southern.]

5. Furthermore, we shall describe these same parallels of the equator in the plane of the astrolabe, together with the equator, even if the analemma has not been separately constructed, in this way. Having described the equator ABCD of any size in the plane of the astrolabe from center E (for the size of this circle can be determined at anyone's discretion), and having drawn two diameters AC, BD cutting one another at right angles at the center, let this circle ABCD be taken for the meridian of the analemma — since the equator of the astrolabe and the meridian of the analemma are equal, as has been said — and AC for the axis of the world, with A the south pole and C the north; finally let BD, extended both ways, be taken as the common section of the equator and the meridian, as in the analemma: just as if the

semicircle BAD stood at right angles to the plane of the equator (or astrolabe) upon the line BD, and the other semicircle BCD stood at right angles to the same plane on the other side, so that the whole circle ABCD occupies the position of the meridian.

[Margin: To describe the equator and its parallels in the astrolabe, without constructing the analemma, if the size of the equator is given.]

Diagram. The same stereographic-projection woodcut as on p.487 is repeated at the foot of this verso (so the reader can see it while reading either side of the leaf). Same construction and labels: concentric circles about center **E**; horizontal diameter **X R I C G E F A H**; vertical axis **S p/b o M n B K l L D N**; upper-left analemma arc **Y r e V q**; right-side points **P Q a**; small interior labels **h[?], z[?], m[?], i[?], o[?]**. See fig-p487-1 for the full description.

And so, if the northern declination of a given parallel is counted from point B toward C — or the declination of a southern parallel toward A — and from A a straight line goes out through the end of the count, the line EB will be cut at the point through which the parallel of the given declination is to be described from center E. For these lines from A will cut the line BD, produced to infinity, at the same points where they would cut it if the circle ABCD stood at right angles to the plane of the astrolabe upon the line BD, as is evident. Thus you see that the greatest declinations of the Sun on either side, BP and BO, of 23°30', are counted off, and that the lines AP, AO cut the line EB at the points K, M, through which the tropic of Cancer (♋) and the tropic of Capricorn (♏) are described.

[Margin: To describe in the astrolabe, from a single arc of declination in the equator, both the southern and the northern parallel of that declination.]

6. And by the same art we shall describe any parallel of a given declination, if we count its declination from point B toward C when it is northern, but toward A when it is southern. The reasoning here is the same as in the analemma. For through the ends, for example, of the declinations PO, the diameters of the parallels of those declinations are to be drawn in the analemma. Therefore their endpoints P, O will appear at K, M, and consequently their apparent semidiameters will be EK, EM, etc.

Moreover it is enough if the given declination is counted to one side only, so that from it we may describe the parallel of that declination, both northern and southern. For if the declination is BO, the radius AO, drawn out from A, the nearer pole, will cut off the semidiameter EM of the southern parallel; while the

radius CO, drawn from C, the more distant pole, will cut off the semidiameter EK of the northern parallel, etc.

7. Conversely, we shall know the declination of any parallel described in the astrolabe if, from the point where it cuts the line EB, we draw a straight line to A.

For this will cut the semicircle ABC at the point of declination; and if indeed it cuts the quadrant BC, the declination will be northern, but if the quadrant BA, southern. Thus the line drawn AK gives, in the quadrant BC, the northern declination BP, while the line AM gives the southern declination BO, in the quadrant BA.

[Margin: To know the declination of any parallel of the equator described in the astrolabe, without constructing the analemma, and whether it is northern or southern.]

8. But when the southern declination of a given parallel — such as the declination BQ — is so great that the points A, Q are little distant from one another, the visual ray AQ is drawn out to its termination only with much difficulty and not without error. For, on account of the nearness of the points A, Q, the ruler with which we draw straight lines can very easily be shifted this way and that from its proper position, and so the point which terminates the apparent semidiameter of the parallel on the line EB cannot be found exactly. In that case one must resort to Lemma 11, where we taught how to produce a straight line to any length through two points little distant from one another, such as A, Q are in the given example. And if perhaps this line should cut the line EB so obliquely that the point of intersection can scarcely be discerned without error, we must also employ Lemma 13, where we taught how to find that point most exactly, however obliquely the lines AQ, EB intersect one another.
9. We shall produce the same line AQ in a straight continuation very accurately in this way. From A, having described an arc RS at any interval AR — which the line AS, perpendicular to AR, cuts at S, so that RS is a quadrant — let there be taken (by the scholium to Proposition 27, Book 3 of Euclid) an arc ST similar to half the arc AQ, that is, one containing half the number of degrees of arc AQ. (This will be done if, by Lemma 3, an arc Sq is taken similar to arc AQ and bisected at T; for ST will then be similar to half of arc AQ.) For the line AT will pass through point Q, since by Lemma 10 the lines AS, AQ cut off from the circle RS an arc similar to half of arc AQ, of

which kind is the arc ST that was taken. But if the perpendicular AS does not cut the arc RS in the plane, a line must be drawn from A through B, cutting the arc RS at V, and an arc VT taken similar to half of arc BQ; for the line AY will again pass through Q, since by Lemma 10 the lines AV, AT cut off the arc VT similar to half of arc BQ. Now the arc RV is half a quadrant, since the half-right angle BAE stands upon it at center A, as is plain. But we shall proceed most conveniently thus also. From E, having described an arc XY whose semidiameter EX is equal to the semidiameter AR, and having bisected the arc CQ at Z, we shall draw the line EZ — having first taken an arc Da equal to arc BZ, so that the line may be drawn more accurately through three points E, Z, a — which cuts the arc XY at Y; and the arc XY will be similar to arc CZ, that is, to half of arc CQ, by the scholium to Proposition 22, Book 3, or by Proposition 33, Book 6 of Euclid. If then, by means of the compasses, we cut off from arc XY an equal arc RT (since these circles are equal), the arc RT will also be similar to arc CZ, and therefore, the line AT being again drawn, it will pass through Q. Indeed, ^a since the lines EZY, AQT are parallel, ^b because the external angle XEY at the center is equal to the internal angle RAT at the center (on account of the equal circles RS, XY): if through A we draw AT parallel to the line aEZ, by Lemma 4 it will pass entirely through Q. Moreover we shall demonstrate that the lines aEZ, AQ are parallel in this way too, even if the circles RS, XY are not described: ^c Since the arcs Aa, CZ are equal (because of the equal vertical angles AEa, CEZ at the center), and the arc CZ is equal to the arc ZQ, the arc Aa will also be equal to arc ZQ; and therefore, by the scholium to Proposition 27, Book 3 of Euclid, the lines aEZ, AQ will be parallel.

[Margin: To find the semidiameters of the parallels of the equator, especially the southern ones, more accurately and exactly.]

[Margin: a — Euclid I.28.] [Margin: b — Euclid III.27.] [Margin: c — Euclid III.26.]

10. You can also, if you please, from any point d on the line AC, taken through A, describe a circle Abe touching the circle ABCD at A. For, its quadrant Ae being divided into 90°, if an arc Ab is taken similar to arc AQ, the line Ab will pass through Q, since by Lemma 9 any line drawn from A cuts off similar arcs from the tangent circles AB, Ae. If you employ these precautions and remedies, it can scarcely happen that an error is committed in drawing the visual rays for southern declinations, however great. But if the quadrant RS is divided into 180 equal parts, so that each corresponds to the individual degrees of the semicircle CBA and can thus

be treated as degrees; if from V, the midpoint of the quadrant RS, the northern declinations are counted toward R and the southern toward S — taking, for example, for the Sun's greatest declination, $23\frac{1}{2}$ of the 180 parts into which the quadrant RS was divided, as if they were $23^{\circ}30'$, and for a declination of $45^{\circ}36'$ taking 45 parts and 36 minutes of one part (how these may be obtained is set forth in Lemma 3), and so with the rest — the semidiameters of the parallels will be found on the line EB, by lines drawn from A to the quadrant RS, much more accurately than if the same declinations were counted on the semicircle ABC from point B, on both sides. For the lines from A to the points of the quadrant RS are drawn more exactly than through the points of the semicircle ABC, since those points are farther from point A than these.

[Margin: To find the semidiameters of the parallels of the equator by another method, and exactly enough.]

11. It must not be passed over here that the semidiameter of the equator in the astrolabe is the mean proportional between the semidiameters of two equal and opposite parallels. For let there be two parallels in the astrolabe, FKGL and HMIN, corresponding to any two parallels in the sphere that are equal to one another and opposite. I say that EB, the semidiameter of the equator, is the mean proportional between their semidiameters EK, EM — that is, that EK is to EB as EB is to EM, or EM is to EB as EB is to EK. For, the lines AK, AM being drawn, the semicircle ABC will be cut at the points of declination P, O, as was demonstrated in numbers 4 and 7; and the arcs of declination BP, BO will be equal, since they belong to opposite and equal parallels; and therefore their complements CP, AO will be equal.^d Consequently the angles PAC, OCA (the line CO having first been drawn) will be equal. Since then the angle COA,^e which is right (being in a semicircle), is equal to the right angle AEK, the triangles COA, AEK will be equiangular. And for the same reason the triangles COA, MEA will be equiangular, since the right angle COA is equal to the right angle MEA, and the angle EAM is common.^f Therefore, as CO is to OA, so ME is to EA; and so EA is to EK:^g and hence, as ME is to EA — that is, to EB — so EA — that is, EB — is to EK; and consequently, by conversion, as EK is to EB, so EB is to EM, which was proposed. And since the arc CO is composed of the quadrant CB and the arc of declination BO, it is itself known; and the arc AO is also known, being the complement of the declination. Therefore the chords CO, OA will be known, and so their ratio will be known. Since then the semidiameters EM, EB, EK are continuously proportional in the ratio of

CO to OA, as we have demonstrated, the continued proportion of the semidiameters will also be known. For their ratio, of the greater to the less, is always the same as that of the chord of the arc composed of a quadrant and a declination, to the chord of the complement of the declination — namely, CO to OA.

[Margin: That the semidiameter of the equator is the mean proportional between the semidiameters of two opposite parallels described in the astrolabe.]

[Margin: d — Euclid III.27.] [Margin: e — Euclid III.31.] [Margin: f — Euclid VI.4.] [Margin: g — Euclid V.[?]] (of the fifth [book]).]

[Margin: What continued proportion the semidiameter of the equator and the semidiameters of two opposite parallels in the astrolabe have.]

12. Since this is so, it will be enough to find, on the line EB, the apparent semidiameters of the northern parallels, by lines sent out from A through the points of declination in the quadrant BC — which is not difficult, since the visual rays drawn from A through the points of the northern quadrant BC cut the semidiameter EB not very obliquely. For if, by Lemma 12, a third proportional is found to the apparent semidiameter of any northern parallel and the semidiameter of the equator...

[Margin: The semidiameter of any [northern parallel...].]

ASTROLABII LIBER SECVNDVS

PROPOS. IV (continued)

[Margin: To derive in the astrolabe the apparent semidiameter of the opposite southern parallel of the equator from the semidiameter of the opposite northern parallel.]

this will be the apparent semidiameter of the opposite southern parallel. Nevertheless, the caution we prescribed in that lemma for finding the third proportional must here be observed in every case: namely, when the semidiameter of the northern parallel is much smaller than the semidiameter of the equator, the latter must be continually halved until a final part is reached — whether a half, a quarter, an eighth, a sixteenth, and so on, always proceeding by successive halving — which is either equal to, or smaller than, the semidiameter of the northern parallel. For by means of this part there will be found a certain fourth proportional to the semidiameter of the northern parallel, to the last part

of the semidiameter of the equator, and to the semidiameter of the equator; and this fourth proportional will be the same part of the third proportional — that is, of the semidiameter of the southern parallel that is sought — as that last part is of the semidiameter of the equator. Therefore, doubled, or quadrupled, or octupled, and so forth, it will give the required semidiameter of the southern parallel. In this way every oblique intersection of straight lines is avoided, and consequently the semidiameters of the southern parallels are found with great precision.

For example: having found the semidiameter EK of the tropic of Cancer [☿], if from it we wish to find the semidiameter of the tropic of Capricorn [♑], we shall bisect the semidiameter EB of the equator at g. And since the half Eg is now smaller than the semidiameter EK, we shall find a fourth proportional to EK, Eg, EB — which, as we said in lemma 12, is now found far more accurately, since the first line, here EK, is greater than the second, Eg. For this fourth proportional will also be the half of the semidiameter of the southern parallel. Hence, even if the semidiameter to be found is that of a southern parallel receding $41^{\circ}30'$ from the equator toward the south, we shall take in the northern quadrant BC the arc Bh of $41^{\circ}30'$, and draw the straight line Ah, which cuts off Ei, the semidiameter of the northern parallel at $41^{\circ}30'$. And since Eg, the half of the semidiameter EB of the equator, is greater than Ei, we shall subdivide Eg in half at l. Then, since El, the quarter of the semidiameter EB of the equator, is now smaller than Ei,...

Diagram. A large square woodcut enclosing the stereographic projection construction of the equator and its parallels (Proposition IV). Concentric circles about the center E, with radial and chord lines and two large quarter-arcs sweeping across the frame. The figure is densely lettered with small labels, several of which do not resolve cleanly. Readable labels:

- E (center)
- A (bottom center, southern pole / projection point)
- H (below A, at foot)
- B, D (ends of the vertical Meridian axis through E)
- C (left, on horizontal axis)
- F (lower center-left)
- G (upper center, on a circle)
- L (near center)
- N (lower left, on a sweeping arc)
- R (upper, on the vertical axis)
- X (top, above R)

- Y (right margin, on outer arc)
- S (lower right, on outer arc)
- f (left, on a horizontal line)
- e (upper right)
- p (right of center)
- Central cluster (crowded, individual positions uncertain): K, B, M, P, Q, m, n, o, z [?], h [?], b/q [?]

we shall find a fourth proportional E_m to the three lines E_i , E_l , E_B ; and to it we shall take three others equal, mn , no , op , so that the whole E_p is quadruple the found E_m , just as E_B was quadruple E_l . For E_p will be the semidiameter of the southern parallel receding $41^\circ 30'$ from the equator toward the south.

But we shall more easily find the third proportional by that second method which we gave at the end of lemma 12. For if the semidiameter of the northern parallel is taken along D as far as L , and a circle is drawn through the three points A , L , C , it will cut the straight line BD at M , and EM will be the third proportional to EL and EB , as was there demonstrated, and so forth. Let the same procedure be kept in the remaining cases. Another method of finding the semidiameter of the opposite parallel you will find in the following proposition, at number 11, where you will also find a method by which you may obtain the semidiameters of two opposite parallels from the southern declination alone.

13. Finally, from what we have said, we shall also easily demonstrate that, of all the points of the sphere, the southern pole alone — where the eye is placed — cannot be projected into the plane of the astrolabe, as we hinted at proposition 1.

[Margin: That the southern pole of the world alone, of all the points of the sphere, cannot be projected into the astrolabe.]

For since E represents the northern pole, and the straight line EB extended to infinity represents the Meridian circle — so that EB and ED stand for its two northern quadrants between the pole and the equator, and the whole BD for its entire northern semicircle, while the remaining parts, running from B toward M and from D toward N , reach the rest, the southern semicircle of the Meridian, in which the southern pole is contained — then, if the southern pole could exist in the plane of the astrolabe, both BM and DN would pass through that pole, and so would meet at the same point, which is absurd. Again, if the southern pole were contained in the astrolabe, it would be projected by the straight line AS , which touches the Meridian at A , the southern pole...

(For other straight lines drawn out from A and cutting the circle ABCD project into the plane of the astrolabe those points through which they are drawn, as is clear from what has been demonstrated) — and consequently the straight line AS would coincide with the straight line EB, which is absurd [a — Eucl. I.28], since they are parallel, on account of the right angles at E and A. [b — Eucl. III.18] For the angle EAS is right, being formed by the tangent AS, and E is right by construction. Finally, if the antarctic pole had a place in the astrolabe: since the straight lines AC, BD, and all others drawn through the center E represent great circles which pass through the poles of the world — whose arctic pole is E, as we said — all those straight lines would necessarily also pass through the antarctic pole, just as they pass through the arctic pole E. Hence they would all meet at the antarctic pole, which cannot be. Therefore the antarctic pole cannot be projected into the astrolabe. Indeed, neither can all the other points of the southern semicircle BAD of the Meridian (even excluding the southern pole A) be conveniently projected into the astrolabe, because the straight lines drawn from A through the neighboring points run out somehow to infinity before they can cut the straight line BD.

[Margin: That not all the southern points of the sphere (even with the southern pole excluded) can be conveniently projected into the astrolabe.]

SCHOLIVM

[Margin: To describe the equator and its parallels in the astrolabe, if the magnitude of the tropic of Capricorn is given.]

1. The method of describing the equator with its parallels in the plane of the astrolabe, which we have explained up to now, assumes that the equator has a fixed and determinate magnitude. Since, then, ordinary and customary astrolabes have as their greatest circle the tropic of Capricorn, it will not be amiss if we briefly teach, along with other astronomers, how, from a given tropic of Capricorn, the equator and the tropic of Cancer, together with the remaining parallels, are to be described in the plane of the astrolabe.

Let the tropic of Capricorn, then, be given as ABCD, of the size of the astrolabe plates, with center E; let the Meridian line, representing the Meridian circle, be BD, which AC cuts at right angles. Taking the greatest declination of the Sun BF, let the straight line AF be drawn, cutting EB at the point G, through which let a circle GI be described from E. On this circle, taking likewise the greatest declination of the Sun GH — which the straight line drawn EF will give, since the

arcs BF and GH are similar, by the scholium to Euclid III.22 — let the straight line IH be drawn, cutting EB at the point K, through which let a circle KL also be described from E. I say that GI is the equator, and KL the tropic of Cancer, if ABCD is the tropic of Capricorn.

For when the straight lines AB and GI are drawn [c — Eucl. VI.2] — they are parallel, since the sides EA and EB are cut proportionally at I and G, inasmuch as equal parts have been taken from equals — [d — Eucl. I.29] the alternate angles BAF and IGO will therefore be equal. And since BF is the greatest declination of the Sun, [the equator GI is] therefore equal to the Meridian of the Analemma; and the semidiameter EA of the tropic of Capricorn — drawn from the pole G through the greatest declination of the Sun — is such that the circle ABCD represents in the sphere that circle which is distant from the equator toward the south by the greatest declination of the Sun, as was demonstrated. Therefore, ABCD being posited as the tropic of Capricorn, GI will be the equator, since the former is distant from the latter by the greatest declination of the Sun, as we said and as the matter requires. Rightly, then, has the equator been found from the tropic of Capricorn, since indeed the same equator, once found, exhibits to us the same tropic of Capricorn as was proposed. It is evident that EK is the semidiameter of the tropic of Cancer, as we taught above — namely by the straight line IH, drawn from I, the southern pole, through the greatest declination of the Sun. And, the equator GI having been found, we shall describe all its other parallels in the astrolabe, as was handed down above.

Diagram. The scholium's projection figure: a circle about the center E (the tropic of Capricorn ABCD), with the vertical Meridian BD and horizontal AC, two inner concentric circles (the equator GI and the tropic of Cancer KL), and a network of chords and radii carrying the construction points. Readable labels:

- E (center)
- B, F (top)
- G, K (upper center, on the inner circles)
- H, Q (upper left)
- R, O (right of center)
- M (center, small)
- C (far left, on horizontal axis)
- L, I (lower center-right)
- N (lower)
- P (below N)

- D (bottom)

[Margin: To describe the equator and its parallels in the astrolabe, if the magnitude of the tropic of Cancer is given.]

2. But what harm will there be if in this place we also teach by what method, from a given tropic of Cancer, the tropic of Capricorn and the remaining parallels may be described? Let the tropic of Cancer, then, be given as KL, of whatever magnitude, about the center E; let the Meridian line, representing the Meridian circle, be BD, which AC cuts at right angles. Taking the greatest declination of the Sun LM, let the straight line KM be drawn, cutting EA at the point I, through which let a circle IG be described from E. And on this, taking likewise the greatest declination of the Sun IO — which the straight line drawn EM will give, since the arcs LM and IO, taken away, are similar, by the scholium to Euclid III.22 — let the straight line GO be drawn, cutting EA at the point A, through which let a circle ABCD also be described from E. I say that GI is the equator, and ABCD the tropic of Capricorn, if KL is the tropic of Cancer.

For when IK is produced to H: since the arcs LM and IO are similar, if the similar quadrants LN and IP are added, the whole arcs NM and PO will also, by lemma 6, become similar. Therefore, by the scholium to Euclid III.22, the angles NKM and PGO will be equal; [e — Eucl. I.29] and on that account the straight lines HI and GO will be parallel; and so, by the scholium to Euclid III.27, the arcs IO and GH will be equal. Since, then, IO is the greatest declination of the Sun, GH will also be the greatest declination of the Sun. If, therefore, GI be established as the equator — and therefore equal to the Meridian of the Analemma — then the southern pole I: the straight line IH will cut off the semidiameter EK of the tropic of Cancer, so that the circle KL represents in the sphere that circle which is distant from the equator toward the north by the greatest declination of the Sun, as we said and as the matter requires. Rightly, then, has the equator been found from the tropic of Cancer, since indeed the same, once found, exhibits to us the same tropic of Cancer as was proposed. Hence it is clearly evident that EA is the semidiameter of the tropic of Capricorn, since it has been found through the equator GI, as we taught above — namely by the straight line GO, drawn from the southern pole through the greatest declination of the Sun IO. And by the same method, the equator GI having been found, we shall describe all its other parallels in the astrolabe, as was handed down above.

[Margin: To describe the equator and its parallels in the astrolabe from the given magnitude of any parallel whatsoever.]

3. But what we have said concerning the tropic — both of Capricorn and of Cancer — is to be understood also of any other parallel whatsoever, whether southern or northern. For if any parallel be described in the astrolabe, if in it there be reckoned its declination from the equator, at the place of the greatest declination...

of the Sun BF or LM, the equator will be found from it, and thence all the other parallels. For the demonstration in that case will be the same as in the tropic of Capricorn and the tropic of Cancer.

[Margin: That no parallel of the equator can be described in the astrolabe from the given magnitude of the opposite parallel, unless the equator be first described.]

4. However, although through a given equator all its parallels, both northern and southern, can be described in the plane of the astrolabe, and through any parallel described in the same plane the equator, and then through this all the other parallels also, can be described — as we have demonstrated in this proposition and its scholium — nevertheless, through no parallel can another, opposite parallel be described, even if in it the distance of the one from the other be computed, unless the equator be first described. This it will be worth the trouble to notice, lest anyone be deceived in this matter.

For let there be, for example, three parallels described in the adjoining figure: the tropic of Capricorn ABCD; the equator GIP; the tropic of Cancer KLN. And since, if the tropic of Capricorn ABCD is given, the semidiameter EG of the equator is found by taking the greatest declination of the Sun BF, which the tropic of Capricorn has from the equator, and by drawing the straight line AF, as has been demonstrated — I say that in this way the semidiameter EK of the tropic of Cancer cannot be found, namely if from B the doubled greatest declination of the Sun be reckoned, and to its end a straight line be drawn from A. For this straight line will not pass through the point K, but either above or below it. This we shall demonstrate as follows.

Let there be, if it be possible, an arc BQ equal to the doubled greatest declination of the Sun — that is, let FQ be the greatest declination, since BF is the other greatest declination, from which the semidiameter EG of the equator was found — and let the straight line AQ pass through the point K. Then, the straight line KL being drawn: since FQ is the greatest declination, as the adversary maintains, and LM is also the greatest declination, as appeared above when we found the semidiameter EI of the equator from the tropic of Cancer, the arcs FQ and LM will

be similar; and consequently, by the scholium to Euclid III.22, the angles FAQ and IKL will be equal. [a — Eucl. I.29] But the whole angle BAQ is also equal to the whole angle AKL, alternate to alternate, [b — Eucl. VI.2] because AB and KL are parallel — on account of the sides EA and EB being cut proportionally at L and K, since equal parts have been cut off from equals. Therefore, those equal angles being taken away, the remaining angles BAF and AKI will also be equal. [c — Eucl. I.29] But BAF is equal to the angle AGI, alternate to alternate, [d — Eucl. VI.2] because AB and GI are also parallel — on account of the sides EB and EA being cut proportionally at G and I, since equal parts have been taken from equals; [e — Eucl. I.29] and the angle AKI is equal to the angle GAK, alternate to alternate, [f — Eucl. I.28] because AG and IK are also parallel, since the angle EKI is equal to the angle EGA, external to internal, by the scholium to Euclid III.22, as they stand upon the arcs MN and OP, which are similar. For since the arcs LM and IO are similar — because each is the greatest declination of the Sun, as appeared above — when the similar quadrants LN and IP are added, the whole arcs MN and OP will, by lemma 6, become similar. Therefore the angles AGI and GAK will be equal to one another; [g — Eucl. I.6] and so the straight lines GR and AR will be equal. Again, [h — Eucl. I.29] since the angles AKI and GIK are equal to the equal angles GAK and AGI, alternate to alternate, they will be equal to one another; [i — Eucl. I.6] and on that account the straight lines IR and KR will also be equal.

Diagram. The projection figure for §4, essentially the same construction as the scholium figure: a circle about the center E (the tropic of Capricorn ABCD), with the vertical Meridian BD and horizontal AC, the concentric equator GIP and tropic of Cancer KLN, and a network of chords carrying the points F, Q, H, M, O, R of the demonstration. Readable labels:

- E (center)
- B, F (top)
- Q, G, H, K (upper center)
- R, O (right of center)
- M (center, small)
- C (left, on horizontal axis)
- L, I (lower center-right)
- N (lower)
- P (below N)
- D (bottom)

Now since the two sides GR and RK are equal to the two sides AR and RI, and contain equal angles at the vertex R, [k — Eucl. I.4] the angles KGR and IAR upon the bases GK and AI will be equal, as also those opposite the equal sides KR and IR. But the angles AGI and GAK were also equal. Therefore the whole angles EGA and EAG will be equal; [l — Eucl. I.6] and so the sides EG and EA will also be equal. And since EG is equal to EI, EI and EA will also be equal — the part and the whole — which is absurd. Wherefore the arc BQ is not equal to the doubled greatest declination of the Sun; and so, though AQ pass through K, a straight line from A will not pass to the end of the doubled greatest declination of the Sun through the point K, but either above or below it — which was to be demonstrated.

From all of which it is clear that, from a given equator in the plane of the astrolabe, any parallel can be described, and from any parallel the equator; but that from no parallel can its opposite parallel be found, unless the equator has first been found.

PROBL. II. PROPOS. V

To describe in the astrolabe any oblique Horizon, its primary Vertical, the Ecliptic, and any other greatest oblique circle which is nevertheless right to the Meridian and has a known inclination to the equator; and to distribute it into degrees — that is, into unequal parts which correspond to the equal degrees in the sphere.

[Margin: An oblique Horizon, its primary Vertical, the Ecliptic, and any other greatest oblique circle nevertheless right to the Meridian — by what method they are to be described in the astrolabe from the Analemma.]

1. If, in the Analemma described at the beginning of proposition 4, from the straight line nX — the seen diameter of the Horizon, of the primary Vertical, and of the Ecliptic, namely nm, SX, LM, which the visual rays from A, through the extreme points of the diameters fg, OR[?], of the same circles set out in the Analemma, cut off, and which are the greatest of all, as we showed in the scholium to proposition 3 (since the Meridian, in whose common section with the equator they appear, is right to these circles): if, I say, these seen diameters be transferred from the straight line nX into the astrolabe, onto the straight line BD (which corresponds to the straight line nX in the Analemma), in the same order and situation which they have in the Analemma, and about them, from their mid-points, circles be described, then the aforesaid greatest circles will be described in the

astrolabe. For instance, since the seen diameter of the Horizon is nm in the Analemma, we shall transfer its greater part En into the astrolabe, from the center E as far as F, and the lesser part Em as far as G; and, the straight line FG being bisected at H, we shall describe from H, at the interval HF or HG, the Horizon AGCF. So also, of the apparent or seen diameter of the Vertical SX, we shall transfer the part ES from the Analemma into the astrolabe — the lesser part from E as far as I, and the greater part EX as far as K; and, the straight line IK being bisected at L, we shall describe from L, through I and K, the primary Vertical AICK. Again, from the Analemma... ..of the apparent diameter of the Ecliptic ML, we shall transfer the greater part EM into the Astrolabe from E out to M, and the lesser part EL out to N. Then, having bisected the diameter MN at O, we shall describe from O, through M and N,

Diagram. The upper woodcut, set in a square frame: a stereographic projection of the sphere onto the plane of the Astrolabe (the meridian/analemma construction combined with the projected result). A large circle has its topmost point at C and its lowest at A (A serving as the southern projection-point). A horizontal diameter runs left to right through the labelled points X, N, D, the center E, and B. An oblique straight line (the Horizon) descends toward the lower left. The Ecliptic diameter is drawn as M–L with its midpoint O, and the projected Ecliptic passes through the points A, M, C, N. Numerous parallel arcs and radiating construction lines fill the figure, carrying many small italic and Greek minuscule labels.

- C (top of the great circle)
- A (bottom; southern projection-point)
- Horizontal diameter, left→right: X, N, D, E (center), B
- Y (bottom, beside A)
- S (just below center E)
- R, Z, I (lower left region)
- Upper right arc: h, K, O, F
- Upper left region: P, ρ [Greek: rho], G
- Ecliptic diameter endpoints: M, L
- Numerous further small italic/Greek minuscule labels along the radiating lines and parallels are not individually resolvable at this resolution. [?]

the Ecliptic AMCN, which will touch the tropic of Cancer ☿ at N and the tropic of Capricorn ♊ at M. Now if, in the Analemma, there are drawn f i and g K parallel

to BD — namely the diameters of the parallels, of which the former is always the greatest of the parallels that remain hidden, while the latter is always the greatest of those that appear — and if, through their apparent semidiameters En, Em, circles are described from E, the center of the Astrolabe, passing through F and G, then the Horizon will touch these; and the one that passes through f will always be the greatest of the hidden circles, while the one that passes through g will always be the greatest of those that appear. [Margin: Which parallels the Ecliptic, Horizon, and Vertical touch.]

In the same way, if in the same Analemma there are drawn OP, QR parallel to the same BD — namely the diameters of the parallels which the prime Vertical touches, one of them being drawn through O, the pole of the Horizon or Zenith, the other through R, the other pole of the Horizon or Nadir — and if, through their apparent semidiameters ES, EX, circles are described from E, the center of the Astrolabe, passing through I and K, then the prime Vertical AICK will touch these; and the one that passes through I will represent the one that in the sphere passes through the upper pole of the Horizon, while the one

Diagram. The lower woodcut on the page: the large, fully drawn Astrolabe plate — a great bounding circle filled with concentric almucantar circles and radiating azimuth/construction lines, with the visual rays converging on a vertex A at the right. A vertical diameter carries, from top downward, S, F, M, ..., and the horizontal diameter runs from R at the left to A. T is at the bottom. Dashed visual rays fan out toward the upper right, where several small Greek digraph labels appear ($\theta\theta$, $\chi\alpha$). The interior is crowded with small italic and Greek minuscule labels.

- S (top)
- F (upper, on the vertical)
- R (left end of horizontal diameter)
- A (right; vertex of the converging visual rays)
- T (bottom)
- Descending toward the foot on the vertical: D, K, P
- Upper-right dashed rays: $\theta\theta$, $\chi\alpha$ [Greek digraphs], and further minuscules
- Many further small italic/Greek minuscule labels are not individually resolvable at this resolution. [?]

that runs through K will represent the one that is drawn through the lower pole of the Horizon. For every oblique great circle to the Equator touches two equal parallels of the Equator. [Margin: 2. Theod.[?] (Theodosius, *Sphaerica* [?])]

By this very same reasoning, any oblique great circle that is perpendicular to the Meridian and has a known inclination to the Equator will be described in the Astrolabe by the same procedure by which the three aforesaid great circles were described. For instance, suppose a great circle is to be described that is drawn through the poles of the Zodiac and is perpendicular to the Meridian — such as the one that is also drawn through the common intersections of the Equator and the Horizon: with the first point of Cancer ϖ placed on the Meridian, it is inclined to the Equator by 66 degrees 30 minutes. We shall draw in the Analemma its diameter hZ (which, to avoid confusion, we have not actually drawn) through the points h, Z, which are 66 degrees 30 minutes distant from BD, the diameter of the Equator; and by means of the visual rays drawn from A through the extreme points h, Z, we shall investigate its apparent diameter on the straight line BD, and so on. Thus you see in the Astrolabe the said circle described from the center P (how P is to be found, even when we do not have the whole apparent diameter, we shall teach a little below at Number 4) through the point Q, which in the Analemma corresponds to the point p, through which the visual ray Ah is drawn. And the same reasoning holds in the other cases. Moreover, all oblique great circles of this kind will necessarily pass through the points A, C, as we shall demonstrate below in the scholium to this proposition, at Number 1.

[Margin: To describe in the Astrolabe — without constructing the Analemma — any oblique Horizon, its prime Vertical, the Ecliptic, and any other oblique great circle that is nevertheless perpendicular to the Meridian and has a known inclination to the Equator. (Euclid I.5)]

Diagram. The large Astrolabe plate: a great bounding circle enclosing concentric almucantar circles and radiating azimuth lines, with the visual rays converging on the vertex A. The vertical diameter carries, from top downward, F, M, H, B, E (center), O, I, G, N, L, D, K, P; S stands at the top; T at the bottom; R at the left end of the horizontal diameter. Dashed visual rays fan toward the upper right, with small Greek digraph labels ($\theta\theta$, $\chi\alpha$) and a tropic sign (ϖ). The interior is densely labelled with small italic and Greek minuscules (among them η , ξ , u , ϕ , χ , π , and paired minuscules dd, aa, cc, bb).

- S (top)
- Vertical diameter, top→bottom: F, M, H, B, E (center), O, I, G, N, L, D, K, P
- R (left end of horizontal diameter)
- A (vertex of the converging visual rays, right)
- T (bottom)
- Upper right: $\theta\theta$, $\chi\alpha$ [Greek digraphs]; ϖ (tropic of Cancer sign)

- Numerous further small italic/Greek minuscule labels are not individually resolvable at this resolution. [?]

[Margin: To describe in the Astrolabe — without constructing the Analemma — any oblique Horizon, its prime Vertical, the Ecliptic, and whatever other oblique great circle that is nevertheless perpendicular to the Meridian and has a known inclination to the Equator. (Euclid I.5)]

2. We shall describe these same oblique great circles in the Astrolabe even when the Analemma has not been separately constructed, in the following way. With the Equator described together with both tropics, as above, let there be described from A, at any interval you please, an arc of a circle SRT, which is cut at S, T by the straight line ST drawn through A perpendicular to AC (or parallel to BD extended in both directions), so that two quadrants RS, RT are formed — by the scholium to Proposition 27 of Book 3 of Euclid — on account of the right angles at A. For by means of this arc SRT we shall find the points in the Astrolabe more precisely than without it. Next, from the poles A, C (for the Equator ABCD, being equal to the Meridian of the Analemma, can be taken for the Meridian, with A for the south pole and C for the north, and the straight line BD, extended both ways, for the common section of the plane in which the Equator lies and of the other plane in which the Meridian ABCD lies — as was stated in Proposition 4, Number 5 — just as if the circle ABCD, like the Meridian, stood at right angles to the plane of the Astrolabe along the straight line BD): let the latitude of the place for which the Astrolabe is being constructed — or, what is the same, the altitude of the pole — be counted off in opposite directions as far as V and X, and let the diameter of the Horizon VX be drawn. Then, with straight lines drawn from A through B and D, the quadrants RS, RT will be bisected at Z, a, if no error has been made. For since the angle AEB is right, and the angles EAB, EBA are equal,

they will each be half a right angle, [Margin: (a) Euclid I.32] because all three angles together equal two right angles. Therefore the remaining angle SAZ too, taken from a right angle, will be half a right angle, and hence equal to the angle RAZ, which is half a right angle; [Margin: (b) Euclid III.26] and consequently the arcs ZR, ZS on which they stand will be equal. In the same way you will show that the arcs aR, aT are equal. Then, with each quadrant RS, RT divided into 180 equal parts, let there be counted off in them, as if they were degrees, from S and R toward R and T, the altitude of the pole — or, what is the same, from Z and a toward S and R, the complement of the altitude of the pole — as far as Y and b; or else, by lemma 3, let there be taken arcs SY, Rb similar to half the arc AV (or CX) of the altitude of the pole, or arcs ZY, a b similar to half the complement of the

altitude of the pole, that is, to half the arc BV (or DX). For the visual rays AY, Ab will cut off the apparent diameter of the Horizon FG, inasmuch as they pass through the extreme points V, X of the Horizon's diameter — because, by lemma 10, the straight lines AS, AV and AR, Ab cut off from the circle SRT arcs similar to the halves of the arcs AV, CX of the altitude of the pole (such as, by construction, are the arcs SY, Rb), while the straight lines AZ, AY and Aa, Ab intercept from the same circle SRT arcs similar to the halves of the arcs BV, DX of the complement of the altitude of the pole (such as are the arcs ZY, a b that were taken). If, then, the diameter FG thus found is bisected at H, the Horizon AFCG will be described from H through F and G. That the apparent diameter FG has been correctly found is clear from this: the rays AV, AX cut the straight line BD in exactly the same points in which they would cut it if the circle ABCD stood at right angles to the plane of the Astrolabe (or of the Equator) along the straight line BD, so as to hold the position of the Meridian, as is evident. You see, therefore, that the arc SRT alone has been described, so that the rays may be drawn out more accurately from A through the points of the circle ABCD (which would otherwise suffice).

3. The center of the apparent Horizon — that is, the point H — [Margin: To find the center of the Horizon in the Astrolabe, even if its apparent diameter has not been found.] will easily be found in this way, even if neither of the extreme points F, G had been found. Let there be drawn from A a perpendicular Ace to VX, the diameter of the Horizon. For this, as was demonstrated in lemma 35, will bisect the base FG of the triangle AFG (cut off by the emitted rays AV, AX) — so much so that a straight line drawn from the south pole perpendicular to the diameter of an oblique great circle described on the Equator of the Astrolabe falls on the center of that same oblique circle. And the perpendicular Ace will be drawn easily thus: let the arc AV, by which the Horizon in the sphere is distant from the south pole — that is, the altitude of the pole — be doubled as far as c; and, to make the matter more accurate, let Ye also be taken equal to the arc SY, which is similar to half the arc AV. For the straight line Ace will be perpendicular to VX, the diameter of the Horizon, in the sphere. For since the arc Ac is bisected at V, the straight line Ac will also be bisected — by the scholium to Proposition 27 of Book 3 of Euclid — at d, [Margin: (c) Euclid III.3] and consequently will meet it at right angles, which is what was proposed. Now if the axis of the Horizon fg is drawn perpendicular to VX, the diameter of the Horizon, then Cf will be equal to the arc VB, that is, to the complement of the arc AV. [Margin: A straight line drawn from the south pole perpendicular to the diameter of an oblique great circle described on the Equator, at right angles, falls on the center of that same oblique circle in the Astrolabe. (Euclid III.3)] For since the quadrants CB, fV are equal, when the common arc fB is taken away, the remaining arcs Cf, BV will be

equal. Therefore AV, Cf will make up a quadrant; and consequently the arcs Vc, fc will complete the remaining quadrant of the semicircle ABC. Hence the arc fc will be the complement of the arc cV, that is, of the arc AV, and therefore equal to Cf itself. For this reason, if the complement of the arc AV — the distance of the Horizon from the pole — that is, if the arc VB (or Cf) is doubled from the other pole C, the same point c will be found, through which the straight line Ac, produced, falls at H, the center of the apparent Horizon. This latter result we have also demonstrated in another way in lemma 35.

4. By this same reasoning the center of any oblique great circle will be found in the Astrolabe, namely if a perpendicular line is drawn from the south pole A to its diameter: which will be done if its distance from the pole is doubled from the south pole A, or the complement of its distance is doubled from the north pole C, and so on, as was done in the case of the Horizon. [Margin: To find the center of any oblique great circle in the Astrolabe, even if its apparent diameter has not been found.]

5. From these things it is established that the center of an oblique great circle in the Astrolabe is different from the center of the Astrolabe — as was also demonstrated in Proposition 3, Number 4; for since the perpendicular drawn from A to the diameter of the oblique circle falls on the center of that same apparent oblique circle, as we showed, that perpendicular will not pass through E, the center of the Astrolabe, since AE, making right angles with BD, cuts the diameter of the oblique circle obliquely and not at right angles. The same will become clear by this reasoning as well: since an oblique great circle cuts the Equator in two points — one extremity of its diameter being inside the Equator and the other outside, as is plain from the finding of its diameter, and as is evident in the diameter FG — its center will be different [?] from E, the center of the Equator, since two mutually intersecting circles cannot have the same center. [Margin: The center of any oblique great circle in the Astrolabe differs from the center of the Astrolabe. (Euclid III.5)]

6. In no other way shall we describe the other oblique great circles perpendicular to the Meridian. For let the diameter of the prime Vertical be fg, cutting VX, the diameter of the Horizon, at right angles and passing through f, g, the poles of the Horizon. If, then, visual rays are drawn from A through f, g, they will cut the straight line BD at I, K, the poles of the Horizon, through which — from L, the midpoint of the apparent diameter IK — the prime Vertical AICK is to be described. But so that the extreme points of the apparent diameter IK may be found more precisely, especially the more remote point K, let there be taken the

arc Zh similar to half the arc Bf (or the arc Rh similar to half the arc Cf); likewise the arc a i similar to half the arc Dg (or the arc Ti similar to half the arc Ag). The center L, too, is found by means of the straight line Ak drawn perpendicular to the diameter fg, which is drawn through l, the end of the arc Al (which is double the arc Ag), and also through k, the end of the arc Tk (which is double the arc Ti), and so on.

7. Again, let the diameter of the Ecliptic be mn, distant from BD, the diameter of the Equator, by the greatest declination of the Sun. If, then, visual rays are drawn from A through m, n, cutting BD at M, N, then MN will be the apparent diameter of the Ecliptic; [Margin: (d) Euclid III.5] and it will be found more accurately if arcs Zo, a p are taken similar to the halves of the arcs Bm, Dn. The center O, too, is found by means of the straight line Ar drawn perpendicular to mn, which is drawn through q, the end of the arc Aq (which is double the arc Am, the complement of the greatest declination), and also through r, the end of the arc Sr (which is double the arc So): these points q, r are also obtained by means of the arcs Cq, Rr, of which the former is double the greatest declination, while the latter is similar to half the arc Cq.

Although the Ecliptic, together with the Colures, is carried around in the sphere by the diurnal motion, its circular figure in the Astrolabe is nevertheless not thereby impaired. For whatever position the Colure of the Solstices occupies, it is always perpendicular to the Ecliptic, and therefore in its common section with the plane of the Equator (or of the Astrolabe) — a section which, under the diurnal motion, coincides with all the straight lines drawn through the center of the Astrolabe — the apparent diameter of the Ecliptic will always be the greatest; and the plane of the Astrolabe (or of the Equator) will always make a subcontrary section in the cone whose base is the Ecliptic, that is, a circle, as was demonstrated in Proposition 3. From this it follows that the Ecliptic is always projected onto the Astrolabe as a circle of the same magnitude, whatever position it occupies in the sphere. [Margin: The Ecliptic always appears as a circle in the Astrolabe, and of the same magnitude, even though it is continually carried around by the diurnal motion in the sphere.]

(Proposition V continues — projecting the Horizon, prime Vertical, and Ecliptic onto the plate. Numbered articles 8–15.)

8. Finally, let s t be the diameter of any oblique circle — one that is, however, at right angles to the Meridian, namely of that circle which is drawn through the poles of the Zodiac s, t, and through the common sections of the Equator and the Horizon, once the same poles have been set on the Meridian. If, then, visual rays

are drawn from A through s and t , the line As will cut the straight line BD at Q , the pole of the Ecliptic, through which the proposed circle is to be described. But so that these rays may be drawn more precisely, one must take arcs Ru and $T\alpha$, similar to the halves of the arcs Cs and At . And because the ray $A\alpha$ meets BD too far off — so that the other pole of the Ecliptic can scarcely be obtained on the plane — only the portion AQC of the proposed circle has been described, from the center P . This center is found by means of the straight line $A\beta$ drawn perpendicular to the diameter st , namely through δ , the endpoint of arc $A\delta$ (which is double the arc At), and through β , the endpoint of $T\beta$ (which is likewise double the arc $T\alpha$).

As for how a greatest circle oblique to the Meridian — not at right angles to it, yet at right angles to the Horizon — is to be described on the astrolabe, we shall teach in Proposition 8; one at right angles to the prime Vertical, in Proposition 10; and finally one at right angles neither to the Horizon nor to the Vertical, in Proposition 12.

Diagram. A large, very dense stereographic-projection construction occupying most of the lower portion of the page, enclosed in a square frame. Within it a large quarter/half-circle (the astrolabe rim, the meridian circle $ABCD$) is drawn, together with the projected Horizon, prime Vertical, Ecliptic, and the oblique great circle through the poles of the Zodiac, plus the analemma arcs used to find their centers. The projection point A stands at the right on the horizontal diameter, and the radii fan out from it across the whole figure; R marks the left (west) end of the horizontal diameter, the center of the plate lies in the crowded knot of lines just left of A , and T lies on the vertical line at the foot. The figure is crowded with dozens of small point-labels, many of them Greek minuscules that do not resolve cleanly at this resolution.

Labels read with confidence (Latin capitals and the principal points):

- N — on the outer arc, upper left
- Y (Aries) — on an inner arc, upper left (appears twice)
- S — top, on the vertical meridian line
- F — on the vertical meridian, below S
- M, H, B, I, O — descending the vertical/central axis
- A — right, on the horizontal diameter (the projection point / pole of projection); radii fan out from here
- R — left end of the horizontal diameter
- D — on the central axis, low

- K, P — lower left, on an inner circle
- T — foot of the figure, on the vertical line
- ξ (xi), $\theta\theta$ (double theta), $\chi\alpha$ (chi-alpha) — on the right-hand arc

Smaller interior labels, mostly Greek minuscules, that are only partly legible: f , η , ε , u/ω , λ , ψ , ς , κ , ρ , φ , dd , cc , χ , π , $\mathfrak{S}$, μ , δ , α , β , $\alpha\alpha$ and others clustered around the center. [Many of these interior labels are illegible or ambiguous at this resolution — see QA.]

[Margin: The method by which the diameter of a given greatest circle that is oblique but at right angles to the Meridian is to be drawn in the Equator of the astrolabe, so that by it the oblique circle may be described on the astrolabe.]

9. But in order that we may know toward which side the diameter of any oblique circle — one at right angles to the Meridian — is to be drawn, its intersection with the Meridian in the sphere must be carefully observed. For in the same way its diameter must cut the circle ABCD on the astrolabe, the circle taken for the Meridian, in such a way that A is the south pole and C the north pole in the upper hemisphere, and B is its intersection with the Equator in the upper hemisphere. Therefore, since the Horizon cuts the Meridian in the sphere between the Equator (in the upper hemisphere) and the antarctic pole, its diameter must be drawn between B and A — this is the diameter VX. But since the prime vertical in the upper hemisphere cuts the Meridian between the Equator and the arctic pole, its diameter must be drawn between B and C, as was done with the diameter of the Vertical fg . So too, since the Ecliptic — its beginning placed at $\mathfrak{V}$ [?], in the Meridian of the upper hemisphere — cuts the Meridian between the Equator and the antarctic pole, its diameter mn must be drawn between B and A, like the diameter of the Horizon. Finally, since the greatest circle drawn through the poles of the Ecliptic in that position and through the poles of the Meridian cuts the Meridian between the Equator and the arctic pole, its diameter $s\ t$ must be drawn between B and C, just as the diameter of the Vertical. And so with the rest, always keeping account of the distance of the oblique circle from pole A, or pole C, or at any rate from B, the intersection of the Equator.

[Margin: a — Theodosius, *Sphaerica* II.8.]

10. Furthermore,^a since any oblique greatest circle touches two parallels of the Equator that are equal and oppo-

-site: once that endpoint of the visible diameter of any oblique greatest circle which lies nearer the center E of the astrolabe has been found (which can conveniently be obtained, since the visual ray displaying it always cuts the

diameter BD within the Equator), the other, more distant endpoint will be found far more accurately as follows. Take two straight lines — one being the portion of the straight line BD between E, the center of the astrolabe, and the nearer endpoint (that is, the semidiameter of the northern parallel which the greatest circle touches at that endpoint), the other being the semidiameter of the Equator — and find a third proportional, as we taught in Lemma 12. For this will give the other endpoint of the visible diameter of the proposed oblique greatest circle, since it is the semidiameter of the southern parallel which that same greatest circle touches, as we demonstrated in Proposition 4, Number 11.

[Margin: To find more accurately the endpoint of the visible diameter of a greatest oblique circle that lies farther from the center A of the astrolabe.]

Thus in the Horizon: G having been found, if for the two lines EG, EB a third proportional EF is found, the other endpoint F will be found. So too in the Vertical: after point I has been found, if to the two EI, EB a third proportional $E\kappa$ is joined, the other endpoint K will be obtained. Likewise in the Ecliptic: once point N has been found, if to the two EN, EB [a third proportional] EM is joined, the other endpoint M will be given. Finally, in the circle AQC: once point Q has been found, if for the two EQ, EB a third proportional is found, it will present the other, more distant endpoint of the visible diameter of the same circle. And so with the rest.

But the finding of this more distant endpoint is not altogether necessary. For if the center of the given oblique circle is found exactly, by a line drawn from the south pole A perpendicular to its diameter on the Meridian of the Analemma (which on the astrolabe is the Equator itself), as we said above in Number 3, then the oblique circle is described on the astrolabe from that center, at an interval equal to the semidiameter between the center and the nearer endpoint already found; and this will at the same time display the other, more distant endpoint. Indeed, not even the nearer endpoint is absolutely necessary. For, as we shall show in the scholium to Number 1, the oblique circle passes necessarily through point A. If, therefore, from the center that has been found a circle is described through A, it will be the greatest circle sought, and will at the same time display both endpoints.

[Margin: To describe a greatest oblique circle on the astrolabe even if its visible diameter has not been found.]

[Margin: To find the semidiameter of any parallel of the southern Equator by another method than the one above, and very exactly.]

11. Indeed, by this very same technique we shall dig out the semidiameter of any parallel of the southern Equator with almost no trouble. For if, say, the semidiameter of a parallel whose southern declination is BV is wanted, we shall draw the diameter of the greatest circle VEX, and to it draw a perpendicular Ad, which will cut the straight line DB, produced, at H. For since we transfer as far as F the diameter of the opposite northern parallel — a diameter equal, by the straight line HG between H and the point G, which bounds the semidiameter of the northern parallel AX; because the northern declination DX is equal to the southern declination BV — EF will be the semidiameter sought, because H is the center of the greatest circle tangent at G and F to those two opposite and equal parallels, whose declinations are DX and BV, as is clear from what has been said.

However, the diameter VX is not even necessary for finding the center H. For if an arc Vc equal to arc AV is taken, the straight line Ac will give the center H, because it is perpendicular to the diameter VX, were it to be drawn. And so, that two opposite parallels of the Equator may be described, it is enough to reckon their declination from B toward A; for if an arc Vc equal to arc AV is taken and the straight line Ac drawn, cutting the meridian line at H, and then from H, at the interval HA or Hc, the meridian line is cut at G and F, then EG will be the semidiameter of the northern parallel whose declination is BV, and EF the semidiameter of the southern parallel of the same declination — because H is the center of the greatest circle tangent to those two parallels at G and F.

[Margin: By what straight lines the pole of any greatest oblique circle at right angles to the Meridian is indicated on the astrolabe in the meridian line.]

12. The pole, too, of any greatest oblique circle at right angles to the Meridian — the pole which in the sphere lies more distant from the south pole — is indicated in BD, the meridian line of the astrolabe, by the visual ray drawn from A to the midpoint of that semicircle which the diameter of the circle cuts off; or (what is the same) by the ray which bisects both the angle formed by the rays drawn through the endpoints of the diameter of the circle itself, and the angle formed by the rays drawn through the center of the astrolabe — that is, through the center of the oblique circle in the sphere — and through its center as drawn on the astrolabe.

For example, the ray Af, falling on f, the midpoint of the semicircle VfX which the diameter of the Horizon VX cuts off, dividing in half both the angle VAX and the angle HAE, presents I. It indicates as the pole of the Horizon the point corresponding, in the sphere, to the pole f, which lies farther from the south pole A. For f, the point equally distant from the Horizon along VX when drawn, is the

pole of the Horizon, and it will therefore appear at I. That the straight line Af bisects both the angle VAX , contained by the rays AV , AX drawn through the endpoints of the diameter VX , and the angle HAE , which the rays AE , AH — drawn through E , the center of the astrolabe (that is, of the Horizon in the sphere), and through H , the center of the Horizon on the astrolabe — form, we shall show thus.

[Margin: What angles the ray drawn from the south pole through the more distant pole of a greatest oblique circle bisects.]

Since the arcs fV , fX are equal,^a the angles fAV , fAX will also be equal. Next,^b since the arc CX is equal to the arc AV — because of the vertical angles at the center — and since an arc Vc was taken equal to the same arc AV , the arcs CX , Vc will also be equal; and when these are subtracted from the quadrants fX , fV , the remaining arcs fC , fc will likewise be equal, and so too the angles standing on them.^c Consequently the angles Eaf , Haf will be equal. And since the poles are opposite through a diameter — the straight line drawn through f and E falling on the other pole g — the ray Ag will accordingly be perpendicular to Af ^d (because the angle fAg , being in the semicircle fAg , is a right angle); and it will indicate on the astrolabe the other pole K , corresponding in the sphere to the pole g , which lies nearer the south pole A . And the same reasoning holds entirely for the other greatest oblique circles. For G , F are the poles of the Vertical; Q [is one pole] of the Ecliptic — the other would be indicated by the ray $At \alpha$, if the narrowness of the plane permitted it — and N , M are the poles of the circle AQC .

[Margin: That the pole of any oblique circle on the astrolabe is distinct from the center of the astrolabe.]

13. From these things it is evident that on the astrolabe the pole of any oblique greatest circle is distinct from the center of the astrolabe. For since the ray drawn from the south pole through the pole of the oblique circle does not pass through the center of the astrolabe — because C , the pole of the world, cannot be the pole of the oblique circle — it is clear that the pole of the oblique circle appears outside the center of the astrolabe, and therefore is distinct from it.

[Margin: To find the center of a greatest oblique circle on the astrolabe by another method.]

14. Therefore, a ray having been drawn from A through f , the pole of the Horizon, cutting the arc RS at h : if an arc he equal to arc Rh is taken, or an arc fc equal to arc Cf , the straight line Ac will fall at H , the center of the Horizon on the astrolabe;^e because the angles RAh , eAh are made equal, and consequently the

angle RAe , contained by two straight lines — of which AR is drawn through E , the center of the astrolabe (that is, the center of the Horizon in the sphere), while Ae is drawn through H , the center of the Horizon on the astrolabe — is bisected. The same happens in the other greatest oblique circles.

It is also to be noted here in passing that the ray Af , falling from the south pole onto the pole of the greatest oblique circle, cuts off from the meridian line and from the diameter of that same greatest oblique circle two equal lines up to E , [the center of the astrolabe]—

[Margin: What equal straight lines are cut off by the ray drawn from the south pole to the pole of the greatest oblique circle.] [Margin: a — Euclid, *Elements* I.32; b — I.5; c — I.6; d — I.32; e — I.5; f — I.6.]

—up to E , the center of the astrolabe: that is, the line EI , up to I , the visible pole, is equal to the segment of the line EV up to the ray Af ; and by the same reasoning the line EK , up to the other visible pole K , is equal to the segment of the line EV , produced up to the visual ray KA , produced toward A .

^a For since the three angles in the triangle AEI , formed by the straight lines Ef , fA , EV set up to the intersection of the lines fA , EV , are equal [to two right angles]; and since the removed right angles AEI , fEV are equal,^b just as the angles EAF , EfA in the isosceles triangle AEf are equal, the remaining angle EIA will also be equal to that which the lines EV , fA form at their common section.^c Therefore the line EI is equal to the segment of the line EV up to the ray Af .

^d Again, since the three angles in the triangle AEK , formed by the straight lines Eg , gA , EV set up to the intersection of the lines gA , EV , are equal [to two right angles]; and since the removed right angles AEK , gEV are equal,^e just as the angles EAg , AgE in the isosceles triangle AEg are equal, the remaining angle EKA of the triangle AEK will also be equal to that which the lines gA , EV produce at their meeting.^f Therefore the line EK is equal to the segment of the line EV , produced up to the ray gA , produced toward A — which was what was proposed.

Diagram. The same large stereographic-projection construction printed on p. 496 is repeated here at the foot of the page (square frame; meridian circle $ABCD$; projected Horizon, prime Vertical, Ecliptic, and oblique great circle; projection point A at the right on the horizontal diameter; R at the left end; T at the foot; the same dense knot of interior Greek-minuscule point-labels around the center). The principal legible labels are as on p. 496: N , Y , S , F , M , H , B , I , O , A , R , D , K , P , T , with ξ , $\theta\theta$, $\chi\alpha$ on the right-hand arc; the smaller interior labels (f , η , ε , u , λ , ψ , ς , κ , ρ , φ ,

$dd, cc, \chi, \pi, \mathfrak{S}, \mu, \delta, \alpha, \beta, \alpha\alpha$) remain only partly legible. [Same figure repeated; interior labels a]

[Margin: That the pole of a greatest oblique circle differs from its center on the astrolabe.] [Margin: g — Euclid, *Elements* I.26; h — I.26.]

15. From these things it is likewise established that the pole of any oblique circle on the astrolabe is distinct from its own center — which can even be seen easily in the examples given. Yet it can be briefly demonstrated as follows. Let s be the pole, say, of the Ecliptic, appearing by the ray As at Q . I say that Q is not the center of the Ecliptic. For since the center is indicated by the ray perpendicular to the diameter of the Ecliptic, as was demonstrated in Number 3: if Q is said to be the center of the Ecliptic, the angles at θ will be right and equal. Moreover the angles $m\theta A, n\theta A$ are also equal, because the ray $s\theta$ drawn through the pole bisects the angle mAn , as was shown in Number 12. Therefore the two angles $m\theta A, n\theta A$ of the triangle $A\theta m$ are equal to the two angles $n\theta A, m\theta A$ of the triangle $A\theta n$. Since, then, the common side $A\theta$ is adjacent to them,^g the sides $m\theta, n\theta$ will also be equal; and consequently, since the line $n\theta$ would be greater than nE — that is, than mE — the line $m\theta$ too would be greater than mE , a part greater than the whole, which is absurd. Therefore Q , the pole of the Ecliptic, is not the center of it.

By like reasoning, let O be the center of the Ecliptic, which the perpendicular $A\mu$ to mn presents. I say that O is not the pole of the Ecliptic. For the pole is indicated by the ray which bisects the angle mAn , as we showed in Number 12; so if O is said to be the pole of the Ecliptic, the angles $m\mu A, n\mu A$ will be equal. But the angles at μ are also equal, since they are right. Therefore the two angles $m\mu A, n\mu A$ of the triangle $A\mu m$ are equal to the two angles $n\mu A, m\mu A$ of the triangle $A\mu n$. Since, then, the common side $A\mu$ is adjacent to them,^h the lines $m\mu, n\mu$ will also be equal; and consequently, since $n\mu$ would be greater than nE — that is, than mE — the line $m\mu$ too, a part, would be greater than the whole mE , which is absurd. Therefore O , the center of the Ecliptic, is not [its] pole—

is the pole of that same circle. The same reasoning holds for the other great circles. This too can be confirmed in the following way. Since it was demonstrated above at Nu. 12 that a radius drawn through the pole bisects the angle contained by the two radii drawn through the centre of the astrolabe and the centre of the oblique circle, the radius drawn through the pole must necessarily differ from the radius drawn through the centre of the oblique circle. And so these two radii will indicate different points on the astrolabe.

16. Now let us show how any oblique great circle drawn on the astrolabe is to be divided into degrees. Since their arcs are not always projected into similar arcs — as we demonstrated at Proposition 3, Nu. 2 and 3 — the arcs of these circles that correspond to the individual degrees of the same circles on the sphere will not be equal to one another; otherwise the arcs projected onto the astrolabe would be similar to the arcs on the sphere from which they are projected. We must therefore adopt another way and method by which we can obtain the degrees of the oblique great circles drawn on the astrolabe. Now although they could be divided into degrees by means of the great circles drawn through their poles — as the Horizon is divided by the vertical circles [azimuths], and the Ecliptic by the great circles drawn through its poles, which are customarily called circles of latitude, and so on for the rest — nevertheless, because we have not yet taught how great circles of this kind are drawn on the astrolabe, and because some of them grow to an almost immense size, so that they can scarcely be delineated without error, we shall most conveniently divide them by means of straight lines. And this can be done in several ways, of which the very first is the finest and easiest of all; and so I judge it the one to be chosen among the rest. Its earlier part — since it contains two [methods], that is, it can be carried out in two ways — is as follows.

[Margin: How to divide the Horizon on the astrolabe into degrees from its upper pole.]

17. Once the pole of the Horizon has been found — or the pole of any oblique great circle whatever (for the reasoning is the same in every case, as will be stated at Nu. 23) — the pole that lies within the Equator (which indeed represents the pole that on the sphere is the more remote from the southern pole): if from that pole straight lines are drawn through the individual degrees of the Equator up to the oblique circle, the oblique circle will be divided into degrees, that is, into arcs which, although unequal to one another, nevertheless correspond to the equal degrees of those oblique great circles which they represent on the sphere. For example, if from I, the pole of the Horizon, through any point σ of the Equator the straight line I σ is drawn, cutting the Horizon at γ [the point marked with a sampi-like symbol], the arc F γ will correspond to as many degrees of the Horizon on the sphere as there are degrees contained in the arc B σ of the Equator. That is, the arc F γ will represent an arc of the Horizon on the sphere equal to the arc B σ of the Equator — so much so that if the arc B σ is 1 degree, the

arc $F\gamma$ too is 1 degree; if the arc $B\sigma$ is 2 degrees, the arc $F\gamma$ too is 2 degrees; and so on.

This we shall demonstrate as follows. A plane drawn in the sphere through the antarctic pole and through the pole of the Horizon more remote from it — that is, through the Zenith — cuts off equal arcs from the Equator and from the Horizon: the reckoning beginning, on the Equator, from the upper semicircle of the Meridian, in which the Zenith lies, and, on the Horizon, from the southern section which it makes with the Meridian; or else, on the Equator, from the lower semicircle of the Meridian, and, on the Horizon, from the northern section — as we demonstrated in lemma 23. Therefore that same plane [note a] — which indeed makes a circle in the sphere — projected onto the astrolabe will be seen to cut off, from the southern pole, those same equal arcs from the Equator and from the Horizon on the astrolabe: namely those which correspond to the arcs cut off on the sphere. Since, then, the plane, or rather the circle, which on the astrolabe (by Proposition 1, Nu. 1) [is represented by] the straight line drawn through the pole of the Horizon I and the point σ of the Equator — for this straight line corresponds to that circle on the sphere, through which that circle is drawn; and it is seen from the southern pole [to be cut] at the Equator at the point σ , and, carried round through all its points, does not depart from it, and therefore [comes to rest] at $I\gamma$ — the arc $F\gamma$ of the Horizon therefore represents that arc on the sphere which is equal to the arc $B\sigma$ of the Equator. The same is to be said of all the other straight lines going out from I , the pole of the Horizon, and cutting both the Equator and the Horizon.

For the straight line $I\varepsilon\sigma$ [?] also cuts off from the Horizon an arc $F\varepsilon$ of as many degrees as are contained in the arc $B\varepsilon$ of the Equator; and the straight line IA cuts off the arc FA of the Horizon of as many degrees as the quadrant BA of the Equator comprises, namely 90 — so that FA represents a quadrant of the Horizon on the sphere. Finally, every straight line drawn out from I , the pole of the Horizon, and extended in both directions if need be, will always intercept in the Equator and the Horizon [arcs] equal in number; the beginning always being taken either from two points B, F , or from two points D, G , of which — of the first two — the point B on the Equator is the upper, and F on the Horizon is the southern; while [of the] latter two, the point D on the Equator is the lower, and G on the Horizon is the northern. This is what we prescribed in lemma 23, when the pole of the Horizon is assumed more remote from the southern pole, such as is the assumed pole I . And by the same reasoning any two straight lines sent out from I enclose in the Equator and in the Horizon two equal arcs, such as are the two arcs $\gamma\varepsilon, \sigma f$ [?] lying between the two straight lines $I\gamma, I\varepsilon$; likewise the two

arcs γ C, σ C intercepted between the two straight lines I γ , I σ [and I C], if it were drawn. Therefore, if from I straight lines were drawn through the individual degrees of the Equator, the Horizon would be divided into 360 arcs, which correspond to the individual degrees of the Horizon on the sphere.

[Margin: How an oblique circle is exactly divided into degrees when the pole I is very close to the circumference of the Equator.]

But since it sometimes happens that the pole I is very close to the point B, and consequently straight lines can scarcely be drawn from it, through the degrees of the Equator near B, without error, so as to display to us the degrees on the oblique circle, we shall provide a very easy remedy for this inconvenience at Proposition 6, near the end, Nu. 25, where we shall teach how another circle of any size whatever can be described from some fixed centre, in such a way that straight lines sent out from I through its degrees indicate the corresponding degrees on the oblique circle, just as do the straight lines going out from I through the degrees of the Equator, as has been demonstrated.

[Margin: How any given degree is found on the Horizon in the astrolabe from its upper pole.]

18. Therefore, if any degree whatever is desired on the Horizon — that is, an arc of any number of degrees whose beginning is either at one of its sections with the Meridian, as at F or G, or at one of its intersections with the Equator, as at A or C — those degrees are to be counted from the corresponding point of the Equator, namely from B or D, or from A or C, toward that part on which the arc is to be cut off. For a straight line sent out from I, the pole of the Horizon, through the end of the reckoning on the Equator will cut the Horizon at the degree which is desired.

[Margin: What is the eastern, western, northern, and southern part on the Horizon of the astrolabe.]

For example, if one desires an arc of 25 degrees having its beginning at the intersection of the Horizon with the eastern part of the Equator — such as is customarily the point C on the astrolabe (although A too may be taken for the eastern, and C for the western) — and tending toward the north, the 25 degrees are to be reckoned from C toward D on the Equator. (For the point G of the Horizon is the northern one, since it represents the extreme point X of the diameter of the Horizon, which is the more remote from the southern pole A; whereas the point F is the southern, since it corre-

-sponds to the extreme point V of the same diameter, which is the nearer to that same southern pole.) A straight line drawn to the end of the 25th degree will present a point on the Horizon corresponding to the 25th degree; and so on for the rest. Likewise, if one wishes on the Horizon an arc of 15 degrees whose beginning is at the south-eastern quadrant and at the 22nd degree from its southern intersection with the Meridian, one must first count 22 degrees from B up to σ , and draw the straight line I σ , cutting the Horizon at the point γ , which is 22 degrees distant from the southern section F. Then from the point σ the given 15 degrees are to be counted, either toward B or toward C, according as the arc of the Horizon to be cut off ought to incline toward the south or toward the north. For the straight line drawn from I through the end of the 15th degree will pass over on the Horizon by 15 degrees, and so forth.

Indeed, by exactly the same reasoning we shall bisect any given arc of an oblique great circle. For let there be given, for example, an arc $a\alpha\epsilon$ of the Horizon, to be divided in two. When from its pole I the straight lines I a , I ϵ have been drawn, cutting the Equator at V, m, we shall bisect the arc V m at t. For the straight line I t will cut the given arc in two at θ ; that is, the arcs $a\theta$, $\theta\epsilon$ will contain an equal number of degrees. This is clear from what has been demonstrated, since these arcs correspond to the equal arcs V t, t m on the Equator. The same can be accomplished by other methods, by which [we may divide] the oblique great circles into degrees; we shall teach these in what follows — which it is enough to have pointed out once.

[Margin: To bisect a given arc of an oblique great circle in the astrolabe.]

Diagram. A large stereographic-projection astrolabe figure occupying most of the page, enclosed within a square plate border (the astrolabe rim). It shows the projected Equator, Horizon, Ecliptic, and the vertical (azimuth) circles, together with the poles I, K, G, N, Q of the several oblique circles and a dense web of straight construction lines (some solid, some dotted) radiating from these poles through the degree-points of the Equator to the oblique circles, illustrating §§17–20 (dividing the Horizon and other oblique circles into degrees, and bisecting a given arc). Many minor points are labelled with Greek letters and doubled Latin letters at a size at the resolution floor.

Clearly readable labels (position approximate):

- Y (upper left, on the outer arc)
- S (top centre)
- F (upper centre — southern section of the Horizon)

- A (right of centre — a radiant node; southern pole / intersection point, with many lines converging)
- B (right centre — upper point of the Equator)
- C (centre region — projected centre / node)
- D, G (lower centre — Equator-lower point and northern Horizon point)
- M (upper centre)
- H (upper centre)
- I (centre — pole of the Horizon)
- O (centre)
- W (centre)
- N (lower centre)
- L (lower centre)
- K (lower centre — pole)
- P (lower left, and a second P lower centre)
- R (left, on the horizontal axis)
- T (bottom centre)
- V (right of centre)
- U (left)
- Greek/minor point labels around and within the circles: ζ , ε , λ , ψ , π , ϕ , ρ , τ , ω , θ , β , and doubled Latin points aa, bb, cc, dd. [several of these are [?]] — see below]

Uncertain / unresolved labels at this resolution: the small Greek letters (ζ , ψ , τ , ω , θ) and the doubled-letter points (aa, bb, cc, dd), several of which cannot be securely distinguished. More than two labels are uncertain.

19. Conversely, if one wishes to know how many degrees are contained in any proposed arc of the Horizon, two straight lines are to be drawn from the extreme points of the given arc to I, the pole of the Horizon, cutting the Equator toward the same part of the Horizon in which the given arc lies. For these will intercept on the Equator just as many degrees as are contained in the given arc. If, then, by lemma 3 one inquires how many degrees are enclosed in that arc of the Equator, the number of degrees contained in the given arc of the Horizon cannot fail to be known. But the latter part of this first [method] is as follows.
20. Once the other pole of the oblique circle has been found — the one outside the Equator (which of course represents on the sphere the pole that is nearer to the southern pole) — if from it straight lines are drawn through

the individual degrees of the Equator, cutting the oblique circle, the oblique circle will again be divided into degrees; but the order of the degrees on the Equator and on the oblique circle must now be taken differently from before. For if on the Equator they begin from the upper point B, then on the Horizon they are to be begun from the northern point G; but if on the Equator they begin from the lower point D, then they are to be begun on the Horizon from its southern section F, with the Meridian, as must be done according to lemma 23. For example, if from K, the pole of the Horizon lying outside the Equator, a straight line K b b is drawn through any point b b, it will cut off from the Horizon an arc F ȳ, begun from the point F, of

as many degrees as are contained in the arc of the Equator between the point D and the point b b assumed, through which the straight line K b b was drawn — because the point D of the Equator is the lower one on the Meridian. So likewise the arc of the Horizon from the point G, the northern point, through C, up to the point [ȳ] is equal (as far as concerns the number of degrees) to the arc of the Equator from the point B, the upper point of the Equator, up to the point b b, in the quadrant C B, through which the straight line A b b was drawn. But if the equal arcs to be cut off must begin from the point A or C, they are always to be taken toward contrary parts, so that the arc of the Equator from C toward B is equal to the arc of the Horizon from C toward G, provided each is intercepted between the same straight line sent out from K and the point C. For in this way the arc cut off from the Equator tends toward the upper point B, while the arc cut off from the Horizon tends toward the northern point G. So too the same straight line will cut off two equal arcs begun from the point A or C, of which the one taken on the Equator tends toward D, the lower point, and the one on the Horizon tends toward F, the southern point, as the reasoning requires.

But since the same straight line, falling outside the points A, C, cuts both the Equator and the Horizon in two points (except when it is tangent to each circle, as will be stated in the scholium at Nu. 15, 16, and 17), those points which are nearer to the point A or C, or more remote from it, will correspond to one another. All these things are demonstrated from the same lemma 23 in this way. A plane in the sphere through the antarctic pole and the pole of the Horizon nearer to it — such as is the one that the pole K represents — drawn [through them], cuts off from the Equator and the Horizon equal arcs begun from the aforesaid points: namely, on the Equator from the upper point, on the Horizon from the northern; or on the Equator from the lower point, and on the Horizon from the southern — as was demonstrated there. Therefore that same plane, drawn on the astrolabe,

will cut off the same arcs, namely those which correspond to the arcs cut off on the sphere. Since, then, by Proposition 1, Nu. 1, that plane passing through the southern pole is projected onto the astrolabe as a straight line passing through the pole K, every straight line going out from the pole K will represent that plane, and therefore will cut off equal arcs from the Equator and the Horizon, as we have said.

Therefore, just as the straight line I σ gave the point γ on the Horizon, so a straight line drawn out from the pole K through the end of the arc of the Equator begun from the point D — which is equal to the arc B σ — will necessarily present the same point γ of the Horizon, provided the circles have been correctly described. And thus it will always be possible to find the same desired point on the Horizon by means of two straight lines, of which one goes out from the pole I, the other from the pole K, provided only that those things are observed which we prescribed as to be considered concerning the beginnings of the arcs cut off from the Equator and the Horizon.

[Margin: To divide the Ecliptic, the prime vertical, and any other oblique great circle that is oblique to the Meridian into degrees on the astrolabe from either of its poles.]

21. All these things are to be understood also of the Ecliptic A M C N, the vertical circle A I C K, and the circle A Q C, since the demonstration in these circles is the same as in the Horizon. For the straight line Q ξ , sent out from Q, the pole of the Ecliptic, within the Equator, will cut off an arc of the Ecliptic M λ equal to the arc B ξ of the Equator. And the same point λ will be found if from the other pole of the Ecliptic (namely from that point of the straight line E K on which the straight line A t α falls, or in which it is cut by the circle A Q C) a straight line is drawn through the end of the arc of the Equator D cc, begun from D, which is equal to the arc B ξ ; or through the end of the arc of the Equator B cc, begun from B, which is equal to the arc D ξ — because by this latter way there will be cut off an arc of the Ecliptic N λ corresponding to the arc B cc of the Equator. By like reasoning the straight line G π , from the pole of the vertical circle G, within the Equator, cuts off an arc of the vertical circle I ρ equal to the arc B π of the Equator; because, if the vertical circle be conceived to be a Horizon, above which the northern pole is raised, the point B of the Equator is the lower, and the point I of the vertical is the northern; whereas the point D of the Equator is the upper, that is, in the upper semicircle of the Meridian, in which of course lies the pole of the vertical C, the more remote from the

southern pole, which indeed lies within the Equator; and the point K of the vertical is the southern. And the same point ρ will be found by means of a straight line drawn from F, the other pole of the vertical circle, through the end of the arc of the Equator D d d, begun from the point D, the upper one, which is equal to the arc B π ; or through the end of the arc of the Equator B d d, begun from the point B, the lower one, which is equal to the arc D π — because by this latter way there will be cut off an arc of the vertical circle K ρ , begun from the southern point K, corresponding to the arc B d d of the Equator. Finally, the straight line N ω too, from N, the pole of the circle A Q C, within the Equator, cuts off an arc Q ϕ equal to the arc B ω of the Equator. And the same point ϕ will be obtained if from M, the other pole of the circle A Q C, a straight line is drawn through the end of the arc of the Equator begun from D, which is equal to the arc B ω , and so on.

[Margin: To divide any oblique great circle that is not perpendicular to the Meridian into degrees on the astrolabe from either of its poles.]

22. The Ecliptic, therefore, will be divided into degrees by straight lines drawn from its pole Q; the vertical circle by straight lines from its pole G; and the circle A Q C by straight lines from its pole N, drawn out through the individual degrees of the Equator — just as we said concerning the Horizon.

23. In exactly the same way any other oblique great circle drawn on the astrolabe that is not perpendicular to the Meridian will be divided into degrees, if its poles are found; but in place of the Meridian line B D another straight line is to be taken, which is drawn through the centre of the oblique circle and the centre of the astrolabe, and is the common section of the Equator (or the plane of the astrolabe) and of the great circle passing through the poles of the world and the poles of the oblique circle — serving as a kind of proper Meridian of the proposed oblique circle. But in what way the poles of any oblique circle are found on the astrolabe, we shall show below at Proposition 8, Nu. 17.

[Margin: An easy rule for determining the beginnings of the arcs cut off in the divisions of great circles into degrees, by means of straight lines sent out from either of the poles of any oblique circle.]

Moreover, in dividing great circles into degrees, there is no reason for us to be anxious and troubled about which of the points on the Equator is the upper or the lower, and which of the sections of the oblique great circle is the southern or

the northern. For since the pole of the oblique circle lying within the Equator lies also within that great circle itself, if the division is set up from that pole, the arcs on the Equator and on the oblique circle, cut off by the straight lines drawn out from that pole, will take their beginning from the points assumed toward the same parts of that pole existing on the astrolabe — that is, from the upper points or the lower ones; or else from one or other of the points at which the Equator and the oblique great circle intersect. So you see it has been done in the higher great circles to be divided: the arcs of [the Ecliptic] and of the Horizon, cut off by the straight lines sent out from the pole I, took their beginning from the points B, I, or D, K, or else from one of the two, C or A. So too, in order that the vertical circle might be divided, there were assumed as beginnings of the arcs the points B, I, or D, K, or else one of the two, A, C, when the division was made by straight lines sent out from G, the pole of the vertical circle lying within both circles. In the same way, when the Ecliptic was divided by straight lines drawn out from its pole Q, the arcs cut off had their beginning from the points B, M, or D, N, or else from C or A. Finally, in the division of the circle A Q C, from its pole N, the beginning is to be made from the points B, Q, or from the point D and the other point at which the same circle would cut the straight line B D extended, or else from one or other of the points A, C.

But when the division is to be made by straight lines going out from the other pole, which lies outside both circles, care must be taken that the beginning be taken from two points toward different parts of the other pole existing on the astrolabe...

so that, when the upper point of the equator is taken, the lower point is taken on the greatest oblique circle, and conversely; or, if one prefers to begin from either of the points A or C, so that the arcs run out in different directions. Here I call "lower" and "upper" — of the equator and of the greatest oblique circle — that point which in the figure occupies the upper or lower position with respect to the center of the astrolabe, and not the point which in the heavens is higher or lower. On this understanding, in the equator, the horizon, the prime vertical [Verticali], the ecliptic, and the circle AQC, the upper points are B, F, I, M, Q, and the lower ones D, G, K, N — together with that other point in which the circle AQC, if fully drawn, would cut the line BD extended.

Diagram. A large geometric woodcut set within a rectangular frame, showing the upper-left portion of a great circle used in the stereographic construction of the divided oblique great circle (the "upper figure" referred to in the text). A quadrant arc runs across the top with a vertical radius rising to it; two vertical

construction lines descend from the arc through the major points F and S. The interior is densely ruled with straight lines converging toward the projection center A (at right of the visible portion) and with several concentric and oblique circles crowded with graduation ticks.

Clearly legible labels (by position):

- Top arc and upper region: Y, o, S (with vertical line to arc), F (with vertical line to arc)
- Right of arc (graduation points): "oo", "αα", plus a small zodiac-like mark [?]
- Central cluster: M, H, c, m, B
- Center / lower vertical axis: A (projection center), G, N, L, D
- Lower region: K, P, T, and along the left arc "B", ρ, a
- [?]: The figure additionally carries a large number of small Greek minuscule graduation labels around the inner circles and along the oblique lines (λ, ψ, ξ, π, χ, ω, cc, and others). Individual Greek minuscules do not resolve reliably at this resolution and are not transcribed here. (many Greek minuscule labels unresolved).

[Margin: An easy rule for recognizing whether one of the two points of the equator is truly the upper or the lower one in the heavens, and whether one of the two points of the greatest oblique circle is northern or southern.]

So that we may easily recognize which of the equator's two points can truly be called the upper or the lower in the heavens — that is, which faces the upper semicircle of the Meridian and which the lower — and likewise which of the points of the greatest oblique circle, where it is cut by the line drawn through the center of the astrolabe and the center of the oblique circle, is northern or southern: this rule must be kept. In the equator, the point that is nearer to the pole of the oblique circle lying within the equator — that is, the point through which passes the line drawn from the center of the astrolabe through that pole — is called the upper point, because it truly lies in the upper semicircle of the Meridian, provided the arctic pole is elevated; and the oblique circle is taken as the horizon, above which [that pole rises]. The other point, more distant from the same pole — that is, the point through which passes the line drawn from the center of the astrolabe through the other pole, lying outside the equator — is called the lower point, for the contrary reason. Thus in the upper figure the equatorial point B is the upper one and D the lower; while with respect to the prime vertical and the circle AQC, the point D is the upper and B the lower. Likewise, in the oblique circle, the point nearer the center of the astrolabe is

northern, the more distant one southern. If this matter is carefully considered, there will be no difficulty at all as to from which of the two poles of the oblique circle the division is set up, provided that what we prescribed in lemma 23 about these same starting-points is observed.

[Margin: An easier rule for fixing the starting-points of the arcs.]

And since, in dividing the oblique circle by lines drawn from a pole lying within the equator, there is no difficulty at all — for any such lines cut off from the equator and from the oblique circle corresponding arcs, which take their start either from the common section of the equator with the oblique circle, as at point C or A, or from the two nearest points at which the line drawn through the center of the astrolabe and the center of the oblique circle intersects the equator and the oblique circle, as at the points B and F, or D and G, as is clear from what we have said — we shall easily understand how we ought to proceed in the division by lines issuing from the other pole, since the arcs must begin in the equator, or from the point opposite to that of the line drawn through the centers; so that, if the division previously began at the upper point, it now begins at the lower...

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lower point, yet advancing toward the same section of the circles, and conversely; or from the same intersection into opposite parts of the circles, so that if in the equator an arc descends from that section, in the oblique circle it ascends, and conversely. You see that all this is observed in the upper figure and in the following one. For in the following figure the line IN will cut off arcs CP, CN of an equal number of degrees, begun from the same section C toward the same side; or arcs BP, FN begun from the nearest points B, F. But the line KN cuts off arcs DQ, FN of an equal number of degrees, begun from the points D, F — of which the former is the lower in the equator and the latter the upper in the horizon; or arcs CQ, CN begun from the same section C, yet tending in opposite directions.

[Margin: To divide, on the astrolabe, into degrees, any greatest oblique circle that is perpendicular to the Meridian, from the center of another greatest circle which, with respect to it, stands in place of the prime vertical.]

24. There is another way by which any greatest oblique circle drawn on the astrolabe may be distributed into degrees, and it is of this kind. Let ABCD be the equator about center E, and let AFCG be the oblique horizon — or any other greatest oblique circle, but one perpendicular to the Meridian, that is, having both its center and its poles I, K on the meridian line, which

is BD extended on both sides. Then let the semidiameter EC be cut, by lemma 8, into unequal parts, which are produced by the perpendiculars let fall from the individual degrees of the quadrant BC to CE. Now, having found L, the center of the greatest circle which in the sphere is drawn through the poles of the oblique circle AFCG and through the common sections of the equator with the oblique circle — (this circle is the prime vertical, if the oblique circle AFCG is the horizon; or the greatest circle through the poles of the zodiac and the common sections of the ecliptic with the equator, with the first points of ♋ [Cancer] and ♑ [Capricorn] placed on the Meridian, if the oblique circle AFCG is the ecliptic) — which center is found by means of the line Ad, drawn perpendicular to its diameter ab, or parallel to the diameter YZ of the given oblique circle in the sphere that the circle AFCG represents: this center L, I say, having been found, if from it lines are drawn through all the points of the semidiameter EC, they will severally cut the oblique circle in two points, which correspond to those degrees of the oblique circle to which the points of the semidiameter EC correspond; so that the parts of the arc CNF correspond to the degrees of the quadrant CB, and the parts of the arc COG to the degrees of the quadrant CD. For the individual points of the semidiameter EC belong to two degrees each — namely to those on which the perpendiculars drawn through the divided points fall. For example: if from L, through the point M (which corresponds to the 60th degree, counted from C in either direction up to P, Q), a line LM is drawn, cutting the oblique circle in N, O, then each of the arcs CN, CO will be of 60 degrees; and so of the rest. But since the lines issued from L through A, C touch the circle AFCG at A, C (as will be proved below, at number 28), this division can be set up more conveniently — especially when the line EC is small, so that it does not easily admit so many points of division — in the following way. Let kl be drawn, parallel to AC, cutting LA, LC at l, k, and...

[Margin: Any greatest oblique circle that is perpendicular to the Meridian: to divide it, on the astrolabe, into degrees, from the center of another greatest circle which, with respect to it, stands in place of the prime vertical.]

at a distance as great as you please from the line AC, so that kl becomes much greater than AC. For if each half of it, tk, tl, is cut — as was taught in lemma 8 (which will also be done if a circle is described about the diameter kl, and from its degrees perpendiculars are let fall to kl, as was done in lemma 7) — there will be obtained on kl points through which, if lines are drawn from L, the circle AFCG will be cut, just as before by the lines from L through the points of the line

AC. For by lemma 7 the lines AC, kl are cut similarly at those points. Since, then, the lines from L likewise cut the same lines AC, kl similarly, it follows — from the scholium of proposition 4, book 6, of Euclid — that a line from L, drawn through any point of the one, passes through the corresponding and similar point of the other. Thus you see that the line LN passes through the corresponding points M, i, since the ratio of CM to ME is the same as that of ki to it, by the aforesaid scholium of proposition 4, book 6, of Euclid. The same remedy will have to be employed in the division of the parallels into degrees, as will be stated at proposition 6, number 26.

That the oblique circle is rightly distributed into degrees in this manner is demonstrated as follows. By lemma 25, a plane drawn in the sphere through the line AL, however extended, cuts off from the oblique circle — to whose diameter YZ the line AL is parallel — two equal arcs begun from the points Y, Z. Therefore that same plane, projected onto the astrolabe, will be seen, from the southern pole, to cut off onto the astrolabe those same equal arcs from the horizon — namely those which correspond to the arcs cut off in the sphere. Since, then, that plane, passing through the southern pole, produces (by proposition 1) on the astrolabe a straight line passing through the center L, the straight line LM — drawn through the center L and the point M of the diameter AC (which is the common section of the oblique circle and the equator, as is evident: for if the Meridian ABCD is conceived to be turned about BD until it becomes perpendicular to the equator, or to the plane of the astrolabe, then both the equator and the oblique circle will be perpendicular to the Meridian; [a] and therefore their common section will also be perpendicular to it, and consequently perpendicular to the line BD lying in the Meridian, at the center E of the sphere. Since, then, AC is perpendicular to BD, AC will itself be the common section of the oblique circle and the equator, or of the plane of the astrolabe) — that plane, drawn through the same points L, M, will represent it; and therefore, produced, it will cut the oblique circle in the points N, O, which correspond to those cut off from the oblique circle in the sphere by that plane; so that the plane, from the southern pole, is seen to cut the oblique circle in the points N, O, when the visual ray...

[Margin: a — 19. unde.]

Diagram. A large geometric woodcut within a rectangular frame (lower-right of the page), showing the complete stereographic construction: a great circle with F at the top (vertical diameter dropping through the center), a horizontal dotted line N–S–T across the upper middle, and the projection center A at the right, with

K at the bottom vertex of a converging pencil of lines. An inner oblique circle and several chords/perpendiculars fill the interior.

Clearly legible labels (by position):

- Top: F
- Horizontal dotted line: N (left), S (center), T (right)
- Upper-middle cluster: H, P, B, Y, I, E (center)
- Left: C, ☉ [Taurus symbol, or the letter "ø"] [?]
- Lower-middle cluster: M, x, G, m, d
- Right: A (projection center)
- Lower / converging vertex: z, ρ, L, D, K (bottom)
- [?]: Numerous small Greek-minuscule and additional graduation labels along the inner circle and chords do not resolve reliably. (several labels unresolved, incl. the C-adjacent zodiac/letter mark).

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visual radius, carried around through all the points of that plane, does not depart from it, and therefore always lies in LN, the common section of that plane with the plane of the astrolabe, or of the equator. Therefore the arc CN of the oblique circle represents, in the sphere, that arc which corresponds to the equatorial arc CP; the arc CO represents that which is equal to the arc CQ; and the remaining arcs FN, GO are equal to the remaining arcs BP, DQ. And the same reasoning holds for all the other lines issued from L. For each cuts two arcs from the oblique circle: the one that tends from C toward F comprises as many degrees as there are in the arc of the equator from C toward B, up to the perpendicular drawn through a point of the diameter AC; and the one that inclines from C toward G contains as many degrees as are contained in the arc of the equator from C toward D, up to the same perpendicular. So that, if from the individual degrees of the equator perpendiculars are drawn to the diameter AC, and through their points lines are thrown from L, the whole oblique circle will be distributed into single degrees. But it is enough to divide one semicircle in this way; for the points of division, transferred to the other semicircle, will give the degrees in that other semicircle.

[Margin: Any proposed degree — how it may be found on the astrolabe in the greatest oblique circle, from the center of another greatest circle which, with respect to it, stands in place of the prime vertical.]

25. And so, if an arc is to be cut off from the oblique circle — from F toward C or A, or from G toward C or A; or from C toward F or G, or finally from A

toward F or G — then, however many degrees are to be counted, those degrees are numbered from B toward C or A; or from C toward B or D; or finally from A toward B or D; and from the end of the counting a perpendicular is to be drawn to AC. For a line from L, drawn through the point of this perpendicular on AC, once produced, will give the arc that is sought.

[Margin: To find how many degrees of a given arc of a greatest oblique circle are contained on the astrolabe, from the center of another greatest circle which, with respect to it, stands in place of the prime vertical.]

26. Conversely, if it is asked how many degrees a given arc of the oblique circle contains, two lines are to be drawn from its ends to L, and from the points where they cut the diameter AC, two perpendiculars are to be raised to AC. For the arc of the equator between these perpendiculars will give the number of degrees that is desired.

[Margin: To divide into degrees any greatest oblique circle that is not perpendicular to the Meridian, from the center of another greatest circle which, with respect to it, stands in place of the prime vertical.]

27. These same things are also to be understood of any oblique circle that is not perpendicular to the Meridian, if in place of the meridian line BD one takes the line drawn through its center and the center of the astrolabe, and if, in place of the center L, one takes the center of another greatest circle which, with respect to the oblique circle, stands in place of the prime vertical, as of some oblique horizon, and so on.

[Margin: The agreement of the second method of dividing the greatest oblique circles with the first.]

Notice, moreover, in the figure the beautiful correspondence and, as it were, agreement of this kind with that other, earlier method. Just as the line LM in this method exhibits to us, in the oblique circle, the arcs FN, GO corresponding to the equatorial arcs BP, DQ, so the lines IP, IQ present the same arcs to us, drawn from the pole I through the same degrees of the equator, as the first part of the first method prescribed; and likewise the lines KQ, KP supply exactly the same arcs, issued from the other pole K in the manner that the latter part of the first method requires.

Diagram. A large geometric woodcut within a rectangular frame, essentially the same complete construction as the figure on the preceding page: a great circle

with F at the top and a vertical diameter through the center; a horizontal (partly dotted) line N–S–T across the upper middle; the projection center A at the right; and K at the bottom vertex of a converging pencil of lines. The interior carries the inner oblique circle, its chords, and perpendiculars.

Clearly legible labels (by position):

- Top: F
- Horizontal line: N (left), S (center), T (right)
- Left margin of circle: R (R)
- Upper-middle cluster: H, P, B, Y, I, E (center)
- Left-lower: C, $\propto$ [Taurus symbol, or "θ"] [?]
- Lower-middle cluster: M, x, G, m, d
- Right: A (projection center)
- Lower / converging vertex: z, ρ, L, D, K (bottom)
- [?]: Numerous small Greek-minuscule graduation labels around the inner circle and along the chords do not resolve reliably. (several labels unresolved).

28. Nor do I wish it to escape the studious reader that the lines issued from L through A and C touch the oblique circle at the points A, C. For the plane passing through AL, and carried around through all the points of the diameter AC — (the circle ABCD being set perpendicular to the plane of the astrolabe or of the equator) — which is the common section of the oblique circle and the equator, always cuts the oblique circle by lines perpendicular to the diameter AC; and these lines cut off equal arcs on either side from the points A and C, as is evident from lemma 25. It follows that, as soon as it reaches the points A and C, it no longer cuts the oblique circle, but touches it at those points — which, however, will also be proved geometrically shortly. Since, then, the line LA or LC is the common section of the same plane with the plane of the astrolabe, and therefore never departs from it but always remains in it, it results that the same line LC or LA touches the same oblique circle on the astrolabe at the point C or A. For if it cut it, the plane drawn through it would also cut the oblique circle in the sphere in two points, corresponding to those in which it would be cut by the line LC or LA — which is absurd, since it touches it only at C or A, as we said, and as we shall also demonstrate geometrically thus.

Let the circle ABCD be set perpendicular to the plane of the astrolabe or of the equator, so that the diameter YZ is the common section of the Meridian and the

oblique circle. If, through AC lying on the astrolabe, a greatest circle is conceived to be drawn perpendicular to the oblique circle of diameter YZ in its proper position, [a] it will likewise be perpendicular to the Meridian, since it passes through A, C, the poles of the Meridian — that is, through the intersections of the equator with the oblique circle, in the oblique circle in the sphere. Therefore, since both the Meridian and the oblique circle are perpendicular to that greatest circle drawn through A, C, [b] their common section YZ will also be perpendicular to it; [c] and consequently AL too, lying in the plane of the Meridian and parallel to YZ, will be perpendicular to the same greatest circle. [d] Therefore the plane drawn through AL — lying in the same plane of the Meridian, and through the point C or A, existing in the sphere — that is, the circle produced by it in the sphere — will make right angles with the same greatest circle. Wherefore, since both this circle drawn through AL and the point C or A, and the oblique circle drawn through AC (if everything is conceived in its proper position in the sphere), are perpendicular to that greatest circle, [e] their common...

[Margin: a — 15, [Theodosius] I.] [Margin: b — 19. unde.] [Margin: c — 8. unde.] [Margin: d — 18. unde.] [Margin: e — 19. unde.]

their common section is perpendicular to the same line; and consequently it is perpendicular also to the diameter AC of the oblique circle, and to the diameter — made by that greatest circle — of the circle drawn through A, L, and C (or through A). (For since that greatest circle, which cuts at right angles the circle drawn through A, L, and C — or through A — bisects it, as has been proved, [Margin: a — Theodosius I.13] and passes through its poles, it will pass through its center and there produce a diameter.) It will be perpendicular, because each diameter lies in that greatest circle. Therefore that same common section of the oblique circle and of the circle drawn through A, L, and through C (or A) will touch each circle at C or at A, by the corollary of Euclid III.16; and accordingly those same two circles will touch one another at C or at A and will in no way cut one another, by the definition in Theodosius, Book II.

29. But that the straight lines drawn from L through A and C are tangent to the oblique circle AFCG we shall prove more easily as follows. The perpendicular An, drawn to YZ (the diameter of the oblique circle in the sphere), falls at H, the center of the oblique circle in the astrolabe, as was demonstrated above in Section 3 of this proposition; and AL is parallel to YZ. [Margin: b — Euclid I.29] Therefore the angle LAH will be a right angle. Hence, by the corollary of Euclid III.16, the straight line LA will be tangent to the circle AFCG at A, and so on.

But at this point a certain difficulty seems to call for resolution, one that might cause someone trouble. For the straight lines FG and NO cut off from the Horizon the equal arcs FN and GO (as regards the number of degrees) — that is, they represent, on the Horizon of the sphere, two parallels, one of which is the common section of the Horizon and the Meridian, while the other is the common section of the same Horizon and of a plane drawn through the southern pole and the point L. (This is the plane which, as we said, is carried around the straight line AL — parallel to the Horizon in its proper position — through all the lines that are drawn in the Horizon parallel to the meridian line.) [Margin: That certain lines converging in the astrolabe represent, in the heavens, parallel and non-converging lines.] It might therefore seem astonishing to someone that the straight lines FG and NO meet at L, whereas the parallels they represent do not meet. For from this it seems to follow that, just as the individual points of the lines FG and NO correspond to certain definite points of those parallels, so too the point L should correspond to a single point common to both parallels — which, however, they cannot have, since they never meet. This doubt is to be met as follows: all the points of the lines FL and NL that lie above the point L correspond to points of those parallels, but the point L itself has no corresponding point among them.

For since AL — lying between the southern pole and the point L, in the plane of the astrolabe or of the Equator — is equidistant from the plane of the Horizon in which those parallels lie, the line AL, even produced to infinity, can never meet them; and therefore no point of theirs will appear at L. But since the ray from the southern pole, passing through any point whatever of the line FL or of the line NL, however close to L itself, does cut the parallel that lies in the Horizon — because it cuts, at A, the line AL that runs equidistant from it, and lies in the plane drawn through AL and that parallel — it comes about that every point above L has a corresponding point on the parallel: namely, that point on which the ray from the southern pole, passing through that point of the line FL or NL, falls. And so, if the circle ABCD is understood to be the Horizon in its proper position — with the point B facing south, D north, C east, and A west — all the points of the parallels BD and PQ that are contained in the northern halves ED and MQ will have corresponding points on the lines EL and ML, up to the point I exclusive; while those contained in the southern halves EB and MP will have their corresponding points on the lines EF and MN extended to infinity, as is plain on a material sphere. It is therefore no wonder that the lines FL and NO, even though they represent parallels, meet at L; for they represent not only those parallels, but also the whole planes that are drawn through AL in its proper position and through those parallels. Therefore, just as those parallels do not exist in every

part of those planes, so too not all the points of the lines FL and NL — which represent those planes — can correspond to points of the parallels; rather, those points of the planes that lie on the line AL, or below it, will necessarily appear outside the parallels in the astrolabe, so that they in no way belong to them.

30. By a THIRD method we shall divide any oblique great circle into degrees on the astrolabe, in this way. [Margin: To divide into degrees, from the southern pole of the Analemma, any oblique great circle that is perpendicular to the Meridian.] Let each half-diameter of the oblique circle in the sphere, EY and EZ, be cut — by Lemma 8 — into the unequal parts made by the perpendiculars dropped from the individual degrees of the quadrants, from Y and from Z, to YZ. It is enough to divide one of them, since its points, transferred to the other, divide it in the same way. Then from A, the southern pole, let straight lines be drawn through all the points of division of the diameter YZ, cutting the diameter FG of the given oblique circle in points; and if perpendiculars are raised to that same diameter FG through these points, the oblique circle AFCG will be divided into degrees. For example: if from A, through the point R (which corresponds to the 30th degree counted from Y, in either direction as far as e, f [?]), the line AR is drawn, cutting FG at S, and through S a perpendicular NT is raised to FG, then each of the arcs FN and FT will contain 30 degrees — that is, it will represent that arc of the oblique circle in the sphere which contains the same number of degrees; and so for the rest.

[Margin: To find, from the southern pole of the Analemma, any proposed degree on an oblique great circle that is perpendicular to the Meridian.] The demonstration of this is as follows. Let the circle ABCD be set at right angles to the plane of the astrolabe, so that YZ, the diameter of the oblique circle, is the common section of the Meridian and of the oblique circle, and let a circle then be drawn through YZ and AC. Since the plane in the sphere — drawn through the southern pole A (in that position of the circle ABCD) and through the straight line which through R is perpendicular to the diameter YZ in the plane of the oblique circle — meets the plane of the astrolabe at S, and makes, by the lemma, [that] either the circle drawn through the poles of the World and the poles of the oblique circle [is represented] [?] by the straight line NT, and it will be seen in the astrolabe to cut off the same [degrees] which in the sphere it cuts off from the same [circle], when the visual ray passes through [it] [?]; and therefore, projecting into the astrolabe as NT the perpendicular drawn through R — the arcs Ye and Yf of the oblique circle are therefore equal; but the arcs CN and AT represent those [degrees] which are equal [to the arcs of the Equator] [?]: so that, if from the individual degrees of the Equator straight lines go out from A to the

diameter [YZ], the line FG, being cut, is seen to be divided into the individual degrees of the oblique circle.

31. Accordingly, if an arc of any number of degrees is to be cut off from the oblique circle, beginning either from F or from G, the proposed degrees must be counted from Y or from Z in both directions — for example, as far as e, f, or g, h — and the line ef must be drawn, cutting EY at R (or the line gh, cutting EZ at V). For the line AR (or AV) will meet the line FG at the point S (or X), through which a perpendicular to FG — drawn as NT (or Om) — will cut off both arcs FN, FT (or GO, Gm), containing the given number of degrees, which are contained in the arcs Ye, Yf (or Zg, Zh).

Diagram. The analemma construction for dividing an oblique great circle perpendicular to the Meridian into degrees from the southern pole (illustrating §§30–31). A large outer great circle carries F at its top and descends to an apex K far below the figure; from F a vertical line drops through S to the center of an inner circle. A horizontal dashed line (the projected equator/almucantar line) runs across the figure through N (left) and S (upper center) to T (right). Within the outer circle sits the inner circle (the Equator/analemma) with center E, carrying B at top, C at left, D at bottom, and division points around it; the southern pole A lies at the far right, with Y and f on the right, Z and Q at the lower left, L below D near the bottom, and the pencil of chords from A (radial construction lines AR, etc.) sweeping across FG.

- F (top of outer circle): pole / summit of the oblique great circle
- N (left, on horizontal dashed line): division point
- S (upper center, on vertical from F): intersection with diameter FG
- T (right, on horizontal dashed line): division point paired with N
- H (center, below S): center of the oblique circle in the astrolabe
- B (top of inner circle): equatorial point (facing south, per text)
- E (center of inner circle): center of the Equator
- I (center, below E): endpoint of the northern semicircles (exclusive limit)
- P (upper left of inner circle): endpoint of parallel PQ
- M (center-left): division point on parallel
- x (center): intersection point
- G (lower center of inner circle): endpoint of diameter FG
- C (left of inner circle): equatorial point (facing east)
- D (bottom of inner circle): equatorial point (facing north)
- Q (lower left): endpoint of parallel PQ

- Z (lower left): quadrant endpoint (half-diameter EZ)
- Y (upper right): quadrant endpoint (half-diameter EY)
- f (right, below Y): counted degree point
- L (bottom, below D): projection vertex (the point where FL, NL meet)
- A (far right): southern pole
- K (extreme bottom apex): vertex of the projecting pencil
- [?] Additional small lowercase labels (i, t, q, η, e, m, d, and letters near C and D) are present but not securely resolved at this resolution.

32. Conversely, if one wishes to know how many degrees are contained in a given arc of the oblique circle, two perpendiculars must be drawn from its endpoints to FG; and from the points where FG is cut, two straight lines must be drawn to A, which will cut YZ in two points; and from these, two perpendiculars must be erected to YZ. For the arc of the Equator lying between those perpendiculars will indicate the number of degrees sought. [Margin: To learn, from the southern pole of the Analemma, how many degrees are contained in a given arc of an oblique great circle perpendicular to the Meridian.]

33. It is evident, moreover, that this method fits every other oblique circle as well — one that is not perpendicular to the Meridian — provided that in place of the meridian line BD one takes the straight line drawn through its center and the center of the astrolabe: the line, that is, which is the common section of the plane of the astrolabe (or of the Equator) and of the great circle drawn through the poles of the world and the poles of the oblique circle, and so on. [Margin: The agreement of the third method of dividing oblique great circles with the first two.]

Here, too, one may observe the agreement of this third method with the two earlier ones; for the very same arcs FN, GO (or CN, CO) are found by this method as we found by those.

34. I should like at this point to explain yet another way of dividing any oblique great circle into degrees. [Margin: To divide any oblique great circle into degrees on the astrolabe from its own center and the center of the astrolabe.] Although its use seems somewhat more cumbersome than the others we have explained — especially if the whole circle is to be divided into degrees — it is nevertheless most convenient when only one degree or another needs occasionally to be found. For in it neither the poles of the oblique circle are required, as in the first method, which we explained in Sections 17 and 20; nor the center of the great circle that stands as the prime vertical with respect to the given oblique circle,

together with the divisions of the diameter AC, as in the second method explained in Section 24; nor, finally, the diameter of the oblique circle divided in the Analemma, as the third method required. Rather it is accomplished solely by straight lines drawn out from the center of the Equator and from [the oblique circle's] own center — in the following way, namely.

Let ABCD be the Equator, whose center is E, and let AFCG be any oblique circle whatever, whose center is K; and let it be required to find the point on the oblique circle corresponding to the degree H of the Equator. From E, the center of the Equator, draw the straight line EH through the given point H; and on this line, when produced, let HI be taken equal to the half-diameter of the oblique circle on which the corresponding point is to be found. (When the whole circle is to be divided into degrees, or several points are to be found, it is convenient to take the line BL equal to the half-diameter FK, and, from E through L, to describe the circle LI. For in this way all the lines drawn out from E as far as that circle will have, between it and the Equator,...)

Diagram. The construction for the fourth method (§34): dividing an oblique circle into degrees from the center of the Equator E and the oblique circle's own center K. The Equator **ABCD** (center **E**) is drawn with its perpendicular diameters and a pencil of radial lines from E; an oblique circle with center **K** cuts across it. A dashed auxiliary arc (the circle LI described from E) sweeps around the lower right, and long straight lines run out from a vertex **R** at the far left and reach a point **I** at the far right.

- F (top): summit of the oblique circle
- I (upper center, and again at far right): endpoint of the produced line EH / circle LI
- K (center): center of the oblique circle
- E (center): center of the Equator
- A (right of E, and at the equatorial rim): equatorial point / western point
- N (upper area and again at right): division point(s)
- o (center, small): intersection point
- P (left of center): division point
- Q (left): division point
- S (lower left): division point
- H (center-left, and again a small h near A at right): the given degree H of the Equator and its projection
- B (center-left): equatorial point

- C (left): equatorial point
- G (lower center): endpoint of oblique diameter FG
- D (bottom center): equatorial point
- R (far left): vertex of the construction lines
- L (lower, from the text): point taken on the equatorial radius ($BL = FK$)
- [?] Additional small lowercase labels (e, s near G, and a second N) are present but not securely resolved.

and the portions added to the Equator equal to the semidiameter FK. For since EL and EI, drawn from the center, are equal to EB and EH, the remainders BL and HI will likewise be equal, and so on for the rest. Now let the straight line IK be drawn to the center K of the circle to be divided; let NO cut it in half and at right angles, cutting EI at O; and through that point O let a straight line KO be drawn from the center K, cutting the circle to be divided at P. I say that the point P corresponds to the given point H — that is, that the arcs BH and FP are equal in their number of degrees.

For since the two sides KN and NO are equal to the two sides IN and NO, and they enclose equal right angles, [Margin: Euclid I.4] the bases OK and OI will also be equal. Moreover, KP and IH are equal, since KP is the semidiameter of the oblique circle, while IH is posited equal to that same semidiameter. Therefore, equals having been taken from equals, the remainders OP and OH will also be equal. Consequently the circle described from O through H and P will touch both circles — because the lines OH and OP belong to (are directed to) the centers E and K — as we showed in Lemma 42; and it will represent, on the sphere, a circle tangent to those same circles at the points corresponding to the points H and P. Accordingly, by Lemma 43, the arcs BH and FP will encompass an equal number of degrees.

The point P will also be found by means of the straight line KP, which forms at the center K an angle equal to angle I. [Margin: Euclid I.6] For in this way the lines OK and OI will again be equal, and so forth. Indeed, if through the point H, given on the Equator, one draws HP parallel to the line KI, the same point P will be found. For since the triangles IOK and HOP are isosceles [Margin: Euclid I.15] and have equal angles at O, the remaining angles will be equal to the remaining ones. [Margin: Euclid I.5] Therefore, since both I and K, and H and P, are equal to one another, the angles OIK and OHP will also be equal; [Margin: Euclid I.27 or 28] and consequently IK and HP will be parallel.

AGAIN, suppose that, given the point P of the oblique circle, the corresponding point on the Equator is to be found. From K, the center of the oblique circle, draw a straight line through the given point P on it; and take PR equal to the semidiameter of the Equator, on which the corresponding point is to be found. (Here too, if several points are to be found, one must describe a circle from K through R, so that all the straight lines drawn from K to that circle have segments between it and the oblique circle equal to the semidiameter PR.) Now draw the line RE from R to E, the center of the Equator, and let it be bisected at right angles by the line SO, which cuts KR at O. For again, the straight line drawn from the center E through O will give on the Equator the point H that was sought. For once more, OE and OR are equal to HE and PR. Therefore, equals being taken from equals, the remainders OH and OP will be equal. Hence the circle described from O through H and P will touch both circles, and so on, because the lines OH and OP belong to the centers E and K. The same point H will also be found if at the center E an angle E is made equal to the angle R; or if, from the given point P, a line is drawn on the oblique circle parallel to RE, and so forth.

AND this method holds for all great circles, even if neither of the two circles is the Equator.

[Margin: To divide any great circle of the Astrolabe into degrees by means of another great circle already divided. — Given an arc on any great circle, to cut off an arc equal to it in number of degrees from any other great circle.]

35. THEREFORE, given two great circles on the Astrolabe: if in one of them any arc whatever is given, beginning from their common intersection, we shall easily cut off from the other an arc equal to it in number of degrees. For suppose the arc CP is given on the circle AFCG (the two great circles ABCD and AFCG cutting each other at A and C). If from its center K a straight line is drawn through the endpoint P, and on that line produced PR is taken equal to the semidiameter of the other circle; and if from R a straight line is drawn to E, the center of that circle, and SO cuts this line at right angles and in half, cutting KR at O — then the line drawn from O to the center E will give, on that circle, an arc CH equal to the arc CP; and so on for the rest.

This can indeed also be done by the first method of dividing oblique circles into degrees, but there one must first have found the poles of the given circles. For if from the endpoint of the given arc a straight line is drawn to its pole, an equal arc will be cut off on the Equator; and if through the endpoint of that arc a straight line is drawn from the pole of the other circle, the equal arc sought will be cut off

on it. But the method explained here seems more convenient, since it does not require the poles of the circles.

36. ANOTHER way of distributing great circles into degrees — a very easy and agreeable one — you will find in the following proposition, No. 36. Here, however, we shall settle this matter by yet another, most elegant method, using straight lines, since by it one and the same point on a great circle can be found by means of many straight lines. It is as follows.

[Margin: Another, most beautiful method of dividing any oblique great circle into degrees.]

LET ABCD be the Equator, with center E; let AFCG be any oblique great circle whatever, with center H and true diameter ik; and let DF be a straight line drawn through its center and through the center of the Astrolabe, representing the great circle drawn through the poles of the world and through the poles of that oblique circle — like a kind of proper Meridian; and let K be the poles of that same oblique circle. And since the line AC represents, in the sphere, the common intersection of the Equator and of the given oblique circle — as will be demonstrated in the following scholium, No. 1 — all the points of that common intersection existing in the sphere will appear on this common line AC (which appears [recte: appareat; printed *apapret*] in the Astrolabe) at exactly the same distances and in the same situation that they occupy in the sphere, since they are the same points, true in the sphere and seen (projected) in the Astrolabe. This is because the visual rays proceeding from the southern pole terminate at those same points and are not extended further, seeing that that common intersection is exactly the same as the one seen.

Now conceive the circle ABCD to be turned about BD until it stands perpendicular to the Equator, and ik, the diameter of the oblique circle, has its own proper position, with the semicircle BAD inclining southward below the plane of the Astrolabe — that is, behind it — and the semicircle BCD [inclining] northward above the plane of the Astrolabe. This being done, all the points of the diameter ik will be projected onto the line FD by the visual rays emitted from A, since the three lines AC, ik, and FD, in that position, lie on the same circle perpendicular to the oblique circle — namely the one which, like a Meridian, belongs to the oblique circle drawn through the diameter ik.

And since the plane in which the oblique great circle of diameter ik exists, when turned about AC, will at some point coincide with the Equator, it comes about that the straight lines emitted from any point of the Astrolabe on the line FD (or

even placed outside it) through the degrees of the circumference ABCD will cut the line AC at the same points at which they would cut it, if from the corresponding points...

Diagram. A large geometric construction: a circle inscribed within a square frame, densely crossed by many chords, diameters, and radiating straight lines, with a smaller inner circle and dotted (projection) arcs, illustrating §36's stereographic projection of an oblique great circle onto the plane of the Astrolabe. A vertical diameter runs from F (top) to D (bottom); a horizontal diameter AC runs across the middle (C at left, A at right), continuing to N at the far right of the frame; center E on AC, with K just above E. Readable labels: F, D, A, C, N, E, K, q, I, m, s, h, M, O, P, B, T, R, Z, V, G, Y, S. Several small interior labels around the central cluster and along the chords (apparent Greek χ , δ , λ , and lowercase u/x near B, K and the inner circle) do not resolve at this resolution.**

at the points of the plane in which the oblique circle of diameter ik — set in its proper position — would be drawn out through the degrees of the oblique circle. For example: the line BS, drawn through the endpoint S (the arc CS being 30°), cuts AC at the point T; and it is at T that this same line is cut by the line drawn from point i (which is in its proper position and corresponds to point B, since both points are equally distant from the center E and lie in the same meridian of the given circle), when that line is carried through the 30th degree of the oblique circle counted from point C. The reason is that, as has been said, the oblique circle of diameter ik , when revolved about AC, necessarily coincides with the Equator (that is, with the plane of the astrolabe); and conversely the plane of the Equator (or of the astrolabe), revolved about AC, necessarily coincides with the oblique circle set in its proper position; and point i coincides with B, and k with D. Now it is established that the line BS always cuts the line AC at the same point T, however far the plane of the circle ABCD is turned about AC. For the same reason, the line that would be drawn from k — k being in its proper position in the plane of the oblique circle — through the point Q (the corresponding point) would cut the same AC at R, where it is cut by the line DQ. Likewise the line IS cuts the same [AC] at e , the point at which it would be cut by the line drawn from the point c (which is equally distant from point I) along the diameter ik from the center E, in the plane of the oblique circle set in its proper position, toward the point corresponding to S. And so for the rest.

With this settled: suppose an arc equal to arc AM is to be cut off. We draw from some point of the line FD — say from B — a line through M cutting AC at N. Now since point i of the oblique circle, which corresponds to point B, appears from the

south pole at F, the whole line BN will appear to pass through the two points F and N (since its point B, or *i*, is seen at F, and N at N). Therefore the line FN, when drawn, will cut the oblique circle at the point O, which will correspond to M — because point M of the oblique circle ABCD, which has its proper opposition, appears at O, the point through which the line BN, passing through the given point M, is seen to pass, as was said. In the same way, when the line BS is drawn cutting AC at T, the line FT, when drawn, will fall on V, the point corresponding to S. Again, since point *k*, which corresponds to D, appears at G: if the line DQ is drawn cutting AC at R, then the line GR, when drawn, will fall on the point X, which corresponds to Q.

[Margin: Which two points of the oblique circle are best suited for the division.] But since the lines drawn from points B and D through points near the circumference ADC cut the line AC (produced beyond the circle) very obliquely, in order to keep all the points inside the circle we shall draw the lines from D through the points of the semicircle ABC. For the lines from G, through the intersection points on the line AC, will give the corresponding points on the oblique semicircle AFC. And through the points of the semicircle ACD we shall draw the lines from B; for the lines from F, through the intersection points on the line AC, will indicate the corresponding points on the oblique semicircle AGC. In this way, by means of the three points F and G — corresponding to the two points B and D — the whole circle will be divided into degrees most conveniently.

[Margin: To draw, from any point of the meridian line of the oblique circle, lines cutting the greatest circle into degrees.] By this same method we shall be able to draw lines from any point of the line BD (other than the center E of the astrolabe) that cut the oblique circle into degrees — provided the radius drawn from A to that point cuts the diameter *ik* (produced too, if need be) and the line DF. Namely, from A we send a ray to that point, and we transfer the point where it meets the diameter *ik* onto the line FD, [measuring] from E. For if from this point — once transferred onto the line FD — we draw, through each degree of the circle ABCD, a line cutting AC, then the line sent out from the assumed point through the intersection point will fall on the opposite degree of the oblique circle. For example: suppose a line is to be drawn from H, the center of the oblique circle, falling on the 30th degree counted from C toward G. We draw the radius AH cutting *ik* at *c*, the point where the center H appears, and we cut off EI equal to the line Ec, so as to have the transferred point I. Then from I we send out toward S — the point marking the 30th degree — a line cutting AC at *e*. For the line drawn from H through *e* will fall on V, the 30th degree sought, since the line IS is

projected onto the line He : because its point c , to which the point I corresponds, appears at H , and the line Ie is seen to pass through the point e .

Now just as the line IS , when produced, cuts the Equator on the other side at t , so the line He , when produced, presents in the oblique circle another point s corresponding to t , such that the arcs Bt and Fs are equal. This is because the line tS — existing in the true oblique circle (once the circle $ABCD$ is set in its proper position, that is, revolved about AC until the diameter BD coincides with the diameter ik , both being set in their proper meridian, and therefore also the point I with the point c) — is projected, as was said, onto the line sV , since it is seen to pass through the points H and e : the point c indeed through H , and e through the very point e , which lies in the common section of the plane of the Equator and of the oblique circle.

Again, suppose from the point h — given on the meridian line, outside the given greatest oblique circle — a line is to be drawn that cuts off from the quadrant AG an arc equal to arc AY . We draw the radius Ah cutting ki (produced) at g , and we transfer the point g from E to u , so as to have the transferred point u . Then from u to Y we join a line cutting AC at Z . For the line hZ will yield the point b corresponding to Y . And the point where the line hY intersects the oblique circle, near F , will correspond to the point where the line uY intersects the circle $ABCD$, near B .

But should it happen that a line drawn from some point transferred outside the circle $ABCD$ — as from u , which corresponds to g — through a given point p must be made to touch the circle $ABCD$ at that given point p , then from h , the visible point, one must draw the line hq tangent to the oblique circle. For the point of contact q will correspond to the given point of contact p . For just as up touches the oblique circle in the sphere, so it will be seen to touch the same visible circle on the astrolabe. Since, then, the point g (to which u corresponds) appears at h , the tangent up will be projected onto the tangent hq .

So too, should it ever happen that a line drawn from some point transferred inside the circle $ABCD$ — as from H , which corresponds to point f — through a given point (say P) is to make a right angle with the line FD , then it must be drawn

Diagram. A large woodcut of the master stereographic-projection construction for this proposition, set inside a square frame. A circle (the Equator / plate, standing for circle $ABCD$) is inscribed in the square and is densely filled with straight construction lines and dotted arcs representing the oblique circle and its

degree-divisions. A horizontal diameter runs left–right through the center E, and a point *h* stands above the circle at the top, with a dotted line descending from it into the figure. The figure carries many small labelled points; the clearly legible ones are listed below, with the dense interior cluster only partly resolvable at this resolution.

- *h* (apex, above the circle, top): external meridian point, dotted line descending
- F (top of circle): projected point
- *s* / *f* (upper right, just below F): projected point
- *q* (upper left, outside circle)
- *l* (far left)
- *x* (upper-left interior)
- *m* (right, outside circle)
- O (right, outside circle)
- N (far right corner)
- A (right end of horizontal diameter, on the circle)
- C (left end of horizontal diameter, on the circle)
- T (left of center, on the horizontal)
- R (center-left, on the horizontal)
- E (center of the circle)
- *z* (right of center, on the horizontal) [?]
- K (just below center E)
- V (lower left)
- G (bottom center)
- Y (lower right)
- B (lower right, near Y)
- S (lower left)
- R (bottom, second R) [?]
- D (bottom of circle)
- *o*, E (left interior, lower) [?]
- interior cluster (upper-center, very dense): B, P, M, *a*, *d*, *i*, *u* — several individually unresolved [?]

through the point *n*, where *f* appears, as the perpendicular *mn**l*. For the point *l* will correspond to the given point P, and the point *m* to the other point where the line PH (produced) cuts the circle ABCD. This we demonstrated above at Number

30: because the line HP corresponds to the line that would be drawn through f in the oblique circle in the sphere, perpendicular to the diameter ik , and it cuts off arcs equal to the arc BP, etc.

Finally, suppose the division is to be made from K, the visible pole of the oblique circle — that is, suppose an arc equal to arc BQ is to be cut off from the oblique circle. We transfer the point a onto the line FD, as far as K, so that the lines Ea and EK are equal (as we demonstrated above at Number 14; you will, however, find this demonstrated more clearly near the end of Number 21 of Proposition 6), so that the transferred point does not differ from the visible one. Then from K, the transferred point (which corresponds to point a), we throw a line through Q cutting AC at r . For the line drawn from K, the visible point — namely at the point of section r , where it appears — which does not differ from the former one, will (because of the same points K and r) indicate the point X corresponding to point P, and when produced will give the other point α corresponding to point β . From this it is also clear that a line drawn from the visible pole through any point of the Equator presents, in the oblique circle, the point corresponding to that point — which we showed above at Number 17, from Lemma 23 of Book 1.

[The auxiliary projected points are labelled here with Greek minuscules: α (alpha), β (beta), γ (gamma), δ (delta), ϵ (epsilon), μ (mu).]

For greater clarity of this method, we find the same points O, V, b , s , q , l , X, α corresponding to the points M, S, Y, t , p , P, Q, β by means of lines sent out from the visible pole K, as was taught at Number 17.

[Margin: To find, for a given point on a greatest oblique circle, the corresponding point on the Equator.] It will not, however, be difficult, conversely, to investigate from a given point on the oblique circle the corresponding point on the Equator (or on the oblique circle in the sphere, whose part the Equator plays). Let the given point be O. From F, which corresponds to B, we send through O a line cutting AC at N. For the line BN will cut the Equator at the point M, which corresponds to the given point O, as is clear from what has been said. We do the same from any other point given on the meridian line, such as H. For, drawing the radius AH cutting the diameter ik at c , let c be transferred onto the line FD as far as I; and suppose the task is to investigate the point of the Equator corresponding to point V. When the line HV is drawn cutting AC at e , the line Ie, issuing from the transferred point I, will fall on the sought point S. And so for the rest.

Now if the division is to be set up from the center of the circle that serves as the proper vertical of the given oblique circle — such as the point L in the upper figure at Number 24 — then, since that [center] has no true corresponding point on the diameter YZ that could be transferred onto the line FD (for the line AL, falling on the said center L, is parallel to the diameter YZ, and therefore lies wholly outside the plane of the given oblique circle, as is clear from Number 4), a line parallel to FD must be drawn through the point given on the Equator, and from that center a line must be drawn through the point of section on AC, and so on, as was taught at Number 24.

[Margin: Given any point on the plane of some greatest circle in the sphere — even outside the circle — to find its point on the astrolabe.] Now indeed, by means of what has been explained here, we shall find, for any point on a given greatest circle (or assigned in its plane produced, outside the circle itself), its position on the astrolabe — that is, the place where, in the plane of the visible circle, it would appear when viewed from the south pole. Let the given point be ϵ . If it lies on the Equator, its position will be at ϵ , since it appears at ϵ . But if it is understood to lie on some greatest circle — say the one represented by the circle AFCG, holding in it the same situation and position as it would in the Equator of the astrolabe — we shall find its visible place as follows. From any point, say B, of the line FD, draw the line B ϵ cutting AC at γ ; then from the point F (which corresponds to B) draw the line F γ . The point ϵ will appear on the line F γ , since the whole line B γ is projected onto the line F γ , as is clear from what has been said. Again, from any other point D draw the line D ϵ cutting AC at T [?]; then from the point G (which corresponds to D) draw the line GT. The same point ϵ will again appear on the line GT, since the whole line D ϵ is projected onto the line GT, as is evident from what has been said. Therefore the point δ , where the lines F γ and GT meet, will be the position of the point ϵ .

But if one of the two lines drawn from B and D through the assigned point ϵ cuts the line AC too far off and too obliquely, then in place of that point from which the line drawn through ϵ falls outside the circle ABCD — such as the point B — one may take any other point Q. For when the line Q ϵ is drawn cutting AC at μ , if the point X on the oblique circle corresponding to the assumed point Q is found, and X μ is drawn, then GT will be cut at the same sought point δ . Indeed, once merely one of the lines F γ , GT, X μ (on which the given point ϵ appears) has been found, if from K, the visible pole of the oblique circle, a line is drawn through ϵ , it will cut that line at the same sought point δ . For since the point ϵ has, in the true circle, the position that the point δ has in the visible one, then, if a parallel of the Equator through ϵ is described equal to the oblique parallel described through δ ,

the line $K\epsilon$ drawn from the pole cuts off from that oblique parallel the corresponding point, namely δ , as will be demonstrated at Proposition 6, Number 21. This can also be established from the fact that, since the point a on the diameter ik corresponds to the visible pole K , and Ea and EK are equal, the transferred point does not differ from the visible one; and therefore on the same line $K\epsilon$ the same point δ will appear — just as on KQ , produced, the visible point X , corresponding to Q , lies, since the line KQ does not differ from the line rK , as was said above.

If the given point lies on the line FD — that is, on the diameter of the oblique circle, with which the line FD coincides (once the plane of the astrolabe is revolved about AC) — as for example the point I , we shall cut off from the diameter ik a length equal to EI , namely Ec , so as to have the true point c . For the radius Ac will indicate the point c , seen at H .

An exception, however, must be made for the points lying in the common section of any oblique circle in the sphere with the plane drawn through the south pole parallel to the Equator. For these have no visible corresponding points on the astrolabe, since that whole common section vanishes on the astrolabe and none of its points appears there — inasmuch as all the visual rays lying in that parallel plane run parallel to the plane of the Equator (that is, of the astrolabe). On this matter we shall write more at Proposition 6, Number 37.

[Margin: Given any point seen on the astrolabe, to find its true position in the plane of any greatest circle.] **Conversely**, given any point δ seen on the astrolabe, we shall find its true position in the sphere — that is, in that circle of the sphere which the circle of the astrolabe (on which the point δ is understood to be seen) represents. For when lines are drawn from the points F and G of the oblique circle through the given point δ , cutting AC at γ and T , let lines be drawn from γ and T to the points B and D (which correspond to F and G); these lines, intersecting each other at ϵ , give the point that is sought, since the lines $B\gamma$ and DT are projected onto the lines $F\gamma$ and GT , etc. In the same way, if another line δX is drawn through δ cutting AC at μ , and the point Q corresponding to X is found, then the line μQ , when drawn, will pass through the same point ϵ .

Only for the points lying on the line drawn perpendicular to FD through the center of the circle that serves as the proper vertical of the given oblique circle — such as the point L in the upper figure at Number 24 — can no true corresponding points be assigned in the plane of the true oblique circle. For since that line represents the plane drawn through the south pole parallel

to the oblique circle in the sphere, as we shall show at Proposition 6, Number 3, the true points corresponding to the points lying on the said line will exist in that parallel plane, but not in that oblique circle. And if anyone should try, by the method we have explained, to find in the Horizon the true point corresponding to the visible point L in the figure at Number 24 — namely by drawing lines from L through two apparent points on the circumference of the Horizon — he will find that the two lines (drawn through the section points of the line AC with those two lines, and through the points of the circle ABCD corresponding to those apparent points of the Horizon) are parallel to the line FD and do not intersect one another. And if he should wish to find the true point, in the same true Horizon, corresponding to any other point of the aforesaid line drawn perpendicular to FD through L, he will find two lines that are likewise parallel to each other, drawn through the intersection points on the line AC, although they are not equidistant from FD, etc.

[Margin: From any point on the astrolabe outside the meridian line to divide the given greatest circle into degrees.] **From this it follows** that a given greatest circle can be divided from any point on the astrolabe outside the meridian line and the line AC. For if, say, the point corresponding to a given point Q is to be found from the point δ , one must first investigate (as was just shown) the true point ε corresponding to δ . Then, through ε — the true point found — a line must be drawn to Q, cutting AC at μ . For the line drawn from δ through μ will fall on X, the point corresponding to the given point Q, since the whole line Q μ is projected onto the line X μ , as is clear from what has been said — for ε is the true point on the circle ABCD, which the oblique circle AECG represents, and which indeed appears at δ , etc. Here too an exception must be made for the points on the line drawn perpendicular to FD through the center of the proper vertical of the given oblique circle. For since, as has been said, those points have no true corresponding points on that oblique circle in the sphere, the visible circle cannot be divided into degrees from those points in the way we have explained.

But in what way this division can be carried out — namely by lines parallel [drawn] from the point that in the sphere corresponds to the point where the line perpendicular to AC at A, the south pole, cuts the diameter *ki* of the oblique circle (produced) — we shall set forth at Proposition 6, Number 37. There you will also find another method, by which the visible oblique circle is divided into degrees by lines sent out through the center E of the astrolabe, so that each line yields two points diametrically opposite. Finally, in the same place, at Number 38, we shall divide the same circles — both the greatest and the lesser — into degrees most conveniently, from any point given in the common section of the plane of the

astrolabe and of the proposed circle in the sphere. For we have deferred these three methods to that place, lest the figure here presented be confused by too great a multitude of lines.

Diagram. The same master construction figure reprinted on this leaf — a circle (standing for the Equator / circle ABCD) inscribed in a square frame, filled with straight construction lines and dotted oblique-circle arcs, with a horizontal diameter through the center E and the external point *h* set above the circle at the top. The labelled points are the same as in the figure on p. 508; the clearly legible ones are listed below.

- *h* (apex, above the circle, top), with dotted line descending
- F (top of circle)
- *s* / *f* (upper right, below F)
- *q* (upper left, outside circle)
- *l* (far left)
- *x* (upper-left interior)
- *m* (right, outside circle)
- O (right, outside circle)
- N (far right corner)
- A (right end of horizontal diameter, on the circle)
- C (left end of horizontal diameter, on the circle)
- T (left of center, on the horizontal)
- R (center-left, on the horizontal)
- E (center of the circle)
- K (just below center E)
- V (lower left)
- G (bottom center)
- Y (lower right)
- B (lower right, near Y)
- S (lower left)
- D (bottom of circle)
- interior cluster (upper-center, very dense): B, P, M, *a*, *d*, *i*, *u*, *z*, *o* — several individually unresolved [?]

SCHOLIVM

1. Now any greatest oblique circle that is at right angles to the Meridian — and therefore has its center on the meridian line of the astrolabe — will

necessarily, on the astrolabe (barring error), pass through the points A and C, where the Equator is cut by the right Horizon AC. For the points A and C are those in which the Horizon, the prime vertical, the ecliptic (with the beginnings of Cancer ☊ and Capricorn ☋ set in the Meridian), and any other greatest circle having its poles in the Meridian — and therefore standing at right angles to it ^a — intersect the Equator; because the line AC represents the right Horizon, or the equinoctial colure, when the colure of the solstices coincides with the Meridian, as we demonstrated at Proposition 4, Number 1. Hence it comes about that, on the plane of the astrolabe, such a greatest oblique circle is necessarily seen to pass through those two points A and C, since through them are represented those points of the sphere through which that same circle is drawn — so that the line AC exhibits, on the astrolabe, that diameter of the oblique circle which in the sphere is its common section with the Equator. For the circles on the astrolabe must be seen to run through the same line and through the same points through which they are drawn in the sphere.

[Margin a: Theodosius I.15.]

This, however, we shall also easily demonstrate geometrically, from the very projection of oblique greatest circles of this kind onto the plane of the astrolabe, as follows. Let ABCD be the Equator, with center E; let the meridian line — that is, the common section of the Meridian and the plane of the Equator (or of the astrolabe) — be BD, which is cut at right angles by FG (itself at right angles to the Meridian), so that the arc AF is the elevation of the pole above that oblique circle. For, as was said above in this proposition at Number 2, and at Proposition 4, Number 5, the circle ABCD is taken for the Meridian of the analemma. From the visual rays AG and AF let the visible diameter HI be found; bisect it at K, and through the line AK, perpendicular to FG at V (as has been demonstrated), describe from K, through H and I, a circle. I say that it passes through A and C. ^b For since the angle FAG, being in a semicircle, is a right angle, the triangle HAI will be right-angled. And since the side HI, opposite the right angle, has been bisected at K, the circle described from K through H and I must necessarily pass through the right angle A, by the scholium to Proposition 31 of Book 3 of Euclid. For the same reason it will pass through the point C.

[Margin b: Euclid I.31.]

For when the lines CH and CI are drawn, the angle HCI is likewise a right angle, which is proved thus. Since the two sides EH, EA are equal to the two sides EH, EC, and contain equal angles (namely right angles), ^c the bases AH, CH will also be

equal. In the same way you will show that the bases IA, IC are equal, in the triangles AEI, CEI. Since therefore the two sides AH, AI are equal to the two sides CH, CI, and the base HI is common, ^d the angles HAI, HCI will be equal; and therefore, since HAI is a right angle, HCI will also be a right angle. Consequently the circle described about HI will pass through C, by the same scholium to Proposition 31 of Book 3 of Euclid.

[Margin c: Euclid I.14.] [Margin d: Euclid I.8.]

This, however, can be shown more easily thus. When the line CK is drawn, since the two sides EK, EA are equal to the two sides EK, EC, and contain equal angles (namely right angles), ^e the bases KA, KC will also be equal. Therefore the circle HMI, described from center K through A, will pass through the point C. Which was the proposition.

[Margin e: Euclid I.4.]

(Continuation of the Scholium to Proposition V. The numbered paragraphs below continue the sequence begun in the previous section.)

2. Hence it is also clear that any great circle drawn on the astrolabe is larger than the equator. For if from K — the center of the oblique great circle, which we showed above (in Number 5 of this proposition) to be distinct from E, the center of the astrolabe — the two radii KA and KC are drawn, these taken together will equal the whole diameter HI. [margin a: *Eucl.* I.20] Since, then, they are greater than AC, the diameter HI too will be greater than the diameter AC; and therefore the oblique circle AHCI will be larger than the equator ABCD. The same reasoning holds for the rest.

[Margin: That any oblique great circle on the astrolabe is larger than the equator. — *a*: Euclid I.20; *b*: Theodosius I.15.]

3. By exactly the same reasoning: suppose any other oblique great circle is drawn on the astrolabe that is not perpendicular to the meridian, and that a straight line is drawn through its center and the center of the astrolabe. (This line is of course the common section of the plane of the astrolabe — that is, of the equator — with the great circle drawn through the poles of the world and the poles of the oblique circle, [margin b: *Theod.* I.15] and it is therefore perpendicular to that great circle; indeed, we showed in the scholium to Proposition 3, Numbers 1 and 3, that the apparent greatest diameter of the oblique circle is projected onto this line.) We will demonstrate that, when the diameter of the equator cuts this line at right angles, the oblique circle passes through the endpoints of that diameter — the

very endpoints that represent the common section of the oblique circle and the equator in the sphere, as we shall shortly show.

For example, suppose the circle AHCI is set on the astrolabe as any great circle oblique to the equator and to the meridian, and that through its center K and the center of the astrolabe E the straight line HI is drawn, which is the common section of the plane of the astrolabe (or of the equator) with the great circle passing through the poles of the world and the poles of the oblique circle. Since in this section lies the center of the oblique circle on the astrolabe, as was demonstrated in the scholium to Proposition 3, Number 4 — for in it lies the circle's apparent greatest diameter — and since the diameter AC of the equator is drawn perpendicular to HI, we will demonstrate that it necessarily passes through the points A and C, just as we showed that the same circle, when it is perpendicular to the meridian (as are the horizon, the prime vertical, the ecliptic when its beginning Cancer [♋] is placed on the meridian, and others), passes through the points A and C.

This will also be demonstrated for the verticals in Proposition 8, Number 16. From this it follows that any great circle bisects the equator on the astrolabe, since it passes through two of the equator's diametrically opposite points.

[Margin: The oblique great circles that are not perpendicular to the meridian — through which points of the equator they are drawn on the astrolabe. That any great circle on the astrolabe bisects the equator, that is:]

The line AC will likewise represent the common section of the equator and that oblique circle in the sphere. We will show this no differently than it was shown that the same line AC represents the common section of the equator with the horizon, or with the prime vertical, or with the ecliptic, if the circle AHCI is taken to be one of these. [margin c: *Theod.* I.15] For since both the equator and the oblique circle are perpendicular to the great circle drawn through the poles of the world and the poles of the oblique circle, [margin d: *Eucl.* XI.19] their common section will be perpendicular to that great circle; and therefore this same line, lying at right angles to HI within that great circle, will be perpendicular at the center of the equator, by Definition 3 of Book 11 of Euclid. Therefore AC, being perpendicular to HI, will be that common section.

[Margin: The common section of the equator and of any oblique great circle in the sphere — by which straight line it is represented on the astrolabe.]

4. Therefore, just as in the sphere every great circle bisects the equator, so too on the astrolabe the equator is bisected by every oblique great circle — whether it is

perpendicular to the meridian or not — since it is cut by that circle at the endpoints of the diameter AC, which is perpendicular to HI, the common section of the plane of the astrolabe with the great circle passing through the poles of the world and the poles of the oblique circle, a line that serves as a kind of proper meridian, as we have demonstrated. And since the equator in turn bisects any great circle in the sphere [margin e: *Theod.* I.11] (because all great circles in the sphere mutually bisect one another), it comes about that on the astrolabe too it is seen to bisect every oblique great circle, so that the arc AHC represents one semicircle and the arc AIC the other, even though these arcs are very unequal to each other. That this must necessarily happen so on the astrolabe, clear reasoning demonstrates.

[Margin: The equator and any oblique great circle on the astrolabe mutually bisect one another, although the segments of the oblique circle are very unequal to each other. — e: Theodosius I.11.]

5. For since one of the two semicircles of any oblique great circle — the semicircles made by its common section with the equator — inclines from the equator toward the south pole, and the other toward the north, the one nearer to the eye (that is, to the south pole) will appear larger than the one that is farther off, as is clear from optics [Perspectiva]. [margin f: *Theod.* III.8] Likewise, since every oblique great circle touches two opposite and equal parallels — one northern, one southern — and since (by Proposition 2) the southern parallel is projected into a circle larger than the equator and the northern into a smaller one, the northern semicircle of the oblique circle will necessarily be projected inside the equator, such as AIC, while the southern falls outside the equator, such as AHC. The latter, then, will be larger than the former, since the semicircle AHC runs out farther from the line AC than does the semicircle AIC.

[Margin: Why the semicircles of any oblique great circle made by the equator are unequal on the astrolabe. — f: Theodosius III.8.]

6. But since each semicircle of the equator, however it is cut by a diameter, is equally distant from the eye (that is, from the south pole), both will appear equal. This too is clearly established from Proposition 2, where it was shown that the equator and its parallels are projected onto the astrolabe in such a way that their equal arcs are projected into equal arcs. For from this it follows that equal semicircles are projected into equal semicircles; and therefore any oblique great circle, since it bisects the equator in the sphere, will necessarily pass on the astrolabe through two diametrically opposite points, so as to cut off from it two

equal semicircles, corresponding to those it cuts off from the same equator in the sphere.

[Margin: Why the equator on the astrolabe is cut by any oblique great circle into two equal semicircles at two diametrically opposite points.]

7. By similar reasoning, any circle — whether great or not great — that bisects one of the equator's parallels in the sphere will necessarily pass, on the astrolabe, through two diametrically opposite points of that projected parallel, so as to bisect it there as well.

[Margin: Any circle, whether great or not great, dividing...]

8. But no non-great circle will be drawn on the astrolabe through two diametrically opposite points of the equator, since it cannot bisect the equator in the sphere. For if it did, it would be a great circle, inasmuch as it would pass through a diameter of the equator and hence through the center of the sphere (that is, of the equator) — which contradicts the hypothesis.

9. From these results it also remains evident that a circle on the astrolabe which cuts the equator at two diametrically opposite points represents a great circle in the sphere, since a non-great circle cannot bisect the equator, as was just said.

[Margin: A non-great circle bisecting some parallel of the equator in the sphere passes on the astrolabe through two diametrically opposite points of that parallel. A non-great circle cannot bisect the equator on the astrolabe. A circle on the astrolabe cutting the equator into two halves represents a great circle.]

Diagram. The first figure of this scholium, an uncaptioned woodcut set within a square frame. A large outer circle represents the oblique great circle AHCI; an inner circle set below and to the left represents the equator ABCD. Numerous straight lines (some solid, some dotted) connect the labelled points, illustrating the projection of the oblique circle and its bisection by the equator. Readable point-labels:

- H (top apex of the figure, on the frame's upper edge; projection vertex)
- B (upper part of the inner/equator circle)
- N (just right of B, near the center-top)
- X (small label just below B)
- R (upper left, on the oblique circle)
- F (upper right, on the oblique circle)
- M (far left)

- O (left)
- C (left, at the intersection of the two circles)
- E (center of the figure)
- V (near the center, below N)
- A (right, at the intersection of the two circles)
- P (lower right)
- L (below and right of E)
- S (lower right)
- I (lower center)
- G (lower left)
- T (bottom left)
- D (bottom center, foot of the vertical line from H)

(Continuation of §9.) ...for a non-great circle cannot bisect the equator, as was just said. But a circle that cuts the equator at two points which are not diametrically opposite represents a non-great circle. For if it represented a great circle, it would bisect the equator, as was shown — which is not the case here.

[Margin: ...a great circle in the sphere; but a circle which does not bisect it represents a non-great circle. — *a*: Euclid III.3; *b*: Euclid III.31.]

We shall now demonstrate this same thing geometrically, by the following reasoning. Let ABCD be the equator, with center E, and let the circle FCGA bisect it at the diametrically opposite points A and C. I say that it represents a great circle in the sphere. For when the diameter AC has been drawn, let the line FD be drawn through E, the center of the equator, and through the center of the circle FCGA. This line will be perpendicular to AC, which it bisects at the center E, [margin a: *Eucl.* III.3] and it will represent the great circle drawn through the poles of the world and the poles of the circle FCGA, as was demonstrated in the scholium to Proposition 3, Number 4. Hence the line AE will be the perpendicular axis of the world, and A and C the poles of the world (if the circle ABCD is understood to be perpendicular to the equator, that is, to the plane of the astrolabe), since they are a quadrant distant from the equator drawn along BD.

Now let the rays AF and AG go out through the ends of the greatest apparent diameter, cutting the equator at H and I; and let HI be joined. This will be the diameter of the circle that FCGA represents, since its ends appear at F and G, the ends of the greatest apparent diameter FG. [margin b: *Eucl.* III.31] And since the angle FAG — that is, HAI — is right, then by the scholium to Proposition 31 of

Book 3 of Euclid, HAI is a semicircle, and therefore HI passes through the center E and is the diameter of the great circle which FCGA represents.

Next, let the circle KLMN cut the equator at L and N — not into halves — below the points A and C, so that the line LN drawn through them does not pass through the center E. I say that it represents a non-great circle. For again let KM be drawn through its center and through E, the center of the astrolabe, as the common section of the astrolabe with the great circle drawn through the poles of the world and the poles of the circle KLMN; and let AC be drawn perpendicular to it as the axis of the world, as before. Then from N let the lines NK and NM be sent out through the ends K, M of the apparent diameter, cutting the equator at O and P; and let OP be joined. [margin c: *Eucl.* III.31] Since the angle KNM — that is, ONP — is right, then by the scholium to Proposition 31 of Book 3 of Euclid, ONP is a semicircle and its diameter is OP. Now the rays sent out from the pole A to the same ends K, M of the apparent diameter KM cut the equator short of the points O, P, at Q and R (for AK lies short of KN, and AM cuts NM at M). Therefore QAR is a segment less than a semicircle; and consequently the line QR, when joined — the diameter of the circle that KLMN represents — will not pass through the center, and hence it will be the diameter of a non-great circle.

Finally, let the circle STVX cut the equator at T and X — not into halves — above the points A and C, so that the line TX drawn through them does not pass through the center E. I say that it too represents a non-great circle. For again let the line SV be drawn through its center and through E, the center of the astrolabe, as the common section of the astrolabe with the great circle drawn through the poles of the world and the poles of the circle STVX; and let AC be perpendicular to it as the axis of the world. Let the lines XS and XV be drawn out through the ends of the apparent diameter SV, cutting the equator at Y and Z — whether Y falls above X or below (for it can happen, when S is far off, that the line XS cuts the equator below X); and let YZ be joined. [margin d: *Eucl.* III.31] And since the angle SXV — that is, YXZ — is right, then by the scholium to Proposition 31 of Book 3 of Euclid, YXZ is a semicircle and its diameter is YZ. Now since the rays sent out from the pole A through the same ends S, V of the apparent diameter SV cut the equator beyond the points Y, Z, at a and b (for AS falls below XS, and AV cuts XV at V), the arc aAb will be a segment greater than a semicircle. And therefore the line ab, when joined — the diameter of the circle that STVX represents — will not pass through the center, and hence will be the diameter of a non-great circle. Which was to be demonstrated.

10. Again, since all the diameters of any oblique great circle in the sphere are drawn through the center of the sphere, and are seen to pass through the same center on the astrolabe, it follows that every straight line drawn through the center of the astrolabe out to the circumference of the oblique circle on both sides expresses that diameter of the oblique circle in the sphere which passes through the points represented on the astrolabe's oblique circle — the points to which the straight line traced through the center of the astrolabe extends. Thus any such line on the astrolabe stands for some diameter of the circle, running through two points that correspond in the sphere to two diametrically opposite points. For example, in the first figure of this scholium, the line LM, projected through E, the center of the astrolabe, represents in the sphere that diameter of the oblique circle represented by AHCI which recedes southward from the common section of the oblique circle with the equator by as many degrees as the arc CM subtends on the astrolabe. (How one may know how many degrees the arc CM contains was set out in Number 19 of this Proposition 5.) So the points L, M express two diametrically opposite points in the sphere.

11. But that any line extended through the center of the astrolabe — namely LM — represents, as we said, some diameter of an oblique great circle (even though it cuts that circle into unequal parts), and marks on the oblique circle two diametrically opposite points L, M, just as the line AC (which we showed represents the common section of the oblique circle and the equator in the sphere) — this we will now demonstrate geometrically in another way, following Ptolemy. In the first figure of this scholium, erect at E a perpendicular EN to LM, and produce it as far as T. Produce ML also as far as P; join the lines MN, ON, LN, PN; and let the equator be cut by MN and LN at R and S. Since, then, in the circle AHCI the two lines AC and LM intersect one another at E, [margin e: *Eucl.* III.35] the rectangle under LE and EM will be equal to the rectangle under AE and EC — that is, to the square on the line AE, and therefore also to the square on the line EN (or ET). [margin f: *Eucl.* VI.17] Therefore each of EN and ET is a mean proportional between EM and EL; and hence the circle described about the diameter LM will pass through the points N and T.

[Margin: That every straight line drawn through the center of the astrolabe marks, on the oblique great circle, two diametrically opposite points, so that it plays the role of some diameter.]

Diagram. The second figure of this scholium, an uncaptioned woodcut in a square frame, illustrating the geometric proof that a circle bisecting the equator (FCGA) represents a great circle, while circles cutting it off-center (KLMN, STVX)

represent non-great circles. Several concentric and eccentric circles are drawn with many chords and radial lines. Readable point-labels:

- H (top apex, on the frame's upper edge)
- F (below H, on the upper part of the outer circle)
- S (upper center)
- B (center-upper)
- Y (upper right, near two small ring-marks)
- T (left)
- X (right)
- E (center)
- V (below center)
- G (below center, under V)
- L (lower left)
- b (small label beside L)
- I (lower left, below L)
- Z (lower left)
- P (lower center)
- R (lower center)
- D (bottom center, foot of the vertical)
- N (right)
- A (right, on the equator circle)
- M (left)
- O (left)
- C (left, on the equator circle)

(Continuation of §11.) For if it passed beyond the point N — say — or fell short of it near N, it would cut off from the perpendicular EN either a greater line or a lesser line EN, which by the scholium to Proposition 13 of Book 6 of Euclid would likewise be the mean proportional between the same segments LE and EM. And so the cut-off line and EN would be equal — the part and the whole — which is absurd. This can also be demonstrated from Lemma 15. Therefore that circle will pass through N, and consequently through T as well, for the same reason; and thus it will represent some great circle in the sphere, as we showed just above in Numbers 6 and 9, since it bisects the equator at N, T. [margin a: *Theod.* II.8] And since an oblique great circle touches two opposite and equal parallels, the circles that would be described from center E with radii EL and EM would touch the circle whose diameter is LM (by the scholium to Proposition 13 of Book 3 of

Euclid) at L and M, as two opposite and equal parallels. [margin b: Corollary of *Theod.* II.6] Therefore, since the points of contact are diametrically opposite in the sphere, L and M will represent two diametrically opposite points in the sphere, and consequently the line LM will represent some diameter of an oblique great circle. Which was the proposition.

Now, in order to understand which points of the sphere are represented by the points L, M, and which diameter the line LM stands for with respect to the poles of the world — for we said a little earlier which points they represent in the case of the true horizon, namely those distant from the common sections of the equator with the horizon by as many degrees as are apparently contained in the arcs CM and AL — we shall proceed as follows. Since the circle described about the diameter LM passes through N, as we demonstrated, [margin c: *Eucl.* III.31] the angle MNL, being in a semicircle, will be right, and therefore equal to the angle ONP, [margin d: *Eucl.* III.31] which is likewise right, being in the semicircle ONP; [margin e: *Eucl.* III.26] and hence the arcs RTS and OTP will be equal. Since, then, OTP is a semicircle — because the line LM was posited to pass through the center E — RTS will also be a semicircle; and consequently the line RS, when drawn, will be a diameter of the circle ABCD.

Therefore, if the circle ABCD is conceived to be a great circle drawn through the poles of the world and through the diameter RS, making in the plane of the astrolabe (that is, of the equator) the section PLEOM — a circle which will be oblique to the circle of diameter FG in the sphere (the one that the circle AHCI represents on the astrolabe), since it does not pass through its poles; for the great circle drawn through the poles of the world and through the poles of the oblique circle of diameter FG makes on the astrolabe, or equator, the section DEH, not PEM — then N and T will be the poles of the world, and NT the axis, since in the great circle ABCD drawn through the poles of the world the points N, T are a quadrant distant from the equator drawn along the line OP.

So, if the antarctic pole is set at N, the endpoints R, S of the diameter RS will appear at the points M, L in the plane of the astrolabe, viewed by the visual rays NR, NS from the south pole N. Therefore the points M, L represent the points R, S — which are diametrically opposite in the sphere and whose distances from the poles of the world are the arcs NR, TS — and the line ML will represent the diameter RS, which is the common section of the oblique circle expressed in the sphere by the circle AHCI and of the great circle ABCD drawn through the poles of the world (a circle oblique to that same oblique circle). And if in the sphere a great circle is conceived to be drawn through the diameter RS, perpendicular to

the circle ABCD in the position we said it has, then ML will be the greatest apparent diameter of that circle drawn through RS; and therefore the circle described about ML will represent that circle drawn through RS, the one perpendicular to the circle ABCD.

And to make the whole matter still clearer, let us suppose the circle AHCI to be some oblique horizon. If, for example, the solstitial colure [Colurus solstitiorum] is carried around in the sphere until its segment between the south pole and the horizon is similar to the arc NR, and its segment between the north pole and the horizon similar to the arc TS, then the circle ABCD will represent the solstitial colure in that position; and RS will be the diameter of the horizon, which is the common section of the solstitial colure in that position with the horizon, and which is projected onto the line ML, at the common section of the astrolabe (that is, of the equator) with that same colure in that same position — the line we called PLEOM. Finally, the opposite and equal parallels of the equator which the circle described about ML touches, as we said, are those whose declinations from the equator are the arcs OR, PS. This matter is not hard to understand if a material sphere is employed; and the same reasoning can be transferred without difficulty to other oblique great circles.

12. Since in Proposition 3, Number 3, I promised that I would here demonstrate that equal arcs of oblique circles are projected onto the astrolabe as unequal arcs in continuous order, this must be shown as follows. Let ABCD be the equator of the astrolabe, with center E; let AFCG be an oblique great circle, with center H, one of its poles I and the other Y. Taking on the equator the equal arcs BK, KL, let the lines IK, IL be drawn from the pole I, cutting the oblique circle at M and P. The arcs FM, MP will correspond to arcs of the oblique circle in the sphere that are equal — equal to the arcs BK, KL — since (as was demonstrated in Number 17 of this proposition, under the first method of dividing oblique circles into degrees) they comprise as many degrees as are contained in the arcs BK, KL. And since, by Lemma 33, FM is greater than MP, and MP greater than the next arc — which corresponds to an arc of the equator equal to KL — and so on down to the end of the semicircle FCG, it is clear that equal arcs of an oblique great circle are projected as unequal arcs in continuous order, the one nearer to the point F being always greater than the more remote, provided they correspond to equal arcs of the equator, as was shown in Lemma 33. Thus if the oblique circle AFCG is divided into 360 degrees, as we taught above, those degrees will continuously decrease from F to G, in each semicircle FCG and FAG, so that the degrees are largest near the point F and smallest near the point G. From this it follows that

the parts of the oblique circle on the astrolabe are not similar to the corresponding parts of the same circle in the sphere.

[Margin: That equal arcs of an oblique great circle are projected into unequal arcs, in continuous order.]

13. It can nevertheless happen that some one part, of any number of degrees — fewer than 180, however — is similar to another single part, which might perhaps seem incredible to someone. For if from the pole I a perpendicular IT is drawn to FG, and on either side of it two equal angles TIM, TIQ are constructed, then by Lemma 34 the arcs MQ, KO will be similar. For when PS is drawn — the diameter of the oblique circle perpendicular to FG — the three points A, T, P will lie on one straight line, as will be shown in Number 14. And since, as we demonstrated in the same lemma, the whole angle MIQ is equal to each of the angles MHQ, KEO, then if the whole angle MIQ, composed of the two equal angles TIM, TIQ, stands upon an arc of 1, or 2, or 3, or 4, or 20, or 100 degrees, and so on, in the circle that would be described from I — that is...

[Margin: That some one arc of an oblique great circle in the sphere can be projected onto the astrolabe into a similar arc, that is:]

Diagram. A re-impression of the first figure of the scholium (the same construction as fig-p159-1), an uncaptioned woodcut in a square frame. A large outer circle (the oblique great circle AHCI) with an inner circle set low-left (the equator ABCD), crossed by many solid and dotted chords and radial lines.

Readable point-labels:

- H (top apex, on the frame's upper edge)
- B (upper part of the inner circle)
- N (right of B, center-top)
- K (small label between B and N)
- R (upper left, on the oblique circle)
- F (upper right, on the oblique circle)
- M (far left)
- O (left)
- E (center)
- V (center, below N)
- C (left, at the intersection of the circles)
- A (right, at the intersection of the circles)
- P (lower right)

- L (lower center)
- S (lower right)
- I (lower center)
- G (lower left)
- T (bottom left)
- D (bottom center, foot of the vertical from H)

that is, if both the angle TIM and the angle TIQ stand — in the semicircle described on I — on an arc of $\frac{1}{2}^\circ$, or 1° , or $1\frac{1}{2}^\circ$, or 2° , or 10° , or 50° , and so on, so that the whole angle MIQ stands on an arc of 1° , or 2° , or 3° , or 4° , or 20° , or 100° , and so on: then the angles MHQ, KEO will likewise stand on the arcs MQ, KO, of the same number of degrees, in their respective circles, because these are similar to those, by the scholium to proposition 22 of Euclid book 3. From this it follows that an arc of any number of degrees in any oblique great circle whatever can be projected into a similar arc — of the same number of degrees, of course — namely the one that corresponds to arc MQ. For that arc in the sphere will be equal to arc KO, which we showed to be similar to arc MQ, however many degrees may finally have been assumed.

For since, by lemma 23, the planes drawn through the southern pole and the straight lines IK, IO cut off from the horizon of the sphere an arc equal to arc KO — and arc KO has been shown similar to arc MQ of the horizon in the astrolabe — that arc of the horizon in the sphere (which is indeed projected into arc MQ by those two planes drawn through the straight lines IK, IO and the southern pole) will also be similar to that same arc MQ. And in the same way, any other two straight lines issuing from I and making an angle either greater or less than the angle MIQ (which is bisected by the straight line IT) will cut off similar arcs from the oblique circle and from the Equator. Never, however, will two or more arcs be found in the oblique circle, one wholly outside the other, that are similar to two or more arcs in the Equator, one of which is likewise wholly outside the other; rather, only several can be similar to several, individually to individuals, when one is included within another — because the straight lines cutting off similar arcs must make equal angles with IT on either side, as has been said. Never, therefore, can two or more equal arcs of the oblique circle in the sphere be projected into two or more equal arcs in the astrolabe: all of which are demonstrated in lemma 34.

[Margin: The various properties of the oblique great circles in the astrolabe. A circle described in the astrolabe through two points diametrically opposite is the greatest. — (a) Theodosius II.8 [?]]

14. But it is worth while here, for the sake of fuller instruction, to demonstrate some further matters — neither unpleasant nor useless — which pertain to the oblique great circles projected into the astrolabe. First, then, through I, Y, the poles of the oblique circle AFCG, let a circle AICY be described about the diameter IY. This circle will be the greatest, since it is described through the points I, Y, which are diametrically opposite in the sphere; and it will represent in the sphere the circle that is drawn through the poles of the oblique circle represented by AFCG and stands perpendicular to it — like the prime vertical with respect to the horizon — as is clear from what has been said in this proposition. For if the points I, Y are diametrically opposite, the two parallels of the Equator described from E through I and Y will be equal and opposite, and will touch the circle AICY at I and Y, by the scholium to proposition 13 of Euclid book 3. Since, therefore, the greatest circle in the sphere touches two opposite and equal parallels (a), the greatest circle AICY will represent it, and will pass through the points A, C, as we have demonstrated. And when a diameter PS is drawn through H, the center of the oblique circle, perpendicular to FG, then the three points A, I, P, as well as the three points C, I, S, will lie in one straight line — that is, a straight line drawn through any two of them will pass also through the remaining one. The same is to be said both of the three points P, C, Y, and of the three points S, A, Y.

Let ZA be the diameter of the oblique circle in the sphere; the visual rays AZ, Aa drawn through its extremities Z, a cut off its apparent diameter FG. Likewise, let the diameter Lb cut the diameter ZA at right angles, so that L, b are the poles of the circle of diameter ZA; and accordingly the visual rays AL, Ab fall on the poles I, Y, and cut off the apparent diameter IY of the circle of diameter Lb. Since, therefore, by lemma 10, the straight lines AL, Aa cut off similar arcs from the circles ABCD, AFCG: and since the arc La cut off is a quadrant — from the scholium to proposition 27 of Euclid book 3, because of the right angle LEa — then AL produced will likewise cut off from the circle AFCG an arc that is a quadrant. Since, therefore, arc PG is likewise a quadrant, from the same scholium, because of the right angle PHG, AIL will pass through the point P, so that it can cut off the quadrant GP.

And since the two sides EI, EC are equal to the two sides EI, EA, and contain equal right angles, the angles ICE, IAE will also be equal (a) [Euclid I.4]; and accordingly (b) [Euclid III.26] the arc on which the angle ICE stands, in the circle AFCG — namely arc CP — will be equal to the arc on which the angle IAE stands in the same circle. Since, then, by the scholium to proposition 27 of Euclid book 3, the arcs CP, AS, lying between the parallels AC, PS, are equal, the straight line CI produced will fall on the point S, so that it may cut off an arc equal to arc CP. Thus

the three points A, I, P, as well as the three points C, I, S, lie in a straight line. Again, when the straight lines CP, CY are joined: since (c) [Euclid III.31] the angles PCS, YCS, being in the semicircles PCS, ICY, are right, the straight lines CP, CY will be joined in one continuous straight line (d) [Euclid I.14]; and the same is to be said of the straight lines AS, AY. Therefore the three points P, C, Y, as well as the three points S, A, Y, lie in a straight line. From this it comes about that the ray Ab, in order to find the other pole Y, can be drawn through the three points S, A, b; since both the straight line SA and the straight line PC, when produced, fall on the pole Y, as we have shown.

It is also worth observing that the quadrant of any oblique great circle in the astrolabe — the southern quadrant, which its meridian line and the diameter perpendicular to that same meridian line enclose — is equal, as far as the number of degrees is concerned, to the arc of the altitude of the world's pole above that circle in the sphere. And the arc of the same circle lying between the diameter perpendicular to its own meridian line and its intersection with the Equator is not only equal — as regards the number of degrees — to the complement of the altitude of the world's pole above that circle in the sphere, but is entirely similar to it as well. For the quadrant FP contains as many degrees as are contained in arc BL, as is established from what was demonstrated in Num. 17 of this proposition 5, since the straight line AIL falls on P, as was demonstrated. Now it is clear that arc BL is equal to arc AZ — the arc of the altitude of the pole above the great circle which circle AFCG represents, and whose true diameter is aZ — on account of the equal quadrants LZ, BA and the common arc BZ. From this it follows that the remaining arc LC is equal to the complement of the altitude of the pole (which arc PC represents), as is clear from Num. 17 of this same proposition; and therefore that arcs PC, LC are equal as far as the number of degrees is concerned. That they are also similar is manifest from lemma 10, where it was demonstrated that the straight lines AP, AC cut off similar arcs PC, VC. This is likewise established from lemma 34. For since the angles ICA, IAC are equal (e) [Euclid I.5], and the angle ICE is equal to its alternate angle CIT, and IAE to the external angle PIT, the angles CIT, PIT will also be equal (f) [Euclid I.29]; and therefore the arcs PC, LC are similar, as was demonstrated in the said lemma 34.

15. Next, since — in the latter part of the first method of dividing the oblique great circle AFCG into degrees — any straight line drawn out from Y cuts off from the oblique circle an arc (between F and that line) corresponding to as many degrees as are contained in the arc of the Equator (between D and that same line) enclosed by it: it comes about that a straight line issuing from Y and touching one

of the circles touches the other as well, so that the arc between F and the point of contact corresponds to the arc between D and the point of contact enclosed in the Equator. This, however, we shall demonstrate geometrically, and at the same time we shall find the points of contact, in the following way. When the straight line EY has been bisected, let a semicircle be described from the point of division through E and Y, cutting the Equator in d. I say that the straight line Yd touches the Equator in d, and that the same line produced touches the oblique circle in T — the point on which falls the straight line IT drawn from I, the pole of the oblique circle, perpendicular to FG. [Margin: By which straight line the Equator and the oblique great circle in the astrolabe are touched, and where. A straight line drawn from the lower pole of the oblique great circle, if it touches the Equator, will touch the oblique circle as well; and if it touches the oblique circle, it will touch the Equator as well.] For when the straight line Ed is joined, the angle EdY, being in the semicircle EdY, will be right (g) [Euclid III.31]; and therefore, by the corollary to proposition 16 of Euclid book 3, the straight line Yd, perpendicular to the semidiameter dE, will touch the Equator in d.

16. Now, in order to demonstrate that the same line produced touches the oblique circle in T, it must first be shown that the perpendicular IT cuts off from the Equator an arc eB similar to the arc TG of the oblique circle, and that any other straight line drawn out from the pole I — such as Ig — cuts off an arc fB dissimilar to arc gG. Each of these we shall establish thus. When the straight lines Ee, HT are joined: the triangles PHI, AEI are equiangular, since the angles at H, E are right (h) [Euclid I.13], and the vertical angles at I are equal — for the straight line AI produced falls on P, as we have demonstrated — (i) [Euclid I.29] as well as the alternate angles P, A; therefore (k) [Euclid VI.4] it will be as PH — that is, as TH — is to HI, so is AE — that is, eE — to EI. Since, then, in the triangles THI, eEI the right angles at I are equal, and the sides about those angles are proportional, as we have shown, and each of the remaining angles T, e is less than a right angle — (l) [Euclid III.31] because the straight lines FT, GY, Be, De make right angles in the semicircles of which they are parts — the triangles THI, eEI will be equiangular (m) [Euclid VI.7], and will have equal angles THI, eEI at the centers H, E; and therefore, from the scholium to proposition 22 of Euclid book 3, the arcs eB, TG will be similar. This is the first point.

That any other straight line Ig cuts off dissimilar arcs fB, gG we shall conclude as follows. If Ig falls above the perpendicular IT, arc fB will be less than eB, and hence too small to be similar to arc TG, since arc eB was shown similar to TG; and much less, then, will arc fB be similar to arc gG, since this arc is greater than TG. But if Ig falls below the perpendicular IT, arc fB will be greater than eB, and

hence too great to be similar to arc TG, to which eB was shown similar; and much more, then, will arc fB be too great to be similar to arc gG, which is less than TG. And accordingly the perpendicular IT alone cuts off similar arcs Be, TG.

17. These things demonstrated, we shall easily show that the straight line Yd produced touches the oblique circle in T. For, when the straight line HT has been drawn parallel to Ed, we shall prove that the straight line Yd produced touches the oblique circle in T, and that the perpendicular drawn out from I to FG falls on T, the point of contact, and accordingly that the same line Yd produced touches the oblique circle in T, the extreme point of the perpendicular IT (n) [Euclid I.28]. For since PH, CE are parallel, because of the right angles at H, E, and the straight line YC produced falls on P, as we have shown, the triangles YHP, YEC will be equiangular by the corollary to proposition 4 of Euclid book 6 (o) [Euclid VI.4]. Therefore it will be as YH is to HP, so YE to EC; and, alternately, as YH is to YE, so HP — that is, HT — to EC — that is, to Ed. Since, then, HT, Ed are parallel, the straight line Yd produced will pass through T, by the scholium to proposition 4 of Euclid book 6. And since the angle YdE, in the semicircle, is right (p) [Euclid III.31], the angle YTH — the external angle being equal to the internal — will also be right (q) [Euclid I.29]; and accordingly YT will touch the circle AFCG in T.

Moreover, when the straight line IT is joined, cutting the Equator in e: since the point T is also found by means of a straight line drawn out from the other pole Y, which cuts off from the Equator an arc, beginning at D, equal to arc Be (as is plain from the first method of dividing the oblique circle into degrees), the arc Dd will be equal to arc Be. For each of the straight lines Ie, Yd cuts off the same arc FT, of as many degrees as are contained in arc Be, or Dd. Now arc Dd is similar to arc TG, by the scholium to proposition 22 of Euclid book 3, on account of the angles DEd, GHT at the centers (r) [Euclid I.29], which are equal, being external and internal in the parallels Ed, HT. Therefore arc Be too will be similar to that same arc TG. Since, then, the perpendicular alone, drawn out from I to FG, cuts off an arc aB (beginning at a) similar to the arc aG (beginning at a), as was demonstrated, IT will be perpendicular to FG; and therefore the straight line Yd produced touches the oblique circle in the point T, on which the perpendicular raised from I to FG falls. Which was what was proposed.

[Margin: (Which arcs, similar to arcs of the Equator and of the oblique great circle, are cut off by straight lines drawn out from the poles of that same oblique circle.) — (a) Euclid VI.4; (b) Euclid I.21; (c) Euclid VI.7; (d) Euclid VI.4; (e) Euclid VI.31 [as printed; context: III.31]; (f) Euclid VI.9]

18. Third: when a straight line YM is drawn out from Y in any way whatever, cutting the Equator in V, N — it has happened by chance that the point V coincides in the figure with the point L — and cutting the oblique circle in M, Q; and when the straight lines IM, IQ are drawn, cutting the Equator in K, O: then the arcs VCM, MCQ will be [equal(?)]; likewise BV, FM and GQ, DN will be similar; likewise the arcs VCN, KCO will be equal; and finally the angles MIF, OID will also be equal.

For when the straight lines HM, HQ and EV, EN are joined: since (a) [Euclid VI.4] it is as YH is to HP, so YE to EC, and HQ is equal to HP, and EN to EC, it will also be as YH is to HQ, so YE to EN; and the sides about the angles H, E will be proportional. Therefore the triangles YHQ, YEN have the angle Y in common. Since, then, each of the remaining angles Q, N is greater than a right angle — (b) [Euclid I.21] for the angle HQY is greater than the right angle HTY, just as the angle at N is greater than the right angle ENY [the trailing "EdY" here appears to be a compositor's stray; (?)] — the triangles YHQ, YEN will be equiangular (c) [Euclid VI.7], and will have equal angles at H, E. Therefore, from the scholium to proposition 22 of Euclid book 3, the arcs GQ, DN are similar.

In the same way (d) [Euclid VI.4], since it is as YH is to HP — that is, to HM — so YE to EC — that is, to EV — the triangles YHM, YEV will have the angle Y common, and the sides about the angles H, E proportional. Since, then, each of the remaining angles M, V is less than a right angle — because, since both stand on the circumferences on the semidiameters HQ, EN only, they are acute (e) [Euclid VI.31, as printed; context suggests III.31]; for they would be right, if straight lines were drawn from M, V to the extreme points of the semidiameters QH, NE produced — the triangles YHM, YEV will be equiangular (f) [Euclid VI.9], and will have equal angles YHM, YEV; and accordingly, from the two right angles, the remaining angles FHM, BEV will be equal. Therefore, from the scholium to proposition 22 of Euclid book 3, the arcs FM, BV are similar; and accordingly, from the same scholium, or from lemma 6, and from the semicircles, the remaining arcs VD, MG will be similar. But DN, GQ were also similar. Therefore, from lemma 6, the remaining arcs VN, MQ will likewise be similar.

It is established, then, that the straight line YM cuts off similar arcs on every side — namely both the upper ones FM, BV and the lower ones GQ, DN, and both those placed to the left, MQ, VN, and those to the right, MAQ, VAN, that is, the remaining arcs out of the whole circles, once the similar arcs MQ, VN are removed. Next, since the same point M is found by means of the straight lines IK, YN, the arcs BK, DN will be equal, as is established from the first method of

dividing the oblique circle into degrees. Likewise, since the same point Q is found by means of the straight lines IO, YV, for the same reason the arcs DO, BV will be equal. Therefore the arcs BK, DO will together be equal to the two arcs DN, BV together; and accordingly, from the semicircles, the remaining arcs KO, VN will be equal. And since VN was similar to arc MQ, arc KO too will be similar to that same arc MQ. Therefore the straight lines IM, IQ, drawn through the points of the oblique circle at which it is cut by the straight line YM, cut off from the Equator an arc KO similar to arc MQ. From which, finally, it follows — by lemma 34 — that they cut off the angles MIT, OIT, and therefore, from the two right angles, that the remaining angles MIF, OID are equal. Which, without a lemma...

Diagram. A stereographic-projection construction for the astrolabe, set within a ruled square frame in the lower portion of the page. The construction is organized about a vertical diameter running from F at the top down through B, H, I, E, G, D to Y at the bottom. Two large circles dominate: the Equator circle (centered near E) and the great circle AICY drawn about the diameter IY through the poles I, Y; the oblique great circle AFCG is also present. Point A lies on the right at the level of E, with S above it on the right and Z near the top right. A dotted line runs from F down toward the right. Numerous straight rays (several dotted) are drawn from I and from E to circumference points, and two large crescent/lens-shaped arcs sweep from the upper left down to the apex Y, representing the projected oblique circles/horizons.

Readable labels:

- F (top of vertical diameter): apex point
- M (upper left, on a large arc)
- K (below M)
- B (right, above H): on/near the inner circle
- H (below B): center of the oblique circle
- P (left)
- L (below K, small)
- V (below L)
- I (center): pole / center of circle AICY
- E (center): center of the Equator
- S (upper right, on the Equator circle)
- A (right, at level of E)
- X (center-left)
- Q (center, below X)

- N (lower center-left)
- G (right of center, low, on vertical diameter)
- D (low, on vertical diameter)
- Y (bottom apex)
- Z (top right, near S)
- lowercase cluster near left-center: s [?], t [?], o [?], d [?], e [?], f [?] (auxiliary projection points, dotted)
- numeral "6" [?] near the right, between A/G

Diagram. The same reference construction for Propositio V, printed again as a distinct woodcut in the lower portion of this verso so that it faces the running demonstration. As on p. 514, it shows the vertical diameter F–B–H–I–E–G–D–Y, the Equator circle and the great circle AICY about diameter IY, the oblique circle AFCG, the points S, A, Z on the right, and the two lens-shaped projected arcs converging on Y, with many straight and dotted rays from I and E.

Readable labels (confirmed on this printing): F, M, K, B, H, P, L, V, I, E, S, A, X, Q, N, G, D, Y, Z; plus the lowercase cluster s [?], t [?], o [?], d [?], e [?], f [?] near the left-center, and a numeral "6" [?] near the right.

ASTROLABII LIBER SECVNDVS — Propositio V (Scholium, §§18–21) and Problema III / Propositio VI (§§1–4)

Propositio V, Scholium (continued)

18 (continued). ...it can likewise be shown by lemma 34. For since PH is to HI as AE is to EI — because the triangles PHI and AEI are equiangular [Margin a: *Elements* VI.4] — MH too will be to HI as OE is to EI. And since the angles MHI and OEI, contained by these sides, are equal (because the remaining angles MHF and OED are also equal, being what is left from two right angles, by the scholium to Proposition 22 of Book 3 of Euclid, on account of the arcs FM and DO, which are similar; for since FM and BV were shown to be similar, DO too will be equal to BV and similar to the same FM), the triangles MHI and OEI will be equiangular [Margin b: *Elements* VI.6], and will have equal angles MIF and OID — which was proposed.

Here too it seems worth noting in passing that the lines KO and VN cut one another on the diameter AC of the Equator at the point X; that is, that the diameter AC passes through their intersection X. For, the line CV having been drawn: since the arcs BK, DN are equal to the arcs BV, DO, as was said, the remaining arcs CK, CN will also be equal, as will the remaining CV, CO; and consequently both the angles COK and CVN, standing on the equal arcs CK, CN, and the angles ACO, ACV, standing on the equal arcs AO, AV [Margin c: *Elements* III.27], will be equal. (For if to the equal arcs DO, BV the equal quadrants AD, AB are added, the whole arcs AO, AV become equal.) They are therefore equal to one another as well. And so, since in the triangles COX, CVX — which are cut off by the line AC, although it is not yet established that it passes through the same point X — the two angles COX, OCX are equal to the two angles CVX, VCX [Margin d: *Elements* III.29], and the adjacent sides CO, CV are likewise equal on account of the equal arcs CO, CV, the sides CX, CX will also be equal [Margin e: *Elements* I.26] — that is, the segments of the line AC between C and the lines KO, VN. Therefore AC passes through X. For if it cut the lines KO, VN in two points, one segment would be greater than the other, because one point would be nearer to point C than the other.

Finally, from what has been said it will also be permissible to infer that, if at the pole I of the oblique circle two equal angles MIF, OID are constructed, the line drawn through the points M, Q — where the lines IM, IO cut the oblique circle — falls, when extended, on the other pole Y; that is, the three points M, Q, Y lie in one straight line. For if the line MY, once drawn, be said not to pass through point Q but to cut the oblique circle at another point, then the line drawn from this point to I, together with ID, will make an angle equal to angle MIF, as we demonstrated a little earlier, and consequently equal to angle OID as well; and thus the part and the whole would be equal, which is absurd. Therefore the line MY passes through point Q — which was proposed.

And let this suffice concerning the various properties of the oblique great circles; now let us return to our purpose.

19. Since in the scholium to Proposition 4, Num. 1 and 2, we described the Equator in the plane of the astrolabe from a given tropic — of Capricorn [♑] or of Cancer [♋] — let us now also show here in what way, from any given oblique great circle that is perpendicular to the meridian (such as the horizon and the prime vertical; the ecliptic, when the beginning of Cancer [♋] is placed on the meridian; and finally every great circle drawn through the poles of the meridian, that is, through the common sections of the Equator and the horizon), and whose

inclination to the Equator is known, the Equator may be drawn in the plane of the astrolabe. For not rarely this affords great convenience, since any oblique circle in the astrolabe is larger than the Equator, as we demonstrated above at Num. 2, and a smaller circle is drawn more accurately from a larger than a larger from a smaller.

Let there therefore be given, in the plane of the astrolabe, an oblique great circle AFCG, perpendicular to the meridian, whose inclination to the Equator contains 30 degrees — that is, such that the altitude of the north pole above that circle, or the complement of its inclination to the Equator, comprises 60 degrees — and let it be required to draw the Equator in the same plane. Let the diameter FG of the circle be drawn through its center K, and from point G let the complement of the inclination (or the altitude of the pole) be counted off in both directions, to A and to C; and let the line AC be drawn, which will be bisected at E — because, by the scholium to Proposition 27 of Book 3 of Euclid, the diameter FG bisects the arc AGC [and so bisects the chord AC at E]. And finally, from E, through A and C, let the circle ABCD be described. I say that this is the Equator.

For, the line AG cutting the circle ABCD at H, the arcs CG, CH will be similar by lemma 10. Since therefore CG measures the altitude of the pole above the given oblique great circle, the arc CH will measure the same. Therefore, a line being drawn from H through the center E [to I], HI will be the diameter of the great circle whose complement of inclination, or altitude of the pole, is CH. And since, the line AI being drawn, the angle HAI is right, being in a semicircle [Margin f: *Elements* III.31], that line, when produced, will fall on point F. For if it fell short of F or beyond it, the line FA, drawn, would make in the semicircle FAG another right angle FAG equal to the former, and thus the part and the whole would be equal — which is absurd.

And so, if ABCD is laid down as the Equator, a circle of the given inclination AFCG will be described; and, the visual rays AH, AI being drawn through the extreme points of its diameter, they will cut off the apparent diameter FG, as is evident from what was demonstrated in this proposition at Num. 2. For HI is the diameter of that circle in the sphere, since the arcs CH, AI measure the altitude of the pole above it, as we said. Conversely, therefore, AFCG being laid down as the oblique circle, ABCD [will be] the Equator, since from this Equator that circle is described, just as we demonstrated — provided the greater part of the given oblique circle must incline toward the lower part, as happens in the prime vertical. [In that case] the complement of its inclination to the Equator, or the altitude of the pole, must be counted off from F in both directions, and so on. For its diameter must

fall between B and C, as is clear from what we described in this proposition at Num. 9, when we declared toward which part the diameter of any oblique circle — provided it be nevertheless perpendicular to the meridian — is to be drawn. By this same method, from any other great circle that is not perpendicular to the meridian, we shall describe the Equator in the astrolabe, as [we shall do] in Proposition 8, Num. 23.

Diagram. Within a square frame, a large circle (the oblique great circle AF CG) and, inscribed within it and offset upward, a smaller circle (the Equator ABCD) centered at E. The vertical diameter runs from F (top) down through B, K (center of the larger circle), E, G, to D (bottom); the horizontal diameter runs from C (left) through E to A (right). Straight lines join F–A and A–I; a dotted line runs from A toward the lower region (toward D). [Margin, beside figure: "To describe the Equator in the astrolabe from a given oblique great circle that is perpendicular to the meridian and has its inclination to the Equator known."]

- F (top): pole/extremity of the oblique circle's vertical diameter
- B (upper center, on the Equator): point of the Equator
- K (center of the larger circle)
- I (upper right, on the Equator)
- C (left): pole of the oblique circle / end of horizontal diameter
- E (center of the Equator ABCD)
- A (right): pole of the oblique circle / south pole of projection
- G (lower center): intersection on the vertical diameter
- H (lower left, on the Equator)
- D (bottom): extremity of the vertical diameter

20. It is established from these things that, if at any point A of the circumference of the Equator a right angle FAG is constructed, from which a line AC is drawn through the center E, and to this line, at the same center E, a perpendicular FG is raised cutting the lines AF, AG (which form the right angle) at F and G — then the points F, G represent two points in the sphere opposite by a diameter; that is, the intercepted line FG is the diameter of the greatest circle. For since, by the scholium to Proposition 31 of Book 3 of Euclid, IAH is a semicircle, the rays AI, AH, drawn through the extremities of the diameter HI, will cut off the apparent diameter FG of the greatest circle, whose diameter is HI, by what was demonstrated at Num. 2 of this proposition; and consequently the points F, G are opposite by a diameter in the greatest circle described about the apparent

diameter FG, since the points I, H represent [points] opposite by a diameter.
[Margin: Which points in the astrolabe are set opposite by a diameter.]

21. Finally, given any oblique great circle described in the astrolabe which is nevertheless perpendicular to the meridian — that is, through whose points A, C the line [AC] passes — we shall come to know its inclination to the Equator, the altitude of the pole above it, and its situation in the sphere, in this way. [Margin: To come to know, of any oblique great circle that is perpendicular to the meridian, its inclination to the Equator and its situation in the sphere.]

From A, the south pole, through G — the point where the oblique circle AFCG intersects the meridian line BD nearer to the center E of the astrolabe — let a line AG be drawn, cutting the Equator at H. For CH will be the arc of the altitude of the pole, and its complement DH the inclination to the Equator; because the line AH falls at H, the extremity of the diameter of the oblique circle, since the radius AH indicates the extremity G of the apparent diameter, as is clear from what has been said.

The other reasoning of this operation is this. Since the arc of a great circle drawn in the sphere through the poles of the world and the poles of the oblique great circle — the arc lying between the pole of the world and the oblique circle — measures the altitude of the pole above that oblique circle, whereas the arc intercepted between the same oblique circle and the Equator measures its inclination to the Equator, it comes about that, when the line BD represents that great circle (as was shown in Proposition 1, Num. 1), the portion EG, set between E (the pole of the world) and the oblique circle, represents the arc of the altitude of the pole, while the portion GD, between the same oblique circle and the Equator, expresses the arc of the inclination of that same oblique circle to the Equator. Wherefore, since the portion EG represents the arc CH, and the portion GD the arc HD (as we showed in Proposition 1, Num. 6), CH will be the arc of the altitude of the pole, but HD the arc of the inclination to the Equator.

But if the point G, being nearer to the center of the astrolabe, falls below the line AC, the great circle in the sphere which AFCG represents will cut the meridian between A (the south pole) and B (the point of the Equator) in the upper hemisphere; whereas if the point G were above the line AC, the oblique circle would cut the meridian between C (the north pole) and B (the point of the Equator) in the same hemisphere. And this same reasoning fits also any oblique great circle that is not perpendicular to the meridian, as we shall tell in Proposition 8, Num. 22.

Diagram. The same construction as on the preceding page, re-cut within a square frame: the large oblique great circle AFCG and, inscribed within it and offset upward, the Equator ABCD centered at E. The vertical diameter runs F (top)–B–K–E–G–D (bottom); the horizontal diameter runs C (left)–E–A (right). The chords F–A and A–I are drawn as straight lines, with a dotted line from A toward the lower right/bottom.

- F (top); B (upper center); K (center of larger circle); I (upper right); C (left); E (center); A (right); G (lower center); H (lower left); D (bottom)

PROBLEMA III. PROPOS. VI. [p. 166]

To distribute into degrees — that is, into unequal parts which correspond to the equal degrees they subtend in the sphere — the parallels of any oblique horizon, of its prime vertical, of the ecliptic, and of any other oblique great circle (whether it be perpendicular to the meridian, with its inclination to the Equator known, or not perpendicular) once it has nevertheless been drawn on the astrolabe.

1. In the first place we shall treat of the parallels of those oblique great circles that are perpendicular to the meridian — although the same method holds in those that are not perpendicular to the meridian, as we shall tell at Num. 25. If, therefore, in the Analemma described at the beginning of Proposition 4, straight lines parallel to the diameters of these circles are drawn through each degree of the circle of the Analemma, these will be the diameters of the parallels drawn through each degree. Hence, if from the south pole A visual rays are emitted through the extreme points of these diameters, there will be cut off — on the line [BD], at X — the apparent, or visible, diameters of the parallels; and if [these are set] in the order and position which they have in the Analemma, and circles are described about them from their midpoints, the parallels of the circle of the horizon, and of the other great circles which we named in the proposition, will have been described. [Margin: To describe from the Analemma, in the astrolabe, the parallels of the horizon and of any other oblique great circle that is perpendicular to the meridian.]

2. We shall describe these same parallels most conveniently in the astrolabe, even if an Analemma has not been separately constructed, if — the diameters of the said great circles having been found in the Equator of the astrolabe, as was taught in the preceding proposition — straight lines parallel [to them] are drawn through each degree of the Equator. For these will again be the diameters of the parallels. Wherefore, if visual rays are emitted through their extreme points from A, the south pole, there will be cut off from them, on the meridian line BD

extended in both directions, the greatest apparent diameters of the parallels, as was shown in the scholium to Proposition 3 — since the meridian, in whose common section with the Equator they appear, is perpendicular to these parallels. If, therefore, from the midpoints of the apparent diameters, circles are described about those same midpoints, the aforesaid parallels will have been described in the astrolabe. [Margin: To describe in the astrolabe, by means of the Equator, the parallels of the horizon and of any other oblique great circle perpendicular to the meridian, even if an Analemma has not been separately constructed.]

That this may be made plainer, let there be, by way of example, in the astrolabe: the Equator ABCD; the center E; the diameter of the horizon HI; [the diameter] of the prime vertical KL; the horizon AFCG; the prime vertical A i C k; the center of the horizon O; the vertical P; the upper pole of the horizon — that is, the vertex of the head, or Zenith — the point i; the lower pole, or Nadir, the point k. If, then, the parallels of the horizon, which the Arabs call almucantars [Almucantarath], are to be described, the Equator must be divided — beginning from the diameter HI of the horizon — into 360 degrees, if all the parallels of the horizon, passing (namely) through each single degree of the prime vertical, are desired. To avoid confusion, we have been content with a division into 12 equal parts, so that each comprises 30 degrees.

Then any two points equally distant from the points H and I are to be joined by straight lines, which by the scholium to Proposition 27 of Book 3 of Euclid will be parallel to HI — of which kind are the lines ST, VX, YZ, ab — and which consequently will be the diameters of the parallels of the horizon drawn through every 30 degrees; that is, the common sections of the meridian (for which the circle ABCD is now taken) and of the parallels of the horizon [Margin a: 16. unde [?]], since all these sections are parallel to one another, being made (namely) by the plane of the meridian in parallel planes. Therefore, if from A, the south pole, rays are emitted through S and T, the parallel ST will cut off the apparent diameter cd, which, being bisected at e, a circle is described from e through c, d, which will represent the parallel of the horizon whose diameter is ST. By like reasoning the rays AV, AX will cut off the apparent diameter fg of the parallel of the horizon whose diameter in the sphere is VX. Thus the extremity Z of the diameter YZ will appear, by the ray AZ, at the point o, while the other extremity Y will be seen by the ray...

2 (continued). ...by the ray AY, at the meeting of this ray with the meridian line DBF, which occurs at a point so far distant that it cannot be marked on the plane.

Therefore, in order that the portion of that parallel passing through o may be described, its center must be found, even if the other extremity is not available — as we shall teach a little below at Num. 9. And all these parallels, whose diameters the line AK — drawn in the Equator of the astrolabe from the south pole A to the pole of the horizon K — cuts (that is, those which in the sphere intersect between the south pole and the zenith of the meridian), have their centers in the astrolabe above the zenith i, toward F, and are described about i, the zenith, or the upper pole of the horizon. [Margin: The parallels of the horizon which in the sphere intersect between the south pole and the zenith of the meridian are described in the astrolabe about the zenith.]

3. But the parallel of the horizon whose diameter passes through the south pole A — such as the line Ab o [?], perpendicular to the axis KL of the horizon, falling at P, the center of the vertical, as was demonstrated above in Proposition 5, Num. 3 — is projected into the straight line PQ, perpendicular to BD at P. That it makes a straight line in the astrolabe is established from Proposition 1, Num. 1, since it is drawn through the south pole. That it makes the line PQ perpendicular to BD at P is proved thus. Since both the plane of the Equator (or of the astrolabe) and the plane of the parallel of the diameter AP are perpendicular to the meridian ([Margin a: 15. [?] Theod.] for the meridian, drawn through their poles, is perpendicular to each, and consequently, conversely, those planes will be perpendicular to the meridian), their common section too will be perpendicular to the same meridian [Margin b: 19. unde], and therefore, by Definition 3 of Book 11 of Euclid, also perpendicular to the line BD existing in the meridian, at the point P where the parallel meets the plane of the astrolabe. Therefore the perpendicular PQ will be that common section, representing the parallel of the horizon drawn through A, the south pole.

4. Finally, the other parallels — whose diameters the line AK, drawn in the Equator of the astrolabe from the south pole A to the pole of the horizon K, does not cut (that is, those which in the sphere intersect between the south pole and the Nadir of the meridian) — have their centers in the astrolabe below the Nadir k, and are described about that same Nadir k, in such a way that their circumferences curve downward, away from the line PQ; just as the circumferences of the former [parallels] tend upward from the same line PQ. Thus you see the ray Ab, through b (the extremity of the diameter ab), indicate one extreme point of that parallel, the visible [point ↓]; while the other extremity will be indicated by the ray Aa o, which through the other extreme... [Margin: The parallels of the horizon which in the sphere intersect between the south pole and the Nadir of the meridian are described in the astrolabe about the Nadir.]

Diagram. A large and very densely worked stereographic projection plate within a square frame — the master figure for the almucantars (parallels of the horizon) and the verticals/azimuths of Proposition VI. An outer dotted circle (the Equator) is graduated, with numeral divisions running around the rim (readable among them: 10, 3, 4, 7, 8, 9 down the left side; 60, 30, 90, 1, 2 in the interior; 12, 14, 10, 14 along the lower edge; 13, 13, 6 and zodiac/planet symbols along the top edge). A dense web of great and small circles (almucantars and verticals) and many straight lines radiate from and converge at the projection center A (lower-right of the interior, where a bundle of rays meets). Because of the extreme density of overlapping arcs, small-type Latin/Greek point-letters, and a brown ink/foxing stain over the lower-center, most of the minor labels cannot be resolved with confidence. (many labels illegible).

High-confidence labels (position within the figure):

- A (interior, right of center): projection center / south pole, where the rays converge
- F (upper right, on a circle); L (right of center); N (right rim); M (upper left interior); R (left interior); O (left-of-center interior); P (center-lower interior); T (center-lower interior); S (upper interior); K (lower left); E (center, implied)
- Doubled letters: AA (lower center), BB (center-right), mm / m (lower center), RR (left-center), pp (center-right)
- Greek/script minuscule labels scattered through the web (not all placeable): α , π , μ , ν , ξ , ψ , Γ , Δ , Σ , Φ , Θ ($\theta\theta$), λ , τ , Ω
- Lowercase Latin near the convergence: a, b, c, d, e, g, o, cc, aa
- [Numerous further small labels: ?]

the extremity a is drawn below Nadir k, at the meeting-point 14 (if it could be marked on the plane), so that the whole apparent diameter lies below the straight line PQ, between whose extremities Nadir k itself is found. But because this other extremity runs out too far, it is better to find the center of the parallel, which is the point 12 (which we shall shortly teach how to find, under Number 9), even though the other extremity of the apparent diameter is not available. Therefore the circle $\text{—}60$ [the point — is a small hook-shaped typographic label used, like the numerals and Greek letters, as a point-name], described from center 12 around Nadir k, will represent the parallel of diameter ab.

And by this same device all the parallels of the Horizon will be drawn — both those in the upper hemisphere above the Horizon, which are represented by the

ones drawn below the Horizon [on the instrument], and those that lie below the Horizon, which are of course represented by the ones marked outside the Horizon. Yet the use of the former is greater than that of the latter in astronomical matters. Hence it has come about that on astrolabes no parallel of it is usually drawn outside the Horizon, except the one that lies 18 degrees below the Horizon, which is called the twilight line [linea crepusculina], of which we shall treat in Proposition 10.

Diagram. A large square-framed stereographic-projection figure filling the lower two-thirds of the page (the master figure for PROPOS. VI): the astrolabe plate showing the Horizon, its parallels (almucantars), the vertical circles (azimuths), and the projected day-circles, with a dotted bounding/twilight circle running around the interior of the square frame. All rays issue from the southern pole. The interior is exceptionally crowded with intersecting solid and dotted lines and a very large number of tiny point-labels. Legible labels, by approximate position:

- Around the dotted peripheral arc (upper and left): 13, 8, 7, 6, 5, 4, 10, 9 (running numerals marking almucantar/twilight steps)
- Upper edge, inside the frame: u, r, F, E, B, N
- Left region: M, V, R, d, C
- Lower/central region: numerals 20, 60, 12, and at the foot "14 · 10 · 14"
- Interior letter-points (mostly on and about the central and right circles): c, O, S, A, Y, ξ, D, L, K, P, T, I, R
- Doubled-letter points: RR, BB, mm, pp, AA, θθ
- Greek and symbol points: α, ω, π, ϖ, θ, and a cross-symbol ⊕ and asterisk * used as point-names; a ligatured "ao"; "ñ" (n with tilde); "II" (=Π)

Many of the smallest interior labels cannot be resolved with certainty at this resolution (more than two are uncertain).**

One must also not pass over, at this point, [the fact] that — when some parallel of an oblique great circle intersects the Equator (which happens when its diameter cuts the meridian line within the Equator, as does the diameter ST) — the two points where the Equator intersects the parallel, and the point where the meridian line intersects the diameter of that same parallel, lie on one straight line: namely, on the common section of the plane of the Equator and the plane of the parallel in the sphere, which [common section] is perpendicular to the meridian line on the Astrolabe. [Margin: That the common section of the Equator and of an oblique parallel is perpendicular to the meridian line on the Astrolabe.] [Margin: a — Euclid XI.19.]

For since both the parallel of diameter ST, in its proper position, and the Equator are at right angles to the Meridian, (a) their common section too will be at right angles to the same Meridian, and therefore also perpendicular to the meridian line BD, by definition 3 of book 11 of Euclid. If, then, through the point where the diameter ST intersects the meridian line a perpendicular is drawn to that same meridian line, it will be the common section of the parallel and the Equator. Since therefore, viewed from the southern pole, the parallel is seen to pass through that common section, the apparent parallel drawn on the Astrolabe will necessarily cut the Equator at the extreme points of that common section; and consequently the two points where the Equator and the parallel meet, and the point where the diameter ST meets the meridian line, will lie on one straight line — that is, on the common section of the parallel and the Equator. By this reasoning you will find that the two intersections of the parallel c30d with the Equator, and the intersection of the diameter ST with the meridian line, lie on one straight line; which must also be said of the two intersections of the parallel BB[?]³⁰ with the Equator, and the intersection of the diameter YZ with the meridian line.

Now I call the *Meridian* of any oblique great circle and of its parallels the great circle drawn through the poles of the world and the poles of that great circle; and [I call] the *meridian line* the common section of the plane of the Astrolabe and of that great circle which passes through the poles of the world and of the oblique circle. [Margin: How the Meridian, and the meridian line, of any oblique circle is to be understood.]

One must also note that the oblique parallel passing through E, the center of the Astrolabe, is equal to the oblique parallel which in the sphere is drawn through the southern pole and is projected on the Astrolabe onto the straight line PQ — the former [reckoned] from the upper pole, the latter from the lower; since each is equally distant in the sphere from its own pole, the distance of each being measured by the arc of its own Meridian intercepted between the pole of the world and its own pole. And it is equal, moreover, to each of two [further parallels]: both to the parallel of the Equator through i, the pole of the oblique circle, and to the parallel of the Equator through k, the other pole of the described oblique circle; because each of these recedes in the sphere from the pole of the world by the arc intercepted between the pole of the world and the pole of the oblique circle — just as each of them is distant from its own pole by the same arc.

5. But just as in the sphere the primary vertical circle, drawn through the poles of the Horizon and of its parallels, cuts all the parallels, and the Horizon itself, into two halves, so also on the Astrolabe the same must

happen. [Margin: a — Theodosius I.15.] [Margin: That the semicircles and quadrants of the Horizon and of its parallels, [formed] by the primary Vertical and the Meridian, exist also on the Astrolabe, which...]

Accordingly, just as on the Horizon the arcs AFC, AGC represent its two semicircles — as we said above, in the scholium of the preceding proposition, Number 4 — so too in the parallels of the Horizon the arcs which the primary Vertical AiCk cuts off represent semicircles. Again, just as the Vertical and the Meridian divide those same parallels of the Horizon, and the Horizon itself, into quadrants in the sphere, so too on the Astrolabe the arcs of the Horizon and of its parallels, contained between the Vertical and the Meridian (the latter expressed by the straight line BD extended in both directions), represent their quadrants: such are the arcs of the Horizon AF, FC, CG, GA, and the arcs of the parallels c30, 30d; f60, 60g; ω30; −60, and so on. Indeed, the diameter of the primary Vertical, cutting the meridian line BD at right angles in P, exhibits the semicircle of the parallel whose diameter in the sphere is Abe, which we said is represented by the straight line PQ; while the semidiameters Pkk, Pmm represent the quadrants of that same parallel — the semicircle, I mean, and the quadrants of it that are farther from the southern pole A — since the rays of the northern quadrants fall on the straight line kkm, but those of the southern quadrants fall outside that same straight line.

6. By another method, and perhaps more accurately, we shall find on the meridian line BD, extended in both directions, the apparent diameters of the parallels of the Horizon, together with their centers — that is, the midpoints of the diameters — as follows. Let the Horizon AFCG be drawn, and through its center O let the diameter MN be drawn perpendicular to FG, and let the Horizon itself be divided into 360 degrees, beginning at the point F, or G (if all the parallels are wanted; but we, to avoid confusion, have divided it into 12 equal parts, each comprising 30 degrees). [Margin: To find, on the Astrolabe, the apparent diameters of the parallels of the Horizon, together with their centers, by the Horizon itself.] Finally, through every two points equally removed from the diameter FG, let hidden [dotted] straight lines be drawn, cutting the diameter MN at u, α, β, γ; these, by the scholium of proposition 27 of book 3 of Euclid, will all be parallel to FG and to one another, and — by the same scholium of proposition 29 of book 3 of Euclid — will all be bisected by the diameter MN.

For once this is done, rays sent out from A through the extreme points of any such [parallel line] will cut off on FG the apparent diameter of that parallel

[circle] which in the sphere is distant from the Horizon by as many degrees as the [line] itself is removed from the diameter FG; and the parallel itself will lie above the Horizon if the [line] inclines toward the point M, but below it if it tends toward the point N — so that the semicircle FCG belongs to the parallels above the Horizon, and the semicircle FAG to the parallels below the Horizon. And a straight line sent out from A through the point at which the diameter MN is cut by the [parallel line] will indicate on the straight line FG the center of that same parallel — that is, it will bisect its apparent diameter.

For example: since the parallel line lp is removed from the diameter FG toward M by 30 degrees, the rays Al, Ap will cut off the apparent diameter cd of the parallel that is 30 degrees away from the Horizon toward the Zenith; and the straight line Au will bisect the diameter cd at e, the center of the parallel c30d — which we shall demonstrate in the following manner. Since the straight lines AF, Al, by lemma 10, intercept similar arcs in the circles ABCD, AFCG, and AF passes through the point H (the extremity of the diameter of the Horizon), because it was through the ray AH that the point F, the extremity of the apparent diameter of the Horizon, was found, Al will pass through S, since the arcs Fl, HS are similar. Therefore, just as the ray AS exhibited the point c, so the same point c will be indicated by the ray Al. Again, since by the same lemma 10 the straight lines AG, Ap intercept similar arcs in the same circles, and the straight line AG passes through I, Ap will pass through T, since the arcs Gp, IT are similar. Therefore the point d will be found by the ray Ap, just as it was found by the ray AT. And since, by the scholium of proposition 4 of book 6 of Euclid, as lu is to up, so ce is to ed, and lu is equal to up, ce will also be equal to ed. Therefore e is the center of the parallel described about cd, found by the straight line Au.

By the same reasoning the rays Am, An will take off the apparent diameter fg, and the straight line Aα will bisect it; because, by the same lemma 10, both the straight lines AF, Am and the straight lines AG, An intercept similar arcs in the same circles. Since therefore the arc HV is similar, by construction, to the arc Fm, and the arc IX to the arc Gn, the straight line Am will pass through V, and An through X, and so on. So too the straight line Aβ will fall on the center of the parallel described through ω; since, by the same lemma [10], in the same circles the straight lines AF, At, etc. [intercept similar arcs]. Finally, the rays Af, Ar too will pass through the points [?a, b]; for the straight lines AN, Af, produced toward A, intercept, by the same lemma, [similar arcs,] on account of the equal vertical angles at A; and NA passes through L. For, as we showed in the scholium of the preceding proposition, Number 4, the four points N, A, L, k lie on one straight line. Therefore sA, produced, [will pass through a, since the arcs N?, La are]

similar. Again, the straight lines AN, Ar, produced toward A, cut off similar arcs by the same lemma 10; and since NA passes through L, as was said, and the arc Lb is similar to the arc Nr, rA, produced, will pass through b. The straight line Ay too, produced toward A, will fall on the point 12, which will be the center of the parallel [described about the apparent diameter $\neg 14$]. For, again, the straight line fr and the apparent diameter $\neg 14$ are cut proportionally at [γ , 12], since fr and $\neg 14$ are parallel; that is, $r\gamma$ is to γf as $\neg 12$ is to 12·14 (taking 14 for the concurrence of the straight lines BD, $A\alpha$) — which will be demonstrated in the same way in which the scholium of proposition 4 of book 6 of Euclid was proved. Since therefore fr is bisected at γ , $\neg 14$ too will be bisected at 12.

7. It happens, moreover, in both the methods set out, that the parallels drawn in the Equator and in the Horizon — those of the same order — intersect one another on the diameter AC, or on it produced. [Margin: The diameters of the parallels of the Horizon drawn in the Equator and the Horizon, where they intersect one another.] Thus you see the parallels ST, lp intersect one another at the point tt of the diameter AC; likewise the parallels VX, mn, produced, cut AC, produced, at one and the same point ææ; and the parallels YZ, tq at the point ss; and finally the parallels ab, sr, produced, meet at the same point gg of the straight line CA produced.

The reason for this is as follows. Since the straight line AO, falling from A (the southern pole) onto O (the center of the Horizon), is perpendicular to HI, the diameter of the Horizon (for if it be supposed not to be perpendicular, then were a perpendicular drawn from A, it would fall — as was demonstrated in the preceding proposition, Number 3 — onto the center of the Horizon, and thus the Horizon would have two centers, which is absurd), (a) the lines AO, KL will be parallel, (b) and therefore the external angle $cc\cdot E\cdot tt$ is equal to the internal angle $O\cdot A\cdot E$. [Margin: a — Euclid I.28.] [Margin: b — Euclid I.29.] [Margin: c — Euclid VI.4.] Since therefore the right angles $E\cdot cc\cdot tt$ and $A\cdot E\cdot O$ are also equal, the triangles AEO and $E\cdot cc\cdot tt$ will be equiangular. (c) Therefore it will be: as AE, the semidiameter of the Equator, is to AO, the semidiameter of the Horizon, so $cc\cdot E$ [is] to $E\cdot tt$ [?] — the sine of the arc HS to $E\cdot tt$. But by lemma 5 the semidiameters have the same proportion as the sines of similar arcs. Therefore $E\cdot tt$ will be the sine of an arc similar to the arc HS — that is, the sine of the arc Fl, which was shown to be similar to the arc HS; and consequently the straight line lp, cutting off from EC the sine of the arc Fl, will fall on the point tt, where the straight line ST cuts the straight line EC. The same demonstration holds in the other cases, since the triangle $E\cdot bb\cdot \alpha\alpha$ is equiangular with the triangle AEO; and likewise the

triangles $E\cdot oo\cdot ss$ and $E\cdot nn\cdot gg$ are equiangular with the same triangle AEO , on account of the alternate angles EAO , $nn\cdot EA$ being equal, and so on.

Diagram. The "separate figure" [figura seorsum] announced in the paragraph that follows below it: a second, square-framed stereographic-projection figure showing the Equator $ABCD$ (center E), the Horizon $AFCG$ (center H), and the parallels of the Horizon with their diameters, together with a dotted bounding circle. As in the first figure, all rays issue from the southern pole A , and the interior is densely filled with intersecting solid and dotted construction lines and small point-labels. Legible labels, by approximate position:

- Upper part of the inner circle: f , F , m , a , and Greek/small points u , v , Y
- Central region: M , C , π , o , and small letters c , δ , E , D
- Lower and right region (the projected apex where rays converge): A (the pole/apex), k , S , T
- Lower right, outside the inner circle on the frame: X , R (endpoints of two straight rays running to the frame corner)

Numerous interior labels are too small to resolve with certainty (more than two are uncertain).**

But since this second method of finding the diameters of the parallels of the Horizon is very convenient and easy, I am pleased to dwell on it a little longer, by demonstrating the various properties that follow from it. [Margin: That a circle described through the extreme points of the apparent diameter of any parallel of the Horizon, and through the southern pole, touches the Horizon at the southern pole.] And that this may be done more conveniently and without confusion of lines, we shall draw a separate figure, in which again let the Equator be $ABCD$, whose center is E , and the Horizon $AFCG$, whose center is H . The parallels of the Horizon, together with their diameters, [lie] on the Horizon itself, as above — except that here the arcs Fl , lm , mM , etc. are not equal, as they were there.

First, then, a circle [drawn] about three points — one of which is the southern pole A , from which all the rays issue, while the other two lie at the extremities of the apparent diameter of any given parallel — touches the Horizon at the southern pole A . Thus you see the circle Acd touch the Horizon at A . For since the apparent diameter cd is found by rays drawn out from A to the extremities of the straight line lp , which is parallel to FG , as was shown here under Number 6, in the triangle Alp the straight line cd will be parallel to the base lp . Therefore, by lemma 40, the circles $AFCG$ and Acd , described about the triangles Alp and Acd , will touch one another at A ; and I , the center of the circle Acd ,...

(Continuation of the Scholium to Proposition VI / Problema III of the Second Book of the Astrolabe: the construction of the parallels of the Horizon — almucantars — by stereographic projection. The opening sentence continues one begun on the preceding page.)

[the center of the circle] Acd will lie on the straight line AH drawn from A through the center of the Horizon. This is found by means of the straight line dI , which — drawn through d , the extremity of the visible diameter of the parallel — makes with the radius Ad an angle IdA equal to the angle IAd [Margin (a): Euclid I.6]; for then the straight lines IA and Id are equal, and consequently the circle described from I through A passes through d . And since through the two points A and d only one circle can be described that touches the circle $AFCG$ — the kind we have shown it to be — it also passes through c . For if through the points A , d another circle could be described touching the circle $AFCG$, it would likewise touch the circle Acd , since its center would lie on the straight line AH ; and that is absurd, since it would have to touch or cut Acd at the same point, or else touch it again at d .

By the same reasoning, if at c — the other extremity of the visible diameter of the parallel — an angle equal to the angle cAI is set up, the straight line making that angle will fall so as to have its center at I . The same holds for the parallels whose visible diameters lie below S , the center of the Vertical, and which are described about the other pole of the Horizon, k .

For let KL be the visible diameter which the rays AP , AQ present, drawn to the extremities of the line PQ , parallel to FG , and extended through A . I say that the circle described about the three points A , K , L likewise touches the Horizon at A . For since in the triangles APQ , ALK the sides PQ , LK are parallel, the circles $AFCG$ and AKL described about those triangles will touch one another at A by Lemma 40; and R , the center of the circle AKL , will be found on the straight line HA extended — either by the straight line LR , which makes the angle ALR equal to the angle LAR , or by the straight line KR , which makes the angle AKR equal to the angle KAR .

Finally, if from the poles of the Horizon i , k there are erected perpendiculars iV , kX to the straight line Fk , then V , X will also be the centers of the circles passing through i , k and touching the Horizon at A [Margin (b): Euclid I.28]. For the straight lines iV , kX will be parallel to MN itself, because of the right angles at H , i , k ; and therefore both the triangles AHM and AVi , and the triangles AHN , AXk , will be similar. Therefore [Margin (c): Euclid VI.4] it will be, as AH is to HM , so AV to Vi ; and as AH is to HN , so AX to Xk . Since, then, the semidiameters AH , HM , HN

are equal, the lines VA, Vi and XA, Xk will likewise be equal. Therefore the circles described from V, X through i, k will pass through the point A, and will touch the Horizon at it. Here too you see that the straight lines iV, kX, which make the angles ViA, XkA equal to the angles VAi, XAk, fall onto the centers V, X [Margin (d): Euclid I.5]; for both those two angles and these are equal.

[Marginal summary] [Margin: To cut off from the meridian line of the astrolabe a straight line that is the visible diameter of some parallel of the Horizon.]

From this it follows that, if the visible diameter of some parallel of the Horizon is required without determining its distance from the Horizon or from its pole, this can be done more quickly than said. From any point I taken on the straight line AH, with the aid of a compass, let two points c, d be cut off at the interval of the straight line IA. For cd will be the visible diameter of some parallel — namely, of the one whose distance from the Horizon the rays Ac, Ad determine at the points l, p. For since the circle described through A, c, d touches the Horizon at A, the straight lines cd, lp will be parallel by Lemma 9. Therefore, as was shown above at Number 6, the straight line cd will be the visible diameter of the parallel distant from the Horizon by the arc Fl or Gp. Likewise, if from the point a taken there are cut off, at the interval aA, two points b, q, then bq will be the visible diameter of the parallel whose distance from the Horizon is the arc Fr or Gs. Again, if from the point R taken there are cut off, at the interval RA, two points K, L, then KL will be the visible diameter of the parallel whose distance from the Horizon is the arc FP or GQ.

[Marginal summary] [Margin: Given one end of the visible diameter of any parallel of the Horizon, to find the other end by means of a circle that touches the Horizon, and to bisect the diameter thus found by a perpendicular line.]

From this, again, a very easy method is derived by which, from one given extremity of the visible diameter of any parallel of the Horizon, the other extremity may be brought out — a matter of great usefulness in investigating the points that run far above the center of the Horizon, since there the rays cut the meridian line very obliquely. Thus we shall proceed. Let the distance of a parallel below the Horizon be the given arc Fr or Gf, whose visible diameter is to be investigated. Drawing the radius Af, cutting the meridian line at q (and all these sections between i, the pole, and S, the center of the Vertical, are less oblique, and therefore more convenient), let the angle Aqa be made equal to the angle qAa, and let the straight line qa cut the straight line AH at a; and finally, from a, at the interval aA or aq, let a point b be taken on the meridian line, which I say is the other extremity of the visible diameter — the one, namely, upon which the ray Ar

falls; because the circle described through a, A, q, b touches the Horizon at A , and consequently, as we have shown, cuts off the diameter of the parallel of the Horizon. Since, then, q is one of the extremities, b will be the other.

But if the straight line qa should happen to cut the straight line AH too obliquely, we shall use this device. From any point of the straight line qa , which makes the angle aqA equal to the angle qAa , let us describe through A an arc of the circle $A\phi\downarrow$ [Margin: the symbol here transcribed “ $\downarrow$ ” is a distinct point-label whose exact typographic form is uncertain [?]], cutting the straight line aq produced at ϕ ; and to the arc ϕA let us take an equal arc $\phi\downarrow$. For if, when the straight line $A\downarrow$ is drawn, an angle $A\downarrow 1$ is made equal to the angle $HA\downarrow$, the straight line $\downarrow a$ will again fall onto a , and its section with AH will be less oblique. That $\downarrow 1$ falls onto a , where Aa and qa meet, is established. For, the straight line $a\downarrow$ being drawn, since the sides $\downarrow\omega, \omega 1$ are equal to the sides $A\omega, \omega 1$ and contain right angles at ω (for the straight line qa , passing through the center of the arc $a\phi\downarrow$ and cutting it in half at ϕ , also — by the scholium to Proposition 27 of Book III of Euclid [Margin (e): Euclid III.3] — bisects the straight line $A\downarrow$, and therefore at right angles [Margin (f): Euclid I.4]), the bases $aA, a\downarrow$, and the angles $aA\downarrow, a\downarrow A$, will be equal. Consequently the straight line making at $\downarrow$, with the straight line $A\downarrow$, an angle equal to the angle $HA\downarrow$, will fall onto a .

So too, if the extremity K of the diameter KL has been found by the ray QAK (which is more easily found than the other extremity L , on account of the section being too oblique), and an angle RKA is made equal to the angle RAK ; and then from R , where the straight line KR cuts the straight line HAR , at the interval RK the meridian line is cut at L : then L will be the other extremity. Once the other extremity has been found in this way, the perpendicular drawn to the meridian line from the point of the straight line AH out of which that found extremity was found will give the center of the parallel — that is, it will bisect the visible diameter.

Thus you see that the perpendicular Ie falls onto the center e of the parallel cd ; and the perpendicular at , onto the center t of the parallel bq ; and the perpendicular RT , onto the center T of the parallel KL . For since the straight lines RK, RL are equal (because from R , at the interval RK , the point L has been taken) [Margin (g): Euclid I.5], the angles K, L are equal; and the angles at T are posited as right angles. Therefore, since the sides RK, RL , opposite to those angles, are equal [Margin (h): Euclid I.26], the sides KT, LT will also be equal. And the same reasoning holds in the others, since both Id, Ic and aq, ab are equal, and so on.

[Marginal summary] [Margin: To find the visible diameters of the parallels of the Horizon by means of a circle that touches the Horizon at the south pole.]

But if the Horizon should sometimes be of such magnitude that, on account of the narrowness of the plane, the parallels lp , mn , and so on, can scarcely be drawn within it, we shall be able most conveniently to make use of any circle $A\gamma\beta\downarrow$ described from some point of the straight line AH through A , and therefore touching the Horizon at A . For if we draw the diameter βx , parallel to the diameter MN or AC , and cut it at right angles by another diameter $\gamma\downarrow$, then the arcs γc , $c\mu$; $\downarrow l$, $d\xi$; $\gamma\theta$, $\theta\pi$; $\delta\epsilon$, $\epsilon\pi$, and so on, are to be taken similar to the arcs of the Horizon Fl , lm ; Gp , pn ; Fr , rP ; Gf , fQ — that is, the circle $A\gamma\beta\downarrow$ is to be divided as the Horizon was divided; and the straight lines cd , $\mu\epsilon$, $\theta\epsilon$, $\pi\rho$, and so on, are to be drawn, since the rays $A\gamma$, Ac , $A\mu$, and so on, fall on F , l , m , and so on, because by Lemma 9 they intercept similar arcs $\gamma\epsilon$, Fl ; $c\mu$, lm , and so on. So that, just as on the Horizon, so in this circle the rays $A\mu$, $A\xi$ will give the apparent diameter of the parallel fg , and the ray $A\nu$ will fall onto the center h , and so on. And so, if the circle $A\gamma\beta\downarrow$ is divided into equal

parts (which has not been done in the figure), the very same parallels will be described that were described above at Number 6 through the Horizon.

[Marginal summary] [Margin: Straight lines drawn from the center of the Vertical to the intersections of the parallels of the Horizon with the Vertical touch the parallels at those same intersections.]

From these things we shall also easily demonstrate that the straight lines drawn from S , the center of the Vertical, to the intersections of the same Vertical with the parallels touch the parallels at those same intersections; such are See , Sc . For, the straight line SA being joined, it will touch the Horizon at A , as we showed at Proposition 5, Number 28. If, then, a circle Acd is described touching the Horizon at A and passing through c , d , the extreme points of the parallel's diameter, the same straight line SA will touch this circle at A , as was shown a little before. [Margin (a): Euclid III.36] Therefore the rectangle under cS , Sd will be equal to the square of the straight line SA , or of See (which is equal to SA itself); and consequently the straight line See will touch the parallel $cecd$ at ee , and so of the other parallels described about the Zenith i . Nor is the reasoning different for the parallels described about the Nadir k . [Margin (b): Euclid III.37] For when the circle AKL has been described, touching the Horizon at A and passing through K , L , the extreme points of the diameter of the parallel KL , SA will touch this circle at A , [Margin (c): Euclid III.18] since it is perpendicular to HAR . [Margin (d): Euclid III.36] Therefore the rectangle under LS , SK will be equal

to the square of the straight line SA — that is, to the square of the straight line drawn from S to the intersection of the Vertical with the parallel KL — [Margin (e): Euclid III.37] and consequently this straight line will touch the parallel at that same intersection. And the same reasoning holds of the other parallels described about the Nadir k.

Diagram. A large square-framed woodcut occupying the lower two-thirds of the page: the master construction figure for the stereographic astrolabe projection of the Horizon and its parallels (almucantars), together with the Vertical (azimuth) circles and hour-lines. Within the square frame is a large circle (the primitive Vertical / tympanum), with the point **A** at the right-center where a dense pencil of straight rays converges (the projection point / south pole of the Horizon). A second large circle at lower center and further circular arcs represent the Horizon and its parallels above and below; numerous straight lines (azimuths, hour-lines, diameters) and dotted construction arcs cross the field.

Legibly readable labels (many small peripheral point-labels cannot be resolved individually at this resolution):

- **A** (right-center): common point of tangency / convergence of the ray-pencil.
- **M** (left), **N** (upper), forming the diameter MN / AC referenced in the text.
- **S** (upper-center-right, near c–O): center of the Vertical.
- **O** (upper-center): center of the Horizon.
- **E, G** (center and lower-center), **I** (left of center-low), **K, L** (lower center-right), **P, T, R** (left of center).
- **c** (upper center, on a small circle), with **B, D** implied on the meridian.
- Doubled-letter labels: **AA** (lower-right, on the large lower arc), **BB** (right, mid), **mm** (right, below BB), **RR / KK** (left-center, "kR"), **PP** (right of A), **ll / pp** (near A, lower).
- Greek point-labels around the periphery and interior: **α, ω, θ, π, ρ, σ, ψ, φ, μ, ξ, δ**, and the doubled **γγ, θθ** (the auxiliary circle $A\gamma\beta\downarrow / \gamma\gamma R \theta\theta$ of the text).
- Graduated numerals along the circles: **2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 20, 60** (degree/division marks), and **10, 14, 10, 14** along the lower frame.
- Several cross-shaped and zodiacal sort-symbols (Aries-type) along the top frame serve as division marks.

[Numerous small interior/peripheral labels (individual almucantar–azimuth intersection letters) are below the resolution of the scan and are not individually committed here.]

[Marginal summary] [Margin: Given one end of the diameter of the Horizon or of its parallel, to find the other end by means of a third proportional — to the straight line between the given end and the center of the Vertical, and to the semidiameter of the Vertical.]

[Marginal summary] [Margin: The semidiameter of the Vertical is the mean proportional between the straight line that lies between the center of the Vertical and one of the two ends of the diameter of the Horizon (or of its parallel), and the straight line placed between the same center of the Vertical and the other end of the diameter of the Horizon or of its parallel.]

Moreover, from this it is again inferred that, if one of the extremities of the diameter of the Horizon or of its parallel has been found, and a third proportional is found to two straight lines — the first of which is between the center of the Vertical *S* and the extremity found, the second the semidiameter of the Vertical — then the extremity of this third proportional is the other extremity of the diameter. For since *SA* touches the Horizon, [Margin (f): Euclid III.36] the rectangle under *SG*, *SF* will be equal to the square of the straight line *SA*. [Margin (g): Euclid VI.16] Therefore it will be, as *SG* is to *SA*, so *SA* to *sF*. By the same reasoning, since *See* touches the parallel *cd* at *ee*, [Margin (h): Euclid III.36] its square will be equal to the rectangle under *Sd*, *Sc*. [Margin (i): Euclid VI.16] Therefore it will be, as *Sd* is to *See*, so *See* to *Sc*. Hence, the extremity *d* having been found, the other extremity *c* will be found if a third proportional *Sc* is found to the two lines *Sd*, *See*; and so of the rest.

8. We shall find the visible diameters of these same parallels of the Horizon — even if their diameters are drawn neither in the Equator nor in the Horizon — by this third method as well. From the point *A*, in the previous figure, let a circle $\gamma\gamma\theta\theta$ of any magnitude be described, and, $\gamma\gamma\theta\theta$ being drawn perpendicular to *AR* so as to make the quadrants *Ryy*, *Rθθ*, let the arc *Rε* be the half of the complement of the altitude of the pole — that is, the half of that arc which is

similar to the arc *CK*; and the straight line *Aε*, being drawn, will pass through *K*, since by Lemma 10 the straight lines *AR*, *AK* cut off the arc *Rε*, the half of the arc that is similar to the arc *CK*. For the same reason, if the arcs *εδ*, *εθ* are the halves of quadrants, the straight lines *Aδ*, *Aθ*, being drawn, will pass through *H*, *I*, because *KH*, *KI* are quadrants.

Now the quadrant *δθ* — which corresponds to the semicircle *HKI* — having been divided into 180 equal parts (that is, each of the arcs *εδ*, *εθ* into 90), if all the almucantars are wanted (but we have distributed each into three parts, so that

the individual parts contain thirty parts each, that is, fifteen degrees), then any two rays drawn from A through two points equally distant from the point ε — which corresponds to the crown of the head — will cut off, from BD, the apparent diameter of that parallel of the Horizon which is as many degrees from the Zenith in the sphere as those two points are half-degrees from the point ε ; or which is as many degrees from the Horizon as those two points are half-degrees away from the points δ, θ toward the Zenith, if the points taken are in the quadrant $\delta\theta$; or toward the Nadir, when the points are taken from the points δ, θ toward $\gamma\gamma$ and $\theta\theta$. So that the quadrant $\delta\theta$ corresponds to the parallels of the Horizon above the Horizon, but the parts from δ and θ toward $\gamma\gamma$ and $\theta\theta$ correspond to the parallels below the Horizon.

For example: the rays $A\lambda, A\xi$ will cut off the diameter cd of the parallel which is 60 degrees distant from the Zenith. For since the straight lines $A\varepsilon, A\lambda$ intercept 60 half-degrees in the circle $R\delta$, the same will cut off 60 degrees from the Equator by Lemma 10, and consequently the ray $A\lambda$ will pass through S, and by the same reasoning the ray $A\xi$ will pass through T; and therefore both will pass through the points c, d, just as before the rays AS, AT did. In like manner the rays $A\mu, A\nu$ will pass through V, X and will cut off the visible diameter fg . And these rays, lying between ε and the points δ, θ , will cut off the diameters of the parallels above the Horizon; whereas the other rays, beyond the points δ, θ , will cut off the diameters of the parallels below the Horizon. Thus the rays $A\sigma, A\pi$ will give the visible diameter of the parallel which is described through ω below the Horizon. And both rays equally distant from the point ε , or from the points δ, θ , if they cut the straight line BD below the point P, will exhibit the diameter of a parallel lying below the antarctic pole, and which is described in the astrolabe below the straight line PQ, about the Nadir k. Of this kind are the rays $A\nu, Ap$, cutting off the visible diameter $\downarrow$ -14. And so, if all three methods that we have handed down are employed, the visible diameters of the parallels of the Horizon will be found most exactly, since for each ray to be drawn from A there will be, besides the point A, three other points through which they must be drawn — namely one in the Equator, another in the Horizon, and a third on the circle $\gamma\gamma R \theta\theta$, as is clear from what has been said.

[Marginal summary] [Margin: To find the visible diameters of the parallels of the Horizon by means of any arc described from the south pole.]

9. For the rest: just as, if an angle equal to the angle CAK — which the straight line AC makes when it cuts the diameter HI of the Horizon, emitted from E, the point where the axis of the Horizon KL cuts the diameter of the Horizon HI, with the

radius AK falling in the Zenith — is made on the other side of that radius, that is, the angle OAK (namely, if from K toward B an arc equal to the arc CK is taken, and through its end the straight line AO is drawn), then the straight line AO falls onto the center of the Horizon in the astrolabe, that is, it bisects the visible diameter of the Horizon FG, as we showed in the preceding proposition at Number 3: — just so also, if a straight line A2 drawn from A through the point cc, where the diameter ST of the parallel of the Horizon cuts the same axis KL, is made so that the angle 5AK is equal to the angle zAK (that is, if an arc K5 equal to the arc zK is taken), the straight line A5, being drawn, will fall onto e, the center of the parallel in the astrolabe whose diameter in the sphere is ST — that is, it will bisect the visible diameter cd of the same parallel — by what we have demonstrated in Lemma 35. [Margin (a): Euclid I.29] For the axis KL is perpendicular to the diameter ST, since it is perpendicular to the diameter HI of the Horizon, from which ST is equidistant.

By the same reasoning, if from A, through the point bb, where the diameter VX intersects the same axis KL, the straight line Abbσ is drawn, and to the arc Kσ an equal arc K15 is taken, the straight line A15, drawn, will fall onto h, the center of the parallel whose diameter is VX. Likewise, if from A, through the point oo, where the diameter YZ divides the same axis KL, the straight line Aooff is drawn, and to the arc Kff an equal arc Kgg is taken — or (what is the same) an arc Lgg equal to the arc Lff — the straight line Agg, drawn, will fall onto the center of the parallel whose diameter is YZ. Finally, for the same reason, if from A, through the point nn, where the diameter ab cuts the same axis KL, the straight line Anndd is drawn, and to the arc Ldd an arc Lee is taken equal, the straight line Aee, produced, will fall onto 12, the center of the parallel whose diameter is ab, and so on. For the demonstration is the same in all cases.

[Marginal summary] [Margin: The line emitted from the south pole bisects the visible diameters of the parallels of the Horizon found in the first and third methods — that is, it falls onto the centers of the parallels.]

This same thing also fits the circle γγ R θθ. For if, for example, the straight line Acc were produced, cutting the circle Rε at some point, and to the arc between this point and the point ε an equal arc were cut off, the straight line drawn through the end of this arc would fall onto e, the center of the parallel whose diameter is ST. For, on account of the arcs equal on either side of the point ε, [Margin (b): Euclid III.27] the angles at A, the center, standing on them, would be equal; [Margin (c): Euclid III.26] and therefore they would also stand, in the circle

ABCD, on equal arcs Kz, K5. Hence, as was demonstrated, the straight line A5 would fall onto the center e, and so on.

[Marginal summary] [Margin: To find the semidiameter and the center of any parallel of the Horizon by means of a single straight line that touches the Vertical.]

10. Besides the three methods set forth, we have devised a fourth — a most beautiful and very easy way of describing the parallels of the Horizon in the astrolabe, by which, namely, through a single straight line that touches the Vertical, both the semidiameter of the parallel to be described and its center are found. It is of this sort. The primary Vertical Ai CK being described, let its quadrant iC be divided into 90 degrees, if all the parallels above the Horizon are to be described, and likewise the quadrant Ck, if all the parallels below the Horizon are wanted. (We have divided each quadrant into three parts, so that the individual parts correspond to thirty degrees each; which division, from what has been handed down, is not difficult.) For if each quadrant of the Equator CB, CD is cut into as many parts as the quadrants of the Vertical are to be divided into, and from G, the pole of the Vertical (for just as KL are the true poles of the Horizon, so HI are the true poles of the Vertical, which appear at the points F and G), through the points of division in the Equator hidden straight lines are drawn, then each quadrant of the Vertical, Ci and Ck, will be divided at the points 30, 60, which correspond to those in the Equator — as was demonstrated in the preceding proposition at Number 17, in the first method of distributing the great oblique circles into degrees. And we have set an example of this here in the straight line G30, which, passing through the 11th, the 30th degree of the Equator counted from C toward D, cuts off the arc C30 of 30 degrees from the Vertical circle.

Then through the points of division of each quadrant on the Vertical let straight lines be drawn touching the Vertical. For these, on the meridian line BD, will indicate the centers of the parallels to be described through those same points of the Vertical, in such a way that the portions of the tangents between the points of contact and the straight line BD are the semidiameters of the parallels. For example: through C, if the straight line CO8 is drawn touching the Vertical at C, it will fall onto O, the center of the Horizon, which is the greatest of all those parallels, and the semidiameter will be OC. Therefore the circle described from O through C will give the Horizon. So the straight line 30e7, touching the Vertical at the point 30 of the quadrant Ci, will fall onto e, the point from which, through that point 30, the circle described will give the parallel of the Horizon which is 30 degrees turned away from

distant from it [the pole] toward the Zenith. And the straight line 60h4, tangent to the Vertical at point 60 of the same quadrant Ci, will yield h, the center for describing the parallel through point 60, which lies 60 degrees from the Horizon toward the Zenith. In the same way the line 30913, tangent to the Vertical at point 30 of quadrant Ck, will cut DB — extended — at the center for describing the parallel through point 30 of the same quadrant Ck, which lies hidden 30 degrees below the Horizon. Finally the line 6012, tangent to the Vertical at point 60 of the same quadrant Ck, will pass through 12, the center for describing the parallel through that point 60, which recedes 60 degrees from the Horizon toward the Nadir. And the same reasoning holds for the rest.

The demonstration of this construction — the one that, of them all, pleases me most — is as follows. The parallels necessarily pass through the points found on the Vertical in this way, since these points represent those points of the primary Vertical on the sphere through which are drawn the parallels that the ones described here on the Astrolabe represent. Now since, as we demonstrated above at Number 7, the straight lines drawn from P, the center of the Vertical, to the points where the Vertical cuts the parallels are tangent to the parallels at those very points, the straight lines drawn from those points to the centers of the parallels will [Margin (a): 2 & 8 [?] of Book 3 (Euclid III)] be perpendicular to the aforesaid lines drawn from P, the center of the Vertical, to the points of intersection of the Vertical with the parallels. Therefore those same lines drawn from the centers of the parallels, since they are perpendicular to the semidiameters of the Vertical — that is, to the lines drawn out from center P — will be tangent to the Vertical at those same points, by the corollary of Proposition 16 of Book 3 of Euclid. Hence the straight lines tangent to the Vertical will pass through the centers of the parallels, since the lines drawn from these centers to the points where the Vertical is cut are tangent to the Vertical, as we have shown. Otherwise two straight lines would be tangent to the Vertical at the same point — namely the one drawn from the point of section tangent to the Vertical, and the one drawn from the center of the parallel to the same point of section — which is absurd.

Diagram. The master construction plate for the almucantars (parallels of the Horizon) and their opposites, occupying the lower two-thirds of the page within a ruled rectangular border. It is a very dense projection: a large circle (the Vertical / prime-vertical projection) with an outer dotted arc carrying degree-marks, a smaller inner circle, the meridian and horizon diameters, and a pencil of many straight lines radiating from a labelled focus at the right (point A / H region). Numerous points are lettered in Latin capitals and minuscules, Greek minuscules,

and astronomical symbol-glyphs. The same woodcut is reprinted on p. 528 (see fig-p528-1). Because of the density, more than two peripheral labels cannot be resolved with confidence

Clearly-legible reference points (as used in the surrounding text):

- A — focus of the radiating pencil of lines (right of center)
- B, C, D — meridian / horizon endpoints
- E — an interior center
- F — near the top of the plate
- G, H, i, k — construction points for the "opposite-point" method
- K, L — pole / axis endpoints (lower center)
- M, N — endpoints of the Horizon diameter
- P — center of the Vertical (left-center, near "T")
- Q — center of the hidden auxiliary circle Q439
- R, RR, T — left-side construction points
- Numeric point-marks around the arcs: 1, 2, 3, 4, 7, 8, 9, 10, 12, 14, 60; and 13, 13 at the top
- Bottom edge marks (inside frame): "14 | 10 | 14"; near the foot Φ , "60" [?], and K
- Greek / minuscule and composite labels visible include: c, ω , α , m, n, u, o, x, π , ψ , q, aa, ff, mm, pp, qq, PP, BB, AA
- Symbol point-labels (used in the compass worked example of §11): $\mathbb{C}$ (crescent glyph), $\boxplus$ (six-point burst glyph) [?], $\text{\textcircled{V}}$ (Venus glyph)

11. Moreover, by this device, if several parallels are proposed for description, we shall draw the lines tangent to the Vertical without great labor. [Margin: An easy practice for drawing several lines that are to be tangent to a given circle at given points.] From P, the center of the vertical circle, describe a circle of any size whatever — kept hidden, however, lest confusion arise — such as Q439. From i, along ik, draw the perpendicular i3, cutting the described circle at 3. For if, with the help of the compass, you take the interval i3 and transfer it from any point of the vertical circle onto the circumference Q439 described from P — whether on both sides or on one side only — then a straight line described from the found point on that circumference, drawn through that point of the Vertical, will be tangent to the Vertical at that very point.

Thus, because at the interval i3, from point 60 of the Vertical in quadrant iC, the compass cuts the circumference on both sides at points 4.4, the line 4 60 4 will be tangent to the Vertical at point 60. By the same reasoning, since the compass with

the same interval, from point 30 of the same quadrant, cuts the circumference on both sides at points 7.7, the line 7 30 7 will be tangent to the Vertical at 30. Again, the same interval from C gives, on both sides on the circumference, the points 8.8; therefore the line 8 C 8 will be tangent to the Vertical at C. Likewise, because the same interval, from point 30 of quadrant Ck, cuts the circumference on both sides at 9.9, the line 9 30 9 will be tangent to the Vertical at 30. Finally, since the same interval yields, on both sides on the circumference, the points 10.10, from point 60 of the same quadrant the line 10 60 10 will touch the Vertical at 60. And so for the rest.

The reason for this operation is that all the tangents between the Vertical iCk and the circle 347 are equal, by Lemma 48. Indeed, since — as was demonstrated in the same Lemma — the arcs placed between two tangents are similar, if to arc i60 the arc 34 is taken as similar, and to arc i30 the arc 37, and to arc iC the arc 38, and to arc iC30 the arc 39, and to arc iC60 the arc 310 (which will easily be done if from P, the center of the Vertical, straight lines are drawn through the points i, 60, 30, C, etc. of the Vertical; for these will cut off, from the circle 347 described, similar arcs, which are to be transferred from point 3 onto the circumference 347), then the same points 4, 7, 8, 9, 10 will be obtained, through which the tangent lines are to be drawn. And in this way the points will be found more reliably on the circle described from P than in the former method, in which the compass cuts the said circle too obliquely.

And since similar arcs are to be transferred downward from point 3, we shall find the other points by this technique. Since the line from P through point 30 of the Vertical below C cuts the said circle at $\mathbb{C}$, and accordingly the arc $\mathbb{C} * [?]$, transferred from 3 downward, will give point 9; and the arc $\mathbb{C} 9$, transferred upward, will show the other point 9. This is because the line P $\mathbb{C}$ cuts the line 9 30 9 in half at 30, by Lemma 48, and therefore also the arc 9 $\mathbb{C}$ 9; and so for the rest. Indeed, when the arc $\mathbb{C} *$ is too large — so that all oblique section may be avoided — its half may be taken and transferred twice from 3 downward: this will happen if a parallel is to be described below the Horizon at 60 degrees. For the line P.60 $\mathbb{Q}$ cuts the circle at $\mathbb{Q}$; therefore, if the half of the arc $\mathbb{Q} \square$ is transferred twice from 3 downward, point 10 will be found, etc. This will happen all the more if the parallel to be described lies below the Horizon at 80 degrees.

From all this it will be easy to gather that no parallel of the Horizon, however small, has its center at the pole i itself. [Margin: That the center of any parallel of the Horizon is distinct from its pole.] For since the line Ai, drawn through the pole i and extended, falls on M, the extreme point of the diameter of the Horizon

— as was shown in the scholium of the preceding proposition at Number 14 — whereas the line drawn from A through the center of any parallel falls on some interior point of the same diameter of the Horizon MN, namely on that point through which passes the line equidistant from FG and corresponding to the diameter of the parallel in the Equator (as we showed a little earlier at Number 6), it is clear that the center of any parallel is distinct from the pole i, since the lines drawn from A through the center and through the pole i differ from one another.

This can also be proved from what we demonstrated at Number 9. For since the center is found by the line drawn from A to the point of the Equator distant from the pole K, toward B, by as much as the line drawn from A through the intersection of the diameter of the parallel with the axis KL is distant, toward C, from the same pole K (as we showed there), it is manifest that the line drawn from A through the center is distinct from the line AK. Finally, the same is established from what was demonstrated at Number 10: namely, because the line tangent to the Vertical at the point where it is cut by the parallel falls on the center of the parallel; and this tangent can in no way fall on the point i, since [Margin (a): Euclid III.2] the line drawn from the intersection of the parallel with the Vertical to i falls within the Vertical and is not tangent to it, as is evident — seeing that the line i3 is tangent to the Vertical at i.

12. It must not, however, be passed over that from any parallel of the Horizon the opposite parallel can be described on the Astrolabe, even if its apparent diameter has not been found. [Margin: To describe, from any parallel of the Horizon described on the Astrolabe, the opposite parallel, even if its diameter has not been found.] [Margin (b): Theodosius II.14] For since through any point of a circle that is not a great circle on the sphere a great circle can be described tangent to it, [Margin (c): Theodosius II.6] that great circle will be tangent to another circle — not a great one — equal and parallel to the former. Since, therefore, by the Corollary of Proposition 6 of Book 2 of Theodosius the points of contact are opposite through the diameter of the sphere, there will correspond, to any assigned point on any parallel of the Horizon, another point opposite through the diameter of the sphere on the opposite parallel — namely that point at which the great circle tangent to the former parallel at the assigned point touches the latter, opposite parallel. Wherefore, if on the described parallel any three points are assigned and their three opposite points are found (as we shall shortly teach), and through these a circle is described, the described circle will be the opposite parallel. And it will be described through those three points if the center is found by the scholium of Proposition 5 of Book 4 of Euclid (which, however, is easily

managed here, since the center always lies on the meridian line BD), or, when the center is too far distant, by the instrument which we constructed in Lemma 14.

13. Furthermore, by this technique the point opposite to any given point on the Astrolabe will be found. [Margin: Given a point on the Astrolabe, to find, through the diameter of the sphere, the point opposite to it.] Draw from the given point a straight line through the center of the Astrolabe; then, by Lemma 12, find a third proportional to two lines — of which the first is the line lying between the given point and the center of the Astrolabe, and the second is the semidiameter of the Equator — and let a segment equal to this third proportional be cut off from that line drawn through the center of the Astrolabe, beginning from that same center. For the endpoint will be the opposite point. For since, as we showed above at Proposition 4, Number 11, the semidiameter of the Equator is the mean proportional between the two semidiameters of opposite parallels of the Equator, it follows that, when the line between the center of the Astrolabe and the given point is set as the semidiameter of one parallel of the Equator, the other line — between the same center of the Astrolabe and the found point — will be the semidiameter of the opposite parallel of the Equator, and accordingly the found point will be opposite to the given point through the diameter. The third proportional will be found without difficulty by the method we explained at the end of Lemma 12. For if, to the line drawn out from the given point through the center of the Astrolabe, the diameter of the Equator is erected at right angles, and through the extreme points of this diameter and the given point a circle is described, it will cut off the third proportional, as we demonstrated there, etc.

We shall find the point opposite to any given point more easily in this way. Let there be given, in the upper figure, the point F outside the Equator; from it, through the center E, draw the straight line FG, and at it erect in E the perpendicular EA;

and to the line EA joined with AF erect the perpendicular AG, cutting FG at G. This will be done if to the arc BH of the Equator the opposite arc DI is taken equal.

[Margin (a): Euclid III.31] For the line AI will be perpendicular to AF — that is, the angle HAI, in the semicircle HAI, will be a right angle — since the point G will be opposite to point F through the diameter, by what was demonstrated in the scholium of Proposition 5, Number 20. Again, let there be given the point i, inside the Equator; from it, through center E, draw the line ik, erect in E the perpendicular EA, and to the line iA joined [with EA] erect the perpendicular Ak; and again k will be the point opposite to point i through the diameter. But if it should happen that the perpendicular Ak cuts the line ik very obliquely, we shall

proceed conveniently thus: extending AE as far as C, we shall describe a circle through the three points A, i, C. For this will cut ik at k, the point opposite to point i through the diameter, [Margin (b): Euclid III.31] since the angle iAk, in the semicircle, is a right angle. But in what way, given a point of a parallel, the point opposite to it through its diameter on the same parallel is found, we shall teach at Proposition 14, Number 4. When the given point lies on the circumference of some great circle, the line drawn from it through the center of the Astrolabe will give, on the circumference of that same circle, the point opposite through the diameter.

14. But since, as we demonstrated in the scholium of the preceding proposition at Number 10, any straight line drawn across through the center of the Astrolabe marks, on any oblique great circle, two points opposite through the diameter, it follows that the straight lines drawn from the points at which the Vertical cuts the given parallel, extended through the center of the Astrolabe, will mark on the same Vertical two points opposite to those. For example: with the parallel of the Horizon c30d described, if from point 30 — where it is cut by the Vertical — a straight line is drawn through E, the center of the Astrolabe, the Vertical will be cut at BB, the opposite point.

Diagram. The same master construction plate reprinted from p. 526 (see fig-p526-1) so that it faces the continuing text, again within a ruled rectangular border occupying the lower part of the page. The projection, the pencil of lines from focus A, the inner and outer circles, and the labelling are identical to the earlier printing. The legible reference points match those catalogued at fig-p526-1 (A, B, C, E, F, G, H, i, k, K, L, M, N, P, Q, R, RR, T; numeric marks 1, 2, 3, 4, 7, 8, 9, 10, 12, 14, 60 and 13, 13; the foot marks "14 | 10 | 14", Φ , "60" [?], K; Greek / composite labels c, ω , α , m, n, u, o, x, π , ψ , aa, ff, mm, pp, qq, BB, AA; and the symbol point-labels $\mathbb{C}$, $\square$ [?], φ). Same density limitation as the first printing

And by the same reasoning the line drawn from the other intersection of the Vertical with the aforesaid parallel, through E, will likewise exhibit on the Vertical the opposite point 30. But if to the two lines Ec, EB a third proportional $E\omega$ is found (which will easily be done if a circle is described through the three points A, c, C; for this will cut off the third proportional $E\omega$, as was shown at the end of Lemma 12), the point ω will be opposite to point c. Through the three points 30, ω , BB, therefore, the parallel opposite to c30d is to be described. And if opposite points are found for several points of the parallel c30d, only slightly distant from one another, the opposite parallel will be described through those several points (provided, of course, that those points are joined by a curved line),

even if the center is not found and the construction is not carried out by the instrument of Lemma 14. Again, if from the two points where the Vertical cuts the parallel f60g lines are drawn out through the center E, the Vertical will be cut at points AA, 60, which are opposite to them. And if it is made that as Ef is to EB, so EB is to another [line], we shall find—

(Continuation of the Scholium to Propositio VI [Problema III] of ASTROLABII LIBER SECVNDVS — the construction of the parallels of the Horizon [almucantars] and their transfer to oblique great circles. Numbered paragraphs 14–21.)

[...the point ↓, opposite to the point f, will be found] (this too will be done easily if a circle is drawn through the three points A, f, c; for it will cut off the third proportional E ↓, as was demonstrated at the end of Lemma 12). And therefore the parallel opposite to f60g must be drawn through the points 60, ↓, AA.

15. But suppose that for some other point — say the point α , on the straight line MN — the opposite point is to be found. Then a straight line must be drawn from α through E. For if we arrange that as E α is to EB, so is EB to a further length, a third line will be found whose terminus, reckoned from the point E, is the point opposite to α . And so with all the rest. This third line will be found without trouble by the method I set down a little earlier at the end of Number 13.

16. From this, in turn, we shall find, in any given parallel of the Equator, the point at which it is cut by a parallel of the Horizon that lies any number of degrees below the Horizon toward the Nadir — even when this Horizon-parallel has not been drawn. The procedure is most convenient when the parallel lies only a little off the straight line PQ; that is, when its distance from the Horizon is nearly equal to the altitude of the pole AH. For a parallel of this kind is very hard to draw, since its center falls too far off and the parallel itself appears in the astrolabe as almost a straight line. We shall therefore proceed as follows. Suppose, for example, that we must find the point at which a parallel of the Horizon lying 40 degrees below the Horizon toward the Nadir intersects the parallel of the Equator whose southern declination is 20 degrees. Draw the opposite parallel of the Equator — the one, that is, whose northern declination is 20 degrees — and, in addition, the opposite parallel of the Horizon, the one that recedes 40 degrees from the Horizon toward the Zenith. Then, if from the points where these two parallels intersect straight lines are drawn through the center E, the given parallel of the Equator will be cut in two points, which are opposite to those two; and these, accordingly, are the points at which

the proposed parallel of the Horizon would cut the given parallel of the Equator, had it been drawn — because opposite parallels pass through opposite points on the sphere. And should it ever happen that the northern parallel of the Equator, opposite to the given southern one, is not cut by the drawn parallel of the Horizon, that is proof that the proposed southern parallel cannot be cut by the named parallel of the Horizon either.

But to make the matter plainer, let the point be sought at which a parallel of the Horizon 30 degrees below the Horizon divides the Equator. Draw, then, the parallel of the Horizon 30 degrees above the Horizon about the diameter cd , and let it cut the Equator at H (for the Equator, being a greatest circle, has no opposite parallel that could be drawn); from H draw the straight line HE through E , cutting the Equator at I ; and I will be the point opposite to H . Now since the parallel of the Horizon 30 degrees below the Horizon — the one, that is, opposite to the parallel about the diameter cd — must necessarily pass through the point opposite to H , it will certainly cut the Equator at the point I , which is opposite to H ; and thus the point I has been found, even though the opposite parallel of the Horizon $BB \omega 30$ has not been drawn. We took the points H, I , the extremities of the diameter of the Horizon, as our example because, although the aforesaid parallels do not intersect the Horizon exactly at these points, the intersections nonetheless occur not far from them, so that the matter is illustrated aptly enough without our being forced to draw another line and produce greater confusion in the figure. And if someone should seek the point at which a parallel of the Horizon 60 degrees below the Horizon [cuts the Equator, one would have to draw] the parallel of the Horizon 60 degrees above the Horizon, about the diameter fg . But since this one does not cut the Equator — rather, it lies wholly within it — we shall say that the parallel of the Horizon 60 degrees below the Horizon in no way cuts the Equator. This is evident in the parallel $AA \downarrow 60$. And so with the rest.

[Margin: To find, in a given southern parallel of the Equator, the point at which it is cut by a proposed parallel of the Horizon below the Horizon — when it is so cut — even if that parallel has not been drawn.]

17. From what has been said, we shall now describe without any difficulty any parallel of the Horizon whose distance is given, whether toward the Zenith or toward the Nadir. Suppose, for example, that a parallel of the Horizon 30 degrees toward the Zenith is to be drawn. In the first method, we shall count in the Equator, from the true diameter of the Horizon HI , toward the Zenith K , 30 degrees, up to S, T , so as to obtain its diameter in the sphere,

ST; for the radii AS, AT will cut off the apparent diameter cd of the proposed parallel. In the second method, we shall reckon the same 30 degrees from the apparent diameter of the Horizon FG, toward M, up to l, p; for the radii Al, Ap will cut off the same apparent diameter cd of the given parallel. In the third method, in the circle $\gamma\gamma R\theta\theta$ we shall count from the points δ, θ — toward ε — 30 parts out of the 90 into which each of the arcs $\varepsilon\delta, \varepsilon\theta$ is divided, up to λ, ξ ; for the radii $A\lambda, A\xi$ will present the same apparent diameter cd. And finally, in the fourth method, in the Equator from the point C, toward B, we shall take an arc of 30 degrees, and through its endpoint, from G, the pole of the Vertical, we shall draw a straight line that cuts the Vertical at 30. For the straight line tangent to the Vertical at 30 will give e, the center of the parallel to be drawn through the point 30, and so on. And if the parallel to be drawn is toward the Nadir, the counting must be set up from the same termini but in the contrary directions: in the first method, from the diameter HI, toward L; in the second, from the diameter FG, toward N; in the third, from the points δ, θ , toward $\gamma\gamma$ and $\theta\theta$; and finally in the fourth, from the point C, in the Equator toward D, and so on.

[Margin: To draw in the astrolabe a parallel of the Horizon given in the sphere.]

18. Conversely, we shall learn how far any parallel of the Horizon drawn in the astrolabe is distant from the Horizon, whether toward the Zenith or toward the Nadir, in this way. Let the parallel of the Horizon that has been drawn cut the meridian line BD at the points c, d; from these points draw the straight lines cA, dA to A, the south pole, cutting the Equator at S, T. For each of the arcs HS, IT contains the distance of the drawn parallel from the Horizon, toward K [?], the Zenith. And it is necessary — provided no error has been committed — that the straight line ST drawn be parallel to the diameter HI; that is, that the arcs HS, IT be equal. Again, let there be a drawn parallel of the Horizon cutting the meridian BD at the point $\downarrow$ — which is enough, even though the other point of section cannot be had, because of its too great distance — and let the straight line $\downarrow A$ be drawn, cutting the Equator at b. For the arc 1b measures the distance of that parallel [from the Horizon] toward L, the Nadir; and so with the rest. And if you wish to be certain whether it is truly a parallel of the Horizon, proceed thus: cut off in the Equator, from C toward D, an arc equal to the arc 1b already found, and from G, the pole of the Vertical, draw a straight line to the endpoint; for if this line falls upon the intersection of the circle AA $\downarrow 60$ with the Vertical, and the arc of the Vertical between C and that

intersection contains as many degrees as are contained in the arc 1b, it will be a parallel of the Horizon; otherwise not.

We shall obtain the same result in this way as well. From G, the pole of the Vertical, let a straight line be drawn through the point of section of the given parallel with the Vertical, cutting the Equator. For the arc of the Equator between this line and the point B will indicate the distance of the parallel from the Zenith i; and consequently its complement will be the distance of the same parallel from the Horizon. For example, the line G [drawn] through the section of the parallel 30 (namely ωBB) with the Vertical cuts the Equator at 11 [?]; therefore the arc B11 is the distance of the parallel from the Zenith i, while the arc D11 shows the distance of the same from the Nadir k. And finally the arc C11 is the distance of the same below the Horizon. And so with the rest. The reason is that the straight lines drawn out from G, the pole of the Vertical, cut off from the Equator and from the Vertical arcs of equal numbers of degrees, as was demonstrated above in Number 17. When, however, it is not established that the proposed circle is one of the parallels of the Horizon, one must use the former method; for by it we learn at the same time whether or not the given circle is one of the parallels of the Horizon, according as its diameter, once found, proves to be parallel to the diameter of the Horizon or not. But which circle in the sphere it represents, when its diameter as found is not equidistant from the diameter of the Horizon, we shall explain in Proposition 17. [recte: Number 17? — printed "proposition. 17."]

[Margin: Given a parallel of the Horizon in the astrolabe, to learn how great its distance from the Horizon is.]

19. Everything we have prescribed for drawing parallels of the Horizon in the astrolabe may be transferred, without any difficulty, to other oblique circles that are perpendicular to the Meridian. In the first method of drawing the parallels, one draws in the Equator, through the degrees of that same Equator, straight lines parallel to the diameter of the oblique great circle to which the circles to be drawn are parallel — just as, parallel to the diameter HI of the Horizon, the lines ST, VX, etc. were drawn. In the second method, in place of the Horizon AFCG one takes the proper oblique great circle, and...

[Margin: How everything that has been said about drawing parallels of the Horizon is to be adapted to drawing the parallels of any other oblique great circles that are nonetheless perpendicular to the Meridian.]

Diagram. The master stereographic-projection plate of the astrolabe construction (Equator ABCD, primary Vertical AKCN, Horizon-parallel FGHq, Equator-parallel OPQR, hour- and vertical-circles radiating from the south pole A). This is the same master plate reprinted the full label list and structural description are given under **fig-p531-1** below. On p. 530 the plate occupies the lower half of the page beneath §19.

(§19 continues below the plate on p. 530:)

[the oblique great circle] is divided into degrees, beginning from the meridian line of the astrolabe BD, etc. Thus, if the parallels of the primary Vertical were to be drawn, one would have to draw, in the first method, lines parallel to the diameter KL; and in the second, the primary Vertical AiCk is to be divided into degrees, taking the beginning from the points i and k. In the third method, for the point ε — which corresponds to the Zenith itself, or to the upper pole of the Horizon — one takes, in the same circle drawn from A, the point answering to one of the poles of the great circle to which the parallels to be drawn are equidistant in the sphere; and for the points δ , θ , which answer to the extremities of the diameter of the Horizon HI, one takes the points corresponding to the extreme points of the diameter of the assumed oblique great circle. Thus, in drawing the parallels of the Vertical circle, one must take for ε either of the points θ , δ — for these answer to the poles of the Vertical — and then the points ε , κ are to be taken in place of the points δ , θ , and so on. Finally, in the fourth method: for the primary Vertical, which is perpendicular to the Meridian and drawn through the poles of the Horizon, one employs the great circle perpendicular to the Meridian and drawn through the poles of the assumed great circle; and for the pole of the Vertical G, one takes the pole of the great circle that plays the part of the Vertical — just as, in drawing these same parallels of the Vertical, one must employ the Horizon and its pole i, and so on.

20. Indeed, by exactly the same means it will be permissible to draw the parallels of any oblique great circle drawn in the astrolabe that is not perpendicular to the Meridian — provided that, in place of the meridian line BD, one takes the line extended through the center of the oblique circle and the center of the astrolabe; that is, the common section of the Equator (or of the plane of the astrolabe) and of the great circle drawn through the poles of the world and the poles of the proposed oblique circle, this line serving as the proper Meridian of that same oblique circle. You will find an example of this matter in Proposition 8, Number 19.

[Margin: How everything said about drawing parallels of the Horizon is to be adapted to drawing the parallels of any other oblique great circle that is oblique to the Meridian as well.]

21. Now we shall divide the parallels of any oblique great circle into degrees — that is, into unequal parts, into which the degrees of those circles in the sphere are projected onto the astrolabe — by the same methods by which, in the preceding section, from Number 17 to the end, we divided the oblique great circles themselves into degrees. In the earlier part of the first method we shall carry out the task thus. Let ABCD be the Equator of the astrolabe, with center E; let kl be the diameter of any oblique great circle — say, the Horizon; XY the diameter of any of its parallels; and FGHq the same parallel drawn in the astrolabe. Let mn be the diameter of the primary Vertical, and AKCN the Vertical itself as drawn, with center L; let K be the upper pole of the Horizon, N the lower, and M the pole of the Vertical, more remote from the south pole in the sphere — that is, the point of intersection of the Meridian and the Horizon on the northern side, through which the Horizon, if drawn, would pass. And since the parallel of the Horizon FGHq, in this earlier part of the first method, is to be divided into degrees from K — the pole of the Horizon found within the Equator, and which in the sphere is the more remote from the south pole — the parallel of the Equator OPQR must be drawn, distant from the south pole by as great an interval as the given parallel of the Horizon is distant from the pole m (which is the more remote in the sphere from the south pole); so that the arc Aε, measuring the distance of the parallel of the Equator from the south pole A, is equal to the arc mX, which measures the distance of the parallel of the Horizon from the more remote pole m. Consequently, when the diameter of the parallel of the Horizon XY recedes from the diameter of the Horizon kl toward m — its pole more remote from the south pole — the diameter of the parallel of the Equator recedes from the diameter of the Equator BD toward the south pole A; that is, the parallel of the Equator is southern. But when that diameter of the Horizon-parallel inclines away from the diameter of the Horizon toward the pole of the Horizon n, nearer the south pole, this diameter of the Equator-parallel inclines away from the diameter of the Equator toward the northern pole C; that is, the parallel of the Equator is northern. This parallel of the Equator can indeed be drawn from E, even if its apparent diameter has not been found, through the point...

Diagram. A large stereographic-projection construction of the astrolabe (the same master plate reprinted from p. 530), enclosed in a ruled rectangular border. It superimposes several great circles and parallels of the astrolabe (the "spider"/rete-like tangle of hour-lines and vertical circles) about the projection center. The construction is extremely crowded, with several dozen small labels — many in minuscule Greek — most of which cannot be resolved at this resolution. (far more than 2 labels flagged [?]).

Confidently read labels, with position:

- P (top of the large outer circle): north/upper apex of the graduated Equator-parallel OPQR; a dotted line runs from P down toward the interior.
- N (bottom of the large outer circle): lower pole of the primary Vertical AKCN.
- B (upper, on the Equator ABCD): top of the Equator.
- D (lower, on the Equator, near L): bottom of the Equator.
- S (upper left, on the large circle).
- V (lower left, on the large circle).
- Q (far left, on the horizontal diameter): left end of diameter OQ.
- O (right, on the horizontal diameter): right end of diameter OQ.
- R (lower, near N): lower end of diameter PR.
- A (center-right, hub of the radiating pencil of straight lines): the south pole / center of projection from which the vertical- and hour-lines emanate.
- F (upper middle, on the Horizon-parallel).
- G (interior, an intersection point of the oblique parallel with AKCN).
- H (right-center region, on the Horizon-parallel).
- K (interior, pole of the Horizon) — also a second **K** appears at the lower edge near the foot.
- L (lower-center, near D): center of the primary Vertical.
- M (lower-center): pole of the Vertical.
- T (left-center, near θ and m).
- AA (lower right), BB (right), mm (right): labels of Equator-parallels on the right flank.
- Φ , $\theta\theta$ (bottom edge): with a cross/dagger-like fiducial mark between them.
- Interior minuscule/Greek cluster (largely unresolved): α , X, z, r, p, e, h, u, n, i, l, d, ψ , and the Greek θ , δ , ε , λ , ξ , $\gamma\gamma$, m — a dense knot at the projection center. [multiple small labels: ?]

- Graduation numerals around the circles: 60, 20, 10, 3, 4, 7, 8, 9, 12, 14, 1, 2, 6 (degree/quadrant marks). [several partly legible: ?]

(§21 continues below the plate on p. 531:)

Q, where the straight line KG — drawn from K, the pole of the oblique circle, through G, the intersection of the oblique parallel with the great circle AKCN — cuts the diameter of the Equator AC. For, as we shall show presently, just as FG represents a quadrant of the parallel, so the line KG must cut off a quadrant from the parallel of the Equator. Now, once this parallel of the Equator has been drawn and, by the two diameters OQ, PR, divided into four perpendicular quadrants: if from K, the pole of the Horizon, straight lines are drawn through the individual degrees of the parallel OPQR, the parallel of the Horizon FGH will be cut into degrees — that is, into arcs that are indeed unequal, but that represent equal degrees of the same parallel in the sphere. For example, if from K the straight line KS is drawn, cutting off the arc PS of 60 degrees, it will cut off the same amount from the parallel of the Horizon, namely the arc FT, answering to the arc of 60 degrees of the same parallel in the sphere. Likewise, if the straight line KV cuts off the arc RV of 60 degrees, the arc Hb of 60 degrees will likewise be cut off from the parallel of the Horizon. Finally, the straight line KQ, cutting off the quadrant PQ, will also cut off the quadrant FG from the parallel of the Horizon — that is, it will pass through G, the point where the Vertical cuts the parallel of the Horizon. For just as in the sphere the Meridian and the Vertical divide the Horizon itself and its parallels into quadrants, so too it must necessarily happen in the astrolabe, so that the arcs FG, GH, Hq, qF represent the quadrants of the same parallel in the sphere — which we declared above in Number 5 of this proposition. Moreover, the beginning of the arcs must be taken in each parallel from two points of the same order — that is, either from the upper points P, F, or from the lower points R, H — and one must proceed toward the same side, or descend...

[Margin: To draw in the astrolabe a southern parallel of the Equator from an equal parallel of the oblique great circle, drawn about that circle's pole more remote from the south pole.]

[Margin: The starting-point of the corresponding arcs on the parallels — from where it is to be taken in this earlier part of the first method, namely from their upper pole.]

descending on each parallel, or ascending. For the point P of the Equator's parallel lies in the upper semicircle of the Meridian — the one, of course, in which the Zenith is contained — while the point F of the Horizon's parallel is

southern; likewise the point R of the Equator's parallel lies in the lower semicircle of the Meridian, and the point H of the Horizon's parallel is northern. Hence, by what was said in Lemma 23, we rightly said that the beginning must be taken either from the upper points P, F, or from the lower points R, H. Here I call "upper" those points that hold the upper place in the figure with respect to the parts of the astrolabe, and "lower" those that hold the lower place — not those that are upper or lower in the heavens. The same beginning can be taken from the straight line KQ, which cuts off quadrants from the parallels, as from the points Q, G, always advancing toward the same side; for in this way one always proceeds toward the points from which we said the start must be made. Thus you see that the corresponding arcs PS, FT begin from the upper points P, F and descend toward the same (left) side, whereas the corresponding arcs RV, Hb begin from the lower points R, H and descend toward the same side, and so on. This, however, must be understood as holding when the pole of the oblique circle, lying within the Equator, is also found within the oblique parallel. For when it lies outside that parallel — as happens in the parallel drawn through the south pole, and in the other parallels lying below it, whose circumferences are drawn in the astrolabe toward the contrary directions and not toward the greatest oblique circle — the upper and lower points cannot be taken in this way. In that case, therefore, one must observe what we wrote in Lemma 23 about the beginnings of the arcs cut off.

[Margin: An easy rule for recognizing which of the points of the Equator's parallel on the astrolabe is called upper in the heavens, or lower, with respect to a given greatest oblique circle; likewise which of the points of the oblique parallel is northern or southern.]

Now, so that on the astrolabe we may easily recognize which of the points of the Equator's parallel is upper in the heavens, or lower — that is, whether it is contained in the upper or the lower semicircle of the Meridian, if the greatest oblique circle, to which the oblique parallels are parallel, is taken as the Horizon, above which the arctic pole is elevated — and likewise which of the points of the oblique parallel is northern or southern, this rule must be kept. The point of the Equator's parallel that is nearer to the pole of the oblique circle contained within the Equator — that is, the point through which a straight line drawn from the center of the astrolabe through that said pole passes — represents the upper point in the heavens; the other one, which is farther from that same pole — that is, through which a straight line drawn from the center of the astrolabe through the other pole passes — is the lower point. Likewise, the point of the oblique

parallel nearer to the center of the astrolabe (which indeed does not differ from the north pole) is northern; the more remote one is southern.

Diagram. A large square-framed woodcut showing the stereographic projection used to divide the oblique (horizon) parallels into degrees. Within the square frame is a large bounding circle; the vertical diameter runs from P at the top down through the center to N at the bottom, and the horizontal diameter runs from Q on the left through the center to O on the right. Nested inside are the projected Equator, the horizon circle, several oblique parallels (almucantars), and the vertical circles, together with many chords drawn from the poles K and (at bottom) the projecting point. Readable labels, by position:

- P (top of vertical axis): projected north pole
- F (upper, on the vertical axis below P)
- B, with X beside it (upper region)
- S (upper left)
- Q (left end of horizontal axis)
- C (left, inside)
- V (lower left, on the bounding circle)
- T (left of center)
- Z (upper center)
- θ , m (center-left cluster)
- A (center-right, a point where several chords converge)
- O (right end of horizontal axis)
- K (center — projected pole of the horizon)
- E (center)
- G (lower-left of center)
- M (lower center)
- L (lower center, above D)
- D (lower center)
- R (script r / ρ , lower center on the axis)
- N (bottom of vertical axis)

A dense cluster of small letters near the center (including d, ρ , α , r, p, ξ , h, u, n, l) could not be resolved individually at this resolution. [More than two peripheral labels illegible]

[Margin: The starting-point of the corresponding arcs on the parallels — from where it is to be taken in this earlier part of the first method, namely from their upper pole.]

If this matter is considered attentively together with what we wrote in Lemma 23 about fixing the beginnings of the arcs, there will be no difficulty in determining beforehand the starting-points of the arcs cut off — whether the division of the parallel is to be made from the pole of the oblique circle lying within the Equator, or from the other pole.

Take, then, the following demonstration of this division of the oblique parallels into degrees. A plane drawn in the sphere through the antarctic pole and through the pole of the Horizon more remote from it cuts off, by Lemma 23, from the Equator's parallel and from the parallel of the Horizon equal to it — such that the latter is as far from the south pole as this one is from its own pole, which is the more remote from the south pole — equal arcs, the beginning being made from the points we mentioned. Therefore that same plane, [note a] which in the sphere produces a circle, when projected onto the astrolabe will be seen, from the south pole, to cut off those same equal arcs from the two parallels drawn in the astrolabe. Since, then, that plane — or rather the circle which, passing through the south pole in the sphere, it produces — makes in the astrolabe, by Proposition 1, no. 1, a straight line passing through the pole K, the straight line KS will represent that circle drawn through the pole of the Horizon K and through the point S of the Equator's parallel. This line, therefore, will cut the Horizon's parallel at T, the point that corresponds to it in the sphere, through which that circle is drawn — so that the circle is seen, from the south pole, to cut the Horizon's parallel at T, and the Equator's parallel at S, because the line of sight, carried around through all the points in the plane of that circle,

[Margin: (a) Theodosius, *Spherics* I.1.]

never departs from that plane, but always lies in KS, its common section with the plane of the astrolabe. The arc FT of the Horizon's parallel, therefore, represents that arc in the sphere which is equal to the arc PS of the Equator's parallel. The same must be said of the straight line KV and of all the others which, going out from K, the pole of the Horizon, cut both parallels. Consequently, if from K straight lines are drawn through the individual degrees of the Equator's parallel, the Horizon's parallel will be cut into 360 arcs, which correspond to the 360 degrees of the same parallel in the sphere — so that any two straight lines sent out from K intercept, on those two parallels, two arcs equal as regards the number of degrees, that is, two arcs which correspond in the sphere to two

entirely equal arcs on the same parallels. Of this kind are the two arcs SQ, TG; likewise the two SV, Tb; and QV, Gb, and so on.

[Margin: To find on the astrolabe any given degree on the Horizon's parallel, from its upper pole.]

22. From this is gathered the method of finding any given degree on the Horizon's parallel — the degree whose distance is measured either from one of the two intersections F, H of the parallel with the Meridian, or from one of the two intersections G, q of that same Horizon's parallel with the primary Vertical circle. For if the given degree is counted on the Equator's parallel from one of the four points P, Q, R, O — which correspond to the four points F, G, H, q of the Horizon's parallel — and if through the end of the count a straight line is drawn from K, it will cut that parallel at the required degree. For example, if from the point F toward G an arc of 60 degrees is to be cut off, or from G toward F an arc of 30 degrees, we shall count from P toward Q 60 degrees, or from Q toward P 30 degrees, as far as S. For the straight line KS will cut the Horizon's parallel at T, at the 60th degree from F, or the 30th from G; and so with the rest. Now the point F faces south; H faces north; G the east, and q the west, just as we said of the Horizon.

[Margin: To determine on the astrolabe how many degrees are contained in a given arc of the Horizon's parallel, from its upper pole.]

23. Conversely, we shall also easily recognize how many degrees any arc on a given Horizon's parallel corresponds to, if from the two extremities of the given arc straight lines are drawn to K, the pole of the Horizon, and to the poles of the Equator's parallel. For the arc of the Equator's parallel comprehended between them will contain just as many degrees as are contained in the given arc, as is clear from what has been said. Therefore, if by Lemma 3 it is inquired how many degrees of the Equator's parallel are contained in that arc, the number of degrees contained in the proposed arc of the Horizon's parallel will become known. For example: if the given arc is γT on the Horizon's parallel, then, drawing from K the straight lines $K\gamma$, KT , which cut the Equator's parallel at β and S, there will be as many degrees in the arc γT as are contained in the arc βS .

[Margin: To divide the parallels of any greatest oblique circle into degrees, from their lower pole.]

24. In the latter part of the same first method, one must proceed as follows. Let a parallel of the Equator uret be described, equal likewise to the given Horizon's parallel FGHq, but opposite to the earlier Equator's parallel OPQR — that is, one as far distant from the south pole as the given Horizon's parallel recedes from its pole n, which is nearer to the south pole — so that the arcs Aθ and nX, which measure the said distances of the parallels, are equal; or, what is the same, so that the diameter of the Horizon's parallel and the diameter of the Horizon kl, and the diameter of the Equator's parallel and the diameter of the Equator, incline toward the same side, not toward opposite sides as before.

[Margin: The starting-point of the corresponding arcs on the parallels — from where it is to be taken in this method of dividing the oblique parallels into degrees, from their lower pole.]

For, when this parallel of the Equator has been described and divided into quadrants by the diameters rt, eu cutting each other at right angles, if from N — the other pole of the Horizon, which lies outside the Equator and is nearer in the sphere to the south pole — straight lines are drawn through all the degrees of that line, the Horizon's parallel will be cut into its degrees, as before. But the order of degrees on each parallel is not to be taken from two points of the same rank — namely from the upper points r, F, or from the lower points t, H — but from contrary points; that is, from the upper point of one and the lower point of the other, so that on the one a descent is made and on the other an ascent, yet toward the same side, left or right. The same beginning can be made from the straight line NG, which cuts off quadrants from the parallels, as from the points e, G, proceeding, however, toward different sides, so that on one parallel an ascent is made and on the other a descent. But since two upper or two lower points cannot always be distinguished in the figure — on account of the oblique parallels whose circumferences incline not toward the parts of the greatest oblique circle to which they are parallel, but toward the contrary parts — it is better to fix the order of degrees beforehand from what we wrote in Lemma 23: namely, that on the Equator's parallel the upper point be taken, and on the oblique parallel the lower point, and on this the southern point. But in what way an upper or lower point on the Equator's parallel, and a northern or southern point on the oblique parallel, is to be taken with respect to the parts of the heavens, was explained a little earlier, in the earlier part of this first method of dividing the parallels into degrees, at no. 21. For example, if from N a straight line Nδ is drawn cutting off the arc tδ of 60 degrees, the same line will take from the Horizon's parallel the arc FT corresponding to an arc of 60 degrees of the same

parallel in the sphere. So too, if the straight line $N\alpha$ takes off the arc ra of 60 degrees, there will also be cut off from the Horizon's parallel the arc Hb of 60 degrees. Finally, the straight line Ne , taking off the quadrant te , will also cut off from the Horizon's parallel the quadrant FG — that is, it will pass through G , the point of intersection of the primary Vertical circle with the Horizon's parallel. For, as was said above, the arcs FG , GH , Hq , qF are quadrants. Here you see that the beginning of the equal arcs is always made from contrary points, as was set forth. This, moreover, will be demonstrated as follows. A plane drawn in the sphere through the antarctic pole and through the pole of the Horizon nearer to it, which the pole N represents, cuts off, by Lemma 23, from the Equator's parallel and from the parallel of the Horizon equal to it — such that this latter is as far from the south pole as the former is from its own pole, which is nearer to the south pole — equal arcs, the beginning being made from the points from which we prescribed, a little earlier and in the said Lemma, that the beginning should be made: such points being r , H ; likewise t , F . Therefore that same plane, drawn in the astrolabe, will be seen to cut off the same arcs — namely those which correspond in the sphere to the arcs cut off. Since, then, by Proposition 1, no. 1, that plane, passing through the south pole, produces in the astrolabe a straight line passing through the pole N , any straight line sent out from the pole N will represent that plane, and therefore will cut off from each parallel equal arcs, as was said.

Thus the same points T , b , G have been found by means of straight lines going out from each of the poles K , N — each point, that is, by a pair of lines. And by the same art it will be possible to find any point on the Horizon's parallel by means of two straight lines, one of which goes out from the pole K and the other from the pole N , provided that this latter is drawn through an arc of the Equator's parallel that takes its beginning from a point of the meridian line BD contrary to the point from which the arc of the Horizon's parallel begins, as was set forth.

From what has been said, moreover, you will easily understand what you must do in order to cut off from the Horizon's parallel an arc of any number of degrees, and in order to know how many degrees are contained in a given arc.

[Margin: How everything said about the division of the Horizon's parallels is to be applied to the other oblique parallels.]

25. In exactly the same way the parallel of any other greatest oblique circle will be divided into degrees, if its poles are found; and when the oblique circle is not perpendicular to the Meridian, then in place of the meridian line BD

[Margin: To divide an oblique parallel into degrees by means of a circle of any size divided into equal degrees, so that there is no need to draw the southern parallel of excessive size, or the northern one of very small size.]

let there be taken the common section of the Equator, or of the plane of the astrolabe, and of the greatest circle passing through the poles of the world and the poles of the oblique circle — that is, a straight line drawn through the center of the astrolabe and the center of the oblique circle.

But since, when the oblique parallel lies near the upper pole *m*, the southern parallel of the Equator that must be drawn equal to it grows to almost immense size; whereas, on the contrary, when it is not far distant from the lower pole *n*, the northern parallel of the Equator that must be drawn equal to it is very small — it comes about that the oblique parallel cannot easily be divided into degrees in this way by means of the Equator's parallel. For this reason the following device must be employed, by which indeed we shall divide the oblique parallel into degrees, without the Equator's parallel, by means of a circle of any size, in this manner. Let *ABCD* be the Equator, whose center is *E*; the semidiameter of the greatest oblique circle *Et*, and its axis *HX*; the diameter of the oblique parallel *FG*, cutting its axis at *f*; the radius *AH*, exhibiting *K*, the visible pole of the oblique circle, cutting *FG* at *e*; the radii *AF*, *AG*, cutting off the diameter *Nq* of the oblique parallel, about which the visible parallel *Niaqk* is described. When the straight line *Et* has been produced, if from *H*, through *F*, a straight line is sent out cutting *Et* at *L*, then *EL* will be the semidiameter of the southern Equator's parallel whose diameter is in the sphere equal to the diameter *FG*. For if *H* is conceived as the south pole of the world, and *HX* as the axis of the world, then *EL* will represent the meridian line — that is, the common section of the plane of the astrolabe, or of the Equator, and of the Meridian. Therefore the radius *HF* will cut off the visible semidiameter *EL* of the parallel whose diameter is *FG*, as is clear from what was demonstrated in Proposition 4, no. 5.

If, then, from *E*, through *L*, a parallel *LdmQR* can conveniently be described in the plane of the astrolabe, we shall by its means divide the oblique parallel *Niaqk*, as was said, by drawing from *K* straight lines through all the degrees of the parallel *Ldm*. But if, on account of its excessive size, the said parallel cannot be described, we shall carry out the same division by means of a circle of any size that can conveniently be described and divided into equal degrees, in this manner. Let the given semidiameter of a circle be *gh*, by whose means the oblique parallel is to be divided into degrees. [note a] Let *gh* be cut at *r*, just as *fF*, the true semidiameter of the oblique parallel, is cut at *e* by the radius *AH*; or as *Ed*, the semidiameter of

the Equator's parallel (when it can conveniently be had), is cut at K, the visible pole of the oblique circle. For, as we shall show presently, Ed is cut at K just as ff is cut at e. Now, taking the straight line KI equal to gr, let a circle blPSMn be described from I with the given radius gh. I say that the straight lines sent out from the pole K through the degrees of this circle cut the parallel Niaqk into degrees — so that, for example, the arc Nk corresponds to as many degrees as are contained in the arc bn; and Ni to as many as in bl; and qa to as many as in SP.

Diagram. A large square-framed woodcut showing the auxiliary-circle construction for dividing an oblique parallel into degrees without drawing the (over-large or over-small) equal Equator parallel. Within the square frame are several concentric and intersecting circles (the Equator ABCD, the projected oblique parallel Niaqk, and the auxiliary circle blPSMn), with numerous radii and chords drawn from the poles, and many degree-marks around the peripheries (shown as dotted arcs). Readable labels, by position:

- m (upper left of center)
- b (upper center)
- L (right)
- M (right)
- K, H (left of center)
- B, E (center)
- N (center)
- C (left)
- G (lower left)
- D (lower center)
- T (lower center)
- P (lower left)
- S (lower center)
- Q (lower left)
- *B* (script B, lower left)
- R (bottom)

A number of small labels around the peripheries and in the central cluster — including Greek letters (φ , ω , π , θ) and numerals (7, 8, and others) used as point-names — could not be resolved individually at this resolution. [More than two peripheral labels illegible]

For, since by construction dK is to KE as bK is to KI , then also, by composition, dE will be to KE as bI is to KI ; and by alternation, dE (a semidiameter) to bI (a semidiameter) will be as KE to KI . Likewise, therefore, the point K (which stands in place of two poles) [is the vertex of] the straight lines going out from the point K by Lemma 21 (each of which stands in place of a pair, forming equal angles at K) — so that, if the circles $LdmQR$ and $blPSMn$ were described separately, [these lines] would cut off [equal arcs]; so that both the arcs dm , bl and the arcs df , bn , and RQ , SP [are similar]. As we demonstrated in Lemma 23, the straight line Kf takes off the arc Nk ... it will also take off, by the straight line Kn (the arc bn being taken, similar to the arc df), inasmuch as it cuts off similar arcs bn , df , as was demonstrated. [The arc Ni will contain] as many [degrees] as are contained in the arc bl ; and in the same way the arc qa [will be similar to the arc SP].

That the semidiameter Ed is cut at K just as fF is cut at e — which we assumed as true — we shall easily show. For since, by the scholium to Euclid VI.4, [note b] as fe is to eF , so is Eu to uL — because [note b] since the triangles AEK , HEu are right-angled [and] [note b] have the angles EAK , EHu equal in the isosceles triangle AEH , the remaining angles EKA , EuH will also be equal; [note c] and therefore the sides EK , Eu will also be equal. And thus always the radius drawn from the south pole to the pole of the oblique circle will cut off, from the meridian line and from the diameter of the greatest oblique circle, equal semidiameters, right up to the center of the astrolabe — which we also proved above, in Proposition 5, near the end of no. 14. And so EL will be equal to Ed ; and it will also hold that as fe is to eF , so is EK to Kd .

[Margin: (a) Euclid VI.10.] [Margin: (b) Euclid I.5.] [Margin: (c) Euclid I.6.] [Margin: What equal straight lines the radius falling on the pole of the oblique circle cuts off.]

But if, from any point O of the semidiameter EH , a line OV parallel to the straight line EL is drawn, cutting AH at ω , then also, by the scholium to Euclid VI.4, the straight line OV will be cut at ω just as fF is cut at e . Hence, if the straight line KI is taken equal to eO , and from I , at the interval OV , a circle $blPSMn$ is described, we shall find in the given parallel the degrees corresponding to the degrees of this circle.

[Margin: When the oblique parallel lies near the lower pole.]

No dissimilar reasoning will hold when the northern parallel of the Equator is to be described near the lower pole, and consequently the parallel [is small]. For example, if the diameter of the oblique parallel is $\phi\xi$, the radius $H\xi$ will cut off

from Et the semidiameter [that represents] the visible diameter E3 of the Equator's parallel. And again, by the scholium to Euclid VI.4, the semidiameter E3 is cut at u — the point that corresponds to the visible pole K — on account of the equality of the straight lines Eu, EK, just as the semidiameter $\omega\xi$ is cut in 4. If, then, the given semidiameter gh is cut at cc, just as $\omega\xi$ is cut at 4 (or E3 at u), and a straight line ccg equal [to it] is to be cut off... K7 [is cut off], and 7 will be the center of the circle to be described at the interval gh, which, drawn in the astrolabe, will be divided into degrees. Again, if the diameter of the oblique parallel is TZ, the radius HZ will cut off from Et the visible semidiameter Ep of the Equator's parallel. And again, by the scholium to Euclid VI.4, as Ep is to Eu, so is the semidiameter YZ to $Y\omega$. If, then, the given semidiameter is YZ, K8 must be cut off equal to ωY , and a circle must be described from 8 at the interval YZ, and so on. But if another semidiameter is given, it must be joined to that straight line, so that the given semidiameter has to the added portion the same ratio as YZ to $Z\omega$, or Ep to $p\omega$, and so on. And in this case, when the semidiameter of the oblique parallel lies wholly below AC — as does TZ — the visible pole K will lie outside the parallel, and outside the circle described from the point 8.

Now, to make it easier to find at will the centre and radius of the circle to be described — the circle from which the parallel is to be divided — the segment fe may be taken from K twice, three times, four times, or five times, and so on, and carried down onto the straight line KD; then, at the end-point of this transferred line, a circle may be described with a radius that is likewise double, triple, quadruple, or quintuple the radius fF , and so forth.

[Margin: To divide a maximum oblique circle into degrees by means of a circle larger than the Equator, of any size whatever.]

The very same device must be employed in dividing the oblique great circles when their upper pole lies only a little away from the circumference of the Equator. Suppose, for example, that the oblique great circle $A\sigma C\gamma$ [?] is to be divided into degrees with the help of a circle larger than the Equator: then a radius of any size whatever is taken and divided, and its segment, corresponding to the segment KE, is carried down from K, so that the centre of the circle to be described is obtained at the interval of the assumed radius. In our figure we doubled the segment KE as far as γ — for BE, the radius of the Equator, has been divided at K — and from γ , at the interval $\gamma\rho$ (which is likewise double the radius EB, since as BK is to KE, so is ρK to $K\gamma$), we described the circle $\gamma\mu\beta\pi$. If this circle is cut into 360 degrees, the straight lines drawn from K through its degrees will divide the oblique circle $A\sigma C\gamma$ into degrees; for the point K stands at an equal

distance from E, the centre of the Equator, and from γ , the centre of that oblique circle, and therefore the straight lines issuing from K cut off similar arcs on the Equator and on the circle $A\sigma C\gamma$, as was demonstrated in the scholium to Lemma 21. Thus you see that the straight line $K\beta$ cuts off the arc $\gamma\theta$, corresponding to the arc $\pi\beta$, or to the arc $D\pi$ of the Equator, which is similar to the arc $\pi\beta$. Likewise the straight line $K\mu$ will cut off the arc $\sigma\lambda$, corresponding to the arc $\gamma\mu$, and the straight line $K\eta$ [?] the arc $\sigma\gamma$, similar to the arc $\rho\lambda$, as far as the number of degrees is concerned. The same would happen if the straight line KE were tripled or quadrupled, and so on, and if from the tripled or quadrupled end-point of the line KE a circle were described at an interval triple or quadruple EB itself, and so forth.

[Margin: To divide any given apparent maximum circle into apparent degrees by means of the equal degrees of the same visible maximum circle [as seen] from its upper pole — the method most excellent of all, and the most expeditious.]

While I was writing this, behold, Christoph Grünberger [Christopher Grienberger], Professor of the Mathematical disciplines in our Roman College — a man most penetrating in devising new demonstrations, and by whose work and diligence not a few things have accrued to this Astrolabe of mine — observed that oblique circles, both great and not great, can be divided into apparent degrees by means of straight lines drawn from the equal degrees of those same circles through either of the visible poles. Since this matter is outstanding and remarkable — although perhaps it may seem utterly incredible to someone — it seems by no means to be passed over here.

Diagram. A large geometric construction within a ruled rectangular frame, illustrating the division of an oblique great circle into apparent degrees by projection through the visible poles. It shows the Equator ABCD (centre E) together with the oblique great circle AFCG (centre H), the two apparent poles I and Y on the vertical diameter, and a dense web of straight lines drawn from the poles through corresponding degree-points, with several auxiliary construction circles. Readable labels:

- F (top of frame, on the vertical diameter)
- M (upper left, on the oblique circle)
- B (upper centre, above H)
- H (centre of the oblique circle)
- P (left, on the horizontal diameter through I)
- K (upper centre)

- L (below K)
- V (below L)
- N (upper right, on a projection line)
- I (centre — apparent pole)
- E (centre — centre of the Equator)
- X (centre-left)
- G (lower centre)
- N (lower centre)
- D (bottom, on the vertical diameter)
- S (right)
- A (lower right of centre)
- 6 (a point so labelled, appearing twice at lower right) [?]
- Y (bottom, on the vertical diameter — apparent pole)
- a cluster of small italic letters at the left among the auxiliary arcs: *c, d, e, f, i, o, a* [several uncertain]

[The oblique-circle centre and several auxiliary points are labelled with small Greek letters ($\sigma, \gamma, \mu, \beta, \pi, \lambda, \theta, \rho$) that recur in the surrounding text; some are at the resolution floor.]

Thus, then, is how it must be done. Let the figure described in the scholium to Proposition 5, No. 12, be repeated, in which ABCD is the Equator, with centre E; AFCG the oblique great circle, with centre H; and I, Y the apparent poles; the diameters of the Equator and of the oblique circle, AC and PS, cutting FG at right angles. And since in the same scholium, No. 14, we demonstrated that both the three points A, I, P and the three points C, I, S lie on one straight line — so that each of the straight lines AP, CS passes through the pole I — if through I any straight line Mlb be drawn, cutting the Equator and the oblique circle at K and i, then, by Lemma 9, the arc BK of the Equator will be similar to the arc Gi of the oblique circle, and the arc Dy [?] of the Equator similar to the arc FM of the oblique circle. Therefore, if from the point F, toward C, an arc of any number of degrees is to be cut off, those degrees must be counted on the opposite part of the oblique circle, from the point G as far as i. For the straight line

drawn from i, through I, and produced, will cut off the arc FM, corresponding to as many degrees as are contained in the arc Gi. For since the arc Gi is similar to the arc BK, and the straight line IK cuts off the arc FM of as many degrees as are contained in the arc BK — as we demonstrated in Prop. 5, No. 17 — the same straight line iIK will cut off that same arc FM of as many degrees as are contained

in the arc Gi. By the same reasoning, the straight line MI will cut off from the oblique circle the arc Gi, corresponding to as many degrees in the heavens as are truly contained in the arc FM. Likewise, when the straight line CIS is drawn, it will cut off the arc FC, corresponding to as many degrees in the heavens as are really contained in the arc GS — namely 90. And conversely, the same straight line will cut off the arc GS, corresponding to as many degrees in the heavens as are contained in the opposite arc FC; and these are indeed more than 90, since GA represents a quadrant, and therefore GS represents an arc greater than a quadrant, just as FC too is greater than a quadrant of its own circle, although it represents an apparent quadrant. And so with the rest. Therefore, if the whole circle AFCG is distributed into 360 equal degrees, from which straight lines are drawn through I, the visible pole, the oblique circle AFCG will be cut into visible or apparent degrees — in such a way, however, that each apparent degree corresponds to a true degree enclosed on the opposite side between the same two straight lines within which the apparent degree is contained.

[Margin: To accomplish the same from the lower pole.]

Again, since in the aforesaid scholium to Prop. 5, No. 18, we demonstrated that, if from Y, the lower pole, any straight line YM be drawn, both the arc BL of the Equator is similar to the arc FM of the oblique circle, and the arc DN of the Equator is similar to the arc GQ of the oblique circle: if from the point F toward C an arc corresponding to any number of degrees is to be cut off, the proposed degrees must be counted in the same semicircle, from the point G opposite, as far as Q. For the straight line drawn from Y, the lower pole,

[The same woodcut described (fig-p535-1) is reprinted here]

and emitted through Q, will cut off the arc FM, corresponding to as many degrees in the heavens as are truly contained in the arc GQ. For since the arc GQ is similar to the arc DN — the straight line YN cutting off the arc FM of as many degrees as are contained in the arc DN, as was shown in Prop. 5, No. 20 — the same straight line YNQ will cut off that same arc FM of as many degrees as are contained in the arc GQ. By the same reasoning, conversely, the straight line YM will cut off the arc GQ, corresponding to as many visible degrees as are really contained in the arc FM. So too the straight line YC will cut off the arc FP, corresponding to as many degrees as are contained in the arc GC; and conversely the same straight line YP will cut off the arc FC, corresponding to the quadrant GP. Finally, the tangent straight line YT will cut off the arc FT, corresponding to as many degrees as are contained in the arc GT; and likewise the arc GT, corresponding to as many degrees as are contained in the arc FT. Therefore, if from Y straight lines are

drawn through all the degrees of the circle AFCG, the circle itself will be cut into all its apparent degrees — in such a way, however, that to each equal degree there corresponds an apparent degree on the same side, between the same two lines issuing from Y.

[Margin: To divide any given apparent oblique parallel into apparent degrees, by means of the equal degrees of the same parallel, from its upper pole.]

Let there again be an oblique parallel KnLC, whose centre is O and whose visible poles are P, Q; let the southern parallel of the Equator equal to it be VXY, and the northern one bkc; and let there be given, through E, the diameter XE, perpendicular to VY. And since — as we shall demonstrate below, in the scholium to this proposition, No. 3 — the straight line drawn from X through P falls on the extremity of the diameter of the oblique parallel drawn through O perpendicular to VY: if through P any straight line be drawn to A, cutting the oblique parallel at f and C, then, by Lemma 9, the arc Va will be similar to the arc LC, and the arc YA similar to the arc Kf. Therefore, if from the point K, toward n, [an arc] is to be cut off

of any number of degrees, those degrees must be counted from the point L, opposite, in the contrary direction as far as C. For the straight line drawn from C through P, when produced, will cut off the required arc Kf, since when extended it cuts off the arc Va similar to the arc LC, as has been said; and it was demonstrated above, in Number 21, that the straight line Pa cuts off the arc Kf corresponding to the arc Va. In like manner the same straight line will cut off the arc LC, corresponding to as many degrees in the heavens as are truly included in the arc Kf. And so with the rest. Therefore, if the whole parallel is to be distributed into apparent degrees, it must first be divided into 360 equal degrees; for the straight lines drawn from these degrees through P will indicate the opposite apparent degrees, as was said concerning the maximum circle.

[Margin: To accomplish the same from the lower pole.]

Next, since in the scholium to this proposition, No. 5, we shall demonstrate that, if from Q, the lower pole, any straight line Qf be drawn, both the arc Kf is similar to the arc b β and the arc Ly is similar to the arc ca: if from the point K, toward n, an arc of any number of degrees is to be cut off, the given degrees must be counted from the point L, opposite, on the same side as far as γ . For the straight line drawn from Q, the lower pole, through γ will cut off the required arc Kf, which of course corresponds in the heavens to as many degrees as are comprehended in the arc Ly. For since the arc Ly is similar to the arc ca, and the straight line Qa,

passing through γ , cuts off the arc Kf of as many apparent degrees as there are equal ones contained in the arc ca — as was shown above in No. 24 — the same straight line Q γ , passing through a, will cut off that same arc Kf. Conversely, the same straight line Qf will cut off the arc L γ , corresponding to as many degrees as are contained in the arc Kf. Therefore, if we are bidden to divide the whole parallel into apparent degrees, we shall distribute it into 360 equal degrees; for the straight lines passing from these degrees through Q will show the apparent arcs, as was said concerning the maximum circle.

[Margin: To find out, by a very easy method, how many degrees are contained in a given arc of an oblique circle.]

Hence we shall very easily understand how many degrees any arc of an oblique circle in the Astrolabe — whether a great circle or not — comprises. For two straight lines drawn from the endpoints of the given arc through either of the apparent poles cut off, on the other part of the circle, an arc of as many equal degrees as the number of degrees to which the given arc corresponds. Thus, in the circle KnL — whether it be a great circle or not — if the arc Kf be given, the straight lines KP, fP will include the arc LC, while the straight lines KQ, fQ include the arc L γ , of as many equal degrees of the same circle KnL as the number of degrees to which the given arc Kf is equivalent, as is clear from what has been demonstrated in this place. So, if the arc L γ be given, the straight lines QL, Q γ will cut off the true arc Kf, to which the apparent L γ is equivalent. And if the straight line γ P were produced, it would in the same way cut off an arc as far as K, to which the given arc L γ corresponds.

Diagram. A geometric construction within a ruled rectangular frame, showing an oblique circle (parallel) KnL with its visible poles and the projection lines used to divide it into apparent degrees. Within a large circle carrying V at the top and Q at the bottom, an inner projected parallel circle is drawn, with a sheaf of straight lines converging on the poles. Readable labels:

- V (top of the large circle, on the vertical diameter)
- ϵ (epsilon, upper left)
- θ (theta, upper left)
- K (upper, on the vertical diameter)
- λ (lambda, mid-left, beside a point)
- f (long-s point, i.e. the point *f*, near λ)
- n (left of centre)
- O (centre-right, upper — centre of the oblique parallel)

- R (centre-right)
- β (beta, centre)
- P (far left, on the horizontal diameter)
- X (far left, on the lower horizontal)
- E (centre)
- c (small, right of E)
- L (centre, below E)
- e (small, below-left of centre) [?]
- ρ (rho, small, near L) [?]
- a (small point, lower centre-left)
- A (lower centre)
- Y (bottom, on the vertical diameter)
- Q (bottom, below Y — the lower visible pole)

[Several of the small Greek/italic auxiliary labels (ε , θ , λ , β , ρ , e, a, ι) are at the resolution floor]

So too, if a given arc Kf of the oblique circle is to be divided into two or more equal parts, this will be done if, once the straight lines KP, fP or KQ, fQ have been drawn, the arc LC or Ly is cut into two equal parts, or into more, and through P or Q straight lines are drawn from these parts, and so forth.

[Margin: To cut a given arc of an oblique circle into any number of equal parts by a very easy method. — [by VI.4].]

But this excellent and remarkable method of dividing oblique circles into apparent degrees — by straight lines drawn from their equal degrees through their own visible poles — we shall also easily demonstrate from what we wrote a little earlier, near the beginning of this No. 25, in the device by which oblique circles are divided into degrees by circles other than the Equator and its parallels. For since, in the upper figure of the scholium to Prop. 5, No. 12 — which is the second [figure] of this No. 25 — as AE, the radius of the Equator, is to EI, so is PH, the radius of the oblique great circle, to HI (for it was demonstrated in the same scholium, No. 14, that the three points A, I, P lie on one straight line): the upper pole I will be equally distant from the centres E, H. Therefore any straight line Mb, issuing from I, will cut off from the Equator and from the oblique circle — by the scholium to Lemma 21 — similar arcs Db, FM, because of the equal angles DIb, FIM set toward their own centres. For since the centres E, H recede from the point I in different directions, on opposite sides — just as, in the figure of the Corollary to Lemma 21, because the centres A, B recede from the point I toward

the same side, similar arcs CK, FM (or EL, HN) are cut off toward the same sides — which was also observed in the first figure of this No. 25: for since the centres E and γ recede from the pole K toward the same side, there were cut off, from the straight line K β , similar arcs

similar, the arcs D δ , $\pi\beta$, toward the same parts. And if the center, having been taken from the pole K upward — that is, not toward the same part as before (with center E) but toward a different one — the same straight line K β would have cut off arcs similar toward the opposite parts. Therefore, since the arcs Db and FM in the first figure of the scholium to Proposition 5, Number 12 — which is the second figure of the present Number 25 — are similar, and since the straight line Ib cuts off the arc Gi of as many apparent degrees as there are equal degrees contained in the arc Db (as we showed at Proposition 5, Number 17), the same straight line IM will cut off the same arc Gi, of as many apparent degrees as there are equal degrees included in the arc FM. And this is the reason why, if the division of the greatest oblique circle is to be set up from the upper pole I, the equal degrees must be counted in the part opposite to the apparent degrees that are to be cut off.

The same reasoning holds for the parallels. For, as appears in the first figure of the scholium to this proposition, Number 2 [Margin (a): Euclid, *Elements* VI.4], XE — the semidiameter of the parallel of the Equator — is to EP as NO — the semidiameter of the oblique parallel — is to OP. For, as we shall demonstrate in the same scholium, Number 3, the three points X, P, N lie on one straight line. Therefore the upper pole P is proportionally distant from the centers E, O. And since the centers E, O recede from the point P in different directions, what was proposed is clear.

Again, since in the aforesaid figure of Number 12 of the scholium to Proposition 5 — that is, in the second figure of this Number 25 — CE, the semidiameter of the Equator, is to EY as PH, the semidiameter of the greatest oblique circle, is to HY (for it was demonstrated in the aforesaid scholium, Number 14, that the three points Y, C, P are set on one straight line), the lower pole Y will likewise be distant from the centers E, H. Therefore, by the scholium to Lemma 21 (since the centers recede toward the same part from the point Y), any straight line YM drawn out from Y will cut off arcs FM, BL as well as arcs GQ, DN that are similar from the same part. Hence, since the straight line YN removes the arc FM of as many apparent degrees as there are equal degrees contained in the arc DN (as we demonstrated at Proposition 5, Number 20), the same straight line YQ [?], passing through N, will cut off the same arc of as many apparent degrees as there are

equal degrees contained in the arc GQ. And so, when the division of the greatest oblique circle is to be set up from the lower pole Y, the equal degrees must be counted from the same part.

The reasoning is no different for the parallels [Margin (b): Euclid, *Elements* VI.4]. For, as is manifest in the first figure of the scholium to this proposition, Number 2, de — the semidiameter of the parallel of the Equator — stands to EQ as MO — the semidiameter of the oblique parallel — stands to OQ. For, as will be demonstrated in the same scholium, Number 4, the three points Q, d, M lie on one straight line. Therefore the lower pole Q is proportionally distant from the centers E, O, and the centers E, O recede from the point Q toward the same part, and so on.

Diagram. A large stereographic-projection figure of the astrolabe occupying the full width of the page within a ruled rectangular frame. A large circle (the primary meridian / bounding circle) carries the pole **P** at its top and **N** at its bottom, joined by the vertical diameter PN (partly shown as a dotted line near the top). A horizontal diameter runs across the figure from a left endpoint (marked at far left, reading **Q/ε** [?]) through **C** (left of center) and **A** (right of center) to **O** at the far right. On the upper-left arc are **B** and **S**; on the lower-left arc, **V**; near the bottom, **R** stands just above **N**, with **D** and **L** on the vertical below the center. Around the crowded center lie a dense cluster of intersecting circles and straight lines (the projected parallels, verticals, and hour-circles) bearing small capital and Greek point-labels: **F** and **x** (upper inner), **Z**, **d** (δ), **K** (center), **M**, and the Greek minuscules **m**, **θ**, **γ**, **α**, **β/ρ**, **r**, **ξ**, **h**, **η**, **τ** (t), with **T** at mid-left and **I** at lower-left inner. The tangent line Lq and the radials from L through the graduation points are visible. The interior labelling is extremely dense and several small Greek labels are only partly legible.

- P (top): pole P
- N (bottom): point N
- O (far right, end of horizontal diameter): O
- Q / ε [?] (far left, end of horizontal diameter): left endpoint
- A (horizontal diameter, right of center): A
- C (horizontal diameter, left of center): C
- B (upper-left arc): B
- S (left arc): S
- V (lower-left arc): V
- R (bottom, above N): R

- D (lower center): D
- L (center-low): L, center of the Vertical
- M (center): M
- F, x (upper inner): F, x
- Z, d (δ) (upper-center inner): Z, d
- K (center): K
- T (mid-left inner): T
- I (lower-left inner): I
- Small Greek cluster near center ($m, \theta, \gamma, \alpha, \beta/\rho, r, \xi, h, \eta, \tau$): several only partly legible [?]

[Margin: To distribute into degrees the parallels of any greatest oblique circle, from the center of the greatest circle which is, as it were, their primary Vertical.]

You see, then, that the oblique circle itself is one of those that we said a little earlier must be described, so that through them the oblique circle — whether it is a greatest circle or not a greatest one — may be divided; since the ratio of the semidiameter of the oblique circle to the straight line between its own center and one or the other of the poles is the same as the ratio of the semidiameter of the Equator (or of its parallel) to the straight line between the center of the Astrolabe and the same pole of the oblique circle. The only difference is this: the center of the oblique circle does not, from the upper pole, tend toward the center of the Astrolabe, but toward the opposite part; and therefore the equal degrees must be counted toward the contrary part, not toward the same part from which the apparent degrees are to be cut off. This is what would also have to be done in the first figure of this Number 25, if the centers I and γ [?] were transferred above the pole K, and from them circles were described at intervals equal to the semidiameters Ib, $\gamma\rho$ [?]. Finally, when the pole of the oblique circle — from which the division of the oblique circle is to be made — lies between the center of the Astrolabe and the center of the described circle (through whose degrees the lines are to be drawn that divide the oblique circle), the equal degrees must be counted toward the part contrary to the apparent degrees that correspond to them; but toward the same part, when that same pole is not found between those two centers. Yet the straight lines proceeding through the equal degrees always cut the oblique circle into apparent degrees, as has been said. And from which part the apparent degrees are to be counted, when the division is made by a circle different from the oblique circle, can easily be understood from the scholium to Lemma 21, or must be gathered from what we have written here.

26. By the second method we shall divide the parallel of the greatest oblique circle into degrees in this way. Since the primary Vertical, when it is drawn through the poles of the parallels of the Horizon, divides the parallel FGHq in half at G and q, the line Gq will be the one representing the diameter of the parallel — that is, the common section of the Vertical and the parallel in the sphere.

Let the semidiameter ·G, then, be cut by Lemma 8 into unequal parts — the parts that the perpendiculars dropped to ·G from the individual degrees of the quadrants of the circle described about Gq produce. And from L, the center of the primary Vertical (which is found by the straight line drawn from A perpendicular to mn, the diameter of the Vertical, as we showed above at Proposition 5, Number 3), let straight lines be drawn through all the points of the semidiameter ·G; for each will cut the parallel in two points, which correspond to those points of the parallel of the Horizon to which the points of the semidiameter ·G correspond. For the individual points of the semidiameter ·G correspond to two points of the circle described about Gq. Therefore, if both semidiameters ·G and ·q are cut at the points that correspond to all the degrees of that circle described about Gq, the parallel will be divided into the whole 360 degrees. But it is enough if in this way the semicircle FGH is distributed into 180 degrees; for the degrees of this semicircle, transferred to the other semicircle FqH, will exhibit the degrees of that other semicircle. For example, if from L, the center of the Vertical, through the point a — which corresponds to the 60th degree counted on either side from the meridian line in the circle described about Gq — the straight line La is drawn across, the parallel of the Horizon will be cut at the points T, b, which are 60 degrees away from the points F, H; and if these are transferred to the other semicircle FqH, as far as Ig, the points I, g will likewise be 60 degrees distant from the same points F, H. Here too, since the straight lines Lq, LG touch the parallel — as we showed at Number 7 of this proposition, and as will be demonstrated again below at Number 30 — if they are produced, and a line parallel to qG itself is drawn between them, a longer line than qG will be obtained, which must be divided in the same way as qG was divided; just as was said in the preceding proposition, at Number 24, concerning the greatest oblique circle.

That the parallels are correctly divided into degrees in this way will be demonstrated by the following reasoning. The straight line AL — in the greatest circle ABCD drawn through the poles of the world and the poles of the Horizon (for we now take the circle ABCD for the Meridian) — is equidistant from kl, the diameter of the Horizon. If, then, planes are understood to be drawn through AL, each of them will, by Lemma 25, cut off from the parallel of the diameter XY two equal arcs beginning at the points X, Y in the sphere. Therefore those same planes

will also be seen to cut off, from the same parallel of the Horizon projected onto the Astrolabe, those same equal arcs, as viewed from the south pole. Since, then, those planes drawn through the south pole produce (by Proposition 1, Number 1) straight lines in the Astrolabe passing through the center L of the Vertical circle, where all those planes meet, the straight lines drawn through L in the Astrolabe necessarily represent those planes. But the same planes, drawn in the sphere through the individual degrees of the parallel of the Horizon, divide both semidiameters of that parallel — that is, the common section of the Vertical and the parallel — in the way in which the semidiameter of any quadrant is usually divided by the perpendiculars dropped to it from the individual degrees of the quadrant; because their common sections with the parallel are parallel to the common section of the Meridian with the same parallel, as is plainly established from the demonstration of Lemma 25 [Margin (a): Euclid, *Elements* I.29], and are therefore perpendicular to each of the aforesaid semidiameters of the parallel, just as the common section of the Meridian and of the same parallel is perpendicular to it. (For since both the Meridian and the parallel are at right angles to the Vertical [Margin (b): Euclid, *Elements* XI.19], their common section too will be at right angles to it; and therefore it will also be perpendicular to the common section of the Vertical and the parallel, by Definition 3 of Book 11 of Euclid.) And since the apparent diameter Gq is divided in the same way as the true diameter of the parallel, as will presently be demonstrated, it plainly follows that the straight lines drawn from L, the center of the Vertical, through the said points of the sections of the apparent semidiameter ·G (if it is divided as we have said) pass through the points of the parallel that correspond to the degrees of the same parallel in the sphere; since these straight lines in the Astrolabe represent those planes passing through the individual degrees of the parallel in the sphere, as has been said.

That the apparent diameter Gq is cut by those planes just as the true diameter of the parallel in the sphere is divided by them, we shall demonstrate in this manner. The true diameter of the parallel (by "the true diameter of the parallel" I mean the common section of the parallel and the Vertical in the sphere) is viewed from the south pole through a triangle whose base is the true diameter itself and whose vertex is at the eye, so that the apparent diameter Gq is the common section of the plane of the Astrolabe (or of the Equator) and of the aforesaid triangle [Margin (c): Euclid, *Elements* XI.9]; and the apparent diameter is parallel to the true diameter, because each is parallel to the common section of the Vertical, of the Equator, and of the Horizon. (For the true diameter of the parallel and that common section of the Vertical and the Horizon, being sections made in planes parallel to the plane of the Vertical [Margin (d): Euclid, *Elements* XI.16], are

parallel to each other. And if through that same section of the Vertical and Horizon a plane is understood to be drawn parallel to the aforesaid triangle — which is drawn through the true diameter [Margin (e): Euclid, *Elements* XI.16] — then that same common section and the apparent diameter will likewise be parallel, since they are common sections made in planes parallel to the plane of the Equator.) Therefore, by the scholium to Proposition 4 of Book 6 of Euclid, the true and the apparent diameter will be cut proportionally by those planes drawn through the straight line AL and through the individual degrees of the parallel in the sphere — that is, by the visual rays, which are the common sections of those planes and of the aforesaid triangle. Since, then, the true diameter is cut by those planes just as the semidiameter of any quadrant is divided by the perpendiculars dropped to it from the degrees, as has been shown, the apparent diameter will be divided in the same way — which was what was proposed.

[Margin: To find any proposed degree in an oblique parallel of the Astrolabe, from the center of the greatest circle which is, as it were, its primary Vertical.]

27. Therefore, if one wishes, say, for the 30th degree in the parallel FGHq — starting from the point G and proceeding either toward F or toward H — a straight line must be drawn from L through a, the point of the apparent diameter Gq that corresponds to the 30th degree of the circle described about Gq; that is, through the point across which passes the perpendicular dropped from the 30th degree of that circle, the start likewise being made in that circle from the point G.

[Margin: How many degrees are contained in a given arc of an oblique parallel — [found] from the center of the greatest circle which is, as it were, its primary Vertical.]

28. Conversely, we shall also learn how many degrees any arc of the parallel of the Horizon contains, if it has its beginning at the point G or q. For if from the endpoint T of the given arc GT a straight line is drawn to L, cutting Gq at a, the perpendicular drawn through a to Gq from the circle described about Gq will cut off an arc of as many degrees as are contained in GT. But if the arc does not begin at G or q, we shall attain what is sought, as we wrote at Number 26 of Proposition 5 [?].

[Margin: In what way everything that has been said about the division of the parallels of the Horizon, from the center of the Vertical, may be adapted to other oblique parallels.]

29. The reasoning is not different for dividing into degrees the parallel of any other greatest oblique circle, provided that in place of L one takes the center of

that greatest circle which is, as it were, the primary Vertical with respect to the greatest circle to which the parallel is equidistant, and which is therefore drawn through the poles of the parallel, and so on.

[Margin: That straight lines drawn in the Astrolabe from the center of any greatest circle to its intersections with the parallels of another greatest circle — which stands to it as the Horizon to the Vertical — touch the parallels there.]

30. From what we have said, it can be gathered with almost no trouble that the straight lines drawn from the center L to G and q touch the parallel at G and q (in the figure the tangent line Lq is drawn); which we also demonstrated above at Number 7. For since those straight lines represent, in the Astrolabe, the planes drawn through AL and through the endpoints of the true diameter of the parallel, and since those planes in no way cut the true parallel in the sphere but only touch it at those endpoints (as we shall presently show), it comes about that those straight lines likewise touch the parallel at the points G , q , which represent those endpoints of the true diameter. For if they cut it, the planes drawn through them would also cut the true parallel in the sphere at two points corresponding to those at which it would be cut by the straight lines LG , Lq — which is absurd, since those planes touch the true parallel in the sphere at the endpoints of the diameter. This is proved as follows.

Since the plane passing through AL , carried around through all the points of the true diameter of the parallel, always cuts the parallel by lines perpendicular to that diameter — or parallel to the common section of the parallel and of the greatest circle drawn through its poles and the poles of the world, as is clear from Lemma 25 — it comes about that when it first reaches the endpoints, it no longer cuts the parallel but touches it at those endpoints. This can also be demonstrated otherwise, and geometrically, as follows.

Let a circle $ABCD$ be posited at right angles to the plane of the Astrolabe (or of the Equator), so that kl is the common section of the greatest oblique circle and of that greatest circle which is drawn through its poles and the poles of the world, in the manner of a proper Meridian. If through the straight line AC , in the plane of the Equator (or of the Astrolabe), one conceives a greatest circle to be drawn at right angles to the greatest oblique circle of diameter kl (of which kind is the primary Vertical with respect to the Horizon, and, with respect to any other greatest oblique circle whatever, the greatest circle drawn through its poles and through its common sections with the Equator) [Margin (a): Theodosius, *Sphaerica* I.15], this circle will be at right angles to the greatest circle $ABCD$, in the position we called "right," since it passes through A , C , the poles of the greatest

circle ABCD — that is, through the common sections of the oblique circle and the Equator; for in these lie the poles of the circle ABCD, which has the said position. (For since the greatest circle ABCD is at right angles to the oblique circle and to the Equator [Margin (b): Theodosius, *Sphaerica* I.13], it will pass through their poles; and therefore these in turn will pass through its poles, by the scholium to Proposition 15 of Book 1 of Theodosius; and thus their common sections, the poles, will belong to the circle ABCD.) Therefore, since both the greatest circle ABCD and the oblique circle of diameter kl are at right angles to that greatest circle drawn through AC — which is itself at right angles to the oblique circle [Margin (c): Euclid, *Elements* XI.19] — their common section kl too will be at right angles to that same greatest circle drawn through AC [Margin (d): Euclid, *Elements* XI.8]; and consequently AL also, being parallel to kl, will be at right angles to the same greatest circle [Margin (e): Euclid, *Elements* XI.18]. Therefore the plane drawn through AL and through one or the other of the endpoints of the diameter of the parallel — which is the common section of the same greatest circle and the parallel — that is, the circle produced by it in the sphere, will make right angles with the same greatest circle drawn through AC.

Therefore, since this circle too — drawn through AL and through the assumed endpoint of the diameter of the parallel in the sphere — and the parallel itself are at right angles to that greatest circle drawn through AC [Margin (f): Euclid, *Elements* XI.19], the common section of these planes will also be at right angles to that same circle; and consequently it will be perpendicular both to the diameter of the parallel (which is the common section of the parallel and of that greatest circle drawn through AC) and to the diameter of the circle passing through AL and the assumed endpoint of the diameter of the parallel — the diameter that the aforesaid greatest circle drawn through AC makes in this circle. (For since that greatest circle, cutting at right angles the circle drawn through AL and the assumed endpoint of the diameter of the parallel, as we showed [Margin (g): Theodosius, *Sphaerica* I.13], cuts it in half and through its poles, it will pass through its center and will therefore make a diameter in it.) This common section will be perpendicular at the endpoints of these diameters, since both of these diameters lie in that greatest circle. Therefore that same common section — of the parallel and of the circle passing through AL and the assumed endpoint of the diameter of the parallel — will touch each circle, both the parallel and the circle drawn through AL and the produced endpoint of the diameter of the parallel, at the assumed endpoint of the diameter of the parallel, by the corollary to Proposition 16 of Book 3 of Euclid. From which it follows, by the definition in Book 1 of Theodosius, that these two circles touch each other at the endpoint of the diameter of the parallel and in no way cut each other — which was what was

proposed. But that the straight lines drawn through L, through G and q, touch the parallel FGHq, we shall demonstrate still otherwise in the following scholium, at Number 3; though the demonstration we adduced in this proposition at Number 7 is easier.

[Margin: That the semidiameter of the Vertical is a mean proportional [between the whole line and its outer segment].]

From this it is inferred that any straight line drawn from the center of the Vertical as far as the concave circumference of the parallel is so divided by the parallel that the semidiameter of the Vertical is a mean proportional between that whole line and its outer segment. For example, if from L, the center of the Vertical, the straight line LT is drawn, cutting the parallel FGHq at b: I say that the semi—

the semidiameter LK, or Lq, is the mean proportional between LT and Lb. For since the semidiameter Lq touches the parallel, as has been shown,[a] the square of the line Lq will be equal to the rectangle contained by LT and Lb.[b] Therefore, as LT is to Lq, so Lq is to Lb — which is what was proposed. And the same ratio holds for all the other straight lines drawn from L.

[Margin: proportional between the straight line by which, from its center, it cuts any parallel of the Horizon whatever, and the outer segment of that same line.]

[Margin: [a] Euclid III.36] [Margin: [b] Euclid VI.17]

From this too is derived the method of finding the other extremity of the diameter of a visible parallel when one extremity is known. For if, between the center of the primary Vertical and the known extremity, one takes the intercepted segment, and then finds a third proportional to it and to the semidiameter of the primary Vertical, and cuts off a length equal to that third proportional, beginning from the same center, the point so found will be the other extremity. Thus, if the extremity F of the parallel FGHq is known, and from the two lines LF, LA a third proportional LH is cut off, then H will be the other extremity of the visible diameter FH. Likewise, if the extremity H is given, and from the two lines LH, LA a third proportional LF is cut off, then F will be the other extremity, and so on. This too we have demonstrated at Num. 7 of this proposition.

[Margin: Given one extremity of the visible diameter of some oblique parallel, to find the other extremity by means of a certain third proportional.]

31. In a third way we shall divide the parallel of any oblique great circle into degrees, by this method. Each semidiameter of the parallel in the sphere, pX and pY , is divided by Lemma 8 into unequal parts — namely, the parts produced by the perpendiculars dropped from the degree-points of the circle around XY . It suffices, however, if only one semidiameter is divided in this way, since its points, transferred to the other, divide it in like manner. Then from A , the south pole, straight lines are drawn through all the points of the divisions, cutting the diameter FH of the parallel in various points; and if perpendiculars are raised through these points to the same diameter FH , the parallel $FGHq$ will be divided into degrees. For example: if from A , through the point Z (which corresponds to the 60th degree counted from X on the circle described around XY), a straight line AZ is drawn, cutting FH at d , and through d a perpendicular TI is raised to FH , then each of the arcs FT , FI will comprise 60 degrees — that is, it will represent the arc of the parallel of 60 degrees, counted from the south point toward each side, both eastern and western. This we shall demonstrate as follows.

[Margin: To distribute the oblique parallels of the Astrolabe into degrees, from the south pole of the Analemma.]

Let the circle $ABCD$ be set at right angles to the plane of the Astrolabe, so that XY , the diameter of the parallel, is the common section of that plane and of the great circle $ABCD$ which passes through the poles of the world and through the poles of the parallel. Since the plane in the sphere

Diagram. A large analemma / stereographic-projection construction enclosed in a heavy ruled square frame, used throughout Proposition VI. The same woodcut is reprinted at the foot of both p. 189 and p. 190 so it can be consulted while reading; it is described here once. Within the square is a large principal circle (the meridian $ABCD$) with a vertical diameter running F (top) to λ (bottom) and a horizontal diameter running X (left) to the right, crossing at the Astrolabe center E , where a small square (right-angle mark) is drawn. Numerous concentric and eccentric circles (parallels of the Equator and oblique parallels), tangent circles, and radiating straight lines from the poles fill the figure; several arcs are dotted. A dense cluster of small labels surrounds the center.

Readable labels (position $\rightarrow$ label):

- Top of vertical diameter: F
- Bottom of vertical diameter: λ (lambda) — appears both at mid-lower and at the very foot
- Left end of horizontal diameter: X

- Center (with right-angle mark): **E**
- Down the upper vertical, below F: **E, I, H, B, b** (successive intersection points)
- Down the lower vertical: **N, G, V, A, D**
- Right of center along/near the horizontal: **Y**, then **A** (bold), then **θ** (theta), then **ς** (Greek stigma/final-sigma) at the far right point of a long straight line
- Upper right, on a tangent line rising from center: **μ** (mu), **B** (bold), with **π** (pi) / **γ** (gamma) and **P, Q** on the dotted circle below
- Upper left: **q**, with **P, K, I, C** descending toward the horizontal
- Lower left: **R, η** (eta), **o, T, Y** (upsilon) on lines running to the lower-left corner
- Central cluster (small, crowded): **S, R, M, N, c, d, ξ** (xi) and several other small letters [?]

per the south pole A — that is, by the straight line AZ, in that position of the circle ABCD — and by the straight line that cuts the diameter XY at right angles at d and lies in the plane of the parallel, when produced meets the plane of the Astrolabe at d, and makes (by Lemma 24) the line to FH which is the common section of the plane of the great circle passing through the poles of the world and the poles of the parallel, and of the parallel itself, a perpendicular; and it will be seen in the Astrolabe to cut off the same degrees on the parallel FGHq that are cut off on the same parallel in the sphere, since the visual ray, passing through all the points, cuts off the perpendicular drawn through Z, and carries it hither and thither to the line TI. The arcs FT, FI therefore represent in the sphere those arcs which comprise 60 degrees on the parallel of the sphere, reckoned from the point X. And so with the rest.

32. If, therefore, from a given parallel an arc of any number of degrees is to be cut off, beginning from the point F or H, one must count as many degrees as are proposed on the circle described around XY, beginning from X or Y, and from the end-point of the count a perpendicular is to be dropped, cutting XY at some point. For if through this point a straight line is drawn from A, cutting FH at some other point, then the perpendicular drawn through that point to FH will, on either side, give an arc — begun from F or from H — which contains the proposed number of degrees.

[Margin: To find any proposed degree on an oblique parallel, from the south pole of the Analemma.]

33. On the other hand, if it is to be inquired how many degrees are contained in a given arc of the parallel, straight lines must be drawn from the endpoints of that arc to FH, as perpendiculars cutting it at two points; and from these to A, the south pole, two straight lines must be drawn, cutting XY, the diameter of the parallel, at two other points. For if from these points two perpendiculars are raised to XY, they will intercept, on the circle described around XY, an arc of as many degrees as are contained in the proposed arc.

[Margin: To ascertain, from the south pole of the Analemma, how many degrees are contained in a given arc of an oblique parallel.]

34. This third method of distributing the parallels into degrees also fits the parallel of any oblique great circle, if at any time it is not perpendicular to the Meridian: in place of the meridian line BD one takes the straight line drawn through its center and the center of the Astrolabe — namely, the common section of the plane of the Astrolabe (or of the Equator) and of the great circle which passes through the poles of the world and the poles of the oblique circle — this line serving as its proper Meridian.

35. Let us add, if you please, yet a fourth method of distributing any oblique parallel into degrees, similar to the one we set out at Num. 24 of the preceding proposition. For this method too will very often be convenient for investigating certain particular degrees which cannot easily be found by other ways. Let, then, the given oblique parallel be IKL, whose center is b. Let a parallel of the Equator, aRZV, be described equal to the given parallel — that is, whose diameter in the Analemma ABCD (for that the Equator of the Astrolabe can be taken for the Meridian of the Analemma has been said at Proposition 4, Num. 5, and elsewhere) is equal to the diameter of the given parallel in that same [Analemma], yet so that it be northern when the given parallel is in the upper hemisphere, but southern when it is in the lower. And we call

[Margin: In what manner all the things that have been said about dividing the parallels of the Horizon, from the south pole of the Analemma, are to be adapted to the other oblique parallels.] [Margin: To distribute any oblique parallel of the Astrolabe into degrees, from its own center and from the center of the Astrolabe.]

the upper and lower hemisphere with respect to the upper or lower pole of the oblique circle, after the manner of the Horizon of some [place] to which the given parallel is equidistant. As for how the upper and lower pole are to be taken, we have declared it in Lemma 23. And in this parallel of the Equator, a point S must be found which corresponds, in the oblique parallel, to a point [M] — that is, so

that the arcs RS, NM contain equal numbers of degrees. (For when the parallel of the Equator and the oblique parallel are equal and tend toward the same part of the sphere, the beginning of the degrees is taken, in the parallel of the Equator, from the upper point R, and in the oblique from the northern point N; or, in the other case, [it is taken] in that [Equator parallel] from the lower point V, and in this [oblique] from the southern point I, as was explained in Lemma 23.) This will be done as follows. From E, the center of the parallel in which the given point lies, with the semidiameter ES, let a straight line Sd be cut off (the line being produced toward the center, if need be) equal to the semidiameter of the other parallel; and let the straight line db be drawn to the center b of this other parallel, in which the point [M] is to be found; let db be bisected at e, and at right angles through the straight line ef, which cuts ES at f, let the straight line bf be drawn through f and the center b, cutting the given parallel at M. I say that the point M corresponds to the point S — that is, that the arcs RS, MN, or ξS , ξM , are equal in the sphere. For since the sides be, ef are equal to the sides de, ef and contain right angles,[a] the bases bf, df will also be equal; but bM, dS are also equal, by construction. Therefore the remaining lines fM, fS will also be equal; and consequently, as we showed in Lemma 42, the circle described from f through M and S will touch both

[Margin: [a] Euclid I.4]

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parallels, and will therefore represent a circle in the sphere touching those same [parallels]. Wherefore, by Lemma 44, the arcs NM, RS will be equal in the sphere. Moreover, the same point M will be found if at b an angle bdS is made equal to the angle dbM, or if a straight line SM is drawn parallel to bd, as we showed at Num. 34 of the preceding proposition — even if the line bd is not bisected, and so on.

Again, let there be assigned, corresponding to the point Y of the parallel of the Equator, a point in the oblique parallel — that is, one must be found such that the arc IO is equal to the arc VY, or the arc ρO equal to the arc ρY . With the semidiameter EY drawn, let Yg be cut off equal to the semidiameter of the parallel; and let the straight line gb be drawn, bisected at i, and at right angles through the straight line ih, which cuts EY, produced to h; and let the straight line hb be joined, cutting the parallel at O. I say that O is the point sought.[a] For again bh, gh are equal. Since therefore Yg, Ob are also equal, the remaining lines hY, hO will be equal. Therefore the circle described from h through O and Y will touch both parallels, and consequently, by Lemma 44, the arcs ρO , ρY will be equal in the sphere, and so on. The same point O will also be obtained if the angle ghO is

made equal to the angle bgY , or if through Y a line YO is drawn parallel to bg , even if the line bg is not bisected, and so on.

[Margin: [a] Euclid I.4]

But if it happens that a point k is given in such a position that, when the semidiameter Ek is drawn and kc is taken equal to the semidiameter of the parallel, the joined straight line cb makes a right angle — so that the line which cuts bc in half at right angles is parallel to kc — then bl must be drawn parallel to ck , so that the point l may be obtained as the corresponding point. Then, if the straight line kl is drawn, since ck , bl are parallel and equal, [b] bc , lk will also be parallel, and cl will therefore be a parallelogram; [c] and the angles at k and l will be right, and consequently the straight line kl [will touch] both [parallels]. The tangent line kl will represent a circle drawn through the south pole, which touches both parallels at k , l . For every straight line in the Astrolabe can, when extended to infinity, represent a circle drawn through the south pole — namely that circle which is produced by the plane drawn through that straight line and the south pole in the sphere. Wherefore, just as the straight line kl touches both parallels, so too the circle drawn through the south pole, which it represents, will touch those same parallels at k , l , and will therefore cut off equal arcs ξk , ξl , by Lemma 44. That the arcs ξk , ξl are equal we likewise show. The tangent line kl , when produced, falls upon the lower pole of the great circle to which the parallel IKl is equidistant, if this parallel faces its upper pole — or the contrary: if the parallel faces the lower pole, the tangent kl will fall upon the upper pole. For, as we shall show in the following scholium near the end, a straight line drawn from either of the poles of the oblique circle, if it touches one parallel, will touch also

[Margin: [b] Euclid I.33] [Margin: [c] Euclid I.34] [Margin: That every straight line in the Astrolabe can represent a circle drawn through the south pole.]

the other. Since, then, one single straight line can touch both on the same side (as is evident) — for if it touched the parallel RkV below k , that line, produced, would fall entirely outside the parallel IKl ; and if it touched IKl above k , it would fall wholly clear of the parallel IKl , as is plain — it will fall as a tangent exactly at k . Now, as was demonstrated at Num. 21 and 24, the straight line from the pole cuts off equal arcs from the parallels: the arcs Rk , Nl are cut off; and for the same reason the arcs $R\xi$, $N\xi$ are also cut off equal, since the line drawn from the pole of the oblique circle to ξ cuts off equal arcs. Therefore the remaining arcs ξk , ξl are also [equal] — which was proposed.

Let there be given, moreover, in the parallel of the Equator a point X, and let there be found [in the oblique parallel a point Q such that] the arc ρQ is equal to the arc ρX , or the arc IQ equal to the arc VX. With the semidiameter EX drawn, and Xt cut off equal to the semidiameter of the parallel, let tb be joined, and let the line that bisects it at right angles cut uL, cutting Xt toward t, and produced to L. (For this perpendicular will cut the semidiameter of the parallel in which the given point lies, either toward the given point — even when produced, if need be — or it does not cut it at all, or finally it is produced in the contrary direction, according as the angle at the end of the line which was cut off equal to the semidiameter of the other parallel is acute, right, or obtuse.) And then, with the straight line drawn from L through the center b, let it cut the parallel at Q. I say that the arc IQ is equal to the arc VX in the sphere.[d] For again the bases tL, bL are equal. Since therefore tX, bQ are also set out equal, the whole lines LX, LQ will be equal. Therefore, by Lemma 42, the circle described from L through Q, X will touch the parallels; and consequently, by Lemma 44, the arcs IQ, VX — or ρQ , ρX — will be equal in the sphere. The same point Q will also be found by the straight line LQ, making the angle tbL equal to the angle btL, or by drawing the straight line XQ parallel to bt, as was demonstrated above — even if bt is not bisected, and so on.

[Margin: [d] Euclid I.4]

Let there also be described a parallel of the Equator, $\theta\xi\kappa\lambda$, equal to the former and opposite to it, by which the same oblique parallel IKl is to be divided. And because the parallels $\theta\xi\kappa\lambda$, IKl are equal and [tend] toward different parts of the sphere, they will take in themselves equal parts, corresponding on the same side and progressing toward the same [direction], as was said in Lemma 23 — namely from the points ξ , I toward κ , l, or from λ , N toward κ , l, and so on. Let there therefore be taken an arc λT similar to the arc RS; and let the arc NM be found from the arc λT to be the same [as before]. With the semidiameter ET drawn, let there be cut off from it, produced, the line Tm equal to the semidiameter of the other parallel; and let the line mb be joined, bisected at n, and through the straight line no drawn at right angles, cutting ET at o, let ob be connected, cutting the parallel at M. I say that the arc NM is equal to the arc λT — that is, to the arc RS — the same as was found before by the arc RS.[e] For since om, ob are equal in the triangles mno, bno, if the equal parts Tm, Mb are taken away, the remainders oT, oM will be equal. Therefore the circle described from o through T, M will touch the parallels T, M, as was shown in Lemma 42; and consequently, by Lemma 44, the arcs λT , NM will be equal in the sphere. And if an angle mbo is

made equal to the angle Emb, or if TM is drawn parallel to mb, the same point M will be found, even if mb is not bisected at right angles.

[Margin: [e] Euclid I.4]

Again, let an arc equal to a given arc ξp be cut off, [equal to] IK. With Ep drawn, let there be taken in it, outside the parallel, a straight line pq equal to the semidiameter of the parallel IKl. Let the straight line qb be joined, bisected at right angles at r, [and let the perpendicular] through the straight line cutting Eq at f be joined by the line fb, cutting the parallel at K; and the arc IK will be equal to the arc ξp in the sphere — which will be demonstrated as was said of the arc NM.

By a like method, if in any oblique great circle AFCG a point a is given, we shall find the corresponding point P in its parallel IKl. And the same will be done if the two said circles are parallel, though neither of them be a great circle. For from the center H of that [circle] in which the point is given, let a semidiameter Ha be drawn; and outside the parallel let βb be taken equal to the semidiameter of the other parallel, and let us join βb , which we cut in half at γ , and at right angles [draw] the line $\gamma\delta$ cutting $H\beta$ at δ . For when δb is joined, it will cut the parallel at P, the point sought — which will also be found if the angle $\beta b\delta$ is made equal to the angle $b\beta H$, or if through δ a line aP is drawn parallel to βb . This will be demonstrated as was just now said. For

[Margin: To divide any oblique parallel of the Astrolabe into degrees, from its own center and from the center of the Astrolabe, or from another parallel to which it is equidistant, or from another parallel divided into degrees.]

[Margin: Euclid I.4] For again $\alpha\beta$ and αb will be equal, in the triangles $\alpha b\gamma$ and $\alpha\beta\gamma$; and if the equal segments Pb and $\alpha\beta$ are subtracted from them, the remainders δP and $\delta\alpha$ will be equal, and so on.

CONVERSELY, from a given point P the corresponding point α in another parallel will be found. For once the semidiameter bP has been drawn, cut off outside the parallel the straight line P μ equal to the semidiameter of the other parallel AFCG. Then, joining μH , the rest is completed as before.

By this method, with Lemma 45 brought to bear, from any point of the Horizon — or of any of its parallels — the corresponding point can be found in any other parallel of it, and conversely.

Diagram. A large square-framed woodcut of the stereographic projection used in the construction of §36 (the Equator and an oblique great circle). A great circle is

drawn with its vertical axis running from F at the top down to λ at the foot, with E on the axis. Chords, radii and several auxiliary circles (some dotted) cross a densely lettered central region. Readable peripheral labels, by position:

- F (top of the vertical axis)
- μ , α , B (top center to upper right)
- E (upper left, on the axis)
- P, C (upper region), with 1, H, B, b clustered at the upper right
- X (left), R (center-left), n, o (center-left)
- T (lower center-left), m (foot, lower left), λ (bottom center)
- G, V, D (right of center, descending), Y, A (right)
- θ , ς (far right)
- A dense central cluster of small labels including Z, M, L, E, N, K and others that do not resolve at this resolution [?].

You see, then, that when equal arcs in two circles proceed in the same direction — upward or downward, as happens in any parallels whatever, or in two circles inclining toward different parts of the sphere — the diameter of the one must be added to the semidiameter of the other; but when in the one circle one must descend and in the other ascend along arcs that answer to equal arcs on the sphere, then the semidiameter of the other must be subtracted from the semidiameter of the one, toward the center. This latter happens when two equal circles incline toward the same part of the sphere, as has been shown in the examples.

[Margin: What must be observed so that a circle may be divided into degrees by means of another divided circle. — To divide oblique great circles, and their parallels, into degrees by means of various circles described each through three points.]

36. Nor indeed should another very easy and agreeable method be passed over — a method for distributing both great and non-great circles into degrees, or rather for locating any given degree in a circle, whether great or not great. It is as follows. Let ABCD be the Equator, whose center is E, and AFCG the oblique great circle, whose pole is R. Take two points on the meridian line FD, equally distant from E (the pole of the Equator) and from R (the pole of the oblique circle), toward D and toward F — but not in the segment ER, lest one fall too close to the other. Of this kind are the points D and F, since the segments ED and RF represent the quadrants lying between the

pole of the world E and the Equator, and between the pole R of the oblique circle and that circle itself.

Now, the straight line FD having been bisected between the assumed points at a, draw through a perpendicular aT to FD, extended without limit in both directions. Given now a point q on the semicircle ABC of the Equator, 60 degrees distant from the point B, we shall find the corresponding point r on the semicircle AGC of the oblique great circle if, through the three points F, q, D — from center T (which, by the corollary to Euclid III.1, lies on the perpendicular aT) — a circle FqD is described, cutting the oblique circle at r. For the circle FqD represents that circle on the sphere which is drawn through the three points corresponding to F, q, D; and since F and D are equally distant on the sphere from the poles R and E, the pole of this circle will lie on the great circle drawn through the pole of the meridian FD and through the midpoint of the arc represented by the straight line FD — as we showed at the end of Lemma 47. Therefore, by the same Lemma, the said circle FqD will cut off equal arcs from the Equator and from the great circle AFCG, to which the arcs Bq and Gr correspond.

But if, through the same two points F and D and through the point b of the Equator (30 degrees distant from the point B), a circle FbD is described, having its center on the same perpendicular aT, then the great circle AFCG will be cut at the point f, 30 degrees distant from the point G.

The same point f will be found in this way. Let the straight line YX cut DG in half and at right angles, and through the points D, G and g (30 degrees distant from D) let a circle GDg be described from center X. For this circle will cut AGC at f. For again, as was shown at the end of Lemma 47, the circle GDg has its poles on the circle which bisects the arc DG on the sphere at right angles. Therefore, by the same Lemma, it will cut off from DC and GC the equal arcs Dg and Gf.

Again, we shall find the same point f by this reasoning. Let two equal arcs Cl and Sp be taken, and let the radii Al and Ap be drawn, so that the points n and m are obtained, equally distant from the poles E and R — since the segments En and Rm answered to the equal arcs Cl and Sp. For if the arc Bb of 30 degrees is taken on the Equator, and through the three points m, b, n a circle is described having its center t on the straight line kZ (which cuts m, n in half and at right angles), then CG will be cut at the point f, which will correspond to b itself, as is clear from Lemma 47.

Moreover, if through the three points B, b, G a circle BbG is described, having its center u on the perpendicular iV (which bisects BG), then CG will be cut at the

same point f — because the points B and G too are equally distant from the poles R and E. For since EB and RG are quadrants drawn from the poles to the great circles, once the common arc RE is taken away, the remaining arcs RB and EG will be equal.

And in this way, if other and still other points are taken equally remote from the poles R and E, and circles are described through each pair together with the given point b, the same point f will be found by several routes. The poles R and E themselves can also be taken as the points, provided their distance is not too small.

So too, if through the points F and B, and through the point b (30 degrees distant from B), a circle Bb is described, having its center on the straight line QP (which is perpendicular to FB and bisects it), the point N will be found, corresponding to the point b. For, as was shown at the end of Lemma 47, the circle FBbN has its poles on the great circle which bisects the arc FB on the sphere at right angles, and which accordingly passes through C and A, the poles of the circle FBD. Therefore, by the same Lemma, the circle FBbN will cut off from the circles BC and FC the equal arcs Bb and FN.

And so, in order that — by means of two points equally remote from the poles R and E — a point may be found on the semicircle AGC that is any number of degrees distant from the point G, one must take the corresponding point on the semicircle ABC of the Equator; but on the semicircle ADC a point must be assigned so that the corresponding point may be found on the semicircle AFC. If, on the other hand, by means of the two points D and G, any point is to be found on the semicircle AGC, then the corresponding point must be taken on the semicircle ADC of the Equator. And if, finally, by means of the two points F and B, a point is to be found on the semicircle AFC, then the corresponding point must be taken on the semicircle ABC. All of which is derived from Lemma 47, and has been observed here in locating the points. For from the point g and the points n, m (equally distant from E and R), the point N was determined by means of the circle gnmN; and likewise, from the point b and the points F, B, the same point N was found by means of the circle FBbN.

Diagram. A square-framed woodcut of the stereographic projection, inset at the right of the text, showing the Equator, the oblique great circle and their parallels with numerous auxiliary circles (several dotted). The vertical axis runs from E at the top to D at the foot. Readable labels, by position:

- E (top), with P, Q, α just below the top center, and H, B in the center just above the middle
- N (upper left), then T, Z, C descending the left side, and I at the lower left, δ at the bottom left corner
- A (right, at the middle), S (lower right)
- a symbol at the right resembling $\mathfrak{h}$ /"h" [?] and a small "r" [?]
- D (bottom center)
- A dense central cluster including R, ξ , C, F, L, K, ς , n, Y, G, O, X, Y and further small labels that do not resolve [?].

The same procedure is to be observed in the case of non-great circles: if, for a given non-great circle, a parallel of the Equator equal to it is described, distant from the north pole by just as much as the given circle recedes from its own upper pole — the pole that lies within the Equator. For example, let HIKL be the oblique parallel, whose pole is R, and let MO be the boreal parallel of the Equator equal to it [reading of "a e MO" uncertain: ?]. Then the point I corresponding to the point M will be found by means of the circle FMD, or by the circle MnmI from center h, or by MGBI from center δ .

But if a non-great oblique circle is nearer to its lower pole than to its upper one, then — provided the dividing circle is to be described through its upper pole and through the north pole — an austral parallel of the Equator equal to it must be described (because in this way both circles, from their poles through which the dividing circle is to be described, will have equal distances). And the straight line between the north pole and the upper pole of the oblique circle, or the straight line between two points equally distant from them, must be bisected, so that on the perpendicular drawn from that midpoint the center may be found of the circle to be described through those two poles or those two points, and so on. But if the lower pole of the oblique circle is assumed, then a boreal parallel of the Equator equal to it must be described (because, this being posited, both circles, from their poles through which the dividing circle is to be described, will be equally distant). And the straight line between the north pole and the lower pole of the oblique circle, or the straight line between two points equally distant from them, must be bisected, and so on.

And if, in the case of great circles, the straight line between the north pole and the lower pole of the oblique circle is bisected, then equal parts will be cut off from the Equator and from the oblique circle in the same order which we said must be observed when the construction is set up by the first method, from the divided upper pole of the oblique circle — but not in the order we prescribed in Lemma

47. That is, the cut-off arcs must take their beginning from the points F, B or D, G, and not from the points F, D or B, G. In the same way, in the case of non-great circles, when the parallel encircles the lower pole, the cut-off arcs must be begun in it, and in the austral parallel of the Equator equal to it, from the points [that lie] on the upper ones or the lower ones; and the circle is to be described through the upper pole and the north pole, so that the curvatures of the cut-off arcs proceed in the same order — that is, tending either upward or downward.

But so that you may also learn by experiment that the proposed points are correctly found in the oblique circles by this method: we found the point N from the upper pole by the straight line RbN; and the point r by the straight line Rrf; and the point f, if the line Rfg were also drawn.

[Margin: The most excellent method for finding a given point on any oblique circle, by means of a parallel on the right sphere (*sphaera recta*).]

Now, since C and A are the poles of the great circles drawn through the poles of the world and through the poles of the oblique circles AFCG, HIKL — which [great circle] the straight line FD represents — then, if about one of them, say about C, there is described (through the given point b on the Equator) a parallel of the circle FED, as will be taught in Proposition 18, Number 5, whose center lies on the straight line AC (as will be evident from Proposition 7), the oblique circle AFCG will be cut at the point N, which corresponds to the point b, as is clear from the same Lemma 47. But if, about the pole A, a parallel is described in the same way through the given point M in a parallel of the Equator, the oblique parallel will be cut at the corresponding point I. Indeed, if the arc FB is bisected at α , as was delivered in Proposition 5, Number 18, and if through A, α , C the great circle A α C is described, and about any point β or γ of it a parallel of that great circle — whose pole is β or γ — is described through the given point b or g on the Equator, as we shall prescribe in Proposition 18, Number 6, then the former parallel will cut the oblique great circle at N, and the latter will cut the same at f, as is evident from the same Lemma 47.

So too, if the arc ER — placed between the pole of the parallel of the Equator and the pole of the oblique parallel — is bisected at ς , then, by what we wrote in Proposition 5, Number 18, and if through A, ς , C the great circle is described, and about any point of it, by the doctrine of Proposition 18, a parallel is described through the given point M in a parallel of the Equator, the oblique parallel will be cut at the point I, which corresponds to M itself.

But the former route — by parallels described about the poles C, A — is preferable, both because parallels about those poles are more easily described through a given point (since they are parallels of the right sphere), as will be delivered in Proposition 18, Number 5, and also because parallels whose poles are A and C cut off, from any oblique great circle and its parallels, arcs corresponding to a given arc on the Equator or on its parallel. Thus the parallel described through the point b will cut the oblique great circle at N and f, and the arcs FN, Gf will be equal to the arc Bb or Dg. You will find an example of this matter in Proposition 18, Number 5. To this is added that, in this method, it is not necessary that the non-great circles have their poles on the great circle FD, equally distant from the middle great circle (as was said in Lemma 47), or in determined places; it is enough that they answer, on the sphere, to equal circles — whether the parallel of the Equator be austral or boreal, wherever the non-great oblique circle has its poles on the circle FD. Thus, as in the figure of Lemma 47, the parallel described about the pole B cuts off [equal arcs] both from the infinitely many great circles drawn through B and from the infinitely many equal non-great circles that have their poles on the great circle ADC, and that have equal arcs, [all] at once.

Diagram. A square-framed woodcut of the stereographic projection, inset at the left of the text; it appears to be the same figure as fig-p545-1, reprinted. The vertical axis runs from E at the top to D at the foot. Readable labels, by position:

- E (top), with P, Q, α just below and H, B in the center above the middle
- N (upper left), then T, Z, C descending the left side, I at the lower left, δ at the bottom left corner
- A (right, at the middle), S (lower right)
- a small "r" [?] at the right, K and V within the central region
- D (bottom center)
- A dense central cluster of small labels (including R, K, and further letters) that does not resolve at this resolution [?].

The same thing will occur in the figure proposed a little earlier. For if, about C or A, a parallel of the great circle FED is described, as we shall teach in Proposition 18, Number 5, it will cut off — from the circles whose centers lie on the straight line FD, and accordingly which have their poles on the same line, whether those circles be great or not great — two equal arcs corresponding to that arc of the Equator, or of the parallel of the Equator, through whose extremity the parallel about the pole C or A was described, provided the parallel of the Equator is equal to the non-great circle from which the arcs to be cut off are proposed, exactly as

happens on the sphere. And this method is inconvenient only when the given point on the Equator, or on its parallel, is little distant from the straight line FD; for then the parallel to be described through it is too large, so that its center can scarcely be had on the straight line AC.

37. Finally, we shall be able to divide any oblique parallel into degrees by that most beautiful method which we set out in the preceding proposition, Number 36, for great circles.

[Margin: Another most beautiful method of dividing any parallel into degrees.]

For let ABCD be the Equator, about center E; let AFCG be the great circle, whose true diameter is ik and whose axis is LE; and let $\alpha P\beta Q$ be its parallel on the astrolabe, whose true diameter is IN, meeting the meridian line at the point S. Through S draw Sp perpendicular to FD; this will be the common section of the plane of the Equator and the plane of the parallel on the sphere. [Margin a: Theodosius I.15] For since both the Equator and the parallel are perpendicular to their proper Meridian — because the Meridian passes through the poles of each — [Margin b: Euclid XI.19] their common section too will be perpendicular to the same [Meridian]; and accordingly, by Euclid XI, definition 3, it will be perpendicular to the straight line FD (which lies in the Meridian) at the point S, where the parallel meets the plane of the Equator. Therefore the perpendicular Sp is the common section of the parallel and the Equator.

Next, from the straight line SM cut off ST equal to it, either downward or upward; and from T describe a circle VXZY with radius equal to the semidiameter of the parallel, MN or MI — which circle will be equal to the parallel on the sphere. And indeed, if the circle ABCD is taken as the parallel's own Meridian and is conceived as perpendicular to the Equator (that is, to the plane of the astrolabe), and finally if the plane in which the circle VXZY lies is rotated about Sp, this circle will coincide with the parallel on the sphere. If, therefore, from the points V, X, Z, Y — and also from the center T, or from any other point of the plane in which that circle lies — straight lines are drawn through any points of the circumference, the common section Sp will be cut at the same points at which it would be cut if, from the corresponding points of the parallel in its proper position, straight lines were emitted through the same points of the parallel's circumference.

Now the point X corresponds to the point P on the apparent diameter (which is obtained if, from θ , the center of the proper Vertical circle — shown by the straight line $A\pi$, perpendicular to LE at a — the Vertical is described through the pole K, cutting the parallel at P, Q; for the straight line PQ will be the apparent

diameter, and R the apparent center, which is also found by the radius AM drawn to M, the true center). And Y corresponds to Q; V to the point β ; and Z to α — namely, left to left, right to right, the more remote from the common section Sp to the more remote, the nearer to the nearer, and center to center.

From any of these points of the visible parallel, therefore, we shall divide the parallel itself into degrees, if from the corresponding point on the true parallel we draw a straight line through the given point on the circumference, and then, through its intersection with Sp, we emit a straight line from the corresponding point on the visible parallel. For this line will pass through the sought point of it. For instance, if from the point V a straight line is drawn through the given point n, cutting Sp at u, then the line βu [?] will give the sought point r — which...

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Grienberger's Scholium (continued, §37–§38) ...which
 corresponds to the point n. The reason is that the line Vnu is projected onto the line βru , since the point V appears at β and u at u. In the same way, if from the point Z a line is drawn through n cutting Sp at γ , the line $\alpha \gamma$ will give the same point r. Again, if from X a line is drawn through n cutting Sp at p, the line Pp will pass through the same point r. Likewise, if the line Yn is drawn cutting Sp at t, the same point r will be found by the line Qt.

But the operation is carried out most conveniently by the lines drawn out from the points V and Z — from V, indeed, through the degrees of the semicircle XZY, but from Z through the degrees of the semicircle XVY. For in this way the points of intersection on the line Sp will not fall far from the point S. And by the lines drawn from V there will be found, on the arc P α Q, points corresponding to the points of the semicircle XZY, if from β lines are made to issue through the intersection-points on the line Sp produced by the lines from V; while by the lines issuing from Z there will be found points on the arc F β Q corresponding to the points of the semicircle XVY, if from α lines are thrown out through the intersections on the line Sp made by the lines drawn from Z.

[Margin: Which two points of the oblique parallel are best suited for the division.]

If a line drawn from the centre T through the given point n can conveniently intersect Sp — such as the line Tn, cutting Sp at q — we shall show, by the line Rq thrown out from the apparent centre through q, two points r and q, of which the former corresponds to the point n and the latter to the point 4, opposite to it through the diameter.

[Margin: Given a point in the apparent oblique parallel, to find the corresponding point in the true oblique parallel.]

CONVERSELY, from any given point on the apparent parallel we shall find the true degree to which it corresponds, if from one of the points α , P, β , Q, R on the apparent parallel we draw, through the given point, a line cutting Sp at some point. For the line drawn from the point of the true parallel that corresponds to the assumed point, carried out to the intersection-point, will pass through the true corresponding point. Thus, because the line βr cuts Sp at u, the line Vu will give the corresponding point n, so that the arcs αr and Zn embrace an equal number of degrees.

[Margin: Given a point in the plane of any oblique parallel in the sphere, to find its position in the astrolabe.]

BY A NOT DISSIMILAR method, if a point be given in the plane of any oblique parallel, we shall find its position in the astrolabe — that is, the place where it appears on the same apparent parallel, viewed from the southern pole. For let the given point be bb, which is to be conceived as having such a position in the sphere, with respect to the circle VXZY, that it lies near the eastern and southern quadrant, outside the circle. For if the parallel VXZY has its proper situation, the quadrant XZ is eastern and southern, and XV is eastern and northern, and so on. Therefore, drawing from any two points whatever — say from T and V — through the given point bb, lines cutting the common section at the points 3 and 7, let there be drawn to 3 and 7, from the corresponding points R and β , the lines R3 and $\beta 7$; for where these intersect each other, at the point 2, there will be the apparent place of the given point bb. This is because the lines T3 and V7, passing through the given point bb, are projected onto the lines R3 and $\beta 7$, as is clear from what we have said.

[Margin: Which true points in the plane of the oblique parallel in the sphere have no corresponding points in the astrolabe.]

BUT WE MUST EXCEPT the points lying on the common section of the oblique parallel with the plane that is drawn through the southern pole parallel to the equator. For these can have no corresponding apparent points in the astrolabe,

since that whole common section vanishes in the astrolabe, and no point of it appears in the astrolabe's plane — inasmuch as all the visual rays lying in that parallel plane, and passing through the points of the said common section, are projected equidistant from the plane of the astrolabe (that is, from the equator). For example: if from A, the southern pole, a line Al be drawn perpendicular to AC, or parallel to the plane of the equator, the plane drawn through Al parallel to the equator will meet the plane of the parallel through ll (drawn to l), and will make a common section through l, perpendicular to ll. If, then, the line Sl [Margin: (a) Euclid XI.16] — which is always equal to the semidiameter of the primary vertical Aθ, on account of the parallelogram AS — be made equal to SG (this is to be cut off below S if the true parallel is above S, but above S if below; for in this way the point G, corresponding to the point l, will have the true distance from the true parallel, as is evident if the situation of the true parallel is rightly conceived, and the plane of the astrolabe is carried around Sp until it coincides with the line ll lying in the plane of its own meridian) — then the said common section is to be drawn through G, perpendicular to FG (it does happen, in one case, that the line SG is equal to the line Sl). And so, if anyone tries to find the apparent point corresponding to the point G, by drawing from G to the point n a line cutting Sp at f, he will find that the line drawn from f through the point r (which corresponds to n) is parallel to the line FG; and he will experience the same thing with the other lines. Consequently the lines drawn through the intersection-points on Sp to the apparent points corresponding to the true points to which lines were drawn from G, will in no way cut one another, so that an apparent point might be had at their intersection. In the same way, if anyone wishes to investigate the apparent point corresponding to any other point on the line drawn perpendicular to FG through G, he will find that the other lines drawn through the intersection-points on the line Sp are parallel to one another, although they are not equidistant from FG itself, and so on.

THE SAME may be observed in the great oblique circles, as was said in the preceding proposition, No. 36. For since the plane parallel to the equator, drawn through the line Al, meets the plane of the great circle at m, if a line Eb be cut off equal to Em [Margin: (b) Euclid I.34] (Em being likewise perpetually equal to the semidiameter of the vertical Aθ, on account of the parallelogram AE), the aforesaid common section of the plane of the oblique circle and the plane of that parallel is to be drawn through b. If, then, anyone wishes to exhibit the apparent point corresponding to the point b, by drawing from b, through some point of the true oblique circle — say through O — a line that cuts AC at e, then the line drawn through e to c (the corresponding point on the apparent oblique circle) will be parallel to that same FG. And in this way other parallel lines too will be found

parallel to the same FG. Therefore these apparent lines will in no way cut one another, so that an apparent point might be had. From other points of the aforesaid common section, drawn through b, other lines will be found parallel to one another, although they are not equidistant from FG. But that the lines drawn from the points of this common section to any points whatever of the true oblique circle are projected into parallel lines will become clearer from what we shall demonstrate shortly.

LET IT THEREFORE be proposed to divide the great oblique circle into degrees from the true point b, which corresponds to m,

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[Margin: To divide the great oblique circle of the astrolabe into degrees by parallel lines.]

and the oblique parallel from the true point G, which corresponds to l — which indeed will be done by parallel lines, in this way. From b, through any given point O, draw a line to the common section of the oblique circle and the plane of the astrolabe (or equator) at e; and through e let a line ec be drawn parallel to FG, cutting the apparent oblique circle at the point c, which I say corresponds to the given point O. For if, through the line Al lying in the plane equidistant from the equator, and through b passing in its proper situation, the plane be carried around, it will project onto the plane of the astrolabe (or equator) lines parallel to Al, so that the plane thus carried around is projected into lines parallel to Al, and hence parallel to one another. Therefore, since the plane drawn through Al and bO meets AC (the common section of the equator and the oblique circle) at e, it will be seen to pass through the parallel ec; and hence, since it is drawn through O, the point O will appear at c, since it appears on that parallel. Here you see that the line drawn from the pole K through O falls upon the same point c, as the matter requires, just as was demonstrated in Prop. 5, No. 17. And that same parallel ec indicates, on the other side, another point f, which corresponds to the point d [Margin: To divide the oblique parallel of the astrolabe into degrees by parallel lines.] — as is also indicated by the fact that if a line of the oblique circle be drawn cutting AC at g, the parallel gh will give the point Kd. Again, if from b, through L, the true pole of the oblique circle, a line be drawn, h corresponds to L, and the line KL also falls upon it; and the point h is at the extremity of the diameter of the horizon hμ, perpendicular to FG, so that the arc hC corresponds to the arc LC — which we also demonstrated in the scholium to Prop. 5, near the end, No. 14. Moreover, the line bL touches the circle ABCD at the pole L, and cuts off a line Eg equal to the semidiameter of the apparent horizon. For since the two

sides bE , EL of the triangle bEL are equal to the two sides mE , EA of the triangle mEA [Margin: (b) Euclid III.27], and contain equal angles (because the arcs Ai , BL , which measure the elevation of the pole above the oblique circle, are equal) [Margin: (c) Euclid I.4], the angles bLE , mAE will also be equal. Since, then, mAE is a right angle, bLE will also be a right angle; and therefore, by the corollary to Prop. 16 of Book 3 of [Euclid], bL will touch the circle at L . And that the line Eg cut off is equal to the semidiameter of the horizon Hh [Margin: (d) Euclid I.34] is clear, on account of the parallelogram gH .

LET the apparent point now be assigned to the point n of the true parallel. Draw from G — the true point corresponding to l — the line Gn , cutting the common section Sp at f . For the line fr , parallel to FG , will present the corresponding point r ; which will be demonstrated in the same way. For if, through the line Al lying in the plane equidistant from the equator and touching the sphere at the southern pole A , and through G passing in its proper situation, the plane be carried around, it will make in the plane of the astrolabe (or equator) lines parallel to Al , onto which that plane thus carried around is projected. Therefore, since the plane drawn through Al and Gn meets Sp (the common section of the plane of the equator and of the parallel) at f , it will be seen to pass through the parallel fr ; and hence, since it is drawn through n , the point n will appear at r , since it appears on that parallel onto which the line Gn is projected.

[Margin: To divide oblique circles — both the great ones and their parallels — into degrees by straight lines drawn through their apparent centres.]

FINALLY, we shall divide any great oblique circle whatever, and its parallels, into degrees by straight lines that pass through their apparent centres, each of which lines exhibits two points opposite through the diameter, in this way. Let an arc $\xi\tau$ be taken equal to the arc $A\xi$, and let the line $A\tau$ be drawn, cutting FD at θ , the centre of the primary vertical, as we showed in Prop. 5, No. 3 and 4; and through θ let $\theta\lambda$ be extended, perpendicular to FD , representing the parallel of the given great oblique circle, which is drawn through the southern pole, as was demonstrated above, No. 3. Then, having described from K , the apparent pole, a circle $\Lambda\varsigma$ of any magnitude whatever (we described one equal to the equator, so that the degrees of the equator might more easily be transferred into it), let lines be drawn through its degrees from K , both through these section-points and through the apparent centre of the great circle, as well as through the apparent centre of the parallel; and each circle will be divided into degrees. The corresponding arc on the apparent oblique circle — whether great or not great — if it is given in the semicircle in which the pole K lies, on the opposite side is taken

equal to [the corresponding arc]; equal to the equator (since the given arc of the equator is given in the other semicircle, in which the pole K is not, a similar or equal arc is to be taken on the described circle $\Lambda\zeta$) — [these are] the lines cutting the line $\theta\lambda$ at [various] points. For if, through these section-points [and through the apparent centre of the great circle] — that is, through E — as well as through R, the apparent centre of the parallel, [lines be drawn, each circle will be divided]. For example: if to the arc BO a corresponding arc is to be found not on [the apparent great circle] but on its parallel, let there be taken [an arc equal to] BO — if it is given in that semicircle [in which the pole K lies] — or equal to it, if the circle $\Lambda\zeta$ is described equal to [the equator; but if it is given] in the other semicircle, in which the pole K is not, the arc is to be taken [on the opposite side] — and let the line $K\theta$ be drawn cutting $\theta\lambda$ at λ . For the line λE , drawn out through E, the apparent centre of all the great circles, will cut off [two opposite arcs], of which one is Fc. Likewise the line from λ through R, [the apparent centre of the parallel, will set] as many degrees as are comprehended in BO. And the same [will be effected by the line from λ through the apparent centre] of the other parallel, whose pole is K. And this we shall demonstrate in the following manner: [if the triangle... whose sides] are both semidiameters of the primary vertical of the oblique circle, [be understood to be moved] downward, toward the southern pole, until it reaches the plane of the astrolabe, and consequently the point A coincides with the southern pole — [and it is understood that about the line $\theta\lambda$] there is likewise moved downward the line $K\theta$, together with the plane of the circle $\Lambda\zeta$, until it reaches the line $A\theta$, [then] K [will fall] into the pole A, and the plane of the circle $\Lambda\zeta$ will be parallel to the oblique circle. [The remainder of this dense demonstration — the two planes through the lines $K\theta$, $K\lambda$ making, in the oblique circle of the sphere, lines parallel to $K\theta$, $K\lambda$; the equality of the angle $\theta K\lambda$; the projection by Prop. 1 onto the lines $E\theta$, $E\lambda$, cutting off the corresponding apparent arcs — follows, invoking Euclid XI.16, XI.10, and III.26. [?: several interlinear clauses at the gutter not fully legible].]

[Margin: (f) Euclid XI.16; (g) Euclid XI.10; (h) Euclid III.26]

AND this way is the most excellent, when several oblique parallels are to be divided into degrees, since

Diagram. A large square-framed stereographic-projection figure for the §37 construction (dividing an oblique great circle and its parallels into degrees by parallel lines). It shows two principal circles (an upper and a lower) inscribed in the square, with the equator ABCD, an oblique great circle, the primary vertical, and the horizon, crossed by numerous straight lines and dotted construction-

lines. This is the same woodcut reprinted across the recto/verso opening; it is described here once. Readable labels:

- 4 (upper left, inside the top circle)
- V (top centre); F (just below V); I (upper right, on a short oblique line outside the circle)
- X (left flank); Y (right flank)
- bb (left, inside the upper circle); T (upper centre); b (centre)
- h (far left, on the horizontal diameter); P (right, on the horizontal); M (far right, outside, on the extended horizontal)
- K (centre); H (centre); R (lower centre); L, Z, 3, 7, 2 (small labels clustered left-of-centre)
- Lower arc / border: R (lower left), G (lower centre), A (lower right, on the horizontal), D (bottom centre), π , λ , θ (bottom); Greek small letters α , β , γ , δ , ϵ , ζ , ξ , ς scattered through the interior
- Numerous additional small Greek/italic point-letters in the crowded interior are not individually resolvable at this resolution [?].

[p.197 — running header "PROPOS. VI."]

this from one and the same point found on the line $\theta\lambda$; when that point has been found, in all the parallels lines are drawn through the apparent centres, as has been said. The only inconvenience is when the points on the line $\theta\lambda$ are too far from the point θ : for then the lines drawn out from K cut the line $\theta\lambda$ too obliquely, so that those points can scarcely be found without error. Therefore in that case other methods must be used, namely those that shall seem more convenient.

38. I do not wish, even here, to pass over another method which, after all the ways I have explained, occurred to me, and which seems among all of them the most convenient — inasmuch as, from any point taken on the common section of the oblique circle and the plane of the astrolabe (or equator), outside the meridian line, one can project any point whatever, such that a line to the given point set in the circle exhibits it; so that one may take at will a point on the equator, if great circles are in question, or on the true parallel, if a point is to be found on the oblique parallel, and, when it is drawn out, it will most conveniently cut the proper meridian line.

[Margin: Another most convenient way of dividing oblique circles — both the great ones and those not great — into degrees, from any point given on the

common section of the oblique circle and the plane of the astrolabe (or equator) outside the meridian line.]

Let there be again, therefore, the equator ABCD, whose centre is E; the great oblique circle AFCG, whose true diameter is HI, and apparent pole i; the true diameter of the vertical of its own oblique circle gh; the true diameter of the parallel of the same oblique circle CK, and the apparent parallel LtE; and finally the true parallel upf, with the common section SX, as was said in the preceding method, No. 37.

Now let the given point be K, first on the equator — that is, on the true great circle — for which the corresponding point on the great oblique circle is to be investigated. From any point N taken on the common section AC of the plane of the astrolabe and the oblique circle in the sphere (most conveniently it will be taken on the part opposite to the given point, as on the line EA, even produced, when the given point is on the semicircle BCD; but on the line EC, even produced, when the point is given on the semicircle BAD), let a line be drawn to the given point K, cutting the meridian line at some point, which for now lies between B and L; and of the line between E and that section-point let a line Ec be cut off equal to it from the true diameter HI; and from A, the southern pole, let a ray drawn through c cut EB at M. For the line NM will fall on the point O, upon which, of course, the line drawn from the pole i through K falls. For if the circle ABCD be imagined carried around AC until it reaches the diameter HI lying in its own meridian, the point of intersection of the line NK and the line EF will coincide with the point c; so that in the sphere the line NK, drawn to the given point K, cuts the diameter at the point c, which, seen through the ray Ac from the southern pole A, appears at M. The line NK, therefore, will be projected onto the line NM, and hence will fall upon O, the point corresponding to the given point K, just as NK falls upon the given point K.

LET the same corresponding point O be sought for the same point K, from the point A taken at the intersection of the circumference of the equator with the circumference of the great oblique circle. The line AK being drawn, cutting EB at L, let there be taken Ed equal to EL, so that d is the point on the true diameter at which the line AK cuts it, if the circles are conceived in their proper position. The point d will appear at P, by the ray Ad; and hence the same line AP will fall upon the sought point O.

MOREOVER, let the same point O be found from the point R. The line RK being drawn, cutting the line EB between B and V, let there be taken a line Ee equal to the line between this section-point and the centre E; and e will be the point at

which the line RK cuts the true diameter HI, if the circles are understood to have their proper situation. And the point e will appear, by the ray Ae, at Q. The line RQ, therefore, will represent the line RK, and hence will pass through the sought point O.

FINALLY, let the point q corresponding to the point Z be sought from the point Y. The line YZ being joined, cutting ED at m, let there be cut off Er equal to Em, so that we have the point r at which the line YZ cuts the diameter HI, if all things have their proper situation. And when the ray Ar is drawn, the point r will appear at o. The line Yo, therefore, will indicate the sought point q, upon which the line iZ also falls.

NEXT, let the given point p be on the true parallel, for which the corresponding point is to be found on the apparent one. From any point S of the common section SX (it will also be most convenient to take the point on the opposite part) let a line be drawn to the given point p, cutting EF at b; and let Va be cut off from the true diameter equal to Vb: then, the ray Aa being drawn, cutting EB at f, the line Sf, joined, will fall at k, the point corresponding to the given point p. For if the circle upf be conceived turned around SX, until it reaches the diameter Vc, which has its proper situation in its own meridian, the point of intersection b will coincide with the point a; so that in the sphere the line Sp, drawn to the given point p, cuts the diameter of the parallel at a, the point which, seen through the ray Aa, appears at f. The line Sp, therefore, will be projected onto the line Sf, and so on. And if the point f were given, the corresponding point t would be found in the same way.

BUT let the same point k, corresponding to the given point p, be found from the assumed point X. The line Xp being drawn, cutting EF at Q, let there be taken a line VQ equal to VT: then T will be the point at which the line Xp cuts the true diameter in its proper position, which, by the ray AT, will appear at n. The line Xn, therefore, will pass through the sought point k. And if the given point were u, the corresponding point l would be found in the same way.

[Margin: Given a point on the apparent oblique circle, to find the corresponding point on the true oblique circle.]

BY THE CONVERSE order, we shall investigate, for a given point on the apparent oblique circle, the corresponding point on the true circle. For if from a given point — say q — on the great circle, a line be drawn to any point Y of the common section, cutting ED at o, and the ray Ao be joined, cutting the true diameter at r,

we shall take a line Er equal to Em. For the line Ym will fall upon the sought point Z.

Diagram. A square-framed stereographic-projection figure for the §38 construction (dividing oblique circles into degrees from any point on the common section, outside the meridian). It shows the equator ABCD with centre E, a great oblique circle AFCG, its vertical and parallel, the common section SX, and a network of rays and dotted lines converging on the southern pole. Readable labels:

- Q (top); O and I (upper centre, close together — "O I"); M (upper-centre inside); P (centre)
- S (left, on the horizontal); X (right, on the horizontal)
- V (centre); E (centre, at the crossing of the diameters)
- R and T (lower left, on the horizontal); A (lower right, on the horizontal); N (far right, outside, on the extended horizontal)
- H (bottom left); Z (bottom left); D (bottom centre)
- Small Greek/italic interior labels (r , m , G , δ , and others near the lower centre) are not individually resolvable [?].

Again, if from a given point k on the parallel a straight line of the common section is drawn to any point S, cutting EB at f, and the radius Af is joined, cutting the true diameter at a, we shall take Va equal to Vb. For the straight line Sb will then indicate the required point p.

In just the same way, when a point is given in the plane of an oblique circle outside its circumference, we shall find the corresponding point on the astrolabe from any two points taken on the common section. Suppose, for instance, that the point p is conceived to lie in the plane of the parallel on the sphere, outside its circumference. From any two points S and X we draw, through p, straight lines cutting EF at b and Q; on the straight lines Vb and VQ we mark off equal segments Va and VT; and we join the radii Aa and AT, which cut EF at f and n. The straight lines Sf and Xn will then pass through the required point k. Conversely, if a point k is given on the astrolabe outside the circumference of the projected parallel, we shall find in the plane of the true parallel the corresponding point p as follows. From k we draw to the two points S and X of the common section two straight lines cutting EF at f and n; through f and n we emit radii from A cutting the true diameter at a and T. Then, if we mark off Vb and VQ equal to Va and VT, the straight lines Sb and XQ will cut one another at the true corresponding point p.

[Margin: Given a point in the plane of a circle on the sphere, to find the corresponding point seen on the astrolabe, and conversely.]

Now among all the methods of distributing the circles of the astrolabe — both the great circles and their parallels — into degrees, the most expeditious is the first, which we set out in Proposition 5, Number 17, and in the present proposition, Number 21. It is carried out, of course, by means of straight lines drawn out from the pole of the oblique circle; and it is especially convenient when, for the Aequator or its parallel, the circle itself is described from another centre, as was taught in Number 25 of this proposition. Indeed, if several circles of this kind are described, each according to one proportion or another, and each is divided into degrees, the several lines drawn from the pole of the oblique circle will pass through [the corresponding points], so that it may seem impossible that any error be committed in drawing them.

[Margin: Which method of dividing the circles of the astrolabe into degrees is the most expeditious of all.]

Diagram. A large square-framed stereographic-projection figure occupying the upper part of the page, showing the construction for locating a point of a parallel (or its plane) and its corresponding projected point. Straight radial lines run from the projection point through the intersecting circles. Clearly readable perimeter and principal labels:

- Q (apex, top)
- O, M, P (upper interior, on the vertical axis)
- S (left), X (right) — endpoints of the upper transverse line
- R, T (lower left)
- A (lower right, on the horizontal axis), N (far lower right)
- H (lower left corner region), Z, D (bottom, on the vertical axis)
- interior points including L, Y and small Greek-letter labels (γ and others): densely crowded and not individually resolvable at this resolution [?]

The interior of the figure carries roughly two dozen additional small letters that cannot be resolved cell-by-cell. Marked as a

Not otherwise than this, given a point in the plane of an oblique circle outside its circumference, we find the corresponding point on the astrolabe.

Yet if several such circles are described, each according to a different proportion, and each is distributed into degrees, the individual lines drawn from the pole of

the oblique circle pass through [all the division points], so that it may seem that no error can be committed in drawing them.

SCHOLIVM.

[Margin: Equal arcs of an oblique parallel are projected into unequal arcs, in continuous order.]

1. From the earlier part of the first method — by which the parallels of the oblique circles are distributed into degrees — it is readily gathered that equal arcs of any oblique parallel are projected into unequal arcs, in continuous order, beginning from the straight line that passes through the centre; just as we demonstrated that the same happens with the greatest oblique circles too, in the scholium of the preceding proposition, Number 12. This we undertook to demonstrate here, in Proposition 3, Number 3. Therefore, in the third figure of this proposition, let there be three equal arcs $P\beta$, βS , SQ in the parallel of the Aequator $OPQR$; and from K , the pole of the oblique parallel $FGHq$ contained within the Aequator, let three straight lines $K\beta$, KS , KQ be drawn, cutting the parallel at γ , T , G . To the arcs $P\beta$, βS , SQ there will correspond the arcs $F\gamma$, γT , TG — that is, as many degrees will be contained in the former as in the latter, as we demonstrated in Number 21 of this proposition. But since, by Lemma 33, the arc $F\gamma$ is greater than the arc γT , and this greater than the arc TG , and so on continually to the end of the semicircle FGH , it clearly follows that equal arcs of the oblique parallel on the sphere are projected onto the astrolabe into unequal arcs, in continuous order, together with the fact that the arc nearer to point F always becomes greater than the more remote one, provided they correspond to equal arcs of the parallel of the Aequator, as was demonstrated in Lemma 33.

Diagram. The "third figure" (*tertia figura*) referred to above: a square-framed stereographic projection of the parallel of the Aequator $OPQR$ with the oblique parallel $FGHq$ inside it, and the radial lines $K\beta$, KS , KQ from the pole K cutting the parallel at γ , T , G . Clearly readable principal labels:

- P (top), β , S (upper left), F , K (upper interior)
- B , θ , m , T , Z , Δ , α , R (crowded upper-central interior)
- Q , C , G (left), V (lower left)
- A (right, on the horizontal axis), N (lower right)

The central cluster of small letters is densely overprinted and only partially resolvable [?]. Marked as a

And so, if the oblique parallel FGHq is distributed into 360 degrees, as we taught above, these degrees will decrease continuously from F to H, in each of the two semicircles FGH and FqH, so that the degrees are greatest near F but least near H. From this it follows that the arcs of the oblique parallel on the astrolabe are not similar to the corresponding arcs of the same parallel on the sphere.

[Margin: Various properties of oblique parallels in the astrolabe.]

2. For fuller instruction, I should like here to demonstrate certain other things — not without use, and which I trust will not be unwelcome to students — concerning oblique parallels projected onto the astrolabe. For from these, among other things, it will be possible to gather how, through a given point on the astrolabe, the parallel of any oblique great circle may be described, as will be evident from Proposition 18; and likewise that it can happen that some arc of an oblique parallel, of however many degrees — provided they are fewer than 180 — is on the astrolabe similar to some arc of the same parallel on the sphere, which perhaps no one would easily believe, as we shall say toward the end of Number 5. This too we demonstrated concerning the greatest oblique circles in the scholium of the preceding proposition, Number 13.

Let, then, the Aequator be ABCD, with centre E, divided by two mutually perpendicular diameters AC, BD into four quadrants; the diameter of any oblique parallel FG, whose poles H, I are equally distant from F and G, and whose axis is HI; the apparent diameter of the parallel KL, found by the radii AF, AG; the parallel on the astrolabe KMLN, described from the centre O; its diameter MN, cutting KL at right angles; its poles P, Q, found by the radii AH, AI; and through these, the great circle described APCQ, at right angles to the greatest circle drawn through the poles of the world and the poles of the oblique circle, and making in the astrolabe the section BD, passing through A, C, as we demonstrated in the scholium of the preceding proposition, Number 1; the southern diameter of the parallel of the Aequator ST, cutting AC at l, and equal to the diameter of the oblique parallel FG, so that the distances AS, HF from the poles A, H are equal; the parallel of the Aequator itself described on the astrolabe VXY, whose semidiameter EY is exhibited by the radius AT; the diameter of the northern parallel of the Aequator, equal to the former, Za; and the parallel itself described bde.

Diagram. The "first figure" of this Number 2: a square-framed projection with the Aequator and the oblique parallel around the upper pole. Clearly readable principal labels:

- V (top), K, B, F (upper interior), S (upper right), H (upper left interior), M, N (mid interior)
- P (left), X (left, on the horizontal transverse), u (lower left)
- C, G (central-left), a (lower central), E (centre)
- I, T (lower right interior), D (bottom interior), Y (lower right), Q (bottom), q (right)

The central region is heavily overprinted with small letters that are not individually resolvable [?]. Marked as a

First, then, we shall demonstrate that YE, the semidiameter of the apparent parallel, is to EP — the straight line between the centre of the southern parallel and its pole — as the diameter of the oblique parallel is to OP, the straight line between its centre and its pole; whether the oblique parallel surrounds the upper pole, as in the first figure of this Number 2, or the lower one, as in the second figure. For the straight line AR being drawn to the intersection of the diameter of the oblique parallel FG with its axis HI, let the angle RAP be made equal to the angle [PAO], as was demonstrated in Number 9 of this proposition. Then AO will fall on O, the centre of the parallel, by what was shown in this proposition, Number 1[?]; and let it cut FG at f, and ST at g.

Since, then, the triangles AFG, AKL are similar — but subcontrarily placed, as was demonstrated in Proposition 3, Number 1 — the angle AGF will be equal to the angle AKL. [Margin: a — Euclid III.27] Moreover, the angles GAP, KAP, standing on equal arcs HG, HF, are equal. Therefore, in the triangles AGf, AKP, the remaining angles AfG, APK will also be equal. Again, from the equal angles GAP, KAP, the equals RAP, OAP being taken away, the remainders GAR, KAO are equal. And since the angles at G and K have been shown equal, in the triangles GAR, KAO the remaining angles ARG, OAK are likewise equal. Likewise, since in the triangles AfR, APO both the angles AfR, APO are equal (as we have shown) and the angles RAf, OAP are equal by construction, the remaining angles ARf, AOP will also be equal — from which it can also be proved that the remaining angles, taken from two right angles, are equal.

[Margin: b — Euclid VI.4] These things demonstrated, it will be, as GR is to RA, so KO to OA; and as RA to Rf, so OA to OP; and therefore, ex aequali, as GR to Rf, so KO to OP. [Margin: c — Euclid III.14] Now since FG, ST are equal and equally

distant from the centre E, the perpendiculars ER, El will be equal — for the axes EH, EA are perpendicular to the parallel diameters FG, ST [Margin: d — Theodosius I.10 (?), "10.1. The."], and consequently also perpendicular to those diameters themselves, by Definition 3 of Book 11 of Euclid. These being taken away from the semidiameters EH, EA, the remaining straight lines HR, Al will be equal; and since in the triangles HRf, Alg equal angles are adjacent — for the angles at R and l are right, [Margin: e — Euclid I.5] and the angles EHA, EAH in the isosceles triangle AEH are equal — [Margin: f — Euclid I.26] the straight lines Rf, lg will also be equal. But GR, Tl, the halves of the equals FG, ST, are equal. Therefore it will be, as GR is to Rf — that is (since it was just shown that as GR to Rf, so KO to OP), as KO to OP — so Tl to lg.

Proportion (continued):

GR,	KO,
RA,	OA,
Rf,	OP,

that is, $GR : RA : Rf = KO : OA : OP$.

Since, then, from the scholium of Proposition 4, Book 6 of Euclid, as Tl is to lg, so YE to EP, it will also be, as KO to OP, so YE to EP; which was to be demonstrated. And this demonstration, together with those that follow, holds whether the oblique parallel surrounds the upper pole, as in the first figure, or the lower one, as in the second, as is evident from the figures.

[Margin: The apparent semidiameter of the parallel of the Aequator is divided at the pole of the oblique circle in the same way as the true semidiameter of the oblique parallel is cut by the radius drawn through that same pole.]

From this demonstration it is gathered that the apparent semidiameter VE of the parallel of the Aequator is cut by the pole P of the oblique circle, as seen, in the same way in which the true semidiameter RF of the oblique parallel is cut equally at f by the radius APH drawn to H, the true pole of the oblique circle; namely because it has been shown that, as GR — that is, as RF — is to Rf, so KO to OP, and as KO to OP, so YE, that is, so VE, to EP, and so on. And the same ratio holds in the others.

3. Next we shall show that the straight line XP, produced, falls on N, the extremity of the diameter MN — that is, that the three points X, P, N lie on

one straight line; which must also be said of the three points q, P, M. Likewise, that the straight line Qh, drawn from the opposite pole Q through h, the intersection of the great circle APCQ with the oblique parallel KMLN, falls on M, the other extremity of the diameter MN; and in the same way that the straight line Qr, produced, falls on N. Finally, that the straight line mh, drawn from m, the centre of the greatest circle APCQ, to h, the intersection of the same greatest circle with the oblique parallel, touches the oblique parallel at the point h. And this last we also showed above, in this proposition, Numbers 7 and 30, otherwise than here. For let XP, produced, cut MN at N. I say that N is the extreme point of the diameter MN.

Diagram. The "second figure" of this Number 2: a square-framed projection with the oblique parallel around the lower pole, showing the great circle APCQ and the diameter MN. Clearly readable principal labels:

- u, a (upper left), X (left, on the horizontal), C (upper central), n, p (left interior), M (lower left)
- I (upper right), B (upper central), T (upper), A (right, on the horizontal axis), E (centre), F, S (central interior), H (central), P (lower central), R, L (interior)
- O (bottom), N (lower right), K (bottom edge), Y (right), Q (right), γ (interior), Z, G (left interior)

The dense central cluster of small letters is not individually resolvable [?].

Marked as a

For since the triangles EPX, OPN are equiangular — having right angles at E, O, [Margin: a — Euclid III.29] the vertical angles at P equal, and finally also the alternate angles X, N equal — [Margin: b — Euclid VI.4] it will be, as XE (that is, as YE) to EP, so NO to OP. But it was shown in Number 2 that, as YE to EP, so KO to OP. Therefore it will be, as NO to OP, so KO to OP; [Margin: c — Euclid V.9] and consequently NO, KO will be equal, and therefore NO will be a semidiameter of the parallel. XP therefore falls on N, the extremity of the diameter MN — that is, the three points X, P, N lie on one straight line. And the same will be proved of the three points q, P, M. Which was the first point.

Now since, as was shown in this Proposition 6, Number 21, the straight line PX, cutting off from the parallel of the Aequator the quadrant VX, also cuts off a quadrant from the oblique parallel; and the greatest circle APCQ, together with the circle which the straight line VQ represents, cuts off a quadrant, so that Kh, hL

correspond to quadrants — NPX will pass entirely through the point h, the intersection of the greatest circle APCQ with the oblique parallel. [Margin: d — Euclid III.31] Therefore the angle PhQ, being in a semicircle, will be right; and consequently, Qh being produced to M, the angle NhM will also be right. [Margin: e — Euclid III.31] Since, then, the angle contained by the arc Kh of the greater segment, together with the straight line hN, is greater than a right angle, Qh produced will fall within the circle KhL; and consequently the arc in which the right angle NhM stands will be a semicircle, by the scholium of Proposition 31, Book 3 of Euclid.

[Scholium to Propositio VI, continued — Num. 5, on similar projected arcs]

Euclid. And so, since MLN is a semicircle, Qh, when produced, will cut the circle at M, the endpoint of the diameter MN, so that that right angle can exist within a semicircle. By the same reasoning Qr, produced, will fall at N; which is the second point.

Finally, let the straight line Oh be joined. Since the angles OhN and ONh are equal [Margin a: Euclid, *Elements* I.5], and since the alternate angle PXE is also equal to the angle ONh [Margin b: *Elements* I.29], and the angle PQh is equal to this PXE — for since the triangles PXE and PQh have the angle P in common and right angles at E and h, as we showed, they will also have their remaining angles X and Q equal — the angle PQh will therefore also be equal to that same angle ONh; and consequently the angles OhN and PQh will likewise be equal to one another. But the angle PQh is equal to the angle mhQ in the isosceles triangle hmQ [Margin c: *Elements* I.5]. Therefore the angles OhN and mhQ will also be equal; and when the common angle mhN is added, the whole angles Ohm and NhQ become equal. But NhQ — that is, PhQ — was just shown to be a right angle. Therefore Ohm too will be a right angle; and for that reason the straight line mh will touch the oblique parallel — by the corollary to *Elements* III.16 — at h, the intersection of the great circle APCQ with the oblique parallel KMLN. In the same way we shall show that the straight line mr, drawn, touches the same parallel at r; which is the third point.

3. In the third place [TERTIO loco], certain things must be demonstrated concerning similar arcs in each parallel, KMLN and VXY. So, with the perpendicular Pn drawn from the pole P to KL, cutting the parallels at n and p, I say that the arc Kn is similar to the arc Yp, and the arc Ln to the arc Vp. For since, as was shown at Num. 2, KO is to OP as YE is to EP, then by conversion OP will be to KO as EP to YE; and by composition, KP to KO as YP to YE; and by permutation, KP — the versed sine of arc Kn — will be to

YP — the versed sine of arc YP — as KO, the whole sine, is to YE, the whole sine. Therefore, by Lemma 5, the arcs Kn and Yp are similar; and accordingly, from the remaining semicircles, Ln and Vp will also be similar, by Lemma 6. From this it is clear that no other straight line drawn from P, except the perpendicular Pnp, cuts off similar arcs in the same correspondence. For if such a line falls to one side of the perpendicular, cutting the parallel of the Equator at X, the arc Kh will be too large to be similar to the arc Yp, since the arc Kn was shown similar to the arc Yp. Much larger, then, will the arc Kh be than would be needed for it to be similar to the arc YX, which is smaller than the arc Yp. But if the straight line drawn from P falls to the other side of the perpendicular Pn, we shall show in the same way that the arc cut off from the parallel KMLN is smaller than would be needed for it to be similar to the arc cut off from the parallel YpV, since the former is necessarily smaller than Kn, while the latter is larger than Yp, which was shown similar to Kn itself. Therefore the straight line drawn from P that cuts off similar arcs in that manner from each parallel will be perpendicular to KL.

Again, let a parallel of the Equator bde be described, opposite and equal to the earlier one VXY, cutting AC at d. I say that the straight line Qh — which we showed, when produced, passes through M — also passes through the point d; or, what is the same, that the straight line Qd, when produced, passes through h. For as we demonstrated in this proposition at Num. 24, the straight line Qd, drawn from the pole opposite to the oblique parallel, cuts off from the opposite parallel an arc, begun from the point K, equal to the arc ed as far as the number of degrees is concerned.

Diagram. A large stereographic-projection figure enclosed in a rectangular frame, occupying the lower two-thirds of the page. A large circle (the projected horizon / bounding circle) is crossed by a vertical diameter (V at top, through the interior to Y and Q at the bottom) and a horizontal diameter running left to right through the interior to A at the far right. Numerous chords, tangents, and dotted arcs represent the projected parallels and their tangent constructions. Readable labels, with approximate positions:

- V (top, on the vertical diameter)
- K (below V, on the vertical diameter)
- B, F (below K, flanking the vertical: B left, F right)
- H (upper-left of centre), S (upper-right of centre)
- M (left of centre), N (right of centre)

- o (centre, small)
- P (left margin of the circle, on the horizontal), X (below P at the left), u (below X at the left)
- C (centre-left), and a small script letter (r / ρ) just below it
- R (script, near centre), n and P (small, centre-left)
- E (centre, at the crossing of the diameters), L (just below E)
- Y (small script, right of centre), and ϵ with a small c to its right
- A (far right, at the right end of the horizontal diameter), q (small, just beyond A to the right)
- a (lower centre-left), e (lower centre, small), m (lower centre), T (lower right), I (lower right, on the circle)
- D (bottom centre), Y (below D), Q (at the very bottom vertex where dotted arcs converge)

Several dotted curves sweep from the upper region down toward Y and Q. The interior lettering is very dense and small; a number of the minuscule labels (the script r/ρ , the small n, P, ϵ , c near the centre) are at the limit of legibility.

Therefore, since ed is a quadrant, that arc too will be a quadrant. Hence, so that Kh may correspond to a quadrant — as we showed a little before at Num. 3 — the straight line Qd will fall exactly on h, so as to cut off the quadrant Kh; and produced further, it will also fall on the point M, onto which we showed the produced line Qh to fall. And so the four points Q, d, h, M will lie on one straight line: which must also be said of the four points Q, t, r, N.

Also, with a semicircle described about the straight line QE, cutting the parallel bde at k, let the straight line Ek be joined, and let On be drawn parallel to it, cutting the oblique parallel at n. I say that the straight line Qk, produced, passes through n and touches each parallel, at k and at n. For since it was shown a little before that the straight line Qd, produced, falls at M, it will be [Margin d: *Elements* VI.4] that QO is to OM — that is, to On — as QE is to Ed — that is, to Ek; and by permutation, as QO is to QE, so On is to Ek. Therefore, by the scholium to *Elements* VI.4, the straight line Qk will pass through n; and the angle Qke will be equal to the angle Qno, external to internal [Margin e: *Elements* I.29]. Since therefore the former is a right angle in a semicircle, the latter too will be a right angle [Margin f: *Elements* III.31]; and accordingly, by the corollary to *Elements* III.16, the straight line Qkn will touch each circle at k and n; which is what was proposed.

But the point of contact n will necessarily be that through which the perpendicular Pnp passes — that is, the straight line nP , drawn from the point of contact to the pole P , will be perpendicular to KL . For with Pn produced as far as p , and Ek as far as u , since...

the point n — that is, the arc Kn — is found by means of the straight line Pp from the arc Vp of the parallel VXY , and by means of the straight line Qk from the arc ek of the parallel bde , as was demonstrated in this proposition 6 at Num. 21 and 24, the arc Vp will be similar to the arc ek , since each contains as many degrees as are contained in the arc Kn . Now the arc Yu is similar to the arc ek , by the scholium to *Elements* III.22. Therefore the arc Vp too will be similar to the arc Yu , and indeed equal to it, since both lie in the same circle. So, adding the common arc pu , the whole arc Vu will be equal to the whole arc Yp [?]. Now, by the scholium to *Elements* III.22, the arc Vu is similar to the arc Kn [Margin a: *Elements* I.29], because, on account of the parallels Eu and On , the angles at the centres — KOn and VEu — external and internal, are equal. Therefore the arc Yp too will be similar to that same arc Kn .

Since therefore it has been demonstrated at the beginning of this Num. 4 that only the perpendicular drawn from P to KL can cut off similar arcs in that order from each parallel, it will necessarily be Pnp that cuts off the said similar arcs, being perpendicular to KL — that is, the straight line On , falling on the point of contact n , falls on the extreme point of the perpendicular Pn , drawn as far as the oblique parallel; and thus the straight line Qk , touching the parallel of the Equator bde at k , when produced will touch the oblique parallel in the perpendicular Pn . From this it follows that a straight line drawn from Q , which touches either of the parallels, also touches the other: because it was shown that the straight line Qk — which alone touches the parallel bde — falls on n , and there touches the parallel KML , etc.

4. In the fourth place [QVARTO loco], it must be shown that any straight line whatever drawn from Q , the opposite pole — whether it touches the parallels bde , $KMLN$, or cuts them — intercepts, together with the straight line QK , similar arcs toward the same parts, etc. For let there be described separately, so that confusion may be avoided, the parallels with their poles and the centres of the parallels, as in the preceding first figure; and let there first be drawn the straight line Qkn , touching each parallel at k and n . I say that both the arcs ek , Ln and the arcs bk , Kn are similar. For with the straight line Pn drawn from the pole P through n , cutting the other parallel at p — which, as we demonstrated just now at Num. 4, is perpendicular to

KL — the arc Vp will be similar to the arc Ln, and the arc Yp similar to the arc Kn, by the things demonstrated at Num. 4. Now the arc Vp is similar to the arc ek, since as many degrees are contained in the one as in the other; for the same arc Kn of the oblique parallel is found by means of them, by the aid of the straight lines Pp and Qk, as was shown in this proposition 6 at Num. 21 and 24. Therefore the arc ek too will be similar to the arc Ln; and accordingly, from the semicircles, the remaining arcs bk and Kn will also be similar.

Diagram. A second stereographic-projection figure, in a rectangular frame, similar in construction to the first but drawn "separately" (as the text says) so that the parallels, their poles, and their centres are shown apart. A large circle is crossed by a vertical diameter (V at top, down through K, o, E, L to Y and Q at the bottom) and a horizontal diameter reaching A at the far right. Two smaller circles (the projected parallels) are drawn interior to the left of centre, with tangents and dotted lines from the poles. Readable labels, with approximate positions:

- V (top, on the vertical diameter); to its upper left, a mark read as "3" and, below it, θ
- K (upper centre, on the vertical diameter), with o and ς close beneath it near the centre
- λ and ς (upper-left, at the small interior circle)
- n (left of centre), B and R (centre)
- E (centre, at the diameter crossing), L (below E)
- α (small, lower centre), e (lower centre)
- A (far right, at the right end of the horizontal diameter)
- Y (bottom, on the vertical diameter), Q (at the very bottom)

The interior of this figure is likewise crowded, and the small minuscule / Greek-letter labels near the centre (o, ς , λ , α , and the upper-left "3"/ θ) are at the limit of legibility.

The same will also be confirmed in this way. Since Qkn touches each parallel, the angles QkE and QnO will be right angles [Margin b: *Elements* III.18]. Since therefore the angle OQn is common, the remaining angles E and O, in the triangles QkE and QnO, will be equal at the centres; and accordingly, by the scholium to *Elements* III.22, the arcs ek and Ln will be similar, etc.

Next, let the straight line Q ς be drawn, cutting the oblique parallel at ς and γ , and the parallel of the Equator bke at α and β . I say that the arcs K ς , b β (as also L ς , e β , and L γ , e α , and Ky, b α) and the arcs $\varsigma\gamma$, $\beta\alpha$ are likewise similar. For, joining the

straight lines $O\zeta$, $O\gamma$, $E\beta$, $E\alpha$, let nO and kE also be joined, which will be perpendicular to the tangent Qn [Margin c: *Elements* III.18], and accordingly parallel to one another [Margin d: *Elements* I.28]; and thus, since the angles n and k are right, the angles O and E — internal and external — are equal, and Q is common [Margin e: *Elements* I.29]. Therefore it will be [Margin f: *Elements* VI.4] that QO is to On — that is, to $O\gamma$ — as QE is to Ek — that is, to $E\alpha$. The triangles $QO\gamma$ and $QE\alpha$ therefore have the angle $OQ\gamma$ in common, and the sides about the angles O and E proportional. Since, then, each of the remaining angles $O\gamma Q$ and $E\alpha Q$ is greater than a right angle [Margin g: *Elements* I.21] (for the former is greater than the right angle n , and the latter greater than the right angle k), the triangles themselves will be equiangular [Margin h: *Elements* VI.7], and will have equal the angles O and E at the centres. Therefore, by the scholium to *Elements* III.22, the arcs $L\gamma$ and $e\alpha$ will be similar; and accordingly, from the semicircles, the remaining arcs $K\gamma$ and $b\alpha$ will be similar, by Lemma 6. By the same reasoning, since the triangles $QO\zeta$ and $QE\beta$ have the angle $OQ\zeta$ in common, and the sides about the angles O and E proportional, and each of the remaining angles ζ and β less than a right angle, by the corollary 3 to *Elements* I.17 — because they stand upon the bases of the isosceles triangles $O\zeta\gamma$ and $E\beta\alpha$ — they too will be equiangular [Margin i: *Elements* VI.7], and will have equal the remaining angles ζOK and βEb , made from two right angles. Therefore, by the scholium to *Elements* III.22, the arcs $K\zeta$ and $b\beta$ are similar; and when there are taken away — both the arcs $K\gamma$ and $b\alpha$, which we just now showed similar, and, from the semicircles $K\zeta L$ and $l\beta$ — the remaining arcs $\zeta\gamma$ and $\beta\alpha$, and $L\zeta$ and $e\beta$ [?], will be similar, by Lemma 6. Which is what was proposed.

Lastly, with $P\zeta$ and $P\gamma$ drawn from the pole P through ζ and γ , cutting the parallel of the Equator at ϵ and δ , I say that the arcs $\epsilon\delta$ and $\zeta\gamma$ too are similar, and that the angles ζPp and δPp are equal. For since the same arc $K\zeta$ is cut off by the straight line $P\epsilon$ and by the straight line $Q\alpha$, the arcs [?] ϵ and [?] α will be...

similar, from the things demonstrated in this proposition 6 at Num. 21 and 24. And in the same way the arcs $Y\delta$ and $l\beta$ will be similar, because the same arc $L\gamma$ is cut off by the straight lines $P\delta$ and $Q\beta$. Therefore, if from the semicircles VXY and KnL there are removed the similar arcs $V\epsilon$ and $e\alpha$, the remaining arcs ϵY and $e\beta$ will also be similar, by Lemma 6. And if from these there are again removed the similar arcs $Y\delta$ and $b\beta$, there will remain, in the same way, $\epsilon\delta$ and $\beta\alpha$, similar. Now the arc $\beta\alpha$ was shown a little before, in this Num. 5, to be similar also to the arc $\zeta\gamma$. Therefore the arcs $\epsilon\delta$ and $\zeta\gamma$ will be similar; which is what was proposed.

Therefore, since the arcs $Y\delta$ and $l\beta$ are similar, as just shown, and a little before the arc $K\varsigma$ was shown similar to the arc $l\beta$, the arcs $Y\delta$ and $K\varsigma$ will also be similar; and therefore, by the scholium to *Elements* III.22, the angles ςOK and δEY at the centres will be equal; and accordingly the remaining angles ςOP and δEP , made from two right angles, will be equal. Since therefore the triangles ςOP and δEP have the angles O and E equal, and the sides about them proportional (for it was shown above at Num. 2 that YE — that is, δE — is to EP as KO — that is, ςO — is to OP), they will be equiangular [Margin a: *Elements* VI.6], and will have the angles ςPK and δPE equal; and accordingly, from the right angles, the remaining angles ςPp and δPp will be equal.

From these things it follows, conversely, that if two straight lines $P\varepsilon$ and $P\delta$ are drawn from P , making equal angles with the perpendicular Pp — or with the straight line KY — the arcs intercepted by them, $\varepsilon\delta$ and $\varsigma\gamma$, are similar. For with the straight line $Q\gamma$ drawn, it will fall on ς , as will be proved; and accordingly, as was shown a little before in the 3rd member of this Num. 5, the arcs $\varepsilon\delta$ and $\varsigma\gamma$ will be similar; which is what was proposed. But if it be said that the straight line $Q\gamma$, when produced, falls not on ς but either to the right or to the left — say at λ — then, with the straight line $P\lambda$ drawn, cutting the parallel of the Equator at θ , the arcs $\theta\delta$ and $\lambda\gamma$ will also be similar, by the 3rd member of this Num.; and accordingly, by the 4th member of the same Num. 5, the angles θPp and δPp will be equal; and for that reason the angles εPp and θPp — or εPV and θPV — will be equal to one another, a part to the whole, which is absurd.

More easily, however, we shall demonstrate that the arcs $\varepsilon\delta$ and $\varsigma\gamma$ are similar if the two angles εPp and δPp are equal, or the angles εPK and δPY , in this way. Since, as we showed above in this scholium at Num. 3, the point P is that through which passes the straight line joining the extremities of the diameters — the perpendiculars, in the parallels VXY and KnL , to the straight line VY — because in the two preceding figures the straight line XP , produced, falls on N , as was there demonstrated: the arcs $\varepsilon\delta$ and $\varsigma\gamma$ will be similar, by Lemma 34 [?]. From this it further follows that the three points Q , γ , ς lie on one straight line — such that a straight line drawn through any two of them also passes through the third — if the two angles ςPK and γPL are equal. For if, say, the straight line $Q\gamma$ does not pass through ς , let it cut the parallel at λ . We shall then show, as before, that the arcs $\theta\delta$ and $\lambda\gamma$ are similar, and that the angles λPK and γPL are equal. Therefore the angles ςPK and λPK will also be equal to one another, the whole and the part, which is absurd. Therefore $Q\gamma$ passes through ς . And by the same reasoning we shall show that the straight line $Q\varsigma$ passes through γ .

It is clear from all these things that it can happen that an arc of some oblique parallel is projected onto the astrolabe into a similar arc — namely one that is equal, say, to the arc $\varepsilon\delta$ on the sphere. For since, by Lemma 23, the planes drawn through the southern pole, together with the straight lines $P\varepsilon$ and $P\delta$, cut off from the oblique parallel on the sphere an arc equal to the arc $\varepsilon\delta$ — that is, equal to an arc of a parallel of the Equator which is similar to $\varepsilon\delta$ itself; and since the arc $\varepsilon\delta$ was shown similar to the arc $\varsigma\lambda$ of the oblique parallel on the astrolabe: that arc of the oblique parallel on the sphere, which is projected into the arc $\varsigma\lambda$ by those two planes drawn through the straight lines $P\varepsilon$, $P\delta$ and the southern pole, will be similar to that same arc $\varsigma\lambda$, etc. — although other arcs of the oblique parallel are projected into dissimilar arcs, etc. And this much concerning the properties of the oblique parallels; now let us proceed to other matters.

[Margin: That any one arc of an oblique parallel on the sphere can be projected onto the astrolabe into a similar arc. That parallels of the same oblique great circle have different centres on the astrolabe.]

6. It is evident from what we have written in this proposition 6 — especially in the second and fourth mode of describing the oblique parallels — that the parallels of the same oblique great circle obtain different centres on the astrolabe. For in the second mode of description, the straight lines drawn from A, the southern pole, through the points of the diameter MN of the oblique great circle — that is, the perpendiculars that fall, drawn down from the degrees of that same oblique circle, upon the straight line BD cutting the oblique great circle at right angles, such as, in the first figure of this proposition, are $A\varepsilon$, Au , etc. — indicate the centres of the parallels on the straight line BD. Since therefore these straight lines are different, the centres indicated by them must also be different. In the fourth mode, moreover, the straight lines touching the great circle $AiCk$ exhibit the same centres of the parallels on the straight line BD. Wherefore, since these tangents differ from one another, they will necessarily show different centres. Ptolemy demonstrates the same thing by a geometrical method in his *Planisphaerium*; but since that is long and difficult, we shall accomplish the same by a shorter and easier demonstration, in this way.

Let ABCD be the Equator, whose centre is E, which is taken for the great circle drawn through the poles of the world and the poles of the oblique parallels; and let AC be the axis, and BD the common section of the said great circle and of the Equator — the line in which the apparent diameters of the parallels must be taken, as was shown in the scholium to proposition 3 at Num. 1 and 2. Let FG, HI,

KL be the diameters of the oblique parallels drawn to the axis, whose apparent diameters MN, OP, QR are cut off by the radii AM, AN, AH, AI, AK, AL; and let MN be bisected at a, so that a is the centre of the parallel of diameter FG to be described about MN. I say that a is not the centre of the parallel of diameter HI to be described about OP — that is, that OP is not bisected at a. For since the diameters of the parallels cut the axis obliquely, their extremities will not be equally distant from the pole of the world C, since C is not the pole of those parallels. Let the points F and H, then, be farther from C than the points G and I — that is, let the arcs CF and CH be greater than the arcs CG and CI; and accordingly the angles CAF and CAH will be greater than the angles CAG and CAI, by the scholium to *Elements* III.27. Since therefore the three angles in the triangle AME are equal to the three angles of the triangle ANE, by the corollary 1 to *Elements* I.32 — and the right angles at E are equal, and the angle EAM is greater than the angle EAN, as we showed — the remaining angle N of the first will be less than the remaining angle N of the second; and therefore the straight line AM will be greater than the straight line AN [Margin b: *Elements* I.19]. In the same way we shall show that AO is greater than the straight line AP; and so on in turn: whenever a diameter of a parallel cuts the axis, we shall demonstrate that the radius toward B, as far as the straight line BD, is greater than the other radius toward D, as far as the same BD. But if some diameter, such as KL, does not cut the axis, the radius AQ will nevertheless be greater than the radius AR: because, since the angle ARQ is greater than the right angle AEQ — external to internal — it will itself be obtuse [Margin c: *Elements* I.16]; and accordingly AQR is acute in the triangle AQR. Therefore the straight line AQ will be greater than AR [Margin d: *Elements* I.19]. Let AS be cut off equal to AN, and AT equal to AP, and AV equal to AR; and let the straight lines ST and TV be joined. And since the two sides AS, AT are equal to the two sides AN, AP, and contain equal angles [Margin e: *Elements* III.27] — angles standing upon the arcs FH, GI, which by the scholium to *Elements* III.27 are equal, on account of the parallels FG, HI — the triangles AST and ANP will be equal [Margin f: *Elements* I.4]: and therefore the triangle AMO will be equal to the triangle...

Diagram. A circle inscribed in a rectangular frame, the figure for the foregoing demonstration that the oblique parallels have different centres. A vertical diameter runs from M at the top down through the interior (H/B, o, Q, R, E, P) to N and D at the bottom; a horizontal diameter reaches A at the right. Various chords (the diameters FG, HI, KL of the oblique parallels and the radii from A) cross the interior, together with the constructed points S, T, V. Readable labels, with approximate positions:

- M (top, on the vertical diameter)
- K (upper left, on the circle); H and B (upper centre, H left of B); F (upper right, on the circle)
- o (below H/B), Q (below o), S (right of centre, below F), T (below S), V (small, below T)
- a and b (centre, below Q), then a small letter read as δ (or d); R (below), E (centre), P (below E), N (below P)
- L and C (left, L above C), I (lower left, on the circle)
- A (far right, on the circle at the horizontal)
- G (lower left, on the circle), D (bottom, on the circle)

Marginal citation-letters keyed to the text stand to the right of the frame: b (*Elements* I.19), c (*Elements* I.16), d (*Elements* I.19), e (*Elements* III.27), f (*Elements* I.4). This figure is cleaner than the two preceding; most labels are legible, the small central δ/d and V being the least certain.

[Continuation of the Scholium to Proposition VI, mid-demonstration.]

[Margin: a. 1. sexti (Euclid VI.1).] the angle ANP, it will be the greater. But as the triangle AMO is to the triangle ANP, so is the base MO to the base NP. Therefore the base MO too will be greater than the base NP. Since, then, Ma is equal to Na, the remainder Oa will be less than the remaining Pa. Hence OP is not bisected at a. But if OP be bisected at b, we shall show in exactly the same way that the straight line QR is not bisected at b. For again the triangle ATV will be equal to the triangle APR, and therefore the triangle AOQ greater than APR; and consequently OQ greater than PR. When these are subtracted from the equal segments Ob, Pb, the remainder Qb will be less than the remainder Rb. Therefore the midpoint d of the diameter QR will fall below b. And so the three parallels of the diameters FG, HI, KL have, in the Astrolabe, different centres a, b, d. The same reasoning holds for the rest.

[Margin: That any parallel of the equator in the Astrolabe is divided by any oblique parallel into parts similar to those into which it is divided by the same [oblique parallel] in the sphere.]

7. Since, in Proposition 2, Number 4, we concluded that the equator and its parallels as described in the Astrolabe are to be divided into equal degrees, just as is customarily done in the sphere, Ptolemy [Ptolemy] demonstrates by a subtle argument that any oblique circle of the Astrolabe cuts any parallel of the equator into parts similar to those into which that same parallel of the equator is divided by that oblique circle in the sphere —

even though the oblique circle itself, in the Astrolabe, is *not* cut by a parallel of the equator into parts similar to those into which it is divided in the sphere by the same parallel of the equator. This is precisely because not all parts of the oblique circle are equally distant from the south pole, from which we view it; for from this it comes about that the more remote part appears smaller than the nearer one, as is demonstrated by the writers on perspective [the Perspectivists]. This cannot be said of a parallel of the equator, since all its equal arcs are equally distant from the south pole, and therefore also appear equal. In roughly this manner, then, Ptolemy demonstrates what was proposed.

Let ABCD be the equator, with centre E, taken as the great circle drawn through the poles of the world and through the poles of the oblique parallel; let AC be the axis of the world, and BD the common section of that great circle and the equator; A the south pole; FG the diameter of a parallel of the equator; HI the diameter of an oblique parallel, cutting FG in S. Now, radii having been drawn from A through the extremities of each diameter, so that the apparent (projected) diameters are KL, MN, let there be described about them the parallels KQL, MQN, cutting each other in Q [and Y]. I say that the arcs KQ, QL, KY, YL [second-position labels soft on the scan] are similar to the arcs into which, in the sphere, the parallel of diameter FG is divided by the oblique parallel of diameter HI. For, a semicircle having been described from O about FG...

Diagram. A large framed geometric figure illustrating Ptolemy's demonstration. It shows the great circle (equator ABCD) with centre E and horizontal axis; from the pole A at the right, visual rays are drawn to the extremities of the several parallel-diameters, generating the projected diameters and the circles described about them. Readable point-labels:

- M (top of the figure)
- K, B, F (upper vertical run, descending)
- T, H (upper right, on the projection lines toward A)
- E (centre, on the horizontal axis)
- O (centre, just right of E)
- A (right end of the horizontal axis — the south pole)
- C (far left end of the horizontal axis)
- S, V, P (lower-middle cluster)
- X, I (small labels near the lower centre) [?]
- R, Z, D, G, N (lower cluster, crowded) [?]

- L (bottom of the figure)

[Margin: a. 6. sexti (Euclid VI.6).] by construction they are equal; and let them have their sides about them proportional, as we have just shown — then the triangles themselves will be equiangular, and will have equal angles at RS; and consequently, since this one is a right angle, that one too will be a right angle.

[Margin: b. 17. sexti (Euclid VI.17).] Therefore, by the scholium to Proposition 13 of Book 6 of Euclid, RQ will be the mean proportional between KR and RL, and so the rectangle contained by KR, RL will be equal to the square on the straight line RQ. Therefore the rectangle contained by MR, RN as well (which was shown to be equal to the rectangle contained by KR, RL) will be equal to that same square on RQ; and consequently RQ will be the mean proportional between MR and RN.

[Margin: c. 17. sexti (Euclid VI.17).]

[Boxed proportion figure:]

LE,	GO,
EA,	OA,
ER,	OS,

Therefore the circle MQN will pass through its extremity, the point Q. For if, on the contrary, it were to cut the straight line RQ short of Q or beyond it, it would (by the same scholium to Proposition 13 of Book 6 of Euclid) cut off another straight line between MR, RN that was likewise a mean proportional in place, smaller or larger than RQ — which is absurd. Wherefore the circles KQL, MQN, since each passes through Q, will cut each other in Q, the extremity of the perpendicular RQ. And since, by the scholium to Proposition 22 of Book 3 of Euclid, the arcs LQ, GP are similar (because the angles at the centres E, O are equal), and consequently so also are the arcs KQ, FP, it is clear that the parallel of the equator KQL is cut by the oblique parallel MQN in the Astrolabe into arcs similar to those into which it is divided by the same in the sphere. Which was the thing proposed.

For the same demonstration will be applied on the other side: if the angle LEY be made equal to the angle SOP, and the straight line EY meet the parallel KYL in Y, and finally the straight line YR be joined. For in the same way it will be shown that the point Y also lies on the oblique parallel MYL.

8. Exactly the same thing occurs if the oblique parallel passes through the south pole A. For let the equator remain, together with its parallel and with the semicircle FPG described about the diameter FG, as before; but let the diameter of some oblique parallel drawn through the south pole be AZ, passing through the pole A and cutting the diameter FG in S. And since, by Proposition 1, Number 1, the parallel of diameter AZ makes, in the plane of the equator or of the Astrolabe, an infinite straight line passing through R (where the diameter meets the plane of the Astrolabe), let that straight line be QRY — namely the common section of the parallel and of the plane of the equator, in Q.

[Margin: d. 15. 1. Theod. (Theodosius, *Sphaerica* I.15).] Now since both the oblique parallel and the equator are at right angles to the great circle ABCD (drawn through their poles) [Margin: c. 19. undec. (Euclid XI.19).], their common section QRY will also be at right angles to that same circle; and so [the section] of the equator or of the Astrolabe, and [the section] of the great circle ABCD, will be perpendicular to the plane of the Astrolabe or of the right equator, by definition 3 of Book 11 of Euclid — that is, the angles at R will be right angles. Let SP also be drawn perpendicular to FG, as was proved above in Number 7; and let the straight lines EQ, OP be joined. Since, then, by the scholium to Proposition 4 of Book 6 of Euclid, as LR is to ER, so is GS to OS, it will follow, by composition, that as LE — that is, as QE — is to ER, so is GO — that is, PO — to OS.

[Margin: f. 7. sexti (Euclid VI.7).] And since the triangles EQR, OPS have equal right angles R, S, and the sides about the angles E, O proportional, [each of the angles at E and O being] less than a right angle by Corollary 1 of Proposition 17 of Book 1 of Euclid, they will themselves be equiangular, and will have the remaining angles Q, P equal; and thus the angles LEQ, GOP will be equal. Therefore, by the scholium to Proposition 22 of Book 3 of Euclid, the arcs LQ, GP are similar, and hence so also, from the remaining semicircles, are the arcs KQ, FP. It is clear, then, that the parallel of the equator KQLY is cut in the Astrolabe into arcs similar to those into which it is divided by the same in the sphere — namely by the oblique parallel represented by the straight line QY. Which was the thing proposed.

For by the same reasoning we shall demonstrate that the arc LY is similar to the arc GP — and therefore also to that which PS, produced, cuts off from the other semicircle — [Margin: (Euclid III.27).] just as, by the same scholium, the arc LY is equal to the arc LQ. And in this way, of two parallels, one oblique and the other equidistant from the equator, cutting each other mutually in the sphere, [the

corresponding thing holds in the Astrolabe], whether the oblique one passes through the south pole or not.

[Margin: To determine whether a circle in the Astrolabe that is not a great circle encloses a portion of the sphere smaller or larger than a hemisphere.]

9. Finally, if anyone should wish to know whether a circle described in the Astrolabe that is not a great circle — that is, one which does not bisect the equator — contains within itself a portion of the sphere smaller than a hemisphere, or larger, he will accomplish this easily, by the following reasoning. When the circle lies wholly within the equator, or wholly outside it without enclosing it, or when it cuts the equator not into two equal parts and the smaller segment of the equator lies within the cutting circle, then the portion of the sphere enclosed within the circle is smaller than a hemisphere. But when the circle encloses the whole equator, or cuts it not into equal parts with the larger segment of the equator lying within the circle, then the portion of the sphere enclosed within the circle is larger than a hemisphere.

For when the whole circle is within the equator, it encloses a smaller portion of the sphere than the equator does. Since, then, the equator cuts off a hemisphere, that circle will cut off a portion smaller than a hemisphere. So too, when the equator encloses the circle — as, in the first figure of this Proposition 6, is the circle *c 30 d* [middle label uncertain — likely a special point-sort] — if through its centre and the centre *E* of the Astrolabe a straight line *c E* be drawn, which the diameter *AC* of the equator cuts at right angles, then through its point *c* (outside the equator) and the two points *A, C* a great circle can be described, which will enclose the whole circle *c 30 d*, since it touches it in the single point *c*, by the scholium to Proposition 13 of Book 3 of Euclid. Since, then, that great circle encloses a hemisphere, the portion within the circle *c 30 d* will be smaller than a hemisphere.

Finally, when the circle lies wholly outside the equator and does not enclose it — such as, in the same earlier figure of this Proposition 6, is the circle *AA ↓* [the sort ↓ is a special printer's character used as a point-label] — if again through its centre and the centre of the Astrolabe a straight line *↓ E* be drawn, which the diameter *AC* of the equator cuts at right angles, then through its point more remote from the equator, on the straight line *E ↓*, and the two points *A, C* a great circle can be described, which — since it contains a hemisphere within itself and encloses the whole prior circle — [makes it so that] the portion lying within it is smaller than a hemisphere. But when the circle encloses the whole equator, it

will comprehend a larger portion than the equator. Since, then, this one comprehends a hemisphere, [that circle] will cut off a portion larger than a hemisphere. So too, when the circle does not indeed enclose the equator, but cuts it not into two equal parts, with the larger segment of the equator lying within it — of which kind, in the same earlier figure of this Proposition, is the circle $BB\omega$ — if through its centre and the centre of the Astrolabe a straight line be drawn, which the diameter AC of the equator cuts at right angles, then through its point ω and the two points A, C a great circle can be described, which will be wholly contained within the circle $BB\omega$, since it touches it in the single point ω , by the scholium to Proposition 13 of Book 3 of Euclid. Wherefore, since this great circle encloses a hemisphere, the circle $BB\omega$ will comprehend a portion larger than a hemisphere. Which was the thing proposed.

PROBL. III. PROPOS. VII.

To describe in the Astrolabe the parallels of any great circle drawn through the poles of the world, and to divide them into degrees.

Although parallels of this kind can be described by the method of the preceding Proposition 6, nevertheless, because in the right sphere their description differs in certain respects from the description of the same parallels in the oblique sphere, it seemed good to describe, in a proposition of its own, the parallels of a great circle drawn through the poles of the world.

[Margin: To describe in the Astrolabe the parallels of any great circle drawn through the poles of the world.]

[Margin: a. 19. undec. (Euclid XI.19).]

1. Since, therefore, all great circles drawn through the poles of the world are projected onto the Astrolabe as straight lines cutting one another at the centre of the Astrolabe, as was demonstrated in Proposition 1, Number 4: let the straight line AC , [drawn] through E the centre of the Astrolabe (in which the equator $ABCD$ is drawn), represent some one of these circles, whose parallels are to be described in the same Astrolabe. And let $ABCD$ be understood as a circle drawn through the poles of the world at right angles to the given circle which the straight line AC represents — such as is the Meridian, if the straight line AC stands for the right Horizon, or the circle of the 6th hour from noon and from midnight; or [such as is] the circle of the 6th hour from noon and midnight, if that same straight line AC represents the Meridian circle. Let this circle make, in the Astrolabe, the straight line

BD extended to infinity on both sides, which will be perpendicular to AC. For since both this circle and the equator (which is at right angles to the given circle) do not differ from the plane of the Astrolabe, [and are] at right angles to the given circle, their common section BD will be at right angles to it; and therefore, by definition 3 of Book 11 of Euclid, it will also be perpendicular to the straight line AC at the centre E. And since...

Diagram. A large framed stereographic-projection figure — the master construction for Problem IIII / Proposition VII. It shows the equator ABCD with centre E; a great circle through the poles projected as the vertical line BD; several pairs of parallel-diameters (FG, HI, KL, MN, drawn at 30° intervals); and the pencil of visual rays converging on the south pole A at the right, from which the projected (apparent) diameters and the circles described about them are generated. The figure is very densely lettered with a mixture of Latin capitals, lowercase italics, and a few special/Greek sorts. Readable point-labels:

- P (top), S (just below P)
- K, T (upper vertical run)
- B, d, b, e, a, i (crowded cluster below T)
- Z, F, Y, U, E (central vertical run)
- A (right, on the horizontal axis — the south pole)
- C (left of centre), R, t, L (mid-lower)
- M, N, c, X (lower cluster)
- h (lower right), V, g (bottom)
- On the left arc: a leftmost horizontal point [α / "cc" ?], and F, Z, E, m, o, u, n (script) along a dotted circle [?]
- Special sorts, upper right: a script ω / ϖ -like character; lower right: a long-f / integral-like sort [?]

[Margin: b. 19. 1. Theod. (Theodosius, *Sphaerica* I.19).] this circle ABCD, [being] at right angles to the given circle, cuts all its parallels into two equal parts and [passes] through the poles B, D (for B, D are the poles of the great circle AC and of its parallels). If, through the several degrees of the circle ABCD, lines parallel to AC be drawn, these will be the diameters of the parallels of the given circle. We have drawn, on each side, two apiece — FG, HI, KL, MN — at intervals of thirty degrees, lest a multitude of lines should breed confusion. Therefore, A having been established as the south pole (for the given circle, which the straight line AC represents, is supposed to be drawn through each pole), if from A visual rays be emitted through the extreme points of the diameters, they will cut off, on BD

produced, the apparent (projected) diameters of the parallels. For, as was demonstrated in the scholium to Proposition 3, Numbers 1 and 2, on the straight line BD — the common section of the plane of the Astrolabe and of the great circle drawn through the poles of the world, and [perpendicular] to the given great circle and its parallels — [these parallels] must be viewed at right angles from the south pole; for that straight line cuts off both the subcontrary triangles and the greatest apparent diameters, as we showed there. Thus the extreme points of the diameter FG will appear in O, P, so that the whole apparent diameter is OP; and the extreme points of the diameter HI will be seen in Q, R; and so with the rest. Therefore, the apparent diameters having been bisected, if circles be described about them, the given parallels will be described, since by Proposition 3 they appear in circular form when viewed from the south pole. And they will all pass through the extremities of the diameters on the equator ABCD, which is the primary Vertical of the right Horizon AC, just as in the sphere they pass through the same points. Yet this,

We may also reach the same conclusion geometrically, as follows. Draw the straight line CO; then the two sides CE, EO are equal to the two sides AE, EO. Since, moreover, they enclose equal angles — namely right angles — [a] the angles ECO, EAO will also be equal to each other; [b] and therefore they stand on equal arcs. Consequently, since the arcs CF, AG are equal, and the angle CAF stands on the arc CF, the angle ACG will stand on the arc AG — that is, the line CO, extended, will fall on the point G. [c] And since the angle AGC, being in a semicircle, is a right angle, the adjacent angle PGO will likewise be a right angle. Therefore, by the scholium to prop. 31 of Euclid book 3, the circle described about OP will pass through G. By the same reasoning it will pass through F, and so on for the rest. But since the rays from A — from the point A of the quadrant AB or AD — run out too far, it will be enough if the centre S of the three points F, O, G is found on the line BD extended, and likewise the centre T of the three points H, Q, I, and so on for the rest; for the parallel must pass through these three points, as we have shown. In this way the parallel FOGP is described more exactly than if the other extreme point P were found, which, on account of the oblique intersection of the line AG with DBP, can scarcely be determined without error.

[Margin: a — Euclid I.24; b — Euclid III.26; c — Euclid III.31.]

Moreover, it can also be gathered from this that every parallel passes through the three points found — as GPFO does through F, O, G. For the parallel of the right horizon and the right horizon itself cut off equal arcs from the verticals of that same right horizon, by prop. 10 of book 9 [?] of Theodosius [Theodosius]. Let the

verticals of this kind be the equator ABCD and the meridian DEB, and let EO represent the arc CF; then, by prop. 1, there will be three equal arcs CF, EO, AG. Therefore the parallel GPFO, since it is seen to pass through O, will also pass through the points F, G. For the same reason the parallel IRHQ will pass through the three points H, Q, I. And so with the rest.

2. In this way, moreover, we shall easily find the centres of the parallels.

From A, through Y — where the diameter FG cuts the line BD — let the line AY be drawn, cutting the equator at Z. For if an arc Ba equal to the arc BZ is cut off, the line Aa will fall on S, the centre sought, as was demonstrated in Lemma 35. So too, if the line Abd is drawn and an arc Be is taken equal to the arc Bd, the line Ae will meet at T, the centre of the parallel described through H, Q, I. Likewise, if the line Afg is drawn and an arc Dh is taken equal to the arc Dg, it will give the line Ah, the centre V of the parallel described through K, L. Finally, if the line Aik is drawn and an arc Dl equal to the arc Dk is taken, the line Al will pass through X, the centre of the parallel described through M, N. It is enough, however, if the centres S, T are found for the parallels of the semicircle ABC. For if EV, EX are made equal to the lines ES, ET, then V, X will be the centres of the opposite parallels to be described about the points K, L and M, N. For opposite parallels in the right horizon are altogether equal in the astrolabe, just as in the sphere.

[Margin: To find, in the astrolabe, the centres of the parallels of the great circle drawn through the poles of the world.]

3. We shall describe the same parallels in another way, even if neither their diameters are drawn in the circle ABCD nor rays are emitted from A. For, as we shall show a little below (Num. 10), any line — such as EK — drawn from the centre to the equator touches at K the parallel described through K; [d] and it comes about that KV, drawn perpendicular to EK (or tangent to the equator), falls at V, the centre of the parallel to be described through K. Accordingly, if to all the points of the equator — which is the primary vertical in the right sphere — lines are drawn from the centre E, and through the extremities of those lines perpendiculars are drawn to those same lines (which, by the corollary of prop. 16 of Euclid book 3, will touch the equator at those same points), the centres of all the parallels will be found, and the semidiameter of each will be the very tangent line from the found centre up to the point of contact. Thus, in the given example, the semidiameter of the parallel KL is VK. And we shall easily draw, for each

single point of the equator, tangents — that is, perpendiculars to its semidiameters — in this way. From B, having drawn to BD a perpendicular Bu of any length, let a hidden circle be described from E through u, and let the line Bu, by means of the compasses, be transferred from the points of the equator HF, KM onto the hidden circumference on either side — as from H up to m, n; from F up to o, p; from K up to q, r; and from M up to s, t. For the lines mn, op, qr, st will touch the equator at H, F, K, M — that is, they will be perpendicular to the semidiameters if EH, EF, EK, EM are drawn. For when the lines Eu, Eq are joined, the two sides EB, Bu will be equal to the two sides EK, Kq. Since, then, the base Eu is also equal to the base Eq, [e] the angle EBu will be equal to the angle EKq, and consequently this too will be a right angle; and therefore it will touch the equator at K. The same reasoning holds for the rest.

[Margin: To describe the same parallels by means of tangent lines. d — Euclid III.19; e — Euclid I.8.]

4. From what has been said it will not be difficult to describe a parallel any number of degrees distant from the right horizon AC, if we count the given distance from the point C or A — toward B, if the parallel to be described is above the horizon, or toward D, if below the horizon — and describe the parallel through the terminus of the counting, as has been taught.

[Margin: To describe in the astrolabe a given parallel of the right horizon.]

5. Conversely, if any parallel has been described, its distance from the right horizon will be known from the arc of the equator between C (or A) and the point of intersection of the parallel with that same equator. Or, if lines are drawn through the intersections of the parallel with the meridian line, the equator will be cut in two points of the same distance; and these lines will necessarily pass through the intersections of the parallel with the equator, since otherwise the given circle would not represent any parallel of the right horizon. Hence, when it is not established that the proposed circle is one of the parallels of the right horizon, the latter method must be used, so that we may at the same time recognize that we are not inquiring in vain and at random into the distance of the given parallel from the right horizon. For if lines drawn from A through the intersections of the proposed circle with the meridian line pass through the intersections of the same circle with the equator, it is certain that it is parallel to the horizon, its diameter being the line that joins these two intersections; otherwise it

will not be parallel to the horizon, but will represent some other circle, as we shall say at prop. 17.

[Margin: To recognize how far a parallel of the right horizon described in the astrolabe stands from the right horizon in the sphere.]

6. Furthermore, so that the rays emitted from A and running out farther may be drawn more precisely, a circle $\alpha\beta$ must be described from A at any interval you please, as was done in the preceding propositions. For if, for example, an arc $\alpha\beta$ is taken similar to half the arc CBG, the ray AG will pass through β ; for indeed, by Lemma 10, the lines $A\alpha$, $A\beta$ intercept two arcs, of which the one that lies in the circle described from A is similar to half the arc in the circle passing through A. So too, if arcs $\alpha\gamma$, $\alpha\delta$ are taken similar to the halves of the arcs CBa, CBI, the rays $A\gamma$, $A\delta$ will pass through a, I, and so on.

[Margin: To draw more accurately the rays that run out farther.]

7. Now the great circle which the line AC represents, and its parallels, will be distributed into degrees in exactly the same ways by which we divided the circles above. For the great circle represented by the line AC extended to infinity will be divided by lines emitted from B, the upper pole, through the degrees of the equator, in the order we prescribed in Lemma 23: namely, to the arc DP cut off, beginning from the lower point D, there corresponds the arc EO, beginning from the northern section; likewise to the arc DQ corresponds the arc ER; likewise to the arc DG corresponds

the arc EL, so that, just as the arc BG begins from the upper point, so there may correspond to it an arc beginning from the southern section (if the southern pole could be marked) up to L. And so, if PQ is a quadrant, OR will also be a quadrant. Again, the same great circle AC will be divided by lines emitted from the lower pole D, yet in such a way that the arcs begun from the upper point B have their correspondents in AC begun from the northern section E, and so on, as was said in the same Lemma 23. Thus you see that to the arc BG there corresponds the arc EX, of which the former takes its beginning from the upper point, the latter from the northern section, and so on.

8. Let there also be some parallel of the great circle AC — namely FGHI — to be divided into degrees by lines drawn from the upper pole B. Let a parallel of the equator KLMN be described, standing off from the southern pole A by as much as the parallel FGHI stands off from the upper pole B, so that the arcs BG, Am, which measure the said distances, are equal. If, then, an

arc KS of any number of degrees is taken on the parallel of the equator, the line BS will give, on the given parallel, the arc FT of the same number of degrees, because KS begins from the upper point K and FT from the southern section F. By the same reasoning, there will be as many degrees in the arc MLS, begun from the lower point M, as are contained in the arc HGT begun from the northern section H. And since FG, GH, HI, IF correspond to the quadrants of the given parallel in the sphere — because the equator ABCD, that is, the primary vertical of the right sphere, and the meridian FD cut the horizon and its parallels into quadrants — it is necessary that the line BL pass through the point G, so as to cut off the arc FG corresponding to the quadrant KE, and so on.

[Margin: To distribute into degrees, from their poles, the parallels of the great circle drawn through the poles of the world.]

Diagram. A large square frame enclosing the stereographic projection used throughout Prop. VII for dividing the parallels of a polar great circle. Within the frame is the equatorial great circle (the primary vertical / equator ABCD) drawn about the centre E, with F at the top and M at the bottom of a larger great circle (the polar great circle FGHI being divided), and a dense web of chords, radii, and auxiliary circles (KLMN, VXYZ, the hidden construction circle pGqI). Point B is the upper pole and D the lower pole; A lies at the right and C (implied on the left) on the equatorial diameter. The figure is extremely crowded with small italic and Greek point-labels; several are only partly resolvable even under magnification.

Readable labels (approximate positions):

- F (top of the large circle)
- M (bottom of the large circle)
- B (upper centre, on the vertical diameter)
- D (lower centre, on the vertical diameter)
- E (centre)
- A (right, on the horizontal diameter)
- N (far right, on the horizontal)
- T (upper left)
- S (left, below T)
- K (upper centre, below the top)
- k (upper centre, on the dashed line running to I)
- I (upper right, at a circle intersection)
- L (left, on the horizontal)

- V (centre, near E)
 - H (centre, near E, below V)
 - P (lower left)
 - Y (centre, lower)
 - Q (centre-right, below A)
 - R (near E, lower right, with z)
 - z (near E, lower right)
 - m (centre-right)
 - h (centre-right, below m)
 - b (left of centre)
 - u (left)
 - n (left of centre)
 - e (centre)
 - c (left of centre, on the horizontal)
 - f (centre)
 - o (centre, lower)
 - x (centre-left, lower)
 - d (lower centre)
 - q (lower centre)
 - r (lower centre)
 - a (lower centre)
 - p (upper centre, near B)
 - s / t (lower right, tangent-point letters)
 - α (alpha, lower left of centre)
 - β (beta, left)
 - γ (gamma, lower left)
 - δ (delta, near I) [?]
 - θ (theta, centre, near E) [?]
 - ϵ (epsilon, appears as a point label in the accompanying text) [?]
9. But if the same parallel FGHI is to be divided into degrees by lines going out from the lower pole D, then a parallel of the equator VXYZ must be described, opposite to the parallel KLMN — one distant from the southern pole A by as great an interval as the parallel FGHI is distant from the lower pole D, so that the arcs DCG, ABn, which measure the said distances, are equal. For if an arc Ya similar to the arc KS (begun from the upper point) is taken — an arc which in the sphere is equal to KS itself, since the parallels

are equal — begun from the lower point, the line Da extended will give the arc FT of the parallel, the same arc begun from the southern section. Likewise it will cut off, for the arc Vxa begun from the upper point V, the arc HGT begun from the northern section H. In the same way DX will cut off the two quadrants YX, FG, as is clear from Lemma 23.

10. In another way we shall divide the same parallel into degrees, thus. Having described about GI the circle pGqI, let arcs pb, qd, equal to each other, be taken; and when the line bd is joined, let it cut GI at e. For the line Ee will cut the parallel in two points T, f, and each of the arcs FT, Hf will contain as many degrees as are contained in the arc pb; likewise each of the arcs GT, Gf will comprise as many degrees as are found in the arc Gb — so that if the arcs KS, pb are made similar, the lines Ee, BS fall on the same point T. Now this reasoning is entirely the same as that by which, in the preceding proposition (Num. 26), we distributed the parallels of the oblique circles into degrees, because E is the centre of the primary vertical, just as the point L was there. Hence it comes about that the lines EG, EI touch the parallel at G, I, the extreme points of the visible diameter GI, just as there we showed the lines Lq, LG to touch the parallel.

[Margin: To distribute into degrees, from the centre of the astrolabe, the parallels of the great circle drawn through the poles of the world.]

We shall, however, demonstrate this geometrically as follows. Since the ray from A through I, extended, cuts off EF, the semidiameter of the parallel of the equator whose southern declination is BI, and the ray from A through G takes off EH, the semidiameter of the parallel whose northern declination BG is equal to the declination BI (as is established from prop. 4), EF, EH will be the semidiameters of the opposite parallels. Therefore, as we showed at prop. 4, Num. 11, EG will be the mean semidiameter of the equator, proportional between them; [a] and consequently the rectangle under EF, EH will be equal to the square on EG, [b] and therefore EG will touch the circle FGHI at G, which is what was proposed.

[Margin: a — Euclid VI.17; b — Euclid III.37.]

11. Thirdly, we shall distribute this same parallel, and others too, into degrees by this method. In the circle described about GI (the true diameter of the parallel), let two equal arcs Ig, Ih be taken; and when the line gh is joined, cutting GI at i, let the line Ai be drawn from A, the southern pole, through i, until it cuts EB extended at k. For the line Tl, drawn through k perpendicular to BF, will cut off the two arcs FT, FL, each containing as

many degrees as are included in the arc Ig — or the two arcs GT, IL, of the same number of degrees as the arc pg comprises — so that if the arc Ig is similar to the arc KS, or equal to the arc pb, the perpendicular kT falls on the very point T, which was shown [to lie] on the lines BS, Ee. And this reasoning does not differ from the third mode of dividing the parallels which we set out in the preceding proposition (Num. 31).

[Margin: To distribute into degrees, from the southern pole of the analemma, the parallels of the great circle drawn through the poles of the world.]

12. In no different way will the parallels below the right horizon AC be divided into their degrees. For let there be a parallel rst below the horizon, altogether equal to the parallel FGHI — that is, let the distance of each from the horizon toward opposite directions be the same. Then from the upper pole it will be divided by means of a parallel of the equator VXYZ, which is distant by as great a space from the southern pole A as the parallel rst is distant from the zenith B: so that the lines falling from B, cutting off arcs begun from the upper point V, cut off from the parallel corresponding arcs begun from the southern section, which lies below the point M; while the lines cutting off from the parallel of the equator arcs begun from the lower point Y cut off, in the given parallel rst, corresponding arcs beginning from the northern section r, as before. But from the lower pole D the same parallel rst will be cut by means of the parallel of the equator KLMN, since this is removed from the southern pole by as great a space as rst recedes from the nadir — that is, from the lower pole of the horizon: so that the lines going out from D, which cut off arcs of the parallel of the equator beginning from K, the upper point, cut off from the parallel rst corresponding arcs taking their beginning from the northern section r; while the lines cutting off from KLMN arcs whose beginning is at M, the lower point, cut off from rst corresponding arcs begun from the southern section below the point M, as before. All of which clearly follows from what we wrote in Lemma 23.

These same parallels can also be divided into degrees, if you please, from their own centres and from the centre of the astrolabe, in the manner we set out in the preceding proposition (prop. 6, Num. 35): a matter which, since it is easy, needs no fuller explanation.

Finally, all those things too will readily be adapted here which we described at Num. 36 and 37 of prop. 6, as is evident.

But before all else let there be transferred here the things we wrote at prop. 6, Num. 25 — that is: if from the point F toward G an arc of any number of apparent degrees is to be cut off from the parallel, let there be counted from the opposite point H, in the same direction toward G, the same number of equal degrees up to ε . For the line drawn out from D, the lower pole, through ε will cut off the sought arc FT, containing as many visible degrees as there are equal degrees contained in the arc H ε . But if the same equal degrees are counted from H in the opposite direction toward I, the line drawn from the end of the counting through B, the upper pole, will give the same arc FT. Conversely, if from F up to T any number of equal degrees are counted, the line TD, drawn to D, the lower pole, from the same side, will cut off an arc H ε of the same number of visible degrees; while the line drawn from T through B, the upper pole, and extended, will take off from the opposite side an arc of the same number of apparent degrees.

Next, since V, the centre of the circle pGqI, and E, the centre of the parallel of the equator KLMN, stand off equally from B, the upper pole ([c] since, as GV — that is, as pV — is a semidiameter to VB, so LE, that is, KE, is a semidiameter to EB), the division of the parallel FGHI by the circle pGqI will come about just as by the parallel KLMN, from the upper pole B. Thus you see that the line Bb (with the arc pb taken similar to KS itself) passes through S and indicates the same point T. Again, since the same centres V, E stand off equally from D, the lower pole, when E is taken as the centre of the parallel of the equator VXYZ ([d] since, as GV — that is, as pV — is a semidiameter to VD, so XE, that is, VE, is a semidiameter to ED), the same division of the parallel FGHI by the same circle pGqI will come about from the lower pole D. Thus you see that the line Dd (with the arc qd taken similar to Ya) passes through a and indicates the same point T. And in this manner, if for the parallels of the equator KLMN, VXYZ other circles are described whose centres stand off equally from B, the upper pole, along with E, the centre of the parallel KLMN, or from D, the lower pole, likewise along with E, the centre of the parallel VXYZ, there will be had other circles through whose degrees the lines extended from the pole B, or D, will divide the parallel FGHI into degrees, as we demonstrated at prop. 6, Num. 25.

[Margin: c — Euclid VI.4; d — Euclid VI.4.]

13. Finally, all those things are true here which were demonstrated in the scholium of the preceding proposition (Num. 2, 3, 4, and 5) — that is: when the line Bu β is drawn perpendicular to BD from B, the upper pole of the parallels of the right horizon, the line Du drawn from D, the lower pole, touches the parallels at u, α , and the arc Fu is similar to the arc M β , and the

arc Hu to the arc Kβ. Likewise the arcs Yα, Hu, and Vα, Fu, which the tangent line Du, drawn out from D the lower pole, cuts off, are similar. Again, if from the same lower pole D a line DT is drawn in any way whatever, the arcs FT, Vθ, as well as Hε, Ya, and as well as Tε, θa, are similar. Moreover, when the lines BT, Bε are drawn cutting the parallel of the equator KL, MN at S, γ, the arcs Sγ, Tε are similar, and the angles TBF, γBM — or TBβ, γBβ — are equal. Finally, if the angles TBF, γBM are made equal, so that the lines BT, Bγ cut the parallels at T, ε, S, γ, then conversely the arcs Sγ, Tε will be similar; and thus the line drawn DT passes through the point ε, where the line Bγ cuts the same parallel of the horizon, and the line drawn Dε passes through the point T, where the same parallel is cut by the line BT — that is, the three points D, ε, T lie on one straight line. For the very same demonstration which was made in the said scholium holds here also, as is clear.

ASTROLABII LIBER SECVNDVS

PROBL. V. PROPOS. VIII.

To describe on the astrolabe the vertical circles — those drawn through the poles of the horizon, which the Arabs call *Azimuth* — together with other great circles that run through the poles of any great circle already drawn on the astrolabe; and to divide them into degrees.

1. In the fifth Proposition we drew the primary vertical, the horizon, the ecliptic, and other great circles that are perpendicular to the meridian but inclined to the equator by a known inclination. The remaining verticals, however — those inclined to the meridian, which the Arabs call *Azimuth* — cannot be drawn by that method. For in the analemma they share the same diameter as the primary vertical, namely the axis of the horizon (since they all pass through the poles of the horizon); the meridian is not perpendicular to them; and therefore along the straight line BD, the common section of the meridian and of the plane of the astrolabe (or of the equator), their diameters do not appear as greatest. (Only those diameters are seen as greatest which lie in the common sections of the plane of the equator, or of the astrolabe, with great circles drawn through their own poles and through the poles of the world, as we showed in the scholium to Proposition 3, Number 1.) Instead, they all appear to share the same visible diameter as the primary vertical, such as is HI in the figure here set out.

For this reason we shall project them onto the astrolabe in the following way. Let the primary vertical AHCI be divided into equal parts by as many diameters as there are verticals to be drawn on the astrolabe, after first drawing through its centre K, at right angles to HI, the perpendicular PQ of unbounded length (for every diameter is drawn through two opposite points). If 180 verticals are wanted, dividing the horizon and its parallels into 360 degrees, then divide it into 180 parts by 90 diameters. If 90 verticals are to be drawn, dividing the horizon into 180 parts so that two degrees fall between each adjacent pair, then divide it into 180 parts by 90 diameters. Or into 120 parts by 60 diameters, so that the single parts

[Margin: To describe the vertical circles on the astrolabe.]

Diagram. A large, very densely ruled stereographic-projection construction for the vertical circles (azimuths) of the astrolabe, enclosed in a ruled rectangular border and spanning most of the page. A central vertical axis runs from a point **F** at the top (marked with a zodiac sign above it) down to the pole-point **I** at the bottom vertex, from which a great many straight rays (radii) fan out; a horizontal diameter through the centre carries **P** at the far left and runs across to the right margin. The interior is filled with numerous mutually intersecting great circles and their chords/diameters (many drawn dotted), forming an interlaced rosette. Numerous points are lettered in roman capitals, roman minuscules, doubled letters, and Greek minuscules, with degree-numbers (60, 30, 90, 20) and zodiac symbols marked along the primary vertical and horizon.

Readable principal labels (position approximate):

- F (top of central axis; zodiac sign above [?])
- I (bottom vertex — pole of the horizon)
- P (far left, on the horizontal PQ); horizontal line continues right toward e / far-right margin
- U (upper left, marked "60")
- RR (left, marked "30" [or "34" ?])
- Aa, C (mid-left)
- M (below Aa C); a second M at lower left
- Bb (left); ξ (below P); L (lower centre-left); X (lower left-centre)
- B with "c b α" (upper centre); "mm 20" (upper centre-right); "60 mm" (top centre-right)
- ΘΘ (ΘΘ, upper centre-left); u (centre)
- A, O (right-centre); Dd [or "bd" ?] (right); e (far right on the horizontal)

- v, Θ , Λ , N (centre-right / lower centre)
- Y, φ (bottom centre)
- G (centre, near "90" and a zodiac sign and "PP"); D (centre); r, s (centre)
- numerous further small doubled-letter and Greek-minuscule labels (cd, R, Q, S, b, etc.) too small/overlapping to resolve individually.

should embrace three degrees each; or into 72 parts by 36 diameters, so that each part contains five degrees; or into 60 parts by 30 diameters, so that six degrees are included between each two adjacent parts; or into 40 parts by 20 diameters, so that nine degrees fall between any two; or into 36 parts by 18 diameters, so that each part contains ten degrees; or into 24 parts by 12 diameters, so that each part embraces fifteen degrees; or into 20 parts by 10 diameters, so that each part comprises eighteen degrees. Or finally into 12 parts by 6 diameters, so that each part embraces thirty degrees, as has been done in our example. For in it 6 verticals are drawn, thirty degrees are intercepted between each adjacent pair, and the horizon together with its parallels is divided by those same verticals into 12 parts.

Next, from either of the poles of the horizon, H or I — say from I — let rays be sent out through all the extremities of the diameters, cutting the straight line PQ at points which will exhibit the diameters and the centres of the vertical circles, in this order. The rays sent through the ends of any diameter cut off from PQ the diameter of that vertical which, in the sphere, is distant from the primary vertical toward the south from the point of rising by as many degrees as the diameter assumed in the primary vertical recedes from the point T (the eastern point) toward I (the southern point); or of that vertical which in the sphere is distant from the primary vertical from the point of setting toward the north by as many degrees as the same diameter, assumed in the primary vertical from the point V (the western point), is removed toward the north (H); or of that vertical which in the sphere recedes from the primary vertical from the point of rising toward the north by as many degrees as the diameter assumed in the primary vertical is distant from the point T (the eastern point) toward the northern point (H); or finally of that vertical which in the sphere is distant from the primary vertical from the point of setting toward the south by as many degrees as the same diameter, assumed from the western point V toward the southern point I, is distant.

For the straight line PQ must be conceived on the astrolabe in such a way that, when we stand at the south pole, the part KP lies to the right and KQ to the left. For when we turn to face the astrolabe (which lies in the plane of the equator),

the eastern part — as it is taken in use by the authors — is placed to the left, such as is the part stretched to the left from the meridian line FI; and the western part lies to the right, such as is the portion extended to the right from that same meridian FI. Hence, when we stand at the antarctic pole, the eastern part of the astrolabe lying in the plane of the equator is set to the right and the western part to the left — so that the south pole must be conceived as behind the plane of the astrolabe.

This matter, attentively considered, greatly helps in conceiving the positions of all the centres of the verticals on the straight line PQ produced to infinity. For all writers take, in the use of the astrolabe, the part which is placed to our left when we face the astrolabe as the eastern part, and the part to the right as the western; but for those standing at the south pole and facing the equator, the east lies to the right and the west to the left. So if anyone should prefer the part KP of the line PQ (extended to infinity) to appear to us, from the south pole, on the left, and the part KQ on the right — which nothing prevents — then the right part of the astrolabe must be taken for the east and the left for the west. But the former consideration is the more usual among astronomers. Therefore, since the equator separates the northern part of the heaven from the southern in the sphere, the point T will be the eastern point of the primary vertical on the astrolabe; V the western; H the northern; and I the southern.

[Margin: The eastern part and the western part in the astrolabe: which they are.]

A ray, then, sent out through a point of the primary vertical, whose distance from the point I is double the distance which the assumed diameter has from that same point I, falls upon the centre of the vertical to be drawn — that is, it cuts the cut-off diameter in half. For example: since the diameter LO recedes from T, the eastern point, toward the southern point I — or from the western point V toward the northern point H — by 30 degrees, the rays IL, IO therefore cut off the diameter PS of the vertical PHSI, which is distant by just as many degrees from the eastern point C of the horizon (on the horizon of the astrolabe the point C is eastern, A western, G northern, and F southern, according as the primary vertical in the sphere separates the northern part from the southern) toward the southern point F; or which is distant by just as many degrees from the western point A toward the northern point G. Its centre is the point R, upon which falls the ray IM, drawn from I to the point M, whose distance IM is double the distance IL. Likewise the rays IX, Id will cut off the diameter of the vertical HaI, which recedes from the eastern point C of the horizon toward the south, or from the western point A toward the north, by 60 degrees; its centre will be P. Again, the

rays IY, Ib will cut off the diameter of the vertical HZI, which is distant from the western point A of the horizon toward the south, or from the eastern point C toward the north, by 60 degrees; its centre will be Q. Finally the rays IN, IM will exhibit the diameter QR of the vertical QHRI, which recedes from the western point A of the horizon toward the south, or from the eastern point C toward the north, by 30 degrees; its centre will be S.

2. That the vertical circles are correctly drawn by this method in the present figure — in which PQ, perpendicular to BD, represents a parallel of the horizon drawn through the south pole A in the sphere, as we demonstrated in Proposition 6, Number 3 — [is shown thus].¹ Since therefore the vertical circles cut the horizon and its parallels into similar parts in the sphere, the same must necessarily happen on the astrolabe. So a vertical passing, for instance, through 30 degrees of the horizon on the astrolabe from the eastern point C toward the south F must be drawn through 30 degrees of the parallel of the horizon which the straight line PQ represents, counted from its eastern point T up to P, toward the southern part, which tends toward P. And because the same vertical cuts the horizon and the parallel PQ at opposite points, it must necessarily also pass through 30 degrees of the same parallel, counted from the point V (the western point) toward the northern point K, up to S. For in the parallel PQ (to explain this too in passing) the eastern point is T, the western V, the northern K; the southern point cannot be marked, since the straight line PQ runs out to infinity — yet its southern parts are the segments tending from the points T and V (eastern and western) toward P and Q. And since the same parallel which the straight line PQ represents on the astrolabe is distant from the south pole A by the straight line AK — that is, by the straight line IK, equal to AK, since each is a radius of the same circle — the parallel PQ will be divided into single degrees by straight lines sent from I (the point) through the single degrees of the circle HTIV described about I, and whose diameter IH is perpendicular to PQ, as is clear from what we demonstrated in Proposition 1, Number 5. Thus the portion TP corresponds to the arc TL, of 30 degrees reckoned from the rising toward the south, and the portion VS corresponds to the arc VO, of 30 degrees counted from the setting toward the north.

[Margin (note a): [Euclid] ... 2. Theod. [?]]

¹ [Margin note "a" — see the marginal citation above; the printed reference reads "... 2. Theod. [?]."]

Furthermore, the parallel of the horizon PQ is also divided into degrees by straight lines sent from either of the poles of the horizon H, I through the degrees of the vertical HTIV, or of any vertical circle tangent to it at H or at I, such as is the circle $\alpha\pi I\omega$ [?] in the figure. (For by the 9th Lemma, straight lines drawn out from I cut off, from the circle HTIV and from $\alpha\pi I\omega$ [?] tangent to it

at I, similar arcs; and hence the same straight lines pass through the degrees of both circles. The same must be said of the straight lines going out from H, if a circle is drawn tangent to the vertical at H.) This can also be demonstrated by yet another argument. Since the parallel of the horizon drawn through the south pole — which on the astrolabe the straight line PQ represents — is divided into degrees by straight lines drawn from the pole H of the horizon through the degrees of the parallel of the equator which is described from the centre E through H, as we demonstrated in Proposition 6, Number 21 from Lemma 23 (this parallel of the equator being as far from the south pole as that parallel of the horizon is from the zenith, or from the northern pole of the horizon, since in both cases the distance is the arc of the meridian intercepted between the south pole and the northern pole of the horizon, one being drawn through the zenith and the other through the south pole in the sphere): it follows that the straight lines sent out from H through the degrees of a vertical, or of any circle tangent to it at H, also cut into degrees that parallel of the horizon represented by the straight line PQ — inasmuch as those straight lines divide into similar arcs the vertical, and any circle tangent to it, and the parallel of the equator described from E through H and tangent to them at H, by Lemma 9. And precisely the same reasoning holds for the straight lines sent out from I, since these divide the straight line PQ in the same way as they are cut by the straight lines drawn out from H, on account of the equal distance of both points H and I from the straight line PQ.

Since this is so, a vertical circle distant from the primary vertical by 30 degrees, from the rising toward the south and from the setting toward the north, will cut the parallel PQ at the same degrees, namely at the points P, S. By like reasoning, a vertical distant by 60 degrees from the primary vertical, from the rising toward the south and from the setting toward the north, will pass through that point of the parallel PQ upon which falls the ray IX (drawn through 60 degrees, counted from T, the eastern point, toward the southern point I, up to X), and through the point a (which answers to 60 degrees, reckoned from the western point V toward the northern point H, up to d). And so it must be said of the rest.

And because all the verticals pass through the poles of the horizon H, I, it is evident — from the corollary to Proposition 1, Book 3 of Euclid — that the centres of all the verticals lie on the straight line PQ, which cuts in half, at K, and at right angles, the line HI common to all the verticals. Therefore the middle points of the diameters found on the straight line PQ will be the centres of the verticals — namely the points upon which the straight lines from I fall perpendicular to the diameters of the circle HTIV, as we showed in Lemma 35 — these being the straight lines drawn from I through those points, whose distances from I are double the distances which the said diameters of the circle HTIV have from that same point I. For these

[Margin: That the centres of the verticals lie on the straight line which is drawn through the centre of the primary vertical perpendicular to the meridian line. (a: [Euclid] III.3)]

(Continuation of Propositio VIII, §2, from the foot of the preceding page.)

for the straight lines drawn to those diameters are perpendicular, since, by the scholium of Euclid III.27, they are bisected by the diameters, just as the arcs are. [Margin (a): Euclid III.3.] For example, since the diameter dX cuts the arc IL in half at X, it will also cut the straight line IL in half at f, and therefore at right angles. In the same way IM is perpendicular, and meets LO in e, IN meets Yb in h, and IO meets NM in g. This being so, the vertical circle PHSI is correctly drawn from center R; the vertical HaI from center P; RHQI from center S; and HzI from center Q.

3. The circles drawn from the centers found on PQ — those centers being found on the same line PQ about the diameters lying there — must necessarily pass through H and I, the poles of the horizon, as the argument requires, since all the vertical circles pass through those poles in the sphere. Hence they truly represent those same vertical circles, because they also pass through the points of the parallel PQ through which we showed they must be described. We will demonstrate this briefly in the following way. [Margin (b): Euclid III.31.] Since, for instance, the angle LIO is a right angle in a semicircle — that is, the angle PIS — the circle described about PS from R, the midpoint of the straight line PS, will necessarily pass through the point I, by the scholium of Euclid III.31. The same reasoning holds for the others. Now on astrolabes it is customary to draw only the segments of the vertical circles that lie between the horizon and the tropic of Capricorn ♎; but we have drawn the whole circles, so that the method of their description might be clearer.

4. So that the rays running out far from the point I may be drawn more easily and without error, we described about center I a circle $\mu\beta\xi$ of any size — the larger it is, the more exactly we shall carry out what is proposed. For, as was shown in Lemma 10, if an arc $\beta\gamma$ similar to half the arc HX is taken, or (with the diameter $\mu\xi$ drawn perpendicular to HI) if an arc $\mu\gamma$ similar to half the arc IX is taken, the ray IX will pass through γ . For this reason the arc $\xi\delta$ was likewise taken similar to half the arc IY, and the arcs $\mu\epsilon$, $\xi\theta$ similar to the halves of the arcs IL, IN, and so on. Thus if the semicircle $\mu\beta\xi$ is divided into 180 equal parts, the straight lines drawn from I through those parts will give, on the line PQ, the centers of all 180 vertical circles that divide the horizon into 360 degrees. [Margin: To find the centers of all the vertical circles cutting the horizon into 360 degrees, by means of the semicircle divided into 180 degrees.] For the straight lines drawn from I through the 180 parts of the whole circle ITHV — to whose halves those parts are similar — yield the same centers of all 180 vertical circles. For the line IL, falling on center P of the vertical HaI, cuts off from the circle ITHV the arc IL of 60 degrees, but from the semicircle $\mu\beta\xi$ the arc $\mu\epsilon$ of 30 degrees, which is similar to half of it, and so on. But if the same semicircle $\mu\beta\xi$ is cut into 90 parts, the centers of the 90 vertical circles that divide the horizon into 180 parts of two degrees each will be found in the same way, and so with the rest. And if the lines were drawn from H, not from I, they would exhibit the centers on the line PQ; then, in place of the circle $\mu\beta\xi$, a circle from H at any interval whatever would have to be described, and so on.

5. Again, so that the vertical circles may be described as exactly as possible, one must find in the horizon — by the methods we set out in Propos. 5, Num. 18 and 25 — the points of 30 and 60 degrees, through which they must pass: namely, both from the eastern point C and from the western point A, toward the south and north. [Margin: To find several points in the horizon and in its parallels, through which the vertical circles are to be described.] This is done not only by straight lines drawn from H, the pole of the horizon — such as Hkll, Hmkk, Hiip, Hhhq, Hppr, Hoof, Hunn, Hæimm — but also by lines from K, the center of the vertical circle, through the points of the line AC divided as taught in Lemma 8, such as the points i, l, n, t, which are found by the lines mp, kq, ai, vs, as appears in the figure; or (which I prefer) such points must be sought by the methods we set out in Propos. 6, Num. 25. For in this way each vertical circle has six points through which it is to be described, so that it cannot happen that the center of each, and its diameter, be correctly found unless the circle itself passes through all six points. And if several parallels of the horizon are drawn, in each of them those two points for describing each vertical circle can be found, if one wishes. But in the horizon it is enough if one point is found for any vertical, since a

straight line drawn from it through the center of the astrolabe will give another point on the same horizon — for every vertical circle cuts the horizon in two points diametrically opposite, as was shown of the two points of the horizon indicated by a straight line passing through the center, in the scholium of Propos. 5, Num. 10.

Indeed, when the vertical circle to be described is little distant from the meridian, and its center therefore lies far off from the point K on the line PQ, and the vertical itself, close to the meridian line BD, differs little from a straight line, it will be worthwhile to seek points on several parallels of the horizon at which that vertical cuts them. [Margin: To describe, through points and without a compass, the vertical circles that are little distant from the meridian.] For if those points are suitably joined by a curved line that nowhere makes an angle, the said vertical will be drawn in the astrolabe, in that portion which is contained between the tropic of Capricorn ϑ and the horizon — the portion in which we said (Num. 3) the vertical circles are to be described in the astrolabe.

6. It will perhaps be understood more easily that the vertical circles must be drawn through the points found on the line PQ, in this way. Conceive the circle HTIV to be equidistant from the horizon, and the point I to lie at the south pole, so that the plane of that circle is the one in which the parallel of the horizon, drawn through the south pole, lies — and let its point ϖ verge toward the rising (east) and ω toward the setting (west); and in the same plane, about the diameter I α equal to the diameter Aff of the parallel of the horizon drawn through A the south pole, let the parallel of the horizon itself be described, $\alpha\varpi I\omega$, from center dd. The common section of this parallel and of the equator, or of the plane of the astrolabe, is the straight line PQ, which represents that same parallel in the astrolabe, as has been said, since its distance KI from the point I is equal, by the definition of a circle, to the line AK, which in the sphere measures the distance of that same line PQ from the south pole. [Margin (c): Theodosius II.10.] And since the vertical circles cut the horizon and the parallel $\alpha\varpi I\omega$ in the sphere into similar arcs, those six vertical circles described in the astrolabe will make six diameters in the same parallel, each 30 degrees apart from the next — so that the primary vertical makes the diameter $\varpi\omega$; the vertical 30 degrees receding from it toward the north, on the eastern side, makes the diameter v[?], and so on. Therefore the points of the vertical circles at which they cut the parallel $\alpha\varpi I\omega$ will appear, from I the south pole, at those points of the line PQ on which fall the rays emitted from I through the ends of the diameters of that same parallel. Since, then, by Lemma 9 the said rays cut off from the circle HTIV — which touches the circle $\alpha\varpi I\omega$ at I — arcs similar to the arcs of the circle $\alpha\varpi I\omega$, and since by

construction the arcs IX, XL, LT, etc. are similar to the arcs $\text{I}\sigma$, $\sigma\rho$, $\rho\varpi$, etc. (both sets comprising thirty degrees each), the same rays will pass through the ends of the diameters of the circle HTIV. Therefore the vertical circles, drawn from the south pole, will be seen to pass through those points of the line PQ at which it is cut by the said rays. Which was to be shown. And so, since the centers of the vertical circles lie on the line PQ, it comes about that the portion of it intercepted between the two rays drawn from I through the ends of any diameter in the circle $\alpha\varpi\text{I}\omega$ is equal to the greatest...

diameter as seen along that diameter. Thus the portion PS is equal to the visible greatest diameter of that vertical which is 30 degrees distant from the primary vertical and passes through the diameter qaa, and so with the rest. Here too the straight line drawn from I perpendicular to any diameter of the circle $\alpha\varpi\text{I}\omega$ falls on the center of the vertical circle — that is, it bisects the diameter found on the line PQ, as is clear from the corollary of Lemma 35. Thus you see that Icc, perpendicular to qaa, meets the line PQ at R, the midpoint of the found diameter PS; and this same Icc is also perpendicular to LO at e, [Margin (a): Euclid I.28.] because qaa and LO are parallel — on account of the angles qddI, LKI, which are equal, by the scholium of Euclid III.22 — because of the similar arcs Iq, IL. And the same reasoning holds for the rest.

7. Since indeed in the scholium of Propos. 3, Num. 1, we demonstrated that the visible greatest diameter of every oblique great circle, and of any of its parallels, must be sought in the common section of the plane of the equator...

Diagram. A large stereographic-projection woodcut occupying the lower two-thirds of the page, enclosed in a square frame (the astrolabe plate). It shows the horizon, its parallels (almucantars), the full set of vertical circles (azimuths) with their construction rays converging on the south pole I at the bottom, and the auxiliary circle $\alpha\varpi\text{I}\omega$ used in the construction. The web of straight rays, dotted lines, arcs, and densely-packed letter-labels is extremely intricate; only the principal labels resolve clearly at this resolution, and many of the small interior Greek and doubled-letter labels cannot be read with confidence.

Principal legible labels (with approximate position):

- F — top apex of the figure, marked 90
- ll (printed as a doubled letter with 60) — upper-left region
- mm — upper-right region, marked 60
- 30 — degree-marks at left (near R) and at right
- P — left end of the horizontal diameter PQ (left frame)

- U / [Q?] — bottom-left, where the diameter meets the frame (accompanied by a symbol)
- I — bottom center, the south pole; apex from which all the construction rays radiate
- A — right of center (western point per the text)
- C — adjacent to A (eastern point per the text)
- B, Bb — left of center
- H — central region (pole of the horizon)
- E — central region (center of the astrolabe / equator)
- R, S — centers of vertical circles on PQ
- D, G, K — central cluster
- N, O, o — right of center
- dd — right (center of the parallel $\alpha\varpi I\omega$)
- V — right of center
- X, Y — lower-central
- L, T — points on the vertical circles
- Greek point-labels scattered through the interior: α , ϖ , ω , π , ρ , σ , ξ , and others [several not resolvable]

Because far more than two labels cannot be resolved, this figure is a for a focused high-resolution pass.

(Text resumes below the diagram, continuing §7.)

of the astrolabe, or of the equator, and of the great circle which is drawn through the poles of the world and the poles of that oblique circle; and (as we showed there, Num. 4) that the straight line traversing the center of the astrolabe and the center of the oblique circle is that common section of the plane of the astrolabe (or of the equator) and of the great circle passing through the poles of the world and the poles of the oblique circle: let us inquire whether the straight line gg ee, drawn through R the found center of the vertical PHSI and through E the center of the astrolabe, is that common section — so that from this too it may appear that the vertical circles have been correctly described by us. [Margin (b): Theodosius I.15.] Now since the vertical circle in the sphere, which the circle PHSI represents in the astrolabe (as we said), makes the diameter qaa in that circle $\alpha\varpi I\omega$, and is at right angles to the circle $\alpha\varpi I\omega$ itself, then, by Euclid, Elements XI, def. 4, the line Icc — which is perpendicular to qaa, the common section of the vertical and of the said circle — will be at right angles to the plane of that same vertical. [Margin (c): Euclid XI.18.] Therefore the great circle drawn through the

south pole I, and through the straight line Icc and the center E of the sphere, will be at right angles to that same vertical circle; [Margin (d): Theodosius I.13.] and so it will pass through the poles of that vertical. Since, then, it makes in the plane of the astrolabe a section which is the line gg ee — because its plane, extended along the line Icc R, meets the plane of the astrolabe at R, the center of the said vertical, and besides is supposed to pass through E, the center of the equator, just as the line gg ee has been drawn through R and E — it is clear that the line gg ee is the common section of the plane of the astrolabe (or of the equator) and...

of the great circle which is drawn through the poles of the world and the poles of that vertical, in the sphere. And because the common section of the said circle passes through the point cc in the sphere, and Icc has been shown to be perpendicular to the vertical — that is, to the diameter of the vertical — at cc, by Euclid, Elements XI, def. 3; and moreover because here too the straight line drawn from the south pole I, perpendicular to the diameter of the oblique great circle (which diameter is the common section of that circle with the great circle drawn through the poles of the world and through its own poles), is such as Icc is — as has been shown, and as is altogether necessary, by Propos. 5, Num. 3 and 4 — it falls at R, the center of the oblique great circle: which is exactly what we demonstrated. In just the same way we shall show that the straight lines traversing the centers of the other vertical circles and the center of the astrolabe are the common sections of the plane of the astrolabe and of the great circles which are drawn through their poles and through the poles of the world.

8. Moreover, since all the vertical circles are drawn through the poles of the horizon, the horizon will in turn pass through their poles, by theorem 1 of the scholium of Propos. 15, lib. 1 of Theodosius; and accordingly, since (by the corollary of Propos. 16, lib. 1 of Theodosius) the pole of every great circle is distant from it by a quadrant of a great circle, i.e. by 90 degrees, the poles of any vertical will be found with little trouble, if from either of the two points at which it cuts the horizon 90 degrees are counted off, on both sides, in the horizon itself. [Margin: To find the poles of any vertical circle in the astrolabe.] And so the points hh, mm will be the poles of the vertical PHSI, because between either of them and either of the points kk, oo — where this vertical intersects the horizon — 90 degrees are intercepted, that is, three arcs of the horizon each comprising thirty degrees. [Margin: The vertical circles divide the horizon and its parallels into degrees.] Here you see that the straight line gg, ee — in which lie the center of that vertical and the center of the astrolabe — passes through both poles hh, mm, as the matter requires, since that line (as has been shown) is the common section of the plane of the astrolabe and of the great circle drawn through the

poles of the world and the poles of the said vertical; that is, it represents that great circle drawn through the named poles. In the same way the points ii, nn will also be the poles of the vertical llHppl, etc. By this method we shall easily find a point in the horizon distant by a quadrant from a given vertical. Let the given vertical be iiHnn, cutting the horizon in the points ii, nn; and from either of them, from H the pole of the horizon, let a straight line be drawn Hii, or Hnn, cutting the equator at p, or u. If then, from p, or u, two quadrants of the equator pk, pr, or uk, ur, are taken on both sides, and the lines Hk, Hr are drawn, the horizon will be cut at the poles ll, pp, [Margin (a): Theodosius II.10.] so that the arcs ii, ll, iipp, or nn, ll, nn, pp of the given vertical iiHnn correspond to the quadrants pk, pr, or uk, ur of the equator — as is manifest from what we demonstrated in Propos. 5, Num. 17, 18, and 19. [Margin: The vertical circles divide the horizon and its parallels into degrees.] Furthermore, just as in the sphere the vertical circles divide the horizon and its parallels into degrees, so too the vertical circles in the astrolabe distribute those same circles into degrees.

9. Therefore, if from either of the poles of any vertical (I judge that the one should be chosen which lies within the equator, that is, in the semicircle of the horizon AGC) straight lines are drawn through the several degrees of the equator, the vertical itself will be divided into degrees, as we demonstrated in Propos. 5, Num. 17 and 20 — provided the order we there prescribed is kept, with the additions we advised (Num. 23 of the same proposition) must be observed, and so on. [Margin: To divide the horizon and its parallels into degrees. To divide any vertical into degrees.]

10. Now any given vertical proposed in the astrolabe we shall describe, from what has been said, with almost no trouble. For if it deflects from the primary vertical, from the east toward the south, or from the west toward the north, by any number of degrees — say 30 — we shall count those 30 degrees from the point T toward I, up to L, and cut off LM equal to the arc IL. For the line IM will cut the line PQ at R, the center of the proposed vertical, to be described through the points H and I. But if it deflects from the primary vertical from the east toward the north, or from the west toward the south — say by 30 degrees — we shall count 30 degrees from the point V toward I, up to N, and cut off NO equal to the arc IN. For the line IO will cut the line PQ at S, the center of the proposed vertical, to be described through the points H and I. [Margin: To describe in the sphere any given vertical proposed in the astrolabe.] And so that the given vertical may be described more exactly, from L or N the diameter LO or NM must be drawn; and by the rays emitted from I through the ends of the diameter, the visible diameter of the proposed vertical PS, or QR, must be cut off from PQ, so

that four points are had, P, H, S, I, or Q, H, R, I, through which the given vertical is to be described.

The same center of the proposed vertical will be found if the doubled declination of the given vertical is counted from H toward T — when the given vertical declines from the primary from the east toward the south, or from the west toward the north; or from H toward V — when the given vertical declines from the primary from the east toward the north, or from the west toward the south. That is, if, the declination being (say) 30 degrees, an arc of 60 degrees is taken, up to M or O; for again the line IM, or IO, will give the center R, or S, which is sought. [Margin: To find in the astrolabe the center of the vertical corresponding, in the sphere, to a given vertical.] For since the declination, e.g. Hb, is equal to the declination TL, with the common arc bT added the arc bL will be equal to the quadrant HT; and accordingly the angle bKL will be a right angle, by the scholium of Propos. 27, lib. 3 of Euclid — that is, the diameter bY will be perpendicular to the diameter LO. Therefore, from what we demonstrated in Lemma 35, if bM is taken equal to the arc Hb, the line IM will bisect the segment PS cut off by the rays IL, IO, at R, and so with the rest. Others, to find the center of each vertical on the line PQ, count its doubled declination from I toward T, or V, and through the end of the counting emit a straight line from H, which cuts the line PQ at the center of the given vertical: which method does not differ from ours. For if the arcs HM, IL are equal, the lines IM, HL will cut off the same line KR from PQ. [Margin (b): Euclid I.26.] For two triangles arise, equilateral with respect to one another, since they have right angles at K, [Margin (c): Euclid III.27.] and the angles at IH equal, resting on the equal arcs HM, IL, and moreover the adjacent sides IK, HK equal, and so on.

Again, the same center on PQ will be found if the declination of the given vertical is counted from the point β , in the semicircle $\mu\beta\xi$, toward μ , if the vertical deflects from the east toward the south, or from the west toward the north; or from β toward ξ , if it deflects from the west toward the south, or from the east toward the north. For the straight line drawn from I through the end of the counting will give the sought center on PQ — because such a declination counted from the point β is similar to that same declination, i.e. to half the doubled declination counted from the point H. Therefore, by Lemma 10, the straight line drawn from I to the end of the declination in the semicircle $\mu\beta\xi$ will pass through the end of the doubled declination in the circle HTIV. Hence, since the line drawn to the doubled declination in the circle HTIV falls on the sought center, as has been shown, the line drawn to the declination in the semicircle $\mu\beta\xi$ will also fall on the same center. Thus you see that the line I ς , drawn from I through the end of the arc $\beta\varsigma$ of

60 degrees, falls at P, the center of the vertical HaI, which deflects from the east toward the south by 60 degrees, and by as many from the west toward the north, and so on.

Indeed, if the arc of the declination of the given vertical is cut off from the horizon — starting from C, or A, toward F, or G, according as the given vertical deflects from the primary from the east or the west toward the south, or the north — three points will be had, through which it is to be described. Two of them in every vertical are H, I; the third is the one found in the horizon by the declination of the vertical, together with the point opposite it through the diameter in the horizon, which the line drawn from the found point through the center of the astrolabe indicates; through it the given vertical will necessarily pass, if no error has been committed in the drawing. But you will do more prudently if you investigate the center by the earlier method, on the line PQ, together with the extreme points of the diameter, since then more points are had through which the vertical is to be described.

11. Conversely, when any vertical has been described in the astrolabe, we shall recognize the degrees of its declination from the primary vertical, and toward which part it deflects, in this way. From H, the upper pole of the horizon, let a straight line be drawn to the point of intersection of the given vertical with the horizon, and let the point of intersection of this line with the equator be noted. For the arc of the equator between this point and whichever of the points A, C is the less distant... [Margin: To recognize the inclination of any vertical in the astrolabe to the primary vertical.]

(Text continues into the following page.)

[the arc of the Equator so found] will measure the declination of the given vertical circle from the prime vertical — from the east toward the south, if the arc of the Equator found runs from C toward B, or toward the north, if that arc inclines from C toward D; but it will deflect from the west toward the south, if the arc of the Equator found inclines from A toward B, or toward the north, if that arc recedes from A toward D. For example, if the given vertical circle is llHppI, we shall draw the straight line Hll, which cuts the Equator at k. The arc of the Equator Ck will then measure the inclination of the given vertical circle to the prime vertical, from the east toward the south. But if instead the straight line Hpp is drawn, cutting the Equator at r, the arc Ar will measure the same inclination, from the west toward the north. For one and the same vertical circle deflects from the prime vertical toward the south on the one side and toward the north on the other, and each inclination comprises the same number of degrees.

The same inclination may be found in this way. From I, to either of the two points at which the given vertical circle cuts the straight line PQ, let a straight line be drawn, and let the point where this line intersects the prime vertical be marked. For the arc between this point and whichever of the points T, V is the nearer will measure the inclination of the given vertical circle to the prime vertical: from the east toward the south, if the arc found runs from T toward I; or from the west toward the north, if that same arc tends from V toward H. But the given vertical circle will deflect from the east toward the north, if the arc found runs from T toward H, or from V toward I. Thus, if the given vertical circle is PHSI, we shall draw the straight line IP or IS, which cuts the prime vertical at L or O. The arc TL or VO will then give the required inclination — the former from the east toward the south, the latter from the west toward the north.

Others investigate the same inclination by this method. From I, or from H, through the centre of the given vertical circle lying on the straight line PQ, they carry a straight line as far as the prime vertical. For half of the arc intercepted between that line and the diameter IH will give the required inclination. Thus, if from I, through R, the centre of the vertical circle PHSI, the straight line IR is drawn as far as M, then Hb — half of the arc HM intercepted between the lines IM and IH — will be the arc of inclination. And if the centre lies to the left of the line IH, the given vertical circle deflects from the east toward the south and from the west toward the north; but if it lies to the right, from the west toward the south, or from the east toward the north. Yet since the vertical circles are not always drawn whole [i.e. as complete...]

Diagram. A large stereographic projection of the astrolabe, occupying the lower part of the leaf within a square frame. A great bounding circle (the outer meridian/tropic) is drawn; the vertical diameter runs from F at the top (marked 90) down to I at the bottom (the projected south pole), and a horizontal diameter runs through P at the left to e at the right. Within the circle is a very dense network of great circles (the verticals/azimuths and the horizon) and small circles (almucantars and parallels), overlaid with many dotted straight tangent-lines converging on the projection poles, producing a crowded star-pattern toward the centre. The degree graduations 90, 60, 60, 30, 30 are marked around the periphery.

Labels legibly read (with approximate position):

- F (top): apex of the vertical diameter, at the 90° graduation
- 90 (top, beside F): degree graduation
- ll (upper left, on the bounding circle, beside "60"): intersection point

- 60 (upper left): degree graduation by ll
- mm (upper right, on the bounding circle, beside "60"): intersection point
- 60 (upper right): degree graduation by mm
- 30 (right of centre-top): degree graduation
- $\theta\theta$ (upper centre): point
- B, c, α (upper-central cluster): points
- cd (upper-central): point
- Aa, C (centre-right, "Aa C"): point
- Bb (left of centre): point
- P (left end of the horizontal diameter): centre of the prime vertical
- T (on the horizontal, right of P): point
- ε (just below P): point
- γ (lower left): point
- M (lower left) with a Cancer-like glyph [$\mathfrak{C}$?]: point
- L (lower centre): point
- π (lower centre): point
- X (lower left): point
- A (centre-right): point
- O (right of A): point
- bd (right): point
- e (right end of the horizontal diameter): point
- R, S, V (central, on/near the horizontal): points
- θ (right, below e): point
- N (lower centre-right): point
- Y (bottom centre): point
- Φ (bottom centre, above I): point
- I (bottom): projected south pole
- G, D, H (central cluster): points
- further doubled-letter and Greek labels in the dense centre (pp, kk, oo, $\delta\delta$, $\gamma\gamma$, and others): mostly unresolvable at this resolution

whole, we shall not always have the points of intersection on the straight line PQ, nor the centres; and for that reason the former method seems preferable to this latter one.

But perhaps we shall obtain the same inclination more easily if, from I, through the centre of the given vertical circle, we draw a straight line as far as the

semicircle $\mu\beta\xi$. For the arc from β up to that line will give the required inclination — from the east toward the south, or from the west toward the north, if the centre is found to lie from K toward P; but from the west toward the south, or from the east toward the north, if the centre is found to lie from K toward Q. Thus you see that the straight line $I\theta$, drawn through Q, the centre of the vertical circle HZI, yields the arc $\beta\theta$ of 60 degrees, by which that vertical circle recedes from the prime vertical — from the east toward the north, and from the west toward the south. Nevertheless the former method, by which the inclination is found on the horizon, is more pleasing, because the centres of vertical circles that lie only a small interval from the Meridian are very far distant from the point K.

[Margin: A most elegant way of investigating the inclination of a given vertical circle to the prime vertical.]

But we shall obtain the same inclination most conveniently — however far the centres of the vertical circles may lie from the point K — in this way. Since any vertical circle cuts the straight line PQ in two points, one inside the prime vertical between the points T and V, and the other outside it, we shall draw from I, through its intersection with the line TV inside the prime vertical, a straight line, until it cuts the prime vertical or the semicircle $\mu\beta\xi$. For the arc of the prime vertical between T (or V) and that line will measure the inclination of the given vertical circle to the prime vertical, as is evident from what we showed just above under Number 2. For, as we demonstrated there, the segments of the straight line PQ — which represents the parallel of the horizon drawn through the south pole — correspond, as regards the number of degrees, to the arcs of the circle HTIV intercepted between the same lines drawn from I. Since, therefore, the vertical circles are distant from one another, as was demonstrated there, the arcs of the circle HTIV will also contain the same degrees by which the vertical circles are distant from one another. And because the same line, together with the semicircle $\mu\beta\xi$, gives half of the arc of the vertical circle by Lemma 10, [the arc] comprised [between Bb, or Dd, and that line] is half of the same inclination, and hence, doubled, will exhibit the whole inclination: from the east toward the north and from the west toward the south, when the given vertical circle cuts the segment KT, or the arc of the horizon CG; but from the west toward the north and from the east toward the south, when the intersection falls in the segment KV, or the arc of the horizon AG. Thus the straight line IR, drawn from I through R, the intersection of the vertical circle HRIQ with the line KT, cuts off from the prime vertical the arc TM of 30 degrees, and from the semicircle $\mu\beta\xi$ the arc Bb, Aa of 15 degrees. Therefore the said vertical circle [deflects] from the prime vertical [toward the north] and from the west toward the south by 30 degrees.

[Margin: To ascertain toward which side a given vertical circle on the astrolabe deflects from the prime vertical.]

By exactly the same reasoning we shall investigate the inclination of any two vertical circles whatever, if through [the centres, or rather through] their intersections with the line KT or KV, we send out straight lines from I, and so forth. For example, the lines IR, IZ intercept [the arc] Mb, the arc of inclination of the vertical circle HRI to the vertical circle HZI, on the prime vertical, or [in the semicircle $\mu\beta\xi$ they intercept] half of the same inclination between the lines IR and IZ; and so with the rest.

[Margin: To ascertain, on the astrolabe, the inclination of one vertical circle to another.]

12. In no other way will the circles of the latitudes of the stars be described — those that pass through the poles of the Ecliptic and, running along the longitudes of the stars, measure their latitudes. For if the Ecliptic, in the position in which it was described under Proposition 5, Number 7, is taken as some horizon, there will be a great circle drawn through its poles and through the intersections of the Ecliptic with the Colure of the equinoxes on the Equator of the astrolabe — the circle represented by the circle AQC in the figure of Proposition 5, Number 7, described from the centre P, after the manner of the prime vertical. Hence the others will be described just as the other vertical circles deflecting from the prime vertical are, provided their centres are found on the straight line drawn through the centre P at right angles to the meridian line PQ. But since the lower pole is too far distant, their centres and diameters will more conveniently be found on that straight line by means of lines drawn from the nearer pole, so as [to cut off] equal parts of the circle AQC — or rather (since it is too large) by means of [a small semicircle] that touches the circle AQC at Q. For these lines will cut off similar arcs from the circle AQC, by Lemma 9; just as, in the figure of this proposition, the lines drawn from I through the arcs of the circle $\alpha \varpi I \omega$ pass through similar arcs of the prime vertical ATIV. Or finally, if from Q, at any interval, a semicircle is described, the lines sent out from Q through the degrees of that semicircle will give the centres on that same perpendicular passing through P, just as was said a little before, at Number 4, concerning the semicircle $\mu\beta\xi$.

[Margin: To describe on the astrolabe a great circle through the poles of any other great circle.]

Finally, by the same reasoning we shall draw the great circles through the poles of any given great circle, provided that we first describe the prime circle, after the manner of the prime vertical, through the same poles — namely such that it too has its own poles and centre on the same straight line on which the centre and poles of the given great circle lie, and that it pass through the intersections of that circle with the Equator; just as the prime vertical of the horizon has its poles and centre on the meridian line, on which the poles and centre of the horizon lie, and runs through the common intersections of the horizon with the Equator, and so forth.

13. Moreover, just as the straight lines drawn from K, the centre of the prime vertical, through the points A, C — where the horizon and the prime vertical cut each other — when produced are tangent to the horizon at A and C, and the lines sent out from B, the centre of the horizon, to the same points are tangent there to the prime vertical (as was shown in Proposition 5, Numbers 28 and 29): so too it happens in the other vertical circles. For the line llHpp, at the point ll where it cuts the horizon, is there tangent to the horizon; and the line Pll, drawn from P, the centre of the vertical circle, is tangent there to the horizon; and conversely the line Bll, drawn from B, the centre of the horizon, to the same point of intersection, is tangent there to the said vertical circle. In the same way, the line Ppp, drawn [from P], would touch the horizon at pp, and there the line Bpp would touch the said vertical circle. Again, the lines Rkk, Roo, when sent out, would be tangent to the horizon at kk, oo, and the lines Bkk, Boo would in turn be tangent there to the vertical circle PHSI; and so with the rest. Moreover, any line drawn from P, the centre of the vertical circle llHpp, cuts off equal arcs to either side of the point of contact ll from the horizon, as regards the number of degrees. Thus you see that the line PkkF cuts off the two arcs llkk, llF of 30 degrees. In like manner the line PC, produced, would fall on the point mm, so as to cut off the two arcs llC, llmm of 60 degrees. And the line PG, produced, would pass through oo, so that to either side of the point of contact pp it would cut off the two arcs ppG, ppoo of 30 degrees. And so with the rest.

[Margin: That straight lines drawn from the centre of any vertical circle to its intersection with the horizon are tangent to the horizon, etc.]

By like reasoning, if from the centre εε a parallel of the horizon δδγγ is described, cutting the vertical circles llHpp, PHSI at δδγγ, the line Pδδ will be tangent to the parallel at δδ, while the line εεΑΑ [is tangent to] the vertical circle llHpp at the

same point $\Lambda\Lambda$. Likewise the line $R\gamma\gamma$ would be tangent to the same parallel at $\gamma\gamma$, while the line $\varepsilon\gamma\gamma$ would in turn be tangent to the vertical circle $PHSI$ at $\gamma\gamma$; and so with the rest. Moreover, any line drawn from P , the centre of the vertical circle $lHpp$, cuts off to either [side...]

[Margin: That straight lines drawn from the centre of any vertical circle to its intersection with the horizon, when drawn out, are tangent to the horizon, etc.]

[cuts off to] either side of the point of contact $\Lambda\Lambda$ equal arcs from the parallel of the horizon, as regards the number of degrees — so much so that the line $P\gamma\gamma$, produced, would fall on $\theta\theta$, since any of the arcs $\delta\delta\gamma\gamma$, $\Lambda\delta\theta\theta\Lambda$ [?] comprises 30 degrees. And so with the rest. Therefore, if the point B is found to be the centre of the horizon described on the astrolabe, and from it any straight line Bll is drawn as far as the circumference, [^a] the line lP dropped perpendicular to Bll at P will fall, [P being] the centre of the vertical circle to be described through ll ; and this because Bll is tangent to that vertical circle at ll , as has been said. Conversely, if from P as centre the vertical circle lH is described, cutting the horizon at ll , and to the drawn line Pll a perpendicular llB is erected, this will fall on B , the centre of the horizon — because Pll too is tangent to the horizon at ll . Again, if from P , the centre of the vertical circle lH , to $\delta\Lambda$ [?], where that vertical circle cuts the parallel of the horizon, a tangent line is drawn (as has been said) touching the parallel at $\delta\Lambda$, then $\delta\delta\varepsilon\varepsilon$ will fall perpendicular to $P\delta\delta$ at $\varepsilon\varepsilon$, the centre of the parallel. And on the contrary, if from $\varepsilon\varepsilon$, the centre of the parallel, to $\delta\delta$, where the vertical circle lH cuts the parallel, a line is sent out, $\delta\delta P$ will fall perpendicular to $\varepsilon\varepsilon\delta\delta$ at P , the centre of the said vertical circle. And the same is to be said of all the other vertical circles and parallels, and of their centres.

[Margin: ^a III.19 (Euclid, *Elements* III.19).]

Now all these things we shall demonstrate as follows. Let the parallel $\alpha\varpi I\omega$ of the horizon, drawn through the south pole, be conceived to have its own proper position in the sphere, such that — the circle $ABCD$, which is now taken for the Meridian of the horizon, standing at right angles to the very plane of the astrolabe — the point I coincides with the south pole A . And because in such a position the straight line qaa [?] is the common section of the said parallel $\alpha\varpi I\omega$ and of the vertical circle deflecting 30 degrees from the east toward the south from the prime vertical (which vertical circle the circle $PHSI$ on the astrolabe represents — a matter easily understood if the south pole is imagined to be placed behind the astrolabe, as we said above at Number 1 of this proposition); and because the great circle drawn through the poles of the world and through the poles of the said vertical circle — which of course stands upright to it after the

manner of its proper Meridian — is drawn through the straight line IccR, and makes on the astrolabe the section gg ee; and because the common section of this same great circle and of the said vertical circle passes through the point cc, in such a way that Icc, drawn from I, the south pole, in that position to that common section (that is, to the true diameter of the said vertical circle) is perpendicular, and falls on R, the centre of the same vertical circle on the astrolabe — all of which were demonstrated just above at Number 7: it comes about that the plane drawn through the straight line IccR in that same position, and revolved about that same line, [b] is always perpendicular to the aforesaid vertical circle, and produces on the horizon common sections parallel to one another, which cut off equal arcs on either side from the common section of the horizon with that same vertical circle, as was demonstrated in Lemma 25 — except when that plane, drawn through the straight line IccR, reaches the extremities of the common section of the horizon with the said vertical circle. For then all section ceases, and the plane itself, at those extremities, [touches] each circle, that [is...]

[Margin: ^b XI.18 (Euclid, *Elements* XI.18).]

[The same astrolabe-projection woodcut described above (fig-p568-1) is reprinted at the foot of this leaf; not re-described.]

ASTROLABII LIBER SECVNDVS — Propositio VIII (continued)

(continuation of §13, from the foot of the previous page)

that is, it will touch both the horizon and the said vertical circle — exactly as was said above (Prop. 5, nn. 24 and 28) about the plane drawn through the line IK or AK. Therefore, since that plane makes, in the plane of the astrolabe, straight lines passing through the centre R (Prop. 1, n. 1), the straight lines drawn out from R will represent that plane in its revolution, and will cut the horizon at the same points at which it is cut by that plane. Consequently, on either side of the vertical circle kkHoo they will cut off equal arcs from the horizon and will touch it at the same points kk, oo, as we also said (Prop. 5, n. 28).

But although the aforesaid plane, carried around the line IR, divides the common section of the horizon and of the said vertical circle in the sphere — at the points through which are drawn the straight lines perpendicular to that section from the individual degrees of the horizon — nevertheless the visible common section kkoo of those same circles in the astrolabe cannot be divided in the same way, since it differs from the former one in the sphere and is not parallel to it. I have said this so that you will not suppose that the horizon can be distributed into

degrees, in the astrolabe, by straight lines drawn from the centre R through the points of the line kkoo when that line has been divided in the same ratio in which we set out [the division] in Lemma 8.

14. But should anyone wish to find the points of the line kkoo through which straight lines drawn out from the centre R divide the horizon into degrees, he should proceed thus. Beginning from the points of contact kk, oo, the line kkoo must be produced through the centre E. This line will be the common section of the plane of the astrolabe (or of the equator) with the great circle drawn through the poles of the world and through the common sections of the horizon and the said vertical circle — the great circle which stands at right angles to the vertical circle hhHmm, being drawn through the poles of the said vertical circle kkHoo; for that circle drawn through kkoo also passes through kk and oo, the poles of the vertical circle hhHmm. Indeed, since this circle passes through the poles of that one, that one will in turn pass through the poles of this one (by the scholium to Prop. 15, Book 1, of Theodosius [Theodosius]) — and all these poles lie on the horizon.

Next, at kkoo one must erect, through the centre E, the perpendicular cbZ, which will represent the axis of the world, if the circle ABCD is taken for that great circle which, drawn through the poles of the world, makes in the plane of the equator the straight line kkoo. Finally, if from the pole cb rays are drawn through the extreme points kk, oo of the visible diameter of the vertical circle, the circle ABCD will be cut at the points cd, cf; and through these must be drawn the true diameter of the horizon — namely cdcf — which is the common section of the horizon itself and of the said vertical circle kkHoo in the sphere. Let this diameter be divided in the same ratio in which the line was divided in Lemma 8 (that is, by that plane which is drawn through the line IR and revolved through the individual degrees of the horizon); and from the points of division let rays be sent out from the pole cb. At the points where these cut [the visible diameter], if straight lines are then thrown across from the centre R, the horizon will be distributed into degrees. I have supplied no worked example of this division, lest too great a confusion of points and lines should arise in the figure, and especially because it is also the longer method and has almost no use — unless one should wish to employ it in order to test whether or not it agrees with the earlier divisions.

[Margin, beside §14: To find the points in the common section of any vertical circle with the horizon, through which — if straight lines be drawn from the

centre of that vertical circle — the horizon is distributed into degrees. (a)
Theodosius I.15.]

15. By exactly the same method, that plane, drawn through the line IR and revolved, will cut the parallels of the horizon into degrees, and will touch them at the points where the said vertical circle cuts them; and the straight lines sent out from the centre R will produce exactly the same result, inasmuch as they represent that plane in its revolution, as has been said. But here the finding of the points on the visible diameter of each parallel of the horizon — through which the straight lines are to be drawn from the centre R, so that the parallel itself may be distributed into degrees — is more difficult. It will nonetheless be set up as follows (should anyone happen to wish to divide the parallels of the horizon in this way, so as to learn by trial how neatly it agrees with the foregoing).

From the centre E, to $\theta\theta\gamma\gamma\eta$ — the visible diameter of the said parallel $\theta\theta\gamma\gamma\eta$, that is, to the common section of the said parallel with the vertical circle $\gamma\gamma\eta\eta$ in the astrolabe, whether produced or not produced — let there be drawn the perpendicular diameter EFG (as appears in the accompanying figure). This will represent the great circle drawn through the poles of the world and through the poles of the non-great circle, which is drawn through the south pole and through the true diameter of the given parallel (that being the common section, in the sphere, of the vertical circle proposed and of the parallel of the horizon), and which therefore makes in the astrolabe the section $\gamma\gamma\eta$. (b) For since that great circle is at right angles both to this non-great circle and to the equator, these two will in turn be at right angles to that great one. (c) Therefore their common section $\gamma\gamma\eta$ will also be perpendicular to it; and hence it will likewise be perpendicular to the common section of that great circle with the equator (or with the plane of the astrolabe), by Euclid, Book 11, def. 3. Consequently the line EF will be perpendicular to $\gamma\gamma\eta$ — [EF being] the common section of the plane of the astrolabe and of the said great circle.

Next, at EG let the perpendicular diameter EI be erected, which will represent the axis of the world if the circle ABCD be understood to be at right angles to the plane of the astrolabe (or of the equator); and I will be the south pole. Hence, if through F the straight line IFO be drawn across, it will be the diameter of that non-great circle [drawn] through the south pole I, and of the true diameter of the given parallel drawn in the sphere, which makes in the astrolabe the visible diameter $\gamma\gamma\eta$ of that same parallel — since that circle meets the plane of the astrolabe at F. (c) For it follows from this that, when that circle is bisected by the great circle drawn through its poles (which makes in the astrolabe the section

EFG), the line IF, lying in that great circle, passes through its centre, because that great circle cuts it along the line IF; and therefore the whole IFO is its diameter.

After this, on EG produced let the line FL be taken equal to FI; from it let LS be cut off equal to the found diameter IO; and about LS let the circle LmSp be described. This will be that non-great circle drawn through the south pole and through the common section, in the sphere, of the vertical circle and of the proposed parallel — as is evident if one conceives it to be moved downward, about $\gamma\eta$, toward the south (the equator ABCD remaining in its own position, so that the surface in which it is described inclines toward the north, and A looks toward the west and C toward the east), until L is joined to the south pole. (e) For then the said circle will pass through the south pole, and will be at right angles to the great circle drawn through the poles

of the world and through its own poles, making the section GE, since it is drawn through $\gamma\eta$, which we showed to be perpendicular to the great circle drawn through EG. Since, therefore, it has its own proper diameter LS, it is clear that it is that very circle we spoke of.

In order, then, that we may find in this circle the true diameter of the given parallel — that is, its common section with the given parallel and with the vertical circle — the rays L $\gamma\gamma$, Ln must be drawn, cutting the said circle at m, p. For the line mp will be that diameter, since the rays drawn through its extremities exhibit the visible diameter $\gamma\eta$. This diameter mp, therefore, is divided by the aforesaid plane drawn through the south pole L, as was stated in Lemma 8. So if it be divided in that way, and from the points of section straight lines go out from L, the south pole, the visible diameter $\gamma\eta$ will be cut at points through which the straight lines sent out from the centre R will cut the parallel $\theta\theta\gamma\gamma$ into degrees, since they represent that plane revolved through the individual degrees of its parallel in the sphere [circumductum, recte: circumductum]. Moreover the found diameter mp, if no error has been made, ought to be equal to the diameter ST of that same parallel in the first figure of Prop. 6 — provided that the equator of that figure is equal to the equator ABCD of the present figure. And the same reasoning holds for the other parallels.

Now, what has been said of the vertical circle PHSI and of the straight lines drawn out from its centre R is also to be understood of the other vertical circles and of the straight lines going out from their centres. Indeed, the same can readily be transferred to other great circles as well — namely to the ecliptic and to the great circles drawn through its poles, and so on. For there too the straight lines sent out from the centre of any great circle drawn through the poles of the

ecliptic will touch the ecliptic, and [will touch] any parallel of it, at the points where they are cut by the said great circle, and so on.

Diagram. A framed stereographic construction for Prop. VIII, showing the equator ABCD as the bounding circle, an oblique vertical/parallel circle inclined across it, and the diameters and chords used in §§14–15. Readable labels (approximate positions):

- $\theta\theta$ (top, on the vertical line): paired theta point-marks
- L (upper right, on the outer circle)
- B (upper middle, on the vertical diameter)
- G (upper middle-right)
- O (upper left, on a dashed circle)
- H (middle, on the vertical diameter above centre)
- E, F (centre): E = centre of the astrolabe, F just to its right
- n (centre-right, on the horizontal diameter)
- A (right end of horizontal diameter)
- C (left end of horizontal diameter)
- I (lower right)
- S (lower left, on a dashed circle)
- K (bottom middle)
- P (bottom middle-right)
- R (bottom left, in the frame corner)
- D (bottom of the vertical diameter)

The $\gamma\gamma$ point-marks named in the text (visible diameter $\gamma\gamma n$, parallel $\theta\theta\gamma\gamma$) sit near the centre-right about n but are not separately legible on the scan. [The paired dashed circle is the non-great circle LmSp / aCbO of the construction.]

16. But since any great circle described in the astrolabe must divide the equator into two equal semicircles, as was shown in the scholium to Prop. 5, n. 6, it must be demonstrated that the vertical circles here described in the astrolabe do the same — so that a straight line joining the two intersections of each vertical circle with the equator is a diameter of the equator, and consequently the vertical circle itself passes through two points of the equator diametrically opposite. Let there be set up, then, for the sake of example, from the previous figure of this proposition, a vertical circle PHSI drawn separately, deviating 30 degrees from the prime vertical, from east toward south, whose centre R lies on the straight line PS, which is drawn from K, the centre of the prime vertical, perpendicular to

the meridian line BD; the equator being ABCD, the horizon AFCG, and its poles H, I.

[Margin, beside §16: That any vertical circle — or any other great circle whatever — cuts the equator, in the astrolabe, at two points diametrically opposite.]

Through R, the centre of the given vertical circle, and E, the centre of the astrolabe, let the straight line ggmm be drawn, cutting the vertical circle at ee — this being the common section of the plane of the equator (or of the astrolabe) and of the great circle drawn through the poles of the world and the poles of the said vertical circle, as we showed in the scholium to Prop. 3, n. 4; and to it let the diameter of the equator LM be erected at right angles. I say that the vertical circle PHSI passes through the points L, M. For (n.) if the circle ABCD be set at right angles, along the line ggee, to the plane of the equator

(or of the astrolabe), (a) and is therefore in that position through the poles of the equator (that is, of the world) — since it is perpendicular to the line ggee lying in the plane of the equator or of the astrolabe, as reasoning requires; (b) (for since the axis is at right angles to the equator and passes through the centre E of the sphere, it will likewise be perpendicular to the line ggee, by Euclid, Book 11, def. 3) — it comes about that the rays sent from the pole M through ee, gg, the extremities of the visible diameter, fall on N, O, the extremities of the true diameter of the aforesaid vertical circle; so that the line NO passes through the centre E, being the diameter of the great circle which the circle PHSI represents in the astrolabe. For if any line other than NO were said to be the diameter of the aforesaid vertical circle (whose visible diameter is eegg), the rays sent from the pole M through the extreme points of that diameter would cut off another visible diameter from the line gg, mm — which is absurd.

By the same reasoning we shall find the true diameter of any circle whatever described in the astrolabe, whether great or non-great, if we draw a straight line through its centre and the centre of the astrolabe, and erect a perpendicular to it at the centre of the astrolabe. For the rays falling from either of the extremities of this perpendicular, through the extremities of the visible diameter of the given circle (which the circle itself cuts off from the line drawn through both centres), pass in the circle ABCD through the extremities of the true diameter — as was done in the vertical circle PHSI; and you have another example in the circle aCbO, a non-great circle. For if, through its centre h and the centre E of the astrolabe, the drawn line hE is cut at right angles by the diameter LM of the equator, and if from M (which is taken for the south pole) rays are sent through a, b, the extremities of the visible diameter ab, the equator will be cut at Y, Z. The line YZ,

therefore, will be the true diameter of the non-great circle aCbO; and the same reasoning holds in the rest.

[Margin, beside this passage: To find the true diameter of any circle described in the astrolabe, whether great or non-great.]

Now let the circle ABCD, with its lines, be conceived to lie once more in the plane of the astrolabe; (c) and the angle NMO will be in a semicircle — that is, the angle ee, M gg is a right angle. Therefore the circle described about the diameter ee, gg will pass through the point M, by the scholium to Prop. 31, Book 3, of Euclid [Euclid]. From L, M let the lines LR, MR be drawn to the centre R. And since the two sides ER, EM are equal to the two sides ER, EL, and contain equal angles — namely (d) right angles — the bases RM, RL will also be equal (Euclid I.4). Since, therefore, RM is a semidiameter of the vertical circle, as it has been shown to pass through M, RL will likewise be a semidiameter of the same, and consequently the same vertical circle will pass through L. The vertical circle PHSI therefore passes through the points L, M, and consequently divides the equator at those same two diametrically opposite points — which was the proposition. The same will be demonstrated of all the other vertical circles — indeed of any great circle whatever described in the astrolabe — as we also noted in the scholium to Prop. 5, n. 3. And since the great circles in the sphere cut one another in half, the same happens with the circles in the astrolabe that represent great circles; and therefore the arcs LeeM, LggM represent the semicircles of the proposed vertical circle, into which of course it is divided by the equator.

17. And since the poles of any great circle are distant a quadrant from it (by the corollary to Prop. 16, Book 1, of Theodosius), if the circle ABCD be understood to be, in the sphere, at right angles to the vertical circle which the circle PHSI represents, (e) and consequently to pass through its poles, then the points Q, T — which bisect the semicircles NQO, NTO (that the true diameter NO, found in n. 16, cuts off) into two quadrants — will be the poles of that same vertical circle, and will appear in the astrolabe, through the rays MQ, MT, at the points hh, mm, which points will lie on the horizon. For since any vertical circle passes through the poles of the horizon, the horizon will in turn pass through its poles (by the scholium to Prop. 15, Book 1, of Theodosius); and so the poles hh, mm lie on the horizon, and on that same horizon the vertical circle ZHmn will intersect [it], being 90 degrees distant from the vertical PSHI, or 60 degrees from the prime vertical, receding toward the north from the east, as appears in the first figure of this proposition.

[Margin, beside §17: To find the poles of any vertical circle, or of any other circle — whether great or non-great — described in the astrolabe.]

We shall find the poles of any other vertical circle, or of any great circle described in the astrolabe (or non-great circle), in no other way [than this]: if we bisect the segments of the equator that are cut off by the true diameter of the circle, found as we taught in n. 16. For these points of section will be the true poles of the given circle; and if rays are sent to them from the south pole (from which the true diameter was found), the line drawn out through the centre of the circle and the centre of the astrolabe will be cut at the apparent poles of that same circle — as was done in the vertical circle PHSI, and as you have another example in the circle aCbO, a non-great one. For the points Q, T, which bisect the arcs YQZ, YTZ cut off by the true diameter YZ found in n. 16, will be the true poles; while the rays MQ, MT will indicate the apparent, or visible, poles hh, mm on the line hE, extended through the centre h of that non-great circle and through E, the centre of the astrolabe. And the same reasoning holds in all other circles, both great and non-great.

[Margin: To find the poles of any vertical circle, or of any other circle — whether great or non-great — described in the astrolabe.]

But if only one of the poles is required — for example, the upper one, which of course falls within the equator (and which alone is generally needed in the use of the astrolabe) — it will be found with almost no trouble on the great circle, even though neither the whole circle has been described nor its true diameter found in the manner [above]. Let there be given only the arc HS, cutting the equator at M. (For if it does not cut it, it must be produced until it does.) From its centre R, through E, the centre of the astrolabe, let the line RE be drawn, cutting the given arc at ee. (And if it does not cut it, it must be produced until it does.) Through ee, and from M — the point where the given arc cuts the equator, or upon which the diameter of the equator LM falls — let the line M ee be drawn perpendicular to R ee, cutting the equator at N; and let the arc NQ be taken equal to a quadrant AB of the equator, in such a way that the line drawn MQ cuts the line R ee within the equator at hh. For this point of section hh will be the pole of the given great circle. For since the line R ee is the common section of the plane of the astrolabe and of the great circle drawn through the poles of the world and the poles of the given circle (as we showed in Prop. 3, n. 4), M can be taken for the south pole, if the circle ABCD is understood to be at right angles to the plane of the astrolabe (or of the equator); and consequently the ray M ee will fall on N, the extremity of the true diameter. Since, then, the pole is distant a quadrant of the circle from it,

Q will be the pole, and so on. If the quadrant NT be taken on the other side, the ray MT will give the other pole mm — the lower one, that is — which falls outside the equator.

[Margin: To find the pole of any great circle, even though it is not wholly described in the astrolabe.]

Otherwise, the pole within the equator will be found, with no regard had to the circumference of the great circle. For if, LM having been drawn perpendicular to RE, the angle EMR be bisected by the line MQ, RE will be cut at the pole hh, as we showed in Prop. 5, n. 12.

18. Moreover, (f) since all great circles in the sphere cut one another in half, it is necessary that the same happen in the astrolabe: so that, when two circles in the astrolabe which represent great circles cut one another, the straight line joining their intersections represents their common diameter, and therefore passes through the centre of the astrolabe — since all the diameters of great circles pass through the centre of the sphere, which (as was shown in Prop. 5, n. 4) does not differ from the centre of the astrolabe. Thus you see, in the immediately preceding figure

Diagram. A large framed figure showing the vertical circle PHSI (deviating 30° from the prime vertical) drawn separately, superimposed on the equator ABCD and horizon AFCG, with the rays, diameters (ggmm, LM, NO) and pole-constructions of §§16–17. The figure is very crowded and several peripheral marks are set with astrological/planetary sorts as point-labels; the clearly legible labels are:

- E (top, on the vertical diameter): centre of the astrolabe
- N, B (upper middle, flanking the vertical line)
- k (kappa, upper-middle-right)
- Y (left), L, P (upper left cluster)
- C (left)
- A, T (right side)
- X (lower-middle-right)
- M (lower middle)
- Z, O, S, D (lower band)
- b (lower left)
- I (bottom)
- a (α , near centre-left)

Several additional peripheral marks (paired/doubled sorts near the upper right, and small sorts along the lower-left arc — some resembling zodiacal/planetary type) could not be resolved on the scan and are left unread. **More than two labels are illegible.** [The paired dashed circle is the non-great circle aCbO of the construction; H, G and the further pole-points named in the text sit on it but are not separately legible.]

that these two greatest circles AFCG and PHSI cut each other along the straight line VX, drawn through E, the center of the astrolabe. That this is altogether necessary I shall demonstrate geometrically as follows. Since each of the two great circles bisects the Equator [*Æquator*] at two points diametrically opposite—as I showed a little earlier in this same Proposition at Num.16, and in the scholium to Proposition 5 at Num.6—both straight lines AC and LM, which join their intersections with the Equator, therefore pass through E, the center of the astrolabe. I say, then, that the straight line VX as well, which connects their intersections, passes through the same center E: that is, that the line VE, when produced, falls upon the other of their intersections, X.

For let the straight line VE, produced, cut one of the circles—say the circle AFCG—at X. I say that the other circle PHSI passes through the same point X, and hence that both circles cut each other there; that is, that the line VE, produced to X, falls upon their common intersection. For since the lines VX and AC cut each other within the circle AFCG at E, the rectangle under VE, EX is equal to the rectangle under AE, EC [a]. But the rectangle under LE, EM is equal to this latter rectangle, for the same reason—because the lines AC and LM also cut each other within the circle ABCD at E. Therefore the rectangle under VE, EX will also be equal to the rectangle under LE, EM; and consequently, by the scholium to Proposition 35 of Book 3 of Euclid [Euclid, *Elements* III.35], the circle PHSI, described through the three points V, L, M, will necessarily pass through the fourth point X as well. And so the point X lies on both circles, AFCG and PHSI. The straight line VE, then, produced to X, falls upon the common intersection of those circles. Which was to be demonstrated.

[Margin: The straight line that joins the intersections of any two great circles in the astrolabe passes, through the center of the astrolabe, into the other intersection.] [Margin: a. Euclid III.35.]

19. Next, so that you may see in what way the parallel of any oblique great circle drawn in the astrolabe is described—as I mentioned at Proposition 6, Num.20—it will not be amiss to make it clear by a single example. Let there be described, then, the parallel of some oblique great circle: for instance, of

the vertical circle PHSI, which recedes 30 degrees from it toward the pole hh. And since this can be done in four ways, we shall proceed by the first way as follows. Once the true diameter NO of the oblique great circle PHSI has been found—as taught at Num.16—we shall count off from it toward Q, from each of its two ends, 30 degrees, up to Y and Z, so that the diameter YZ of the proposed parallel can be drawn. For if from M, the south pole, rays are drawn through Y and Z, the apparent diameter ab of the parallel will be cut off; and this, bisected at h, [gives the center] from which, describing through a and b, the proposed parallel is drawn, as appears in the next figure.

[Margin: To describe the parallels of any vertical circle, or of any other great circle, in the astrolabe.]

Diagram. A large square frame enclosing the stereographic construction for the parallel of a vertical circle (oblique great circle). A great circle is drawn with a vertical diameter running from E at the top down to I at the foot; within it lies the oblique great (vertical) circle PHSI together with its proposed parallel and the analemma-construction lines (dotted and dashed rays from the south pole). The figure is crowded with small labels, several at the paleographic limit of the woodcut.

- E (top of the vertical diameter): north pole / top of primitive circle
- I (foot of the vertical diameter): bottom point
- B (upper interior, just below E): point on the inner circle
- N (upper left, near B): equatorial point
- Y, L, P (upper-left cluster on the oblique circle): construction points
- C, Q (left): points on the arc
- T, A (right side): points on the arc
- M (lower right): south pole
- X (interior, right of center): intersection point
- D, O (bottom, near the horizon line): points
- Z (lower left): point
- S (right end of the horizontal/horizon line): cardinal point
- F, G (lower-left interior): points
- hh (interior left): projected pole of the parallel [?]
- a, b (interior): ends of the apparent diameter of the parallel [?]
- k (interior, lower): point [?]
- mm (upper right, near T): projected pole-pair [?]

- ☿ (Cancer symbol, near the foot): tropic marker [?]

[Numerous small peripheral labels; hh, mm, a, b, k and the zodiac marker are uncertain]

By a second way we shall accomplish the matter thus. Having drawn the diameter cd of the oblique great circle perpendicular to the straight line ee, gg, we shall count off from the points ee, gg 30 degrees up to f, e, and shall draw the line e f cutting cd at g. For the rays Me, Mf will cut off the same apparent diameter ab, while the line Mg will exhibit the center h, and so on.

By a third way the same parallel will be described if, from the south pole M, a circle of any size is described, and the rest is done as I prescribed at Proposition 6, Num.8.

By a fourth way we shall delineate the same parallel if, first, about the diameter hh, mm through the poles hh, mm of the oblique great circle, a great circle is described, which will serve as a primary vertical of the given oblique circle. For if in its quadrant intercepted between hh and L there is taken the 30th degree, beginning from the point hh—as I taught at Proposition 5, Num.18—and we draw through that degree a line tangent to that circle, it will fall at h, the center of the parallel, and so on.

In passing it should also be noted that all these points—the center of the astrolabe (or of the world); the center of any oblique great circle whatever, or even of any of its parallels; and its two poles—lie on one and the same straight line: so much so that a straight line drawn through two such points passes entirely through the remaining two points. Thus you see in the next figure that on the straight line gg mm there lie E, the center of the astrolabe; R, the center of the vertical circle PHSI [printed PHMI (?)]; h, the cen-

[Margin: The center of the astrolabe, the center of any greatest oblique circle, the centers of its parallels, and its poles all lie on one straight line in the astrolabe.]

—ter of the parallel aCbO of the same vertical circle; and its two poles hh, mm. The reason is that a straight line drawn through the center of the astrolabe, or through the center of an oblique circle, represents the common section of the plane of the astrolabe with the Equator, or with the great circle that is drawn through the poles of the world and the poles of the described oblique circle in the manner of its proper meridian—as I showed in the scholium to Proposition 3, Num.4.

20. Now the parallels of any oblique great circle whose apparent diameters, contained within the circle, lie within its apparent diameter eegg, look toward the north, on account of the north pole E, which lies within that same circle. For hence it comes about that this whole face of the oblique circle is called northern; whereas the parallels described outside the oblique great circle belong to the south, for the contrary reason. From this it further follows that the diameters of the parallels in the semicircle NQO look toward the northern parallels, while in the semicircle NTO they look toward the southern; because the former are projected upon the apparent diameter eegg in such a way that their several parts are parts of the diameter eegg, and the parallels themselves are described within the oblique great circle; whereas the latter are either projected upon diameters greater than eegg—so that the circles described upon them surround the oblique circle, of which kind are the diameters of the parallels whose distance from the diameter NO is less than the arc OM—or else are found upon diameters wholly outside the oblique circle, along the line eegg produced southward toward the parts mm, of which kind are the diameters of the parallels whose distance from the diameter NO is greater than the arc OM.

[Margin: To distinguish the northern from the southern parallels of any oblique great circle.]

21. Conversely, if some parallel of an oblique circle has been described in the astrolabe, we shall easily learn by how great an interval it lies distant, toward the north or toward the south, from the great circle itself on the sphere. For let there be described the parallel aCbO of the oblique circle PHSI, from the center h. Through h and the center of the astrolabe E let the straight line hE be drawn, and to it let there be raised a perpendicular diameter of the Equator LM, which will represent the axis of the world, as was said above at Num.16. Then from M, the south pole, through a and b, the extreme points of the apparent diameter of the parallel, let straight lines Ma, Mb be drawn, cutting the Equator at Y and Z. For the line YZ (which will be entirely parallel to NO, if no error has been made) will be the diameter of the given parallel on the sphere, and the arcs NY, OZ will measure its distance from the diameter NO of the great circle, either toward the north or toward the south, according as the said arcs are found to lie toward Q or toward T.

[Margin: To know how far a parallel of any oblique great circle described in the astrolabe lies distant from the great circle itself, and toward which part it inclines.]

22. Furthermore, if the straight line RE is drawn through the center of the oblique great circle described in the astrolabe and through the center of the astrolabe, and if to it there is erected the diameter of the Equator at right angles, LM, and through the rays M ee, M gg the true diameter NO of the given oblique circle on the sphere is found, then OM (or NL) will be the arc of the pole's altitude above that same oblique great circle. For if the circle ABCD is taken as the circle of the Analemma [circulus Analemmatis], drawn through the poles of the world and the poles of the oblique circle represented by the circle PHSI, the poles of the world are L and M, as was said at Num.16, and NO is the common section of that oblique circle and of the circle of the Analemma ABCD, as I showed in the same place. The inclination of that same oblique circle to the Equator will be the arc Nk—namely, the complement of the pole's altitude LN—since the complement of the pole's altitude above any great circle is the inclination of that same circle to the Equator, as is evident.

[Margin: To investigate the altitude of the pole above any oblique great circle, and the inclination of that same circle to the Equator.]

But more briefly we shall investigate both the altitude of the pole above any given great oblique circle and its inclination to the Equator—even if its true diameter has not been found—in this way. Having drawn through its center and the center of the astrolabe the straight line RE, and having raised to it at the center E the perpendicular LM, we shall draw through ee, the intersection of the given circle with the line RE, the line M ee, cutting the Equator at N. The arc Nk, between this point N and the intersection of the line RE with the Equator, will be the inclination of the given circle to the Equator, since to it corresponds the portion eek, as I showed at Proposition 1, Num.5—which portion indeed represents the arc of the great circle that is drawn through the poles of the world and the poles of the given circle, and which the line gg mm expresses. Now it is established that the arc of this great circle intercepted between the Equator and the given circle—namely, eek—measures the inclination of the given circle to the Equator. From which it follows that the arc Nk, which is equal to the arc eek, measures the same inclination. And the altitude of the pole above that same given circle will be the arc NL, the complement of the arc Nk. And by this same reasoning we shall find

the altitude of the pole above any oblique great circle whatever described in the astrolabe, and its inclination to the Equator.

[Margin: An easier finding of the pole's altitude above a given great circle in the astrolabe, and of its inclination to the Equator.]

23. Finally, given any great circle whatever, oblique both to the Equator and to the Meridian—whether it be one of the verticals or not—we shall describe from it the Equator of the astrolabe, provided that the altitude of the pole above it, or its inclination to the Equator, has been made known, in this way. Let the given great circle, oblique in any manner, be $L ee, M gg$, whose center is R , through which let a diameter $gg ee$ be drawn in any way. If, then, from ee , in either direction, the altitude of the pole above the said circle—or the complement of its inclination to the Equator—is counted off up to LM , and the straight line LM is joined, it will be bisected at E , by the scholium to Proposition 27 of Book 3 of Euclid; and it will be the diameter of the sought Equator, so that the circle $ABCD$, described from E about LM , will be the Equator in the astrolabe, if the given circle $L ee, M gg$ is taken to be one of the oblique great circles. The demonstration is easy. For when the straight line $M ee N$ has been drawn, the arcs $ee L$ and NL are similar, by Lemma 10; and the arc NL will also measure the altitude of the pole above the given circle, and hence its complement Nk will measure the inclination of that same circle to the Equator. Since, then, with the Equator $ABCD$ set down, the arc NL measures the altitude of the pole above the given circle $L ee, M gg$, and the arc Nk measures the inclination of that same circle to the Equator—as was demonstrated at Num.22—it is clearly established that the Equator has been rightly found from the given altitude of the pole eeL .

And so, by this device, if there is offered any circle in the plane which is supposed to be some determinate great circle in the astrolabe, we shall find through it the Equator itself in the same astrolabe.

[Margin: To describe the Equator from any circle that is said to represent some oblique great circle in the astrolabe.]

PROBL. VI. PROPOS. IX.

To describe the hour-circles and the declination-circles in the astrolabe.

1. There are four kinds of hours. Equal hours, taking their beginning from noon or from midnight, in the manner of astronomers, which the Germans,

Spaniards, and French use [emittantur (?); the verb is doubtfully printed]. Unequal hours, dividing any day or night into 12 equal parts, which were in use among the Hebrews and among nearly all the ancients. Equal hours whose beginning is taken from sunrise, which the Babylonians used. And finally equal hours begun from sunset, whose use was once among the Athenians, but today has remained among the Italians.

The hour-circles begun from noon or midnight will be described in the astrolabe as follows. Let the Equator, or any of its parallels, be divided into 24 equal parts, and let straight lines be drawn through the center of the astrolabe and through the points of division. For these will represent those [hour-]circles in the astrolabe. Since, as I showed in our *Gnomonica*, Book 1, Proposition 9, circles of this kind pass through the poles of the world and cut both the Equator and its parallels into 24 equal parts, they will be projected—by Proposition 1, Num.1 and 4—into straight lines cutting each other at the center of the astrolabe, and will accordingly divide the Equator and all its parallels into 24 equal parts, exactly as happens on the sphere, since equal arcs of the Equator and of its parallels are projected into equal arcs upon the astrolabe, as was demonstrated at Proposition 2, Num.1, 2, 3, and 4. And if the several hours on the Equator, or on its parallels, are bisected, and through the sections straight lines are again drawn from the center of the astrolabe, there will also be described circles indicating half-hours; and if these are again bisected, and so on, there will be had circles showing the quarter-hours, and so forth, if smaller parts of the hours are desired.

[Margin: To describe in the astrolabe the circles of the hours begun from noon or midnight.]

2. These straight lines, representing the hour-circles begun from noon or midnight, are usually drawn in the common astrolabes only below the Horizon, as appears in the figure, yet in such a way that they do not pass beyond the tropic of Cancer ☿, lest the part of the astrolabe above the Horizon, in which the vertical circles and the parallels of the Horizon are described, be confused by too great a multitude of lines. Others, however, mark the same hours only on the limb of the astrolabe, writing beside the points on which the said lines fall the numbers of the hours, beginning from the meridian line BD, and proceeding in the upper part toward the right, but in the lower part toward the left. Then at the center of the astrolabe they fix a certain revolving rule, one of whose lines, at its extremity, passes through that same center, and is called the line of trust (the fiducial line) [linea fiduciæ]. For this rule, when carried round,

performs the office of all the hour-circles of which we are now speaking. The same office that this rule performs can also be performed by a very fine thread going out from the center of the astrolabe and carried round the limb through the several hours.

Diagram. A square-framed astrolabe plate showing the hour-circles and declination-circles. From the center E radiate the hour-lines to the outer limb, which carries the hour-numbers; a revolving rule (the fiducial line) runs as a dotted diagonal from K at the upper left to L at the lower right. The meridian runs vertically through B (top) to D (foot), with the oblique circle and its intersections marked. The plate is very dense with small numerals and tally-mark tick figures, several at the limit of legibility.

- K (upper left, with crossed lines): end of the fiducial rule
- L (lower right): opposite end of the fiducial rule KL
- M (upper region, on the oblique line): point
- N (just below M): intersection [?]
- B (top, on the meridian): meridian point
- D (foot, on the meridian): meridian point
- E (center): center of the astrolabe
- G (upper interior, on a dotted circle): point
- A (right of center): point
- Y (upper left): point
- P, V (left of center): points
- F (lower left interior): point
- R (bottom center): point
- S (upper right, near the limb): point [?]
- Outer-limb hour numerals: legible are 3, 4, 5, 6, 7 (left arc) and 5, 6, 7, 8, 9, 10, 11, 12, with a repeated 12 (lower right), interspersed with tally-mark tick numerals [remaining limb numerals not securely legible]
- Named arcs referenced in the text: QRS (tropic of Capricorn), CDA (Equator), TVX (tropic of Cancer ☼)

[Dense plate with many small numerals and tick-labels; multiple labels uncertain]

3. The greatest circles of declination, since they too are drawn through the poles of the world, will be described in the astrolabe in the same way, if through the center and the several degrees of the Equator straight lines are

drawn—which, however, on the limb of the astrolabe are usually shown by degrees only. For that revolving rule, or the thread hanging from the center, if it is carried round through the several degrees, will perform the office of the declination-circles drawn through the several degrees.

[Margin: To describe the declination-circles in the astrolabe.]

4. The circles of unequal hours, which divide each of the diurnal and nocturnal arcs into twelve equal parts, are projected by the authors [of astrolabes] into the plane of the astrolabe in this way. Having divided the nocturnal arcs of the tropic [of Capricorn ♎ (?)] QRS, of the Equator CDA, and of the tropic of Cancer ♋ TVX into 12 equal parts (for the unequal hours are usually described only below the Horizon, for the reason stated in the case of the hours begun from noon or midnight), they describe, through the three points corresponding to the same unequal hour, circles that would pass in the Equator through points diametrically opposite, if they were produced. For they think that these circles show the unequal hours, wherever the Sun may be in the Zodiac. This is by no means true, since these circles represent the greatest circles on the sphere, as [I show] in the scholium to Proposition 5, Num.

[Margin: To describe in the astrolabe the circles of the unequal hours according to the authors of the astrolabe.] [Margin: The circles of the hours...]

[continuation of Problema VI / Propositio IX, §4]

In Section 9 we demonstrated that although circles may be described through two diametrically opposite points of the Equator, still no great circles can be given in the sphere that pass through the unequal hours of all the parallels — that is, which divide the diurnal and nocturnal arcs of each parallel into twelve equal parts — as was demonstrated by us in Lemma 39. It is therefore clear that those described circles do not truly mark the twelfths in each of the diurnal or nocturnal arcs, with the exception of those three arcs which are [themselves] divided into 12 equal parts. [Margin: The [circles] of the unequal [hours] commonly described do not truly indicate the unequal hours throughout the whole year.]

Now although circles of this kind divide the diurnal and nocturnal arcs of all the parallels into roughly 12 equal parts on that Horizon above which the pole is elevated by no more than 45 degrees — so that scarcely any discrepancy can be perceived by the senses — nevertheless, at a greater obliquity of the sphere, if they divide the diurnal or nocturnal arcs of three parallels into 12 equal parts,

they will never divide the diurnal or nocturnal arcs of the other parallels into equal parts, but will make them so unequal that their discrepancy can be perceived by the senses; and the greater the altitude of the pole, the greater the inequality found among them, just as the smaller the pole's altitude above the Horizon (below 45 degrees), the smaller the inequality discerned among them. [Margin: The unequal hours are more truly described through the twelfth parts of several diurnal arcs.]

Therefore, in order that the unequal hours be more truly described on the Astrolabe, more parallels must be described between the Equator and each tropic, their nocturnal arcs must be distributed into 12 parts, and finally the points of the individual hours — which are by no means situated on the circumference of a circle, as is commonly supposed — must be joined suitably by curved lines, so that nowhere do they form angles, exactly as is done in describing hyperbolas and the other conic sections. If, however, anyone should wish to represent the unequal hours on the Astrolabe entirely by means of circles, treating as negligible that slight discrepancy we spoke of, so that he may describe such circles more easily and more expeditiously, he must find their centres on the straight lines that cut the Equator into 24 equal parts — that is, on the lines of the hours reckoned from noon or from midnight, if these are produced. [Margin: To find the centres of the unequal hours.]

For the centre of any such circle lies on the line that cuts the diameter of the Equator FG into two equal parts and at right angles [i.e. the line through the points] through which that circle must pass. For example, the arc — or circle — HFP, to be described through the points F, G of the Equator, has its centre on the straight line EYM drawn through Y, the point of the Equator that is 6 hours distant from the points F, G. For since the line EYM is equidistant from the points F, G, it follows that a circle described from any point of it through either of the points F, G passes also through the other — just as the Horizon, having its centre on the meridian line BD (which is a quadrant distant in the Equator from the points A, C), passes through both points A and C, as we showed in the scholium to Proposition 5, Section 1. This can also be demonstrated as follows. Since the straight line EM cuts the diameter of the Equator FG into two equal parts and at right angles — because, by the scholium to *Elements* III.27, the angles at the centre E standing on the quadrants YF, YG are right — the same line EM will pass through the centre of the circle to be described through the points F, G (by the corollary to *Elements* III.1), such being the circle of a given unequal hour. It will therefore suffice to find on this line EYM the centre of the circle passing through either of the points corresponding [to it] in the tropic of ♊ [Capricorn] or ♎

[Cancer]: which is easily done by opening or closing the compasses more or less, as the case requires. Geometrically, however, you will find the same centre if from G and H, with any equal interval taken on either side, you describe two arcs cutting one another at K, L; likewise others from the points H, P, with any interval, cutting one another at N, O. For the straight lines LK, ON drawn through those intersections will cut the straight line EYM at M, the centre of the arc HEP [recte HFP?], as is evident from what we demonstrated in the scholium to *Elements* III.25. And exactly the same procedure holds for finding the centres of the other arcs.

§5. Finally, we shall project onto the Astrolabe the circles of the hours from sunrise or sunset of the Sun in this way. About E, the centre of the Astrolabe, through F, the centre of the Horizon, let a circle FG be described and distributed into 24 equal hours (which, if desired, may be subdivided into halves and quarters of hours), and from the points of division, as centres, with the interval always the same — namely the semidiameter of the Horizon FH — let circles be described. I say that these circles indicate the hours from sunrise or sunset of the Sun; that is, they represent the great circles in the sphere which divide into equal parts all the parallels of the Equator lying between the greatest of the ever-visible and the greatest of the ever-hidden [circles], the reckoning beginning from the Horizon. [Margin: To describe the circles of the hours from sunrise and sunset on the Astrolabe.] For since, by Proposition 10 of Book 1 of our *Gnomonics*, circles of this kind touch the greatest of the ever-visible and of the ever-hidden parallels at the points where they are cut by the circles of the hours from noon and midnight, it is necessary that they do the same on the Astrolabe.

Since, then, the circles described from the points of division of the circle FG, with the interval of the semidiameter of the Horizon, touch the two parallels KL, HI — which the Horizon touches, and of which the one here is the greatest of the ever-visible — at the points where these are cut by the straight lines drawn through the centre of the Astrolabe representing the circles of the hours from noon or midnight (as has been shown, and as we shall demonstrate), it is clear that the circles described are the circles of the hours from sunrise and sunset of the Sun. For let the straight line EG be drawn through E, the centre of the Astrolabe, and the point G, cutting the parallels KL, HI at L, I. And since both EK, EL are equal to one another and EH, EI [are equal to one another], the whole lines KH, LI will be equal; and again, since EF, EG are equal, the whole lines FH, GI will also be equal. Since, then, FH is the half of KH, and GI the half of LI, the circle LhI described from G at the interval GI, or GL, has its semidiameter equal to the semidiameter of the Horizon FH, and (by the scholium to *Elements* III.13) touches the parallels

KL, HI at the points L, I, at which the straight line LI — representing one of the circles of the hours from noon and midnight — cuts them.

By the same reasoning we shall show that the other circles described from the other points of division of the circle FG, at the interval of the semidiameter of the Horizon, touch the parallels KL, HI at the points where they are cut by the straight lines drawn out through the centres — that is, that their diameters, lying between each parallel, are cut in two by the circle FG, and that the circles themselves are equal to the Horizon. And indeed that the circles of the hours from sunrise and sunset are projected onto the Astrolabe as equal circles can also be made manifest from this: since in the sphere they touch the greatest of the ever-visible and the greatest of the ever-hidden parallels at the 24 points at which the [circles] dividing the said parallels into 24 equal hours cut them (as is clear from Proposition 10, Book 1 of our *Gnomonics*), they are themselves — by the scholium to Proposition 21, Book 2 of Theodosius — equally inclined to the Equator, and consequently their poles are equally distant from that same Equator. From this it follows that all of them, together with the Horizon, are equally distant from the antarctic pole, and therefore, viewed from that pole, appear equal to one another. Hence it is also established that the said circles are rightly described, since all are equal to the Horizon (because of the equal semidiameters), and represent great circles, inasmuch as they touch the two opposite parallels KL, HI — namely those which the Horizon touches [Margin: a — Theodosius, *Sphaerica* II.8 [?]] — while it is evident that the Horizon touches two opposite parallels. From this it will also be permissible to infer that any of these circles passes through two hours in the Equator that are diametrically opposite and 6 hours — that is, a quadrant — distant from the straight line drawn through its own centre,

just as the Horizon passes through the hours A, C, which are diametrically opposite and 6 hours distant from the straight line drawn through the centre F. For every great circle on the Astrolabe cuts the Equator into two equal parts at diametrically opposite points, as was shown in the scholium to Proposition 5, Section 6, and will be demonstrated more clearly in the scholium to Proposition 12. Thus you see that the circle described from G passes through the hours M, N, which are diametrically opposite in the Equator and 6 hours distant from the straight line drawn through the centre G.

§6. Now the circles of the hours from sunrise or sunset are, on ordinary Astrolabes (on which they are usually drawn — for they are not drawn on all), customarily described only below the Horizon, yet in such a way that they do not

pass beyond the tropics, for the reason brought forward a little earlier in the case of the circles of the hours from noon and midnight; as appears in the figure, where the outer numbers pertain to the hours from sunset and the inner ones to the hours from sunrise. These arcs, however, are not sufficient for investigating the hours from sunrise and sunset in both daytime and nighttime, as we shall say in Book 3, Canon 8, Section 3. [Margin: In what manner the hours from sunrise and sunset are customarily described on ordinary Astrolabes, and what order they observe.] In fact, however, if circles of this kind were to be described complete, the arc of the circle described through the points O, P from [centre] Q, above the Horizon on the eastern side C, would look to the 1st hour from sunrise of the Sun, while the same arc, below the Horizon on the western side A, would pertain to the 1st hour from sunset of the Sun; just as the arc passing beneath the Horizon through M ought to look to the 23rd hour from sunrise, and the arc passing above the Horizon through N to the 23rd hour from sunset — and so of the remaining hours. This too we shall point out in its proper place, in the use of the Astrolabe, and we shall now explain it in some fashion. [Margin: Through which points of the Equator the arcs of the hours from sunrise are truly to be described, and through which those of the hours from sunset — that is, which hour from noon or midnight in the Equator pertains to the hours from sunrise, and which to the hours from sunset.]

Diagram. A large stereographic-projection plate of the astrolabe within a square border, occupying the middle-to-lower portion of the page. It shows the Equator, the tropics, the Horizon (drawn as a dotted circle), and the family of hour-circles from sunrise and sunset, densely interwoven about the centre. A ring of Arabic numerals (the outer set, for hours from sunset) runs 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24 around the arcs, with a corresponding interior set of Roman-numeral hour marks (for hours from sunrise). The point-letters read, by region:

- E (centre of the Astrolabe)
- A (right of centre; a horizon/hour point)
- F (centre of the Horizon), G (upper right, on the dotted Horizon circle)
- Inner cluster about the centre: H, I, Y, M, R, N, P, S, V, f
- O, c (left of centre)
- d, D, Z (lower centre)
- b (upper, near E), g [?]
- Top of the plate: L, K, n [?], e [?], a
- T (foot of the plate)

§7. If the circle of a proposed hour from sunrise or sunset is to be described (whether it be whole, without minutes, or whether some minutes are attached to it), it will be accomplished in this way. Let the given hour be reckoned off (the hours, and their minutes if present, having been reduced to degrees and minutes of degrees, by assigning to each hour fifteen degrees, and to each minute of the hour fifteen minutes of one degree, etc.) in the Equator from the point C toward B if the given hour is from sunrise, or from the point A toward D if the hour given is from sunset. [Margin: To describe on the Astrolabe the circle of a proposed hour from sunrise or sunset.] For by the terminus of the reckoning the circle of that hour is to be described; and its centre will be found on the parallel FG, [the parallel] described from the centre of the Astrolabe through F, the centre of the Horizon. Taking, by means of the compasses, the semidiameter of the Horizon FH, or FK, let one of its feet be set at the point found on the Equator, and with the other let the parallel FG be cut in two places. For one of these two intersections will be the centre sought; but which of them is to be taken, you will learn from what follows. [Margin: a — [Theodosius] II.13 [?]. To recognize which of the semicircles of the hours from sunrise or sunset pertain to the hours from sunrise, and which to the hours from sunset.] Since all the circles of the hours from sunrise or sunset are equal on the Astrolabe, and touch the two parallels HI, KL at the 24 points where they are cut by the circles of the hours from noon or midnight (as we said above in Section 5), and are divided in two at those points of contact — since on any one of them the two points of contact are diametrically opposite, by the corollary to Proposition 6, Book 2 of Theodosius — the semicircles comprised between the points of contact will pertain to the same kind of hours, not meeting, or not intersecting one another, since they cut off similar arcs from the parallels of the Equator. Of this kind are the semicircles HAK, INL, RST, VMX, YZa. And since the first, HAK, being a semicircle of the Horizon, looks toward the western parts of the Astrolabe, toward the sunset of the Sun, the other four named semicircles [likewise pertain] to the hours from sunset. In the same way the remaining semicircles HCK, IML, RZT, VNX, YSa are non-meeting; and since the first is a semicircle of the Horizon toward the eastern parts of the Astrolabe and looks toward the sunrise of the Sun, the other four named semicircles will indicate the hours from sunrise of the Sun. Here you see that of any circle of the hours from sunrise or sunset, the one semicircle intercepted between the two points of contact pertains to the hours from sunset, and the other to the hours from sunrise. From these it will not be difficult to judge which of the two intersections on the parallel FG is to be taken

as the centre of the hour-circle to be described through the point found on the Equator; for that one must be chosen from which the semicircle destined to

indicate the hour from sunset — enclosed between the two points of contact — does not meet, when described, the semicircle of the Horizon HAK, or any other looking toward the hours from sunset. And by the same reasoning, the semicircle destined to indicate the hour from sunrise, when described from the intersection assumed, must not meet the semicircle of the Horizon HCK, or any other looking toward the hours from sunrise.

For example, if the semicircle of the 15th hour from sunset, or from sunrise, is to be described, we shall reckon in the Equator from A, the point of sunset, toward B, 15 hours as far as S; or from C, the point of sunrise, toward B, 15 hours as far as Z. For through S will pass the semicircle of the 15th hour from sunset, and through Z the semicircle of the 15th hour from sunrise. And since the semidiameter of the Horizon HF, or FK, taken with the compasses from the point S as well as from Z, gives us on the parallel FG two points b, d, the centre to be adopted will be d, not b: because neither the semicircle RST described from d meets the semicircle of the Horizon HAK, nor does the semicircle RZT meet the semicircle of the Horizon HCK; whereas the semicircle YSa described from b meets the semicircle of the Horizon HAK at the point e, and the semicircle YZa meets the semicircle of the Horizon HCK at the point f. And consequently neither the former will pertain to the 15th hour from sunset, nor the latter to the 15th hour from sunrise; rather the former will indicate the 3rd hour from sunrise, and the latter the 3rd hour from sunset — because the point S is 3 hours distant from the sunrise C toward B, and the semicircle YSa does not meet the semicircle of the Horizon HCK, while likewise the point Z is 3 hours distant from the sunset A toward D, and the semicircle YZa does not meet the semicircle of the Horizon HAK.

For the same reason, the semicircle of the 11th hour from sunset will pass through the point M, and the semicircle of the 11th hour from sunrise through the point N, and the centre of each will be the point g, not G. For neither does the semicircle VMX described from g meet the semicircle of the Horizon HAK, or the semicircle RST of the 15th hour from sunset; nor does the semicircle VNX meet the semicircle of the Horizon HCK, or the semicircle RZT of the 15th hour from sunrise. But the semicircle IML described from G cuts the semicircle of the Horizon HAK between the points H, I, or the semicircle RST of the 15th hour from sunset at the point h; and the semicircle INL cuts the semicircle of the Horizon HCK at the point k, or the semicircle RZT at the point m. And consequently neither the semicircle IML will pertain to the 11th hour from sunset, nor the semicircle INL to the 11th hour from sunrise; rather the former will show the

23rd hour from sunrise, and the latter the 23rd hour from sunset. And so of the rest.

§8. By a not dissimilar procedure, through any point given within the parallels HI, KL on the Astrolabe, both the semicircle looking to some hour from sunset and the semicircle looking to some hour from sunrise will be described. [Margin: Through a given point between the two parallels touching the Horizon, to describe on the Astrolabe both the semicircle looking to some hour from sunrise and the semicircle looking to some hour from sunset.] For if the point *n* be given, there will be found, by means of the semidiameter of the Horizon (with the compasses, from *n*), two centres *G*, *b* on the parallel *FG*. From the former, through *n*, will be described the semicircle *INL*, pertaining to the hours from sunset, since — advancing from *A* through *D*, namely in the order of the signs — we meet its concave side at the point *N*; from the latter, through the same point *n*, [will be described] the semicircle *YSa*, looking to the hours from sunrise, because, advancing from *C* toward *B*, we meet its convex side at the point *S*. [Margin: To recognize to what hour from sunrise or sunset any semicircle described for some hour from sunrise or sunset pertains.] Now the arc of the Equator reckoned from sunset toward *D*, or from sunrise *C* toward *B*, as far as the semicircle of the hour from sunset or sunrise, will indicate which hour from sunset or sunrise the described semicircle signifies; [and it will show] to which hour from sunrise or sunset any described semicircle looks — namely, if from *A*, the point of sunset, toward *D*, the arc of the Equator be reckoned up to it (if it pertains to the hours from sunset); or if from *C*, the point of sunrise, toward *B*, the reckoning be made up to it (if it looks to the hours from sunrise), and so forth.

§9. Moreover, this too should not be passed over in silence: that the altitude of the pole above all the circles of the hours from sunrise or sunset is the same as it is above the Horizon. [Margin: That the altitude of the pole above all the circles of the hours from sunrise or sunset is the same as it is above the Horizon.] For since they touch the same parallel *HIR*, all the arcs of the pole's altitude drawn from the pole to the points of contact will fall [upon them], and will therefore be equal; and these are represented in the figure by the straight lines *EH*, *EI*, and the others drawn out from the centre of the Astrolabe to the points of contact, which are portions of the straight lines drawn through their centres and representing the great circles that pass through their poles and the poles of the world. Since, then, *EH* measures the altitude of the pole above the Horizon, the proposition is established.

PROBL. VII. PROPOS. X.

To describe on the Astrolabe the circles of the celestial houses, or of the positions, and the line of twilight or dawn. [Margin: To describe on the Astrolabe the celestial houses, as they are constituted by Johannes Regiomontanus.]

§1. The circles of the celestial houses — which are also called circles of positions — passing through the common intersections of the Horizon and the Meridian, and dividing (as Johannes Regiomontanus [Regiomontanus] would have it) the Equator into 12 equal parts, beginning from the eastern semicircle of the Horizon (which is itself one of their number) and proceeding toward the lower hemisphere, will be described on the Astrolabe in this way. The Equator having been divided into 12 equal parts, let [the circles] be describ-

Astrolabe, Book II — Problem VII / Proposition X (continued); Problem VIII / Proposition XI

[Proposition X, continued — finding the centers of the celestial-house circles]

[Margin: To find the centers of the celestial-house circles.]

let them be described through the points of intersection; and through the points F, G — where the horizon cuts the meridian line — once a center has been found for any three points, of which two are F, G and the third lies on the equator. For these circles will run through the beginnings of two celestial houses, in the way that Regiomontanus arranges them; and each of them, being a great circle (since it is drawn through the two points F, G, which are diametrically opposite on the sphere), will pass through two points diametrically opposite on the equator, as we showed in the scholium to Proposition 5, Num. 6, and shall demonstrate more clearly in the scholium to Proposition 12. Thus you see that the circle FKG, of the 3rd and 9th houses, is drawn through the points K, L, which are diametrically opposite on the equator.

From this it follows that the center of any such circle lies on the straight line which, at the center E, cuts at right angles the diameter of the equator drawn through those two opposite points — that is, the line which bisects the semicircle of the equator lying between those two opposite points. For that perpendicular, since it bisects the said diameter of the equator at right angles, will pass through

the center of every circle passing through the extreme points of that diameter (by the corollary to Proposition 1, Book 3 of Euclid), and such is the circle of the proposed celestial house. Thus the center of the circle F K G L will lie on the straight line E N, which bisects the diameter K L at E and bisects the semicircle KNL at N, and is perpendicular to the diameter KL — since all the points of this line are equally distant from the points K, L, through which the circle must be drawn, just as was said of the centers of the unequal hours in the preceding Proposition, Num. 4. And since, by the same corollary to Proposition 1, Book 3 of Euclid, the same centers lie also on the straight line O P — which cuts the meridian line F, G at right angles at the center of the horizon H, and bisects it, because all the points of this line too are equally distant from the points F, G, through which the circles of the houses [are to be drawn] —

Diagram. The same large woodcut plate is printed at the foot of p. 580 and again at the foot of p. 582, with the text flowing around it; it is described once here. Within a ruled rectangular border it shows the astrolabe plate for the celestial houses (Regiomontanus's and Campanus's position circles) constructed by stereographic projection. The center of the astrolabe **E** lies near the middle; **A** is below it and **C** to its left, with **H** (center of the horizon) above **E** and **B**, **D** nearby. A horizontal fiducial line runs from **O** (far left) through **M** across to the right, and the oblique lines **LINEA ... CREPVSCVLI** (the twilight line) sweep from lower-left to upper-right, meeting **P** and **θ** at the right. The dense web of arcs are the position circles of the houses; those drawn after Regiomontanus carry Roman ("antique") numerals, those after Campanus carry Arabic numerals.

Readable labels (positions approximate):

- **E** (center): center of the astrolabe
- **A** (below center); **C** (left of center); **H** (upper center — center of horizon); **B**, **D** (near center)
- **O** (far left, on the horizontal line); **M** (on the horizontal line); **N** (upper-center); **P** (far right, on line OP); **θ** (far right); **μ** (upper left) [?]
- **Q**, **R**, **S**, **T** (points along line OP) [some uncertain]
- Roman-numeral houses (Regiomontanus): I, II, III, IIII, VII, VIII, X, XI, XII
- Arabic-numeral houses (Campanus): 1, 2, 3, 4, 6, 8, 9, 12; 15 [?]
- Greek letters: π , ϵ , γ , ρ [?], θ
- Lowercase: a, b, f, h, m, u [?], v
- Words along curves: "LINEA" (lower left) and "CREPVSCVLI" (right) — the twilight line

- F, f (top): the poles/points at the top of the plate

[The plate is exceptionally dense; more than two peripheral labels could not be resolved with certainty**]

—[the centers thus] should be drawn equally distant, in the way said [of the centers of the verticals lying on the line PQ] in Proposition 8, Num. 2. So it comes about that the center of the circle FKGL is the point M, where the lines EN, OP cut each other; and the same reasoning holds for the rest. For the centers of the other circles too are the points Q, R, S, at which the lines drawn from the center E through the points of division of the equator cut the line O P. And so, if straight lines are drawn from E through the several degrees of the equator, the line OP will be cut at the centers of the position circles that pass through the several degrees of the equator and divide the individual celestial houses into thirty degrees each — just as the line EN, drawn through N (30°) reckoned from the point C, gave the point M, the center of the circle F K G L, which was described through K, the 30th degree of the equator reckoned from the meridian.

2. [Margin: To describe, through any given point of the equator, a position circle.] But if a position circle is to be described through any degree of the equator distant from the meridian, we shall count the same degree from C toward B, if the given degree of the equator lies on the western side, or from A, if it lies on the eastern side. For the straight line sent out from E through the endpoint of the count will give, on the line OP, the center of the sought circle. Thus, if a position circle is to be described through the point β , 60° distant from B, the point of midday, toward the west, we shall reckon 60° from C up to ϵ . For the line E ϵ will give the center Q, from which the circle is to be described through the given point β and through the points F, G — and so with the rest. That the circles of the celestial houses are correctly described as Regiomontanus constitutes them is plain, since they appear in circular form and are described through those points through which they are drawn in the heavens by Regiomontanus — namely through the twelfth-parts of the equator and through the points F, G of the intersections of the horizon and meridian.

3. [Margin: To describe the celestial houses as Campanus constitutes them in the astrolabe.] The circles of the celestial houses as Campanus constitutes them in the heavens — namely dividing the primary vertical into 12 equal parts and passing through the same points F, G of the intersections of the horizon and meridian — will be described in the astrolabe in the same way, if instead of the twelfth-parts of the equator one takes the twelfth-parts of the primary vertical: not indeed twelfth equal parts of the vertical itself, as was done on the equator, but unequal

parts corresponding in the sphere to the twelve equal parts of the primary vertical, and found

[p. 581 — Proposition X, continued]

by the straight lines drawn from either of the poles G, F of the vertical through the 12 parts of the equator, as taught in Proposition 5, Num. 17 and 20 — or by other means, which we explained partly in Proposition 5, partly in Proposition 6, and especially in Proposition 6, Num. 25. For once these twelfth-parts of the vertical have been found, if circles are described through each of them and through the points F, G — whose centers lie on the line O P — they will run through the beginnings of the celestial houses as Campanus conceives them; and each of them will pass through two points of the vertical diametrically opposite through the diameter of the world (which is drawn through E, the center of the astrolabe), since each represents a great circle and therefore bisects the other great circles. Thus you see that the circle F a G b, of the 3rd and 9th houses, is drawn through the points a, b of the vertical, which are diametrically opposite.

4. [Margin: To describe the celestial houses as Campanus imagines them in the astrolabe, after the manner of the verticals of the primary vertical itself taken as a horizon.] These same latter circles of the celestial houses we shall likewise describe as follows. Since they are drawn through the poles of the primary vertical in the sphere — that is, through the intersections of the horizon and meridian — and divide the primary vertical into equal parts, they will stand to the primary vertical exactly as the vertical circles stand to the horizon, passing through the poles of the horizon (that is, through the intersections of the primary vertical and the meridian) and dividing the horizon into equal parts. Therefore, just as in Proposition 8, Num. 1 and 2 the centers of the verticals were found on the line P Q — which is drawn through the center of the primary vertical perpendicular to the meridian line, in the first figure of that proposition — so here too the centers of the circles of the celestial houses which Campanus devised for himself will be found on the line O P, which is carried through H, the center of the horizon, perpendicular to the meridian line, and is the common section of the equator (or plane of the astrolabe) and of the parallel of the primary vertical drawn through the antarctic pole, whose diameter in the first figure of Proposition 5 is the line A c. Just so, the line P Q, in the first figure of Proposition 8, is the common section of the same equator (or plane of the astrolabe) and of the parallel of the horizon drawn through the antarctic pole, whose diameter in that same first figure of Proposition 5 is the line A l. For the demonstration will be the same in both cases. For if the primary vertical is understood to be some

oblique horizon, then its horizon will be the primary vertical, and the points F, G its poles. And so, since through these latter circles of the celestial houses the primary vertical — taken as some oblique horizon — must be divided into 12 equal parts (which are indeed only six in number, since each passes through two points of the vertical), we shall divide the horizon A F C G, as if it were the primary vertical of the vertical AaCb taken as some horizon, into 6 parts wholly equal among themselves. Then from the point F, or G, through these divisions we shall draw straight lines cutting the line O P in the points O, T, H, V, P, which will be the centers of the circles of the celestial houses to be described through F, G — after the manner of the verticals with respect to the vertical A a C b taken as a horizon, as was demonstrated in Proposition 8. In the figure the earlier circles, described after Regiomontanus's view, have attached the antique numerals, thus: I. II. III. &c.; the later ones, after Campanus, have the ordinary numeral characters affixed, thus: 1. 2. 3. 4. &c. And all these circles are usually drawn so as not to exceed the tropic of Capricorn γ — which we too have observed. But if from F, at any interval, a circle $\delta\gamma\theta$ [?] is described and distributed into 360° , beginning from the point γ , then the straight lines drawn from F through the several degrees of that circle will give, on the line O P, the centers of all the position circles that pass through all the degrees of the primary vertical and divide the individual celestial houses into thirty degrees each. For just as the line F μ , through the point μ , 120° distant from the point G of the meridian, falls on O, the center of the position circle FaG, 60° removed from the horizon, so the line F δ , drawn through the point δ , 60° distant from the point γ of the meridian, falls on the same center — because the same line passes through both points μ , δ (by Lemma 10), since the arc $\gamma\delta$ is like the half of the arc G μ , and so on.

5. [Margin: To describe a position circle through any given degree of the vertical.] But if a position circle is to be described through any degree of the primary vertical distant from the horizon, we shall count the same degree from γ toward δ , if the given degree of the vertical lies on the western side, or toward θ , if it lies on the eastern side. For the straight line sent out from F through the endpoint of the count will give, on the line O P, the center of the sought circle. Thus, if a position circle is to be described through a point of the vertical which is 60° distant from the horizon on the eastern side toward the zenith, we shall take the arc $\gamma\theta$ of 60° . For the line F θ will give the center P, from which the circle described through the points F, G will pass through π , the point of the vertical 60° distant from the horizon-point C toward the zenith. But if a point on the vertical is proposed below the horizon, distant from the horizon by any number of degrees, whether toward the east or the west, we shall describe, through the opposite point which lies above the horizon, a position circle toward the contrary side, as

was said; for this too will pass through the given point. Thus, if a position circle is proposed through the 60th degree of the vertical below the horizon on the eastern side, we shall describe, as was said, the circle through 60° above the horizon on the western side — that is, we shall count 60° from γ up to δ on the eastern side, so that the line $F\delta$ yields the center O , and so on. [Margin: To describe a position circle through any given point outside the equator and the vertical.] We shall do the same whether the given point of the vertical be above the horizon or below it, if, having found that point on the vertical, from its distance from the horizon (as taught in Proposition 5, Num. 18) [we describe through it and through the two points] [?] F, G a circle (by the scholium to Proposition 5, Book 4 of Euclid) whose center [will lie on the line OP] [?].

6. [Margin: To know how far any position circle is from the horizon, whether on the equator or on the vertical.] Now if, through any point in the astrolabe assigned outside the equator and the primary vertical, a position circle is to be described, one must find on the line OP the center of the three points, of which two are F, G and the third is the one proposed. The arc of the equator between the point A (or C) and the intersection of the described circle with the equator will measure the distance of the position circle from the horizon on the equator. Likewise the arc of the vertical between A (or C) and the described position circle will measure the distance of that same circle from the horizon on the vertical, provided that beforehand — by what we demonstrated in Proposition 5, Num. 19 — [the arc of] the vertical is made equivalent. And by this same reasoning, from the arc of the equator or of the vertical between A (or C) and any position circle so placed, we shall know how far that position circle is from the horizon, whether on the equator or on the vertical, according as it is understood to be described after the view of Regiomontanus or of Campanus; and thence we shall understand how large a portion each position circle cuts off from the celestial house.

7. [Margin: To describe the twilight line in the astrolabe.] The twilight line, or the line of dawn, will be described if a parallel of the horizon rp is drawn 18° distant from it toward the nadir — because, when the Sun (wherever it may be on the ecliptic) touches the parallel of the horizon lying 18° below the horizon, the morning twilight begins and the evening twilight ends. And so, in the manner stated by what was demonstrated in Proposition 6, we shall describe the parallel rp . On the equator, once the diameter de of the horizon and its axis fg have been drawn, let there be taken below de two arcs dh, eL of 18°, so that the straight line drawn hL is the diameter of the parallel bounding both twilights; and from A , the south pole, through h, L , let rays be sent out that cut off from the meridian line

the apparent diameter of the same parallel. But since the ray Ah runs out too far, it will be enough to find the endpoint r of its diameter, in order to describe through r the center of the parallel of the horizon; which will be done thus. [Margin: To find the center of the twilight line.] Through the point l, where the diameter drawn hL cuts the axis fg of the horizon, let a straight line be drawn from A, the south pole, cutting the equator in m; and let there be taken f n equal to the arc m f. For the ray An will cut the meridian line in q, the center for describing the parallel of the horizon through r — that is, the twilight line — as we demonstrated in Lemma 35 and in Proposition 6, Num. 9. Or we shall proceed thus. Having taken the arc Af of 18° on the equator, we shall draw from G, the pole of the vertical, through f a straight line cutting the vertical in p; and the arc of the vertical A p will be 18° below the horizon, by what was demonstrated in Proposition 5, Num. 17; and therefore through p the parallel of twilight is to be drawn. If, then, through p a line is drawn tangent to the vertical, it will cut the meridian line in q, the center for describing the parallel through p, by what we demonstrated in Proposition 6, Num. 10. Or, finally, on the horizon let there be taken two arcs Ft, Gu of 18° on the semicircle FAG, which in Proposition 6, Num. 6 we said looks toward the parallels of the horizon below the horizon; and let the straight line t u be joined, cutting the diameter of the horizon in α . For the straight line sent from A through α will fall on q, the center of the parallel lying 18° below the horizon, as we demonstrated in Proposition 6, Num. 6. Moreover, the points h, L, which bound the diameter of the twilight parallel, we shall find without the help of the horizon's diameter de, in this way. From C toward D, let there be reckoned the arc composed of the pole's altitude and 18° , up to L, which on the Roman horizon [i.e. latitude of Rome] comprises 60° . Likewise from B toward A, the arc [numerus?] [?] composed of the complement of the pole's altitude and 18° , up to h, which on the same Roman horizon comprises 66° . For the straight line drawn h L will be the diameter of the twilight parallel — because the arc C L is composed of C e, the arc of the pole's altitude, and e L, the arc of 18° ; whereas the arc Bh is composed of Bd, the arc of the complement of the pole's altitude, and dh, the arc of 18° . [Margin: The error of Johannes Stöffler in describing the twilight line.] From this it is plain that Johannes Stöffler [Stoflerinus] — and therefore certain others too who follow him — errs when he prescribes that the pole's altitude, together with 18° , is to be reckoned both from C toward D and from B toward A. For this is true only where the pole's altitude is 45° . For there the complement of the pole's altitude B d is equal to the pole's altitude Ce (or d A), as is evident.

PROBL. VIII. — PROPOS. XI.

[Margin: To construct the rete of the astrolabe. / To find the center of the ecliptic. / To find the poles of the ecliptic.]

To construct the rete of the astrolabe — that is, the figure in which the ecliptic, divided into signs and degrees, together with the fixed stars, is contained.

1. Let there be described about E, the center of the astrolabe, the equator ABCD together with the tropics, as taught in Proposition 4; and let the ecliptic A F C G be described — tangent to the tropic of Capricorn ϑ at F, and to the tropic of Cancer $\var�$ at G — as we taught in Proposition 5, about the center H. This center is found by the straight line sent out from A, the south pole, through the endpoint of the arc AIK, which is double the complement of the greatest declination (the greatest declination being B I, or C L, and its complement A I, or B L) — or (which is the same) through the endpoint of the arc CK, which is double the greatest declination CL — as we showed in Proposition 5, Num. 3 and 4. For the diameter of the ecliptic is drawn through I, N, and is distant from the south pole by the arc A I, whose complement is the greatest declination CL, or BI. And since L, P — the points at a quadrant's distance from the ecliptic, drawn through I, N — are the poles of the ecliptic, those poles will appear, by the rays A L, A P, at the points M, R; of which the southern and more remote, R, will be found more accurately as follows.

ASTROLABII LIBER SECUNDUS — Proposition XI (Problem VIII), continued

[the preceding sentence ends]. Draw from A, through the end of the arc AO — which is double the greatest declination AP — the straight line AO, falling on Q, the center of the great circle drawn through the poles of the ecliptic and through the first points of Aries γ and Libra ϱ , as though it were the prime vertical, supposing the ecliptic were the horizon. For if from Q, through M, a circle is described, passing necessarily through A and C, the meridian line will be cut at R, the pole of the ecliptic. And on the straight line ST — which is drawn through Q perpendicular to MR — will lie all the centers of the other great circles of latitude drawn through the poles of the ecliptic M and R. Thus, when the circle AMCR is cut into six equal parts, and straight lines are drawn from M through the points of division, the perpendicular ST is cut at the centers of those circles which divide the ecliptic into the 12 signs — as is clear from what we demonstrated in Proposition 8, Number 2, concerning the centers of the verticals.

[Margin: To divide the ecliptic into 12 signs and into 360 degrees.]

Thus you see that the circle MT, described from center S, runs through the first points of Pisces ♉ (and Virgo ♍; while the circle MS, described from center T, passes through the first points of Scorpio ♏ [?] and Taurus ♉. And if each of the six parts of the circle AMCR is cut into thirty parts, the straight lines drawn from M through those divisions onto the line ST will give the centers of the other great circles, which divide each of the 12 signs of the ecliptic into thirty degrees.

But because the lower semicircle of the circle AMCR runs out too far, and cannot always be drawn within the plane proposed, the same centers on the line ST will be found more conveniently in this way. Let the semicircle XVY, described from M at any radius, be cut into 6 equal parts. For the straight lines drawn out from M through the several divisions will give the centers of pairs of signs — namely, of those signs which are marked against those very divisions. And if each of those parts is divided into thirty degrees, the centers of the individual degrees will be found, and so on, as is evident from what we demonstrated in the aforesaid Proposition 8, Number 4, concerning the centers of the verticals.

Diagram. A large square-framed stereographic-projection figure for constructing the division of the ecliptic and for placing a star on the rete. At the upper right sits the projected sphere as a set of nested circles about a center **H**; from H radiate many straight lines to the periphery. Two concentric graduated arcs sweep across the upper part, lettered along their curves **LAT. BOR.** (*latitudo borealis*, northern latitude) and **LAT. AVS.** (*latitudo australis*, southern latitude), each carrying the degree graduations **30 · 60 · 90**; a zodiac symbol (Capricorn ♐ [?]) stands at the 90° end. From the projected sphere the large ecliptic and equator curves descend and swing down to a common apex vertex lettered **R** [?] at the foot of the frame. On the horizontal mid-axis are points **S** (left) and **T** (right); a short tick at upper left is lettered **X** and a matching tick at right **Y** (the ends of the semicircle XVY), the latter beside a "90".

Readable interior labels (many small peripheral and rotated labels are abraded or too fine to resolve):

- H (center of projection)
- P, Q, S, I (central cluster near H)
- A, B (on the vertical axis just below the inner circle)
- D (lower left of the inner circle)
- O (beside B)
- M, R and lowercase m, r (the ecliptic poles and their neighbours)
- K (left-middle of the inner circle)

- T (right end of the horizontal axis), S (left end)
- X (left tick), Y (right tick)
- R (bottom apex where the outer curves meet)
- lowercase e, ε, f, i, d, o, u; Greek θ (near the star construction)
- numerous faint peripheral graduation labels and zodiac glyphs around the projected sphere [several illegible — ?]

:* more than two peripheral labels (rotated latitude-arc glyphs, small zodiac symbols, and graduation numbers) are illegible at this resolution.

(Text resumes below the woodcut on p. 583:)

The ecliptic, however, will be divided into signs and degrees more easily if straight lines — both from the ecliptic pole M and from the other pole R (if R is marked on the plane of the astrolabe) — are drawn out through the twelfth-parts of the equator and through its individual degrees, all the way to the ecliptic, as was shown in Proposition 5, Numbers 17 and 20. Or if, through the twelfth-parts of the equator and each of its degrees, lines are drawn parallel to the meridian line so as to cut the straight line AC at points, through which straight lines are then projected from Q, the center of the circle AMCR — and so on, as was shown in the same Proposition 5, Number 24. Thus you see that the line Za, parallel to BD, stands 60 degrees from A and cuts the line AC at b; and finally that the line Qb passes through the first points of two signs [zodiac glyphs faint — ?; both lie 60 degrees from Aries, hence Gemini ♊ and Aquarius ♒], at a distance of 60 degrees from Aries ♈, and so on. Here too, if one wishes, may be transferred the other methods of dividing the great circles into degrees, which we set out in Propositions 5 and 6, and especially in Proposition 6, Number 25.

[Margin: To place the fixed stars on the rete of the astrolabe by their longitudes and latitudes.]

2. The fixed stars will be placed on the rete of the astrolabe most exactly, by their longitudes and latitudes, in this way. Once a parallel of the ecliptic has been described passing through the proposed star in the sphere, with account taken of the star's latitude — whether northern or southern — let there be counted along that parallel the longitude of the star, that is, its distance from the first point of Aries ♈.

Let this counting begin from the parallel's eastern intersection with the circle AMCR (which passes through the first points of Aries ♈ and Libra ♎) and run toward the parts of C, as was taught in Proposition 6, Number 22 and following.

For the end-point of the counting will be the place of the proposed star. Any parallel of the ecliptic, moreover, will be described and divided into degrees by the same methods by which the parallels of the horizon were described and divided into degrees in Proposition 6.

[Margin: To prepare a figure by which any parallel of the ecliptic may easily be drawn on the astrolabe.]

But so that the matter may be carried out more easily by the method we prescribed in Number 8 of that proposition, a figure must be prepared in this way. From A, with a circle def described at any radius, let radii AI and AN be drawn, passing through the extremities of the apparent diameter of the ecliptic FG and cutting the circle def at d and f; and df will be a quadrant, since by Lemma 10 it is similar to half of the semicircle ILN of the equator, or to half of the quadrant FCG of the ecliptic circle. Let there also be drawn the radius AL, passing through L, the true pole of the ecliptic, and through M, the apparent pole, and cutting the circle def at e; and the arcs de and ef will be equal, since by the same Lemma 10 they are similar to the halves of the quadrants IL, LN of the equator, or Fi, iG of the ecliptic. For the straight line Ae, drawn through the pole of the ecliptic, passes through k, the [other] extremity of the ecliptic diameter ik, being perpendicular to FG, as [shown] in the scholium of Proposition 5, Number 14.

(Text resumes below the reprinted woodcut on p. 584:)

we demonstrated. Then, taking the arcs dg and fh equal to the arcs de and ef — which arcs the radius APR, passing necessarily (by the same scholium, Proposition 5, Number 14) through k, cuts off at the other extremity of the ecliptic diameter ik — [we proceed] because both the lines Ak, AF and the lines AP, AN, by Lemma 10, intercept the arc dg similar to half the quadrant of the ecliptic FK [?], and the arc fh similar to half the quadrant PN of the equator. Let each of the arcs dg, de, fh, fe be divided into 90 equal parts, which will be half-degrees, always beginning from the points d and f. For through the parts of the arcs de and fe will be found the apparent diameters of the parallels of northern latitudes, while through the parts of the arcs dg and fh will be found the diameters of the parallels of southern latitudes. Therefore to the former is assigned northern latitude, and to the latter southern latitude, so that it may be known at once toward which side the proposed latitude is to be counted. And in what way the parallels are to be described from the circle def thus divided, was explained in Proposition 6, Number 8; it can again be understood from the following examples. By what method, likewise, such parallels are to be divided into degrees, was taught in the same Proposition 6, Number 21 and following.

[Margin: To place Spica of Virgo on the rete.]

3. Let there be placed on the rete, for example, Spica of Virgo $\mathfrak{M}$, whose longitude from the first star of Aries Υ is 170 degrees, while its true longitude from the first point of Aries Υ is 197 degrees 55 minutes, and its latitude 2 degrees toward the south. Reckon the latitude of 2 degrees, that is, take two of the parts out of the 90 into which each of the arcs dg and fh was divided, as though they

as though they were degrees; and to the [two] limits let there be drawn from A two radii cutting off, on BD, the apparent diameter of the southern parallel of the ecliptic by 2 degrees. These two radii will cut off 2 degrees both from the equator, at I and N, toward A, and from the ecliptic, at F and G, toward k — because the arcs of the circle def cut off by the radius Ad and by that radius which passes through the latitude of Spica, and likewise by the radius Af and by that which passes through the latitude of Spica, are similar to the halves of the arcs cut off both from the equator and from the ecliptic, as we demonstrated in Lemma 10. And accordingly, since each of the two former arcs embraces two half-degrees, that is, 1 degree, each of the two latter will contain 2 degrees. Next, let there be marked the intersection of the ecliptic diameter ik with the straight line joining the two points of the ecliptic that are 2 degrees distant from F and G toward k — the points through which, of course, the two aforesaid radii pass. For the radius drawn from A through that point of intersection of the diameter ik will indicate, on the line FG, the center for describing the parallel about the apparent diameter cut off, from what was demonstrated in Proposition 6, Number 6.

This parallel therefore having been described, let the star's true longitude be counted along it, that is, 197 degrees 55 minutes — its distance from Aries Υ — following the succession of the signs; for the star must be placed at the end of the counting on the said parallel. And we shall find, on the said parallel, the point that terminates the degree of longitude $197^{\circ}55'$ in this way. Since the parallel of the ecliptic recedes toward the south from the ecliptic by 2 degrees, we shall describe a parallel of the equator receding by that same number of degrees from the equator toward the north, and along it we shall count the aforesaid longitude, beginning from its eastern intersection toward the parts of C, with the line EC, advancing toward D and A up to l. In the given example this is done as follows: taking a semicircle from the $197^{\circ}55'$, let the remaining 17 degrees 55 minutes be counted from the line EA, on the western side, up to l. For the line Ml, drawn from the pole of the ecliptic, will give on the parallel of the ecliptic the point m, terminating the degree of longitude $197^{\circ}55'$.

Furthermore, once the parallel of the ecliptic has been described, the parallel of the equator by which the longitude is to be found on it will be described just as easily, even if its declination on the equator is not computed. From M, the ecliptic pole, let a straight line be drawn through the point of the circle AMCR where it is divided by the parallel of latitude. For this line will cut off, from the line EA or EC, the semidiameter of the equatorial parallel. Conversely, if the parallel of the equator is described first — as we taught in Proposition 4, Number 6 — standing as many degrees from the southern pole as the ecliptic parallel stands from the northern, the parallel of the ecliptic will be described in this way too. Draw from M, the ecliptic pole, through the point of intersection of the equatorial parallel with the line EA or EC, a straight line; it will cut the circle AMCR at the point through which the parallel of the ecliptic is to be described. Its center will be found if through that point a straight line is drawn tangent to the circle AMCR, as was demonstrated in Proposition 6, Number 10.

[Margin: To describe the parallel of the equator from the opposite parallel of the ecliptic, and conversely the latter from the former.]

We shall find that same degree m of longitude more easily — even if neither the circle AMCR nor the parallel of the equator has been described — from what we set out in Proposition 6, Number 25. For since the longitude amounts to $197^{\circ}55'$, if we subtract it from three quadrants, that is, from 270 degrees, there will remain 72 degrees 5 minutes, by which the star on the ecliptic parallel is distant from the meridian line above F, toward A. If, then, from the opposite point below G, in the opposite direction toward C, we count 72 degrees 5 minutes on the same ecliptic parallel, the line drawn out from the pole M through the end of the counting will fall on the sought point m — because the arc of the said parallel contained between the meridian line and the drawn line holds as many apparent degrees as there are equal ones contained in the arc counted from the meridian line below G toward C, as we demonstrated in the place cited.

[Margin: The easiest way of finding the longitude-point of Spica of Virgo on the parallel of its own latitude.]

We shall find that same place of the star m, that is, of longitude $197^{\circ}55'$, by means of the great circle of latitude drawn through the poles of the ecliptic, in this way. Since the star has a true longitude of $197^{\circ}55'$, that is, stands at $17^{\circ}55'$ of Libra ϱ , we shall count from the point V, the first point of Libra ϱ , toward Scorpio $\mathfrak{M}$ [?], along the circle XVY, $17^{\circ}55'$, up to θ ; and from M, through θ , we shall extend a straight line cutting the line ST at μ , the center of the great circle [?]Mm which

passes through $17^{\circ}55'$ of Libra ϱ and through Aries γ , and which cuts the ecliptic parallel at m, the point of the same longitude.

[Margin: To set on the rete the star called Hircus [Capella].]

4. Let there again be placed on the rete the star called Hircus [Capella], shining on the left shoulder of Auriga [the Charioteer], whose longitude from the first star of Aries γ is 48 degrees 20 minutes, its true longitude from the first point of Aries γ 76 degrees 25 minutes [?], and its latitude — northern — 22 degrees 30 minutes. Having therefore counted the latitude from the points d and f toward e, and having drawn radii through the ends of the countings, FG will be cut at the extremities of the apparent diameter of the parallel of latitude; and if the points n and o, at which those radii cut the ecliptic, are joined by a straight line, the diameter ik will be cut at the point p — [the diameter] of the parallel δ_{ref} passing through the star, and cutting the circle AMC at r and f. Let there be described in addition a parallel of the equator $\alpha\beta$, whose declination is southern and equal to the northern latitude of the parallel δ_{ref} , $22^{\circ}30'$; its semidiameter $E\alpha$ is cut off by the line Mr, produced. Now, the longitude of the star having been counted from α up to β , the line $\mu\beta$ will cut the parallel of latitude at δ , the point of the same longitude. At δ , therefore, will be the place of the proposed star — which we shall likewise find as follows.

Having described, about the apparent diameter ref of the parallel of latitude, the circle ref (which of course represents the common section of the parallel and of the great circle drawn through the poles of the ecliptic and through the first points of Aries γ and Libra ϱ), let the longitude of the star be counted along the circle ref from r, toward either side, up to the point t, from which a line drawn parallel to BD cuts the same diameter rf at u. For the line Qu will cut the parallel of latitude at two points, δ and ε , each of which is distant from the point r by $76^{\circ}15'$ — as was demonstrated in Proposition 6, Number 26 — the same amount by which the point t is distant from that same point r. And because the star is in the northern half of the ecliptic (since its longitude from Aries γ is less than 180 degrees), the place of the star will be the point δ , in the lower half of the parallel of latitude, the half that inclines to the north. But if some star should have the same latitude and the same distance from Aries γ , yet against the succession of the signs — so that its true longitude amounts to 283 degrees 45 minutes — its place will be at the point ε , looking toward the south.

In this example, moreover, there is no need to labor at finding the star's place by means of the great circle drawn through the poles of the ecliptic, since that would be very inconvenient, because its center lies too far off on the line ST, from the

point Q toward T — inasmuch as the star has a longitude of $76^{\circ}15'$, that is, stands at $16^{\circ}15'$ of Gemini Υ .

[Margin: The easiest way of finding the longitude-point on the parallel of its own latitude.]

But here too, without the circle AMCR and the equatorial parallel $\alpha\beta$, we shall more easily find the point δ of the star's longitude, $76^{\circ}15'$. For since this distance is taken from Aries Υ toward Cancer $\mathfrak{S}$, the same star will be distant from Cancer $\mathfrak{S}$, toward Aries Υ , by $13^{\circ}45'$. If, therefore, on the parallel of latitude $\delta\epsilon\zeta$, an arc of $13^{\circ}45'$ is cut off from the meridian line below the pole M, toward r, that arc will terminate at δ , the place of the star. And we shall proceed by the method we described in Proposition 6, Number 25. On the said parallel $\delta\epsilon\zeta$, counting from the meridian line above the pole M... toward f, $13^{\circ}45'$. For the line extended from the end of the counting through the pole M will cut the said parallel at δ , because, as [we demonstrated in the place cited]...

ASTROLABII LIBER SECVNDVS — Propositio XI (Scholium) and Propositio XII

[continuation of §4] ...as we showed in the passage cited, the parallel arc contained between the drawn line and the meridian below the pole M holds as many degrees as appear equal in the opposite arc between the same straight lines above the pole M.

In exactly the same way any other star, whose longitude and latitude are known, will be drawn on the astrolabe.

5. If the declinations, right ascensions, and culminations [mediationes coeli] of the stars that are to be set on the rete are at hand, the same stars will be placed on the astrolabe without much labor, in this way. From the center of the astrolabe draw a straight line through the degree of the ecliptic with which the star culminates — that is, with which it reaches the meridian — or through the endpoint of its right ascension on the equator; where the parallel of latitude, or of the star's declination, cuts that line, there will be the place of that same star on the rete or astrolabe. So too the place of the same star will be at the point where the parallel of latitude intersects the parallel of declination. But the earlier method, by the star's longitude and latitude, which we explained, is the more reliable, since tables that contain the stars' declinations, right ascensions, and culminations without error are rarely found, whereas the longitudes of these

same stars, reckoned from the first star of Aries, remain always the same. Thus, once the distance of the first star of Aries from the beginning of Aries is known, the distances of all the others become known, as we shall say shortly.

[Margin: To place the fixed stars on the rete of the astrolabe by their declinations, right ascensions, and culminations. — *Stellas fixas reti Astrolabij per earum declinationes, ascensiones rectas, & coeli mediationes, imponere.*]

SCHOLIVM

1. Since the chief use of the fixed stars on ordinary astrolabes is that by them the hours may be investigated at night, care must be taken that some few stars be included in the whole circuit of the rete — as few as possible, lest their multitude breed confusion — yet in such a way that, however the rete is turned, always one or another is at least above the horizon. Once these have been set on the rete, the superfluous parts are to be cut away, and only the stars retained, and the ecliptic divided into degrees, to this end: that any degree of the ecliptic and the point [cacumen] of any star can be set at any point of the plane of the astrolabe on which the circles of the sphere, always keeping the same situation, have been drawn — such as the equator, the tropics, the vertical, the horizon and its parallels, the hour-circles, [the circles] of the celestial houses, and so on. This matter is to be accomplished by one's own industry, in imitation of some other astrolabe, rather than belabored with many words. But because we shall also teach by what method the declination of any star, its ascension both right and oblique, and its culmination can be found from its known longitude and latitude, the precept we gave above concerning the placing of stars on the astrolabe must be carefully committed to memory, so that the place of any star may be found on the plane of the astrolabe whenever use requires. But now, so that stars may be set on the astrolabe for exploring the hours at night, we have here set out the true longitudes of several stars, reckoned from the beginning of Aries — that is, their places in the zodiac; then their latitudes, declinations, right ascensions, and finally their culminations, or the points of the zodiac with which they reach any meridian, both above the horizon and below; where the letter S signifies northern latitude and declination, and the letter M southern. Finally, the numbers prefixed to the stars themselves denote of what magnitude they are. Moreover, the longitudes of the stars, from the [Prutenic] tables... [continues on p. 235]

[Margin: What the chief use of the stars on ordinary astrolabes is. — *Vsus praecipuus stellarum in Astrolabiis vulgaribus quis.*]

[Margin: What is taught in this astrolabe concerning the fixed stars. — *Quid in hoc Astrolabio de stellis fixis tradatur.*]

A table of certain fixed stars, computed for the year of the Lord 1600
completed [TABELLA FIXARVM ALIQVOT stellarum ad annum
Domini 1600. completum supputata]

M ag ni tu de	Star name	Place in Zodiac (G · M)	Latitu de (G · M)	La t. di r.	Declinat ion (G · M)	De cl. dir .	Right ascension (G · M)	Culminati on [Mediatio coeli] (G · M)
3	Cornu Υ , praecedens (precedin g horn of Aries)	Υ 28 · 5	7 · 20	S	17 · 39	S	23 · 20	Υ 25 · 11
2	Caput Medusae (head of Medusa) [Algol]	♄ 21 · 5	23 · 0	S	40 · 5	S	40 · 55	♄ 13 · 23
1	Oculus ♄ (eye of Taurus) [Aldebara n]	♄ 4 · 5	5 · 10	M	15 · 56	S	63 · 6	♄ 5 · 3
1	Dexter humerus Orionis (right shoulder of Orion) [Betelgeus e]	♄ 23 · 25	17 · 0	M	6 · 21	S	83 · 41	♄ 24 · 12
1	Hircus (the Goat)	♄ 16 · 25	22 · 30	S	45 · 9	S	72 · 6	♄ 13 · 30

M ag ni tu de	Star name	Place in Zodiac (G · M)	Latitu de (G · M)	La t. di r.	Declinat ion (G · M)	De cl. dir .	Right ascension (G · M)	Culminati on [Mediatio coeli] (G · M)
	[Capella]							
1	Canis maior (Great Dog) [Sirius]	♋ 9 · 5	39 · 10	M	15 · 54	M	97 · 19	♋ 6 · 43
2	Lucida Hydrae (bright star of Hydra) [Alphard]	♏ 21 · 25	20 · 30	M	5 · 4	M	137 · 19	♏ 14 · 51
1	Cor ♏ (Heart of Leo) [Regulus]	♏ 23 · 55	0 · 10	S	13 · 44	S	146 · 19	♏ 23 · 59
1	Cauda ♏ (Tail of Leo) [Denebola]	♏ 15 · 55	11 · 50	S	16 · 26	S	171 · 49	♏ 21 · 5
1	Spica ♍ (Spica of Virgo)	♍ 18 · 5	2 · 0	M	8 · 58	M	195 · 55	♍ 17 · 16
1	Arcturus	♍ 18 · 25	31 · 30	S	21 · 49	S	209 · 23	♍ 1 · 33
2	Cor ♏ (Heart of Scorpius) [Antares]	♏ 4 · 5	4 · 0	M	24 · 57	M	241 · 16	♏ 3 · 19
1	Lyra [Vega]	♌ 8 · 45	62 · 0	S	38 · 40	S	275 · 15	♌ 4 · 49
1	Vltima	♐ 28 · 25	23 · 0	M	33 · 24	M	339 · 56	♐ 8 · 17

M ag ni tu de	Star name	Place in Zodiac (G · M)	Latitu de (G · M)	La t. di r.	Declinat ion (G · M)	De cl. dir .	Right ascension (G · M)	Culminati on [Mediatio coeli] (G · M)
	aquae ♒ (last star of the Water, Aquarius)							
2	Cauda Cygni (Tail of the Swan) [Deneb]	✕ 0 · 35	60 · 0	S	44 · 8	S	307 · 22	≈ 5 · 0
2	Crus Pegasi (Leg of Pegasus)	✕ 23 · 35	31 · 0	S	25 · 44	S	341 · 1	✕ 9 · 26

[Scholium §1, continued] ...the [Prutenic] tables, diligently and accurately, for the year 1600 completed. Then from these longitudes we hunted out the declinations, right ascensions, and culminations by the doctrine of sines. The method we followed in this matter we shall disclose in Book 3, when we treat the same subjects in connection with the use of the astrolabe, so that anyone, when he pleases, may examine our calculation. For I think one ought not rely wholly on any tables of declinations, ascensions, culminations, and other things that depend on long computations, since an error may easily creep into them without our noticing. And in this our calculation, account has always been taken of the proportional part in the sines and minutes, as I advised in the use of the table of sines. But in the earlier table we neglected the seconds when they were fewer than 30, and for more than 30 we added one minute. Therefore, in order that right ascensions be computed from the declinations, they are not to be taken as they are set down in the table, but as they were found by the doctrine of sines, together with the seconds. But of this matter we shall write more in Book 3.

2. Furthermore, the places of the stars in the zodiac are found as follows. If to their longitudes — which we noted in our Commentary on the Sphere from approved authors — there be added the true precession of the equinoxes (which,

computed from the Prutenic tables for the year of the Lord 1600 completed after the Gregorian correction, comes to $28^{\circ}5'$), and then the number so composed be divided by 30: for the quotient will indicate how many signs the star has passed through, while the remaining number will show the degrees of the following sign in which it stands, and if the leftover minutes, if any, be added, the true place of the star in the zodiac will be had. For example, the first star of Aries, which is in the preceding and right[-hand] horn, has no longitude in the table of fixed stars which we compiled in the Commentary on the Sphere, since from it the longitudes of the others are numbered. So, adding the true precession of the equinoxes, $28^{\circ}5'$, the true longitude of that star becomes $28^{\circ}5'$; and since in this longitude no whole sign is contained, the first star of Aries will stand at $28^{\circ}5'$ of the first sign, which is Aries. Again, Spica of Virgo has a longitude of $170^{\circ}0'$; if the $28^{\circ}5'$ of true precession of the equinoxes be added, the true longitude becomes $198^{\circ}5'$. Dividing 198° by 30 gives a quotient of 6, with 18 remaining. The star has therefore passed through these six signs — Aries, Taurus, Gemini, Cancer, Leo, Virgo — and stands at $18^{\circ}5'$ of the next following sign, Libra. The same reasoning holds for the rest. But if the number composed by adding the true precession is greater than the whole circle of 360° , the whole circle of 360° must be cast out before the division is made, or the whole zodiac of 12 signs after the division is made. For example, the star of the second magnitude which is in the navel of Pegasus and in the head of Andromeda has a longitude from the first star of Aries of $341^{\circ}10'$; adding the true precession of the equinoxes, $20^{\circ}5'$ [recte: $28^{\circ}5'$ — the printed value cannot be right, since $341^{\circ}10' + 28^{\circ}5' = 369^{\circ}15'$, the sum stated next], the sum becomes $369^{\circ}15'$. Casting out the whole circle of 360° , there remain $9^{\circ}15'$ of the first sign, Aries, for the place of the star. Or, dividing the true longitude $369^{\circ}15'$ by 30, there result 12 signs and $9^{\circ}15'$; and casting out the 12 signs, the same place of the star is found at $9^{\circ}15'$ of Aries. Now this precession of the equinoxes, $28^{\circ}5'$, can be retained for many years following the year 1600, because, on account of the slowness of the motion of the stars from west to east, their places in the zodiac are not perceived to change so quickly. Yet whoever wants their exact places must find the true precession of the equinoxes for at least every 20 years, and for the same intervals compute again the declinations, right ascensions, and culminations of the stars. For these must necessarily change when the places of the stars in the zodiac change.

[Margin: To find the places of the fixed stars in the zodiac from their longitudes.
— *Loca stellarum fixarum in Zodiaco reperire ex earum longitudinibus.*]

But that we may relieve students, in this part, from the trouble of calculating the true precession of the equinoxes, we have computed the following table, from

which the true precession of the equinoxes for any year — from the beginning of the Olympiads, which falls in the year 774 before Christ the Lord, up to the year 3000 after Christ — may be drawn out with the greatest ease. For if the year proposed is found in the table, the true precession of the equinoxes in degrees and minutes will appear at once opposite it. Now years by hundreds have been placed in the table, except sometimes when, for the notable memory of some event, certain years have been inserted among the hundreds — namely years in which notable astronomers flourished, or from which, as from roots, the astronomers computed the celestial motions; such as the epoch of King Nabonassar, who is also Nebuchadnezzar or Shalmaneser [Salmanassar], from whom Ptolemy computed the motions. But if the given year is not found in the table, one must take the difference between the two true precessions of the two nearest years — one of which is less than the year proposed, the other greater — together with the difference of these years. For as the difference of these years is to the difference of the precessions, so is the difference between one of those years and the year proposed to another quantity, which will be the difference of precession to be added to the precession of the lesser year of the table (if the difference was taken between that lesser year and the year proposed), or to be subtracted from the precession of the greater year (if the difference was taken between that greater year and the given year). By this method the precession of any year will be found quite exactly, just as if it were drawn from the Prutenic tables, and there will be a difference only occasionally in certain seconds, which may deservedly be neglected. For example, let the true precession of the equinoxes be sought for the year 880, in which Albategnius [al-Battānī] flourished. Subtract the precession of the year 800, $16^{\circ}44'$, from the precession of the year 900, $18^{\circ}33'$; and let it be done as 100 years is to the difference of the precessions, $1^{\circ}49'$, so 80 years [is] to (the difference of the years 800 and 880) another quantity — and there will be found the proportional part, $1^{\circ}27'$. If therefore $1^{\circ}27'$ be added to $16^{\circ}44'$ (the precession of the year 800), it will make the precession of the year 880, $18^{\circ}11'$, as before. [Or: as 100 years is to the difference of the precessions $1^{\circ}49'$, so 20 years is to (the difference of the years 880 and 900) another quantity, and there will be found the proportional part $22'$, nearly agreeing for that time with 20 years; which, subtracted from $18^{\circ}33'$ (the precession of the year 900), will make the remaining precession of the year 880, $18^{\circ}11'$, as before.] The same reasoning holds for the rest. Now the years of this table are to be understood as complete and whole, both after Christ and before; and the precession of each can be taken as the root of the precession of the following years. So if anyone should wish to compute the precession from the

Prutenic tables for the year 1638, he could draw out the precession for the [38] years, and add to it the precession of the year 1600 of this table, as a root.

[Margin: To elicit the true precession of the equinoxes for very many years from the table. — *Praecessionem veram aequinoctiorum ex tabella ad plurimos annos elicere.*]

Table of the precession of the equinoxes [TABVLA PRAECESSIONIS AEQVINOCTIORVM]

The table is printed in three side-by-side column-blocks; it is transcribed here in one continuous sequence of epochs (earliest to latest), which is the block reading order. Heads: TEMPVS (epoch) | Anni ante/post Christum (years) | Praecessio: S · G · M. In the later blocks only praecessio G · M is printed (the S-column value is 0 throughout). Here "S" is the *signa* column of the trepidation argument (1 signum = 60°, so the argument runs 0–6 signa = 0–360°); "G" = degrees, "M" = minutes. This explains the apparent wrap between the epoch of Thales (357°40' □ 5 signa 57°40') and that of Meton (0 signa 0°41') — a genuine feature of the model, not a misprint.

Epoch [Tempus]	Year	S	G	M
Ab Olympiadibus (from the Olympiads)	774 BC	5	5	4
			4	4
Ab Vrbe condita (from the founding of the City)	750 BC	5	5	4
			5	6
A Nabonnasaro (from Nabonassar)	746 BC	5	5	5
			5	0
Thaletis (of Thales)	637 BC	5	5	4
			7	0
Metonis (of Meton)	431 BC	0	0	4
				1
A morte Alexandri (from the death of Alexander)	324 BC	0	1	5
				9
Timocharis (of Timocharis)	292 BC	0	2	2
				1
Hipparchi (of Hipparchus)	126 BC	0	4	3
Iulij Caesaris (of Julius Caesar)	45 BC	0	4	5
				0
CHRISTI (of Christ)	0 (post Christum)	0	5	3

Epoch [Tempus]	Year	S	G	M
				2
Menelai (of Menelaus)	100 AD	0	6	1
				6
Ptolemaei (of Ptolemy)	138 AD	0	6	4
				0
—	200 AD	0	7	2
				1
—	300 AD	0	8	3
				4
Concilij Nicaeni (of the Council of Nicaea)	325 AD	0	8	4
				4
—	400 AD	0	9	5
				6
—	500 AD	0	1	2
			1	8
—	600 AD	0	1	8
			3	
—	700 AD	0	1	5
			4	4
—	800 AD	0	1	4
			6	4
Albategnij (of al-Battānī)	880 AD	0	1	1
			8	1
—	900 AD	0	1	3
			8	3
—	1000 AD	0	2	1
			0	8
—	1100 AD	0	2	5
			1	8
—	1200 AD	0	2	2
			3	8
Alphōsi Reg (of King Alfonso)	1251 AD	0	2	1
			4	1
—	1300 AD	0	2	4

Epoch [Tempus]	Year	S	G	M
			4	9
—	1400 AD	0	2	1
			6	
—	1500 AD	0	2	6
			7	
—	1582 AD	0	2	5
			7	5
—	1600 AD	0	2	6
			8	
—	1700 AD	0	2	3
			9	
—	1800 AD	0	3	3
			0	
—	1900 AD	0	3	7
			1	
—	2000 AD	0	3	1
			2	9
—	2100 AD	0	3	3
			3	9
—	2200 AD	0	3	1
			5	0
—	2300 AD	0	3	4
			6	8
—	2400 AD	0	3	3
			8	4
—	2500 AD	0	4	2
			0	3
—	2600 AD	0	4	1
			2	2
—	2700 AD	0	4	5
			3	9
—	2800 AD	0	4	3
			5	9
—	2900 AD	0	4	1

Epoch [Tempus]	Year	S	G	M
—	3000 AD	0	4	3
		7	1	8

PROBL. IX. PROPOS. XII.

To describe in the astrolabe any great circle whose position and situation in the sphere is not unknown, together with its parallels and verticals.

[Margin: To describe in the astrolabe a great circle through two points, one given on the horizon and the other on the meridian, or expressed by degrees. — *Circulum maximum per duo puncta, quorum vnum in Horizonte, et alterum in Meridiano datum sit, vel per gradus expressum, in Astrolabio describere.*]

[Margin: To describe a great circle through two points, one given on some great circle of the astrolabe and the other on some other great circle, or expressed by degrees. — *Per duo puncta, quorum vnum in circulo aliquo maximo Astrolabij, & alterum in alio quopiam circulo maximo sit datum, vel per gradus expressum, circulum maximum describere.*]

[Margin: To describe in the astrolabe a great circle whose declination from the vertical and inclination to the horizon are known, by means of the vertical [circle] that measures its inclination. — *Circulum maximum, cuius declinatio à Verticali, & inclinatio ad Horizontem nota sit, in Astrolabio describere, beneficio Verticalis eius inclinationem metientis.*]

1. Let there be in the astrolabe, whose center is E, the equator ABCD, the horizon AFCG, and the vertical AHCI. (In what follows it will be of great use if on some plane or paper the principal circles of the sphere are drawn, as on an astrolabe — such as the equator, the ecliptic, the horizon, the primary vertical of the proposed region, and the two tropics — to this end, that we may have ready at hand the magnitude and situation of each of them.) And let it be proposed that a great circle be described, cutting the horizon at a point which is 30° distant from the eastern equinoctial point C toward the south, and accordingly by as many degrees from the western equinoctial point A toward the north G; but cutting the meridian at a point which, above the horizon, inclines from the equator toward the south [by a stated amount] — that is, at the point on the horizon which is 30° distant from C, and likewise at the point P, which inclines 24° from the equator toward the south. This will be done as follows. Having found the point N — and this toward the north — likewise the point P, which recedes by as many degrees

from A, that one toward the south, 30° , as was shown in Proposition 5, Number 17: it is enough to have found one of the points N and P, since a straight line drawn from it through the center E will exhibit the other, because those points are opposite through the diameter. These points found here are [given] by the straight lines HM, HO, which cut off from the equator the arcs CM, AO. Next, on the meridian let the point R be sought, distant 24° from B; which will be done if the arc BQ of 24° be taken on the equator, and the straight line AQ be drawn, cutting the meridian at R. And if an arc DS equal and opposite to BQ be taken on the equator, the straight line AS will give, on the same meridian, the point T opposite to the point R, as is clear from what we demonstrated in Proposition 6, Number 13. And since a great circle in the sphere passes through two opposite points, we shall have four points, N, R, P, T, through which the proposed great circle is to be described. Therefore, having found V, the center [equidistant] from any three of the points — which indeed lies at the meeting of the two perpendiculars that bisect the straight lines NP, RT, by the corollary of Proposition 1, Book 3 of Euclid — the circle NRPT described from V through those three points will be the very great circle we were bidden to describe, since it will also pass through the fourth, seeing that it passes through the given points of the horizon and meridian, which are opposite through the diameter. And by this method, through any two given points — one on one great circle, the other on another great circle — we shall describe a great circle, if the points opposite to them be investigated, so that four points be had through which it is to be described. Thus if the point N be given on the horizon and the point R on the meridian, we shall seek the opposite points P, T, and so on. But if those points are not assigned, but only their degrees are expressed — namely 30° on the horizon from the east toward the south, and 24° on the meridian from the equator toward the south — then those degrees, namely the points N, R, must be investigated, just as was done a little before.

2. But if a great circle is to be described representing some plane declining from the south — for example, 30° toward the west — and inclined to the horizon 26° on the southern side (how the declination and inclination of any plane may be found, we taught in the Gnomonics, Book 1, Proposition 23): that circle will again cut the horizon at the points N, P, of which the one inclines from the east toward the south, the other from the west toward the north; and these will be found as before. And the poles of the vertical circle — the circle passing through the poles of the horizon and of the given circle, and measuring its inclination to the horizon — will be [those points]. For since this vertical must be perpendicular both to the horizon and to the given circle, ^a it will pass through the poles of both, and accordingly each will in turn pass through the poles of the other, by the

scholium of Proposition 15, Book 1 of Theodosius; and therefore the points N, P, where they intersect, will be its poles. And since the poles are distant a quadrant of a great circle from their own great circle, by the corollary of Proposition 16, Book 1 of Theodosius, if there be found on the horizon points X, Y distant 90° from the poles N, P — or, what is the same, 30° from the points G, F (which will be done by straight lines drawn from H through the points a, b of the equator, which are 30° distant from the points D, B) — the said vertical must be described through the points X, H, Y, from the center Z, which lies on the straight line LZ (perpendicular to the meridian line at L, the center of the primary vertical), and will be found in this way. Since that vertical recedes 60° from the primary vertical — from the east toward the north, or from the west toward the south — we shall take the arc *de* of 60° on the vertical, and we shall double the arc *Ie* up to *f*; or from H we shall take... [continues in the next installment]

[Margin: (footnote marker ^a) Theodosius, [*Sphaerica*] I.13. — 13.1. *Theod.*]

[Margin: To describe the vertical, because it measures the inclination of the proposed circle to the horizon. — *Verticale, quia propositi circuli inclinationem ad Horizontem metitur, describere.*]

Astrolabe, Book II — **Problema IX / Propositio XII** (construction of a great circle through four points), continued; its **Scholium**; and the opening of **Problema X / Propositio XIII**. ... we shall take the arc of 60 degrees, doubled, up to f. For the straight line If will cut LZ at Z, the center of the given Vertical, as was taught in the same Prop. 8, No. 10. The straight line NP, produced, will also yield the same center Z, since the poles of that Vertical, and its center, lie on the same line NP — drawn through the center and its poles — as was shown in the same Prop. 8, No. 19.

Now once the Vertical XHY has been drawn, if the arc Yk of 26 degrees is cut off from it (as was taught in Prop. 5, No. 17), we shall have three points N, k, P, through which the proposed circle is to be drawn; and it will necessarily pass through a fourth point i, opposite to point k along the diameter iEk.

We shall cut off the arc Yk of 26 degrees as follows. From P, the pole of the Vertical XHY, draw to Y the straight line PY, cutting the Equator at g; then take the arc gh of 26 degrees. For the line Ph will cut off the required arc Yk of 26 degrees. Or else, from the other pole N, draw the line NY, cutting or touching the Equator at ϕ (in this example it touches and does not cut, and therefore also touches the Vertical at Y, as was shown in the scholium of Prop. 5, No. 15); and take the arc $\phi\omega$ of 26 degrees. For the line N ω will give the same point k.

Here you see that the arcs of the Equator γg and $[?]\phi$, which yield the same point Y, are equal, being begun from opposite points of the Equator; likewise the arcs γh and $[?]\omega$; and also that the arcs xg , $x\phi$, xh , $x\omega$ are equal, all having their beginning at the same intersection x [Greek: the intersection-point is labelled with a small mark, read as x/κ], though they run in opposite directions. This is what we advised must be observed in Prop. 5, No. 23.

Or indeed let a parallel of the Horizon $\beta k\phi$, distant 26 degrees from the Horizon, be drawn in this way. Having taken two arcs Fl and Gm of 26 degrees, draw the line lm, cutting the diameter of the Horizon Kn at n. For when the lines Al, Am, An are joined, cutting the meridian line at β , ϑ , p, then $\beta\vartheta$ will be the diameter of that parallel, and p its center — as is clear from what we demonstrated in Prop. 6, No. 6.

Diagram. A large stereographic-projection figure enclosed in a ruled square frame, occupying the lower two-thirds of the page. It shows the astrolabe plate for Prop. XII: a great outer circle (the Equator / primitive), an obliquely placed great circle NRPT, a Vertical XHY, and numerous auxiliary circles, diameters, and rays, drawn in solid and dotted lines. The same woodcut is repeated on p. 590 (see fig-p590-1). Readable labels, by approximate position:

- I (upper left), F (top), N (left margin of circle), Y (upper right), A (right), e (bottom)
- interior capitals: M, C, R, B, K, P, S, T, G, X, D, L
- Greek / small letters at interior intersections: ψ , θ , δ , and small marks read as ω/ϖ , "ii", "aa", plus lowercase b, o, q, a, h at various nodes
- Several peripheral and interior labels are too small to resolve under magnification.

Therefore the parallel drawn from p through β , ϑ will cut the Vertical XHY at the point k, which cuts off the arc Yk of 26 degrees. Indeed, if the parallel $\beta\vartheta$ is drawn, and on it, from the point ϕ , 60 degrees are counted off (as we taught in Prop. 6, No. 22) up to k, the point k will be found through which the proposed great circle must pass, even if the Vertical XHY has not been drawn. This method is in fact most convenient when that Vertical is only slightly distant from the Meridian, so that its construction becomes very difficult because of the excessive distance of its center, on the line LZ, from point L. At the end of the scholium of Prop. 15, too, you will find a most easy and most elegant procedure by which, without the Vertical and without the parallel of the Horizon, a third point bb may be found, through which the proposed circle is to be drawn.

But it is necessary — if no error has been made — that the points q, r , where the great circle that has been drawn cuts the Equator, be diametrically opposite; that is, that the line q, r pass through the center E . [Margin a: Theodosius I.11.] This is because the great circles on the sphere bisect one another, as we also noted in the scholium of Prop. 5, No. 6. For from this it follows that all circles drawn in the Astrolabe in any manner whatever — such as, in the present example, the points N, P and R, T — bisect the Equator, since they represent the great circles of the sphere. On this matter we shall write more in the scholium of this proposition.

[Margin: To cut off the arc of the given inclination from the drawn Vertical that measures the inclination of the proposed circle. — To draw in the Astrolabe the same great circle, whose declination from the Vertical and inclination to the Horizon are known, by means of the parallel of the Horizon which measures the inclination, without the Vertical. — The convenience of this latter construction: to draw the same great circle by an easiest procedure, through the teaching of the scholium of Prop. 15.]

3.[?section digit faint] In order to draw the parallels of this great circle $NRPT$ that has been drawn, its true diameter must be found in the Equator, taken as the Meridian of the Analemma, as we prescribed in Prop. 8, No. 16 — namely, in this way. Through E , the center of the Astrolabe, and V , the center of the circle drawn, let the line st be drawn. [Margin b: Euclid III.3.] Being drawn from center V , it will be perpendicular to q, r (which lies in circle $NRPT$ and which it bisects at E), and it will represent the common section of the Astrolabe — or of the Equator — and of the proper Meridian of that same great circle, as was said in the scholium of Prop. 3, No. 4. Then from r , as if from the south pole, let straight lines going out through s, t , the extremities of the apparent greatest diameter, cut the Equator at u, ϖ . For the line $u\varpi$ will be the true diameter of the said great circle in the sphere, so that ru is the altitude of the pole above it.

[Margin: To find the true diameter of the great circle that has been drawn, its poles, and the altitude of the pole above that same circle.]

... And if another diameter $\theta\mu$ is drawn perpendicular to $u\varpi$, it will be the axis of that same circle and [the line of] its proper poles θ, μ , of which θ will appear at λ — all of which was demonstrated in Prop. 8, Nos. 16 and 17. You see, therefore, that the Vertical XHY passes through λ , the pole of circle $NRPT$, just as here it is drawn through N, P , the poles of that Vertical, as Theorem 1 of the scholium of Prop. 15 of Book 1 of Theodosius [Theodosius] requires.

And so, if lines parallel to the true diameter $u\varpi$ are drawn through the individual degrees of the Equator — or if lines parallel to st itself are drawn through the individual degrees of circle NRPT — and from r rays are cast out through their extremities, the line st will be cut at the extreme points of the apparent diameters; and the lines from r to the intersections of the parallels of st with the diameter of circle NRPT (which cuts st at right angles) will indicate, on that same st , the centers of the parallels, as we said in Prop. 6, No. 6 concerning the parallels of the Horizon.

[Margin: To draw, in the Astrolabe, the parallels of the great circle that has been drawn.]

4. Finally, the Verticals of this same circle NRPT, taken as a Horizon, will be drawn in no other way than the Verticals of the Horizon, of which we spoke in Prop. 8. For the primary [Vertical] will be $q\lambda r$, whose center q is found on the line st , if an arc $\mu\varpi$ is made equal to the arc $r\mu$ and the line $r\varpi$ is drawn; or if [an arc] $a\varpi$ is taken equal to the arc $q\varpi$, as was demonstrated in Prop. 5, No. 4. The centers of the other Verticals will be found on the line through q perpendicular to st , just as we prescribed in Prop. 8.

[Margin: To draw the Verticals of the same great circle that has been drawn, taken as a Horizon of some sort.]

This proposition, moreover, has an outstanding use beyond the others in gnomonics. For by it are found the altitudes of the Sun, the "latitudes" of shadows (that is, the horizontal circumferences), and the hour-arcs on a given great circle, for each hour, in any region, wherever the Sun may be in the Zodiac — provided that first, from Prop. 23 of Book 1 of our *Gnomonics*, the declination from the Vertical and the inclination to the Horizon of that plane on which the dial is to be drawn are found; and that a great circle is drawn in the Astrolabe by means of this proposition, representing the great circle in the sphere to which the plane of the dial is parallel; and finally that the parallels and Verticals of that same circle are drawn, as we have said here. But these matters will become clearer in Book 3, Canons 16 and 21.

[Margin: The usefulness of this proposition.]

Diagram. The same stereographic-projection woodcut as on p. 589 (see fig-p589-1), printed again on the facing page so that it can be consulted while reading either leaf. Same labels: I, F, N, Y, A, e on the periphery; M, C, R, B, K, P, S, T, G, X, D, L in the interior, with Greek and small-letter node labels (ψ , θ , δ , ω/ϖ , "ii", "aa", and lowercase b, o, q, a, h). [Many small labels [?]]

SCHOLIUM.

1. Since it has been stated — both in this proposition (No. 3), and in Prop. 8 (No. 16), and in the scholium of Prop. 5 (No. 6) — that all great circles in the Astrolabe bisect the Equator, it has seemed good to demonstrate this very thing in another way, and geometrically, by setting out the following theorem.

[Margin: If, in a circle that bisects a given circle, a straight line is fitted passing through the center of the given circle, then all circles drawn through the extremities of that line will likewise bisect the given circle.]

If another circle cuts a given circle into two equal parts — that is, in diametrically opposite points — and in this [second circle] a straight line is fitted in any way, passing through the center of the given circle, then all circles drawn through the extreme points of this line will likewise bisect the given circle.

Let a given circle ABCD, with center E, be cut into two equal parts, at A and C, by a circle AFCG whose center is Q; and let a straight line HI be fitted in any way, passing through the center E, lying in circle AFCG but not passing through its center Q; and through H and I let circles HLIM, HOPV be drawn as you please. I say that these bisect the given circle ABCD, in the points L, M and O, P.

For let the center K of the first circle HLIM lie, first, on the line HI; and through the center E let a perpendicular LM be raised to HI, cutting the given circle at the points L, M, through which I say the circle HLIM passes. For, the diameter AC of the given circle being joined (since the given circle is assumed to be bisected at A, C by circle AFCG), then, because the lines HI and AC cut one another within circle AFCG at E, [Margin a: Euclid III.35.] the rectangle under HE, EI will be equal to the rectangle under AE, EC — that is, to the square of the line AE, or of the line LE. Since, then, LE is perpendicular to HI, the semicircle HLI will pass through L (by Lemma 15), and for the same reason the semicircle HMI will pass through M. Therefore the circle HLIM cuts the given circle at the points L, M, opposite along the diameter LM, and hence bisects it — which was to be proved.

Next, let N, the center of the latter circle HOPV, lie outside the fitted line HI; and let its diameter VX be drawn through E, the center of the given circle, and to VX let there be drawn OP, the perpendicular diameter of that same given circle. I say that the circle HOPV passes through the points O, P. For, since the lines HI and AC cut one another within circle AFCG at E, [Margin b: Euclid III.35.] the rectangle under HE, EI will be equal to the rectangle under AE, EC — that is, to the square

of the line AE, or of OE. But equal to the rectangle under HE, EI is the rectangle under VE, EX, [Margin c: Euclid III.35.] since the lines HI and VX also cut one another at E within the circle HOPV drawn through H, I. Therefore the square of the line OE will also be equal to the rectangle under VE, EX. Since, then, OE is perpendicular to VX, the semicircle VOX will pass through O (by Lemma 16), and for the same reason the semicircle VPX through P. Therefore the circle HOPV cuts the given circle at the points O, P, opposite along the diameter OP, and hence bisects it — which was to be proved.

But if in the circle AFCG a line FG is fitted, passing through its center Q and through E, the center of the given circle, and through F, G a circle FaYS is drawn as you please, from center R, cutting the given circle at a, S, then I say again that the given circle is bisected at a, S. For, when the diameter TY of the circle drawn has been passed through E, the center of the given circle, and the perpendicular diameter aS of the given circle has been raised to it, we shall demonstrate in the same way that the circle FaYS passes through a, S. For, since the lines FG and AC cut one another within circle AFCG at E, [Margin d: Euclid III.35.] the rectangle under FE, EG will be equal to the rectangle under AE, EC — that is, to the square of the line AE, or of aE. But equal to the rectangle under FE, EG is the rectangle under TE, EY, [Margin e: Euclid III.35.] since the lines FG and TY also cut one another at E within the circle FaYS drawn through F, G. Therefore the square of the line aE will also be equal to the rectangle under TE, EY. Since, then, aE is perpendicular to TY, the semicircle TaY will pass through a (by Lemma 16), and for the same reason the semicircle TSY through S. Therefore the circle FaYS cuts the given circle at the points a, S, opposite along the diameter aS, and therefore bisects it — which was to be proved.

Diagram. The geometric figure of the scholium, enclosed in a ruled square frame in the lower-left of the page. It shows a given circle ABCD (the Equator) with center E; a circle AFCG (center Q) bisecting it at A and C; and the several secant circles of the proof — HLIM, HOPV, and FaYS — with their diameters (LM, OP, VX, TY, aS, HI, FG) drawn through the common intersection E, in solid and dotted lines. Readable labels, by approximate position:

- F (top), H (left), O and B (upper center), L (upper right), with small a and d beside L
- R, K (left of center), C (left), N (lower left of center), E (center)
- X, A (right), Y, G, I (lower right)
- V (left), M, S, D, P (along the bottom), T (upper right), Q (interior center of AFCG)

- A few small interior nodes are unresolved under magnification.

2. And since all great circles are drawn through some two diametrically opposite points, while the straight line connecting two such points is the diameter of some oblique great circle that bisects the Equator — for just as the Horizon, the Vertical, and the Ecliptic bisect the Equator, because the extreme points seen on the diameter of each of them represent two diametrically opposite points on the sphere (as we showed in the scholium of Prop. 5, Nos. 1 and 3), so too a circle described from the midpoint of any straight line joining two diametrically opposite points bisects the same Equator (as was demonstrated in the same scholium, No. 3) — it follows from the theorem of this scholium that all great circles in the Astrolabe, since they are drawn through two such diametrically opposite points, bisect the Equator, exactly as happens in the heavens. From this it follows that all Verticals, circles of position, hour-circles, and great circles drawn through the poles of the Ecliptic cut the Equator in diametrically opposite points. This too was shown above in its proper places.

[Margin: That all great circles in the Astrolabe bisect the Equator.]

PROBL. X. PROPOSITIO XIII.

To draw a great circle through two given points in the Astrolabe, or through one alone.

[Margin: To draw a great circle through two points given in any way in the Astrolabe.]

1. Theodosius demonstrates this same thing, as regards the two points, in Book 1, Prop. 20. And this proposition differs from the preceding one in that in this thirteenth [proposition] neither the situation and position of the circle to be drawn, nor two points on two great circles, are given (as in that twelfth one), but only two points are assigned in any way whatever. Let it be conceived, then, in the figure of the preceding scholium, that the Equator of the Astrolabe is ABCD, and that the given points are F, d, through which a great circle is to be drawn.

Having found the point G diametrically opposite to one of them — namely to F itself — by the means we demonstrated in Prop. 6, No. 13 [Margin f: Euclid III.31.] (which will indeed be done if, to the line drawn from F through the center E, a perpendicular EA is erected at the center E; and to the joined line AF a perpendicular AG is raised — which will be drawn without any trouble if, equal to the arc Be that the line AF cuts off on the Equator, an opposite arc Db is taken, and the line Ab is joined, making in the semicircle eAb a right angle at A; or if,

having drawn on the Equator the diameter AC perpendicular to FD, a circle is drawn about the three points A, F, C, with its center Q on FD — for this will cut off the point G opposite to point F) — let a circle FdG be drawn through the three points F, d, G, with its center R lying on the line QR that is perpendicular to the line FG at its midpoint Q. For this circle will be a great circle, since it passes through the opposite points F, G; and it will bisect the Equator at a, S, as we showed in the scholium of the preceding proposition.

[Margin: To draw a great circle through two points, one of which is given on the circumference of the Equator.]

2. When one of the given points is on the circumference of the Equator, the problem will be completed if another, opposite point is taken on the Equator, and through three points — two of which are opposite on the Equator — the third ...

so that a circle may be drawn through the third given point as well. For instance, if the two given points are F and a, then, having drawn the diameter aS of the Equator, we shall describe through the three points F, a, S the circle FaS.

[Margin: To describe a great circle through two points that lie on one straight line drawn through the center of the astrolabe.]

3. But if two given points lie on a straight line with E, the center of the Equator — as, for example, if the given points are F and B, or F and G — then the straight line FB (or FG) itself, extended to infinity, will represent a great circle drawn through the poles of the world, as is clear from Proposition 1. Nor can any other great circle be drawn through those two points, unless they are diametrically opposite, as F and G are. For in that case not only does the straight line FG, extended to infinity, represent a great circle drawn through those points, but through the same points infinitely many other great circles can also be drawn — such as FAGC and FYGT — the centers of all of which will lie on the straight line QR, which bisects FG at right angles, as is clear from the corollary to Euclid III.1.

[Margin: To describe a great circle through two given points on the circumference of the Equator.]

4. Again, if the given points lie on the circumference of the Equator, such as B and L, then the Equator itself will be the great circle drawn through them, and no other circle can be drawn through those same points, except when they are diametrically opposite. Thus if the given points are O and P, there can be drawn through O and P, besides the Equator, infinitely many other

great circles, such as OHVP — just as we said a little earlier concerning opposite points outside the circumference of the Equator. The centers of all of these will lie on the straight line EN, perpendicular to OP, as is clear from the corollary to Euclid III.1.

[Margin: To describe any number of great circles through any given point on the astrolabe.]

5. Now, if a circle is to be drawn through a single given point, it will be done more quickly than one can say, provided the circle is drawn through the given point together with any two other points on the Equator that are diametrically opposite. From this it follows that through any given point infinitely many great circles can be drawn, since two opposite points on the Equator can be chosen in infinitely many ways.

Diagram. A large square-framed stereographic projection of the sphere as laid down on the astrolabe. The Equator is the principal circle, with its cardinal points A (right), B (upper), C (left), D (lower) and center E; within and around it a dense web of great circles is drawn through the various point-pairs discussed in the text (e.g. FAGC, FYGT, OHVP, HOP, HLM, HAC, HCIF, HMIL, HVIO), with their centers marked on the perpendicular bisectors Q–R and N–Q.

- Readable labels: A (right of circle), B (top), C (left), D (bottom), E (center), F (upper edge), G (lower-right), H (upper-center), I (lower-right), K (center, an intersection), L (center-right), M (bottom), N (center-left), O (upper), P (bottom-right), Q (center-right), R (upper-center-left), S (bottom), T (left), V (lower-left), X (right), Y (lower-right).
- The figure is extremely crowded; several small interior letters could not be resolved with certainty. Marked as a

Thus you see that through the point H three great circles HOP, HLM, HAC have been drawn, since the pairs O and P, L and M, and A and C are all diametrically opposite on the Equator.

[Margin: To describe any number of great circles through two diametrically opposite points.]

6. Finally, if two diametrically opposite points are given, infinitely many great circles can be drawn through them, the centers of all of which lie on the straight line that bisects, at right angles, the straight line joining those points. Thus in the same figure, through the diametrically opposite points H and I, three great circles HCIF, HMIL, HVIO have been drawn, whose

centers lie on the straight line NQ, which cuts the straight line HI at right angles in K, as is clear from the corollary to Euclid III.1. And in this way infinitely many other great circles can be drawn through the same points, by taking other centers on the straight line NQ. This too we asserted in passing a little earlier, toward the end of nos. 3 and 4.

PROBLEM XI. PROPOSITION XIV.

Given two points on the astrolabe a quadrant of a great circle apart, to describe through one of them a great circle whose other point is the pole; likewise, given any point, to describe a great circle whose pole is that given point; and moreover to describe a lesser circle whose distance from that pole is given.

[Margin: Given two points a quadrant of a great circle apart, to describe through one of them a great circle whose other point is the pole.]

1. On an astrolabe whose Equator is ABCD about center E, and on which the two diameters AC and BD cut one another at right angles — AC representing the right horizon, BD the meridian — let there first be given two points F and G, one of which is distant from the other by a quadrant of a great circle; and let the great circle whose pole is G be the one to be drawn through F. Draw the straight line GE through G, the pole of the circle to be drawn, and through E, the center of the astrolabe; let the diameter HI cut GE at right angles; and describe, from center K, the circle through the three points F, H, I. (By the corollary to Euclid III.1, K necessarily lies on GE, because of the right angles at center E.) This circle FHI cuts the straight line GE at L; and, by what we demonstrated in the scholium to Proposition 5, §9, it is a great circle, since it bisects the Equator at H and I. I say that its pole can be a quadrant of a great circle distant [from F], and hence that it can be the pole of some great circle drawn through F, as was posited. For since the great circle represented by the straight line KL passes through G — the pole of some great circle drawn through F — that great circle drawn through F, whose pole is G, will in turn pass through the poles of the great circle which the line KL represents, by the scholium to Theodosius I.15. Since, then, H is a pole of the circle KL — being equally distant from it and separated by the quadrants HI, Hi — FHLI will be that great circle drawn through F whose pole is G. For no other great circle drawn through F and different from the circle FHI passes through H and I, the poles of the circle KL; and this, we said, is necessary, by the scholium to Theodosius I.15, if G is the pole of some circle drawn through F.

And so that you may see how neatly these results agree with what has been demonstrated, from H, the pole of circle KL, draw the radii HM, HN. For if G is the pole of the circle FHI, GL must be a quadrant of a circle — that is, the arc of the Equator MN, to which GL corresponds, must be a quadrant. Likewise, if through the points F and G a great circle FGO is drawn by the preceding proposition (which is done as follows: find the point O opposite to point G, either by the circle GHOI drawn from center Q through the three points H, G, I, or by the right angle MHO, once the straight line HM drawn to H has been established — which will be constructed more quickly than said, if the diameter MP is drawn and the straight line HP is projected, cutting GL at O; then from center R let the circle be described through the three points F, G, O) — it is necessary that the arc FG be a quadrant, which you may test thus. If, through E, the center of the astrolabe, and R, the center of the circle FGO, the straight line ER is drawn, cutting the circle FHI at S, then S will be the pole of the circle FGO. For since FGO is posited to pass through G, the pole of the circle FHI, then, by the scholium to Theodosius I.15, the circle FHI will in turn pass through the poles of the circle FGO. Since, therefore, the pole of this latter lies on the straight line ER, as was shown in Proposition 8, §19, S will be its pole. Consequently, if FG is a quadrant, the radii SG, SF must cut off from the Equator the quadrant TV.

[Margin: To describe a great circle whose pole is a given point on the astrolabe.]

2. It is not, however, necessary that the circle drawn through the given point F should enclose the other given point which must be the pole — that is, that the pole be contained within the circle described, of which it is the pole — since on the astrolabe one pole always lies inside the circle of which it is the pole, and the other outside, as is evident in the horizon and its parallels. For if the other given point is O, then, drawing the straight line OE and raising the perpendicular HI to it, the circle FHI will be the great circle whose pole is O, even though it does not enclose O. For since the great circle represented by the straight line OE passes through O, the pole of some great circle drawn through F, then by hypothesis, by the scholium to Theodosius I.15, that great circle drawn through F, whose pole is O, will in turn pass through the poles of the great circle OE — that is, through H and I. The circle FHI is therefore that great circle whose pole is O; for no other circle drawn through F passes through H and I, the poles of the circle OE.

Here too you see that the radii SF, SO, projected from S, the pole of the circle FGO, cut off from the Equator the quadrant VX; and hence that the arc OYF of the circle

FGO represents a quadrant, as the hypothesis requires. For O is posited to be distant from F by a quadrant of the great circle drawn through those points. The remaining arc OGF, moreover, contains three quadrants, just as does the arc XIV of the Equator, to which it corresponds.

3. Suppose next that any point G is given, and that a great circle is to be drawn whose pole is the given point G. Having drawn the straight line GE through the given point and the center of the astrolabe, we shall raise to it the perpendicular HI. Then from H, the pole of the great circle GE, draw the straight line HG cutting the Equator in M, and take the quadrant MN, whether toward the right or toward the left. (In the given example it would be inconvenient to take the quadrant Mk toward the left, because the straight line Hk would cut the line EG too far off.) Then draw the straight line HN, which cuts GE in L. For the circle drawn through the three points H, L, I will be a great circle, since it bisects the Equator; and its pole will be G, since G is distant from it by the quadrant GL of a great circle.

[Margin: To describe a lesser circle whose pole is a given point on the astrolabe.]

By like reasoning, if the given point is O, the pole of the great circle to be drawn, we shall likewise draw the straight line OE and raise to it the perpendicular HI. Then from H, the pole of the great circle OE, draw the straight line HO cutting the Equator in P, take the quadrant PN, and project the straight line HN, cutting OE in L. For again the circle drawn through the three points H, L, I will be a great circle, and its pole will be O, since O is distant from it by the quadrant OL of a great circle.

The center of the great circle to be drawn will be found from what we demonstrated in Proposition 5, §3, as follows. Draw the straight line from H through the pole G (or O), cutting the Equator in M (or P); then, taking the two quadrants MN, Mk (or PN, Pk), the radii HN, Hk will give, on the straight line KO, the apparent diameter of the great circle — for the straight line drawn kN [gives its true diameter, since its pole is M] [?]. But if, at an arc equal to Hk, a length is cut off from point k toward N [?], the straight line drawn from H through the end of the arc so taken falls in K, the center of the circle, bisecting in K the diameter that has been cut off. And so, even though the whole diameter cannot conveniently be had — because sometimes one of the radii, such as Hk here, runs out too far — the great circle can nevertheless be drawn from the center found by way of the other end of the diameter, such as point L here.

[Margin: To describe a lesser circle whose pole is a given point on the astrolabe.]

4. Finally, let a lesser circle be drawn whose pole is G, and from which its circumference recedes by any given number of degrees. Having drawn the straight line through G and center E, which HI cuts at right angles, we shall draw from H, through G, the straight line HG meeting the Equator in M; and M will be the pole of the circle to be drawn, since the radius HM exhibits its pole G on the astrolabe, and ME will be the axis of that same circle. If, then, we count off from M on both sides the proposed degrees, so as to have the ends of the true diameter of the circle to be drawn, and if from H radii are projected through those ends, there will be cut off from GE the diameter of the circle to be drawn, which, bisected, gives the circle. But if at any time the whole diameter cannot conveniently be had — as when one of its ends lies too far from G — the center of the circle to be drawn must be found by what we demonstrated in Proposition 6, §9, in this way. Having counted off the proposed degrees from M on both sides, join the extreme points by a straight line, which (as we said) will be the true diameter of the circle to be drawn, and mark the point where that diameter intersects the axis ME. For if, from H, a straight line is projected through this point, and a length equal to the arc intercepted between M and that straight line is cut off on the other side, the straight line drawn from H through the extreme point of the arc so cut off will fall in the center, and so on.

We shall proceed in the same way if the point O is posited as pole. For, drawing the straight line HO cutting the Equator in P, PE will be the axis of the circle to be drawn, and so on. We do not set out an example of a lesser circle to be drawn, lest the figure be thrown into confusion by so great a multitude of lines.

PROBLEM XII. PROPOSITION XV.

To find the magnitude of a spherical angle contained by any two great circles on the astrolabe, or (what is the same) the inclination of two great circles on the astrolabe.

[Margin: To investigate the magnitude of a spherical angle standing on the circumference of the Equator, or the inclination of two great circles — one of which is the Equator, or which both intersect one another on the circumference of the Equator.]

1. In the figure of the preceding proposition, let the great circle HOIG first cut the Equator ABCD in the opposite points H, I; or let two great circles HGI, HLI cut one another on the circumference of the Equator in those same

points H, I. Let it be proposed to determine the magnitude of the angle OHA (or OIA) — that is, the inclination of the great circle HOI to the Equator, and so on. Having drawn the diameter HI of the Equator, let another diameter li, extended as far as one pleases, cut it at right angles, cutting the Equator and the given circle in i and O; and let the straight lines HO, Hi be joined, cutting the Equator in P and i.

I say that the arc Pi measures the angle OHI, that is, the inclination of the great circle HOI to the Equator. For since li bisects the straight line HI in E at right angles, it will pass through the center of the circle HOI, by the corollary to Euclid III.1; and therefore also through the poles of that same circle, as we showed in Proposition 8, §19. Since, then, by Proposition 1, it represents a great circle drawn through the poles of the world, the arcs Hi, HO will be quadrants, by the corollary to Theodosius I.16; and hence the arc iO will be the arc of the angle OHi, or of the inclination of the circles. Wherefore, since by Proposition 1, §5, the segment Oi is equal to the arc Pi as regards the number of degrees, Pi too will be the arc of the angle OHi, or of the inclination of the circle HOI to the Equator. In the same way, the arc of the angle GHi (which is the supplement of the angle OHi to two right angles) is the segment Gi, to which the arc Mi corresponds. Likewise Li (or Ni) is the arc of the angle LHi; and Ll (or Nl) the arc of the angle LHL. Finally GL (or MN) is the arc of the angle GHL, which the two great circles GHI, HLI form, cutting one another on the circumference of the Equator. From which it follows that the magnitude of that angle is to be investigated in the same way.

[Margin: To investigate the magnitude of a spherical angle standing outside the periphery of the Equator, or the inclination of two great circles that cut one another outside the Equator's periphery.]

2. Next, let two great circles FGZ, FHZ cut one another in the opposite points F, Z, outside the periphery of the Equator, forming the angle GFH, which is to be investigated. Having drawn their diameter FZ through E, the center of the astrolabe (but if the circles cut one another only in F, they must be extended until they cut one another in Z; or else the straight line FE must be extended, and the point Z, opposite to point F, found, as was taught in Proposition 6, §13), let some straight line cut it at a in half and at right angles — such as the straight line KR, passing through K, R, the centers of the circles. Hence it suffices to draw the straight line KR through the centers of those circles, even if their common section FZ has not been drawn; and this will be most convenient when one of the points of intersection lies far off. Indeed, if one of the centers is too far from the

straight line EF, it suffices to drop from the nearer one, R, a perpendicular Ra to EF. For this will bisect the straight line FZ, if it were drawn, and so on. Then, from any point m on the straight line FZ — whether it be the same as the midpoint a or not — describe through F the circle Ffe; or from point F, at any interval, the circle gh. Lastly, through the points b, d, where the given great circles cut the straight line KR, let straight lines go out from F cutting the circle Ffe in f, e, or the circle gh in g, h. I say that the arc ef is the arc of the angle GFH — that is, of the inclination of the circles — and that the arc gh is half of that same arc.

Diagram. A large square-framed stereographic figure for this proposition, showing two great circles FGZ and FHZ (and FLZ) cutting one another in the opposite points F and Z outside the Equator, with their centers K, R on the bisecting line, the small measuring-circles Ffe and gh about F, and the various arcs (ef, gh, bd, Pi, etc.) by which the spherical angle GFH is measured.

- Readable labels: F (upper-left, on the outer circle), Z (lower-right), H, E, R (upper-center), P (right), X (right), Y (upper-right), C (left), A (right), I (center-right), S (center), L (center) and l (bottom), e and n (center), K (far lower-left), T (lower-left), M (bottom), D (bottom-center), N (lower-right), G (lower area), b and d (interior).
- The figure is very densely ruled; several small interior letters (including some lower-case measuring-points) could not be resolved with certainty. Marked as a

For if the opposite points F, Z are posited as the poles of some oblique horizon, the circles FGZ, FLZ will be two verticals, of which the principal one is to be drawn from center a through F, Z; while the straight line KR will represent the parallel of that horizon through the pole of the world, at which the eye is placed, as we showed in Proposition 8, §2. Therefore, as was proved in the same proposition, §11, the segment bd of the straight line KR corresponds to as many degrees of its parallel as are contained in twice the arc ef (or twice the arc gh). Since, then, the arc of that same parallel between the circles FGZ, FLZ [note a: Theodosius II.10] is similar to the arc of the angle GFH, it is clear that the arc ef too is the arc of that same angle. And since, in the preceding proposition, the circle FHZ was described about the pole G, the circle FGZ will pass through its poles; [note b: Theodosius I.15] and hence the angle GFH will be a right angle. It is necessary, therefore, that its arc ef be a quadrant of the circle Ffe, and the arc gh be half a quadrant of the circle gh.

Moreover, even if a circle is drawn through the point F in any manner whatever — although its center does not lie on the straight line FZ — such as, for instance, either of the two arcs containing the given angle GFH, namely FG, cutting the two straight lines Fb, Fd in b, p: its arc bp will measure the proposed angle GFH, since by Lemma 10 it is similar to the arc ef; and hg, half of that arc, which is similar to the arc bp, and so on.

[Margin: To investigate the same, when one of the circles is drawn through the poles of the world.]

3. But if one of the circles constituting the spherical angle passes through the center of the astrolabe — that is, if it represents a great circle drawn through the poles of the world — then only a single straight line must be drawn from the angle. Thus, if the spherical angle is contained by the great circle FEZ, represented by a straight line, and the great circle FGZ, then en will be the arc of that angle, and hm its half. In the same way, the arc of the angle EHL will be IN, and so with the rest.

[Margin, note c: Theodosius III.3. — An easy way to find the magnitude of a spherical angle, neither of whose arcs passes through the center of the astrolabe.]

Indeed, even if no straight line is drawn from the angle, and neither the circle Fen nor hm is described, the arc bZ will nonetheless measure the angle bFZ, and the arc LI the angle EHL; because, by Lemma 10, both the arcs bZ, en and the arcs LI, NI are similar, and so on. From which it comes about that, since the arcs FbZ, HLI are bisected by the perpendiculars ab, EL, the arcs Fb, HL also measure the same angles — so that one point of intersection need not [be had].

This method can also be adapted to any angle, even though neither of the circles passes through the center of the astrolabe. For let there be given the angle bZd, such that the point of intersection F can scarcely be had. Having drawn the straight line ZE through the center of the astrolabe, draw to it, from R, the center of the circle bZ — which is the nearer — a perpendicular cutting both circles in b, d. Since, then, the arc bZ measures the angle bZa, and the arc dZ the angle dZa, if to the arc bZ there be added an arc similar to the arc dZ, the arc of the whole angle bZd will be composed. And the same will be obtained if to the arc dZ there be added an arc similar to the arc bZ. Again, let there be given the angle hLK, in the figure of the following proposition. Having drawn the straight line LE through the

the center of the astrolabe, let a perpendicular be drawn to it from the center of one or the other circle, cutting both circles at h and K. Once this is done, the arc

Lh will measure the angle hLN, and the arc LK will measure the angle KLN. If, therefore, from the arc Lh an arc similar to the arc LK is subtracted, the remainder becomes the arc of the angle hLK.

4. We shall solve this same problem if, by the preceding proposition, a great circle is described about the given angle as pole. For the arc of this circle intercepted between the circumferences that contain the given angle will measure the angle itself; and the straight lines drawn from the angle through the extreme points of this arc will cut off from the Equator an arc equal to it as far as the number of degrees is concerned, as we demonstrated in proposition 5, Number 17. That arc of the Equator will therefore indicate the size of the given angle. Thus you see in the figure that, with the point H taken as pole of the angle iHO, the great circle KO has been described — represented by the straight line KO — and that the arc iO, intercepted between the circumferences Hi, HO which contain the angle, measures the said angle; and the magnitude of this arc is exhibited by the arc of the Equator Pi, cut off from the straight lines Hi, HO by the extremities of the arc iO drawn out. The same reasoning holds for the others.

[Margin: Another solution of the problem.]

SCHOLIUM

1. It should be observed here in passing that, when several great circles passing through the same opposite points are inclined to some other great circle, then — with one exception, namely the circle that stands at right angles to it — the circle that stands at right angles to *this* one is the most inclined to that other circle; of the rest, those nearer to the most inclined one are more inclined than those farther off; and finally, two circles equally distant on either side from the one that stands upright are equally inclined. I say, moreover, that a circle is more inclined to another when it makes the smaller acute angle with it.

For let E be the pole of the great circle ABCD, and let there be drawn through it any number of great circles AEC, EF, EG, EH, EI, inclined to some great circle AHC, with the exception of EH, which stands at right angles to it; and let AEC in turn also stand at right angles to EH. I say that AEC is the most inclined to AHC, and that EF is more inclined than EG; and finally that EF and EI, equally distant on either side from AEC (the most inclined) at the points A, C, are equally inclined.

Since E is the pole of the circle ABCD, then by the corollary of proposition 16, Book 1, of Theodosius, the arcs EA, EK, EL, ED, EM, EC are quadrants, and therefore EF, EG, EH, EI are less than a quadrant.

Therefore the arcs EA, EF, as well as EF, EG, and EG, EH, are each less than a semicircle, since no two of them are equal to two quadrants. Hence, by proposition 14 of our *Spherical Triangles*, the exterior angle EHC, being a right angle, will be greater than the interior opposite angle EGH; and this greater than the interior opposite angle EFG; and this greater than the interior opposite angle EAF. Therefore EGH is acute, EFG *a fortiori* more acute, and EAF much more acute still. Hence the circle EA is the most inclined to AHC, and EF more than EG.

Next, since the two sides AE, AF are equal to the two sides CE, CI (for EA, EC are quadrants, and the arcs AF, CI are equal, because the circles EF, EI are supposed to lie equally distant in the circle AHC from the points A, C), and they contain equal angles A, C by proposition 13 of our *Spherical Triangles*, then by proposition 7 of the same *Triangles* the angles AFE, CIE will also be equal; and consequently, being the remainders from two right angles, EFH, EIH will be equal — and these are precisely the angles of inclination. Therefore EF, EI are equally inclined to AHC, which was what was proposed.

[Margin: When several great circles are drawn through the same opposite points, which of them is more or less inclined to another great circle, and which are equally inclined.]

And because all the Verticals are inclined to the Equator except the Meridian — to which the prime Vertical stands at right angles — it follows that the prime Vertical is the most inclined to the Equator, and that the others are inclined the more, the less they depart from the prime Vertical. Likewise, since all the circles of position are inclined to the Equator except the Meridian — to which the Horizon stands at right angles — it is inferred that the Horizon is the most inclined to the Equator, and that the other circles of position are inclined the more, the less they are distant from the Horizon.

Diagram. A circle representing the great circle ABCD, projected onto the plane of the astrolabe, with center E, and a family of great circles drawn through the poles A and C.

- A (top of the circle): pole
- C (bottom of the circle): pole
- E (center): common pole E of the circle ABCD
- B (left extremity)

- D (right extremity)
- K (upper right arc)
- L (right arc, below K)
- M (lower right arc)
- F (upper middle, on an inclined circle)
- G (middle, on an inclined circle)
- H (lower middle, on the reference circle AHC)
- I (lower middle, on an inclined circle)

The chords from E to F, G, H, I represent the inclined great circles EF, EG, EH, EI meeting the great circle AHC.

[Margin: The prime Vertical is the most inclined of all the Verticals, and the Horizon of all the circles of position, to the Equator.]

2. Now a most beautiful and easy way is opened to us by this proposition 15: by it, through the inclination to the Horizon given in proposition 12, Number 2, the third point may be found through which the proposed great circle is to be described. We shall proceed as follows. Since the circle there proposed declines from south toward west, and since in this way the two points N, P (in which the circle must cut the Horizon) have been found in the figure of proposition 12, and since it has an inclination to the Horizon on the southern side of 26 degrees — from which the point k was found, whether by the Vertical XHY or by the parallel of the Horizon [label: $\beta\epsilon\delta$?] — we shall now find, without these circles, from the same inclination, a third further point, in this manner. In the figure of proposition 12, let a perpendicular cc aa be drawn through cc, the midpoint of the straight line NP; it will pass entirely through K, the center of the Horizon, by the corollary of proposition 1, Book 3, of Euclid, since it cuts the straight line NP in the Horizon into two equal parts and at right angles. Then, with a circle also described from N at any interval of the arc of the circle ee ii, let a straight line be drawn from N to aa [and] from the Horizon cutting the described arc at ee. And from ee, toward the center of the Horizon, let there be cut off an arc ee ii containing half the inclination, that is, 13 degrees. Or, if minutes cling to the inclination, let the arc of the whole inclination be taken, and its half [gives] the arc ee ii. For the straight line drawn N ii will cut the straight line cc aa at the point bb, through which the proposed great circle is to be described. For once a circle has been described through the three points N, bb, P, the angle bb Naa will contain

26 degrees of the given inclination, as was demonstrated in this proposition, Number 2.

[Margin: A most elegant procedure, belonging to proposition 12, for finding the third point of the great circle to be described from its given inclination to the Horizon, without a Vertical and without a parallel of the Horizon.]

PROBL. XIII. PROPOSITIO XVI

At a given arc of a great circle in the astrolabe, and at a given point in it, to construct an angle equal to the given angle of any two great circles described in the astrolabe, or whose arc is given in degrees; or (what amounts to the same) to describe, through the given point, a great circle in which the given point lies, having an inclination equal to the inclination of any two great circles in the astrolabe. Likewise, to bisect a given angle of two great circles.

[Margin: Given a spherical angle in the astrolabe, to construct at a given point an equal spherical angle together with a given arc.]

1. We demonstrated the first part of this proposition in proposition 12 of the *Spherical Triangles*. Let there be, then, in the astrolabe the Equator ABCD about the center E, and let there be given the spherical angle EFG, contained by the great circle FEH drawn through the poles of the world and by another great circle FGH, to which an equal angle is to be constructed at the arc IKL, at the point I. Let the diameters FH, IL be drawn through the center E, so that the opposite points are F, H and I, L; let these be bisected at M, N, and to them let the perpendiculars GM, KN be drawn, which pass through the centers of all circles passing through the points F, H and I, L, by the corollary of proposition 1, Book 3, of Euclid. Let there be described through F, I — from centers taken on the straight lines FH, IL — any equal circles FQOP, ITRS, or, from the centers F, I, equal circles of any size XY, ab. Also, drawing from F, I through the points G, M, K (where the perpendiculars are intersected by the arcs) straight lines cutting the circles FQOP, ITRS at Q, O, d, and the circles XY, ab at x, V, e; then QO will be the arc of the given angle EFG, and VX half the arc of the same angle, as we showed in the preceding problem. If, therefore, an arc dT equal to the arc OQ is taken (should the angle be constructed on the left of the given arc IK), or an arc df (if on the right), or an arc eb equal to the arc VX, or eg; and if the straight line IT, or Ib, or If, or Ig be drawn, cutting KN at h or i: then both the arc described through the three points I, h, L will make the angle hIK, and the arc described through the three points I, i, L the angle iIK,

equal to the given angle EFG. That is, the inclination of the arcs IhL, IiL to the arc IKL will be equal to the inclination of the arc FGH to the circle FEH, because of the equality of the arcs OQ, dT, df, and so on.

By the same reasoning we shall construct, at the great circle IEL, at the point I, an angle NIK equal to the angle EFn, if — having drawn the straight line Fn cutting the circle described through F at P, and the circle described from F at Y — we take Rd equal to the arc OP, or Ze equal to the arc VY, and draw the straight line Ied cutting KN at K. For the circle described through the three points I, K, L will make with the circle IEL an angle equal to the angle EFn, as is evident.

[Margin: Given a spherical angle in degrees, to construct one equal to it at a given point together with a given arc of a great circle.]

If the magnitude of some angle is given in any number of degrees, we shall construct such an angle at the arc IKL, at the point I, by numbering from d the proposed degrees up to T, or F; or by taking half the arc of the proposed degrees, eb, or eg. Likewise, if we take a quadrant dS, or half a quadrant ea, and draw through S, or a, a straight line cutting KN at k, the arc IkL will make with IK the right angle KIk.

Diagram. A dense astrolabe/stereographic-projection construction enclosed in a square frame, showing the Equator and several great and lesser circles with their poles, together with the auxiliary equal circles used in the angle-construction of this proposition. The figure carries a large number of small point-labels, several of which do not resolve at the available resolution; it is marked a

- E (center): center of the astrolabe / Equator ABCD
- A (right), B, C, D: extremities of the Equator ABCD
- I, L: opposite points of one diameter (arc IKL)
- F, H: opposite points of the other diameter
- K, N: bisection points and perpendicular feet
- T, S, R: on the auxiliary circle ITRS
- P, Q, O: on the auxiliary circle FQOP
- Greek/lower-case labels visible near the perimeter: h, d, δ, Z, a, f, n, m, Y, X
[several labels [?] — see QA]

We shall construct a given angle no differently at a given point of the Equator. For instance, if an angle is to be constructed at D with the great circle DEB of 70 degrees, or with DCB of 20 degrees, we shall number the arc Bl of 70 degrees, or

the arc Cl of 20 degrees, and draw the straight line Dl cutting AC at m. For the circle DmB will complete what was proposed.

[Margin: When two great circles in the astrolabe contain a right angle, a straight line drawn from the center of the astrolabe through the center of one cuts the other at the pole of that former circle. (a Theodosius 13.1)]

2. And because the two arcs IKL, IkL contain the right angle KIk, as has been said, ^a the one will pass through the pole of the other. Since, then, the pole of each great circle also lies on the straight line drawn through the center of the astrolabe and the center of that circle, as was said in proposition 8, Number 19: the straight line Eq, drawn out through q, the center of the circle IK, will cut the circle Ik at p, the pole of the circle IK; and the straight line Ef, drawn out through f, the center of the circle Ik, will cut the circle IK at r, the pole of the circle Ik. And by this same reasoning, when any two great circles in the astrolabe cut one another at right angles, the straight line connecting the center of the one with the center of the astrolabe will cut the other at the pole of that former circle. From which it follows that the pole of each circle may then easily be found, namely if straight lines are drawn from the center of the astrolabe through their centers; for these will cut the circles at the poles.

[Margin: To find the poles of two great circles that contain a right angle.]

3. Now, by a not dissimilar reasoning, we shall bisect the angle that two great circles in the astrolabe comprehend. Let the angle to be bisected be hIi. With IL drawn through the common intersection of the arcs Ih, Ii — passing through the center of the astrolabe, and itself bisected and at right angles at N by the straight line hk — let there be described from I any arc a b, or through I any circle ITS having its center on the common intersection IL, for example Z. Then, drawing the straight lines Ih, Ii cutting the described circles at b, g and T, f, let the arc g b, or f T, be bisected at e, or d, and let the straight line I e, or Id, be joined, cutting hk at K. For the circle described through the three points I, K, L (which will be a great circle, since it passes through the opposite points I, L) will bisect the given angle h Ii, as is clear from what has been demonstrated.

[Margin: To bisect a given angle of two great circles in the astrolabe.]

PROBL. XIV. PROPOS. XVII

To explore the position in the sphere of any circle described in the astrolabe, or of any straight line drawn in it.

This proposition contains nothing but a certain application to the various circles of the astrolabe of the things that have already been demonstrated, especially proposition 8, Numbers 16 and 17. Let there be, then, in the astrolabe the Equator ABCD, the Horizon of the given region AFCG whose center is H and whose true diameter is IK, and consequently let the altitude of the pole above it be the arc AI, or CK.

[Margin: To explore the position in the sphere of various circles described in any way in the astrolabe.]

Let there first be described the circle LMNO, from the center Λ [?: printed as a small symbol], whose position in the sphere is to be investigated. Through its center Λ , and E, the center of the astrolabe, let the straight line LEN be drawn, which the diameter of the Equator OM cuts at right angles, falling on the points O, M, where it is cut by the given circle. Then, sending out from O the radii OL, ON through the extreme points L, N of the apparent diameter, cutting the Equator at P, Q: the joined line PQ will be the true diameter of the proposed circle, as is established from what we showed in proposition 8, Number 16. And because it is a great circle, and its true diameter PQ passes through the center, its pole [will be raised above it, as] was said in the same proposition 8, Number 22. It happens, moreover, that the altitude of the pole OP is here equal to the altitude of the pole AI above the Horizon. From which it follows that this circle is one of the hour-circles reckoned from rising or from setting, since the altitude of the pole is the same above all such circles, as was handed down in proposition 9, Number 9. And since it cuts the Equator at O and M, we shall easily recognize to which hour it belongs, as we taught in the same proposition 9, Number 8. Again, because the same circle cuts the Meridian at R, we shall recognize how far the point R is distant from the Horizon, if we investigate how many degrees are contained in the segment FR, from the teaching of proposition 1, Number 6. Finally, if through the pole of the Horizon and through the pole of the same circle a Vertical were described, the arc of the inclination of the same circle to the Horizon would become known — a Vertical which, however, we did not describe, in order to avoid greater confusion in the figure. Indeed, by proposition 15 the same inclination can be investigated from the angle of inclination FTR. Likewise, by the same proposition, you will find the inclination of the same circle both to the Meridian, from the angle ERO, and to the Equator, from the angle NOV. For

example (that you may see how the matter is carried out by proposition 15), with YZ drawn perpendicular to the straight line TX from its midpoint Y, and with any arc b e described from T: if the straight lines TZ, Ta are sent out to the points of intersection of the straight line YZ with the circle Ta and with the Horizon, cutting the arc be at d, b, then bd will be half the inclination, and the arc b e, being double bd, will give the whole inclination of the circle to the Horizon, as is clearly established from what was demonstrated in proposition 15. The straight line NV, moreover, will show the arc of inclination of the same circle to the Equator — namely the arc of the Equator QV corresponding to the straight line NV — and so on. Thus the circle LMNO has been found to be a great circle, above which the pole is raised through the arc OP, and it cuts off from the Meridian above the Horizon, on the southern side, the arc FR; and finally its inclination to the Horizon on the side of west and south is measured by the arc b e, and so on.

Diagram. A large and dense stereographic-projection figure enclosed in a rectangular frame, showing the Equator ABCD, the Horizon, the great circle LMNO and the lesser circles of the proposition, with a small separate circle drawn below (labelled θ, v) connected by a line to the main figure. The figure carries very many small point-labels, most of them Latin and Greek letters at the perimeter and interior; a large number do not resolve at the available resolution. Marked a

- E (center): center of the astrolabe / Equator ABCD
- A, B, C, D: on the Equator
- L, M, N, O: the circle LMNO whose position is sought
- F, R, T, H, P, Q: perimeter and Meridian points
- v (top), z (upper right), V, A (right)
- Greek/lower-case interior labels visible: d, e, c, m, i, s, δ , ϵ , ρ , μ , ν , q [several labels [?]]
- θ, v : in the small inset circle at the foot of the figure

Astrolabe, Book II — Propositio XVII (Problema XIV), continued; and Problema XV / Propositio XVIII

(...continuing §4, from the catchword "modo" of the previous leaf:) ...cutting the equator in such a way that it is not a great circle. When the rays A r, A q are drawn, cutting the equator at γ [?] and ρ , the line $\gamma\rho$ [?] so drawn will be its true

diameter. Since this does not lie parallel to I K, the diameter of the horizon, it indicates that the circle does not represent a parallel of the horizon, but of that great circle whose true diameter $u \zeta$ — drawn through the centre E — is parallel to $\Upsilon \rho$ [?], and above which the pole is elevated by the arc A u, or C ζ . And indeed the position of this great circle, as it stands perpendicular to the meridian, will be recognized in the sphere if the circle itself is described in the line F D, once its apparent diameter has been found by means of the rays A u, A ζ , and so on.

5. Further, let there be presented a circle $\alpha\beta$, having its centre on the same straight line L N as the great circle L M N O, which M O cuts at right angles. When the rays O α , O β are drawn, cutting the equator at ϱ [?] and ς , the line $\varrho\varsigma$ [?] so drawn will be the true diameter of the circle, and it does not lie parallel to P Q, the true diameter of the circle L M N O. From this you will infer that the circle $\alpha\beta$ does not represent a parallel of the great circle L M N O, but of the one that has for its true diameter the line drawn through E parallel to $\varrho\varsigma$ [?], and so on.

Diagram. A large stereographic-projection figure enclosed in a ruled square frame, occupying the lower right of the page. It shows the equatorial disk of the astrolabe with an inscribed projected sphere: a great outer circle, several oblique great circles and their parallels, numerous radii from a projection point, and dotted construction lines. A small separate circle (the projected nadir-parallel) hangs below the main disk. The figure is very densely lettered and lightly foxed; many peripheral labels are only partly legible.

- Upper region: T, L, F, r, d, e, Z (across the top arc)
- Central cluster: R, ς , H, B, a, u, h, o, P
- Left/lower-left: γ , δ , c, m, ϵ , i [?], ς [?]
- Right of centre: A (large), V, D, e, μ
- Below the main disk (nadir-parallel circle): q, "2" [?], and at its foot θ , V
- Additional faint labels near the disk margin: [?] (several unresolved)

6. In addition, let a circle $q\theta$ [?] be described, lying wholly outside the equator and therefore not a great circle, whose centre lies farther out, on the same straight line as the centre of the horizon. When the rays A q [?], A θ are drawn, cutting the equator at V and μ , its true diameter will be the line V μ , parallel to I K, the true diameter of the horizon. Therefore the circle $q\theta$ [?] represents a parallel of the horizon, described below the horizon around the nadir, whose distance from the horizon toward the nadir amounts to the arc I V, or K μ , and so on.

[Margin: When the true diameter of the circle that has been found is very small, what is to be done.]

When the true diameter of the circle that has been found is very small — as the last one, $V\mu$, was — so that a parallel to it cannot easily be drawn through the centre E , we shall bisect the arc $V\mu$ at the point ξ , which will be one of the poles of the circle; and, the axis ξp having been drawn, we shall draw to it a perpendicular diameter IK , to serve as the true diameter of the great circle to which the given circle is parallel.

[Margin: What is to be observed in investigating the position of a circle described on the astrolabe.]

[Margin: To investigate the position in the sphere of any straight line drawn on the astrolabe.]

[Margin: *19. unde (see Number 19).]

7. By this technique, then, you will investigate the position of any other circle described on the astrolabe, and its intersections with the other circles it cuts, and so on — provided that you first draw, through its centre and the centre of the astrolabe, a straight line to serve as the common section of the plane of the astrolabe and of the great circle that passes through its poles and the poles of the world; and then cut this line at right angles by [a line along] the diameter of the equator. One extreme of that diameter — namely the one nearer to the south pole of the astrolabe's projection for the given region — is taken as the south pole, A , from which the rays were emitted in laying out the astrolabe, and the other for the point from which the rays are to be emitted, and so on.

8. Finally, let a straight line FG be given, and let it be required to determine what it represents in the sphere — for it can represent many things. If it is imagined extended to infinity, it will correspond to a circle drawn through the south pole, as was stated in Proposition 6, Number 35; and we shall find its position in the sphere as follows. From E , the centre of the astrolabe, draw EH perpendicular to FG , cutting the equator at L ; to EH draw the perpendicular semidiameter EI , and join $I H$, cutting the equator at K . Now suppose the circle $ABCD$ is conceived as standing perpendicular to the plane of the equator (or of the astrolabe) upon the line $E H$, in such a way that I inclines toward the south — the equator remaining in its own position, that is, with A facing west and C facing east. Then the line $E I$ represents the axis of the world, and I the south pole. The plane drawn through $I H$, standing perpendicular to the circle in that position, will meet the plane of the astrolabe at H and will make the section $F H$. For since both the plane of the equator and that plane drawn through $I H$ stand perpendicular to the circle $ABCD$ in that position, their common section too will be perpendicular to it; and

therefore, by Definition 3 of Euclid, Book XI, F H — being perpendicular to E H, which lies in the same circle — will be that common section of the plane of the astrolabe and of the plane drawn through I H.

(...continuing §8:) ...drawn. * And so, since this plane makes in the sphere a circle whose diameter is I K, the given line F G, extended to infinity, will correspond to that circle — the one, namely, that passes through I, the south pole, and stands perpendicular to the great circle drawn through the poles of the world, and is inclined to the meridian of the given region (which is represented by B D) by as many degrees as are contained in the arc B L: in the upper part of the equator toward the west, A, but in the lower part toward the east, C.

[Margin (a): Theodosius I.1.]

But if the line F G is understood as terminated at the points F, G, it can represent the chord of a great circle described through those points — such as F G M N — or the chord of innumerable non-great circles described through the same points, whose situation and position in the sphere can be investigated from what we have written in this proposition; or, finally, the diameter of some non-great circle that is parallel to some oblique great circle. This last we shall investigate as follows. Since F G represents the diameter of some circle, that circle will be bisected by the great circle F G M N, [Margin (b): Theodosius I.14.] and therefore this great circle will pass through its poles. Hence the midpoint of the arc F G will be its pole, which is found thus. Having found O, the pole of the great circle F G M N contained within the equator — (this we shall find, as we described in Proposition 8, Number 17, in this way: through its centre P and the centre of the astrolabe we draw a straight line meeting the circle within the equator at Q, [Margin (c): III.3.] cutting the joined diameter M N at right angles; for the line M N will be a diameter, since it is the common section of two great circles. Then, the line M Q being drawn, cutting the equator at R, we take the arc R S equal to a quadrant; for the line M S will cut E P at the pole O) — let the lines O F, O G be drawn, cutting the equator at T, V; and, the arc T V having been bisected at X, let the line O X be drawn, cutting the arc F G at Y. For Y will be the midpoint of that arc, since the arcs F Y, G Y correspond to the equal arcs V X, T X, as we demonstrated in Proposition 5, Number 17; and therefore Y will be the pole of the circle whose diameter the line F G represents. But when the pole O lies close to the point X, so that the line O X can scarcely be extended without error, we shall perhaps find the same pole Y more accurately in this way. Let a point Z be taken opposite the point X, and, a straight line being drawn through the three points Z, E, X, let X a be taken equal to the semidiameter P Q of the circle F G M N; and, the

line a P being joined, let it be bisected at b and cut at right angles by the line b d, which cuts E a at d. For the line P d, extended, will give the point Y corresponding to the point X, as we demonstrated in Proposition 5, Number 34 — which the line X Y, parallel to a P, will also yield, or the line Y P making the angle Y P a equal to the angle P a X, as was shown in the same place.

You will conveniently find this same pole Y by the method we described in Proposition 6, Number 36. For suppose you describe a circle through three points — two of which are those where the line E P cuts the equator and the circle G Y F on the same side of the centre (namely either the arcs M Q N, M I N, or the arcs M F N, M B N), the third being the point X — whose centre lies on the line that bisects at right angles the line lying between the equator and the circle G Y F: that circle will pass through the point Y, as was demonstrated in the place cited. Or, by the method we shall hand down in the following Proposition 18, Number 5, if through the point X given on the equator you describe a parallel of the great circle represented by the line P Q, it will cut the circle F Y G at the same pole Y, as we showed in the same Proposition 6, Number 36.

Diagram. A stereographic-projection figure in a ruled square frame, at the right of the page. An upper open region carries the chord/pole construction (point q at top, with F, Y, G, H spanning a wide arc), and a lower circle (the equatorial disk) carries the projected sphere with many radii and dotted lines. Densely lettered; several small labels are indistinct.

- Top and upper arc: q, Y, F, a (α), T, b, δ [?], G, H, P
- Equatorial disk (clockwise): B, L, M, K, O, E (centre), S, c, h, δ [?], A, I, k [k?], N, R, D, Z, C, ς
- Faint interior/marginal labels: [?] (several unresolved)

To find this same pole Y, moreover, the other methods set out in Proposition 5 can also be used, especially the one we gave in Proposition 6, Number 25. For if the lines F O, G O are produced toward the pole O, and the arc of the oblique circle F G Q intercepted between those lines opposite the arc F G is bisected, then a straight line drawn from that midpoint through the pole O will fall on Y, the apparent midpoint of the arc F G, and will therefore pass through the point X, bisecting the arc T V of the equator — so that now three points are had through which the line bisecting the arc F G must be drawn: namely X, O, and that midpoint of the aforesaid arc of the oblique circle F G Q, opposite the arc F G, which is intercepted between the produced lines F O, G O. And if, in place of the equator, other circles are described whose semidiameters on the line P Q are cut at O just as the semidiameter of the equator is cut at that same point O, other

points will be found through which the same line O X must be drawn — provided, that is, that on those circles arcs similar to the arc T X are cut off from the line O T, starting from the beginning and proceeding toward the line P Q.

[Margin: Given a finite straight line, to inquire of how great an arc of a great circle it is the chord.]

Furthermore, the arc T V of the equator will indicate how great an arc of a great circle the arc T V is equal to, in number of degrees, with respect to the arc F Y G that the given line F G subtends, as we demonstrated in Proposition 5, Number 17. And in this way, given any terminated straight line, we shall investigate how great an arc of a great circle it subtends — if, that is, we describe a great circle about its end points and, from its pole once found (as we described a little earlier), two straight lines are drawn to those same ends. For these will cut off from the equator an arc equal — as regards the number of degrees — to the arc of the great circle that the given line subtends. And if the lines F O, G O are produced, they will also intercept, in the lower part of the same great circle F G Q, an arc of as many equal degrees as are contained in the apparent arc F Y G, as we showed in Proposition 6, Number 25. But in the following proposition, Number 3, we shall teach once again how to investigate of what arc of a great circle a given line is the chord, even if no great circle is described about its ends.

Therefore, once Y has been found — the pole of the circle, one of whose diameters the line F G represents — if the line E Y is drawn, both the centre of that circle and the centre of the great circle to which it is parallel will lie on it, as was shown in Proposition 8, Number 19. For this reason the line r q [?], cutting F G in half and at right angles at q, will fall on the centre of the circle F G, of which one of the diameters is the line F G. Moreover, the great circle to which the circle described from q is parallel will be described in this way. When the diameter g h is drawn perpendicular to E Y, the radius g Y will cut the circle A B C D at the pole i, and therefore i E k will be the axis of the sought great circle, and l m, perpendicular to it, its diameter. Therefore g n, perpendicular to l m at p, will fall at e, the centre of the great circle h o g, to which the circle described from q is parallel, since it has the same pole Y. This great circle will pass entirely through O, the pole of the great circle F G M N, since the latter passes through Y, the pole of the former. The other pole of the circle F G M N is the point ç, and the other pole of the circle g o h is the point f. Now, by what has been said above, the position of the great circle g o h will easily be investigated — and of its parallel, one of whose diameters is the given line F G.

Diagram. A stereographic-projection figure in a ruled square frame, repeating the construction of the preceding leaf (the F G chord / pole determination). Point q stands at the top, with F, Y, G, H across a wide upper arc, and the equatorial disk below carries the projected sphere with numerous radii and dotted lines. Densely lettered; several small labels indistinct.

- Top and upper arc: q, F, a (α), Y, T, b, δ [?], G, H, P
- Equatorial disk: B, L, M, K, O, E (centre), S, c, h, A, I, k [k?], N, R, D, Z, C, ς
- Faint interior labels: [?] (several unresolved)

[Margin: That a straight line drawn through the centre of the astrolabe can represent various things.]

But if a straight line is given that, when extended, passes through the centre of the astrolabe, it will represent a great circle drawn through the poles of the world; or, if its end points are diametrically opposite, the diameter of the infinitely many great circles that can be described through those end points; or, if those end points are not diametrically opposite, it will represent either the chord of the many non-great circles that can be described through them, or the apparent diameter of a non-great circle described about it.

PROBL. XV. PROPOS. XVIII.

Through a given point, to delineate on the astrolabe a parallel to a given great circle. Likewise, about a given pole, to describe a circle — whether a point is given through which it must pass, or not.

[Margin: Through a given point on a straight line drawn through the centre of the astrolabe and the centre of some great circle, to describe a parallel of that great circle.]

1. Let there be, on the astrolabe, the equator A B C D, whose centre is E; and any oblique great circle whatever, A F C G, whose pole is I; and let there first be given a point L on the line F G — drawn through H, the centre of the great circle, and E, the centre of the astrolabe — through which a parallel of the given great circle is to be described, having its centre on that same line F G. Now infinitely many circles can indeed be described through L, taking infinitely many centres on the line F G; but only one will correspond to a parallel of the given circle A F C G, and this we shall find from the given point L as follows. When the diameter A E C is drawn perpendicular to F G — a diameter that will fall on the intersections of the equator with the given circle — and when the true diameter P Q of the given great circle has been found by means of the rays A F, A G cutting the equator at P,

Q, let the radius A L be drawn, cutting the equator at the point O; through O let O q be drawn parallel to P Q, and this will be the true diameter of the parallel passing through L, because the ray from A, cast out through its extremity O, falls on L, the extremity of the apparent diameter — since the parallel to be described is assumed to pass through L. But if the pole I is given, we shall find the true diameter of the sought parallel, without the true diameter of the great circle, in this way. When the radius A L is drawn, cutting the equator at O, let a radius be drawn through the pole I, which will fall on the true pole b. Then let an arc b q be taken equal to the arc b O. For the line O q will be the true diameter, since the points O, q are equally distant from the pole b, and the true diameter passes through O, on account of the radius A L cutting the equator at O. Therefore, the radius A q being drawn through the other extremity q of the true diameter, the other apparent extremity M will be obtained — which will also be found by this method, even if no account of the true diameter is had. Having found the pole I of the given great circle by means of the radius A b drawn to b — the midpoint of the semicircle P b Q that the true diameter P Q cuts off, that is, the outermost point of the axis of the given circle — let an arc b q be taken equal to the arc O b, and let the radius A q be drawn, cutting F G at M; and the segments I L, I M of the great circle F G will be equal, since they correspond to the equal arcs O b, b q, as is clear from Proposition 1, Number 5. Since, therefore, F G represents one of the verticals of the given circle...

Diagram. A second stereographic-projection figure in a ruled square frame, at the foot of the page, illustrating Probl. XV / Propos. XVIII: the equator A B C D with the oblique great circle A F C G and the construction of its parallel through a given point. Two large intersecting circles (equator and oblique great circle) with an inner projected sphere, many radii from the projection point, and dotted construction lines. Densely lettered; several small labels indistinct.

- Top / upper: F, L, T, Z, H, Y
- Upper interior: B, N, O, P
- Centre and interior: I, C, k [κ?], h, E (centre), m, G, A
- Right / lower right: A, d
- Left / lower left: S, Q [?]
- Lower: R, δ [δ?], D, and at the foot K

Astrolabe, Book II — Proposition XVIII (continued)

(Batch covers = printed pp. 249–251, continuing the scholium/commentary to Propositio XVIII.)

a great circle, as of some horizon, then the very same parallel will run through the points L and M, equally distant from the vertex I. Therefore, once the visible diameter LM has been bisected at N, this N will be the center of the parallel that is to be described through the given point L.

[Margin: To describe, through a given point on the prime vertical of some given great circle, the parallel of that great circle.]

2. Now let a point h also be given on the prime vertical AICK of the given great circle, as of the horizon. From the center of the vertical draw the straight line Rh; and at h erect a perpendicular hN. This will fall on N, the center of the parallel to be described through h, as is clear from Proposition 6, Number 10 — because the line hN touches the vertical at h, by the corollary to Euclid III.16. And if an arc Ik is taken equal to the arc Ih, and from FG the segments IL and IM are cut off, equal (as regards the number of degrees) to the arcs Ih and Ik, we shall have four points h, k, L, M, through which the parallel is to be described, whose center lies on the line FG.
3. Next, let a point T be given outside the line FG drawn through the center of the given great circle and the center of the astrolabe, and outside the prime vertical.

[Margin: To describe the parallel of that great circle through a given point lying outside the line drawn through the center of the great circle and the center of the astrolabe, and outside the vertical.]

Having found the other pole K of the great circle given by means of the radius Ad — drawn through d, the midpoint of the other semicircle PdQ, or more accurately through the prime vertical AICK — that is, the circle described from the center R (which the radius drawn from A to the point f indicates, the arc Af being double the arc Ad): from this other pole K draw the straight line KT. Then, having drawn the straight line TI to the other, prior pole I, make an angle KIe equal to the angle TIF, and let the straight line Ie cut the line KT at e. The parallel drawn through T will then pass through the point e. For if we conceive the sought parallel as

described through T, the line KT will cut that parallel at e, the intersection of the line Ie with the parallel, on account of the equality of the angles TIF and KLe — as is plain from what we demonstrated in the scholium to Proposition 6, near the end, Number 5. For if the line KT cut the parallel at some point other than e, the line drawn from that point to I, together with IK, would make an angle equal to the angle TIF, and hence also to the angle eIK, as we showed in the same scholium, Number 5; and so a part and the whole would be equal, which is absurd. Therefore, the line iN drawn so as to bisect Te at right angles will pass through the center of the parallel passing through T and e, by the corollary to Euclid III.1. Since, then, the center lies on the line FG, N will be the center of the sought parallel, which necessarily also passes through the point Y, if the line TZ is drawn perpendicular to FG and ZY is taken equal to TZ.

But should it happen that the given point T lies in such a position that the line TI makes a right angle with FG (as was shown in the scholium to Proposition 6, Number 4), the line KT will be tangent to the parallel described through T at T; and then the line erected perpendicular to KT from T will fall on the center of the parallel to be described.

Again, if the given point lies below the line RS, which is drawn through the center of the prime vertical perpendicular to FG, the line must be drawn from the pole I through that point, and at the other pole K two angles must be constructed equal, in place of the angles TIF and eIK — because the parallel then to be described will encompass the pole K, and hence the line drawn from I through the given point will cut the parallel at points at which the lines making equal angles at K meet the same, etc.

Finally, if the point T lies in such a position that it is equally distant from both poles I and K (which will easily be recognized with the compass — for if, one foot being set at T and the other at I, the compass swung round passes through K, T will be equidistant from the points I and K, and not otherwise), that is, if it is found on the line RS which is drawn through the center of the prime vertical perpendicular to the meridian line, then the line RS itself will represent the parallel described through T; that is, the parallel in the sphere corresponding to it will be carried through the south pole, and hence will be projected onto a straight line, etc.

The same thing can be accomplished in this way. From the given point T, having drawn the perpendicular TZ to FH, take ZY equal to ZT, and the parallel will also pass through Y. Then from either of the points T, Y — namely from Y, through either of the poles I, K, namely through I — let the line YIe be drawn, which the

line TK will cut at e (TK being drawn from the other point T to the other pole K). For the parallel to be described will pass through e as well, as is clear from what we demonstrated at Proposition 6, Number 25. For if the parallel through T and Y is conceived to be described, there will be as many apparent degrees seen in the arc LY as there are equal degrees contained in the arc cut off on the arc by the lines LI, YI produced, as was there shown. Since, then, the line KT likewise cuts off from the arc LT as many apparent degrees as there are equal degrees included in the arc cut off by the lines KL, KT — as we demonstrated in the same place — and since the arc LT is equal to the arc LY (for the line KF, drawn through the center of the parallel so as to cut the line TY in two and at right angles, also bisects the arc TLY, by the scholium to Euclid III.27) — there will be cut off entirely the same arc by the lines KL, KT as by the lines LI, YI; and hence the parallel TLY will pass through e, the intersection of the lines YI and KT. Otherwise the lines LI, YI and KL, KT would not cut off the same arc. The circle, then, described through the three points T, Y, e will be the sought parallel. And exactly the same reasoning holds if the given point T lies below the line RS, and hence the parallel through T is to be described about the lower pole K. Thus, if in the third figure of the scholium to Proposition 6, in the parallel LMN described about the lower point P, the given point be N, we shall draw from N the perpendicular NO to the meridian line, and take OM equal to ON. For if from N, through the lower pole P, a line be drawn, the line drawn from M to the other pole Q will cut it at h, a point of the parallel, as is clear from what was demonstrated in the cited place, i.e. at Proposition 6, Number 25. For each of the arcs KN, KM includes as many apparent degrees as there are equal degrees contained in the arc Lh, etc.

Diagram. A large stereographic-projection figure of the astrolabe, enclosed in a square frame, occupying the lower-left of the page. It shows the outer bounding circle with a vertical diameter running from **F** (top) to **K** (bottom), and a network of intersecting great circles, verticals, horizon-parallels and radii filling the interior. Readable labels, by position:

- F (top of the vertical diameter)
- K (bottom of the vertical diameter)
- L (upper, on the vertical diameter, below F)
- Y (upper right of center)
- H (center; a small **z** stands just above/beside it)
- B (just left of H)
- N (center, below H)
- O (right of N)

- P (right, further out)
- C (lower left)
- A (right, on the bounding circle)
- G (below center)
- Q (lower left)
- R (lower center, on the horizontal line)
- D (below R)
- S (left, on the horizontal diameter/line)
- Several small interior letters in the densely crossed region (apparently including lowercase e, f, h, m, and Greek-like λ and a θ/δ near the foot) could not be resolved individually [?].

This figure is heavily crowded with construction lines; more than two interior labels are unresolved.**

[Printed p. 250]

But there is a way no less expeditious — namely, that by which the diameter of the parallel to be described is found on the very meridian line itself — and it is this. When lines have been drawn from the point T, outside the vertical AICK, to both poles I and K: if the acute angle ITK is bisected, the line dividing it will fall on the point M, an end of the diameter through which the parallel is to be described; and if at the line MT thus drawn a perpendicular is erected at T — or (what is the same) if the obtuse angle which the line KT, produced beyond T, makes with TI is bisected — that perpendicular, or this dividing line, will fall on the point L, the other end, so that the whole diameter is LM. This being bisected at N, N will be the center of the parallel to be described through T, L, M. This will be demonstrated as follows.

[Margin: The most expeditious method for finding, on the meridian line, the diameter of the parallel to be described through the given point.]

Conceive the parallel LTMY described. And since, as we demonstrated at Proposition 6, Number 25, there are as many apparent degrees in the arc LT as there are equal degrees contained both in the arc Me — cut off by the lines TK, LK — and in the arc on the other side cut off by the lines TI, LI produced: these cut-off arcs will be equal to one another. [Margin: a — Euclid III.27.] Therefore the angles which the line MT makes with the lines TK, TI, standing on those arcs, will be equal; and consequently the line bisecting the angle ITK will fall on the point M. [Margin: b — Euclid III.31.] Since, then, the angle at T constructed in the

semicircle LTM is a right angle, the perpendicular to the line TM drawn will fall on the point L. That the line TL drawn bisects the obtuse angle which TI makes with KT produced — and hence that the line dividing that angle in two falls on the point L — we shall show in this way. [Margin: c — Euclid III.31.] Since the line LT drawn makes right angles — i.e. equal angles — with MT produced (the angle LTM being in a semicircle): [Margin: d — Euclid I.15.] and since also the angle MTI is equal to MTK, the vertical angle which MT and KT produced make, being equal to it — the remaining angle ITL will likewise be equal to the remaining angle which the line LT makes with KT produced. Which was proposed.

In like manner, if a point e is given within the vertical AICK, and lines are drawn from e to both poles I, K, and the acute angle TeI is bisected, the dividing line will fall on the point L, an end of the diameter. And if at the line eL thus drawn a perpendicular is erected at e — or (what is the same) if the obtuse angle IeK is bisected — that perpendicular, or the dividing line, will fall on the point M, the other end. For conceive the parallel LTM described. And since, as was shown at Proposition 6, Number 25, there are as many apparent degrees in the arc Me as there are equal degrees existing both in the arc LT — cut off by the lines KT, KL — and in the arc on the other side cut off by the lines eI, MI produced: these cut-off arcs will be equal to one another. [Margin: e — Euclid III.27.] Therefore the angles which the line Le makes with the lines eT, eI, standing on those arcs, will be equal; and hence the line bisecting the angle Tel will fall on the point L. [Margin: f — Euclid III.31.] Since, then, the angle at e constructed in the semicircle LeM is a right angle, the perpendicular will fall on the line eL drawn, at the point M. Further, that the line eM drawn bisects the obtuse angle IeK — and hence that the line dividing it falls on the point M — will be proved thus: since the line Me drawn, together with the line Le drawn, makes equal angles, namely right angles, [Margin: g — Euclid III.31.] the angle Lem being a right angle in the semicircle. [Margin: h — Euclid I.15.] And since also the angle LeI is equal to LeT, the vertical angle which Le, Te produced make, being equal to it — the remaining angle MeI will likewise be equal to the remaining angle MeK. Which was proposed.

Now this way is the most convenient. For if a straight line bisecting an acute angle cuts the meridian line too obliquely, the other line bisecting the obtuse angle will cut the same less obliquely. Hence, by this line the point in the meridian line is then to be found — for example the point L, by the line which bisects the obtuse angle that the line IT makes with KT produced. For when the radius AL is drawn from the south pole A, cutting the Equator at O, the line Oq drawn parallel to the diameter PQ of the greatest oblique circle will be the true diameter of the parallel; and hence the radius Aq will exhibit M, the other end. Or

else, if the line TL joined is bisected at right angles, the center N will be found, on the meridian line, by the line dividing it. And in order that what has here been demonstrated may be more easily understood, the lines TM, eL are to be drawn and, together with the line KT, produced; likewise the lines TL, eM are to be joined — which in this figure has not been done, so that confusion of lines might be avoided.

[Margin: To find how great an arc of a great circle a given straight line subtends, even if that great circle is not drawn.]

From these things we shall also easily explore of how great an arc of a great circle a given straight line, terminated at its ends, is the chord — even if the great circle in which the chord lies is not drawn, as was done in the preceding proposition, Number 8. For let there be, on the astrolabe on which the Equator ABCD lies about the center E, a given straight line TI. Let us feign either of the ends — namely I — to be a pole, about which a circle is to be described through the other end T; which will be done thus. From E, the center, through the point I (which is to be the pole) draw the line IEK, and find the point K opposite to the point I through the diameter, as we taught at Proposition 6, Number 13; this will be the other pole. Then, drawing from this other pole K to the other end T the line KT, bisect the acute angle ITK by a line which cuts the line IK at M; or, if you prefer, having produced the line KT, bisect the obtuse angle constructed at T by the line IT and KT produced, by a line cutting IK at L: and either M or L will be an end of the diameter of the circle to be described through T, as was shown. Now since, by the definition of a pole, the lines falling from the pole to the circumference of the circle are equal, [Margin: i — Euclid III.28.] the arcs of the great circles placed between the pole and the same circle — of which those lines are the chords — will also be equal. Therefore the arcs of the meridian IM, IL and the arc of the great circle through the points I, T described (of which the line TI is the chord) will be equal. Therefore, having drawn from E, perpendicular to IK, the diameter AC, if from either end — say from A — radii be sent out through IM or I, L, cutting the Equator at b, q, or b, O, then the

Diagram. A second impression of essentially the same large stereographic astrolabe figure as on p. 601, enclosed in a square frame, in the lower part of the page. It again shows the outer bounding circle with the vertical diameter from **F** (top) to **K** (bottom) and the same dense network of great circles, verticals and radii. Readable labels, by position: **F** (top), **L**, **Y**, **H** (with adjacent small **z**), **B**, **N**, **O**, **P**, **C**, **A** (on the circle, right), **G**, **Q**, **S** (left), **R**, **D** (lower center), **K** (bottom). As on p. 601, numerous small interior letters in the crowded central region (apparently

including e, f, h, m, and Greek-like λ , θ/δ) could not be individually resolved [?]. (more than two interior labels unresolved).

[Printed p. 251]

apparent arc IM, or IL, will be equal to the true arc bq, or bO — since these true arcs are projected into the apparent arcs IM, IL. Therefore TI will represent the chord of the arc of the great circle which is equal to the arc bq or bO.

In the same way, if T is set up as the pole about which a circle is to be described through I, a line must be drawn from T through the center E, and on it the point opposite to T through the diameter must be found, for the other pole; then from this pole a line must be drawn to I, and the angle — whether acute or obtuse — which this line makes with the given line IT must be bisected, so that on the line TE drawn the extreme point may be found through which the circle about the pole T is to be described. For when, through E, the diameter perpendicular to the line TE joined has been drawn, if from either of its ends radii are sent out through T and through the point found on the line TE joined, they will cut off from the Equator an arc equal to that of which the given line TI is the chord, etc.

For the rest, if a point can conveniently be found, on the line RS drawn perpendicular to FG at R (the center of the prime vertical), which is the center of the vertical passing through T and I, let such a vertical TI be described; and from the center S let the line SE be drawn, which will cut the given great circle at V, the pole of the vertical TI. For since the circle TI passes through I, the pole of the given circle, the same given circle will pass through the pole of TI, by the scholium to Theodosius I.15. Since, then, the pole of the vertical TI lies on the line SE, as was demonstrated at Proposition 8, Number 19, V will be the pole of the vertical TI. Therefore, when the lines VI, VT are drawn cutting the Equator at aX, aX will be an arc equal to the arc TI as regards the number of degrees, as is clear from Proposition 5, Number 17. If, then, we cut off from the greatest circle FG arcs IL, IM equal to this, we shall have three points T, L, M through which the sought parallel is to be described, whose center lies on the line FG. Now the points L, M will be found in this way. Having drawn the line AI cutting the Equator at b, take on either side the arcs bO, bq equal to the arc aX. For the lines AO, Aq will cut off segments IL, IM of as many degrees as are contained in the arcs bO, bq, and hence also in aX or TI, as is clear from what was demonstrated at Proposition 5, Number 23 and Proposition 1, Number 6.

Likewise, if an arc aA is made equal to the arc aX, the line VA drawn will cut off from the vertical TI an arc Im equal to the arc aA, or aX, or TI, and the parallel to

be described will pass through m . If, then, the line Tm drawn is bisected at right angles, the dividing line will fall on N , the center of the sought parallel, by the corollary to Euclid III.1 — since the line Tm lies in that parallel. In the same manner, the line cutting the joined line TL , or TM , in two and at right angles will fall on the same center N , since each of the lines TL , TM lies in the same parallel.

Indeed it is not necessary that the points L , M be found. For if, from S , the center of the vertical Tlm (which is found by the line that bisects at right angles the line TI , or TK , drawn from the given point T to either of the poles of the oblique circle), a line ST is drawn to the given point T , and the right angle STN is made, TN will fall on N , the center of the sought parallel, as was demonstrated at Proposition 8, Number 13. Hence the circle described from N through T will be the sought parallel.

But most conveniently we shall describe the parallel of the given oblique circle through the given point T by this other method. Having drawn from the given point T to R (the center of the prime vertical) the line TR , let a third proportional be found to the two lines TR , RI (of which the former is the line drawn, the latter the semidiameter of the vertical), and let RI be cut off equal to it. Finally, TI being bisected at p , erect at TI the perpendicular pN . I say that the circle described from N through T and l , namely Thl , is the parallel of the greatest oblique circle $AFCG$. For if it is not, let a parallel be imagined described through T , cutting the line RT (if that is possible) at some point other than l , say at r . Then, from what we demonstrated at Proposition 6, Number 30, the semidiameter of the vertical RI will be the mean proportional between RT and Rr — which is absurd, since RI is by construction the mean proportional between RT and Rl . So too, if a point l is given, the line drawn from R through l , and RT taken as third proportional to the two lines RI , RI , the parallel to be described through l , T will be found, as has been said.

Now it must be known that, when the given point lies outside the vertical (as was the point T), the third proportional RI is smaller than the line RT ; whereas when the given point lies inside the vertical (as is the point l), the third proportional RT is larger than the line Rl which is drawn from the center of the vertical to the given point.

This same reasoning applies also to a point that is given on the line drawn through the center of the given oblique great circle and the center of the astrolabe. For example, if the given point is L , and a third proportional RM is found to the two lines RL , RI , the parallel is to be described through L , M from the midpoint of the line LM . Likewise, if the given point is M , having found the third

proportional RL to the two lines RM, RI, the same sought parallel is to be described through M, L, etc.

[Margin: Another construction, when the given point lies on the line drawn through the center of the given oblique great circle and the center of the astrolabe. — When the given point lies on the circumference of the Equator. — To describe the parallel of the Equator through any point whatever given. — Another construction of the former oblique parallel through a given point.]

But if the given point lies on the circumference of the Equator, a line must be drawn from it perpendicular to the meridian line. For the line which will be drawn through the intersection of that perpendicular with the meridian line, parallel to the diameter PQ of the great circle to whose parallel it must be equidistant, will be the diameter of the sought parallel in the sphere: from which the parallel will be described, as was set forth at Proposition 6. The reason for this is that the intersections of that parallel with the Equator, and the point of intersection of its true diameter with the meridian line, lie on one straight line — namely in the common section of the plane of the parallel with the plane of the Equator, as we showed at Proposition 6, Number 4. Since, then, that perpendicular to the meridian line drawn from the given point is that common section (inasmuch as, as was there demonstrated, the common section is perpendicular to the meridian line, and passes by hypothesis through the point given on the circumference of the Equator, since the parallel must pass through it), the point of intersection of the said perpendicular with the meridian line will be that through which the diameter of the proposed parallel is to be drawn. Thus, if either of the intersections of the parallel LTM with the Equator were given, the line from that point to FG would cut FG perpendicularly at the point through which the diameter Oq of the said parallel is drawn.

4. Lastly, let the parallel of the Equator be described through the given point T, wherever it may lie. This will be done without any trouble, if from E, the center of the astrolabe, the circle TYg is described through T — since all parallels of the Equator possess the same center as the astrolabe, as we demonstrated at Proposition 2, Number 6.

By means of this parallel of the Equator described through the given point T, we shall describe in another way the oblique parallel through the same point. For if from A, the south pole, a line is drawn to the intersection of the parallel of the Equator with the line FG, it will cut the Equator at the declination of that parallel — as, for instance, in the given ex-

in the given example — namely at the point a, through which the line drawn parallel to FG will be the diameter of that same parallel. Next, through the given point T, draw TZ perpendicular to FG, and from A let the visual ray be emitted to Z. For where this ray cuts the diameter of the parallel of the equator that passes through the point a (in the given example, Oq, which is parallel to PQ, the diameter of the greatest oblique circle), through that point of intersection the line Oq must be drawn — parallel to the diameter PQ of the greatest oblique circle — to serve as the true diameter of the oblique parallel to be described.

For since TY is the common section of the parallel of the equator TYg and of the oblique parallel to be drawn through T — as is clear from what we demonstrated in Prop. 6, near the end, Num. 4 — the point Z will lie both on the parallel of the equator and on the oblique parallel. Therefore, since the point Z as seen corresponds to the true point on the meridian — and indeed to that point of the parallel's diameter through which the ray AZ is projected, since this point appears at Z — the oblique parallel will pass through that same point on the meridian, and consequently the diameter of the oblique parallel must be drawn through it. Once the true diameter Oq of the oblique parallel has been found, the rays AO and Aq will cut off its apparent diameter LM, about which the oblique parallel is to be described.

[Margin: To describe, through a given point, a parallel of the greatest circle drawn through the poles of the world.]

5. It is done more easily to describe, through a given point, the parallel of the greatest circle drawn through the poles of the world. For let the straight line BED represent the greatest circle drawn through the poles of the world, and let the diameter AEC cut it at right angles; this AEC will represent its meridian, in which all the centres of the parallels of the greatest circle BD will lie, as is established from what we demonstrated in Prop. 7.

So let the given point F first lie on the equator. Draw the line DF cutting AC at G, and take the arc DH equal to the arc BF. For the circle FGH, drawn through the three points F, G, H from the centre I, will represent the parallel of the greatest circle BD, as is clear from what we demonstrated in Prop. 7.

Next, let the given point K lie inside the equator. Having drawn from E, through K, the parallel of the equator KLM, let its opposite POQ be described. This is easily done: through L draw the radius CLN cutting the equator at N, and from A, through N, draw the ray ANO cutting DB at O. For EO will be the semidiameter of

the opposite parallel, as is established from what was demonstrated in Prop. 4, Num. 6. For the arc BN is equal to the one that the ray AL would cut off, had it been drawn. Then, having drawn the line EK cutting the parallel POQ at P — so that the arcs OP and LK are similar — if the arcs RM, SQ are taken equal to the arcs RK, SP, the circle PKMQ, drawn from the centre I, will be the parallel that is sought. The reason is that on the sphere a parallel of this kind cuts off equal arcs from opposite parallels of the equator, since the arcs it cuts off have equal right sines — namely the perpendiculars that are dropped onto the plane of the greatest circle from the intersections of that parallel with the equal and opposite parallels of the equator; for these perpendiculars lie between parallel planes, as we demonstrated at the end of Lemma 48. Therefore, since the four arcs OP, LK, TM, VQ represent equal arcs on the sphere, the parallel drawn through K will also pass through P, M, Q — which is what was proposed.

Again, let the given point P lie outside the equator. Having drawn from E, through P, the parallel of the equator POQ, let its opposite KLM be described. This will be done by drawing through O the radius AO cutting the equator at N, and then drawing the radius CN cutting BD at L. For EL will be the semidiameter of the opposite parallel. Then, having drawn the line EP cutting the parallel KLM at K, if the arcs VQ, TM are taken equal to the arcs OP, LK, the sought parallel will pass through P, K, M, Q, and so on.

But if a parallel is to be described through the point R, which lies a quadrant distant on the parallel of the equator KLM from the greatest circle BD, it will necessarily pass also through the point S, which lies a quadrant distant on the parallel POQ from the same greatest circle BD. So, having bisected the line RS at X, the circle drawn from X through R and S will be the parallel that is wanted, and it will touch the two parallels KLM and POQ, just as happens on the sphere. In this way a parallel to be drawn through S will pass through R, and so on.

[Margin: By what method the greatest circles and the oblique parallels are divided into degrees by means of the parallels of the greatest circle drawn through the poles of the world.]

Finally, let the given point G lie on the line AC. Draw the line DG cutting the equator at F, and take the arc DH equal to the arc BF. For the circle FGH, drawn through the three points F, G, H, will be the sought parallel.

Now, so that you may see how conveniently the oblique parallels are divided into degrees by means of such parallels — as we wrote at the end of Prop. 6 — let YZ be an oblique parallel, distant from its lower pole by as great a space as the

parallel of the equator KLM is from the north pole, or POQ from the south; and let its primary vertical be aCb, cutting off from it the quadrant YZ. Here you see that the parallel RZS, drawn through the end of the quadrant LR (or OS), which touches both parallels of the equator, cuts off the same quadrant YZ and touches the parallel YZ itself at Z — just as on the sphere the same parallel RZS touches the three equal circles KLM, POQ, YZ. Likewise you discern that the line aR, drawn from a, the upper pole of the parallel YZ, through the end of the quadrant TR of the northern parallel of the equator, passes through the end of that same quadrant YZ; and similarly that the line bS, drawn out through the end of the quadrant OS of the southern parallel of the equator, also passes through the end of the same quadrant YZ, as reason requires — just as was demonstrated in Prop. 6, Num. 21 and 24. Again it appears that the parallel PGQ cuts off an arc Ye equal — as far as the number of degrees is concerned — both to the arc TM and to the arc OP; for the same arc Ye is cut off both by the line aM drawn from the upper pole and by the line bP drawn from the lower pole. And it is established, from what was demonstrated in Prop. 6, Num. 21 and 24, that the arc Ye is equal to the arcs TM and OP.

By the same reasoning, the same parallel PGQ cuts off, from the greatest oblique circle AaCb (which has its poles on the line OV), two equal arcs ad and bf, corresponding of course to the equal arcs BF, DH of the equator. And thus a parallel whose pole is C or A always cuts off two equal arcs — both from a greatest oblique circle and from a non-greatest one having its poles on the line OV — the arcs taking their beginning from the line OV drawn through the centre of the oblique circle from the centre of the astrolabe.

Diagram. A dense stereographic-projection figure enclosed in a square frame, showing the equator, several parallels of the equator and their opposites, an oblique parallel and its vertical, together with the projecting rays used throughout §§4–5. The peripheral labels that read clearly are:

- Q (top of frame)
- P (upper left, on the outer circle)
- B (top centre)
- N (upper right)
- F (left of centre)
- a (small, beside F)
- L (centre)
- E (centre right, on the horizontal diameter)
- I (far left, on the horizontal diameter)

- A (far right, on the horizontal diameter)
- S (lower left)
- X (left of centre)
- R, G (near centre)
- M, Y, T (lower centre)
- H, D (lower centre-right)
- V (bottom of frame)
- Additional lower-arc labels near the foot that read as C and 6 [?]

I do not, however, think it should be passed over in silence that this method of dividing the oblique circles into degrees by means of various circles drawn through sets of three points — which we explained in Prop. 6, Num. 36 — virtually contains the first method, by which both the greatest oblique circles and their parallels are distributed into degrees by means of straight lines issuing from one or the other of the poles of the proposed oblique circle. That first method we set forth in Prop. 5, Num. 17 and 20, and in Prop. 6, Num. 21 and 24, and it was demonstrated from Lemma 23.

[Margin: Another easy demonstration of the first method of dividing the oblique circles into degrees, which depended on Lemma 23.]

For suppose that, on the sphere, one conceives an arc of the proper meridian of the given oblique circle — the arc lying between the pole of that oblique circle (whether the upper or the lower) and the south pole of the world — to be bisected by a greatest circle at right angles to that same meridian. Then in this greatest perpendicular circle there will lie the pole of a certain non-greatest circle. This non-greatest circle passes through the assumed pole of the oblique circle and through the south pole of the world, and through any given point on the equator or on its parallel; and it cuts off, from the given greatest oblique circle (or from its parallel that is equal to a parallel of the equator, as was said in Prop. 6, Num. 21), an arc equal to the one it cuts off from the equator or its parallel, as was demonstrated in Lemma 47 — since its pole lies on that greatest perpendicular circle, from which the pole of the oblique circle and the south pole of the world are equally distant along the proper meridian. Therefore this same circle, when drawn on the astrolabe, will accomplish the same thing. Since therefore it is projected into a straight line — as we showed in Prop. 1, because it is drawn through the south pole — a straight line will represent that circle: a line extended through the assumed pole of the oblique circle (that is, through the upper or lower pole) and through the given point of the equator or of its parallel. And therefore it will cut off, from the given greatest circle (or from its parallel that

corresponds to the assumed parallel of the equator), arcs equal as far as the number of degrees is concerned, just as we taught should be done in the aforesaid first method. The beginnings of the arcs cut off are to be taken as we wrote in Lemma 47. These things ought to have been said in Prop. 6, Num. 36; but since they first occurred to us here, we thought they should not be omitted.

6. But now, in the earlier figure, let a circle be described about the given pole I and through the given point T — a circle that will be a parallel of the greatest circle whose pole is likewise I.

[Margin: Theodosius II.2.] [Margin: To describe a circle about a given pole, whether or not a point is given through which it must pass.]

Draw the line through I and through E, the centre of the astrolabe; on this line will lie the centre of the circle to be described, as we showed in Prop. 8, Num. 19; and let the diameter AC cut this line at right angles. Then, once the other pole K has been found, if the line TK is drawn, and the line TI drawn, and the angle KIE made equal to the angle TIF, the sought circle will pass through e; and the line iN, bisecting Te at right angles, will fall on N, the centre, as was demonstrated in Num. 3. Again, if — having found R, the centre of the circle AIC, that is, the midpoint of the line IK — the line TR is drawn, and a third proportional RI is found to the two lines TR, RI, the same circle will pass through I; and the line pN, bisecting TI at right angles, will fall on the centre N, as we showed in the same place.

If the given point is L, through which the line EI extended passes, we shall draw the radius AI falling on the true pole b; and, drawing the radius AL cutting the equator at O, we shall take an arc bq equal to the arc bO. For the line Aq, when drawn, will cut FK at the point M, through which the sought circle will pass, since the arcs IL, IM correspond to the equal arcs bO, bq, and so on. Therefore the point N, the midpoint of the apparent diameter LM, will be the centre.

But if only the pole I is given, about which a circle of any size whatever is to be described, no point being given through which it must pass, we shall draw the radius AI falling on the true pole b. For if two arcs bO, bq are taken equal in any way, the rays AO, Aq will give the apparent diameter LM of the circle to be described, and so on. And if the line Oq (which is the true diameter of the sought circle), when drawn, passes through the centre E, the circle described will be a greatest circle and will pass through A and C, since its true diameter passes through the centre; but if it does not pass through E, the circle described will not be a greatest circle.

When the given pole is on the circumference of the equator — namely C, in the later figure — a parallel of the greatest circle BD must be described, through any assumed point P, or F, or K, or G, etc., at will, as we taught in Num. 5.

If perchance the other pole K, outside the equator, is given, its opposite I, inside the equator, must be sought, and the rest carried out as has been said.

In the later figure the matter will be completed, as we said in Num. 5, since all those parallels have been described about the poles C and A.

7. Now, if — given a point on an oblique parallel, whether that parallel has been drawn or not — one wishes to find the point diametrically opposite to it on the same parallel (which task we undertook, in this place, to perform, as promised in Prop. 6, Num. 13), one will accomplish it in the following way.

[Margin: Given a point on any parallel, to find the point diametrically opposite to it on the same, through the apparent diameter, even if the parallel has not been drawn.]

First, in the drawn parallel LTM (in the earlier figure), let the given point be T, whose opposite must be found — that is, the point that on the sphere is diametrically opposite the given point T. Let the line hk be joined, which will represent that diameter of the parallel which on the sphere is the common section of the parallel and the primary vertical. And since on the sphere all the diameters of the same parallel intersect one another in the plane of the meridian, all its diameters will be seen to pass through n, the point of the meridian through which hk is seen to be drawn. Therefore the line Tn, when drawn, will fall on the opposite point m; that is, Tm will represent the diameter of the parallel drawn through the opposite points T, m. This can also be demonstrated geometrically as follows. Since the line RI, bisecting the arc hIk at I, also bisects the line hk at n (from the scholium to Prop. 27 of Euclid, Book 3), the same RI will cut that same hk at right angles [Euclid III.3].

Diagram. A large stereographic-projection figure within a square frame, combining the "earlier" and "later" figures of Prop. XVIII: two large overlapping principal circles, several parallels and verticals, and a dense sheaf of construction lines and dotted rays. The peripheral labels that read clearly are:

- F (top of frame)
- L (below F)
- Y (upper right)

- H, B (upper centre)
- O, P (right of centre)
- N (centre)
- C (left)
- A (right)
- G, Q (lower left of centre)
- d (lower centre-right)
- S (left, on the horizontal)
- R, D (lower centre)
- K (bottom of frame)
- Y (foot, below frame)

By the scholium to Prop. 27 of Euclid, Book 3 [the line hk being bisected at n], the same RI will cut that same hk at right angles. Since therefore the rectangle under Tn , nm is equal to the rectangle under hn , nk [Euclid III.35], it will be equal to the square of the line nh . But the rectangle under In , nK is also equal to that same square, because — from the scholium to Prop. 13 of Euclid, Book 6 — the line nh is a mean proportional between In and nK [Euclid VI.17]. Therefore the rectangles under Tn , nm and under In , nK are equal. Or else, because — by Euclid III.35 — the rectangle under In , nK is equal to the rectangle under hn , nk (since the lines IK , hk cut one another at n in the circle $AICK$), and hence to the rectangle under Tn , nm , therefore — from the scholium to Prop. 35 of Euclid, Book 3 — a circle $TImK$ can be described through the four points T , I , m , K . And since this circle is a vertical (being drawn, of course, through the poles of the horizon I , K), it will cut the parallel at opposite points [Theodosius I.15], since it bisects it. Therefore the point m is diametrically opposite the point T on the parallel.

The same opposite point will be found more easily by means of the vertical that is drawn through the given point and through the poles I , K , whenever such a vertical can conveniently be drawn. For this, as we said just now, will cut the parallel at the opposite point.

Next, let the given point be Y , on a parallel that has not yet been drawn, whose opposite point must be found. Draw YT perpendicular to FG , and take ZT equal to ZY ; then the point T will lie on the same parallel. Then, having joined the line RT , let Rl be a third proportional to the two lines RT , RI . I say that the point l is diametrically opposite the given point Y . For when the parallel LTM has been drawn, it will necessarily pass through l , because — as was shown in Prop. 6, Num. 30 — the parallel cuts off from the line RT a third proportional to the two

lines RT, RI, such as RI was. And since the arcs hl and hT are equal — as regards the number of degrees, as is clear from Prop. 6, Num. 26 — and the arcs hM, hL represent quadrants, the arcs lM and TL will also be equal. But TL is equal to the arc YL. Therefore lM will also be equal to YL; and, adding the common arc YM, the whole arcs LYM and lMY will be equal. Therefore, since LYM is a semicircle as far as the number of degrees is concerned, lMY will also be a semicircle; and hence the point l is diametrically opposite the point Y on the parallel LTM — which is what was proposed. In the same way, if the point m is given, and the perpendicular mt is drawn, take tl equal to tm; and, extending the line RI through l, let RT be a third proportional to the two lines RI, RI. Then T will be the point diametrically opposite the given point m.

But the same opposite point will be found more easily if — when this can conveniently be done — the vertical TIK is drawn through the given point T and through the poles I, K of the parallel. For this vertical will pass through the opposite point. Therefore, if an arc Im equal to the arc TI is cut off — by the means we wrote of in Prop. 5, Num. 18 — then m will be the sought opposite point.

PROBL. XVI. PROPOS. XIX.

Through a given point on the circumference of a given non-greatest circle in the astrolabe, to describe a greatest circle that touches the given circle.

[Margin: Through a given point on a non-greatest circle, to describe a greatest circle that touches it.]

1. This is Prop. 14 of Theodosius, Book 2, which we shall carry out on the astrolabe as follows. Let ABCD be the equator of the astrolabe, about the centre E, and let FGH be any non-greatest circle, whose centre is I, with the given point F on it. Draw the line IF through F and through the circle's centre I, produced as far as you like; and draw another line FEK through F and through E, the astrolabe's centre, on which the point K, opposite to F, is to be found, as we taught in Prop. 6, Num. 13. This is easily done: draw the diameter AC perpendicular to FK, and describe a circle about the three points A, F, C. For this circle will cut FEK at the opposite point K. Next, let the angle FKL be made equal to the angle KFL; then the lines FL, KL will be equal [Euclid I.6]. Therefore the circle drawn from L through F will pass through K and will touch the given circle at F, because the straight line at F, making right angles with both radii IF and LF, touches both circles at F, by

the corollary to Prop. 16 of Euclid, Book 3. And this circle is also a greatest circle, since it is drawn through the two opposite points F, K.

In the same way, if the point H is given, we shall draw through it and through the centre I the line HI; likewise, through H and the centre E, the line HEM; and we shall find M as the point opposite to H. This too will be done if, having drawn the diameter BD perpendicular to HM, a circle is described through the three points B, H, D. For this will cut HM at the opposite point M. Next, we shall make the angle HMN equal to the angle MHN; then again the lines HN, MN will be equal [Euclid I.6]. Therefore the circle drawn from N through H will pass through M and will touch the given circle at H, by the scholium to Prop. 13 of Euclid, Book 3 — or else because the line making right angles at H with HI touches both circles, by the corollary to Prop. 16 of Euclid, Book 3. And this circle too is a greatest circle, since it is drawn through the two opposite points H, M.

2. But if it should ever happen that the given point P, or T, is so situated that a line drawn through it and through the centre I passes through the centre E — as is the case with the line TIPE — we shall complete the problem by drawing the diameter RS perpendicular to TE and describing, through the three points R, P, S, a circle RPSQ from the centre V.

[Margin: When the given point lies on the line drawn through the centre of the given circle and the centre of the astrolabe, to do the same.]

For this will be a greatest circle, by the scholium to Prop. 5, Num. 9, and it will touch the given circle at P. In the same way the circle RTSV, drawn through the three points R, T, S from the centre X, will be a greatest circle, and will touch the given circle at T.

Diagram. A large stereographic-projection figure within a square frame, for Prop. XIX: a great outer circle, a non-greatest circle FGH with several concentric/overlapping circles, and a dense web of tangent constructions and radii. The peripheral labels that read clearly are:

- e (top, on the outer circle)
- K (upper centre)
- N (upper centre, below K)
- e (upper right, on the outer circle)
- R (upper left of centre)
- C (left of centre)
- Y (left of centre)

- G (lower left)
- P (lower centre)
- F (lower centre)
- N (lower centre)
- I (lower left of centre)
- H (bottom left)
- T (bottom)
- L, A (right of centre)
- S (right of centre)
- P (lower right)
- O (right, on the outer circle)
- e (lower right, on the outer circle)

Astrolabe, Book II — Propositio XX (Problema XVII) and the Scholium on constructing the instrument

[Conclusion of Propositio XIX (Problema XVI), §3]

3. Finally, if the given circle is one of the parallels of the Equator, such as Yd, with the given point Y, we shall draw from Y, through the centre E, the straight line YEb, and cut it at right angles by the diameter Za. Then the circle described from the centre L through the three points a, Y, Z as aYZb will be a greatest circle, by proposition 13 of Euclid, Book 3, and will touch the parallel at Y, by the scholium of that proposition. Likewise, given the parallel of the Equator be and the point b, we shall draw from b, through the centre E, the straight line bE, and upon it erect the perpendicular diameter aZ. For again the circle abZY, described from L through the three points a, b, Z, will be greatest, and will touch the parallel mb — which was the thing proposed.

But we shall accomplish this more easily as follows: having drawn the straight line Yb through the centre E, either from the given point Y in the parallel Yd, or from b, the given point in the parallel be, let us describe opposite the parallel Yd the parallel be, or opposite the parallel be the parallel Yd. For, the straight line Yb having been bisected at L, the circle abZY described from L, through Y or b, will touch both parallels.

PROBL. XVII. PROPOS. XX.

Through a given point outside the circumference of a given non-greatest circle — the point lying between that circle and its opposite parallel — to describe a greatest circle which touches the given circle.

[The body statement appears to read "between that circle and another circle equal and parallel to it," which is equivalent to "its opposite parallel."]

[Margin: *Per datum punctum extra circumferentiam circuli non maximi, inter ipsum circumulum, et eius oppositum parallelum, ita ut recta coniungens datum punctum et centrum Astrolabii transeat per dati circuli centrum, circumulum maximum describere, qui eum tangat* — "Through a given point outside the circumference of a non-greatest circle, [lying] between that circle and its opposite parallel, in such a way that the straight line joining the given point and the centre of the Astrolabe passes through the centre of the given circle, to describe a greatest circle which touches it."]

1. This is proposition 15 of Book 2 of Theodosius [Theodosius, *Sphaerica* II.15], which is solved on the Astrolabe as follows. Let the Equator of the Astrolabe be ABCD, whose centre is E, and let HN be the given non-greatest circle — whether it be a parallel of the Equator or of some other greatest circle — and first let the portion of the sphere comprehended within it be smaller than a hemisphere. (This will be the case when the circle either lies wholly within the Equator, or is wholly contained outside it without however surrounding it; or when it does not bisect it, provided the smaller portion of the Equator lies within the same circle — as we showed in the scholium of proposition 6, number 9.) And let the given point F, lying outside the circumference of the given circle, be between the given circle and its opposite parallel — F being the point through which a greatest circle is to be described tangent to the given circle.

Having drawn from F, through E, the centre of the Astrolabe, the straight line FG, let there be found, by proposition 6, number 13, the point G opposite to F, which will necessarily lie outside the given circle, provided F lies outside it and between it and its opposite parallel. For if G were within it, the point F would lie within the opposite parallel, and not between those two opposite parallels — which is contrary to the hypothesis. For if G were in the portion of the sphere smaller than a hemisphere (namely the portion which the given circle HN cuts off), its opposite point F would be in the opposite portion of the sphere — a portion likewise

smaller than a hemisphere — namely the one which the opposite parallel comprehends within itself.

Now let the straight line FG first pass through the centre of the given circle — which indeed always happens with the parallels of the Equator, since the centre of the Equator and of its parallels is one and the same; but with other non-greatest circles this does not always occur. And since the greatest circle to be described through F passes also through G, the opposite point, we shall describe — by what we demonstrated at the beginning of Lemma 41 — through the two points F, G, which lie outside the given circle, a circle tangent to it, namely in this way.

The straight line FG having been bisected at L, let the perpendicular LK be erected to FG; and the centre of the circle to be described will lie on the straight line KL, by the corollary of proposition 1 of Euclid, Book 3. This we shall find as follows. Having described the semicircle FBG about FG from L, we shall erect at E, the centre of the given circle, the perpendicular EB. (This semicircle FBG will necessarily pass through the intersection of the straight line EB with the Equator, by the scholium of proposition 31 of Euclid, Book 3; for, the straight line FB having been drawn, the other pole G is found by the line perpendicular to FB, as we taught in proposition 6, number 13.) Then, the straight line BN having been drawn from B to either extremity of the diameter of the given circle — namely to N — we shall make the angle NBQ equal to the angle BNF [?]; and EQ will be greater than the straight line EL, as was shown in the aforesaid Lemma 41.

Therefore, having described from the centre E of the given circle, through Q, an arc of a circle cutting the perpendicular KL at K and P: KEH [?] will be the semidiameter, and K the centre of the circle passing through F, G, [H] and touching the given circle at H on one side of the straight line FG. But P will be the centre of the other circle touching the given circle on the other side of the straight line FG, at the extreme point of the straight line drawn from P, through E, as far as the circumference of the given circle — as was demonstrated in the aforesaid Lemma 41.

We shall solve the problem in no other way if the given point be G, outside the circumference of the given circle HN, under the same condition. For, having again drawn from G, through E, the centre of the Astrolabe, the straight line GF, and having found the opposite point F — which will likewise lie outside the circle, provided G lies outside it and between it and its opposite parallel — we shall describe about GF, from its midpoint L, the semicircle GBF, and from L and E we shall erect the perpendiculars LK, EB. And again let the straight line GF pass

through the centre of the given circle. Then, having drawn from B to the extremity N (for example) the straight line BN, we shall complete the rest as before.

Diagram. A circular figure enclosed in a square border. The Equator ABCD is the outermost circle, with A at right, B at top, C at left, and D at bottom on the two perpendicular diameters through the centre E. A horizontal diameter carries (from left to right) C, F, N, E, L, Q, A, G, O; a smaller given circle and the constructed tangent circle FHG are drawn near the centre, with dotted construction arcs radiating from B. The vertical diameter BD passes through E.

- A (right, on the horizontal diameter): endpoint of the Equator
 - B (top): endpoint of the Equator / apex of the construction arcs
 - C (left, on the horizontal diameter): endpoint of the Equator
 - D (bottom): endpoint of the Equator
 - E (centre): centre of the Astrolabe / Equator
 - F (left of centre, on the horizontal): given point
 - N (right of F, on the horizontal): extremity of the given circle's diameter
 - G (far right, near the circumference): point opposite to F
 - O (just beyond G): extreme point on the produced line
 - I (upper right, near circumference): construction point
 - K (upper centre, above L): centre of one tangent circle
 - L (centre, on the horizontal near E): midpoint of FG
 - Q (below L): construction point
 - P (below centre): centre of the other tangent circle
 - H (lower left): point of tangency
2. Let there next be a given non-greatest circle MIO, with the portion of the sphere within it — that is, toward the arctic pole E — greater than a hemisphere. (This will occur when the circle either embraces the whole Equator, or does not bisect it, provided the greater portion of the Equator is included within the same circle, as we showed in the scholium of proposition 6, number 9.) And let the given point F be outside the circumference of the given circle, yet lying within its parallel.

Again let a straight line be drawn from F, through E, the centre of the Astrolabe, through the centre of the circle, and let the point G be found opposite to F, which will also lie within the given circle. For if it fell outside, the point F would lie within the opposite parallel, and not within the given circle and its opposite parallel — which is contrary to hypothesis. For if G were outside the circle MIO —

that is, in the portion smaller than a hemisphere, namely the one contained outside the circle — its opposite point F would be in the opposite portion of the sphere, namely the one which lies within the opposite parallel.

Therefore, the straight line FG having been bisected at L, and the semicircle FBG having been described about FG, let there be erected from L the perpendicular LK to FG, and [from E] EB. Then, having drawn from B to the extremity O (for example) of the diameter of the given circle the straight line BO, let the angle OBQ be made equal to the angle BOF; and again EQ will be greater than EL, as was shown in Lemma 41. And having described from E, the centre of the given circle, through Q, an arc of a circle cutting the perpendicular KL at K and P: K will be the centre of the circle FIG touching the given circle at I — I being the extreme point of the straight line EK produced as far as the circumference of the given circle on one side of the straight line FG. But P will be the centre of some other circle touching the given circle on the other side of the straight line FG, at the extreme point of the straight line EP, produced as far as the circumference of the given circle, as we showed in the aforesaid Lemma 41.

In the same way we shall proceed if the given point be G, within the given circle which contains the portion greater than a hemisphere. For, having again drawn from G, through E, the centre of the Astrolabe, the line GF, and having found the opposite point F — which will also lie within the circle, provided G lies between the circle itself and its opposite parallel — we shall complete the rest as before, provided the straight line FG also passes through the centre of the given circle.

3. Furthermore, let there be a given non-greatest circle IMO, enclosing a portion of the sphere smaller than a hemisphere, whose centre is H. And let the straight line drawn from F, a given point outside the circumference of the given circle, through E, the centre of the Astrolabe, not pass through the centre H — whether that same line cuts the circle or not. Having then found the point G, which is opposite to the given point F, a greatest circle is to be described through the two opposite points F, G, tangent to the given circle. This will be done, by Lemma 41, as follows.

Having drawn from the given point F to the centre H the straight line FH, let there be described, from its midpoint S, through H, an arc of a circle cutting the given circle at I; and, the straight line FI having been drawn, let a third proportional FK be found to the two straight lines GF, FI; and the point K will fall either short of G or beyond G. Wherever at length it falls, the straight line KH having been drawn, let there be described, from its midpoint L, a circle through H cutting the given circle at O, N.

If, then, there be drawn from the given point F, through O — the point nearer to point I — the straight line FO, as far as the circumference, to the point M: the circle described through the three points F, G, M, from the centre Q (which lies on the perpendicular QR bisecting FG), will touch the given circle at M, as was demonstrated in Lemma 41. But if from the same given point F, to the point N — farther distant from I — the straight line FN be drawn, cutting the circumference of the given circle at P: the circle described through the three points F, G, P, from the centre R (which also lies on the perpendicular QR bisecting FG), will touch the same given circle at P, as was shown in the same Lemma 41.

Diagram. A crowded circular construction enclosed in a square border, with several intersecting circles and a number of dashed and dotted straight lines. The principal circle is drawn about the centre E, which lies on the horizontal diameter running C–E–A; a vertical line drops through B, E to D. The given circle (centre H) and the auxiliary circles cut one another at the lettered points.

- H (top centre): centre of the given circle IMO
 - B (centre, upper): construction point on the vertical
 - L (right of B): construction point
 - I (upper left, on the outer arc): intersection of the auxiliary arc with the given circle
 - Q (upper left): construction point
 - M (upper right): point of tangency
 - N (right, below M): second point of tangency
 - S (left of centre): midpoint of FH
 - O (centre-left): intersection point (nearer I)
 - P (centre-left, below O): construction point
 - C (lower left, on the horizontal): endpoint of the diameter
 - E (centre, on the horizontal): centre of the Astrolabe
 - G (right of centre): point opposite F
 - K (right): construction point
 - A (far right, on the horizontal): endpoint of the diameter
 - T (lower left): given point F region [?]
 - R (bottom): centre on the perpendicular QR
 - D (bottom right): endpoint of the vertical diameter
4. Not otherwise, by the same Lemma 41, shall we describe the tangent circle if the given non-greatest circle includes the greater portion of the hemisphere, and consequently — as we showed a little before, in number 2

— both the given point and its opposite lie within the same circle; as was demonstrated in that Lemma, when two points within the circle are given.

For let the given non-greatest circle be KLMN, whose centre is O, enclosing a portion of the sphere greater than a hemisphere. And let the straight line drawn from F, a point given within the circle, through E, the centre of the Astrolabe, again not pass through the centre O. Having then found the point G, which is opposite to point F through the diameter, G will also lie within the given circle, as we said in number 2. Therefore a greatest circle is to be described through the two points F, G, opposite through the diameter, tangent to the given circle. This will be done, by Lemma 41, as follows.

To the three straight lines FG, FH, FE, a fourth proportional FI having been found, the point I will necessarily fall [outside the given circle, as we there demonstrated]. Having drawn from I to the centre O the straight line IO, and it having been bisected [and a circle described from the point of bisection] [?] cutting the given circle at L, M: if, then, from L, through F, a straight line be drawn cutting the given circle at N, the circle described through the three points F, G, N (whose centre Q will lie on the straight line QR, cutting the straight line FG in half and at right angles) will touch the given circle inwardly at N, as was demonstrated in Lemma 41. In like manner, if from M, through F, a straight line be extended cutting the given circle at K, the circle described through the three points F, G, K, from the centre R (which lies on the same straight line QR, cutting FG in half and at right angles), will touch the given circle at K, as we showed in the same Lemma 41. Which was the thing proposed.

Diagram. A second circular construction (the lower figure on the page), enclosed in a square border and partly running off the foot. Several concentric and intersecting circles are drawn about the centre E, which lies on the horizontal diameter C–E–A(–G); the vertical B–E–D runs top to bottom, with a dotted construction line descending from K at the upper left.

- K (upper left): construction point
- B (top centre): point on the vertical near the top
- R (centre, near O): centre on the perpendicular
- O (centre): centre of the given circle KLMN
- Q (centre, below R): centre of the tangent circle
- P (centre-left): construction point
- C (left, on the horizontal): endpoint of the diameter
- E (centre, on the horizontal): centre of the Astrolabe

- F (lower centre): given point within the circle
- L (lower left): point of intersection
- M (bottom, left of centre): point of tangency
- N (bottom right): point of tangency
- D (bottom): endpoint of the vertical diameter
- G (right, on the horizontal): point opposite F
- A (far right, on the horizontal): endpoint of the diameter
- H (upper right, near circumference): construction point [?]

SCHOLIVM [Scholium]

[Margin: *Materia Astrolabii quae esse debeat* — "What the material of the Astrolabe ought to be." *Facies Astrolabii quae* — "What the Face of the Astrolabe is." *Dorsum Astrolabii quod* — "What the Back of the Astrolabe is." *Faciei Astrolabii constructio* — "Construction of the Face of the Astrolabe."]

1. Let us now explain in what manner the instrument on which the Astrolabe is described is to be constructed. Let there be prepared, then, from orichalc [brass], or copper, or some other solid material, a circle ABCD with centre E, of as great a size as we wish the instrument to have. On one side let it be hollowed out circularly, a rim [*limbus*] being left, so that on it the number of the hours and of the degrees may be described; but on the other side let it be flattened very accurately. Then let there be prepared several circular plates of brass or copper, of such size that they can conveniently be placed within the hollowed-out part, and enough of them to fill the concavity. This hollowed part, together with the rim and the plates — which they are accustomed to call *tympana* — is called by writers the Face of the Astrolabe; and its concave part contained within the rim is called the Mater [Mother], while the other part is called the Back [*Dorsum*] of the Astrolabe.

[Margin: *Limbi constructio in facie Astrolabii* — "Construction of the rim on the face of the Astrolabe." *Tympanorum in facie Astrolabii constructio* — "Construction of the tympana on the face of the Astrolabe."]

2. The Face, then, is to be constructed as follows. Let the rim be divided, by four circles described from the same centre of the face, into three spaces. In the outer space, divided into 24 equal parts, let the number of the hours be described, as appears in the figure. Let the middle space be cut into 360 degrees, beginning from the straight line BD. In the third and innermost space, finally, let there be placed the numbers of the degrees, whose

beginning is at the straight line BD, so that 90 degrees terminate on either side at the straight line AC.

Diagram. A woodcut of the complete Face of the Astrolabe. At the top is the suspension ring and the ornamented throne (*kursi*); below it the circular instrument with cardinal points A (top), C (bottom), B (left), and D (right) on the two perpendicular diameters through the centre E. The rim carries three concentric bands: an outer band numbered 1–12 twice around (the 24 hours), a middle band numbered by tens (10, 20, 30 ... 80) for the 360 degrees, and an inner degree band. Within the rim the stereographic projection for a pole-elevation of about 42° is drawn, showing the concentric tropics and Equator, the horizon and its parallels, the prime vertical and the verticals, and the hour-lines.

- A (top): cardinal point on the vertical AC
- C (bottom): cardinal point on the vertical AC
- B (left, on the horizontal): cardinal point on BD
- D (right, on the horizontal): cardinal point on BD
- E (centre): centre of the instrument
- K, V, X, Y (upper interior): projection points of the Equator / verticals
- Q, Z (upper centre-right): projection points
- R, T (on the horizontal, right of centre): projection points
- L, U (centre-left): projection points
- P, S (below centre): projection points
- M, H (lower centre): projection points
- G (left, on the horizontal near B): projection point
- Numerals 30 and 60 (repeated within the vertical circles): degree graduations; a small "44" [?] near the centre
- Rim: hour numbers 1–12 (twice) in the outer band; degree numbers 10–80 in the middle band

Below the figure: the degree-numbers begin from the straight line BD; and in the third, innermost space let the numbers of the degrees be placed, so that 90 degrees terminate on either side at the straight line AC.

3. Then, on the bronze plates prepared for this purpose, let there be described: the tropic of Cancer [☿] FGHI; the Equator KLMN; and the tropic of Capricorn [♋] QRST — from the given magnitude of the tropic of Capricorn [♋], as we taught in the scholium of proposition 4, number 1 — unless you first wish to describe the tropics from the given magnitude of

the Equator, and then from the tropic of Capricorn [♊], once described, to define the magnitude of the Mater.

After this, let there be described on one plate, for a given elevation of the pole, as many of the remaining circles of the sphere as can conveniently be drawn. We, for example, in the figure below, for a pole-elevation of 42 degrees — such as Rome roughly has — have described the Horizon LPN, with only two of its parallels VX, YZ; and about the Zenith O, [almucantars] which are 30 degrees apart from one another; the prime Vertical LON, with only four other Verticals aO, bO, dO, eO, likewise 30 degrees apart; and finally, below the Horizon, the circles of the unequal hours only, dividing the portions, both of the tropics and of the Equator below the Horizon, into 12 equal parts.

On the same plate could also be described, if desired, the circles of the celestial houses, as was taught in proposition 10, and the circles of the hours from the rising or setting of the Sun — which we have judged should not be described here, lest we confound the figure with so great a multitude of lines. And just as on one plate the aforesaid circles are described for a given pole-elevation, or for a given latitude of place, so on the others the same are to be delineated for other pole-elevations, namely those believed more likely to be of use.

Lastly, [there should be] one plate alone, on which only the Equator and the tropics are described, together with the Ecliptic most carefully distributed into signs and degrees, and with some of the stars — the superfluous parts having been cut away, in the manner of a net [*rete*], so that there remain only the Ecliptic with the names of the signs, and the numbers of the degrees, and the tips of the stars. Now, in each of the plates it is customary to leave a certain small tooth [*denticulus*] near the upper part F, which is inserted into a hole of the rim next to the same point F, so that the plates themselves are not carried round by the motion of the rete, but always keep the same position. The plate of the rete alone will lack this tooth.

so that it can revolve freely about the centre E. To this end, a small circle is to be cut out around the centre E — one that just fills that round hole — and is carried around within it. If, moreover, an armilla [suspension-ring] is fixed to the upper part of the astrolabe next to the point A, so that the astrolabe hangs freely suspended from it, and if a revolving rule is set at the centre of the astrolabe, one of whose outermost lines — the line they call the fiducial line — passes through the centre, then the whole face of the astrolabe is complete. This rule is called the index [ostensor]. It either reaches only from the centre to the edge of the limb, or else is twice as long, as the figure below shows. This rule is also usually divided

from the centre out to the tropic of Capricorn ♄ into degrees, in the following way.

[Margin: Construction of the suspension-ring and of the index.]

First, from the centre are transferred the semidiameters of the equator, of the tropic of Cancer ☊, and of the tropic of Capricorn ♄, out to A, B, C, taken from the astrolabe. Then, the semicircle LKN of the equator having been divided into 180 degrees, straight lines are drawn from N to each single degree, cutting EF, the semidiameter, into degrees; these are transferred onto the rule from the centre, and their numbers begin from A, the point of the equator, and run out toward each of the two tropics, as appears in the figure, where they proceed by tens of degrees. The purpose of these degrees is to indicate the declinations of the points of the astrolabe from the equator, and thus to serve the office of all the parallels of the equator.

Diagram. The index-rule (*ostensor*/alidade for the face), shown in two forms. The upper rule is a graduated straight rule ending in a circular boss (with an offset sighting notch at the left end) at the right; along its length runs a scale of degrees.

- Left half of scale: 40, 30, 20, 10 (declination degrees running toward the centre)
 - Right half of scale: 10, 20, 30, 40, 50
 - C (left end of graduated portion), A (mid, at centre), B (to the right of A) — the three points named in the text
 - "OSTENSOR." (engraved caption beneath the upper rule)
 - Below: a second, plain rule (the simple index without graduation) terminating in a circular boss with a central hole/boss at its right end
4. The **back of the astrolabe** [dorsum] is to be constructed as follows. First, the outer limb is to be divided by five circles into four spaces, and in the outermost the numbers of the degrees into which the next space is divided are to be placed. Beginning is made at the points B and D, and one proceeds toward A and C, so that at A and at C "90 degrees" is written. In the third space are to be written the numbers of the degrees running by 30s for the signs, whose beginning is at the point D. Finally, in the last space the signs themselves are to be painted, as you see in the figure.

[Margin: Construction of the back of the astrolabe. Construction of the limb on the back of the astrolabe.]

5. **Next**, three further spaces are to be prepared by four circles: for the days of the months in the uppermost space, and for their names to be placed in the lowest, which is usually done in two ways. For these four circles are either concentric with the previous five, or eccentric. Those who make them concentric apply the rule to the centre E and to the 10th degree of Capricorn ♐, and draw the line SK through those three spaces for the beginning of January — because, as the ephemerides teach, the Sun on the first day of the year stands at the 10th degree of Capricorn ♐. Then, from the same ephemerides, they draw another straight line for the position of the Sun on the 5th of January. They do the same for the 10th, 15th, 20th, and so on, until they reach the end of the year, and produce 73 spaces, which, subdivided into 5 equal parts each, will give the 365 days of the whole year. Finally, in the third space they inscribe the names of the months, and the number of days according to the succession of the signs, assigning to January 31 days, to February 28, to March 31, to April 30, and to the remaining months their proper numbers of days. We have not appended an example of this division, both because it is easy and because it is generally found among the writers on the astrolabe, especially in Johannes Stöffler [Ioannes Stophlerinus].

[Margin: Description of the months and days on the back of the astrolabe by concentric circles.]

6. **Those, however, who prefer to draw eccentric circles** — so that they need not investigate the position of the Sun every five days — follow this method. They look for the position of the Sun's apogee [aux], which at this time is at the 9th degree of Cancer ♋, and from it they draw the semidiameter RE, and bisect it at F; again they bisect EF at G, again EG at H, again EH at I, and finally EI at t, so that Et is one of the 32 particles into which the whole of RE is divided. For it thus comes about that the ratio of Rt to tE — namely 31 to 1 — is very nearly the same as 60 to 1 $\frac{14}{15}$, which is the ratio that the semidiameter of the Sun's eccentric has at this time to the eccentricity, since the eccentricity contains 1 part and 56 minutes, of which 60 are contained in the semidiameter of the eccentric. In reality, however, the ratio 31 to 1 is a little smaller than 60 to 1 $\frac{14}{15}$; but since the discrepancy is very slight, the particle Et may fairly be taken for the eccentricity at this time. When the quantity of the eccentricity is found to have changed, the straight line ER is to be divided at t so that the ratio of Rt to tE is the same as 60 to the eccentricity — as at this time to 1 part and 56 minutes. This is done as follows. The straight line ET being drawn, let 116

equal particles be taken off with the compass from E as far as a — that is, 1 part and 56 minutes,

[Margin: Description of the months and days on the back of the astrolabe by eccentric circles.]

Diagram. The construction diagram for the calendar side and shadow-square on the back of the astrolabe: a large circle enclosed in a square frame, with concentric graduated rings carrying degree scales (numbered 10, 20, 30, 40, 50, 60, 70, 80, 90) and a ring of month names, together with a shadow-square (*umbra recta/umbra versa*) built into the lower-central region.

- A (top of the vertical diameter)
- B (left end of the horizontal diameter)
- C (mid-left, on the horizontal diameter, at the graduated scale)
- D (lower right)
- E (centre)
- T (upper left, at the end of a straight line running to the rim)
- F, G, H, I (points descending along the upper vertical radius toward the centre — the successive bisection points of §6)
- Month-name ring (reading around): IVL. [printed "IAI"], AVG., SEP., OCT., NOV., DEC., IAN., FEB., MAR., APR., MAI., IVN.
- Shadow square, lower centre: "VMB. VER." (*umbra versa*) on the vertical sides with scale 3, 6, 9; "VMB. REC." (*umbra recta*) on the horizontal side with scale 9, 6, 3; corner/side points M, N, and below Q, S, P
- Inner numeral fields at the rim (degree/day subdivisions): 10, 20, 30, 40, 50, 60 [several small edge numerals [?]]

1 part and 56 minutes, which make 116 minutes. First, then, let 10 be taken. This little line taken six times will give 60; and with the same little line added once more, there will be 116 particles. After this, from these particles [the division is made] — namely first by tens, then this line of 10 particles six times over.

[Margin: (Euclid, *Elements* VI.2)] Let there be, therefore, in aT, 60 parts, of which aE contains 1 and 56 minutes; and when the straight line TR has been drawn, let a parallel to it be drawn through t, and the eccentricity will be [as aE is to aT]. For since Ta is to aE — that is, 60 to the eccentricity — so is Rt to tE. But since it cannot be that the straight line ET should take up in the assumed [figure] exactly so many particles that Ea contains 116 and aT 360, you will do better if on another plane you draw a line long enough [Margin: (Euclid, *Elements* V.15)] that you may cut it into those parts. For if you take some aliquot part of it — say a half,

a third, a fourth, a fifth, and so on — which can conveniently be transferred from E as far as T, and transfer the same aliquot part of that segment which [contains] 116 particles from E onto a, and, having joined the straight line TR, draw a parallel to it through t, you will have the point t as before. [Margin: (Euclid, *Elements* VI.2)] For it will be that, as that whole line is to the segment of 116 particles — so its fifth part, for example ET, is to Ea, a fifth part of the said segment. Therefore, by division, as the semidiameter of the eccentric is to the eccentricity, so is Ta to aE; and by the same proportion so is Rt to tE. From the centre t, then, at the interval tR, they describe the eccentric circle, and below it three others, and they divide the uppermost space into days in this way. They find the beginning of January at K, just as those do who describe concentric circles. Then they apply the rule to the centre E and to the 4th degree and 40th minute of Capricorn ♑ — that is, to a point which is distant a little from the 10th degree of Capricorn ♑ toward the beginning [of the sign] — [namely at] 5 degrees 20 minutes, and they mark the point L on the eccentric, because the little space KL corresponds to the $5\frac{1}{4}$ days on which the Sun stands opposite the apogee. The remaining arc KRL takes in the remaining 300 days of the year. The arc [KL is divided?] into $5\frac{1}{4}$ — that is, into 21 parts, of which 20 are due to the five days, and [the rest to the quarter part of a day]; thus the whole eccentric is distributed into 365 days and 6 hours. Finally, the months are inscribed as before.

7. **For all this an altimeter scale is to be constructed** in this way. From E a circle is drawn tangent to the last eccentric at V; two semidiameters EO, EP are drawn to the 45th degree of the limb, cutting the described circle at O and P. When OP has been joined, cutting EC at Q, let EM and EN be cut off equal to QO and QP, and let the straight lines OM, PN be joined. The four lines MO, OQ, QP, PN being then divided into 12 equal parts, and three sets of lines being drawn equidistant from them and enclosing three spaces, let there be painted in the outermost space twelve parts running toward the centre E; in the middle space let the number of the parts be set down, so that 12 occupies the corners O and P; and in the last space let the shadow, right and reversed, be written — the right shadow [umbra recta] on the side OP, the reversed [umbra versa] on the sides OM and PN.

[Margin: Construction of the altimeter scale on the back of the astrolabe.]

8. **Likewise, when the two quadrants XY and XZ have been divided** each into six equal parts, and arcs of circles have been described through the centre E and through pairs of points equally distant from the diameter CD — whose centres lie on the diameter AC — and the last is described whole

about the diameter EX, there will be had on the back the 12 unequal hours, as appears in the figure.

[Margin: Description of the unequal hours on the back of the astrolabe.]

9. **Lastly**, at the centre E is set the revolving alidade [mediclinium], which is nothing other than the whole index just described, but with square perforated plates fixed at its ends, from which the pinnules [pinnacidia] are drawn. And this whole alidade the astronomers are also accustomed to call the dioptra.

[Margin: Construction of the alidade, or dioptra, on the back of the astrolabe.]

Diagram. A circular astrolabe plate for the right sphere [sphaera recta], set within a square frame. An outer ring carries the hour numerals; within it lies a network of straight lines and arcs (horizon, meridian, almucantars, verticals/azimuths, and the projected hour-circles), with the shadow/twilight lines drawn below the horizon.

- Hour numerals around the rim: X (top), XI, IX, XII, VIII, VII, VI (foot), V, IIII, III, II
- Rim degree numerals: 90 (at top), 60, 30 (both sides)
- E (centre)
- A, C (on the horizontal diameter, the right horizon AC)
- B, D (the meridian / vertical points)
- F, G, H (near centre, on the vertical, the almucantar/twilight construction points)
- "LINEA" (labelling the diagonal line running down to the lower left)
- "CREPVSCVLI" (labelling the diagonal twilight line running down to the lower right — i.e. "LINEA CREPVSCVLI", the twilight line)
- Scattered small numerals 1–17 and near-centre figures (24, 12, 6, 20, 60) marking the hour-circle divisions [several [?]]

[Margin: What things in the astrolabe are common both to the sphere that is oblique and to the sphere that is right and most oblique, under the pole.]

11. **Next, in the right sphere:** since the horizon passes through the poles of the world, and is projected into a straight line drawn through E, the centre of the astrolabe — which is also the pole of the world — as was shown in Proposition 1, let the straight line AC be the right horizon, and let the straight line BD, standing at right angles to it, represent the meridian circle.

And because in that sphere the equator ABCD is the primary vertical, the point B — receding 90 degrees on either side from the horizon AC — will be the vertex of the head, that is, the pole of the horizon and the point opposite the zenith; or [alternatively] the other pole of the horizon is D.

The **almucantars** [Almucantarath] — that is, the parallels of the right horizon — are described as we set forth in Proposition 7, numbers 2 and 3. As you see in the figure, two are drawn about the zenith B, one of which is distant from the horizon, and the other from it and 30 degrees from the zenith.

The **azimuths** [Azimuth], or vertical circles, are described as in the oblique sphere. For if the equator ABCD — that is, the primary vertical — is cut into as many equal parts as there are verticals to be drawn, and through the points of division straight lines are drawn from B or from D,

[Margin: Construction of the astrolabe in the right sphere.]

the straight line AC will be cut at the centres of the verticals that are to be drawn through B and D, and these will cut the right horizon AC into degrees, just as in the oblique sphere in Proposition 8. The vertical circles are divided into degrees by the parallel to the horizon represented by the straight line PQ, as was there demonstrated. In this figure we have drawn four verticals, 30 degrees distant from one another.

The **hour-circles** of every kind are here represented by straight lines drawn from the centre E through every fifteenth degree of the equator and of its parallels. For since the right horizon, and the circles of the hours from noon and midnight, are drawn through the poles of the world, the circles of the hours from rising and setting, and of the unequal hours, also pass through the same poles — the former [i.e. the horizon] because there is no parallel touching the horizon which they might touch, but the latter so that both the semicircles of the diurnal parallels and those of the nocturnal may be distributed into 12 equal hours; and these indeed can have their beginning either from noon and midnight, or from rising and setting. Since, therefore, all the great circles falling through the poles of the world are projected into straight lines, drawn as we said, [these lines] represent the hour-circles of every kind. These lines we have drawn only below the right horizon AC, and within the tropics, lest the multitude of lines above the horizon should present us with confusion. Furthermore, the numbers next to the tropic of Capricorn ♎ pertain to the hours from noon and midnight; those next to the equator, to the hours from rising and setting; and those next to the tropic of Cancer ☊, finally, to the unequal hours.

[Margin: In the right sphere the same greatest circles indicate both the hours from noon and midnight and those from rising and setting, and also the unequal hours.]

The **celestial houses** [domus coelestes], whether according to the opinion of Johannes Regiomontanus [Ioan. Regiom.] or according to Campanus [of Novara], are projected like the hour-circles. For their circles too pass through the poles of the world — namely through the common sections of the horizon and the meridian — and are therefore projected into straight lines: these we have drawn through the whole astrolabe, dividing both the equator, as Regiomontanus wishes, and the primary vertical, as Campanus prefers (which here does not differ from the equator), into 12 equal parts.

Diagram. A circular astrolabe plate for the right sphere within a square frame, of the same type as the plate on the preceding page, showing the horizon, meridian, four verticals, almucantars, and the projected hour- and house-lines, with the twilight and index lines drawn below the horizon.

- Hour numerals around the rim: X (top), XI, IX, XII, VIII, VII, VI (foot), V, IIII, III, II
- Rim degree numerals: 90, 60, 30
- E (centre)
- A, C (horizontal diameter — right horizon AC)
- B, D (vertical / meridian points)
- F, G, H (near-centre construction points)
- "LINEA" (diagonal line to lower left)
- "CREPVSCVLI" (twilight line, diagonal to lower right)
- Scattered internal hour-division numerals 1–17 and near-centre figures (24, 12, 6, 20, 60) [several [?]]

The **line of twilight** [linea crepusculi], finally, will be described no otherwise than the circles of altitude, that is, the parallels of the horizon — since the circle in which morning twilight begins and evening twilight ends is parallel to the horizon, distant from the horizon toward the [nadir] by 18 degrees. And so, if from A and from C, on the equator below the horizon, 18 degrees are reckoned as far as G and F, and from A a straight line is drawn through F, cutting the meridian line at H, there is to be described a parallel — that is, the line of twilight, or of dawn [aurora] — through the three points F, H, G, having its centre on the meridian line ED, produced.

[Margin: Construction of the astrolabe in the most oblique sphere.]

12. **But in the most oblique sphere** — which has the vertex of the head at the arctic pole — the parallels of the equator are to be drawn as far as the [equator] only, that is, only the northern ones; for since the equator is there the horizon, the parallels between the equator [and the pole] have no use, except the one in which morning twilight begins and evening twilight ends. In the following figure the equator is ABCD; the tropic of Cancer ☿ and the arctic circle are two circles marked off by points — that is, the one nearest the equator and the one nearest the centre E.

The **horizon**, as has been said, does not differ from the equator, and therefore its parallels are described like the parallels of the equator: so that when the quadrant BC is divided into 90 degrees, if from A straight lines are drawn through each single degree, the line AC will be cut at the points through which, from the centre E, the almucantars are to be described. In the figure two are described, whose semidiameters are cut off by the rays AF, AG.

The **verticals**, since they run through the poles of the world — namely through the poles of the horizon — are projected into straight lines passing through the centre E, as was shown in Proposition 1. For which reason straight lines drawn through the centre E, dividing the equator (that is, the horizon of the astrolabe) into 360 equal parts, will stand for all the verticals. In the figure we have drawn the verticals fifteen degrees distant from one another.

[Margin: In the most oblique sphere the greatest circles are not properly the hour-circles from noon or from midnight, or from rising and setting, or [the] unequal [hours].]

The **hour-circles** are likewise straight lines, dividing the equator and its parallels into 24 equal hours, since they also run through the poles of the world; and they can have their beginning at any point whatever, as at the straight line BD, which we have assumed in the astrolabe for the meridian line. Now hours of this kind indicate the twenty-fourth parts of one whole revolution of the equator, begun from some fixed point, and not from rising or setting, since there is perpetual day there, the Sun being in the upper hemisphere, and so neither rising nor setting, nor noon nor midnight, can be assigned — if we wish to speak properly. Nevertheless one may, at pleasure, take the straight line BD for the meridian line, and AC for the primary vertical, and accordingly [take] the point C, in a manner, for the rising, and A for the setting, and so on.

[Margin: In the most oblique sphere...]

The **circles of the celestial houses** cannot be inscribed in this astrolabe, because neither the true rising and setting [can be assigned], nor can the equator be divided by great circles through the common sections of the meridian — even taken at pleasure — and the horizon...

of the horizon, which is the same as the equator, running along, as is evident. But if we call the points C and A the rising and the setting, and the line BD the meridian, then — following the view of Campanus — the circles of the celestial houses will be described, just as the verticals are in the right sphere. For if the primary vertical is conceived to be ABCD, perpendicular to the plane of the astrolabe and making in the astrolabe the line AC, and if straight lines are drawn from B or from D through 12 equal parts of it in that position, the vertical line AC will be divided at the centers of the circles of the celestial houses, all of which will pass through the points B and D. For just as in the right sphere the circles that have their centers on the line AC — that is, on the right horizon — and run through the points B and D (namely through the zenith of the head and the point opposite to it) divide the right horizon into its degrees, so too these circles, passing through B and D, being the common sections of the horizon ABCD and of the assumed meridian, divide the vertical line AC into the 12 celestial houses, and so on.

[Margin: ... there are properly no circles of the celestial houses.]

Diagram. A circular astrolabe face (the plate for the most oblique sphere), shown with the twilight line. The outermost ring is inscribed with the legend **LINEA CREPVSCVLI** ("the twilight line"), the words running around the circumference (reading from the left, up and over: LINEA · CRE · PVSCV[LI]). The circumference is divided into 24 numbered divisions, labeled 1 through 24 (the twenty-four hours). Within, concentric rings carry degree-numbers in the radial sectors — repeatedly **90, 75, 60, 45, 30, 15**. A set of radial lines runs from the rim to the central point.

- Center: **E**
- Upper-left, near the "90" division: **B**, with **H** adjacent
- Lower division: **D**
- Lower semicircle: the Roman numerals **I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII**
- Small interior letters **F** and **G** are also present
- Several of the small interior numerals are only partly legible [?]

DENIQVE — Finally, since the twilight line represents a parallel of the equator — that is, of the most oblique horizon — inclining toward the opposite pole and 18° distant from the equator, it is projected onto the astrolabe in this way. From B, toward the antarctic pole A (because the parallel described through the beginning of morning twilight and the end of evening twilight is southern in this most oblique sphere), let 18° be counted off up to H; and from A let a ray be drawn through H, cutting the straight line BD at I. For the circle described from center E through I will yield the twilight line — that is, the parallel depressed 18° below the horizon — as is clear from what has been demonstrated.

13. Moreover, this same astrolabe will also serve those who live under the antarctic pole, if the center E is taken for the antarctic pole, the tropic of Cancer ☿ for the tropic of Capricorn ♑, and the arctic circle for the antarctic; likewise if the individual signs of the zodiac are interchanged with their opposites, so that from Aries ♈ comes Libra ♎; from Taurus ♉, Scorpio ♏; Sagittarius ♐ [?] from Gemini ♊; Capricorn ♑ from Cancer ☿; and so on. For with the eye placed at the opposite pole — namely at the arctic (for the eye must be set there in order that the astrolabe be described for the southern sphere) — the antarctic pole is seen at E, and the tropic of Capricorn ♑ in the same form in which the tropic of Cancer ☿ is seen from the arctic pole, and so on.

14. In the same way the astrolabe will be adapted to any oblique sphere of the antipodes of those people, for whom the antarctic pole is raised above the horizon, provided the same exchange of the northern signs into the southern (and conversely) is made. But the stars are to be placed differently on the rete — the southern ones, namely, near the center, that is, near the antarctic pole; and so on. What has been said about the rete must also be said for the astrolabe of the most oblique southern sphere: because in constructing an astrolabe of this kind the eye is set at the north pole, so that the southern pole appears at the center E, as has been said.

[Margin: How the astrolabe of the most oblique northern sphere is fitted to the most oblique southern sphere.]

[Margin: How the astrolabe of any oblique northern sphere is fitted to the oblique southern sphere.]

15. Now, just as up to this point we have described in the plane of the equator all the celestial circles in the form, and with the mutual distance of one from another, in which they are seen from the southern pole, so the same

circles can be described in the plane of any great circle whatever, in the form and distance in which they appear from its lower pole, provided the circle of the analemma — in which the diameters of the circles are contained — is taken as the proper meridian of that great circle, that is, as the circle through the poles of the world and through the poles of that great circle. For example: if in the first figure of Proposition 4 the line BD is taken as the diameter of the horizon; A as its lower pole, or the nadir; and C as the upper pole, or the zenith; fg as the diameter of the equator; O as the north pole of the world (since it lies nearer to the vertical point C), and R as the southern pole; and so on — then the horizon will appear at the size of the circle ABCD, and the zenith at the center E; and its parallels will be described just as the parallels of the equator were described before; while the equator, with its own parallels, will be projected...

[Margin: Description of the astrolabe in the plane of any oblique great circle.]

Diagram. A rectangular framed figure (an analemma / orthographic projection of the oblique sphere). Within the frame an upper semicircle stands on a horizontal base-line, with numerous straight and dotted construction lines converging on a point below the base at bottom center. The following labels are legible:

- Top of the semicircle: **C**
- Upper-left arc: **P**, with **G** below it
- Upper-right arc: **K**, **O**, and **F** (descending along the right side)
- On the base-line, from left to right: **X**, **M**, **D**, a cluster **L · T · m**, the center **E**, a cluster **L · K · S**, and **B**
- Base-line continuing to the right: **M**, then further labels toward the margin
- Lower region (below the base, toward the converging point): **I**, **R**, **Z** (left), **A** (bottom center, the pole toward which the lines converge), **Y** (bottom right)
- Several small lowercase and Greek-letter labels along the upper arc and the right base (including apparent zodiac symbols and letters α , η , d , ρ) are too small to resolve with confidence [?]

will be projected onto the plane of the astrolabe, just as before the oblique horizon [was projected] with its own parallels, so that mn is the apparent diameter of the equator, and the north pole O appears at S, and the southern pole R at X; while all the verticals will be projected into straight lines running through the center E; just as before the hour circles and the circles of declination, passing through the poles of the world, and so on. And in this way indeed the astrolabe will be described in the plane of the horizon, but not in the plane of the equator.

This is readily understood from what has been demonstrated, and will be perceived more clearly in Book 3, Canon 12, and in several other places that follow, in which the circle ABCD — which up to now in the astrolabe was the equator — will represent the horizon, and so on. But in Canon 15, Number 8, we shall describe the astrolabe in the plane of the ecliptic.

16. But neither should this be omitted: that the terrestrial globe, with all its circles and towns, can be described in the fashion of an astrolabe — namely in the form which the astrolabe had in Number 12 for the most oblique sphere. For the equator will be ABCD; the circles of longitude, or of the meridian, will be represented by straight lines drawn through the center E; and finally the non-greatest circles of latitude will be described as parallels of the equator. And so, if the position of some city is sought, we shall take, for example, the line ED as the meridian of the [Fortunate] Isles, from which the cosmographers take the beginning of longitudes, and from it, toward the right, we count off the longitude of the proposed city, and draw a line for the meridian of that city. Then we describe a parallel of the equator for the latitude of that same city — which, if it is northern, we count from B toward C; but if it is southern, from B toward the opposite side. For where this parallel intersects the meridian, or the line drawn from E through the longitude of the city, there will be the location of the proposed city.

[Margin: The description of the earth in the form of an astrolabe.]

But since the more southern places — namely those that run out beyond the tropic of Capricorn ♎ — can only with difficulty be described on the astrolabe, we shall do conveniently if we draw two maps: one from the equator toward the north pole E, as we have said thus far; and the other from the equator toward the south pole, which the center E then represents, and so on. But these matters will become clearer in Book 3, Canon 15, where we shall investigate the distances of places.

Diagram. A circular astrolabe face of the same design as the figure on p. 261, here illustrating the earth described in the form of an astrolabe. The outer ring is inscribed **LINEA CREPVSCVLI** ("the twilight line"), the circumference divided into 24 numbered divisions (1–24), with inner concentric rings carrying the degree-numbers **90, 75, 60, 45, 30, 15** in the radial sectors, radial lines converging on the central point, and the Roman numerals **I–XII** along the lower half. The small interior numerals are only partly legible [?].

FINIS SECVNDI LIBRI.

[END OF THE SECOND BOOK.]

BOOK THREE OF THE ASTROLABE.

ASTROLABII LIBER TERTIVS.

By the author CHRISTOPH CLAVIUS [Christoph Clavius] of Bamberg, of the Society of Jesus.

[Margin: The argument of the third book.]

There remains the third book, and the last, in which we must treat of the manifold use of the circles that we described on the astrolabe in the preceding book. In this matter all our care and effort must be applied, so that what others investigate by means of a material instrument, we may inquire into by compass and rule alone — and indeed far more surely and accurately. Yet we shall not entirely neglect the common use of the material astrolabe, but shall explain it, where it can conveniently be done, in the principles of the Canons. (For we shall not always be able to do so, since a great many more things can be found without the instrument, by the benefit of the astrolabe.) [We do this] in order to satisfy especially those who take delight in the astrolabe. And so, that what we have set before ourselves in this third book may be understood more plainly, let there be set before the eyes some globe so carefully turned that nothing rounder could be made. Then, just as on it we might measure out all the intervals of the points, the magnitudes of the arcs and of the angles, the inclination of one circle to another, and other things of that kind, so we shall attempt exactly the same thing on some plane surface — so that there is nothing at all which anyone might wish to know in the *primum mobile* that he could not, by our precepts, ascertain most perfectly in the plane. Thus whatever the astronomers extract, even from the science of spherical triangles (which is boundless and well-nigh infinite) by the most tedious multiplications and divisions of numbers — with wonderful skill, indeed, and industry — that same thing it will be no less exactly permitted us to draw out, to trace, and to examine in some plane space, by the benefit of the circles that were described in the preceding book. And that this may be rendered more complete and perfect, we shall in most places also add the use of the analemma, by which not a few astronomical problems are solved with a sometimes marvelous ease and pleasantness. Nor indeed shall we omit to teach how some of the things we have proposed may be investigated by the doctrine of sines as well. But what can

be acquired by this new use of our astrolabe, the Canons that follow will show far more clearly than many roundabout words could explain. Therefore let us proceed at once to the Canons themselves.

CANON I.

To determine the altitude of the Sun and of the other stars at any moment of time.

[Margin: How the altitude of the stars is to be explored by means of the astrolabe.]

1. Let the astrolabe be suspended from its ring [armilla] so that it hangs freely, and let the point B be directed toward the Sun or a star; and let the alidade [mediclinium] on the back of the astrolabe be turned up and down about the center E for so long, until the Sun's ray passes through the corresponding holes of the sights [pinnacidia], or until the eye, through the same holes, catches sight of the star — or even, at times, of the Sun itself, when it is covered by clouds — and the alidade holds, say, the position of the line FG. I say that the degrees contained in the arc BF indicate the altitude of the Sun, or of the star — that is, that just as many degrees as are included in the arc BF, so many are intercepted between the Sun (or the star) and the horizon on the vertical circle drawn through the Sun or star at the time of observation. For since, as we demonstrated in [the treatise on] the sphere, the earth, when compared with the heavens, is as a point, the center E of the astrolabe will be the same as the center of the earth, or of the heaven itself; and the instrument is therefore placed in the plane of the vertical which passes through the Sun (or star) at that time. Since, then, the line BD is equidistant from [parallel to] the horizon, and the straight lines drawn from concentric circles cut off similar arcs — as we showed in the scholium to Proposition 22 of Book 3 of Euclid — the straight lines EB, EF, produced all the way to the heaven, will intercept as many degrees as are contained in the arc BF. Therefore, since EF, drawn to the Sun or the star, reaches it, the arc BF will indicate the degrees intercepted between the star and the horizon on the said vertical.
2. But since it is troublesome to raise and lower the alidade so many times, until the Sun's ray penetrates through the holes of the sights, or the eye catches sight of the star, a more convenient and better-suited instrument for taking the altitudes of the stars will be a quadrant of the circle EHG, on whose side EG two sights [pinnacidia] are affixed, and whose numbering of

90 degrees begins from H toward G, proceeding onward, and at last from the center E a thread with a plummet hangs. For...

[Margin: The quadrant, a more convenient instrument for taking the altitudes of the stars than the astrolabe.]

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*(Printed pages 264–266; *

For if the side EH of such a quadrant is aimed at the Sun or a star, and the quadrant itself is raised and lowered about the centre E as about a hinge — the plumb-thread grazing its flat surface — until the Sun's ray enters through the holes of the sighting-vanes [pinnacidia], or the line of sight looks at the star through the same holes (which is in fact done more easily and expeditiously with a quadrant than with an astrolabe, as experience teaches), the plumb-thread will cut off the arc HC of the star's altitude. For since the ray GE, when produced, reaches the star, the arc BF will show its altitude, as has been demonstrated. Since therefore BF and HC are equal — because the whole quadrants FH, BC are also equal, and the arc BH removed [from each] is common — HC too will be the arc of the star's altitude. There is also another reason why in this matter I prefer the quadrant to the astrolabe: namely that, for the altitude to be taken by the astrolabe, the instrument must have uniform weight, so that, whatever position the alidade [mediclinium] is in, the line AC points exactly toward the centre of the world — which usually does not happen, since an instrument can easily have more weight on one side than on another.

[Margin: How the sighting-vanes are to be constructed.]

Diagram. A square-framed woodcut of the sighting apparatus and the graduated instrument. Across the top are the two sighting-vanes [pinnacidia] flanking a central suspension ring, from which the instrument hangs; below them is the graduated circle inscribed in the square frame.

- K, K (top, the twisted suspension ring at the apex)
- I, I (lower inner corners of the two square vane-tablets)
- L (small central hole in each vane, on the thin supporting diameter)
- M (small hole on the rotating tablet of each vane, corresponding to L)
- A (top of the circle, at the suspension point)

- F (upper left of the circle)
- B (left end of the horizontal diameter)
- E (centre of the circle)
- D (right end of the horizontal diameter)
- G (lower right of the circle)
- H (lower left of the circle)
- C (bottom of the circle)
- Straight lines run from the centre E to A, B, C, D, F, G, and H; the diameters BD (horizontal) and the line through A–E–C, together with the radii EF and EG, mark off the altitude arcs described in the text.

3. Moreover, when the altitude of a star must be investigated by the line of sight, two sighting-vanes [pinnacidia] should be constructed in this way. In a square tablet IK, make a large round hole, in the middle of which let a very tiny hole L be left, supported by some thin diameter [wire]; and about I let another square tablet, equal to the first, be turned, in the middle of which is a very tiny hole M, corresponding to the hole L. If two sighting-vanes of this kind are made, it can hardly be said how conveniently one may observe any star, or any other object whatever. For the vane that is nearer the eye must be closed with that square tablet, while the other is opened. For thus it comes about that the line of sight, admitted through the hole M near the eye, at once looks through that hole L in the more distant vane at the star or other proposed object: because that large open hole lets us easily view the object itself, and, without any trouble, direct the tiny hole L at the object.

[Margin: To know whether a star is before the meridian, or after it, or stands on the meridian itself.]

4. But to know, when a star is near the meridian, whether it is found before it, after it, or actually on the meridian itself, the star's altitude must be taken twice or three times, with a short interval of time between two successive observations. For if the later altitude is found greater than the earlier, you may know that the star has not yet reached the meridian; but if less, that it has passed the meridian; and, when it was found to have its greatest altitude, that it was on the meridian itself. But how great the meridian altitude of the Sun is on any day, and of the stars in any clime, we shall teach below in Canon 3, Num. 8.

SCHOLIUM

[Margin: In what way, in the altitude of the stars, minutes are taken besides the degrees.]

Since on a quadrant or an astrolabe only whole degrees are marked, it happens that the altitude of the stars is determined exactly only when the plumb-thread, or the fiducial line of the alidade [Mediclinium], falls on some whole degree. For when the thread, or the fiducial line, falls within the part of a degree, there must be added to the whole degrees of altitude as many minutes as by estimation may be judged, more or less, to be cut off by the thread or the fiducial line: so that if half a degree seems to be cut off, add 30 minutes; if a third part, 20 minutes; and so on. Or else the number of minutes must be found by the compass from the cut-off portion, as we taught in Lemma 3, and in chapter 2 of book 1 of the Practical Geometry.

CANON II

To find the true place of the Sun in the Zodiac.

[Margin: To find the Sun's place on any day by means of the astrolabe.]

1. On the back of the astrolabe are marked the days of the months with the corresponding degrees of the Zodiac in which the Sun stands on those days, more or less. So if the fiducial line of the alidade [Mediclinium], or a fine thread from the centre E, is drawn out to a proposed day of the month, that same fiducial line, or thread, will indicate on the circle of signs the sign and degree in which the Sun stands on that day. Thus you see on the back of the astrolabe, which we constructed in the scholium of the last proposition of the preceding book, that a line drawn from the centre E through 20 July indicates degree 27 of Cancer ☊, and some minutes besides. We shall therefore say that on 20 July the Sun is found beyond the 27th degree of Cancer. Conversely, from the known degree of the Sun we shall learn the day of the month. For the same line, extended from the centre through the Sun's degree, will pass through the corresponding day of the month. Thus, with the Sun at 27° Cancer ☊, if one wishes to know on what day of the year this occurs, let him extend a line from the centre through the said degree, and this will indicate approximately 20 July.

2. How we may find the Sun's place in the Zodiac from memory, we have explained in the *Sphere* [*In Sphaeram Ioannis de Sacro Bosco*] and in the *Roman Calendar* [*Romani Calendarii*], to which we refer the reader; for we think it not worthwhile to repeat the same things here.

CANON III

To investigate the declination of the Sun on any day, or of any point of the Ecliptic, and of the stars. And, conversely, from a given declination of the Sun, to find the corresponding arc or point of the Ecliptic; and hence to determine how great the meridian altitude of the Sun, or of any star, may be.

[Margin: To find, by means of the astrolabe, the declination of a proposed degree of the Ecliptic, or of any star.]

1. If the pointer [ostensor] on the face of the astrolabe is divided into degrees, as we taught in the scholium of prop. 20 of the preceding book, the declination of any point of the Ecliptic, or of a star, will be found by means of the astrolabe in this way. Let the fiducial line [of the pointer] be placed over the proposed degree of the Ecliptic, or over the tip of the star. For the degree of the pointer, falling on that degree

or star, will at once show the sought declination of that point: northern, if the degree of the Ecliptic, or the star, lies within the Equator — that is, if the degree found by the pointer inclines from the Equator toward the centre of the astrolabe; but southern, if the degree of the Ecliptic, or the star, lies outside the Equator — that is, if the degree found by the pointer recedes from the Equator toward the tropic of Capricorn ♎.

2. But if there is no pointer divided into degrees, turn the rete about until the proposed degree of the Ecliptic, or the tip of the star, falls on the meridian line. For when the rete holds such a position, the circles of the almucantars [Almucantarath] — that is, the parallels of the Horizon interposed between the degree of the Ecliptic (or the tip of the star) and the Equator — will count the degrees of declination: northern from the Equator toward the centre of the astrolabe, but southern from the same Equator toward the tropic of Capricorn ♎.

[Margin: Which points on the astrolabe have a northern declination, and which a southern.]

3. Conversely, to find, from a given declination, the corresponding arc or point of the Ecliptic, count the given declination along the meridian line, between the parallels of the Horizon, from the Equator either toward the north or toward the south. Then turn the rete about until the Ecliptic exactly coincides with the end of the count. For that degree of the Ecliptic, or point, will have that declination; and, besides, three other points, which share an equal distance from the equinoctial points with it, will have the same declination. Thus, if the beginning of Pisces)

(had been found, the beginning of Scorpio [♏? — glyph small] would have the same declination, together with the beginnings of Taurus ♉ and of Virgo ♍. For always four points of the Ecliptic — two northern and two southern — have the same declination, as we showed in Lemma 49, Num. 5, and shall also demonstrate in another way a little later, in Num. 6.

[Margin: To investigate, from the astrolabe, the arc or point of the Ecliptic corresponding to a given declination.]

You will obtain the same by means of the Index, or of the pointer divided into degrees. For if you turn it about until the point terminating the declination touches the Ecliptic — whether this be done toward the north or toward the south — the given declination will fit that point of the Ecliptic, and, moreover, three others, as has been said.

4. But since a pointer accurately divided into degrees is rarely found, nor an astrolabe on which the parallels of the Horizon are drawn through the individual degrees with the requisite care, it necessarily follows that by either method the true declination cannot be found exactly, but only more or less, or approximately. Therefore, without an instrument, we shall find the arc of the true declination exactly — provided great care and diligence are applied in drawing the circles — by this device.

Let ABCD be the Equator of the astrolabe, of any size, about the centre E, with the tropics RT, QS; and the Ecliptic AQCR, tangent to the tropics at Q, R, with its centre H and pole G. Now let it be proposed to find the declination of the beginning of Pisces ♓. And since the sign Pisces ♓ is southern, and accordingly is contained in the southern semicircle AQC, and its beginning is 30° distant from Aries ♈, we shall count, from the point C, which is assigned to the beginning of Aries ♈, toward B, 30° up to a; and from the pole of the Ecliptic G, through a, we shall draw the straight line Ga, which cuts the Ecliptic in I; and I will be the beginning of Pisces ♓, since — as we demonstrated in prop. 5, Num. 17 of the preceding book — the arc CI is equal to the arc Ca, so far as degrees are concerned. Now, drawing from the centre E, through I, a straight line cutting the Equator in F, we shall take an arc BK equal to the arc CF, and we shall draw the straight line KI, which cuts the Equator in L. I say that FL is the arc of declination of the point I of the Ecliptic. For since the straight line EI represents the declination-circle drawn through I, the beginning of Pisces ♓, as we demonstrated in prop. 1, Num. 4 of the preceding book, the portion IF will correspond to the arc of declination — to which indeed the Equator's arc FL is equal. For if the circle ABCD is imagined to be the meridian, and to stand on the plane of the astrolabe along the line EI at

right angles, K will be the southern pole, since it is distant from the plane of the Equator, or of the astrolabe, by a quadrant FK; because, if to the equal arcs CF, BK the common arc FB is added, the whole arc FK becomes equal to the whole quadrant CB. From this it follows that the arcs FL, FI are equal, as was shown in prop. 1 of book 2, Num. 5.

[Margin: To find with certainty, without an astrolabe, the declination of a proposed degree of the Ecliptic, or of any star.]

Again, let it be required to find the declination of the star called Canis Maior [Sirius]. Its place M having been found on the astrolabe, as we taught in prop. 11 of book 2, Num. 2, by its longitude and latitude, let the straight line EM be drawn, representing the declination-circle, so that NM measures the southern declination of the star. Then, taking an arc DN equal to the arc AO, let the straight line OM be drawn, cutting the Equator in P; and, as was just demonstrated, NP will be the arc of the sought declination — that is, the arcs NM and NP will be equal.

5. Furthermore, we shall obtain the declination both of a given point of the Ecliptic and of a star in this way also. Through the found point I on the Ecliptic, from the centre E let an arc Ib be described, cutting the meridian line in b; and from A, or C, to b let a straight line be extended, cutting the Equator in d. For Bd is the arc of declination of the parallel bI, as we showed in prop. 4, Num. 7 of the preceding book, and hence also of the point I given on the Ecliptic. Which is what was proposed. Again, from the same centre E, through the centre of the star M, let an arc Me be described, cutting the meridian line in e; and from A, or C, to e let a straight line be drawn cutting the Equator in f: and, as was said, Df will be [equal] to the arc of declination of the parallel Me — that is, of the star M.

[Margin: A general rule for finding the declination of any point of the astrolabe.]

[Margin: To find the declination otherwise, without an instrument.]

6. By this same reasoning we shall find the declination of any point placed on the astrolabe; namely, if through that point we draw a straight line from the centre E, and from the point where it cuts the Equator we take a quadrant along the same Equator, and from the end of this we draw a straight line to the given point. For this line, and the former one drawn through the same given point, will intercept on the Equator the arc of declination. Thus you see that the straight line EM, drawn from the centre through the point M, together with the straight line OM, drawn from the point O, the end of the quadrant NO, to the same point M, intercepts NP, the arc of declination of the point M, as we showed. And the quadrant will be cut off on the Equator without any trouble, if

Diagram. A square-framed astrolabe/declination diagram: the Equator ABCD as a full circle about centre E, with concentric circles for the two tropics and the eccentric Ecliptic tangent to them; a network of straight lines and dotted construction-lines running to the several lettered points. An engraved cursive label "Canis maior." runs along the lower-right arc.

- Q (top of the outer circle; tropic-tangent point)
- b (on the meridian, just below Q)
- d, B (upper region, near the top of the vertical diameter)
- E (centre)
- H, G (near the centre, upper of the two just below B)
- K (upper right)
- Y (right)
- O (right, below Y)
- L, I (upper left / left)
- T (left of centre, below L)
- S (far left, on the horizontal diameter)
- C (left, inner from S, on the horizontal diameter)
- A (right, on the horizontal diameter)
- V, X (lower right)
- R (lower centre)
- N, P (lower centre and lower centre-right)
- D (bottom of the vertical diameter)
- F (lower left)
- M (at the foot, tip of the star Canis Maior)
- [Several interior labels are very small; the readings b, d, K, Y, O, T, V, X are given as read but flagged as the least certain for the peripheral labels.]

drawing the two diameters AC, BD cutting each other at right angles, we cut off — starting from the other diameter — an arc equal to the arc intercepted between one of them and the point where a straight line drawn from the centre E cuts the Equator; just as in the preceding examples the arc AO was taken equal to the arc DN, and the arc BK equal to the arc CF, so that the quadrants NO, FK might be obtained. The same quadrants will be had if the quadrant AD, or AB, or BC, or CD is transferred from N and F to O and K.

Diagram. A repetition of the same astrolabe/declination woodcut as on the preceding page: the Equator ABCD about centre E, with concentric tropic circles,

the eccentric Ecliptic, and the construction lines to the lettered points; the cursive label "Canis maior." along the lower-right arc.

- Q (top)
- b (below Q, on the meridian)
- d, B (upper middle)
- L, I (upper left/left)
- K (upper right)
- Y (right)
- H, G (centre, below B)
- O (right of centre, below Y)
- T (left of centre)
- S (far left on the horizontal diameter)
- C (left of centre on the horizontal diameter)
- E (centre)
- A (right on the horizontal diameter)
- V, X (lower right)
- R (lower centre)
- N, P (lower centre)
- D (bottom, on the meridian)
- F (lower left)
- M (at the foot, tip of Canis Maior)
- [Peripheral small labels flagged as least certain]

Or indeed we shall find the declination of any point if, from the centre E, we describe through the given point a parallel of the Equator, and to the point where it cuts the meridian line BD we send out a straight line from A, or C. For this will cut off from the Equator the arc of declination, beginning at the meridian line, as we said concerning the point I and the star M.

And so, if the Ecliptic is divided into signs and degrees, it will not be necessary that the distance of the given degree of the Ecliptic from the nearest equinox be counted on the Equator, in order that its position on the Ecliptic may be found by a straight line sent out from the pole G — the way in which the position I, the beginning of Pisces)(, was found, by the straight line Ga; but it is enough that from the centre E a straight line be drawn through the proposed degree, and, beginning from this, a quadrant be cut off on the Equator, and so on. Or indeed let a parallel of the Equator be described from the centre E through the proposed

degree, and so on. It is likewise enough that the declinations of the points of one quadrant of the Ecliptic — say the quadrant CQ — be investigated. For these declinations are equal to the declinations of the points in the other three quadrants, which, although it has been shown by us in Lemma 49, Num. 5, we shall nevertheless demonstrate here in this way. Let there be taken, in another southern quadrant AQ, the beginning of [Scorpio ♏? — glyph small]; and let the straight line EY be drawn, so that ZY is the arc of declination, which I say is equal to the arc FI.

[Margin: The declinations of the points of one quadrant of the Ecliptic are equal to the declinations of the points of the other quadrants.]

[Margin (source references keyed to the superscripts): a. "29. tert." (Euclid III.29); b. "27. tert." (Euclid III.27); c. "4. pri." (Euclid I.4); d. "11. 1. The." (Theodosius, *Sphaerica* I.11).]

For, the straight lines CI, AY being drawn, the two sides EC, CI will be equal to the two sides EA, AY (for EC, EA are the semidiameters of the Equator [a], and CI, AY are equal [b], because of the equal arcs they subtend); and the angles ECI, EAY too, standing on the circumference upon the equal arcs AQI, CQY, are equal [c]. Therefore the bases EI, EY will also be equal. Hence, the equals EF, EZ being taken away, the remainders FI, ZY will be equal; and these, since they are equally distant from the centre E, will correspond to equal arcs of the Equator; and accordingly the declinations of the points I and Y will be equal. In the same way we shall show that the declination of any other point in the quadrant CQ is equal to the declination of the point in the quadrant AQ whose distance from the equinox A is equal to the distance of the other point from the equinox C. Again, IE being produced as far as X, cutting the Ecliptic in V, the lines IV, FX will represent semicircles [d], because the greatest circles bisect one another; and, the common arc FV being taken away, the remaining arcs of declination FI, VX will be equal, so far as degrees are concerned. Since therefore the points I, V of the Ecliptic are diametrically opposite, as we showed in book 2, in the scholium of prop. 5, Num. 11, it is clear that opposite points of the Ecliptic have equal declinations. For the same demonstration holds in the other opposite points, which are at F, V, as is evident.

[Margin: From a given declination, to elicit — without an instrument — the corresponding point or arc of the Ecliptic.]

7. Furthermore, from a given declination we shall obtain the corresponding point, or arc, of the Ecliptic by this reasoning. Let the given declination be

counted on the Equator from the point B up to d, either toward A or toward C; and from A, or C, through d let a straight line be drawn cutting the meridian line in b; and finally, through b, from E let a parallel of the Equator be described, cutting the Ecliptic in I; and the point I will be that which is sought. How far the found point I is distant from the equinoctial point C will be shown by the straight line drawn from the pole of the Ecliptic G to I. For this will cut off an arc Ca of the Equator equal to the arc CI of the Ecliptic, as we showed in book 2, prop. 5, Num. 17.

[Margin: To determine the meridian altitude of the Sun, or of any star.]

8. Finally, from the known declination of the Sun, or of a star, we shall find its meridian altitude in this way. If the declination is northern, let it be added to the complement of the pole's altitude; but if southern, let it be subtracted from the same. For the number thus made up, or left over, will indicate how great is the meridian altitude of the Sun or of the star.

But when, by adding a northern declination to the complement of the pole's altitude, a number greater than 90° is made up, the Sun or star will be on the meridian between the zenith of the place and the arctic pole. Therefore that number, subtracted from the semicircle [180°], will show the meridian altitude. This happens whenever the altitude of the pole is less than the northern declination.

Again, when the altitude of the pole is greater than the complement of the northern declination — or (what is the same) when the complement of the pole's altitude is less than the northern declination — the Sun or star will have two meridian altitudes, namely a greatest and a least, and will never rise or set. The greatest is found as stated; the least, if from the altitude of the pole the complement of the northern declination is removed, or if the complement of the pole's altitude is subtracted from the northern declination.

Finally, when the complement of the pole's altitude is less than the southern declination, the Sun or star will always lie hidden below the Horizon, and will have no meridian altitude. All these things are clearly evident from the material sphere. And these are to be understood of the northern region; but in the southern region, what has been said of northern declination is to be understood of southern, and conversely.

In the scholium of Canon 22 we shall investigate the declination of a given point of the Ecliptic, even though the Ecliptic itself is not drawn on the astrolabe, and the declination of any star, even though its place has not been found on the

astrolabe: a matter which truly seems to me excellent and outstanding, since the finding of the place of any star on the astrolabe is not easy, as is clear from prop. 11 of book 2, because the ecliptic-parallels of some stars are either too wide or too narrow.

SCHOLIVM.

1. From the analemma we will investigate the declination of any point of the ecliptic in two ways. First, as follows. Draw the line AB, and from A describe an arc of a circle CD with any radius, on which the arc of greatest declination CD is taken — that is, the angle CAD of greatest declination is constructed. Then drop from D the perpendicular DE onto AB, and from E, through D, describe the quadrant of a circle DB. If, therefore, one now counts from the point B up to F the degrees by which the given point of the ecliptic is distant from the nearest equinox, and drops onto DE the perpendicular FG (which is also parallel to BA), cutting the arc CD at H, then CH will be the arc of declination of the given point.

For since it has been demonstrated in Lemma 18 that the whole sine is to the sine of greatest declination as the sine of the arc counted from the nearest equinoctial point is to the sine of the declination of the point terminating that arc, it plainly follows that the arc CH measures the declination of the point which lies as far from the nearest equinox along the ecliptic as the arc BF stands with respect to its own circle.

[Margin: To find the declination of any given point of the ecliptic from the analemma.]

For since ED, the whole sine of the circle BD, is to EG, the sine of the arc BF of that same circle, as ED, the sine of the greatest declination of the circle CD, is to EG, the sine of the arc CH of that same circle; and since, by Lemma 5, as ED, the whole sine, is to EG, the sine of the arc BF, so is the whole sine of the ecliptic to the sine of the arc similar to BF — it will likewise follow that as the whole sine of the ecliptic is to the sine of the arc by which the given point recedes from the nearest equinox, so is ED, the sine of greatest declination, to EG, the sine of the declination CH. And permuting: as the whole sine of the ecliptic is to the sine of greatest declination, so is the sine of the distance of the given point from the nearest equinox to the sine EG. From which it is gathered that EG is the sine of the declination of the given point, and therefore that the arc CH measures the declination itself.

Now this method does not differ from the earlier procedure by which, in Lemma 19, we described the parallels of the Sun in the analemma, except that here the full circles are not drawn. For the sector ACD of this figure corresponds to the sector EHM of the analemma in Lemma 19; and in that same Lemma 19 we also taught, at the end, by what method the declination is to be investigated from the analemma. I therefore think the reader should be referred there, so that what is delivered in this place may be more fully understood.

2. By the latter method we shall obtain the same result as follows. Let ABC be the meridian, or solstitial colure [colurus solstitiorum], about the centre D; its section with the equator AC, with the ecliptic ED; the axis of the equator DB, and the ecliptic DN. Let DF be the right sine [sinus rectus] of the arc of the ecliptic counted from the nearest equinox (which is found if the given arc is counted from N and the perpendicular OF is dropped). Through F draw GH parallel to AC. I say that AG is the arc of declination.

[Margin: (a) Euclid XI.18.]

From I as centre, describe the semicircle GKH, and to GH erect the perpendicular FL. If, then, the semicircle is moved about EP until it stands upright to the plane of the colure, it will — by Definition 4 of Book 11 of Euclid — [stand at right angles to that plane]. By the same reasoning, if the semicircle GKH is revolved about GH until it too stands upright to the same plane, the perpendicular will coincide with OF. Therefore the plane through the line GH and through the line OF will be at right angles to the colure. So, since the parallel of the equator drawn through the given point O makes in it a section parallel to AC, the semicircle GKH, in that position passing through PO [i.e. through F], will make with the colure the section GH parallel to AC. Consequently AG will be the arc of declination.

This latter method does not differ from that by which, in Lemma 19, we described the parallels of the Sun in the analemma; for there too, from the point k, the extremity of the arc Ik, we dropped the perpendicular ku onto the ecliptic's diameter MP, and through u drew a line for the parallel of the equator passing through the ecliptic point k — though in that Lemma 19 we demonstrated it otherwise.

3. Now, in two ways likewise, we shall assign to a given declination the arc and point of the ecliptic that corresponds to it. First, thus: in the arc CD described from A in the first figure, count off the declination up to H, and through H draw FG parallel to AB; it will cut off from the quadrant BD the arc BF, which [gives] the distance of the sought point from the nearest [equinoctial] point. In the latter

way, thus: in the second figure count off the given declination from A and C up to G and H, and draw the straight line GH cutting the diameter of the ecliptic in F. For the perpendiculars DN, FO, erected upon EP, will intercept the sought arc NO, reckoned from the nearest equinoctial point, as is clear from what has been said.

4. The declination of any star whatever, whose longitude and latitude are known, we shall find in this way. Again let ABC be the meridian, or solstitial colure, about the centre D; its common section with the equator AC; with the ecliptic EF; the axis of the equator DB; [the diameter of] the ecliptic DG, and the northern pole B. From the ecliptic take the two arcs of the star's latitude EI, FH — toward the northern pole B if the latitude is northern, but in the contrary direction if it is southern — and draw the straight line IH for the diameter of the parallel of the ecliptic passing through the star. Then let Ea be the versed sine [sinus versus] of the arc by which the star is distant from the beginning of Cancer [♋] — that is, from the semicircle of the colure passing through the beginning of Cancer — whether according to the succession of the signs or against it; this versed sine will be found if that distance is counted from E in the semicircle EGF, and from the end of the counting a perpendicular is dropped to EF, falling at a. Now cut the semidiameter IK in the same ratio [at O] in which the semidiameter ED is cut when the point a lies on ED; or cut the semidiameter KH as the semidiameter DF is cut when the point a falls on DF — which is easily done thus.

5. Having drawn the semidiameter DI, take Db equal to Da, and draw bO perpendicular to IK (which will easily be done if, from any point L taken on IK, an arc is described through b, and nd is cut off equal to the arc nb; for the line bd will then be perpendicular, as is evident from the construction of Proposition 12 of Book 1 of Euclid). I say that IK is cut at O just as ED is cut at a. For since Da is to aE as Db is to bI, on account of the equality of the lines Da and Db (etc.), it will likewise follow that KO is to OI as Db is to bI, and hence that KO is to OI as Da is to aE. And in this way the half of a line parallel to the diameter of the circle will always be cut just as the semidiameter has been cut.

[Margin: (b) Euclid XI.16. To divide the half of a line parallel to the diameter of the circle, as the semidiameter has been divided. — (c) Euclid VI.2.]

6. Conversely, we shall also cut the semidiameter ED just as the half IK of its parallel has been cut at O, in this way (for we shall have need of this too in what follows). Draw the semidiameter DI again, and let the perpendicular Ob, erected to IK, cut it at b (this will easily be drawn if De is taken equal to the line KO — for Oe will then be perpendicular to IK, [Margin: (a) Euclid I.29] since it is parallel to KD [Margin: (b) Euclid I.33]); and cut off Da equal to Db. I say that ED is cut at a

just as IK is cut at O. [Margin: (c) Euclid VI.2] For since KO is to OI as Db is to bI, and since Db is to bI as Da is to aE (on account of the equality of the lines Db and Da, etc.), it will also follow that KO is to OI as Da is to aE.

[Margin: To divide the semidiameter of the circle, just as its parallel half has been divided.]

7. Now, once the point O has been found (which will also be found if from K, about IH, you describe the semicircle ImH, and in it count from I the distance of the star from the beginning of Cancer up to m, and from m drop the perpendicular mO onto IH; for IO will thus likewise be the versed sine of the said distance), draw MN through O parallel to the diameter AC of the equator. I say that AM is the arc of declination of the proposed star. For from Q, about MN, describe the semicircle MPN, and to MN erect the perpendicular OP. If, then, the semicircle ImH is conceived to be revolved about IH until it stands upright, [Margin: (d) Euclid XI.18] mO will be perpendicular to that same colure, and m will be the place of the star. By the same reasoning, if the semicircle MPN is moved about MN until it stands upright to the same colure and is parallel to the equator, the line PO will be perpendicular to that same colure and will coincide with mO. [Margin: (e) Euclid XI.16] Therefore the plane through the line PO, or through MN, will be at right angles to that colure. Since, then, the parallel of the equator through the star at the point [m] makes in it a section parallel to AC, the semicircle MPN, in that position passing through PO, will — being parallel to the equator — make on the colure the section MN parallel to AC. Therefore AM is the arc of the star's declination.

8. This declination will be northern when the versed sine IO of the star's distance from the beginning of Cancer is less than the segment of the diameter of the star's parallel lying between the colure near Cancer and the intersection of that parallel with the diameter AC of the equator; but southern, if greater; and finally the star will have no declination if it is equal — and this is always true, whether the star's latitude be northern or southern, or whether, finally, it have no latitude. Thus, if the star's latitude is northern, EI, and the versed sine of the distance from the colure in its own parallel [equals] IS, the star will have no declination [?: the printed form appears to read *latitudinem*, "latitude," but the four-way parallel of cases requires *declinationem*, "declination"]. But if the versed sine is IT, it will have southern declination. So too, if the star has southern latitude EV and the versed sine VX, it will have northern declination; but if it has the versed sine VR, it will have no declination, and so forth.

9. Again, when a star lies on the solstitial colure — that is, at the beginning of Cancer [♋] or of Capricorn [♐] — its declination is found by this method. When the declination of the tropic point at which the star lies, and the star's latitude, are of the same denomination — that is, both northern or both southern — they are added together, and there results the star's declination, of the same denomination as the declination of the tropic point or the latitude.

But when the declination of the tropic point and the star's latitude are of different denomination — that is, the tropic point is northern and the star's latitude southern, or the reverse — subtract the smaller from the greater, and there remains the star's declination, of the same denomination as the greater quantity, from which the subtraction was made.

When from the addition a number greater than 90 results, the remainder from 180 will give the star's declination, of the same denomination as the tropic point. Likewise, when nothing remains from the subtraction, the star will have no declination. And when, finally, the latitude is nil, the star will have the same declination as the tropic point.

Diagram. A composite analemma figure accompanying §§1–9, comprising three parts. (1) A right-triangle construction (§1): apex D at top; baseline B (lower left) to A (lower right); near the top, F, H, and G, with the perpendicular foot and short arc at C and E on the baseline. This is the sector ACD, with CH the arc of declination and BF the arc measured from the equinox. (2) A large circle at the lower left (the second/third figure of §§2–8): readable labels include K, L, O, B, N along the upper rim (with a dotted arc), E, I, G, F and d toward the centre, A and Q at the lower left, and P, H, M, C along the right. (3) A large circle at the lower right (the analemma for the star, §§4–8): readable labels include P, m (dotted arc at top), B, z, G, f (upper right, with star-marks), L, o, d, Q (centre), N and K (right), e, b (centre-left), S, C, T, H (right and lower right), X, R, A, Y (lower left), D and F (bottom), with a comet symbol at the foot. Many of the small peripheral labels on the two circles are at the resolution floor.

Diagram. The same composite analemma figure as on p. 619, here enclosed in a ruled rectangular frame: the §1 triangle (D, F, H, G, B, C, E, A) at top, and beneath it the two large analemma circles (§§2–8), with the same label sets — left circle K, L, O, B, N, E, I, G, F, d, A, Q, P, H, M, C; right circle P, m, B, z, G, f, L, o, d, Q, N, K, e, b, S, C, T, H, X, R, A, Y, D, F, and the comet symbol at the foot. Many peripheral labels are at the resolution floor.

Example. A star lying at I will have the northern declination AI, compounded from the northern declination AE of the tropic point E and from the northern latitude EI. Likewise the declination of a star at g will be southern, compounded from CF, the southern declination of the tropic point F, and from the southern latitude Fg. Again, a star lying at V will have northern declination, and a star lying at H southern declination: the former remains when the southern latitude EV is subtracted from the northern declination AE of the tropic point E, while the latter is what remains when the northern latitude FH is subtracted from the southern declination CF of the tropic point F. But a star at Aries [γ] will have southern declination, and a star at f northern; for the former remains after the subtraction of the northern declination AE from the southern latitude EY, while the latter remains after the subtraction of the southern declination CF from the northern latitude Ff. Further, since from the northern declination AE and the northern latitude EZ an arc greater than the quadrant AB results, the remainder CZ from the semicircle will give a northern declination. Moreover, a star at A or at C has no declination, since after subtracting one from the other nothing remains, the declination being in each case equal to the latitude. Finally, a star at E will have the same declination as the tropic point E, namely northern; while a star at F will obtain southern declination, the same as the tropic point F.

10. By sines, finally, the declination of any point of the ecliptic will be investigated. Since, in the second description of this figure, as the whole sine DF is [to] the line DI [the whole sine], DI is the sine of the angle DFI [Margin: (f) Euclid I.29] (which is equal to the alternate angle ADF, the greatest declination); so DF, the sine of the arc of the ecliptic NO reckoned from the nearest equinox N up to DI, [is to] the sine of the declination of the point O — which we also demonstrated in Lemma 19. If, then, it be made that as the whole sine is to the sine of greatest declination, so is the sine of the distance of the given point of the ecliptic from the nearest equinox to another [number], the sine of the declination will be produced, and the declination itself will become known.

[Margin: To find the declination of any point of the ecliptic by sines.]

Conversely, if it be made that as the sine of greatest declination is to the whole sine, so is the sine of a given declination to another, the sine of the arc of the ecliptic reckoned from the nearest equinox, to which that declination belongs, will be produced. For since, as the sine of greatest declination is to the whole sine, so is the sine of the arc of the ecliptic reckoned from the nearest equinox to the sine of the declination of that same arc (as has been said), it will follow, by conversion, that as the sine of greatest declination is to the whole sine, so is the

sine of the given declination to the sine of the arc of the ecliptic to which it is due, reckoned from the nearest equinox.

[Margin: From a given declination, to find by sines the corresponding point of the ecliptic.]

And in order that the declination of any star whatever may be found by numbers, let ABCD be the solstitial colure; BD the equator, and its pole F;

[Margin: To trace out the declination of any star by numbers.]

[the equator's] pole F; AC the ecliptic, and its pole G; E the beginning of Aries [γ] or Libra [$\vartriangle$]; A the beginning of Cancer [ϑ]; C the beginning of Capricorn [φ]; H the place of the star; FH the great circle of the star's declination, cutting the equator at L and the ecliptic at M; GH the great circle of the star's latitude, cutting the ecliptic at I and the equator at K; HL the star's declination, and FH its complement; HI the star's latitude, and GH its complement; and finally AI the arc of the ecliptic, [the measure of] the star's distance from the beginning of Cancer, counted either according to the succession of the signs or against it — as appears in the twelve circles described in this place.

Since, therefore, in the spherical triangle FGH the two sides GF, GH are known (FG being the arc of greatest declination, and GH the complement of the star's latitude), and the angle FGH contained by them is also known — (for in the first six circles, in which the star's latitude is northern, the arc AI measuring that angle, the star's distance from the beginning of Cancer, is known; while in the latter six circles, in which the star has southern latitude, the arc CI subtending the said angle is the star's distance from the beginning of Capricorn, which is left when the arc AI, the distance from the beginning of Cancer, is subtracted from the semicircle) — the third side FH, that is, the complement of the star's declination, will be found by Problem 22 of spherical triangles in the last Lemma, in this manner. Let it be made that as the whole sine is to the sine of the greater given side — that is, to the sine of the greatest declination FG, or of the complement of latitude GH — so is the sine of the lesser given side to another; and a certain fourth number will be found. Then again let it be made that as the whole sine is to the fourth number just found, so is the versed sine of the given angle FGH to another; and there will be produced the difference between the versed sine of the third side FH, which is sought, and the versed sine of the arc by which the two given sides FG, GH differ from one another. This difference, added to the versed sine of the arc by which the said two given sides FG, GH differ from one another, will yield the versed sine of the sought side FH; from which the side

FH itself — that is, the complement of the star's declination — will become known.

Now the declination is always of the same name as the latitude — that is, northern if the latitude is northern, but southern if southern — except when the versed sine of the sought side FH turns out greater than the whole sine, as in the 6th and 8th circles, where the found side FH is not the complement of the sought declination but rather its complement HL [in which case the found side FH is greater than the quadrant]. For in this position the star has a declination contrary to its latitude: so that, when the latitude is northern, the declination is southern, as in the 6th circle; and when the latitude is southern, the declination [is northern]. The complement HL is the sought declination; and the side itself [is] contrary to the latitude — so that, when the latitude is northern, the declination is southern, and when southern, the declination is northern, as in the 8th circle.

But if it should happen that the given sides FG, GH are equal (which occurs when the star's latitude comprises $66^{\circ} 30'$, that is, is equal to the complement of greatest declination): let it be made that as the whole sine is to the sine of greatest declination — that is, to the sine of the side FG — so is the sine of half the angle FGH, [the distance] of the star from the beginning of Cancer if its latitude is northern, or from the beginning of Capricorn if southern, to another; and there will be found the sine of a certain arc which, doubled, will make known the whole sought side FH — as we said at the end of the aforesaid Problem 22 of spherical triangles.

Again, if it happen that the given angle FGH is a right angle (which occurs when the star's distance from the beginning of Cancer is a quadrant, as in the 4th and 9th circles): let it be made that as the whole sine is to the sine of the complement of greatest declination, FG, so is the sine of the complement of the side GH — that is, the sine of the star's latitude — to another; and there will be found the sine of the complement of the sought side FH, as is clear from the first mode of Problem 15 of spherical triangles in the last Lemma.

The same declination of the star may also be computed by yet another method. When the star lies at the beginning of Aries [γ] or Libra [ϱ] — that is, its distance from the beginning of Cancer contains 90° , as in the 4th and 9th circles — then, in the triangle EHL whose angle L is right, by the first mode of Problem 8 of spherical triangles explained in the last Lemma, let it be made that as the whole sine is to the sine of the star's latitude HE, so is the sine of the angle HEL, the complement of greatest declination, to another; and there will be begotten the sine of the sought declination HL, of the same name as the latitude.

But when the star is outside the beginnings of Aries [Υ], Libra [♎], Cancer [♋], and Capricorn [♏], as in the other 10 circles (excepting the 4th and 9th), then, by the first mode of Problem 4 of spherical triangles explained in the last Lemma: in the triangle EIK, whose angle I is right, let it be made that as the whole sine is to the sine of the angle IEK, the greatest declination, so is the sine of the complement of the arc EI (which measures the star's distance from the nearest equinox) to another; and there will be produced the sine of the complement of the angle EKI, which subtends the arc of declination HL, in the triangle HKL.

[Margin: To know whether a star's declination is northern or southern.]

[Margin: Alternatively, when the star is at the beginning of Aries or Libra.]

[Margin: When the star is outside the beginnings of Aries, Libra, Cancer, and Capricorn.]

Diagram. The principal construction figure for the star-declination problem (upper right of the page): a large circle representing the solstitial colure, with the equator and ecliptic diameters and the two great circles (of declination FH and of latitude GH) drawn as oblique arcs. Readable labels around and within it include N, F, T, G (upper rim, with star-marks at T and near F), A and P (upper left), H, I, M, E (centre), B (left), K, L (lower left), R (lower left, star-mark), S and e (right), C and o (lower right, star-marks), D (right). Beneath it is a second, plainer circle of the same construction with labels A, F, G (top), H, I (centre), E, L (centre), B, K (left/lower), D (right), M, C (lower). Several small peripheral labels are at the resolution floor.

Diagram. The array of twelve small case-circles referred to in the text ("as appears in the twelve circles described in this place"), showing the spherical triangle FGH (star at H) in the twelve configurations — the first six with northern latitude, the latter six with southern latitude, and special cases at the beginnings of Aries/Libra and Cancer/Capricorn. They are printed in ruled rectangular frames, each holding four circles (2×2). In each circle A, B, C, D mark the cardinal points of the colure/equator/ecliptic, and F, G, H, I, K, L, M, E mark the interior points of the construction (H, the star, is regularly marked with a star-glyph near the centre-left or foot). Representative readable label sets: one circle A–F–G / B–H–K–E / K–L–E / M–C; another A–F–G / B–L–K–H–I–M–C–D–E; others A–M–I–E–K–H–G–F–B–C and A–B–E–K–D–L–M–H–I–C. The individual point-labels of the small circles are largely at or below the resolution floor and cannot each be resolved with confidence; the structure (twelve labelled case-circles in three 2×2 framed groups) is secure, but the per-circle label assignments are not.

[Continuation of the Scholium to Canon III — computing declination]

[Margin: The argument of a star's declination.]

Next, in the same triangle EIK: if, by the first method of Problem 11 of spherical triangles, you make the whole sine to the sine of the arc EI (which measures the star's distance from the nearest equinox) as the tangent of the angle IEK, [the angle] of the greatest declination, is to a fourth term, the tangent of the arc IK will be found. By this arc IK the latitude HI differs from the arc HK, which we may call the argument of declination.

This difference IK is northern — that is, it extends from the Equator toward the north — when the star's place lies in some northern sign; but it is southern when the star lies in some southern sign. Therefore, when the difference IK and the star's latitude HI have the same denomination, namely both northern or both southern, the sum formed from them will give the argument HK, of the same denomination as the latitude or the difference. But when the difference IK and the star's latitude HI are of different denominations — that is, one is northern and the other southern — then, subtracting the smaller from the greater, the remainder becomes the argument of the same name as the arc from which the subtraction was made. Thus in circles 1, 2, 3, 5, and 8 you see the argument HK to be northern, whereas in circles 6, 7, 10, 11, and 12 it is southern.

Lastly, in the triangle HLK, which has the angle L a right angle: if, by the first method of Problem 8 of spherical triangles, you make the whole sine to the sine of the argument HK just found as the sine of the angle HKL — found in the first place in the triangle EIK — is to a fourth term, the sine of the declination HL will be produced, of the same denomination as the argument. But so that the star's declination may be found more precisely, the angle EKI must be found by proportional parts most accurately; and likewise the difference IK, between the argument and the star's latitude, so that in a third pass a truer sine of the argument may then be drawn out by proportional parts. Finally, the declination HL too must be sought from its sine by proportional parts, so that [its sine and its complement's sine may be found] and, in the scholium of the following Canon, [its sine] may be computed still more precisely.

Diagram. A rectangular frame enclosing four circles arranged two-by-two, each a projection of the sphere carrying a spherical triangle used in the declination construction of the passage above (the differing configurations by which the argument HK comes out northern or southern). In every circle the vertical

diameter runs A (top) to C (lower), the horizontal diameter runs B (left) to D (right), E is the center, and H marks the star (drawn with a small star-burst). The auxiliary points I, K, L, M, F, G recur in each.

- Upper-left circle: A (top), B (left), D (right), C (lower right), M (upper, on the AE line), I (left of center), E (center), K (center, just below E), H (below E, star-mark), G (lower left), F (lower center).
- Upper-right circle: A (top), B (left), D (right), C (lower right), L (upper right), K (upper right, near D), E (center), M (center, right of E), H (below E, star-mark), I [?] (right of H), G (lower left), F (lower center).
- Lower-left circle: A (top), B (left), D (right), C (lower right), M (upper center), L (center left), E (center), k (center right — lower-case as printed), I (below center), H (lower, star-mark), G (lower left), F (lower center).
- Lower-right circle: A (top), B (left), D (right), C (lower left), E (center), K (center right), I (center), M (center, near L), L (lower center), H (lower, star-mark), G [?] (lower left).

[Several of the small peripheral single-letter labels are at the resolution floor of the woodcut; the readings marked [?] are uncertain.]

Now this too must be observed in all computations: unless the sines and arcs are drawn out by proportional parts, it may happen that, in finding the last arc, [an error not slight is committed].

[Margin: When a star is at the beginning of Cancer or of Capricorn.]

As to how, when a star lies in the solstitial colure, its declination is to be found, we taught a little before, at Number 9 of this scholium; and you have worked examples of that precept in the stars N, O, P, Q, R, [S, T, B, D, A, C of the first circle], in the order of the places of the stars I, g, V, H, Y, f, Z, A, C, E, F, which in the third figure of the first drawing of this scholium correspond.

CANON IV.

[Margin: To find the right ascension of a given point of the ecliptic, or of a star, from the astrolabe.]

To determine the right ascension and descension of any point of the ecliptic or star; and conversely, to assign to a given right ascension or descension its corresponding [point]; and finally, to determine the point of the ecliptic with which a proposed star rises in the right sphere, or sets, or culminates.

[Margin: Which degree of the ecliptic rises with a given star in the right sphere, or culminates.]

1. Let the rete of the astrolabe be turned about until the degree of the ecliptic, or the proposed star, is set on the right horizon on the eastern side — that is, in the diameter of the astrolabe which cuts at right angles the meridian line (the diameter, that is, which runs to the suspension-ring). For when the rete holds this position, the arc of the Equator from the first point of Aries [Y], reckoned according to the succession of the signs up to that same right horizon on the eastern side (which lies to the left), will measure the right ascension of the given point of the ecliptic or star; since in the right sphere such an arc rises simultaneously with the given point — that is, with the arc of the ecliptic from Aries [Y] up to that point — as the star ascends above the right horizon. This arc of ascension is also given by the degrees intercepted on the limb between the right horizon and the pointer (or index) that passes through the first point of Aries [Y] in that position of the rete: the degrees, I say, counted from the index-line according to the succession of the signs, that is, toward Taurus [♉], Gemini, Cancer, and so on, up to the right horizon. Now with the star placed on the right horizon on the eastern side, the point of the ecliptic then present on that same horizon is the one with which the star rises, or culminates, or (which is the same) reaches the meridian.

[Margin: To find the right descension of a given point of the ecliptic, or of a star, from the astrolabe.]

2. In no other way will you explore the right descension of any point of the ecliptic or star: place the given point, or the star, on the right horizon on the western side. For with the rete holding that position, the arc of the Equator from the first point of Aries [Y], reckoned according to the series of the signs up to the right horizon on the western side, will give the descension in the right sphere — which is also exhibited by the degrees of the limb intercepted between the pointer through the first point of Aries [Y] and the right horizon on the western side, if they are counted according to the series of the signs. But it is enough to investigate the right ascension of any point or star, since this is equal to the descension of the same in the right sphere, as was said for the sphere. And with the star placed on the right horizon on the western side, the point of the ecliptic then present on that same horizon is the one with which the star sets. This point, moreover, is always the same as that with which the same star rises in the right sphere and culminates.

[Margin: The right ascension of any point is equal to the descension of the same.]

[Margin: Which degree of the ecliptic sets with a given star in the right sphere.]

3. But if the right ascension or descension of some point or star is known, we shall find the corresponding arc of the ecliptic — that is, the point of the ecliptic which reaches the horizon together with the star (whose ascension or descension is given), or to which the given ascension or descension belongs — in this way. Let the rete of the astrolabe be turned until the arc of the Equator lying between the first point of Aries [Y] and the right horizon on the eastern side, [reckoned] according to the series of the signs, is equal to the given right ascension of the point of the ecliptic sought; or until the summit of the star is found on the right horizon on the eastern side, so that then the arc of the Equator set between the first point of Aries [Y] and the right horizon on the eastern side measures the given ascension of the star. For with the rete holding that position, the point of the ecliptic then present on the right horizon on the eastern side will be the one to which the given ascension belongs; or the one which reaches the right horizon together with the star whose right ascension was given.

[Margin: Given a right ascension, or descension, to find the corresponding arc of the ecliptic from the astrolabe.]

The same will hold if on the limb the degrees of the given [right ascension]

right ascension are counted against the succession of the signs, beginning from the right horizon on the eastern side; and if, at the end of the counting, the index-line of the pointer is applied. For when the rete is then turned about until the first point of Aries [Y] reaches the index-line, there will be present on the right horizon on the eastern side that point of the ecliptic to which the given ascension belongs, or which ascends above the horizon together with the star to which that ascension is due. And the arc of the ecliptic set between that point and the first point of Aries [Y] will be the one sought, provided that arc be taken from Aries [Y] up to the point found, according to the series of the signs. Exactly the same is to be said of the point, or arc of the ecliptic, to be found which corresponds to a given descension, if for the eastern side of the right horizon the western part be taken. Indeed the same point, or arc found, also fits an equal descension in the right sphere, since, as was said, the ascension of any point in the right sphere is equal to the descension of the same.

[Margin: To find, from the astrolabe, the right ascension and descension of any arc of the ecliptic not beginning from Aries.]

4. From these things we shall easily find the right ascension and descension of any arc of the ecliptic not begun from the first point of Aries [Y]. For the

difference between the ascension of the first point and the ascension of the last point of the arc proposed will be the right ascension of the said arc. Or we shall proceed thus. With the last point of the given arc placed on the right horizon on the eastern side, let the index-line of the pointer be set upon the first point of the same arc. For the arc of the Equator, or of the limb, between the index-line and the right horizon on the eastern side, reckoned according to the succession of the signs, will measure the right ascension of the given arc. The same you will say of the descension of that arc. Here we do not teach how to investigate an arc not begun from Aries [Y] that corresponds to a given right ascension, because various arcs of the ecliptic can have equal ascensions, as is plain in the material sphere, and as we shall say at the end of Number 8.

[Margin: To investigate, without the astrolabe, the right ascension and descension of any point of the ecliptic or star.]

5. Without the instrument we shall hunt this same right ascension and descension by this method. Let the figure of the preceding Canon be repeated, in which the Equator is ABCD, the Ecliptic AQCR, its center H and its pole G; and let it be proposed to investigate the right ascension or descension of the first point of Pisces [♊, ♋]. This point of the ecliptic having been found — let it be I — through the straight line Ga drawn from the pole G, ending at the point a of the ecliptic which marks the distance of the first point of Pisces [♊] from Aries [Y], let there be drawn from E, the center of the astrolabe, to I, a straight line cutting the Equator in F. I say that the arc of the Equator CDABF, reckoned according to the succession of the signs, is the right ascension — or the descension — of the point of the ecliptic I, or of the arc CRAQI begun from Aries [Y]. For since EI is a certain right horizon, in that it carries a great circle drawn through the poles of the world — as we showed in Proposition 1, Number 4 of the earlier book — the two points I and F will rise together in the right sphere, and will set together. Therefore at the time at which the first point of Aries [Y] completes the arc FBADC by the motion of the first mobile, by the same [motion] the whole arc of the ecliptic CRAQI will have ascended or descended; and the point I will have reached the right horizon — that is, the whole arc of the ecliptic [CRAQI].

[Margin: To find, without the astrolabe, the right ascension and descension of any arc of the ecliptic not beginning from Aries.]

6. In the same way we shall explore the right ascension and descension of an arc [not begun] from Aries [Y], if from E, the center of the astrolabe, two straight lines are drawn through the two extreme points of the arc given on the ecliptic. For these will include on the Equator the arc of the right ascension or descension.

Thus, [if the arc of the Equator included is] BF, the right ascension or descension will be the arc of the ecliptic QI, which is intercepted between the beginning of Capricorn [♑] and the beginning of Pisces [♈, ♉].

[Margin: To construct the figure of the right ascensions of all arcs.]

7. Therefore, if the ecliptic AQCR be distributed into the 12 signs, as we taught in Proposition 5 of Book 2, Number 17, and to their points straight lines be drawn from the center E, there will be constructed a figure containing the right ascensions and descensions of all the signs. For the arc of the Equator from the point C, toward D, up to each such line, will give the ascensions and descensions of the points which mark the beginnings and ends of the signs. And the arc of that same Equator comprehended between any two such straight lines will exhibit the ascension and descension of that arc of the ecliptic (not begun from Aries [♈]) which is enclosed between the same two straight lines. And if the individual signs be subdivided into degrees, and lines be sent out from E to them likewise, we shall have also the ascensions and descensions of all the degrees of the ecliptic. Thus in the aforesaid figure you see that the arc CD is the right ascension of the arc CR, set between the beginning of Aries [♈] and the beginning of Cancer [♋]; that the arc CDA is the ascension of the arc CRA, between the beginning of Aries [♈] and Libra [♎]; likewise that the arc CDAB is the ascension of the arc CRAQ, from the beginning of Aries [♈] up to the beginning of Capricorn [♑]; that the arc FCD, moreover, is the right ascension of the arc ICR, set between the beginnings of Pisces [♈, ♉] and Cancer [♋]; and so of the rest. And a figure of this kind is represented by the earlier figure of Andreas Schöner [Andreas Schöner], which we described in the Scholium of Proposition 9 of Book 2 of the *Gnomonics*, and an example of which we shall set down in the following Canon, Number 10.

Diagram. A large circle within a square frame — a projection of the celestial sphere used for the right-ascension construction of §§5–7, worked out here for the constellation Canis Maior (the Dog / Sirius). E is the center; the horizontal diameter runs S (left) – C – E – A (right), and the vertical diameter Q (top) – E – ... – M (bottom, the star). The Equator is ABCD and the Ecliptic AQCR (Q = summer solstice on the ecliptic, R lower; A and C the equinoctial points); G marks the pole of the ecliptic and H its center, both near the middle. The dotted radial line from E runs to the star at M, labelled **Canis maior** (with a star-burst).

- Q (top): apex of the vertical diameter
- b, B, d (upper center): points near the top of the inner circles
- K (upper right); Y (right, near K); L, o (upper left, near I); I (upper left)

- F, T (left of center); S (left end of horizontal diameter); C (on horizontal, left of center); E (center); A (right end of horizontal diameter)
- H, G (center): center and pole of the ecliptic
- N, O (right of center); V, X (lower right); R (lower center); N, P, D (lower center); f [?] / ʌ (lower left)
- M (bottom, star-burst): the star of Canis Maior (Sirius); c (small, beside M)

[Several small peripheral labels (the lower-case a/o/n cluster and the lower-left f/ʌ) are at the resolution floor.]

Also this same figure of the right ascensions will be constructed if the ecliptic be divided into degrees by straight lines drawn through the center of the astrolabe, as we taught in Book 2, Proposition 6, up to the end of Number 37: namely, if points be found on the straight line which, at the center of the greatest circle, stands as the Vertical of the ecliptic (such as is the line ST, in the figure of Proposition 11 of Book 2) and is perpendicular to the meridian line — points through which straight lines are drawn through the center of the astrolabe. For these straight lines both distribute the ecliptic into degrees, as we show in Book 2, Proposition 6, up to the end of Number 37, and indicate the right ascensions of those same degrees, as has here been shown.

[Margin: From a given right ascension or descension, to extract the corresponding arc of the ecliptic.]

8. Conversely, from a given right ascension or descension we shall elicit the corresponding arc of the ecliptic, if from the center E, through the terminus of the ascension or descension, a straight line be sent out. For this line will cut the ecliptic in a point to which the given ascension belongs; and the corresponding arc will be the one which extends from the first point of Aries [Y], according to the succession of the signs, up to that point. Thus to the right ascension CDABF corresponds the arc of the ecliptic CRAQI, and so of the rest. It is manifest, however, from the figure itself, that to a given ascension which does not begin from Aries [Y] no corresponding arc of the ecliptic can be assigned. For to the ascension BF corresponds both the arc QI and the arc QY, since the ascension BF is equal to the ascension BZ; and thus, if an arc equal to the arc BF be taken elsewhere on the Equator, some other arc of the ecliptic will correspond to that ascension.

9. ASCEN-

9. The right ascension and descension of any star will be found with the same ease. For if from the center of the astrolabe a straight line be drawn through the

place, or center, of the star, the arc of the Equator intercepted between the first point of Aries [Y] and that line, according to the series of the signs, will measure the right ascension or descension of the star. Thus the right ascension or descension of Canis Maior will be the arc of the Equator CDN. Now the point of the ecliptic which rises above the right horizon EM together with the proposed star, or which sets, or which reaches the meridian — that is, culminates — will be the one through which that same line EM passes on the ecliptic. And at how great an interval that point is distant from the first point of Aries [Y] will be indicated by the straight line drawn from G, the pole of the ecliptic, through that same point of the ecliptic. For just as many degrees are contained in the arc of the ecliptic between the said line and the first point of Aries [Y] as are comprehended in the arc of the Equator between the same line and the first point of Aries [Y], as we demonstrated in Book 2, Proposition 5, Number 17. For example, if the straight line EI were drawn through the center of some star, that star would rise above the right horizon EI, or descend below it, or culminate together with the point of the ecliptic I, which recedes from the first point of Aries [Y] toward Capricorn [♑] by just as many degrees as are contained in the arc of the Equator Ca; and the right ascension or descension of that same star would be the arc CDAF.

[Margin: To find, without the astrolabe, the right ascension and descension of any star, together with the point of the ecliptic that rises or sets with it.]

Diagram. The same figure as on the preceding page (repeated here for Number 9): a large circle in a square frame, a projection of the sphere with the Equator ABCD and the Ecliptic AQCR, worked for Canis Maior (the Dog Star at M). E is the center; horizontal diameter S (left) – C – E – A (right); vertical diameter Q (top) – E – ... – M (bottom). G and H mark the pole and center of the ecliptic.

- Q (top); b, B, d (upper center); K (upper right); Y (right); I, L, o (upper left); F, T (left of center)
- S (left end), C, E (center), A (right end) on the horizontal diameter
- H, G (center); N, O (right of center); V, X (lower right); R (lower center); N, P, D (lower center); f [?] / ʌ (lower left)
- M (bottom, star-burst) with **Canis maior** labelled beside it; c (small, beside M)

[Same labelling as fig-p623-1; small peripheral labels at the resolution floor.]

SCHOLIUM.

[Margin: To obtain the right ascension or descension of a given point of the ecliptic from the analemma. (10 of Theodosius, Book 2.)]

1. From the analemma we shall in this way obtain the right ascension or descension of any point of the ecliptic. Let the figure of the scholium of the preceding Canon be repeated; and in the second drawing let the arc NO be taken, equal to the distance of the given point from the nearest equinoctial point, and let there be dropped to the diameter of the ecliptic the perpendicular OF; and through F let there be drawn GH, parallel to the diameter of the Equator, cutting BD in I; and finally, to GH let the perpendicular FL be raised, cutting the circle described about GH in L. I say that the arc KL is the right ascension or descension of the given point O. For, as we showed in the scholium of the preceding Canon, GH is the diameter of the parallel which the given point describes, and GKE is its semicircle, and AG is the declination of the given point.

And since the equinoctial colure, drawn through D — the beginning of Aries [Y] — and the circle of declination (which is then the right horizon) cut off similar arcs from the Equator and the parallel, the arc KL will be similar to the arc of the ascension or descension in the Equator which the circle of declination, passing through the point L, cuts off, as though it were the right horizon. And that this may become plainer, let there be conceived the semicircles ENP, GKH (of the ecliptic and of the parallel), erected at right angles to the colure, in which position, being opposite, the points L and O mutually coincide, as we said in the scholium of the preceding Canon. Since therefore the circle of declination passes through the point O of the ecliptic as though the right horizon, the same [circle] will pass through the point L. And because the point K is then in the equinoctial colure with IK — the common section of the parallel and of the said colure being perpendicular to the solstitial colure, as the reasoning requires (for since both the equinoctial colure and the parallel are perpendicular to the solstitial colure, [Note b: from 19.] there will also be their common section perpendicular to the same, and therefore also perpendicular to GH, the common section of the parallel and of the solstitial colure) — therefore KI [?], [being] the common section of the parallel drawn perpendicular to GH, will [be the common section] of the equinoctial colure and of the parallel; [Note c: 10 of Theodosius, Book 2.] the arc KL will be similar to the arc of the Equator which the circle of declination passing through L cuts off, which arc is indeed the right ascension or descension, being that which is intercepted between the right horizon — which is then the circle of declination — and the equinoctial colure.

Diagram. A compound analemma figure. At the top, a plane right-angled construction: a triangle with apex D above the base B–C–E–A, with F on the arc at the left, and H, G marking the feet of a small perpendicular between the arc BD and its chord (F, H, G clustered at the upper left of the base; A at the right end of the base). Below, two overlapping projection circles set side by side:

- Left circle: center D (labelled center), with radial diameters; peripheral labels K (top), L, O, B, N (upper); E, I (left of center); G, F (left); A, Ω (lower left); C (lower center); P (bottom); H, M (right of center); dotted arcs of a projected semicircle.
- Right circle: center D (labelled), peripheral labels P, m (top, dotted arc); B, Z (upper, Z with a star-burst); I (upper left); G (upper right, with a small star / ʌ-like mark); N (right); o, d (center dotted); n, e (center, lower-case cluster); K (right of center); E, V, X (lower left); R (bottom); O, S, C, T, Y (right and lower right); star-bursts at Z, at lower left (ϣ/Y), and at lower right (ʌ).

[This compound figure carries many small lower-case construction letters and three star-marks; the lower-case cluster (o, d, n, e, q) and the peripheral single letters are at the resolution floor and cannot all be resolved with confidence.]

[Margin: Note b: from 19. Note c: 10 of Theodosius, Book 2.]

Therefore, if the given point O lie between Aries [Y] and Cancer [♋], its right ascension or descension will be KL, less than a quadrant. If it lie between Cancer [♋] and Libra [♎], the ascension or descension will be an arc compounded of the quadrant KG and the arc GL, because then the ascension or descension KL, since it is reckoned against the succession [of the signs] from Libra [♎], must be taken away from the semicircle, so that the ascension or descension begun from Aries [Y] may be left. If it lie between Libra [♎] and Capricorn [♏], the ascension or descension will be an arc compounded of the semicircle and the arc KL, because then the ascension or descension KL takes its beginning from Libra [♎] and tends toward Capricorn [♏]. If finally [the point lie] beyond Capricorn [♏], the right ascension or descension will be an arc of three quadrants, [compounded of the semicircle and the] arc GL, because then the ascension or descension KL agrees with the remaining arc of the ecliptic up to Aries [Y], and accordingly must be taken away from the whole circle, so that the ascension or descension begun from Aries [Y] may be left over. But if the given point be E, the beginning of Cancer [♋], its ascension or descension will be a quadrant; if the beginning of Libra [♎], a semicircle; if finally the beginning of Capricorn [♏], an arc compounded of three quadrants.

[Margin: To find, from the analemma, the right ascension or descension of any star.]

2. We shall know in the same way the right ascension or descension of any star, if its declination be found, as was said in the scholium of the preceding Canon. For in the third drawing the straight line QO will be the sine of the right ascension or descension in the parallel MPN, so that the line DB, produced, and the perpendicular OP, intercept the right ascension or descension. For the reasoning here is the same as that which was set forth a little before concerning the ascension and descension of a given point of the ecliptic.

If therefore the star's distance Im from the beginning of Cancer [♋] be reckoned against the succession of the signs, and be less than a quadrant, its right ascension or descension will be less than a quadrant — namely the arc corresponding to the sine QO; but if that distance [be reckoned] against the order [of the signs]

Throughout this passage the four cardinal points of the ecliptic are written with their signs: ♈ = the first point of Aries (the vernal equinox), ♋ = the first point of Cancer (the summer solstice), ♎ = the first point of Libra (the autumnal equinox), ♏ = the first point of Capricorn (the winter solstice). ...[the length] is greater than a quadrant, then the right ascension or descension will exceed three quadrants by the complement of the arc that corresponds to the sine QO. For since the ascension or descension then found begins at ♈ and tends toward ♏, it must be subtracted from the whole circle, so that there remains the right ascension or descension reckoned from ♈ in the order of the signs.

But if the distance Im, [reckoned] from the first point of ♋ in the order of the signs, is less than a quadrant, then the right ascension or descension found will take its beginning from ♎, tending toward ♋, and therefore must be subtracted from a semicircle, so that the star's right ascension or descension may remain, begun from ♈. If, however, that distance, [reckoned] in the order of the signs, is greater than a quadrant, [it tends] toward ♏, and so must be added to the semicircle, so that the star's ascension or descension begun from ♈ may be produced. And if the distance from ♋ is nil, its right ascension or descension will contain a quadrant; if it equals a quadrant in the order of the signs, a semicircle; and if finally it equals a semicircle — whether counted in the order of the signs or against it — three quadrants. All of which are plain on the material sphere.

3. If the right ascension or descension of any arc of the ecliptic *not* begun from ♈ is desired, one must investigate the ascensions or descensions of the two extreme

points of the given arc. For if the lesser ascension or descension is subtracted from the greater, the remainder becomes the right ascension or descension of the given arc.

[Margin: To find, from the analemma, the right ascension or descension of a given arc of the ecliptic not begun from Aries.]

4. Now, from a given right ascension or descension, we shall gather the corresponding arc of the ecliptic — that is, the arc to which the given ascension or descension belongs — in this way. If the right ascension or descension is less than a quadrant, take it as it is proposed. If it is greater than a quadrant but less than a semicircle, subtract it from a semicircle. If greater than a semicircle but less than three quadrants, take a semicircle away from it. And if finally greater than three quadrants, subtract it from the whole circle. For by this procedure one will always have a right ascension or descension reckoned from the nearest equinoctial point and less than a quadrant. Of this ascension or descension ... take, in the second figure, the right sine DQ — which is easily done if, from B toward A, the ascension or descension itself is counted off, and from the end of the counting a perpendicular is dropped to AD; for this will cut off the sine DQ that we want. One must therefore find the parallel GI (which is parallel to the ecliptic's diameter DE) so divided at F that IF is to FG as DQ is to QA. For then, if a semicircle GKH were described about it and a perpendicular FL were erected, the arc KL would be similar to the arc of the given ascension or descension, whose sine is DQ, by Lemma 5; and consequently that right ascension or descension would belong to the arc of the ecliptic whose sine is DF, and to the declination AG of the last point. In what manner, moreover, from the point F thus found the arc of the ecliptic to which the given ascension or descension corresponds is to be elicited, we shall teach at Num. 6.

[Margin: From a given right ascension or descension, to seek out through the analemma the corresponding arc of the ecliptic.]

Now, how the parallel GI is to be divided in that manner will be found thus. By Lemma [52?] the ellipse must pass [through the point], the semi-axis of whose major axis is DB and of the minor DQ. For the straight line by which [the division] is sought is such, by Lemma 51, that as DQ is to QA, so IF is to FG. Furthermore, the point F represents that upon which fall the perpendiculars dropped from the common section of the declination circle and of the parallel onto the plane of the colure of the solstices — since, from the perpendiculars dropped from all the points of that [circle], by Proposition 24 of Book 1 of our *Gnomonics* [Clavius, *Gnomonices* I.24], [they] cut the parallel, in its proper position, at the point L; and

therefore KL is an arc similar to the arc of right ascension or descension on the equator, which that same circle cuts off, and whose sine is DQ, which the perpendicular dropped from the intersection of the said declination circle with the equator onto the colure of the solstices cuts off.

5. We shall find that same point F of the ecliptic, and the declination AG, without the aid of the ellipse, in this way. Since, by Proposition 44 of our *Spherical Triangles* [tract], in the spherical triangle ELM — which is contained in the twelve circles of the Scholium of the preceding Canon — it holds that as the whole sine is to the sine of the arc of right ascension or descension EL, so the tangent of angle MEL, of the maximum declination, is to the tangent of the arc of declination LM; then by permutation, as the whole sine is to the tangent of the maximum declination, so the tangent of the given right ascension or descension is to the tangent of the declination of the point to which the ascension or descension belongs. But by Proposition 18 of the tract on sines and tangents of our [work], the tangent of the maximum declination is also the [same ratio], so that the sine of the complement of the maximum declination is to the sine of the maximum declination as the whole sine is to the tangent of the maximum declination. a Therefore it will also hold that as the sine of the complement of the maximum declination is to the sine [of the maximum declination], so the sine of the right ascension or descension is to the tangent [of the declination] to which that ascension or descension belongs.

[Margin: (a) the eleventh of the fifth (Euclid, *Elements* V.11).]

Let, then, ANCM be the meridian, or the colure of the solstices, whose centre is D; let AC be the diameter of the equator, EP that of the ecliptic, and gh the axis of the world. To AC drop the perpendicular EB, and from A to that same AC erect the perpendicular AK, which will be tangent to the circle, by the corollary of Proposition 16 of Book 3 of Euclid [*Elements* III.16 coroll.]. Finally, let De be the sine of the given right ascension or descension, and from e to AC erect the perpendicular eI. b And since, as BD — the sine of the complement of the maximum declination AE — is to BE, the sine of that same maximum declination, so De, the sine of the given right ascension or descension, is to eI; then, as we have just demonstrated, eI will be the tangent of the sought declination. Therefore, taking AK equal to eI, draw from K through the centre D the straight line KDY, cutting the circle at G; and AK will be the tangent of the arc AG, and so AG will be the sought declination — such that then the ecliptic, with the colure or meridian, makes the common section GY. And GH, drawn parallel to AC, will cut the ecliptic at the point F, which is what is sought.

[Margin: (b) the fourth of the sixth (Euclid, *Elements* VI.4).]

6. With the point F found, draw from D and F to EP the two perpendiculars Dr and Fi; and ri will be the arc of the ecliptic between Υ (or $\triangle$) and the declination circle, which plays the part of the right horizon. If, then, the given right ascension or descension is less than a quadrant, the arc ri ... and it will take its beginning from Υ . But if the given ascension or descension is greater than a quadrant, [it will tend from] $\triangle$ toward $\boxminus$. Therefore, with that removed from a semicircle, the remainder becomes the sought arc from Υ , taking [its beginning there]. If it is greater than a semicircle but less than three quadrants, the arc ri will incline from $\triangle$ toward $\boxplus$, [and once added to the semicircle] the sought arc begun from Υ will be assembled. If finally the given ascension or descension [is greater than three quadrants], the [arc ri] will run from Υ toward $\boxplus$. Therefore, with that subtracted from the whole circle, the sought arc will remain. Moreover, it is manifest that if the right ascension or descension is a quadrant, the corresponding arc of the ecliptic is [a semi]circle; a semicircle [gives] a semicircle; and if finally three quadrants, three quadrants.

Diagram. A square frame enclosing the great meridian circle — the colure of the solstices — with centre D, drawn with the construction of §§5–6. The equator diameter is AC and the ecliptic diameter EP; the axis of the world is gh; the perpendiculars EB and AK, the secant line KDY cutting the circle at G, the parallel GH cutting the ecliptic at F, and the perpendiculars Dr, Fi with the arc ri all appear as a dense pencil of chords radiating from the centre D. The rim and interior carry many small point-letters.

- Left side: A, V, B (upper left, near the ecliptic pole/perpendicular)
- Upper left: K, G, F cluster (with the tangent point)
- Top: q, L, E, i, and a small sign-mark [?]
- Upper right: M, P
- Middle right: O; L (right of centre)
- Centre: D
- Lower right: E, H, N, C, Y [?], K
- Lower left: R, z, δ [?]
- Along the lower rim: a row of small letters — N, F, K, [?], m, E, [?] — several illegible at this resolution [?]

7.

7. By the aid of the sines we shall track down all these things by this method. Let the twelve circles described at the end of the Scholium of the preceding Canon be

taken up again; in all of them (the third and twelfth excepted) the right ascension, computed from the nearest equinoctial point — which corresponds to the point m of the ecliptic — is the arc EL , since the circle FL plays the part of the right horizon, being drawn through the poles of the world and forming right angles with the equator at L . If, then, in the right-angled spherical triangle ELM , by the first mode of Problem 9 of the spherical triangles of the last Lemma, it be made that as the whole sine is to the sine of the complement of angle MEL , of the maximum declination, so the tangent of the arc EM of the ecliptic — begun at the nearest equinoctial point and reaching to another — [is to a fourth]: there will be produced the tangent of the sought right ascension EL . And if the point M falls between the first point of Υ and $\mathfrak{S}$, the right ascension will be the very arc EL found, less than a quadrant. But if it falls between the first point of $\mathfrak{S}$ and $\mathfrak{A}$, the ascension found, which is reckoned from $\mathfrak{A}$ toward $\mathfrak{S}$, must be subtracted from a semicircle, so that the sought right ascension, begun from Υ , may remain. If between the first point of $\mathfrak{A}$ and $\mathfrak{V}$, a semicircle must be added to the ascension found, since this is reckoned from $\mathfrak{A}$ toward $\mathfrak{V}$, so that the sought right ascension begun from Υ may be assembled. And if finally between $\mathfrak{V}$ and Υ , the ascension found — which is reckoned from Υ toward $\mathfrak{V}$ — must be taken away from the whole circle, so that the right ascension begun from Υ and computed in the order of the signs, which is sought, may remain. And in the very same way the right descension of any point of the ecliptic will be computed, since it is equal to the right ascension.

[Margin: To compute the right ascension or descension of a given point of the ecliptic by the benefit of the sines.]

In turn, from a given right ascension or descension, the corresponding arc of the ecliptic will be computed in this way. In the same triangle ELM , if by the first mode of Problem 13 of the spherical triangles of the last Lemma it be made that as the whole sine is to the sine of the complement of angle LEM , of the maximum declination, so the tangent of the complement of the given right ascension or descension EL is to another, there will be brought forth the tangent of the complement of the sought arc EM . But here too, as we said at Num. 4, if the given right ascension or descension is less than a quadrant, it must be assumed as proposed; if greater than a quadrant but less than a semicircle, subtracted from a semicircle; if greater than a semicircle but less than three quadrants, a semicircle must be taken away from it; and if finally greater than three quadrants, it must be subtracted from the whole circle. For by this procedure one will always have a right ascension or descension less than a quadrant and begun from the nearest equinoctial point. Again, when the given right ascension or descension is less

than a quadrant, the arc of the ecliptic EM will be the one sought, begun from Υ . But if greater than a quadrant, yet less than a semicircle, the arc EM found must be subtracted from a semicircle, so that the sought arc may remain, counted from Υ . If, however, greater than a semicircle but less than three quadrants, a semicircle must be added to the arc EM found, so that the sought arc, taking its beginning from Υ , may be assembled. And if finally greater than three quadrants, the arc EM found must be subtracted from the whole circle, so that the sought arc, counted from the beginning Υ , may remain. Which we also said in the preceding Num. 6.

[Margin: From a given right ascension or descension, to find by numbers the corresponding arc of the ecliptic.]

The right ascension and descension of any star will be found by this art through numbers thus. In all twelve circles the right ascension or descension of the star is the arc BL, counted from the semicircle of the colure of the solstices in which the first point of $\odot$ lies; or the arc DL, computed from the semicircle of that same colure in which the first point of $\sphericalangle$ lies; which arc we shall investigate from angle BFL or DFL as follows. Since in the spherical triangle FGH the three sides are known — FG being the arc of the maximum declination, GH the complement of the star's latitude, and finally FH the complement of the declination of that same star, found in the Scholium of the preceding Canon at Num. 10 — then, if by Problem 21 of the spherical triangles of the last Lemma it be made that as the whole sine is to the sine of the arc FH, the complement of the declination (or of the arc FH compounded of the declination HL and the quadrant FL, when indeed the star's latitude and declination are of different denomination, as happens in the 6th and 8th circle), so the sine of the arc FG, of the maximum declination, is to another, a certain fourth number will be found. Then if again it be made that as the fourth number just found is to the whole sine, so the difference between the versed sine of the third arc GH — which measures the complement of the star's latitude — and the versed sine of the arc by which the two arcs FG, FH differ from each other, is to another, there will be generated the versed sine of angle GFH, whose arc DL, together with BL, [is the sought right ascension or descension], to be counted, indeed, on the equator from the semicircle of the colure of the solstices drawn through $\sphericalangle$, if the star's latitude is northern, as in the first 6 circles; but from the semicircle

[Margin: To hunt out by numbers the right ascension and descension of any star.]

Diagram. The array of twelve small spherical figures ("the twelve circles") repeated from the end of the preceding Canon's Scholium, arranged in three

framed boxes of four each. Each little sphere shows the same construction: the horizon/meridian frame lettered A (left-upper), B (left), C (lower right), D (right), with the interior spherical-triangle points E, F, G, H, I, K, L, M and a star-mark, illustrating the successive cases in which the star lies in the ascending or descending semicircle with northern or southern latitude. The point-letters recur identically across all twelve cells; individual small labels are at the resolution floor.

- Recurring cardinal labels per cell: A, B, C, D
- Recurring interior labels: E, F, G, H, I, K, L, M (positions vary case by case)
- Several interior labels per cell are ambiguous at this resolution [?]

but from the semicircle of that same colure described through $\mathfrak{S}$, if the latitude is southern, as in the latter 6 circles. Moreover, the versed sine itself found will indicate whether that ascension is greater or less than a quadrant, or indeed a quadrant — according as it is greater or less than the whole sine, or equal to it. And whether the ascension or descension found must be counted in the order of the signs, or against it, from $\mathfrak{V}$ or from $\mathfrak{S}$, will be shown by the star's place in the zodiac. For if the star lies in the ascending semicircle of the ecliptic and has northern latitude, the ascension or descension found must be counted from $\mathfrak{V}$ in the order of the signs; but on the contrary, if it lies in the descending semicircle and has northern latitude. But when the star has southern latitude, the ascension or descension found must be counted from $\mathfrak{S}$ against the order of the signs; and, on the contrary, in the order, when the star lies in the descending semicircle and has southern latitude. But the arc FH is greater than a quadrant when the star's latitude and its declination have different denomination, as happens in the 6th and 8th circle; and it is then made up of the quadrant and the star's declination. Therefore in that case the arc FH must not be taken for the complement of the declination, but for the arc compounded of the quadrant and the declination.

From these things we shall find, with no trouble, the right ascension or descension of the star begun from $\mathfrak{V}$. For when it is counted from $\mathfrak{V}$ in the order of the signs, three quadrants must be added, and from the resulting number a whole circle must be cast off, if it can be cast off, so that the ascension or descension begun from $\mathfrak{V}$ may be produced. But when it is counted from $\mathfrak{V}$ against the order of the signs, it must be subtracted from three quadrants, so that the ascension or descension begun from $\mathfrak{V}$ may remain. When, however, it is computed from $\mathfrak{S}$ in the order of the signs, a quadrant must be added, so that the ascension or descension begun from $\mathfrak{V}$ may be assembled. And finally, when it is counted from $\mathfrak{S}$ against the series of the signs, it must be subtracted from a

quadrant, a whole circle having first been added when the subtraction cannot be made, so that the ascension or descension counted from Υ may remain. All of which are plain on the material sphere.

Now if it should ever happen that the complement of the declination is equal to the maximum declination, so that the sides FG, FH, which contain the sought angle GFH, are equal: then, if it be made that as the whole sine is to the sine of half the complement of the latitude — that is, to the sine of half the side GH — so the secant of the complement of the arc FG, of the maximum declination, is to another, there will be generated the sine of half the angle GFH, and so on; as is established from the second mode of Problem 1 of the spherical triangles [of the last Lemma].

Again, if the angle GFH is found to be a right angle, the first point of Υ or of ϖ will lie in the right horizon, as is clear in the 3rd and 12th circle. For which reason the right ascension or descension is either nil or equal to a semicircle. For when the ascension found (which then equals a quadrant) must be counted from φ in the order of the signs, or from ϑ against the order, the ascension or descension is nil; but when it must be computed from φ against the order, or from ϑ in the order, the ascension or descension equals a semicircle.

The right ascension and descension can also be computed by this other method. When the star is at the first point of Υ or of ϖ , as in the 4th and 9th circle, if in the triangle K LH, having the right angle L, by the first mode of Problem 9 of the spherical triangles of the last Lemma it be made that as the whole sine is to the sine of the complement of angle HKL — that is, to the sine of angle LKM [?], since the latter is the complement of the former — [and to the sine] of the maximum declination [?], so the tangent of the star's latitude HK is to another, there will be brought forth the tangent of the right ascension or descension KL, begun at the nearest equinoctial point. This — if the star is northern and lies at the first point of Υ — must be counted from Υ against the order of the signs, and consequently, being subtracted from the whole circle, it leaves the ascension begun from Υ ; but if it is northern and lies at the first point of ϖ , it must be counted from ϖ in the order of the signs, and so, added to the semicircle, it makes the ascension begun from Υ . But if the star is southern and lies at the first point of Υ , it must be counted from Υ in the order of the signs; and if southern and at the first point of ϖ , it must be reckoned from ϖ against the order of the signs, so that, subtracted from the semicircle, it leaves the ascension begun from Υ .

[Margin: Another way, when the star is at the first point of Aries or of Libra.]

When, however, the star lies at the first point of $\mathfrak{S}$, its right ascension or descension will comprise a quadrant; but at the first point of $\mathfrak{V}$, three quadrants.

[Margin: When the star is at the first point of Cancer or of Capricorn.]

But when the star lies outside the first points of $\mathfrak{V}$, $\mathfrak{A}$, $\mathfrak{S}$, or $\mathfrak{V}$, then in all the circles (except the 4th and 9th) there will be a right ascension or descension EL to be computed from the nearest equinoctial point, which will be found thus. In the triangle EIK, whose angle I is right, if by the first mode of Problem 13 of the spherical triangles of the last Lemma it be made that as the whole sine is to the sine of the complement of angle IEK, of the maximum declination, so the tangent of the complement of the arc EI — which measures the star's distance from the nearest equinoctial point — is to another, there will be produced the tangent of the complement of the arc EK, which we may call the argument of the right ascension.

[Margin: The argument of the right ascension.]

Next, in the triangle HLK, whose angle L is right, if by the first mode of Problem 7 of the spherical triangles of the last Lemma it be made that as the whole sine is to the secant of the declination HL — found in the Scholium of the preceding Canon — so the sine of the complement of the argument of the declination HK, found in that same Scholium, is to another, there will be produced the sine of the complement of the arc KL, which is the difference between the right ascension EL and its argument found, EK. When the star has northern declination and lies in the northern semicircle of the ecliptic, as in the 1st, 2nd, 3rd, and 8th circle; or has southern declination and lies in the southern semicircle of the ecliptic, as in the 6th, 10th, 11th, and 12th circle, let the argument of the ascension and the difference between it and the ascension be compared with each other; and if they are found unequal, let the lesser be taken from the greater. For the remaining number will give the sought right ascension or descension EL, to be computed from the nearest equinox — toward the same part in which the star's place is found, when the argument is greater than the difference, as in the 1st, 6th, 8th, and 10th circle; but toward the part contrary to the star's place, when the argument is less than the difference, as in the 2nd and 11th circle. But if the argument is found equal to the difference, the star will lie on the equinoctial colure, as in the 3rd and 12th circle. Wherefore, if the star lies near $\mathfrak{V}$, its right ascension or descension will be nil; but if near $\mathfrak{A}$, it will be equal to a semicircle. When, however, the star's declination is northern and its place is in the southern semicircle of the ecliptic, as in the 5th circle; or its declination is southern and its place is in the northern semicircle of the ecliptic, as in the 7th circle: the sum of

the argument and the difference will give the ascension or descension, to be computed toward the same part into which the star's place inclines.

Now indeed, in all the circles (except the 3rd and 12th, in which the star rises above the right horizon and culminates together with the first point of Υ or of ϖ — according as it lies next to Υ or ϖ — since it is then on the equinoctial colure), the point M of the ecliptic, with which the star rises in the right sphere and culminates, will be computed in this way. In the triangle ELM, whose angle L is right, if by the first mode of Problem 13 of the spherical triangles of the last Lemma it be made that as the whole sine is to the sine of the complement of angle LEM, of the maxi[mum declination] ...

[Margin: To compute by numbers the point of the ecliptic with which a star rises in the right horizon and culminates.]

of the greatest declination, so is the tangent of the complement of the right ascension EL (found and reckoned from the nearest equinox) to another tangent; and there will result the tangent of the complement of the arc of the ecliptic EM, inclining toward the same side to which the ascension tends. The point M of the ecliptic that was sought, therefore, cannot remain unknown.

But if the star has no latitude, its declination and right ascension — or descension — will be found from its distance from the nearest equinox, exactly as the declination and right ascension of a given point of the ecliptic were computed.

CANON V.

To investigate the oblique ascension and descension of any point of the ecliptic, or of a star; and conversely, to a given oblique ascension and descension, to assign the corresponding arc of the ecliptic; and finally, to determine the point of the ecliptic with which a proposed star rises or sets in the oblique sphere.

[Margin: Any star culminates with the same point of the ecliptic in the oblique sphere as it does in the right sphere.]

1. We do not here propose the problem of determining the point of the ecliptic with which a given star culminates — that is, reaches the meridian — since every star touches the meridian in the oblique sphere with the same point as it does in the right sphere. This is shown on the ecliptic by laying the sighting-line of the alidade over the star's tip, or by a straight line drawn from the center of the astrolabe through the star, as we said in the preceding Canon, Number 9.

[Margin: To find the oblique ascension of a given point of the ecliptic, or of a star, by the instrument.]

Let a given point of the ecliptic be placed — that is, the last point of an arc begun from Aries [♈], or the tip of a proposed star — on the oblique horizon of the given region, on the eastern side. For, the rete being so set, the arc of the equator reckoned from the beginning of Aries [♈] in the order of the signs up to the oblique horizon — that is, up to the eastern intersection of the equator with the horizon, right and oblique — will give the oblique ascension that is sought. This same arc will also be given by the arc similar to it on the limb, intercepted between the sighting-line of the alidade passing through the beginning of Aries [♈] and the right horizon. For that arc of the equator rises fully together with the arc of the ecliptic reckoned from Aries [♈] up to the given point above the oblique horizon; and it will have completed its rising at the moment when the star reaches the oblique horizon, as is clearly apparent from the instrument. And when the star is set upon the oblique horizon on the eastern side, the point of the ecliptic then found on that same horizon is the one with which the star rises.

[Margin: With which degree of the ecliptic a given star rises in the oblique sphere.]

2. In the same way, if the given point, or the star, is placed on that same oblique horizon on the western side, the arc of the equator reckoned from the beginning of Aries [♈] in the succession of the signs up to the oblique horizon — that is, up to the intersection of the equator with the horizon, oblique and right — will give the oblique descension of the given point or star. Similar to this arc is the arc of the limb intercepted between the right horizon and the sighting-line of the alidade passing through the beginning of Aries [♈]. For that whole arc of the equator will be seen to have descended below the oblique horizon as soon as the star, or the given point, reaches the oblique horizon. And when the star is placed on the oblique horizon on the western side, the point of the ecliptic then found on that same horizon is the one with which the star sets. And this point is always different from the one with which the same star rises in the oblique sphere.

[Margin: To find the oblique descension of a given point of the ecliptic, or of a star, by the instrument.]

3. Given an oblique ascension or descension — whether it belongs to some point of the ecliptic or to a star — you will find the corresponding arc of the ecliptic as follows. Turn the rete until the arc of the equator running from the beginning of Aries [♈] toward Taurus [♉] and Gemini [♊], up to the oblique horizon on the

eastern side, encompasses as many degrees as are contained in the given ascension. For the point of the ecliptic that then touches the oblique horizon on that same side marks the end of the sought arc of the ecliptic, to which of course the given ascension corresponds. And if the given ascension belongs to some star, then the star must at that time be found on that same horizon. Hence, to obtain the point of the ecliptic with which the star rises, it is enough to place the star on the oblique horizon; for the point of the ecliptic touching the same horizon will be the one sought. You will easily count the ascension on the limb, proceeding from the right horizon on the eastern side toward the armilla. For if you lay the sighting-line of the alidade to the endpoint, the rete must be turned until the beginning of Aries [♈] is found exactly under the sighting-line; then the arc of the equator between Aries [♈] and the right horizon will be similar to the one counted on the limb. In no other way will you find the arc of the ecliptic descending together with the oblique descension, provided that for the eastern side you take the western.

[Margin: With which degree of the ecliptic a given star sets in the oblique sphere.]

Moreover, when the given point of the ecliptic, or the star, is placed on the oblique horizon and the sighting-line is laid over that point or star, the arc of the limb intercepted between the sighting-line and the right horizon is the ascensional difference of that point or star; for the right ascension terminates at the sighting-line, which stands in place of the right horizon, whereas the oblique ascension terminates at the right horizon, as was said in Number 1.

[Margin: To find, by the instrument, the arc of the ecliptic that rises together with a given oblique ascension or descension.]

4. From these it will not be difficult to infer the oblique ascension or descension of any arc of the ecliptic not begun from Aries [♈]. For the difference between the ascension, or descension, of the first and last points of the proposed arc will be the oblique ascension or descension of that arc. Or we may proceed thus. With the first point of the given arc placed on the oblique horizon, note on the limb — by the sighting-line of the alidade passing through that same point — the degree on which the sighting-line falls. Then turn the rete until the last point of the same given arc touches the oblique horizon, and again note the degree on the limb shown by the sighting-line passing through the first point [?]. For the arc set between those two points will be the oblique ascension or descension of the given arc, according as the eastern or the western part of the oblique horizon has been taken.

[Margin: How the ascensional difference is found by the astrolabe.]

5. We shall determine the oblique ascension and descension of any point of the ecliptic, or of a star, without an instrument, by this method. Let ABCD be the equator, whose center is E; let FLM be the tropic of Capricorn [♑] and GNO the tropic of Cancer [♋]; let AFCG be the ecliptic, whose center is H and pole I; let LCPAM be the oblique horizon drawn for the given region, whose center is K and pole Q; and through K, the center of the horizon, let KTR, a parallel of the equator, be drawn. Then, taking with the compass the semidiameter of the horizon KP, place one foot of the compass on the given point of the ecliptic, or on the center of the star — for example at d, the beginning of Virgo [♍], or at the center of the star V — and with the other foot let the center T be taken on the

[Margin: To investigate without an instrument the oblique ascension or descension of a given point of the ecliptic, or of a star.]

[Margin: In what way the oblique horizon is to be laid down for oblique descensions.]

Diagram. A large stereographic astrolabe/horizon projection enclosed in a square frame, showing the equator, the two tropics, the ecliptic, and an oblique horizon with its center-parallel and hour-circles, densely overlaid with lettered points, two star-markers, and the twelve zodiac glyphs set around the ecliptic circle. The same woodcut is repeated on the following page (see fig-p630-1 and fig-p630-2). Readable labels include:

- F (top apex, at the tropic of Capricorn [♑])
- K (upper center)
- E (center of the figure, center of the equator)
- L (just above center)
- M (right), and a small lowercase m near the left-center
- B (two positions, upper and center)
- Y (right of center), P and A (center-right, A on/near the eastern end of the horizontal diameter)
- C (left end of the horizontal diameter)
- N (lower-left of center), O (center-right, lower)
- G (bottom center, below L), D (lower center), R (bottom center)
- T (upper-left interior, and lower-left), S (left and lower)
- X (bottom-left, marked with a star ☐), V (bottom-right, marked with a star ☐)
- ∂/d (left side), H (ends of the horizontal diameter)

- Zodiac glyphs distributed around the ecliptic circle: ♋ (near F), and the succession ♊ ♉ ♈ ♇ ♆ up the right side, ♅ near the bottom (G), and ♄ ♃ ♂ ♁ ♀ down the left side.

Many peripheral single-letter and Greek labels could not be resolved with certainty at this resolution (well over 2 labels flagged).

circle KTR; and from it, through d or V, let a horizon Vdm be drawn similar to the given horizon, so that its concave side faces, from the given point, the preceding parts of the ecliptic, or the western signs — as from Virgo [♍] it faces Leo, from Scorpio [♏] Libra, and so on. For the arc of the equator CDI, from Aries [♈] up to the said horizon, will be the oblique ascension of the point d, or of the arc of the ecliptic CGd, and of the star V; because the point of the equator i rises above the oblique horizon dV together with the point of the ecliptic d and the star V. That dV is a horizon similar to the given one — that is, of the same inclination to the equator as the given horizon APC — is plain, since it is one of the hour-circles reckoned from rising or setting, as is established from what we demonstrated in Book 2, Proposition 9, Number 5. For all those circles have the same inclination to the equator as the horizon to which they are equal, by Theorem 1 of Proposition 21 of Book 2 of Theodosius, since they touch the same parallels that the horizon touches. Since, therefore, the signs and stars rise in the same way above all horizons of the same inclination — even though one is more easterly than another — it is evident that the arc of the equator CDi is the ascension of Virgo [♍] and of the star V on the given horizon, since the ascension takes place above the horizon passing through Virgo [♍] and through the star V. Thus too, if through the beginning of Scorpio [♏] — that is, through the point Z — a horizon is drawn from the center S cutting the equator at Y, the arc of the equator CDY will be the oblique ascension of the point Z, or of the arc of the ecliptic CGZ. And so for the rest. But the degree d of the ecliptic, cut off by the horizon drawn through the star V, is the one with which the star rises.

The oblique descension is found in the same way, if through the given point or star a horizon is drawn having its center on the aforesaid parallel KTR, described through the center of the horizon, but so that its convex side faces the preceding, or western, parts of the ecliptic. Thus, if through f — the beginning of Taurus [♉] — or through the star X, a horizon fX is drawn from the center S cutting the equator at l, the arc of the equator Cl will be the oblique descension of the point of the ecliptic f, or the arc Cf, and of the star X. And the degree f of the ecliptic, cut off by the horizon drawn through the star X, is the one with which the star sets.

6. If from the center E a straight line is drawn through the given point of the ecliptic, or the star, cutting the equator, the arc of the equator intercepted between that line and the horizon drawn in the way we described will be the ascensional, or descensional, difference. Thus pY will be the ascensional difference of the first point of Scorpio [♏], since its right ascension is CDp and its oblique ascension CDY. In the same way ln will be the ascensional difference of the first point of Taurus [♉]; and ki the ascensional difference of the star V.

[Margin: With which degree of the ecliptic a given star rises in the oblique sphere.] [Margin: In what way the oblique horizon is to be laid down for oblique descensions.] [Margin: With which degree of the ecliptic a given star sets in the oblique sphere.]

7. The oblique ascension of a given arc of the ecliptic not begun from Aries [♈] is the arc of the equator between the two horizons drawn through the extreme points of the given arc, so that the concave side of each faces the preceding sign — that is, the one which rises before the given point. For an arc of this kind will be the difference of the ascensions owed to the extreme points of the given arc. Thus the oblique ascension of the sign Libra [♎] is AY, that of the sign Virgo [♍] is Ai; and finally the arc dZ — between the beginning of Virgo [♍] and the end of Libra [♎] — has as its oblique ascension iAY [?]. By no different reasoning, the oblique descension of a given arc begun elsewhere than from Aries [♈] will be the arc of the equator between the two horizons drawn through the extreme points of the given arc, so that the convex side of each faces the preceding parts of the ecliptic — those, namely, which rise first. Thus the oblique descension of the sign Aries [♈] will be Cl; that of the sign Pisces [♓] will be Cq; and finally the oblique descension of the arc fm, set between the beginnings of Taurus [♉] and Pisces [♓], will be the arc of the equator lq.

[Margin: How the ascensional (or descensional) difference is found without an instrument.]

8. From a given oblique ascension or descension of some arc, or of a star, we shall come to know the corresponding arc of the ecliptic in this way. On the equator, from the beginning of Aries [♈] — namely from the point C, toward Taurus [♉], Gemini [♊], and so on — count off the given oblique ascension, and through the endpoint of the counting let a horizon be drawn, as was said in Number 5: that is, so that for an ascension its concave side, and for a descension its convex side, faces the western parts of the ecliptic. For a horizon of this kind will pass through the sought point of the ecliptic. Thus, if the given ascension of some point or star is the arc CDi, the sought point of the ecliptic will be d — namely the beginning of

Virgo [♍] — to which the aforesaid ascension corresponds; while to the ascension CDY there will correspond the arc CGZ. Likewise, to the descension Cl there will correspond the point f, or the arc Cf, from the beginning of Aries [♈]; and to the descension CDBq the arc CGFm will correspond.

[Margin: To detect, without an instrument, the oblique ascension or descension of any arc of the ecliptic not begun from Aries.]

9. There are also two other ways of investigating oblique ascensions and descensions without drawing horizons, of which the first is this. From the center E, through the given point or star, let an arc of a parallel of the equator be described against the succession of the signs, up to the horizon on the eastern side; for it will measure the oblique ascension. Thus the arc aVb will give the ascension of the beginning of Aries [♈], or of the arc of the ecliptic CGa. For, since similar arcs of the equator and of any of its parallels ascend above any given horizon in equal times, on account of the uniform motion of the primum mobile — and since the arc aVb ascends in the same time in which the point a reaches the horizon by the motion of the rete, inasmuch as the point a describes the said arc by the motion of the primum mobile — it is clear that this arc is similar to the arc of the equator which ascends above the horizon together with the aforesaid arc of the ecliptic CGa, and measures its oblique ascension. By the same reasoning the arc VXb will be the oblique ascension of the star V, similar namely to the arc of the equator Ci; likewise the arc Xb the oblique ascension of the star X, similar to the arc of the equator Ci; and the arc dfe the oblique ascension of the beginning of Virgo [♍], and the arc fe the ascension of the beginning of Taurus [♉]. Further, the arc fb is the ascensional difference of the first point a and of the stars V, X, since their right ascensions are af, Vf, Xf; likewise the arc et is the ascensional difference of the points d, f, since their right ascensions are dft, fet [?]. All this stands out more clearly than daylight from what we demonstrated in the said Lemma 49, Number 8. For the line Eb — that is, the great circle drawn from the pole of the world E through b, the point of intersection of the horizon with the parallel through the given point of the ecliptic a — cuts off from the equator the ascensional difference Cα, to which fb is similar; whereas another great circle drawn from the pole E through the given point a, namely the line Ea, makes the arc of the equator γDα the oblique ascension of the point a, to which the arc aVb is similar. So, since the parallel

[Margin: To detect, without an instrument, the ascension (or descension) obliquely; and to assign, without an instrument, the arc of the ecliptic that rises or sets together with a given oblique ascension or descension.] [Margin: Another,

twofold method of finding oblique ascensions and descensions without an instrument.]

drawn through u , the beginning of Aquarius [ϖ], would cut the horizon at b , the lines Eb , Eu — representing great circles — would cut off from the equator the arc $\beta D\alpha$, namely the oblique ascension of the arc of the ecliptic CGu . And so it is not necessary to describe a parallel through the given point of the ecliptic, but it is enough to mark on the horizon the point where it would be cut by that parallel. For a line drawn through this point, and a line drawn to the given point, will intercept on the equator the arc of the oblique ascension of the given point, as was demonstrated in the said Lemma 49, Number 8.

But if from the center R , through C and A , an oblique horizon gCA is drawn, turned opposite to the horizon of the given region, the arc aVg will be the oblique descension of the point a ; and Vg the oblique descension of the star V ; and Xg the oblique descension of the star X . Likewise dfr the oblique descension of the point of the ecliptic d , and fr the oblique descension of the point f . Finally tr will be the descensional difference of the points of the ecliptic d , f , and so on.

But the other way — which I approve of more, because in it there is no need to describe a parallel, and the ascension and descension are found at once on the equator itself — is this. Let $ABCD$ again be the equator, about the center E ; let Gee be the tropic of Cancer [ϖ], and $FL[?]$ the tropic of Capricorn [φ]; let $AFCG$ be the ecliptic, whose pole is M ; let AQC be the oblique horizon, whose pole is L and center K ; and let the oblique ascension of the beginning of Taurus [ϑ] be sought. Draw from the center E , through μ , the beginning of Taurus [ϑ], the straight line Em , passing through the point u — where, on the eastern side, the parallel described from E through the given point μ of the ecliptic cuts the oblique horizon — this line Em cutting the equator at m . With the compass take the arc ξC on the equator, from the point ξ up to the beginning of Aries [γ], reckoned against the order of the signs, and cut off from it an equal arc mq , from the point m , likewise proceeding against the order of the signs. I say that the arc qC is the oblique ascension of the beginning of Taurus [ϑ]. For if the ecliptic is imagined to move against the order of the signs — that is, from east to west — until μ , the beginning of Taurus [ϑ], reaches u , the line $E\xi$ will coincide with the line Em , and C , the beginning of Aries [γ], will come to be at q , because of the equal arcs ξC , mq . Hence it comes about that the arcs ξm , Cq are equal, and are therefore traversed in equal times: so that when the point ξ has been advanced to m , the point C will have reached q . Therefore the arc of the equator qC , reckoned from the beginning of Aries [γ] up to the horizon in the succession of the signs, is the

oblique ascension of the beginning of Taurus [γ], now existing at u, the eastern point of the horizon.

Again, let the oblique ascension of the beginning of Capricorn [ν] be sought. Draw the line EF from the center E to F, the beginning of Capricorn [ν], cutting the equator at B; and the line Ef to the eastern intersection of the horizon with the parallel described through F, which cuts the equator at t. Take an arc tBr equal to the arc reckoned [from B], toward the same side. I say that the arc rABC [?], reckoned from the beginning of Aries [γ] against the order of the signs, is the oblique ascension of the beginning of Capricorn [ν]. For when the ecliptic is moved against the succession of the signs until F, the beginning of Capricorn [ν], reaches f, the line EF will coincide with the line Ef, and C, the beginning of Aries [γ], will be at r, because of the equal arcs BACt, CtBr; and therefore at the same time B reaches t and C reaches r. Hence the arc of the equator rABC, from the beginning of Aries [γ] up to the eastern horizon, reckoned [against the order of the signs], is the oblique ascension of the beginning of Capricorn [ν], now at f, the eastern point of the horizon. And in the same way we shall find the oblique ascension of the star Z. For, the lines EZ, Ed being drawn to the star and to the intersection of its parallel with the horizon on the eastern side, if to the arc of the equator from the line EZ up to C, the beginning of Aries [γ], an equal arc is taken against the succession of the signs, up to ff, the arc ffBC will be the oblique ascension of the said star.

[Margin: Another, very easy method.]

The oblique descensions will be investigated in no different way, if instead of the eastern intersection of the horizon with the parallel described through the given point or star, the western intersection is taken. Thus, if the oblique descension of the beginning of Taurus [γ] be sought, the intersection α is to be taken; and through α a line is to be drawn from E, cutting the equator at β , and another line from E through μ , the beginning of Taurus [γ], cutting the equator at ξ . For if, to the arc of the equator ξC , an equal arc $\beta\gamma$ is taken, the arc γA will be the oblique descension of the beginning of Taurus [γ]. For when the ecliptic is moved from east to west until μ , the beginning of Taurus [γ], reaches α , and the line E ξ coincides with the line E β , the beginning of Aries [γ] will come to be at γ , on account of the equality of the arcs ξC , $\beta\gamma$. Hence it comes about that the arcs $\xi\beta$, $C\gamma$ are equal, and therefore at the same time ξ reaches β and C reaches γ ; and consequently the arc of the equator γA , reckoned from the beginning of Aries [γ] up to the western horizon in the succession of the signs, will be the oblique descension of the beginning of Taurus [γ], now at α , the western point of the

horizon. So too, if the oblique descension of the beginning of Aquarius [ϖ] be desired, let the line $E\delta$ be drawn to δ , the beginning of Aquarius [ϖ], cutting the equator at θ , and another line El to the western intersection l of the horizon with the parallel of the beginning of Aquarius [ϖ]. (Nor is it necessary that the said parallel be described; it is enough if, at the interval $E\delta$, the point l on the horizon cutting the equator at oo is noted. For if, to the arc of the equator θAC , taken against the succession of the signs up to Aries [γ], an equal arc $ooDq$ is taken, qDA will be the oblique descension of the beginning of Aquarius [ϖ], because Aries [γ] would then be at q , and so on.)

10. Now indeed we shall construct a certain figure — which we described in the second place in Book 2 of the *Gnomonics*, in the scholium to Proposition 9, from Andreas

Diagram. The same astrolabe/horizon projection woodcut as fig-p629-1, repeated at the top of this page (equator, tropics, ecliptic, oblique horizon with center-parallel and hour-circles; lettered points F, K, E, L, M, B, Y, P, A, C, N, O, G, D, R, S, T, X (star δ), $\square$, $\diamond$, ∇ , star), $\square$, $\diamond$, ∇ (the twelve zodiac glyphs set around the ecliptic). Numerous peripheral single-letter and Greek labels are unresolved at this resolution

Diagram. A second impression of the same astrolabe/horizon projection woodcut, at the foot of the page (serving the "other way" and the descension demonstration), with the same repertory of lettered points, two star-markers (X, V), and zodiac glyphs. Peripheral labels largely unresolved

Astrolabe, Book III — Canon V (continued)

(continuation of the demonstration begun in §10)

which we also described together with Andreas Schöner. In that figure, however, the circle drawn from L must **not** be divided into 12 equal parts — as, through an oversight, we there said it should be — but into the 12 right ascensions of the [twelve] signs, just as in the present figure the circle ABCD has been divided. (I mention this so that the diligent reader may correct that figure.) In the present figure the right and oblique ascensions of all points of the Ecliptic are contained, so that, given any point of the Ecliptic whatever, we can with very little trouble exhibit both its right ascension and its oblique ascension for the given elevation of the pole — the elevation, that is, for which the figure was constructed. Likewise, from the given right ascension of any point we can draw out its oblique ascension, and conversely from a given oblique ascension its right ascension; and

finally, from either one known, we can assign the corresponding point of the Ecliptic.

[Margin: To construct a figure containing the right and oblique ascensions of all points of the Ecliptic.]

Therefore, from the centre H, let a circle KLMN of any size be described, with two diameters KM, LN cutting each other at right angles. Now, taking the arc MP equal to twice the greatest declination — that is, 47° — let the straight line KP be drawn, cutting HL in Q. And since, when the straight line PH is joined, the angle PHM is twice the angle HKQ [Margin: Euclid III.20], the angle HKQ will be the angle of the greatest declination, and consequently HQK the angle of the complement of the greatest declination. Now since KH — the sine of the angle HQK, the complement of the greatest declination, in terms of the total sine KQ — is to HQ, the sine of the angle HKQ, the greatest declination, in the same terms, as KH, the whole sine, is to the sine HQ in terms of the total sine KH, it will follow — from what we demonstrated in Lemma 49, No. 19 — that HQ is the sine of the ascensional difference of the beginning of ♋ [Cancer] or ♑ [Capricorn] (that is, of the point of the Ecliptic that has the greatest declination from the Equator) at latitude 45° , comprising 43481 parts of the total sine KH, and a little more, as is gathered from the stated proportion. This sine, moreover — as we showed in that same place, and as here too appears — is equal to the Tangent of HQ, the greatest declination, with respect to the sine of the same total KH (since HQ is the tangent of the angle HKQ when KH is set as the whole sine). To this Tangent of 43481, found in the table of sines — that is, to the sine of the ascensional difference of the beginning of ♋ or ♑ at latitude 45° — there correspond $25^\circ 46'$. From this it follows that if from K, M one numbers off $25\frac{3}{4}^\circ$ and a little more, up to R, then the straight line Ra, when joined, exhibits the same point Q, inasmuch as it cuts off the straight line HQ equal to the sine of $25\frac{3}{4}^\circ$ and a little more — the amount, that is, of the ascensional difference of the beginning of ♋ or ♑ at latitude 45° which the Tangent HQ of the greatest declination, found in the table of Sines, yields (even though the sine of that ascensional difference is not itself computed from the aforesaid proportion), namely $25^\circ 46'$, as we said.

Once the point Q has been found, let there be set up the angle of the elevation of the pole [equal] to the given greatest declination; and QEH will be the angle of the complement of the elevation of the given pole HQE, which [angle HQE] is made no greater than the complement of the elevation of the pole. From the centre E, then, let there be described the Equator ABCD of any size; and, the diameter BD having been drawn at right angles to AC, let the arcs CS, ST be taken equal to the

greatest declination. The joined hidden straight line AT will cut BD in O, the centre of the Ecliptic, as we showed in Book 2, Prop. 5, No. 4; while the joined hidden straight line AS will cut the same BD in I, the pole of the Ecliptic, as we demonstrated in the same place, No. 12. The Ecliptic AFCG having therefore been described from O through C and A, let it be divided into 12 signs by straight lines drawn from its pole I through the equal twelfth-parts of the Equator, as you see done in the figure; and from the centre E let straight lines be projected through the 12 signs of the Ecliptic, each of which will pass through two opposite signs. For these cut the Equator at the right ascensions of the signs, as was stated in Canon 4, No. 7 — so that the arc of the Equator lying between C and the straight line drawn through any point of the Ecliptic (proceeding from the point C, which is the beginning of ♈ [Aries], toward D, that is, in the order of the signs) measures the right ascension of that point of the Ecliptic; while the arc intercepted between any two of the straight lines is the right ascension of the arc of the Ecliptic lying between those same two lines. These same straight lines will then, in the same way, cut the circle KLMN described at the outset at the oblique ascensions, so that the straight lines projected from the centre H through the points where those lines cut the circle KLMN form, at the centre H, the angles of the oblique ascensions. This we shall demonstrate as follows.

Diagram. A large circle enclosed in a square frame, representing Clavius's combined figure for the right and oblique ascensions of every point of the Ecliptic. An outer dotted circle carries the twelve zodiac sigils around its rim. Concentric with it are the Equator ABCD (centre E), the Ecliptic AFCG (centre O, pole I), and the oblique-ascension circle KLMN (centre H). A dense web of straight lines runs from the centres through the sign-divisions. Readable labels include:

- L (top of frame): top point of circle KLMN
- K (right / east): right point of circle KLMN
- M (left): left point of circle KLMN
- N (bottom): bottom point of circle KLMN
- A (right-of-centre): equatorial / Ecliptic point
- B (upper-left), C (left-of-centre), D (lower area): equatorial points
- E, H, O (near centre): centres of Equator, circle KLMN, and Ecliptic
- P (upper-left), Q, R (right), S, T (upper), Y (left) [?]: auxiliary construction points
- V (lower-right), I (lower-centre): auxiliary points / pole of Ecliptic
- Numerous small peripheral zodiac sigils and Greek point-letters (ξ, β, and others) around the rim — several not resolvable at this resolution.

[More than 2 peripheral labels are illegible]

DESCRIBATVR — Let there be described from E a circle $d\beta\xi$ [?], entirely equal to the circle KLMN, so that it may be similarly cut by the straight lines issuing from E as is the circle ABCD, and $d\beta\xi$ [Margin: from the scholium to Prop. 22, Euclid Book 3]; it too will be cut into the right ascensions, since both circles ABCD and $d\beta\xi$ [are equal].

First, then, that Mb is the oblique ascension of the beginning of $\mathfrak{S}$ [Cancer] at the assumed elevation of the pole — whose angle is of course HQE — will be made clear thus. Draw the straight line EY parallel to Hb; since Hb and EY are equal (being semidiameters of equal circles), HE and bY will likewise be parallel and equal [Margin: Euclid I.33]. Now since [QE], the [sine of the complement] of the elevation of the pole, is to the total sine QE as the straight line QH, the sine of the complement of the elevation of the pole, is to HE, the sine of the elevation of the pole — which [QH] we showed a little earlier to be the sine of the ascensional difference of the beginning of $\mathfrak{S}$ at latitude 45° with respect to the total sine KH — [so] with respect to HE: it will follow, from what we demonstrated in Lemma 49, No. 20, that HE is the sine of the ascensional difference of the beginning of $\mathfrak{S}$ at the proposed latitude. Therefore Yb also, shown equal to HE, will be the sine of the ascensional difference of the beginning of $\mathfrak{S}$ at the given latitude. Since, then, Yb is the sine of the arc $Y\beta$, the arc $Y\beta$ will be the ascensional difference of the beginning of $\mathfrak{S}$ in the given region. But $d\beta$ is a quadrant, the right ascension of the beginning of $\mathfrak{S}$. Therefore, the ascensional difference $Y\beta$ having been subtracted (for the oblique ascensions from $\mathfrak{V}$ [Aries] up to $\mathfrak{L}$ [Libra] are smaller than the right ascensions, as we showed in Lemma 49, No. 12), the remaining arc dY will give the oblique ascension of the beginning of $\mathfrak{S}$ [Margin: Euclid III.26] — to which the arc Mb is equal, on account of the angles at the centres dEY, NHb [Margin: Euclid I.29], which are equal because of the parallels EY, Hb.

But that the arc M ϵ is the oblique ascension of the beginning of $\mathfrak{II}$ [Gemini] we shall make plain thus. Draw Eu parallel to H ϵ [Margin: Euclid I.33]; then again [the joined line u ϵ will be]...

[the joined line u ϵ] will be equal and parallel to HE. Likewise, dm and uk having been dropped perpendicular to E ϵ , the triangles Edm, ϵ uk will be equiangular, because the angles m, k are right [Margin: Euclid I.29], and dEm, u ϵ k are the internal and external [angles], equal [Margin: Euclid VI.4]; for u ϵ and HE have been shown to be parallel. Therefore, as Ed, the whole sine, is to dm, the sine of the right ascension d ϵ of the beginning of $\mathfrak{II}$ [Gemini], so ϵ u, the sine of the ascensional difference of the beginning of $\mathfrak{S}$ [Cancer] in the given region, is to uk;

and consequently, as was shown in Lemma 49, No. 18, uk will be the sine of the ascensional difference of the beginning of Π in the given region, and the arc ue the ascensional difference — and therefore du is the oblique ascension of the beginning of Π [Margin: Euclid III.26], to which the arc Me is equal.

Likewise, that the arc Mi is the oblique ascension of the beginning of ϑ [Taurus] will be proved thus. Draw Eg parallel to Hi [Margin: Euclid I.33]; then again gi , when joined, will be equal and parallel to HE . Likewise df , ge having been dropped perpendicular to Et , the triangles Edf , ige will be equiangular, because of the right angles f , e [Margin: Euclid I.29], and the angles dEf , gie , internal and external, are equal [Margin: Euclid VI.4]. Therefore, as Ed , the whole sine, is to df , the sine of the right ascension dt of the beginning of ϑ , so ig , the sine of the ascensional difference of the beginning of ϑ in the given region, is to ge ; and therefore, as we showed in Lemma 49, No. 18, ge will be the sine of the ascensional difference of the beginning of ϑ in the given region, and the arc gt the ascensional difference, and dg the oblique ascension of the beginning of ϑ [Margin: Euclid III.26], to which the arc Mi is equal.

Again, that the arc MV is the oblique ascension of the beginning of $\mathfrak{M}$ [Virgo] we shall demonstrate in the same way. For, Ep having been drawn parallel to HV [Margin: Euclid I.33], the joined straight line pV will, as before, be equal and parallel to HE . Likewise dq , pn having been dropped perpendicular to EV , the triangles Edq , Vpn will be equiangular, because the angles q , n are right [Margin: Euclid I.29], and dEq , pVn are equal — the external and the internal [Margin: Euclid VI.4]. Therefore, as Ed , the whole sine, is to dq , the sine of the right ascension dn of the beginning of $\mathfrak{M}$, so Vp , the sine of the ascensional difference of the beginning of $\mathfrak{M}$ in the given region, is to pn . It follows, then, from what we showed in Lemma 49, No. 18, that pn is the sine of the ascensional difference of the beginning of $\mathfrak{M}$ in the same region; and therefore the arc $p[?]$ is the ascensional difference, and dp the oblique ascension of the beginning of $\mathfrak{M}$ [Margin: Euclid III.26], to which the arc MV is equal.

Finally (for in every case the same method of demonstration will be employed), that the arc $K\theta$ is the ascension of the beginning of $\approx$ [Aquarius — reading of woodcut sigil uncertain] numbered as an oblique [ascension] from the beginning of $\triangle$ [Libra], and consequently — the semicircle MNK being added — that the whole arc $MK\theta$ is the oblique ascension of the same beginning of $\approx$ [?] numbered from the beginning of γ [Aries], we shall demonstrate in exactly the same manner. For, $E\theta$ having been drawn parallel to $H\theta$ [recte: $H\zeta$? — reading uncertain] [Margin: Euclid I.33], the joined straight line $\theta\zeta$ [?] will again be equal

and parallel to HE. Likewise $\xi\mu$, ζr [?] having been dropped perpendicular to E θ , the triangles F $\xi\mu$, $\theta\zeta r$ [?] will be equiangular, on account of the right angles μ , r [Margin: Euclid I.29], and the alternate [angles] $\xi[?]\mu$, $\zeta\theta r$ [?] equal. Therefore [Margin: Euclid VI.4], as F ξ , the whole sine, is to $\xi\mu$, the sine of the right ascension $\xi\pi$ [?] of the beginning of $\approx$ [?] numbered from the beginning of $\triangle$ [Libra], so $\theta\zeta$ [?], the sine of the ascensional difference of the beginning of $\mathfrak{S}$ [Cancer] or $\mathfrak{C}$ [Capricorn] in the given region, is to ζr [?]. From what has therefore been demonstrated in Lemma 49, No. 18, ζr [?] will be the sine of the ascensional difference of the beginning of $\approx$ [?] numbered from the beginning of $\triangle$ in the same region; and therefore the arc ζl [?] is the ascensional difference. And since, as was shown in Lemma 49, No. 12, the oblique ascensions from $\triangle$ up to $\mathfrak{V}$ are greater than the right ones [Margin: Euclid III.26], if to the right ascension $\xi\pi$ [?] the said difference ζl [?] is added, ξl [?] will be the oblique ascension of the beginning of $\approx$ [?], to which the arc K θ is equal.

[Margin: To find the right and oblique ascension of any point of the Ecliptic, and — given either one — the other, together with the corresponding point of the Ecliptic, from the upper figure.]

11. Now let there be given any point Z of the Ecliptic — for example, the beginning of ϱ [Leo] — and let it be proposed to find its right ascension from the upper figure. From E, the centre of the Equator, through the given point Z, let the straight line EZ be drawn, cutting the Equator in X; and CX will be the right ascension of the given point, as was demonstrated in Canon 4, No. 5. But if the oblique ascension of the same point be desired, for the region whose angle of pole-elevation is HQE, we shall again draw from E, the centre of the Equator, a straight line through the given point Z. For this cuts off from the circle KLMN the oblique ascension M λ , as we showed just now. Accordingly, if we are bidden to draw out the oblique from a given right ascension, we shall number the given right ascension on the Equator from C up to X. For the straight line projected from E, the centre of the Equator, through X will cut off from the circle KLMN the oblique ascension M λ . But if the right ascension be sought from the oblique, let the given oblique ascension be numbered on the circle KLMN from M up to λ ; for the straight line E λ will take off from the Equator the right ascension CX. Lastly, if from a given ascension — whether right or oblique — the point of the Ecliptic to which it belongs is to be found, the given ascension is to be numbered (the right on the Equator from C up to X, the oblique on the circle KLMN from M up to λ), and through the end of the numbering and the centre E a straight line is to be drawn cutting the Ecliptic in Z. For the straight line drawn from the pole of the

Ecliptic I through Z will cut off from the Equator the arc Cl, to which the arc of the Ecliptic Cz is equal in the sphere, as far as the number of degrees is concerned.

[Margin: How the oblique descension is found from the preceding figure.]

12. Concerning, further, the descensions of arcs and points of the Ecliptic to be sought from the aforesaid figure, we give no [separate] instruction. For, as was said in Lemma 49, No. 14, the descension of any arc is equal to the ascension of the opposite arc; and so the ascension of the opposite arc is to be sought as the descension of the proposed arc.

Diagram. The companion figure to that on the previous page (the same construction of the Equator ABCD, Ecliptic, and oblique-ascension circle KLMN), enclosed in a square frame, with the outer dotted circle carrying the twelve zodiac sigils and a dense interior web of straight lines. Readable labels include:

- L (top), K (right / east), M (left), N (bottom): points of circle KLMN
- A (right-of-centre): equatorial point
- E, H, O (near centre): the centres
- P (upper), R (right), S, θ (or ϑ , upper-right), Y (left), V (lower-right), D (lower): auxiliary points
- Numerous small peripheral zodiac sigils and Greek point-letters (ξ , μ , ζ , β , and others) — several not resolvable at this resolution.

[More than 2 peripheral labels are illegible]

[Margin: That four equal arcs of the Ecliptic equidistant from the equinoctial or the tropic points have equal right ascensions.]

13. From this same figure we shall easily demonstrate that four equal arcs of the Ecliptic, of which two on each side are equidistant from the equinoctial points, or [equidistant] from the tropics, have equal right ascensions — which we also demonstrated in Lemma 49, No. 6. For since the arcs of the Equator C ϖ , A ϱ , comprising (say) 30° apiece, are equal, and through their end-points ϖ , ϱ straight lines are projected from I, the pole of the Ecliptic (these straight lines are not actually drawn, to avoid confusion), and [these] exhibit the arcs of the Ecliptic C ϖ , A ϱ — arcs, for instance, of γ [Aries] and ζ [Libra] — and since the point I lies on the diameter of the Equator BD, off its centre E, the angles which those straight lines would make with BD will, by Theorem 5 of the scholium to Prop. 29 of Book 3 of Euclid, be equal. Therefore, since those same two straight lines reach to ϖ , ϱ and make at the point I...

angles equal, off the centre O of the Ecliptic, as has been shown; then, by the same Theorem, the arcs of the Ecliptic $C\varpi$, $A\varrho$ will be equal. Consequently, since the straight lines $E\varpi$, $E\varrho$, falling from E — a point off the centre O of the Ecliptic — cut off equal arcs $C\varpi$, $A\varrho$, the angles $FE\varpi$, $FE\varrho$ will, by the same theorem, be equal; and hence the remaining [angles] ϖEd , $\varrho E\xi$ [?] will also be equal, at the centre E — of the Equator, or of the concentric circle $d\beta\xi$ [?] [Margin: Euclid III.26]. Wherefore the arcs $d\varpi$ [?], $\xi\varrho$ [?] — that is, the right ascensions of the equal arcs of the Ecliptic $C\varpi$, $A\varrho$ — will be equal. And since the straight lines ϖE , ϱE , when produced, pass through the opposite points of the Ecliptic, that is, through the beginnings of $\mathfrak{M}$ [Virgo] and $\mathfrak{S}$ [Taurus] [Margin: Euclid III.26], and since the arcs $\xi\varrho$ [?], dt are equal to the arcs $d\varpi$ [?], $\xi\pi$ [?], on account of the angles at the vertex E being equal, all four right ascensions $d\varpi$ [?], dt , $\xi\delta$ [?], $\xi\gamma$ [?] of the four equal arcs of the Ecliptic — namely of the four signs $\mathfrak{X}$ [Pisces], $\mathfrak{Y}$ [Aries], $\mathfrak{M}$ [Virgo], and $\mathfrak{L}$ [Libra], equidistant from the equinoctial points C, A, or from the tropics F, G — will be equal.

By exactly the same reasoning we shall show that the angles $FE\approx$ [?], $FE\mathfrak{L}$ [?] are equal; and these being taken away from the equal [angles] $FE\varpi$, $FE\varrho$, the remainders $\varpi E\approx$ [?], $\varrho E\mathfrak{L}$ [?] will be equal. Therefore, as before, the four right ascensions of the four equal arcs — namely of the signs $\approx$ [Aquarius, sigil uncertain], $\mathfrak{S}$ [Taurus], $\mathfrak{L}$ [Leo], and $\mathfrak{M}$ [Scorpio? — sigil uncertain] — will again be equal. And so of the rest.

[Margin: That equal arcs of the Ecliptic equidistant from either of the equinoctial points have equal oblique ascensions.]

14. It is inferred from the same figure that the oblique ascensions of two equal arcs of the Ecliptic equidistant from either of the equinoctial points are equal to each other. For let there be equal arcs of the Ecliptic $A\varrho$, $A\mathfrak{M}$ [?], equidistant from the beginning of $\mathfrak{L}$ [Libra] — that is, let them correspond to arcs equal in the sphere and equidistant from the beginning of $\mathfrak{L}$. I say that their oblique ascensions $K\theta$, KV are equal. For since their right ascensions are equal, as we showed in No. 13, the angles θEH , VEH will be equal. Since, then, the point E lies off H, the centre of the circle KLMN, on its diameter, the arcs $K\theta$, KV will, by Theorem 5 of the scholium to Prop. 29 of Book 3 of Euclid, be equal. By the same argument we shall conclude that the oblique ascensions $K\omega$, $K\lambda$ of the equal arcs of the Ecliptic $A\mathfrak{M}$ [?], AZ are equal; and consequently, the equals $K\theta$, KV being taken away, the remaining ascensions $\theta\omega$, $V\lambda$, of the equal arcs $\mathfrak{M}Z$ [?], $\mathfrak{M}Z$ [?], are also equal. And so of the rest.

[Margin: That arcs of the Ecliptic in the ascending semicircle have oblique ascensions smaller than their right ascensions by as much as the oblique ascensions of the equal opposite arcs — or of arcs equidistant from the same tropic point and lying in the descending semicircle — are greater than their right ascensions.]

15. Moreover, from the same figure it will be permissible to gather that equal arcs of the Ecliptic equidistant from either of the tropic points, or opposite through the diameter, have unequal oblique ascensions — smaller indeed in the ascending semicircle from ♊ [Capricorn] through ♈ [Aries] up to ♋ [Cancer], but greater in the descending semicircle from ♋ through ♎ [Libra] up to ♊; and that those [oblique ascensions] are smaller than the right ascensions of the same arcs by as much as these [in the descending semicircle] are greater. For let there be two equal arcs $\gamma\pi$, $\eta\varrho$ [?], equidistant from the tropic point G [and lying, one in the ascending, the other in the descending semicircle]. Since their right ascensions are equal, as was shown in No. 13, the angles $\tau\epsilon\alpha$ [?], $\nu[\epsilon\eta]$ will be equal; and since [E lies] on the diameter of the circle KLMN, off its centre H, it will follow, by Lemma [32], that the arc $[\tau\alpha]$ is smaller than the arc $\nu\lambda$. And by the same reasoning it will be proved that the oblique ascension of any arc in the ascending semicircle of the Ecliptic is smaller [than that of] the arc equal to it in the descending semicircle GAF — [an arc] which is equally distant, from the same tropic point G, as that first arc. For since the arcs $\gamma\pi$, $\eta\varrho$ [?] are equal, and equidistant from the tropic point G — and distant [also] from the [equinoctial] point A — the arc $\eta\varrho$ [?], taken with the equal and equally-moved arc $\approx\upsilon$ [?], has an equal oblique ascension, as was shown in No. 14;... [and it has] a smaller oblique ascension [than] the equal arc $\approx\upsilon$ [?] which is opposite to it,... so that C, A are distant [from them] in the order of the signs. And by the same reasoning the [arc] of the Ecliptic FCG will have a smaller oblique ascension than the equal arc [opposite it].

Next [Margin: Euclid I.5], since in the isosceles [triangle] $iH\theta$ the angles i , θ are equal [Margin: Euclid I.29], the ascensional differences gt , $\xi\pi$ [?] of the opposite arcs will also [be equal] [Margin: Euclid III.26]; [and] by as much as the oblique ascension dg , or Mi , is smaller than the right ascension dt , by so much [the oblique ascension is greater] than the right ascension $\xi\pi$ [?]. Since, then, the oblique ascension $K\theta$ has been shown equal to the oblique ascension Mi , the arc $C\gamma$ [?] is smaller than the right [ascension] by as much as [the oblique ascension of the arc] equally receding, together with it, from the tropic point G is greater than the right ascension. And exactly the same reasoning holds for the other equal arcs, whether opposite or equidistant.

16. Finally, from all this it follows that the oblique ascensions of two opposite arcs of the Ecliptic — or of arcs equidistant from the same tropic point — taken together are equal to the right ascensions of the same arcs taken together: since, evidently, by as much as the ascension of the one is smaller than its own right ascension, by so much the [ascension] of the other is greater.

[Margin: The oblique ascensions of two equal opposite arcs of the Ecliptic — or of arcs equidistant from the same tropic point — taken together are equal to their right ascensions.]

SCHOLIUM

[Margin: To elicit the oblique ascensions and descensions from the Analemma.]

1. By the Analemma we shall investigate the oblique ascensions and descensions of points of the Ecliptic and of the stars in this way. Let the figure be required which we described in the scholium of the preceding Canon, No. 5, in which the Meridian is ANCM, its centre D; the diameter of the Equator AC; the Ecliptic EP, or kl; and the axis of the world gh. If, then, the point of the Ecliptic whose oblique ascension is sought lies in the descending semicircle, let the complement of its distance from the beginning of ♎ [Libra] be numbered from E, the beginning of ♋ [Cancer], up to i; and from i let a perpendicular iF be dropped to EP; and through F let GH be drawn parallel to the diameter AC of the Equator — GH will be the diameter of the [small circle] parallel [to the Equator] described through the point at which the numbering ended. Let GH cut the diameter of the Horizon aZ in b, and the axis of the world gh in d. Finally, from d, through G, H, let the parallel semicircle GpH be described; and from b, F let the perpendiculars bp, Fq be drawn to GH. Then the arc pq will be the oblique ascension of the arc of the Ecliptic numbered from the beginning of ♎ toward ♋, whose sine is of course DF — equal to the arc ri intercepted between the perpendiculars Dr, Fi, as was shown in Book 1, Lemma 49, No. 17. If, then, you subtract the arc pq from the semicircle, the remainder will be the oblique ascension of the arc numbered from the beginning of ♈ [Aries] up to the point of the Ecliptic corresponding to the point F, in the order of the signs. And since the same oblique ascension is had by the arc numbered from the beginning of ♎ toward ♏ [Capricorn] — an arc equal to that whose sine is DF, numbered from the same beginning ♎ toward ♋, as was shown a little before in this Canon, No. 14 — if the ascension pq found is added to the semicircle, there will result the oblique ascension of the point of the Ecliptic that recedes from the beginning of ♎ toward ♏ by as great an interval as the point corresponding to F is distant from the same beginning ♎ toward ♋.

But if the point of the Ecliptic whose oblique ascension is to be found lies in the ascending semicircle, then the complement of its distance from the beginning of ♈ [Aries] is to be numbered from k, the beginning of ♐ [Capricorn], up to m; and from m a perpendicular mn is to be drawn to kl; and again...

Astrolabe, Book III — Canon V, Scholium (continued)

the line VX is to be extended again through n, parallel to AC, the diameter of the equator — namely the diameter of the parallel circle passing through the point at which the counting terminated — cutting the diameter of the horizon at T and the axis of the world at f. For if a semicircle of the parallel, V $\square$ X, is described from f through V and X, then, as we demonstrated in Book 1, Lemma 49, No. 17, the arc $\pi\xi$ of that semicircle — intercepted between the perpendiculars T ϖ [?] and r ξ drawn from T and n to VX — is the oblique ascension of the ecliptic arc; that arc being sm, intercepted between the perpendiculars Ds and nm. If, therefore, the oblique ascension so found is subtracted from the whole circle, the remainder will be the oblique ascension of the ecliptic arc counted, according to the succession of the signs, from the beginning of Aries ♈ up to the point corresponding to n. And since the arc counted from the beginning of Aries ♈ toward Cancer ♋ has the same oblique ascension — being equal to the arc whose sine is Dn, counted from that same beginning of Aries ♈ toward Capricorn ♐, as was shown in No. 14 of this Canon — the ascension so found will fit the ecliptic point that is as far distant from the beginning of Aries ♈ toward Cancer ♋ as the point corresponding to n is removed from that same beginning of Aries ♈ toward Capricorn ♐.

[Margin: Finding the ascensional difference of a given ecliptic point or star from the analemma.]

ANOTHER WAY. Once the declination of the given ecliptic point or star has been found, as taught in Canon 3, let it be counted from A and C, in whichever direction, up to G and H; and let the diameter GH of the parallel be drawn, passing through the given ecliptic point or star, cutting the axis of the world at d and the diameter of the horizon at b. Since Gb is the versed sine of the semidiurnal arc, db will be the right sine of the difference between the semidiurnal arc of the parallel and the semidiurnal arc of the equator, to which the whole sine Gd corresponds. Now since, as we showed in Book 1, Lemma 49, No. 15, this difference is the same as the difference between the semidiurnal arc

of the point (or star) and the semidiurnal arc of the equator, db will likewise be the sine of the ascensional difference of the star or given ecliptic point. Therefore, if the given point or star declines to the north, let the ascensional difference so found be subtracted from the right ascension of that same star or point, found by Canon 4; but if the star or given point declines to the south, let it be added to the right ascension. For thus the oblique ascension will be left over, or made up, as is clear from what we said in Book 1, Lemma 49, No. 15. It makes no difference whether the declination is reckoned to the north or to the south from the points A, C, since opposite points have the same ascensional difference, as was taught in the same place.

[Margin: To ascertain, from a known oblique ascension, in which part of the heavens the beginning of Aries lies.]

2. In order to elicit, from the known oblique ascension of some ecliptic point, the corresponding ecliptic arc, several things must first be explained. First, one must know that when the oblique ascension is less than a quadrant, the beginning of Aries Υ lies between the eastern point and the meridian above the horizon; when it is exactly a quadrant, it lies on the meridian itself above the horizon; when greater than a quadrant but less than a semicircle, between the meridian above the horizon and the western point; when greater than a semicircle but less than three quadrants, between the western point and the meridian below the horizon; when it comprises three quadrants, on the meridian itself below the horizon; and finally, when greater than three quadrants, between the meridian below the horizon and the eastern point.

[Margin: To ascertain the position of an ecliptic point — both on the meridian above the horizon and on the eastern horizon — from the position of the beginning of Aries.]

Next, it must not be overlooked that: when the beginning of Aries Υ is between the eastern point and the meridian above the horizon, the ecliptic point on the meridian is southern, but on the eastern horizon northern; when it is on the meridian above the horizon, the point on the eastern horizon is northern; when it is between the meridian above the horizon and the western point, both the point on the meridian and the one on the eastern horizon are northern; when it is at the western point, the point on the meridian is northern; when it is between the western point and the meridian below the horizon, the point on the meridian is northern and the one on the eastern horizon southern; when it is on the meridian itself below the horizon, the point on the eastern horizon is southern; and finally, when it is between the meridian below the horizon and the eastern point, both

the point on the meridian and the one on the eastern horizon are southern. All of this is clear on a material sphere.

Diagram. A large analemma figure — a circle inscribed in a square, representing the plane of the meridian, densely crossed by lines of the equator, ecliptic, parallels, and horizon, with the center marked D. The peripheral and interior points are lettered with a mixture of Latin capitals, Latin minuscules, and Greek minuscules; the figure is very crowded and several small labels do not resolve even under magnification. Readable labels:

- q (top of the circle)
- P (upper region, near q); P also appears again on the right of the circle and once near the bottom
- L, K (upper-left interior, clustered)
- A (left, on the circle)
- V, B (left interior)
- R [R-form] (left interior)
- Z (lower-left, on the circle)
- δ (delta, lower-left interior)
- τ , γ [?], s, f (center-left interior cluster)
- N (center-left, below D)
- D (center)
- E (center-right, below D)
- O (center-right interior)
- d (center-right interior)
- M (right interior)
- H, N, C (right, outside / on the circle, stacked)
- Y, X (lower-right, on the circle)
- a cluster of Greek/minuscule labels along the lower rim (μ , ν , ξ and neighbours) that do not resolve [?]

(more than two labels flagged [?]).

[Margin: To exhibit, by means of the analemma, the ecliptic arc corresponding to a given oblique ascension.]

3. With these things known, we shall investigate, in this way, the ecliptic arc — counted from Aries Υ according to the succession of the signs — that agrees with a given oblique ascension. If the oblique ascension is greater than a quadrant but less than a semicircle, let a semicircle be subtracted from it; if greater than a

semicircle yet less than three quadrants, let a semicircle be taken from it; and finally, if greater than three quadrants, let the whole circle be taken away. For in this way we shall always have an arc of the equator, lying between the beginning of Aries Υ and the horizon — whether eastern or western — that is less than a quadrant. Of this remaining arc (or of the oblique ascension itself, if it is less than a quadrant), let the right sine $D\alpha$ be taken on the diameter AC of the equator. This is easily done: if from g, toward A, the oblique ascension (less than a quadrant) or the remaining arc is counted up to β , and from β a perpendicular $\beta\alpha$ is dropped to AD, this will cut off the right sine $D\alpha$ that we want. And the point α will be that on which the perpendicular dropped from the beginning of Aries Υ onto the plane of the meridian falls, since the beginning of Aries Υ then lies at β — if the semicircle $\Upsilon\beta C$ is thought of as being perpendicular to the meridian, that is, the same as the semicircle of the equator. And this indeed holds when the given oblique ascension is less than a semicircle. For when it is greater, the point α will be that on which the perpendicular dropped from the beginning of Libra $\underline{\alpha}$ onto the plane of the meridian falls, because by however much the beginning of Aries Υ is depressed below the horizon on one side, by that much the beginning of Libra $\underline{\alpha}$ is raised above it on the opposite side.

This being posited, the remaining arc βA will be the one lying in the equator between that same beginning of Aries Υ (or Libra $\underline{\alpha}$) and the meridian above the horizon — that is, the right ascension of that ecliptic point which then occupies the meridian above the horizon, whose right sine is $\alpha\beta$; the right ascension, I say, begun from Aries Υ or Libra $\underline{\alpha}$. From this right ascension the declination of that point — which is then found on the meridian, and to which that right ascension belongs — is to be found, as was taught in the scholium of the preceding Canon, No. 5, namely in this way. Let a line De equal to the sine $\alpha\beta$ be taken, and at AD let a perpendicular el be raised, from which, out of the tangent AK, a length AK equal to it is cut off; for the line KD will cut off the sought arc of declination AG, as we demonstrated in the place cited. This declination will be northern when the given oblique ascension is greater than a quadrant and less than three quadrants, but southern when the given oblique ascension is less than a quadrant or greater than three quadrants, as we said in No. 2, and as is plainly gathered from a material sphere. Moreover, the line drawn from G through the center D will then be the common section of the ecliptic and the meridian. And since the ecliptic is inclined to the meridian — except when one of the tropical points is on the meridian above the horizon and the other below (for then the ecliptic is perpendicular to the meridian, because the meridian passes through its poles [Margin: a — Theodosius I.15 (marginal note reads "15.1.The." [?])]) — all the perpendiculars dropped from the ecliptic points onto the plane of the meridian

will fall on an ellipse, by Proposition 24 of Book 1 of our *Gnomonics*; one of these is α , on which falls the perpendicular from the beginning...

of Aries Υ or Libra ϖ . The major axis of this ellipse is GT, while the minor axis lies on the diameter MN, perpendicular to GT, and it is found thus. With the interval DG (half the major axis), let a point O be marked on MN from α by means of the compass, and let the line αO be drawn, cutting the major axis GT at Q. For αQ is half the minor axis; and if it is carried from D on both sides along the line MN, up to R and S, then RS will be the minor axis, by Lemma 50 of Book 1. If, then, by Lemma 52, the points T and t (through which the said ellipse passes) are found on the diameter Za of the horizon, a perpendicular erected from one of them to the meridian — namely from T, if the ecliptic cuts the horizon on its southern side — will fall on the ecliptic point then lying on the eastern horizon. But if, the line T α having been drawn, an equal length T δ is taken, and perpendiculars T ϵ [?] and $\delta\theta$ are raised to ZD such that $\delta\theta$ is equal to $\alpha\beta$, then the line $\theta\epsilon$ drawn will be equal to the chord of the ecliptic arc lying between the horizon point T and the beginning of Aries Υ (or Libra ϖ) — since it is equal to the line intercepted between the perpendiculars dropped from T and α to the plane of the meridian, which line is indeed the chord of the said arc. And thus, if by means of the chord $\theta\epsilon$ an arc $\theta\mu$ is cut off from some point, say from a, this arc will be equal to the aforesaid ecliptic arc; and so, if the said arc is counted from the beginning of Aries Υ or Libra ϖ (according as the point α corresponds to the beginning of Aries Υ or Libra ϖ), the count will terminate at the point that is then found on the horizon, and from which the perpendicular dropped onto the plane of the meridian falls at T. In the same way, if the ecliptic cuts the horizon on its northern side, the ecliptic point then lying on the horizon and corresponding to the point t will be found — if, the line ta having been drawn, an equal line is taken on Za, and so on.

Once the ecliptic point corresponding to point T or t has been found — that is, the arc intercepted between the beginning of Aries Υ (or Libra ϖ) and the eastern horizon — we shall find the ecliptic arc corresponding to the given oblique ascension in this way.

When the given oblique ascension is less than a quadrant, the point α will correspond to the beginning of Aries Υ , and the declination of the point on the meridian will be southern; the northern ellipse point t is to be assumed, and the arc found — namely the one intercepted between the perpendiculars dropped from t and α onto the plane of the meridian — will be the one sought.

When the oblique ascension is greater than a quadrant but less than a semicircle, again the point α will correspond to the beginning of Aries Υ , but the declination of the point on the meridian will be northern, as will also the point then found on the eastern horizon; and therefore the point on the western horizon, to which the beginning of Aries Υ is nearer, will be southern, and so the southern ellipse point T is to be assumed. Hence the ecliptic arc found — which is intercepted between the perpendiculars dropped from T and α onto the plane of the meridian — subtracted from a semicircle, will leave the arc sought, to be numbered from the beginning of Aries Υ according to the succession of the signs.

When the oblique ascension is greater than a semicircle but less than three quadrants, the point α will correspond to the beginning of Libra $\underline{\alpha}$, and the declination of the point on the meridian will be northern; the southern ellipse point T is to be assumed, and the ecliptic arc found — which is enclosed between the perpendiculars dropped from T and α onto the plane of the meridian, and is equal in the figure to the arc $\alpha\mu$ — must have a semicircle added to it so that the arc sought, begun from Aries Υ , is completed.

Finally, when the oblique ascension is greater than three quadrants, again the point α will correspond to the beginning of Libra $\underline{\alpha}$, but the declination of the point then on the meridian will be southern, as will also the point on the eastern horizon; and therefore the point on the western horizon, to which the beginning of Libra $\underline{\alpha}$ is nearer, will be northern, and so the northern ellipse point t is to be assumed. Wherefore the ecliptic arc found — which is set between the perpendiculars raised from t and α to the plane of the meridian (and to which is equal the opposite arc set between the beginning of Aries Υ below the horizon and the eastern horizon) — subtracted from the whole circle, will leave the arc sought, to be numbered from the beginning of Aries Υ according to the succession of the signs.

But if the proposed oblique ascension is a quadrant, the beginning of Aries Υ will lie on the meridian above the horizon, at point A; the major axis of the ellipse will be AC, and the minor axis will be the segment gh of the axis of the world, cut off from the diameters of the parallels of Cancer $\mathfrak{C}$ and Capricorn $\mathfrak{C}$, as is established from Proposition 24 of Book 1 of our *Gnomonics* — because the inclination of the ecliptic to the meridian is then equal to the complement of the greatest declination. Therefore, having again found the points at which the ellipse cuts the horizon, the northern one is to be assumed. For the arc found — namely the one set between the perpendicular erected from that northern point to the meridian and the point A — will be the one sought.

But if the ascension comprises three quadrants, the beginning of Libra ϱ will lie on the meridian above the horizon, that is, at point A; the same ellipse as before is formed, but its southern horizon point is to be assumed, and to the arc found — which is intercepted between the perpendicular raised from that southern point to the meridian and the point A — a semicircle is to be added, so that the arc sought comes out to be numbered from Aries γ .

Finally, if the ascension is a semicircle, the ecliptic arc corresponding to it will also be a semicircle. All of which is easily gathered from what we said in No. 2, and from a material sphere.

[Margin: To investigate, by sines, the oblique ascension of a given ecliptic point or star.]

4. By the doctrine of sines we shall obtain the same result, in this way. If, through the ecliptic point (or the center of the star) as it rises or sets, a great circle is drawn, like a kind of right horizon, then (as is evident from a material sphere) the arc of the equator lying between that circle and the horizon will be the ascensional (or descensional) difference — the right ascension (or right descension), begun from Aries γ and proceeding according to the succession of the signs, terminating on that great circle, while the oblique one terminates on the horizon. [Margin: Finding the ascensional difference.] This difference is to be computed in a right-angled spherical triangle, of which one side is the difference itself; another is the arc of the said great circle intercepted between the equator and the ecliptic point (or star), measuring the declination of that point or star; and the base, finally, is the arc of the horizon enclosed between the equator and the ecliptic point (or star), measuring the rising (or setting) amplitude — namely in this way.

Let Figure 1 of this Canon be repeated, in which the right ascension of the first point of Cancer $\var�$ [?] is the arc CDp, the oblique one CD γ (CDY), the ascensional difference p γ (pY), and pZ the arc of declination. If, then, by the first method of Problem 10 of the [treatise on] spherical triangles of the last Lemma, it is made that as the whole sine is to the tangent of the complement of the angle pYZ — which the equator makes with the horizon, and which in the present case is always acute (for since all the arcs are less than a quadrant, inasmuch as they measure the declination, the ascensional difference, and the rising amplitude, all of which comprise fewer than 90 degrees, the two angles Y and Z will be acute, by Proposition 28 of our [treatise on] spherical triangles) — that is, to the tangent of the pole's altitude, so is the tangent of the declination pZ to a fourth term, there will be produced the sine of the ascensional difference pY. That the ascensional

difference can be found in this way we also demonstrated, without spherical triangles, in Lemma 49, No. 17.

[Margin: Another way of finding the ascensional difference.] But if you do not wish to use tangents, the same difference will be found — as was demonstrated in the same Lemma, No. 18 — if it is made that as the whole sine is to the sine of the right ascension of the given ecliptic point, so is the sine of the ascensional difference of the beginning of Cancer ♋ (or Capricorn ♐) in the given region (which sine is found by the first method of Problem 10 of the [treatise on] spherical triangles, as was said, so that this sine alone is to be sought by tangents) to a fourth term. For in this way the sine of the ascensional difference of the given ecliptic point is found.

[Margin: Yet another way of finding the ascensional difference.] The same difference is found — as we showed in the same Lemma, No. 20 — in this way: let it be made that as the whole sine is to the tangent of the altitude of the given pole, so is the sine of the ascensional difference of the given ecliptic point at pole-altitude 45 degrees (which difference the tangent of the declination will supply in the table of sines, as we proved in No. 19 in the same Lemma 49) to a fourth term. For the fourth number will be the sine of the sought ascensional difference.

The ascensional difference of any star will be computed no differently, as is clear in the case of star V: for again, by the first method of Problem 10 of the [treatise on] spherical triangles, in the spherical triangle kiV — whose angle k is right — let it be that as the whole sine is to the tangent of the complement of the angle kV , that is, to the tangent of the pole's altitude, so is the tangent of the declination kV to the sine of the ascensional difference ik , and so on. And the same procedure holds for all ecliptic points and stars, whether they have southern or northern declination.

[Margin: Finding the descensional difference.]

Exactly the same procedure applies to computing the descensional difference of any ecliptic point or star. Thus, in the same figure, the right descension of the beginning of Taurus ♉ is the arc Cn of the equator, the oblique descension is Cl , and the descensional difference is ln . And finally, by the first method of Problem 10 of the [treatise on] spherical triangles, as the whole sine is to the tangent of the complement of the angle fn — that is, to the tangent of the pole's altitude — so is the tangent of the declination fn to the sine of the descensional difference ln , and so on. But there is no need to compute the descensional difference, since it is equal to the ascensional difference: because the oblique ascension is smaller than

the right ascension by just as much as the oblique descension is greater than the right descension of the same point, or the reverse, as was shown in Lemma 49, No. 12.

Diagram. A second large analemma figure — a circle inscribed in a square, again the plane of the meridian, crowded with the equator, ecliptic, several parallels (some drawn as dotted circles), horizon and hour-circles, plus two small burst/star marks near the lower rim (at X and V). Labels are mixed Latin and Greek minuscules and several do not resolve. Readable labels:

- F (top of the circle)
- K, B (upper interior, on the vertical)
- U [?], T (upper region, flanking K)
- L (left, on the circle); M appears twice at the right (M, M) on and outside the circle
- Z (upper-right interior)
- Y, P (right interior)
- A (right interior), O, E (center-right), D (lower center, on the vertical)
- H, R (right interior)
- S, N (left interior), G (lower center)
- T (lower-left, on the rim), X (bottom-left, at a burst mark), V (bottom-right, at a burst mark)
- δ (delta, lower-left), and a cluster of Greek minuscules along the lower rim that do not resolve [?]
- the center of the figure (labelled E on the vertical axis)

(more than two labels flagged [?]).

Once the ascensional or descensional difference has been found, we shall elicit the oblique ascension or descension in this way. If the ecliptic point or star declines to the north, let the ascensional difference found be subtracted from the right ascension of that same point or star; but let it be added to the right ascension if the point or star has southern declination. For the number remaining, or the number made up, will give the oblique ascension sought, as was taught in Lemma 49, No. 15, and is plainly gathered from the figure set out. For a point — say, the northern point d, namely the beginning of Virgo ♍ [symbol reads "np"; ?] — has the oblique ascension CDi, smaller than the right ascension, which terminates beyond i, namely at the point where a right horizon thrown out from E through d would fall; and the same reasoning holds for the other points and stars northern of the equator. From which it follows that the ascensional

difference must be subtracted from the right ascension for the oblique ascension to be left over. But the southern point Z — namely the beginning of [♊? Pisces; symbol illegible] — has the oblique ascension $CD\Upsilon$ (CDY), greater than the right ascension CDp ; and in the same way the star V, southern of the equator, has the oblique ascension CDi , greater than the right ascension CDk ; and so for the other points and stars southern of the equator. From which it comes about that the ascensional difference must be added to the right ascension for the oblique ascension to be produced.

[Margin: How the oblique ascension is elicited from the ascensional difference.]

The exact opposite must be done in investigating the oblique descension. For in the case of ecliptic points and stars northern of the equator, the descensional difference must be added to the right descension; but in the case of points and stars southern of the equator, the same difference must be subtracted from the right descension, so that the oblique descension is made up or left over — because northern points have oblique descensions greater than their right ones, while southern points have smaller. Thus, in the same figure, the oblique descension of the beginning of Taurus ♉ — that is, of the northern point — is the arc Cl , greater than the right descension Cn . But the oblique descension of the beginning of Pisces ♊ — which is southern — is measured by the arc $CDAq$ [?], smaller than the arc of the right descension $CDA\alpha$; and so for the rest.

[Margin: To elicit the corresponding ecliptic arc from a given oblique ascension or descension by numbers.]

Now, given the oblique ascension or descension of some ecliptic point or star, we shall find the corresponding ecliptic point — the one that rises or sets together with the star, or to which the given ascension or descension belongs — in this way. When the oblique ascension or descension is greater than a semicircle, let a semicircle be subtracted from it, so that we always have an oblique-angled spherical triangle, two of whose sides (one on the equator, the other on the ecliptic), begun from the beginning of Aries ♈ or Libra ♎, terminate on the horizon, while the third, on the horizon itself, is the arc of the rising or setting amplitude of the ecliptic point sought. And since in this triangle one side is given — namely the arc of the equator measuring the ascension or descension begun from Aries ♈ or Libra ♎ — together with the two angles adjacent to it: one being the angle of greatest declination, which the equator makes with the ecliptic; the other, which the equator makes with the horizon — this latter obtuse (the one that remains when the complement of the pole's altitude is subtracted from a semicircle) when the given oblique ascension begins from Aries ♈ and the

descension from Libra ϱ , but acute (equal to the complement of the pole's altitude) when the ascension begins from Libra ϱ and the descension from Aries Υ , as is clear on a material sphere — the ecliptic arc sought, begun from Aries Υ or Libra ϱ and terminated on the horizon, will be found by Problem 23 of the [treatise on] spherical triangles of the last Lemma.

To make this clearer, let there be a triangle of this kind, ABC (with the perpendicular AD), in which the arc of the equator measuring the oblique ascension or descension is AB; the ecliptic arc sought is BC, so that the angle of greatest declination is ABC; the arc of the horizon measuring the rising amplitude is AC; and BAC is the angle which the equator makes with the horizon. From this angle let a perpendicular arc AD be dropped to the ecliptic BC — whether it falls inside or outside the triangle ABC, the operation itself will soon show. Since, then, in the spherical triangle ABD the angle D is right, and AB — the arc of the given ascension or descension, opposite the right angle — is given, together with B, the angle of greatest declination, then, by the first method of Problem 8 of the [treatise on] spherical triangles: let it be made that as the whole sine is to the sine of the arc AB (the oblique ascension or descension), so is the sine of the angle B (of greatest declination) to a fourth term, and the sine of the arc AD will be produced.

Diagram. Two small triangles side by side, each with apex A, illustrating the two positions of the perpendicular AD relative to the triangle ABC. Labels:

- Left triangle: apex A; base lettered (left to right) B, D, C — the foot D falling between B and C, so the perpendicular AD lies inside the triangle.
- Right triangle: apex A; base lettered (left to right) B, C, D — the foot D falling beyond C, so the perpendicular AD lies outside the triangle.

Again, since in the same triangle ABD the arc AB (opposite the right angle) is given — as it measures the given oblique ascension or descension — and moreover the angle B of greatest declination is given, then, by the first method of Problem 3 of the [treatise on] spherical triangles: let it be made that as the whole sine is to the sine of the complement of the arc AB (the given oblique ascension or descension), so is the tangent of the angle B (of greatest declination) to a fourth term, and there will be produced the tangent of the complement of the angle BAD. If this is found to be smaller than the angle BAC — which the equator and the horizon contain — the perpendicular arc AD will fall inside the triangle; but if greater, outside. Therefore, subtracting the found angle BAD from the given angle BAC (or the latter from the former), the angle CAD will also be known.

Next, since in the same triangle ABD the arc AB, opposite the right angle — which of course measures the oblique ascension...

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the [rising] or the setting [of the star], together with the angle B, [it computes], along with the greatest declination — by way of the first method of Problem 9 of the spherical triangles. Set the whole sine to the sine of the complement of the angle B, the greatest declination, as the tangent of the arc A B, the [given] right ascension, is to the tangent of the side B D; and thus the arc B D will be known.

[Margin: Which point of the ecliptic rises or sets together with a given star.]

LASTLY, since in the triangle C A D the angle D is right, proceed by the first method of Problem 11 of the spherical triangles. Set the whole sine to the sine of the arc A D found in the first operation, as the tangent of the angle C A D, known from the second operation, is to another; this will yield the tangent of the arc C D, and so the arc C D will be known. Now if the perpendicular arc A D falls *within* the triangle A B C, the sum of the sides B D and C D, both known, will give the whole side B C — the arc of the ecliptic that answers to the given oblique ascension or descension. But if it falls *outside* the triangle, then C D subtracted from the side B D will give the remaining side B C that is sought. The extreme point C on the ecliptic is that point which rises or sets together with the star whose oblique ascension or descension was given. Far more easily, in the scholium to Canon 22, we shall find the same arc of the ecliptic answering to a given oblique ascension or descension without numbers — by only four operations, as you see, whereas here it was found by [numerical computation].

[Margin: How the declination of a star is found from its meridian altitude.]

NOW that we have taught by what method the declination of any star and its right ascension and culmination are to be found, let us also teach by what device, from the declination of a star and the culmination of the heavens, its latitude and its true place in the zodiac may be discovered; likewise by what art, from the declination of a star together with its latitude, the same true place may be investigated. The declination of a star is easily known from its meridian altitude taken with the instrument. For when its meridian altitude is southern: if it be found smaller than the complement of the pole's altitude, subtract it from the complement of the pole's altitude; but if greater, subtract on the contrary the

complement of the pole's altitude from it. The remainder in either case will be the star's declination — southern by the former case, northern by the latter. But when the meridian altitude of the star is northern: if it be smaller than the pole's altitude, take it away from the pole's altitude; if greater, subtract on the contrary the pole's altitude from it. The remaining number will indicate the complement of the star's declination, which will be northern.

[Margin: To find with which point of the ecliptic a given star culminates, even if its place in the zodiac is unknown.]

The culmination of the heavens too — that is, the point of the ecliptic which reaches the meridian together with the star — will be known thus. When the star is on the meridian, let the hour then instant be sought through the altitude of some other star whose place in the zodiac is not unknown, as we shall teach in Canon 8 and its scholium. For by means of this hour, once found, we shall arrive at the point of the ecliptic that is on the meridian at that moment, as will be demonstrated in Canon 11 and its scholium. The latitude of the star, finally, is plain from the tables of the fixed stars, since it does not change.

[Margin: The finding of a star's latitude and true place from its declination and the culmination of the heavens.]

AND SO, if in the twelve circles set out at the end of the scholium to Canon 3 the point M be known — the culmination point of the star H — together with the declination H L, then we shall hunt out the star's latitude and its true place as follows. Find the arc L M, the declination of the point M, as we taught in the scholium to Canon 3. Then by the first method of Problem 3 of the spherical triangles, in the triangle E L M, set the whole sine to the sine of the complement of the arc of the ecliptic E M — reckoned from the nearest equinox to the culmination point — as the tangent of the angle L E M, the greatest declination, is to another; and there will be found the tangent of the complement of the angle E M L. To this angle the angle H M I, vertically opposite, is equal in the first circle, and it is opposite the arc H I, the star's latitude. In the third and twelfth circles such an angle, opposite the star's latitude H I, is the complement of the greatest declination A E B or C E D — which happens when the star culminates with the beginning of ♈ [Aries] or ♎ [Libra].

Next let the declination of the star and the declination of the point M, the culmination point, be compared with each other. If they are of the same denomination — as in the 1st, 6th, 8th, and 10th circles — subtract the smaller from the greater; but if of different denomination — as in the 2nd, 4th, 5th, 7th,

9th, and 11th circles — add them into a single sum, so that there results, or is made up, the arc H M between the star and the ecliptic. When the culmination point is the beginning of Υ or ϱ , as in the 3rd and 12th circles, such an arc is equal to the star's declination H L. After this, in the triangle H I M, whose angle I is right, proceed by the first method of Problem 8 of the spherical triangles: set the whole sine to the sine of the arc H M just found, as the sine of the angle H M I, found in the operation above, is to another; and there will be recovered the sine of the arc H I, the star's latitude. When the culmination point is the beginning of Υ or ϱ , as it is in the 3rd and 12th circles, then by the first method of the said Problem 8 set the whole sine to the sine of the star's declination H L, as the sine of the angle H L I — which is equal to the complement of the greatest declination — is to the sine of the star's latitude H I. Having found the star's latitude H I, we shall proceed to the knowledge of its true place in the way we shall presently subjoin, a method which indeed assumes the star's declination and latitude to be known.

Diagram. First of two framed panels, each holding four small spherical (analemma-type) projections that illustrate the successive cases of the demonstration above. Each little sphere carries the meridian frame lettered A (upper left), B (left), C (lower right), D (right), with the centre E, the top points F and G, the star H (marked with an asterisk), and the interior points I, K, L, M variously placed; the first sphere additionally shows rim points N, R, O, P, S, T. The labels are small and, on several spheres, abraded. Confidently read across the four spheres of this panel: A, B, C, D, E, F, G, H (starred), I, K, L, M, and (sphere 1 only) N, O, P, R, S, T. Individual interior-point placements are not all separately resolvable. (many labels [?]).

Diagram. Second framed panel, four further small spherical projections continuing the same series of cases. Same lettering scheme: meridian A, B, C, D; centre E; top points F, G; star H (asterisk); interior points I, K, L, M. Labels small and partly abraded; per-sphere interior placements not all resolvable. (many labels [?]).

LET the star's declination H L, then, be known, and its latitude H I, and consequently their complements F H and G H. Since the arc F G, the greatest declination, is also known, all three sides in the spherical triangle F G H will be known. Therefore, by Problem 21 of the spherical triangles, the angle F G H will become known, and with it its arc A I, which measures the star's distance from the beginning of ϱ [Cancer] when its latitude is northern (as in the earlier six circles); or else the arc C I, which measures the star's distance from the beginning of Υ [Capricorn] when its latitude is southern (as in the later six circles). Whether

this distance from $\mathfrak{S}$ or $\mathfrak{V}$ is to be numbered with, or against, the succession of the signs, the culmination point M will teach us. For from it we shall learn whether the star is on the descending semicircle of the ecliptic or on the ascending one, since that point and the star lie in the same semicircle of the ecliptic. Or else the same thing will be known from the position of the star...

the star. For if it is nearer to the beginning of $\mathfrak{V}$ than to the beginning of $\mathfrak{A}$, it will be on the ascending semicircle; but on the descending, if it stands nearer to the beginning of $\mathfrak{A}$ than to the first point of $\mathfrak{V}$. Therefore, with the star on the descending semicircle, the numbering must be made from $\mathfrak{S}$ following the succession of the signs, but from $\mathfrak{V}$ against it; whereas with the star on the ascending semicircle, the numbering must be made from $\mathfrak{S}$ against the succession of the signs, and from $\mathfrak{V}$ following the order of the signs. In this way, by the aforesaid Problem 21, the angle F G H will be found.

Set the whole sine to the sine of the greater side F G, the greatest declination, or of G H, the complement of the latitude, as the sine of the smaller side is to another; and there will be found a certain fourth number. Then again set that fourth number found to the whole sine, as the difference between the versed sine of the side F H, the complement of the star's declination, and the versed sine of the arc by which the two sides F G and G H differ from each other, is to another. For there will be found the versed sine of the angle F G H. Therefore the angle F G H, and with it its arc A I or C I, will be known — the arc that measures the star's distance from the beginning of $\mathfrak{S}$ or $\mathfrak{V}$.

BUT if the complement of the latitude be equal to the greatest declination — that is, if the sides F G and G H be equal — the same angle F G H will be found more easily. For proceed by the second method of Problem 1 of the spherical triangles: set the whole sine to the sine of half the side F H, as the secant of the complement of the greatest declination F G is to another; this will yield the sine of half the angle F G H, and so on.

Diagram. A framed panel of four small spherical projections completing the twelve-circle series begun on the preceding leaf. Same lettering scheme throughout: meridian A (upper left), B (left), C (lower right), D (right); centre E; top points F, G; star H (asterisk); interior points I, K, L, M variously placed. Labels small and partly abraded; individual interior placements not all separately resolvable. (many labels [?]).

CANON VI.

To explore the rising or setting latitude of the Sun, or of any point of the ecliptic, or of a star, on any day of the year. And conversely, to find the point of the ecliptic that answers to a given rising or setting latitude.

1. The *rising* or *setting latitude* of the Sun, or of a degree of the ecliptic, or of a star, is called the arc of the horizon intercepted between the equator and the Sun (or the degree of the ecliptic, or the star) as it rises or sets. Some call this the zenith of the rising or setting of the Sun or of a degree of the ecliptic or of a star; others call it the rising or setting *amplitude*. You will explore it thus. Place the degree of the ecliptic in which the Sun stands, or the point of the proposed star, upon the horizon, whether on the eastern side or the western. For the vertical circles intercepted between the degree of the ecliptic (or the star) and the intersection of the horizon with the equator or with the prime vertical will indicate the rising or setting latitude — that is, how many degrees are contained in that arc of the horizon which lies between the degree of the ecliptic (or the star) and the said intersection. And if indeed the degree of the ecliptic or the star, when on the horizon, lies between the equator (or the prime vertical) and the meridian line of the astrolabe, the latitude will be northern; but southern if it lies between the equator and the limb.

[Margin: What the rising or setting latitude is.] [Margin: To investigate the rising or setting latitude by means of the astrolabe.]

2. Now the rising latitude of any point is equal to the setting latitude of the same point. For since the horizon touches the greatest of the ever-apparent parallels, ^a its two arcs between the equator and any parallel it cuts (of which one determines the rising latitude, the other the setting) will be equal to each other. From which it follows that it is enough if either the rising latitude be found (since it is equal to the setting), or the setting (since it is equal to the rising), as we have shown. Indeed, since four points of the ecliptic have equal rising latitudes — as we showed in Lemma 49, number 5 — it is enough if the rising latitudes of the degrees of a single quadrant of the ecliptic be found.

[Margin: ^a Theodosius II.13.] [Margin: That the rising latitude is equal to the setting latitude.]

NOW when the degree of the ecliptic, or the point of the star, does not fall precisely upon some one of the vertical circles — as most often happens — the quantity of the rising latitude cannot be known except by estimation, more or less, by dividing in thought the space between the two nearest verticals, between which the degree of the ecliptic or the star lies, into as many degrees as are intercepted between any two verticals on the astrolabe.

[Margin: From a known rising or setting latitude, to find the corresponding point of the ecliptic.]

3. CONVERSELY, from a known rising or setting latitude of the Sun, the degree of the ecliptic to which it belongs will be known thus. Turn the rete about until some degree of the ecliptic falls precisely at the end of the known latitude. That is indeed the degree sought — or else another, one that stands off at an equal space with it from the same tropic point; for two points equally distant from the same tropic point have the same rising latitude, as was shown in Lemma 49, number 3. Further, the known rising latitude must be taken on the horizon from the equator toward the limb if it is southern, but toward the tropic of ♋ if it is northern.

[Margin: To inquire the rising latitude without an instrument.]

4. WITHOUT an instrument we shall know the same rising latitude more surely thus. Let the first figure of the preceding Canon be repeated, in which the equator is A B C D about the centre E; the tropic of ♋ is F L M; the tropic of ♏ is G N O; the ecliptic is A F C G, whose centre is H and pole I; the horizon, oblique and drawn for the given region, is L C P A M, whose centre...

Diagram. A large square-framed circular (stereographic astrolabe-type) projection for Canon VI — the "first figure" referred to in the text: the equator A B C D about centre E, the two tropics, the oblique ecliptic, and the oblique horizon, with the zodiacal signs and numerous point-letters around and within. Confidently read: the outer meridian/frame letters A (upper left), B (left), D (right), C (lower right); the centre region and several interior letters (E, H, I, K, L, M, N, O, P, R, S, T, V, Y). A ring of zodiac signs runs round the rim (♈, ♉, ♊, ♋, ♌, ♍, ♎, ♏, ♐, ♑, ♒, ♓). Many small interior labels are abraded/overlapping and not all separately resolvable. (many labels [?]).

is K, and pole Q. If then, through a given point of the ecliptic, or through a given star — that is, through its place found on the astrolabe, as was delivered in Book

2, Proposition 11, numbers 2 and 3 — a parallel of the equator be described from the centre E, it will cut off from the horizon the arc of the rising latitude as far as C, and of the setting latitude as far as A, since at that point of the horizon which it cuts off, that degree of the ecliptic, or the star, rises or sets. And if from Q, the pole of the horizon, through the point where the said parallel cuts the horizon, a straight line be drawn, the arc of the equator intercepted between this line and the point C (or A) will indicate the quantity of the latitude — so that the latitude contains as many degrees as are comprised in that arc of the equator; because that arc of the equator and the cut-off arc of the horizon contain an equal number of degrees, as we demonstrated in Book 2, Proposition 5, number 19. For example: the rising latitude of the beginning of ♈ is the arc of the horizon C N, the setting latitude is A O, and both are northern; the rising latitude of the beginning of ♋ is the arc C L, the setting latitude is A M, and both are southern; the latitude of the beginning of ♎ [Libra — symbol uncertain] is the arc C b, which also suits the star V or X, and is southern. And if from Q, the pole of the horizon, a line be drawn to b, the arc of the equator between this line and the point C will give the quantity of the latitude C b. And so with the rest.

BUT if it seem too troublesome to seek the place of that star whose latitude is wanted, take its declination from the table of some astronomer in which the declinations of the stars have been computed for this present time — such as Giovanni Antonio Magini [Magini] also composed in his *Ephemerides*. For a parallel of that declination, described from the centre E, will cut off from the horizon the arc of the rising latitude of that star; but the latitude is found more exactly by the former method, because the tables of stellar declinations are scarcely ever found without some error.

5. GIVEN, then, the rising or setting latitude, we shall find the point of the ecliptic to which it belongs by this method. Number the proposed latitude on the equator from the point C, toward D if it is northern, toward B if it is southern. Through the end of the numbering, from Q, the pole of the horizon, let a straight line be sent out, which will cut off from the horizon the same latitude, as is clear from what we wrote in Book 2, Proposition 5, number 18. Lastly, from the centre E, through the end of the latitude found on the horizon, let a parallel of the equator be described. For this will cut the ecliptic in two points, to which the proposed latitude belongs. What degrees those two points represent, you will learn from number 19 of Proposition 5, Book 2 — namely if from I, the pole of the ecliptic, you cast straight lines through those points. For these will cut off from the equator similar arcs, as far as the number of degrees is concerned. For example: if,

from a given northern rising latitude, the arc C e (northern) has been found on the horizon, the parallel of the equator described from E through e will pass through f, the beginning of ♉ [Taurus], and through d, the beginning of ♍ [Virgo]. Likewise, if from a given southern latitude the southern arc C b has been found on the horizon, the parallel described from E through b will pass through a, the beginning of ♊ [Pisces], and through u, the beginning of ♒ [Aquarius]. The former latitude, then, belongs to the beginnings of ♉ and ♍, the latter to the first points of ♊ and ♒.

[Margin: From a known rising or setting latitude, to find without an instrument the point of the ecliptic that answers to it.]

HOW GREAT the arc of the horizon intercepted between C (or A) and the vertical drawn through the centre of the Sun may be, at any hour of the day — not only at rising or setting, as here delivered — we shall teach in Canon 16.

SCHOLIUM.

[Margin: To find, from the analemma, the rising latitude of any point of the ecliptic or star.]

1. That we may teach by what method we can grasp from the analemma the rising latitude of any point of the ecliptic or of a star, let the analemma itself be described, together with the diameters of the parallels passing through the beginnings of the signs, as was delivered in Lemma 19 of Book 1: in it the meridian is A B C D about the centre E; the axis of the world is F G; the diameter of the equator is H I; the diameter of the horizon is B D; the vertical is A C; the tropic of ♈ is M O; the tropic of ♋ is N P; and the diameters of the other parallels passing through the beginnings of the signs have been described by means of the circle M K N divided into 12 equal parts, as we wrote in the said Lemma 19, cutting the diameter of the horizon at L, R, S, T, V, Y.

I say that the straight line between E and any parallel is the sine of the rising and setting latitude of that point through which the diameter of that parallel passes — namely E L, the sine of the rising latitude of ♈; E R of ♊ [Gemini] and ♌ [Leo]; E S of ♉ and ♍; E T of ♏ [Scorpio — first symbol uncertain] and ♎; E V of ♏ [Sagittarius — first symbol uncertain] and ♒; and lastly E Y of ♋ — so that the straight lines drawn from these points perpendicular to B D intercept, upon A C, the arcs of the rising latitudes on the meridian. For example, the arc A q (or C b) — with b q and Y d drawn perpendicular to B D through L and Y — is the rising or

setting latitude of $\mathfrak{S}$, and C d that of $\mathfrak{V}$. For since the horizon and the parallel of $\mathfrak{V}$, drawn through the lines B D and M O, are at right angles to the meridian ^a (because the meridian, drawn through their poles, is at right angles to them), ^b their common section through L, passing to the same, will be a straight line, and therefore, by Definition 3 of Book 11 of Euclid, perpendicular to B D lying in the plane of the meridian. If then the circle A B C D be conceived in the plane of the horizon, q b will be the common section of the horizon and the parallel of $\mathfrak{S}$, provided the line B D holds the position of the meridian line. And in the same way A C will be the common section of the horizon and the equator or prime vertical; and Y d the common section of the horizon and the parallel of $\mathfrak{V}$. Therefore A q (or C b) will be the latitude of the rising or setting of $\mathfrak{S}$, and C d that of $\mathfrak{V}$. And the same reasoning holds for the intermediate parallels. For by the same argument we shall show that the perpendiculars to B D drawn through R, S, T, V are the common sections of the horizon and the intermediate parallels. By this method you will find the rising latitude of any point of the ecliptic, if by means of the circle M K N you find that point's declination...

[Margin: ^a Theodosius I.15. ^b 19, from which.]

Diagram. A large square-framed analemma for the scholium — two intersecting great circles (meridian and its projections) filled with a dense lattice of parallels, perpendiculars, and dotted construction lines. Confidently read of the principal lettering: the meridian frame A, B, C, D; the centre E; the axis endpoints F and G; the equator-diameter endpoints H and I; the tropic points M, N, O, P; the horizon-diameter division points K, L, R, S, T, V, Y; and the ring letters running round the rim. In addition many point-labels are set in Greek minuscule and lowercase italic (e.g. φ , ψ , μ and small letters a, b, d, e, q, etc.) together with zodiac signs around the circumference; these small labels are numerous, overlapping, and largely abraded, and are not individually resolvable at this resolution. (many labels [?]).

Astrolabe, Book III — The Use of the Astrolabe

(Canon VI, Scholium §§2–4, continued; Canon VII and its Scholium §1)

so that you find it — that is, draw the diameter of the parallel passing through that point, as we taught in the said Lemma 19. For a diameter of this kind will cut off from BD the sine of the latitude sought, in such a way that a perpendicular to BD, erected at the end of that sine, marks off the arc of latitude you seek, reckoned from A or C.

You will obtain the rising or setting latitude of any star in no different way, if — having found its declination either from Canon 3 or taken from the table of some astronomer — you draw in the analemma the diameter of the parallel that the star describes. Thus, suppose some star has a northern declination HM, so that the diameter of its parallel is MO; then its rising or setting latitude will be Aq, or Cb, and so on.

[Margin: *Given the rising latitude, to find the corresponding point of the ecliptic.*]

[Margin: *Another way of finding rising latitudes from the analemma.*]

2. From a given rising or setting latitude, on the other hand, we shall obtain the corresponding point of the ecliptic in this way. Reckon the given latitude from A or C toward D if it is northern, or, if it is southern, toward B, as far as σ ; and, having dropped from σ a perpendicular σR to BD, draw through R the line R ϕ parallel to the equator's diameter HI, cutting the circle M κ N at ϕ . For the point of the ecliptic to which the northern latitude Ab belongs recedes from the beginning of Υ or $\underline{\alpha}$ toward $\mathfrak{S}$ by as many degrees as are contained in the arc $\kappa\phi$ — as is clear from what was explained at the end of Lemma 19 of Book 1 and in the scholium to Canon 3, No. 3.

3. Now, just as by means of the circle M κ N, described about the greatest declinations of the Sun, the declinations of all points of the ecliptic are found (as we set out at the end of Lemma 19 of Book 1 and in the scholium to Canon 3, No. 1), so by means of another circle, described about the rising latitudes of $\mathfrak{S}$ and $\mathfrak{V}$, we shall hunt out the latitudes of all points of the ecliptic; and this in the following way. When the latitudes of $\mathfrak{S}$ and $\mathfrak{V}$ — Cb and Cd — have been found, as was said, let the straight line bd be joined, cutting EC at f; and bd will be bisected at f, by the scholium to Proposition 27 of Book 3 of Euclid [Euclid III.3], and therefore also at right angles. So, having described from f, through b and d, the circle bpd, and having divided it into 12 equal parts, if two points equally distant from the points b and d are joined by hidden lines, the arc bCd will be cut into the rising latitudes that correspond to the beginnings of the signs. Thus Cb is the latitude of $\mathfrak{S}$; Cg, of Π and ϱ ; Ch, of Υ and $\mathfrak{M}$; Ci, of $\mathfrak{M}$ and $\mathfrak{X}$; Cl, of $\mathfrak{X}$ [?] and $\mathfrak{A}$; and Cd, finally, of $\mathfrak{V}$ — which is proved as follows.

In the triangle ELf, the sides EL, Ef are cut proportionally at S, R, θ , ε [Euclid VI.2]; and the segments E θ , $\theta\varepsilon$, εf are equal to the segments Qe, ea, aM [Euclid I.34]. Therefore the segments ES, SR, RL are also proportional to the segments Qe, ea, aM. By the same reasoning the segments ET, TV, VY will be proportional to the segments Qn, nr, rN; and consequently the whole line LY is cut as the whole MN.

But by Lemma 7 of Book 1 the line bd too is cut as the line MN . Therefore the lines LY and bd are also cut proportionally. Since, then, they are equal [Euclid I.34], the segments of the one will also be equal to the corresponding segments of the other [Euclid V.9]; and for this reason the parallels drawn through the two points of the circle bpd will fall upon the points R, S, T, V , since these parallels cut off equal segments from the lines bd, LY , and hence will cut off the rising latitudes from the arcs Cb, Cd , just as the parallels through the points R, S, T, V mark off the same, as was demonstrated in No. 1. Furthermore, the lines drawn from the centre E to the points b, g, h, i, l, d may be called the radii of the rising and setting latitudes, just as the lines drawn from E to the outer points of the parallels MO, au , and so forth, are called the radii of the signs, as we said in the *Gnomonics*.

[Margin: b — Euclid VI.2; c — Euclid I.34; d — Euclid I.34; e — Euclid V.9.]

Diagram. A large square-framed orthographic analemma (the same figure recurs on p. 290). Several overlapping great circles — the meridian, the horizon, the equator, the ecliptic, and the tropic parallels — are projected within the square, densely crossed by construction lines, chords, and dotted radii. The principal legible labels are:

- A, dd (upper centre, on the meridian/ecliptic)
- B (upper right, with the sign group near $\mathfrak{S}$)
- Λ (Lambda), F, X (right periphery)
- D, δ (delta) (right)
- K, H (upper left)
- ρ (rho), E, Y (left and centre-left)
- E (centre), S, R, L, T, θ (theta), f, O (central cross of lines)
- M, O with parallel MO ; the semicircle MXO below centre
- N, G, m (lower left)
- I (lower right); P (bottom centre)
- Greek letters Φ (phi), μ (mu), ψ (psi), π (pi), ε (epsilon) scattered among the construction points
- Zodiac degree-marks ($\mathfrak{S}$ and neighbouring signs) run around the outer circumference.

Numerous small construction letters (qq, cc, ee, ce, ll, hc , and the doubled/minuscule points around the centre) and the peripheral degree-glyphs are too fine to resolve individually. (more than two labels unresolved).

Therefore, if the distance of any given point of the ecliptic from the nearest equinoctial point is reckoned on the circle bpd, from p in either direction, and a parallel to CE is drawn through the end of the reckoning, the arc Cb or Cd will be cut at the rising latitude of that point of the ecliptic. Thus, if the distance from either equinoctial point is 30 degrees, and from p one counts 30 degrees as far as ω , the parallel ωh will cut off the rising latitude Ch of the point that is 30 degrees from the beginning of γ or α — such as the beginning of δ , or ϵ , or ζ , or η .

Conversely, if a given latitude of rising or setting is reckoned from the point C toward b or d, as far as h, and a parallel h ω is drawn, the arc p ω will give the distance of the point of the ecliptic from γ or α , to which the given latitude belongs.

From this it is also clear that four points of the ecliptic — apart from the beginnings of δ and η — have the same rising latitude, two of them northern and two southern, just as they also have the same declination. This too we demonstrated in Lemma 49 of Book 1, Nos. 2 and 3. For the two latitudes Ch, Ci, which are equal, correspond to four points of the ecliptic, namely two northern and two southern, and so on.

[Margin: *To investigate the rising latitude by numbers.*]

4. By the calculation of sines the rising or setting latitude of any point of the ecliptic, or of any star, will be found, in this way. The great circle of declination, drawn through the poles of the world and the given point of the ecliptic (or through the centre of the star) while it is on the eastern horizon, forms with the equator and the horizon a spherical triangle. In this triangle the angle that the circle of declination makes with the equator is right; the arc of declination of the point of the ecliptic or star is known, together with the angle of the complement of the pole's elevation, which the equator makes with the horizon. Thus, in the figure of No. 4 of this Canon, when the straight line EZ has been drawn from the centre through the beginning of δ [?], representing the circle of declination of that beginning, there is formed the spherical triangle pYZ, whose angle p is right, and whose arc of declination pZ is known, together with the angle pYZ (the complement of the pole's elevation) that is contained — an acute angle, by Proposition 28 of our *Spherical Triangles*. For the angle contained by the horizon and the equator is always acute, since in that triangle all the arcs are less than a quadrant. Therefore, by the first mode of Problem 14 of the last Lemma on spherical triangles, let it be made: as the whole sine is to the secant of the complement of the pole's elevation, so the sine of the arc of declination pZ to a fourth term; and the sine of the arc of the rising latitude YZ will be produced. Or,

if you wish to use sines only, proceed by the third mode of the same problem: as the sine of the angle pYZ is to the whole sine, so the sine of the arc of declination pZ to a fourth term; for there will again be produced the sine of the arc of the rising or setting latitude YZ .

Each of these operations can also be plainly demonstrated in the figure of this scholium. For in the right-angled rectilinear triangle ELf , by Problem 5 of the last Lemma on rectilinear triangles, it holds that: as the whole sine Ef is to Ef in so far as it is the sine of the declination of the parallel MO , so is EL , the secant of the angle LEf (the elevation of the pole), to EL in so far as it is the sine of the rising latitude. (For, with Ef posited as the whole sine, the sine of the declination stands to EL , the sine of the rising latitude. Likewise, as the sine of the angle ELf , the complement of the pole's elevation, is to the whole sine, so is Ef , the sine of the declination, to EL , the sine of the rising latitude.)

Exactly the same reasoning holds for finding the rising or setting latitude of any star whatever. For thus you see, in the case of the star V , that the very same triangle ikV is formed, whose angle k is right, and whose arc of declination kV is known, together with the angle kiV (the complement of the pole's elevation), and Vi the arc of the rising latitude that is sought — as is evident in the figure of this same Canon, and so on.

[Margin: *Given the rising latitude, to find the corresponding point of the ecliptic by numbers.*]

Conversely, given the rising or setting latitude of some point of the ecliptic, we shall find that point of the ecliptic to which it belongs, if, in the same triangle pYZ , by the first mode of Problem 8 on spherical triangles, it be made: as the whole sine is to the sine of the arc YZ of the given rising latitude, so the sine of the angle pYZ (the complement of the pole's elevation) to a fourth term. For the fourth number produced will be the sine of the arc of declination sought, pZ . Then, by what we wrote in Canon 3 and its scholium, the point of the ecliptic will be found to which the declination so found belongs. But since four points have the same declination, we must know in which quadrant of the ecliptic it is contained, in order to elicit the point sought. This same operation will be demonstrated in the right-angled rectilinear triangle ELf , the figure of this scholium. For by Problem 2 on rectilinear triangles of the last Lemma it holds that: as the whole sine is to the sine of the base EL in so far as it is the sine of the known rising latitude, so is the sine of the angle ELf (the complement of the pole's elevation) to Ef , the sine of the declination sought, in the parts of the sine EL .

CANON VII.

To investigate the semidiurnal and seminocturnal arc of any point of the ecliptic or of any star; and, in turn, to inquire what point of the ecliptic corresponds to a given semidiurnal or seminocturnal arc.

1. This is nothing other than to seek out the tarrying of the Sun (as it stands at any degree of the ecliptic), or of any star, from the eastern horizon up to the meridian, or from the meridian down to the western horizon — that is, how many degrees of the equator rise together with any given degree of the ecliptic, or with a star, or descend from the meridian to the horizon, and so on. If, therefore, the rete of the astrolabe is turned about until the degree of the ecliptic that the Sun occupies on the proposed day, or the point of the proposed star, is set on the eastern horizon, and the fiducial line of the sighting-rule (or of the Index) is laid over the same degree or over the point of the star, then the arc of the limb contained between the fiducial line and the meridian line on the upper side, near the suspension ring, will be the semidiurnal arc of that degree or star; while the remaining arc of the limb, from the same fiducial line down to the meridian line on the lower side, will be the seminocturnal arc. And if each of these is doubled, the whole diurnal and nocturnal arc will result.

[Margin: *To investigate, by the instrument, the semidiurnal or seminocturnal arc of any degree of the ecliptic or of any star.*]

Now you may easily reduce such an arc, once found, to hours, if you assign to each hour fifteen degrees, and four minutes of an hour to each single degree. Or else let all the degrees contained in the semidiurnal or seminocturnal (or diurnal or nocturnal) arc be reduced to hours by the little table that we described in Chapter 2 of the *Sphere*, at the end of the explanation of the equator. Indeed, the hours marked on the limb, which are contained between the meridian line and the fiducial line holding the aforesaid position, will give the quantity of the semidiurnal or seminocturnal arc in hours, and so on.

It is not, however, necessary that all the degrees of the limb placed between the fiducial line and the meridian line be counted; it is enough if the few degrees that are contained between the fiducial line and the right horizon — those, namely, that give the ascensional difference of the given point of the ecliptic or star — are counted, as we said in No. 3 of Canon 5. For these, added to a quadrant (that is, to 90 degrees) if the point of the ecliptic or star inclines northward, or subtracted from the same quadrant if the point of the ecliptic or star is southern, will produce the semidiurnal arc; and this being taken from a semicircle (that is, from

180 degrees), the remaining arc will be the seminocturnal. This too will be obtained if, the point of the ecliptic or star being northern, the ascensional difference found — that is, the arc interposed between the fiducial line and the right horizon — is taken from a quadrant; but it is to be added to the quadrant when the point of the ecliptic or star inclines to the south.

[Margin: *From a given semidiurnal or seminocturnal arc, to find the corresponding point of the ecliptic on the astrolabe.*]

2. Given, on the other hand, a semidiurnal or seminocturnal arc, we shall search out the corresponding point of the ecliptic in this way. Let the semidiurnal arc be reckoned on the limb from the meridian line on the upper side, and the seminocturnal from the same meridian line on the lower side, and let the fiducial line of the sighting-rule be applied to the end of the reckoning. Then let the rete be turned about until some point of the ecliptic falls upon the point of intersection of the fiducial line with the horizon. For to that point, and to the other that corresponds to it on the other side of the tropic point, the given semidiurnal or seminocturnal arc belongs.

3. Without the instrument we shall proceed thus. Let the earlier figure of Canon 5 be repeated, and from the centre E, through the given point of the ecliptic or through the star, let a parallel of the equator be described. For its arc between the oblique horizon LPM and the meridian line EF, above the centre E, will be the semidiurnal arc sought; while the arc of the same between the oblique horizon and the meridian line Ef, below the centre E, will be the seminocturnal. Thus LF will be the semidiurnal arc of ♌, and Lf the seminocturnal. Likewise the semidiurnal arc of the equator, or of the beginning of ♈ and ♎, will be CB, and the seminocturnal CD. So the semidiurnal arc of ♊ will be the arc NH (H being taken for the intersection of the tropic of ♊ with the meridian line), and the seminocturnal NG. Again, the seminocturnal arc of the beginning of ♏(?) or ♐ is the segment of the parallel aVb, between b and the meridian line Ef; while the semidiurnal arc of the same is the segment between b and the meridian line EF, if the whole parallel were described. Finally, the seminocturnal arc of the star V or X is the arc of the same parallel between b and the line Ef, and the semidiurnal arc of the same is the arc between b and the line EF, if the whole parallel is described.

Diagram. A square-framed astrolabe-projection (the same figure recurs at the top of p. 290). It shows the meridian, the equator, the tropics, the ecliptic, the oblique and right horizons, and almucantar circles, with radial lines through the centre E. The principal legible labels are:

- F (top, on the meridian)
- K (upper centre), B (centre-upper), Y
- L (left), M (right), S (lower left), H
- E (centre), O, L (inner)
- P, A, R (right side)
- N, G (lower left), D (lower centre)
- R (bottom centre), X (bottom left), V (bottom right), T
- Zodiac glyphs (♈, ♉, ♊ and neighbours) run around the periphery.

Several small peripheral zodiac-glyphs and minuscule construction letters are not resolvable at this resolution. (more than two labels unresolved).

[Margin: *From a given semidiurnal or seminocturnal arc, to investigate the corresponding point of the ecliptic without the instrument.*]

Or thus. Through the point where the parallel described through the given point of the ecliptic or star cuts the horizon, let a straight line be drawn from the centre E. For this will cut the eastern semicircle of the equator into two arcs, of which the upper is semidiurnal and the lower seminocturnal. Thus, because the parallel described through the beginning of ♉ or ♊, or through the star V or X, cuts the oblique horizon at b, if the straight line Eb is drawn from E, cutting the equator at α , then αB will be the semidiurnal arc of the beginning of ♉ or ♊, or of the star V or X; and αD the seminocturnal.

Otherwise. When the oblique horizon has been described through the given point of the ecliptic or the star (its centre being always on the parallel KZR described through the centre of the horizon K, and its semidiameter PK), let a straight line be drawn from the centre E to the same point or star, which will cut off from the equator the ascensional difference between that line and the described oblique horizon, as was said in Canon 5, No. 6. This difference, then, when the given point or star is northern, added to a quadrant, will make the semidiurnal arc; but the same, subtracted from the quadrant when the given point or star is southern, will leave the semidiurnal arc. For example, if the oblique horizon is described through the beginning of ♉ and through the beginning of ♊, cutting the equator at l, Y, and the straight lines Ef, EZ are drawn to the beginnings of ♉ and ♊, cutting the equator at n, p, then the ascensional differences will be ln, Yp. And since the beginning of ♉ is northern, the difference ln, added to a quadrant, will yield the semidiurnal arc of the first point of ♉. But since the beginning of ♊ is southern, the difference Yp, taken from the quadrant, will leave the semidiurnal arc. Finally, when the horizon is described through the

star V, cutting the equator at i, and the straight line EV is drawn, cutting the equator at k, the ascensional difference of the star will be ik, which, taken from the quadrant, will leave the semidiurnal arc of the star V — since the star is southern, being placed beyond the equator.

The same ascensional difference, when the point of the ecliptic or the star is northern, subtracted from the quadrant makes the remaining arc the seminocturnal; but added to the quadrant it makes the seminocturnal arc, when the star or point of the ecliptic is southern.

Given, moreover, a semidiurnal or seminocturnal arc, we shall find the point of the ecliptic to which it belongs, in this way. Let the given semidiurnal arc be reckoned on the equator from the point B, or the seminocturnal from the point D, in either direction, and through the end of the reckoning let a straight line be drawn from the centre E, until it cuts the horizon. For the parallel of the Canon described on the horizon will cut the ecliptic at two points equally distant from the tropic point, to which the given semidiurnal or seminocturnal arc belongs. Thus, if the semidiurnal arc is $B\alpha$ or the seminocturnal $D\alpha$, the straight line $E\alpha$ drawn will cut the horizon at the point b, through which the parallel of the equator — described from E through that point of section on the horizon — passes; and this parallel cuts the ecliptic at the beginnings of ♊ and ♋. To these points, then, the given semidiurnal or seminocturnal arc belongs.

Diagram. A repeat of the astrolabe-projection figure of Canon VII (see fig-p641-1): the same square-framed diagram with meridian, equator, tropics, ecliptic, oblique and right horizons, and radial lines through E, bearing the labels F, K, B, L, M, S, H, E, O, P, A, R, N, G, D, X, V, T and peripheral zodiac glyphs. Small peripheral labels unresolved

[Margin: *From a given semidiurnal or seminocturnal arc, to search out the corresponding point of the ecliptic without the instrument.*]

SCHOLIVM.

1. We shall investigate the same semidiurnal or seminocturnal arc of a given point of the ecliptic, or of any star, by the analemma, in this manner. When the declination of the proposed point or star has been found from the scholium to Canon 3, let there be drawn in the analemma the diameter of the parallel that the given point or star describes. For its upper portion, between the meridian and the diameter of the horizon, is the versed sine of the semidiurnal arc, while the lower portion is the versed sine of the seminocturnal arc sought. For example, in the

analemma of the scholium of the preceding Canon, the declination of the beginning of $\mathfrak{S}$ is HM, and the diameter of its parallel is MO, cutting the diameter of the horizon at L. ML, then, will be the versed sine of the semidiurnal arc of the beginning of $\mathfrak{S}$, and OL the versed sine of the seminocturnal arc — so that, if the circle MXO is described about the diameter of the parallel MO, and the perpendicular LX is drawn from L to MO, the semidiurnal arc of $\mathfrak{S}$ will be MX, and the seminocturnal OX. For since both the horizon and the parallel MXO, in their proper position, are perpendicular to the meridian, their common section too will be perpendicular to the same [Euclid XI.19], and therefore, by Definition 3 of Book 11 of Euclid, perpendicular to MO lying in the meridian. So the line LX, perpendicular to MO, will be the common section of the horizon and the parallel MXO; and therefore MX will be the semidiurnal arc, and OX the seminocturnal. By the same reasoning NZ will be the semidiurnal arc of $\mathfrak{V}$, and PZ the seminocturnal. And so for the rest. But if HM were taken as the declination of some star, MX would be its diurnal arc, and OX the seminocturnal of the same.

[Margin: *a* — *Euclid XI.19.*]

Now both $\mathfrak{f}$ L and $\mathfrak{t}$ Y are the right sine of the ascensional difference, so that, in northern points of the ecliptic and northern stars, the arc $\mathfrak{[?]X}$, added to a quadrant, makes up the semidiurnal arc, while the arc $\mathfrak{mZ}$, in southern ones, subtracted from the quadrant, leaves the semidiurnal arc, and so on.

Diagram. A repeat of the orthographic analemma of Canon VI's scholium (see fig-p640-1): the same square-framed figure with overlapping meridian, horizon, equator, ecliptic and tropic circles, bearing (among the legible labels) A, B, D, F, K, H, L, M, O, E, S, R, T, X, I, N, G, P and Greek $\Phi, \Lambda, \theta, \delta, \mu, \psi, \rho, \varepsilon$, with the parallel MO and the semicircle MXO below centre. Numerous small construction letters unresolved Printed beneath the frame: *...tractus arcum semidiurnum relinquat, &c.* ("...subtracted, leaves the semidiurnal arc, etc.").

[Margin: *To learn the semidiurnal or seminocturnal arc of a given point of the ecliptic, or of a star, from the analemma.*]

[Scholium of Canon VII, continued]

2. From the known semidiurnal arc, in turn, we shall elicit the point of the Ecliptic that corresponds to it, in this way. From the points F and G let the difference between the given semidiurnal arc and the semidiurnal arc of the Equator — that is, the quadrant — be counted off in either direction; and a straight line joining the ends of the numeration will (by the scholium of Prop. 27,

Book 3 of Euclid) be parallel to the axis FG, because the counted arcs are equal. Let it cut the diameter of the Equator at ee, so that Eee is the right sine of the said difference. Then, erecting Haa perpendicular to the same diameter of the Equator, where it cuts the produced diameter of the Vertical at aa, and taking aabb equal to Eee, let bbdd be drawn parallel to HI, cutting AC at dd; and finally let Hcc be cut off equal to bbdd. For the straight line Ecc, when drawn, will cut off HM, the arc of declination of the sought point: this will be northern if the given semidiurnal arc is greater than a quadrant, but southern if less. And to this declination, once found, will be assigned the corresponding point of the Ecliptic, as was explained in the scholium of Canon 3, No. 3.

This is demonstrated as follows. Since, as we demonstrated in Lemma 49, Book 1, No. 17, as the whole sine is to the tangent of the pole's altitude, so is the tangent of the declination of any point of the Ecliptic to the sine of the ascensional difference; by inversion it will be: as the tangent of the pole's altitude is to the whole sine, so is the sine of the ascensional difference to the tangent of the declination. Therefore, since Haa is the tangent of the arc AH, the pole's altitude, and aabb is equal to Eee, the sine of the ascensional difference (for it is the same as the difference of the semidiurnal arc, and so on, as was said in the same Lemma 49, No. 15); and since [Margin: a — *Elements* VI.4] as aaH, the tangent of the pole's altitude, is to HE, the whole sine, so is aabb, the sine of the ascensional difference, to bbdd — that is, to Hcc, which is equal to bbdd — Hcc will be the sought tangent of the declination, and consequently HM will be the arc of the declination.

Otherwise. By Lemma 52, Book 1, on the diameter BD of the Horizon let the points L, Y be found, at which the ellipse described about the axes FG, eeff (taking Eff equal to Eee) intersects it. For if through L — when the given semidiurnal arc is greater than a quadrant — or through Y — when it is less — a line be drawn parallel to HI, the diameter of the Equator, namely MO or NP, this will be the diameter of the parallel through the sought point of the description, and consequently it will cut off the sought declination from the Meridian. For since, by Lemma 51, Book 1, it holds that as EI is to Eee, so is sO to sL; or as EH is to Eff, so is tN to tY; and since (by Lemma 5) the sines of similar arcs are proportional to their whole sines, sL or tY will be the sine of the ascensional difference in the circle of diameter MO or NP, just as Eee or Eff is in the greatest circle ABCD.

Moreover, the ellipse described about the axes FG, eeff represents the circle of declination (or the hour-circle) through the poles of the world and through the point of the Horizon at which it is cut by the parallel of the given semidiurnal arc;

since the perpendiculars dropped from its points onto the Meridian produce it, and that point of the Horizon falls at L or Y.

But from the given semidiurnal arc of any parallel we shall likewise elicit the corresponding declination, in the manner we handed down in the scholium of Prop. 33, Book 1 of the *Gnomonics*, and demonstrated at the end of Book 8, and which finally we have repeated in the little book on the Construction and Use of the horary instrument, chapter 12.

Diagram. A horary construction figure (the "instrument of horologes," ch. 12, here repeated). A gnomon-pointer rises at the top, at the apex **A**. From **A** a fan of radial (hour-) lines descends to a graduated outer arc. Below and left is the point **C**, with **E** near the center, **B** below **E**, and **D** at the right; a quarter-circle arc **CBD** runs across the lower part, itself divided, with dashed perpendiculars dropped from **A**'s radial lines. The outer arc carries a numeral scale of hour-divisions.

- **A** (top apex): gnomon/pointer origin of the radial lines
- **C** (left): terminus of the quarter arc **CBD**
- **E** (center): center of the construction
- **B** (below center): point on the quarter arc
- **D** (right): terminus of the quarter arc **CBD**
- Outer-arc scale, reading down the left side then along the bottom: 24, 21 [?], 20, 19, 18, 17, then 16 (at **G**), 15, 14, 13, 12, 11, 10, 9, 8, 7 [numerals: several ?]
- Right-side scale near **D**: 0, 3, 4, 5, 6 [numerals: ?]

For if, in the figure we have here appended, the semidiurnal arc is counted from **D**, in the circle described about the straight line **CD** and divided into 24 equal parts (or into 360 degrees); and through the end of the numeration a line is drawn parallel to **AB**, the radius of the Equator; then **CD** will be cut at a point through which a straight line drawn out from **A** will cut off, from the arc **CBD**, the sought arc of declination, beginning from the point **B** — which will be southern if it is contained in the arc **BD**, but northern if in the arc **CB**, and so on.

3. Finally, by sines, we shall proceed thus. Since it was demonstrated in Lemma 49, No. 15, that the ascensional difference of any point of the Ecliptic is the same as the difference between the semidiurnal arc of the parallel described through that point and the semidiurnal arc of the Equator (which is always a quadrant), it suffices to seek the ascensional difference of the given point of the Ecliptic or of the proposed star. For this difference, if the point of the Ecliptic or the star recedes northward from the Equator, added to the quadrant makes up the

semidiurnal arc, but taken from the quadrant leaves the seminocturnal arc; whereas if the point or star declines southward, the same difference taken from the quadrant makes the remaining semidiurnal arc, but added to the quadrant makes up the seminocturnal arc. This too was demonstrated by us in the aforesaid Lemma, No. 15, at the same place.

This ascensional difference is to be computed as we handed down in the scholium of Canon 5, No. 4. Other methods of computing the semidiurnal arc may also be employed, if one wishes, which we demonstrated in Book 1 of the *Gnomonics*, Prop. 34, and in the scholium of Prop. 35; one of these we shall bring forward in the scholium of Canon 10, No. 2.

[Margin: To find, by sines, the semidiurnal and seminocturnal arc of a given point or star. To investigate, by numbers, the point of the Ecliptic corresponding to a given semidiurnal or seminocturnal arc.]

Conversely, given the semidiurnal or seminocturnal arc, we shall find the point of the Ecliptic corresponding to it, in this way. Subtracting the given arc from the quadrant, or the quadrant from it, so that the difference is obtained between the given semidiurnal (or seminocturnal) arc and the semidiurnal arc of the Equator, which is a quadrant —

— if the sought point is conceived as set on the Horizon, and through it a great circle of declination is drawn from the pole of the world, there will be formed a right-angled spherical triangle whose right angle is contained by that circle of declination and the Equator; and the arc of the Equator between the Horizon and the said circle of declination is known, since it is the difference between the given semidiurnal (or seminocturnal) arc and the quadrant of the Equator; and finally the angle that the Equator makes with the Horizon is the complement of the pole's altitude, which is opposite, in the said triangle, to the arc of declination that we seek. If, therefore, by the first method of Problem 11 of spherical triangles, it is made that as the whole sine is to the sine of the difference between the given semidiurnal (or seminocturnal) arc and the quadrant of the Equator, so is the tangent of the complement of the pole's altitude to a fourth term, there will be produced the tangent of the sought declination.

A triangle of this kind is found in the first circle of figure 1 of Problem 49, which we have repeated in this place. For there the semidiurnal arc of the northern point of the Ecliptic is MN, to which the arc AR of the Equator is similar; and ER is the difference between the semidiurnal arc AR and the quadrant AE, which is the semidiurnal arc of the Equator; and the aforesaid triangle is ENR, in which, by

the first method of Problem 11 of spherical triangles of the last Lemma, as the whole sine is to the sine of the arc ER (the aforesaid difference), so is the tangent of the angle REN (the complement of the pole's altitude) to the tangent of the arc of declination NR. A similar triangle is ELQ, when KL — or the arc AQ of the Equator similar to it — is the semidiurnal arc of the southern point H of the Ecliptic, and so on.

Once the declination has been found in this way, the point of the Ecliptic corresponding to it must be sought, as we wrote in the scholium of Canon 3. And if the given semidiurnal arc is greater than 6 hours, or the seminocturnal arc less than 6 hours, there will be two points of the Ecliptic, northern, equidistant from the beginning of ♋ (Cancer), to which it agrees; but southern points, equidistant from the beginning of ♑ (Capricorn), if the semidiurnal arc is less than 6 hours, or the seminocturnal greater than 6 hours. If, however, the declination found is equal to the greatest declination, then the first point of ♋ (Cancer) will correspond to a semidiurnal arc greater than 6 hours and a seminocturnal arc less than 6 hours; while the first point of ♑ (Capricorn) will match a semidiurnal arc less than 6 hours and a seminocturnal arc greater than 6 hours.

Diagram. A single ruled frame enclosing four circular projections (a 2×2 block), repeating figure 1 of Problem 49. Each circle is a stereographic-type projection showing the Equator, Ecliptic, Horizon and declination/vertical circles with intersection points lettered. The labels read as follows.

- Upper-left circle: K, A, M across the top; S; P; F, H (upper left); Y (right); B (left), B/D (right); V, Q, b [?] (center); Z, L, E, N (across the middle); O, R, X, I (lower); T, G (bottom flanks); C (bottom)
- Upper-right circle: A (top); F, I (upper); H (left); E, G (center); D (right); K (lower center); F (lower right); C (bottom)
- Lower-left circle: A (top); N (upper right); L, F (center); I (left); B (left); D (right); E (center); G (lower left); M (lower center); K (lower center); O (lower right); H (bottom); C (bottom)
- Lower-right circle: F (top); A, H (upper center); N (upper left); L (center); I (center); B (left); M, K, E, O, G (across the lower middle); D (right); Q (lower left); R (lower center); C (bottom)

CANON VIII.

To ascertain the hour: by day from the Sun's altitude, and by night from the altitude of any star.

1. Since there are four kinds of hours — three of them equal, namely those reckoned from noon, or from midnight, or from sunrise, or from sunset, and one of unequal hours, about all of which we wrote quite fully at the beginning of our *Gnomonics* — the present Canon is to be understood as applying to all of them.

[Margin: To hunt out, by the astrolabe, the hour from noon or midnight, by day.]

So, in the daytime, if you want the hour elapsed since noon or since midnight, take the Sun's altitude by Canon 1, and turn the rete about until the degree of the Ecliptic in which the Sun then stands reaches the parallel of the Horizon — that is, the almucantar of the altitude found — on the eastern side if the time is before noon, but on the western side if it is afternoon. For the fiducial line of the Index [Ostensor], laid over that same degree of the Sun, will indicate on the Limb the hour from midnight, or from noon, according as the time was before noon or afternoon. But if the hours are not marked on the Limb, the hour must be elicited from the arc of the Limb intercepted between the fiducial line in that position and the meridian line, assigning to every fifteen degrees a single hour, and to each degree four minutes of an hour — yet so that before noon that arc begins from the meridian line on the lower side, but after noon from the upper side.

[Margin: To find, by the astrolabe, the hour from noon or midnight, by night.]

2. But if, at night, you wish to find the same hour from noon or midnight, observe by Canon 1 the altitude of some star marked on the rete, and turn the rete about until the point [cacumen] of that star reaches the parallel of the Horizon — on the eastern, or left, side if the star has not yet reached the Meridian, but on the right, or western, side if it has passed the Meridian. For the fiducial line, laid over the Sun's degree, will show on the Limb the hour from noon or from midnight, according as the Sun's degree lies in the right or left half of the Astrolabe. But if the hours are not marked on the Limb, the degrees of the Limb between the fiducial line and the meridian line must be reduced to hours, beginning from the upper side if the Sun's degree is in the western, or right, part of the Astrolabe, but from the lower side if it is in the eastern, or left, part. For the former arc will give the hours elapsed since noon, and the latter the hours since midnight.

[Margin: To know, by the astrolabe, the hour from sunrise or sunset.]

3. To find the hour from sunrise or sunset, you will proceed thus. Note the point of the hour from noon or midnight already found, whether by the Sun's altitude in the daytime or by a star's altitude at night, as has been said. Then, placing the Sun's degree on the Horizon — on the eastern side if the hour from sunrise is sought, or on the western if the hour from sunset is wanted — count the arc of

the Limb between the point that the fiducial line of the Index, laid over the Sun's degree, then indicates, and the point of the hour from noon or midnight previously noted, always advancing from the later noted point against the succession of the signs toward that earlier one (that is, advancing from east to west up to the point of the hour from noon or midnight noted), namely toward the right: for the hour from sunset, indeed, from the western side toward the lower part of the Astrolabe; but for the hour from sunrise, from the eastern side toward the upper part. For if the degrees comprehended in this arc of the Limb are reduced to hours, the number of hours elapsed since sunset or sunrise will be had.

But if, on the lower part of the Astrolabe, the arcs of the hours from sunrise and sunset are drawn, as we said in Book 2, Prop. 9, No. 6, then, in the daytime, having placed the Sun's degree above the almucantar circle of the Sun's altitude found — the rete being nonetheless moved from left toward right, so that the left is the part before noon and the right after noon — the degree opposite, set among those arcs, will indicate the hour from sunset.

— the degree opposite, set among those arcs, indicates the hour from sunset. But if the same degree of the Sun is placed above the almucantar circle of the Sun's altitude found, the rete being this time moved from right toward left, so that the right part looks to the forenoon and the left to the afternoon, the same opposite degree, among the same hour-arcs, will indicate the hour from sunrise — as the hour-numbers in the figure of the said Prop. 9, Book 2 show. At night, however, the hours from sunset cannot be found from the altitude of the stars by this method, unless other hour-arcs are drawn that intersect the earlier ones. For this reason the former method set out above seems the more commendable.

[Margin: To find the unequal hour by the astrolabe.]

4. Finally, the unequal hour will be shown, in the daytime, by the degree opposite the Sun in the lower part of the Astrolabe, once the Sun's degree itself has been placed on the parallel of the Horizon — that is, on the almucantar of the Sun's altitude found; while at night the Sun's own degree will do the same, once the star has been placed on the almucantar of its altitude found.

[Margin: When the altitude of the Sun or star has no corresponding parallel of the Horizon: in what way the Sun or star is to be placed between the next-smaller and the next-larger parallel, so that it may have its proper altitude.]

5. When the parallels of the Horizon are not drawn for each single degree, but are two, three, or five degrees apart from one another, and the altitude of the Sun

or star found has no corresponding parallel, but must be placed between two such parallels: then, in order that it may be more accurately placed at its proper altitude, the proportional part must be found in this way. Let the Sun's degree, or the star's point, be placed on the parallel of the next-smaller altitude, and let the point on the limb, shown by the fiducial line laid over that degree or star, be noted. Then let the same degree, or the star's point, be moved together with the fiducial line up to the parallel of the next-greater altitude, and let the point on the limb be noted again, and let the degrees of the limb between those two points be carefully counted. After this, let it be made that: as the number of degrees enclosed between the two neighboring parallels in the Astrolabe is to the number of degrees of the limb noted between those two points, so is the number of degrees of the altitude of the Sun or star — the number of degrees of the parallel of the next-smaller altitude having first been subtracted — to a fourth term. For a fourth number of degrees will be found; and if this is counted off from the earlier point noted on the limb toward the later point, and the fiducial line is brought to the end of the reckoning, then the Sun's degree or the star's point is to be placed precisely under the fiducial line holding that position, so that it may have the proper position of its altitude.

For example, let us suppose that one parallel is 5 degrees distant from another, and that the altitude found is 33 degrees. Having then noted the points on the limb that are shown by the fiducial line laid over the Sun's degree or the star's point when it is placed now on the parallel of 30 degrees, now on the parallel of 35 degrees, let us imagine that between those two points there lie 16 degrees. If, then, we say: if a difference of 5 degrees requires 16 degrees on the limb, what will a difference of 3 degrees require — namely, between the altitude of 30 degrees and that of 33 degrees? — there will result 9 degrees 36 minutes. And if we count these off from the earlier point on the limb, and apply the fiducial line to the end of the numeration, and finally set the Sun's degree or the star's point under the fiducial line in that position, then the Sun's degree or the star's point will be placed at the altitude of 33 degrees.

[Margin: To investigate the hour without a material instrument.]

6. Without an instrument we shall search out the hour in this way. Let the second figure of Canon 5 be repeated, in which ABCD is the Equator about center E; F^o [the tropic of Capricorn] and Gee are the tropics; AFCG is the Ecliptic, whose pole is M; AQC is the oblique Horizon, whose center is K and whose vertex or pole is L, through which the primary Vertical ALC is described, whose center is ϕ and pole is Q — namely the intersection of the Horizon with the Meridian; and finally Kg is

the parallel described through K, the center of the Horizon, on which lie the centers of all the hour-circles from east or west, as we demonstrated in Book 2, Prop. 9, No. 5.

[Margin: The hour from noon or midnight, in the daytime.]

So, in the daytime, one who is to investigate the hour should take the Sun's altitude. Then let him seek the intersection of the parallel of that point of the Ecliptic which the Sun then occupies with the parallel of the Horizon described through the degree of the altitude found. For a straight line drawn from the center E through that point of intersection will cut the Equator at the point of the Sun's distance from noon or midnight, so that the arc of the Equator between that point and the lower meridian line, reduced to hours, gives the hour from midnight if the time is before noon; but the arc between the same point and the upper meridian line gives the hour from noon if the time is afternoon.

For example, with the Sun at the beginning of ♈ (Aries) or ♒ (Aquarius) [?], let the Sun's altitude be observed as 20 degrees, whether before or after noon. Let the parallel of the Equator ♈♂Zd [?] be described through ♈, the beginning of ♒ [?], or through ♂, the beginning of ♈ [?]. Then, the Sun's altitude AO of 20 degrees having been counted on the Equator, whether on the eastern or the western side, let the straight line QO be drawn from Q, the pole of the Vertical, through O, cutting the Vertical at a; and let it comprehend the arc Aa of 20 degrees, the Sun's altitude, as was shown in Book 2, Prop. 5, No. 17 and following; and accordingly through a the parallel of the Horizon then passing through the Sun must be described. Therefore, drawing through a the straight line aP tangent to the Vertical at a — that is, perpendicular to aφ, the semidiameter of the Vertical, if it were drawn — P will be the center of that parallel, and Pa its semidiameter, from what we demonstrated in Prop. 6, Book 2, No. 10; yet this parallel can also be described by the other methods we handed down in Book 2, Prop. 6, if one wishes.

Now let this parallel of the Horizon, described from P through a (which will necessarily pass through the point R on the meridian line, onto which falls the straight line drawn out from A to the end-point n of the arc Cn of 20 degrees, the Sun's altitude, as is clear from what we showed in the same proposition, No. 2), cut the parallel of the Equator ♈♂ at S and I; and from the center E let the straight line ES or EI be drawn, cutting the Equator at N. If, then, the Sun's altitude was taken before noon, the degrees contained in the arc DN will indicate the hours elapsed since midnight; but if after noon, the degrees comprehended in the arc BN will show the hours passed since noon — because at that time the

given point of the Ecliptic, Υ or X , stands at S or I, and the straight line ES or EI represents the fiducial line, just as if the rete were rotated.

Diagram. A large, densely ruled circular projection within a square frame (the second figure of Canon 5, repeated): the Equator, both tropics, the Ecliptic, the oblique Horizon, the primary Vertical, and a full network of parallels and hour-circles, with solid and dotted lines. Readable lettered points include: E (top center), K, B, H, P, N, L, M (center), C (left), A (right), D (right), Q (lower left), R, S, I, X; together with many small lowercase and repeated labels around the rim (including a Sun-star glyph at the lower right) that are not individually legible.

Now, if the hour from sunrise is required before noon, one must draw through S — the point where the parallel of the Sun intersects the parallel of the Horizon — the circle SV, at the interval of the semidiameter of the Horizon KQ, taking the center h on the parallel Kh, so that its convex side at V, the point of the Equator, inclines toward the eastern (or hinder-rising) parts; that is, so that we meet its convex side as we advance from C, the first point of Υ [Aries], against the succession of the signs. For the arc CV will give the hour reckoned from sunrise, as is clear from what we wrote in Book 2, Proposition 9, Numbers 7 and 8.

[Margin: The hour from rising or setting, in daytime.]

But if the hour from setting is sought before noon, one must draw through the same point S the circle ST, at the interval of the semidiameter of the Horizon KQ, taking the center l on the parallel Kg, so that its concave side at T, the point of the Equator, meets us as we advance from A against the succession of the signs — that is, so that it inclines toward the eastern parts. For the arc ADCT will indicate the hour from setting, as we showed in the same place. And if, after noon, the hour is to be found either from rising or from setting, the said two circles must be drawn through I — such as are Ib and Ie, whose centers are i and g. For the arc Cb, reckoned against the series of the signs up to the convex side of the circle Ib, will give the hour from sunrise; and the arc ACe, computed against the succession of the signs up to the concave side of the circle Ie, will present the hour from setting.

Diagram. An analemma/astrolabe projection set in a ruled square frame, occupying the left and center of the page. A large primary circle is crossed by two crossing sets of concentric arcs (the parallels of the Sun and of the Horizon) and by radiating straight lines from the center, producing a dense net. Readable labels: F (top), K (upper center), B and P (upper center, below F), L and M (center), A (right side), S (both at the right flank and at the far left), T and V (upper left), H

(upper left). Along the lower rim: X, Y, R. Numerous smaller peripheral labels around the two crossing circles are too fine to resolve at this resolution.

- F (top): rim point
- K (upper center): rim/parallel point
- B, P (upper center): net points
- L, M (center): net points near center
- A (right): equatorial point
- S (right flank; also far left): intersection of the Sun's parallel with the Horizon's parallel
- T, V (upper left): equatorial points of the two auxiliary circles
- H (upper left): parallel point
- X, Y, R (lower rim): net points
- (many small labels around the crossing circles): [?]

[Margin: The hour from rising or setting, at night.]

At night, however, let the altitude of some star be observed — namely of the one whose position is at Z — and let us suppose the altitude found to be 20 degrees, that the star has not yet reached the Meridian, and that the Sun stands at $\aleph$, the first point of $\approx$ [?]. Now let the parallel drawn from the star (through $\pi\phi$ [?] to) Z and the parallel of the Horizon RS at 20 degrees cut one another on the eastern side at S. Then draw the straight lines EZ, ES, $E\aleph$, cutting the Equator at f, N, θ . Taking arc $f\theta$ reckoned according to the succession of the signs, let Nc be taken equal to it, starting from point N and likewise advancing in the order of the signs; and through its endpoint c let the straight line EX be drawn equal to $E\aleph$, so that the parallel described through $\aleph$, the first point of ♊ [Capricorn], passes through X. And since — when the rete is turned until the star Z reaches S, and the line EZ coincides with the line ES — the line $E\aleph$ coincides with the line EX, and the point $\aleph$ with the point X (because of the equality of the arcs $f\theta$ and Nc), it follows that, with the star Z at S, the Sun, occupying the first point of ♊ , stands at X. Consequently the arc Dc gives the hour from midnight. And if through X, at the interval of the semidiameter of the Horizon KQ, taking the centers H and k on the parallel KH, two circles are described cutting the Equator at ξ and Y, then the arc AD ξ will give the hour from setting, and the arc CBADY the hour from sunrise, as is clear from what we wrote in Book 2, Proposition 9, Numbers 7 and 8. Moreover the arc BN indicates the star's distance from the meridian at the time of observation.

With the Sun standing at the first point of ♊ [Capricorn] and having the same altitude of 20 degrees: if the line E $\approx$ [?] is drawn to the intersection of the parallel of ♊ with the parallel of the Horizon at 20 degrees, cutting the Equator at $\approx$ [?], then the arc B $\approx$ will give the hour from noon, if the time is afternoon, and the arc DA $\approx$ the hour from midnight, if the time is before noon. So too, when the Sun holds the first point of ♋ [Cancer] and has an altitude of 20 degrees: if the line Eee is drawn through the intersection of the parallel of ♋ with the parallel of the Horizon at 20 degrees, cutting the Equator at ce, then the arc Bcc will give the hour from noon in the afternoon, and the arc Dcc the hour from midnight in the forenoon. And if through $\approx$ ee [?] two circles are described at the interval of the semidiameter of the Horizon KQ, whose centers lie on the parallel Kg, the hour will likewise be found, both from sunrise and from setting, just as in the preceding cases.

[Margin: To catch the unequal hour without an instrument.]

7. Finally, we shall know the unequal hour if we divide the semidiurnal or seminocturnal arc of the parallel described through the given point of the Ecliptic into six equal parts, for the unequal hours. For a straight line drawn from the center E to the place of the Sun at the time of observation — as to S or X — will indicate which unequal hour has elapsed.

SCHOLIUM.

[Margin: To determine the hour from noon or midnight, by day, from the analemma.]

1. If an analemma is described for a given elevation of the pole, as we set forth in Lemma 19 of Book 1 and in the scholium of Canon 6, we shall find the hour by day from the Sun's altitude in this way. In the analemma of the scholium of Canon 6, draw the diameter MO (or NP) of the parallel passing through the Sun's degree, and describe about it the semicircle MXO (or NZP). At the point L (or Y), where it is cut by the diameter of the Horizon, erect a perpendicular LX (or YZ) to that diameter, so that MX (or NZ) is the semidiurnal arc, and OX (or PZ) the seminocturnal. Then, from D and B, the Sun's altitude having been computed up to $\nearrow$ and ♊ [?], let $\nearrow$ ♊ [?] be joined — the diameter of the parallel of the Horizon of the altitude found; and from the point ξ (or π), where it divides the diameter of the Sun's parallel, erect a perpendicular $\xi\mu$ (or πg) to that same diameter of the Sun's parallel. For the arc M μ (or Ng) will indicate the hour from noon or from midnight, according as the time of observation is afternoon or forenoon;

because the Sun, at the time of observation, stands at the point μ (or g). For since the parallel of the Sun (whose diameter is MO or NP) and the parallel of the Horizon (whose diameter is $\Upsilon\chi$ [?]) are perpendicular to the Meridian, [a] their common section too will be perpendicular to it, and therefore — by Definition 3 of Book 11 of Euclid — perpendicular to the line MO (or NP) lying in the plane of the Meridian. [Margin: a. Euclid XI, [proposition] 19.] Wherefore $\xi\mu$ (or πg), being perpendicular to MO (or NP), will be that common section; and thus, since the Sun then stands in that common section — namely at the point where those two parallels drawn through the Sun intersect — the Sun will be at the point μ (or g), and consequently the arc $M\mu$ (or Ng) will measure its distance from the Meridian.

But the arc $X\mu$ (or Zg) will be the Sun's distance from the Horizon, since LX (or YZ) is the common section of the Horizon and the Sun's parallel, as was demonstrated in the scholium of the preceding Canon, Number 1. From this distance $X\mu$ (or Zg) we shall know the hour from sunrise thus. If the time is before noon, the arc $X\mu$ (or Zg) itself will give the hour from sunrise; but if it is after noon, the arc compounded of XM and $M\mu$ (or of ZN and Ng) will make the same hour manifest — because the Sun has then moved from X (or Z), the point of rising, up to M (or N), the point of noon, and from noon up to μ (or g). From the same distance $X\mu$ (or Zg) we shall recognize the hour from setting thus. If the time is before noon, the arc compounded of XO and $O\mu$ (or of ZP and Pg) will indicate the hour from setting — because the Sun has then moved from X (or Z), the point of setting, up to O (or P), the point of midnight, and from midnight up to μ (or g). But if the Sun is after noon, the arc compounded of XO and the semicircle OM , together with $M\mu$ (or of ZP and the semicircle PN , together with Ng), will make the same hour from setting known; because the Sun has then moved from X (or Z), the point of setting, up to O (or P), the point of midnight, and thence up to M (or N), the point of noon, and finally thence up to μ (or g).

If the semidiurnal arc XM (or ZN) is divided into six equal parts for the unequal hours, the same perpendicular $\xi\mu$ (or πg) will indicate the unequal hour, and so on.

[Margin: To hunt the unequal hour by day by the analemma.] [Margin: To explore any hour whatever of the night by the analemma.]

2. At night, however, it will be possible to hunt the hour from the altitude of some star by this method. Let the star's distance from the Meridian be sought, as we said of the Sun — namely by a perpendicular line drawn to

the diameter of the star's parallel from the point where it cuts the diameter of the Horizon passing through the star's found altitude. For example, let a star whose declination is HM be northern, the diameter of its parallel MO, and the parallel itself MXO; let it have the altitude Dβ [?] (or BH), so that the line Hβ [?] drawn is the diameter of the parallel of the Horizon carried through the star, cutting the diameter of the star's own parallel at k. Then the perpendicular kA [?] will show the star's distance MA [?] from the upper semicircle of the Meridian toward the east or the west, according as the star is found in the eastern or the western part.

[Margin: That the star's distance from the superior Meridian, reckoned eastward, must be taken for investigating the hour.]

Then, to avoid a multitude of rules in seeking the hour from the star's distance from the Meridian once found, we shall always take its distance from the upper Meridian toward the east — that is, according to the succession of the signs — so that, when the star is western, we subtract its found distance from the whole circle, so that the remainder becomes the distance of the same star reckoned from the upper Meridian toward the east, even though it be greater than a semicircle. For example, if the distance of some star from the upper Meridian toward the west is found to be 70 degrees, we shall subtract 70 from 360 degrees, so that 290 degrees remain for the distance of the same star reckoned from the upper Meridian toward the east.

[Margin: How the Sun's distance from the star, from west to east, is investigated from the star's distance from the superior Meridian reckoned eastward.]

Next, from this distance of the star from the upper Meridian reckoned toward the east, let the Sun's distance from the star — likewise from west to east — be investigated by this art. Let the right ascension of the star, found from the scholium of Canon 4, Number 2, be subtracted from the right ascension of the Sun, known from the same scholium, Number 1 (a whole circle being first added, if the subtraction cannot be made). For the remaining number will give the Sun's distance from the star reckoned according to the succession of the signs. For example, let the circle ABCD in the adjoining analemma be conceived as the Equator, in which the said distances are to be reckoned, and let D be the first point of ♈ [Aries], and A the point of the upper Meridian; and let AM be set as the star's distance from the upper Meridian toward the east, and AN the Sun's distance from the same Meridian toward the east. If DM, the right ascension of the star, is subtracted from DN, the right ascension of the Sun, the arc MN will result — the Sun's distance from the star according to the order of the signs.

Again, if the star's distance from the Meridian toward the west is Aq, and the Sun's distance from the Meridian toward the same part is ABCDq [?], and the right ascension of the star Dq is taken (a whole circle being first added) from the right ascension of the Sun DA [?], and to the remaining arc qBCD the right ascension of the Sun Dq [?] is added, the remainder will result, reckoned according to the succession of the signs.

Diagram. The scholium's analemma, set in a ruled square frame: a large circle (the Equator/meridian projection), center E, crossed by many chords, diameters, and dotted construction lines, with an inner hatched lune-shaped figure toward the lower center and a scalloped series of small points around the outer rim. Readable peripheral and interior labels: at and near the top — aa (dotted, upper center), Φ , cc, K (upper left), λ (lambda), A, B, μ (upper right), ψ , M; upper left rim — H, and small n, θ points; left rim — N, B, Y; center — E, with S, R, L to its right, and T, V to its left; right rim — F, X, D; lower field — G, Z, m (lower left), the hatched lune, I, and a cluster of small doubled letters (dd, ff, ee, cc, ss, gg); foot of the circle — P, W, O. Center E is the common vertex of the radiating straight lines. The figure is extremely crowded; several small interior and rim labels cannot be resolved at this resolution. (more than 2 labels flagged [?]).

- E (center): common center / vertex of the radial lines
- K, H (upper left rim): parallel points
- A (top): point of the upper Meridian
- B (upper center; also left rim): net points
- λ , μ , ψ (upper right): net/parallel points
- M (upper right rim): star / meridian point
- F, X, D (right rim): equatorial and parallel points
- N, Y (left rim): equatorial points
- S, R, L (center-right): net points
- T, V (center-left): net points
- G, Z, m (lower left): rim points
- I, O, P, W (lower rim / foot): midnight and net points
- aa, cc, dd, ee, ff, gg, ss (scattered, small): auxiliary construction points, several [?]

But this same distance of the Sun from the star may also be found in this way. When the right ascension of the Sun is found greater than the right ascension of the star, then, this being subtracted from that, there will remain the Sun's distance sought from the star. Thus, since DM, the right ascension of the star, is

less than DN, the right ascension of the Sun, by subtracting the arc DM from the arc DN there is left MN, the Sun's distance from the star, from west to east. But when the right ascension of the Sun is less than the right ascension of the star, if that [of the Sun] is subtracted from this [of the star], and the remaining number from the whole circle, the remainder will be the Sun's distance sought from the star. Thus, with the star placed at M and the Sun at ϱ [Leo]: if D ϱ , the right ascension of the Sun, is taken from DM, the right ascension of the star, there remains the arc ϱ M; and this being taken away from the whole circle, the remaining arc is MC ϱ , the Sun's distance from the star, from west to east.

[Margin: To gather the Sun's distance from the superior Meridian eastward, from the star's distance from the superior Meridian eastward and the Sun's distance from the star, found in the same order.]

Now the arc compounded of the star's distance from the upper Meridian reckoned toward the east and the Sun's distance from the star reckoned likewise according to the order of the signs (a whole circle being cast off, if the compounded arc is greater) will indicate the Sun's distance from the upper Meridian, to be reckoned likewise according to the succession of the signs. This distance, subtracted from the whole circle, will leave the Sun's distance from noon known. Thus, in the same analemma, from AM, the star's distance from the upper Meridian toward the east, and MN, the Sun's distance from the star M toward the east, AN is made up — the Sun's distance from the upper Meridian toward the east; and this, taken from the whole circle, leaves ADN, the Sun's distance from noon. Therefore, when the arc ADN is reduced to hours, the hour elapsed since noon cannot fail to be known; and if more than 12 hours are found, then, 12 being subtracted, the remaining hours will be those from midnight. Again, with the star placed at q and the Sun at ϱ : if from the arc compounded of ABCq and qABC ϱ the whole circle — namely that made up of ABCq and qA — is taken, there will be left ABC ϱ , the Sun's distance from the upper Meridian reckoned eastward. So too, with the star placed at q and the Sun at N: if from the arc composed of ABCq and qAN the whole circle — namely that compounded of ABCq and qA — is removed, there will remain AN, the Sun's distance from the upper Meridian reckoned toward the east. But if the right ascension of the Sun happens to be found equal to the right ascension of the star, the Sun and the star will be equally distant from the Meridian toward the same part; and then the star's distance from the Meridian, once found, will indicate the hour. Or, if the difference of the right ascensions of the Sun and the star happens to equal a semicircle, then the Sun's distance from the upper Meridian will equal its distance from the lower Meridian according to the succession of the signs, and

conversely; and so the Sun's distance from noon will be known. All of which are clear from the same analemma.

[Margin: How the Sun's distance from the star toward the west is sought.]

Otherwise. When the star's distance from the Meridian — whether eastward or westward — has been found, as we said, let the right ascension of the Sun be subtracted from the right ascension of the star (a whole circle being first added, when the subtraction cannot be made). What is left will be the Sun's distance from the star toward the west. From this distance let the star's distance from the Meridian, once found, be subtracted, if the star is eastern; or let the star's distance from the Meridian be added to the Sun's distance from the star, if the star is western. For what is left, or is compounded,

will be the Sun's distance from noon toward the west; and therefore the hour cannot lie hidden. For example, let the star be placed at N and the Sun at α : the right ascension of the Sun $D\alpha$ being subtracted from the right ascension of the star DN , there will be left $N\alpha$, the distance of the Sun α from the star N, toward the west. And since the star N inclines from the Meridian toward the east, if from $N\alpha$ the distance NA of the star from the Meridian is taken away, there will remain $A\alpha$, the Sun's distance from noon toward the west. Again, with the star placed at q and the Sun at α : if the right ascension of the Sun $D\alpha$ is subtracted from the right ascension of the star Dq , there remains $q\alpha$, the distance of the Sun α from the star q, toward the west. And since the star q inclines from the Meridian toward the west, if its distance from the Meridian Aq is added to $q\alpha$, the Sun's distance from the star, there will be made up $A\alpha$, the Sun's distance from noon toward the west. Likewise, with the star placed at H and the Sun at G: if the right ascension of the Sun DAG is taken from DAH , the right ascension of the star (a whole circle being first added) — that is, if the right ascension of the Sun DAG is subtracted from the whole circle, and to the remaining arc GD the right ascension of the star DH is added — there will come out HAG , the Sun's distance from the star toward the west; and if from this HA , the eastern star's distance from the Meridian, is subtracted, there will remain ADG , the Sun's distance from noon toward the west. Finally, with the star set at q and the Sun at M: if DM , the right ascension of the Sun, is subtracted from the whole circle, and to the remaining arc MCD is joined Dq , the right ascension of the star (that is, if the right ascension of the Sun is subtracted from the right ascension of the star, a whole circle being first added), there will come out qDM , the distance of the Sun M from the star q, toward the west; and if to this is added the western star's distance from the Meridian Aq , there will be compounded ADM , the Sun's

distance from noon toward the west. Moreover the Sun's distance from the star toward the west, converted into time, indicates the hour from noon at which the star reaches the upper Meridian; because, the star being placed under the Meridian, that same distance is then the Sun's distance from noon toward the west.

Diagram. The same analemma woodcut as fig-p647-1, reprinted here so that the text of this page may refer to it (see the label list under fig-p647-1). The block carries the identical dense net of chords, the inner hatched lune, and the peripheral labels (A, B, K, H, N, Y, F, X, D, M, λ , μ , ψ , E, S, R, L, T, V, G, Z, m, I, O, P, W, and the small doubled-letter points aa, cc, dd, ee, ff, gg, ss). (same crowded figure; more than 2 small labels [?]).

[Margin: To find the hour at which the star reaches the Meridian.]

Now, the hour from noon or midnight being known, we shall easily find the hour from sunrise or from setting as well. For, reckoning that hour from noon M, or from midnight O, up to ss [?], according as the Sun has been found before or after midnight: if indeed the Sun has not yet reached midnight, the arc compounded of the arcs XM and Mss will give the hour from sunrise, and the arc Xss the hour from setting. But if it has passed midnight, the arc compounded of the arcs XM, MO, Oss will give the hour from sunrise, and the arc composed of the arcs XO, Oss will indicate the hour from setting.

If the seminocturnal arc XO is cut into six equal parts for the unequal hours, the unequal hour into which the point ss [?] falls will likewise be known.

3. Now, since we have said much about finding the hours, it will be worthwhile to teach by what method, from a given hour from noon or midnight, one may elicit the hour both from sunrise and from setting; and conversely, by what means, from a given hour from sunrise or setting, the given hour from noon or midnight may be found; and likewise how, from a given hour from sunrise, the hour from setting may be found, and conversely the hour from sunrise from the hour from setting. For by this method it will come about that, once the hour from noon or midnight is found (which finding is very easy by the Astrolabe or the analemma), the hour from sunrise or setting is immediately known.

[Margin: Reduction of the hour from noon or midnight to the hour from sunrise.]

Therefore, if the seminocturnal arc is subtracted from the given hour from midnight (24 hours being first added, if the subtraction cannot be made; likewise,

to the given hour from noon, 12 hours being first added, so that we may have the distance from midnight), the remaining number will give the hour reckoned from sunrise. Thus, the seminocturnal arc containing five hours, if the given hour is 8 from midnight, let 5 be taken from 8, and there will be left the hour 3 from sunrise. But if the given hour is 3 from midnight, let 24 hours be added (because 5 cannot be taken from 3), and from the compounded number 27 let 5 be removed, and the remainder will be the hour 22 from sunrise. Finally, if the given hour is 6 from noon, let 12 hours be added, so that it becomes the hour 18 from midnight; and from the compounded number 18 let 5 be subtracted, and there will remain the hour 13 reckoned from sunrise. The reason for this is clear from the adjoining analemma. For if the hour μ is reckoned from the point O of midnight, if the seminocturnal arc OX is subtracted, the remainder will be the distance $X\mu$ from the point of rising X. But if the same hour μ is reckoned from the point M of noon, if 12 hours are added so that the distance from midnight $OM\mu$ is had, and the seminocturnal arc OX is taken away, the remainder will be the distance $XM\mu$ from the rising point X. Finally, if the hour ss [?] is given from midnight, from which the seminocturnal arc OX cannot be subtracted, let 24 hours be added, so that the distance from midnight $OMOss$ is had; from which, if the same seminocturnal arc OX is removed, the remaining distance $XMOss$ from the point of rising X will result. But if the same hour ss is reckoned from noon, 12 hours being added, the distance from midnight $OMss$ will be had; from which, if the seminocturnal arc OX is taken, there will be left the distance $XMss$ from the point of rising X, as is manifest.

[Margin: Reduction of the hour from noon or midnight to the hour from sunset.]

If, on the other hand, the seminocturnal arc is added to the given hour from midnight (12 hours being first added to the hour from noon, so that the distance from midnight is had), the hour begun from setting will be compounded — 24 hours being nevertheless cast off, if they can be cast off. Thus, if the given hour is 8 from midnight, and the seminocturnal arc of 5 hours is joined, the hour 13 from setting will be made. But if the given hour is 6 from noon, let 12 be added, so that the distance from midnight becomes 18 hours; to which, if the same seminocturnal arc of 5 hours is added, the hour 23 from setting will be composed. The reason for this too is not obscure from the same analemma. For if the hour μ is reckoned from midnight O, the seminocturnal arc XO being joined, the distance from setting $XO\mu$ will become known. But if the same hour μ is computed from noon, the semicircle OM of 12 hours being joined so that the distance from midnight $OM\mu$ is had, then, if the seminocturnal arc XO is added to it, the whole distance from setting $XOM\mu$ will be known. But if the hour ss [?] is reckoned from

noon, the semicircle being joined so that the distance from midnight is OM_{ss} , if the seminocturnal arc XO is added, the distance from setting XOM_{ss} will result, greater than the whole circle; therefore, the whole circle $XOMX$ being cast off, the remainder will be the hour from setting X_{ss} .

[Margin: Reduction of the hour from sunrise to the hour from noon or midnight.]

Conversely, if the seminocturnal arc is added to the hour from sunrise, the hour from midnight will come out — 24 being nevertheless cast off, if they can be cast off. And if the compounded number is greater than 12, then, 12 being cast off, the hour reckoned from noon will remain. Thus, if the given hour is 4 from sunrise, the seminocturnal arc of 5 hours being added, the hour 9 from midnight will be made. Likewise, if to the hour 22 from sunrise we join the seminocturnal arc of 5 hours, the number 27 will be compounded; and 24 being cast off, the hour 3 from midnight will remain. Finally, if to the hour 10 from sunrise the same seminocturnal arc of 5 hours is added, the hour 15 from midnight will arise; 12 therefore being cast off, the remainder will be the hour 3 from noon. For in the same analemma, if to X_{μ} , the hour begun from sunrise X , the seminocturnal arc XO is added, the distance O_{μ} [?] from midnight will be compounded. But if to XM_{μ} , the distance from sunrise X , the seminocturnal arc XO is added, the distance OM_{μ} from midnight will be produced,

[Continued from the preceding scholium of Canon VIII — the analemma reduction of the hours.]

by night, [it will be] greater than a semicircle. Therefore, once the semicircle OM has been discarded, the remainder is the distance M_{μ} from the meridian. Finally, if to XMO_{ff} — the distance from rising, reckoned from X — the seminocturnal arc XO is added, the result is the distance OMO_{ff} from midnight, greater than a whole circle; and so, once the whole circle OMO has been discarded, the distance O_{ff} from midnight will remain.

[Margin: Reduction of an hour reckoned from the Sun's setting to an hour reckoned from noon or from midnight.]

But if instead the seminocturnal arc is subtracted from an hour reckoned from the Sun's setting — 24 having first been added if the subtraction cannot otherwise be made — the remainder will be the hour reckoned from midnight. And if the remaining number is greater than 12, then, once 12 is discarded, the hour reckoned from noon will remain. For example: if from hour 16 reckoned from setting we subtract a seminocturnal arc of 5 hours, hour 11 reckoned from midnight is left. Likewise, if from hour 23 reckoned from setting 5 are cast off, the

remainder is hour 18 reckoned from midnight, that is (discarding 12) hour 6 reckoned from noon. Finally, if hour 3 reckoned from setting is given, we shall add 24, and from the sum 27 we shall reject 5, so that the remainder becomes hour 22 reckoned from midnight, that is (discarding 12) hour 10 reckoned from noon. For in the same analemma, if from the distance-from-setting $XO\mu$ the seminocturnal arc XO is subtracted, the distance from midnight μO will be left. So too, if from the distance-from-setting $XOM\mu$ the seminocturnal arc XO is taken away, the remainder is the distance-from-midnight $OM\mu$; and once the semicircle OM has been subtracted, the remainder is the distance $M\mu$ from noon. Finally, if from the distance Xff reckoned from setting — a whole circle $XOMX$ having first been added — the seminocturnal arc XO is removed, the distance-from-midnight $OMff$ will be left, that is (with the semicircle taken away) the distance Mff from noon.

[Margin: Reduction of an hour reckoned from rising to an hour reckoned from setting.]

MOREOVER, if the whole nocturnal arc is added to an hour reckoned from rising, the result (24 having first been rejected, if it can be rejected) will be the hour reckoned from setting. For example: if to hour 8 reckoned from rising a nocturnal arc of 10 hours is added, hour 18 reckoned from setting is produced. Likewise, if to hour 19 reckoned from rising the same nocturnal arc of 10 hours is joined, hour 29 arises, reckoned from setting, that is (discarding 24) hour 5 reckoned from setting. For in the same analemma, if to the hour-from-rising $X\mu$ the nocturnal arc XOX is added, the hour-from-setting $XO\mu$ is completed. Likewise, if to the hour-from-rising $XMff$ the nocturnal arc XOX is added, the hour-from-setting $XOMff$ is formed; and, the whole circle $XOMX$ being discarded, the hour-from-setting Xff remains.

[Margin: Reduction of an hour reckoned from setting to an hour reckoned from rising.]

FINALLY, if the whole nocturnal arc is subtracted from an hour reckoned from setting — a whole circle having first been added if the subtraction cannot be made — the remainder will be the hour reckoned from rising. For example: if from hour 20 reckoned from setting a nocturnal arc of 10 hours is taken away, hour 10 reckoned from rising is left. Likewise, if from hour 9 reckoned from setting, that is (24 having been added) from hour 33 reckoned from setting, 10 are removed, hour 23 reckoned from rising will remain. This is plain from the same analemma. For if from the hour-from-setting $XO\mu$ you take away the nocturnal arc XOX , you will have the hour-from-rising $X\mu$. Likewise, if from the hour-from-

setting Xff — a whole circle ffOMff having first been added — the nocturnal arc XOx is subtracted, the remainder will be the hour-from-rising XMff.

[Margin: To know the magnitude of the unequal hours, both by the instrument and without the instrument.]

4. FURTHERMORE, in order that the unequal [temporal] hours may be reduced to equal hours, and conversely, one must first find, for any given day, the magnitude of the unequal hour, both diurnal and nocturnal, in this way. Place the degree of the ecliptic opposite to the one the Sun occupies — that is, the Sun's Nadir (for so they call the degree opposite the Sun) — upon any line of the unequal hours, and mark on the limb the point indicated by the fiducial line of the index-rule (Ostensor) as it then passes through the Sun's degree. Do the same, placing the same degree upon the immediately following, or preceding, hour-line. For the degrees of the limb intercepted between the two marked points will contain the quantity of one unequal diurnal hour. When those degrees have then been converted into time, the magnitude of one unequal diurnal hour will be known. And if the same is done with the Sun's own degree, the quantity of the unequal nocturnal hour will be found — which you will also obtain if you subtract the quantity of the diurnal hour from 30 degrees.

Without the instrument we shall attain the same thing more surely in this way. Divide the semidiurnal or seminocturnal arc — the arc, that is, which is displayed by the arc of the parallel drawn through the Sun's degree and intercepted between the horizon and the meridian line of the astrolabe, or, in the analemma, by the arc of the parallel described about its own diameter and intercepted between the meridian and the perpendicular drawn to the diameter from its intersection with the diameter of the horizon, as we set out in Canon 7, Numer. 3, and in its scholium, Numer. 1 — divide this, I say, into 6 equal parts; then each of them will be the magnitude of one unequal hour: a diurnal hour if it was the semidiurnal arc, but a nocturnal hour if it was the seminocturnal arc that was divided into 6 equal parts. How many degrees and minutes are contained in each sixth part you will learn from Lemma 3 of Book 1. By this method you will find that, with the Sun at the first point of Cancer (♋), one unequal diurnal hour comprises about 18 degrees 50 minutes — that is, one equal hour together with a little more than 15 minutes, and so on.

[Margin: Reduction of an unequal hour to an equal hour.]

Given, therefore, any unequal diurnal hour, if its number is multiplied by the quantity of one unequal diurnal hour, the Sun's distance from rising will be produced; but if the number of any unequal nocturnal hour is multiplied by the quantity of one unequal nocturnal hour, the Sun's distance from setting will be produced. And in this way any unequal diurnal hour will be reduced to an hour reckoned from the Sun's rising, and any nocturnal hour to an hour reckoned from the Sun's setting. From this, in turn, by the reduction of an hour reckoned from rising or setting to an hour reckoned from noon or midnight, there will also be known the hour reckoned from noon or midnight that corresponds to the given unequal hour.

[Margin: Reduction of an equal hour to an unequal hour.]

ON THE CONTRARY, if by day the Sun's distance from rising, or by night its distance from setting, is divided by the quantity of one unequal diurnal or nocturnal hour, the number of the unequal diurnal or nocturnal hour will result. And if, given an hour reckoned from noon or midnight, the corresponding unequal hour is to be found, it must first be reduced — by day to an hour begun from rising, by night to an hour begun from setting — and so on.

[Margin: To investigate the equal hour by means of the sines.]

5. BY the calculation of sines, the equal hour is likewise found in this way — from the Sun's altitude by day, and by night from the altitude of some star. (I do not wish to repeat here the methods explained in the last proposition of Book 1 of our *Gnomonice*; the most expeditious of them all is the one that immediately precedes the method carried out by spherical triangles.) Let the first 4 circles be taken from those 12 which we set out at the end of the scholium of Canon 3, in which let ABCD be the meridian; DEB the horizon and F its pole; AEC the equator and G its pole, that is, the pole of the world; FL the vertical circle drawn through the Sun or star H, so that HL is its altitude above the horizon; and GI the hour-circle, or circle of declination, so that HI is the declination, whether northern or southern. Since, then, in the spherical triangle FGH the three sides are known — FG being the complement of the pole's altitude [the co-latitude]; FH the complement of the altitude of the Sun or star; and GH the complement of the declination when the declination is northern (but when the declination is southern, the arc GH will have the same sine as the remaining arc which ends at the other pole across the semicircle, and which is the complement of the southern declination) — the angle FGH will be found from Problem 21 on spherical triangles in the last Lemma, in this way. Let it be as the whole

sine is to the sine of the arc FG, the complement of the pole's altitude, so is the sine of the arc GH, the complement of the declination, to a fourth [quantity]; and a certain fourth number will be produced. Again, let it be as this fourth number found is to the whole sine, so is the difference between the versed sine of the arc FH, the complement of the Sun's or star's altitude, and the versed sine of the arc by which the two sides FG, GH differ from each other, to a further [quantity]; and there will be generated the versed sine of the sought angle FGH. From this, the distance of the star AI, reckoned from the meridian, will be known; whether it is to be counted toward rising or toward setting the position of the star itself will show, according as it lies in the eastern or the western hemisphere. This distance of the Sun from the meridian, once found, will not allow the hour to be unknown; and from the distance of a star from the same meridian the hour is to be drawn out, as we taught in Numer. 2.

Diagram. A large, densely inscribed analemma set within a square frame, occupying the upper-right of the page (the "upper analemma" of the scholia of Canons 6, 7 and 8, to which the text refers). Two large overlapping circles (the meridian and the horizon/parallel systems) are crossed by numerous straight and dotted chords and by several inner circular arcs, so that the plate carries many small labels in Latin capitals, Latin minuscules, and Greek/script sorts. Legible principal labels include (approximate positions): A (top-center), K (upper-left), H (left), F (upper-right), X (right), B (left-center), D (right-center), L (right of center), V, T (left of center), M (center-left), N (upper-left rim), S/R (center), Z (lower-left), G (lower-left), m (lower-left rim), C (bottom-center-left), I (lower-right), P (bottom-center), and the double-long-s point ff (right rim). Many further small peripheral labels (Greek minuscules and doubled-letter sorts such as θ , ϕ , and cc/dd/ff/gg pairs) are present but cannot be resolved individually at this resolution.

Diagram. A framed group of four small circles arranged in a 2×2 grid (the "first 4 circles" of the sine-calculation for the equal hour). Each circle shows the same construction at successive configurations: a bounding circle (meridian ABCD) with center E, a horizontal diameter (horizon DEB), an oblique great circle (equator AEC), the pole F (upper) and pole G, and the spherical triangle FGH with points H, I, L, K, M marked. In the fully labelled upper-left circle additional star-marked points appear on the rim: N (top-left), T, S (right), R (lower-left), O and Q (bottom). Labels legible: A, B, C, D, E, F, G, H, I, K, L, M (all four circles); plus N, P, T, S, R, O, Q on the upper-left circle. [The same four-circle block is impressed again at the head of p. 298 — see fig-p298-1.]

à Solis occasu.

Diagram. The same framed 2×2 group of four small circles as fig-p297-2, repeated here beside the sine-calculation text of §5. Each circle carries the construction of the spherical triangle FGH: bounding circle (meridian ABCD) with center E, horizon DEB, equator AEC, poles F and G, and points H, I, K, L, M; the upper-left circle additionally shows the rim points N, T, S, R, O, Q. Labels legible: A, B, C, D, E, F, G, H, I, K, L, M across all four circles.

[The scholium of Canon VIII concludes with §5 above; Canon IX now begins.]

CANON IX.

To find out at what hour the Sun, or any star, rises and sets, or reaches the meridian; and which days and nights are equal to one another; and finally which days have their diurnal and nocturnal arcs alternately equal.

[Margin: To investigate, by the astrolabe, the hour of the rising and setting of the Sun, or of any star.]

1. TURN the rete about until the Sun's degree, or the tip [cacumen] of the proposed star, is found on the eastern horizon, whether right or oblique; then the fiducial line of the index-rule (Ostensor), laid upon the Sun's degree, will indicate on the limb the hour at which the Sun or star then rises. For when the Sun's degree, or the star, is on the horizon — that is, rising above the horizon — the heaven holds that same position which the astrolabe then displays. In the same manner you will find the hour of setting, if you place the Sun's degree, or the star's tip, on the western horizon and lay the fiducial line upon the Sun's degree.

[Margin: To learn from the astrolabe the hour at which a star culminates.]

2. NO otherwise will you find the hour at which a proposed star culminates — that is, reaches the meridian (for the Sun is always at midday, that is, at hour 12, in the upper meridian, and at midnight in the lower meridian) — if you set its tip on the meridian line, whether above the horizon or below it, and lay the fiducial line upon the Sun's degree.

[Margin: To learn from the astrolabe which days and nights are equal to one another.]

3. NOW if there are taken on the rete any two degrees of the ecliptic equally distant from the first point of Cancer (♋) or of Capricorn (♐), and if on the

back of the astrolabe two days corresponding to those degrees are found, those two days will have equal diurnal and nocturnal arcs, and the same hour of rising and of setting.

[Margin: Which days have their diurnal and nocturnal arcs alternately equal.]

4. BUT if any two degrees of the ecliptic equally removed from the first point of Aries (γ) or of Libra (ϱ) are taken on the rete, and on the back of the astrolabe two days corresponding to those degrees are taken, the diurnal arc of the one will be equal to the nocturnal arc of the other, and the nocturnal arc of the one equal to the diurnal arc of the other.
5. WITHOUT the instrument we shall proceed in this way. Through the Sun's degree, or through the star, we shall describe from the center E a parallel, until it cuts the horizon and the meridian. For its arc placed between the horizon and the meridian will measure the distance of the Sun, or star, from the meridian at the moment of rising; and this distance, if it is the Sun's, converted into time, will show how many hours before noon the Sun rises, and how many hours after noon it sets. Hence, if the said hours be subtracted from 12, the remainders will be the hours after midnight at which the Sun rises. For example, with the Sun at the first point of Capricorn (φ), whose parallel cuts the horizon at f and the upper meridian at F, the arc Ff is the distance from noon of the Sun standing at f , and so on.

But the hour of the rising of a star situated, say, at Z — whose parallel cuts the horizon at d (for its distance from the meridian does not indicate the hour) — you will hunt out thus. Draw the line EZ to the star's position; the line Ed to the intersection of the star's parallel with the horizon; and the line E δ to the Sun's degree, which we now suppose to be the first point of Cancer ($\var�$). Then, to the arc of the equator $f\theta$ lying between the lines EZ and E δ , take an equal arc, [measured] from the point of intersection of the line Ed with the equator up to the point cd , in such a way that the point cd is removed from the point [where Ed meets the equator] toward the same side to which the point θ is removed from the point f . For the arc BCcd will be the distance of the Sun, or of the first point of Cancer ($\var�$), before noon, when the star at d rises: because, if the rete be conceived to move until the line EZ [coincides with] the line Ed — that is, until the star Z stands at d — the line E δ will cut the equator at cd , on account of the two equal arcs taken, and so on.

No otherwise will you investigate the hour at which the same star sets. For if, from the point of intersection of the equator with the line drawn from E to the

intersection of the star's parallel with the western horizon, an arc equal to the aforesaid arc $f\theta$ be taken along the succession of the signs (namely, receding from that point of intersection toward the same side to which the point θ recedes from the point f), the terminus of this arc will be that point to which the Sun's degree arrives at the moment when the star sets. Therefore the arc of the equator between that same point and the meridian line EF will be the distance of the Sun before noon, or after, according as that point lies in the eastern or the western part of the astrolabe. So too the hour at which the star reaches the meridian will be found, if BC be taken equal to the arc $f\theta$. For as soon as the line EZ arrives at the line EB , the line $E\delta$ will coincide with the line EC , and therefore the arc BC will be the distance of the Sun before noon. But if to the same arc $f\theta$ an equal arc DA be taken, BA will be the distance of the Sun after noon, the star standing on the meridian below the horizon: because, the line EZ having been moved to the line ED , the line $E\delta$ coincides with the line EA , since the arcs $f\theta$ and DA are equal. Finally, there is no other method of investigating the hour when a star is on the horizon or on the meridian than when it is found at some other point of the heaven. For it was by this same method above, in Canon 8, Numer. 6, that — from the position of the star Z at the point S , which we found from its altitude and its parallel — the arc Bc was found, of the distance of the Sun from the meridian at the first point of Cancer ($\mathfrak{C}$); because, namely, we took the arc Nc equal to the arc $f\theta$, and so on. From which it is plain that, if on the line EC a line equal to the semidiameter of the Sun's parallel $E\delta$ is taken, and through its far end, with the interval of the semidiameter of the horizon KQ , two hour-circles are described whose centers lie on the parallel Kg , then the hour, both from rising and from setting, at which the star Z culminates has likewise been found. Likewise, if from the line Ecd , produced, a line equal to the same $E\delta$ is cut off, and through its far end two hour-circles are described in the same way, then the hour, both from rising and from setting, at which the same star rises above the horizon at d has likewise been found — and so on. This condition being observed, however: that the hour-circle to whose convex side we come, proceeding from the point C toward B , indicates the hour from the Sun's rising; but the hour-circle to whose concave side we come, proceeding from the point A toward D , demonstrates the hour from the Sun's setting. Which is plain from what we have demonstrated in Book 2, Proposition 9, Numer. 7.

6. TWO other [days] will be found, as was said in Numer. 3 and 4, except that the days corresponding to the degrees of the ecliptic are to be sought not from the back of the astrolabe, but from the table of the scholium of Canon 2.

Diagram. A large analemma within a square frame at the foot of the page, illustrating Canon IX. Two large overlapping circles are crossed by many straight chords, diagonals, and dotted arcs, with a fan of dotted radial lines toward the lower part; several nested inner circles carry a graduated dotted rim. Legible principal labels include (approximate positions): F (top), K (upper-center), B (center), T (center-left), P (center), N (center-left), H (left), V (left), L (center, and again at right rim), M (below center), S (left rim and right rim, star-marked), C (left rim), X (lower-left), I (right), Z (right rim), cd (bottom-center), ϕ (bottom-center), and several star-marked points (★) on the outer rims. Numerous small Greek minuscule and doubled-letter labels along the arcs cannot be individually resolved at this resolution.

SCHOLIVM.

[Margin: To investigate, by the analemma, the hour of the rising and setting of the Sun, or of a star.]

1. IN the analemma, the line that is drawn perpendicular to the diameter of the horizon from the intersection of that diameter with the diameter of the Sun's parallel cuts off, from the semicircle described about the diameter of that same parallel, the arc of the Sun's distance from noon or from midnight — that is, the arc dividing the semidiurnal from the seminoturnal arc. Thus in the upper analemma of the scholium of Canons 6, 7, and 8, with the Sun at the first point of Cancer ($\mathfrak{C}$), its distance from noon is the arc MX, and from midnight the arc OX, and so on. But the hour of the rising or setting of a star is investigated with more difficulty by the analemma. For first one must find its distance from the meridian when it rises or sets — that is, its semidiurnal arc — as we taught in the scholium of Canon 7, Numer. 1. Then, from this distance, one must seek the Sun's distance from the meridian, as we wrote in the scholium of the preceding Canon, Numer. 2. For from this distance the hour will be gathered without trouble, as was set forth in that place.

[Margin: In what way the hour of the rising and setting of the Sun, or of a star, is to be sought by means of the sines.]

2. IN order that, by the doctrine of sines, the hour of the rising and setting of the Sun or of a star may be drawn out, one must investigate the semidiurnal arc from what was written in the scholium of Canon 7, Numer. 3. For this will make plain the distance of the Sun, or star, from the upper meridian when it rises or sets. Therefore the hour of the Sun's rising and

setting cannot be unknown; and from the distance of a star from the meridian the hour of its own rising and setting is to be extracted, as we wrote just above in Numer. 1.

CANON X.

To search out the beginning, end, and duration of each twilight, both morning and evening.

[Margin: The morning and evening twilight.]

1. PLACE the Sun's degree upon the twilight-line on the eastern side; then let there be noted on the limb the hour, or part of an hour, which the fiducial line of the index-rule (Ostensor), laid upon the Sun's degree in that position, indicates. For this will give the beginning of the morning twilight...

Astrolabe, Book III — Canon X (continued) and Canon XI (beginning)

Canon X (continued)

[Margin: The instrument tells you how long it lasts, and at what hour it begins and ends.]

of the morning [twilight]. Then, when the Sun's degree has been advanced up to the Horizon, the same fiducial line placed over the Sun's degree will indicate on the limb the hour, or the fraction of the hour, at which the morning twilight ends or ceases. The time enclosed between the beginning and the end will fix the duration of the whole morning twilight. In just the same way you will look for the beginning, the end, and the duration of the evening twilight. For, with the Sun's degree placed above the Horizon on the western side, the fiducial line laid over the Sun's degree will show, among the hours of the limb, the beginning of the evening twilight. Then, when the Sun's degree has been advanced up to the crepuscular line, the same fiducial line placed over the Sun's degree will show on the limb the hour, or the fraction of the hour, at which the evening twilight vanishes. The time enclosed between the beginning and the end will then exhibit the magnitude of the whole evening twilight — which is always found equal to the magnitude of the morning twilight. Moreover, the degrees of the limb lying between the points that are marked by the fiducial line of the sight [Ostensor]

laid over the Sun's degree while it stands on the crepuscular line, and again while it stands on the Horizon, converted into time, likewise give the duration of each twilight.

[Margin: Another, more reliable, way of finding the twilight.]

2. But since the crepuscular line cannot easily be drawn without error — because its center runs out too far from the center of the astrolabe — the same twilight can be investigated, even if the crepuscular line has not been drawn, more accurately in the following way. Let the degree of the Ecliptic opposite the Sun's place be set on the parallel of the Horizon distant 18° on the western side. (For the parallel of the Horizon distant 18° from it toward the Zenith is drawn much more reliably than its opposite receding from the same by 18° toward the Nadir.) And because the Sun's degree is then necessarily set at the opposite point — namely on the crepuscular line itself on the eastern side, that is, the crepuscular line must pass through the Sun's degree in that position — the fiducial line of the sight laid over the Sun's degree will show on the limb the hour of the beginning of the morning twilight, as before. Then, when the Sun's degree is advanced up to the Horizon, the same fiducial line laid over the Sun's degree will indicate on the limb the hour of the end of that same twilight. In the same way, with the degree of the Ecliptic opposite the Sun's place set on the parallel of the Horizon distant 18° on the eastern side, the fiducial line resting on the Sun's degree will show the hour of the end of the evening twilight on the limb. And when the Sun's degree is restored to the Horizon, the same fiducial line, running through the Sun's degree, will give the beginning of that same twilight on the limb. The time set between the beginning and the end of each twilight will measure the twilight's duration.

[Margin: How, from one twilight, the beginning and end of the other twilight of the same day may be derived.]

But once either twilight has been found, the other is had as well, since it is equal to the first: the hour of the beginning of the one, subtracted from 12 hours, leaves the hour of the end of the other; and the hour of the end of the one, taken from 12 hours, leaves the hour of the beginning of the other.

[Margin: To find how far we are from the beginning or the end of the twilight.]

Now if, by night, the hour is found from the altitude of some star — as we taught in Canon 8, nos. 2 & 6 — you will at once know how far you are from the beginning or end of the twilight, whether morning or evening, provided you

compare the hour found with the hour of the beginning or end of the twilight, as is plain.

[Margin: To investigate both twilights without a material astrolabe.]

3. **Without the instrument** we shall proceed thus. Let ABCD be the Equator about center E; let FHK, GRS be the tropics; KAC the oblique Horizon; and the crepuscular line — that is, the parallel of the Horizon distant 18° from it in the lower hemisphere — Rab, whose center is L; and finally let AFCG be the Ecliptic, whose pole is I, divided into 12 signs by straight lines drawn from I through 12 equal parts of the Equator, at the points C, c, Z, G, f, g, A, N, P, E, d, e. If then a parallel of the Equator is described through a given point of the Ecliptic, its arc intercepted between the crepuscular line and the Horizon — whether on the eastern or the western side — will be the magnitude of the twilight, both morning and evening. The beginning of the morning [twilight] will be measured by the arc of the parallel counted from the meridian line below AC up to the crepuscular line; while its end will be measured by the arc of the same parallel counted in the same way up to the Horizon. But the beginning of the evening [twilight] will be measured by the arc of the parallel counted from the meridian line above AC up to the Horizon; and its end will be given by the arc counted in the same order up to the crepuscular line.

For example: with the Sun at the first point of Cancer [♋], the magnitude of each twilight will be the arc RS; the hour of the beginning of the morning twilight will be given by the arc GR, and the hour of its end by the arc GS, to be counted from midnight; while the hour of the beginning of the evening twilight will be counted by the arc fS, and the hour of its end by the arc fR, begun from noon. Again, with the Sun at the first point of Capricorn [♐], the magnitude of each twilight will be the arc tK of the tropic of Capricorn [♐], between the Horizon and the crepuscular line; and the arc ut, computed from midnight, will give the beginning of the morning twilight, and the arc uK its end; whereas the arc FK, counted from noon, will indicate the beginning of the evening twilight, and the arc Ft its end. Likewise the arc aT will be the duration of each twilight when the Sun is at the first point of Gemini [♊] and Leo [♌]. And the arc bV will measure each twilight when the Sun is at the first point of Taurus [♉] and Virgo [♍]. Finally the arc kC will count its duration when the Sun is at the equinoctial points; and so for the rest. The beginning [of the parallel] will be bounded by the arc of the proper parallel up to the meridian line produced; and the beginning and end of any

twilight will be determined by the arc of the proper parallel produced, as has been set out.

Or, if you prefer, the beginning and end of any twilight can be taken on the Equator, from the meridian line up to the straight lines emitted from the center E through the ends of the twilight's arc: so that, since RS is the twilight-arc of Cancer [♋], if through R and S straight lines are drawn from E cutting the Equator at h, k, the arc Dh will give the beginning of the morning twilight and Dk its end; while the arc Bk will show the beginning of the evening twilight, and Bh its end — because the arcs Dh, Dk are similar to the arcs GR, GS, and the arcs Bk, Bh to the arcs fS, fR, from the scholium to proposition 22 of Book 3 of Euclid, etc.

[Margin: To find the twilights in another way, without a material astrolabe.]

4. But **when the crepuscular line has not been drawn, or cannot easily be drawn**, we shall explore the twilight of any point of the Ecliptic most exactly in this other way. Let there be described above the Horizon its parallel distant 18° from it, opposite to the parallel bounding the twilight, HIMm. For here more easily than the paral-

Diagram. A large square-framed stereographic astrolabe projection illustrating the twilight construction of Canon X, no. 3. It shows the Equator ABCD about center E, the two tropics (FHK and GRS), the oblique Horizon (KAC), the crepuscular line / parallel of the Horizon distant 18° in the lower hemisphere (Rab, center L), and the Ecliptic AFCG (pole I) divided into signs by radial lines from I. Concentric circles and radial/chord lines are crowded with small labels. Readable labels, by approximate position:

- L — top, on the vertical meridian
- F — upper center, below L
- H — upper right of F
- B — center-upper, below F
- P, M — right of center, on radial/chord lines
- K — far right (Horizon/tropic intersection)
- N, o — right of center
- e — right, near lower loop
- A — lower right, on the Horizon
- C — lower right (with the engraved word "Crepus[culina]" along the lower-right arc)
- d — upper left

- α , ρ — left of center [Greek-letter labels; ρ [?], α]
- I — center-left (Ecliptic pole)
- x — below I
- C — left, on the Horizon (with the engraved word "Linea" along the lower-left arc)
- v, s — lower left
- T — lower left
- b, z — bottom left
- G — bottom center (tropic)
- D — bottom, on the meridian
- u — bottom, below D
- E — center
- Additional small peripheral point-letters near the lower loop (ω [?], ξ [?], ζ [?], κ [?]) are present but not securely legible.

The engraved arc-captions "Linea [Crepusculina]" (lower left) and "Crepus[culina]" (lower right) label the crepuscular line. **More than two peripheral labels are unresolved**

CANON X (continued) — running header "CANON X."

[*continuing no. 4*] ...parallel bounding the twilight will be drawn, since the whole of it is contained within the Horizon, and hence its diameter, which appears, and its center, can conveniently be had. Then, through the point of the Ecliptic opposite the point whose twilight is sought, let a parallel of the Equator be described from E. For its arc between the Horizon and its parallel HIMm will exhibit the magnitude of the sought twilight; and the beginning and end of it will be shown by the arcs of the Equator emitted between the meridian line and the straight lines drawn from the center E through the ends of the aforesaid twilight-arc, as was said a little before.

For instance: the arc HK of the tropic of Capricorn [φ], between the Horizon and its parallel 18° , will be the magnitude of the twilight, both morning and evening, when the Sun is at the first point of Cancer [ϑ]. And the beginning of the morning [twilight] will be determined by the arc FH, and its end by the arc FK, begun from midnight; whereas the beginning of the evening [twilight] will offer the arc uK, and its end the arc uH. Or, by drawing the straight lines EH, EK cutting the Equator at r, m: the beginning of the morning [twilight] is measured by the arc Br, and its end by the arc Bm, up to the line EK; but the beginning of the evening

[twilight] will give the arc Dm up to the line EK, and its end the arc Dr — because the arc Br is similar to the arc FH, and Bm to FK, and Dm to uK, and Dr to uH, from the scholium to proposition 22 of Book 3 of Euclid.

By the same reasoning, the arc IO, drawn through the first point of Libra [$\cap$?] and Aquarius [$\approx$?], will be the twilight of the first point of Libra and Aquarius; and the beginning of the morning [twilight] will be recognized by the arc Bn, and its end by the arc Bo; while the evening twilight's beginning will be exhibited by the arc Do, and its end by the arc Dn. So too the arc MQ, drawn through the first point of Sagittarius [$\times$?] and Pisces [$\times$], will be the twilight of the first points of Sagittarius and Pisces: and the morning's beginning will be exhibited by the arc Bp, and its end by the arc Bq; while the evening's beginning will be given by the arc Dq, and its end by the arc Dp. Likewise the arc Am of the Equator, drawn through the first point of Libra [$\cap$?], between the Horizon and its parallel 18° , will be the twilight of the first point of Aries [γ]; and the beginning of the morning [twilight] will be given by the arc Bm up to the parallel of the Horizon, and its end by the arc BA.

Conversely, the arc SX of the tropic of Cancer [☿], between the Horizon and its parallel 18° , will be the twilight of the first point of Capricorn [♑]. But the arc Ti, drawn through the first point of Gemini [♊] and Leo [♌ ?], will be the twilight of the first points of Libra [$\cap$] and Aquarius [$\approx$]; and the arc VY, drawn through the first point of Taurus [♉] and Virgo [♍], will be the twilight of the first points of Sagittarius [$\times$?] and Pisces [$\times$]. Finally the arc Ca of the Equator, drawn through the first point of Aries [γ], will be the twilight of the first point of Libra [$\cap$]. Now the beginnings and ends of these twilights will be found, as before, if from E straight lines are drawn through the ends of the arcs set between the Horizon and its parallel 18° — with this observed: that the beginning and end of any morning twilight be counted from midnight, and of the evening from noon. Likewise, that the beginning of the morning twilight begin, on the Equator, from the point through which the straight line drawn from E through the end of the twilight-arc passes; while the end [begins] from the point through which the line is drawn from E through the [other] end of the same twilight-arc emitted to the Horizon. But the beginning and end of the evening twilight are sought in the contrary way [to] the crepuscular line: so that the beginning is [counted] from the point B, and the evening's [is counted] from the point D.

[Margin: What is to be observed in determining the beginning and end of any twilight.]

We shall demonstrate that the twilight of any point of the Ecliptic is found by the arc drawn through the opposite point, as follows. [a] Since through any point of a non-great circle on the sphere — for example through H — a great circle tangent to it can be described, [b] that great circle will touch another non-great circle equal, parallel, and opposite to the former. Since, therefore, HE is the diameter of that great circle, where it meets the crepuscular line at R, there the same great circle will touch the parallel of the Horizon baRt, opposite to the parallel HIMm. And so, since by the corollary to proposition 6 of Book 2 of Theodosius the points of contact are opposite through the diameter of the sphere, the points H, R will be opposite through the diameter. Therefore, with the first point of Capricorn [♑] at H, the first point of Cancer [♋] will be at R, the point of the crepuscular line; and thus, with the Sun there, the morning twilight will begin. But when, by the sweep of the primum mobile, the first point of Capricorn [♑] has reached K, the first point of Cancer [♋] will be at S — since the points K, S on the Horizon are likewise opposite through the diameter, namely the setting of Capricorn [♑] and the rising of Cancer [♋]. Therefore the arc HK, which the first point of Capricorn [♑] traverses in the same time in which the first point of Cancer [♋] completes the twilight-arc RS (since it is similar to it, by the scholium to proposition 22 of Book 3 of Euclid, on account of the equal vertical angles HEK, RES at the center), will measure the duration of the twilight of the first point of Cancer [♋]. In no other way we shall show that the arc IO is similar to the twilight-arc aT, because for the same reason, with the first point of Libra [♎] or Aquarius [♒] at I, the first point of Gemini [♊] or Leo [♌] is at a, the point of the crepuscular line; and with the same first point of Libra [♎] or Aquarius [♒] advanced from I to O, the point of the Horizon, the first point of Gemini [♊] or Leo [♌] is then advanced to the point T of the Horizon; and so for the rest.

[Margin: a — Theodosius II.14; b — Theodosius II.6]

SCHOLIUM

[Margin: To find the twilights by means of the Analemma.]

1. **We shall also learn the twilights readily from the Analemma.** For let ABCD be the meridian of the Analemma, about center E; let AC be the diameter of the Horizon; BD the vertical diameter; FG the axis of the world; HI the diameter of the Equator; KL the diameter of the parallel of the Sun (whether northern or southern), about which the semicircle KPL is described; and finally ab the diameter of the parallel of the Horizon [distant] 18° in the lower hemisphere, in which all twilights begin and end. If then, from N, O — the intersections of the diameter KL with AC and with

ab — the perpendiculars NP, OQ are drawn to KL, the arc PQ will be the magnitude of the twilight. And if it is a morning twilight, its beginning will be distant from midnight by the arc LQ, and its end by the arc LP; but if it is an evening twilight, its beginning will be distant from noon by the arc KP, and its end by the arc KQ — because NP is the common section of the parallel of the Sun and of the Horizon, as was shown in the scholium to Canon 7, no. 1; and for the same reason OQ is the common section of the same parallel of the Sun and of the parallel of the Horizon. In like manner, with YZ drawn perpendicular to HI, the arc GZ will be the length of the twilight when the Sun is at the equinoxes; and the beginning of the morning [twilight] will be distant from midnight by the arc IZ, and its end by the arc IG; while the beginning of the evening [twilight] will be distant from noon by the arc HG, and its end by the arc HZ.

Diagram. A square-framed figure (the Analemma) composed of two circles, one above the other. The **upper circle** is the meridian analemma (meridian ABCD about center E; horizon diameter AC; vertical diameter BD; axis of the world FG; equator diameter HI; parallel-of-the-Sun diameter KL with semicircle KPL; parallel-of-the-Horizon-18° diameter ab), crowded with radial and chord lines; the **lower circle** is a sphere showing the Horizon, Meridian, Equator, parallel of the Sun, and the parallel of the twilights. Readable labels:

- Upper circle: H, K (top left and top), B (top), F, P, Q (upper right), M (center-upper), R, V, N, T (center), E (center), Y (center), A, b (right of center), O, x (center-lower), S (lower center), G, z (lower left), P, Q, e (left), S, D, L, I (bottom of upper circle)
- Lower circle: B (top), I, G, K (upper right), H (left), F, O (center), A (left), L, C, N (right of center), I, E (center), N, R (lower left), M (bottom center), D (bottom), H (lower right)
- Right margin caption: "Crepuscula ex Analemme inquirere" (To inquire the twilights from the Analemma).

Several small interior point-letters (including a Greek-letter label ξ [?] in the upper circle) are not securely legible. (dense small labels; this figure is repeated on p. 302).

LIBRI III. ASTROLABII — running header "302 / LIBRI III. ASTROLABII"

[Margin: a — Euclid VI.2. To explore, by numbers, the versed sine of the semidiurnal arc, and thereby the semidiurnal arc itself. To investigate the twilights by numbers.]

2. **By sines the twilights will be computed thus**, if first we find the versed sine of the semidiurnal arc in this way. In the Analemma, from the extreme points K, L of the diameter of the parallel, let straight lines KS, LS be drawn parallel to the vertical diameter BD and to the horizon-diameter AC, cutting one another at S; and from M, the midpoint of the diameter of the parallel, where the axis of the world cuts it, let another parallel MR be drawn to the same horizon-diameter AC. Then the straight line KS will be cut in two equal parts at R — [a] since, as KM is to ML, so is KR to RS. Now KS itself will be composed of KT, the sine of the meridian altitude of the said parallel, and of TS, the sine of the meridian depression of the same parallel opposite [to it] — which depression is equal to the meridian altitude of the opposite parallel. Therefore, if it be made that, as KR, half the straight line KS composed of the sine of the meridian altitude and the sine of the meridian depression, is to KT, the sine of the meridian altitude, so is KM, the whole sine, to another [term], there will be produced KN, the versed sine of the semidiurnal arc KP. From this versed sine will be drawn out the semidiurnal arc itself, as we taught in the exposition of the table of sines.

Now if, again, it be made that, as KR — half the straight line KS composed of the sine of the meridian altitude and the sine of the meridian depression — is to the sine of the arc of 18° (that is, to the segment of the straight line KS between AC and ab), so is KM, the whole sine, to another [term], there will be found the straight line NO; which, added to the versed sine KN of the semidiurnal arc, makes up KO, the versed sine of the arc KQ, composed of the semidiurnal arc KP and the twilight-arc PQ. If, then, from this arc KQ the semidiurnal arc is subtracted, the remainder will be the twilight-arc PQ.

But the same twilight can also be investigated by spherical triangles. For let ABCD be the Horizon; BD the meridian; AFC the Equator; GIH any parallel of the Sun; E the pole of the Horizon; AEC the prime vertical; KK the parallel of the twilights, distant 18° below the Horizon, cutting the parallel of the Sun at K, so that KG is the twilight-arc as the Sun runs along the parallel GIH; [b] to which is similar the arc NO of the Equator, which the great circles MG, MK, issuing from

M, the pole of the world, intercept. This, then, we shall find by the following reasoning. Through K — the center of the Sun at the beginning of the morning, or the end of the evening, twilight — the vertical EK being drawn, cutting the Horizon at L: since in the spherical triangle EKM all three sides are known (for EM is the arc of the complement of the Sun's declination in the northern parallel, but in the southern one an arc composed of the quadrant and the declination; and MK is composed of the quadrant EL and the arc LK of 18°), the [triangle/angle] EMK will be found, and hence its arc FN, in this way. Let it be made: as the whole sine is to the sine of the side [EM, the complement] of the declination — or of the arc composed of the quadrant and the declination — (thus, if [the sine of the side EM, complement of the altitude] of the pole [be taken], to another [term], and a fourth number will be found). And if it be made again: as the whole sine is to the whole sine, so is the difference between the versed sine of the side EK, composed of [18° and the arc] by which the two sides ME, MK differ from one another, to another [term], there will be produced the versed sine [of the sought angle EMK]; and so the angle itself, and its arc FN, will become known: from which, if the semidiurnal arc [FO] be taken, the twilight-arc NO will remain.

[Margin: a — Theodosius II.10]

Diagram. The Analemma figure of the Scholium, **repeated from p. 301** (same two-circle construction: upper meridian-analemma, lower sphere), here set at the upper left with the text of no. 2 wrapping around it. Readable labels match those of fig-p653-1: upper circle — H, K, K, B, F, P, Q, M, R, V, N, T, E, Y, A, b, O, x, S, G, z, P, Q, e, S, D, L, I; lower circle — B, I, G, K, H, F, O, A, L, C, N, I, E, N, R, M, D, H. Right margin caption: "Crepuscula ex Analemmate inquirere." (dense small labels; see fig-p653-1).

CANON XI.

To explore which points of the Ecliptic are on the Meridian, and on the Horizon, or on any other circle cutting the Ecliptic; and in which celestial house any proposed star, or point of the Ecliptic, is found at any moment of time.

[Margin: By the material astrolabe, to investigate the points of the Ecliptic that lie on any circle cutting the Ecliptic.]

1. **By day**, let the Sun's altitude be taken, and let it be counted among the almucantars [Almucantarath] on the eastern or western side, according as the time is before or after noon. For if the Sun's degree is advanced to the

almucantar of the altitude found, the Ecliptic will represent the position which it then has in the sky; and hence the points of the Ecliptic which are then found on the meridian line, on the Horizon, and on any other circle — whether it be the vertical, or the circle of position, or the parallel of the Horizon, or any other circle whatsoever, whether great or not great — will be those that at that time actually lie on the said circles in the sky. Indeed, the stars described in the rete will [also] indicate the position which they then hold in the sky.

By night, let the altitude of some star be observed, and let the star's summit be found among the almucantars on the eastern or western side, according as the star is eastern or western. For by this reasoning the Ecliptic will again have the position it then has in the sky; and therefore not only will it appear which points of the Ecliptic lie on any circle, but also in which circle this or that star is found, or what position it has in the sky.

2. **If** the same is to be investigated for any given hour, the fiducial line of the sight must be moved to that hour, whether before noon or after, according as the time given is before or after midday. For when the rete has then been turned round until the degree of the Ecliptic which the Sun occupies is set beneath the fiducial line, the Ecliptic will again have its proper position, etc.

So too, if anyone wishes to know what hour it is when any sign, or degree of the Ecliptic, or any star described in the astrolabe rises — the Sun occupying any given degree of the Ecliptic — that degree, or star, must be set on the eastern Horizon. For the fiducial line of the sight, running through the Sun's degree at that time, will show on the limb the hour, or the distance of the Sun from the meridian circle, etc.

3. **Without a material astrolabe** we shall attain the same in this way. Let ABCD be the Equator about center E; and let the Eclip-

[Margin: By what hour any degree, or sign, of the Ecliptic rises, to be known.]

[Margin: By the material astrolabe, to investigate the points of the Ecliptic that lie on any circle cutting the Ecliptic.]

Astrolabe, Book III — Canon XI (continued) and Canon XII

(Continuation of Canon XI: which points of the ecliptic lie on the meridian, horizon, and any hour-circle, and the celestial house of a star or point.)

the ecliptic AFCG, whose center is δ and pole a ; the horizon AqC; the tropic of Cancer (♋), GI; the tropic of Capricorn (♎), FH. Let the first case proposed be this: the Sun stands at point O of the ecliptic, and the Sun's altitude is found, before noon, to be 20 degrees (the almucantar being θMi). Having drawn the parallel of the horizon for 20 degrees, θMi , trace the parallel of the equator through the given point O, cutting the parallel θMi at M. From E, draw through O and M straight lines cutting the equator at L and N; take an arc BP equal to arc LN, and draw the straight line EP, cutting the tropic of Capricorn at Q and the tropic of Cancer at I.

Now suppose the rete is imagined carried around until the given point O has [the given altitude] before noon, and the line EL coincides with the line EN. Then the line EB will coincide with the line EP, because of the equal arcs LN and BP; the beginning of Capricorn, F, will stand at Q, and the beginning of Cancer at I. Therefore the line QEI cuts the parallel of the equator δRQ (drawn through δ , the center of the ecliptic) at R, and cuts the parallel abh (drawn through a , the pole of that same ecliptic) at b ; and so the center of the ecliptic will then be at R, and its pole at b . Hence the ecliptic QSRic, described from R through Q and I and touching the tropics at Q and I, will then have its proper position; it will cut the meridian at S and X, and the horizon at K and c .

To which degrees of the ecliptic these points correspond will be shown by straight lines drawn from b , the pole of the ecliptic, to the points themselves, as we showed in Book 2, prop. 5, no. 19. For S will be distant from the beginning of Capricorn — that is, from point Q — according to the succession of the signs, by as many degrees as are contained in the arc PT of the equator. Point K will be removed from the same beginning of Capricorn, according to the succession of the signs, by as many degrees as are contained in arc PBY, or by as many degrees from the beginning of Cancer, I, against the order of the signs, as are found in arc μY . Finally, the points X and c are diametrically opposite the points S and K, whose distances from Cancer and Capricorn are likewise measured by the arcs μV and Pd; but the former is to be counted according to the succession of the signs, the latter against the series of the signs.

But if an hour is given — that is, a distance from the meridian — by which we must investigate these same points, then a straight line must be drawn from the center E through the given hour; that is, a line that cuts off from the equator the arc of the Sun's distance from the meridian circle. Such is the line EN, cutting the parallel of the given point O of the ecliptic — the point at which the Sun stands — at point M. For at point M the Sun will stand at the proposed hour, exactly as if the Sun's parallel were to intersect the parallel of the horizon θM . The rest, therefore, is to be carried out as before.

Now if, with the Sun standing (for example) at point 9 of the ecliptic, it is to be found at what hour point 3 of the same ecliptic rises — that is, cuts the eastern horizon at K — then draw from E through 3 the arc that cuts the eastern horizon, and drawing from E through 3, K, 9 straight lines cutting the equator at 4, 2, e, take an arc e7 equal to arc 42. Then the arc B7 will be the Sun's distance from the meridian when point 3 ascends above the horizon. For when point 3 is advanced as far as K, the line E4 will coincide with E2, and point e will have been advanced to 7, on account of the equality of the arcs 42 and e7, and so on.

Diagram. A large stereographic-projection figure (the "astrolabe without a material instrument") enclosed in a square frame, occupying the lower-right of the page. It shows the ecliptic, the two tropics, the equator, the horizon, a system of almucantars/parallels and hour-lines, and the rete's ecliptic in a displaced (observation-time) position, all projected on the plane. The construction is that described in the surrounding text (points A–Y, Greek θ , μ , π , σ , ρ , ϕ , γ , the center E, ecliptic center 8 and pole a, and the zodiac glyphs strung around the ecliptic circle). Readable labels:

- θ (top center): almucantar/parallel point at the top
- F (upper center, on the fixed ecliptic): beginning of Capricorn
- Q (upper right): tangent point on the tropic of Capricorn
- I (right, near the sun-mark): tangent point on the tropic of Cancer
- H (far left, on a dotted horizontal): horizon point
- A (right, beside a sun symbol): horizon point (AqC)
- C (lower left): horizon point
- Interior points visible near the center: N, M, and small numerals (e.g. 6, 8) and the center E
- Right-hand margin of the figure: P, R, d, c, and Greek π , σ

- Zodiac glyphs (♈ ♉ ♊ ♋ ♌ ♍ ♎ ♏ ♐ ♑ ♒ ♓) and hour-numerals (1–12) are distributed around the ecliptic and along the projected hour-lines; many are small and abraded.

Numerous peripheral labels (small Greek letters and interior lower-case letters b, e, f, g, h, i, k, l, m, n, p, q, r, s, t and the smaller numerals) could not be resolved with confidence at this resolution. **More than two labels are unresolved**

4. Next, let the same points of the ecliptic be sought when the star Z has, at night, an altitude of 20 degrees. Having described through Z, the center of the star, a parallel of the horizon for 20 degrees, i, draw straight lines through Z and i from E, cutting the equator at l, k; cut off Be [equal to lk], and draw the line Ee, cutting the tropics at H and f, and the parallels R8g and bah at g and h. Then, with the star Z at i, the beginning of Capricorn will be placed at H, the first point of Cancer at f, the center of the ecliptic at g, and finally the pole at h. So the ecliptic described from g through H and f will cut the meridian at m, r, and the horizon at p, n; and the distances of these points from the beginning of Capricorn, H, and from the beginning of Cancer, f, will be found by lines drawn from the pole h, as before.

5. By the same reasoning we shall learn which points of the ecliptic, at the time of observation, are found on any circle whatever — whether a great circle or not, provided it cuts the ecliptic. Thus you see that the parallel of the horizon θMi is cut by the ecliptic $QSXe$ at M. And if a circle of position be described through γ [and through q, δ] — namely through γ , the beginning of the 5th house — it will be cut by the ecliptic $AFCG$ at s, t, and by the ecliptic $Hrfm$ at β , ε : and how far all these points are distant from Capricorn and Cancer, both according to the series of the signs and against it, will be indicated by lines drawn from the poles a, b, h to the points themselves. In no other way are had the points that lie on any hour-circle at the given hour. Thus, if the line $Q\mu$ represents some hour-circle drawn from the meridian or from midnight, then with the ecliptic holding the position of circle $AFCG$, the points π , σ will be on that hour-circle; and how far they are from the beginnings of Capricorn and Cancer — that is, from the points F, G — will be shown by lines drawn from the pole a to π , σ . But with the ecliptic in position $QSXc$, the first points of Capricorn and Cancer — namely Q and I — will be found on the hour-circle $Q\mu$. And finally, with the ecliptic holding the position of circle $Hrfm$, the same hour-circle will pass through the points ε , ϕ ; and the arcs of the ecliptic $f\varepsilon$, $H\phi$, counted from the beginnings of Cancer and Capricorn according to the succession of the signs, will be known through the arcs of the equator cut off by lines drawn from the pole h to ε , ϕ .

6. Now if, with the rete or the ecliptic holding any position whatever, one wishes to know in which celestial house — and in what part of that house, as described according to the view of Johannes Regiomontanus [Regiomontanus] — any given star or point of the ecliptic stands (the place of that star with respect to the ecliptic in that given position having first been found, as was taught in Book 2, prop. 11, nos. 2, 3, and 4), then one must describe, through the center of the star and through the two points in which the horizon intersects the meridian line, a circle of position whose center lies on the line perpendicular to the meridian line at the center of the horizon, as was stated in Book 2, prop. 10, no. 6. For if the star or point of the ecliptic stands above the horizon, the degree of the equator through which the circle of position passes will at once show the distance of the given star or point from the meridian line — that is, from the beginning of the 10th house — and in which house above the horizon it is found, since thirty degrees of the equator constitute each celestial house. And you will say the same
(p. 304)

of the houses below the horizon, if the star or point stands below the horizon. For example, if the given point is u , and the ecliptic $Q SXc$ is described above the horizon, let a circle of position uq be drawn through u , cutting the equator at z . And because the arc $B[?]$ comprises 30 degrees, we shall say that point u stands at the beginning of the 11th house. But the given point α , below the horizon (if through it a circle of position αq be described, cutting the equator at $\varkappa[?]$), we shall say to be at the beginning of the 5th house, because the arc $D\alpha$ also comprises 30 degrees. In like manner we shall pronounce the star o [to be as many degrees distant from the beginning of the 5th house] as are contained in the arc $\varkappa\lambda$; but the star $\varpi[?]$ to be in the 11th house, as many degrees from its beginning as are included in the arc $\gamma\omega$. We shall proceed in no other way if one prefers to describe the celestial houses according to Campanus, by numbering the unequal degrees of the prime vertical circle — as was taught in Book 2, prop. 5, no. 17 — for the equal degrees of the equator, and so on.

Scholium

[Margin: To investigate, by right and oblique ascensions, the points of the ecliptic that lie at any hour on the meridian, on the horizon, and on any hour-circle, reckoned from noon or from midnight.]

1. We shall also find, with the greatest ease and by right and oblique ascensions, the points of the ecliptic that lie at any hour on the meridian, on the horizon, and on any hour-circle whatever, reckoned from noon or from midnight — by this

method, namely. To the Sun's distance from noon, proceeding westward (this distance is gathered from the hour reckoned from noon, if 15 degrees are assigned to each hour; and from the hour reckoned from midnight the same distance is found, if a semicircle is added to the distance from midnight), add the right ascension of the point of the ecliptic that the Sun then occupies — which is either taken from a table of right ascensions or worked out, as we taught in Canon 4. The number so compounded, after a whole circle has first been cast away (if it can be cast away), will be the right ascension of the point of the ecliptic then standing on the meridian above the horizon. Therefore, either from a table of right ascensions or from what we wrote in Canon 4 and its scholium, the point of the ecliptic standing on the meridian — the point to which the found right ascension belongs — is to be extracted; and the point opposite to it will stand on the meridian below the horizon. But if a quadrant is added to the said right ascension, there will be compounded (after a whole circle has first been cast away, if it can be) the oblique ascension of the point of the ecliptic standing on the horizon in the eastern part; and this will be known either from a table of oblique ascensions computed for the given elevation of the pole, or from Canon 5 and its scholium. The point opposite to this will stand on the horizon in the western part.

The reason for this rule of ours is clear from the material sphere, and it can also easily be shown in this way from the figure above. Let the distance from noon be set as Bd, so that the hour-circle passes through d, in the manner of a certain right horizon on which stands the point of the ecliptic at which the Sun then is. If, then, Ad is the right ascension of that point — that is, of A, the beginning of Aries — there will be compounded AB, the right ascension of the point of the ecliptic then on the meridian; and if the quadrant BC is added, as far as the oblique horizon, there will be compounded ABC, the oblique ascension of the point of the ecliptic standing on the horizon. But if the right ascension of the point of the ecliptic standing on the hour-circle drawn through d is PBd, there will be compounded the arc PBdB, and, casting away the whole circle PBdP, the remainder PB will be the right ascension of the point of the ecliptic on the meridian, and so on.

Likewise, if the right ascension of the said point of the ecliptic is γ Dd, so that the beginning of Aries γ is the beginning, there will be compounded γ DdB, the right ascension of the point of the ecliptic on the meridian; and, the quadrant BC being added, there will be made γ DBC, the oblique ascension of the point of the ecliptic standing on the horizon; and, the whole circle γ DB γ being cast away, the remainder will be the oblique ascension γ C, and so on.

For example. With the Sun standing at the beginning of Taurus (♉), at the elevation of the pole 42 degrees, let the four points of the ecliptic be sought at 3 hours before noon — that is, at 9 hours after midnight, or 21 hours after noon. This time gives 315 degrees of elapsed distance from noon. Then to the right ascension of the beginning of Taurus, which contains 27 degrees 54 minutes, let us add 315 degrees; we shall make 342 degrees [54 minutes] for the right ascension of the point then culminating, to which ascension corresponds nearly 341 degrees 27 minutes. Therefore 11 degrees 27 minutes of Pisces (♊) then culminates; and consequently the opposite point, namely 11 degrees 27 minutes of Virgo (♍), stands on the same meridian below the horizon. But if to the right ascension 342 degrees 54 minutes of the culminating point a quadrant is added, the number 432 degrees 54 minutes is made; and, the whole circle being cast away, the remainder is the oblique ascension of the point ascending above the horizon (which they call the Horoscope), 72 degrees 54 minutes, to which, at the elevation of the pole 42 degrees, are due 95 degrees 20 minutes, a little more, as is established from the tables of oblique ascensions, or from what we wrote in Canon 5 and its scholium. Therefore 5 degrees 20 minutes of Cancer (♋) then ascends above the horizon; and thereby the opposite point, namely 5 degrees 20 minutes of Capricorn (♏), is found to descend below the horizon.

2. By this very same reasoning, for a given hour — that is, for a given distance of the Sun from noon — we shall explore the point of the ecliptic standing on any hour-circle whatever drawn through the poles of the world, if the given hour-circle is conceived to be a certain meridian, and from the given hour the distance of the Sun from that same given hour-circle, proceeding westward, is sought. This is done if the distance of this circle from noon is subtracted from the distance of the given hour from noon (a whole circle being first added, if the subtraction cannot otherwise be made). Or, more surely, let all the hours from the given hour-circle be counted westward up to the given hour; for the hours counted will give the Sun's distance from the given hour-circle, as from some meridian, westward. For example: with the Sun still standing at the beginning of Taurus (♉), at 3 hours before noon — that is, at 21 hours after noon — let the point of the ecliptic be investigated that lies on the hour-circle 10 hours 35 minutes from the meridian. Subtracting the distance of this given circle from noon (which comprises 10 hours 35 minutes) from the given distance of the Sun — that is, from 21 hours — the remainder will be the Sun's distance from this circle, 10 hours 25 minutes, proceeding westward. This same distance is also found if, from the hour-circle at 10 hours 35 minutes, all the hours are run through up to hour 11 (giving 25 minutes); then follow hour 12, midnight, and hours 1, 2, 3, 4, 5, 6, 7, 8, and 9 after midnight, up to 3 hours before noon (or 9 after midnight), reckoned

from the hour-circle at 10 hours 35 minutes. This distance contains 156 degrees 15 minutes. If, then, the right ascension of the beginning of Taurus, 27 degrees 54 minutes, is added, there will be compounded the arc 184 degrees 9 minutes for the right ascension of the point of the ecliptic standing on the hour-circle 10 hours 35 minutes from the meridian, to which are due 184 degrees 31 minutes 38 seconds. Therefore 4 degrees 31 minutes 38 seconds of Libra (♎) then stands on the given circle.

If, with the same data, the point of the ecliptic is to be sought that lies on the hour-circle 11 hours after midnight — that is, standing 23 hours after noon — let us subtract the distance of this circle from noon, namely 23 hours, from 21 hours, a whole circle of 24 hours being first added, so that from the compounded number of 45 hours the subtraction can be made. In this way the remainder will be 22 hours, by which the given hour of 21 after noon recedes westward from the given circle of 23 hours after noon — which distance comprises 330 degrees. And the same distance is obtained if, after the hour 23 after noon of the given circle, all the hours are run through up to the given hour 21 after noon: for again there will be found 22 hours, which are these — hour 12 [midnight],... after noon, and moreover hours 1, 2, 3, 4, 5, 6, 7, 8, and 9 after midnight, all of which are 22, as before. Therefore, the right ascension of the beginning of Taurus, 27[?] degrees 54 minutes, being added, there will be made the right ascension of the point of the ecliptic standing on the hour-circle 23 hours after noon,

(p. 305)

357 degrees 54 minutes, to which corresponds nearly 357 degrees 42 minutes 33 seconds. Therefore 27 degrees 42 minutes 33 seconds of Pisces (♓) stands on the hour-circle 11 hours after midnight. And so of the rest.

We taught how to investigate this same point on any hour-circle in prop. 9 of the *Gnomonics*, if the point that ascends above the horizon at the given hour, and its oblique ascension, is [known]; or the point on the hour-circle then standing 6 hours after midnight, and its right ascension. But the method here proposed is more expeditious, since it needs neither of those points, but requires only the right ascension of the point of the ecliptic in which the Sun stands at the time of observation (which right ascension is always the same at every elevation of the pole).

Indeed, if the same thing is to be investigated with any point of the ecliptic whatever set on the eastern horizon, we shall take the semidiurnal arc of that point then ascending above the horizon in place of the hourly distance from the

meridian circle, and complete the rest as has been said. For example: when the beginning of Leo (♌) ascends above the horizon of latitude 42 degrees, let the point of the ecliptic on the hour-circle 5 hours from noon be sought. Subtract this distance of 5 hours from 16 hours 43 minutes — that is, from the distance of the first point of Leo from the meridian, proceeding westward (since the semidiurnal arc of Leo comprises 7 hours 17 minutes) — so that the distance of the beginning of Leo, then rising, from the hour-circle 5 hours from noon may remain, namely 11 hours 43 minutes, that is, 175 degrees 45 minutes. To this distance, if the right ascension 122 degrees 12 minutes (which belongs to the beginning of Leo) is added, there will be produced the right ascension of the point of the ecliptic on the hour-circle 5 hours from noon, 297 degrees 57 minutes, to which correspond 295 degrees 57 minutes, a little more. Therefore 25 degrees 57 minutes of Capricorn (♐) stands on the hour-circle 5 hours from noon; and consequently 25 degrees 57 minutes of Cancer (♋) will be found on the hour-circle 5 hours from midnight, since the beginning of Leo [rises].

[Margin: A more accurate way of finding the point of the ecliptic standing on a given hour-circle — in degrees and minutes, or in hours, minutes, and seconds — for use when the semidiurnal arc is not had in degrees and minutes, or in hours, minutes, and seconds.]

But unless the semidiurnal arc is taken in hours, minutes, and seconds, or in the degrees and minutes in which it was found by sines, an error of some minutes may occur — which we shall make clear by the next example. The semidiurnal arc of the beginning of Leo contains 109 degrees 21 minutes, that is, 7 hours 17 minutes 24 seconds; and this being subtracted from the whole circle of 360 degrees, or 24 hours, there remains the distance of Leo, standing on the eastern horizon, from the meridian, proceeding westward, 250 degrees 39 minutes, or 16 hours [42 minutes 36 seconds]; from which, [subtracting the quadrant] 75 degrees, the remainder will be the distance of Leo (♌) from the hour-circle 5 hours from noon, [11 hours 42 minutes 36 seconds], to which hours and minutes are due the same 175 degrees [39 minutes]. And [adding the right ascension] 122 degrees 12 minutes, there will be compounded the right ascension of the point of the ecliptic on the hour-circle [5 hours from noon], 297 degrees 51[?] minutes, that is, 25 degrees 51 minutes of Capricorn (♐) — so that the difference between this [and the point first found] amounts to 6 minutes. Since this is so, when the semidiurnal arc is not had in degrees and minutes, or in hours, minutes, and seconds, the point on the given hour-circle will be found more exactly by that method which we explained in the *Gnomonics*: namely, by subtracting the degrees of the equator counted from the sixth hour of the morning [westward] up

to the given hour-circle, from the oblique ascension of the given point emerging above the horizon, [a whole circle being first added] if the subtraction cannot [otherwise be made]. For thus the remainder will be the right ascension of the point of the ecliptic on the given circle. As in the same example: from the sixth hour of the morning up to the fifth hour after noon are reckoned 11 hours, that is, 165 degrees. If, then, [the oblique ascension is added...] of the beginning of Leo [is] 102 degrees 51 minutes — that is (a whole circle being added), from 462 degrees 51 minutes the remainder [will be the right ascension] of the point of the ecliptic on the hour-circle 5 hours from noon, as above.

[Margin: To find, by oblique ascensions, the hour at which any point of the ecliptic whatever rises, wherever the Sun may stand.]

3. Finally, the hour at which any sign or point of the ecliptic rises, the Sun possessing any degree of the ecliptic whatever, we shall explore in this way. Let the oblique ascension of the arc of the ecliptic lying between the Sun's place and the ascending point (both taken according to the series of the signs), reduced to hours, be subtracted from the semidiurnal arc of the point that the Sun occupies; or, conversely, let the semidiurnal arc be subtracted from the said oblique ascension reduced to hours — the smaller number, that is, from the greater. For in the first way there will remain the hour before noon, in the second the hour after noon, at which the point of the ecliptic emerges above the horizon. The reason for this is clear from the parallel of the point whose oblique ascension was taken, [being] on the eastern horizon; and with the sphere moved until the ascending point reaches the same horizon, the arc of the Sun's parallel between the Sun's place and the horizon measures the oblique ascension of the arc of the ecliptic intercepted between that same place of the Sun and the ascending point, since that arc of the parallel rises together with this point of the ecliptic. Therefore, subtracting that arc of the parallel from the semidiurnal arc, or this from that, the remainder will be the Sun's distance from the meridian, either before noon or after noon, as we said.

By way of example. With the Sun standing at the beginning of Leo (♌), at latitude 42 degrees, let the hour be sought at which the beginning of Libra (♎) rises. The oblique ascension of the arc between the beginning of Leo and Libra contains 77 degrees 9 minutes, that is, 5 hours 9 minutes; and these being subtracted from 7 hours 17 minutes — that is, from the semidiurnal arc of the beginning of Leo — there remain 2 hours 8 minutes. Therefore the beginning of Libra rises 2 hours 8 minutes before noon. Again, if it is to be investigated at what hour the beginning of Cancer (♋) rises: the oblique ascension of the arc from the beginning of Leo up

to the beginning of Cancer, computed according to the succession of the signs, comprises 324 degrees 6 minutes, that is, 21 hours 36 minutes; from which, if the semidiurnal arc of Leo, 7 hours 17 minutes, is taken away, there remain 14 hours 19 minutes after noon — that is, 2 hours 19 minutes after midnight — at which the beginning of Cancer emerges above the horizon. And so of the rest.

Canon XII

To find the meridian line, and thereby also the line of true rising and setting, on a plane parallel to the horizon.

[Margin: To investigate, by a material astrolabe, the meridian line and the points of true rising and setting.]

1. Having found the altitude of the Sun, whether before noon or after noon, let the degree that the Sun then occupies be set on the parallel of the horizon of that altitude, and let the vertical circle on which that same degree falls be noted. For by however many degrees that vertical circle recedes from the prime vertical — that is, from the intersection of the equator, the horizon, and the prime vertical — toward the south or toward the north (which degrees are measured by the arc of the horizon lying between the prime vertical and the vertical on which the Sun's degree falls), by so many degrees are we to count on the back of the astrolabe from the horizontal diameter (which, namely, cuts the meridian line — extended through the center and the suspension-ring — at right angles), on the side of the east or of the west, according as the Sun's altitude was found before noon or after noon: upward indeed, toward the ring, if the Sun was found on the southern vertical; but downward, if on the northern. For, with the fiducial line of the alidade [Mediclinium] set upon the last degree of the numeration, if the astrolabe is then laid parallel to the horizon and turned this way and that until the shadow of one edge of the sight-vane is extended along the edge of the alidade, and the shadow of the other edge is parallel to the fiducial line, then the diameter of the given back of the astrolabe passing through the ring will indicate the position of the meridian line — so that its part toward the ring points straight south, and the other part north; while the other diameter, cutting the former at right angles, will show the true points of rising and setting.

ASTROLABE, BOOK III — Canon XII (continued) and Canon XIII

[Margin: *Lineam meridianam sine Astrolabio materiali certius inuenire.* — To find the meridian line more reliably, without a material astrolabe.]

2. More reliably, however, we shall find the meridian line — and thereby the true points of rising and setting — without a material astrolabe, by the method we prescribed in the scholium to Proposition 23 of Book 1 of our *Gnomonics*; and we do not think it needs repeating here. I wish only this to be noted about it: it is not necessary that the vertical HO be drawn through O, the point of intersection of the Sun's parallel with the parallel of the horizon, in order to derive its declination from the primary vertical. It is enough that, from that point O and from the point of intersection of the primary vertical with the parallel of the horizon — which in the figure of the aforesaid scholium lies a little below the point O — two straight lines be extended through H, the pole of the horizon. For these lines, beyond H and in the same parallel of the horizon, will intercept the arc of the required declination: that is, [the arc] will comprise as many equal degrees of the parallel as there are apparent degrees between O and that other intersection, as we demonstrated in Book 2, Proposition 6, Number 25.

[Margin: *Lineam meridianam sine instrumento materiali ex declinatione Solis, & altitudine poli cognitis, per vnicam obseruationem inuestigare.* — To investigate the meridian line without a material instrument, from the Sun's declination and the pole's altitude, both known, by a single observation.]

3. Perhaps we shall attain the same thing more conveniently by a single observation, from the same data — namely, the Sun's declination and the known altitude of the pole (which were given there too) — in this other way. In a plane parallel to the horizon let the circle ABCD, representing the horizon, be described from center E; in whose plane several circles of the sphere are to be drawn, as they are seen in it from the Nadir, or its lower pole, just as was explained in the scholium to Proposition 20 of Book 2, Number 15. Then, at any hour, by some fine thread — or by the instrument we constructed at the beginning of the scholium to Proposition 23 of Book 1 of the *Gnomonics* — let the Sun's shadow be observed; through its two points let the straight line FG be extended through the center E, and at the same time (with no delay interposed) let the Sun's altitude be taken, which the arc FN measures. Or else, with the instrument we shall construct in the following scholium, Number 3, let the shadow and the Sun's altitude be observed at one and the same operation.

Now, having erected HL perpendicular to the diameter FG, let there be counted from L the complement of the pole's altitude up to K — or the pole's altitude itself from G up to K; and, the radius HK being drawn to cut EG at M, the segment EM of the vertical FG will contain as many degrees as are contained in the arc LK, as is clear from what was shown in Book 2, Proposition 1, Number 5. For from the Nadir H the point K will appear at M. Hence the parallel of the horizon described from E through M will pass through the pole of the world, since it recedes from the Zenith E by the complement of the pole's altitude; and it is described from the point E, just as before the parallel of the Equator was described from the same center when the circle ABCD represented the Equator.

Diagram. A large circular analemma-type projection inscribed in a square frame, densely crossed by straight lines and containing several concentric and oblique circles (horizon, equator, parallels of the Sun, meridian, and hour circles). Many small letter-labels sit around and within the figure. The readable labels, by position, are:

- B, L, Q (spaced along the top of the outer circle)
- K (upper right), A (right, on the horizontal diameter), G (lower right)
- S, q [?] (upper left, inside)
- F, T (left of center)
- P, f [?] (lower left, inside)
- a [?], l, m (lower-left / lower area)
- d [?] (upper center), E (center), R, M (right of center), I, s [?] (center)
- H, D (lower center)
- a small u [?] in a little box at the foot of the figure
- I and C: end-labels of two nearly-horizontal lines running out to the left of the frame
- the top-left symbol (ħ-like) and the letters N/M [?] near the top are not securely resolvable

But that we may know which point of this parallel is the pole of the world, we shall draw from H a radius to the Sun's center at N — N being, as is evident, situated (when the circle ABCD is conceived in the straight line FG, in the plane passing through the Sun) at right angles to the plane of the horizon, that is, in the position of the vertical; and the Sun will appear at the point O. And since, in the sphere, a circle described from the Sun's center as pole at an interval equal to the complement of the Sun's declination — yet, when the Sun is southern, the interval to be taken is the one composed of a quadrant plus the declination —

passes through that same pole of the world: if such a circle be described around O as pole, it will cut the parallel previously described, and will be drawn in the manner [set out] before. From N let there be counted on both sides the [arc] composed of a quadrant and the declination, up to P, Q. For, the radii HP, HQ being drawn, there will be cut off from that circle the visible diameter SR; and this being bisected at T, let the aforesaid circle be described, cutting the parallel of the horizon in two points. Of these, the one that is more northerly — namely, the one which for us lies between the Sun and the center E — is the pole, if the observation is made after noon; such is the point I. Therefore, the straight line IE being drawn, it will be the meridian line, that is, the line drawn through the pole of the world and the Zenith; and if the diameter AC cut it at right angles, C will be the point of true rising and A the point of true setting.

[Margin: *Lineam meridianam sine Astrolabio materiali ex sola declinatione Solis cognita, per duas obseruationes indagare.* — To find the meridian line, without a material astrolabe, from the Sun's declination alone (known), by two observations.]

4. But if the pole's altitude is unknown, we shall investigate the same thing by two observations, in this way. In the morning let the Sun's shadow make the straight line *a b*, when its altitude above the horizon is the arc *a e*. Drawing *Eg* perpendicular to *a b*, let a ray be sent out from *g*, the Nadir. (For if the circle ABCD be conceived to be revolved about *a b* until it stands upright to the horizon and the point *g* points downward, *Eg* will be the axis of the horizon and *g* its lower pole.) The radius *g e* will cut *a b* at *f*, the point where the Sun appears. From *e* let there be counted on both sides the complement of the Sun's declination, up to *n*, *m*, and from *g* let the radii [*g n*, *g m*] go out, cutting *a b* at *i*, *l*; and *i l* being bisected at K, the circle *h i* will be described from *k* through *i*, *l*, around *f*, as from the Sun's center at the interval of the complement of the declination — that is, it represents in the sphere the parallel described from the Sun's center at the interval of the complement of the declination, which passes through the pole of the world. This center *k* we found by what we taught in Book 2, Proposition 6, Number 9 — even though the radius *g m* runs out so far that its intersection with *a b* can scarcely be obtained.

[Margin: *Meridianam lineam sine Astrolabio materiali per tres obseruationes, etiamsi declinatio Solis, & altitudo poli ignorentur, inquirere.* — To find the meridian line without a material astrolabe by three observations, even when the Sun's declination and the pole's altitude are unknown.]

After some further interval of time, let the Sun's shadow make the straight line FG, and let its altitude be measured by the arc FN. Drawing EH perpendicular to FG, let the radius HN be produced from the Nadir H, cutting FG at *o*, the point where the Sun appears from the Nadir. Counting likewise from N, on both sides, the complement of the declination up to P, Q, let the radii HP, HQ go out, cutting FG at SR; and RS being bisected at T, let the circle be described from T through R, S, around O as pole — it will represent in the sphere the [parallel] described around the Sun through the pole of the world. Where this circle intersects the former, toward the north, at I, there will be the apparent pole of the world. Therefore the straight line IF being drawn will be the meridian line. And if, after some interval, a third observation be made (which, however, is not necessary), and in the same way a third circle be described around the Sun as pole, it will necessarily pass through the same point I, if no error has been committed.

5. Indeed, by three observations we shall find the meridian line even if neither the pole's altitude nor the Sun's declination is known — which we have also accomplished, by nearly the same method, in the booklet *On the Construction and Use of the Instrument for Sundials* [*de Fabrica & usu instrumenti Horologiorum*], Chapter [Cap.] 19...

by nearly the same method we accomplished it. So, in the first observation, let the Sun's shadow make the straight line *a b*, and its altitude [be *a e*]. Drawing *Eg* perpendicular to *a b*, the Sun's center will appear at *e*, established through the radius *g e* at *f*. In the second observation let the shadow make the straight line FG, and let the Sun's altitude be FN. Drawing EH perpendicular to FG, the Sun's center will appear at N, [situated] through the radius HN at O. In the third observation, finally, let the shadow-line be VX and the Sun's altitude VZ. Drawing EY perpendicular to VX, the Sun's center will appear at Z, [situated] through the radius YZ at the point *p*. Since therefore the Sun is set, in those three observations, to lie in the same parallel of the Equator — because its declination in them does not change perceptibly — if the center *t* of the three points *f*, O, *p* be found, the straight line *tE* will be the meridian line; for the center of the Sun's parallel *f O p* and the center of the horizon lie on the meridian line, as is manifest from what we demonstrated in Book 2, Proposition 6.

SCHOLIUM

[Margin: *Lineae meridianaee inuentio ex Analemmate per declinationem Solis & altitudinem poli cognitae*. — Finding the meridian line from the analemma, when the Sun's declination and the pole's altitude are known.]

By what method the meridian line may be derived from the analemma, when both the pole's altitude and the Sun's declination are known, we have set forth in Book 1 of the *Gnomonics*, in the scholium to Proposition 23, and in the booklet *On the Construction and Use of the Instrument for Sundials*, Chapter 18 — so that it would be superfluous to repeat it here.

[Margin: *Lineae meridianae inuentio in plano Horizontali per tres obseruationes, etiamsi declinatio Solis, & altitudo poli cognita non sint.* — Finding the meridian line in the horizontal plane by three observations, even when the Sun's declination and the pole's altitude are not known.]

2. But we shall also accomplish the same, by a pleasant operation, through three observations of the shadow and three altitudes of the Sun — of which let two be before noon and one after noon, or two after and one before — even if neither the Sun's declination nor the pole's altitude is known. For let the circle ABCD, whose center is E, lie in a plane equidistant from [parallel to] the horizon; and in the morning, at different hours, let the Sun's shadow make the straight lines DE, CE extended through the center E, and in those same hours let the Sun's altitudes be caught as DF, CB. But in the evening let the shadow be cast along the straight line AE, and let the Sun's altitude be AI, less than the altitude CB, since this observation is nearer to noon; and therefore [nearer the] meridian too. From the Sun's altitudes let the perpendiculars FG, BH, IK be dropped to the proper shadow-lines. Now, extended from H through G along the straight line HG, let HM be made parallel to FG and equal to HB, and let the straight line MF be joined, which will cut the line HG at O. For, HS being cut off equal to GF — (for the Sun's altitude DF is less than the altitude CB, and the sine FG is less than the sine BH) — and the straight line FS being joined, [a — Euclid I.33 (33. *primi*)] it will be parallel to GH, since the exterior [angle equals] the interior. [c — Euclid I.17 (17. *pri*.)] Since therefore in the triangle FSM the two angles S, M are less than two right angles — [that is,] the [angles] M are less than two right angles — the lines HG, MF will accordingly meet: that is, the line MF, produced, will cut the line HG in some point, namely at O, through which I say the Sun's parallel passes. For since, if GF and HM (or HB) be conceived perpendicular to the plane of the horizon, the Sun in the two observations stood at the points F, B, the Sun's parallel will pass through F, B, and its plane, extended through the straight line MF, will meet the plane of the horizon at O. [d — Euclid VI.4 (4. *sexti*)] For since, as HM is to GF, so is HO to GO, it will also be that HM, making right angles with the plane of the horizon at GF, [and] the line likewise making right angles with the same plane, so is HO to GO; and therefore, by the scholium to Proposition 4 of Book 6 of Euclid, the straight line drawn from M on high through F, extended on high (differing

from the straight line MF in the triangle HMO, even though it be moved about HO until it becomes upright to the horizon — since in it the lines HM, GF are not perpendicular to the horizon, as is plain) will fall on the point O; and therefore the plane of the Sun's parallel, drawn through that straight line, will meet the plane of the horizon at O.

In the same way, if from H through K the straight line HK be extended, and from H a line HN be drawn parallel to KI and equal to HB, the joined straight line MI will cut the line HK — namely at the point L, at which the same parallel of the Sun meets the plane of the horizon. Therefore, the straight line OL being added, it will be the common section of the Sun's parallel and the horizon. Hence the straight line PE, drawn through the center perpendicular to OL, will be the meridian line — that is, the common section of the Meridian and the Horizon. For since both the Sun's parallel and the horizon [stand] to the common section OL as to the same straight line, and therefore, by Definition 3 of Book 11 of Euclid — [e — Euclid XI.19 (19. *undecimi*)] and since [a line standing upright upon it] will make right angles — accordingly PE, perpendicular to OL, will be the meridian line. But because for the most part the straight lines MF, NI cut the lines HG, HK very obliquely, the points O, L (at which MF, NI meet the lines HG, HK) must be found by Lemma 17 of Book 1, especially by the last method handed down there.

Diagram. A circle inscribed in a square frame, crossed by many straight lines representing shadow-lines, perpendiculars, and the constructed lines MF, NI, OL, PE of the demonstration. Readable labels by position:

- M (top left), B (top center)
- S (upper left, inside), C (just left of S) [?]
- H (left of center), R (center), E (center)
- K, A (right of center, on a line running out to the right)
- L (far right, at the end of a nearly-horizontal line outside the frame)
- F (far left)
- D, G (lower left), N (below them)
- O (bottom-left corner), Q (bottom center), P (bottom, right of Q)
- I (lower center)

3. If it should happen that two of the Sun's altitudes are equal — namely one before noon and the other after noon, as if the altitudes DF, AI were equal — the angle DEA must be bisected. The bisecting line will be the meridian line, because the Sun in those two observations had equal distances from noon, and the two verticals drawn through the Sun make equal angles with the Meridian, and so on.

4. But if it should happen that all three observed altitudes of the Sun were equal, it would be an argument that the Sun's parallel is equidistant from [parallel to] the horizon, and consequently that the pole of the world is at the upper pole of the horizon [the Zenith], and its altitude above the horizon is 90° . From which it follows that then no line in that plane can properly be a meridian line.

All three observations may also be made either before noon or after; but in that case the two points O, L will be found on the same side, differing so little from one another that the straight line OL cannot easily be drawn without error. For which reason the matter will be carried out more exactly if one observation be made after noon and two before noon, or one before noon and two after, as we have said.

[Margin: *Instrumentum quo simul vmbra, & altitudo Solis deprehendetur.* — An instrument by which the shadow and the Sun's altitude are caught at once.]

5. But since in every observation the Sun's altitude must be taken immediately, so that no delay be interposed between observing the shadow and taking the Sun's altitude, we shall construct, together with Pedro Nunes [Petrus Nonius], in Book 2 of the *On Navigation*, Chapter 6, an instrument by which, at one and the same operation, both the shadow and the altitude of the Sun may be observed — namely, in this way. On some square flat tablet ABCD, let a quadrant BF be described from E, and let it be divided into 90 degrees, beginning at B; and through F let FH be drawn parallel to the side CD of the square. And upon the semidiameter EF let there stand, at right angles to the square tablet itself, a set-square, that is, a right-angled triangle EFG whose two sides EF, FG are equal, with hypotenuse EG. This triangle can be fitted so that it may be lowered and raised, yet so that, when raised, it is always upright to the square ABCD. And that the instrument may be made less heavy or cumbersome, the superfluous parts within the quadrant EBF, and outside it, must be cut away; likewise the interior parts of the triangle EFG — so that only the arc BF, the straight line FH, and the hypotenuse EG are left untouched. Alongside the side GF of the triangle a thread with a plumb-bob may also be hung, so that the plane on which the instrument is to be set for the observation may easily — or at least the square ABCD itself — be set parallel to the horizon.

[Margin: *Vsus instrumenti.* — The use of the instrument.]

The **use** of this instrument is as follows. The instrument being placed in a horizontal plane (which will only then have been achieved, when the thread [of the plumb-line]...

[when the thread] of the plumb-line lies flush against the side GF), and the triangle EFG being turned this way and that toward the Sun, let it be rotated until the shadow of the side FG — which makes right angles with the square ABCD — falls exactly on the straight line FH. For then the straight line drawn parallel to CD, described near the side CD in the plane on which the instrument is set, will indicate the shadow; while the shadow EH, cast from the hypotenuse EG, will cut off the arc BI of the Sun's altitude above the horizon. For since the side FG is upright to the tablet ABCD, the angle GFH will, by Definition 3 of Book 11 of Euclid, be a right angle. Since therefore the two sides GF, FH of the triangle FGH are equal to the two sides EF, FH of the triangle EFH, and they contain right angles, [**a** — Euclid I.4 (4. *primi*)] the bases GH, EH will be equal, and [so will] the angles GHF, EHF. Now GHF is the angle of the Sun's altitude above the horizon, since the lines HF, HG, produced, intercept in the vertical drawn through the Sun the arc between the Sun and the horizon. Therefore EHF too will be the angle of the Sun's altitude; [**b** — Euclid I.29 (29. *pri.*)] and since the alternate angle BEH, at the center, is equal to it, BEH also will be the angle of the Sun's altitude, and therefore the arc BI will measure the same altitude — which was the thing proposed.

Diagram. The observing instrument just described: a square tablet with an inscribed right-angled set-square and a graduated quarter-circle. Readable labels:

- Outer/large right-angled triangle with apex **G** at the upper right; the lines EG (hypotenuse), GF, GH, and EH are shown.
- Inner square tablet with corners **A** (upper left), **D** (upper right), **B** (lower left), **C** (lower right).
- **E** (left side of inner square), **F** (right side).
- **H** (below F, lower right); **I** (center, where the shadow-lines cross).
- A quarter-circle arc from B up to F, graduated by tens and labelled **10, 20, 30, 40, 50, 60, 70, 80, 90** (beginning at B near the foot, running up to F).

[Margin: 4. *primi*. (Euclid I.4) · 29. *pri.* (Euclid I.29)]

The instrument, however, ought not to be too large, because the tip of the shadow EH, cast from the hypotenuse, would nearly vanish on the plane ABCD, owing to the excessive distance of the hypotenuse from that plane; but it should have a certain moderate size, so that the tip of the shadow can easily be discerned — a thing which practice and experience will teach you.

CANON XIII.

To determine the altitude of the pole of any town or place — that is, its latitude, or distance from the Equator — and its longitude, that is, its distance from the Fortunate Isles.

[Margin: *Altitudinem poli reperire per vnam obseruationem, quando declinatio Solis, & situs lineae meridianae dantur.* — To find the pole's altitude by a single observation, when the Sun's declination and the position of the meridian line are given.]

1. When the Sun's declination on the day the pole's altitude is sought is known, and the position of the meridian line is known, we shall find the pole's altitude by a single observation in this way. In a plane parallel to the horizon let the circle ABCD be described from center E, with the meridian line BD extended through the center. The shadow FG having been observed, and the Sun's altitude FN, let the perpendicular EH be erected to FG; and the radius HN being drawn to cut FG at O, the place of the Sun at the time of observation, let there be counted the complement of the declination — when the Sun is northern, or [when it is] southern, the arc made up of the quadrant and the declination — from the point N on both sides, up to P, Q; and the radii HP, HQ being drawn to cut FG at S, R, let there be described about RS, from its midpoint T, the circle SIR, which represents in the sphere the parallel described around the Sun's center at the interval of the complement of the declination, and which therefore passes through the pole of the world. For where this circle, on the northern side, intersects the meridian line — as at I, the point which, for us who are placed [in the plane] with F between the Sun and the center E, lies to the right if the observation is made before noon, or to the left if the observation is made after noon — there will be the apparent north pole.

Therefore, having drawn AC perpendicular to the meridian line as a diameter, if a radius AIf be drawn from A through I, the seen pole, the arc Df will be the pole's altitude, since to it corresponds the seen arc ID of the Meridian, between the horizon and the pole; and the arc fC [will be] the complement of the pole's altitude, since to it corresponds the arc EI of the Meridian, which appears between the vertex [Zenith] and the pole — as is clear from what was demonstrated in Book 2, Proposition 1.

Diagram. The same dense analemma-type projection as fig-p658-1 (p. 306), repeated here to accompany Canon XIII: a circular projection in a square frame, crossed by many straight lines and containing horizon, equator, parallels,

meridian and hour-circles. The readable labels match the earlier figure — B, L, Q along the top; K, A, G at the right; S, F, T, E, R, M, P, I toward the center; H, D below; and I, C as the end-labels of two lines running out to the left, with a small u [?] in a box at the foot.

If it should happen that the circle described around the Sun, at the interval of the complement of the declination, touches [is tangent to] the meridian line — which can happen only at the sixth hour, before or after noon (as, for instance, if the shadow had been ab and the Sun's altitude ae ; Eg erected perpendicular to ab , and the radius ge drawn to cut ab at f , the complement of the declination would have to be counted from e up to m, n , so that the radii gm, on would cut off the diameter il , [and] the circles hi , whose center is k , touching the meridian line at D) — then the very point of contact will be the north pole. For since any circle described through the pole from any point of the six-o'clock circle touches the meridian at the pole (because the six-o'clock circle is upright to the meridian), it comes about that a circle described from the Sun's center — with [a point] on the six-o'clock circle serving as pole — at the interval of the complement of the declination, touches the meridian at the pole, since it necessarily passes through the pole, on account of the interval of the complement of the declination.

It sometimes happens, when the Sun is northern, that the circle described around the Sun's center as pole, at the interval of the complement of the declination, cuts the meridian at two points beyond the vertical point, toward the north. When, therefore, the Sun's distance from the meridian is greater than six hours, the intersection [that is] less northerly will be the north pole; but if the distance is less, the more northerly intersection will be the north pole — because in the former case the [six-o'clock] circle...

CANON XIII. ...the hour circle drawn through the Sun and the pole makes with the meridian, toward the south, an obtuse angle — the very one that the great circle in the sphere, drawn through the Sun and the less-northern intersection, produces. In the latter case, by contrast, the hour circle drawn through the Sun and the pole makes with the meridian, toward the south, an acute angle — the very one that the great circle in the sphere, drawn through the Sun and the more-northern intersection, establishes. This is because two great circles drawn through the Sun and those two intersections form an isosceles triangle whose two base angles are acute. All of this is plainly evident on a material sphere.

But if it is not known whether the Sun's distance from the meridian is greater than six hours or less, another observation must be made. For the point of the meridian line at which the circle described in the later observation — drawn,

with the pole as center, at the interval of the complement of the declination — cuts the circle of the earlier observation will be the north pole. For the later circle must necessarily intersect the earlier one on the meridian, since each passes through the pole; and yet the later circle will not pass through both intersections of the earlier one with the meridian, but through one only. Otherwise there would be two straight lines in the sphere from the Sun's center in the first observation to those two intersections, equal to two straight lines drawn from the Sun's center in the second observation to those same two intersections [Margin a: Euclid, *Elements* I.7] — which is absurd. Read, if you like, chapter 13 of book 2 of Pedro Nunes' [Pedro Nunes] *De Navigatione*, where all these cases are demonstrated more fully.

2. WHEN, however, the position of the meridian line is unknown, we shall find the pole's altitude and the meridian line from the given solar declination by two observations, in this way. [Margin: To investigate the pole's altitude and the meridian line by two observations, from the Sun's declination alone once it is known.] From the two shadows a b, FG, and the altitudes of the Sun a e, FN, let the north pole I be found, at the intersection of the circles h i I, S I R, as was done in the preceding Canon, No. 4. For once the straight line IE is drawn, it will be the meridian line; and if the diameter AC is erected perpendicular to it, and from A a ray goes out through the pole I, then the arc Df will be the pole's altitude, and the arc Cf its complement, as was said a little before.
3. WHEN, finally, both the position of the meridian line and the Sun's declination are unknown, we shall find that same pole altitude, together with the Sun's declination — and therefore also with its place on the ecliptic and the position of the meridian line — by three observations, in this way. [Margin: To ascertain the pole's altitude and the meridian line by three observations, even though the Sun's declination is unknown.] From the three shadows a b, FG, VX, and the altitudes of the Sun a e, FN, VZ, let t be sought, the center of the circle described through the three centers of the Sun f, O, p, as was done in the preceding Canon, No. 5. For once the straight line t E is drawn, it will be the meridian line; and if the diameter AC is drawn perpendicular to it, and from A, through d and u — the intersections of the meridian line with the circle f O p, which represents the Sun's parallel — rays are emitted, as we said in No. 5 of the preceding Canon, the circle ABCD will be cut at q, r, the extremities of the true diameter of the Sun's parallel, represented by the apparent diameter d u — as is evident if A is placed at the Nadir and the circle ABCD is understood to

be perpendicular to the horizon. Then, having bisected the arc $q r$ at f , f will be the true pole of the world, and the ray sent out from A through f will indicate the same apparent pole at I . Therefore, as before, the arc Df will measure the pole's altitude, and the arc Cf its complement. Finally, the arc $f q$ (or $f r$) will be the complement of the Sun's declination — that is, of the Sun's parallel — whose true diameter would be the straight line $q r$, if drawn.

Diagram. A large, very densely inscribed woodcut set in a rectangular frame: an analemma-type orthographic projection for the three-observation construction of §3, showing two large overlapping circles (the meridian/horizon circle $ABCD$ and the Sun's-parallel circle $f O p$), crossed by a dense network of straight construction lines, chords, and radial rays, with a central point E . Because the labels are small, closely packed, and set against the crowded line-work, only part of them resolve cleanly even under magnification. Legible labels, by position:

- E (center): the central point (Sun's/circle center)
- d (just above E): intersection point on the meridian
- I (below E): apparent pole
- R (below I , center): construction point
- T (center-left): construction point
- f (lower-left of center): the true pole (bisection point of arc $q r$)
- q (upper-left of center): extremity of the true diameter of the Sun's parallel
- r (lower-right of center): the opposite extremity [?]
- P (left, below center): construction point
- F, c (left): points on the left circle
- S, V (upper left): points on the outer circle
- h (upper-left, outer circle)
- M, Z, B, L (across the top): points on the upper arc
- Q (upper right)
- K (upper right), with an adjacent curved symbol [δ ?] [?]
- A (right): the point placed at the Nadir
- G (right, below A)
- M (center-right, below R)
- R (far left, large): outer construction point
- a (lower-left, script): construction point
- S (lower-left): construction point
- m (lower-center, script), H (lower-center), D (bottom center)

- a planet-like symbol near D at the bottom [?]
- Y (lower-right)
- u (bottom): the apparent-diameter intersection
- I, C (far-left margin, partly cut by the gutter) [?]

SCHOLIUM.

1. IN scholium 2 of proposition 28, book 1 of the *Gnomonics* [Clavius, *Gnomonices libri VIII*] we showed how the pole's altitude may be elicited from the analemma by two observations, even if the Sun's declination is not given, provided the position of the meridian line is not unknown. [Margin: How to find the pole's altitude from the analemma by two observations, even though the Sun's declination is unknown, provided the position of the meridian line is given.] So there is no need to repeat it here, since it can be learned from that scholium. But we shall be content to explore the same pole altitude by three observations, even though neither the Sun's declination nor the position of the meridian line is known. [Margin: To ascertain the pole's altitude and the meridian line by three observations, even though the Sun's declination is unknown.]
2. THEREFORE, by three shadows DE, CE, AE on the horizon, and three altitudes of the Sun DF, CB, AI — of which two are observed before noon and the third after, or the reverse, as appears in the first figure of the preceding Canon's scholium — let OL be found, the common section of the plane of the horizon and of the Sun's parallel, as was done in No. 2 of the scholium of the preceding Canon 12. For the perpendicular PE will give the meridian line — or indeed any other perpendicular HQ. And if HR is drawn parallel to OP, or to the meridian line, and equal to HB, and the straight line QR is joined, then QRH will be the angle of the pole's altitude. For if the triangle QHR is conceived as perpendicular

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to the horizon upon the straight line HQ, the Sun's center will be at R, at the time when the shadow CE and the Sun's altitude CB were observed. Since, then, the Sun's parallel passes through OL, it will also pass through the straight line RQ, so that RQ is the common section of that same parallel and of the meridian. Therefore RQH will be the angle of the complement of the pole's altitude — namely the one that the equator and the planes of its parallels make with the horizon; and hence QRH will be the angle of the pole's altitude.

3. THE SAME pole altitude — whether northern or southern — can be found without difficulty, without drawing any figure: by day from the meridian altitude together with the known declination, and by night from the meridian altitude of any star together with its ascertained declination — provided we first teach how it may be known whether the vertex of the head (the pole of the horizon) lies between the arctic pole and the Sun (or a star) placed on the meridian, or whether, on the contrary, the Sun itself (or a star), when it occupies the meridian, lies between the arctic pole and the zenith of the place. [Margin: Whether the zenith of the place lies between the arctic pole and the Sun or star placed on the meridian, or whether, on the contrary, the Sun or star placed on the meridian lies between the arctic pole and the zenith of the place — in what way this is ascertained.] This is made plain as follows.

When it is established toward which quarter north or south lies — which the magnetized needle reveals in an instant — what is proposed is easily perceived. For if the shadows of bodies, when the Sun has its greatest altitude, are cast toward the north — or if, with us turned to face south, the greatest altitude of a star is to be observed — then the zenith of the place will be fixed between the arctic pole and the Sun or star. But if the shadows of bodies are cast toward the south — or if the greatest altitude of a star is to be observed with us turned toward the north — then the Sun or star will be fixed between the arctic pole and the zenith of the place.

But if it is unknown on which side north or south lies: with our face turned toward the Sun or star when it is near the zenith, if we see the Sun or star revolve with the heavens from east to west, from left to right, the zenith of the place will lie between the arctic pole and the Sun or star; but if from right to left, the Sun or star will be set between the arctic pole and the zenith of the place.

4. ACCORDINGLY, if the declination of the Sun or star, when it is northern, is subtracted from the quadrant intercepted between the arctic pole and the equator — or, when it is southern, is added to that same quadrant — there will remain, or be produced, the distance of the Sun or star from the arctic pole. [Margin: The pole's altitude, once found from the declination and the meridian altitude.] Then, having observed the altitude of the Sun or star several times around midday until the greatest is found, take the complement of the greatest altitude found. (If a meridian line is present, the Sun will have its greatest — that is, its meridian — altitude when the shadow of a stylus set on the meridian line is cast onto the meridian line

itself; a star, in turn, will attain its meridian or greatest altitude when it is on the meridian, which happens when a plane erected perpendicular to the horizon along the meridian line passes, if produced, through the star.) This complement is to be subtracted from the already-found distance of the Sun or star from the arctic pole, if the zenith of the place lies between the south and the arctic pole; or added to that same distance, if the heavenly body lies between the zenith of the place and the arctic pole of the world. For the number that remains, or that is produced, will indicate the distance of the zenith of the place from the arctic pole of the world.

If this distance turns out equal to a quadrant, the zenith point will be on the equator, and the pole will have no altitude above the horizon. But if it is found less than a quadrant, then, subtracting it from the quadrant, the remainder will be the altitude of the north pole; and if it comes out greater than a quadrant, then, taking the quadrant from it, the remainder will be the altitude of the south pole — as is easily understood if a material sphere is used.

If the Sun or star is found at the zenith of the place — that is, if its greatest altitude is found to be 90° — then the declination of the Sun or star will itself be the altitude of the pole above the horizon: northern if the declination is northern, southern if southern.

AGAIN, if the Sun or star in northern regions neither rises nor sets (which can happen with the Sun when it is in the northern signs, and the zenith of the place lies between the north pole and the arctic circle), it will have, within the span of 24 hours, two meridian altitudes — one greatest and the other least. From the greatest, the altitude of the arctic pole is found, as has been said; from the least, in this way. The distance of the Sun or star from the arctic pole, found as we said at the beginning of this No. 4, is added to the least altitude; for the resulting number gives the altitude of the arctic pole. In the same way, if the Sun or star in southern regions neither rises nor sets (which can happen with the Sun when it runs through the southern signs, and the zenith of the place lies between the south pole and the antarctic circle), it will have, within the span of 24 hours, two meridian altitudes — one greatest and the other least. From the greatest, the altitude of the antarctic pole is derived, as we prescribed at the beginning of this No. 4; from the least, by this method. The distance of the Sun or star from the antarctic pole — which is obtained if its distance from the arctic pole, found as taught above, is subtracted from the semicircle, or if its southern declination is subtracted from the quadrant — is joined to the least altitude; for the resulting number will exhibit the altitude of the south pole.

FINALLY, if it ever happens that the altitude of the Sun or star, over some span of time, neither increases nor decreases, the pole's altitude will amount to 90° — that is, the pole will be located at the very zenith of the place: the north pole, if the declination of the Sun or star is northern; the south pole, if southern.

5. WE shall attain the same result by another, somewhat different method, namely this. First let it be learned, roughly, where the northern part of the world lies and where the southern — which the magnetized needle readily teaches us. [Margin: Another way. Where the northern part and the southern lie — in what way it is ascertained.] But if we lack such a needle, then around midday — that is, when the Sun or star has very nearly its greatest altitude — we shall turn our face toward the Sun or star. If indeed it is seen to move from left to right, our back will face north and our face south; but if from right to left, the northern part will lie over against us, and the southern in the opposite quarter.

Once this is known, we shall observe the greatest altitude of the Sun or star. If the shadows of bodies are cast toward the same quarter to which the heavenly body declines (in the case of a star, since it casts no shadow, we shall take for its shadow the line of sight drawn from the eye to the star), then its complement, added to the declination, will make the altitude of the pole of the same name as the declination — that is, of the arctic pole if both shadow and declination are northern, but of the antarctic pole if southern. But if the shadows of bodies are cast toward the opposite quarter — that is, toward the north if the declination is southern, or toward the south if northern — then: if the complement of the greatest altitude is found equal to the declination, the zenith of the place lies under the equator, and the pole will have no altitude; but if the complement

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of the greatest altitude is found less than the declination, then, subtracting the former from the latter, the remainder will be the altitude of the pole of the same name as the declination — that is, the arctic pole if the declination is northern, the antarctic if southern. If, finally, the complement of the greatest altitude turns out greater than the declination, their difference will be the altitude of the pole of the name opposite to the declination — namely the antarctic pole if the declination is northern, the arctic if southern.

WHEN the Sun or star has no declination, the complement of the greatest altitude will give the pole altitude of the same name as the shadow — namely the arctic pole if the shadow is northern, the antarctic if southern.

WHEN, finally, the Sun or star stands at the zenith of the place, its declination, if it has any, will be the altitude of the pole of the same name as the declination — the arctic pole, that is, if the declination is northern, the antarctic if southern.

6. WHEN it is established that the arctic pole is raised above the horizon, astronomers usually investigate its altitude by this easy route. [Margin: Another and easier way, if it is established that the arctic pole is raised above the horizon.] If the Sun or star has no declination, the complement of the meridian altitude gives the altitude of the arctic pole. But if there is a northern declination and the body inclines from the zenith toward the south, the arc composed of the declination and the complement of the meridian altitude reveals the altitude of the arctic pole. If, however, the declination is southern, then, subtracting it from the complement of the meridian altitude, the remaining arc measures the altitude of the north pole. But if the body tends from the zenith of the place toward the north, the complement of the meridian altitude, subtracted from the northern declination, gives the remaining altitude of the north pole. Finally, if the Sun or star neither rises nor sets, so that it has two meridian altitudes: if at its greatest altitude it inclines from the zenith toward the north, then half the sum of both meridian altitudes indicates the altitude of the north pole; but if at its greatest altitude the body tends from the zenith toward the south, then, subtracting it from the semicircle, half the sum of the remainder and the least altitude is the altitude of the arctic pole itself.

WE shall proceed no differently in the southern regions, provided that what has been said about northern declination and the northern quarter is transferred to southern declination and the southern quarter, and vice versa.

Diagram. A woodcut set in a rectangular frame: a single large circle (the meridian) with center E, crossed by many diameters and chords, illustrating the determination of §3 — the position of the zenith of the place relative to the arctic pole and the Sun/star. Two straight lines run out well beyond the frame to the right, toward the labels L and u. Legible labels, by position:

- E (center)
- M (top left, outside the circle)
- B (top, on the circle)
- S (upper left, on the circle)
- C (small, just left/below S) [?]
- R (upper right of center)

- H (left of center)
- F (left, on the circle)
- K (right of center)
- A (right, on the circle)
- I (lower right, on the circle)
- D, G, N (lower left)
- O (lower left, on the circle)
- Q, P (bottom, on the circle)
- u (far right, at the foot of a projected line)
- L (far right, at the end of a projected line)

CANON XIV.

To ascertain, in whatever part of the world we may be — even at sea — in which Zone and climate we are situated.

1. I would in no way explain this Canon — since it contains nothing new, but requires only the finding of the pole in the place where we are — were it not set down by all the writers on the astrolabe. Once the pole's altitude, or the latitude of the place, has been found by Canon 13 or its scholium, then: [Margin: To know in which Zone a given place is located.] if it is less than $23^{\circ}30'$, the place will lie in the torrid Zone; and if it has no latitude, it will have its zenith under the equator itself — that is, it will lie in the middle of the torrid Zone. But if the latitude contains precisely $23^{\circ}30'$, the place will be located precisely under the tropic of Cancer ☿ or under the tropic of Capricorn ♎, according as the place is northern or southern — that is, it will lie at the end of the torrid Zone and at the beginning of the temperate. But if the latitude is greater than $23^{\circ}30'$ yet less than $66^{\circ}30'$, the place will be in the temperate Zone, whether northern or southern, according as it inclines toward the north or the south. But if the latitude of the place comprises precisely $66^{\circ}30'$, it will be under the arctic (or antarctic) circle — that is, it will be located at the end of the temperate Zone and at the beginning of the frigid. And if, finally, the latitude of the place is greater than $66^{\circ}30'$, its position will be found in the frigid Zone; and if the latitude contains 90° , it will have its zenith under the pole itself, and will occupy the middle of the frigid Zone.

THE SAME pole altitude, once found, will teach in which climate the place where we are is located. [Margin: To perceive in which climate a given place is located.] For if the pole altitude found is sought in the table of climates which we

described most fully at the end of chapter 3 of the *Sphaera*, according to the more recent authors: if it is found precisely, it will at once be established at the beginning, or the middle, or the end of which climate our place is situated. But if it is not found precisely, we shall understand — from the pole altitude entered in the table that differs least from our own altitude — near the beginning, or the middle, or the end of which climate we are. For example: suppose some sailor has been carried to the port of Mozambique in eastern Africa. Since the southern latitude is found to be roughly 15° , we shall say that he is near the middle of the first southern climate, since climate 1 has, at its middle, a southern pole altitude of $16^\circ 43'$. Again, suppose someone has been carried to the Orkney Isles beyond Scotland. Since the latitude of those islands comprises about $61^\circ 50'$, we shall declare that they lie in the 13th northern climate, and indeed near its end, and hence next to the beginning of climate 14 — since the pole altitude at the end of climate 13 and the beginning of 14 comprises $61^\circ 53'$.

CANON XV.

To measure the distance of any two cities on the earth, or of two stars in the heavens, whose longitudes and latitudes are known — that is, to investigate the arc of a great circle described through them.

This distance is to be taken along the arc of the great circle intercepted between the two places on the earth, or between the two stars, because that arc is smaller than all arcs of non-great circles described through the same places, as has been demonstrated in the *Cosmographia*. [Margin: To find out the wayfaring distance of two places on the earth situated under the equator. To measure the distance of two places of the same longitude.]

1. WHEN, therefore, two places lie under the equator — that is, they have no latitude — then, subtracting the lesser longitude from the greater, the remainder will be the difference of longitude, and this same will measure the distance sought.
2. WHEN, however, two places have the same longitude — that is, they lie under the same semicircle of the meridian, intercepted between the two poles of the world — and each inclines toward the north, or toward the south: then, subtracting the lesser latitude from the greater, the remainder will be the difference of latitudes, and this same will measure the distance sought. But if one of the places inclines toward the north and the other toward the south, then, adding the one latitude to the other, there will be produced the arc of the meridian that measures the distance sought.

Finally, if one of the places is under the equator, and the other inclines either toward the north or toward the south, then the latitude of the latter place will itself measure the distance that is desired.

3. WHEN

(Continuation of Canon XV — the great-circle distance between two places or stars.)

[Margin: To find the distance of two places whose difference of longitude is 180 degrees.]

3. When two places have a difference of longitude of 180 degrees — that is, they lie under two different semicircles of the same meridian — and each extends toward the north (or each toward the south), then subtract the sum of the two latitudes from a semicircle; the remainder will be the arc of the meridian that measures the distance we are seeking. But if one of the places inclines toward the north and the other toward the south from the equator, then the difference of the latitudes subtracted from a semicircle leaves the arc of the meridian that will measure the required distance. Alternatively, the arc of the meridian formed from the latitude of one of the two places, the complement of the latitude of the other place, and the quadrant that lies between the pole and the equator, will measure the desired distance, provided that it is less than a semicircle; but if it exceeds a semicircle, then when this arc is subtracted from the whole circle, the remaining arc measures the distance of the places. Finally, if one of the two places lies on the equator, the latitude of the other subtracted from a semicircle leaves the arc of the meridian that will measure the distance we are inquiring after.

[Margin: To investigate the distance of two places of different longitudes and different latitudes.]

4. When, finally, the two places stand in none of the aforesaid conditions — whether one of the two lies on the equator or neither does, and whether they have the same latitude or not — we shall explore their distance in this way. On the astrolabe, let the equator be ABCD, with center E, and let the two diameters that cut each other at right angles be AC and BD, of which let AC represent the meridian drawn through the Fortunate Isles [insulas Fortunatas], from which the longitudes of places begin. Suppose that there are two given places: of the first, let the longitude be 60 degrees and the northern latitude 30 degrees; of the second, let the longitude comprise 150 degrees and the northern latitude 60 degrees. Reckon the longitudes off from A toward B — that is, from west toward east — as far as F and G, and draw the diameters FE and GE, which represent the meridians

passing through the given places. Again, count the latitudes off from B as far as L and G [recte: as far as L for the one and to the corresponding point for the other]; then, drawing the radii AL and AG, which cut BE in M and N, describe from E, through M and N, the parallels of latitude that cut the meridians FE and GE in P and I. Then P will be the position of the first place, and I of the second. If, therefore, by Book 2, Proposition 13, a greatest circle is described through the places P and I, the arc PI will measure their distance. So, when the pole O of that circle has been found — as we taught in Book 2, Proposition 8, Number 17 — the straight lines OP and OI, drawn out, will cut off from the equator an arc QR equal to the arc PI. Therefore, however many degrees are contained in the arc QR, by that many degrees will the one place be distant from the other.

Now we shall describe the greatest circle through P and I, and find its pole, as follows. Draw the straight line EK perpendicular to FE. (A perpendicular could also have been drawn to GE, but it is preferable to choose the line FE, since P is the point drawn farther from the center E; for in that way the point opposite P will be less distant from the center than the point opposite I.) From K, through P, draw the straight line KPB, to which let the perpendicular KD be raised — which will be done if we take the arc gD equal to the arc FB, and so on — cutting FE produced in H. Then the point H will be opposite P, as is clear from what we wrote in Book 2, Proposition 6, Number 13. If, therefore, a circle is described through the three points P, I, H from center X (which will lie on the line fX that cuts PH in f, bisecting it and at right angles), that circle will be the greatest, since it passes through P and H, which are diametrically opposite. Now, from center X, through E, draw the straight line XE, cutting the described circle in c, and to XE erect a perpendicular — or, what is the same thing, join the straight line YZ. (For this YZ will be perpendicular to XE: it will pass through the center E, since it is a diameter of the greatest circles (a) that cut each other in Y and Z into two equal parts. Therefore the line XE will bisect YZ at the center E, (b) and hence also at right angles.) From Y, through c, let a straight line be sent out cutting the equator in T, and take the arc TV equal to a quadrant. (The quadrant TV is to be taken toward that side toward which the radius YV, drawn out, cuts the line XE within the equator.) For the radius YV will cut the line XE — which represents the meridian of the circle PIH — in O, the pole of the circle PIH, as we demonstrated in Book 2, Proposition 8, Number 17.

[Margin: (a) "11.1. The." — Theodosius I.11 (prop. 11 of the first book). (b) "3. tert." — Euclid III.3 (prop. 3 of the third book).]

[Margin: Another way, even if a greatest circle is not described through the given places.]

5. We shall learn this same distance more briefly, even without describing a greatest circle through the given places, and so on — if, after drawing the straight line PI, we inquire, by what has been handed down by us in Book 2, Proposition 18, Number 3, how great an arc of a greatest circle that chord subtends. This will be done as follows. Having found the point H, which is opposite P (the point farther from the center E), join the straight line HI, and let the angle PIH be bisected by the straight line Ia, which cuts PH in a — the point through which the non-greatest circle passing through the point I, about the pole P, would have to be described, as we showed in Book 2, Proposition 18, Number 3. So the arc Pa of the greatest circle PEH, drawn through the pole E, is equal to the arc of the greatest circle described through P and I, intercepted between P and I, since both fall from the pole P onto the circumference of the non-greatest circle described through a and I about the pole P. Therefore, when EK is erected perpendicular to PH, the radii KP and Ka will cut off from the equator an arc BS of as many degrees as the arc subtended by the straight line PI comprises; and this arc BS will be equal to the former arc QR, which we found, if there is no error.

6. Again, let there be a place whose longitude is 150 degrees and northern latitude 60 degrees, and another place whose longitude is 240 degrees and southern latitude 30 degrees. When the longitudes have been counted off from A toward B as far as G and g, the straight lines GE and gE will have been drawn as the meridians of the given places. Then, taking the northern latitude BG of the first place, and sending out the radius AG, which cuts BD in N, describe from E, through N, the parallel of that latitude, cutting the meridian GE in I; and I will be the position of the first place. And if the southern altitude Dd of the second place is taken, and the radius Ad is sent out, cutting BD in b, and finally from E, through b, a parallel of this latitude is described, cutting the meridian gE in H, then the position of the second place will be at H. Therefore, if a greatest circle is described through I and H (having first, of course, found the point p opposite H, and so on), and its pole is found to be O, then the radii sent out from O through I and H will give on the equator an arc Re equal to the arc IH, which measures the distance of the places I and H.

Or, more briefly, as in Number 5, we shall proceed thus, without describing a circle through the places I and H. Having found the point P opposite H, and drawing the straight lines HI and PI, let the angle PIH be bisected by the straight line Ia, which cuts PH in a, the point through which

[Continuation:] the non-greatest circle passing through the point I, about the pole H, would have to be described, as we showed in Book 2, Proposition 18, Number 3. So the arc Ha of the meridian HP is equal to the arc of the greatest circle described through H and I, intercepted between the places H and I, since both fall from the pole H onto the circumference of the non-greatest circle described through a and I about the pole H. Therefore, when EK is erected perpendicular to HP, the radii KH and Ka will cut off from the equator an arc DS of as many degrees as are contained in the arc Ha, and hence also in the arc of the greatest circle subtended by the straight line HI; and this arc, if there is no error, will be exactly equal to the former arc eR that we found.

By this technique you will find the distance of any two given points on the sphere — the distance that the arc of the greatest circle described through them measures — whether both incline northward from the equator, or southward, and whether one tends north and the other south, and whether both lie on the same parallel of the equator or not, and whether, finally, one lies on the equator ABCD while the other departs from it either north or south.

[Margin: How the distance between a northern and a southern place may more conveniently be found.]

7. Since, however, southern places are described on the astrolabe less exactly than northern ones — because the semidiameters of the southern parallels are found by radii sent out from A, which cut the line BD very obliquely — when one of the places is southern and the other northern, the business will be carried out most conveniently if, in place of the southern place, one takes the northern place diametrically opposite to it (the one, that is, inhabited by the Antipodes), whose northern latitude is equal to the southern latitude of the other, but whose longitude differs from that longitude by a semicircle. Thus, if the longitude of the southern place is less than a semicircle, a semicircle must be added to it; but if greater, a semicircle must be taken away from it, so that the longitude of the opposite northern place is either made up or left over. For if the distance found between the given northern place and this other northern place — opposite the southern one — is subtracted from a semicircle, the remainder will be the distance of the given northern place from the given southern place.

For example: Let there be given a northern place I, whose longitude contains 150 degrees and whose latitude is 60 degrees; and a southern place whose longitude is 240 degrees and whose latitude is 30 degrees. For this southern place we shall take a northern place P, whose longitude is 60 degrees (which is what remains when a semicircle is subtracted from the given longitude of 240 degrees, which is

greater than a semicircle), but whose latitude is 30 degrees, the same as the southern place. For if the distance found between the places I and P is subtracted from a semicircle, the remainder will be the distance of the place I from the southern place opposite to the place P. For since the greatest circle described on the sphere through the places P and I necessarily passes through opposite places, and the place P is distant from its opposite place by a semicircle, it plainly follows that when the arc of that greatest circle set between P and I (that is, the distance between the places P and I) is taken away from the semicircle, it leaves the arc of the same greatest circle set between the place I and the southern place opposite to the place P — the arc which measures the distance of the place I from that southern place. So you see in the figure the arc PI, subtracted from the semicircle PIH, leaving the arc IH. And if the given southern place has a longitude of 40 degrees and a latitude of 50 degrees, then the northern place to be taken will be one whose latitude is likewise 50 degrees, but whose longitude is 220 degrees — which is made up from the longitude of 40 degrees of the southern place (a longitude less than a semicircle) and a semicircle.

[Margin: How the distance between two southern places should be sought from the opposite northern places.]

In like manner, if the distance of two southern places is to be investigated, one must find the distance of two northern places opposite to them — places having, that is, the same latitudes as they, but longitudes differing from their longitudes by a semicircle. These longitudes are obtained if a semicircle is either added to them (namely if the given longitudes are less than a semicircle) or, if they are greater than a semicircle, subtracted from them, as was said just a little before. For this distance, once found, will be exactly equal to the distance of the given southern places.

Or else, on the astrolabe, the center E must be taken as the southern pole, in such a way that the eye is placed at the northern pole. For in this way the astrolabe, between the equator and the center, will represent the southern hemisphere; and in it all the southern places will be described, if their longitudes — as noted by the geographers — are counted off from A toward B, and the latitudes from B toward C, so that the parallels of southern latitude are described within the equator, just as before the parallels of northern latitude were. This is what we advised at the end of Book 2.

[Margin: To investigate the distance of any two stars.]

8. The distances of the fixed stars will be investigated by exactly the same method. For if on the astrolabe the positions of any two proposed stars are found — as we taught in Book 2, Proposition 11, Numbers 2, 3, and 4 — and through those positions a greatest circle is described, we shall learn the magnitude of the arc intercepted between those positions, by means of radii sent out from its pole through the extreme points, that is, through those very positions. Or, if on the straight line drawn from the star more remote from the center of the astrolabe through the center, a point is found opposite to that same more remote star, we shall learn the arc whose chord is the straight line set between those same stars, as we handed down in Book 2, Proposition 18, Number 3, and as an example was set out just now in Number 5 concerning the line PI, and in Number 6 concerning the line HI. Finally, just as the distance of two places on the earth, so also the distance of two stars in the heavens — if their positions are found on the astrolabe, as we handed down in Book 2, Proposition 11 — is to be inquired after.

But, to find the positions of the stars more easily for the purpose of eliciting their distances, we shall assume, in the figure of this Canon, that the circle ABCD is not the equator but the ecliptic, and that its northern pole is E; so that we describe the circles of the sphere on the plane of the ecliptic in the form in which they appear from its southern pole. For in this way the longitude-circles of the stars, passing through the poles of the ecliptic, will be projected as straight lines drawn through the center E; and the parallels of the same ecliptic, drawn through the stars, will be described on the astrolabe from the center E, like the parallels of the equator. From which it follows that the position of any star can be found on the astrolabe through its longitude and latitude, no otherwise than a place on earth was found on it above. For if, say, some star has a longitude of 60 degrees from the first star of Aries, and a northern latitude of 30 degrees, we shall count off its longitude from A toward B as far as F.

[Continuation:] For the straight line FE will be the circle of its longitude. Then we shall reckon its northern latitude off from B as far as L, so that by the radius AL the semidiameter EM of the parallel passing through the star is cut off. For this parallel, described from E through M, will cut FE in P, the position of the star. By the same method the point I will be found — the position of a star having a longitude of 150 degrees from the first star of Aries and a northern latitude of 60 degrees — and so for the rest.

Therefore the distance of the star P from the star I will be found just as if P and I were places described on the earth. But if, of the two stars, one has a southern latitude, we shall find the distance between the point opposite to it and the other,

northern, star, and we shall take this away from the semicircle, so that the distance between those two stars remains — just as we said above concerning two places on the earth, one of which is northern and the other southern. The point that is set opposite to the star of southern latitude will have a northern latitude equal to it, but a longitude equal to that which is made up either by adding a semicircle to the longitude of the southern star, or which is left after subtracting a semicircle (if it can be subtracted), as was said concerning the places on the earth in Number 7. So too, if two stars of southern latitude are presented, we shall track down the distance of the two opposite points; for this will be equal to the distance between the two given stars.

[Margin: When the other place, or star, is southern, to find the same distance, even if its opposite point is not assumed.]

But in the scholium of Canon 22 [Canonis 22 — Arabic numeral, forward cross-reference] we shall investigate this same distance even if one of the places, or one of the stars, is southern — where, namely, we shall teach how, from two given sides of a spherical triangle together with the angle contained by them, the third side may be elicited on the astrolabe without a calculation of sines; so that it is not necessary to take the place diametrically opposite to the southern place or star.

[Margin: To examine the distance of two places on the earth by means of the analemma.]

SCHOLIVM

1. Besides that method of Francesco Maurolico, Abbot [Francisci Maurolyci Abbatis — Francesco Maurolico], for investigating the distance of any two places by means of the analemma [ex Analemmate] — which we set out in Chapter 2 of the *Sphere*, when we were treating of the offices of the meridian circle, and confirmed by geometrical demonstrations (which method is indeed the easiest and most exact) — we shall offer here two others, nearly as easy, which Pedro Nunes [Petrus Nonius] suggests in Book 2 of his *On Navigation* [de Nauigatione], Chapter 20. But, to demonstrate the former, it must first be shown that the chords of the arcs of two parallels between two meridians are parallel, and hence that, together with the chords of equal arcs of the same meridians which the said parallels cut off, they form a quadrilateral figure lying in one plane.

For let two meridians ABC, ADC cut each other at the poles A and C, and let the straight line BD be the chord of the arc of the equator between those meridians;

let FG and HI be the chords of the arcs of the parallels between the same meridians; and let FH and GI be the chords of the equal arcs which the parallels cut off. For that the arcs FH and GI are equal (a) is evident. I say that HI and FG are parallel, and so on. For let AC be the axis and E the center of the sphere; and, having taken the arc BN equal to the arc BF, let the straight line FN be joined. And since the remaining arcs of the quadrants FA and NC are likewise equal, by the scholium of Proposition 27 of Euclid Book 3, AC and FN will be parallel. (b) Therefore, when the semidiameter of the sphere FE is drawn, the angles AEF and EFO are equal to two right angles; and hence AEF and EFH are less than two right angles. The straight lines EA and FH will therefore meet outside the sphere at K. By the same reasoning one will show that the straight line GI meets the same axis EA, produced, at some point, which I say is the same point K. For, when the semidiameter of the sphere GE is joined, (c) the angles AEF and AEG, standing at the center on the equal arcs AF and AG, will be equal; and likewise the angles EFH and EGI, standing on the circumferences, will be equal, standing on equal arcs — the arcs, namely, that are left over if the equal arcs FH and GI are subtracted from the semicircles of the meridians which the semidiameters FE and GE, produced, cut off. Since, therefore, the sides EF and GE adjacent to them are also equal, (d) the remaining sides FK and EK of the triangle EFK will also be equal to the remaining sides of the triangle whose base is GE and whose sides are the straight lines drawn from the point E through A, and from the point G through I, extended to their meeting. Therefore EA and GI meet in K, since the side EK of the triangle EFK is equal to the side of the other triangle from E to the meeting of the straight lines EA and GI. So the triangle is KFG, (e) and hence it lies in one plane; and therefore the straight lines FG and HI will also be in one plane, namely in the plane of the triangle KFG. (f) From which it follows that these same straight lines FG and HI are parallel — being, that is, the common sections in the plane FGIH made by the planes of the parallels of the equator, which are parallel. This will also be shown in this way: since the sides KF and KG of the triangle KFG are cut proportionally (g) (because the chords FH and GI are equal, and hence also the remaining lines HK and IK), (h) FG and HI will be parallel.

The very same demonstration will hold if the parallels, whose chords are FG and HI, incline toward different poles, provided they are not equal

[Margin: (a) Theodosius II.10. (b) Euclid I.29 (primi). (c) Euclid III.27 (tertii). (d) Euclid I.26 (primi). (e) Euclid XI.2 (undecimi). (f) Euclid XI.16 (undecimi). (g) Euclid III.29 (tertii). (h) Euclid VI.2 (sexti).]

Diagram. A large square-framed woodcut occupying the middle of the page. It shows the stereographic construction of Canon XV, Number 4 (the great-circle distance between two places of different longitude and latitude). Within an outer square is the astrolabe's equator, a circle with two perpendicular diameters: A at the right and C at the left (horizontal diameter AC, the meridian through the Fortunate Isles), B at the top and D at the bottom (vertical diameter BD), with center E. A large tilted oval represents the greatest circle PIH through the two places, together with numerous meridian lines (FE, GE), parallels of latitude, and radii (AL, AG, KP, etc.); the described greatest circle is centered at X (lower right). Readable point-labels: A (right), B (top), C (left and a second inner c near center), D (bottom), E (center), F (upper right), G (upper left), H (lower left), K (lower right), L (upper left), M (upper center), N (center), O (lower center, pole), S (left), T (lower left), X (lower right), Y (upper right), Z (lower left); and lowercase b (bottom center), d (lower right), e (near D at bottom), f (lower center), and a delta δ (lower left). The construction points P, I (the two places), Q, R (arc on the equator), V, and a are named in the text but sit among the dense line-work and cannot all be individually resolved on the scan. More than two peripheral labels are unresolved.

Diagram. The same greatest-circle-distance woodcut of Canon XV reprinted (right side of the page, beside the text of Numbers 6–8), with the same construction and the same point-labels as fig-p664-1 (A, B, C, D, E, F, G, H, K, L, M, N, O, S, T, X, Y, Z; lowercase b, c, d, e, f, δ). Reprinted so the reader need not turn back. Small interior labels not fully resolvable

Diagram. The same greatest-circle-distance woodcut of Canon XV reprinted a third time (top of the page), identical in construction and labelling to fig-p664-1 and fig-p665-1. Small interior labels not fully resolvable

Diagram. A second square-framed woodcut, illustrating the Scholium (Number 1), the proof that the chords of the arcs of two parallels between two meridians are parallel. It shows, in the upper part, two spherical figures side by side: a left sphere (two meridians ABC, ADC crossing at poles, with the axis AC, center E, and points K at the top, H, I, O, F, B, N, M, G, P, and C at the bottom) and a right sphere (with A at top, H, I, E at center, D at right, G and C below). Below them is a large analemma-style semicircular construction resting on a horizontal base line. Readable labels in the lower construction include: V, X, d, h, f, R, k, p, T, Y, P, S, and the base-line letters c, e, i, and (Greek-letter geometric labels) δ , β , α , η near the center and base, with m and Z toward the right. This figure carries many small and Greek point-labels in dense line-work; several (in the lower analemma

especially) cannot be individually resolved on the scan. ...are equally distant from the equator. For example, suppose the chord of the southern parallel is Nu and that of the northern parallel HI, and that point N is less distant from point B than point H is. Then, taking an arc BF equal to BN itself, the lines FN and AC will again be parallel, by the scholium to Euclid III.27, because the arcs AF and CN are equal. Therefore, once the radius of the sphere NE has been joined, the two angles AEN and ENF will be equal to two right angles [Margin: Euclid I.29]; and so EA and NH, extended toward H, will meet. By the same reasoning uI will meet EA, and indeed at the same point as the line NH, on account of the equal triangles [Margin: Euclid III.27]. For here too the angles AEN and AEu, standing at the center on the equal arcs AN, Au, are equal, as are the angles ENH and EuI, standing on the equal arcs of the circumferences that remain when the equal arcs NH, uI are subtracted from the semicircles of the meridians cut off by the radii NE, uE when produced. And so on.

But if the parallel drawn through Nu lies farther from the equator (drawn through BD) than the parallel drawn through HI, then the lines HN and Iu will meet the axis AC when it is produced toward C.

If, however, the parallels drawn through FG and HI lie at equal distances from the equator (drawn through BD), as in the second figure, we shall show that HFGI is a rectangular parallelogram lying in a single plane. For the lines HF and AC will be parallel, because the arcs AH and CF are equal, and likewise IG and AC, because the arcs AI and CG are equal, by the scholium to Euclid III.27 [Margin: Euclid XI.9]; and therefore HF and IG are also parallel to each other [Margin: Euclid XI.7], and in the same plane with them [Margin: Euclid XI.16]. HI and FG too are in the same plane with them, and indeed parallel to each other, since they are the common sections in the plane HFGI made by the parallel planes of the parallels of the equator [Margin: Euclid I.33]; or because they join the parallel lines HF and IG [Margin: Euclid III.29], which are equal on account of the equality of the arcs FH and GI. Therefore HFGI is a parallelogram lying in one plane [Margin: Euclid XI.10 [?]; Theodosius]. And since the axis AC is perpendicular to the planes of the parallels drawn through FG and HI, and passes through their centers and through the center of the sphere [Margin: Euclid XI.8], the lines HF and IG, being parallel to the axis, will also be perpendicular to those same planes, and therefore, lying in those same planes, perpendicular to the lines FG and HI as well, by Euclid XI, def. 3. Therefore the parallelogram HFGI is rectangular.

2. Having demonstrated these things, we shall by this method investigate the distance of one place from another. [Margin: Another way of finding the distance

of places by means of the analemma.] Let PQR be a meridian, PR the diameter of the equator, and QS the axis of the world; and let there first be two places, both northern or both southern, one of latitude $PT = 20^\circ$, the other $PV = 60^\circ$. Let TY and VZ be the diameters of the parallels drawn through those places, and let PX be the difference of longitude — that is, let the arc PX be equal to the arc of the equator lying between the meridians of the places — and let it contain, say, 50° . When this difference is greater than a semicircle, its complement to the whole circle must be taken: for instance, if it contains 310° , then 50° is to be taken as the difference of longitude, or rather as the arc of the equator intercepted between the meridians drawn through the given places.

Now draw the line SX, and from center S, at a radius equal to one of the two semidiameters — say $\beta\Gamma$ — describe an arc cd, which, being similar to the arc PX, will be equal to the arc of the parallel of diameter TY lying between the meridians of the given places; and once the line cd is joined, it will be the chord of that same arc. If the difference of longitude were greater than a quadrant — namely the arc RX — then the arc of the parallel would have to be described from the radius SR up to the line SX, and the line drawn from point d to the intersection of the parallel with the radius SR would be the chord of the arc of the parallel lying between the meridians. After this, through the points T, V — or, as has been done here, through the points Y, Z — draw a line cutting the axis SQ, produced to q; from Y, at a radius equal to the chord cd, describe an arc, and let it be cut at α by another arc described from q at radius qY; and join the line αZ , which I say is the chord of the arc that measures the required distance of the places — so that when a line Rm equal to αZ is applied [to the circle], the arc Rm measures the said distance. [Margin: Euclid XI.10 [?]; Theodosius] For since the axis QS is perpendicular to the plane of the parallel of diameter TY at its center, all the angles it makes with the radii are right angles, by Euclid XI, def. 3. Therefore the two sides $q\beta$ and $\beta\Gamma$ of the triangle $q\beta\Gamma$ are equal to the two sides of any triangle whose one side is $q\beta$ and the other any radius of the parallel going out from β . And since they contain equal angles too, namely right angles, as has been shown [Margin: Euclid I.4], the bases too will be equal — namely qY and the line drawn from q to the circumference of the parallel, that is, to the point that terminates the radius of the parallel taken as the latter side of the triangle. By the same reasoning all the lines drawn from q to that same circumference will be shown equal to qY, and hence equal to one another. Therefore, if the triangle $q\alpha Y$ is imagined to revolve about qY, the point α will at length fall — because of the equality of the lines $q\alpha$ and qY — onto the circumference of the parallel; and $Y\alpha$ will be the chord subtending the proposed arc of that same parallel between the two meridians of the places, because it was taken equal to cd. Hence α will be the

vertex of the place through which the parallel of diameter TY is drawn. Since, then, Z is the vertex of the other place, αZ will be the chord of the arc measuring the distance of one place from the other.

By the same reasoning, if at a radius equal to the semidiameter aV an arc ef is described, and from Z an arc is drawn at a radius equal to the chord ef, cut at t by another arc described from q at radius qZ, then the line tY drawn will be the chord of the same distance; because, when the triangle qtZ is revolved about qZ, the point t falls onto the vertex of the place through which the parallel of diameter VZ is drawn. And so on.

But if one place lies toward the north and the other toward the south, then, provided their latitudes are unequal, their distance will be investigated in exactly the same way. For then too the line extended through the two points of intersection of one meridian with the diameters of the parallels will meet the axis when it is produced toward the parallel of the place of greater latitude, as was made clear in the first figure for the places whose latitudes were BH and BN, and so on.

But if the latitudes of those same places are equal, the chords of the two meridians between the parallels of the places will form, together with the chords of the parallels between those same meridians, a rectangular parallelogram, as was shown in the second figure. Therefore, if a right triangle is constructed one of whose sides about the right angle is equal to the chord of the arc of the meridian composed of the two equal latitudes, and the other equal to the chord of either of the parallels between the two meridians — which chord is found from the difference of longitude, just as the chord cd in the third figure was found from the difference of longitude PX — then it will give, as the side opposite the right angle (such as the line GH in the second figure), the chord of the required distance on the greatest circle.

Diagram. A single framed woodcut containing three related figures for the analemma method of Section 2.

First figure (upper left — the "prima figura," latitudes BH, BN): a lens/vesica shape formed by two arcs.

- K (apex, top): summit
- A (just below K): point on the axis
- H (upper left) / I (upper right): endpoints of the upper chord
- O (far left) / F, B, N (down the left side): points on the left arc / meridian

- L (mid, left of center) / M (mid, right of center): intersection points
- E (center): center
- C (lower apex): base of the vesica
- G (below C): lowest point

Second figure (upper right — the "secunda figura," the rectangle HFGI): a vesica with an inscribed quadrilateral and diagonals.

- A (apex, top)
- H (upper left) / I (upper right)
- E (center)
- D (right)
- C (lower center) / G (lower right); a small i (lower right)

Third figure (lower — the "tertia figura," analemma for the worked case): a large domed semicircle crowded with construction lines (several dotted).

- C (upper left), a, q, Z, m (across the top), Q (top)
- V, X, d, h, f (down the left); T, o (left)
- R and p (center), β and α (center-bottom)
- P (lower left corner); c, e, i, s, n (along the base); R (lower right corner); Y (right)

[Several of the small lowercase labels in the third figure are at the resolution floor]

Finally, if two places lie toward the same pole and have the same latitude, the chord of the arc of the parallel between the two meridians will be the required chord of the distance on the greatest circle.

3. However, since a line through the extreme points of the diameters of the parallels — such as the line YZ was — does not always cut the produced axis conveniently, but sometimes too far off and hence too obliquely, we shall proceed more conveniently if, in the plane of the second figure, or rather the triangle HFG, we describe [the construction] as follows, which will be done thus. Since the opposite sides HI, LM and HL, IM in the rectangular parallelogram HM are equal — HL and IM being the perpendiculars dropped from H and I to FG [Margin: Euclid I.34] — and since FH and GI are chords of equal arcs of the meridians and therefore equal [Margin: Euclid III.29], it follows that, just as the square on FH together with the squares on HL, LF, so the square on GI together with the squares on IM, MG [Margin: Euclid I.47]; the square on LF will therefore equal the

square on MG, and hence the lines FL and GM will be equal, and so each will be half the difference of the lines FG and HI. Therefore, if a right angle is made, such as QSR in the third figure, and from center S arcs cd, ef are described at a radius equal to the semidiameters $\beta\Gamma$, aV, so that the lines cd, ef are the chords of the parallels between the meridians, take cg equal to the chord ef, and bisect the remainder gd at h, so that gh (or hd) is half the difference gd of the lines cd, ef — the chord, namely, of the arc of the meridian between the two parallels TV or YZ. If then a line il is taken equal to the chord cd of the larger parallel, the triangle kil that is drawn reproduces exactly the triangle HFG, with iS being half the difference of the chords HI, FG in the first figure, the line ik [equal] to the chord FH, and the perpendicular kS [equal] to the perpendicular HL; so that, taking In equal to iS, erecting the perpendicular np equal to Sk, and joining the lines kp, pl, the trapezium kilp corresponds to the trapezium HFGI in the first figure, or to the trapezium t α YZ in the third figure.

4. Lastly, it will be permitted to explore by yet another method the distance of two places lying toward the same pole. [Margin: Another method of investigating the distance between two places, both northern or both southern.] In the following [figure] let ABC be a meridian with center D, the first place under the vertex A, the diameter of its horizon BC, the pole of the world E, and the diameter of the equator FG. Let the latitude of the second place be GH (or FI), and let HI be the diameter of the parallel of the equator drawn through its vertex, about which a semicircle HKI is described. Now count the difference of longitude from I up to K, whether it be less than a quadrant or greater, yet not greater than a semicircle. (For when the difference of longitude is greater than a semicircle, one must take for it the arc that remains when the difference of longitude is subtracted from the whole circle.) Drop to HI the perpendicular KL — namely the right sine of the difference of longitude — from which it follows that the line LI is the versed sine of the same difference. Through L draw MN parallel to BC, the diameter of the horizon of the first place; I say that the arc AM (or AN) is the distance of the given places. For if the semicircle HKI is imagined to revolve about HI until it is perpendicular to the plane of the meridian ABC, then, by Euclid XI, def. 4, the point K will fall onto the vertex of the second place, since the parallel of the equator HKI passes through that same vertex in that position, and the arc IK is the interval of the two meridians. Therefore, if a circle is understood to be drawn in the sphere through the vertex K of the second place [Margin: Theodosius I.1], and if a plane is understood to be drawn through the lines KL, MN, that plane will make in the sphere a circle whose pole is A; and hence, by the scholium to Euclid XI.18, it will be parallel to the horizon of the first place, whose diameter is BC — since both this circle, called a horizon, is perpendicular to the meridian ABC, and

their common sections with that same meridian, MN and BC, are parallel. Since then, by the definition of a pole, the pole A is equally distant from all the points of the circumference of the diameter MN, and the line drawn between A and K (KL being perpendicular to the meridian ABC) is the chord of the distance of the places, the arc AM (or AN) too will be [the arc of] the distance of the given places.

You will find the same distance even if you do not draw the parallel MN. For if, from the line HI, you cut off a line LR equal to the interval LA toward whichever side, then the line RK drawn will be the chord of the required distance. For if to the joined line AL you erect a perpendicular LQ equal to LK, then the line AQ drawn will be the chord of that distance, since, when the triangle ALQ is revolved about AL until it is perpendicular to the meridian ABC, the point Q falls onto the vertex of the second place. Since then the line AQ is equal to the line RK [Margin: Euclid I.4] — because the sides AL, LQ are equal to the sides RL, LK, and contain equal angles, namely right angles — RK too will be the chord of the required distance.

But should it happen that the perpendicular KL falls on S, the intersection of the lines BC and HI, then the distance of the places will be equal to the quadrant AB (or AC), because in that case the parallel MN does not differ from the diameter BC.

So too, when the two proposed places have the same latitude — that is, when the line HI falls onto the point A — the chord of the difference of longitude in the parallel HKI will subtend, in the meridian ABC, the arc of the distance of the places.

Diagram. A framed woodcut showing two circles, both illustrating the meridian ABC construction of Section 4 (the second differing in where the perpendicular falls).

Upper circle:

- A (top), F (upper left), I (upper right)
- N (left), B (lower left)
- P (upper center), D (center), O (lower center), L (center right)
- E (upper right), K, M (right), S (lower center), H, C (lower right), R (right, dotted)
- G (bottom)

Lower circle:

- A (top), I (upper left), F (left), R (upper center)

- E (upper right), K, Q (right)
- B (left), O, D, S (center)
- C, M (lower right), L, P (lower center), N (lower left), H (bottom right)
- G (bottom)

[Point-labels are numerous and small]

5. When one place is northern and the other southern, one must inquire into the distance between one of the places and the point diametrically opposite the other, taking for the difference of longitude (once it has already been reduced to an arc less than a semicircle, as was said in no. 4) that which remains when the difference of longitude is subtracted from the semicircle. For the distance found, subtracted from the semicircle, will leave the required distance, as was said above in no. 7 of this Canon.

6. Now we shall track down the aforesaid distance of the places by a calculation of sines, as follows. [Margin: When one place is northern and the other southern. To seek out the distance of places by sines.] Let the first figure of this scholium be repeated, in which — as in the two previous constructions — let the first place be set at H, so that its latitude is BH and its complement AH; and let the second place be at G, less northern than the first, or even southern, as in the second construction; and let the difference of longitude be the angle BAD, or the arc of the equator (or of the parallel drawn through one of the places) intercepted between the two meridians ABC, ADC, if it is less than a semicircle. For if it is greater than a semicircle — that is, if it completes the whole circle — let there be understood, drawn through the two places H, G, the arc of the greatest circle HG measuring their distance; its magnitude we shall find thus. In the spherical triangle AHG, the two sides AH, AG are given, since they are the complements of the latitudes when both places are northern or both southern — taking the point A for the arctic pole when both are northern, but for the antarctic pole when both are southern. But when one place is northern, namely H, and the other, G, southern, AH will indeed be the complement of the latitude of the northern place, but AG will be an arc composed of the quadrant AD and the southern latitude DG. Moreover, the angle HAG — comprehended by the said sides and known, since it is the difference of longitude — is the difference of longitude, or at least what remains when that difference is subtracted from the whole circle. Therefore, by problem 22 of the spherical triangles of the last Lemma, we shall find the third side HG in this way. Let it be made that as the whole sine is to the sine of the complement of the latitude of the less northern place, so the sine of the complement of the latitude of the more northern place is to another [number]:

and a certain fourth number is generated. If then it is again made that as the whole sine is to this fourth number found, so the versed sine of the angle HAG (the angle of the difference of longitude) is to another, there will be produced the difference between the versed sine of the arc by which the two given sides AH, AG differ from each other, and the versed sine of the third arc HG, which is sought. This difference, added to the versed sine of the arc by which the given sides differ from each other, will make up the versed sine of the required arc HG.

When the latitudes of the places are equal, so that an isosceles triangle AFG (or AHI) is formed, [proceed] by the first method of problem 8 of the spherical triangles. Let it be made that as the whole sine is to the sine of the complement of the latitude of either place, so the sine of half the given angle is to another: there will be produced the sine of half the required side FG (or HI). Its half having been found, the whole side will be known.

Alternatively. Let the second figure of this scholium be repeated, in which the meridian ABC is about center D; the vertex of the first place is A, the diameter of its horizon BC, the pole of the world E, the diameter of the equator FG; the latitude of the second place is GH (or FI), and HI is the diameter of the parallel of the equator drawn through its vertex, about which a semicircle of the parallel HKI is described. Now, the difference of longitude having been counted from I up to K, if it is less than a semicircle — (for if it is greater than a semicircle, its complement must be counted, which remains when it is subtracted from the whole circle, as was said in no. 4) — let the perpendicular KL be dropped from K to HI, and through L let MN be drawn parallel to BC, the diameter of the horizon of the first place. Since, if the semicircle HKI is imagined to revolve about HI until it is perpendicular to the meridian, the point K falls onto the vertex of the second place (IK being the difference of longitude between the two meridians), MN will be the diameter of the parallel of the horizon of the first place drawn through the vertex K of the second place. Since then all the points of this parallel are equally distant from their pole A, the arc AM (or AN) will be equal to the arc intercepted between the two places A, K (the semicircle HKI being perpendicular to the meridian); which arc we shall fish out in this way. Having drawn the perpendicular IO from I to BC, cutting MN at P, IO will be the sine of the arc CI in the first circle, or of the arc BI in the second circle — which is the complement of the arc AI, the arc of the difference of latitude of the two places, since the latitude of the first place is AF and that of the second IF.

Therefore, since by Lemma 5 it holds that as the whole sine of the equator is to the sine of the complement of the latitude of the second place, so the versed sine

of the difference of longitude counted in the equator is to IL — the versed sine, I say, of the difference of the same longitudes counted in the parallel HKI — in the same parts of the greatest circle in which the whole sine of the parallel is the sine of the complement of the latitude of the second place; and likewise, since by proposition 1 of our plane triangles it holds, in the right triangle IPL, that as the whole sine of the right angle P is to the sine of the angle L, the complement of the latitude of the first place — (for the complement of the latitude of the first place is the arc BF, to whose angle BDF the internal angle DSI is equal [Margin: Euclid I.29], and to this likewise the external angle ILP is equal) — so IL, in the parts of the whole sine of the greatest circle, is to IP in the same parts: the same proportion will be compounded of the proportions of the whole sine to the sine of the complement of the latitude of the second place, and of the whole sine to the sine of the complement of the latitude of the first place, which [proportion] is [compounded] of the proportions of the versed sine of the difference of longitude to IL, and of IL to IP (always taking these sines in the parts of the whole sine in the greatest circle), since these compounding proportions are equal to those compounding ones. Now the proportion of the versed sine of the difference of longitude to IP is compounded of the proportions of the same versed sine to IL, and of IL to IP. Therefore the same proportion of the versed sine of the difference of longitude to IP will be compounded of the proportions of the whole sine to the sine of the complement of the latitude of the second place, and of the whole sine to the sine of the complement of the latitude of the first place [Margin: Euclid VI.23]. Since then out of these same two proportions there is also compounded the proportion of the square of the whole sine (that is, of the rectangle contained by the whole sine and the whole sine) to the rectangle contained by the sines of the complements of the latitudes of the given places, the proportion of the square of the whole sine to the rectangle contained by the sines of the complements of the latitudes of the given places will be the same as [the proportion] of the versed sine of the difference of longitude to IP.

Wherefore, if it is made that as the square of the whole sine is to the rectangle [contained] by the sines of the complements of the latitudes of the proposed places, so the versed sine of the difference of longitude is to another, there will be produced the line IP, which we shall call the argument of the distance of the places, since by it the distance itself is elicited. [Margin: Another way of finding the distance of places by numbers.] For when the argument IP is found equal to the line IO — that is, to the sine of the complement of the difference of latitude — so that the parallel MN does not differ from the diameter BC, the distance of the places will embrace the quadrant AB (or AC). But when the argument found is less than IO, the sine of the complement of the difference of latitude, as in the first

circle: when that [IP] is subtracted from this [IO], the remainder will be PO, the sine of the arc CM, which is the complement of the distance of the places AM (or...

[The composite three-figure woodcut (first, second, and third figures — see fig-p315-1) is reprinted on this page, and the two-circle woodcut (see fig-p316-1) is likewise reprinted, so that the reader need not turn back; the labelling is identical.]

(Continuing the Scholium of Canon XV — the great-circle distance of two places, and its several methods of numerical computation.)

AM, or AN. When, finally, the argument IP is found to be greater than IO, the sine of the complement of the difference of latitudes — as in the second diagram — then, this being subtracted from that, the remainder becomes OP, the sine of arc CM, which, added to the quadrant AC, produces the distance of the places AM. And in this way the distance of two places will always be found, provided the latitude of each is northern, or [each] southern.

But when the latitude of one place is northern and of the other southern, the distance must be investigated between the northern place and the place opposite to the southern one. For this distance, subtracted from the semicircle, will yield the remaining sought distance, as was said in Num. 5.

But if the latitude of each place is the same, so that the point I falls on A, it was already said above by what means their distance is found through a spherical triangle; nevertheless we shall track it down from this same second figure in the following way. Since in that case the versed sine IL of the difference of longitudes — reckoned in the parallel of the second place — is the chord of the distance, we shall find the versed sine IL in parts of the whole sine [radius] of the greatest circle by this method. Let it be: as the whole sine of the Equator is to the whole sine of the parallel HKI (that is, to the sine of the complement of the latitude of the second, or first, place — since the latitude of each place is posited the same), so the versed sine of the difference of longitudes reckoned on the Equator, to a fourth. For IL will be produced, the versed sine of the said difference in parts of the whole sine of the greatest circle; since, by Lemma 5, the same proportion holds of the whole sine to the versed sine.

[Margin: *Another way to find the argument for the distance of places.*] Moreover, the argument IP will also become known by this other method. Let it be: as the whole sine IL is to IP, the sine of the angle ILP — the complement of the latitude of the first place (for, IL being set as the whole sine, the line IP is the sine of the angle ILP, as we said in the treatment of sines) — so IL, the versed sine of the

difference of longitudes, to a fourth. For a number will be produced that will give the line IP in parts of the whole sine of the parallel HKI, in which IL was given. Again, let it be: as the whole sine of the parallel HKI is to what has been written [i.e. the fourth term], insofar as it is the sine of the complement of the latitude of the second place in the greatest circle, so IP, now known in parts of the whole sine of the same parallel, to a fourth. For IP will be produced in parts of the same whole sine in the greatest circle, in which the sine of the complement of the latitude of the second place was taken.

[Margin: *The error of certain writers in investigating the distance of places.*] We shall explore this same distance of places by numbers, no less accurately, in the earlier figure of this scholium — once I have first exposed two errors of certain writers in the investigation of this distance. For there are some (among whom is Apianus [Peter Apian] in his *Cosmography*, and Johannes Stöffler [Ioannes Stoefflerinus] in his book on the Astrolabe) who, when two places differ in longitude alone — that is, are situated under the same parallel — teach that their distance is found when the arc of that parallel lying between the two meridians is converted into degrees of the greatest circle (a conversion of which we shall speak a little below). But they are deceived, because by this method the distance is found in the arc of the parallel reduced to degrees of the greatest circle; and this arc is larger than the arc of the greatest circle described through the same places, as we have demonstrated elsewhere — whereas it is the arc of the greatest circle that measures the true distance of places.

Then there are others who inquire into the distance of two places set under different meridians by means of a right triangle, of which one side about the right angle is the arc of the meridian of the more northern place placed between the two parallels; the other, the arc of the parallel of the less northern place enclosed between the two meridians (though this is said improperly, since arcs of parallels do not constitute a spherical triangle, even if they are reduced to degrees of the greatest circle); and the third side, or base, is the arc of the greatest circle described through the two places. A triangle of this kind is in the first drawing, and the second, of the first figure of this scholium: HFG, consisting of three arcs. For they take this triangle just as though it were rectilinear, and reason thus.

[Margin: ^a Elements I.47] The two squares on the arcs HF, FG — as if they were straight lines — taken together are equal to the square on the arc HG as [if] a straight line. Therefore, if the square root of the sum of those two squares is extracted, it will give the magnitude of the arc HG as a straight line. But indeed, by this method, for places little distant from one another — especially near the Equator — the distance will be found without an error of any moment; whereas

for places whose distance is not small, it will not. Therefore another way must be held to.

[Margin: *Werner's method for finding the distance of places.*] Johannes Werner of Nuremberg [Johann Werner], then, carries out the matter thus. The chords HI, FG of the arcs of the parallels having been reduced to parts of the diameter of the greatest circle (as we shall teach a little below), he drops from H and I, to the line FG, the perpendiculars HL, IM. The squares of the lines HF, IG — which are equal, [Margin: ^b Elements III.29] because the arcs of the meridians are equal — are equal; [Margin: ^c Elements I.47] and the square on the line HF is equal to the squares on the lines HL, LF [together], and the square on the line IG equal to the squares on the lines IM, MG [together]: those two squares will therefore also be equal to these two. Therefore, the equal squares on the lines HL, IM being removed [Margin: ^d Elements I.34] — which are equal because of the parallelogram HLMI (for it was shown in Num. 2 that the chords HI, FG are parallel; and since HI, IM are also parallel, because of the right angles at L, M, HLMI will be a parallelogram) — the remaining squares on the lines FL, GM, and hence also the sides themselves, will be equal. [Margin: ^e Elements I.28] Since therefore HI is equal to LM itself, the sum of the lines FL, GM will be the difference of the chords HI, FG, and both FL and MG [will be] half of that same difference. Now that difference is known, because the chords are known. [Margin: ^f Elements I.34] Therefore the halves will be known; and hence LG — from MG, half of the smaller chord — will be known. But HL too will become known. For when the square on the line FL, known, is removed from the square on the line HF, known, the remainder will be the square on the line HL, known. If therefore the squares on the known lines HL, LG are reduced into one sum, the square on the line HG will become known, and therefore its square root will exhibit the chord of the sought distance of the places. But because in this way too many multiplications and operations are made, we shall proceed with Pedro Nunes [Petro Nonio] far more easily, by this method.

[Margin: ^g Elements II.6] [Margin: *Pedro Nunes's method, easier than Werner's.*] The chords HI, FG having been reduced to parts of the diameter of the greatest circle, their difference is conceived as cut in two into the parts FL, GM, and to it is added in a straight line the line LM, or the smaller chord HI. Therefore the rectangle under the whole FG and the added LM (or the smaller chord HI), together with the square on the half-difference FL, will be equal to the square on the line LG, composed of the other half GM and the added LM. Therefore, the common square on the line HL being added, the rectangle [?] under FG, HI (HI being now taken for LM), together with the squares on the lines FL, LH [?]

[Margin: ^h Elements I.47] — that is, together with the square on the line FH — will be equal to the squares on the lines GI, LH, [Margin: ⁱ Elements I.47] that is, equal to the square on the line HG. Wherefore, if the rectangle contained under the chords HI, FG (reduced to parts of the diameter of the greatest circle) and the square on the chord FH — subtending the arc of the meridian between the two parallels — are gathered into one sum, there will arise the square on the chord HG subtending the sought distance; and therefore the square root of this square will render the distance itself known.

When two places have equal latitudes, but one inclines to the north and the other to the south, as in the second drawing of this figure, the distance HG is found more easily. For since, as we demonstrated in Num. 1, HIGF is a rectangular parallelogram, [Margin: ^k Elements I.47] the triangle HFG will be right-angled, [Margin: ⁱ Elements I.47] and therefore the square on the line HG will be equal to the squares on the lines HF, FG. Since therefore those two...

are known, because the sides too are known. For HF is the chord of the arc of the meridian between the two parallels, compounded from the two equal latitudes BH, BF; and the chord FG becomes known through reduction to parts of the diameter of the greatest circle; the square on the line HG too will be known, etc.

[Margin: *Reduction of the circumference of a parallel to parts of the diameter of the greatest circle.*] Now the arc of any parallel having a known declination will be reduced to degrees of the greatest circle in this way. Since the diameters of circles — [Margin: ^a Elements V.15] and therefore also the semidiameters — have the same proportion as their circumferences, as was demonstrated by Pappus [Pappus of Alexandria], and by us too in the *Practical Geometry*: if it be made, as the whole sine of the Equator is to the sine of the complement of the declination of the parallel — that is, to its semidiameter — so 360 degrees of the Equator to a fourth, there will be produced the number of degrees of the greatest circle to which the 360 degrees of the parallel are equivalent. And because similar arcs have the same proportion as the whole circumferences, if it be made, as the whole sine is to the sine of the complement of the declination of the parallel, so the degrees contained in the arc BD of the Equator (or even one degree, that is, 60 minutes) to a fourth, there will be generated the number of degrees of the Equator, or of minutes, to which the arc HI of the parallel, or one degree, is equivalent.

[Margin: *Reduction of the chord of an arc of a parallel to parts of the diameter of the greatest circle.*] With the same ease the chord of any arc of a parallel will be reduced to parts of the diameter of the greatest circle. For if it be made, as the

whole sine of the parallel is to itself insofar as it is the sine of the complement of the declination, so the chord of the given arc to a fourth, there will be produced the chord in parts of the diameter of the greatest circle, in which the whole sine of the parallel is the sine of the complement of the declination, etc.

Finally, I do not wish to pass over in silence how, just as the distance of two places was found from the second figure of this scholium, so from the same figure — and indeed by the same method — the declination of any star can be found. This we took over from Pedro Nunes, [it having been] demonstrated in our commentaries on the *Sphere*. Let the said second figure therefore be repeated, in which let the Colure of the solstices be ABC, about the centre D; the diameter of the Equator BC, and its pole A; the diameter of the Ecliptic FG, such that FA is the latitude of the pole of the world from the Ecliptic, taken as [that of] the first place. Next let two circles be conceived as drawn through the given star: one parallel to the Ecliptic, whose diameter is HI, and the other parallel to the Equator, whose diameter is MN; and IL will be the versed sine of the distance of the star from the Colure of the solstices, and FI its latitude, taken as [that of] the second place. We shall now show, as above, that the square on the whole sine has to the rectangle contained under the sine of the greatest declination (that is, under the sine of the complement of the latitude of the first place, which is equal to the greatest declination BF) and under the sine of the complement of the latitude of the star, as [of] the second place (which sine is the semidiameter of the parallel of the star's latitude, whose diameter is HI) — the same proportion as the versed sine of the said star from the Colure of the solstices reckoned on the Ecliptic (that is, IL, the versed sine of the difference of longitudes) has to the line IP, which we may also rightly call the Argument of the star's declination. [Margin: *The argument of a star's declination.*] Wherefore, if it be, as the square on the whole sine to the rectangle contained under the sine of the greatest declination and under the sine of the complement of the star's latitude, so the versed sine of the longitude of the star from the Colure of the solstices, to a fourth, there will be produced IP, the argument of declination.

From this argument IP we shall find the declination of the star BN thus. When the argument IP is found equal to the sine of the complement of the difference between the greatest declination and the complement of the star's latitude (or the difference between the complement of the greatest declination and the latitude of the star — for each difference is the same as the other, since between EA the greatest declination and EI the complement of the star's latitude the difference is AI, the same as [that] between FA the complement of the greatest declination and FI the latitude of the star) — that is, [equal to] the line IO, such that the diameter

of the parallel MN does not differ from BC — the star will lack declination. But when it is found less, subtracting it from IO, the sine of the complement of the aforesaid difference, the remainder becomes the sine OP of the star's declination, of the same denomination as the star's latitude. When, finally, the argument is found greater than the sine IO, the complement of the aforesaid difference, subtracting this from that, the remainder will be the sine OP of the star's declination, of the contrary denomination to the star's latitude. [Margin: *The declination of a star, by what means it is otherwise found, was said in the scholium of Canon 3.*] On which matter consult Proposition 6 of the book of Pedro Nunes *On Twilights* [*De Crepusculis*], where in 6 figures he has embraced every variety.

[Margin: *Another way to find the argument of longitude.* [?]] Furthermore, the longitude of the star from the Colure of the solstices is to be reckoned from the beginning of Cancer [♋], if the star's latitude is northern, and indeed according to the succession of the signs if the star is in the descending semicircle of the Ecliptic, but the contrary if in the ascending semicircle. The same longitude is to be reckoned from the beginning of Cancer [♋ [?]] — the printed symbol is small; the parallel construction suggests it may be ♊ Capricorn] when the star has southern latitude, and indeed according to the succession of the signs if the star is in the ascending semicircle, but the contrary if in the descending semicircle. For by this method the longitude of the star taken will always be less than a semicircle.

The same argument of declination IP we shall compute by this other method. Let it be: as the whole sine IL to IP, the sine of the angle ILP of the greatest declination, so IL, the versed sine of the longitude of the star from the Colure of the solstices, to a fourth. For a number will be produced that will give the line IP in parts of the whole sine of the parallel HKI, in which the aforesaid versed sine is given. Again, let it be: as the whole sine of the parallel HKI to itself, insofar as it is the sine of the complement of the star's latitude reckoned in the greatest circle, so IP, just found, to a fourth. For the argument IP will be generated in parts of the whole sine in the greatest circle, etc.

But if the star lacks latitude, its declination will be found if it be made: as the whole sine to the sine of the greatest declination, so the sine of the distance of the star from the nearest equinoctial point, to a fourth. For the number produced will be the sine of the sought declination — just as the Sun's declination is found, as we wrote in the scholium of Canon 2, at the beginning of Num. 10.

CANON XVI.

To investigate the altitude of the Sun above any greatest circle, and its Horizontal distance, for each hour.

By the *Horizontal distance* of the Sun we mean the arc of any greatest circle — [taken] in the manner of some Horizon — intercepted between its primary Vertical (that is, the point of its intersection with the Equator) and the Vertical of the same [circle] which, for the proposed hour, is drawn through the centre of the Sun.

1. Let there be, then, in the Astrolabe, the Equator A B C D about the centre E; the tropic of Cancer [☿] P, c, e; the tropic of Capricorn [♊] s, b, Q; the Horizon AF CG, and its centre H; the primary Vertical AICK, and its centre L, and the poles of the Horizon I, K. Let the given hour from midnight be counted from point D toward C; from noon, from point B toward A; the hour from setting, from point A toward D; and finally the hour from rising, from point C toward B. And let N be the terminus of the 10th hour from midnight, the 16th hour from setting, and the 4th hour from rising. The line EN, therefore, will indicate in all the parallels of the Equator the 10th hour from midnight — namely, in the tropic of Capricorn [♊] at point b, and in the tropic of Cancer [☿] at point e. And the circle QNP equal to the Horizon — described through N from the centre h, which lies on the parallel drawn through H, the centre of the Horizon, in such a way that from A toward D we meet its concave side — will cut all the parallels of the Equator at the 16th hour from setting, namely the tro-

-pic of Capricorn [♊] at Q, and the tropic of Cancer [☿] at P. Finally, the circle FNe equal to the same Horizon — described through N from the centre i, which lies on the same parallel drawn through H, the centre of the Horizon, in such a way that from C toward B we meet its convex side — will cut the same parallels of the Equator at the 4th hour from rising, namely the tropic of Capricorn [♊] at f and the tropic of Cancer [☿] at e; as is clear from what we demonstrated in book 2, proposition 9, Number 7.

Diagram. A square-framed astrolabe projection illustrating Canon XVI (the Sun's altitude and horizontal distance above any greatest circle for a given hour). Within the frame is a large bounding circle crossed by numerous intersecting circles and arcs — the Equator (ABCD about centre E), the two tropics, the Horizon AF CG (centre H), the primary Vertical AICK (centre L, poles I, K), and several further Verticals (RNIK, etc.) and hour-circles equal to the Horizon (QNP,

FNe). The many small peripheral point-labels are set in fine type on a busy ground and several cannot be resolved with confidence; the readable labels, with approximate positions, follow. **This diagram** (more than two labels flagged [?]).

- F (top center): pole of Horizon / top of meridian line FG
- G (bottom, on meridian line FG): opposite pole on meridian line
- A (left, on Equator): Equator point / node of primary Vertical
- C (right, on Equator): opposite Equator point
- B (upper region, on Equator): [?]
- D (lower region, on Equator): [?]
- E (center): centre of the Astrolabe / Equator
- H (upper center): centre of the Horizon
- L (center): centre of the primary Vertical AICK
- I (center-right): pole of the Horizon / vertex
- K (bottom center): opposite pole of the Horizon
- M (far left): centre of the Vertical RNIK
- N (lower left): hour-terminus point
- R (left): intersection of Vertical with Horizon
- S (right of center): [?]
- T (right of center): [?]
- P (lower left): tropic point
- Q (lower): tropic point
- d (upper left, inside): [?]
- V (upper right, inside): [?]
- Y (right): [?]
- m, n, p (small, center region): auxiliary points cited in the text ([?] — not individually resolvable)
- α (Greek alpha, center): [?] auxiliary label
- [several further small letters near the lower-center and left margin: illegible [?]]

[Margin: *The altitude of the Sun for a given hour, by what means it is found without a material astrolabe.*] Therefore, if we wish to inquire into the altitude of the Sun above the Horizon and its horizontal distance for the given 10th hour from midnight (or 16th from setting, or 4th from rising), the Sun being in the Equator: we shall describe, through the hour-point N and the poles of the Horizon I, K, the Vertical RNIK, cutting the Horizon at R, whose centre M lies on the line LM perpendicular — at L, the centre of the primary Vertical — to the meridian line

FG. For NR will be the arc of the Sun's altitude above the Horizon, and IN [the arc] of its horizontal distance, inclining to the south; the magnitude of which arcs we shall know thus. When the line ME is drawn from M, the centre of the Vertical RIK, to E, the centre of the Astrolabe, cutting the Horizon — that is, the circle AFCG, above which the Sun's altitude is sought — at m, then m will be the pole of the Vertical RIK. For since this Vertical passes through the poles of the circle AFCG, it will in turn pass through that circle's poles, by the scholium of proposition 15, book 1, of Theodosius [Theodosius], etc. Therefore the lines mN, mR drawn will cut off from the Equator an arc Nn equal to the arc NR of the Sun's altitude; and the lines mN, mI will intercept in the same Equator an arc pN equal to the complement of the same altitude, as is established from what we demonstrated in book 2, proposition 5, Number 17.

Again, when the lines IR, IC are drawn from I, the pole of the Horizon, cutting the Equator at [?], C, the arc [?]C will be equal to the horizontal distance CR, as we showed in the same place.

By the same reasoning, if a Vertical is described through b, I, K, having its centre on the same line ML, the altitude of the Sun and the horizontal distance will be found for the 10th hour from midnight, the Sun being in the first point of Capricorn [♑]. And if a Vertical is described through e, I, K, its arc from point c up to the Horizon will be the altitude of the Sun, and the arc of the Horizon placed between K and the same Vertical [will be] the horizontal distance, for the same hour, the Sun being at the beginning of Cancer [♋]. Thus the same two — namely the altitude of the Sun and the horizontal distance — will be found for the 16th hour from setting, the Sun being at the beginning of Cancer [♋], if a Vertical is described through P, I, K, and the Sun occupying the beginning of Capricorn [♑], if a Vertical is described through Q, I, K. Not otherwise shall we attain the proposed [result] for the 4th hour from rising — both at the beginning of Cancer [♋] and at the beginning of Capricorn [♑] — if a Vertical is described both through e, I, K and through f, I, K, and its pole is found, etc.

[Margin: *To find the altitude of the Sun and the horizontal distance, without a Vertical described through the Sun.*] We shall recover the altitude of the Sun above a given greatest circle — [treated] as some Horizon — and the horizontal distance, even if the Vertical (which is sometimes described not without labor, especially when the hour lies near the meridian line) has not been described for the given hour, in this way. Let there be given, for example, the 16th hour from setting, the Sun holding the beginning of Cancer [♋] at point Q. When the lines QI, QK are drawn from Q to the poles I, K of the given greatest circle AFCG, let the

angle IQK be cut in two by the line QS cutting FG at S; and S will be the point through which the parallel of the circle AFCG described through Q passes, as was shown in book 2, proposition 18, Number 2; and hence the arc IS of the meridian will be equal to the arc of the Vertical described through Q, between the vertex I and the point Q at which the Sun is placed. Therefore the lines emitted from A through I, S will cut off from the Equator an arc equal to the arc IS; or [these lines] will measure, on that arc of the Vertical, the complement of the Sun's altitude.

[Margin: ^a Theodosius II.10] But if, the line QS being joined, it is cut in two and at right angles by a line cutting FG at a, then a will be the centre of the parallel to be described through Q, S. Therefore, the parallel QTS being described from a, cutting the Vertical at T, the arc TQ will represent an arc similar to the horizontal distance — because Verticals of the same circle cut the Horizon and its parallels in similar arcs. The same parallel will be described if an angle equal to FIQ is constructed to the line GI at I, etc., as we said at the beginning of Num. 3, proposition 18, book 2. But we shall know the quantity of the arc TQ of the horizontal distance if from T, Q, through I, the pole of the Horizon, we extend two lines. For these, beyond the pole I, will cut off from the same parallel an arc of as many equal degrees as are represented by the arc TQ, as we demonstrated in book 2, proposition 6, Number 25.

3. What has been said about investigating the altitude of the Sun above the Horizon and its horizontal distance is also to be understood for other greatest circles. For any greatest circle plays the role of some Horizon. Wherefore, if such a circle — known by its proper situation in the sphere — is described on the Astrolabe, as we taught in book 2, proposition 12: one must take the line drawn through its centre and the centre of the Astrolabe as its meridian line, in which its poles are to be investigated, and [likewise] the centre of its primary Vertical, through which a line perpendicular to its own meridian is to be raised, so that in it the centres of all the Verticals may be found. And the line drawn out from the centre of each Vertical through the centre of the Astrolabe cut the described greatest circle at the pole of that same Vertical, etc.
4. The meridian line of the primary Vertical AICK is FK, and its primary Vertical is the Horizon AFCG, since it passes through its poles F, G and through A, C, the poles of the Meridian. But all the other Verticals of the circle AICK itself — [treated] as a Horizon — will have their centres on the line which is drawn perpendicular to FG through H, the centre of the

Horizon AFCG (which is the primary Vertical of the Vertical circle AICK). Thus, a Vertical being described through F, Q, G, its arc between Q and the circle AICK will measure the altitude of the Sun above that same circle AICK, and the arc of the same circle AICK between C and the said Vertical described through F, Q, G will be the horizontal distance. The magnitude of the former arc will be known by the arc of the Equator which the lines emitted from the pole of the said Vertical to the extreme points of that arc cut off; and the magnitude of the latter will be measured by the arc of the Equator cut off by the lines drawn from G, the pole of the circle AICK, through the extreme points of its arc. But if through Q it is descri-

(Canon XVI continues — the enumerated paragraphs of its commentary.)

let a parallel of the circle AICK be described: its arc between Q and the circle AFCG — which we called the primary vertical of the vertical AICK — will then represent an arc similar to the horizontal distance, and so on.

5. For the **meridian** circle FK, the meridian line is AC, which represents the great circle drawn through the poles of the world and through A, C, the poles of that meridian. Its primary vertical, however, will be the equator ABCD, drawn through A, C, the poles of the meridian FK, and through B, D, the poles of the great circle AC (which is the proper meridian of the meridian FK); and on the line FK, perpendicular to AC at E, the centre of the equator — which is the primary vertical of the meridian — will lie the centre of all the verticals of the meridian that are to be described through A, C. Therefore, if a vertical is described through A, Q, C, its arc Q g will measure the Sun's altitude above the meridian at hour 16 reckoned from the west [ab occ.], when the Sun occupies the beginning of Capricorn [♑]. We shall know this arc by the arc of the equator cut off by the lines drawn from q, the pole of the vertical CQg, to g and Q. (The pole q will be found thus: if the line Ag is drawn cutting the equator at V, let us take the quadrant VX; for the line AX will cut FK at the sought pole q — so that the segment g q of the line FK, which represents the great circle drawn through the poles of the world, corresponds to the quadrant VX.) The arc B g, moreover, will be the horizontal distance, and the lines sent out from A to g and B will cut off from the equator an arc equal to it. But if a parallel to the meridian FK is described through Q, as we taught in Book 2, Proposition 18, No. 5, its arc between Q and the equator will represent an arc similar to the horizontal distance. And if the angle contained by the lines drawn from Q to A, C, the poles of the meridian, is bisected by a line, that line will cut AC at the point through which the parallel of the meridian to be described through Q passes. The segment

of the line CA, then, between C and that point, will represent the complement of the Sun's altitude, and so on.

6. Finally, for the **equator** ABCD, the meridian line is BD, and its primary vertical is the line AC, representing the great circle drawn through the poles of the world and through A, C, the poles of the meridian. The Sun's altitude above the equator on any day, at each hour, is equal to the Sun's declination for that day. The horizontal distance, however, is the arc of the equator between C (or A) and the straight line drawn from the centre E through the hour in any given parallel, since this line represents the vertical of the equator drawn through the centre of the Sun.

7. **Therefore**, if the circles of all the hours — both those reckoned from the meridian and midnight and those reckoned from rising and setting — are described in the astrolabe, as was handed down in Book 2, Proposition 9, and the great circle above which the Sun's altitudes and in which the horizontal distances are to be sought is drawn, as we taught in Book 2, Proposition 12, it will at once be evident at what points the hours of every kind cut the parallels of the equator. Hence, if the true diameter of the given great circle is found, as was said in Book 2, Proposition 8, No. 16, and its poles are found, as we prescribed in the same proposition, No. 17, we shall find, for any hour of any parallel whatever, the Sun's altitude and the horizontal distance — provided we describe, through the hour in the given parallel, either the vertical of the proposed great circle or a parallel of that same great circle, and so on.

But we shall find the Sun's altitudes and the horizontal distances by another method in the scholium to Canon 22, even without describing the verticals, or the parallels of the oblique great circle.

Scholium

1. In Book 6 of our *Gnomonics* we called the **complement of the Sun's altitude** above a given great circle the *descensive circumference*, and the horizontal distance the *horizontal circumference*; and both, following Ptolemy, we investigated as much from the analemma as from the calculation of sines. The horizontal circumferences determine the latitudes of the shadows, while the descensive circumferences, or the Sun's altitudes, determine the lengths of those same shadows. And from the latitudes of the shadows, together with their lengths, in a plane equidistant from the great circle above which the altitudes of the Sun and the horizontal distances have been found, sundials are described, as we taught abundantly in Book 5 of the *Gnomonics*, Proposition 5, and in Book 6,

Chapters 9 and 10. The Sun's altitude too — above the horizon indeed we proposed to investigate in Book 1 of the *Gnomonics*, Proposition 36, but above any other great circle in Book 5, Proposition 1, by other means than in Book 6. But if what we have written in this Canon is attentively considered, we shall not much miss those methods described in the *Gnomonics*, since we have taught how to investigate by this Canon, without great labour, both circumferences — the one that measures the Sun's altitude and the one that measures the horizontal distance — for any hour, with the Sun occupying whatever parallel, in any circle whatever; so that by this Canon alone everything is found that is required for the determination of the hours in any sundial.

[Margin: The descensive and horizontal circumference, and their use.]

2. But in order that in planes which are equidistant neither from the horizon, nor from the primary vertical, nor from the meridian, nor from the circle of the sixth hour reckoned from noon and midnight, nor from the equator, sundials may be described by the precepts of Proposition 5, Book 5 of the *Gnomonics*, we shall need the arc of the great circle to which the sundial is equidistant, interposed between the proper meridian of that circle and the meridian of the City in which the sundial is described. Likewise, we shall sometimes need the inclination of the proper meridian to the meridian of the horizon of that place in which we lay out the sundial. We shall treat of these, and of certain other problems (which we have explained partly in the *Gnomonics*), in the Canons that follow.

[Margin: The usefulness of this Canon in describing sundials.]

3. But it is fitting first to expedite this Canon by numbers in a way other than in the *Gnomonics*. Let the former four circles, then, be repeated out of those twelve which we described in the scholium to Canon 3, No. 10: in which the meridian is ABCD; the equator is AC; and the pole of the world is G; the horizon, or any other oblique great circle whose position in the sphere is known, is BD, with its pole F, and whose proper meridian is ABCD, drawn through its own pole and through the pole of the world. Let the Sun be placed at H, occupying whatever parallel; and through H, from the pole of the world G, let the hour-circle GI pass, so that the angle AGI measures the Sun's distance from the meridian. Finally, through H, from the vertex F, let the vertical FL descend, so that HL is the arc of the Sun's altitude above the circle BD — which we shall call the horizon, since it truly performs the office of a horizon in some place —

Diagram. A rectangular frame enclosing four separate spheres (each drawn in projection), arranged two above and two below. They illustrate the four cases

treated in the numbered paragraphs (horizon, vertical, meridian, equator). Each sphere carries lettered points, several marked by small stars/asterisks.

- Upper-left sphere: N (top, starred), F and T and G (upper arc, T starred), A (upper left), P (left, starred), I, H (upper centre, starred), M, E (centre), B (left), K, L (lower centre), R (lower left, starred), D (right), S (right, starred), C (lower right), Q [?] (right, near a star — reading uncertain, possibly "Θ"), O (bottom, starred).
- Upper-right sphere: A (left), F (top, starred), G (top right), H (upper centre, starred), I (left centre), L (centre) and L (right centre), B (left), K (lower left), M (lower centre), D (right), C (lower right).
- Lower-left sphere: A (left), F (top, starred), G (top right), H (centre), B (left), I (left centre), L (centre), D (right), K (lower left), E (lower centre), C (lower right).
- Lower-right sphere: A (left), F (top, starred), G (top right), H (centre, starred), B (left), E and I (centre), L (right centre), D (right), K (lower centre), M (lower right), C (lower right).

Diagram. The identical four-sphere figure of the preceding page is repeated here (same rectangular frame, four spheres, same lettering — A, B, C, D, E, F, G, H, I, K, L, M, N, O, P, Q[?], R, S, T), positioned at the upper left with the text wrapping to its right. It supports the "Quoniam igitur" derivation on this page.

[Margin: To render known, by numbers, the Sun's altitude above any oblique great circle at any hour.]

Since, therefore, in the spherical triangle FGH the two sides FG, GH are known — the former being the complement of the pole's altitude above the given circle (taken as horizon), the latter the complement of the declination, or, if the Sun is southern, the arc compounded of the declination and a quadrant — and since the angle contained by them, FGH, which measures the Sun's distance from the proper meridian of the given horizon, is also known: then, by Problem 22 of the spherical triangles in the last Lemma, let it be made that as the whole sine is to the sine of the arc GH, the complement of the declination (or the arc compounded of the southern declination and a quadrant), so the sine of the arc FG, the complement of the pole's altitude, is to a fourth; a certain fourth number will be generated. And if it is again made that as the whole sine is to the fourth number just found, so the versed sine of the angle FGH, of the Sun's distance from the meridian, is to another, there will be produced the difference between the versed sine of the third side FH and the versed sine of the arc by which the given sides FG, GH differ from each other.

This difference, added to the versed sine of the said arc by which the given arcs FG, GH differ, will make up the versed sine of the third side FH; and hence the arc FH itself, the complement of the Sun's altitude, and therefore also the arc HL, the altitude, will become known. But if the complement of the pole's altitude is equal to the complement of the declination, so that the triangle FGH is isosceles, the third side FH will be found more easily, as was said in the same problem. For if, by the first method of Problem 8 of the spherical triangles, it is made that as the whole sine is to the sine of the complement of the pole's altitude, so the sine of half the angle FGH, of the Sun's distance from the meridian, is to another, there will be produced the sine of half the side FH. Half the side FH being thus known, the whole side too — namely the complement of the Sun's altitude — will be known.

Next, in the same triangle FGH we shall find the angle GFH, by Problem 21 of the spherical triangles, in this way. Let it be made that as the whole sine is to the sine of the arc FG, the complement of the pole's altitude, so the sine of the arc FH, the complement of the Sun's altitude, is to another, so that a fourth number is generated. And again let it be made that as the fourth number just found is to the whole sine, so the difference between the versed sine of the arc GH, the complement of the Sun's declination (for when the Sun is southern, the arc GH, compounded of the arc of declination and a quadrant, has the same sine as the arc of the complement of the declination, since these two arcs make up a semicircle), and the versed sine of the arc by which the two sides GF, FH differ from each other, is to another. For the number produced will be the versed sine of the sought angle GFH. The angle itself, then, will be known, and hence also its arc DL, of the horizon between the meridian toward the boreal pole and the vertical FL, which is drawn through the Sun at the hour of observation. And if the arc DL is greater than a quadrant, the quadrant being subtracted from it, the remainder will be the horizontal distance from the proper primary vertical toward the south; but if less than a quadrant, this being subtracted from the quadrant, the horizontal distance from the same vertical toward the north will remain. But if the complement of the pole's altitude is equal to the complement of the Sun's altitude, so that the triangle GFH is isosceles, the angle GFH will be found far more easily, as we wrote in the same problem. For if, by the second method of Problem 1 of the spherical triangles, it is made that as the whole sine is to the sine of half the side GH (which is the complement of the declination when the Sun runs through the boreal signs, or the arc compounded of the declination and a quadrant when the Sun holds the southern signs), so the secant of the complement of the arc FG — that is, the secant of the pole's altitude — is to another, there will be produced the sine of half the sought angle GFH, and so on.

[Margin: To search out, by numbers, the horizontal distance at any hour.]

The Sun's altitude too, above the horizon or above any great circle whatever, we can compute with **Pedro Nunes** [Petro Nonio], just as in the scholium of the preceding Canon we computed the distances of places and the declinations of the stars. Let the second figure of that scholium be repeated. In its first circle let ABC be understood as the meridian about the centre D; the diameter of the horizon BC, with its pole A; the diameter of the equator FG, with the pole of the world E; the diameter of any parallel of the Sun HI, about which the parallel IKH is described, in which the Sun's place is put at K; and, a perpendicular KL being dropped to IH, let there be drawn through L, parallel to the diameter of the horizon, MN, which will be the diameter of the parallel of the horizon drawn through the Sun — as is evident, if the semicircle IKH is set upright to the meridian. For then KL will be perpendicular to that same meridian, by Definition 4 of Book 11 of Euclid; [Margin: (a) Euclid XI.18] and therefore the plane drawn through KL and MN will be perpendicular to the meridian. Since, then, the horizon too is perpendicular to the meridian, and since BC, MN are the common sections of the meridian with the horizon and with the plane drawn through KL, MN, and are parallel, then by the scholium to Proposition 18 of Book 3 of Euclid the plane of the horizon and the plane drawn through KL, MN will be parallel; and therefore the circle [Margin: (b) Theodosius (1.1)] which the latter plane makes in the sphere will be parallel to the horizon. Finally, a perpendicular IO being dropped from I to BC, IC will be the right sine of the meridian altitude; PO the sine of the Sun's altitude at the time of observation; and IL the versed sine of the Sun's distance from the meridian. Now if A is conceived to be the vertex of the first place, so that its latitude is FA, and the parallel of the second place is HKI, so that its latitude is FI, and the difference of the latitudes is AI, then IO will be the sine of the complement of this difference. Therefore, as we demonstrated in the scholium of the preceding Canon, No. 6, as the square of the whole sine is to the rectangle contained by the sine of the complement of the declination FI and the sine of the complement of the pole's altitude AF, so...

Diagram. Two stacked circular figures within a rectangular frame (the "second figure" of the preceding scholium, the Nunes construction), each a circle with inscribed chords, a diameter, and dotted auxiliary lines. Many peripheral labels; several are small and hard to resolve.

- Upper circle: F (left), A (top), I (top right), N (left), P (upper centre), L (centre), E [?] (right, near a star), K (right), M (right lower), B (left), D (lower

centre), O (lower centre), S (lower centre), H (lower centre), C (lower right), R (right, dotted), G (bottom).

- Lower circle: I (top), A (top), F (left), R (centre, dotted), E (upper right), K (right), Q (right), B (left), O (centre), D (centre), S (centre), L (lower centre), M (right lower), C (right), N (lower left), P (bottom centre), H (lower centre), G (bottom).

AF, so IL, the versed sine of the Sun's distance from the meridian, is to IP, the difference between IO, the sine of the meridian altitude, and PO, the sine of the Sun's altitude at the time of observation.

Wherefore, if it is made that as the square of the whole sine is to the rectangle contained by the sine of the complement of the pole's altitude and the sine of the complement of the declination, so the versed sine of the Sun's distance from the meridian above the proposed circle [is to a fourth; and again, as the square of the whole sine is to that rectangle with] the sine of the complement of the declination, so [the versed sine of the Sun's distance from] the proper [meridian] of the given circle, is to another [?]: there will be produced a number which, subtracted from the sine of the meridian altitude, leaves the sine of the sought altitude of the Sun. And this method holds good for every position of the Sun, even if its whole parallel stands above the great circle and hence it has two meridian altitudes — provided that in the calculation the greater meridian altitude is assumed. On this matter, read, if you please, Proposition 12 of the book of Pedro Nunes *On Twilights [de Crepusculis]*.

[Margin: Another way of finding the Sun's altitude by numbers.]

The same difference IP, between the sine of the meridian altitude and the sine of the Sun's altitude at the hour of observation, will nevertheless be computed by this method too. Let it be made that as the whole sine IL is to IP, the sine of the angle ILP, the complement of the pole's altitude, so IL, the versed sine of the Sun's distance from the meridian, is to another. For the number produced will give the line IP in the parts of the whole sine of the Sun's parallel IH, in which IL is given. If, then, it is made again that as the whole sine of the Sun's parallel is to itself, in so far as the whole sine belongs to the complement of the declination in the great circle, so IP, known in the parts of the whole sine of that same parallel, is to another: there will be produced IP in the parts of that same whole sine in the great circle, in which the sine of the complement of the declination was taken.

[Margin: Another way of finding the difference between the sine of the meridian altitude and the sine of the required altitude.]

Conversely, if it is made that as the rectangle contained by the sine of the complement of the pole's altitude and the sine of the complement of the declination is to the square of the whole sine, so the difference between the sine of the meridian altitude and the sine of the Sun's altitude known from elsewhere at the time of observation is to another, there will be produced the versed sine of the Sun's distance from the meridian. From this distance the hour at the time of observation will easily become known.

This versed sine of the Sun's distance from the meridian we shall also find thus. Let it be made that as IP, the sine of the angle ILP, the complement of the pole's altitude, is to IL, the whole sine, so IP — in so far as it is the difference between the sine of the meridian altitude and the sine of the known Sun's altitude — is to another. For the number that will be generated will give the line IL in the parts of the whole sine in the great circle, in which, namely, the sine of the meridian altitude is given. If, then, again it is made that as the sine of the complement of the declination is to itself, in so far as it is the whole sine of the Sun's parallel, so IL, just found, is to another, there will be produced the same IL, in so far as it is the versed sine of the Sun's distance from the meridian, in the parts of the whole sine of that same parallel. Therefore the distance from the meridian — namely the arc IK — will be known, and so on.

[Margin: To search out by numbers the altitude of a star from its distance from the meridian, and conversely its distance from the meridian from its altitude.]

All these things hold good also for any star whose declination is known. For by exactly the same method, from its distance from the meridian its altitude above the horizon will be found; and, from its altitude known by some instrument, its distance from the meridian — provided, of course, that in place of the declination and the parallel of the Sun the declination and parallel of the star are taken, as is plain. And from the star's distance from the meridian, once found, the hour will be elicited, just as we taught in the scholium of Canon 8, No. 2. But how to compute the hour by day from the Sun's altitude, and by night from the altitude of some star, we have shown above as well, though by another method, at the end of the scholium to Canon 8.

Canon XVII

Given a great circle in the sphere inclined to the meridian, to investigate how great is the arc of that circle intercepted between the meridian of the horizon and its own proper meridian; and how great is the inclination of this proper meridian to the meridian of the horizon.

[Margin: To investigate the arc of any great circle whatever between its proper meridian and the meridian of the given region.]

1. This is Proposition 30 of Book 1 of the *Gnomonics*, which we there worked out by sines; but here we shall expedite the same more easily by the things demonstrated by us in this astrolabe (which method we shall keep also in what follows). Let there be, then, in the figure of the preceding Canon, the great circle whose position and situation in the sphere is given, described by Proposition 12 of Book 2 in the astrolabe as RNIOK, whose centre is M, and which cuts the meridian of the horizon at I and the equator at N, O. From M, the centre of the proposed circle, let the line ME be drawn through E, the centre of the astrolabe, cutting the same given circle at t; this line will represent the proper meridian of the given circle, as we demonstrated in Proposition 3 of Book 2, No. 4, and hence It will be the arc of the proposed circle between the two meridians EI, Et, which is sought. The pole m of the given circle having been found, within the equator, by the things shown in Book 2, Proposition 8, No. 17 (which will be done thus: if the line NO is joined, which will pass through E, the centre, since it is the section of two great circles, [Margin: (a) III.3] and will be perpendicular to Mt, since Mt, drawn from M, the centre of the circle NIO, cuts it in two at E; then from either of the points N, O — namely from N — through t let us send out the line Nf, and let us take the quadrant f a. For the line Na will cut the line Mt at the sought pole m, and so on): the lines mt, mI will cut off from the equator an arc up equal to the sought arc It, as far as the number of degrees is concerned.

2. The arc B u, moreover, will measure the angle BEu, of the inclination of the meridian MEu to the meridian BED: which inclination in the upper hemisphere is western, but in the lower eastern. And so always the arc of the equator lying between the two meridians will measure the inclination of the meridians.

[Margin: To find the inclination of the meridian of any oblique circle whatever to the meridian of the horizon.]

3. When a circle inclined to the meridian passes through the poles of the world — such as, for example, is NEO — no arc of it will be intercepted between the two meridians, since it cuts each meridian in the very poles themselves.

Diagram. A large, very dense stereographic astrolabe projection within a rectangular frame: several intersecting great circles and their images, with numerous straight construction lines (many dotted) and a crowded field of both capital-letter and Greek-minuscule labels. Clearly legible major points: R (left), N and O (equator intersections), I, K (upper), M (lower left and the pole point), E

(centre), t, A (right), C (left of centre), D (lower centre), F (top), B and u [?] on the lower arc. Interspersed small Greek/minuscule labels are visible (including forms resembling δ , ζ , θ , λ , α , β , and others) but most are below the resolution needed to commit individually.

SCHOLIUM

[Margin: The way in which the great circles that sundials lie parallel to are to be drawn in the astrolabe.]

In laying out sundials, a given great circle is either perpendicular to the horizon — that is, it is one of the verticals. In that case, once its declination has been found (as we set forth in Book 1 of the *Gnomonices*, Proposition 23), we shall draw it as a vertical circle in the astrolabe by the method we wrote out in the preceding book, Proposition 8, Number 10 — provided that, for a declination found from south toward east or from north toward west, we take an equal declination from the prime vertical [*a Verticali primario*] on the eastern side toward the north, or on the western side toward the south; and for a declination from south toward west or from north toward east, we take the declination from the prime vertical on the eastern side toward the south, or on the western side toward the north. Or else the given great circle is inclined to the horizon; and in that case, once its declination from the prime vertical and its inclination to the horizon have been found (as we explained in Book 1 of the *Gnomonices*, Proposition 23), that circle is drawn in the astrolabe as we taught in the preceding book, Proposition 12, Number 2.

CANON XVIII

To determine the inclination of a given great circle in the sphere, both to the meridian and to the equator.

[Margin: By what method the inclination to the meridian of a given great circle, whose position in the sphere is marked, may be found.]

1. The earlier part of this Canon was worked out by means of sines by us in Proposition 27 of Book 1 of the *Gnomonices*; the same part will here be carried through by means of the astrolabe, out of what we wrote in Book 2, Proposition 8, Number 11, and Proposition 15; and the later part we shall dispatch out of what we demonstrated in Proposition 8, Number 22. So, in the same figure as Canon 16, let a great circle having a known position in the sphere be drawn in the astrolabe as RNIOK, from the centre M, cutting the meridian at I and K, and the equator at N and O. Then, if the line IK is

bisected and cut at right angles by the line ML — which cuts the given circle at O (for I intend the same letter O to belong both to the intersection of the circles QNO and fNO with the equator, and to the intersection of the line ML with the circle RIK) — and if from I, or from K, a straight line is drawn through O, the intersection of the line ML with the circle RIK: then the arc of the circle AICK described from L through I and K, lying between that straight line and the line IK, will measure the magnitude of the angle LIO (or LKO), the inclination of the given circle to the meridian. Or else, if from K an arc of a circle is described with any radius, the arc of it intercepted between the straight lines drawn from K through L and O will measure half of the same angle LKO, and so on. The straight lines drawn from I through L and O will do the same, if from I an arc of a circle is described with any radius; for these lines too will cut off from that arc half the magnitude of the angle LIO, and so on — as was demonstrated in Book 2, Proposition 15.

[Margin: In what way the inclination to the equator of an oblique great circle, whose position in the sphere is known, may be found.]

2. Next, if the line NO is joined — which the line ME cuts at E at right angles and bisects, ME cutting the given circle at t and the equator at u — then, if straight lines are drawn out from N through t and u, they will cut off from the equator the arc su, which measures the magnitude of the angle tNu, the inclination of the given circle to the equator.
3. When the given circle is perpendicular to the prime vertical — that is, when it passes through the common intersections of the horizon and the meridian — it will yield, as the complement of its inclination to the horizon (found by Proposition 23 of Book 1 of the *Gnomonics*), the inclination of that same circle to the meridian.
4. But when the given circle has no declination, and therefore runs through the poles of the meridian, it will be perpendicular to the meridian and will have no inclination to it.
5. When, finally, the given circle is perpendicular to the horizon — that is, is one of the verticals — it will yield, as the complement of its own declination from the prime vertical (found by Proposition 23 of Book 1 of the *Gnomonics*), the inclination of that same circle to the meridian.

CANON XIX

[Margin: To find the arc of the meridian enclosed between a given oblique circle, whose position in the sphere is known, and both the horizon, the pole of the world, and the pole of the horizon.]

Given an oblique great circle in the sphere, to explore the arc of the meridian enclosed between it and, on the one hand, the horizon, and on the other, the pole of the world and the vertex of the head — that is, the pole of the horizon.

We also solved this problem in Proposition 28 of Book 1 of the *Gnomonics*, both by means of the ellipse and by the calculation of sines. So, in the same figure as Canon 16, let the oblique great circle QIO β be drawn — the one, that is, indicating the 16th hour from setting — cutting the meridian at I and β , in such a way that both β G and IF are arcs of the meridian between the given circle and the horizon, each less than a quadrant (since KG and IF[?] are quadrants from the poles of the horizon up to its circumference). But IE, the arc of the same meridian between the given circle and the pole of the world E, is also less than a quadrant, since EB is a quadrant. And finally there is the arc II[?], between the given circle and the vertex of the place. Now all these arcs will be known through the arcs of the equator intercepted between the straight lines drawn from A through the endpoints of the said arcs; for these arcs of the equator correspond to the said arcs of the meridian, as we demonstrated in Book 2, Proposition 1, Number 6.

Diagram. A large square-framed astrolabe projection (the master figure carried over from Canon 16), extremely crowded with intersecting great and small circles, straight chords, and many point-letters. It serves Canons XVIII and XIX. The clearly legible peripheral capital labels are:

- F (top, on a small circle near the upper edge)
- R (upper left, on the outer great circle)
- A (right)
- C (left of centre)
- L (centre)
- M (lower-left, at the frame)
- D (bottom centre, on a dotted arc)
- K (bottom centre, below D)

Named in the text and belonging to this figure: the great circle RNIOK (with N and O on the equator, I and K on the meridian); the centre M; the poles A and C; points E (pole of the world), t, u, s, f, and the oblique circle QIO β with points Q, β ,

γ, and G. The interior carries numerous small italic/Greek point-letters (including δ, θ, η and several lowercase letters) that do not resolve individually at this resolution. (more than two interior labels unresolved).

CANON XX

Given an oblique great circle in the sphere, to find the altitude of the pole above it.

[Margin: To find the altitude of the pole above a given great circle, whose position in the sphere is known.]

1. This problem too was solved in Book 1 of the *Gnomonics*, Proposition 29, both by means of the ellipse and by the reckoning of sines. So, in the same figure as Canon 16, let there be an oblique great circle, whose position in the sphere is known, drawn as RNIOK, whose centre is M and whose proper meridian is MEt; and let the diameter of the equator, NO, cut Mt at right angles at the centre E[?], which [line] will fall exactly upon the points N and O, since the great circle RNIOK runs through the extreme points N and O, as we demonstrated near the beginning of the scholium to Proposition 5 of Book 2. Therefore, when the radius Nt is drawn, cutting the equator at f, the true diameter of the oblique great circle that RNIOK represents will pass through f. Hence Of will be the arc of the altitude of the pole above the proposed great circle, as is clear from what we demonstrated in Book 2, Proposition 8, Number 22.
2. Let there again be drawn an oblique great circle AgC, whose position in the sphere is known — namely, perpendicular to the meridian, passing through its poles A and C, and oblique to the horizon. When the radius Ag is drawn, cutting the equator at V, then AV will be the arc of the altitude of the pole above it, since its true diameter passes through V — inasmuch as its endpoint V appears at g.

SCHOLIUM

[Margin: To find the arc of an oblique great circle, whose position in the sphere is marked, lying between the great circle that is drawn through its poles and the poles of the horizon, and both the proper meridian and the meridian of the horizon.]

1. We shall solve most of the other problems of the *Gnomonics* in no different a way. For, first, if the given oblique great circle is drawn in the

astrolabe from its known position, and a great circle is drawn through its pole and the pole of the horizon, there will at once appear the arc of the given oblique circle lying between the great circle drawn through the said poles and both the proper meridian of the given circle and the meridian of the horizon; its magnitude will be exhibited by the arc of the equator cut off by the straight lines drawn from its pole through the endpoints of that arc. This arc, too, we investigated in Book 1 of the *Gnomonics*, Proposition 31, by the reckoning of sines.

[Margin: In what way there may be found the arc of the great circle that passes through the poles of the horizon and the poles of a given oblique great circle, lying between the horizon and the circle of the 6th hour from noon or midnight; and how many hours, and which, lie above each face of the oblique great circle, and at what hour it begins to be illuminated; and finally what arcs of the parallels that great circle cuts off.]

2. Next, one will immediately see the arc of the great circle that is drawn through the poles of the given oblique great circle (which has a known position in the sphere) and through the poles of the horizon, lying between the horizon and the circle of the 6th hour from noon or midnight — which in the astrolabe is represented by the straight line AC. Its size we shall learn from the arc of the equator cut off by the straight lines drawn, from the pole of the circle that passes through the said poles, through the endpoints of the said arc. This arc, too, we investigated in Book 1 of the *Gnomonics*, Proposition 32, likewise by sines.
3. Again, for any oblique great circle whose position in the sphere is not unknown, once it is drawn in the astrolabe, we shall find — quicker than it can be told — the arcs of the parallels of the equator cut off by it, and from these we shall soon know how many hours of any given parallel, and which, lie above each face of that great circle, and finally at what hour the Sun begins to illuminate one or the other face. This matter is of exceptional use in laying out sundials, as is clear from our *Gnomonics*. For it was on this account that, in the scholium to Proposition 40 of Book 3 of the *Gnomonics*, we inquired by means of sines at what hour the Sun, when placed on the equator, reaches any given vertical — that is, how large an arc of the equator a given vertical cuts off. Likewise, in the scholium to Proposition 1 of Book 5 of the same *Gnomonics*, we examined, both by sines and by means of the ellipse, how great are the arcs of any parallel of the equator that are cut off by a given oblique great circle, and at what

hour one or the other face of that circle begins, or ceases, to be illuminated by the Sun; and we repeated the same in Book 6, Chapter 10.

But, so that it may appear how readily all of this is learned from the layout of our astrolabe, let, for example, the circle of the 4th hour from rising, $rN\gamma$, be drawn in the foregoing astrolabe. It is inclined to the horizon, since it does not pass through its poles — inasmuch as it cuts the meridian at k , between I (the pole of the horizon) and the horizon itself on the southern side. Let this circle cut the tropic of φ [Capricorn, the winter tropic] at f and γ , the equator at N and O , and the tropic of ϑ [Cancer, the summer tropic] at e and δ . Now, since the upper and northern face of the circle $rN\gamma$ is illuminated by the Sun when it traverses the circumferences $f\theta\gamma$, NAO , $eP\delta$, but its lower and southern face while the Sun runs through the arcs γQf , $OCN\delta e$: then, if the individual parallels are divided into 24 hours — the start being made from their intersections with the meridian FK , if the reckoning is of hours from noon and midnight, or, if the hours are proposed from setting or rising, from their intersections with the horizon on the western or eastern side — the hours contained above each face of the proposed circle, and the hour at which each face begins to be illuminated by the Sun, will be seen at once, and so on.

Thus you see that the upper face of the said circle begins to be illuminated at the 4th hour from rising and ceases to be illuminated at the 4th hour from setting, wherever the Sun may stand in the Zodiac; and that it begins to be illuminated as many hours before noon, when the Sun is at the beginning of φ [Capricorn], as there are hours contained in the arc θf ; when the Sun is on the equator, as many hours as are found in the arc BN ; and finally, when the Sun is describing the tropic of ϑ [Cancer], as many hours as the arc le embraces (the point l being taken for the intersection of the tropic of ϑ with the meridian line), and so on — since the Sun rises above that circle at the points f , N , e , and sets below it at the points γ , O , δ . The same may be observed in any other circle. For, by way of example, above the northern face of the vertical RIK there lie all the hours of the tropic of φ found in the arc going from the point ξ , through Q , up to the intersection of the tropic of φ with the said vertical [and the hours of the arc extending from the point ξ , through θ , up to the same intersection] — the intersection lying between the points β and λ ; while above the southern face lie the hours of the arc extending [from the corresponding point through θ] that intersection. And the Sun, when it is on the equator, will rise above the northern[?] face of that same given vertical at the point N , at the 10th hour from midnight, the 4th from rising, and the 16th from setting; and it will likewise set at the point O , at the 10th hour from noon, the 16th from rising[?], and the 4th from

setting. [Above the northern face it will rise for such-and-such number] of hours and will set at the point N: so that the southern face begins to be lit by the Sun at the 10th hour from midnight, the 16th from rising, and the 4th from setting, while the northern face is lit from the end [of the 10th hour from noon] up to the end of the 10th hour from midnight, and so on.

[Margin: To find the angles that the ecliptic makes with the meridian, the horizon, and each vertical, when it is drawn through the Sun at any given hour.]

4. Finally, with almost no trouble we shall find the magnitudes of the angles that the ecliptic makes, at each of its points, with the meridian, the horizon, and any vertical — angles about which Ptolemy, Johannes Regiomontanus [Regiomontanus], Copernicus, and Geber Hispalensis [Jābir ibn Aflāḥ] wrote much. For if, through a given point of the ecliptic, a straight line representing the meridian is drawn from the centre of the astrolabe, the angle that this meridian makes with the ecliptic will at once appear, and its magnitude will be known from what was set forth in Book 2, Proposition 15. In the same way, if through the degree of the Sun on the ecliptic a parallel is drawn from the centre of the astrolabe, cutting the horizon on the eastern side (if the eastern angle that the ecliptic makes with the horizon at that degree is sought) or on the western side (if the western angle), and then, through that point of the horizon, the ecliptic is drawn in its proper position, the angle that the ecliptic at the given degree makes with the horizon will be obtained. But because through the same point two ecliptics can be drawn — whose centres always lie on the parallel through the centre of the ecliptic that we drew in Book 2, Proposition 5 — in order that the one be drawn which has the proper position, one must consider whether the solstitial point nearest to the given point of the ecliptic precedes the rising of the given point, or rather follows it. For once this has been observed, the one that has the proper position will easily be drawn out of the two ecliptics.

This angle, too, we shall know from what [we set forth] in Book 2, Proposition [15?]. If, [at a given hour from noon or mid]night, a straight line is drawn on the equator from the centre of the astrolabe, which line the parallel of the equator drawn through the point of the ecliptic that the Sun occupies cuts; and if through the point of section the ecliptic is drawn in its proper position (regard being had to the nearest tropic point); and if, finally, through the same point of section a vertical circle is drawn: then we shall find, by the same Proposition 15 of Book 2, the magnitude of the angle that this vertical makes with the ecliptic in that

position. And in this way we shall investigate any arcs or angles whatever of great circles in the sphere — as will become clear from the following Canon, which we propose concerning the hour-arcs in any great circle, since these arcs are of outstanding use in the laying out of sundials.

CANON XXI

To determine the hour-arcs in any great circle whatever.

[Margin: What the hour-arc in any great circle is. The finding of the hour-arcs in any great circle.]

1. We call the hour-arc in any great circle that arc which is enclosed between any hour-circle and the great circle drawn through the poles of the world and the poles of the proper meridian (like the circle of the 6th hour from noon and midnight in the horizon). Now we explored all the hour-arcs of the hours from noon and midnight in Book 5 of the *Gnomonices*, Proposition 4, by means of sines. So, in the foregoing astrolabe of Canon 16, let there be, for example, the great circle Horizon AFCE, which the circle of the 10th hour from noon and midnight, dEA, cuts at d; the circle of the 16th hour from setting cuts at μ ; and the circle of the 4th hour from rising cuts at r. And since A and C are the poles of the meridian, the line AC will represent the circle of the 6th hour from noon and midnight. Therefore Cd will be, in the horizon, the eastern hour-arc of the 10th hour from noon and midnight; C μ the eastern hour-arc of the 16th hour from setting, likewise; and finally Ar the western hour-arc of the 4th hour from rising. All these arcs we shall know through the arcs of the equator cut off by the straight lines drawn from I, the pole of the horizon, through the endpoints of those arcs. For the lines IC and Id, if drawn, will intercept on the equator an arc equal to the hour-arc Cd, and so on.
2. Next, since A and C are also the poles of the meridian of the prime vertical AICK itself, and therefore the line AC likewise represents the circle of the 6th hour from noon and midnight with respect to the vertical (as of some horizon), the hour-arcs in the prime vertical will be those intercepted between A (or C) and the intersections of the hour-circles with that same vertical; and their magnitudes will be known in the same way, through the arcs of the equator cut off by the straight lines drawn from G, the pole of the vertical, through the endpoints of those arcs.

3. Again, since the line Mu is the proper meridian of the vertical circle RIK, and the line NO the circle of the 6th hour from noon and midnight, if the said vertical is set up as some horizon, the hour-arcs in that vertical will be those interposed between N (or O) and the intersections of the circle RIK with the hour-circles; and their magnitudes will be determined on the equator through the arcs that the straight lines drawn from m, the pole of the vertical RIK, through the extremities of the hour-arcs cut off. And so the hour-arcs of the 10th hour from noon or midnight, and of the 16th hour from setting, and of the 4th hour from rising, come to nothing, since these three hour-circles cut the vertical RIK at N, the pole of its proper meridian.
4. Moreover, since AC is the meridian of the meridian FK — inasmuch as it runs through E (the pole of the world) and through A and C (the poles of the meridian FK) — and since B and D are the poles of the circle AC itself, and since, finally, that meridian is itself like the circle of the 6th hour from noon and midnight (being six hours distant from its own meridian AC): there will be intercepted on the meridian FK the hour-arcs between B (or D) and the points at which the hour-circles cut the meridian FK. Thus the arcs of all the hours from noon or midnight will be represented on the quadrant BE, since all the meridian circles of those hours cut the meridian FK at E. But the arc of the 16th hour from setting will be Bl, on the north, and that of the 4th hour from rising Bk, on the south; and arcs on the equator equal to these arcs will be intercepted by the straight lines drawn from A, the pole of the meridian FK, through B, l and B, k.
5. Finally, since the meridian of the equator is FK, having the poles A and C, and AC is the circle of the 6th hour from noon [and midnight], there will be intercepted on the equator the hour-arcs between C (or A) and the individual hours of the equator: thus CN will be the arc of the 10th hour from noon or midnight, and of both the 16th hour from setting and the 4th from rising.

SCHOLIUM

[Margin: The drawing of the hours on any plane whatever, by means of the hour-arcs.]

1. By means of the hour-arcs from noon and midnight, sundials of those same hours can be drawn on any proposed plane, as was treated at length by us in Proposition 5 of Book 5 of the *Gnomonics*, so that it would be superfluous to repeat it here. Therefore in this place we shall only give

brief notice of the method by which the hours from rising and setting are to be drawn by means of the hour-arcs of those same hours. On the plane of the dial, then, from the place of the style, let a circle be drawn equal to the equator of the astrolabe, in which the hour-arcs were found; and in it let a diameter be drawn perpendicular to the proper meridian line — that is, to the line of the style — so that the common section of the proper vertical and of the dial-plane is obtained. From this diameter, the hour-arcs being counted off toward the side on which they were found to decline in the astrolabe, let straight lines be drawn through their endpoints and through the place of the style: these will be parallel to the common sections of the hour-circles and of the great circle to which the dial lies parallel. For if the verticals of that great circle to which the dial lies parallel are conceived as drawn through the style and these common sections, then — [there will be cut off, in the circle we drew on the dial-plane, arcs similar to the hour-arcs in that same great circle to which the dial lies parallel; and in the said circle of the dial-plane lines will come to be parallel to those common sections in the great circle to which the dial lies parallel]. Since, then, by construction, in the circle drawn on the dial-plane the arcs taken are similar to the hour-arcs in the great circle to which the dial lies parallel, those straight lines drawn from the place of the style, extended through the hour-arcs counted off in that same circle of the dial, will be parallel to those [parallels] which the said verticals — passing through all the intersections of the hour-circles and of the great circle to which the dial lies parallel — produce on the plane of the dial. And since the hour-circles, on the plane of the dial and in the great circle to which it is parallel, likewise produce parallel common sections: if there are found, on the plane of the dial, points on the equinoctial line, or elsewhere, through which the hours from rising and setting are to be drawn (that is, through which the hour-circles themselves are drawn), and if through those points, parallel to the aforesaid straight lines drawn in the circle described from the place of the style through the hour-arcs, [further lines] are drawn, then the drawn...

[The scholium to Canon XXI continues.]

Astrolabe, Book III — Canons XXI (Scholium, continued) and XXII

(Continuation of the Scholium to Canon XXI, begun on the preceding leaf.)

there will be described the hours from rising and setting [*ab ortu & occasu*]: for those straight lines drawn from the position of the style through the hour-arcs are parallel to these common sections — that is, to the hour-lines — inasmuch as both the former and the latter have been shown to be equidistant from the common sections of the hour-circles made on the greatest circle to which the dial is parallel. [Margin a: 9. undec.] In the case of astronomical hours, since they all pass through the center of the dial, it is enough to draw through the center of the dial lines parallel to the common sections of the hour-circles from noon or midnight [*à mer. vel med. noc.*] and of the greatest circle to which the dial is equidistant; such are the straight lines drawn from the center of the dial through the hour-arcs in a circle described about that same center of the dial, as we did in Proposition 5 of Book 5 of the *Gnomonics*.

2. Therefore, if all the hour-circles have been drawn on the astrolabe, the hour-arcs in a given oblique circle will appear at once. As regards the number of degrees, the magnitudes of all these arcs are equal to the arcs of the Equator that are cut off by the straight lines drawn from the pole of the given oblique circle through the extreme points of the hour-arcs.
3. In the Canon we said, further, that the hour-arcs lie between any hour-circle whatever and the circle that is drawn through the poles of the world and the poles of the proper Meridian — in the manner of the circle of the 6th hour from noon or midnight — and not between the proper primary Vertical, which nonetheless runs through those same poles of the proper Meridian. For in laying out dials the arcs of the hours from noon or midnight are reckoned from the common section of the plane of the dial and of that circle which plays the role of the circle of the 6th hour from noon or midnight on the greatest circle to which the dial is equidistant; whereas the arcs of the hours from sunrise and sunset are counted from the common section of the plane of the dial and of the proper and primary Vertical. And if the complements of the hour-arcs are taken, they must be counted — both for the hours from sunrise or sunset and for those from noon or midnight — from the proper meridian line, namely the one on which the style is set.
4. Now, since in Proposition 4 of Book 5 of the *Gnomonics* we computed the hour-arcs of the hours from noon and midnight by two operations, using sines, we shall now find the same by a single operation, in this way. A right triangle is always formed from the arc of the proper Meridian measuring the elevation of the nearer pole above the given greatest circle; from the

arc of the hour-circle running from that same nearer pole up to the given greatest circle; and from the arc of the given greatest circle enclosed between the proper Meridian and the hour-circle — which arc is the complement of the hour-arc sought. Therefore, by the first method of Problem 11 of the spherical triangles of the last Lemma: let it be, as the whole sine is to the sine of the arc of the Meridian [measuring] the elevation of the pole, so the tangent of the angle that the hour-circle makes with the Meridian at the pole, to a fourth quantity; and there will be found the tangent of the arc of the given greatest circle enclosed between the Meridian and the hour-circle, and so forth. [Margin: To compute the hour-arcs for the hours from noon and midnight.]

CANON XXII

To solve all problems of spherical triangles without the aid of numbers.

The use of this Canon extends very widely. For by it we shall investigate geometrically, by means of arcs of the Equator, the magnitudes of all the angles and of all the sides of spherical triangles, and so we shall dispatch — with wonderful ease, from the drawing of merely two or three circles of the astrolabe — all the problems that are usually solved by the laborious calculation of such triangles: a result that until now has seemed incredible to not a few. The whole business, however, consists in the construction of the spherical triangles, as will appear. We shall proceed in the same order that we observed in Lemma 53 of Book 1. And although in the previous 16 problems of the right-angled spherical triangle either the angle alone, or the side alone, or finally the base alone is usually investigated, by means of sines, from two given quantities, here too, in any triangle, we shall by means of the astrolabe explore simultaneously the remaining two that are not given. In the right-angled spherical triangle, then, the following will be investigated from certain given quantities.

I. THE ANGLE

Together with the other angle and the side, which are not given.

From the base, and the side that is opposite the angle sought.

[Margin: Probl. 1.] [Margin: To investigate the angle, together with the rest, from the given base and the side opposite the angle sought.]

Let ABCD be the Equator on the astrolabe, about the center E, with the two diameters BD, AC cutting each other at right angles. Let the given side be counted

from point B up to F, and the base from point F up to G. Now, taking arcs BM, CK, DN equal to arc AF, let the diameters FK, MN be joined, also cutting each other at right angles, since FM, MK, KN, NF are quadrants. For the arcs BM, CK, DN must be taken on the side toward which arc AF tends, so that the said quadrants result. Then, drawing the straight line MG, which cuts the straight line FK at H, let an arc MI be taken equal to arc NG, and let a circle be described through the three points G, H, I — its center will lie on the straight line FK produced, and will be marked out by the straight lines tangent to the Equator at G and I, that is, by the straight lines perpendicular to the joined semidiameters EG, EI, as was shown in Proposition 7 of Book 2. This circle cuts the straight line BD at L, within the Equator, and will be parallel to the greatest circle MEN, whose poles are F and K, since it recedes equally from this circle MEN — because arc EH is equal to arc NG, as is established from what we demonstrated in Book 2, Proposition 1, Numbers 5 and 6, and arc MI was taken equal to arc NG. Indeed, from what we wrote in Book 2, Proposition 18, Number 5, it is likewise clear that GHI is parallel to the greatest circle MEN. Finally, through the three points F, L, K let a circle FLK be described, whose center *f* lies on the straight line MN, cutting EB produced at O.

Therefore BFL will be the right-angled spherical triangle that is proposed, since angle FBL is right, the side BF is given, and the base FL is given — because the arcs FL, FG, falling from pole F onto the parallel GHI, are equal. Its sought angle FLB, opposite the given side BF, we shall investigate as follows, by what was demonstrated in Book 2, Proposition 15, Number 3.

Bisect the straight line LO and draw through it, at right angles, the line PR, cutting the circle LFO at R; the arc RO will measure the magnitude of the sought angle FLB. And if, about the vertex L, a circle QS is described with any radius, and the straight line LR cuts it at S, the arc QS will measure half of that same angle FLB [Margin b: 3. tertij.]; and therefore the arc QS doubled will measure the whole angle. But if the section-point O is too far distant, it will be enough to draw from *f*, the center of circle KLF, a perpendicular to LB, cutting the circle KLF at R;^a for this perpendicular bisects the straight line LO. Or we may proceed without the center *f*, thus: having found P, the center of the three points A, L, C (this will be the midpoint of the two points L, O, since the greatest circle described through A, L, C passes through O, the point opposite L), let there be erected the perpendicular PR to BD. For again P will be the midpoint of the straight line LO, since the greatest circle described through A, L, C passes through O, the point opposite L. Hence the arc QS of the circle described from L, lying between the straight lines LQ, LR, will measure half the angle BLF; and if any circle whatever is described through L, its arc between those same straight lines will measure the

whole angle. All this was demonstrated at the end of Number 2 of Proposition 15, Book 2.

Indeed, the straight line LR itself will measure that same sought angle BLF, as we demonstrated in Number 3 of the same Proposition 15, Book 2.

Now by the same reasoning the other angle BFL, not given, will be found. For, drawing the radius FT cutting the Equator at e, the arc Me will measure the angle BFL, since the arc measuring it is MT, to which the arc Me is equal, as was shown in Book 2, Proposition 1.

Finally, the remaining side BL, not given, will be made known by the arc of the Equator that the straight lines drawn from A, the pole of circle BED, through the points B, L (produced) intercept — such is the arc Bg, as is clear from the same Proposition 1, Book 2.

But suppose the diameter FK is drawn from the extreme point of the given side BF, and the diameter MN cuts it at right angles, MN representing the greatest circle drawn through the poles of the world, whose poles are FK. If the parallel GHI of this greatest circle MN, described through the extreme point G of the given base FG, does not cut the diameter BD within the Equator, then the problem will be impossible; for in that case no arc of a greatest circle equal to the base FG — such as was FL, the arc let fall as far as the parallel GHI — can be drawn down from F to BD so as to cut off the side BL less than a semicircle, as the reasoning requires. And so, when the given side BF is less than a quadrant, the base must be proposed greater than the side itself (because, by Proposition 34 of our spherical triangles, the angle opposite the given side is acute, and therefore, by Proposition 11 of the same spherical triangles, the given side is less than the base, which is opposite the right angle), yet in such a way that the base together with the side make up an arc less than a semicircle, such as was the base FG. For if the point G were beyond D, the parallel GHI would not cut the straight line BD. But when the given side is greater than a quadrant, the base must be proposed less than the side itself (because, by Proposition 34 of our spherical triangles, the angle then opposite the given side is obtuse, and therefore, by Proposition 11 of the same spherical triangles, the given side is greater than the base, which is opposite the right angle), yet in such a way that the base is greater than the complement of the given side up to a semicircle. Thus, if the given side is BN, the base must be greater than the arc ND; otherwise the parallel of the greatest circle FK — FK cutting the diameter NM, which is drawn at right angles from the extreme point of the given side — described through the extreme point of the base, would not cut BD within the Equator. But there is no need of this caution, since spherical

triangles in fact exist on a spherical surface. This is also to be understood in the problems that follow.

Diagram. A square frame enclosing the Equator, drawn as a circle ABCD about its center E, with the two perpendicular diameters. The figure carries the full construction of Problem I (the right-angled spherical triangle BFL and the auxiliary parallel circle GHI, the circle FLK/LRO, and their poles). A large arc sweeps across the upper part of the frame outside the circle. Many chords, tangents, and dotted construction lines fill the interior. Readable labels, by position:

- A (right end of the horizontal diameter, on the circle)
- B (top of the circle, on the vertical diameter)
- C (left end of the horizontal diameter, on the circle)
- D (bottom of the circle, on the vertical diameter)
- E (center)
- F (right, on the circle, below R)
- G (lower center, near D)
- H (left of center, interior, below C)
- K (lower left, on the circle)
- L (lower center, interior)
- M (upper left, on the circle)
- N (lower right, on the circle)
- P (upper center, between B and E)
- R (right, on the circle, above F)
- S (upper-right corner of the frame)
- T (lower center-right, interior)
- V (right, near the circle, below A)
- X (upper center, right of P) [?]
- Y (left of center, above H)
- f (script minuscule, left of center, below M)
- h (minuscule, just left of A) [?]
- b, e (minuscules in the bottom cluster, near D) [?]
- O (top, on the outer arc)
- a second letter on the outer arc just below O, read as Z (the vertex Z of Problem III's triangle) [?]
- Q (upper center, above B, underlined) [?]

- a minuscule glyph at the lower left interior, read as δ [?]

Owing to the crowding of construction lines and the small peripheral minuscule labels, several labels could not be resolved with confidence.

II. THE ANGLE

Together with the other angle and the side, which are not given.

From the base, and the side that is adjacent to the angle sought.

[Margin: Probl. 2.] [Margin: To find the angle, together with the rest, from the given base and the side adjacent to the angle sought.]

Let the spherical triangle BFL be constructed from the data, as in the preceding problem — the triangle in which the angle BFL, adjacent to the given side BF, is set to be found. Since the arc TK measures the angle KFL, as was demonstrated in Book 2, Proposition 15, Number 3, if to this angle the right angle KFM is added, the whole sought angle BFL becomes known. But if from F a straight line is drawn through M until it cuts the circle FTK produced, then the arc of that circle intercepted between that line and the point T will give the magnitude of the whole angle BFL, as we demonstrated in Book 2, Proposition 15, Number 2. And if from F a circle is described with any radius, its arc lying between the straight lines FT, FM will measure half of that same angle. Indeed, the arc Me of the Equator measures the same angle.

The other angle, not given, BLF, will be recognized as in the preceding problem — namely either by the arc LR, or by the arc QS doubled, and so forth.

The remaining side BL will here too be found by the arc Bg, which the straight line AL cuts off from the Equator, as in the preceding problem.

Diagram. The same construction figure as fig-p679-1, repeated on this leaf: the square frame, the Equator ABCD about center E with its two perpendicular diameters, the auxiliary parallel circle GHI, the circle FLK/LRO, and the large sweeping arc above. The same set of labels appears (A, B, C, D, E, F, G, H, K, L, M, N, P, R, S, T, V, Y, and the minuscules f, h, b, e, together with O, and the upper labels Q [?], X [?], the outer-arc letter read as Z [?], and the lower-left glyph read as δ [?]). Because it is the same crowded figure, the same peripheral labels remain uncertain.

Canon XXII, continued

III. THE ANGLE

Together with the two sides, which are not given in this place.

From the base, and one non-right angle.

[Margin: Probl. 3.] [Margin: The angle, together with the others, from the given base.]

Count the base from B toward C up to g; and when the visual ray Ag is drawn, cutting BD at L, then BL will be the base of the proposed triangle, since arc BL contains as many degrees as arc Bg, as was demonstrated in Book 2, Proposition 1. Next, at L construct the given angle by Proposition 16 of Book 2, in this way. On the straight line LB, having found the point O opposite L, bisect LO at P, and draw through it, at right angles, the straight line PR. Or, if the point O is too remote, find P, the center of the three points A, L, C (for this will be at the midpoint of the two points L, O, since the circle described from P through A, L, C is a greatest circle and therefore passes through O, the opposite point), and at P erect the perpendicular PR to BL. Now, having described from L a circle QS of any size, count off on it half the given angle, from point Q up to S; or else (if the minutes it contains are an odd number) count the whole angle, and take QS as half of the counted arc. For when the straight line LS is drawn, cutting PR at R, then if a greatest circle LRO is described through the three points L, R, O — or through the two points L, R, if O is too remote (since it passes through opposite points) — having its center f on the straight line PR, the angle BLF will be equal to the given angle, since the arc QS measures its half, as we showed in Proposition 15, Book 2, Number 2.

Now, drawing from f, the center, through E, the center of the astrolabe, the straight line MN — which the diameter FK will cut at right angles, if there is no error — let the radius KT be sent out, cutting the Equator at V, and let the quadrant VX be taken. For the straight line KX will cut fE at Y, the pole of the greatest circle LRO, as we showed in Book 2, Proposition 8, Number 17. If, then, through the three points D, Y, B a circle is described from a center found on the straight line EA, cutting LRO at Z — which will be a greatest circle, since it is drawn through the opposite points D, B — the angle BZL will be right,^a because the greatest circle DYB passes through Y, the pole of the greatest circle LRO. [Margin a: 15.1. Theod.] And so the proposed right triangle will be BZL, since BL is the given base opposite the right angle Z, and BLZ is the given non-right angle.

Therefore the other non-right angle LBZ will be found thus. Draw the straight line Ba through a, the point of intersection of circle ZBD with the straight line AC, cutting the Equator at b; then Db will be the magnitude of the angle aBE, as is established from what we showed in Proposition 15, Book 2. If this is subtracted from the two right angles — to which the two angles aBE, EBZ are equal, by Proposition 5 of our spherical triangles — the remainder will be the sought angle LBZ. This whole angle will also be found in this way whenever the circle DBZ can conveniently be described entire, so as to intersect the straight line EA. Draw a straight line from B through the intersection of circle DBZ with the straight line EA. For both the arc of the Equator and the arc of circle DBZ intercepted between this line and the diameter BD toward D — or else the arc of circle DBZ lying between B and that same line — will measure the sought angle LBZ, as is clear from what we demonstrated in Proposition 16, Book 2, Number 3.

Now the side LZ [the print appears to read "LRZ"] will be equal to the arc of the Equator that the straight lines drawn from Y, the pole of circle KFZ, through the points L, Z cut off.

By the same reasoning the other side BZ will be indicated by the arc of the Equator cut off by the straight lines drawn from h, the pole of circle DBZ, through B, Z. Now the pole h will lie at the intersection of circle KFZ with the straight line AC. For since the greatest circle DBZ passes through Y, B, the poles of the greatest circles KFZ, CA, these in turn will pass through its poles, by the scholium to Proposition 15 of Book 1 of Theodosius [Theodosius]; and therefore the point h will be the pole of circle DBZ. This too will be found if a radius is sent out from B through a, cutting the Equator at b, and the quadrant bV is taken; for the radius BV will cut the straight line AC at h, the sought pole, as was shown in Proposition 8, Number 17, Book 2.

But if the base DL, less than a quadrant, is given, and the same things are done, the proposed triangle DLi will be constructed on the other side, since the angle DLi is equal to the angle BLF, being vertical to it, and so forth.

IV. THE ANGLE

Together with the side and the base, which here are not given.

From the side that is opposite the angle sought, and the other non-right angle.

[Margin: Probl. 4.] [Margin: To seek the angle, together with the rest, from the given side that is opposite it, and the other non-right angle.]

Let BF be the given side; and at F let there be constructed with it an angle equal to the given angle, by Proposition 16 of Book 2, in this way. Draw the diameter FK, which the diameter MN cuts at right angles; count the degrees of the given angle from point M up to e; and when the straight line Fe is drawn, cutting MN at T, describe through the three points F, T, K — from a center f lying on the straight line MN — the circle FTK, which will be a greatest circle, since it passes through the opposite points F, K. Let this circle cut the straight line BD at L; and the angle BFL will be given, since its arc is Me. And so the spherical triangle BLF will be the one sought, having namely the right angle LBF, the given side BF, together with the given non-right angle LFB. Therefore the angle BLF, opposite the given side, will be found as in the first problem. For, bisecting the straight line LO and drawing through it, at right angles, the straight line PR, the arc RO, or LR, will measure the sought angle BLF. Or, if from f, the center of circle KTF, a perpendicular to LB is erected, and from L a circle QS of any size is described, and the straight line LR is drawn, the arc QS will measure half of that same angle, and so forth.

The side BL, moreover, will be known from the arc Bg of the Equator, which the straight line AL cuts off.

And the base FL will be exhibited by the arc FG of the Equator, which is cut off by the straight line drawn from Y, the pole of circle FLK, through L.

Astrolabe, Book III — Canon XXII (continued)

(Solving every spherical-triangle problem geometrically, without numbers, by means of the astrolabe.)

V. ANGLE.

[Margin: Probl. 5.]

With the base, and one other side, not given:

From a side which is adjacent to the sought angle, and from another angle which is not right — provided it is established whether the sought angle is greater than a right angle or smaller, or whether the base, or the other side which is not given, is greater or smaller than a quadrant.

[Margin: To elicit the angle, together with the rest, from a given side which is adjacent to the sought angle, and from another non-right angle.]

Again let ABCD be the equator, with two diameters AC, BD cutting one another at right angles at the center E. Count the given side from A up to F, and join the straight line BF, cutting AC at G: then arc AG will be equal to the given side AF, as was shown in Proposition 1 of Book 2. Likewise count the magnitude of the given angle from A up to H, and join BH, cutting AC at I: then arc AI will be equal to arc AH of the given angle, and necessarily greater than AG, if the given angle is acute, as will be demonstrated. Therefore, with the circle BID described through the three points B, I, D — having its center d on the line AC, and being a great circle since it passes through the opposite points B, D — the angle ABI will be equal to the given angle, since arc AI (or AH) measures it.

Let there also be described, from E through G, a parallel of the equator, GK, cutting the circle BID at L; and, the straight line EL being drawn out and cutting the equator at M, let arc BN be taken equal to arc AM. Now, drawing the diameter NQ, let it cut [this] at right angles at RS — which will easily be done if arcs AS, CR are taken equal to arcs BN, DQ, so that in this way four quadrants NS, SQ, QR, RN are produced. With the circle NOQ now described through the three points N, G, Q — which is a great circle, since it passes through the opposite points N, Q, and has its center P on the line ER, as far distant from E as the center d of circle BID is distant from that same center E, because (as we shall show below) the two circles BID, NGQ [touch the same parallel] — [we obtain] the proposed triangle.

For since arcs AM, BN [are similar] to arc GL by [Proposition 16 of] Book 3 of Euclid, BN too will be similar to the same GL; and since they are arcs cut off from the parallels GK, AB, they are similar arcs, and, not passing through the pole E — namely [the parallel] which is described from E through I — since BID touches it at L, from the scholium of Proposition 13 of Book 3 of Euclid, and consequently, from the scholium of Proposition 21 of Book 2 of Theodosius, they are equally inclined to the greatest of the parallels ABCD; that is, the angles ABI, ANG will be equal — which is also established from this, that their arcs AI, SO are equal.

[Margin: a — 16, [prop.] 2, Th. (Theodosius, II.16)]

Since, therefore, ABI is equal to the given angle, ANG too will be equal to the given angle, which indeed is opposite the given side AG. And so, if it is established that the sought angle at G is acute, the triangle ANG must be taken; but if it is established that the sought angle at G is obtuse, the triangle AGQ must be taken, and so on. We shall recognize the sought angle thus. From P, the center of circle NGQ, let a perpendicular PT be dropped to AC, cutting that same circle at V. For arc GQV will measure the angle CGQ, and hence also the angle AGN of triangle AGN, as we showed in Book 2, Proposition 15, Number 3 — which angle,

subtracted from two right angles, will give the remaining angle AGQ in triangle AQG. The same angle CGQ will be obtained if, from G, any arc whatever XZ is described cutting GC at Y; for arc XY will measure half the angle CGQ, and double the arc XZ will measure the whole angle.

Diagram. A square frame enclosing a great circle (the equator ABCD) crossed by many chords, diameters, and secondary circles, illustrating the construction for Problem V. It is a densely worked figure with roughly thirty labels, several of them small lowercase letters that do not fully resolve.

- B (top of circle) / D (bottom) / A (right, on the horizontal diameter) / C (left, on the horizontal diameter): the equator's cardinal points.
- E (center): center of the equator.
- d (left, on the horizontal line AC): center of circle BID.
- P (upper left, outside the circle): center of circle NGQ.
- R (upper left, below P); S (below R): points of the quadrant division RS.
- T (on the horizontal line, left of center): foot of perpendicular PT.
- Y (left of center, near C): auxiliary point on GC.
- N (upper right, outside the circle); M (far upper right); [?] p (between N and M): points of the NGQ construction.
- K (upper middle): point on the parallel GK.
- L (right of center): intersection of parallel GK with circle BID.
- I (right of center); [?] k (right of L): intersection points.
- X (lower left); V (lower left, on the circle): points of arc XZ / secant PT.
- Ω [?], m, q (bottom, left of D): points near the foot of the figure.
- H, F (lower right, on the circle): F is the foot of the given side; H the foot of the given angle.
- G (bottom center): intersection of BF with AC.
- Small lowercase labels a, b, e, n, o, s, t are also present in the interior but are not all separately legible.

[Several interior lowercase labels are uncertain]

But if the given side is greater than a quadrant — and consequently the opposite given angle is obtuse, yet smaller than the given side, as will be demonstrated — count the given side from C up to F, and send out the radius BF, cutting AC [at G], so that the given side is CG. Likewise count the magnitude of the given obtuse angle from C up to H, and send out the radius BH, cutting AC at I, so that CI is the arc of the given angle. Then, with the circle BID described through the three

points B, I, D from the center d lying on the line AC, CBI will be the angle equal to the given angle. Let the parallel GK cut this circle at L; and, the semidiameter ELM being drawn out, let arc BN be taken equal to arc AM, and let a circle be described through the three points N, G, Q as before: then again the angle GNC will be equal to the angle GBC, which will be proved as before. Therefore, if it is established that the sought angle at G, adjacent to the given side CG, is obtuse, the proposed triangle will be CGN. For if it is acute, the triangle offered will be CGQ. The sought angle CGQ, moreover, will be recognized through arc GV as before; and when this is subtracted from a semicircle, the angle CGN will remain, and so on.

From the construction it is plainly established that, when the given side is smaller than a quadrant, the opposite given angle is acute, yet greater than the given side itself; but when the given side is greater than a quadrant, the opposite angle is obtuse, yet smaller than the given side. For since, by theorem 4 of the scholium of Proposition 21 of Book 2 of Theodosius, arc GA is smaller than arc GN, the angle ANG in triangle AGN will, by Proposition 11 of our *Spherical Triangles*, be smaller than the right angle A — that is, acute — and hence GNC will be obtuse. By the same reasoning, in triangle AGQ the angle GQA will be smaller than the right angle A, which holds, by the same theorem 4 of the said scholium, because arc GA is smaller than arc GQ, and so on. That the angle given as opposite the side AG must be greater than the side AG — as was the angle ABI — is clear. For if it were smaller (such as is the angle ABb, when the circle BbD, making the angle ANG equal to that same ABb, touches the same parallel a b), then — because the circles BbD, NGQ, on account of the equal angles at B, N, are equally inclined to the equator, and so on — [it would follow] that NGQ [touches] the parallel a b, which [is what] the circle BbD

[CANON XXII, continued]

cuts. Hence it follows that the given obtuse angle, opposite the given side CG, is smaller than the side CG itself — such as was the angle GNC. For if it were greater (such as is CBb), the circle NGQ would touch, a second time, the parallel a b which the circle BbD touches — which is absurd. But concerning the angles of a spherical triangle, both right-angled and non-right-angled, we shall demonstrate more in the scholium of this Canon.

[Margin: Another solution of the problem.] It is also established that if — the angle ABI having been constructed equal to the given angle — through the point G there is described, by Proposition 20 of Book 2, the great circle NGQ touching the same parallel IO which the circle BID touches, the proposed triangle is

likewise constructed. For from Theorem 1, Proposition 21 of Book 2 of Theodosius, the circles BID, NGQ will be equally inclined to the equator; that is, the angles ABI, ANG will be equal, and so on.

[Margin: An easier solution of the problem.] We shall solve the same problem more easily in this way. Let Ah be the magnitude of the given angle, and, the radius Bh being drawn cutting AC at b, then Ab will be equal to arc Ah. Therefore, with the circle BbD described through the three points B, b, D, having its center Y on the line AC, ABb will be the given angle. Next let arc Ag be made equal to the given side — and first smaller than a quadrant — and let the radius Bg be drawn cutting AC at k, so that Ak too is an arc equal to the given side. Then, with the parallel of the equator described through k, cutting the circle BbD at i, let the straight line Ei be drawn cutting the equator at N: and the proposed triangle will be BiN, or DiN, [Margin: a — 15, [prop.] 1, Th. (Theodosius, I.15)] since the angle at N is right, and the side Ni is given (inasmuch as it is equal to that same Ak, and hence also to arc Ag), and opposite the given angle NBi, or NDi. Therefore, if it is established that the sought angle i is acute, the triangle BiN must be taken. For since all three arcs are smaller than a quadrant, by Proposition 28 of our *Spherical Triangles* the two angles B, i will be acute. But if it is established that the sought angle is obtuse, DiN must be taken. Now if, from Y, the center of circle BbD, a perpendicular Ye is dropped to iE produced, cutting the circle at f, arc if will give the magnitude of the acute angle BiN, as was shown in Book 2, Proposition 15, Number 3; and when this is subtracted from a semicircle, the obtuse angle DiN too will become known.

But if the given side is greater than a quadrant, and it is counted from C up to g, the drawn radius Bg will give arc Ck equal to that same side. The given angle likewise being counted from C up to h, and the radius Bh being drawn cutting AC at b, if a circle is described through B, b, D, the given angle will be the obtuse CBb. Therefore, with a parallel described through k, cutting the circle BbD at i, and a straight line extended through i and E, iEQ, the proposed triangle will be either BQi, if the sought angle is obtuse, or DQi, if acute; because the angle at Q is right, and the side iQ — opposite the given angle iBQ (or iDQ) — is equal to that same Ck, that is, to arc Cg. The angle at i will be found as before.

From these it is also clear that the angle given as opposite the given side must be greater than the given side itself, and acute, when the given side is smaller than a quadrant; but smaller than the side itself, and obtuse, when the given side is greater than a quadrant. For it has been shown that the angle NBi (or NDi) is acute, and hence QBi (or QDi) obtuse. And unless Ab, the arc of the given acute

angle, were greater than the given side Ak — or unless Cb, the arc of the given obtuse angle, were smaller than the side Ck — the parallel would not cut the circle BbD, and consequently the problem could not be solved.

Again, since the parallel ik cuts that same circle BbD also on the other side of the line AC, at the point I, if a straight line is extended from I through E, the same two triangles will be constituted, as is evident.

Now the base GN will become known through the arc of the equator which the straight lines drawn out from the pole of circle NOQ through the points N, G cut off; and this pole will be found thus. Drawing the straight line NOq, take the quadrant qr. For the line Nr will cut the line PS at f, [which is] the sought pole.

The remaining side AN, moreover, is known of itself, since it is an arc of the equator. Exactly the same procedure holds in the other triangles AGQ, CGN, CGQ, and so on.

VI. ANGLE.

[Margin: Probl. 6.]

With the base, and one other non-right angle, which are not given:

From both sides about the right angle.

[Margin: To elicit the angle, together with the rest, from both sides.]

In the figure of the first problem, about the right angle ABE, let one side be given, BF, and the other, BL, which will be found if it is counted from B up to g, and the radius Ag is sent out, cutting BD at L. For arc Bg is projected onto arc BL, as we demonstrated in Proposition 1 of Book 2. Then, with arc DK taken equal to arc BF, so that the points F, K are opposite, and with the great circle FLK described through the three points F, L, K from the center f lying on the line MN, arc FL will be the base of the proposed triangle BFL. We shall find the angle BLF thus. With the perpendicular fP dropped from f, the center, to BL, cutting arc LF at R, arc LR will measure the sought angle BLF, as we showed in Book 2, Proposition 15, Number 3. We shall find the angle BFL, moreover, in this way. Arc FT measures the angle TFE, and arc FM the angle MFE. Therefore the whole angle BFL will become known if to arc FM is added an arc similar to arc FT. Or rather, the straight line FTe being drawn, the whole arc MKe will measure the whole sought angle BFL. But if the greater side Bg were assumed, and an arc equal to the smaller BF were cut off from BE, the circle to be described

[LIBRI III. ASTROLABII, continued]

would be the great circle through g and its opposite point, together with the extreme point of the side, cut off on the line BE. For in this way exactly the same triangle would be constructed.

The base FL, moreover, will be made known by the arc of the equator which the straight lines drawn out from Y, the pole of circle FLK, through the points F, L intercept — such as is arc FG.

Diagram. A square frame enclosing a great circle crossed by two perpendicular diameters, several chords, and a secondary great circle FLK, illustrating the construction for Problem VI. Signed "Ee 4" at the foot.

- O (top, on the vertical diameter produced above the circle).
- N (upper right, on the circle).
- Q (center-top, with a bar over it); B (just below Q).
- M (upper left, on the circle); X (upper right).
- S (right, outside the circle); R (right, on the circle); F (right, on the circle, just below R).
- I (left, on a dotted line).
- Γ (gamma, center-upper); f [?] (center-left, near Γ).
- Y (left of center); C (left, on the horizontal diameter).
- G (center, below B); E (center).
- h (right of center, between E and A); A (right, on the horizontal diameter).
- V (lower right, on the circle); K (lower left, on the circle).
- s, t (center-low); T (lower center-right).
- ϑ [theta, lower left]; b, e [or q], D (bottom center); G (bottom center, second occurrence); N (lower right, on the circle).

[Several small lowercase/Greek interior labels are uncertain]

VII. SIDE.

[Margin: Probl. 7.]

With both non-right angles, of which neither is given:

From the base, and one other side.

[Margin: To explore the side, together with the rest, from the base and one other side.]

In the same figure, let the side BF and the base FG be given. With the two diameters FK, MN drawn cutting one another at right angles, let the straight line MG be drawn cutting FK at H; and let arc MI be taken equal to arc NG; and through the three points I, H, G let [a circle] be described on the great circle MN, whose pole is F — the parallel GHI cutting BD at L, as was done in problem 1. For if a great circle is described through the three points F, L, K, the proposed triangle will be BFL; since FL, the base equal to the assumed base FG, from the definition of the pole [makes] the angle FBL right, and the side BF is given. But the sought side BL will be equal to arc Bg, which the radius AL cuts off, as is manifest from Proposition 1 of Book 2. And each angle, BLF and BFL, will be recognized, as in the preceding problem.

VIII. SIDE.

[Margin: Probl. 8.]

With one other side, and a non-right angle, not given:

From the base, and the angle which is opposite the sought side.

[Margin: To inquire into the side, together with the rest, from the base and the angle which is opposite the sought side.]

In the figure of problem 5, let Ah be the arc of the given angle; and, the radius Bh being drawn cutting AC at b, let the greatest circle be described through B, b, D, so that ABb is the given angle. Then, the quadrant hm being taken, and the radius Bm drawn cutting AC at n — the pole of circle BbD, as was shown in Book 2, Proposition 8, Number 17 — let the given base be counted from B up to p, the point from which a straight line is drawn to n, the pole of circle BbD, cutting the same circle at i: and arc Bi will be equal to the base Bp, through the things demonstrated in Book 2, Proposition 5, Number 17. Therefore, with the straight line Ei drawn cutting the equator at N, the proposed triangle will be BiN, since the angle N is right, and the base Bi is given, together with the angle iBN, which is opposite the sought side iN. That side iN will be recognized if, from R, the pole of the great circle NEQ, a straight line is drawn through i. For this will cut off from the equator an arc, equal to arc iN, beginning at the point N. Or, if through i a parallel is described cutting AE at k, then arc Ak is equal to arc Ni, and it will become known through the straight line Bk, since this cuts off Ag equal to that same Ak (or Ni), as is clear from Proposition 1 of Book 2.

The other side, BN, moreover, is known of itself, since it is an arc of the equator.

Finally, the remaining angle BiN will be made known if, from Y , the center of circle BbD , a perpendicular is drawn to iE produced, cutting the same circle at f . For arc if will measure the angle eif — that is, the vertical angle equal to it, BiN — as was shown in Proposition 15, Number 3, of Book 2.

But although this problem has been solved by us for the case when the given angle is acute and the given base smaller than a quadrant, it will be solved in the same way if the given angle is acute and the given base greater than a quadrant; or if the given angle is obtuse and the given base smaller, or greater, than a quadrant. For if, to the given acute angle, ADb is made equal, and — the base Dp being assumed greater than a quadrant — there is cut off from n , the pole of circle BbD , [an arc] Di equal [to it], by the radius np , then the straight line Ei will constitute the proposed triangle DiN . By the same reasoning, if the given obtuse angle is counted from C toward D , up to h , and the radius Bh is drawn cutting AC at b , the greatest circle BbD will constitute the given obtuse angle CBb . If therefore the given base is also counted from B up to p , smaller than a quadrant, the straight line iE — extended through the point i , cut off from the line np drawn from the pole n — will constitute the produced triangle BiQ ; and the sought side iQ , to which the given obtuse angle is opposite, will be recognized through the arc of the equator intercepted between Q and the straight line sent out from R , the pole of circle iQ , through i . Finally, if the obtuse angle CDB is given, and the base Dp greater than a quadrant, the line np will cut off for it an equal arc Di . The straight line Ei , therefore, will constitute the proposed triangle DiQ , whose sought side Qi will be found as before.

Diagram. A square frame enclosing a great circle (the equator) crossed by many chords and secondary circles — a re-engraving of the same densely worked figure used for Problem V (the text explicitly refers back to "the figure of problem 5"). Roughly the same thirty labels appear.

- B (top) / D (bottom) / A (right, on the horizontal diameter) / C (left, on the horizontal diameter): equator cardinal points.
- E (center); d (left, on line AC): center of the auxiliary circle.
- P (upper left, outside the circle); R (below P); Z [?], S (upper-left region).
- T (on horizontal line, left of center); Y (left of center, near C).
- N , p [?], M (upper right, outside the circle).
- K (upper middle); a (near B); L (right of center); I , k [?] (right of L).
- n (center-left); o (center); e (lower center-left).
- X (lower left); V (lower left, on the circle).
- Ω [?], m , q (bottom, left of D); G (bottom center).

- H, F (lower right, on the circle); further small labels s, ϑ [?] in the lower-right interior.

[Several interior lowercase labels are uncertain]

IX. SIDE.

[Margin: Probl. 9.]

With one other side, and a non-right angle, which are not given:

From the base, and the angle which is adjacent to the sought side.

Let there be constructed, in the figure of problem 1, the triangle BLZ, from the given base BL and the given angle BLZ, exactly [as before...]

(Continuation of Problem IX. LATVS)

exactly the same as was constructed in Problem 3. The required side will then be LZ, adjacent to the given angle BLZ; and LZ will be made known by the arc of the Equator cut off by the straight lines drawn from Y, the pole of circle LZ, through the extreme points L and Z, as was shown in Book 2, Proposition 5, Number 17. But if the base DL is less than a quadrant, and the same construction is carried out, the triangle DLi will be set up, whose required side Li will again be found by the arc of the Equator which the straight lines drawn from Y, the pole of circle Li, through the extreme points L and i, cut off.

The other SIDE, BZ, will be exhibited as known by the arc of the Equator enclosed by the straight lines drawn from h, the pole of circle BZ, through B and Z, and so on.

The remaining ANGLE, LBZ, will be found as we described in Problem 3, and so on.

X. LATVS. [Problem X — a Side]

With the base and the other angle not given.

From the other side and the angle adjacent to the required side: provided that the kind (species) of the required side, or of the angles which are not right and not given, or finally of the base itself, is established.

[Margin: Probl. 10. — To investigate a side, together with the rest, from the base and the angle adjacent to the required side.]

Here too let there be constructed, in the figure of Problem 5, the very same triangle AGN which was set up in that problem, namely from the given side AG and the given angle ANG, which is adjacent to the required side AN.

For when the given side is less than a quadrant: if it is established that the required side is less than a quadrant, the required side will be AN, in triangle AGN; but if it is established that the required side is greater than a quadrant, the required side will be AQ, in triangle AGQ. And when the given side is greater than a quadrant: if it is established that the required side is less than a quadrant, the required side will be CQ, in triangle CGQ; but if it is established that the required side is greater than a quadrant, the required side will be CN, in triangle CGN, and so on. And, as you see, the required side is always an arc of the Equator, and hence is known.

The BASE, however, GN, will be known from the arc of the Equator which the straight lines drawn through the points N and G intercept, from f , the pole of circle NOQ (found near the end of Problem 5). And we shall find the remaining ANGLE, AGN, as was taught in the same Problem 5, and so on.

Diagram. A square frame enclosing a large circle (the Equator) with center E and two perpendicular diameters, overlaid with numerous great-circle arcs, chords, and construction lines forming the stereographic "net" used for the spherical-triangle problems. This is the figure of Problem 5, reused for Problems X and XI. Readable labels (positions approximate):

- P (upper left, on a line outside the circle)
- B (top of circle)
- R, Z, S (upper-left interior arcs)
- N (upper right of circle)
- M (right, just outside the circle)
- *a* [?] (small italic, upper center-interior)
- K, L (central region)
- *b*, R (right of center)
- A (right end of the horizontal diameter)
- Y, C, Y (left region, about the horizontal diameter near C)
- *d*, T (far left, outside the circle on the horizontal line)
- *n*, I (left and right of center)
- E (center)
- α [?] (small italic, just below E)

- e, o (lower center-left)
- X (lower left)
- Ω [?], m (lower left)
- q, D (bottom)
- H, F (lower right)
- Greek edge-labels φ , ρ , χ , ζ [?] (lower-right circumference)
- V (bottom-left corner, outside)

Many peripheral labels are small and only partly legible. (more than 2 labels flagged [?]).

XI. LATVS. [Problem XI — a Side]

With the base and the other, non-right, angle not given.

From the other side and the angle which is opposite the required side.

[Margin: Probl. 11. — To investigate a side, together with the rest, from the other side and the angle, adjacent [to the required side], opposite the required side.]

In the same figure of Problem 5 let the given angle be set up (if it is acute) as ABb, as in Problem 8. Then, taking the given side BN, let there be drawn from N, through E, the pole of the Equator, the greatest circle NEQ, cutting circle BbD in i; and BiN will be the proposed triangle, since the angle BNi is right, and the given angle NBi is opposite the required side Ni. This side will be made known by the arc of the Equator between N and the straight line drawn from R, the pole of circle NEQ, extended through i; or by the arc between A and the straight line Bg, which is drawn through k, where the parallel drawn through i cuts the straight line AC, as is clear from Proposition 1, Book 2.

The BASE Bi will be equal to the arc of the Equator Bp, cut off by the straight lines nB, np, drawn out from n, the pole of circle BbD.

The other ANGLE, BiN, will be made known as was stated in Problem 5.

And thus indeed the matter will stand when the given side is less than a quadrant and the given angle is acute. But if the given side is indeed less than a quadrant, yet the given angle is obtuse, the required side Qi will be greater than a quadrant, in triangle DiQ; which will be set up if the given obtuse angle CDb is made — from its arc Ch and the radius Bh, cutting AC in b, the point through which circle BbD is described — making the given angle CDb; then let the given side DQ be assumed, from whose extremity the straight line QEi is drawn, and so on.

But if the given side is greater than a quadrant, and the given angle acute, let that angle ADb, that is ABb, be set up (its arc Ah having first been taken, and the radius Bh drawn, cutting AC in b), and so on. Then, taking the given side DN, let the straight line NE be drawn, cutting circle BbD in i. For the proposed triangle will be DiN, since the angle at N is right, and the given angle iDN is opposite the required side Ni, and so on; and that side Ni will be found as before.

Finally, if the given side is greater than a quadrant, and the given angle obtuse, let the given angle CBb be set up, from its arc Ch, and so on. Then, taking the given side BQ, let the straight line QE be drawn, cutting circle BbD in i, and standing for the greatest circle drawn through the poles of the Equator. The proposed triangle will therefore be BiQ, whose required side is Qi, which indeed will be known by the arc of the Equator between Q and the straight line drawn from R, the pole of circle NEQ, extended through i, and so on.

XII. LATVS. [Problem XII — a Side]

With the base and the other side not given.

From both angles, neither being right.

[Margin: Probl. 12. — To explore a side, together with the rest, from both non-right angles.] [Margin: a — Theodosius, *Sphaerica* I.15.]

Let ABCD again be the Equator about center E, with two diameters cutting each other at right angles, AC and BD, and let there first be proposed a right triangle of two obtuse angles. Let the arc of one obtuse angle be AF, and, the radius BF being drawn and cutting AC in G, let the greatest circle be described through B, G, D, so that the given obtuse angle ABG is set up. Then, taking the quadrant FH, and drawing the radius BH, AC will be cut in I, the pole of circle BGD, as is established from what was demonstrated in Book 2, Proposition 8, Number 17. If, then, through I a greatest circle is described making with the Equator the other given obtuse angle, the proposed triangle will be constructed,^a since the angle which this latter circle makes with the former, BGD, is right. And this will be done as follows.

Let CY be the arc of the other given obtuse angle. And since — as will be demonstrated in the scholium of this Canon — in every right spherical triangle either of the non-right angles is less than the arc by which the supplement of the other non-right angle differs from a semicircle; and since the arc AI is equal to the arc EG, that is, to the complement of angle ABG (because GI and EA are quadrants, by the Corollary of Proposition 16, Book 1 of Theodosius); and hence

AI, the complement of angle ABG, differs from the semicircle AC by the arc CI: therefore CY, the arc of the other obtuse angle, will be less than the arc CH, which is equal to arc CI. Therefore, the radius BY being drawn and cutting AC in M, the point M will lie between E and I; and hence, the parallel MN being described, a greatest circle can be described through I, tangent to circle MN, as we taught in Proposition 20, Book 2; which we shall describe as follows.

The straight line between I and the other pole of circle BGD is bisected at K — or, when that other pole runs off too far, K being found as the center of the three points B, I, D, which bisects the aforesaid straight line (since a circle described through B, I, D passes through the other pole, because, being a greatest circle, it proceeds through the opposite points B, D) — let there be erected, perpendicular to AC, the perpendicular KL, on which the center of the greatest tangent circle must necessarily lie, as we there demonstrated. After this, let there be made an angle MBO equal to the rectilinear angle BMC. For since a semicircle described about the line placed between I and the other pole of circle BGD passes through the point B, the extremity of the perpendicular EB, as was demonstrated in the cited place: therefore at B, on the straight line BM, the angle to be set up is equal to angle BMC; [and it will fall/cut] necessarily at the point O, as was there shown, beyond K. Then, a circle being described from E through O, KL will be cut in the points L, Z, either of which can be the center of the greatest circle drawn through I and tangent to circle MN: point L, indeed, will be the center if the tangent circle must make an obtuse angle with the Equator toward angle ABG, and the point of contact will be N, into which the straight line LE falls; but if the tangent circle must make an acute angle with the Equator toward B, its center will be Z, and the point of contact will be indicated by the straight line ZE drawn, as was there shown.

[Margin: b — Theodosius, *Sphaerica* I.15.]

Therefore, a greatest circle being described from L (since we desire the obtuse angle) through I, which will touch circle MN in N and pass through the other pole of circle BGD, and cut the Equator in the opposite points Q, S — in such a way that the straight line QS is perpendicular to LN — if no error has been made, the proposed triangle will be BPQ:^b since the angle P is right, and angle ABG is one of the given obtuse angles, and BQP is the other, because its arc RN is equal to arc CM, that is, to the arc CY assumed. And if the radius SNT is drawn, and the quadrant TV is taken, so that the radius SV exhibits X, the pole of circle QNS (which will necessarily lie in the common section of the straight line EL with circle BGD — for since circle QNS passes through I, the pole of circle BGD, it will

in turn pass through the poles of that circle; since, then, the pole of circle QNS lies on the straight line EL, as the common section of the straight line EL with circle BGD, the pole of circle QNS): we shall know the side PQ by the arc of the Equator between Q and the straight line drawn from the pole X, extended through the extreme point P.

The side BP will be known by the arc of the Equator between B and the straight line drawn from the pole I, sent through the extreme point P, as we demonstrated in Book 2, Proposition 5, Number 17.

Let there then be proposed a right triangle of two acute angles. If, therefore, a right triangle of two obtuse angles is constructed, whose angles are the supplements to a semicircle (or to two right angles) of the given acute ones, as was just now stated — namely triangle BPQ — the proposed triangle will be DPS, since the angle P is right, and the others acute, whose supplements to two right angles are the obtuse angles ADG (or ABG) and RSN (or RQN). The side DP, then, will be equal to the arc of the Equator which the straight lines ID, IP (if they are drawn) cut off; and the side PS will be equal to the arc of the Equator cut off by the straight lines XP, XS (if they are drawn).

Thirdly, let the proposed triangle be right-angled, one of its remaining angles being acute and the other obtuse. Let there therefore again be constructed the triangle BPQ, right-angled and having two obtuse angles, of which one, ABG, is given, and the other, RQN, is the supplement to two right angles of the given acute angle. For the proposed triangle will be DPQ, having the right angle P, the given obtuse angle PDQ, and the acute angle DQP, whose supplement to two right angles is the angle PQB set up. The side PD, then, will be made known by the straight lines drawn from I, the pole of circle BGD, through P and D; and the side PQ by the straight lines drawn from the pole X of circle QNS, through P and Q.

In all these triangles, however, the base BQ (or DS, or DQ) is known of itself, since it is an arc of the Equator.

Diagram. A square frame enclosing a large circle (the Equator ABCD) with center E and the two perpendicular diameters AC (horizontal) and BD (vertical), overlaid with the great-circle arcs and chords of the construction for Problem XII.

Readable labels (positions approximate):

- B, Y (top, close together)
- L (upper left)
- R (upper left-center)
- Q (upper right)

- X (center-left)
- O, C (left; O just above C, C at the left end of the horizontal diameter)
- K (left-center)
- E (center)
- I (right of center)
- A, T (right end of horizontal diameter, T just below A)
- G, M, N (lower center)
- Γ [?] (bottom center)
- M [?], Σ [?] (lower left)
- D (bottom of circle)
- Y (lower right)
- H (right, lower)

Several small peripheral labels are only partly legible. (more than 2 labels flagged [?]).

XIII. BASIS. [Problem XIII — the Base]

With the other side and the angle not given.

From a side and the angle adjacent to it.

[Margin: Probl. 13. — To find the base, together with the rest, from a side and the angle adjacent to it.]

In the figure of Problem 1 let the given side be BF, and at F let there be constructed the angle BFL equal to the given angle, as in Problem 4: which will be done if, taking the arc Me of the given angle, a radius goes out from F through e, cutting MN in T (the two diameters FL, MN having first been drawn dividing each other at right angles), and through the three points F, T, K a circle is described from center f, which will be a greatest circle, since it proceeds through the opposite points F, K. The proposed triangle, therefore, will be BFL; whose base FL will be found by the straight lines drawn from Y, the pole of the base (which will be found if, the radius KTL being drawn, the quadrant VX is taken; for the radius KX will cut the straight line MN in Y, the pole), drawn out through F, L.

The other SIDE, BL, will be equal to the arc of the Equator Bg, cut off by the radius AL.

The remaining ANGLE, BLF, will be known either by the arc LR, or by the arc QS doubled, and so on.

XIV. BASIS. [Problem XIV — the Base]

With the other side and the angle not given.

From a side and the angle opposite it: provided it is established whether the base is greater or less than a quadrant; or whether the angle not given is acute or obtuse; or finally whether the other side not given is less or greater than a quadrant.

[Margin: Probl. 14. — To investigate the base, together with the rest, from a side and the angle opposite it.]

Let there be made, in the figure of Problem 5, from the given side and the opposite angle, the triangle AGN, as in Problem 5: which will be done if, taking the given side AF and the arc AH of the given angle (which will be greater than the arc AF, as was stated in Problem 5), the rest are also constructed as was there done. For the proposed triangle will be AGN, if it is established that the base is less than a quadrant; or AGQ, if it is established that the base is greater than a quadrant. But if the given side is greater than a quadrant, the proposed triangle will be either CGN or CGQ, according as it is established that the base is less or greater than a quadrant. The base, however, GN (or GQ), will be made known by the arc of the Equator cut off by the straight lines drawn through the points G, N (or G, Q) from f, the pole of circle NGQ; which pole will be found if, the straight line NOq being drawn, we take the quadrant qr. For the radius Nr will indicate f, the required pole, on the straight line PS, as is clear from Book 2, Proposition 8, Number 17.

The other SIDE — AN, or AQ, or CN, or CQ — will be known of itself, since it is an arc of the Equator.

The remaining ANGLE at the point G will be known, as was stated in Problem 5.

XV. BASIS. [Problem XV — the Base]

With both angles, neither being right, not given.

From both sides.

[Margin: Probl. 15. — To hunt out the base, together with the rest, from both sides.]

In the figure of Problem 1 let there be two given sides, BF and Bg; and from Bg let there be taken away, by the radius Ag, the arc BL equal to it. Then, the diameter FK being drawn — which MN cuts at right angles — let the greatest circle be

described from center f through the three points F, L, K . For the required base will be FL , in the triangle BFL of given sides, which we also set up in Problem 6. One might also assume the greater side Bg , and cut off from the lesser side BF an arc equal to it out of the straight line BE , and so on, as was stated in the said Problem 6. Further, the base FL will be known by the arc of the Equator cut off by the straight lines drawn through the points F, L from the pole Y of circle FLK — which pole will be found on the straight line MN if, the radius KTV being drawn, the quadrant VX is taken, and the radius KX is drawn out cutting MN in Y .

Each ANGLE, BLF and BFL , will be known, as in Problem 2.

Diagram. A square frame enclosing a large circle (the Equator) with center E and the two perpendicular diameters, overlaid with the great-circle arcs, chords, and dotted construction lines of the figure of Problem 1, used for Problems XIII–XV. Readable labels (positions approximate):

- O (top)
- N [?] / Z [?] (top, near O)
- Q (upper center)
- M (upper left of B)
- B (upper center)
- X (upper right of B)
- S (right, upper)
- R, F (right; R above F , on the circumference)
- f (left of center)
- Γ [?] / r [?] (center)
- Y (center-left)
- C (left end of horizontal diameter)
- A (right end of horizontal diameter)
- h (right of center, on horizontal)
- H, E (center; H left of E)
- V (right, below A)
- K (lower left)
- I (upper left, on a dotted line)
- L, T (lower center)
- δ [?] / ϑ [?] (lower left)
- b, D, G, N (bottom row)

Several small peripheral labels are only partly legible. (more than 2 labels flagged [?]).

(Problem XVI. BASIS begins at the foot of the page and continues on the following leaf.)

ASTROLABE, BOOK III — Canon XXII (continued)

XVI. THE BASE, with neither side given.

From both non-right angles.

[Margin: To investigate the base, together with the rest, from both non-right angles.]

Construct exactly the same triangle of given angles that was constructed in Problem 12 — BPQ, or DPS, or DPQ — according as each given angle turns out to be obtuse, or acute, or the one acute and the other obtuse. In all of these cases the base, BQ or DS or DQ, is known, since it is an arc of the equator.

Each of the two sides, moreover, will be made known, as we taught in Problem 12.

And so all the problems of right-angled spherical triangles are now dispatched. There follow now the oblique-angled triangles — those, that is, in which none of the angles is right.

Diagram. A square frame enclosing a large circle with two perpendicular diameters and numerous intersecting great-circle arcs and chords, the construction figure for Problem XVI (the base). Readable point-labels, by position:

- B, V (top, adjacent)
- L, R (upper left)
- Q (upper right)
- X (upper middle)
- O, K (left, on the horizontal diameter)
- C (left, just below O)
- E (center)
- I (right of center)
- A, T (right edge, A above T)
- G (lower-left of center)

- M (lower-right of center)
- N (below center)
- S, Z, F (lower left)
- P (lower middle)
- Y (lower right)
- H (right, lower)
- D (bottom)

XVII. ALL THE SIDES of an oblique-angled triangle.

[Margin: Problem 17.]

From all the angles.

[Margin: To find the angles that a perpendicular arc, let fall onto the opposite side in a spherical triangle, makes at the opposite angle.]

In any triangle of this kind at least two of the angles will be acute, or [two] obtuse, if not all three are acute or obtuse. Let there be, then, an oblique-angled spherical triangle ABC of given angles, two of whose angles, B and C, are obtuse or acute; and let a perpendicular arc AD be understood to be dropped from the remaining angle A, whatever it may be, onto side BC — an arc which, by Proposition 57 of our *Spherical Triangles*, will fall inside the triangle. First, then, one must investigate the two angles BAD and CAD, in this way. In some circle take the sines of the complements of angles B and C, having the ratio that the straight line E has to the straight line F. Then in the circle GHI, whose center is K, take GI, the arc of angle A, and let its chord GI be cut at N, by the scholium to Proposition 10 [recte: by Euclid VI.10], so that GN is to NI as E is to F; and from the center K, through N, draw the straight line KNH. I say that GH is the arc of angle BAD, and HI the arc of angle CAD. [Margin: a — Euclid VI.4.] For, drawing from G and I the perpendiculars GL, IM to KH — that is, the sines of the arcs GH, HI — since the angles L and M are right, and therefore equal, and likewise the vertical angles at N are equal, the triangles GLN, IMN will be equiangular. ^aTherefore GN will be to GL as IN is to IM; and, alternately, GN is to IN as GL is to IM. But GN is to NI as E is to F, that is, as the sine of the complement of angle B is to the sine of the complement of angle C. Therefore GL is also to IM as the sine of the complement of angle B is to the sine of the complement of angle C. And so, since by Proposition 61 of our *Spherical Triangles* the sines of the complements of angles B and C have the same ratio that the sines of angles BAD, CAD have, GL will likewise be to IM as the sine of angle BAD is to the sine of angle CAD. Since, then, GL and IM are the

sines of the arcs GH and IH, GH will be the arc of angle BAD and IH the arc of angle CAD, because the sines of the angles are the same as those of the arcs that measure the angles. The angles BAD and CAD will therefore be known, since their arcs GH and IH are known.

Diagram. [Margin: A more general demonstration of Proposition 61 of our *Spherical Triangles*.] The lower figure comprises three parts:

- A circle with readable labels C (left), H (top), M and N (center, adjacent), L, K (center-low), I (right).
- Below it, a curvilinear (spherical) triangle: A (apex), B (lower left), D (bottom), C (right).
- Two short straight reference segments, labelled E (upper) and F (lower), representing the two given lengths.

But because it can happen that, with angle BAC being obtuse, the perpendicular arc AD makes one or the other of the angles BAD, CAD right — whereas Proposition 61 of our *Spherical Triangles* was demonstrated by means of Proposition 42 of the same, which has place only in a triangle having a single right angle, and not two, such as BAD, ADB, or CAD, ADC could be — it will be possible to demonstrate the same by Proposition 61 when one of the angles at A is right, in this way. Let angle BAD be right first. Then, since ADB is also right, AB and DB will, by Proposition 25 of our *Spherical Triangles*, be quadrants, and therefore AD will be the arc of angle B. Therefore, as the sine of the right angle BAD is to the whole sine, so the sine of the complement of angle B is to the sine of the complement of arc AD, since in both cases the ratio is one of equality. For the complement of angle B and [the complement] of arc AD are the same, since AD is the arc of angle B. But by Proposition 42 of our *Spherical Triangles*, as the sine of angle DAC (which is always less than a right angle, since the whole angle BAC is less than two right angles, and BAC is here posited as right) is to the whole sine, so the sine of the complement of angle C is to the sine

—	—
sine of the right angle BAD	sine of the complement of angle B
whole sine	sine of the complement of arc AD
sine of angle DAC	sine of the complement of angle C

of the complement of arc AD. And, conversely, as the whole sine is to the sine of angle DAC, so the sine of the complement of arc AD is to the sine of the

complement of angle C. Therefore, *ex aequali*, as the sine of the right angle BAD [printed "BAD, B" — [?]] is to the sine of angle DAC, so

the sine of the complement of angle B is to the sine of the complement of angle C — which was what was to be demonstrated in the said Proposition 61.

Let angle CAD next be right. Then, as we have just demonstrated, it will follow that, as the sine of the right angle CAD is to the sine of angle BAD, so the sine of the complement of angle C is to the sine of the complement of angle B. And, converting, as the sine of the complement of angle B is to the sine of the complement of angle C, [so the sine of the right angle CAD is] to the sine of angle BAD; [and so] the sine of the complement of angle C is to the sine of the complement [of angle B] [?]; [so, alternately,] as the sine of angle BAD is to the sine of the right angle CAD, so the sine of the complement of [angle B is to the sine of the complement of angle C] [?] — which is what was proposed.

[Margin: To elicit the three sides from the three angles.]

The arcs of angles BAD and CAD having been found — the arcs which the arc AD, perpendicular onto side BC, makes — let the equator of the astrolabe, ABCD, about the center E, be constructed equal to the circle GHI above, so that the arcs of the angles that have been found may more easily be transferred onto it; and let two diameters BD, AC be drawn, cutting each other at right angles. Now, taking the arc AF of angle BAC, which is of course to be equal to the arc GHI of the figure above (or at least similar to it, if the equator ABCD has not been drawn equal to the circle GHI), and drawing the radius BF, which cuts AC at I, let the greatest circle BID be described through B, I, D, so that angle ABI is made equal to the given angle BAC. Then, taking the arc AG of angle CAD, which is of course to be equal or similar to the arc HI of the figure above, and drawing the radius BG, which cuts AC at H, let the greatest circle BHD be described through B, H, D, so that angle ABH is made equal to angle CAD, and consequently the remaining angle IBH equal to the remaining angle BAD. Taking also the quadrant GP, the radius BP will give, on the straight line AC, the pole K of circle BHD. And since arc CK is equal to arc EH — that is, to the complement of angle ABH, or CAD, of the figure above (because CE and KH are quadrants) — the complement of angle ABH, or CAD, of the figure above will differ from the semicircle AC by the arc AK. If, then, one takes AL, the arc of angle ACD given in the right-angled triangle ACD of the figure above, and draws the radius BL, which cuts AC at M, the point M will lie between A and K; because, as we shall show in a scholium, in the right-angled triangle ACD the angle C is less than the arc AK, by which the complement of angle CAD of the figure above, or of angle ABH in this figure, differs from the

semicircle. (But if angle C were acute — whose arc would of course be AN, if CM were taken equal to it — the point M would still fall between A and the pole of the circle described through B, N, D, because, as will be demonstrated in the same scholium to this Canon, in every right-angled triangle either of the two remaining angles — here the acute angle C, and therefore also CM, the arc of that same angle C — is greater than the complement of the other, that is, than the arc of the circle CEA, from the point C up to the pole of the circle described through B, N, D.) The parallel described from E through M will therefore be contained wholly between the points A and K; and consequently, if through K a parallel MON is described tangent to the circle KS, we shall have the proposed triangle BKS of the given angles, as will be proved. And in this way, by Proposition 20 of Book 2, or by Lemma 41, we shall describe the tangent circle through K. Bisecting the straight line between K and the other pole of circle BHD at Y — or, when the other pole is too far distant, finding the center Y of the three points B, K, D, which will bisect the said line, since the circle described through B, K, D also passes through the other pole — let the perpendicular YV be erected upon AC, on which the center of the circle to be described will lie; and this is found in the following way. Upon the rectilinear angle BMA let MBT be made equal, and the point T will necessarily fall beyond Y; and the parallel described from E through T will cut the straight line YV at the points V, X, each of which can be the center of the tangent circle — all of which we demonstrated in the said Proposition 20 of Book 2 and in Lemma 31. The point V will indeed be the center if each of the given angles C, B is obtuse; otherwise the point X will be the center. Let us suppose, then, that both angles are obtuse; and, drawing the straight line VEO, let there be described from V, through K, a circle which, as we showed in the said Proposition 20 of Book 2, will touch the circle MON at O, and will cut the circles BID, ABC at two points, namely R and S. I say that BRS is the proposed triangle. For since the greatest circle ZEO passes through the poles of circle KOS and of the equator — because its center and poles, and the center of the astrolabe (that is, the pole of the equator), lie in the same straight line, as was shown in Book 2, Proposition 8, Number 19 — the circle KOS and the equator EO will in turn pass through its poles; and therefore S will be the pole of the greatest circle ZEO drawn through the poles of the world; and consequently ZO will be the arc of angle BSR. Wherefore, since arc ZO is greater than a quadrant, and equal to the arc AM of angle C, the angle OSR will be obtuse and equal to angle C. [Margin: a — Theodosius I.13.] ^aAnd since angle BQS is right, and therefore equal to the right angle ADC, the three angles of triangle BQS will be equal [to the three angles of triangle ADC of the figure above]; and therefore, by Proposition 19 of our *Spherical Triangles*, side BS will be equal to side AC, side PQ to side AD, and side QS to side DC. Again, since in triangle BQR the two [angles

B, Q are equal to the two angles A, D in triangle ADB, and the adjacent side BQ] has been shown equal to the adjacent side AD, by Proposition 20 of our *Spherical Triangles* side BR will also be equal to side AB, and side QR to side DB, and angle R to angle B. The whole triangle BRS is therefore equilateral and equiangular to the whole given triangle ABC. Now the side BS is known, as being a part of the equator; the other two, however, will be known by means of straight lines drawn from their poles through the extreme points: which poles will be found thus. Taking the quadrant Fa, the radius Ba will give, on AC, the pole N of circle BR. Then, since the greatest circle KS is drawn through K, S, the poles of the greatest circles BHD, ZO (for it was shown that S is the pole of ZO itself), these will in turn pass through its poles; and therefore the pole of circle KS will be the point b, where the circles ZO, BQ intersect each other.

But if the angles C, B are posited as acute, the circle to be described will be one from X, through K, which will touch the circle MON at the extreme point of the straight line extended from X through E, and will make with the equator an acute angle equal to angle C, since its arc is then less than a quadrant, and so on.

Diagram. A large square frame enclosing the equatorial circle ABCD about center E, with two perpendicular diameters and a dense web of intersecting great circles, chords, and one dotted arc — the principal construction figure for Problem XVII. Readable point-labels, by position:

- A (right), B (top), C (left), D (bottom) — the four quadrant points of the equator
- E (center)
- V, f, z, b (upper right, along a dotted arc)
- K, i, d, N (center, left of E)
- T (right, below A)
- R, M, o, i (center-left cluster)
- H, Y (below center)
- S (right, lower)
- X (lower right)
- P, F, L, e (lower left)
- Q, G (bottom center)

Small lower-case and Greek-form labels in the crowded interior (a "8"-shaped mark upper left, an "a"/ α near G, and the cluster i, o, d, e about the center) are only partly legible; several cannot be resolved with confidence.

XVIII. ALL THE ANGLES of an oblique-angled triangle.

[Margin: Problem 18.]

From all the sides.

[Margin: To extract the three angles from the three sides.]

In the equator ABCD, whose center is E and [whose] two diameters AC, BD cut each other at right angles, take three arcs equal to the three given sides: BF, BH, FG. About the pole B, or D, by Proposition 18 of Book 2, describe — parallel to the greatest circle AC drawn through the poles of the world — the parallel HYP through the point H, which is done as follows. Drawing the straight line AH, which cuts BD at Y, take the arc AP equal to arc CH. For the circle described through the three points H, Y, P, having its center on the straight line BD, will be a parallel of the greatest circle AC, [drawn] through H. Again, drawing the diameter FL, which MZ cuts at right angles, describe about the pole F, or L — parallel to the greatest circle MZ drawn through the poles of the world — the parallel OIG through the point G, in this way. Drawing the straight line MG, which cuts FL at I, take the arc MO equal to arc ZG; and through the three points O, I, G describe the circle OIG, having its center on the straight line FL, which will be a parallel of the greatest circle MZ — all of which we demonstrated in Book 2, Proposition 18, Number 5. Now these two parallels will cut each other mutually, if the problem is possible, at the point K. If, then, through the three points F, K, L a greatest circle FKL is described, and through the three points B, K, D another, BKD, the proposed triangle BFK will result, since the side BF is one of the given sides, and BK, by the definition of a pole, is equal to another given side BH, and FK is equal to the third given side FG. The angles of this triangle will be found thus. Drawing the radii FaR, Bbs, the arc MR will give the magnitude of angle BFK, and the arc AS the quantity of angle FBK. Finally, drawing the straight line KE, which the diameter NQ cuts at right angles, if the center T of the three points N, K, Q is found, and upon KT the perpendicular TV is erected, the arc KV will measure angle VKT, and the arc KX will measure angle XKT, as was shown in Book 2, Proposition 15, Number 3. If, then, to the arc KV there is added

Diagram. A large square frame enclosing the equatorial circle about center E, with two perpendicular diameters, several great-circle arcs and two parallels intersecting at K, and dotted construction lines — the construction figure for Problem XVIII. Readable point-labels, by position:

- B (top), C (left), A (right), D (bottom)

- E (center)
- V, o (top, along a dotted arc from B)
- N (upper left), M (upper right)
- L (left), T (right)
- I, f (center, near E)
- R, H, G, S (lower left, with a dotted line)
- Y (below center)
- P, X (lower right), F (lower right)
- Z (lower left, near foot)

Small lower-case and Greek-form marks in the interior (a "δ"/"θ"-shaped label upper left, "α"/α near the lower center, and e, d, t about the center) are only partly legible; more than two cannot be resolved with confidence.

an arc similar to arc KX, the arc of the whole angle BKF will be obtained.

XIX. A SIDE WITH THE TWO ADJACENT ANGLES, in an oblique-angled triangle.

[Margin: Problem 19.]

From the remaining two sides and the angle contained by them.

[Margin: To gather a side together with its two adjacent angles, from the remaining two sides and the angle they contain.]

In the figure of the preceding problem let one of the given sides be BF. Taking the arc AS of the given angle, and drawing the radius BS, which cuts AC at b, let a greatest circle be described through the three points B, b, D, such that the given angle is ABb. Then, taking the quadrant Sd, let the radius Bd be drawn, cutting AC at e, the pole of circle BbD, as is clear from what we demonstrated in Book 2, Proposition 8, Number 17. If, then, one takes the arc BH equal to another given side, and from e, the pole [of circle BbD], a straight line eH is emitted, there will be cut off from the circle BbD an arc BK equal to arc BH — that is, to the other given side. Finally, drawing the diameter FL, which the diameter MZ cuts at right angles, and describing through the three points F, K, L the greatest circle FKL, having its center on the straight line MZ, the proposed triangle BKF will have been constructed, since the two given sides are BF, BK, together with the angle FBK contained by them. Now, drawing the straight line FaR and taking the quadrant Rg, if the straight line Fg is drawn, MZ will be cut at f, the pole of circle FKL. The straight line fK will therefore cut off an arc of the equator FG equal to

the sought side FK. And the angles BFK, BKF will be known, as in the preceding problem.

The problem will be solved no differently if the given angle is acute. For let one of the given sides be BL, and CS the arc of the given acute angle. Then, drawing the radius BS, which cuts AC at b, the circle described through the three points B, b, D will constitute the given acute angle LBb. Then, taking the other given side [BH, and if from e, the pole of circle BbD, a straight line] eH is drawn, there will be cut off an arc BK equal to this other given side BH. Finally, drawing the diameter LF, which MZ cuts at right angles, if through the three points L, K, F a greatest circle is described, the proposed triangle BLK will have been constructed. And the straight line fK, emitted from f, the pole of circle KL, will cut off the arc LH equal to the sought side LK. The angles at L, K will be found as before, in the manner set out in Book 2, Proposition 15. For the angle BLK is equal to the angle BFK once found; and if the found angle BKF is taken from two right angles, the remainder will be the sought other angle BKL.

XX. TWO SIDES WITH THE ANGLE CONTAINED BY THEM, in an oblique-angled triangle.

[Margin: Problem 20.]

From the remaining side and the two angles adjacent to it.

[Margin: To investigate two sides and the angle contained by them, from the remaining side and the angles adjacent to it.]

In the same figure of Problem 18 let the given side be BF, from whose ends the diameters BD, FL are drawn, which the other diameters AC, MZ cut at right angles; and let AS be the arc of the angle to be constituted at B, and MR the arc of the angle to be constituted at F. Then, drawing the radii BS, FR, which cut AC, MZ at b, a, if a greatest circle is described both through the three points B, b, D and through the three F, a, L, the proposed triangle BFK will have been constructed, since it has the given side BF, together with the two given adjacent angles FBK, BFK. For these angles are measured by the arcs AS (or Ab) and MR (or Ma), as we showed in Book 2, Proposition 15. And when e, f, the poles of the circles BbD, FaL, have been found (which is done if, taking the quadrants Sd, Rg, the radii Bd, Fg are drawn out, cutting AC, MZ at e, f, the poles, as is clear from Book 2, Proposition 8, Number 17), the straight line eK will cut off the arc BH equal to side BK, and the straight line fK the arc FG equal to side FK. The angle BKF, moreover,

will become known, as in Problem 18, since the arc KV measures the one part of it, VKT, and the arc KX its other part, XKV, and so on.

XXI. TWO SIDES WITH ONE ANGLE OPPOSITE ONE OF THEM, in an oblique-angled triangle.

[Margin: Problem 21.]

From the remaining two angles and the remaining side, which is opposite one of them — provided the kind of the sought side opposite the other given angle is established.

Let ABCD be the equator, about center E, with the diameters AC and BD cutting each other at right angles. Take the arc AF equal to one of the two given angles — which for the present let be assumed obtuse — and, drawing the radius BF, which cuts AC at G, let a circle be described through B, G, D, so that the given angle is ABG. Take also the quadrant FH, and join the radius BH, which cuts AC at I, the pole of the circle BGD. Again let the arc BK be equal to the given side, and join the straight line IK, which will cut off the given side BL — equal, that is, to the arc BK. After this, taking the arc BY of the other given angle, and drawing the radius AY, which cuts BD at Z, let a circle be described through A, Z, C, so that the angle BAZ is made equal to this other given angle. Then, describing from E, through Z, the parallel ZR, the point L will necessarily lie outside it, if the problem is possible.

Now if the given side BL is greater than a quadrant, and the parallel cuts the circle BGD (as at d, e), then the latter given angle must be obtuse, and greater than the former given angle ABG; and the species of the arc opposite the angle ABG must be entirely established. But if it does not cut it, the latter given angle may be either obtuse or acute, and the problem will be solved even if the species of the arc opposite the angle ABG is not established. On the other hand, if the given side is less than a quadrant — namely DL — and the parallel cuts the circle BGD, then the latter given angle must be acute, and the species of the arc opposite the given obtuse angle ABG must be established. But if it does not cut it, the latter given angle may be either acute or obtuse, and it will not be necessary that the species of the arc opposite the angle CDG be given.

[Margin: To investigate two sides and the angle opposite one of them, from the two remaining angles and the remaining side opposite one of them. Various cases of the problem.]

But if the given angle set up in the first place — CBG — is acute, and the given side BL greater than a quadrant, and the parallel intersects the circle BGD, then the

other given angle will necessarily be obtuse, and the species of the side opposite the given angle CBG must be established. But if the parallel does not cut the circle, the latter given angle may be either obtuse or acute, and it will not be necessary that the species of the side opposite the given angle CBG be given. But if the given side is less than a quadrant — namely DL — then, if the parallel cuts the circle, the latter given angle must be acute, and the species of the side opposite the given angle CBG must be established. But if it does not cut it, the latter given angle may be obtuse or acute, and the problem will be solved even though the species of the side opposite the given angle CDG is not given. All of which we shall demonstrate.

So let the parallel described through Z first cut the circle BGD. Then the other given angle BAZ will necessarily be obtuse, and greater than the former given angle ABG. For the arc described through L, making with the equator an angle equal to this latter given angle, must be tangent to the parallel described through Z — which the circle AZC also touches, as is established from the first theorem of the scholium on proposition 21 of Book 2 of Theodosius [Theodosius] — because these two circles, making equal angles, are equally inclined to the equator. Since, then, through L two great tangent circles can be described — one touching at P, the other touching at R, as we shall shortly show — each of them, when produced, will cut the semicircle BAD at the points Q, S, below the point L; for neither of them, when produced above the point L toward B, can cut the arc BL before the point B — otherwise BL would be an arc greater than a semicircle — (a) since great circles bisect each other. Therefore both the angle BQL and the angle BSL will be obtuse, and equal to the obtuse angle BAL, but greater than the given obtuse angle ABG [?], inasmuch as the arc BZ of the angle BAZ is greater than the arc AG of the angle ABG; because the portion of the straight line AC, between A and the parallel near G (which is equal to BZ itself), is greater than AG.

[Margin: (a) 11. 1. Theod. [?]]

Diagram. A large woodcut set in a square frame: the equator drawn as a circle about center E, with several great circles and their parallels superimposed for the oblique-triangle construction of this problem, together with faint zodiacal symbols marking points on the rim. Readable labels (positions approximate):

- B (top center), with an adjacent symbol resembling "R" or a zodiac sign [?]
- Zodiac symbol at upper left, near B [?]
- K (upper middle, below B)
- V (upper right of center)
- M (right of center, below V)

- O (right)
- T (upper left)
- G (left of center)
- E (center)
- I (right of E)
- A (right), with Q just below it
- C (left)
- R (lower left) [?]
- X (lower center-left)
- L (lower center)
- P, Z (lower center, below L)
- K (lower left)
- F, Y (lower left)
- H (lower right)
- h (right, lowercase)
- I (lower left, bottom)
- S (bottom center)
- D (bottom, on semicircle BAD) [?]

[Diagram note: many small rim labels and the zodiacal symbols are hard to resolve at this resolution — more than two are flagged [?]]

Since, then, two triangles have been set up, BQL and BSL, of which two angles — at B and Q, or at B and S — are given, together with the side BL opposite the angle Q or S: unless the species of the side LQ or LS is established (of which the one is greater than a quadrant and the other less), we shall be uncertain which of the two triangles we ought to take. (For since the external angle BQL is equal to the internal BSL, LQ and LS will together be equal to a semicircle, by proposition 15 of our *Spherical Triangles* [Clavius, *Triangula Sphaerica*]. And since, by the 4th theorem of the scholium on proposition 21 of Book 2 of Theodosius, the arc LS is less than the arc LQ — because, L being a point within the periphery of the equator, the three arcs LK, LS, LQ falling from it to the periphery of the equator are unequal, LK [?] being the least, and LS less than LQ — the arc LS will be less than a quadrant, and LQ greater.)

But if it is established that the side opposite the given angle ABL must be greater than a quadrant, then through L a tangent circle LPQ is to be described, through the point P, toward Z; but if the same side ought to be less than a quadrant, a tangent circle RLS is to be described, through the point R, tangent toward the

opposite parts — matters demonstrated in Book 2, proposition 20, and in Lemma 41. And we shall describe each tangent circle in the following way. Draw the straight line from L through E, and, having found on it the point opposite to L, let the line between those opposite points be bisected at M; or else let the diameter Th be drawn perpendicular to EL, and let the center M of the three points T, L, h be found, which will bisect the said line, since the great circle described through T, L, h necessarily passes through the opposite point. And from M let the perpendicular MN be erected; for on this lies the center of each tangent circle, which will be found as follows. Joining the straight line TX, let the angle XTV be made equal to the angle TXE; the point V will necessarily fall beyond M, as was shown in Lemma 41. So, describing from E, through V, a parallel cutting MN at N and O, N will be the center of the circle described through L and tangent to the parallel ZR at P, the endpoint of the joined straight line NEP; while O will be the center of the circle described through L and tangent to the same parallel ZR at R, the endpoint of the joined straight line OER, as we demonstrated in the said Lemma 41.

Next, in the second figure of problem 17, again setting the arc BR equal to the given side, and constructing the obtuse angle B*Ai* (or the acute angle D*Ai*) equal to the other given angle, let the parallel described through *i* not cut the circle BID. I say that in this case the latter given angle may be either acute or obtuse; because the two circles tangent to the parallel, toward the center E, cut the semicircle BAD, and the one nearer to point B makes, toward B, the acute angle B*fR*, while the more remote one makes the obtuse angle BSR. And so it is

not necessary to give the species of the side opposite the angle ABR. For if the other given angle is obtuse, the great tangent circle ROS is to be described; but if the given angle is acute, the tangent circle R*df* is to be described. For both the angle BSR will be equal to the obtuse angle B*Ai*, and the angle B*fR* to the acute angle D*Ai*, since the circles *AiC*, *fRe*, ROS are similarly inclined to the equator. Nor indeed can any arc other than R*f* be drawn making an obtuse angle equal to B*Ai* that would form, with the arc BR at R, an angle toward E. For the tangent circle *fRe* cuts the equator in the other semicircle BCD, as at the point *e*; and likewise the tangent circle SR cuts the equator at *g*. Therefore, when the latter given angle is obtuse, the proposed triangle will be BRS; but if acute, the triangle BR*f*.

Now let the given obtuse angle be constructed, in the first figure of this problem, as ADG, and the given side DL less than a quadrant. If, then, the same things are done as before: if the parallel ZR cuts the circle BGD, the proposed triangle will be either DLQ or DLS, and the latter given angle LQD (or LSD) will always be acute,

and equal to the given acute angle DAZ. Therefore the species of the side opposite the given obtuse angle ADL must be known, so that when it is greater than a quadrant the triangle DLQ is taken, but when less than a quadrant the triangle DLS.

But if, in the second figure of problem 17, the given obtuse angle is constructed as ADR, and the given side DR less than a quadrant, and the parallel does not cut the circle BLD, then the proposed triangle will be either DRf, having the other given angle DfR obtuse, or the triangle DRS, having the angle DSR acute. And here too it is not necessary to give the species of the arc opposite the given obtuse angle ADR.

But now, in the first figure of this problem, let the given acute angle be constructed as CBG, and the given side BL greater than a quadrant, and let the parallel cut the circle BGD, and so on. The proposed triangle will then be either BLf or BLg, always having the latter given angle BfL (or BgL) obtuse. Unless, therefore, the species of the side opposite the given acute angle CBL is given, we shall be in doubt whether the side Lg (greater than a quadrant) or Lf (less than a quadrant) is to be taken, and so on.

Diagram. A woodcut set in a square frame: the equator as a circle about center E, with the great circles, parallels, and tangent circles of the several cases of this problem superimposed, and faint zodiacal symbols on the rim. Readable labels (positions approximate):

- B (top center)
- Zodiac symbol at upper left [?]
- V (upper right)
- C (left)
- K, I (left of center)
- a mark resembling "4" near the center-left [?]
- E (center), with N to its right
- A (right), T (far right)
- M (below center-left), H (center), Y (right of center)
- O, i (below center)
- S (lower right)
- P (lower left)
- F, L (lower left)
- Q (lower left, foot)

- G (lower center), with X
- D (bottom center), with dotted arcs descending

[Diagram note: rim labels and the zodiacal symbols are only partly legible — more than two flagged [?]]

But if the same are assumed, yet the parallel does not cut the circle — as in the second figure of problem 17, in which the given acute angle CBI is set up, and the given side BR greater than a quadrant — then the proposed triangle may be either BRe, having the latter given angle ReB acute, or the triangle BRg, having the other given angle BgR obtuse; and it is not necessary that the species of the side Re or Rg be given.

But if, in the first figure of this problem, the acute angle CDG is again given, but the given side DL less than a quadrant, and the parallel cuts the circle, then the proposed triangle will be DfL or DgL, always having the latter given angle DfL (or DgL) acute. It must therefore be established whether the arc Lg (greater than a quadrant) or Lf (less than a quadrant) is to be taken.

Finally, if in the second figure of problem 17 the given angle is acute — CDI — and the given side DR less than a quadrant, and the parallel does not cut the circle, then the proposed triangle will be either DRe, having the latter given angle DeR obtuse, or the triangle DRg, having the latter given angle DgR acute; nor is it required that the species of the side Re or Rg opposite the given acute angle CDR be given.

[Margin: In which cases the problem is ambiguous, and in which not.]

From all this it is clear that, when one of the given angles is set up either at B or at D, whether obtuse or acute: if the complement of the other given angle is greater than the complement of the former (as it is in the first figure of this problem), the species of the side opposite the former given angle must be given; but if it is less, this is not necessary — as is evident in the second figure of problem 17. For in the first figure of this problem, EZ, the complement of the latter given angle, is greater than EG, the complement of the former; whereas in the second figure of problem 17 the complement of the latter angle — namely Ei — is less than the arc EI, which is the complement of the former angle.

Now in all the aforesaid cases, one of the sought sides is an arc of the equator, and hence known; the other will be known if its pole is found, as was said in the preceding. The third angle too will become known, just as in the other problems. For example, in the first figure of this problem the angle BLQ will be known:

since the arc La measures its part BLE, and the arc Lb the other part QLE — the point b being set at the intersection of the straight line drawn from M with the arc LQ. And so, if to the arc La there is added an arc similar to the arc Lb, the arc of the whole sought angle BLQ will be composed, and so on.

PROBLEM XXII. Two angles, with one side opposite to one of them, in an oblique triangle.

[Margin: Probl. 22.]

[Margin: To find two angles, and one side opposite to one of them, from the two remaining sides and the remaining angle opposite to one of them.]

From the remaining two sides, and the remaining angle which is opposite to one of them, provided only that the species of the sought angle opposite the other given side is established.

Let ABCD be the equator, about center E, as before; and let one given side be BF. At F let the given angle be constructed — which for the present let be obtuse — and this is done as follows. Drawing the diameter FG, which HI cuts at right angles, take the arc HK of the given obtuse angle, and, drawing the radius FK, which cuts HI at L, the great circle described through the three points F, L, G will make the given angle HFL. Let also the other given side be BQ, greater than a quadrant, and through Q let there be described, parallel to the great circle AC, the parallel XVQ — as was delivered in Book 2, proposition 18, at the beginning of Number 5 —

namely in this way. Drawing the radius AQ, which cuts BD at V, take the arc AX equal to the arc CQ. For the circle described through the three points X, V, Q will be the said parallel, which will cut the circle FLG at the points O, P. So the great circle described through the three points B, O, D — as well as that through the three points B, P, D — will accomplish the problem. For in the triangle BOF two sides, BF and BO, are given (since the arc BO is equal to the arc BQ by the definition of the pole), together with the angle BFO opposite the given side BO. Likewise in the triangle BPF two sides, BF and BP, are given (because the arc BP too is equal to the arc BQ by the definition of the pole), together with the same angle BFP opposite the given side BP. Unless, therefore, the species of the angle opposite the other given side BF is established, we shall be in doubt which of the given triangles we ought to take. For since the sides BO, BP are equal, by the definition of the pole, and greater than a quadrant, the two angles BOP, BPO will be obtuse, by proposition 25 of our *Spherical Triangles*, and hence BPF acute. If,

therefore, it is established that the angle opposite the given side BF must be obtuse, the larger triangle BOF is to be taken; but the smaller, BPF, if it is established that the same angle is acute. But if the second given side were less than a quadrant, the two angles BOP, BPO would become acute, and hence BPF obtuse, and so on. And thus, whenever the parallel described through the extremity of the second given side cuts, within the equator, the circle that forms the given angle with the equator at the extremity of the first given side — cutting it in two places — the problem will be ambiguous, unless the species of the angle opposite the first given side is known.

But if the said parallel cuts (or touches) the said circle in only one point within the equator, the problem will not be ambiguous, since only one triangle can then be constructed. For instance, let the first given side be BF, as before, and the given angle acute, to which BFN is constructed equal (which is done if, taking HM the arc of the given angle, the radius FM is joined, cutting HI at N, and a circle is described through the three points F, N, G); and let the second given side be BR, less than a quadrant, and through its extremity R let a parallel RYZ be described to the great circle AC, cutting the circle FNG within the equator at only one point S; and finally let a great circle be described through the three points B, S, D. Then only one proposed triangle, BFS, will have been constructed. For at the other point where the parallel RYZ intersects — outside the equator, toward Z — no triangle will be constructed, because the side from the point F, through N, up to that intersection is greater than a semicircle. So too, if the first given side is BF, greater than a quadrant, and the given angle obtuse BFL, and the second given side BR, less than a quadrant, the parallel RYZ will cut the circle FLG at only one point T. And so only a single given triangle, BFT, will then be constructed.

In the same way, if the first given side is less than a quadrant — BG — and the given angle acute BGN, and the second given side BZ also less than a quadrant, the parallel ZYR will again cut the circle GNF at only one point S, and a single proposed triangle, BGS, will be constructed. But if the first side BG is less than a quadrant, yet the given angle obtuse BGL, and the second given side BX greater than a quadrant, the parallel XVQ will cut the circle GLF at two points, O and P, within the equator, and hence two triangles, BGO and BGP, will be constructed. Therefore, unless the species of the angle opposite the given side BG is given, it will be unknown which of the triangles is to be taken.

If it should ever happen that the parallel described through the extremity of the second side does not cut, within the equator, the circle that produces the given angle, then the problem is impossible, because the second side has been taken too

great or too small. For example, if the first given side is BF and the second Bd, and the given angle either obtuse BFL or acute BFN, the problem cannot be solved, because the parallel dab cuts neither of the circles FLG, FNG within the equator. For the same reason the problem will be impossible if the first given side is BG or BF, and the second Bg, whether the given angle constructed at G or F is obtuse or acute; because the parallel gfe intersects neither of the circles within the equator.

[Margin: When the problem is impossible.]

Diagram. A woodcut set in a square frame: the equator as a circle about center E, with the great circles, parallels, and tangent circles for the several cases of Problem XXII superimposed, and dotted arcs and zodiacal symbols marking rim points. Readable labels (positions approximate):

- B (top center)
- M (upper left), H (upper right)
- e (upper right, on a dotted arc)
- two zodiac symbols at upper left [?]
- f (below B, upper center)
- N (center)
- S (center-right), R (right)
- G (left), Y (center-left)
- T, Z, N (left cluster)
- C (left), A (right)
- E (center), V (below E)
- O (center-left), L (below O)
- K (left, lower)
- F (lower right), with P, X, K adjacent
- Q (lower left)
- a (lower center); b (lower left, dotted); d (lower right, dotted)
- I (bottom left), D (bottom center)

[Diagram note: the rim carries many small labels, dotted construction arcs, and zodiacal symbols; more than two are flagged [?]]

The sought remaining side — namely FO or FP, in either of the triangles BFO, BFP — will become known, as in the preceding, if the pole is found of the circle of which the said side is a portion. And the remaining two angles will be known likewise by what we wrote in Book 2, proposition 15, just as was said in the preceding.

SCHOLIUM.

Since the angles and sides of spherical triangles must have a certain definite magnitude, such that a spherical triangle can be constituted from them (as is gathered from the preceding problems) — although in astronomical matters such triangles are always proposed as actually exist in the sphere, and are not feigned at will — it seems good in this place to demonstrate a few theorems on this subject, so that we may judge whether some proposed triangle is fictitious, or truly exists in nature. Beginning here:

[Margin: Various theorems on the magnitude of the angles and sides of spherical triangles.]

1. In every right-angled spherical triangle, none of whose arcs is a quadrant: the angle opposite the side that is less than a quadrant is acute, and greater than that side; but the angle opposite the side that is greater than a quadrant is obtuse, and less than that side.

[Margin: Theorem 1.]

Let the figure of problem 5 be repeated; and let the first two sides, AG and AN, about the right angle BAE, be less than a quadrant, and, drawing the diameter NQ, let a great circle be described through the three points N, G, Q, so that the spherical triangle AGN is constituted. Then the angle ANG, opposite the side AG, will be acute; because its arc SO, which the straight line RS perpendicular to NQ measures, is less than a quadran-

is less than a quadrant — which is also established from the scholium of Proposition 28 of our [book on] spherical triangles. For since the two sides AG and AN are less than a quadrant, by that scholium each of the angles G and N is acute. I say that this same angle — that is, its arc SO — is greater than the side AG. For the parallel OI, drawn through O from E, since it touches the circle NOQ at O (by the scholium of Proposition 13 of Book 3 of Euclid), will cut AE between E and G. Therefore, since AI is equal to SO itself, it is established that SO, the arc of the angle ANG, is greater than the side AG.

Next, let the side AG be less than a quadrant, but AQ greater than a quadrant, about the right angle DAE; and once the diameter QN has been drawn, let a great circle be described through the three points Q, G, N, so that the spherical triangle AGQ is constructed. In it the angle AQG, opposite the side AG, will be acute, because its arc SO is less than a quadrant. We shall now show, as before, that this same angle — that is, its arc SO — is greater than the side AG.

Again, let the two sides CG and CN, about the right angle BCE, be greater than a quadrant, and once the diameter NQ has been drawn, let the same construction be made as before. The angle CNG in the triangle CGN, opposite the side CG, will be obtuse, on account of its arc RO being greater than a quadrant; but its arc RO — that is, CI — will be less than the opposite side CG.

Finally, let the side CG be greater than a quadrant, and CQ less, about the right angle DCE, and let the same be done. Again the angle CQG, opposite the side CG, will be obtuse, on account of its arc RO being greater than a quadrant; but its arc RO — that is, CI — will be less than the side CG.

Diagram. An orthographic ("analemma") projection of the sphere onto the plane of a great circle, drawn within a square frame with an inscribed circle and a quarter-arc, the standard astrolabe-construction figure carried over from the scholium to Canon 3. A dense mesh of chords, diameters, and projected small circles fills the disk. Readable labels: **B** (top of the circle), **D** (bottom), **P** (upper left, on the frame), **V** (lower-left corner region), **C** and **d/T/Y** (along the left horizontal), **E** (center), **G** (right of center), **A** (right margin, on the frame), **N/Ψ** (upper right), **M, p** (upper right, outside the circle), **K, L, k, l, I** (interior/right), **X** and **Ω** (lower left interior), **m, q** (near the bottom), **H, F, S, δ, Z** (lower-right periphery). Numerous small italic peripheral labels could not be resolved with confidence even under magnification.

- **B** (top of disk): endpoint of principal diameter
- **D** (bottom of disk): opposite endpoint
- **E** (center): center of the projection
- **P** (upper-left frame), **V** (lower-left corner), **A** (right frame): frame reference points
- **C, G**: points on/near the horizontal diameter
- [numerous small peripheral labels [?]]

And so, if in some right spherical triangle one side about the right angle contains 40° , the opposite angle must be acute, yet greater than 40° . And if the angle is said to be 40° , the opposite side must be less than 40° . But if one of the sides comprises 130° , the opposite angle will necessarily be obtuse, yet less than 130° . And if one of the non-right angles is put at 130° , the opposite side will be greater than 130° .

[Theor. 2.] 2. In every right spherical triangle all three angles together are greater than four right angles — that is, the two non-right angles are less than three right angles, or 270° .

In the triangle ABC, let the angle A be right. I say that the two remaining angles ABC and ACB are less than three right angles. For when the sides AB and AC have been produced about the right angle until they meet in D, and the semicircles ABD and ACD are formed, then by Proposition 13 of our [book on] spherical triangles the angle D will also be right. Since, then, both the pair ABC and DBC, and the pair ACB and DCB, are (by Proposition 5 of those same triangles) equal to two right angles, all six angles A, D, ABC, DBC, ACB, DCB will be equal to six right angles. Therefore, since the three angles in the triangle DBC are (by Proposition 31 of those same triangles) greater than two right angles, the remaining three angles in the triangle ABC will be less than four right angles; and consequently, since A is right, the remaining two, ABC and ACB, will be less than three right angles — that is, than 270° . And so, if in a right spherical triangle one of the non-right angles is set at 150° , the other must necessarily be less than 120° .

[Theor. 3.] 3. In an isosceles right spherical triangle, if the two equal angles are acute, each will be greater than half a right angle; but if they are obtuse, [each will be] less than a right angle and a half.

First, in the isosceles triangle DBC, whose angle D is right, let the two angles B and C be acute. I say that each is greater than half a right angle. For since all three are greater than two right angles (by Proposition 31 on spherical triangles), the two B and C will be greater than one right angle. Since therefore they are equal, each will be greater than half a right angle.

Next, in the isosceles triangle ABC, whose angle A is right, let the two angles B and C be obtuse. I say that each is less than a right angle and a half. For since all three are (by Theorem 2) less than four right angles, the two B and C are less than three right angles; and since these two are equal, each will be less than one right angle and a half. And so, in any isosceles spherical triangle each of the equal angles will be greater than 45° but less than 135° .

[Theor. 4.] 4. In every right spherical triangle, either of the non-right angles is greater than the complement of the other.

First, in the triangle DBC, whose angle D is right, let the two angles B and C be acute. I say that the angle B is greater than the complement of the angle C. For since the two angles B and C are greater than one right angle (seeing that all three are greater than two right angles), and the angle C together with its complement amounts to only one right angle, it is clear that the angle B is greater than the complement of the angle C. And for the same reason the angle C will be greater than the complement of the angle B.

Next, in the triangle DBE, let the angle D be right, DBE obtuse, and DEB acute. Here it is plainly established that the obtuse angle is greater than the complement of the acute angle E, since this complement is an acute angle. I say that the angle E too is greater than the complement of the obtuse angle DBE. For through the pole of the arc DB let there be understood to be drawn an arc of a great circle BC; then the angle DBC will be right, and hence the angle CBE acute — and it is the complement of the obtuse angle DBE, than which I say the acute angle DEB is greater. For since the two angles D and DBC are right, DC and BC will be quadrants (by Proposition 25 of our [book on] spherical triangles), and hence the arc CE is less than a quadrant, so that the side DE is (by Proposition 2 of those same triangles) less than a semicircle. Therefore, in the triangle BCE, since the side BC is greater than the side CE, the angle DEB will (by Proposition 11 of those same triangles) be greater than the angle CBE. [Margin: a. 13.1.Theor.]

Now, if both of the angles ABC and ACB in the triangle ABC (whose angle A is right) are obtuse, it is clear that either is greater than the complement of the other, since a complement of this kind is an acute angle. And so, if in a right triangle both non-right angles are acute, and one is set at 50° , the other must necessarily be greater than 40° . But if one is acute and the other obtuse: if indeed the acute one is put at 50° , the obtuse one will certainly be less than 140° , because the complement of 140° amounts to 40° , than which complement the given angle of 50° must be greater. Likewise, if the obtuse angle is put at 140° , the acute one must necessarily be greater than 50° , so that it can be greater than the complement of the obtuse angle.

[Theor. 5.] 5. In every right spherical triangle, either of the remaining non-right angles is less than the angle by which the complement of the other differs from two right angles — that is, from a semicircle.

In the triangle DBC, let the angle D be right. If, then, one of the angles — namely B — is acute, whatever be the case with the other, C, it is plainly established that the angle B is less than the amount by which the complement of the angle C differs from a semicircle. For since this complement is less than a quadrant, the difference between it and the semicircle will be greater than a quadrant.

But if in the triangle ABC the angle A is right and both B and C are obtuse, then both B and C in the triangle DBC will be acute. And since the acute angle DBC is (by Theorem 4) greater than the complement of the acute angle DCB — that is, than the complement of the obtuse angle ACB, since the two angles at C have the same complement — and since this complement together with the difference by which it falls short of a semicircle, and likewise the acute angle DBC together with

the obtuse angle ABC, make up a semicircle, that is two right angles: if from the one there be taken away the complement of the obtuse angle ACB, and from the other the acute angle DBC (which is greater than that complement), the remainder, the obtuse angle ABC, will be less than the difference by which the complement of the other obtuse angle ACB differs from a semicircle. And by the same reasoning the obtuse angle ACB is shown to be less than the difference between the complement of the obtuse angle ABC and a semicircle.

Diagram. A small figure showing a lens-shaped region formed by two great-circle arcs meeting at top and bottom, with a chord and further arcs producing the right and isosceles triangles used in Theorems 4–5. Readable labels: **B** (top), **F** (upper right), **D** (right), **A** (left), **E** (lower left), **C** (bottom), **G** (lower right). No labels flagged.

- B (top vertex): meeting point of the arcs
- F (upper right): produced point
- D (right): produced meeting point of the sides
- A (left vertex): right-angle vertex
- E (lower left): interior point
- C (bottom): opposite vertex
- G (lower right): produced point

Finally, if in the same triangle ABC the angle B is acute — so that DBC is obtuse — and C is obtuse — so that DCB is acute: it was already said at the beginning of this theorem that the acute angle ABC is less than the difference between the complement of the obtuse angle ACB and a semicircle. That the obtuse angle ACB too is less than the difference between the complement of the acute angle ABC and a semicircle will be shown thus. Since the acute angle DCB is (by Theorem 4) greater than the complement of the obtuse angle DBC — that is, than the complement of the acute angle ABC, because the two angles at B have the same complement — and this complement, together with the difference by which it differs from a semicircle, makes two right angles, or a semicircle: then, since the acute angle DCB together with the obtuse angle ACB makes only two right angles, the obtuse angle ACB will be less than the aforesaid difference between the complement of the acute angle ABC and a semicircle.

And so, if in a right triangle [both non-right angles are] obtuse, and one is 130° , the other must necessarily be less than the complement of this (which must be less than 50°) and [less than the difference from] a semicircle [?]; [thus] the other is less than 130° . For since the complement of this [is 40° , the difference between

it and a semicircle is] 140° , [and] that [angle] would not be less than this difference — which is absurd. [But if the acute is] put at 50° , the obtuse must necessarily be less than 140° ; otherwise [it would not be less than the difference between the complemen]t, which is 40° , and a semicircle. By the same reasoning, if [one is made] obtuse [...] 50° [?].

[Theor. 6.] 6. In any spherical triangle the two angles, however taken, are together greater than the difference between the remaining [angle] and a semicircle.

In the triangle ABE, let any two angles A and ABE be taken at will. I say that they are together greater than the angle BED, by which the third angle AEB differs from two right angles. For since the two A and ABE, together with AEB, make up more than two right angles (by Proposition 31 of our [book on] spherical triangles), while the angle BED together with that same AEB makes only two right angles, it follows that the two A and ABE are together greater than the angle BED.

[Margin: Coroll.] From this it is gathered that in every spherical triangle, when one side is produced, the external angle is greater than the two internal and opposite angles taken together.

And so, if two angles are set at 40° and 70° , the third must necessarily be greater than 70° ; otherwise those two, making up 110° , would not be greater than the 110° by which the third differs from a semicircle. Likewise, if one is set at 60° , the remaining two together must be greater than the 120° by which that one differs from a semicircle.

[Theor. 7.] 7. In every spherical triangle the two angles, however taken, are together less than the difference between the angle or arc by which the remaining [angle] differs from a semicircle (or two right angles) and a whole circle (or four right angles).

Let there be any spherical triangle ABC. I say that the two angles B and C are together less than the difference between the arc by which the remaining angle A differs from a semicircle and the whole circle, or four right angles. For when the arcs AB and AC have been produced until they meet in D, then by Proposition 13 of our [book on] spherical triangles the angle BDC will be equal to the angle A, and CDG will be the angle by which that same angle BDC (or A) differs from two right angles; and the difference between this angle CDG and four right angles, or the whole circle, will comprise the three angles CDB, BDF, FDG. It is to be proved, then, that the two angles ABC and ACB are together less than the three angles CDB, BDF,

BDF, FDG — which will be done thus. Since, by Theorem 6, the two angles DBC and DCB are together greater than the angle CDG, by which the remaining angle BDC differs from two right angles; and since both the two angles DBC and DCB together with the two ABC and ACB, and the angle CDG together with the three CDB, BDF, FDG, are equal to four right angles: if from the one side the two DBC and DCB are removed, and from the other the angle CDG (which was shown to be less than they), the remainder will be the two angles ABC and ACB, less than the three angles CDB, BDF, FDG — which is what was proposed. And so, if in any spherical triangle two angles together are set to contain 300° , the third must necessarily be greater than 120° ; because then the difference between it and two right angles will be less than 60° , and consequently the difference between that difference and the whole circle greater than 300° , and therefore the two angles set down will together be less than this difference.

[Theor. 8.] 8. In any spherical triangle the difference between the sum of two angles, however taken, and the whole circle (or four right angles) is greater than the difference between the remaining angle and a semicircle (or two right angles).

Again let there be the triangle ABC. I say that the difference between the two angles ABC and ACB and four right angles is greater than the difference between the remaining angle A and two right angles. For, the same construction being made, the two angles DBC and DCB together will make up the difference between the two angles ABC and ACB together and four right angles; and the angle CDG will be the difference between the remaining angle A — that is, the angle BDC (which by Proposition 13 of our [book on] spherical triangles is equal to A itself) — and two right angles. Since therefore, by Theorem 6, the two angles DBC and DCB together are greater than the angle CDG, what is proposed is evident. And so, if in any spherical triangle two angles together are set to make 300° , the third angle must necessarily be greater than 120° ; because then the difference between 300° and 360° is 60° , while the difference between the third angle — which is greater than 120° — and two right angles, or 180° , will be less than 60° .

From these considerations, then, we shall easily gather whether or not, from three spherical angles proposed in the sphere, a triangle can be constructed in the sphere.

Now that these matters have been set out and demonstrated, so that the diligent reader may understand how agreeable a use the doctrine of spherical triangles inscribed on the astrolabe possesses, I should like here to work through afresh, by means of spherical triangles, most of the problems which in the earlier Canons

we solved by means of the circles of the sphere drawn on the astrolabe. Let us therefore begin here.

QUÆSITUM I. [Question I]

[Margin: Quæsitum 1.]

To find the declination of any point of the Ecliptic, or of a star, whose longitude and latitude are known; and conversely, from a given declination, to determine the point of the Ecliptic to which it corresponds.

[Margin: The declination of a given point on the Ecliptic — how it may be found, without calculation, by means of spherical triangles. The arc of the Ecliptic corresponding to a given declination — how it may be determined, without calculation, by means of spherical triangles.]

The arc of the Ecliptic lying between the given point and the nearest equinoctial point, together with the arc of declination (which is a portion of the great circle drawn through the poles of the world and the given point of the Ecliptic) and the arc of the Equator intercepted between that same equinoctial point and the arc of declination, forms a right spherical triangle. In this triangle, from the base (that is, from the arc of the Ecliptic between the nearest equinoctial point and the given point whose declination is sought) and the angle of maximum declination (which the Equator and the Ecliptic contain), the side opposite this angle — namely the arc of declination — is to be found. If, then, such a triangle is constructed as was taught in Problem 8, the sought arc of declination will be found.

But if the declination is given and the arc of the Ecliptic to which it corresponds is to be sought, this will be done by Problem 14, where the base (which is the sought arc of the Ecliptic) is found from a given side (namely the arc of declination) and the angle opposite it (which here is the angle of maximum declination); which in the given case will easily be done, since it is established that the base is less than a quadrant.

Next, if from the pole of the world and from the pole of the Ecliptic two great circles are conceived as drawn through the center of the star — the former measuring the star's declination, the latter its latitude — a spherical triangle is formed in which two sides are known: namely the arc of the solstitial Colure enclosed between the two poles (equal to the maximum declination), and the complement of the latitude (that is, the arc of the latitude-circle between the pole of the Ecliptic and the center of the star), together with the angle contained by

them — the angle, that is, which the star's distance from the beginning of Cancer [♋] measures, when its latitude is northern, or from the beginning of Capricorn [♐], when the latitude is southern. This distance from Cancer is to be counted in the order of the signs if the star lies in the descending semicircle, but the reverse if in the ascending; while from Capricorn it is to be counted in the order [of the signs] if it lies in the ascending semicircle, but the reverse if in the descending. A triangle of this kind is FGH, among those 12 circles which we described at the end of the scholium to Canon 3.

Diagram. The same orthographic ("analemma") projection of the sphere within a square frame with inscribed circle and quarter-arc that appears on p. 694, again densely filled with chords, diameters, and projected small circles (the astrolabe-projection master figure). Readable labels: **B** (top of circle), **D** (bottom), **P** (upper-left frame), **R**, **E** (upper-left interior), **N/Ψ** and **p**, **M** (upper right, outside circle), **K**, **L**, **l**, **k**, **i** (interior right), **A** (right frame), **C**, **T**, **Y**, **Y** (along the left horizontal), **n**, **E** (center), **o**, **G**, **q** (center-right), **e**, **X** (lower-left interior), **I**, **ζ** (lower right), **Ω**, **m**, **V** (lower-left corner), **q**, **D** (bottom), **H**, **F**, **S**, **δ**, **Z** (lower-right periphery). Many small italic peripheral labels could not be resolved with confidence under magnification.

- B (top of disk) / D (bottom of disk): endpoints of the principal diameter
- E (center): center of the projection
- P (upper-left frame), V (lower-left corner), A (right frame): frame reference points
- F, G, H: the vertices of the triangle FGH named in the text (among the projected circles, lower-right region)
- [numerous small peripheral labels [?]]

If, then, by Problem 19 the third side in that triangle is sought — the side that is the complement of the star's declination — from the two remaining sides (of which one is equal to the maximum declination and the other to the complement of the star's latitude) and from the angle contained by them (which is equal, as we said, to the star's distance from Cancer or Capricorn), the complement of the declination cannot remain hidden. Yet when the third side of the said triangle, once found, is greater than a quadrant, then, a quadrant being subtracted, the remainder becomes the star's declination, of the denomination contrary to the latitude. In all other cases the third side is the complement of the declination, and of the same denomination as the latitude.

[Margin: An easier way of finding the declination.]

This question will be more easily solved as follows. In the figure of Problem 5, let the angle of maximum declination ABb be constructed, which [angle] the arc Ab , that is, [measures], and therefore

and so let Ab measure the arc of greatest declination. Then, taking the quadrant hm , let the radius Bm mark out n , the pole of the circle BbD . If, therefore, the arc Bp is taken equal to the given arc of the ecliptic, the straight line np will cut off the arc Bi equal to it. So when the straight line EiN is drawn, marking out the circle of declination, iN will be the required arc of declination — to which the arc Ag is equal, once, from E through i , the parallel ik has been drawn, so that Ni and Ak are equal, and so on. In this way, given the arc of the ecliptic, its declination has been found.

[Margin: The [finding] of the declination of a given point of the ecliptic.]

AGAIN, if the declination Ag is given, form once more the angle ABb of greatest declination. Then, drawing the radius Bg so that it equals Ak — so that this too is the arc of the given declination — and describing from E , through k , the parallel ki , cutting the circle BbD at i , then Bi will be the required arc of the ecliptic. For when the straight line EiN is drawn, the arc iN , equal to Ak (or to Ag), will measure the declination of the point i . And this arc Bi is equal to the arc Bp of the equator, which the straight line ni cuts off — the line drawn from n , the pole of the circle BbD (which is found by means of the quadrant hm , as above), extended through i .

[Margin: An easier way to find the point of the ecliptic that corresponds to a given declination. — An easier way to find the declination of a star.]

MOREOVER, in the same figure, form the angle ABb equal to the distance of the star from the beginning of Capricorn (♑), if the star is southern — whether that distance is to be counted in the order of the signs or against it, as was said above. Then take the arc BN equal to the arc of greatest declination between the pole of the world and the pole of the ecliptic. Likewise, cut off from the circle BbD an arc equal to the complement of the star's latitude, by means of the straight line drawn from its pole n through the endpoint [of the arc of that same complement on the equator]; and finally, through the end of this arc, and through the point N and its opposite Q , let a circle be described. For the position of this circle's [arc, cut off] by the circle BbD from the star's latitude, will give the complement of the arc between N and the endpoint of the arc of the complement of the star's latitude from the circle of the star's declination — provided that intercepted arc is less than a quadrant; or, if it is greater than a quadrant, it will give the arc

compounded of a quadrant and the declination, as we said above. This arc, moreover, will be known by means of the straight lines drawn from its pole, and so on. For in this way a triangle is produced entirely similar to the triangle FGH in those 12 circles of the scholium to Canon 3, since BN corresponds to the arc FG, the arc of the complement of the star's latitude cut off from the circle BbD corresponds to GH, and finally the third arc found corresponds to FH, and so on.

WHEN the distance of the star from Cancer (♋) or Capricorn (♏) is greater than a quadrant, its angle CBb must be constructed, and the arc BR must be taken, for example, equal to the greatest declination, and so on.

Diagram. An orthographic-projection ("analemma") construction — the recurring "figure of problem 5" used throughout the Canon XXII quæsitæ — occupying the foot of the page. A square frame encloses a large circle crowded with intersecting straight construction lines and secondary arcs, meeting at many lettered points. The frame's horizontal and vertical diameters carry the outer reference letters; the interior is a dense web of chords and radii from the principal points B (top), E (center), and D (bottom). Readable labels, by position:

- P (upper-left, on frame): pole
- B (top of circle): summit point / vertex of the declination angles
- R, Z, S (upper-left interior): construction intersections
- K (upper interior)
- N [?] (upper right interior)
- M (right, on frame)
- d, T (left frame, horizontal diameter)
- Y (left, on circle); C (left-center); Y (repeated, interior)
- A (right end of horizontal diameter, on frame)
- n (left-interior): pole of circle BbD
- a, i, k (upper-central interior): the points i, k of the construction
- e, E (center): E is the center / eye-point
- o, L, l, G (central-lower interior)
- X (lower-left interior)
- I (lower-right interior)
- Ω, ♄ [m], q (lower interior, near foot)
- D (bottom of circle)
- H, F (lower-right, on circle): the points of triangle FGH
- ♄, ♌ [?] (planetary/zodiac symbols, lower-right margin of circle)

- V (lower-left corner of frame)

QVÆSITVM II

[Margin: Quæsitu[m] 2]

TO INVESTIGATE the right ascension and descension of a given point of the ecliptic, or of a star; and conversely, from a given right ascension or descension, to recognize the corresponding point of the ecliptic; and finally, to determine the point of the ecliptic which, together with the star, rises, sets, and culminates in the right sphere.

If, by problem 9, a right-angled spherical triangle is constructed whose base is the arc of the ecliptic between the nearest equinoctial point and the given point, and [whose] angle of greatest declination is adjacent to the required side — namely to the arc of the equator that measures the right ascension or descension — then this arc of the equator will be found, as was said in that problem. For the said arc of the ecliptic, the arc of declination, and the arc of right ascension or descension form a triangle of this kind, one of whose non-right angles is equal to the greatest declination.

[Margin: How the right ascension or descension of a point of the ecliptic may be recognized by a spherical triangle, without numbers.]

CONVERSELY, if, given a right ascension or descension, the corresponding arc of the ecliptic is to be found, then in the same right triangle just mentioned there will be given one side — namely the arc of the equator that measures the right ascension or descension — and the same angle of greatest declination adjacent to that side. From these the base, that is, the corresponding arc of the ecliptic, will be investigated, as was said in problem 13. But for the arc of ascension or descension, one must always take the arc of the equator less than a quadrant, as we did in the scholium to Canon 4, Num. 6.

[Margin: How to find, by a spherical triangle without numbers, the point of the ecliptic corresponding to a given right ascension or descension.]

NEXT, let it be understood that from the pole of the world and the pole of the ecliptic two great circles are drawn through the star, so that the triangle FGH is formed in those 12 circles of the scholium to Canon 3. And since in this triangle two sides are known — namely the arc of the solstitial colure between the two poles, which is equal to the greatest declination, and the complement of the star's latitude, together with the angle contained by them, since that angle is measured

by the distance from the beginning of Cancer (♋) or Capricorn (♏) — then, if by problem 19 a triangle of this kind is constructed (such as is the triangle BKF in the figure of problem 18), the angle will be found which the circle of declination makes with the colure at the pole of the world, namely the angle GFH, in the aforesaid 12 circles, which the right ascension reckoned from the beginning of Cancer (♋) or Capricorn (♏) measures, and so on.

[Margin: How the right ascension or descension of a star may be found, known by a spherical triangle without numbers. — An easier way to find the right ascension of a given point of the ecliptic.]

BUT we shall perhaps carry out this problem more easily as follows. In the figure of problem 5, form the angle of greatest declination ABb, and let the arc Bi be equal to the arc of the ecliptic counted from the nearest equinoctial point — which is easily cut off if an equal arc Bp is taken on the equator and the straight line np is drawn from n, the pole of the circle BbD, through p, and so on. For the straight line Ei, representing the right horizon, will cut off the arc BN of right ascension or descension.

[Margin: An easier way to find the right ascension of a given point of the ecliptic.]

ON THE CONTRARY, if the right ascension is given, form again the angle ABb of greatest declination, and let the arc BN, which measures the given ascension, be taken; then the straight line EN will cut off the corresponding arc Bi of the ecliptic — which the straight line ni, drawn from the pole n, will make known.

[Margin: An easier way to find the point of the ecliptic corresponding to a given right ascension.]

THEN, if the angle ABb equal to the distance of the star from Cancer (♋) or Capricorn (♏) is formed, and the arc BN equal to the greatest declination is taken, and an arc equal to the complement of the star's latitude is cut off from the circle BbD by the straight line drawn from n, its pole, as far as the point [terminating the arc of the equator equal to that same complement of the star's latitude]; and finally, through the endpoint of this arc and through N and its opposite point Q a circle is described — then its arc, enclosed between N and the circle BbD, will correspond to the arc FH, in...

[Margin: An easier way to find the right ascension of a given star.]

FH, in the triangle FGH of the 12 circles of the scholium to Canon 3. Therefore the angle which the same arc makes with the arc BN at the mundane pole (which is

now N) will give the right ascension reckoned from Cancer (♋) or Capricorn (♑), and so on.

AND if by chance the distance of the star from Cancer (♋) or Capricorn (♑) is greater than a quadrant, its angle CBb must be made greater than a right angle, and in the quadrant BC the arc of greatest declination must be taken, and so on.

THE POINT of the ecliptic which agrees with this right ascension will be the one with which the given star rises, sets, and culminates in the right sphere.

[Margin: The point of the ecliptic that rises, sets, and culminates together with the star. — Quæsitum] 3]

QVÆSITVM III

TO INVESTIGATE the oblique ascension and descension of a given point of the ecliptic, or of a star; and conversely, to determine the point of the ecliptic answering to a given oblique ascension or descension; and finally, to find the point of the ecliptic with which the given star rises and sets in the oblique sphere.

The arc of the ecliptic, counted from the beginning of Aries (♈) or Libra (♎) up to the given point rising in the order of the signs, forms with the equator and the oblique horizon an oblique-angled spherical triangle in which two angles are given: namely the angle of greatest declination, which the ecliptic makes with the equator; and the angle which the equator makes with the horizon — which, from Aries (♈) up to Libra (♎), is always obtuse and inclines toward the north, and is what remains when the complement of the pole's altitude is subtracted from a semicircle; but from Libra (♎) up to Aries (♈) it is acute, being precisely the angle of the complement of the pole's altitude, and inclines toward the south. Given, moreover, is the arc opposite the latter given angle — namely the arc of the ecliptic counted from Aries (♈) or Libra (♎) up to the given point. If, therefore, by problem 21, the arc of the equator measuring the oblique ascension is sought — from the given arc of the ecliptic, which is opposite one of the given angles, and from the two aforesaid angles — since it is established that the third arc, that of the horizon, which is opposite the other given angle, is less than a quadrant (namely equal to the rising latitude), then the oblique ascension of the given point of the ecliptic will be found.

[Margin: To investigate, by spherical triangles without numbers, the oblique ascension or descension of a given point of the ecliptic.]

The oblique descension of a given point of the ecliptic is investigated in no other way — for an entirely similar triangle is constructed beneath the western horizon — except that the angle which the equator makes with the horizon is acute from Aries (γ) up to Libra ($\underline{\alpha}$), but from Libra ($\underline{\alpha}$) up to Aries (γ) is obtuse.

[Margin: To assign, by spherical triangles without numbers, the point of the ecliptic answering to a given oblique ascension or descension.]

BUT if the oblique ascension or descension is given, there will be in the same triangle just mentioned the same two angles, together with the arc of the equator adjacent to them which measures the given ascension or descension. Therefore, by problem 20, from these givens the required arc of the ecliptic will become known — namely the one to which the given ascension or descension corresponds. Now the given ascension or descension must always be taken less than a semicircle; so that, when it is greater, the semicircle is subtracted, so that the ascension or descension is reckoned as beginning from Libra ($\underline{\alpha}$).

[Margin: An easier way to find the oblique ascension or descension of a given point of the ecliptic.]

WE SHALL, however, perhaps accomplish both of these more easily by this other method. In the figure of problem 5, construct the angle of greatest declination, and from the semicircle BbD cut off the arc Bi (or Bl) equal to the given arc of the ecliptic, by means of the straight line drawn from n, the pole, to the point of the equator that terminates the equal arc begun from B. For if, through the endpoint i (or l), an arc of the horizon is described whose center lies on the parallel drawn through the center of the horizon, and whose concave side faces toward B, then this arc will cut off from the equator the oblique ascension of the point i (or l), as is evident. But if the convex side of the arc of the horizon described through i (or l) faces toward B, it will cut off from the equator the oblique descension.

[Margin: An easier way to find the point of the ecliptic answering to a given oblique ascension or descension. — How the ascensional difference of a star, or of a given point of the ecliptic, may be found by spherical triangles without numbers.]

ON THE CONTRARY, if the oblique ascension or descension is reckoned on the equator from B, and through its endpoint the horizon is described in such a way that its concave side looks toward the parts B if the matter concerns ascension, but its convex side if it concerns descension, then this horizon will indicate on the circle BbD the point of the ecliptic — to be counted from the beginning of Aries

(♎) or Libra (♎) — with which the given ascension or descension agrees, and so on.

NOW, in order that the oblique ascension or descension of any star may be found, its ascensional difference must be examined by this method. The arc of the circle of declination, drawn from the pole of the world through the star as it rises, lying between the star and the equator; the arc of the horizon measuring the rising latitude; and the arc of the equator measuring the ascensional difference — these constitute a right-angled spherical triangle in which the arc of declination is given by quæsitum 1, together with the opposite angle which the equator makes on the horizon, that is, together with the angle of the complement of the pole's altitude. Therefore, from these givens, by problem 10, the arc of the ascensional difference will be obtained — the arc that is adjacent to the given angle — since it is established that this required arc is less than a quadrant.

[Margin: An easier way to find the ascensional difference.]

WE shall perhaps find this ascensional difference more easily thus. In the figure of problem 5, form the angle ABb of the complement of the pole's altitude, and let the arc Ak measure the star's declination, cut off by the radius Bg from B to g, the end of the arc Ag of declination: and Ak will be less than the arc Ab, which measures the complement of the pole's altitude — since here we are speaking of a pole's altitude that is not greater than $66^{\circ} 30'$. So, describing from E, through k, the parallel cutting the arc Bb at i, the straight line Ei will cut off BN, the required ascensional difference — because the triangle BiN is the very one just discussed, since iN is an arc equal to the arc Ak of declination, and so on. But the star's declination must be less than the complement of the pole's altitude; otherwise it would neither rise nor set, or would certainly touch the horizon, and thus would have no ascensional difference, as we taught in the treatise on the sphere.

BUT how the oblique ascension or descension itself is derived from the ascensional difference, we taught in the scholium to Canon 5, near the end, Num. 1.

Diagram. The same orthographic-projection ("figure of problem 5") woodcut reproduced at the foot of this page: a square frame enclosing a large circle filled with intersecting construction lines and secondary arcs meeting at lettered points. The principal points and reference letters repeat those of the figure on p. 697 (P upper-left; B top; M and A on the right frame; d, T, Y, C on the left; n, a, i, k, e, E, o, L, l, G in the interior; X lower-left; Ω, ♯, q lower interior; D bottom; H, F

lower-right; V lower-left corner; with the same crowded small interior labels). See fig-p697-1 for the full label list; several interior readings remain uncertain

IN JUST the same way, the ascensional difference of any point of the ecliptic will be found, if in place of the star we set the point of the ecliptic itself on the horizon.

FINALLY, the point of the ecliptic to which the oblique ascension or descension of a star agrees is the one with which the star rises or sets in the oblique sphere; but the point with which it culminates is the one with which it rises, or culminates, in the right sphere.

QVÆSITVM IV

[Margin: The point of the ecliptic rising or setting together with the star in the oblique sphere. — Quæsitu[m] 4]

TO EXPLORE the rising and setting latitude (amplitude) of any point of the ecliptic, or of a star; and, on the contrary, given the rising or setting latitude, to find the corresponding point of the ecliptic.

In the right-angled spherical triangle spoken of at the end of the preceding quæsitum, the base must be sought — that is, the arc of the horizon, or of the rising latitude — from the arc of declination known by quæsitum 1, and from the angle of the complement of the pole's altitude, which is opposite the arc of declination: just as was handed down in problem 14, since it is established that this base is less than a quadrant.

AND if the rising latitude is given, there must be investigated, in the same triangle, the arc of declination from the base (which is the rising latitude) and from the angle of the complement of the pole's altitude, which is opposite the required arc, as we wrote in problem 8, and so on.

OR we shall proceed more easily thus. In the figure of problem 5, form the angle ABb of the complement of the pole's altitude. Then, taking the arc of declination Ag of the given point or star, let Ak be cut off equal to it by the radius Bg (Ak will be less than the arc Ab of the complement of the pole's altitude — otherwise the Sun or star would neither rise nor set, as we said in the treatise on the sphere); and describing from E, through k, the parallel cutting BbD at i, draw from E, through i, the straight line Ei. For thus the aforesaid triangle BiN will be constituted, and the arc Bi will measure the rising latitude, which will be known by means of the straight line ni, and so on.

BUT if the latitude is given, then, having constructed the angle ABb of the complement of the pole's altitude, cut off the arc Bi of the rising latitude by the straight line ni drawn from the pole n to the point p, terminating the arc Bp of the rising latitude. For the straight line extended from E through i will give iN, the arc of declination, and so on.

[Margin: To determine the rising latitude of a given point of the ecliptic, or of a star, by spherical triangles without numbers; and conversely. — An easier way to find the rising latitude.]

QVÆSITVM V

TO INVESTIGATE the semidiurnal and seminocturnal arc of a given point of the ecliptic, or of a star.

Once the ascensional difference of the given point of the ecliptic (or of the star) has been found, as was said in quæsitum 3, the semidiurnal and seminocturnal arc will be found from it, as we handed down in Canon 7, Num. 3.

[Margin: To define, by spherical triangles without numbers, the semidiurnal and seminocturnal arc of a given point of the ecliptic, or of a star. — Quæsitu[m] 5]

QVÆSITVM VI

TO SEEK OUT the distance of the Sun, or of a star, from the meridian by means of its altitude.

If, as problem 18 taught, a spherical triangle is constructed from three known sides — one of which is the arc of the complement of the pole's altitude placed on the meridian between the pole of the world and the pole of the horizon; the second, the arc of the circle of declination or of the hour circle enclosed between the pole of the world and the center of the Sun or star (which, if the heavenly body is northern, measures the complement of the declination, but if southern, is compounded of a quadrant and the declination); and the third, the arc of the vertical circle drawn through the heavenly body, measuring the complement of the known altitude — if, I say, a triangle of this kind is constructed, it will give the angle which the arc of the meridian and the arc of the circle of declination contain — the distance of the heavenly body from the meridian; which angle will become known by proposition 15 of Book 2.

[Margin: To scrutinize, by spherical triangles without numbers, the distance of the Sun or a star from the meridian. — Quæsitu[m] 6]

QVÆSITVM VII

To thoroughly investigate the magnitude of twilight.

By the same method, if by problem 18 a spherical triangle is constructed [whose sides are] the arc of the complement of the pole's altitude on the meridian between the pole of the world and the zenith of the place; secondly, [the arc of the circle of] declination between the pole of the world and the center of the Sun standing in the parallel 18° below the horizon (which, if the Sun is northern, is the complement of the declination, but if southern, is compounded of a quadrant and the declination); and finally, the arc of the vertical circle described through that same center of the Sun, made up of a quadrant and the arc of 18° : if, I say, a triangle of this kind is formed, it will give the angle which the arc of the circle of declination makes with the meridian — an arc compounded of the semidiurnal arc and the arc of twilight; which angle will become known by proposition 15 of Book 2. If, therefore, the semidiurnal arc is subtracted from this arc, the remainder will be the arc of twilight.

[Margin: To explore the magnitude of twilight by spherical triangles without numbers. — Quæsitu[m] 7]

QVÆSITVM VIII

To measure out the distance of two places on earth, or of stars in the sky.

Let there be made, by problem 19, a spherical triangle from two known sides, together with the angle contained by them. Its two known sides are the complements of the latitudes of the places, if the latitude of each is northern; or arcs compounded of a quadrant and the latitudes, if the latitude of each is southern, and so on. The angle contained by them, which is given, is the difference of longitudes — that is, it is determined by the arc of the equator, less than a semicircle, placed between the meridians of the places. For the third side, which will become known by means of the straight lines extended from its found pole through its endpoints, will make manifest the distance between the two places.

THE SAME is to be said of the distance of stars, if, in place of the circles that measure the latitudes of places, the circles of the latitudes of the stars are taken.

[Margin: To measure the distance of two places on earth, or of stars in the sky. — Quæsitu[m] 8]

[The worked example continuing from the previous page (Quaesitum VIII, on finding the distance between two places):]

For example: let there be two places, both in the northern hemisphere, and let the angle their meridians make be CBS; let the complement of the latitude of the one be BG, and of the other BS, as appears in the figure of Problem 22. If, then, through G and its opposite point F, and through S, a great circle is described, the arc GS (which the straight lines drawn from its pole will make known) will measure the distance of place G from place S. In the same way, if the two places are both in the southern hemisphere, so that the angle formed by the meridians is FBO, and the arcs of the meridians between B — the arctic pole — and the places themselves are BF, BO, and so on, the arc FO will give the distance of the places. Finally, if one place is northern and the other southern, so that their meridians make the angle GBP, and the arcs of the meridians between the places themselves and the arctic pole are BG, BP, and so on, their distance will be the arc GP. And this method, as you see, is much more convenient than the one we set out in Canon 15; for in this one the lines do not run out far, as they do in that one, even when one of the places is northern and the other southern.

Diagram. An orthographic (analemma-type) projection of the sphere within a square frame: a bounding circle crossed by numerous great-circle and small-circle traces (both solid and dotted) representing the meridian, equator, oblique circles, and their parallels used throughout the spherical-triangle solutions; the same figure the text calls "the figure of Problem 22." Readable labels, by position:

- B (top of circle): upper pole / apex of the vertical meridian
- D (bottom of circle): opposite point, foot of the vertical meridian
- M (upper left, on the circle)
- H (upper right, on the circle)
- f (interior, just below B)
- Y (interior, left of centre)
- N (interior, right of centre, above the middle)
- S (interior, right of N)
- R (right edge, on a dotted arc)
- G (left, on the circle, mid-upper)
- T (left interior, just below G)
- C (far left, on the circle)
- K (far left, on the circle, below C)
- O (interior, left of centre, lower)

- E (centre)
- V (centre, just below E)
- L (interior, below O)
- A (right edge, mid-lower, on the circle)
- F (right, lower, on the circle)
- P (right, lower, below F)
- X (right, lower, below/right of P)
- a (bottom centre, on a dotted arc)
- b (bottom left, on a dotted arc)
- I (bottom left, on the circle)
- d (bottom right, on a dotted arc)
- Two small cursive letters set on the dotted arcs at the upper-left and upper-right corners (near M and near H): [?] — not resolvable at this resolution

[Margin: Quæsitū 9 (Quaesitum 9)]

QVÆSITVM IX.

To find the altitude of the Sun above any great circle, and its horizontal distance from it, for each hour.

[Margin: To determine the Sun's altitude above a given great circle, and its horizontal distance, by spherical triangles, without numbers.]

Although the method set out in Canon 16 is easy and ready to hand, nevertheless, when the Sun's altitude and horizontal distance are to be found for only one hour or another, we shall accomplish it with almost no trouble by this technique. Having found, by Canon 20, the altitude of the pole above the given great circle, and, by Canon 17, the inclination of its own meridian to the meridian of the horizon of the place in which these things are being investigated — so that the hour-distances from that meridian may be known — let there be made, in the same figure as Problem 22, the angle CBS equal to the distance of the given hour from its own meridian; and let BG be the arc of the proper meridian between B, the pole of the world, and G, the pole of the given great circle; and let the arc BS be the complement of the Sun's declination, or else be composed of a quadrant and the declination, when the Sun's distance from the pole visible above the given circle is greater than 90°. For if through G and its opposite point F, and through S, a great circle is described, its arc GS, between the pole of the given circle and the Sun, will be the complement of the Sun's altitude that is sought. But if the angle of the Sun's distance from its own meridian is GBO, and the arc

between the pole visible above the given circle and the Sun is BO, and so on, then GO will be the complement of the Sun's altitude. The former case, moreover, has been brought forward only as an example; for it is impossible that, when the complement of the declination is BS, the angle of the Sun's distance from its own meridian could be GBS, because then the complement of the Sun's altitude, GS, would be greater than a quadrant, which cannot happen.

The horizontal distance will be shown by the angle BGS, or BGO, which is measured by the arc of the given circle — an arc which, taken as that of the horizon, HN or HL, is begun from the proper meridian toward the region of the pole visible above the given circle, that is, the horizon, and so on.

And in this way all the questions concerning the *primum mobile* [*primum mobile*] — those which are solved by sines and by numbers, that is, by spherical triangles — can be dispatched by drawing one or two arcs on the astrolabe; and, provided the greatest care is taken, as is fitting, so reliably that scarcely an error of a few minutes can arise. This is indeed a splendid thing, and one that to this day, so far as I know, has been attempted or demonstrated by no one. It remains that — just as the waters that have flowed out from the Ocean return to it again by long windings — so, since this good thing, whatever it is, has flowed from the Fountain of all good things, God the Best and Greatest, the greatest thanks that mortals can offer be rendered and held to that best Author and most generous Giver.

END OF THE THIRD BOOK.

INDEX OF THE LEMMAS OF THE FIRST BOOK.

Those printed in a different typeface pertain to the Scholia and Corollaries.

1. To divide a given line, straight or circular, into any number of equal parts, even the smallest, by means of a compass whose legs have a spread between them greater than the given line. p. 7.
2. To lay out a given quadrant, or circle, into degrees by means of a compass whose leg-spread embraces more than two or three degrees. 8.
3. From a given circumference, to cut off an arc embracing any whole number of degrees, or any number of degrees and minutes; and conversely, to determine how many degrees and minutes are contained in

any arc of a given circumference, even if the given circumference has not been divided into degrees and minutes. *ibid.*

4. Through a given point, to draw a line parallel to a given straight line. 13.
5. Whatever proportion the whole sines — that is, the radii of any circles — have, the sines, both right and versed, of similar arcs have the same. And conversely, arcs whose sines, both right and versed, have the same proportion as the whole sines, are similar. *ibid.*
6. If similar segments are added to similar segments of unequal circles, or similar segments are taken away from similar ones, the wholes too, or the remaining segments, will be similar. 14.
7. If two unequal quadrants are similarly divided, whether into equal parts, and through the points of division lines are drawn parallel to one radius, or perpendicular to the other radius: then the segments of the radius made in one quadrant by the parallels or perpendiculars will be proportional to the segments of the radius made by the parallels or perpendiculars in the other quadrant. And conversely, if the segments of the radii are proportional, the quadrants will have been similarly divided. *ibid.*
8. To cut a given straight line just as the radius of some quadrant is cut by the perpendiculars dropped to it from any points whatever of the quadrant. 16.
9. If two or more circles touch one another internally, or two touch externally, a straight line drawn through the point of contact cuts off similar circumferences; and the straight lines joining the two pairs of points at which two straight lines cut the circles are parallel.

The same holds for two circles that do not touch one another, if in place of the point of contact one takes a point on the line joining their centres, through which passes the line connecting the alternate ends of the diameters perpendicular to the former line. But when the circles do not touch internally, the similar arcs are the alternate ones, not those taken in the same order as in the former case. 17.

10. If two or more circles cut one another, straight lines drawn through the point of section — whether they cut the circles, or both are tangent, or one of them is — intercept similar circumferences, begun from one of those straight lines and proceeding toward the same side, up to the point of section, or of contact, of the other straight line. And if from the same point of section any circle whatever is described, its circumference comprised

between those same two straight lines will be the half of that arc — in the circle described from the point of section — which is similar to the arc of any of the former circles intercepted between the same straight lines. 19.

11. To extend a very short straight line into a continuous one, or (what is the same) through two points little distant from each other to produce a straight line to any length whatever. 22.
12. Given two straight lines, to find a third proportional, and given three, a fourth proportional. 24.
13. Given two straight lines inclined to each other, to find the point at which they would meet, even if neither is produced. 27.
14. To construct an instrument by which, through three given points — even if they lie almost along a straight line — an arc of a circle can be drawn, without the aid of a compass. 29.
15. A curved line which is subtended by a straight line, and such that the squares of all the perpendiculars dropped from the points of the curve to the subtending straight line are equal to the rectangles contained by the segments of that same subtense made by the perpendiculars — that is, such that all the perpendiculars are mean proportionals between the segments of the subtense made by them — is a semicircle, and its diameter is that subtending straight line; that is, a semicircle described about that subtending straight line will coincide with the given curve, or (what is the same) will pass through the extreme points of all the perpendiculars. 30.
16. If a cone is cut by a plane parallel to the base of the cone, the section made in the conical surface is the circumference of a circle having its centre on the axis of the cone. *ibid.*
17. If a scalene cone is cut by a plane through the axis that is perpendicular to the base, and is cut by a second plane perpendicular to the triangle through the axis made by the former plane, which second plane cuts off from the triangle through the axis a triangle similar indeed to the triangle through the axis but placed subcontrariwise: then the section is a circle, whose diameter is the common section of the triangle through the axis and of the plane that produced the section in the conical surface. And let a section of this kind be called subcontrary [subcontraria]. 31.

That the diameter of a subcontrary section can be equal to the diameter of the base of the cone, and can be unequal. 32.

That the diameter of a subcontrary section and the diameter of the base of the cone never bisect each other. ibid.

That the diameter of a subcontrary section and the diameter of the base of the cone, when they are equal, neither is bisected. 33.

When the diameter of a subcontrary section is unequal to the diameter of the base of the cone, and one of them is bisected, the other is the greater. ibid.

When the diameter of a subcontrary section is unequal to the diameter of the base of the cone, and the smaller is bisected, the greater part of the greater diameter inclines toward the smaller angle of the triangle through the axis which that diameter makes with the side of the same triangle. 34.

18. Whatever proportion the whole sine has to the sine of the greatest declination of the Ecliptic from the Equator, the right sine of the arc of the Ecliptic intercepted between any point of it and the nearest equinoctial point has that same proportion to the right sine of the declination of that same point of the Ecliptic from the Equator. ib.

19. To describe an analemma for any given altitude of the pole whatever. ibid.

The declinations of all points of the Ecliptic, and of any given point — in what manner they may be found geometrically. 36.

20. If two planes cut one another, and in one of them, at the points of the common section, two straight lines make with it two internal angles of any kind that are equal, and in the other, at the same two points, two other straight lines likewise make with that same common section two internal equal angles of any kind: then these two latter straight lines will make, with the two former, two equal angles. 37.

21. If on the diameters of equal circles points are taken equally distant from the centres, and from them straight lines go out to the circumferences making equal angles with the diameters toward the same side: then those straight lines will both be equal and will cut off equal arcs. And if the lines are equal, those straight lines will make equal angles with the diameters toward the same side, and will again cut off equal arcs. And finally, if equal arcs are cut off toward the same side, those straight lines too will be equal, and will make with the diameters equal angles toward the same side. 38.

If on the diameters of unequal circles points are taken similarly distant from the centres, so that their distances from the centres have the same proportion as the radii, and from those points straight lines go out making equal angles with the diameters toward the same side: then similar arcs will be cut off by them. And if the arcs cut off are similar toward the same side, the cutting straight lines will make with the diameters equal angles toward the same side. 40.

If from two centres lying on the same straight line two circles are described under this condition, that outside both a point can be taken similarly distant from the centres: then a straight line tangent to one of the circles will also be tangent to the other; and a straight line cutting both will cut off similar arcs. 41.

22. If in an underlying plane a transverse straight line falls between two straight lines, making with them internal angles on both sides equal to one another — whether all are right angles, or two obtuse and two acute — and on those two straight lines two planes stand perpendicular to the underlying plane: then a plane drawn through the transverse line in any manner will make with the perpendicular planes common sections, straight lines, which will contain, with the two given straight lines in the underlying plane, equal angles. *ibid.*

23. A plane in the sphere, drawn in any manner through either of the poles of the world and either of the poles of any great circle — whether oblique or perpendicular to the Equator — cuts off, both from the Equator and from that oblique or perpendicular great circle, and likewise from any parallel of the Equator and the parallel of that oblique or perpendicular great circle (which parallel, however, is equal to the parallel of the Equator, and is as far distant from its assumed pole as the parallel of the Equator is distant from the assumed pole of the world), two equal arcs, intercepted between the cutting plane and the great circle described through the two assumed poles. 42.

24. If in the sphere there is an oblique circle, whether great or not great, and through any point of its diameter — the diameter which the great circle drawn through its poles and the poles of the world makes — a line perpendicular to that diameter is drawn: then a plane drawn through either of the poles of the world and that perpendicular will make, in the plane of the Equator, a common section, a straight line perpendicular to the diameter of the Equator which that same great circle drawn through the said poles makes. 49.

25. If in the sphere a great circle is drawn through the poles of the world and the poles of any oblique great circle — or of its parallels — and in it, from either pole of the world, a line is drawn parallel to the diameter of the oblique circle, and through this a plane is extended in any manner: then the two arcs — both of the oblique great circle and of any of its parallels — intercepted between the cutting plane and the great circle drawn through the poles of the world and of the oblique circle will be equal to one another. *ibid.*
26. If a circle in the sphere passes through either pole of the world, its diameter drawn from that pole will be perpendicular to the common section of the plane of that circle and the plane of the Equator. 50.
27. In a right cone all the straight lines drawn from the vertex to the circumference of the base are equal to one another; but in a scalene cone they are unequal — the least being that which is drawn to the extremity of the base of the triangle through the axis (which is perpendicular to the base of the cone) on the side of the angle of inclination of the axis, and the greatest that which is drawn to the other extremity of the base of the same triangle through the axis. And the one nearer to the least is always smaller than the one more remote. But only two will be equal [?] to each other, one on each side of the least, or of the greatest. *ibid.*
28. If in a cone there is a circle parallel to the base, straight lines drawn from the vertex in the conical surface will cut off similar arcs from the base and from the parallel circle. 51.
29. If two straight lines touch each other at one point, and from any point outside them in the same plane several straight lines are drawn which cut them: then the segments of the more remote line from the assumed point — proceeding toward the point of section of the proposed lines — will have a greater proportion than the segments of the nearer line. *ibid.*
30. If two isosceles triangles have equal bases, but the sides of one are greater than the sides of the other, the smaller sides will contain the greater angle. And if the sides of one are greater than the sides of the other and contain a greater angle, the base of the former will be greater than the base of the latter. 52.
31. If in a scalene cone a circle is placed subcontrariwise to the base, straight lines drawn from the vertex in the conical surface — one of which is a side

of the triangle through the axis that is perpendicular to the base — will cut off dissimilar arcs from the base and from that circle. And if in the one two opposite equal arcs are cut off, in the other two unequal arcs will be cut off — the greater indeed toward the smaller angle of the triangle through the axis, the smaller toward the greater angle. 53.

32. If on the diameter of a circle, apart from the centre, some point is taken, and from it straight lines are drawn out which intercept two equal arcs on the circumference of the circle: then the angles contained by them will be unequal, and the greater will be that whose lines are farther from the centre. And if the drawn straight lines contain equal angles, the intercepted arcs will be unequal, and the greater will be that whose lines are nearer to the centre. 55.

33. If in circles that cut one another, or that do not cut one another but have different centres, some point is taken on their common diameter drawn through both centres — apart from the centres — a point which lies both between the two centres and within both circles: then straight lines drawn out from that point, cutting the circumference of either circle into equal arcs, will cut the circumference of the other into unequal arcs, and the greater will always be that whose lines are nearer to the centre. Likewise any arc of that circle whose centre lies between the assumed point and its circumference, intercepted between the common diameter and any straight line drawn out from that same point, if it is less than a semicircle, is greater than would allow it to be similar to the arc of the other circle intercepted between the same straight lines. 57.

34. If a circle cuts a circle in half, or not in half, or does not cut it at all, and through the centres two diameters are drawn perpendicular to the straight line drawn out through those same centres: then two straight lines going out from the point on the straight line drawn through the centres — the point through which passes the straight line that joins the extremities of the two drawn diameters, and which lies in both circles — making, with the straight line equidistant from each diameter on both sides, or with the straight line passing through the centres, equal angles, will intercept...

The Index of Lemmas continues (the closing lemmas of Book I, with their appended scholia and corollaries). Each entry gives the lemma-statement followed by its page reference; "ibid." repeats the page of the preceding entry.

- **[34, concluded]** ...they will intercept similar arcs in each circle. That same straight line, equidistant from both diameters, will likewise cut off alternate similar arcs from each circle. And conversely, if two straight lines intercept similar arcs, they make equal angles on both sides with the same equidistant straight line. **58**
- **35.** If in a circle two diameters cut one another at right angles, and in the same circle a straight line is drawn inclined to both diameters, or parallel to one of them; and if from one end of either diameter—through the ends of the inclined straight line, or from the end of the diameter to which the straight line is equidistant—two straight lines are extended forming a triangle whose base is the inclined line (or that parallel): then the other diameter will cut off from the sides of this triangle a similar triangle, but oppositely (subcontrarily) placed. And if the inclined straight line passes through the centre, the perpendicular drawn to it from that same end of the diameter will bisect the base of the triangle cut off by the other diameter, and that perpendicular will equal half of the same base. But if the straight line does not pass through the centre—whether it is inclined or parallel to one of the diameters—and a diameter is drawn perpendicular to it, and through the point where it cuts that line a straight line is drawn from that same end of the diameter as far as the circumference, and finally an arc equal on the other side is cut off from the arc between this point on the circumference and the perpendicular diameter last drawn: then the straight line drawn from that said end of the diameter to the end of this arc will likewise bisect the base of the triangle cut off by the other diameter. **60**
- If in a circle two diameters are drawn cutting one another at right angles, then the straight line drawn perpendicular to some other, oblique diameter from the end of either of the diameters cutting at right angles bisects the segment of any straight line equidistant from the other diameter that is intercepted between the straight lines drawn from that same end-point through the ends of the oblique diameter. **62**
- **36.** If in a circle two diameters cut one another at right angles, and in the same circle two other diameters are drawn inclined to them, and from one end of either of the first diameters two straight lines are extended through the ends of the latter: then the straight lines cut off on the other of the first diameters by the two lines will be greater than the diameter of the circle, and they will also be unequal to one another—the greater being the one

whose inclined diameter makes the larger angle with the other of the first diameters. *ibid.*

- **37.** The circles of position, in an oblique northern sphere, that cut the semidiurnal arc of the Equator into equal parts, cut the semidiurnal arcs of the parallels into unequal parts. In the southern parallels, any part between the Meridian and any circle of position is smaller, relative to its own semidiurnal arc, than the same part in the Equator relative to the semidiurnal arc of the Equator; but in the northern parallels it is larger. Nevertheless the same circles of position cut the parallels that touch the Horizon into equal parts as well. **63**
- **38.** In an oblique northern sphere, the circles passing through the unequal hours of the Equator and of any parallel cut the Meridian on the southern side below the Horizon, between that Horizon and the southern pole; but on the northern side above the Horizon, between that same Horizon and the northern pole. **64**
- **39.** Great circles passing through the unequal hours of the Equator and of two opposite parallels do not necessarily pass through the unequal hours of the intermediate parallels in an oblique sphere. **65**
- That there are no great circles which pass through the unequal hours of all the parallels—against most writers on sundials. *ibid.*
- What the lines of the unequal hours in sundials represent. **66**
- **40.** If in a triangle a parallel to one side is drawn, or if—when two sides are produced toward the angle contained by them—a parallel to the third side is drawn, so that two triangles are formed: the circles described about them touch one another at the angle, or common point. *ibid.*
- Two circles which are described from two centres lying on the same straight line, through the same point, touch one another externally at that point. **67**
- **41.** Through two given points, to describe a circle that touches a given circle. *ibid.*
- **42.** Given two circles, to describe through a given point on the circumference of one a circle that touches both given circles. **73**

- **43.** If in a sphere a circle touches two great circles on the same sides, between the point of section and the great circle drawn through their poles: then the arcs of those two great circles intercepted between the points of contact and the intersection of the circles—or [between the points of contact and] the great circle drawn through their poles—are equal. **74**
- **44.** If in a sphere a circle touches two equal non-great circles, the arcs of those two non-great circles lying between the points of contact and the great circle drawn through their poles—or the point of section, when they intersect one another—are equal. **75**
- **45.** If in a sphere a circle touches two parallel circles on the same sides of the great circle drawn through their poles: then their arcs intercepted between the points of contact and any great circle drawn through their poles are similar. **76**
- **46.** If in a sphere two circles cut one another: the great circle that bisects the segment of the one, and passes through the poles of that circle, also passes through the poles of the other circle. *ibid.*
- **47.** If in a sphere three great circles are drawn through the pole of any great circle, making two equal angles at the pole: then any circle described from any point of the middle circle as pole cuts off—both from the other two great circles, and from two equal circles (whether great or not great) that have their poles on the first great circle at equal distances from that middle great circle—equal arcs beginning on the same sides from that first great circle; yet in the great or non-great equal circles that have their poles on that first great circle, [these arcs are reckoned] from the points that lie short of, or beyond, their poles. **77**
- **48.** If from the same centre two circles are described, and from any number of points on the inner circumference equal straight lines are drawn to the outer circumference, and one of them is supposed to touch the inner circle, then the rest will touch it as well. And if several lines touching the inner circle are drawn toward the same side—namely toward the left, or the right—they will be equal to one another, and the arcs contained between each pair will be similar. **78**
- **49.** To demonstrate a few things concerning declinations, rising latitudes, and right and oblique ascensions. **80**

- Any parallel passes through two points equidistant from one or other of the tropic points. *ibid.*
- Two parallels drawn through two points of the Ecliptic equidistant from one or other equinoctial point—or from the two equinoctial points, or even from the two tropic points—have equal declinations. *ibid.*
- Those same two parallels have equal rising latitudes. *ibid.*
- Those same two parallels are equal. *ibid.*
- Four points of the Ecliptic have equal declinations and rising latitudes. *ibid.*
- That it is sufficient to find the declinations and rising latitudes of all the points of one quadrant of the Ecliptic. *ibid.*
- Which arcs of the Ecliptic are called opposite, and which equidistant from some point of the Ecliptic. **81**
- Four equal arcs of the Ecliptic have equal right ascensions and descensions. *ibid.*
- That it is sufficient to find the right ascensions of all the arcs of the first quadrant of the Ecliptic. *ibid.*
- Which arcs of the Ecliptic are greater than their right ascensions, and which are smaller. *ibid.*
- The right ascension of any arc or point is equal to the right descension [descensioni (?): the printed glyph is abraded; the sense requires "descension"] of the same arc or point. **82**
- A great circle drawn from the pole of the world through the intersection of the parallel of any point of the Ecliptic with the oblique Horizon intercepts, together with the Horizon, on the Equator the arc of the ascensional difference of that point of the Ecliptic; but with another great circle drawn through that point of the Ecliptic, [it intercepts] the oblique ascension of the arc of the Ecliptic lying between that point and the Horizon. *ibid.*
- Two equal arcs of the Ecliptic, begun from one or other equinoctial point, or equidistant [from it], have equal oblique descensions. *ibid.*

- Two equal arcs of the Ecliptic equally removed from the same tropic point, and likewise two opposite arcs, have their oblique ascensions taken together equal to their right ascensions taken together. **83**
- Arcs of the Ecliptic begun from Aries and smaller than a semicircle are greater than their ascensions in an oblique sphere; but those begun from Libra are smaller. *ibid.*
- Arcs of the Ecliptic begun from Aries have oblique ascensions smaller than their right ascensions by as much as the oblique ascensions of the equal arcs begun from Libra are greater than the right ones. **84**
- Opposite points of the Ecliptic have ascensional differences equal to one another. *ibid.*
- Of two equal arcs of the Ecliptic equidistant from the same tropic point, or opposite, the oblique ascension of the one is smaller than the right ascension by as much as that of the other is greater. *ibid.*
- Two equal arcs of the Ecliptic equidistant from the same tropic or equinoctial point, or opposite, have the same ascensional difference. *ibid.*
- Any arc of the Ecliptic bisected from the same tropic point has, in all locations, its oblique ascension equal to its own right ascension. **85**
- The descension of any arc of the Ecliptic is equal to the ascension of the opposite arc. *ibid.*
- That it is sufficient to compute the oblique ascensions of the arcs of the first quadrant of the Ecliptic in order to construct the whole table of oblique ascensions. *ibid.*
- The ascensional difference of any point of the Ecliptic is also the difference between the semidiurnal arc of that point and the semidiurnal arc of the Equator, which is always a quadrant. *ibid.*
- How the semidiurnal arc of any point of the Ecliptic is derived from the ascensional difference of that point. *ibid.*
- When the ascensional difference is to be added or subtracted in order to obtain the semidiurnal arc, or the oblique ascension, of a given point or star. *ibid.*
- That four points of the Ecliptic have the same ascensional difference. **86**

- The whole sine has to the sine of the complement of the declination of any point of the Ecliptic the same ratio that the secant of the arc between that point and the nearest equinoctial point has to the secant of the right ascension of the same arc. *ibid.*
- The whole sine has to the tangent of the pole's altitude the same ratio that the tangent of the declination of a given point of the Ecliptic has to the sine of the ascensional difference of that point. *ibid.*
- How the difference between the longest (or shortest) semidiurnal arc and the semidiurnal arc of the Equator is computed for any elevation of the pole. **87**
- The whole sine is to the sine of the right ascension of any point of the Ecliptic as the sine of the ascensional difference of the beginning of Cancer or Capricorn is to the sine of the ascensional difference of that point. *ibid.*
- The sine of the complement of the declination of any point of the Ecliptic is to the sine of the declination of that point as the whole sine is to the sine of the ascensional difference of that point, at latitude 45°. *ibid.*
- The arc that corresponds, as a sine, to the tangent of the declination of any point is the ascensional difference of that point at latitude 45°. **88**
- The sine of the complement of a given pole's altitude is to the sine of the pole's altitude as the sine of the ascensional difference of any point of the Ecliptic at latitude 45° is to the sine of the ascensional difference of that point at the former given pole altitude. *ibid.*
- The whole sine is to the tangent of a given pole's altitude as the sine of the ascensional difference of any point of the Ecliptic at latitude 45° is to the sine of the ascensional difference of that point at the given pole altitude. *ibid.*
- **50.** Given two axes of an ellipse cutting one another at right angles: if from any point of the minor axis—produced too, if need be—a straight line equal to half the major axis is drawn cutting the major axis, so that its segment beyond that major axis equals half the minor axis, then its end will fall on the ellipse. And if from any point of the ellipse a straight line equal to half the major axis is drawn as far as the minor axis—produced too, if need be—yet cutting the major axis, then its segment between the given point and the major axis will equal half the minor axis. *ibid.*

- Given the axes, to describe an ellipse. **89**
- Given one or other of the axes, and a point on the ellipse to be described about that axis, to find the other axis. *ibid.*
- Given the two axes of an ellipse and any point, to determine whether this given point lies on the ellipse, or outside, or inside it. *ibid.*
- Given two unequal straight lines and any point, to describe an ellipse through this given point, whose centre is likewise given, and whose axes are equal to the given straight lines. **90**
- **51.** If circles are described about the axes of an ellipse, and ordinates are applied to them as far as the peripheries of the ellipse and of the circles: then the ordinates [drawn] to the ellipse are proportional to those [drawn] to the proper circle—namely to whose diameter they are applied. *ibid.*
- Ordinates are cut proportionally by the ellipse and by the circles described about the axes. *ibid.*
- **52.** Given the axes of some ellipse cutting one another at right angles: to find on any given straight line the points through which the ellipse, if it be described, must pass. **91**
- **53.** To resolve all questions that are usually solved by sines, tangents, and secants by prosthaphaeresis alone—that is, by addition and subtraction alone—without the laborious multiplication and division of numbers. **94**
- The manifold calculation of spherical and rectilinear triangles. **100**

INDEX OF THE PROBLEMS AND THEOREMS DEMONSTRATED IN THE PROPOSITIONS OF THE SECOND BOOK AND IN THEIR SCHOLIA

The prefixed numbers indicate those [items] which have been inserted at various places in the propositions and their scholia.

(The numbers heading each entry below are non-sequential and repeat — 4, 4, 5, 5, 5, etc. — as printed; they are the source's own cross-reference numbers into the marked passages of each proposition, not a running count.)

In the Proem

- **1.** A sphere can be depicted in the plane in various ways. p. **125.**

- 2. How the Universal Astrolabe of Gemma Frisius [Gemma Frisius] is to be constructed, and where the eye is to be placed in the sphere. *ibid.*
- 3. On what basis the Universal Planisphere of Juan de Rojas [Juan de Rojas] is constructed. *ibid.*
- 4. How the Astrolabe (or Planisphere) of Ptolemy [Ptolemy] is to be constructed for a given altitude of the pole, and where the eye is to be placed in the sphere. *ibid.*
- 4. What plane Jordanus [Jordanus de Nemore] assumes in constructing the same Astrolabe, or Planisphere of Ptolemy. *ibid.*
- 5. What things are chiefly to be depicted in the Astrolabe. *ibid.*
- 5. The regions comprised between the points, lines, and circles of the sphere need no special depiction in the Astrolabe. *ibid.*
- 5. Which parts of the heavens the individual parts of the Astrolabe correspond to. *ibid.*
- 6. Where any point of the sphere appears in the Astrolabe. **126**
- 7. When a straight line in the sphere appears as a point in the Astrolabe, and when as a straight line. *ibid.*
- 8. How any circle of the sphere is viewed in the Astrolabe. *ibid.*
- 9. What it is to construct an astrolabe. *ibid.*
- 9. What an astrolabe, or planisphere, is. *ibid.*

In Proposition 1

- 1. Any circle of the sphere drawn through the southern pole is projected onto the astrolabe as an infinite straight line, which is the common section of that circle and of the plane of the astrolabe (or of the Equator). The parts of that straight line answering to equal arcs are unequal, and the greater the farther they are from the visual ray drawn through the centre of the circle; yet two parts, one on either side, equally distant from that ray and answering to equal arcs, are equal. **127**
- 4. In the astrolabe the northern pole, the axis of the world, and the centre of the sphere (or of the world) are the same as the centre of the astrolabe. **128.**

- 4. All great circles drawn through the poles of the world are projected as straight lines intersecting one another at the centre of the astrolabe. *ibid.*
- 5. How the circles drawn through the poles of the world—where they are straight lines in the astrolabe—are divided into degrees. *ibid.* [The print reads "Ciculi" [recte: Circuli].]
- 6. How any arc or degree of a circle drawn through the poles of the world is found on the straight line that represents that circle in the astrolabe; and how many degrees are contained in a given segment of that straight line, and how this is known. *ibid.*

In Proposition 2 [printed "IN PROPOS, 2."]

- 1. The Equator and all its parallels are projected onto the astrolabe as circular shapes. **129**
- 3. The arcs of those same circles are projected as similar arcs, and thus equal arcs into equal ones. *ibid.*
- 4. The Equator and its parallels in the astrolabe must be divided into equal parts so that their degrees may be had, just as in the other circles in the sphere. **130**
- 5. In the astrolabe the southern parallels of the Equator are larger than the Equator, and the northern ones smaller. *ibid.*
- 6. In the astrolabe the Equator and its parallels have the same centre as the astrolabe. *ibid.*

In Proposition 3

- 1. Any circle of the sphere oblique to the Equator—or even right but not great—is projected onto the astrolabe as a circular figure. **130**
- 2. The arcs of that same circle, beginning from a certain point, are projected as dissimilar arcs, and thus equal arcs into unequal ones. **131.**
- 4. Any circle oblique to the Equator—or even right but not great—has in the astrolabe a centre different from the centre of the astrolabe. *ibid.*

In the Scholium of Proposition 3

- 1. Any oblique great circle and its parallels—or even a non-great circle right to the Equator—must be viewed from the southern pole in the

common section of the Equator (or the plane of the astrolabe) and of the great circle drawn through the poles of the world and the poles of the oblique or right circle, both so that they may be projected into a circular shape, and so that their greatest diameters may be had as seen. **131**

- **1.** The diameters of any oblique circles—or even of right circles that are not great—in the astrolabe, seen in the common section of the Equator (or the plane of the astrolabe) and of the great circle drawn through the poles of the world and the poles of the oblique (or even right) circles, are the greatest of all. **132**
- **4.** The centres of any oblique circles—or even of right circles that are not great—in the astrolabe must be taken in the common section of the plane of the astrolabe (or the Equator) and of the great circle drawn through the poles of the world and the poles of the oblique or right circles. *ibid.*
- **4.** The straight line drawn through the centre of the astrolabe and the centre of any circle described in the astrolabe is the common section of the plane of the astrolabe (or the Equator) and of the great circle that is drawn through the poles of the world and the poles of the described circle. **133**
- **6.** Jordanus's demonstration that oblique circles—or even right circles that are not great—are projected as circular figures. *ibid.*

In Proposition 4

- **1.** The Equator and its parallels in the astrolabe, from the ana-[lemma...] (*entry continues on the following page*)

Index of the Problems and Theorems of the Second Book (continued)

(*Index Problematum ac Theorematum Secundi Libri* — continuation. The entries below run on from the preceding page; page references stand verbatim as printed, with "ibid." meaning "the same page." The item numbers, including their repetitions and gaps, are reproduced exactly as in the source.)

[The following entries fall under **IN PROPOS. IIII** (On Proposition 4), whose heading stands on the preceding page.]

to describe [it] by means of a lemma, if the magnitude of the Equator is given. **134**

1. Meridian and the right horizon: by what straight lines they are represented on the astrolabe. **135**
2. That the Equator and its parallels must be divided into equal parts, so that their degrees may be had. **ibid.**
3. That straight lines drawn through the centre of the astrolabe, dividing any circle described from that same centre into 360 equal parts, represent the great circles of the sphere drawn through the poles of the world, and the individual degrees of the Equator. **ibid.**
4. To describe on the astrolabe, from the Analemma, any parallel of the Equator whose declination is given. **ibid.**
5. To find, from the Analemma, the declination of any parallel of the Equator drawn on the astrolabe, and whether it is northern or southern. **136**
6. To describe the Equator and its parallels on the astrolabe without constructing the Analemma, if the magnitude of the Equator is given. **ibid.**
7. To describe on the astrolabe, without constructing the Analemma, any parallel of the Equator whose declination is given. **ibid.**
8. From a single arc of declination on the Equator, to describe both the southern and the northern parallel of that declination. **ibid.**
9. To find, without constructing the Analemma, the declination of any parallel of the Equator drawn on the astrolabe, and whether it is northern or southern. **ibid.**
10. To find more accurately and more precisely the semidiameters of the parallels of the Equator, especially the southern ones. **137**
11. That the semidiameter of the Equator is the mean proportional between the semidiameters of two opposite parallels of the Equator drawn on the astrolabe, and what proportion they have. **ibid.**
12. To derive on the astrolabe the semidiameter of any southern parallel of the Equator from the semidiameter of the opposite northern parallel. **ibid.**
13. That, of all the points of the sphere, the southern pole of the world alone cannot be projected onto the astrolabe. **138.**

14. That not all southern points of the sphere (even with the southern pole excluded) can be conveniently projected onto the astrolabe. **139.**

IN SCHOLIO PROPOS. 4.

(On the Scholium to Proposition 4)

1. To describe the Equator and its parallels on the astrolabe, if the magnitude of the Tropic of Capricorn [♋] is given. **ibid.**
2. To describe the Equator and its parallels on the astrolabe, if the magnitude of the Tropic of Cancer [♋] is given. **ibid.**
3. To describe the Equator and its parallels on the astrolabe from the given magnitude of any parallel of the Equator. **ibid.**
4. That no parallel of the Equator can be described on the astrolabe from the given magnitude of the opposite parallel, unless the Equator is first described. **140**

IN PROPOS. 5.

(On Proposition 5)

1. To describe on the astrolabe, from a construction of the Analemma, any oblique horizon, its prime vertical, the ecliptic, and any other oblique great circle whatever that is nevertheless perpendicular to the meridian and has a known inclination to the Equator. **140**
2. Which parallels the ecliptic, the horizon, and the prime vertical touch. **141**
3. To describe on the astrolabe, without constructing the Analemma, any oblique horizon, its prime vertical, the ecliptic, and any other oblique great circle whatever that is nevertheless perpendicular to the meridian and has a known inclination to the Equator. **142**
4. To find the centre of the horizon on the astrolabe, even if its apparent diameter has not been found. **143**
5. That a straight line drawn from the southern pole at right angles to the diameter of an oblique great circle described on the Equator falls upon the centre of that same oblique circle on the astrolabe. **ibid.**

6. To find the centre of any oblique great circle on the astrolabe, even if its apparent diameter has not been found. **ibid.**
7. That the centre of any oblique great circle on the astrolabe is different from the centre of the astrolabe. **ibid.**
8. That the ecliptic always appears as a circle on the astrolabe, and of the same magnitude, even if it is carried round continuously with the diurnal motion in the sphere. **ibid.**
9. In what manner the true diameter of a given oblique great circle that is perpendicular to the meridian is to be drawn on the Equator of the astrolabe, so that the oblique circle itself may be described on the astrolabe by means of it. **144**
10. To find more accurately the extreme point of the apparent diameter of an oblique great circle, the one that is more remote from the centre of the astrolabe. **ibid.**
11. To describe an oblique great circle on the astrolabe, even if its apparent diameter has not been found. **145**
12. To find, by another method than the one above, and very precisely, the semidiameter of any southern parallel of the Equator. **ibid.**
13. By what straight lines the poles of any oblique great circle on the astrolabe are indicated on the meridian line. **ibid.**
14. Which angles the radius drawn from the southern pole through the more remote pole of the oblique great circle bisects. **ibid.**
15. That the pole of any oblique circle on the astrolabe is different from the centre of the astrolabe. **ibid.**
16. To find by another way the centre of an oblique great circle on the astrolabe. **ibid.**
17. That the radius drawn from the southern pole to the pole of the oblique circle cuts off, from the meridian line and from the true diameter of the oblique circle, equal straight lines. **ibid.**
18. That the pole of an oblique great circle differs from its centre on the astrolabe. **146**

19. To divide the oblique horizon into degrees on the astrolabe from its upper pole. **147**
20. When the upper pole of an oblique great circle is only slightly distant from the circumference of the Equator, by what method it may be divided into degrees more precisely. **ibid.**
21. To find any proposed degree on the horizon of the astrolabe from its upper pole. **ibid.**
22. Which are the eastern, western, northern, and southern parts on the horizon of the astrolabe. **ibid.**
23. To bisect a given arc of an oblique great circle on the astrolabe. **148**
24. To know, from its upper pole, how many degrees are contained in a given arc of the horizon of the astrolabe. **ibid.**
25. To divide the oblique horizon into degrees on the astrolabe from its lower pole. **ibid.**
26. To divide into degrees on the astrolabe, from either of its poles, the ecliptic, the prime vertical, and any other oblique great circle whatever that is perpendicular to the meridian. **149**
27. To divide into degrees on the astrolabe, from either of its poles, any oblique great circle that is not perpendicular to the meridian. **ibid.**
28. An easy rule for determining the starting points of the arcs cut off in dividing great circles into degrees, by means of straight lines drawn from either of the poles of any oblique circle. **ibid.**
29. An easy rule for knowing which of the points of the Equator is higher in the heavens, or lower; and which of the points of an oblique great circle is northern, or southern. **150**
30. An easier rule for fixing in advance the starting points of the arcs. **ibid.**
31. To divide into degrees on the astrolabe any oblique great circle that is perpendicular to the meridian, from the centre of another great circle which, with respect to it, serves as a prime vertical. **151**
32. To find any proposed degree on an oblique great circle that is perpendicular to the meridian

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25....perpendicular to the meridian on the astrolabe, from the centre of another great circle which, with respect to it, serves as a prime vertical. **152**

26. To know how many degrees are contained in a given arc of an oblique great circle perpendicular to the meridian, from the centre of another great circle which, with respect to it, serves as a prime vertical. **ibid.**

27. To divide into degrees any oblique great circle that is not perpendicular to the meridian, from the centre of another great circle which, with respect to it, serves as a prime vertical. **ibid.**

28. Which lines touch an oblique great circle on the astrolabe. **ibid.**

29. That certain lines converging on the astrolabe represent lines parallel and non-converging in the heavens. **153**

30. To divide into degrees, from the southern pole of the Analemma, any oblique great circle that is perpendicular to the meridian. **ibid.**

31. To find, from the southern pole of the Analemma, any proposed degree on an oblique great circle perpendicular to the meridian. **ibid.**

32. To know, from the southern pole of the Analemma, how many degrees are contained in a given arc of an oblique great circle perpendicular to the meridian. **154**

33. To divide into degrees, from the southern pole of the Analemma, any oblique great circle on the astrolabe that is not perpendicular to the meridian. **ibid.**

34. To divide any oblique great circle on the astrolabe into degrees from its own centre and from the centre of the astrolabe, that is, of the Equator. **ibid.**

35. To divide any great circle of the astrolabe into degrees by means of another great circle already divided. **155**

36. To cut off, from any given arc on any great circle, an arc equal to it as regards the number of degrees, taken from any other great circle. **ibid.**

37. To cut an oblique great circle into degrees in various ways, by means of various circles described through three points, as will be taught in Proposition 6, No. 36. **ibid.**
38. To divide an oblique great circle into degrees in various ways by means of various straight lines. **ibid.**
39. To draw, from any point of the meridian line of the oblique circle, straight lines cutting the oblique circle itself into degrees. **156**
40. Given a point on an oblique great circle, to find the corresponding point on the Equator. **157**
41. Given any point on the plane of some great circle in the sphere, even outside the circle, to find its position on the astrolabe. **ibid.**
42. Which true points on the plane of a given oblique circle in the sphere do not have corresponding points on the astrolabe. **ib.**
43. Given any point on the astrolabe, to find its position on the plane of any great circle in the sphere. **ibid.**
44. Which apparent points of the astrolabe do not have true corresponding points on the plane of a given oblique circle in the sphere. **ibid.**
45. From any point given on the astrolabe outside the meridian line, to divide a given great circle into degrees. **158**
46. To divide any oblique great circle into degrees by three other ways, as in Proposition 6, Nos. 37 and 38. **ib.**

IN SCHOLIO PROPOS. V.

(On the Scholium to Proposition 5)

1. Through which points of the Equator oblique great circles that are nevertheless perpendicular to the meridian are drawn on the astrolabe. **158**
2. That any oblique great circle on the astrolabe is greater than the Equator. **159**
3. Through which points of the Equator oblique great circles that are not perpendicular to the meridian are drawn on the astrolabe. **ibid.**

4. That any great circle on the astrolabe passes through two diametrically opposite points of the Equator, and therefore bisects the Equator. **ibid.**
5. By what straight line the common section of the Equator and of any oblique great circle in the sphere is represented on the astrolabe. **ibid.**
6. That the Equator and any oblique great circle on the astrolabe mutually bisect one another, although the segments of the oblique circle are very unequal to each other. **ib.**
7. Why the semicircles of any oblique great circle, formed by the Equator, are unequal on the astrolabe. **ib.**
8. Why the Equator on the astrolabe is cut by any oblique great circle into two equal semicircles at two diametrically opposite points. **ib.**
9. That any circle, whether a great circle or not, dividing some parallel of the Equator in the sphere into two equal parts, passes on the astrolabe through two diametrically opposite points on that parallel. **ibid.**
10. That a circle which is not a great circle cannot bisect the Equator of the astrolabe. **ibid.**
11. That a circle on the astrolabe which bisects the Equator represents a great circle in the sphere; but one that does not divide it into two equal parts represents a circle that is not a great circle. **ib.**
12. That any straight line drawn through the centre of the astrolabe indicates, on any oblique great circle, two diametrically opposite points, so that it takes the place of a certain diameter. **160**
13. That equal arcs of an oblique great circle are projected into unequal arcs, in continuous order. **161**
14. That it can happen that some one arc of an oblique great circle in the sphere is projected onto the astrolabe into a similar arc. **ibid.**
15. Various properties of oblique great circles on the astrolabe. **162**
16. That a circle described on the astrolabe through two diametrically opposite points is a great circle. **ib.**
17. Which arc of an oblique great circle on the astrolabe is equal, as regards the number of degrees, to the arc of the Equator measuring the altitude of

the pole above that same oblique circle; and which is not only equal in number of degrees to the complement of that altitude, but also similar to it. **ibid.**

18. Which straight lines touch the Equator and an oblique great circle on the Equator, and where. **163**
19. That a straight line drawn from the lower pole of an oblique great circle, if it touches the Equator, will also touch the oblique circle; and if it touches the oblique circle, it will also touch the Equator. **ibid.**
20. Which similar arcs a straight line perpendicular to the meridian line at the pole of an oblique great circle cuts off from the Equator and from the oblique great circle. **ibid.**
21. Which similar arcs the straight lines drawn from the poles of an oblique circle take away from the Equator and from the oblique great circle. **164**
22. To describe the Equator on the astrolabe from an oblique great circle that is perpendicular to the meridian and has a known inclination to the Equator. **165**
23. Which points on the astrolabe represent in the sphere two diametrically opposite points. **ibid.**
24. To know the altitude of the pole above an oblique great circle on the astrolabe that is perpendicular to the meridian, and its inclination to the Equator, and its position in the sphere. **ibid.**

IN PROPOS. VI.

(On Proposition 6; running head: INDEX)

1. To describe on the astrolabe, from the Analemma, the parallels of the horizon and of any other oblique great circle that is nevertheless perpendicular to the meridian. **166**
2. To describe those same parallels by means of the Equator, even if the Analemma has not been constructed separately. **ibid.**
3. That the parallels of the horizon which in the sphere cut the meridian between the southern pole and the zenith surround the zenith itself on the astrolabe. **ibid.**

4. That the parallel of the horizon which in the sphere is drawn through the southern pole is projected on the astrolabe into a straight line, which is perpendicular to the meridian line at the centre of the prime vertical. **167**
5. That the parallels of the horizon which in the sphere cut the meridian between the southern pole and the nadir surround the nadir itself on the astrolabe. **ibid.**
6. What the common section of the Equator and of a parallel of the horizon is on the astrolabe. **168**
7. In what manner the meridian and the meridian line of any oblique circle are to be understood on the astrolabe. **169**
8. Which are the semicircles and quadrants of the horizon, and of its parallels, cut off by the prime vertical and by the meridian on the astrolabe. **ibid.**
9. To find the apparent diameters of the parallels of the horizon, together with their centres, by means of the horizon itself on the astrolabe. **ibid.**
10. That a circle described through the extreme points of the apparent diameter of any parallel of the horizon, and through the southern pole, touches the horizon at the southern pole. **170**
11. To cut off from the meridian a straight line which is the apparent diameter of some parallel of the horizon. **171**
12. Given one extremity of the apparent diameter of any parallel of the horizon, to find the other extremity by means of the circle touching the horizon. **ibid.**
13. To find the apparent diameters of the parallels of the horizon by means of the circle touching the horizon at the southern pole. **ib.**
14. That straight lines drawn from the centre of the prime vertical to the intersections of the parallels of the horizon with that same prime vertical touch the parallels there. **172**
15. Given one extremity of the diameter of the horizon, or of its parallel, to find the other extremity by means of a certain third proportional. **ibid.**
16. That the semidiameter of the prime vertical is the mean proportional between the straight line lying between the centre of the prime vertical

and either of the extremities of the diameter of the horizon (or of its parallel), and the straight line placed between the same centre of the prime vertical and the other extremity of the diameter of the horizon (or of its parallel). **ibid.**

17. To find the apparent diameters of the parallels of the horizon by means of an arc of any magnitude described from the southern pole. **ibid. & seq.**
18. To find the centres of the parallels by means of straight lines drawn from the southern pole. **173**
19. To find the semidiameter and the centre of any parallel of the horizon by means of a single line that touches the prime vertical. **ibid.**
20. An easy practical method for drawing several lines that touch a given circle at given points. **ibid.**
21. That the centre of any parallel of the horizon is different from its pole. **175**
22. From any parallel of the horizon described on the astrolabe, to describe the opposite parallel, even if its diameter has not been found. **ibid.**
23. Given a point on the astrolabe, to find the point diametrically opposite to it on the sphere. **ibid.**
24. To find a given point on a southern parallel of the Equator, at which it is cut by a proposed parallel of the horizon below the horizon **177**

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it is cut, when it is cut, even if it has not been described. **177**

17. To describe on the astrolabe a given parallel of the horizon in the sphere. **ibid.**
18. Given a parallel of the horizon on the astrolabe, to know how great its distance from the horizon is. **ibid.**
19. In what manner all that has been said about describing the parallels of the horizon may be applied to describing the parallels of other oblique great circles, whether they are perpendicular to the meridian or not. **178**
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21. To describe on the astrolabe a southern parallel of the Equator from the equal parallel of an oblique great circle described about its pole more remote from the southern pole. **ibid.**
22. Whence the starting point of the corresponding arcs on the parallels is to be taken, in this method of dividing oblique parallels into degrees from their upper pole. **ibid.**
23. An easy rule for knowing whether a point of a parallel of the Equator is said to be higher in the heavens or lower, with respect to a given oblique great circle; likewise whether a point of the oblique parallel is northern or southern. **180**
24. To find on the astrolabe any proposed degree on a parallel of the horizon from its upper pole. **181**
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27. Whence the starting point of the corresponding arcs on the parallels is to be taken, in this method of dividing oblique parallels into degrees from their lower pole. **ibid.**
28. In what manner all that has been said about dividing the parallels of the horizon, from its poles, may be applied to other oblique parallels. **ibid.**
29. To divide into degrees an oblique parallel by means of a circle of any magnitude divided into equal degrees; so that there is no need to describe a southern parallel of excessive size, or a northern one of very small magnitude. **182**
30. That the radius drawn from the southern pole to the pole of the oblique circle cuts off, from the meridian line and from the true diameter of the oblique circle, equal straight lines. **ibid.**
31. To divide a great oblique circle into degrees by means of a circle of any magnitude greater than the Equator. **183**
32. To divide any apparent great circle into apparent degrees by means of the equal degrees of that same apparent great circle. **ibid.**

33. To divide any apparent oblique parallel into apparent degrees by means of the equal degrees of that same parallel. **184**
34. To know by a very easy method how many degrees are contained in a given arc of an oblique circle. **185**
35. To cut a given arc of an oblique circle into any number of equal apparent parts by a very easy method. **ibid.**
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38. To know how many degrees are contained in a given arc of an oblique parallel, from the centre of the great circle which is, as it were, its prime vertical. **ibid.**
39. In what manner all that has been said about dividing the parallels of the horizon from the centre of the prime vertical may be applied to other oblique parallels. **ibid.**
40. That straight lines drawn from the centre of any great circle on the astrolabe to its intersections with the parallels of another great circle, which is, as it were, its horizon, touch the parallels there. **ibid.**

ASTROLABIVM — Index Problematum ac Theorematum (continued)

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- 3. To search out, without an instrument, at what hour any given point of the Ecliptic rises, wherever the Sun may be. — *ibid.*
- 6. To determine in what celestial house a given star or point of the Ecliptic lies at the hour of observation. — *ibid.*

On the Scholium to Canon 11 [IN SCHOLIO CANONIS XI.]

- 1. To investigate, by right and oblique ascensions, the points of the Ecliptic lying on the Meridian, the Horizon, and any hour-circle whatever, reckoned from noon or from midnight. — 304
- 2. A more accurate finding of the point of the Ecliptic lying on a given hour-circle, with any sign rising, when the semidiurnal arc is not available in degrees and minutes, or in hours, minutes, and seconds. — *ibid.* & 305.
- 3. The finding, by oblique ascensions, of the hour at which any point of the Ecliptic rises, wherever the Sun may be. — 305.

On Canon 12 [IN CANONE XII.]

- 1. To investigate by the material Astrolabe the meridian line and the points of the true rising and setting. — 305
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- 2. To find the meridian line more reliably without the material Astrolabe. — 306.
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- 3. To investigate the meridian line without the instrument of the Astrolabe, by a single observation, given the Sun's declination and the altitude of the pole. — *ibid.*
-
- 4. To find out the meridian line without the material Astrolabe, from the Sun's declination alone, by two observations. — *ibid.*
-
- 5. To investigate the meridian line without the material Astrolabe, by three observations, even if the Sun's declination and the altitude of the pole are unknown. — *ibid.*

On the Scholium to Canon 12 [IN SCHOLIO CANONIS XII.]

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- 1. The finding of the meridian line from the Analemma, given the Sun's declination and the altitude of the pole. — 307
-
- 2. The finding of the meridian line on a horizontal plane by three observations, even if the Sun's declination and the altitude of the pole are not known. — *ibid.*
-
- 5. The construction and use of an instrument by which both the shadow and the altitude of the Sun are ascertained at once. — *ibid.*

On Canon 13 [IN CANONE XIII.]

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- 1. To find the altitude of the pole above the Horizon by a single observation, when the Sun's declination and the position of the meridian line are given. — 308
-
- 2. To investigate the altitude of the pole and the meridian line by two observations, from the Sun's declination alone. — 309
-
- 3. To search out the altitude of the pole, the meridian line, and the Sun's declination by three observations. — *ibid.*

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- 4. In what way the longitudes of places may be explored by lunar eclipses. — *ibid.*

On the Scholium to Canon 13 [IN SCHOLIO CANONIS XIII.]

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- 1. The finding of the altitude of the pole from the Analemma by two observations, even if the Sun's declination is unknown, provided the position of the meridian line is given. — 309
-
- 2. To determine the altitude of the pole and the meridian line by three observations, even though the Sun's declination is unknown. — *ibid.*
-
- 3. In what way it may be known whether the zenith of the place lies between the arctic pole and the Sun (or a star placed on the Meridian), or rather whether the Sun or star placed on the Meridian lies between the arctic pole and the zenith of the place. — 310
-
- 4. In what way the altitude of the pole is to be hunted out from the declination of the Sun or a star and from its meridian altitude. — *ibid.*
-
- 5. In what way it may be detected where the northern and southern part lies. — *ibid.*
-
- 6. Otherwise, and more easily, if it is established that the arctic pole is raised above the Horizon. — *ibid.*

On Canon 14 [IN CANONE XIV.]

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- 1. To determine in what zone a given place is situated. — 311
-
- 2. To perceive in what climate a given place is situated. — *ibid.*

On Canon 15 [IN CANONE XV.]

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- 1. To search out the road-distance of two places on the earth situated under the Equator. — *ibid.*
- 2. To measure the distance of two places of the same longitude. — *ibid.*
- 3. To find the distance of two places differing by 180 degrees of longitude. — 312
- 4. To investigate the distance of two places of different longitudes and latitudes. — *ibid.*
- 7. In what way the distance between a northern and a southern place may be found more conveniently. — 313
- 7. In what way the distance between two southern places is to be inquired into from opposite northern places. — *ibid.*
- 8. To investigate the distance of any two stars whatever. — *ibid.*

On the Scholium to Canon 15 [IN SCHOLIO CANONIS XV.]

- 1. To determine from the Analemma the distance of two places on the earth. — 314
- 2. By another method, to inquire into the distance of places from the Analemma. — 315
- 3. Another method of finding the distance of two places. — 316
- 4. Another method of investigating the distance between two northern or southern places. — *ibid.*
- 6. To search out the distance of places by calculation. — *ibid.*
- 6. Another finding of the distance of places by calculation. — 317
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- 6. The errors of certain writers in investigating the distance of places. — 318
- 6. Werner's method for searching out the distance of places. — ibid.
- 6. Pedro Nunes' [Petrus Nonius] method, easier than Werner's. — ib.
- 6. Reduction of the circumference of a parallel to the degrees of a great circle. — 319
- 6. Reduction of the chord of an arc of a parallel to the parts of the diameter of a great circle. — ibid.
- 6. In what way the declination of a star may be found by calculation, otherwise than was stated in the scholium of Canon 3. — ibid.

On Canon 16 [IN CANONE XVI.]

- 1. What the horizontal distance of the Sun is, on any great circle whatever. — 319
- 1. In what way the altitude of the Sun for a given hour above any great circle whatever may be found without the material Astrolabe. — 320
- 1. In what way the horizontal distance for a given hour above any great circle whatever may be determined without the material Astrolabe. — ibid.

On the Scholium to Canon 16 [IN SCHOLIO CANONIS. XVI.]

- 1. What the descensive and horizontal circumference are [text abbreviated: "que" [?]]. — 321
- 3. To make known by calculation, at any hour, the altitude of the Sun above any oblique great circle whatever. — 322
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- 3. To search out by calculation the horizontal distance above any oblique great circle whatever. — ib.
-
- 3. Another finding of the altitude of the Sun by calculation. — 323
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- 3. To observe the hour by calculation from the altitude of the Sun. — ibid.
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- 3. To search out by calculation the altitude of a star from its distance from the Meridian; and, conversely, its distance from the Meridian from its altitude. — ibid.

On Canon 17 [IN CANONE XVII.]

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- 1. To investigate the arc of any great circle whatever between one's own Meridian and the Meridian of a given region. — 323
-
- 2. To find the inclination of the meridian circle of any oblique great circle whatever to the Meridian of the Horizon. — ibid.

On the Scholium to Canon 17 [IN SCHOLIO CANONIS XVII.]

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- 1. In what way the great circles, from which the [hour-]clocks are equidistant, may be described on the Astrolabe. — 324

On Canon 18 [IN CANONE XVIII.]

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- 1. By what method the inclination of a given great circle, having a known position on the sphere, to the Meridian may be known. — 324
-
- 2. In what way the inclination to the Equator of an oblique great circle, whose position on the sphere is known, may be found. — ibid.

On Canon 19 [IN CANONE XIX.]

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1. To inquire into the arc of the Meridian between a given oblique great circle, whose position on the sphere is known, and both the Horizon and the pole of the world and the pole of the Horizon. — 325.

On Canon 20 [IN CANONE XX.]

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1. To inquire into the altitude of the pole above a given great circle whose position on the sphere is known. — 325

On the Scholium to Canon 20 [IN SCHOLIO CANONIS XX.]

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1. To find the arc of an oblique great circle, having a known position on the sphere, between the great circle drawn through its poles and the poles of the Horizon, and both one's own Meridian and the Meridian of the Horizon. — 325

•

2. By what method the arc of the great circle passing through the poles of the Horizon and the poles of a given oblique great circle, placed between the Horizon and the circle of the 6th hour reckoned from noon or midnight, may be known. — *ibid.*

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3. How many hours, and which, exist above each face of the oblique great circle, and at what hour it begins to be illuminated. Finally, what arcs of the parallels that great circle cuts off. — *ibid.*

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4. To find the angles that the Ecliptic makes with the Meridian, the Horizon, and the Vertical drawn through the Sun at any hour. — *ibid.*

On Canon 21 [IN CANONE XXI.]

•

1. What the hour-arc is, on any great circle whatever. — 326

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1. The finding of the hour-arcs on any great circle whatever. — *ibid.*

On the Scholium to Canon 21 [IN SCHOLIO CANONIS XXI.]

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1. The description of the hours on any plane whatever, by means of the hour-arcs. — 326
-
4. To compute the hour-arcs for the hours reckoned from noon and midnight. — 327

On Canon 22 [IN CANONE XXII.]

- All 22 problems of spherical triangles — those treated in Lemma 53 of Book 1 — are constructed and explained in the plane, without the aid of calculation, with wonderful ease. — 327

On the Scholium to Canon 22 [IN SCHOLIO CANONIS XXII.]

- By eight theorems, the various determinations of the magnitude of angles in spherical triangles are demonstrated. — 341 & following.
- Then the principal canons expounded above are once again explained more easily, by means of certain problems, with the help of spherical triangles described in the plane. — 344 & following.

FINIS.

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