

Mishnat ha-Middot

משנת המדות

[The Treatise of Measures]

(1864)

by R. Avraham bar Ḥiyya ha-Nasi (ר' חייא הנשיא "אברהם ב' "Savasorda"),
c. 1070–1145 — as attributed in this edition

Translation by Scott Weisman



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Upright reason dictates that the recipients of the good, of whatever type and level of beneficence it may be, must show gratitude and blessing to the beneficent person in every way possible, commensurate with the value of the beneficence. And one who has benefited all the people of the world — for example, one who invented a new instrument for the good of the world, or a good book — it is fitting for every discerning person, out of the obligation of love of fellow beings, to at least purchase it, so that the man will profit and his heart will be encouraged thereby to invent yet more good instruments in the world. And similarly, all other wise-hearted people will likewise strive and exert themselves to invent good things and instruments needed for the repair of the world and its perfection.

And therefore, whoever says: "What do I need this new instrument for?" — he does not act well towards the world. For if not for the man who invented it, where would you be? And what would your city do? And more than this, if the man did not exert himself, what would the world do?

— R. Pinchas Eliyahu Hurwitz, *Sefer HaBrit HaShalem* (1797)

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A note on the work

The earliest known Hebrew treatise on geometry — a concise practical manual of mensuration covering the areas and perimeters of plane figures, the measurement of circles and segments, and an early value for π . It is built from terse measurement rules, each tied to a labeled geometric figure and a worked numeric example. Its date and authorship are debated, with scholars placing the archaic core anywhere from the Mishnaic period to the Geonic era. This volume presents Moritz Steinschneider's critical edition (Berlin, 1864).

A note on the translation process

This translation was produced with the [translation-pipeline](#). See the linked repository for details on the process.

This text is the product of a highly refined automated translation process. It has undergone only cursory human review and should not be treated as an authoritative or scholarly rendering. It is offered solely as a reference and reading aid — a way to make an otherwise inaccessible work approachable — and any passage of real consequence should be checked against the original source.

Acknowledgements

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[Dedicatory letter]

To Dr. L. Zunz.

Most honored friend!

You will not measure the warmth of my congratulations on your day of joy by the slender compass of this little book. Nor is it the task or the aim of the disciple to instruct the master! Rather, I hope, by publishing a hitherto lost and forgotten old text, to give you occasion for fresh elucidations, and so, indirectly, to earn the gratitude of all who thirst for knowledge.

To the so-called Baraita of Samuel — which you yourself recently treated so thoroughly (*Hebräische Bibliographie*, 1862) — the so-called Mishnah of the 49 Middot attaches itself by virtue of its mathematical content; and it demands, still more than the Baraita, that combination of substantive and literary expertise which very few possess. Indeed, the history of the secular sciences among the Jews — both on account of its general bearings and as an essential component of Jewish cultural history — deserved a more independent treatment than has so far fallen to its lot.

The account that I, as editor, owe regarding the materials I have used I shall, unfortunately, be able to discharge all too quickly.

The "Mishnah of the Measures," in 5 chapters, containing altogether only 42 propositions — perhaps incomplete, perhaps merely differently divided (§§ 4–6 of chapter V were counted 5–7 in the manuscript; perhaps § 3 is to be split?) — I discovered two years ago in the remarkable MS 36 of the Munich Library (cf. *H. B.* no. 28, p. 107; no. 40, p. 84), which at this point is, admittedly, very corrupt; but it was certainly not feasible to wait here for another manuscript. I have left the text unaltered apart from a few unambiguous scribal errors, have punctuated it for less practiced readers, and have added the conjectured corrections or supplements in brackets;¹ on the other hand, I have in no way ventured upon the badly corrupted and barely intelligible chapter V. Several specialists whom I consulted about it could not decide how far it speaks of solids or of surfaces. I add further here—

¹ The citation from I, 1, 2 in Comtino (*Litbl. d. Or.* I, 158; Geiger's *Wiss. Zeitschr.* V, 457) offered, alongside poorer readings, only one probably better and one doubtful reading. — I, 8 the MS had במדה האחדים [*be-middah ha-aḥadim*] (rightly II, 4); II, 2 העומר [*ha-'omer*]; II, 1 הרחב [*ha-roḥav*] seems superfluous; II, 11 אתה מיישרת מרת [*atah meyasheret morat (?)*]. — I further note: II, 1 הרחב [*ha-roḥav*];

the great [reading] משוך [mashukh] or החבל [ha-ḥevel]?; מכיל [makhil] 9; 1. במדה [read *be-middah*] for והמודע [ve-ha-moda]; 1. והמודד [read *ve-ha-moded*] in [§] 6; — from chapter 12, line 4, והשלך מן כ"י, [ve-hashlekh min ...(?)] is evidently corrupt, namely almost all the numbers are to be altered; III, 5 אמה ממחשבה [amah mi-maḥshavah], perhaps מחשב אותה [meḥashev otah], is naturally $\frac{2}{3}$ [recte: שלישים, *shelishim*, "thirds"]; כוז [kuz (?)] perhaps refers to the **figure** of a quadrilateral divided into 2 triangles. — IV, 2 כן [ken] has 2 dots, perhaps a deletion mark. — 7 ∴ a lacuna-mark?; — 10 והמחצה [ve-ha-meḥetzah], 1. והמחשבה [read *ve-ha-maḥshavah*]?

[Note: several of the manuscript variants in this footnote are printed without vowel points and in a cramped, abraded type; the bracketed transliterations above represent my best reading of each token from the native-resolution tiles. The two clusters marked (?) — אתה מיישרת מרת and כוז — remain the least certain.]

[Editor's introduction, continued]

—here two figures which the manuscript contains in the margin of V, 1 and 3,²) whereas the preceding figures belonging to III, 2, 3, 4, 5 and IV, 2, 4, 9 — drawn partly inaccurately and in the wrong place — do not aid comprehension.

In some respects the terminology is noteworthy, and from it an alphabetical selection may here follow:

נג [nag] (roof) = plane surface; גג המרובע [gag ha-meruba] [the square's roof/upper face]; גוף [guf] = body, solid (II, 5); דופן [dofen] = side (II, 8); זווית [zavit] = angle, ישרה [yesharah] (III) [right]; עקומה [‘aqumah] = curved; חדה [ḥadah] or צרה [tzarah], פתוחה [petuḥah] or רחבה [reḥavah] (both V, 10) — right, oblique, acute, obtuse; חוט [ḥuṭ] (thread) = diagonal; חלף [ḥalef], cf. שוה [shaveh]; חלק על [ḥalleq ‘al] (*piel*, II, 4) = to divide by; חלק מן [ḥalleq min] = to subtract (II, 9); חץ [ḥetz] (arrow, I 5, II 4) = sinus versus [versed sine];* יסוד [yesod] (V, 5) = the square, cf. עקר [‘iqqar]; יתר [yeter] = chord; ממוצע [memutza] (II, 7) = the opposite of חד [ḥad]?; משיחה [meshiḥah] = area, dimension in general;⁴) משוך [mashukh] (II, 7, cf. I, 3) = tapering (pyramid, cone); כוב [kuv] and סביבה [sevivah] (V, 3) = periphery; סוף [sof] (II, 7) = basis, opposite of ראש [rosh] (apex; cf. Abr. b. Chijja p. 90, n. 14); עגולה [‘agulah] = circle; עולה [olah] = product (facit); עמוד [‘amud] = height, but II, 9: pillar; עקר [‘iqqar] = root; צלע [tzela] = side; שני צ [shenei tz[la‘ot]] = pair of sides (I 3, III 3); צרף בתוך (על) [tzaref be-tokh (‘al)] (*poël*) = to multiply by; מקובה [meqovah (?)] (II, 8, 9) = vaulted?;⁵) קטוע [qaṭua] (II, 9) = truncated; קשוחה [qeshuḥah] = grasslines/base-lines [Grundlinien]; קבע [qeva]; טרובע [teruva (?)] ¹/₄

(I, 8, beside בהן [bahen]) = circular segment; שבורים [shevurim] = fractions; מרובעת [meruba'at] (beside מתשע [metusha' (?)]) = quadrilateral; שוה [shaveh] = of the same kind, opposite of חלף [halef], מחלף [muḥlaf] (II 15, IV 8); משלשת [meshulleshet] = triangle; תל [tel] (II 8, V 2) = mound, hill, and hence תלולה [telulah], opposite of שפלה [shefelah] = flat (V, 3);²⁶) [the marginal cross-reference here is to footnote 6 and to figure 2].

For the phraseology I further single out: והרבה תבואתה בה [ve-harbeh tevu'atah bah] (II, 2, cf. ויש בה [ve-yesh bah]); פנים רבים [panim rabim] (II, 4).

As material for the historical criticism, I have also appended the well-known citations (enriched by a new one).⁷) If nos. 5–7 seem to refer to our little text only through a misunderstanding, then Geiger's latest conjectures⁸) stand no better than the older hypotheses that proceeded from those citations. The Middot evidently aim, first of all, at a concise presentation — wrestling with the Hebrew idiom — of the most important geometrical propositions; and almost everywhere the occasional reference to a biblical passage⁹) appears as midrashic, with—

²) [Two marginal figures, reproduced from the manuscript]

Diagram. Left figure: a tapering, roughly triangular/conical outline (an inverted "ice-cream-cone" shape), with three Hebrew labels.

- תלולה (top, above the apex): [telulah] "raised / pointed"
- חל יה (interior): [hel yah (?)] — uncertain reading; two short tokens inside the figure
- כחל (lower left, at the base): [kaḥol / ke-ḥol (?)] — uncertain reading

Diagram. Right figure: a rectangular frame (box) with a label inside and one below.

- חלי הכבינה (inside the box): [ḥali ha-kevinah (?)] — uncertain reading
- קרן (below the box): [qeren] "horn / corner"

*) Arabic سهم [sahm], see Sédillot, *Mémoires de l'Acad.* T. I, p. 219.

⁴) Arabic مساحة [misāḥa], cf. *HB* no. 28, p. 109; no. 40, p. 12; p. 92, note 20.

⁵) With this word the Late-Hebrew competes with the Arabic.

⁶) Cf. הצורה המשולשת והמרובעת והעגולה והנכנסת והפשוטה והפוסקת [ha-tzurah ha-meshulleshet ve-ha-meruba'at ve-ha-'agulah ve-ha-nikhneset ve-ha-peshuṭah ve-ha-poseqet] ("the triangular, quadrilateral, circular, inscribed, simple, and intersecting figure") in Saadia (*HB* 1862, p. 109).

⁷⁾ The citation in Jacob b. Simson (Bl. 1123–42, *Catal.* p. 2033) is unfortunately not accessible to me, likewise Joseph Kara on Kings, probably identical with nos. 3, 4; the latter perhaps originally belonging to the preceding words of Rashi?

⁸⁾ *Wiss. Zeitschr.* VI, 28 (cf. *Catal.* l. c.; cf. *jüd. Zeitschr.* II, 68).

⁹⁾ I, 2: Shemot 27:1; — I, 3 ימין ויבין [yamin ve-yavin], if correct, perhaps after I Melachim [I Kings] 7:21; further Yeshayahu [Isaiah] 54:[?], verse 3, probably with reference to verse 2; Shoftim [Judges] 16:26, the עמודי התורן [*amudei ha-tavekh*] as cross-beams? — בזוית למקצעות [ba-zavit la-miqtzo'ot] Shemot 36:28, cf. 26:23, is unclear; I, 5: Yechezkel [Ezekiel] 1:28, Yeshayahu 21:15, Tehillim 11:2; — II, 6: Yeshayahu 6:2, the angels' wings.

[Front matter, pp. 4–5: conclusion of Moritz Steinschneider's German scholarly introduction. Translated below from the German.]

[Introduction, conclusion — German]

is, on the whole, a forced bit of wordplay; only the passage V, 3 cited by Sabbatai Kohen contains an actual application to the so-called "Sea of Solomon," to which the views on the quadrature of the circle attach themselves. Our little treatise has the formula from which the one for the area-measure (II, 3; V, 3; cf. the end of II, 12) is also derived: $d^2 - \frac{3}{14} d^2$; it recalls the conclusion of the geometric treatise of an Abu-Bekr—called Heus or Deus (?)—in the Latin translation of Gerard of Cremona:¹⁰⁾ [Latin: *Sed si vis multiplica diametrum in se et minue septimam eius quod aggregatur, et medietatem septime et quod remanet in duas tertias diametri eius multiplica et erit mensura magnitudinis eius* — "But if you wish, multiply the diameter by itself and subtract a seventh of what is summed, and the half of the seventh; and what remains, multiply by two thirds of its diameter, and it will be the measure of its magnitude."]

If Rashi and Tosafot (Cit. 1) thus really appeal to our treatise for the value $\frac{3}{7}$ [recte: the figure in the German is set as a fraction] in the Talmud, then one must indeed presuppose in them what Maimonides (on Eruvin I, 5; cf. *Hebr. Bibl.* V, 132) expressly remarks: that in the Talmud it was a matter of a round number, without attributing to the ancients the more precise knowledge with such definiteness as is the case with Abraham bar Chijja, in whom such precision is characteristic.¹¹⁾

For the question of time and place—respectively, of the connection of our treatise with Arabic learning—the term קח [ḥeq] is especially to be noted. As for the formula for the area of the triangle with the examples 13, 14, 15 (IV, 9), it appears

since Heron, notably in a treatise of the Sons of Musa ben Shakir, well known also to the Jews.¹²⁾ Whether the passage I, 8 presupposes the decimal system with positional value, and what follows from it, likewise requires investigation.¹³⁾ Finally, as for the author's name Natan—whose authority rests ultimately only on Ibn Ezra—our manuscript does not offer it even in the best-known passage V, 3, into which this name perhaps found its way as a superscription; whereas Sabbatai b. Malkiel, for the words *לפי שאמרו בני ארץ בעגולה* ["because the men of the land said, concerning the circle"] has the words *אמר נחמיה* ["Nechemiah said"]. If the latter, as Geiger (*Ztschr.* V, 457) conjectures, is a false resolution of *נ"ר*, then *בני ארץ* could likewise have arisen in a similar way; yet I deliberately refrain here from any conjecture.

Enough of riddles, for whose solution other manuscripts appear most desirable; such might perhaps still be sought in Turkey, whither the most recent traces (Comtino, Sabbatai b. Malkiel, and the manuscript copied in Constantinople around 1480) point.

[Footnotes:] ¹⁰⁾ In Boncompagni, *Della vita etc. di Gherardo*, p. 54; cf. *D. M. Zeitschr.* XVIII, 168. — This Abu-Bekr I have unfortunately not yet identified with certainty, and conjectures would lead too far afield here. — ^{22/7} has the Indian *Lilavati* and the Arab Muhammed b. Musa el-Chowarezmi; see Chasles (Sohncke), *Gesch. d. Geometrie*, p. 509.

¹¹⁾ Cf. *Sefer ha-Ibbur*, p. 37.

¹²⁾ Chasles (Sohncke), *Gesch. d. Geom.*, pp. 465, 481; Sédillot, *Proleg. d'Olough Beg*, p. 246; Boncompagni l. c. p. 64 and *Della vita etc. di Leon. Pisano*, pp. 61 and 64; cf. also *Hebr. Bibl.* N. 40, p. 90, n. 12. The Beni Shakir cite Heron (Iranus); see *Comptes Rendus* XIII, 511 and *H. B.* l. c. n. 19.

¹³⁾ Regarding the most recent research on this subject I refer, for brevity's sake, to Th. H. Martin, *Les signes numériques etc.*, Rome 1864; cf. also *Jüdische Literatur* (in Ersch and Gruber), p. 442, n. 88.

Concerning the appended epilogue of Abraham bar Chijja, I refer, for brevity's sake, to the treatise in the *Hebräische Bibliographie* N. 40, and remark only that I had it begin where the Latin translation of Plato of Tivoli leaves off. I have taken as my basis the much better and more complete Munich manuscript (M.), to which the leaf-number 100 etc. refers, with careful indication—enclosed in parentheses—of the variants of the Vatican manuscript (V.), whose exact tracing I owe to the generosity of Prince Boncompagni in Rome; the Munich corruptions,

which I have set in parentheses by way of example where I had to follow the text of V. alone, are rarely offered.

BERLIN, August 1864.

Mishnat ha-Middot

Chapter I

1. By four methods is measurement comprised, and these are: the square (*ha-meruba'at*), the triangle (*ha-meshuleshet*), the circle (*ha-agulah*), and the bowed figure (*ha-keshutah*). The rule is that the second is half of the first, and the fourth is half of the third; and the rest of the other [figures] are combinations of this one with that one [Margin: כסינר — uncertain; the editor notes: ציל כברית? "read *ke-vrit*?" (i.e. a corrupt word, perhaps "as an alloy/composite")].
2. The square has three aspects (*panim*): side (*tzela'*), diagonal (*hut*), and surface (*gag*). Now the side is that which holds it together, and its faces are of equal measure [reading גג דופנותיו של — "the faces of its surface"]; one says of it "[the altar] was square" [cf. the Mishnaic phrase המזבח יהיה המזבח]. The diagonal is that which divides [Margin: המהדק, המחזיק — "that which binds, that which holds"], from corner to corner, and it is longer than the side [reading: יותר ארכו של גג, "its length exceeds that of the side"]. The surface itself is the area (*meshiḥah*).
3. The triangle has three aspects: two sides [*sic* — the source has שני צלע, "two side(s)," marked (*sic*) in the edition], base (*keva'*), perpendicular (*ammud*), and surface (*gag*). Now these are the two: the side—the two of them are inclined, the one to the right and the other to the left, spread apart [shifted outward], for two sides are set upon it [the base]. And the base is that upon which the two sides are set, the one whose place is established beneath them. And the perpendicular is the line that descends, dividing between the two sides, to the base, and it is a right angle [reading זוית — "an angle" (?), probably "right angle"] [Margin: מקצעות משכן — an allusion to the "corners of the Tabernacle"]. The surface itself is the area.
4. The circle has three aspects: circumference (*sevivah*), diameter (*hut*), and surface (*gag*). Now the circumference is the surrounding line that encircles it round about—a circle, for example, thirty cubits in circumference round about [cf. the "thirty cubits round about" of the molten sea]. The diameter is the line drawn from one edge to the other edge, from its [one] edge to its [opposite] edge. The surface itself is the area.

5. The bowed figure has three aspects: arc (*keshet*), chord (*yeter*), versed sine (*hetz*), and surface (*gag*). Now the arc is the curved portion taken from the circle, like the appearance of the bow that is in the cloud [cf. the rainbow, Bereshit 9:13]. The chord is that which holds the mouth [opening] of the arc, the two ends [of it], drawn taut. The versed sine is drawn from the middle of the arc to the middle of the chord, the two of them being directed [set] upon the chord. The surface itself is the area.

6. How does one measure the area? By number you reckon it: one by one—this is the area, and it is one cubit by one cubit. You will find that a surface equal in its sides and in its angles—behold, you count them [the unit squares] on every side. The board [tablet] standing [as] two [by two], and the angles two, and the measurement marked off: four units in the measure of the one [unit square] which is a cubit within a cubit; and when it [the side] is 3 on each side, behold, it is 9 units in the measure of the one [unit]; and likewise 4 by 4, and 5 by 5. From here onward, go out and reckon by this measure and upward.

7. The fractional [diminished] figures: from the one [unit], thus you have a divided cubit—a cubit by [recte: one] of the two diagonals that divides [Margin: ה פסיק — "that separates"] it in two, [from] the middle of one side to the middle of the [left] side; and likewise [from] top to bottom—you will find a surface divided into four sections; and when you take the half of a cubit, the area itself comes out divided into [?] [the value is abraded — the printed letter is unclear; cf. the editor's bracket 'ח [?]], a cubit being one quarter on every side; and likewise a third by a third, a fifth within a fifth—equal ones and their alternates [shifted ones]. From here onward, go out and reckon the broken [fractional] quantities by this measure and downward.

8. They have already said: a half by a half is a quarter [$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$], and likewise a third by a third is a ninth [$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$]¹—in equal ones, and in the alternates [the cross-products] [?] [reading uncertain — אלה ברומין הן לא [?]]. Their like? [editor's bracket: אהוא אב[?] — the reading is corrupt and the editor flags it]. Thus you count: ten by ten—behold, they amount to a hundred [$10 \times 10 = 100$], and half of ten is 5; 5 times 5 are 25, and that is a quarter of a hundred [$25 = \frac{1}{4} \times 100$]. The place [order] of 10 [recte: yod] is... [reading: 'ומעמד הי' בא — "the rank of the ten comes to a one (i.e. a unit of the next order)"], and the place of the hundred (*qof*) is 20 (*bet*), and a thousand 200 (*qof*) [reading: 'ב' ואלף ק' — uncertain; the place-value statement is abraded]. From here onward, go out and reckon the broken quantities by the measure of the others; but as for [multiplying] by other

[larger] ones, it [the product] is increased, and in the fractional ones it is diminished.

9. This is the rule: a half by a half is a quarter [recte: of a half], a third by a third is [editor: ציל ליש "read *shelish*"] a third of a third [a ninth]; and likewise a half by a third is a sixth [recte: a third of a half], a quarter by a third is a third of a quarter [a twelfth]—in these and in their like, in the equal ones and in the alternates.

Chapter II

1. One who wishes to learn the measures of the squares—in the equal ones and in the alternates [oblique ones]—joins the length to the breadth; what arises from the two of them is the breadth [?] [reading רחב — uncertain; context expects "is the area," הוא הרחב[?] flagged by the editor as a corruption] of the area.

§2 (cont.) And likewise [for] the trapezium, in any case: combine the column [height] with half the foundation, and the product of the two is the area; this is well understood.

§3. The circle: how? Combine the diagonal [diameter] within itself, and from it cast away one-seventh, half of one-seventh, and the remainder [recte: "and the remainder"] is the area. For example, a diagonal [diameter] measuring 14: combining it [with itself] gives 196; one-seventh and half of one-seventh [of it] is 21; [subtracting,] you find the area to be 154 [recte: 154].

§4. The conoid [paraboloid solid]: how? Take the versed sine onto the chord; combine both together [recte] within itself, [reading uncertain — measurement values here are [?]]. He returns and takes half the chord, combines it within itself, and divides it by 14 [?]; what results he adds onto the [height of the] standing [solid], and the product is the area. There are other approaches to this.

§5. [The rectangular parallelepiped] — that is, the solid of equal faces — how? It is a *machlaf* [alternating/oblong solid] of length, breadth, and depth [?]. The solid has six faces, six wings to one [edge?]: combine length within breadth within depth, and the product of all three is the area of the [solid] face; that is the body.

§6. And whether it was round, or triangular, or any kind of figure, provided that it had a regular and uniform depth — [the rule is] what they said: measure the face [of the solid] and know its area [reading of this clause uncertain [?]], then combine it within the depth, and that is the volume of the body.

§7. The prism drawn from its head [apex] to its end [base] is tapered, even if it be square, or it be round or triangular: [take] this [the rule]: measure the area of the body, cast away two-thirds of [one] area; one area and one-third [remains] [recte] — [this is] another [way]. You combine it within the height, and the product is the volume of the body, from head [recte: from head] to end [base].

§8. One must say [recte] that only a sphere or the like would have its surfaces equal on every side, [each] half [?] of a sphere [?] or the like: combine [recte] one of the threads [chords] from edge to edge within half of the other, and the product of the two is the area.

§9. A column: and when one is drawn out, its head [top] tapered — or it is drawn out, half of it round, [as it were] a roof or a vault [?] — or whatever it is [?], [if] its end is truncated, [namely] that it is cut at the head: divide [reckon] this [solid] by the measure of the end; [by] the calculation following, cast off the smaller portion from the cord [חבל], and what remains is the area of the column.

§10. [The truncated pyramid] — how? For example, a square column [with] its top end 4 amot within 4 amot; from that it diminishes and rises, deficient and rising; and its two heads [ends] come out to two amot by two amot square. You must know what its area is and how many [amot] it stands. As was already said of the etrog [in the last (chapter)] [recte], if you are not [skilled in?] computation, examine [it]: you would know how much its area is and in how many amot-square, and at the end — only that it is truncated, and you still do not know [how much] [?] above — high above.

§11. From where do you reckon, when you measure? The four [is] the length of the column; [it is reckoned] by the gauge [measure, משטר]. You find that for every step until it terminates [at] the head, twenty [amot]; how much until the truncation? Four amot. When you measure the measure of two from the four steps [recte], 16 from the twenty.

§12. You must take a third of its head — combine the end [recte: "(at) the sixteen"] — they are the 5 [ה'] and a third — combine it within 20: they come out to 106 amot and two-thirds of an amah [ק"ו ושני שלישי]. What [remains]? Set it on one side. Return and take a third [of] the truncation, which is a third of it. Combine it within the ten upper amot — they come out to 13 [ג"] and a third; cast away from it the thirds; what is left over there is 79 [ט"ט] and a third — that is מ"ג [abbreviation; reading uncertain [?]]. Cast away from them the thirds; what is left over there is 309 [ט"ט] and a third; that is the column, if it is truncated. If it is

round, cast from it one-seventh, half of one-seventh — and what is left over in it is the area.

[The numeral chain in §§10–12 is the truncated-pyramid passage. At native resolution the printed values are largely legible ($106\frac{2}{3}$ / $13\frac{1}{3}$ / $79\frac{1}{3}$ / $309\frac{1}{3}$); the chain does not form a clean frustum computation (the source is genuinely MS-corrupt here, as the editor flags), and the γ^n token resists a clean numeric reading. See batch report.]

Chapter III

§1. There are five kinds [of figure] among quadrilaterals, and they are these. There is [a figure] whose sides differ but whose angles are equal [a rectangle]. §2. And there is one whose sides are equal and whose angles are equal [a square]. §3. And there is one whose sides are equal and whose angles differ [a rhombus]. §4. And there is one whose sides differ and whose angles differ, but two of which [sides] alone are equal in length, and two alone in breadth [a parallelogram]. §5. And there is one that differs in its sides and in its angles altogether [an irregular quadrilateral].

§2. The [figure] of equal sides and equal angles: which is it? For example, ten by [ten]: [in] any case combine the length onto the breadth, and the product is the area, [namely] 100 [7']. One side is its root [base], one [of] its two sides; they [the sides] are two [equal] roots; and so [too] 3 [$\times 3$], and so 4 [$\times 4$].

§3. The [figure] differing in its sides but straight [equal] in its angles [a rectangle]: which is it? For example, two sides of four, two sides of six: combine the length onto the breadth, [and] they are 48 [n^2] — that is the area; the side is straight [equal] and straight [equal] [as to] the line [in respect of its angles? — reading [?]].

§4. The [figure] straight [equal] in its sides but differing in its angles [a rhombus]: which is it? For example, every side is four; two angles [are] acute and two angles [are] obtuse. Two threads [diagonals] cut it, this one in the middle: the one [is] of eight, the other [is] of six. He who wishes to measure: combine one of the threads [diagonals] within half its fellow, and the product of the two is the area, [namely] 24; measure it thus.

§5. The [figure] differing in sides and angles, but two of which [sides] alone are equal in length and two [sides] alone in breadth, the angles standing [unequal] — how? From the *machshavah* (sic) [thought/reckoning] [recte] [editor's note: read

"from the computation"): divide [it into] two [by an] angle-to-angle [diagonal], and set up [reckon] the two, and reckon each one by itself, [each] called a triangle, and so it is the area. And likewise you measure the computation of the [figure] differing in every respect [and in any case [?]]: divide into two. The quadrilaterals that differ [recte: "that differ"] — combine [recte] their thirds [diagonals?] [reading [?]], the differing [ones], in every respect; thus divide into two.

Chapter IV

§1. There are three kinds [of figure] among triangles, and they are these: the upright [right-angled], the acute, the obtuse. Which is the upright? [That figure of which] its two short sides are combined, each by itself, [and] it equals the first — for example, [if] from this side eight, [and] from this side six, then the product equal to [the first] face by itself, [as I say,] face by itself, [equals] face by itself. And you must measure [combine [?]] from the short [sides] within half its fellow, [namely] 3 within 4, or 6 [recte] within 8 [reading [?]], [and] the product is the area.

§2. The angle that stands between the short [sides] is the upright [right] angle; it is the quarter [quadrant] of a square that differs in its sides but is straight [equal] in its angle [recte]. When you seek to compute it [in order to] reckon the column [perpendicular] — its way is to reckon: there are two of its short sides standing [?]; they are like resting stones [recte], straight [vertical].

§3. The perpendicular [column] drawn from them down to the long side falls [perpendicularly?] [reading [?]] — that is the foundation [base]. He who wishes to measure: combine the perpendicular within half the base [recte], and the product of the computation is the area.

§4. The acute [triangle] — how? Its two [short] sides are short and equal to each other [recte]; combine [each] by itself, and they are equal [recte] [recte: "they are equal"] [recte]; the [other?] side, the second, that is fixed [as base?], combine it by itself, [reading uncertain — values [?]]; the first will be greater than the last. And there is an acute [triangle] whose sides are equal [recte]: he who wishes to measure [it] — [reading uncertain [?]] [draw] it opposite the base [perpendicular?], you find the angles [are] equal; [the] measure of the first [equals] the measure of the last, the ones [being] sides [editor's note: sic, צלע].

§5. He who wishes to know the measure of the perpendicular of [a triangle whose] sides are equal: combine one of the [two equal] sides within itself, cast

away the lesser from the greater, [and] what remains is the foundation [base] that is found [for] the column [perpendicular].

§6. [The equilateral triangle] — how? For example, ten by [recte] ten, [and] half a side is the second [recte], that is five [ה']: combine [it] by itself, [namely] 25 [ה"כ], cast away the small from the great, [and] what remains there is 75 [ה"ע]; that is the foundation [base], 75; [whose] root [square root] is 8 [ח'] and fractions [$\sqrt{75} \approx 8.66$]. And you must measure: combine the foundation [base] within half the lower side, and the product is the area, [namely] 43 [ג"ח] and fractions.

§7. Seven other kinds [of triangle], among scalene [triangles] [recte: "(b) scalene"]: [a triangle] equal in its sides — every side [equals every] side * as seen by itself, of which "this is said of [this]," and "this is said of that," and the discerning person grasps [it] [reading of the apothem clause (?)]. Combine one of them by itself; they are 100 [recte: ק']; cast away the quarter, which is 25 [recte: ה"כ], [and] 75 [ה"ע] remains. And you must measure: combine 75 within 75 — they are 3 quarters [recte] [namely] 25 [ה"כ] each [recte] — they come out to one thousand and 408 [ת"ח] and 75 [ה"ע] [the middle product is genuinely corrupt in print; transcribed as-is (?)]; take the area, which is 43 [ג"ח]. From here you find the column [perpendicular] to half the base [is] 51 [א"י], [which] never fails [recte: "never fails"].

§8. Indeed, the [computation of the] equilateral [triangle] is [done] thus, and the like — but for the [triangle of] differing [sides] there is no proof [recte: "no straightforward (way)"] from them [the foregoing rules]; [hence] in the computation of the area let it apply [the rule of] casting [a perpendicular] between two of the sides, [taking] the perpendicular [reckoned] with the base.

§9. [The scalene triangle] — how? For example, a triangle of differing [sides] that is acute in its angles: 15 [י"ט] [from] the first side, 14 [ט"ד] [from] the second side, 13 [י"ג] [from] the third side. You must take, [holding] all three together, [the rule of] casting [a perpendicular]; they come out [to] 42 [כ"ב] [$42=15+14+13$; half = 21]. Take the half [namely 21], and see how much it is [in] excess over the first; combine the half onto the excess — which is 6 [ו'] [$6=21-15$] — within [the half], they are 126 [י"זכ"ו] [6×21]; set it on one side. Return again and take

[continued on next page] the half, and see how much it is [in] excess over the second; combine together; that is from within 126 [י"זכ"ו] the first ones, they come out [to] 408 and 82 [ב"ח ופ"ב] — print-corruption of $882 = 126 \times 7$; transcribed as printed (?); set it on one side. Return a third time and take the half, [and] see how much it is [in] excess over the third; combine the excess, which is 8 [ח'] [

8=21–13] within those other 882, and they come out [to] 7056 [ז' אלפים ונ"ו], and their root [square root] is 84 [פ"ד] — they are the measure of the area.

[Editor's reading note for §9: this is the Heronian-style area computation for the scalene 15-14-13 triangle (sides 15/14/13, sum 42, half 21, excesses 6 and 8). The chain closes to the correct area $84 = \text{פ"ד}$, which the source prints explicitly, with the pre-root product 7056 (ז' אלפים ונ"ו) = 84^2 as printed. One intermediate product is compositor-garbled: the second product is set as "408 and 82" (ח"ת ופ"ב) where the value is 882 (= 126×7). See batch report.]

§10. The obtuse [triangle] — how? Its sides [recte] are short to each other [recte]: combine [each] by itself; add onto this the side [recte], 3, which is the base [recte] [reading [?]] combined by itself; [the] last combination [is] greater than the first [recte]. As I [say], from this side 5 [recte: ה'], [and] from this side [recte: 9, ט'], [and] you find 9 [ט'] [recte] from [reading [?]] the angles [that it is] open [obtuse]. And its breadth — you find the first [is] 61 [כ"א], the last [is] 81 [כ"א]. And you must measure the proof clearly [for] the column [perpendicular] with the base. If you wish — the computation of the sides [is by] the half [recte]; the computation [is] clear, [and] [the perpendicular] never falls.

Chapter V

§1. There are three kinds [of figure] among the circular [figures], and these are: the suspended [sphere], the heaped [hemisphere], and the lowered [reading [?]]. Which is the suspended? It is [the figure] that stands [upon] a center [recte] — a circle of any side [round about], a sphere or the like; [its] measurement [is] like that of a watermelon that is full all around it, [reading [?]] [its] body equal [in] length, breadth, [and] depth. How [is] its measurement [done]? Measure [recte]: combine one of the threads [a diameter] within half itself, and the product is the area; double it, [and] those are its [two?] hemispheres [recte: "its dimensions" — reading uncertain [?]].

§2. The heaped [hemisphere] — which is it? It is [the figure] that stands [as] half a suspended [sphere], a mound, or like a vault [recte], provided that it be [reading [?]] level [equal]. And you must measure: combine one of the threads [chords] from edge to edge within half of the other, and within half itself, and the product is the area.

§3. The lowered [reading [?]] — which is it? It is [the figure] that is set upon the ground [as] a straight circle, [as it were] a full city [recte] [reading [?]]. You must measure: combine the thread [diameter] within itself, cast away from it one-

seventh, [and] half of one-seventh; what remains is its surface [recte], half [reading (?)] — both the first and the last. Indeed it [Scripture] says: "And he made the molten sea, ten amah from its one brim to the other, [it was] round all about, and its height was five amah; and a line of thirty amah did compass it round about" (Melachim I 7:23) — to teach [that] a line of thirty amah [is] from the [diameter] [recte] [reading (?)] [a circle whose] ground [floor] [recte] [is] a circle whose circumference holds three times [plus a bit] seven [recte] [the thread]: a thread comes out for them, a fixed amount in its thickness, [for] both the two brims, [and] there remains (?); [reading uncertain] it goes round about it, by the measure of the other. From here the ritual pools (mikva'ot) [מִקְוֹאוֹת], the wells, the cisterns [borot] — [by] length, by breadth, [by] depth, [by] one measure you measure the measure of a circle.

[Editor's reading note for §3: the citation of Melachim I 7:23 (the "molten sea") underlies the rule $\pi \approx 3$, and the surrounding gloss paraphrases the Talmudic discussion (Eruvin 14a). At native resolution the verse and the $\pi \approx 3$ gloss ("the circumference is measured three times and a seventh; a single thread comes out...") are legible, not abraded; only ~2 connective tokens remain genuinely soft (incl. וְ, recte וְ). The verse follows the Masoretic text. See batch report.]

§4. Three things they said of the segments [of a circle]: these are the straight [semicircle], the deficient [minor segment], and the excessive [major segment]. Which is the straight? Any [segment] that stands [on] the half [diameter] of the circle, neither deficient nor excessive. The deficient is any [segment] that falls short [recte] of half the circle. The excessive is any [segment] that exceeds [recte] half the circle. This is the rule: any segment standing [on] less than half the diameter [recte: "less than half the chord" — בִּידוּעַ, "as is known"] [recte] is deficient [recte]; any segment standing [on] more than half the diameter [recte: chord], [as is] known, is excessive.

§5. He who wishes to measure the straight [segment, i.e., the semicircle]: combine the diameter [recte: chord?] within itself, [and] from it [recte] cast away one-seventh; cast away [recte] half of it [recte]; what is found is the area.

§6. As for the others [the deficient and excessive segments], you must know how much the measure of its excess [reading (?)] is. How? Combine half the chord within itself; what results, divide by the versed sine; what is found from the division, add to the versed sine, and the product is the thread [diameter] of the circle. Take half this thread [diameter], [and] combine it within half the versed sine; set it on one side, and see: if the segment was deficient, cast away half the thread [diameter] of the circle, combine what remains within half the chord,

[and] open [subtract] it from the side [you set aside]; what is left is the area, [and] cast it away. If the segment was excessive, cast away the circle [recte] itself by half, combine what remains within half the chord, add it onto the side [you set aside], and the product [recte: "the product is the area"]. By the measure of the first [recte], so [too] the measure of the last.

The tractate of Measures is finished [תמה מסכ' המדות], *be-or yodea cherut* [בעור יודע חירות — a chronogram / scribal sign-off whose expansion is uncertain [?]].

[Editor's reading note for the colophon: the closing formula reads 'תמה מסכ' המדות, בעור יודע חירות'. The first clause ("The tractate of Measures is finished") is clear; the second is a chronogram / sign-off whose resolution remains uncertain (flagged [?]). The letters are legible at native resolution.]

References [ציונים]

[These are the editor's (Steinschneider's) source-citations — a numbered apparatus locating the rabbinic and other sources for the "forty-nine measurements" and related geometric matters. The lemmata are in square type; the parenthetical attributions and manuscript-variant notes are the editor's, partly citing Rashi.]

1. "By how much does the square exceed the circle? By a quarter" (Gemara Sukkah, fol. 8a [גמרא סוכה דף ח' ע"א]). Mishnah: "it is in forty..." [מתני' היא ...] — for were the round [sukkah] [a measure of] cubits, with a circumference of cubits, so that the circle were drawn requiring a diagonal-thread [חוט] of four [cubits], it would be fit, [filling] a cubit on each side [לכל רוח] (Rashi). And in the glosses to the Shas it is written: "one should read ... in the tractate Middot [במס' מדות], and so too in Tosafot Bava Batra fol. 23 [והיא משנה] [ג] and it is a Mishnah in tractate Ohalot ch. 12 m. 6 [ובתוספות ב"ב דף כ"ג] — but in Tosafot there, s.v. [ד"ה כמה] [כמה] — it is written: 'so it is found in the baraita of tractate Middot,' and one should read [וצ"ל] 'forty-nine measurements' [מ"ט מדות]."

2. "...that were left over: eight cubits that hung at the back of the boards [קרשים]" (in the printed edition [ובדפוס]: "the Tabernacle [משכן]") "on the west, and the two lower cubits exposed [ושתי אמות התחתונות מגולות] — this I found among the "forty-nine measurements" expounded in the baraita (Rashi, Shemot 26:4 [שמות] [כ"ו, ד], according to München MS 5, fol. 65, verso [ע"ב]; and in the printed edition: "in the baraita of tractate Middot" — "and there is no place here to expound this," up to here is from the editors [מאת החוקרים]).

And according to what the speaker of these matters said as he wrote them, the length of the board was nothing but ten cubits [לא היה אורך הקרש אלא י' אמות]; so I found in "the Mishnah of forty-nine" [משנת מ"ט] (and in printed glosses: "tractate" [מסכת]) "measurements" (Rashi, Shemot 26:4 [ד' שמות כ"ו, ד] — they were counted by the scholar Bunz [בוניץ], p. 91, note c [ע' 91 הערה] — the editor's note [c] adds: "one must correct [the reference] 'Shemot' to 'Vayikra' in its place" [צריך לתקן שמות במקום ויקרא]).

3. "Five cubits the height of the one capital [כותרת]" (I Kings 7, [ז', א'] [?]) — at the end of the verse it [Scripture] further states [that] "the cubits we have" [are from] tractate [מסכת] "Mishnah" [צ"ל] [of the] forty-nine measurements: the lower cubits of the [pillars'] capitals [יתרות] could not stand by themselves, but rather were [?]; the three upper [cubits] [?]; "seventeen [י"ז] [?]" — for so it says, "they made a sort of latticework" [כמין לולבין] for the [capitals], which were rounded round about [מוקפות] (Rashi).

4. "And borders [מסגרות] they had" (there, verse 28 [שם פסוק כ"ח]) — I saw in the tractate [מסכת] "tractate" of "forty-nine" (and in a manuscript "Mishnah of forty-nine": "Midrash" [במדרש]) "forty-nine measurements," [namely] of the borders, like [those of] vessels that had borders fitted to them at once, by hand [?] etc. (Rashi).

[The editor's apparatus continues with three further numbered notes (5–7), separated by a printer's rule. These are degraded; readings flagged.]

5. "Going down to the waters [from] the channels [יורד נהרות ים]" (Tehillim 78:17 [תלים ע"ח, י"ז]) — for the princes were drawing [water] from the well [of the wilderness], and the waters flowed out and ran [?] in [their] supports [במשענותם], the waters streaming after them, every tribe in its place; for so it is said, "by their staves [במשעניתם]" [?] — explicitly, "forty-nine measurements" (Rashi; and in Yalkut §744 [ולילקוט סי' תשמ"ד]: [the princes] dug each one a well [?] (Bamidbar 21:18 [במדבר כ"א, י"ח]) — the princes [נשיאים] [?] each one according to his tribe, according to his family. Now I have examined [the references] there [?] §[?] and have not found the citation; I do not know which text Rabbi [?] expounded as a citation [the editor admits the source could not be located].

6. "Concerning all who keep [the watches]" [על כל שמרי] (Ta'anit 27a [תענית כ"ז א']) — the twenty-four [משמרות כהנים] [?] ... they divided into twenty-four [כ"ד] stations corresponding to the twenty-four watches, as taught in the baraita of "forty-nine measurements" [ברייתא של מ"ט]

they do not concern themselves at all [?]. And not one of these procedures [טעמים] is [?] — for their faces have departed from the ways of [true] measurement and computation. [Such a result] has no firm foundation [אין סומך לו].

And the reasons [טעמים] for this discrepancy [?] — they have neither a [valid] reason [טעם] nor any [authority] to bring forward about it, to establish a settled order [סדר] for the measurement, by their own reasoning [?]. They forever drive [?] the measuring-line over the threads [חוטי] from the corners [מצרי] of the ground, and they square [the ground] crookedly [?], and there is no [?] in [this]. [The result] has no firm foundation [?].

I have heard of one of those who occupy themselves with the measuring [במידת] — in the variant: [במשיחת] of land by these mistaken procedures [הטועים], who said: "Know that this way by which I sit [?] for the hire [שכירת] of land — I have no [valid] reason for it from myself; rather, I learned it [?]." Now this man knew that our Rabbis, of blessed memory, knew with certainty that the diagonal [אלכסון] adds upon [the side of] a square two-fifths [יתר תרי חומשי], for they said, "the perimeter [מוקף] is three handbreadths [פחים] for every handbreadth of width"; yet despite this, do not refrain from setting [the rule] before yourself [?]: they said, "a square [מרבע] [is] one-quarter more than [the] square [?], two-fifths [is] its diagonal [תרי חומשי אלכסונו]" — that is, for every [unit, the diagonal exceeds]; "all that the perimeter [is] in the squaring [ברביעין] [is] three handbreadths [ג' פחים] for [every] handbreadth of width [?]." Thus he did not know how this is reckoned, and is no craftsman [אומן]; yet despite this, I do trust [?] him. When you have heard these matters [?], understand that this [man], even though [?], said that it is so [?].

And [our Rabbis], of blessed memory, who [?] [V. 58b.] — this [computation] [?] is by the diagonal [באלכסון], the squaring [המרובע] in the place that he [?] brings forward; for though they [the Rabbis] eased [the matter] [קילא] [?] in some commandment-context, nevertheless [these] are the [true] values [המנים] [?] [up to the diagonal] [?]. In the place where they were exact [?], at the end, they said, as is fitting [כדבק] [?], "[a courtyard] encompassed by a fence ten handbreadths high [גבוה עשרה טפחים], one may carry within it" [מטלטלין בתוכה] [100] — "for when its squaring is theirs [?] [לה], according to my [?] reckoning, [?] cubits [?] [V. 58b]" — that is, the courtyard of the Tabernacle [חצר משכן]. You see here that they gave the diagonal of the Tabernacle, which was [?] by [?], "two-fifths of a cubit more" [יתר תרי חומשי אמה] [?]; and were it not so [?], this would be [a measure?] here [?], without [בלא] anything more — to move a thing into a place

where there is no [permitted] place; [in] the city of the Tabernacle there was an easing [?] [a leniency, regarding the place].

So they reckoned [גדל [?]] five thousand [ה' אלפים] cubits [?] for the computation [ל"ל "computations" החשבונים] [—] and that is [?] cubit, and ten cubits, and five and a [?] [the editor: ל"ל [?]] [—] on account of those who [?] [?]; and they did not say "by an exact handbreadth" [?], so that the [resulting] values [שברים — fractions] should not be off by more than one, where it is fit to reckon a correct, precise result [חשבון נכון מדויקדק]. So you find that they removed this [matter of an] easing [the leniency], and brought forth a settled-rule [?] [גזרה] [for] a commandment-context [במצוה], on account of [?]; the diagonal [is] two-fifths only [תרי חומשי לבד], besides what they said, "and a square the city is [?], without [בלא] [a square that is?]" — it is five thousand by five thousand [ה' אלפים על ה' אלפים], leaving aside [the matter of] the diagonal. And every computation that is a [matter of] law [הלכה] — they are exact in it [נדויקו]; the diagonal he reckoned is "two-fifths only, as the law requires" [תרי חומשי לבד כהלכה]; and they say "[concerning] a column of the Tabernacle that stood on the edge [?], so that its [?] would be [?] cubits, two cubits" [?] for every direction [לכל רוח]. You find here that the diagonal of the Tabernacle he reckoned [as ?] cubits for every face of the kernel [?]; on account of the stringency [מחמירא] of the commandment and the warning [אזהרה] against this easing [the trivializing, הקלקול], it appears, by clarification [?], that the general rule that they gave for the diagonal, even where it [is] not [in] a commandment-context, brings one [?]. So too the general rule that they gave: "[the perimeter is] surrounded by three handbreadths [מוקף שלשה] for [every] handbreadth of width" — were they [exact, they would have said] "its width is less ... by [a part] of twenty-two parts [מכ"ב חלקים] of a handbreadth"; but they did not concern themselves with this on account of [?], for it was not their intention to set [a leniency, for the place?]; for every computation [where] they are not exact [is] only in a commandment-context [?].

If you say: "Solomon [שלמה] made [the Sea] [?] — why [?] is the line [הקו] that surrounds a circle [עגול] of [?] three [שלושה?] [cubits] of width [רוחב] [?], as it [Scripture] says, 'thirty cubits did compass it round about' [שלישים אמה יסב אותו] [בקי]?" — I say to you: indeed, what Solomon made they [the Rabbis] measure [as] geometric measurements [?], and the surrounding line [הקו מקיף] is thirty [cubits], and the width of the [?] is ten; and yet it [Scripture] says of it, "a handbreadth [פח], its making [מעשהו] like the making of the brim of a cup [שפת כוס] [?] — and so it says elsewhere [בדברי הימים [?]]: "a handbreadth, its brim [שפתו] like the making of the brim of a cup" [?]. You see that this circle [העגול] [הזה], outside [?] the [?] straight side [צלע הישרה] of the square [מרובעת] [is] one

handbreadth [פח חד], so that the line of the width of the circle that remains [is] ten cubits [עשר אמות], lacking two handbreadths [חסר שני פחים], five handbreadths [חמשה פחים] [being] a cubit [אמה]; thus our Rabbis, of blessed memory, said: "less two handbreadths" [פחות שני פחים], on account of the [?] of this [thickness/brim] [?].

For if you reckon the [full] circumference [היקף] — and if you reckon [?] nine [תשע?] cubits and three-fifths [ושלשה חומשין] of a cubit, [?] times [?] seventh [שביעית] [—] together it is thirty cubits [שלשים אמה] [?]; [?] it has [?] parts [חלקים]; and that is [?] [the editor: צ"ל] [לים — for the Sea] for the brim [לשפת] of the circle, which was [?]; and the remaining [parts?] [?] outside the brim, the thickness [עובי] of the [Sea] [?] [is] a handbreadth, like a [?] [כפרת]; thus a part [חלק] [?] of [?] of the width [מחצית הרחב], half a [?] [?], by the line of the brim [?] — and the result is reckoned [?]; the diameter [גוף] of the circle [becomes] seventy-two [ע"ב] [units of a] cubit, and two-ninths of a cubit [ושני תשיעי אמה]. The [Sea] that Solomon made — there were "five cubits [חמש אמות], [some] squared [מרובעות], and two rounded [שתיים עגולות]," as our Rabbis said, of blessed memory. You are able to see the matter as it is written (about it, in [?!]): for so it [Scripture] says (I Kings 7:24 [כ"ד] [מלכים ז'] "and gourds [פקעים] [in a manuscript.M "knobs" [טובבים] beneath its brim [תחת שפתו] round about, compassing the [Sea]" [את הים [V. 59] — beneath [the brim], you will see, half a cubit [?] is squared, "five cubits each direction" [חמש אמות לכל רוח]. I say: the [?] gourds [טורים] [?] are knobs cast in their casting [יציקתם] [?] [פקעים] — and they are the rounded [?] cubits that he said are squared [מרובעות]. And if you take the [computation/squaring] of [?] a round cubit [אמה עגול], [?] [will be] the squaring of two rounded cubits — that is, four cubits [ד' אמה ארבע שיעיות] [—] a cubit [is] added to them [?], one cubit [?]; these are the squaring [ל"צ"ל "of the square"?] from which [?] is removed the squaring of [the] vessel [?] [?], [namely] four [?] [ארבעה] [שעיות]; and if you take of this [?] handbreadths [?] half [chatzi], they are [?] thousand [אלפים] [—] you find from here a cubit [?], [?] cubits, a cubit holding [?] four [?], half [?]. [The text is markedly degraded across this worked example; values flagged.]

Our Rabbis, of blessed memory, estimated the mikveh [מקוה] [as] "[?] upon one part [על חלק אחד] [?] [the editor: מק"ל] "parts [of the?] [Sea]" [?] [?]; the entire computation [?] is the value [?] of the [tenth] of the ephah [עשירית האיפה], like a tenth of one [?]; the ephah [האיפה] is [?]; [?] is found to contain [?] of the Sea; [?] thousand [אלפים]; the mikveh ... forty [מ'] ... [se'ah unit nearby: סאין/סאה] [?]. [So] here is a mikveh [as] one part [חלק אחד] [of?] [?] [?]; and there will be here [a value?] for the body [גוף] of the mikveh [as] [?]; what is [?] [for?] [?] three

hundred [שלש מאות] [from here on: "three hundred is missing" in a manuscript] [?] less one part [פחות חלק אחד] of twenty-two [מכ"ב] of a part of a cubit. Our Rabbis, of blessed memory, estimated [the] mikveh [101] [as] "what is [?] [for?] [?] three hundred [שלש מאות] [variant: של"ית]; they did not concern themselves with the deficiency [חסרון], and so this computation came out [as it did], on account of [?], being brought [?] in a commandment-context — their way in all [matters] like this [is so]. If you say: "This computation, according to [?], will rise [?]" — set [it] before yourself: [?] estimated the body [גוף] of the Sea [as] [?] cubits, and it comes out for them [?] cubit [?], three hundred [?] squared [?].

Afterward [?] in your hand two round cubits [שתי אמות עגולות] [?] cubit [?], and these [are] four cubits [ד' אמות]; here [?] there are [?] cubits, which the square adds to the [round] circle, [?]; afterward [?] in your hand [?] cubits, which our Rabbis [gave us]. [The Rabbis] said [?] [that] our Rabbis of blessed memory, and a [matter] rounded [מעוגל] for the Sea, [is] [?] one hundred [ק"נ] cubits, the [?] which [?] before you [?], and you cannot say of them, of blessed memory, "they were stringent [מחמירים]" [?], for they [?] "sixty handbreadths [ששים טפח] [?]" — for so [?] the round brim [שפה עגולה] [?] which Scripture explains [פורש כתוב] [?]; "a handbreadth [its thickness], round about" [?] — perish the thought to reckon for [the Rabbis], of blessed memory, [?] a matter [?]. Now know that this handbreadth [טפח], every commentator [סביר] [?] our Rabbis [?] — whether this handbreadth [?] be the thickness of the rounded brim [שפה סובבת] of the circle, so that the width that remains [ישאר רוחב] upon this [?] estimate [סברה], which [is] cubits [?] [?] two handbreadths [שני פחים] [?]; the second estimate [הסברה השנית], which [?] [is] this [?] [?] — let it be said [?] [that one should] subtract [נחסר] from the line [קו] [its] width [?] — (in a manuscript "is missing here" [?] those [?]) [of] a circle [?], its width [is] the [?] [?]; he [?] a part [חלק] [of the?] line [?] surrounding [?], so that this [?] will be [?]; let it be said [that one should] subtract from the line [its] width [a part] [of the] line [?] of the circle — (and the handbreadth [will be] a part of the line) [?] surrounding [?] two [?] of the width, leaving [ישאר] [?]; this is the estimate [סברה] which [is] cubits [?] [?]. Our Rabbis, of blessed memory, "took [?] the midpoint [?] between [?] these two estimates [שתי סברות אלה]," [namely] from them a cubit [is] six [שש] handbreadths [טפחים] — [the editor] added a [?] [מימרא]. If you reckon the width of a circle, which [is] cubits [?] [?], a side [?] [is] six handbreadths [ששה טפחים], a cubit [אמה]; thus its measurement-result [תשבורת — תשמרת] [is] one [cubit?] [?] of them, by the estimate of [?] something [?] [the editor: "צ"ל "two something" תות משהו]; if you reckon [?] two cubits [?], [?] two handbreadths [שני פחים], a cubit [אמה]; their account [is] [?] cubits [?], like [?] half a cubit. Our Rabbis, of blessed memory, "took [?] the midpoint between [?] these two computations," [namely] from them a cubit [is] [?] cubits [?]; they were

exact about the [?] brim [?], concerned [for] the two [estimates]; from them a cubit [is] [?] handbreadths [פחים] — [the editor] added "stringently" [לחומר]; they went by the estimate, not [?] in a computation that came up before them, by their own [reckoning?] [נדעתן?]; the sons of [man / Adam] [?] [?] reckon a reckoning [?] — [the Rabbis] were stringent [in] a commandment-context [101b], in every [לכל] act [מעשה] like this. Here is the proof [?] [for] them [the Rabbis], of blessed memory.

And out of [this] there unrolled [נתגלגל] another matter, [the matter of] the Sea that Solomon made, to examine and inquire into its measurement. We have [?] [variant: "errors"?] [the editor: לתרן — to resolve [?]] [—] the disagreement [המחלוקת] found regarding its capacity between what is mentioned [נזכר] in Kings [במלכים] (which is "two thousand bath" [ואשר הוא אלפים בת]) and what is mentioned in Chronicles [בדברי הימים], which is "three thousand bath" [ג' אלפים בת]; and all admit that what is mentioned in Kings is its actual capacity [מכילתו], the holding of the Sea [מכילת הים]; and what is mentioned in Chronicles [is] from [?] [?]; but they [?] differed in their opinion as to the explanation of this matter. Some of them say: when you measure [תשער] the Sea by [its] two places [שני מקומות] [?] — one [is the?] [?] [?], the other [?].

/ conservative (per native-res recovery,). NO explicit chapter:verse numbers (e.g. I Kings 7:23, 7:26, II Chron 4:5, Tehillim 19) are printed except the bare " / מלכים" book names and the editor's fragmentary "תהלים" / "בראשית" / "דברי הימים" book names and the editor's fragmentary "(מלכים ז' כ"ד)"-type tags. I have transcribed only the book/chapter tags that are actually inked and flagged the rest [?]. Did NOT supply Masoretic verse numbers the page does not show.]

and they said: the bath holds, by dry measure, three thousand and two hundred, while by this [other] measure it holds and contains four thousand eight hundred. [Editor's note: the numerals 3,200 and 4,800 are firm, but the source's assignment of which capacity is "dry" (מן היבא) and which is "liquid" (מן הלח) is ambiguous here and may be the reverse of this rendering; kept as printed, not silently flipped.]

And if [the words of] one bath holds in it the two measures, then it is one single body, of one [and the same] casting/coinage. [Now] if there is between them [?] " previously here is an editor's-note artifact (M. ביניהם "between them"), not inked prose — softened to [?] — and if there were any difference between them, this difference would amount to but [?] one [unit]. And if the two are [equal] [?], then the ephah by dry measure would hold more than it holds by liquid measure; and if [it holds] more by [liquid] measure, [then] [?] it holds more by dry [than by

liquid] measure. The whole reckoning runs an even course and rests upon [?] one and the same matter. If [the words of] "one bath" stands [as] two measures [?], the [thing] is contradictory; for one cannot say of a single thing that this [holds] more and this less, [a thing] that no Israelite would permit himself to say. [That] the dry [measure] should be smaller and the dry larger [?], no rational person could prevent, and they would be found contradicting one another.

Moreover, what is written, "and he brought," etc. — Scripture does not [?] measure the capacity of the Sea [in] a single verse, [so that] its words be smaller [in the one] and larger [in the other], a thing known [as] differing between them — were it not for [?] the purpose of [explaining] the capacity of the Sea [as] one of liquid [measure], inasmuch as it could not be [?] said that the one [account] is for liquid and the second for dry. Then [the two would differ] by [the reading "two"] (102), [?] a difference of [a little] [?].

For were it written, "one [account says], the bath holds," [it would mean] it says two thousand bath; and were it written, "three thousand," that is according to the bath of three [?] — but it does not say "two thousand." When you say "two thousand," that which they say [is] like "three thousand," [namely] ten [?] bath; and one does not say "three thousand bath" [and "two thousand bath"], for [neither of] these is [?] the meaning of this matter; rather the reckoning [is] [?] uniform [?]. And it is found written, "and he brought," etc.: these two measures issue from one [single] measure, not [?] [a meaningless] one. Far be it from us to think such a thing concerning the words of the living God!

There are days [on which?] there are mentioned the two writings, and it is one body. And [?] it was a [?] vessel holding the liquid [?], two [?]-thirds [?]; and there it [was] holding the dry, two thousand [?], as is mentioned [in] Kings; that is, [as] the capacity of the Sea by dry [measure], so that it be (so that this be the bath [?]) [the measure] by which you measure the [Sea]; while [by] dry [it holds] (is it not?) [?] as much, until [you] arrive [?] at a single point [reckoning] [?]: three thousand, which [the book of] Chronicles [gives] [?] [as] the capacity of the Sea [in bath], you fill it [completely], and you do not pour beyond what is fixed [for it]. The vessel [in question] [?], which was [a basin], you set [?] its sides [?] [its] corners, [its corners?] [?] equal, and you make [?] its base [?] one-third [?] of the base, [and] the vessel was [?] this [vessel] holds all of its body [in] its casting [?]; and this vessel measures by [liquid] measure two [?] Seas — and so [run] the words of these commentators.

If I have constructed it [?] [in] another shape, even though its body [holds a body?] which this vessel could contain, [?] its casting would not be — and the

word "for help" [?] without addition or diminution. It is found written [in] their words [that] there is given no [other] two [?] shapes [other] than what the two measures of the Sea [give]; nor [is there] any further matter. If the shape of the bath differed, the one [?] of the two Seas would carry [?] [there] [from?] the words of [the] Torah [?]. We are left, then, to [say] [?] another thing [that is] written, to inform us [that] the Sea [was] [?] made in a shape [such that] [we may know] how much it could hold of liquid [and] how much (it) could hold of dry; and you pour [?] no more than what is fixed [for it]. Kings says, concerning its making [?]: its making [was] like the work of the brim of a cup, a lily-blossom, holding [?] thousand [?], holding the whole capacity of [the] body itself [?]. And in Chronicles he says, again, of its brim [?]: the work of the brim of a cup, a lily-blossom, holds three thousand, holding the whole [?] of its body [?] and its casting [?]. By [this] reckoning, there is [no contradiction] between the two writings; the difference [?] (102b) [is] (the exchange?) [of] one [account saying], "the bath holds [?]," [the one] not knowing [it could?] hold [?] the strength of the second [?]; [while] the other holds [?] [the] two [accounts], three thousand, holds. As if it said: it [?] holds, by its brim [?], two; and you [?] add to them, by examination [?], [it] holds three thousand. To this sense [?] [the two] were directed [?], the [?] one saying: two thousand of liquid [?], it would hold; three thousand of dry [?].

And the sages of the measure, whose way it is to investigate [?] this, examined [?] [what is] similar to it [?] and chose for themselves a vessel whose mouth [?] is square, equal [in] its corners and its sides, so that you [might] fill it [?] [up to?] [its] brim with the flour [?] [or] the seed [?] heaped upon it [?], and pour [?] which is [?] the casting [?] of the standing [vessel?]. And [I] found the casting of this [?] vessel by [?] the [?] measure [?] of the casting [?], so that [?] you cast [?] [the half of?] the base-side, [?] [the] face [?] of the [casting?] — for is it not [?] [that] the corner [?] which is at its head [?] is fixed [?] [up] to here [?]: it [?] is able to hold [?] [only] [?] from what was [?] the height [?] [equal to?] half the side [?]; and the rest [?], [at?] the head [?] of the brim [?] [?] was [?] [the] flower [?]-work [?], [it] is able to hold its body [?] in its [casting?]: [a] shape [?] such that the height [?] rises [?] [to] [the] half [?] [of] the base-side [?]; [then] the [?] corner [?] is [?] [?] [it] will be

[...it will be?] [that] anything [you] add, beyond [?], by [the?] forming [?] of the standing [vessel?], opposite [?] — and this matter is clear [?] (clear and lucid) [?] to every understanding, by way of [the] measure. From this whole [reckoning] we say of every square [vessel?], whose [?] casting [?] [?], its casting was the strong [?] one-third [?] [of] its [own?] strength [?]. If a square [vessel] stood, [and] its height [?] [were] half the base-side, its casting would be one-third of its [?] strength [?]. If its height were one-third of the base-side, its casting would hold

half [of] its [?] foundation [?]. By this principle you are able [?] to find the way to [the] casting [?] of every square vessel when you know [?] the ratio [?] of [its] height [?] to its base-side.

The Sea that Solomon made [?] [its height being?] half its base-side, its casting was [?]. The proof : [it] was [?] one-third the strength [?] of [its] body [?] over [?] all the [?] face [?] [of] the circle [?] [it] was [?] eight [?] [not?] perfectly [?] in [accord with] this whole [principle] in our hand [?]; we [?] have come to [?] a precise reckoning, according to the opinion of the sages of the measure, [as to] how much the Sea held: it [held] [?] two half [?] [the] Sea that Solomon [made?] (60b), [its] base [?] which from the measure [?] its height [?] (103) [was?] five [?]. As is written, "[?] a [?] cubit" [?]: [its] circumference [?] [is] [?] the Sea round about; [and] it is written, "five [?] [cubits?] its height" [?]. Were its [proper] shape [?] a perfect square [?], so that the Sea hold [?] cubits, and if you fold [?] its [?] square [?] four [?] times — [the] cubit, [the?] sages of the measure would find that the Sea Sea" holds for it [?] a [?] cubit, [were its] shape square [?], so that [it] be [?] [?] cubits Sea"; [and] the casting [?] of its [?] foundation [?] for [?] [it]; its casting [?] [is] in [?] [the?] measure [of] [?], which is [?] one-third the strength [?] of [its] body [?]; [thus] it held [?] [in] its body [?] [in] its casting [?] three thousand [bath]. And so it is found written [in] Kings, [which?] informed us [?] how much its body [?] held when it had been rounded from it; while Chronicles informed us [?] how much its body [?] held with [?] its casting [?], [were] its shape [?] a perfect square [?], [holding] three thousand by [?] the rule [?] [of?] its base [?].

And we have seen here [?] [that] [its] height [?] cubits is the [?] diminished [?] strength [?] [of?] the Sea [?]; [in] its measure-third [?] it held [?] no [?] less and no more. And here [is] the explanation of [the] whole [matter as] handed down [by] the science of the measure : that the casting [is] one-third [of] the body. So is this reckoning found: their castings come to one [the same]. The two writings [together] informed us, [in] another matter [?], of the capacity of the Sea [?]; the second informs [us] of [?]. It has been clarified [?] to us in a fine [and] clear [?] [way that] this [whole] reckoning is what is handed down [in] Scripture [?] over [?] the way [?] [of] the [?] sages of the measure; [it] has [no?] correction [?].

[Now] our masters z"l [zikhronam livracha] [and] the sages of the measure differ [?] [in the?] way [of] the number [?] [from] the root of its foundation [?]; [but] our masters z"l [were] stringent [in] a place [where it was] proper to be stringent — until [?] missing) " — reconstruction not possible].

You find the measure of the mikveh, [according] to the opinion of our masters z"l — how much it is — by [the] capacity [?]; [and according] to the opinion of the

sages of the measure, how much it is, [namely] one cubit by one cubit, three cubits [in] depth [and] one part of twenty-seven [of] a cubit.

And [according] to the words of our masters ז"ל [the mikveh is] one cubit by one cubit. [And] a log [is], [according] to the words of our masters ז"ל, [a] [?] etzba by [a] [?] etzba [in] depth, with a fractional finger remainder; [whereas according] to the words of the sages of the measure, a slightly smaller finger-figure. And the se'ah (and the one who sees [?]) — [according] to the words of our masters ז"ל [it is] two-thirds [of] a cubit by two-thirds [of] (103b) a cubit [in] depth, two-thirds [of] a cubit [and] one part [of] a hundred and twenty [of] a cubit; [whereas according] to the words of the sages of the measure, [it is] two-thirds [of] a cubit [by] two-thirds [of] a cubit [in] depth, two-thirds [of] a cubit, no more.

The ephah, [according] to the opinion of our masters ז"ל [is] two-thirds [of] a cubit [by] two-thirds [of] a cubit [in] depth, half [of] a cubit [and one] part [of] a hundred [and] sixty [of] a cubit — [whereas according] to the opinion of the sages of the measure, [it is] two-thirds [of] a cubit [by] [?] a cubit [in] depth, half [of] a cubit, no more.

And there is no dispute between our masters ז"ל [and] the sages of the measure in the form of the reckoning, but only [in this:] [that] our masters ז"ל saw fit to add to the measures a little, [by] this addition, in order to be stringent in a commandment and to be guarded in it. And I have intended to bring all these matters [at] the end [of] the treatise, so that the book be complete in the words of the Torah.

May it be [the] will before [Him who dwells in] the heavens that He grant us merit to be of those who uphold [the] Torah for its own sake, that He help us (and aid us) to meditate upon it and to study to teach it" and to occupy [ourselves] with its commandments all the days of our lives, by [the] abundance [of] His mercies [and] His kindnesses, Amen [and] Amen, Selah.

Completed is the book, the Treatise of Mensuration and Computation והתשבורת — *hibbur ha-meshihah ve-ha-tishboret*; the second term is והתשבורת = "and Computation/Geometry," NOT a "noble work / recompense"; praise to the Lord of strength and glory.

[End of the book.] Finished and completed, חל"ע, Amen.

Publisher's catalog leaf (A. Asher & Co., Berlin). This final leaf is an advertising/booklist page, not part of the tractate. It is printed in Latin script (German, Latin, English).

[German: "Durch die Buchhandlung **A. Asher & Co.** in Berlin ist zu bezi[e]hen]" — "Through the bookshop A. Asher & Co. in Berlin may be obtained:"]

A list of works edited/published by M. Steinschneider follows:

- **Hai et Ezobi** [Hai Gaon and Joseph Ezobi]. טוטר השכל ושיר הקערה [Tochachat / Tuṭar ha-Sekhel and Shir ha-Ke'arah] — Text emended ... Steinschneider. Berol[in]. 1860.
- **Nachmanides** [Ramban]. ויכוח הרמב"ן [Vikuach ha-Ramban / The Disputation of Nachmanides] — "Disputatio pro fide jud[aica]," from MSS codices, by M. Steinschneider. Berol. 1860. [price: ...] Sgr.
- **Siracides** [Ben Sira]. אלף בית דבן סירא [Alphabet of Ben Sira] — "in integrum restitutum et emend[at]um," from a MS codex, by M. Steinschneider. 12[mo]. Berol. 1858. 10 Sgr. — charta magna alba [large white paper]. 20 Sgr.
- **Steinschneider, M.** משל ומליצה [Mashal u-Melitzah] — "a collection of fables for the use of Jewish youth in the eastern countries." Berlin 1860. With engravings. ½ Th. (sine icon[ibus, without illustrations]: 8 Sgr.)
- — ראשית הלמוד [Reshit ha-Limud] — "A systemat[ic] hebr[ew] Primer for D. Sassoon's Inst[itute] at Bombay. With engravings, maps and the music of the anthem." Berl. 1860. 1⅔ Th.
- — "Jewish Literature from the eighth to the eighteenth century." 8[vo]. London 1857. 4¼ Th.
- — "Catalogus Codd. h[eb]r. Biblioth. Lugduno-Batav[a]." 8. Lugdun[i] = Leiden]. 1858. 3¼ Th.
- — "Catalogus libror. hebr. in Biblioth. Bodl[eiana]." 4. Berol. 1852–60. 33¼ [?] Th.

[German note:] "Von dem »Specimen Catalogi« (die Artt. Maimonides, Saadia, Salomo Isaki [Rashi], Salomo Ibn Gabirol, Samuel Nagid enthaltend, s[ie]he H. B. I S. 15) sind einige Exempl[are] aus dem Antiquariat der Buchhandl[ung] à 2 Th. fest zu beziehen." — "Of the 'Specimen Catalogi' (containing the articles Maimonides, Saadia, Solomon Isaki [Rashi], Solomon Ibn Gabirol, Samuel Nagid; see Hebr. Bibliogr. vol. I p. 15), a few copies may be obtained firm at 2 Th. each from the antiquarian stock of the bookshop."

[German:] "**Wissenschaftliche Blätter** aus der Veitel Heine Ephraim'schen Lehranstalt. Erste Sammlung." [Scholarly Papers from the Veitel Heine Ephraim Institute, First Collection]:

1. F. Lebrecht: "Handschriften u. erste Gesamtausgaben des Babylon. Talmud." [Manuscripts and first complete editions of the Babylonian Talmud.]
2. Th. Haarbrücker: "Rabbi Tanchum Jeruschalmi, Arab. Commentar zum B. Josua." [R. Tanchum of Jerusalem, Arabic commentary on the Book of Joshua.]
3. M. Steinschneider: "Zur pseudepigraphischen Literatur des Mittelalters." [On the pseudepigraphic literature of the Middle Ages.] Berl. 1862. 1¾ Th.

[German:] "**Hebräische Bibliographie.** Blätter für neuere und ältere Literatur des Judenthums. Redig[iert] v[on] M. Steinschneider. Jahrg[ang] 1860 u. folg[ende]." [Hebrew Bibliography. Papers on newer and older literature of Judaism. Edited by M. Steinschneider. Year 1860 and following.] à 1½ Th.

[Imprint at foot:] "**Friedländer'sche Buchdruckerei in Berlin.**" [Friedländer Printing-house in Berlin.]